PENDER

HANDBOOK
FOR
ELECTRICAL ENGINEERS

WORKS OF
HAROLD PENDER, Ph.D., Sc.D.

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HAROLD PENDER
Editor-in-Chief

AND

WILLIAM A. DEL MAR
Associate Editor-in-Chief

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Second Edition, Thoroughly Revised and Enlarged. By Harold Pender, William A. Del Mar, Chief Engineer, Habirshaw Electric Cable Company, and Forty-three Associate Editors. xxiii+2263 pages. 4¼ by 7. Fully illustrated. Leather or flexible binding.

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HANDBOOK
FOR
ELECTRICAL ENGINEERS

A REFERENCE BOOK FOR PRACTICING ENGINEERS AND STUDENTS OF ENGINEERING

COMPILED BY A STAFF OF SPECIALISTS

HAROLD PENDER
EDITOR-IN-CHIEF

WILLIAM A. DEL MAR
ASSOCIATE EDITOR-IN-CHIEF

SECOND EDITION
Revised and Enlarged

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The Publishers and the Editor-in-Chief will be grateful to readers of this volume who will kindly call attention to any errors of omission or commission therein. It is intended to make our publications standard works of study and reference, and, to that end, the greatest accuracy is sought. It rarely happens that the early editions of books are free from errors; but it is the endeavor of the Publishers to have them removed, and it is therefore desired that the Editor-in-Chief may be aided in his task of revision, from time to time, by the kindly criticism of readers.

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PREFACE TO SECOND EDITION

The steady demand for this book since its first appearance in 1914 has shown conclusively that an encyclopedic handbook of Electrical Engineering is an indispensable tool to the engineer. Its use in a number of large universities has also shown that the encyclopedic arrangement is well adapted to the purposes of the class room.

Although in many respects electrical engineering practice to-day is much the same as it was in 1914, there have nevertheless been noteworthy developments in several branches of the industry, for example, in radio-communication, electric welding, electric ship propulsion, etc. Electrical science has also made considerable progress, especially in the development of the electron theory and its application to engineering.

In order to keep this Handbook abreast of the times, the publishers requested the editor in 1920 to undertake a thorough revision of the book. In line with this request, each article was submitted to the orginal contributor or other qualified person with the request that he go over it carefully, add new matter where the development of the art warranted, and bring the article up to date.

The result of this review was that nearly every article in the book has been revised, the amount of revision varying from a few lines to the entire article, according to the desires of the authors. While the greater part of the revisions appear in paragraphs interspersed throughout the book, new articles have been added on the following subjects:

Frequency Changers.
Ignition, Electric.
Phase Converters and Balancers.
Ships, Electric Propulsion of.
Starting and Lighting Systems for Automobiles.
Welding, Electric.

The following articles have been completely rewritten:

Ammeters.
Ampere-hour Meters.
Corona, Electric.
Dams.
Demand Meters.
Depreciation.
Electron Theory.
Heating and Cooking by Electricity.
Hydraulics.
Hydrology.
Insulating Materials.
Lighting of Trains by Electricity.
Oil, transformer.
Radio Communication (replacing the articles on Detectors, Electric Wave; Waves, Electromagnetic; Wave meters; Wireless Telegraphy and Wireless Telephony.
Standarization Rules and Standard Specifications.
Water Wheels and their Settings.
Water Wheels, Speed regulation of.

In addition to the above, extensive changes have been made in the following articles:

Alloys.
Buildings, Allowable Unit Stresses in.
Cross-Arms.

Electrolysis.
Frequency Indicators.
Generators, Alternating Current.
Grounding for Electric Circuits.
Heat and Thermal Properties.
Insulators, Overhead.
Lamps and Illumination.
Motors, Alternating-current Commutator.
Oscillographs.
Power Factor Indicators and Reactive Volt-ampere Indicators.
Resistance and Conductance.
Rubber.
Signalling, Railway.
Synchronizers and Synchroscopes.
Transformers, Instrument.
Voltmeters.
Watt-hour Meters.
Wattmeters.
Wires and Cables, Bare.
Wires and Cables, Insulated.
Wiring of Buildings.

The Standards of the American Institute of Electrical Engineers (formerly known as the Standardization Rules) are given in full at the end of the book. These standards are a verbatim copy of the 1922 edition and are reproduced with the permission of the Board of Directors of the Institute. It is the plan of the publishers to revise these Standards whenever a new edition is published by the Institute, which in the past has been about every two years.

While the encyclopedic arrangement of the book especially commends it for reference purposes, no difficulty attends its use for consecutive study, if the order given in the *Topical List of Articles* is followed. This list is arranged in the order usually followed in the course of class studies. Some teachers may prefer to vary the order of reading, and will find it easier to do so by the aid of such a list, than where the text itself is arranged in some arbitrary order.

Care has been taken to make each article a readable compendium on its subject and not merely a series of disconnected paragraphs containing miscellaneous data, a too common fault in books of this class.

In the preparation of the Second Edition, the editor has been fortunate in having associated with him, as Associate Editor-in-Chief, Mr. William A. Del Mar. Mr. Del Mar has supervised all articles dealing with Electric Railways, Wires and Cables, Transmission and Distribution, Switch Gear and Control Apparatus, Electric Meters and Instruments for commercial use, Mathematics, and Mathematical Tables. These articles make up about one-third of the entire book.

A glance at the make-up of the "Editorial Staff" will show the care with which experts in each line have been selected. In several cases men were selected because of notable investigations or researches which they were conducting at the time along the lines of the articles assigned to them.

The editor takes this opportunity to express his sincere appreciation of the earnest effort of all members of the Editorial Staff to make this new edition of the Handbook throughly representative of modern engineering practice.

HAROLD PENDER

PHILADELPHIA, PA.,
September 1, 1922.

FROM THE PREFACE TO THE FIRST EDITION

1. This Handbook has been prepared primarily for the practicing engineer. With this end in view the matter has been so arranged as to be most readily found, and all *theoretical discussions have been segregated into separate articles.* Fundamental or "theoretical" principles are fully but concisely treated in articles dealing with such matters and nothing else, e.g., such article as *Electricity and Magnetism, Principles of; Alternating Currents; Electrochemistry, Principles of; Mechanics, Principles of; Hydraulics; etc.* Therefore, in articles dealing with practical matters only enough is said of theory to indicate the general principle of which the matter in hand may be a specific application.

2. This book is primarily for Electrical Engineers, but the general arrangement of the subject matter and the method of treatment adopted will make the book a useful reference book for mechanical, civil, mining and other engineers who have occasion to utilize any of the numerous applications of electricity in their special fields. Considerable space has been devoted to those matters pertaining to the applications of motors in all branches of modern industry; see the article on *Motors, Industrial Applications of*, and the numerous cross references there given.

3. Although this book deals primarily with electrotechnical matters, a large amount of space has been devoted to those mechanical and civil engineering subjects which are closely related to electrical engineering practice.

4. Numerous mathematical tables and relations are given. Among these may be mentioned the articles on *Logarithms, Trigonometric Functions, Hyperbolic Functions, Exponential Functions, Derivatives, Integrals, Indeterminate Forms, Equations*, etc.

5. In spite of the fact that this book contains upwards of 2000 pages, many of the articles have been greatly condensed. However, it has been the consistent endeavor of the editorial staff to make each article sufficiently complete so that the information given therein should be of the greatest practical value.

6. Although this book has been prepared primarily for the practicing engineer, it is believed that the method of treatment adopted will render many of the articles suitable as the bases for courses of lectures in technical schools. A teacher usually has his own method of presenting a subject, and sometimes a textbook proves more of a handicap than a help; yet every teacher recognizes the desirability of putting in the hands of his students some sort of synopsis or syllabus which will serve as a general guide.

7. Experience has shown that the alphabetical arrangement is eminently fitted for an encyclopedia of general information; a like arrangement is equally applicable to a reference book dealing only with engineering topics. The subject matter of this book has therefore been disposed in 266 articles alphabetically arranged. The user of the book will undoubtediy find it more convenient at first to refer directly to this detailed index, but as soon as he becomes acquainted with the main article headings, he will be able to turn directly to the article containing the information he may be seeking without referring to the index.

8. One feature of this Handbook which should prove particularly helpful is that the same plan of treatment has been consistently followed in all the articles, at least wherever possible. This applies particularly to articles dealing with apparatus or machinery, the plan of treatment being:

a. General Description and Definitions.
b. Brief Statement of Application.
c. Principle of Operation.
d. Design.
e. Testing.
f. Performance.
g. Specifications.
h. Installation.
i. Operation.
j. Dimensions, Weights and Costs.
k. Bibliography.

In some instances it was found advisable to vary this scheme more or less, but the general sequence of topics was adhered to as closely as possible. At the beginning of each article occupying more than fifteen pages is given a brief table of contents.

9. It is believed that the cost data given in the various articles will prove particularly valuable when properly used. These data in most instances are given as unit costs, and usually the ordinary *range* of cost is given. It has been the experience of the writer that students and recent technical graduates are sadly lacking in even the roughest idea of the cost of apparatus and structures. It is primarily to supply a rough idea of such costs that the cost curves and figures are given in the various articles. The cost data given will also be found suitable for preliminary estimates; but for close estimating, current prices should of course be obtained from the makers of the apparatus, or, in the case of construction work, some one having personal experience in similar work should be consulted.

10. The bibliography given at the end of each article is intended to direct the reader to more extended information in treatises and current periodicals. The space available for these bibliographies has made it necessary to omit many important works and technical articles. The references are usually those in which the writer of the article in question is most familiar and which he has found most useful. They are therefore in no sense complete, but will be found a very useful guide in the search for additional information. In many instances a blank space of a half page or more is left after the bibliography; this space may be advantageously utilized by the insertion of references to new books and articles as they appear.

11. The articles in this Handbook have been prepared by an editorial staff of experts; this staff is given on pp. iii and iv.

The Editor-in-Chief wishes to take this opportunity of expressing his sincere appreciation of the hearty coöperation of the entire editorial staff and of the spirit of coöperation and generosity shown by the publishers throughout the preparation of this book.

HAROLD PENDER.

East Bluehill, Me.,
September 5, 1914.

TOPICAL LIST OF ARTICLES

The articles listed under each main heading, except those of a statistical nature, are in an order appropriate for consecutive reading or assignment for class-ro^m instruction.

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(See also Mechanical Engineering)

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Standards of the American Institute of Electrical Engineers

Issued, September, 1922

Index

C. L. Bogert

ABBREVIATIONS AND SYMBOLS. — (*See also Units and Conversion Factors.*) At the meeting of the International Electrotechnical Commission in Berlin, in 1913, a list of 36 symbols for electrotechnical and related quantities was adopted. The American Institute of Electrical Engineers has also adopted (1918) a list of such symbols, which for the most part is in agreement with the list of the I.E.C., but contains certain additional symbols not included in the I.E.C. list. In the following table both lists are given. See the *Standardization Rules of the A.I.E.E.* for symbols for photometric quantities.

STANDARD SYMBOLS AND ABBREVIATIONS

Name of quantity	Symbol for the quantity		Name of Unit, adopted by A.I.E.E.	Abbreviation for the unit to be used only after numerical values	
	Adopted by I.E.C. (*See Note 1*)	Adopted by A.I.E.E.		Adopted by I.E.C.	Adopted by A.I.E.E.
Acceleration due to gravity	g	g	centimeter per second per second	—	cm. per sec. per sec.
Acceleration due to gravity, Standard (=980.665 cm. per sec. per sec.) (*Note 5*)	—	g_0	centimeter per second per second	—	cm. per sec. per sec.
Admittance	—	Y, y	mho	—	—
Angles	α, β, γ, etc.	—	—	—	—
Angular velocity ($2\pi f$)	ω	ω	radian per second	—	—
Capacity (Capacitance)	C	C	farad	F	—
Conductance	G	g	mho	—	—
Conductivity (*Note 6*)	—	γ	mho per centimeter	—	mho per cm.
Current	I	I, i	ampere	A	—
Dielectric constant	ϵ	ϵ or k	—	—	—
Dielectric field intensity or electric force	—	F	—	—	—
Dielectric flux	—	ψ	—	—	—
Dielectric flux density	D	D	—	—	—
Efficiency	η	η	per cent	—	(*Note 7*)
Electromotive force (e.m.f.)	E	E, e	volt	V	—
Energy	W (U)	U or W	joule or watt-hour	—	—
Frequency	$f(\nu)$	f	cycle per second	—	~
Impedance	Z $(\mathfrak{Z})$	Z, z	ohm	(*Note 2*)	—
Inductance (or coefficient of self-induction)	L $(\mathfrak{L})$	L	henry	H	—

STANDARD SYMBOLS AND ABBREVIATIONS (*Continued*)

Name of quantity	Symbol for the quantity: Adopted by I.E.C. (*See Note 1*)	Symbol for the quantity: Adopted by A.I.E.E.	Name of unit adopted by A.I.E.E.	Abbreviations for the unit to be used only after numerical values: Adopted by I.E.C.	Abbreviations for the unit to be used only after numerical values: Adopted by A.I.E.E.
Inductance, mutual (or coefficient of mutual induction)	M ($\mathfrak{M}$)	M	henry	H	—
Length	l (*Note 3*)	l	centimeter	cm.	cm.
Magnetic flux (*Note 4*)	Φ ($\mathcal{F}$)	Φ, ϕ	maxwell	—	—
Magnetic flux density	B ($\mathcal{B}$)	B, $\mathcal{B}$	gauss (*Note 4*)	—	—
Magnetization, intensity of	J ($\mathcal{J}$)	J	—	—	—
Magnetizing force or magnetic field intensity	H ($\mathcal{H}$)	H, $\mathcal{H}$	gilbert per centimeter or gauss	—	gilbert per cm.
Magnetomotive force (m.m.f.)	—	$\mathcal{F}$	gilbert (*Note 4*)	—	—
Mass	m (*Note 3*)	m	gram	—	g.
Period	T	—	—	—	—
Permeability	μ	μ	—	—	—
Phase displacement (or phase angle)	ϕ	θ, ϕ	degree or radian	—	(*Note 8*)
Potential difference (p.d.)	—	V, v or E, e	volt	V	—
Power	P	P, p	watt	W	—
Quantity of electricity	Q	Q, q	coulomb or ampere-hour	C	—
Reactance	X ($\mathcal{X}$)	X, x	ohm	(*Note 2*)	—
Reluctance	S ($\mathcal{R}$)	$\mathcal{R}$	—	—	—
Resistance	R	R, r	ohm	(*Note 2*)	—
Resistivity (*Note 6*)	ρ	ρ	ohm-centimeter	—	ohm-cm.
Revolutions per unit time	n	n	revolution per second	—	rev. per sec.
Susceptance	—	b	mho	—	—
Susceptibility	κ	κ	—	—	—
Temperature, absolute	T, Θ	—	degree centigrade	—	—
Temperature, centigrade	t (θ, ϑ)	T, t, θ	degree centigrade	—	deg.cent. (*Note 8*)
Time	t (*Note 3*)	t	second	—	sec.
Turns or number of conductors	—	N	convolution or turn	—	—
Voltage	—	E, e or V, v	volt	V	—
Work, mechanical	A (W)	W or A	joule or watt-hour	—	—

NOTES. — (1) The symbols in brackets are recommended in case the principal symbol is unsuitable; instead of the script letters heavy-faced or other special type may be used. (2) One or other of the symbols O and Ω is recommended provisionally to represent the ohm. The symbol Ω should no longer be employed for the megohm. (3) In dimensional equations the capital letters L, M and T for length, mass and time respectively are to be employed. (4) An additional unit for m.m.f. is the "ampere-turn", for flux the " line ", and for magnetic flux density " maxwell per sq. in." (5) This has been the accepted standard value for many years and was formerly considered to correspond accurately to 45° latitude and sea level. Later researches, however, have shown that the most reliable value for 45° and sea-level is slightly different; but this does not affect the standard value given above. (6) The numerical values of these quantities are *ohms resistance* and *mhos conductance* between two opposite faces of a cm. cube of the material in question, but the correct names are as given, not ohms and **mhos** per cm. cube as commonly stated. (7) The symbol % is commonly used for per cent, but is not recommended by the A.I.E.E. (8) The symbol ° is commonly used for degree, ° C. for degree centigrade and ° F. for degree Fahrenheit; these symbols are not recommended by the A.I.E.E.

In addition to the symbols for the units given in the above table, the I.E.C. adopted (1913) the following signs to be used only after numerical values.

Volt-coulomb	VC	Kilovolt-ampere	kVA
Watt-hour	Wh	Kilowatt-hour	kWh
Volt-ampere	VA	Sign for milli-	m
Ampere-hour	Ah	Sign for kilo-	k
Milliampere	mA	Sign for micro- or micr-	μ
Kilowatt	kW	Sign for mega- or meg-	M

SPECIAL RULES IN REGARD TO SYMBOLS. — The following rules were also adopted at the Berlin meeting (1913) of I.E.C., and are concurred in by the Standards Committee of the A.I.E.E. The latter committee also recommends that vector quantities be printed in bold-face capitals.

Instantaneous values of electrical quantities which vary with the time to be represented by small letters. In case of ambiguity they may be followed by the subscript "t."

Virtual (*i.e., effective or r.m.s.*) or constant values of electrical quantities to be represented by capital letters.

Maximum values of periodic electrical and magnetic quantities to be represented by capital letters followed by the subscript "m."

In cases where it is desirable to distinguish magnetic quantities from electric quantities, magnetic quantities should be represented by capital letters of either script, heavy-faced or other special type. Script letters should not be used except for magnetic quantities.

Angles should be represented by small Greek letters.

Dimensionless and specific quantities should be represented wherever possible by small Greek letters.

Ordinary numerals as exponentials shall exclusively be employed to represent powers. (In consequence, it is desirable that the expression $\sin^{-1}x$, $\tan^{-1}x$, employed in certain countries, be expressed by arc sin x, arc tan x.)

The comma and the full-stop shall be employed for separating decimals according to the custom of the country, but the separation between any three digits constituting a whole number shall be indicated by a space and not by a full-stop or a comma(1 000 000).

For the multiplication of numbers and geometric quantities indicated by two letters, it is recommended to use the sign × and the full-stop only when there is no possible ambiguity.

To indicate division in a formula it is recommended that the horizontal bar and the colon be employed. Nevertheless the oblique line may be used when

there is no possibility of ambiguity; when necessary, ordinary brackets ( ), square brackets [ ], and braces { } may be employed to obtain clearness.

INSTITUTE (A.I.E.E.) STYLE. — The Editing Committee of the American Institute of Electrical Engineers issues a little pamphlet called "Suggestions to Authors," in which are given the rules of the Institute in regard to manuscript submitted for publication. The following list of abbreviations is taken from the last edition (1919) of these rules.

Name	A.I.E.E. style	Name	A.I.E.E. style
Alternating current	spell out, or a-c. as adjective.	Kilowatts	kw.
		Kilowatt-hours	kw-hr.
		Magnetomotive force	m.m.f.
Amperes	spell out	Mean effective pressure	spell out
Brake horse power	b.h.p.	Miles	mi.
Boiler horse power	boiler h.p.	Miles per hour per second	mi. per hr. per sec.
British thermal units	B.t.u.		
Candle power	c.p.	Millimeters	mm.
Centigrade	cent.	Milligrams	mg.
Centimeters	cm.	Minutes	min.
Circular mils	cir. mils	Meters	m.
Counter electromotive force	counter e.m.f.	Meter-kilograms	m-kg.
		Microfarad	spell out
Cubic	cu.	Ohms	spell out
Diameter	spell out	Per	spell out
Direct current	spell out, or d-c. as adjective.	Percentage	per cent, or % in tabular matter.
Electric horse power	e.h.p.	Pounds	lb.
Electromotive force	e.m.f.	Power-factor	spell out
Fahrenheit	fahr.	Revolutions per minute	rev. per min., or r. p. m. in tabular matter.
Feet	ft.		
Foot-pounds	ft-lb.		
Gallons	gal.		
Grains	gr.	Seconds	sec.
Grams	g.	Square	sq.
Gram-calories	g-cal.	Square-root-of-mean-square	effective, or r.m.s.
High-pressure cylinder	spell out		
Hours	hr.	Ton-mile	spell out
Inches	in.	Tons	spell out
Indicated horse power	i.h.p.	Volts	spell out
Kilogram	kg.	Volt-amperes	spell out
Kilogram-meters	kg-m.	Watts	spell out
Kilogram-calories	kg-cal.	Watt-hours	watt-hr.
Kilometers	km.	Watts per candle power	watts per c.p.
Kilovolts	kv.	Yards	yd.
Kilovolt-amperes	kv-a.		

1. Use "Fig.," not "Figure." Example: "Fig. 3" and not "Figure 3."

2. In all decimal numbers having no units, a cipher should be placed before the decimal point. Example: "0.32 lb." not ".32 lb."

3. Use the word "by" instead of "x" in giving dimensions. Example: "8 by 12 in." not "8x12 in."

4. Never use the characters (′) and (″) to indicate either feet and inches, or minutes and seconds as period of time.

5. Do not use the expression "rotary" or "rotary converter"; use "converter" or "synchronous converter."

6. Do not use a descriptive adjective as a synonym for the noun described. Example: a "spare transformer," not a "spare"; a "portable instrument," not a "portable"; "automatic apparatus," not "automatics"; a "short circuit," not a "short."

7. Do not use the words "primary" and "secondary" in connection with transformer windings. Use instead "high-tension" and "low-tension."

8. Avoid dividing mathematical expressions into two or more parts if possible. If division is necessary, divide *after* a plus, minus or equality sign.

9. Use the English plural where there is a choice. For example: abscissas, not abscissae; formulas, not formulae.

10. Data is the plural form and takes plural verb, etc.

11. Make compounds of two-word adjectives, as direct-current motor, three-phase circuit, etc.

ACIDS. — The properties of some of the more important acids used in the arts are given in the following paragraphs.

Aqua Regia is a mixture of one part of nitric acid and three parts of hydrochloric acid. It obtains its name from the fact that it will dissolve gold. It is used as an oxidizing agent and for dissolving metals, such as platinum, gold, etc., which are insoluble in other acids. It should be used as soon as possible after being prepared since it loses its characteristic properties after standing a short time. It is distinguished by its reddish yellow color and its chlorine-like odor.

Hydrochloric Acid or Muriatic Acid (HCl) is an aqueous solution of hydrogen chloride, a colorless, pungent gas. While the pure acid is colorless, the commercial acid is a yellowish liquid, the color being due to impurities. The concentrated acid has a specific gravity of 1.16 and contains 32 per cent of HCl. The dilute acid has a specific gravity of 1.09 and contains 18.4 per cent of HCl. Hydrochloric acid is used extensively in the manufacture of chlorine, hydrogen and bleaching powder and is prepared as a by-product of the soda manufacture. The acid is recognized by its odor and by the dense fumes it makes with ammonia.

Hydrofluoric Acid (HF) is an aqueous solution of anhydrous hydrogen fluoride, a colorless, fuming liquid. The acid, when saturated, has a specific gravity of 1.25. Its gas is poisonous if inhaled and causes swellings and pain if applied to the skin. It readily dissolves glass and must be kept in platinum, lead, rubber or wax vessels. The acid is most commonly used for etching glass.

Nitric Acid (HNO_3) is a colorless fuming liquid with a specific gravity of 1.55 at 0° C. Commercial nitric acid has a specific gravity of 1.414 at 15° C. and contains 68 per cent of the pure acid. The commercial acid is yellowish in color and the pure acid becomes yellow if exposed to the light. Nitric acid is a powerful oxidizing agent and decomposes organic substances. The acid is used in the manufacture of many chemical substances. Concentrated nitric acid is best recognized by the red fumes, which are given off when the acid is acting upon a metal.

Sulphuric Acid or Oil of Vitriol (H_2SO_4) is a colorless oily liquid. The concentrated acid has a specific gravity of 1.854 at 0° C. and contains 1.5 per cent of water. Commercial sulphuric acid, sometimes known as "brown acid," has a specific gravity of 1.720 and usually contains arsenic as an impurity. Sulphuric acid boils at 290° C., the temperature increasing to 338° C. as the boiling continues. The acid has a strong affinity for water and is frequently used as a drying agent. When diluted with water, the mixing should be performed gradually as great heat is evolved. The acid should be added to the water as the addition of water to the acid may produce violent explosions due to the ebullition of the water.

Sulphuric acid is used in the manufacture of many chemical substances, in dyeing and in refining petroleum. It is a fairly good conductor of electricity and is used in the lead type of storage batteries (q.v.). The acid is recognized by its weight, by its carbonizing action upon organic bodies and by the white precipitate formed by the addition of barium chloride.

ALLOYS — (*See also Wires, Resistor.*) In this article is given a brief description of some of the more common alloys used in engineering work, together with specific data on some of their more important properties. It should be noted that there may be a considerable departure from the quantitative values given. Traces are not included in the chemical compositions given. The various alloys are listed alphabetically. Additional data on their various physical properties will be found in the articles on *Heat and Thermal Properties; Resistance and Conductance; Strength and Elasticity; Weight of Materials, etc.*

Acid-resisting Alloys. — Alloys of iron and chromium and of iron and silicon are used for containers, partly replacing stoneware, in the manufacture of acids. Ferro-chrome is not attacked by sulphuric or nitric acid, but is readily attacked by hydrochloric acid. Silicon-iron containing 14 to 15 per cent silicon (with a small amount of Mn) is likewise not attacked by sulphuric or nitric acid. Such an alloy has a specific gravity of about 7, a melting point of about 2500° F., a tensile strength of about 15,000 pounds per square inch, and a compressive strength of about 70,000 pounds per square inch. Acid-resisting silicon irons are sold under various trade names, such as "duriron," "tantiron," etc. (*W. C. Carnell, Chem. News, 116, p. 92, Aug. 24, 1917.*)

Aluminum Brass (70.5 Cu, 26.4 Zn, 3.1 Al). — The tensile strength is about 21 tons per square inch, the elastic limit 8.5 tons per square inch and the elongation 50 per cent. Aluminum brass is used when very accurate castings are desired, e.g., for pumps, valves, pinions and propellers. It can be rolled and forged while hot but is not easily worked when the aluminum content exceeds 4 per cent.

Aluminum Bronze (95 Cu, 5 Al). — The tensile strength is about 28 tons per square inch, the elastic limit 12 tons per square inch and the elongation 75 per cent. Aluminum bronze containing less than 7.5 per cent Al is very ductile. With more than 7.5 per cent Al the alloy becomes brittle but increases in tensile strength. Tubes, propellers and propeller shafts are sometimes made of aluminum bronze. It has not been found to withstand intense heat for any length of time without fracturing. It is sometimes drawn into wire for use as an electrical resistance.

Amalgams are alloys of mercury and other metals. When newly made amalgams are plastic but harden in a short time without appreciable expansion or contraction. The common metals combined with mercury to form amalgams are tin, copper, cadmium, bismuth, silver and gold. Amalgams are used for silvering glass and as a cement for metals and porcelain.

Anti-friction Metals are alloys of copper, tin, zinc, lead and antimony in various combinations. These alloys are commonly used in bearings for revolving shafts and for valve packings. See *Babbitt and Bearing Metals* below.

Babbitt Metal (4 Cu, 69 Zn, 19 Sn, 5 Pb, 3 Sb) is used extensively for bearings, the composition stated being that used for car bearings of the Pennsylvania Railroad. See also the article on *Bearings*.

Bearing Metals for use in specific cases are illustrated by the following examples: Locomotive (82 Cu, 8 Zn, 10 Sn), railway car (90 Cu, 10 Sn), low-speed bearing (16 Sn, 84 Pb). There are a large number of anti-friction metals in use, the object in the composition of each of them being to procure a metal which is as hard as possible but plastic enough to be moulded by the shaft into a shape offering a minimum friction.

Brass is an alloy of copper and zinc and often contains small percentages of lead, tin, arsenic, antimony, bismuth and iron. *Cast brass* usually consists of 66 per cent of copper and 34 per cent of zinc. *Low brasses*, suitable for hot

rolling contain from 55 per cent to 63 per cent of copper. *High brasses*, suitable for cold rolling and drawing, contain from 60 per cent to 70 per cent of copper. In drawing brass it must be annealed and cleaned in acid at frequent intervals to prevent fracture. The ductility of brass is impaired if the lead content exceeds 0.1 per cent, but in the case of brasses intended for turning about 2 per cent of lead is often added so that the brass may be turned at higher speed and possess a better finish. Brass is made harder by the addition of about 1 per cent of tin and is found to better withstand corrosion due to salt water. The presence of small quantities of arsenic, antimony or bismuth in brass is liable to cause it to crack when rolled. The addition of 1 per cent to 3 per cent of iron to brass produces a harder and stronger alloy. The tensile strength of commercial brass ranges from 15 to 40 tons per square inch and the elongation varies from 10 per cent to 40 per cent depending upon the composition.

Bronze is a name used to designate various alloys of copper and tin with other metals. Ordinary bronze is an alloy of copper. tin, and zinc, and melts about 980° C. See also *Aluminum Bronze*, *Manganese Bronze*, and *Phosphor Bronze* in this article.

Duralumin is an alloy whose composition varies between the limits Cu, 3 to 4.5 per cent; Mg, 0.4 to 1.0 per cent; Mn, 0 to 0.7 per cent; Fe, 0.4 to 10 per cent; Si, 0.3 to 0.6 per cent; and the balance aluminum. It has a specific gravity of 2.85 and possesses the remarkable property of being appreciably hardened by quenching from temperatures below its melting point and subsequent ageing. (*Amer. Inst. Min. Eng., Bull. No. 150, 1919.*)

Ferro-Alloys. — See above under *Acid-resisting Alloys* and the article on *Steel*.

Fusible (Low Melting Point) Metals consist of various alloys of bismuth, lead, tin and cadmium. The chemical composition of the common fusible metals and their melting points are given in the following table:

Alloy	Composition				Melting point °C.
	Bi	Pb	Sn	Cd	
Newton's alloy	50.0	31.25	18.75	.....	95
Rose's alloy	50.0	26.00	24.00	.....	100
Darcet's alloy	50.0	25.00	25.00	.....	93
Wood's alloy	50.0	24.00	14.00	12.00	66–71
Lipowitz' alloy	50.0	27.00	13.00	10.00	60

Lower melting points are obtained by the addition of mercury.

German Silver is an alloy of copper, nickel and zinc in various combinations. It is sometimes called nickel silver, argentan, packfong, silveroid, silverite or electrum. German silver is extensively used because of its ductility; it can be readily rolled, hammered and drawn. It is hard, tough and not easily corroded. Its composition varies from (50 Cu, 30 Ni, 20 Zn) to (57 Cu, 7 Ni, 36 Zn), the nickel content decreasing as the zinc increases. The usual impurities found in German silver are iron, lead and tin. The presence of iron in the alloy makes it stronger, harder and more elastic. Tin makes the alloy brittle. Lead is sometimes added to render the alloy more workable but should not be added when the alloy is to be rolled. German silver is drawn into wires for use as an

electrical resistance and has been drawn into tubes for use in locomotive boilers. See *Wires, Resistor.*

Gun Metal is a bronze consisting of about 90 per cent of copper and 10 per cent of tin. The tensile strength varies from 12 to 16 tons per square inch, depending upon the method of working. Melting point 980 to 995° C. It is mechanically strong and elastic and withstands severe shocks without fracture. If a small amount of lead is added to gun metal, the alloy is more easily turned or filed. Better castings are made by the addition of a small amount of zinc and the alloy is made harder by adding a small percentage of iron.

Manganese Bronze is an alloy containing copper, tin, zinc, lead, iron and manganese in various combinations. Alloys containing from 4 per cent to 6 per cent of manganese possess high tensile strength at high temperatures and are therefore used for fire-box stays. High tensile strength at ordinary temperatures is obtained, however, by the addition of very small quantities of manganese. The composition of manganese bronze for certain specific uses is as follows: For hydraulic machinery (82 Cu, 8 Sn, 5 Zn, 3Pb, 2Mn); for forging (58.6 Cu, 38.4 Zn, 1.6 Fe, 0.02 Mn). The tensile strength ranges from 27 to 38 tons per square inch but if the alloy is cold-rolled, a tensile strength of 50 tons per square inch may be obtained. Manganese bronze is often used in place of brass or copper, when a higher tensile strength is required. It is not easily corroded and may be bent when hot or cold.

Magnetic Alloys. — See the article on *Magnetic Properties of Iron and Other Metals.*

Monel Metal is described in the article on *Wires, Resistance.* It has a tensile strength of 40 tons per square inch at air temperature and 29 tons per square inch at 500° C. It is used for pump rods and valves as well as for resistance wires. (*J. Arnott, Engineering, 106, p. 451, Oct. 25, 1918.*)

Non-expansive Alloys are composed of iron, nickel and carbon. *Platinite* contains 46 per cent of nickel and 0.15 per cent of carbon. *Invar* contains 36 per cent of nickel and 0.2 per cent of carbon. Its coefficient of expansion ranges from 8×10^{-7} to 25×10^{-7} per degree centigrade. These alloys are used extensively in scientific instruments for standard measures of length and in incandescent lamps where the wire connections fused into the glass must not expand enough to fracture the glass.

Phosphor Bronze consists of copper, tin and phosphorus in various proportions, the phosphorus content rarely exceeding 2 per cent. The most useful property of phosphor bronze is its hardness and resistance to wear. Phosphor bronzes containing (A) 8 to 10 per cent of tin and 0.5 to 0.7 per cent of phosphorus are used for valves, pumps, propellers and boiler fittings, (B) 10 to 12 per cent of tin and 0.7 to 1 per cent of phosphorus are used for worms and gears and (C) 10 to 12 per cent of tin and 1 to 1.5 per cent of phosphorus are used for worms, gears and bearings where the wear is excessive. The tensile strength ranges from 10 to 15 tons per square inch, the elastic limit from 5 to 7 tons per square inch and the percentage elongation from 2 to 6 per cent.

Platinum Substitutes. — Alloys of gold and silver with palladium have been recently developed as a substitute for platinum for contacts and sparking points in magnetos.

Pyrophoric Alloys are certain alloys of rare earths, especially ferrocerium, which give a shower of sparks when filed or struck. Sb and Bi are added (2 per cent) to harden the alloy and copper (5 per cent) to produce a low melting point and a smooth casting product. (*Engineering and Mining Journ., p. 212, 1917.*)

Resistance Alloys. — See article on *Wires, Resistor.*

Solder is an alloy of tin and lead. Cadmium may be used as a substitute for part of the tin, 80 Pb, 10 Sn, 10 Cd being a satisfactory solder. *Hard* or tin solder contains 50 per cent of tin and 50 per cent of lead and melts at 370° F. *Soft* or plumber's solder contains 33.3 per cent of tin and 66.6 per cent of lead and melts at 441° F.

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ALTERNATING CURRENTS. — (*See also Electricity and Magnetism, Principles of; Generators, Alternating-current; Motors, Alternating-current; Resistance and Conductance, Electric; Skin Effect; Transformers; Wave Analysis, etc.*).

GENERAL DEFINITIONS. — (*See also Standardization Rules of the A.I.E.E.*). To avoid repetition the following definitions are given in terms of electric current; they also apply to electromotive forces, potential differences or to any other functions of time.

An alternating current is defined as a current which varies continuously with time from a constant maximum value in one direction to an equal maximum value in the opposite direction and back again to the same maximum in the first direction, repeating this cycle of values over and over again in *equal* intervals of time.

Period, Frequency and Alternations. — The period of an alternating current is the time taken for the current to pass through a complete cycle of positive and negative values.

The frequency or number of cycles per second is the number of periods per second.

The number of alternations per minute is the total number of times per minute that the current changes in direction, from positive to negative and from negative to positive. In engineering practice the number of cycles is usually referred to the second as the unit of time and the number of alternations is referred to the minute as the unit of time.

Let T be the period, f the frequency or number of cycles per second and a the number of alternations per minute, then

$$f = \frac{1}{T} \qquad \text{and} \qquad a = 120f = \frac{120}{T}. \tag{1}$$

The constant

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{1a}$$

is sometimes called the angular velocity or angular frequency of the current. The name periodicity for this quantity is not recommended.

Instantaneous, Maximum and Average Values. — The instantaneous value of an alternating current is the value of the current at any instant. Instantaneous values of current, potential difference and electromotive force will be designated by small letters throughout this article, viz., i, v and e.

The maximum value of an alternating current is the numerical value of its maximum instantaneous value. Maximum values will be designated by capital letters with the subscript "m."

The average value of an alternating current for which the positive and negative half cycles are equal, which is usually the case, is defined as the numerical value of the average of its instantaneous values for a half cycle; the average over a complete cycle is of course zero. The general expression for the average value of a symmetrical current wave over a half cycle is

$$I_{\text{aver.}} = \frac{2}{T}\int_{t_0}^{t_0+\frac{1}{2}T} i\,dt, \tag{2}$$

where t_0 is a value of t at which $i = 0$; similarly for a voltage wave.

R.M.S. or Effective Values. — The square root of the mean of the squares of the instantaneous values of an alternating current over a complete period is called the r.m.s., or effective, value of the alternating current. In specifying the value of an alternating current as so many amperes this r.m.s. value is always meant unless specifically stated otherwise. In the same manner the square root of the mean of the squares of the instantaneous values of an alternating

potential difference over a complete period is called the r.m.s. value of the alternating potential difference. When the value of an alternating potential difference is specified as so many volts, this r.m.s. value is always meant unless specifically stated otherwise.

The reason for selecting this particular function of the instantaneous values of an alternating current or potential difference as the measure of the current or potential difference is that the deflection of all instruments used in alternating-current measurements is a function of this r.m.s. value. See *Ammeters*, *Electro-dynamometers*, *Voltmeters*, *etc.* Moreover, the average power dissipated as heat in a resistance, r, when an alternating current of r.m.s. value I flows through it, is rI^2.

R.m.s. values will be designated throughout this article by capital letters without subscripts.

The general expression for the r.m.s. value of an alternating current is

$$I = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}, \tag{3}$$

and similarly for an alternating potential difference.

Form Factor. — The form factor of an alternating current is defined as the ratio of its r.m.s. to its average value, viz.,

$$\text{Form factor} = \frac{I}{I_{\text{aver.}}}, \tag{4}$$

and similarly for an alternating potential difference.

Crest or Peak Factor. — The crest factor, also called the peak factor or amplitude factor, of an alternating current is defined as the ratio of its maximum to its r.m.s. value, viz.,

$$\text{Crest factor} = \frac{I_m}{I}. \tag{5}$$

Instantaneous and Average Power. — Let v be the value at any instant of the potential drop from any point 1 to any other point 2, and let i be the instantaneous value of the current from 1 to 2 at this same instant; then the power input at this instant is

$$p = vi. \tag{6}$$

When v and i are *both* positive (i.e., in the direction from 1 to 2, say) or when they are *both* negative, the power input is positive, but when v is positive and i negative or vice versa, the power input is negative, i. e., there is an actual power output.

The average value of the product vi over a complete period for both v and i (or over any whole number of periods) is the average power input or output, usually called simply the power input or output (input when the average of vi is positive, output when the average of vi is negative), the word average being understood. That is, the average power input is

$$P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T vi\,dt, \tag{7}$$

T being a complete period. For the actual measurement of alternating-current power see *Wattmeters*.

Power Factor. — Only in certain special cases (*see below*) is the average power input P equal to the product of the r.m.s. value V of the potential difference by the r.m.s. value I of the current; it can never be greater and as a rule is less. The ratio of the average power P to the product of the r.m.s. value V of the

potential difference by the r.m.s. value I of the current is called the power factor of the circuit between the terminals considered, i.e.,

$$\text{Power factor} = \frac{P}{VI}. \tag{8}$$

When V is expressed in volts and I in amperes then P must be in watts; when V is expressed in kilovolts and I in amperes P must be in kilowatts.

Volt-Amperes, Kilovolt-Amperes (kv-a.) — The product of the r.m.s. volts across the terminals of a circuit by the r.m.s. amperes through it is called the volt-amperes taken by the circuit; this product divided by 1000 is called the kilovolt-ampere input. Or, when V is in volts and I in amperes

$$\text{volt-amperes} = VI, \tag{9}$$

$$\text{kilovolt-amperes} = \frac{VI}{1000}. \tag{9a}$$

Kilovolt-amperes are usually abbreviated kv-a. or K.V.A., the former abbreviation being that recommended by the American Institute of Electrical Engineers and used in this book.

Effective Resistance (r) **and Conductance** (g). — The effective resistance of any portion of a circuit to an alternating current is the quotient of the average rate P_h at which heat is developed by this current, either directly in the substance through which it passes or indirectly as a consequence of the hysteresis and eddy-current losses produced by its magnetic field (*see Magnetic Properties of Iron*), divided by the square of the r.m.s. value I of the total current (conduction plus displacement or charging current) through this portion of the circuit, viz.,

$$r = \frac{P_h}{I^2}. \tag{10}$$

Similarly, calling V the r.m.s. value of the potential difference across the given portion of the circuit the effective conductance g of this portion of the circuit is defined by the relation

$$g = \frac{P_h}{V^2}. \tag{11}$$

In general, both r and g depend upon both the frequency and the wave-shape (*see below*) of the current and voltage but in many instances they may be considered as practically constant irrespective of the frequency or wave-shape. See the articles on *Resistance and Conductance* and *Skin Effect* for further discussion.

Impedance (z) **and Admittance** (y). — Let V be the r.m.s. value of the potential drop through any portion of a circuit due to its effective resistance, self-inductance and capacity (*see Electricity and Magnetism, Principles of*), i.e., if there is any other source of e.m.f. in the given portion of circuit (e.g., a generator or motor) V is the r.m.s. value of the potential drop which the same current would produce through this portion of circuit were this external source of e.m.f. removed. Then the quotient of the potential drop V by the r.m.s. value of the current I in this portion of the circuit, viz.,

$$z = \frac{V}{I}, \tag{12}$$

is defined as the impedance of this portion of the circuit.

The reciprocal of the impedance, viz.,

$$y = \frac{1}{z} = \frac{I}{V}, \tag{13}$$

is defined as the admittance of the given portion of the circuit.

Impedance and admittance, as thus defined, both depend upon the frequency and wave-shape. Impedance is expressed in the same units as resistance (e.g. ohms), and admittance in the same units as conductance (e.g. mhos); see *Units and Conversion Factors*.

Reactance (x) **and Susceptance** (b). — The square root of the difference between the square of the impedance and the square of the effective resistance of a given portion of an electric circuit is defined as the reactance x of this portion of the circuit, viz.,

$$x = \sqrt{z^2 - r^2}. \tag{14}$$

The reactance of a coil of inductance L to a sine-wave current of frequency f is

$$x = 2\pi fL. \tag{14a}$$

Similarly, the susceptance b of the given portion of the circuit is defined by the relation

$$b = \sqrt{y^2 - g^2}. \tag{15}$$

The susceptance of a condenser of capacity C to a sine-wave voltage of frequency f is

$$b = 2\pi fC. \tag{15a}$$

The simple relations expressed by (14a) and (15a) hold only for *sine-wave* currents and voltages; see Pender, H., *Electricity and Magnetism for Engineers, Part II*, N. Y., 1919. See also the articles in this book on *Capacity and Charging Current* and *Inductance and Inductive Reactance.*

Inductive and Condensive (or **Capacity**) **Reactance and Susceptance.** — The reactance of a circuit may be due either to the back e.m.f. set up as a consequence of the varying magnetic field of the current or to a back e.m.f. set up by a condenser or its equivalent, or to both. In the first case the reactance and susceptance are said to be "inductive" and in the second case "condensive." A condensive reactance or susceptance is equivalent to a *negative* inductive reactance or susceptance; e.g., the inductive susceptance of a condenser to a sine-wave voltage is $-2\pi fC$.

Equivalent* Resistance, Impedance and Reactance. — Sometimes in calculating alternating-current circuits it is convenient to consider a motor or other load developing a back e.m.f. as equivalent to a single resistance and reactance. Let P be the total power taken by the load, V the voltage between its terminals, I the current; then the equivalent resistance is defined as

$$R = \frac{P}{I^2}, \tag{16}$$

the equivalent impedance as

$$Z = \frac{V}{I} \tag{16a}$$

and the equivalent reactance as

$$X = \sqrt{Z^2 - R^2}. \tag{16b}$$

The difference between the *effective* resistance and the *equivalent** resistance is

* The distinction here made between equivalent and effective is not always observed; the term equivalent resistance is frequently used in the same sense as effective resistance.

that the first takes into account only the power dissipated as heat, whereas the latter takes into account the total power, of which only a part is heat, the rest being converted into some other form, e.g. mechanical power.

SIMPLE HARMONIC OR SINE-WAVE CURRENTS AND VOLTAGES. — A simple harmonic or sine-wave current is one which varies with time according to the sine formula

$$i = I_m \sin(\omega t + \theta),$$

where t represents time in seconds, measured from any arbitrarily chosen instant, I_m the maximum value of the current, $\omega = 2\pi f = \frac{2\pi}{T}$, where f is the frequency in cycles per second and T the period as a fraction of a second, and θ a constant, called the "phase angle," which depends upon the instant chosen as the zero of time. See the section on *Harmonic Motion* in the article on *Mechanics, Principles of*, for a full discussion of this equation and its physical significance.

Difference in Phase Between a Sine-wave Current and a Sine-wave Voltage of the Same Frequency. — In general, when a sine-wave electromotive force is impressed on a circuit the resulting current is likewise a sine function of time (after a very brief interval, see *Transient Electric Phenomena and Oscillations*) having the same frequency, but the e.m.f. and current do not reach their maximum values simultaneously. Let the current and the potential drop in the direction of the current be represented respectively by the two equations

$$i = I_m \sin \omega t,$$
$$v = V_m \sin(\omega t + \theta),$$

where t is the time measured from the instant when $i = 0$ and is increasing in the *positive* direction. The current reaches its maximum value when $t = \frac{\pi}{2\omega}$, while the potential drop reaches its maximum value when $t = \frac{\pi}{2\omega} - \frac{\theta}{\omega}$. Hence when θ is positive the potential drop reaches its maximum value $\frac{\theta}{\omega}$ seconds *before* the current reaches its maximum, or the current reaches its maximum value $\frac{\theta}{\omega}$ seconds *after* the potential drop reaches its maximum; when θ is negative the current reaches its maximum value $\frac{\theta}{\omega}$ seconds *before* the potential drop reaches its maximum. In the first case the current is said to "lag" the potential drop, and in the second case the current is said to "lead" the potential drop. The angle θ is called the "difference in phase," or simply the phase angle, between the current and potential drop.

In general, when i and v are expressed by the formulas

$$i = I_m \sin(\omega t + \theta_i),$$
$$v = V_m \sin(\omega t + \theta_v),$$

i reaches its first maximum at an interval of time $\frac{1}{\omega}(\theta_i - \theta_v)$ ahead of v, and therefore i leads v by the angle $\theta_i - \theta_v$. Note the order of the subscripts: *i leads v* by the angle $\theta_i - \theta_v$; or *v leads i* by $\theta_v - \theta_i$. A negative lead is of course equivalent to an actual lag, and a negative lag is equivalent to an actual lead.

Currents and Voltages in Phase, in Quadrature and in Opposition. — When the phase difference is zero the current and potential drop are said to be "in phase"; when the phase difference is $\frac{\pi}{2}$ radians or 90° the current and potential drop are said to be "in quadrature"; when the phase difference is π radians or 180° the current and potential drop are said to be "in opposition."

R.M.S. and Average Values, Form Factor and Crest Factor. — For a *sine wave*:

$$\text{R.m.s. value} = \frac{\text{Maximum value}}{\sqrt{2}}; \tag{17}$$

$$\text{Average value} = \frac{2}{\pi} \times (\text{maximum value}); \tag{17a}$$

$$\text{Form factor} = \frac{\pi}{2\sqrt{2}} = 1.11; \tag{17b}$$

$$\text{Crest factor} = \sqrt{2} = 1.414. \tag{17c}$$

For any other relation between the instantaneous values of the current or voltage and time, i.e., for any other shape of current or voltage wave, *these relations do not hold.* The form factor of a rectangle is 1.00; of an isosceles triangle, 1.15; of a semi-circle or semi-ellipse, 1.04. The corresponding crest factors are 1.00, 1.73 and 1.22. See also article on *Wave Analysis*.

Power and Power Factor for Sine-wave Current and Voltage. — Let the voltage drop from terminal No. 1 to terminal No. 2 through any piece of apparatus be $v = \sqrt{2}\,V \sin(\omega t + \theta_v)$ and the current from terminal No. 1 to terminal No. 2 be $i = \sqrt{2}\,I \sin(\omega t + \theta_i)$, where V and I are the r.m.s. values and therefore $\sqrt{2}\,V$ and $\sqrt{2}\,I$ are the maximum values. Then the *instantaneous* power input is

$$p = vi = VI[\cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)]. \tag{18}$$

A study of Fig. 1 will show the physical meaning of this expression. The *average* power input is

$$P = VI \cos(\theta_v - \theta_i), \tag{18a}$$

where $(\theta_v - \theta_i)$ is the difference in phase between the current and voltage. Putting θ for this difference in phase, viz., $\theta = \theta_v - \theta_i$, equation (18a) may be written

$$P = VI \cos\theta. \tag{18b}$$

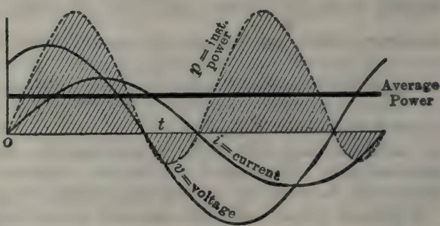

Fig. 1.

Whence the power factor of the load supplied to the apparatus is, from equation (8),

$$\cos\theta = \frac{P}{VI}. \tag{19}$$

Power-factor Angle. — Since in the case of sine-wave currents and voltages the power factor is equal to the cosine of the angle which expresses the difference in phase between them, this difference in phase is frequently called the "power-factor angle." When the wave shape is not a pure sine

curve, the power factor cannot be interpreted as the cosine of the phase difference, for phase difference has no definite meaning except in reference to sine waves; see definitions above. A non-sinusoidal voltage and current may both reach their zero values at the same instant, as in the case of an arc (*see Arc, Electric*), and in a sense may be said to be "in phase," but the power factor as defined by equation (8) may be far from unity.

Leading and Lagging Power Factor. — The power factor is always a positive quantity, but the power-factor angle may be either positive or negative, i.e., the current may lag or lead the voltage drop by any angle between 0° and 90°, or lag or lead the potential drop by any angle between 0° and 90°.* When the current lags the reference voltage by an angle between 0° and 90° the power factor is stated as such a fraction or percentage, *lagging*, e.g., a power factor of 80% lagging, and when the current leads the power factor is stated as such a fraction or per cent, *leading*. In the first case the power-factor angle is taken as positive, and in the second case negative.

Equivalent Sine-wave Currents and Voltages. — In very few instances are the actual currents and voltages in a circuit pure sine-waves, but many of the ordinary calculations of alternating-current circuits may be made with sufficient accuracy by assuming them as sine-waves of the same r.m.s. values as the actual waves, and differing in phase by the angle whose cosine is equal to the actual power factor, i.e., by the angle

$$\theta = \cos^{-1} \frac{P}{VI}, \tag{19a}$$

where P is the average power, V the r.m.s. value of the actual voltage and I the r.m.s. value of the actual current.

Vector Representation of Sine-wave Currents and Voltages. — Consider any sine function

$$i = I_m \sin \omega t.$$

The value of i at any instant may be represented graphically, see Fig. 2, by the vertical projection (i.e., the vertical distance from P_1 to OX) of a point P_1 at the end of a radius $OP_1 = I_m$ which revolves† at an angular speed ω about a fixed point O, the angle ωt being measured from the horizontal fixed line OX. Similarly, any other sine function

$$v = V_m \sin (\omega t + \theta)$$

may be represented by the vertical projection of the point P_2 at the end of a radius $OP_2 = V_m$ also revolving about O with an angular speed ω, the angle between OP_1 and OP_2, *when the frequency of both i and*

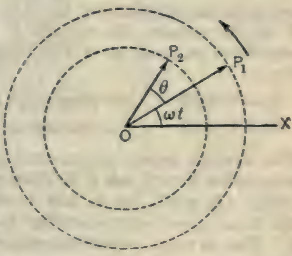

Fig. 2.

* When there is an actual power input into a portion of a circuit (e.g., a motor) it is most convenient to refer the current to the voltage *drop* through this portion of circuit in the direction of the current; when there is an actual power output (e.g., a generator) it is most convenient to refer the current to the voltage *rise* through this portion of the circuit, i.e., to the electromotive force in this portion of the circuit, in the direction of the current. When the potential *rise* is used as the reference, then for a power-factor angle between − 90° and 0° and between 0° and + 90°, the apparatus gives out power, and for a power-factor angle between − 90° and − 180° and between + 90° and + 180° the apparatus absorbs power.

† Counter-clockwise rotation has been adopted (1911) as standard by the International Electrotechnical Commission.

v is the same, remaining fixed in value and equal to the difference in phase θ between v and i. That is, v and i may be represented by rotating vectors (*see article on Vectors*) and when of the same frequency the relative position of the two vectors remains fixed. Similarly any number of currents and voltages of the same frequency may be represented by rotating vectors which remain fixed with respect to one another.

Instead of referring the various rotating vectors to a *fixed* line OX, this line of reference may also be considered as rotating with the same speed ω as the various vectors, or any one of the vectors may be chosen as the line of reference, for example, the vector OP_1 in Fig. 2. The rotating vectors referred to this rotating line of reference are then fixed with respect to this line of reference, and the entire diagram may be considered as fixed, as in Fig. 3, the originally chosen fixed line of reference OX rotating in the opposite direction with an angular speed ω.

Instead of making the vectors equal in length to the maximum values of the sine functions they may be chosen equal in length to their r.m.s. values. This merely introduces a factor $\sqrt{2}$ so that when any vector is considered as rotating the instantaneous value of the quantity which it represents is equal to $\sqrt{2}$ times the perpendicular distance from its end to the fixed line of reference.

Addition of Sine-wave Currents or of Sine-wave Voltages. — Since the r.m.s. values and phase relations of sine-wave currents and voltages may be represented by vectors, sine-wave currents are added in exactly the same manner as vectors, or forces, are added, and similarly for sine-wave voltages. The addition of vectors is fully treated in the article on *Vectors*, q.v. To add any two sine-wave currents or voltages not only must their effective values be known but also their phase relation; *the resultant of two alternating voltages of r.m.s. values V_1 and V_2 is never the arithmetical sum of V_1 and V_2*, except when the two voltages are exactly in phase, and similarly for alternating currents.

In-phase and Quadrature Components. — In Fig. 3, considering OP_1 as equal to the r.m.s. value I of the current and OP_2 as representing the r.m.s. value V of the voltage, the voltage V may be considered as made up of two components, viz.:

$V_1 = V \cos\theta$ in phase with I,

$V_2 = V \sin\theta$ in quadrature with I.

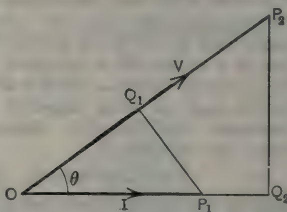

Fig. 3.

The average power corresponding to the component $V_1 = V\cos\theta$ is, from equation (18b), $IV_1 = IV\cos\theta$, and is equal to the total power corresponding to V and I. The average power corresponding to the component $V_2 = V\sin\theta$, since the angle between the current and this component of the voltage is $90°$, is equal to zero. The voltage component $V_1 = V\cos\theta$ is therefore frequently called the "power" component of the voltage, and the component $V_2 = V\sin\theta$ is frequently called the "wattless" component of the voltage. These terms, however, are not recommended. It is preferable to refer to these two components as the in-phase and quadrature components respectively. The terms active and reactive components are also used.

Similarly, the current I may be considered as made up of two components, viz.:

$I_1 = I\cos\theta$ in phase with V,

$I_2 = I\sin\theta$ in quadrature with V.

The first component is called the in-phase component of the current and the second the quadrature component of the current.

Resistance Drop and Reactance Drop. — When a sine-wave current of r.m.s. value I is established in a coil which has an effective * resistance r and inductance L, the drop of voltage V through the coil is represented by the vector diagram shown in Fig. 4. The reactance of such a coil, from the definition given by equation (14a), is then $x = 2\pi fL$. The voltage drop rI, due to the resistance of the coil, is in phase with the current I, whereas the voltage drop $2\pi fLI = xI$, due to the reactance of the coil, is 90° ahead of the current. A "resistance drop" i.e., the drop of potential due to a current through a resistance, is always in phase with the current which causes it, and an *inductive* reactance drop is always 90° ahead of the current which produces it. (A *condensive* reactive drop is always 90° behind the current which causes it, and is therefore directly opposite in phase to an inductive reactance drop produced by the same current.)

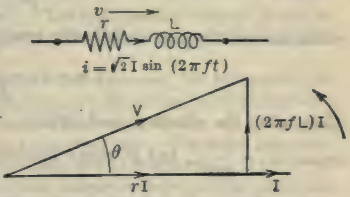

Fig. 4.

Impedance of a Coil to a Sine-wave Current. — From Fig. 4 and the definition of impedance, equation (12), it is evident that the impedance of a coil of resistance r and inductance L is

$$z = \sqrt{r^2 + (2\pi fL)^2}. \tag{20}$$

Leakage Current and Charging Current of a Condenser. — When a sine-wave voltage of r.m.s. value V is established across a condenser which has an effective† conductance g and capacity C, the total current I through it is represented by the vector diagram shown in Fig. 5. The *condensive* susceptance of a condenser from the definition given in equation (15a) is then $b = 2\pi fC$, or the *inductive* susceptance is $b' = -b = -2\pi fC$. The conduction or leakage current gV is in phase with the voltage drop through the condenser, whereas the charging or capacity current (or displacement current) $2\pi fCV = bV$ is 90° ahead of the voltage.

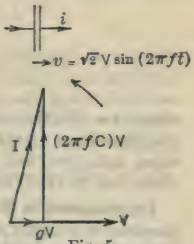

Fig. 5.

Admittance, Impedance, Effective Resistance and Reactance of a Condenser. — From Fig. 5 and the definitions given by equations (12) to (15) it follows that the admittance of a condenser to a sine-wave voltage is

$$y = \sqrt{g^2 + (2\pi fC)^2}; \tag{21}$$

the impedance is

$$z = \frac{1}{y} = \frac{1}{\sqrt{g^2 + (2\pi fC)^2}}; \tag{21a}$$

* When the coil has a non-magnetic core and the frequency is low, say 60 cycles or less, the effective resistance is practically equal to its ohmic or d-c. resistance; see article on *Skin Effect.*

† Even for moderate frequencies, 25 cycles or more, the effective conductance of a condenser is in general many times its ohmic conductance; see article on *Condensers, Electric.*

the effective resistance is

$$r = \frac{g}{g^2 + (2\pi fC)^2}; \tag{21b}$$

and the effective *condensive* reactance is

$$x_C = \frac{2\pi fC}{g^2 + (2\pi fC)^2}, \tag{21c}$$

which is equivalent to an *inductive* reactance of

$$x = -x_C = \frac{-2\pi fC}{g^2 + (2\pi fC)^2}. \tag{21d}$$

Only when the effective conductance g is zero is the reactance of a condenser equal to $-\frac{1}{2\pi fC}$, but in many instances the value of g is so small compared with $2\pi fC$ that the inductive reactance may be taken as $-\frac{1}{2\pi fC}$; the error in this approximate expression is less than 1 per cent for g less than 10 per cent of $2\pi fC$.

Relations between Effective Resistance, Reactance and Impedance and Effective Conductance, Susceptance and Admittance for Sine-wave Currents and Voltages. — From the definitions given above, equations (12) to (15), it may be shown that for sine-wave currents and voltages of a given frequency the following relations hold for any portion of a circuit:

$$\left.\begin{aligned}
z &= \sqrt{r^2 + x^2}, & y &= \sqrt{g^2 + b^2},\\
z &= \frac{1}{y}, & y &= \frac{1}{z},\\
r &= \frac{g}{y^2}, & g &= \frac{r}{z^2},\\
x &= \frac{b}{y^2}, & b &= \frac{x}{z^2},
\end{aligned}\right\} \tag{22}$$

where r = effective resistance, x = reactance (taken positive when inductive), z = impedance, g = effective conductance, b = susceptance (taken positive when inductive) and y = admittance, all for the given portion of circuit. Equations (20) to (21d) are special cases of equation (22).

Impedances in Series. — Let z_1, z_2, z_3, etc., be the impedances of the several portions of a circuit all connected in series (same current through each). Then the resultant impedance is

$$\left.\begin{aligned}
Z &= \sqrt{R^2 + X^2},\\
\text{where}\quad R &= r_1 + r_2 + r_3 + \text{etc.},\\
\text{and}\quad X &= x_1 + x_2 + x_3 + \text{etc.},
\end{aligned}\right\} \tag{23}$$

where r_1, r_2, r_3, etc., are the effective resistances and x_1, x_2, x_3, etc., the reactances (condensive reactances to be considered as negative) of the several impedances. When there is no external source of e.m.f. in any portion of the circuit, then the resultant power factor is $\cos\theta$ where

$$\tan\theta = \frac{x_1 + x_2 + x_3 + \text{etc.}}{r_1 + r_2 + r_3 + \text{etc.}}. \tag{23a}$$

Example. — An alternating current of 100 amperes is to be supplied to a receiver which has an equivalent resistance r_1 of 2 ohms and an equivalent

reactance x_1 of 0.5 ohm. The line has a resistance r_2 of 0.1 ohm and an inductive reactance x_2 of 1.5 ohms. The equivalent resistance of the line and receiver is then $R = 2 + 0.1 = 2.1$ ohms and the equivalent reactance of the line and receiver is $X = 0.5 + 1.5 = 2.0$ ohms. Hence the equivalent impedance of the line and receiver is $Z = \sqrt{(2.1)^2 + (2.0)^2} = 2.90$ ohms. The impedance of the receiver alone is $z_1 = \sqrt{(2)^2 + (0.5)^2} = 2.06$ ohms and the impedance of the line alone is $z_2 = \sqrt{(0.1)^2 + (1.5)^2} = 1.50$ ohms. The arithmetical sum of z_1 and z_2 is 3.56, which is 23 per cent greater than the true impedance of the line and receiver.

When the current supplied to the receiver is 100 amperes, the voltage at the receiver is $V = 100 \times z_1 = 100 \times 2.06 = 206$ volts and the voltage at the generator is $V_0 = 100 \times Z = 100 \times 2.90 = 290$ volts, see Fig. 6, that is, the voltage at the receiver is $290 - 206 = 84$ volts less than at the generator. The total potential drop in the two wires forming the line, however, is $100 \times z_2 = 100 \times 1.50 = 150$ volts, which is 79 per cent greater than the true difference between the potential drops across the generator and across the receiver terminals.

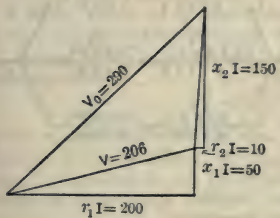

Fig. 6.

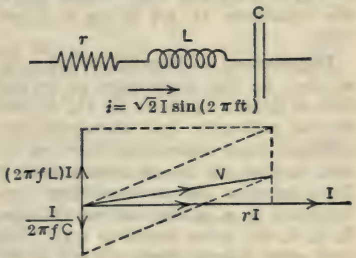

Fig. 7.

Resonance of a Coil and Condenser in Series. — Consider the circuit shown in Fig. 7. When a sine-wave current of effective value I is established in such a circuit the voltage drop across the resistance is $V_r = rI$ and is in phase with I. The voltage drop across the inductance is $V_L = (2\pi fL)\,I$ and *leads* I by 90.° The voltage drop across the capacity (a condenser with *negligible* conductance) is $V_C = \frac{I}{2\pi fc}$ and *lags* I by 90°. Hence the resultant voltage across the entire circuit is

$$V = \overline{V_r + V_L + V_C} = I\sqrt{r^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}. \tag{24}$$

The reactance is therefore

$$x = \sqrt{z^2 - r^2} = \left(2\pi fL - \frac{1}{2\pi fC}\right).$$

When the inductance L, capacity C and frequency f are so related that

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{25}$$

the reactance of the circuit is zero, the impedance is equal to the resistance, the power factor is unity and the current corresponding to a given voltage V is $I = \frac{V}{r}$; that is, the current is a maximum and depends only upon the resistance

of the circuit. The frequency corresponding to this condition is the same as the frequency with which the current and p.d. would oscillate were the condenser short-circuited by the inductance; i.e., this frequency corresponds* to the free period of such a circuit (*see Transient Electric Phenomena and Oscillations*). In general, a condenser and coil in series are said to be in resonance with the impressed frequency when the current for a given impressed voltage is a maximum, and the power factor therefore unity. When the conductance of the condenser is negligible the resonant frequency is given by equation (25).

When resonance obtains in a series circuit the voltage across the coil and that across the condenser may be many times the impressed voltage. For example, when the inductance L is 1 henry, the capacity C is 7.04 microfarads, and the frequency is 60 cycles, the inductive reactance is $x_L = 2\pi \times 60 \times 1 = 377$ ohms, the condensive reactance is $x_c = \dfrac{1}{2\pi \times 60 \times 7.04 \times 10^{-6}} = 377$ ohms, and the circuit is in resonance. For a resistance of 1 ohm in the coil and an impressed e.m.f. of 100 volts, the voltage across the coil is $\sqrt{(100)^2 + (377 \times 100)^2} = 37{,}700$ volts and the voltage across the condenser is $377 \times 100 = 37{,}700$ volts.

Impedances in Parallel.— Let z_1, z_2, z_3, etc., be the impedances of several branch circuits as shown in Fig. 8, and let the resistances and reactances constituting these impedances be r_1, r_2, r_3, etc., and x_1, x_2, x_3, etc., respectively.† First calculate the corresponding conductances g_1, g_2, g_3, etc., and susceptances b_1, b_2, b_3, etc., using equations (22). Then the resultant admittance, *provided there are no external sources of e.m.f. in any of the branches*, is

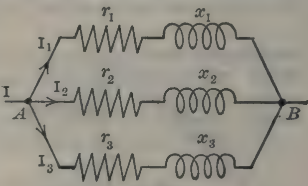

Fig. 8.

$$\left.\begin{aligned} Y &= \sqrt{G^2 + B^2}, \\ G &= g_1 + g_2 + g_3 + \text{etc.}, \\ B &= b_1 + b_2 + b_3 + \text{etc.}, \end{aligned}\right\} \tag{26}$$

where (the second and third lines above)

and the resultant power factor is $\cos\theta$ where

$$\tan\theta = \frac{b_1 + b_2 + b_3 + \text{etc.}}{g_1 + g_2 + g_3 + \text{etc.}}. \tag{26a}$$

Resonance of a Coil and Condenser in Parallel. — When a coil of negligible resistance and a condenser of negligible conductance are connected in parallel, the resultant current established through the circuit by a given impressed voltage will be *zero* (infinite impedance) when the inductance L, capacity C and frequency f bear the following relation:

$$f = \frac{1}{2\pi\sqrt{LC}}. \tag{27}$$

Compare with equation (25). Under these conditions the coil and condenser are said to be in resonance with the impressed frequency. When the coil has an appreciable resistance and the condenser an appreciable conductance they are said to be in resonance when the resultant current is a minimum for a given impressed voltage, and the power factor therefore unity.

* Approximately only when r is large; see *Transient Electric Phenomena and Oscillations.*

† Condensive reactances to be considered negative.

NON-SINUSOIDAL CURRENTS AND VOLTAGES. — The general expression for any alternating current, or in fact of any continuous periodic function, is

$$i = \sqrt{2}\,I_1 \sin(\omega t + \theta_1) + \sqrt{2}\,I_2 \sin(2\,\omega t + \theta_2) + \sqrt{2}\,I_3 \sin(3\,\omega t + \theta_3) + \ldots,$$

where I_1, I_2, I_3, etc., represent the r.m.s. values of each of the terms. That is any current or voltage wave may be considered as made up of a "fundamental" sine-wave, having the same frequency as that of the actual wave, and "harmonic" sine-waves having frequencies which are integral multiples of the frequency of the fundamental. Alternating currents and voltages in practice usually contain one or more of the *odd* harmonics; *even* harmonics practically never occur in ordinary electric circuits supplied from commercial forms of generators. However, as noted above, it is permissible in most instances to assume sine-wave currents and voltages, since the harmonics present are usually relatively weak compared with the fundamental. In certain instances, however, it is necessary to analyze a wave into its fundamental and harmonics. For methods of experimentally determining the shape of current and voltage waves see the articles on *Oscillographs* and *Braun Tube;* for the analysis of the curves themselves see the article on *Wave Analysis.*

R.M.S. Value of a Non-sinusoidal Wave. — The r.m.s. value of such a wave can be obtained directly from equation (3), or if the r.m.s values I_1, I_2, I_3, etc., of the harmonic and fundamentals are known the r.m.s. value of the resultant wave is

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \ldots} \qquad (28)$$

A like relation holds for the r.m.s. value of a voltage wave. Similarly, if for example a 25-cycle e.m.f., say E_{25}, and a direct e.m.f., say E_d, are acting in series in the same circuit, the resultant e.m.f. of the combination is

$$E = \sqrt{E_{25}^2 + E_d^2}.$$

Power Corresponding to Non-sinusoidal Currents and Voltages. — Let I_1 be the r.m.s. value of the fundamental (first harmonic) of the current, V_1 the r.m.s. value of the fundamental of the voltage and θ_1 the difference in phase between these two fundamentals, both being of the same frequency; let I_2, V_2 and θ_2 be the corresponding quantities for the second harmonic; let I_3, V_3 and θ_3 be the corresponding values for the third harmonic, etc. Then the average power is

$$P = V_1 I_1 \cos\theta_1 + V_2 I_2 \cos\theta_2 + V_3 I_3 \cos\theta_3 + \text{etc.}$$

That is, each harmonic contributes an amount to the total power equal to the power it would develop were the other harmonics not present. If, for example, the third harmonic is not present in the current wave, then this harmonic contributes nothing to the average power even though there may be a large third harmonic in the voltage wave. Again, when a 25-cycle alternating electromotive force E_{25} and a direct electromotive force E_d are acting in series on the same circuit the power developed is the sum of the powers which each would develop if they acted separately, but the resultant e.m.f. of the combination is not $E_{25} + E_d$, but, as noted above, is $\sqrt{E_{25}^2 + E_d^2}$.

Calculation of Networks when the Currents and Voltages are Non-sinusoidal. — See below under *Symbolic Notation,* equations (31) and (32)), and the accompanying text.

SYMBOLIC NOTATION FOR EXPRESSING SINE-WAVE CURRENTS AND VOLTAGES. — Since sine-wave currents and voltages may be represented by vectors equal in length to the r.m.s. values of these quan-

tities and making definite constant angles with one another, two mutually perpendicular axes of references may be chosen, and each current and voltage resolved into two components, one along the X-axis or horizontal axis and one along the Y-axis or vertical axis. The component along the X-axis may be expressed as an ordinary algebraic quantity, and the component along the Y-axis may also be expressed as an algebraic quantity with the symbol "j" written in front of it to indicate that it is perpendicular to the X-component, That is, any current may be written

$$\underset{\cdot}{I} = I_1 + jI_2,$$

where I_1 is the horizontal or X-component of the current and I_2 the vertical or Y-component. Until one gets familiar with this notation it is best to indicate the symbolic nature of the currents, voltages, impedances, etc., by writing dots under them; but these dots may be dispensed with in the actual solution of problems by this method when one keeps clearly in mind that in all operations the various quantities are to be treated as *complex* quantities throughout.

A voltage drop in symbolic notation is expressed in a similar manner, viz.,

$$\underset{\cdot}{V} = V_1 + jV_2,$$

and an electromotive force as

$$\underset{\cdot}{E} = E_1 + jE_2,$$

the *same* axis of reference being used for the currents, voltage drops and electromotive forces.

Impedance and Admittance in Symbolic Notation. — In this notation an impedance of resistance r and reactance* x is represented by the complex quantity

$$\underset{\cdot}{z} = r + jx, \qquad (29)$$

and an admittance of conductance g and susceptance* b by the complex quantity

$$\underset{\cdot}{y} = g - jb. \qquad (29a)$$

These expressions are independent of the axes of reference chosen.

Currents, Voltages, Impedances and Admittances as Complex Numbers. — In all operations involving the addition or subtraction of currents or voltages and in all operations involving products of currents and impedances or products of voltages and admittances, these quantities when written in symbolic notation may be treated as complex quantities (*see article on Complex Quantities*), *provided all the currents and voltages are referred to the same axis of references*. In all such operations the symbol "j" may be considered as mathematically equivalent to $\sqrt{-1}$. Hence when a term of the form $A_1 + jA_2$ occurs in the denominator of any fraction, the fraction may be "rationalized" by multiplying numerator and denominator by $A_1 - jA_2$, which gives

$$\frac{1}{A_1 + jA_2} = \frac{A_1 - jA_2}{A_1^2 + A_2^2}. \qquad (30)$$

In any resulting expression for a current or voltage when thus rationalized the "real" part represents the actual component of the current or voltage in the direction of the X-axis and the "j" part, i.e., the sum of the terms which are multiplied by j, represents the actual component of the current or voltage along the Y-axis.

* Taken positive when inductive, negative when condensive, e.g., the admittance of a condenser is $g + j\,(2\pi fC)$.

Solution of Alternating-current Networks. — When all the currents, voltages, impedances and admittances are expressed in symbolic notation, the following relations hold:*

1. The sum of all the currents flowing to any point in any network of conductors is zero. That is, at any point

$$\underset{\cdot}{I} + \underset{\cdot}{I}' + \underset{\cdot}{I}'' + \ldots = 0, \tag{31}$$

where the currents are all expressed in symbolic notation and are all referred to the *same* line of reference.

2. The sum of all the impedance drops in a given direction around any closed loop in any network of conductors is equal to the sum of all the *externally induced e.m.f.'s* acting in this loop in this direction. That is, around any closed loop

$$\underset{\cdot}{z}\underset{\cdot}{I} + \underset{\cdot}{z}'\underset{\cdot}{I}' + \underset{\cdot}{z}''\underset{\cdot}{I}'' + \ldots = \underset{\cdot}{E} + \underset{\cdot}{E}' + \underset{\cdot}{E}'' + \ldots, \tag{32}$$

where the currents, impedances and electromotive forces are all expressed in symbolic notation, and the *currents and e.m.f.'s are all referred to the same axis of reference*. That is, the currents are to be expressed as $\underset{\cdot}{I} = I_1 + jI_2$, $\underset{\cdot}{I}' = I_1' + jI_2'$, etc., and the e.m.f.'s as $\underset{\cdot}{E} = E_1 + jE_2$, $\underset{\cdot}{E}' = E_1' + jE_2'$, etc., where all the components of the currents and e.m.f.'s with the subscript 1 are parallel to one another and all those with the subscript 2 are parallel to one another and lead the first set of components by 90 degrees.

The electromotive forces due to inductance and capacity are taken account of by the impedance; the electromotive forces represented by the E's in equation (32) are the externally induced electromotive forces such as those due to generators or motors or to the mutual inductance of the two windings of a transformer.

In applying equations (31) and (32) to the calculation of the currents and potential drops in any network of circuits, care must be taken to designate clearly the sense of the vectors representing the currents and e.m.f.'s. This is most conveniently done by numbering all the junction points in the network, and designating each current and e.m.f. by a double subscript written in the order corresponding to the assumed direction of the current or e.m.f. vector. For example, in Fig. 9, the e.m.f. from 0 to 1 is represented by $\underset{\cdot}{E}_{01}$ while the e.m.f. from 1 to 0, which is equal to $-\underset{\cdot}{E}_{01}$, is represented by $\underset{\cdot}{E}_{10}$. In the figure, then, the net electromotive force from 1 to 2 is

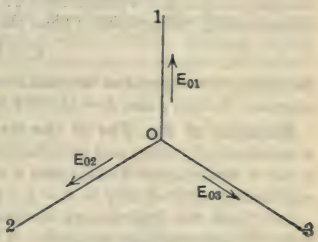

Fig. 9.

$$\underset{\cdot}{E}_{12} = \underset{\cdot}{E}_{10} + \underset{\cdot}{E}_{02},$$

or

$$\underset{\cdot}{E}_{12} = -\underset{\cdot}{E}_{01} + \underset{\cdot}{E}_{02}.$$

Each equation of the form (31) or (32) is in reality equivalent to two equations, since the sum of all the "real" terms on one side must be equal to the sum of all the real terms on the other side, and similarly, the sum of all the "j" terms on one side must be equal to the sum of all the "j" terms on the other side, the denominators of all fractions having been cleared of "j" terms by the transformation given by equation (30). This is merely another way of stating the fact that the component in any direction of the *resultant* of any number of vectors

* These relations are simply Kirchhoff's Laws (*see Electricity and Magnetism, Principles of*) expressed in a convenient form for alternating currents.

must be equal to the algebraic sum of the components in this direction of all the individual vectors. Applying these two laws to any network enables one, therefore, to calculate both components of every p.d. and every current, when the impedances and the electromotive forces are known.

These equations hold only when *the currents and e.m.f.'s are all simple harmonic functions* of the *same* frequency *and the resistances and reactances are constant.* When, however, the resistances, inductances and capacities are constant, a similar set of equations holds for each frequency that may be present. Since the equations are all linear in the I's and E's, the currents and e.m.f.'s of any given frequency will be uninfluenced by the presence of currents or e.m.f.'s of any other frequency. Hence, when the harmonics present in each e.m.f. are known, the harmonics present in each current may be calculated by solving the equations corresponding to the frequency of this particular harmonic, these equations being exactly the same as would hold were all the other harmonics absent.

Network problems can often be simplified by various expedients, see for example, *Analytical Solutions of Power-Circuit Network Problems*, by R. D. Evans, in the *Electric Journal, Vol. 16, pp. 345–349, 1919.*

Difference in Phase between a Current and Voltage in Symbolic Notation. — Let the current be represented by

$$\underset{\cdot}{I} = I_1 + jI_2$$

and the voltage drop in the same sense as the current be represented by

$$\underset{\cdot}{V} = V_1 + jV_2.$$

Then the current lags behind the voltage drop by the angle θ, where

$$\tan\theta = \frac{V_2I_1 - V_1I_2}{V_1I_1 + V_2I_2}. \tag{33}$$

Power Corresponding to a Current and Voltage in Symbolic Notation. — Let the current and voltage drop be represented by the same expressions as in the preceding paragraphs. Then the average power is

$$P = V_1I_1 + V_2I_2. \tag{34}$$

Note that this expression is equal to the real part of the product of the complex numbers $V_1 + jV_2$ and $I_1 + jI_2$ *with the sign between the two terms reversed.*

Example of the Use of the Symbolic Method. — An impedance z_1 has a resistance of 3 ohms and an inductive reactance of 4 ohms; a second impedance z_2 has a resistance of 8 ohms and a condensive reactance of 6 ohms. z_1 is then represented symbolically as $\underset{\cdot}{z_1} = 3 + j4$ and z_2 as $\underset{\cdot}{z_2} = 8 - j6$.

Let these two impedances be connected in series, and let an e.m.f. of 100 volts be impressed across them. Choosing the vector representing the over-all potential drop as the axis of reference, and calling the current I, then $(3 + j4 + 8 - j6)\,\underset{\cdot}{I} = 100 + j0$, whence

$$\underset{\cdot}{I} = \frac{100}{11 - 2j} = \frac{1100 + j200}{121 + 4} = 8.8 + j1.6.$$

Hence the effective value of the current is

$$I = \sqrt{(8.8)^2 + (1.6)^2} = 8.94 \text{ amperes}$$

and the current leads the over-all potential drop by the angle

$$\tan^{-1}\frac{1.6}{8.8} = 10.3°.$$

The potential drop across the first impedance is

$$\underset{\cdot}{V}' = (3 + j4)\,(8.8 + j1.6) = 26.4 - 6.4 + j\,(35.2 + 4.8) = 20 + j40,$$

which has the effective value

$$V' = \sqrt{(20)^2 + (40)^2} = 44.7 \text{ volts},$$

and leads the over-all potential drop V by the angle

$$\tan^{-1}\frac{40}{20} = 63.5°.$$

The potential drop across the second impedance is

$$V'' = (8 - j6)(8.8 + j1.6) = 70.4 + 9.6 - j(52.8 - 12.8) = 80 - j40,$$

which has the effective value

$$V'' = \sqrt{(80)^2 + (40)^2} = 89.4 \text{ volts},$$

and lags behind the over-all potential drop V by the angle

$$\tan^{-1}\frac{40}{80} = 26.5°.$$

The power input into the first impedance is

$$W' = 8.8 \times 20 + 1.6 \times 40 = 240 \text{ watts}.$$

The power input into the second impedance is

$$W'' = 8.8 \times 80 - 1.6 \times 40 = 640 \text{ watts}.$$

The total power input is

$$W = 8.8 \times 100 + 1.6 \times 0 = 880 \text{ watts},$$

which of course is the sum of W' and W''.

POLYPHASE SYSTEMS. — A polyphase alternating-current system is a network (i.e., combination of circuits) supplied from a generator or generators which develop two or more electromotive forces differing in phase from one another by a constant angle; see *Distribution of Electric Energy; Generators, Alternating-current; Transformers.* The two kinds of polyphase circuits commonly employed are the two-phase and three-phase circuits.

Star and Mesh Connections. — Consider n separate coils or windings, which may be mounted on a common armature or be entirely distinct, as for example n groups of lamps. When these n windings are connected end to end so that they are all in series, forming a closed chain, as in Fig. 10, and terminals

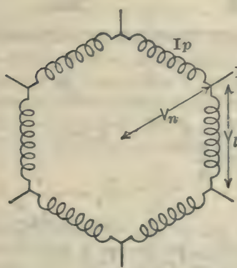

Fig. 10.

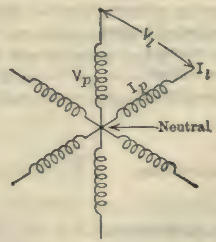

Fig. 11.

are brought out from the n junctions, they are said to be connected in "mesh." When one terminal of each of these windings is connected to a common junction point, as in Fig. 11, and terminals are brought out from the free ends, the windings are said to be connected in "star," and the common point is called the "neutral point."

***n*-Phase System.** — When such a group of n windings, as shown in Fig. 10 or 11, is connected to a generator or other source of e.m.f., having n separate windings and therefore developing n different e.m.f.'s which differ in phase from one another, the system is called an n-phase system, each winding being called a "phase." For example, when there are three separate windings on the generator, three line wires and three windings constituting the load, the system is a three-phase system.

Balanced Systems. — When the e.m.f.'s, if any, in the n-phases of any system of connection are all equal in r.m.s. value and differ successively in phase by $360/n$ degrees and if the currents in these windings are also equal in r.m.s. value and differ successively in phase by $360/n$ degrees, the system is said to be a "balanced" n-phase system. When the e.m.f.'s in the various parts of a system are equal in r.m.s. value and differ successively in phase by $360/n$ degrees, and the impedances in the various windings or phases are all equal, both as regards resistance and reactance, then the currents are necessarily equal in r.m.s. value and differ successively in phase by $360/n$ degrees, and the system is balanced.

Phase and Line Currents and Voltages. — The current in any winding or phase of an n-phase system, see Figs. 10 and 11, is called the "phase current," and the drop (or rise) of potential *through* this winding is called the "phase voltage." The current in the line leaving any terminal of an n-phase system is called the "line current" and the voltage between *adjacent* line wires or terminals is called the "line voltage," except in the special case of a two-phase connection, see below, when the voltage between diametrically opposite terminals is called the line voltage. In the case of a star-connection the voltage between any terminal and the neutral point is called the "voltage to neutral"; in the case of a balanced mesh connection by voltage to neutral is meant the voltage which would exist between any terminal and the neutral of a star-connection connected to the terminals of the actual device, the impedance of all the legs of the star-connection forming this "artificial neutral" being equal and sufficiently large not to take an appreciable current.

The relations between these various currents and voltages for a balanced n-phase system are as follows:

	Mesh	Star
Number of phases	n	n
Line current	I_l	I_l
Line voltage	V_l	V_l
Phase current	$I_p = \dfrac{1}{2\sin\frac{\pi}{n}} I_l$	$I_p = I_l$
Phase voltage	$V_p = V_l$	$V_p = \dfrac{1}{2\sin\frac{\pi}{n}} V_l$
Voltage to neutral	$V_n = \dfrac{1}{2\sin\frac{\pi}{n}} V_p$	$V_n = V_p$
Total volt-amperes	$nV_pI_p = \dfrac{n}{2\sin\frac{\pi}{n}} V_lI_l$	$nV_pI_p = \dfrac{n}{2\sin\frac{\pi}{n}} V_lI_l$

Two-phase or Quarter-phase System. — Strictly, the so-called single-phase system is a star-connected two-phase system, since the currents *from* the two terminals are in opposite directions at any instant, the current leaving by one and entering by the other. However, in practice the name two-phase system is used for a system supplied from a generator or other source of e.m.f. having two windings in which are developed two e.m.f.'s differing in phase by 90°; i.e., a two-phase system is in reality two distinct single-phase systems each with two terminals. Two of the four terminals may be connected to each other, in which case but three line wires are required. Or, the two single-phase systems may be connected at their middle points; in this case the two-phase system may be considered as a four-phase, or, as it is usually called, a "quarter-phase" system. See the articles on *Distribution of Electric Energy* and *Transformers*.

Three-phase System. — Delta and Y-Connections. — For a three-phase system the generators and motors are designed with three windings or phases which may be connected either in mesh, usually called a "delta-connection" in this case, since the diagram of the three windings forms a Greek delta, or the three windings may be connected in star, usually called a "Y-connection" in this case, since the diagram of the three windings forms a Y. The relations between line and coil currents and voltages for a *balanced* three-phase system are as follows:

	Delta	Y
Line current	I_l	I_l
Line voltage	V_l	V_l
Phase current	$I_p = \frac{I_l}{\sqrt{3}}$	$I_p = I_l$
Phase voltage	$V_p = V_l$	$V_p = \frac{V_l}{\sqrt{3}}$
Voltage to neutral	$V_n = \frac{V_l}{\sqrt{3}}$	$V_n = V_p$
Total volt-amperes	$3V_pI_p = \sqrt{3}V_lI_l$	$3V_pI_p = \sqrt{3}V_lI_l$

Calculation of Balanced Three-phase Circuits. — Any problem in regard to a *balanced* three-phase circuit may be solved by reducing all parts of the circuit to an equivalent Y-connection, provided the currents and e.m.f.'s are sine-waves. The transformations are made as follows:

Any Δ-connected motor or generator is considered as equivalent to a Y-connected generator or motor in which

$$E_y = \frac{E_\Delta}{\sqrt{3}}, \qquad r_y = \frac{r_\Delta}{3}, \qquad x_y = \frac{x_\Delta}{3},$$

where the quantities E_y, r_y and x_y are the e.m.f., resistance and reactance per phase of the Y-connected machine equivalent to the e.m.f., resistance and reactance per phase of the actual Δ-connected machine.

Each of the line wires is in series with a corresponding phase of the equivalent Y-connected machine.

When all parts of the circuit have thus been reduced to equivalent Y's, each of the three-phases may be treated as a single-phase circuit, each circuit considered completed by a wire having *zero* impedance connecting all the neutrals together, since all the neutrals are at the same potential.

The voltages thus calculated are the voltages to neutral and the currents are line currents. To find the line voltage multiply the calculated voltage by $\sqrt{3}$; similarly, to find the actual phase current in the Δ-connected generator or load divide the calculated current by $\sqrt{3}$.

Example of Three-phase Calculation. — Energy is supplied from a generating station to a substation 50 miles away at a rate of 20,000 kilowatts. The system is a balanced three-phase system and operates at a frequency of 25 cycles. The transmission line consists of three No. 0000 B. & S. copper wires spaced six feet between centers. It is desired to find (1) the voltage between wires at the generating station when the voltage between wires at the substation is 60,000 volts, and the power factor at the substation is 80 per cent, with the current *lagging*, (2) how much power is lost in the transmission line and (3) what is the power factor at the generating station. The electrostatic capacity of the line may be neglected.

The current per wire is

$$I = \frac{20{,}000{,}000}{\sqrt{3} \times 60{,}000 \times 0.8} = 241 \text{ amperes.}$$

The voltage to neutral at the substation is

$$V_n = \frac{60{,}000}{\sqrt{3}} = 34{,}600.$$

The component of this voltage in phase with the line current is $0.8 \times 34{,}600 = 27{,}700$ volts and the component 90° ahead of the line current is $0.6 \times 34{,}600 = 20{,}800$ volts, since $\cos\theta = 0.8$ and $\sin\theta = 0.6$.

The resistance per mile of a No. 0000 wire is 0.258 ohm; its reactance per mile at 25 cycles is 0.303 ohm. Hence the total resistance of each wire is 12.9 ohms and the total reactance of each wire 15.2 ohms. The resistance drop in each wire is then $12.9 \times 241 = 3110$ volts and is in phase with the line current and the inductive drop in each wire is $15.2 \times 241 = 3660$ volts and is 90° ahead of the line current.

At the generator end the voltage to neutral in phase with the line current is then $27{,}700 + 3110 = 30{,}810$ volts and the voltage to neutral 90° ahead of the current is $20{,}800 + 3660 = 24{,}460$ volts. The resultant voltage to neutral at the generator end is then $\sqrt{(30{,}810)^2 + (24{,}460)^2} = 39{,}300$ volts, and therefore the line voltage at the generating station is

$$V_l' = \sqrt{3} \times 39{,}300 = 68{,}000.$$

The power lost in the line is equal to $3RI_l^2$, where R is the total resistance of each wire and I_l the line current. Hence the power lost in the line is

$$3 \times 12.9 \times (241)^2 \text{ watts} = 2250 \text{ kilowatts.}$$

The total power delivered to the line and substation is then 22,250 kilowatts. Hence the power factor at the generating station is

$$\frac{22{,}250{,}000}{\sqrt{3} \times 68{,}000 \times 241} = 0.784 = 78.4 \text{ per cent.}$$

Measurement of Power in Three-phase Circuits. — See *Wattmeters*.

Calculation of Unbalanced Three-phase Circuits. — When a three-phase system is unbalanced, the currents and voltages can always be calculated by applying Kirchhoff's laws to the junctions and loops formed by the various parts of the system. For example, referring to Fig. 12, what will be the currents in each line wire and what will be the readings of the two watt-meters connected as shown when the three impedances forming the unbalanced star

have the values $z_1 = 1 + j\,2$, $z_2 = 3 - j\,4$ and $z_3 = 5 + j\,6$, and the voltages between the terminals 1, 2 and 3 are balanced and numerically equal to 100 volts? What is the voltage from each terminal to the neutral?

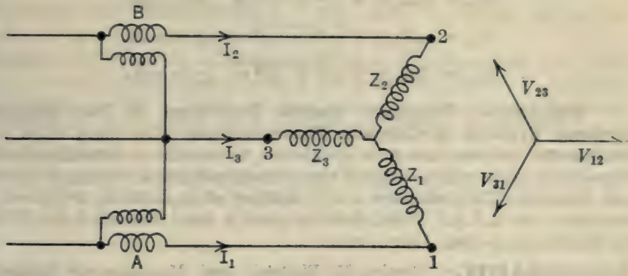

Fig. 12.

The following relations then hold, taking V_{12} as the reference vector:

$$\dot{V}_{12} = 100, \tag{a}$$

$$\dot{V}_{23} = -\,50 + j\,86.6 \tag{b}$$

$$\dot{V}_{31} = -\,50 - j\,86.6 \tag{c}$$

$$\dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 0 \tag{d}$$

$$100 = (1 + j\,2)\dot{I}_1 - (3 - j\,4)\dot{I}_2 \tag{e}$$

$$-\,50 + j\,86.6 = (3 - j\,4)\dot{I}_2 - (5 + j\,6)\dot{I}_3 \tag{f}$$

Substituting in (f) the value of $\dot{I}_3$ from (d) there results.

$$-\,50 + j\,86.6 = (8 + j\,2)\dot{I}_2 + (5 + j\,6)\dot{I}_1 \tag{g}$$

Substituting in (g) the value of $\dot{I}_2$ from (e) and solving for $\dot{I}_1$ there results,

$$\dot{I}_1 = 25.36 + j\,5.90. \tag{h}$$

Substituting this value of $\dot{I}_1$ in (e) there results,

$$\dot{I}_2 = -\,19.43 - j\,7.04 \tag{i}$$

Substituting (h) and (i) in (d) there results,

$$\dot{I}_3 = -\,5.93 + j\,1.14 \tag{j}$$

The r.m.s. values of these three currents are respectively,

$$I_1 = 26.04; \quad I_2 = 20.64; \quad I_3 = 6.04.$$

Wattmeter A indicates the power P_A corresponding to the voltage V_{13} and the current I_1, and wattmeter B indicates the power P_B corresponding to the voltage V_{23} and the current I_2. Note that from (b) and (c) and (h) and (i),

$$\dot{V}_{13} = 50 + j\,86.6, \qquad \dot{V}_{23} = -\,50 + j\,86.6,$$

$$\dot{I}_1 = 25.36 + j5.90, \qquad \dot{I}_2 = -\,19.43 - j\,7.04.$$

Whence, from equation (34),

$$P_A = 1779 \text{ watts} \quad \text{and} \quad P_B = 362 \text{ watts}.$$

Note that $(P_A + P_B)$ is equal to the total power in-put to the three phases, viz., to $(r_1I_1^2 + r_2I_2^2 + r_3I_3^2)$, and compare with the article on *Wattmeters.*

The r.m.s. voltage from each terminal to the neutral is the product of the current in that particular phase of the Y by the impedance of that phase, viz.,

$$V_{10} = 26.04 \times \sqrt{(1)^2 + (2)^2} = 58.23 \text{ volts},$$

$$V_{20} = 20.64 \times \sqrt{(3)^2 + (4)^2} = 103.20 \text{ volts},$$

$$V_{30} = 6.04 \times \sqrt{(5)^2 + (6)^2} = 47.17 \text{ volts}.$$

Unbalanced Polyphase System Equivalent to Two Balanced Polyphase Systems of Opposite Phase-rotation. — In many cases it is advantageous, for the purpose of calculation, to consider an unbalanced polyphase system as the superposition of two balanced polyphase systems of opposite phase rotation. The procedure and a number of applications are given by Fortescue in the *A.I.E.E. Proc., June 18, 1918, p. 629.* See also Lyon, W. V., *Elec. World, 75, p. 1304, June 5, 1920.*

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ALUMINUM. — (*See also Electrochemical Processes, Industrial; Lightning Protectors; Wires and Cables, Bare.*) The method of manufacturing aluminum wire is similar to that employed in making copper wire (*see Copper*).

MECHANICAL PROPERTIES. — The more important mechanical properties of aluminum are discussed below.

Necessity of Stranding Aluminum Wire. — Aluminum wire (i.e., solid wire) is little used for aerial spans owing to the danger of failure where accidental abrasions have occurred. Cables not only reduce the above danger, but have greater breaking load, owing to the superior tensile strength of small wires.

Tensile Strength. — The tensile strength of aluminum has not been studied so thoroughly as that of copper, but it is well known that like copper, aluminum is much stronger in small, than in large, wires. The following table by H. M. Hobart, which is for solid wires, illustrates this point.

Diameter, inches	0.05	0.10	0.15	0.20	0.25	0.30	0.35
Tensile strength, lb. per sq. in.	33,300	28,800	26,200	24,800	23,800	23,200	23,000

The Standard Electric Co., of California, uses a solid aluminum wire, 0.294 inch diameter, having a tensile strength of 22,800 pounds per square inch, and a conductivity of 59.9 per cent.

Elongation. — The ultimate elongation of hard-drawn aluminum wire varies from 2.0 per cent to 3.7 per cent (in 1 meter length) as the diameter is increased from 1 to 4½ mm.. (*H. M. Hobart.*)

Modulus of Elasticity. — Different authorities give values from 9×10^6 to 10×10^6 in pounds per square inch, the former being given by the Aluminum Company of America.

Elastic Limit. — The Aluminum Company of America in their booklet on "Properties of Aluminum," give the following figures for elastic limit in tension, the metal being 99 per cent pure.

	Lb. per sq. in.
Castings	8,500
Sheet	12,500 to 25,000
Wire	16,000 to 33,000
Bars	14,000 to 23,000

Density. — A density of 2.70 grams per cubic centimeter is given by the U. S. Bureau of Standards (*Circ. No. 31*) as a good average value for commercial hard-drawn aluminum. This is equivalent to 0.0975 pound per cubic inch. Hobart gives 2.71 for the density. F. J. Brislee (*Lond. Elec. Review, Jan. 5, 1912*) gives 2.708 as the density of cast aluminum and 2.705 for hard-drawn rod.

THERMAL PROPERTIES. — See article on *Heat and Thermal Properties.*

CONDUCTIVITY AND RESISTIVITY. — The Aluminum Company of America give as the average of many thousands of separate determinations, the figure of 2.828 microhms per centimeter cube at 20° C. as the resistivity of commercial aluminum wire. This corresponds to a conductivity of 61 per cent of the annealed copper standard (see *Resistance and Conductance, Electric*).

Effect of Hardness on Conductivity. — The resistance of aluminum depends upon its hardness; for example, the resistance of aluminum wire of hard-

ness corresponding to a tensile strength of about 20 kg. per mm. is approximately 2 per cent greater than that of the same wire when thoroughly annealed, and consequently soft. (*H. M. Hobart.*)

Temperature Coefficient of Resistivity. — From the data given by G. Grassi (*Elletrotecnica, 6, p. 10, Jan. 5, 1919*), the 20° C. temperature coefficient of 61 per cent conductivity annealed aluminum wire is 0.00408, per degree centigrade. corresponding to a 0° C. coefficient of 0.00444. Grassi also found that the temperature coefficient of aluminum is proportional to its per cent conductivity (as is also the case with copper; see article on *Copper*). American investigators have found a lower coefficient, the figure 0.0039 being most commonly used.

COMPARISON OF ALUMINUM AND COPPER. — Aluminum is usually cheaper than copper of the same length and resistance and is said to be cheaper to install (*C. B. Smith, El. W., 1912, Vol. 59, p. 96*). It is, however, subject to the disadvantage of lower tensile strength which often makes it more expensive than copper, except for very high voltages.

The table below compares the various items for wires having the same length and same resistance and is based on the following assumptions:

	Copper	Aluminum
Per cent conductivity	98	61
Tensile strength, lb. per sq. in.	55,000	25,000
Density	8.89	2.70
Price per pound	P	p

COMPARISON OF COPPER AND ALUMINUM WIRES FOR EQUAL RESISTANCES PER UNIT LENGTH

Item	Copper	Aluminum
Cost	1	$0.488 \times \frac{p}{P}$
Cross-section	1	1.63
Diameter	1	1.28
Weight	1	0.488
Breaking strength	1	0.731
Carrying capacity	1	1.13

Disadvantage of Low Tensile Strength. — The lower tensile strength of aluminum for equal length and conductance as compared with copper affects the cost of an aerial line in two ways: 1st, by making it necessary to erect the spans with a greater sag or less length in order to reduce the stresses, thereby either increasing the height or the number of poles, and 2nd, by making it necessary to increase the distance between wires on account of the increased sag. The increase in the height of poles for the same spacing amounts to about 10 per cent. (*C. L. Johnson*). Aluminum cables with steel core are now used and overcome these difficulties. See article on *Wires and Cables, Bare.*

Effect of Large Elongation of Aluminum. — The extraordinarily great elongation of aluminum enables it to withstand severe mechanical overloads by stretching and thus increasing the sag. However, in dealing with a single

solid wire this cannot be relied on, as a scratch or a single imperfection will often cause the wire to break without any appreciable elongation. This is one reason why cables are to be preferred to single-wire conductors.

Effect of Low Melting Point of Aluminum. — The distance between aerial wires is also influenced by the fact that aluminum has a lower melting-point than copper, making it necessary to keep wires at different potentials well separated in order to avoid the danger of short-circuits through foreign bodies. This danger has, perhaps, been exaggerated in the past as the author has the record of a case where a copper wire was thrown across two aluminum cables with 60,000 volts difference of potential between them and an arc was formed between the cables forming an arch 12 to 15 feet high, but with no injury to the cables except the formation of a few minute beads on their surface.

Formation of Sleet. — While for equal length and resistance, aluminum wires are larger than copper wires, they do not necessarily collect a proportionally greater load of sleet, as it is found that a film of grease left over from the drawing process often entirely prevents the adhesion of sleet. Conclusive information on this subject is, however, lacking, especially with regard to wire that has been in use for several years.

Corona Formation. — At very high potentials, such as 100,000 volts, aluminum conductors possess a marked advantage over copper in the lower corona loss due to their greater diameter for the same conductance.

Corrosion. — E. Huber-Stockar gives the results of a number of tests on the corrosion of aluminum in moist air, salt air, salt water, various gases, acids, etc., which indicate that, in respect to its chemical stability, aluminum is in general the equal of copper. There has been much said about the tendency of electrostatic repulsion to reduce impure condensation on aluminum conductors, but this author rejects such statements as unworthy of the slightest credence.

Aluminum wire exposed to the atmosphere increases in resistance. E. Wilson (*Electrician, 75, p. 886, Sept. 17, 1915*), reports a case where commercial aluminum increased in resistance 17.2 per cent in 13 years. Wilson also gives the results of tests on various aluminum alloys.

Bare aluminum conductors laid in the earth have been tried for railway return feeders, with the result that they were rapidly corroded by galvanic action.

Mr. Dusaugey says that aluminum containing over 4 per cent of impurities is very liable to corrode rapidly. This is especially the case if sodium is present.

At ordinary temperatures aluminum is covered with a layer of oxide which protects it against the influence of the weather and of many chemicals.

Example of Relative Cost. — According to the official publications of the Ontario Hydro-Electric Commission on a line consisting of two three-phase circuits, each comprising three 0000 A.W.G. cables, the six cables cost $1450 per mile as compared with $2050 per mile for copper cables (copper being at 16 cents per pound and aluminum at 23.5 cents per pound), showing a saving of nearly 30 per cent on the cables alone. This saving was reduced to 5.6 per cent only on the total cost of the line, partly because the actual towers weighed 1.72 tons against 1.57 tons for towers for an equivalent copper line, and partly because the cost of cables was only 30 per cent of the total cost of the line, including erection but excluding rights-of-way. (*C. L. Johnson.*) Owing to a tariff of 3½ cents per pound the price of aluminum is higher in the United States than in Canada and Europe, so that the saving would have been considerably less at United States prices.

SOLDERS FOR ALUMINUM. — (*Bureau of Standards, Circ. No. 78*, Washington, D. C., 1919). Aluminum can be welded in a satisfactory manner by the oxyacetylene blowpipe flame, but soldering is often desirable. Alumi-

num solders consist usually of mixtures in varying proportions of Zn and Sn or of Zn, Sn and Al. The surfaces to be soldered are carefully cleaned and "tinned" or coated with a layer of the solder by heating the surface and rubbing the solder into it. The joint between the tinned surfaces may then be made in the usual way with a soldering iron and the solder, no flux being used. A soldered joint is rapidly attacked when exposed to moisture. The joints should never be made by soldering unless they are to be protected against corrosion by a paint or varnish or unless they are heavy, as in the case of repairs in castings.

The composition of the solder may vary within wide limits. For tin-zinc solders, zinc 15 to 50 per cent, the remainder Sn, is recommended. The higher the temperature at which the tinning is done the better is the adhesion of the tinned layer, therefore the solders high in Zn and Al give better results. The joint between previously tinned surfaces may be made by ordinary methods by ordinary soft solders; only the tinning mixture need be special for Al. The tensile strength of a good solder is about 7000 pounds per square inch. The strength of a joint depends on the type and on the workmanship, but much dependence should not be placed upon it.

SPREE-ALUMINUM. — E. Huber-Stodkar gives considerable data on an aluminum alloy, known under the name of spree-aluminum. It has nearly double the strength of pure aluminum. Its breaking length is 10.7 kilometers, as compared with 10.3 kilometers for steel, 6.7 kilometers for aluminum and 5.1 kilometers for hard copper. The use of spree-aluminum cable shows a saving in cost of about 7.3 per cent as compared with copper cable of equal conductance.

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AMMETERS. — (*See also Balances, Current; Electricity and Magnetism, Principles of; Electrodynamometers; Galvanometers; Voltmeters.*) An ammeter is essentially a galvanometer (q.v.) or electrodynamometer (q.v.) of rugged construction, provided with a pointer moving over a scale forming an integral part of the instrument and calibrated to read directly in amperes. The moving element is usually supported in jeweled bearings.

CLASSIFICATION AND USE OF AMMETERS. — Ammeters which have found general commercial application may be classified as follows:

Moving-magnet Type, in which a permanent magnet or polarized vane is caused to move under the influence of a winding which carries the current to be measured. The principle of operation is similar to that of a Thomson galvanometer (*see Galvanometers*). The restoring force is usually a fixed permanent magnet. This type of instrument measures average values and is therefore suitable for direct currents only.

Moving-iron Type, in which there is relative motion between a piece of soft iron and a fixed coil carrying the current to be measured. There are a number of subclasses which are distinguished chiefly by differences of mechanical construction. In the plunger type a soft iron core is drawn into the coil. The core is usually in the form of a rod, and the length of the coil is several times its diameter. In the repulsion type, as usually constructed, a piece of iron is repelled by a fixed piece of iron, both being within the coil carrying the current to be measured. The Weston moving-iron ammeter is of this type. In the Thomson inclined-coil ammeter, the coil is fixed at an angle of about 45° with the shaft which carries one or more iron vanes, the latter also making the same angle with the shaft. As the current through the coil increases the vanes turn so as to approach the condition of parallelism with the magnetic field within the coil. The restoring force of all these forms of ammeter is usually that of a spring. This type of ammeter measures effective (r.m.s.) values * and may be used for either alternating or direct currents, but such ammeters are subject to slight errors in the latter case, due to the tendency of the vane or plunger to hold its magnetism.

Permanent-Magnet Moving-coil Type, in which a permanent magnet maintains a strong field in a fixed location, and the current to be measured is sent through a moving coil, entering or leaving by springs that furnish the restoring force or through auxiliary spirals. The principle of operation is the same as that of a d'Arsonval galvanometer (*see Galvanometers*). This type of instrument measures average values and is suitable for direct currents only. Precision d-c. ammeters are usually of this type.

Electrodynamic Type, in which the operation depends upon the force exerted by one current-carrying circuit upon another, or by a portion of one circuit upon another portion of the same circuit. (See *Electrodynamometers.*) The usual form of construction includes a pivoted coil which moves in the field of a fixed coil or set of coils. The restoring force is usually due to springs, and the current is led into and out of the moving coil through these springs or through flexible conductors constructed to carry relatively large currents and to have a minimum restoring force. This type of instrument measures effective (r.m.s.)

* Moving-iron instruments do not measure r.m.s. values exactly, on account of the lack of exact proportionality between the instantaneous torque and the square of the instantaneous current, but by careful attention to details in the design such nstruments can be made nearly perfect in this respect, so that all things considered, they are sometimes to be preferred to electrodynamic instruments. For precise work, calibration of such instruments made with direct current should not be used on alternating current.

values and may be used to measure either alternating or direct currents. Precision a-c. instruments are usually of the electrodynamic type.

Hot-wire Type, in which the expansion of a wire or strip due to the heat developed within it, caused by the current passing through it, is made to move an indicating pointer. The "thermocouple" ammeter, described below, may also be classified as a hot-wire ammeter, although the principle of operation is quite different. Hot-wire ammeters measure effective (r.m.s.) values and are useful for both alternating- and direct-current measurements.

Induction Type, in which a metal cylinder or disk with pointer attached is caused to rotate under the action of a rotating magnetic field, usually produced from a single-phase source by means of a phase-splitting device. The principle of operation is similar to that of an induction watthour meter (*see Watthour Meters*), the motion of the moving element being opposed by a spring. This type of instrument measures effective values of alternating currents only and generally for only a limited range of frequency and wave form.

Thermocouple Ammeters. — For measuring small alternating currents, particularly when the frequency is high (500 cycles per second or more), a thermocouple (*see Pyrometers*) may be used; the current to be measured is passed through a wire resistor (*see Wires, Resistor*) to which is soldered one junction of the thermocouple, the free ends of which are connected to an ammeter (or galvanometer) of any of the types noted above. The heating of the resistor wire sets up a direct current in the thermocouple circuit which, in turn, depends upon the effective value of the current through the heater wire. The limitations and errors in the use of such ammeters are discussed at length in a paper on *High Frequency Ammeters* in Scientific Paper No. 206, Bull. Bur. Stds., 1913, Vol. 10. See also article in the Gen. El. Rev. 17, p. 981, Oct., and p. 1210, Dec., 1914.

Other Types of Ammeters. — Successful instruments for special needs have been made that would not ordinarily be classed under the above headings, and other instruments might be referred to which combine in a single instrument more than one of the subdivisions.

Portable and Switchboard Instruments. — Switchboard instruments are usually of more rugged construction than portable instruments and are somewhat less precise in their indications. Portable instruments are provided with suitable binding posts on their bases and are generally used in a horizontal position. Switchboard instruments are usually provided with back connections and are used in a vertical position. In the so-called "edgewise" instruments the instrument mechanism is horizontal but the pointer is so designed that it indicates on a vertical scale.

Recording (Graphic) Ammeters. — A continuous record of current may be obtained by using an ammeter provided with a clock mechanism which slowly rotates a chart (disk-shaped or mounted on a drum) or a continuous paper ribbon under the end of the pointer which carries a pen kept continuously inked. The moving element is comparatively heavy since the deflecting force must be great enough to overcome the friction of the pen and prevent "sticking."

INSTRUMENT SCALES. — A uniform scale is usually desirable in any kind of instrument, and with permanent-magnet moving-coil types such a scale is readily obtained, since the deflecting torque is proportional to the current. In moving-iron, electrodynamic and other types in which the deflecting torque depends upon the square of the current, very small deflections are obtained for low values of the current and the scale divisions are crowded for low values.

DAMPING. — For ordinary purposes it is essential that the pointer of the instrument come quickly to rest. The moving coil of the permanent-magnet moving-coil instrument is usually wound on a light aluminum frame which is mounted between the poles of the permanent magnet, and a soft-iron core fills the air gap between the poles, other than that necessary to allow free motion of the coil. The metal frame moves through a very strong magnetic field and eddy currents are set up in it which effectually damp its motion. Although it is not difficult to make the instruments perfectly aperiodic, a slight underdamping is often preferred because it gives opportunity to observe at all times that the movement of the instrument is perfectly free.

Magnetic (eddy current) damping or air damping by means of light vanes moving in a more or less perfectly closed air chamber is used with the other types of instruments.

ELIMINATION OF EFFECT OF STRAY FIELDS. — Due to the low reluctance of iron compared with air, a soft-iron shield placed around the instrument mechanism will greatly reduce the disturbing effect of an external magnetic field on the readings of the instrument. Many types of switchboard and portable instruments are thus shielded. Permanent-magnet moving-coil instruments are affected by only comparatively strong fields.

Astatic Instruments. — Another means of avoiding errors due to stray fields is by making the moving element astatic. The principle of this construction is to divide the moving element into two parts, so arranged that the deflecting torques acting on the two parts due to the field of the instrument are additive, whereas any stray field will produce equal and opposite torques on these two parts.

Thomson Astatic Instruments. — The Thomson astatic ammeters and voltmeters are unique in that the deflecting torque and the controlling torque are both caused by the same magnetic field. This field is produced by an electromagnet wound for any specified voltage and provided with binding posts separate from the current posts of the instrument, or by means of a permanent magnet. Two moving coils are mounted diametrically opposite each other upon an aluminum disk. The disk and coils cut across the field which is directed parallel to the shaft carrying the disk. Two small pieces of soft iron are rigidly mounted on the shaft, the magnetic field entering these pieces of iron radially tending to hold the shaft and moving coils in their initial position.

When current passes through the coils of the moving element, the lines of force parallel to the shaft produce a torque which tends to turn the shaft and cause the needle to travel across the scale. This action is opposed by the magnetic field at right angles to the shaft acting on the two pieces of soft iron. The instrument has no controlling springs and since both the deflecting and controlling force are due to the same electromagnet, the accuracy is not affected by changes in magnetic strength. These instruments are particularly suited to service where very strong external fields are present.

USE OF SHUNTS AND CURRENT TRANSFORMERS. — *See also articles on Shunts; Transformers, Instrument.*) The current which may be led into the moving element of an ammeter (moving-coil or electrodynamic type) is limited by the current-carrying capacity of the springs or special leading-in wires, which must necessarily be light; this current seldom exceeds 5 or 10 amperes. To increase the range of such instruments, and also to avoid the use of heavy conductors in the instruments, shunts or current transformers (the latter for alternating currents only) are used. Ammeters for use with current transformers are usually designed to take a maximum current of 5 amperes, but

their scales are calibrated to read directly the current in the primary of the transformer.

Millivoltmeter Used as Ammeter. — Direct-current ammeters designed for use with shunts are usually designed to carry but a very small current (about 0.025 ampere). A wire resistor having practically zero temperature coefficient is permanently connected, within the case of the instrument, in series with the moving coil. The combined resistance of the moving coil and resistor is thus made large compared with the resistance of the shunt, and the error, due to resistance variation, is not large for ordinary changes in temperature. The ammeter proper is then essentially a millivoltmeter (*see Voltmeters*) and measures the drop of potential through the shunt to which it is connected; the scale of the ammeter, however, is often calibrated to read directly in amperes for one or more standard shunts designed for use with it.

Voltage Drop Across Shunts. — Shunts for use with switchboard direct-current ammeters of the permanent-magnet moving-coil type are usually designed for a drop of 50 or 60 millivolts at full load, although for some special requirements, where the indicating instrument must be located at a considerable distance from the shunt, double-drop shunts having 120 millivolts are used, and even higher drops are used to meet very special requirements. For precision use with portable instruments the resistance coil is so designed that a drop of 100 or 200 millivolts is required for full-scale reading, the high resistance with zero temperature coefficient practically eliminating temperature errors.

Use of Portion of Bus Bar as Shunt. — Sometimes the shunt is dispensed with altogether and the millivoltmeter is connected directly to a portion of the bus bar or main conductor. This arrangement is usually not desirable, because the variation of temperature in the bus bar due to the current flowing through it causes larger errors than the variation due to changes in room temperatures. Some arrangements have been made to compensate for these, but are not very generally used.

Voltmeter and Shunt for Measuring Alternating Currents. — An a-c. voltmeter with shunt is not in general suitable for measuring an alternating current, since such instruments are not readily constructed to read low voltages and therefore a relatively large drop in the shunt would be required, also the division of current between the shunt and voltmeter depends (1) upon the inductances of the shunt and instrument, (2) upon the "skin effect" (q. v.) in the shunt. However, hot-wire voltmeters with shunts are sometimes used for alternating-current measurements and are satisfactory at commercial frequencies.

PRECAUTIONS IN USE OF PORTABLE AMMETERS. — The following precautions, which in the main also apply to other portable electrical instruments, should be observed:

An ammeter should always be connected in series with the load. Never connect it across the mains.

All contact surfaces of nuts, terminals, etc., must be clean.

Transformers and wires carrying heavy currents should be kept at a safe distance from all instruments.

Portable instruments should be always used in a horizontal position, unless specifically designed for use otherwise.

Tapping an instrument while it is being read is usually unnecessary if it is properly constructed and in good order, but when slight friction is present from any cause its effect can be eliminated by gently tapping the instrument. Hard tapping defeats its object.

It is a good plan in testing to place a low-resistance jumper or short-circuit block * or switch around the terminals of the ammeter and to connect a rheostat in series with it and the load. The plug of the short-circuit block should be gradually withdrawn, or the switch gradually opened, and the needle of the ammeter watched to be sure that the current is in the right direction and does not exceed the rated current of the instrument.

A heavy short-circuit in nearby conductors, even though of extremely short duration, is liable to demagnetize the magnet of a permanent-magnet instrument. After such a short-circuit the instrument should be recalibrated, even though the short-circuit causes no mechanical injury.

When it is desired to measure the average value of a pulsating current (e.g., from a rectifier), permanent-magnet moving-coil instruments should be used. The average value is to be used when the current is used for an electrochemical operation, such as plating, or charging storage batteries. Power measurements in rectified-current circuits should in general be made with an electrodynamic wattmeter.

The power of a rectified current in a noninductive circuit, such as incandescent lamps or electric heating apparatus, may be correctly determined by multiplying the effective current by the effective voltage, both being measured by electrodynamic or hot-wire ammeter and voltmeter. For a discussion of this subject see article by Todd. Elec. Rev. (Chicago) Vol. 77, p. 683, 1920.

Instruments containing iron should not be used for a frequency differing largely from that for which they are designed. For high-frequency measurements (above 500 cycles per second), some type of hot-wire or thermocouple instrument should be used. Instruments containing iron are also subject to errors due to distorted current waves; for precise work on badly distorted waves they should be compared with an electrodynamic or hot-wire instrument, using the same kind of current in the comparison as that to be measured.

CHECKING OF AMMETERS. — The readings of two or more ammeters may be compared by connecting them in series with a rheostat (q.v.) and suitable source of e.m.f. An ammeter may thus be compared with a standard ammeter, and a curve drawn showing the true amperes corresponding to the dial readings. It is usually more convenient to plot the correction to be made to the reading against the actual reading. Corrections to be added to the reading are usually designated as + and corrections to be subtracted from the reading as −.

Instead of using a standard ammeter, a standard millivoltmeter and a standard resistor (*see Resistors, Standard*) may be employed for direct current. The standard resistor is connected in series with the ammeter to be checked and the millivoltmeter is connected across the potential terminals of the resistor. Let V be the reading of the millivoltmeter in volts, R the resistance of the resistor and r the resistance of the millivoltmeter; then the current measured is

$$I = V/R + V/r.$$

The last term V/r is usually negligible.

An accurate and convenient way of checking an ammeter on direct current is to use a potentiometer (q.v.), when such an instrument is available.

Checking of A-C. Ammeters. — Alternating-current indicating instruments, whose deflections are independent of frequency and wave form, when calibrated on direct current indicate correctly effective values when used on

* For small currents, two blocks of metal mounted on an insulating base with a tapered hole between the two, into which fits a tapered metal plug, make a convenient short circuit block; for heavy currents a switch should be used.

alternating current. Electrodynamic instruments usually give readings of equal accuracy on alternating or direct current at commercial frequencies and wave forms. Eddy currents in the metal supports and conductors, however, may cause slight errors in some makes, even at commercial frequencies, and for high frequencies errors are liable to occur in all types unless special precautions are taken to avoid eddy currents. Moving-iron and induction-type ammeters should be compared with an electrodynamic ammeter which has been previously calibrated on direct current.

ACCURACY OF AMMETERS AND OTHER ELECTRICAL INSTRUMENTS. — Various methods of expressing the accuracy of electrical instruments have been used. A customary and reasonably satisfactory method is to state that the instrument is correct to within a given percentage of the full scale value. This is understood to apply throughout the scale, and in uniform-scale instruments amounts to saying that no division mark is out of its proper position by more than a certain linear distance.

It is difficult to make general statements of the accuracy of electrical instruments, because this depends on many things, such as operating principle, workmanship, range, and conditions of use. It may be said that 0.1 to 0.2 per cent is about the limit of accuracy attainable in d.-c. instruments, and to get it requires high-grade instruments carefully used in connection with calibration curves obtained by regular and sufficiently frequent comparisons with standards of higher class, such as potentiometers. The accuracy attainable in a.-c. instruments is in general less than in d.-c. instruments.

Portable instruments of high grade are obtainable having a stated accuracy of 0.25 per cent of full scale value; intermediate grade instruments, 0.5 per cent; small portable instruments, 1 per cent. Switchboard instruments of the best grade are usually stated to be accurate to within 1 per cent of full scale value.

The accuracy obtainable in service depends not only on the inherent accuracy of the instrument, but also to a very great extent on the conditions of use. This subject is discussed in detail in a paper on "The Accuracy of Commercial Electrical Measurements" by H. B. Brooks, Trans. A.I.E.E., Vol. 39, 1920.

INSTALLATION OF SWITCHBOARD INSTRUMENTS. — The same general considerations apply to all switchboard instruments, i.e., ammeters, voltmeters, wattmeters, etc. Instruments previous to installation should be kept in a dry, cool place. Switchboard instruments should not be put in place until all the work on the boards or in the vicinity of their location has been done, so that they may not be damaged by violent shocks.

During shipment moving parts, etc., are sometimes blocked and instructions furnished with instruments by makers should be carefully followed in unpacking and assembling.

It is not customary to ground the cases of d-c. instruments or of a-c. instruments above 650 volts; these latter are usually protected by insulated covers. The cases of instruments on secondary circuits below 650 volts are usually grounded.

Determination of Capacity of Instruments. — In providing instrument equipment for switchboard service the relation between ammeters, voltmeters, wattmeters and the circuit conditions should be carefully considered so that proper sizes may be provided. The size should be so chosen that a good indication can be obtained under all ordinary load conditions, and at the same time the highest loads can be accurately indicated.

REPAIRS OF INSTRUMENTS. — Most portable and switchboard instruments in use at the present time are constructed with the idea of having

their parts readily accessible for repair. Instruments, however, are usually very delicate in comparison with other apparatus, particularly with reference to the pivots and jewel bearings. Any extended repairs should, therefore, not be attempted unless equipment and experience suitable for the work in hand are available.

COST OF AMMETERS AND VOLTMETERS. — The following figures are approximate only, and should be used only as a rough guide. Shunts and instrument transformers are not included; see articles on *Shunts* and *Transformers, Instrument.*

COST OF AMMETERS AND VOLTMETERS (1920–21)

Type of instrument	Switchboard instruments	Portable instruments
Permanent-magnet moving-coil type:		
Small instruments	\$6–\$10	\$12–\$17
Medium-size instruments	\$13–\$24	....
Regular-size instruments, high-grade	\$23–\$34	\$43–\$100
Polarized-vane type	\$2–\$4	....
Moving-iron type:		
Unshielded, portable, single-range	....	\$25–\$48
Shielded, portable, two-range	....	\$54–\$100
Round pattern	\$20–\$29	....
Horizontal edgewise pattern	\$36–\$51	....
Electrodynamic type:		
Medium-size instruments	....	\$33–\$42
Regular-size instruments, high-grade	....	\$60–\$104
Induction type:		
High-grade	\$24–\$40	\$54–\$65
Low-grade	....	\$32
Thermocouple type:		
Small instruments	\$16–\$19	....
Regular size instruments	\$53–\$76	....

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See also an extensive bibliography on electrical instruments and meters in the 1914 report of Meter Committee of the National Electric Light Association.

AMPERE-HOUR METERS. — (*See also Ammeters; Wattmeters.*) Ampere-hour or quantity (coulomb) meters may be divided into practically two classes, i.e., electrolytic meters and electromagnetic or motor meters. They are used in this country chiefly for storage-battery work; small capacity ampere-hour meters are also used, particularly abroad, as watthour meters, being calibrated in this case to read directly in kilowatt-hours at some definite voltage.

ELECTROLYTIC METERS. — This type of meter is used extensively in Europe, but is rarely used in this country. The meter generally consists of some form of glass container or "U" tube containing an electrolyte, such as water, or an aqueous solution of a salt of mercury, or of caustic soda. The action of the current passing through the solution decomposes it and since the rate of decomposition is proportional to the strength of the current, the deposit of metal or the number of cubic centimeters of gas evolved can be calibrated to measure the quantity of current. (*See Electrochemistry, Principles of.*) Such meters can also be used as watthour meters (q.v.) in which case they are usually calibrated to read directly in kilowatt-hours at some assumed voltage. In some types the entire current to be measured is passed through the electrolyte and in others the main current passes through a shunt and only a very small part of the current passes through the electrolyte. In a satisfactory design of electrolytic meter the drop at full load does not exceed $1\frac{1}{2}$ to 2 per cent on a 200-volt circuit, and the meter is accurate to within 2 per cent or better (depending on the design) at all loads between 10 per cent and 150 per cent load.

MOTOR METERS. — The essential parts of the motor ampere-hour meter are the motor element, the brake or speed-regulating device and the register or gear train which records the integrated current passed through the meter. The motor consists of an armature rotating in a magnetic field produced either by a permanent magnet or by an electromagnet. The meters are generally arranged so that only a shunted portion of the current flows through the armature and when electromagnet fields are used they are connected *in series* with the armature. The construction of these motors is otherwise similar to that of a watthour meter (q.v.). The commutator and other electromagnetic types of motor meters are all designed with the idea of producing a device which will be cheaper and if possible more rugged than the ordinary watthour meter. The absence of a constantly excited potential circuit in the meter also avoids the constant loss of energy which takes place in such a circuit irrespective of the load. In the motor meter the drop across the shunt varies in meters of different manufacture from about 50 to 100 millivolts in the mercury meter, and from 0.8 to 1.5 volts in the commutator meter.

Commutator Ampere-hour Meter. — The commutator type of meter in its simplest form usually consists of one or two permanent magnets which provide a driving field within which rotates an armature with three coils and a three-part commutator. In some designs the armature coils are mounted flat on an aluminum disk which in addition to supporting the armature coils produces the retarding or braking action in connection with the field magnet. In another design there is a stationary iron core between the poles of the field magnet and a drum armature rotating in the gap between the stationary iron core and the pole faces of the field magnet. No braking device is used in this latter type, and the motor runs at a speed proportional to the impressed e.m.f., taking only sufficient power to overcome the friction in the bearings and in the gear train.

To compensate for friction at light loads a slight auxiliary potential difference is sometimes maintained across the brushes by placing the armature in series with a high resistance across the line. This arrangement is adjusted so that the meter will not run with no current through the main shunt, but at light load the

extra potential difference at the brushes will be sufficient to provide a torque which will compensate for the frictional torque of the moving parts. In a recent type of A.E.G. ampere-hour meter the light-load compensation is effected by movable brushes automatically controlled (*Elekt. Zeits. 40 p. 213, May 8, 1919*).

Oscillating Motor Ampere-hour Meter. — A special form of the electromagnetic motor meter differing from all other designs is the oscillating meter. In principle, this meter is a clock which is controlled by the current. The hair spring of the clock is replaced by a disk carrying a few small iron wires. This special balance wheel is supported between two coils which carry the current to be measured or a shunted portion of it. The magnetic field from the coils magnetizes the iron wires and in conjunction with the main spring of the clock causes the special balance wheel to oscillate at a rate proportional to the strength of the magnetic field and therefore proportional to the current passing through the meter. The movement of the clock is transferred to a registering mechanism geared to register ampere-hours, or kilowatt-hours at a constant voltage.

Mercury Ampere-hour Meter. — The motor meter of the mercury type utilizes mercury to carry current to the armature, the mercury really taking the place of the brushes and commutator of the ordinary direct-current motor. One form of the meter consists in principle of a copper disk or copper thimble armature mounted in jeweled bearings and immersed in the mercury. The current is led into the mercury chamber through suitable terminals and since mercury has about fifty-five times the resistance of copper, the greater part of this current goes through the copper (approximately diametrically across the disk) and out the other terminal by way of the mercury. This arrangement applies to meters with permanent-magnet fields. In case electromagnet fields are used, the driving torque is approximately proportional to the square of the current, hence a braking system must be used to produce a retarding force varying with the square of the speed, such as air or fluid friction. When this is done the rate of revolution of the armature when running at a constant speed is proportional to the current passed through the meter. When permanent-magnet fields are used, a device is sometimes used to compensate for the increase of fluid friction with increase of speed; this device takes some such form as an auxiliary coil, in series with the motor element, which produces a flux which augments the flux cutting the driving disk.

The mercury motor meters utilize iron or in some cases moulded compound for the mercury well, and in the design care must be taken to provide non-spillable joints at the point where the spindle of the armature enters the mercury chamber. All metal in the meters should be of such composition as not to be readily affected by mercury even though not in direct contact with it.

SPECIAL FEATURES. — Ampere-hour meters used for showing the state of charge of storage batteries are sometimes provided with special features. For example, a "compensation for overcharge" causes the meter to run more slowly on a given value of charging current than on the same value of current during discharge. The percentage difference in the two cases is chosen to provide for the excess that must be put into the battery because it is not 100 per cent efficient. This is accomplished by a "variable resistor element" in parallel with the main mercury chamber. A pivoted copper bar in this element is immersed in mercury, and is cut by some of the flux from the driving magnets. It will thus change its position with the reversal of current direction, and will vary the resistance of the element. The "compensation for light-load accuracy and internal battery losses" consists of a thermocouple heated by a potential winding and delivering a small current continually to the motor element in the direction corresponding to battery discharge. This action may be adjusted to cause the meter to record the loss of charge when the battery is not in use. The "thermal

shunt method" of compensation for high discharge rates consists in making the shunt to the motor element of iron or other material of high temperature coefficient, and so designing it that it will heat up at the heavier loads. This causes the ampere-hour meter to have a higher percentage registration as the load increases.

Ampere-hour meters are sometimes provided with contacts which actuate auxiliary devices, such as circuit breakers, bells, or other signaling devices. This feature is used in connection with battery charging and electro-plating.

CALIBRATION OF AMPERE-HOUR METERS.— Ampere-hour meters are adjusted and calibrated as follows:

Electrolytic Meters. — As noted above, in the modern types of electrolytic meters mercury is deposited in a glass tube or water is decomposed, the gases passing off into the atmosphere. In the first type a marked scale is placed beside the registering tube into which the electrolyzed mercury falls. This mercury is run back into the mercury reservoir after the reading is taken, by tipping up the cell which is hinged at the top and held vertical by a removable clip at the bottom. To calibrate the instrument an ordinary ammeter may be placed in series with it and the time that the current passes noted on a watch. In testing the types in which gases are evolved and pass off into the atmosphere, it is only necessary to obtain the number of cubic centimeters of liquid per division of the scale, which can be done with a graduated burette. Only the water is decomposed by the passage of the current. To renew the electrolyte after the reading is taken at the end of a period, it is only necessary to add water.

Motor Meters. — In testing ampere-hour meters several adjustments are necessary, since a shunt, a series adjusting resistance and the gear train are involved. In addition, care must be taken to see that there is no excessive friction in any of the moving parts. The general method is to pass a measured constant current through the meter and to determine the time with a stop watch for a certain number of revolutions. Knowing the shunt constant (ratio of total to shunt current) and the register or gear-train ratio the correct speed at a certain current can be calculated and the shunt or adjusting resistance in series with the motor element adjusted to give the correct speed. To determine the accuracy for any set of readings the following formula can be used.

$$\frac{(\text{No. of rev.}) \times (\text{meter constant})}{\text{No. of seconds}} = \text{current.}$$

The "meter constant" is marked on each meter; it is the number of ampere-seconds per revolution of the disk

WEIGHTS AND COSTS. — Mercury ampere-hour meters of American manufacture are made in different sizes for different types of services. Exclusive of shunts, the larger meter weighs about 16 lb. and the smaller meter 8 to 10 lb. In small capacities with self-contained shunt, the larger meter sells for $30 to $40, and the smaller meter for $12 to $20. External shunts are furnished with the larger meter above 100 amperes, and above 40 amperes with the smaller meter. These shunts range in price from $25 to $30 for 300 amperes and from $65 to $75 for 3000 or 4000 amperes. These prices are only approximate, and should be used only as a rough guide.

BIBLIOGRAPHY. — See the Bibliographies in the articles on *Ammeters* and *Watthour Meters*.

ANGLES. — (*See also Trigonometric Functions; Trigonometry.*) Plane angles may be expressed in degrees or in radians. The degree is arbitrarily taken as $1/360$th of the plane angle about a point; the radian is defined as the angle subtended by an arc of unit length in the circumference of a circle having unit radius. The angle in radians subtended by any arc of any circle is

$$\text{Angle in radians} = \frac{\text{arc}}{\text{radius}}.$$

The relation between radians and degrees is

$$1 \text{ radian} = 57.30 \text{ degrees.}$$
$$1 \text{ degree} = 0.01745 \text{ radians.}$$

Angle of Curvature. — In railroad practice the angle of curvature is the angle subtended by a chord 100 feet in length, the curve being the arc of a true circle. The relation between degree of curvature D and radius of curvature R is

$$R = \frac{50}{\sin \frac{D}{2}} \text{ feet.}$$

For D small, the chord is very approximately equal to the arc, consequently under these conditions

$$R = \frac{5730}{D} \text{ feet,}$$

where D is in degrees. The error involved in this approximate formula is less than 2 parts in 1000 for D less than $10°$, and to less than 1 per cent for D less than $30°$. (*See also Railways, Location and Permanent Way for.*)

Solid Angles. — The solid angle at any point P subtended by any surface S is equal to the portion of the surface of a sphere of unit radius which is cut out by a cone having its apex at the point P and its base coinciding with the perimeter of S. The total solid angle about a point is then equal to 4π. The unit of solid angle as thus defined is called a steradian. A solid angle of 2π steradians is called a hemisphere and a solid angle of $\frac{\pi}{2}$ steradians is called a spherical right angle. See table in *Units and Conversion Factors.*

ARC, ELECTRIC. — (*See also Distribution Systems; Furnaces, Electric; Illumination, Interior and Street; Lamps, Arc; Rectifiers.*) An electric arc is an incandescent vapor bridge consisting of material electrically impelled from a negative to a positive electrode. A spark is also an incandescent vapor bridge, but it differs from an arc, in not depending upon the electrodes for its material medium. The establishment of an arc requires the expenditure of energy for the latent heat of evaporation of the electrode and for the motion of the vapor stream. As no energy can be expended for these purposes until the current flows, an arc cannot start spontaneously. The following expedients are, therefore, adopted to start arcs.

(1) Bringing the conductors into contact with each other and separating them after the current has commenced to flow.

(2) Stressing the dielectric between the conductors until it breaks down electrically and becomes conducting.

(3) Using a subsidiary arc to furnish the initial vapor bridge. The first of these methods is used in carbon-arc lamps and the last in the mercury-arc rectifier.

CARBON ARC. — Until recently the only arc used for illumination was the arc between carbon electrodes, though at the present time there are several materials used, such as magnetite, mercury and the mixtures of various substances with carbon.

Crater. — The vapor column of the carbon arc, impinging on the positive electrode, raises the end of the latter to a very high temperature and volatilizes the carbon. The effect of this is to cause a hollow to be burned in the tip of the positive electrode. This hollow is called the crater.

Temperature. — The maximum temperature of the arc cannot be measured by direct means and the various estimates, which have been made, range from 3200° C. to 6000° C., absl. Using Wien's law connecting temperature and wave length corresponding to maximum radiated energy, Lummer and Pringsheim found that this wave length was 0.7 μ and the corresponding temperature somewhere between 3750° C. and 4200° C., absl., the latter figure being the crater temperature at atmospheric pressure. By raising the pressure to 22 atmospheres, a temperature of 6000° C., absl. may be attained. (*W. Mathiesen, Arcs under Pressure, Elekt. Zeits. 37, pp. 549 and 567, Oct., 1916; 38, p. 573, Dec., 1917.*)

Sources of Luminosity. — The light of the arc is derived either from the vapor column or from a body heated to incandescence by the vapor column.

In the old carbon electrode arc no special effort was made to utilize the luminosity of the vapor column, the incandescent spot or crater on the positive carbon being relied upon for practically the entire light. For this reason the positive carbon was always placed above the negative in order that the crater might better shed its light downward.

The flame arc, on the other hand, owes the greater part of its luminosity to the vapor column and very little or none to the heat of the positive electrode.

Hissing. — When the crater is surrounded by the hot gases constituting the arc, the arc is silent, but if the crater extends over the tip of the carbon and comes into contact with cool air, a peculiar hissing sound is evolved.

Characteristic of Arcs between Carbon Electrodes. — A peculiarity of the solid carbon arc is that, with any particular length of arc, if the current be increased, the difference of potential across the carbons will decrease. This occurs continuously until a certain point, when in the open arc the voltage

drops quite suddenly. If the current is still increased the voltage will again become steady at a much lower value. Between the values before and after the drop, the arc is unstable and hisses. The beginning of the hissing period is indicated by the dotted line in Fig. 1, and the hissing continues throughout the lower part of the curve.

Fig. 1

The hissing point being absent in inclosed arcs, the curve for such arcs is quite continuous.

Steadying Resistance. — In order to maintain a steady current in an electric circuit it is necessary that any decrease in the voltage across the terminals of the circuit shall produce a decrease in the current. Hence, as the arc characteristic shows an increase of current with decreasing voltage there must be placed in circuit with the arc a resistance great enough to compensate for this tendency, if the arc is to be kept in a stable condition. The method of calculating the resistance required to maintain a stable arc for a given current is as follows:

Let e = generator terminal volts $= OE$,
v = arc terminal volts,
x = resistance in circuit, exclusive of arc and generator resistance,
i = current in amperes,

then, since $e = v + ix$,

the resistance x must be such that

$$x = \frac{e-v}{i}.$$

On the arc characteristic (Fig. 2) find the point P corresponding to the current i and set off $OE = e$ along the vertical axis. Then join EP and continue EP until it cuts the horizontal axis at X. The resistance x is then equal to tan $\angle EXO$. If EX cuts the curve not only at P, but at S above P, it would appear that the e.m.f. e could support either of two arcs. The point S, although affording a mathematical solution, is not physically possible, as it would involve an increase of voltage, producing a decrease of current, as may be seen by moving EX upward, parallel to its original position. Hence the arc corresponding to P is the only one possible. If EX cuts the curve not only at P, but at some point below P, say T, the same reasoning shows the arc at P to be unstable, while that at T is stable.

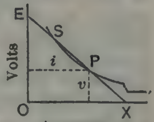

Fig. 2

For any point P, therefore, the arc is stable only if the line EX does not cut the curve below P. Hence, to find the proper value of the resistance x, for a given current i, draw a line tangent to the arc characteristic curve at the point having the abscissa i, then x must be equal to or greater than the tangent of the angle made by this line with the axis of abscissas.

Generator Voltage and Hissing Current. —

Let E = generator voltage,
V = potential difference between carbons just before current is increased to hissing point,
I = maximum amperes which will not produce hissing
D = drop in volts from silent to hissing,
i = rise of current from silent to hissing,

then $E = V + \frac{I}{i}D$ or $i = \frac{D}{E-V}I.$

Thus, if the generator voltage is great compared to the arc voltage, and therefore the steadying resistance great, the rise of current at hissing will be less than when the voltage is small.

Resistance of the Arc. — The relation between the drop of potential across the arc and the current flowing through it depends upon whether the arc is in the steady state corresponding to the current flowing or whether the current has been changed without giving time to the vapor column and electrodes to accommodate themselves to the new strength of current. Let V be the drop of potential across a direct-current arc and A the current flowing. If the current be increased by a small amount dA for so short a time that it produces no effect upon the arc itself, it will be accompanied by a rise in potential difference dV, and the ratio $\frac{dV}{dA}$ will be the resistance of the arc. This may be accomplished experimentally by superimposing upon the direct-current arc a small rapidly-alternating current and measuring the value of the alternating p.d. and current. Unless a very high frequency (about 100,000 cycles) is used, the ratio $\frac{dV}{dA}$ will be negative, due to the fact that the power factor of the arc is not unity for low frequencies and this ratio, therefore, does not represent the resistance (*see Duddell's papers in Bibliography*).

Back Electromotive Force of the Arc.— The product of the direct current of an arc and its resistance, measured as described above, is the ohmic drop in the arc. This is usually less than the actual p.d. by 7 to 15 volts, which represent a back electromotive force. This back electromotive force really consists of two, the larger near the positive electrode, opposing the flow of current, and the smaller near the negative, helping the flow. By varying the direct current carried by the arc, the back e.m.f. is not noticeably affected, but the resistance tends to become infinite for very small currents, and small for large currents. A variation in the length of the arc between solid carbons produces no effect upon the back e.m.f., whereas with cored carbons the back e.m.f. decreases with increase of length. The value of the back e.m.f. depends upon the make of carbons employed. In his experiments which established the existence of a true back e.m.f. in the arc, Duddell used an alternating current of about 1 per cent of the strength of the direct current and having a frequency of 100,000 ~ per second.

ARC AS TELEPHONE RECEIVER. — A direct-current arc may be used as a telephone receiver which can be clearly heard in a quiet room at a distance of 10 or 12 feet. The arrangement of apparatus, which gives the best results, is shown in Fig. 3, which is Duddell's improvement of the H. Simon circuit. In Duddell's experiments AB was a solenoid about 30 cm. long, wound with about 1000 turns of No. 18 D.S.C. wire and having an iron core of about 15 mm. diameter. Six hundred turns of the solenoid were connected to the microphone M and battery B and four hundred to the arc and condenser C.

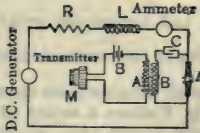

Fig. 3

The arc was between cored carbons of 11 to 13 mm. diameter separated 20 to 30 mm. and took 10 to 12 amperes. The condenser capacity was 2 or 3 microfarads. The resistance R was the usual ballast resistance and the choke coil L served the purpose of confining the telephonic currents to the arc circuit.

ARC AS A TELEPHONE TRANSMITTER. — The arc may be used as a telephone transmitter, but it is unsatisfactory on account of hisses and spits

due to the irregularities of the carbons and to the access of air to the crater. The arrangement of circuits is shown in Fig. 4, the apparatus R, L and C being the same as in the receiver circuit.

MUSICAL ARC. — A direct-current arc of suitable length and current between solid carbons will give a musical note, if it be shunted with a con-

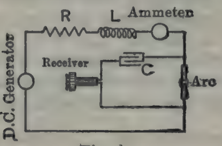

Fig. 4

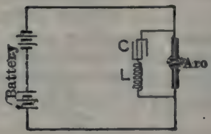

Fig. 5

denser in series with a choke coil, as shown in Fig. 5. The pitch corresponds to the periodic time

$$T = 2\pi\sqrt{LC},$$

where

C = capacity of condenser, farads,
L = inductance of choke coil, henrys.

The choke coil must not have an iron core, or the hysteresis and eddy currents will destroy the effect. A closed circuit, such as a ring of wire placed near the inductance coil, has the same effect. Further details of the musical arc are given below.

THE ARC AS A SOURCE OF HIGH-FREQUENCY CURRENT. — The musical arc owes its pitch to the action of the arc in transforming a part of the direct current into alternating current, the frequency of which can be varied between very wide limits by altering the self-inductance and capacity with which it is shunted. The inclosed arc works just as well as the open arc. In some of Duddell's experiments, the alternating current through the condenser circuit was as large as 5 amperes. Only condensers suitable for high voltages should be used, as, although the arc p.d. may be quite low, the condenser p.d. rises to several hundred volts. Alternating currents of frequencies of from 500 cycles to 500,000 cycles per second are easily obtainable.

Arcs between solid carbons always work well as converters, while those between cored carbons will not work under any conditions. In fact the general conditions under which the arc works as a converter are that $\frac{dV}{dI}$ must be negative and numerically greater than the resistance of the condenser circuit, exclusive of the condenser itself. The expression $\frac{dV}{dI}$ should be taken as referring to small instantaneous changes of the arc voltage and current.

Fig. 6 shows a means of operating several arcs together so as to obtain a greater output than would be possible with a single arc. Direct current is supplied to the arcs from the mains, through the choking coils L_1 and L_2, the ballast resistance R_1 and R_2 and ammeters A_1 and A_2. Alternating current is delivered by the arcs to the circuit containing the capacity C and inductance L. A form of arc used for generating high-frequency currents for use in radiotelephony is the Poulson arc, which is described in the article on *Telephony, Wireless.*

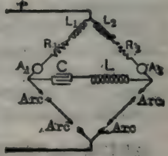

Fig. 6

OTHER APPLICATIONS OF THE ELECTRIC ARC. — In addition to its application for welding and electrochemical manufactures (q.v.), the thermal effect of the arc is utilized for burning steel. An application of this was made by the contractors of the Pennsylvania Railroad (East River tunnel), who used a carbon rod one foot long and one inch diameter, bolted to a copper rod, provided with an asbestos shield, one foot in diameter. The operators worked with asbestos masks and aprons and dark-colored eye glasses.

Direct-current was used, the carbon electrode being connected to the positive and the steel to be cut to the negative feeders. The current was varied by means of water rheostats, the voltage at the tool varying between 45 and 60 volts. The current varied from 250 to 400 amperes per tool for burning off rivet heads and light section plates and from 600 to 800 amperes for burning plates 4 inches thick. The best results were obtained with 40 volts, 600 amperes and a ½-inch to ¾-inch arc. A fair day's work (8 hours) removed 300 rivet heads, although a record of 350 was reached. In the same time 4 feet 6 inches of 4-inch plate could be burned off. (*H. Japp, Proc. A.S.C.E. 1909, Vol. 35, No. 9, p. 1230.*)

Portable plants, consisting of a gasoline engine-driven generator of about 25 kw. capacity, mounted on a truck, are now in use for wrecking buildings. Such an outfit is capable of burning off a 15-inch I-beam in 20 minutes, using the full-rated output of the generator. The arc has also been used for cutting scrap iron and steel in rolling mills (*G. Kearney, Gen. El. Rev., 20, p. 876, Nov.* 1917).

Experiments on rock removal for tunnel projects have been carried out by the Southern California Edison Co. of Los Angeles. Both direct-current and alternating-current (three-phase) arcs were tried. With direct-current, 500 amperes at 30 volts, 1 pound of rock per minute was removed at a cost of ¼ cent per pound, and with a three-phase arc, 450 amperes per phase with 40 volts between terminals 1 pound of rock per minute was removed at a cost of ½ cent per pound (*Engineer, 128, p. 57, July 18, 1919*).

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AUTOMOBILES, ELECTRIC. — (*See also Batteries, Storage, Alkaline Type and Lead Type; Batteries, Storage, Applications of; Trucks, Industrial Electric*). There are two types of electric automobiles, namely the battery car and the "gasoline-electric" car. The former type carries a storage battery from which energy is delivered to the driving motor; the latter type, in most instances, carries no battery but electric energy is supplied to the motor by a small generator which is itself driven by a gasoline engine. In other cases storage batteries are used in combination with a motor generator connected to the engine. Thus the machine may be driven by energy from either battery or engine. The battery car only is discussed in this article. A battery car is commonly referred to as "an electric," regardless of whether it belongs to the passenger or the commercial class.

HISTORICAL DEVELOPMENT. — The first battery-driven automobile in this country was built by Fred M. Kimball in Boston in 1888. This machine had 6 cells of lead storage battery and could travel 10 miles on good roads at an average speed of 5 m.p.h. In 1891 there was exhibited at the Mechanics' Fair in Boston an electric surrey built by the Holzer-Cabot Electric Co. Little more was done in the development of electric automobiles until about 1897 when the passenger vehicle was developed in two directions: one for pleasure took the forms of Victoria runabouts, dos-a'-dos and surreys, and the other for business purposes in the form of public taxicabs or vehicles for hire. The latter development expanded during the following four years to the use of fleets of 100 to 400 cabs and broughams in New York, Boston, Philadelphia, Washington, Buffalo and Chicago for public service on a hired basis; these disappeared from use about 1902 or shortly afterwards, largely for financial reasons, but also on account of the fact that large solid or pneumatic tires were not well developed at that date and storage batteries had not progressed beyond the heavy types used for stationary equipment in power stations.

From 1900 to 1903 the commercial type of electric truck began to be employed for merchandise delivery and progressed gradually but steadily until 1910, when the electric light companies recognized in it a desirable "off-peak" power consumer. The Electric Vehicle Association of America with considerable activity materially furthered the use of electric vehicles, both passenger and commercial, until 1914, when there were 37,000 in use; 25,000 passenger cars and 12,000 trucks (*El. World, Jan. 3, 1914*). The average price of the former at that date was $2200 and the latter $2800; this represented $88,000,000 invested by the public in this type of automobile. Following 1914 there was a rapid decline in the rate of production of electric vehicles due largely to general business conditions, but principally to the competitive influence of mass production in the gasoline vehicle industry; although the electric still maintains its superiority in economic operation. In 1920 there were but three manufacturers of each type aggressively conducting production in their respective fields with encouraging reaction, principally in a renewed demand for the commercial type. Another class of the electric vehicle, in the form of indoor industrial material and freight handling trucks, has been in progressive development since 1904; *see Trucks, Industrial Electric.*

APPLICATIONS OF ELECTRIC TRUCKS.— The three types of vehicles which are at present available for urban street haulage are the horse-drawn wagon, the gasoline motor car and the electric battery car. In comparing these three types for use in any given service, the chief factors to be considered are relative speed, distance between stops, length of stops required by the service, expense per vehicle and reliability (i.e., number of days per year upon which

the wagon can be depended for operation). The first three of these factors may be determined by trial or by observation.

In considering the matter of speed it should be remembered that with motor cars the wear and tear on the machine increases approximately as the square of the speed, so that the speed should be such as to give minimum cost of service. The last two of the above factors, namely expense and reliability, can be absolutely determined for any set of operating conditions only by several years of use of each type of vehicle. It is possible, however, to closely estimate the "expected" values of expense from the experience of others in similar kinds of work. See section below on *Costs*.

The electric truck has found its widest commercial application in city services where the hauls are of moderate length, say from two to ten miles, and where the daily service does not exceed 30 miles on the average, with occasional daily mileages of 40 to 50. For very short hauls where the standing time of the wagon is a large proportion of the working time, so that the work per day of each delivery unit is not largely influenced by the speed of the unit, the horse-drawn wagon can perform the work cheaper than any motor vehicle. For the so-called "long hauls" the gasoline truck would be expected to work to better advantage than an electric truck because of higher running speed or greater distance capacity in a day, or both. The rated values of speed and distance on a single battery charge of electric trucks are given in Table V below. The effect of hills or poor pavements would be equivalent to a decrease in the rated values of both speed and mileage. Systems of operating in which discharged batteries are exchanged for charged batteries during the day in order to increase the mileage capacity beyond the rated value have been used with signal success in a few instances, but in general the battery equipment is usually of sufficient capacity to perform the daily service required, or is in some cases supplemented by high-rate boosting charge during the noon hour or other convenient period.

DESIGN. — The necessity of producing cars which would travel at moderate speeds with a low rate of energy consumption has forced the designers of electric automobiles to use equipment which would meet this requirement, even though in some instances less efficient equipment would cost less or might wear longer than that used. Thus tires of low-energy consumption, batteries with large capacity per unit of weight and "anti-friction" bearings in transmission and axles are used almost universally. Consequently an electric automobile is essentially a high-grade machine, for any inefficient equipment directly influences the operating qualities.

Weight and Speed. — In the design of an electric battery car the two primary features are the total weight and the speed under normal conditions. In special cases a car may be built to meet unusual requirements of distance traveled per charge, in which case either the speed or weight must depart from the limitations of common practice. Any decrease in the total weight of the car will result in an increase in either speed or distance capacity or both; or if the weight is brought back to the original value by the use of additional battery capacity, the speed or distance may be still further increased. This has been one of the chief considerations leading to the adoption of lighter batteries, such as the Edison and "thin" paster lead plate, in place of the heavier type lead plate batteries commonly used where their additional weight is not a critical factor. General practice as regards weight and speed for various types of electric automobiles is indicated in Table I.

TABLE I.—SPEEDS AND WEIGHTS OF ELECTRIC AUTOMOBILES

Class of car or rated capacity of truck	Approximate weight in lb., including empty body	Approximate speed in miles per hour	Weight allowance for body, lb.
2-passenger runabout *	2,200	15 to 25	....
2-passenger Victoria *	2,400	15 to 21	....
2-passenger coupe *	2,600	15 to 20	....
4-passenger brougham	3,780	23 to 25	1010
5-passenger coach	3,860	22 to 24	1060
1,000-lb. truck	3,500	12 to 14	500
2,000-lb. truck	4,500	10 to 12	600
4,000-lb. truck	6,000	8 to 10	800
7,000-lb. truck	8,500	6.5 to 8	1100
10,000-lb. truck	9,500	6 to 7	1400
12,000-lb. truck	11,500	6 to 6.5	1600

* These vehicles are not now (1920) manufactured, but many of them are in use.

Similar data for a representative line of electric trucks are given in Table V below.

Load Efficiency. — By load efficiency of a motor truck is meant the ratio of the load on the machine to the gross weight of the machine, including chassis, body and load. Thus, from the above table, a representative value for the rated load efficiency of a 5-ton capacity electric truck is $\frac{10,000}{10,000+9500} = 0.51$. The rated load efficiencies for gasoline trucks of large capacity, 2 tons or more, are practically the same as the values for electric trucks, but small capacity gasoline cars of good design may have a rated load efficiency as high as 30 per cent as compared to about 24 per cent for electric cars with the customary lead battery equipments.

Motors. — Four-pole series motors are now used almost exclusively for electric automobiles. In the majority of cases but a single motor is used on each vehicle. The field coils are usually arranged in two groups which may be connected first in series and then in parallel. All manufacturers do not rate their motors on the same basis, but good practice is indicated by that of the General Electric Co., whose motors will carry their normal rated load continuously with a maximum temperature rise of 65° C. above the surrounding air at 25° C. or will carry 2½ times their rated load for 1 hour with a maximum temperature rise of 75° C. Ratings are based on bench tests, so that when the motor is installed in a car the overload capacity is usually increased due to the improved ventilation. Large overload capacity is necessary to meet severe street and grade conditions, and unusually good commutating characteristics are required.

An automobile motor must also operate satisfactorily over the varying range in the voltage furnished by the battery in the charged and discharged conditions. The motor speeds which are used with single reduction drives range from 750 to 1000 r.p.m.; the speeds which are used with double reduction drives range from 1000 to 2000 r.p.m.

Number of Motors. — All electric passenger cars and most electric trucks are now being equipped with a single motor. It was formerly common

practice to use two motors on the larger trucks on account of flexibility in control, but a single motor is now being used on account of space limitations, saving in weight and better electrical efficiency. The peculiar construction of the Couple-gear (*see below*) equipment requires a departure from this practice. Otherwise, either two or four motor equipment is used now only when unusually high tractive effort is required.

Motor Characteristics. — Along with the tendency during recent years to standardize battery equipments at 42 to 44 lead cells and 60 Edison cells, the tendency among motor manufacturers has been to develop a line of motor frames, for each of which two windings could be provided. Thus an 80- or 85-volt motor is usually installed with a 42- to 44-cell lead battery and a 60-volt motor with a 60-cell Edison battery. The windings are so proportioned that a motor has approximately the same speed and torque at both voltage ratings. This development has made it unnecessary to alter either the motor suspension or the mechanical transmission system in changing a car's battery equipment from lead to Edison or vice versa. 24-, 36- and 48-volt motors can also be obtained.

TABLE II.—G. E. STANDARD AUTOMOTIVE MOTOR RATINGS

FOR ROAD VEHICLES

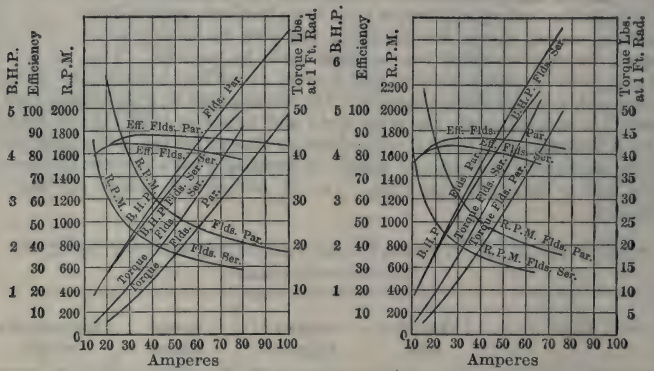
Type	Rated volts	Rated amperes	Speed r.p.m.	Approximate weight
G.E. 1065	60	17	1600	130
G.E. 1065	85	12	1600	130
G.E. 1028	60	28	2000	150
G.E. 1028	85	20	2000	150
G.E. 1043	60	28	1500	218
G.E. 1043	85	20	1500	218
G.E. 1048	60	40	1600	265
G.E. 1048	85	30	1600	265
G.E. 1039	60	40	1200	300
G.E. 1039	85	30	1200	300
G.E. 1022	60	60	1200	425
G.E. 1022	85	40	1200	425
G.E. 1030	60	70	850	530
G.E. 1030	85	50	850	530
G.E. 1027	60	85	900	660
G.E. 1027	85	60	900	660
G.E. 1087	60	160	900	890
G.E. 1087	85	120	900	890

FOR PLEASURE VEHICLES OR INDUSTRIAL TRUCKS

Type	Rated volts	Rated amperes	Speed r.p.m.	Approximate weight
G.E. 1090	24	45	1200	170
G.E. 1090	48	40	2000	170
G.E. 1091	24	65	1750	200
G.E. 1091	48	45	1750	200

The characteristic curves for a typical Westinghouse motor with the 60-volt and 80-volt windings are given in Figs. 1A and 1B. The relatively small change in efficiency for overload with the field coils in parallel is noteworthy. The "torque ratio" of this motor, i.e., the ratio of torque at 2½ times rated current to torque at rated current, is approximately 5 with both windings.

Table II on page 57 gives a partial list of General Electric automobile motors with their normal ratings.



A. — 60 volts, 33 amperes   B. — 80 volts, 25 amperes

Fig. 1. Characteristic Curves of Typical Automobile Motors

Before recommending motor equipment for automobile service the motor manufacturers usually require that the purchaser submit data upon the weight, speed, battery, wheel dimension, etc., for the car on which the motor is to be used.

Size of Motor Required. — In selecting the size of motor the voltage should be chosen on the basis of 1.9 volts per cell for lead batteries and 1.0 volt per cell for Edison batteries. The average discharge voltages of both types of batteries at their normal rates are higher than these values, but it has been found desirable to install motors on the basis of these values so that at high rates of discharge the terminal voltage may not fall too far below the rated motor voltage.

The current rating of the motor (and also of the battery) should be approximately the value of the current required to drive the car at the rated speed on hard level asphalt. The current input into a vehicle motor when driving a car at constant speed is given by the expression

$$I = \frac{2\,vWr}{E\epsilon},$$

where v = speed of car in miles per hour,
W = total weight of car and load in tons,
r = car resistance in pounds per ton of total car weight,
E = motor terminal voltage,
ϵ = over-all efficiency of motor, controller and transmission.

The value of ϵ at the rated speed is the product of the efficiencies of motor and transmission, usual values falling between 60 and 75 per cent. For value of r see next paragraph.

Car Resistance. — The car resistance depends upon the following conditions: speed, grade, diameter of wheels, load per square inch of tire contact

or per inch of tire width, construction and composition of tire, method of attaching tire to wheel rim, and road surface. The effect of speed is to change r in a manner dependent upon the type of tire, the value of r having a minimum usually at a speed between 6 and 10 m.p.h.; air resistance varies approximately as the square of the speed and need not be considered for speeds of less than 20 m.p.h. The effect of up-grades is to increase r by 20 lb. per ton for each per cent grade. The car resistance with pneumatic tires on hard, level asphalt is given by Churchward (*Soc. of Auto. Eng. Handbook, 1913*) as ranging from 15 lb. per ton for special tires designed for electric runabouts up to 35 lb. per ton for the standard type of tire used on gasoline touring cars; the resistance is greatly affected by the air pressure, increasing rapidly as the air pressure is reduced. Churchward also gives the car resistance with solid tires on hard level asphalt as ranging from 18 to 26 lb. per ton. Other tests have shown a car resistance of 30 lb. per ton for a small capacity electric truck equipped with solid tires (*El. W., May 17, 1913, Vol. 61, p. 1040*).

Effect of Road Surface. — Three sets of values of relative car resistance on various level road surfaces as compared to the value on hard, level asphalt are given in the following table. One set of values is given by Churchward in the *Soc. of Auto. Eng. Handbook* (1913) the set is based on tests conducted at the Massachusetts Institute of Technology in 1913 (*El. W., May 17, 1913, Vol. 61, p. 1040*), and the third set is the result of tests by Kennelly and Schurig, Proc. A.I.E.E., Vol. 35, 1916.

TABLE III.—RELATIVE VALUES OF CAR RESISTANCE

Road surface	Churchward	M.I.T.	Kennelly and Schurig
Asphalt, hard	1.0	1.0	1.0
Wood block	1.15	1.1	1.18
Macadam	1.15 to 3	1.15	1.2
Granite block	1.75	2.0	1.41
Dirt road	1.1 to 2.0	...	1.31
Brick	....	1.4	1.25
Snow, packed	....	1.3	....
Snow, fairly hard, without chains	....	1.7	....
Snow, fairly hard, with chains	....	1.9	....
Snow, soft, about 3 in. deep	....	2.1	....
Sand	20	...	....

Transmission Systems. — The transmission systems between motor and wheel which are in common use may be grouped in two classes, known as the "chain" and the "shaft" or gear drives. A single-speed reduction is used in a few makes of cars, but a double reduction is used on a majority of machines. If the first reduction is made by chain and the second by gear, or vice versa, the transmission system is usually designated according to the means employed for the final reduction; e.g., if the final reduction is accomplished by a chain, the system is referred to as a chain drive although the first reduction may be by gears.

Chain Drive. — With chain drive the motor is mounted transversely and a double-speed reduction is used almost without exception. The connection between motor and countershaft is usually by silent chain (Morse or Rey-

nold type, see article on *Chains and Chain Drive*), although spur and herringbone gears are also used; see article on *Gears and Gearing*. The final drive is usually by roller chain, either (*a*) between countershaft and differential in the rear axle, or (*b*) between countershaft and a sprocket on each of the rear wheels. Chain-driven passenger cars are usually equipped with (*a*) and chain-driven commercial cars with (*b*). The chain method of drive possesses the advantages of low first cost and ease of adjustment; its disadvantages are noise, rapid wear due to collection of dirt unless properly housed, poor efficiency when not properly adjusted, and unequal wear on the two sides of the car. The chain drive has been losing favor during recent years among both designers and operators and is now (1921) practically abandoned.

Shaft or Gear Drive. — With shaft drive the motor is usually mounted lengthwise of the car, and either a single- or double-speed reduction is used. For single reduction both bevel and worm gears are used as the connection between propeller shaft and differential in the rear axle. For double reduction the motor is usually connected to the propeller shaft by spur gear, herringbone gear or silent chain, and the propeller shaft is connected to the differential by a bevel gear. The propeller shaft is usually fitted with universal joints. The shaft drive has become standard in passenger cars because it runs quieter than chain drive, and because it can be easily protected from dirt by housing. The shaft drive has the disadvantage of greater weight below the springs than in the case of the chain drive, which tends to increase the wear on tires.

Worm Drive. — There has recently been a decided tendency among designers of passenger cars and light trucks to use the single-reduction worm gear instead of the double-reduction bevel or chain arrangements, the advantages claimed for the worm being better transmission efficiency and less trouble from adjustment.

Walker Balanced Gear. — In the trucks built by the Walker Vehicle Co. the motor is built into the rear axle, which is made hollow and of a sufficient diameter to contain the motor. The armature shaft is also hollow, so that the drive shafts may extend through from the sockets of the differential to the center of each rear wheel. The wheels are driven by a spur-gear reduction mounted inside the wheels. The wheels have steel-plate sides instead of spokes, so that the gears may run in oil and are thoroughly protected from dirt.

Couple-Gear Wheel. — A device built by the Couple-Gear Freight Wheel Co. has the electric motor mounted inside the wheel. The motor armature carries a pinion on either end which engages with a gear ring on either side of the wheel. The sides of the wheels are dished steel plates. These wheels are being used on both front-wheel-drive and four-wheel drive machines. Being located in the wheels, they do not interfere with the use of the ordinary steering knuckle arrangement.

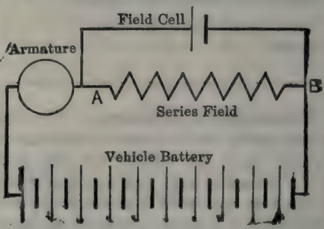


Fig. 2. Diagram of Steinmetz Motor System

Steinmetz System.—A new method of motor excitation consists of connecting a storage battery cell across the field terminals of a series motor employed in vehicle operation. At normal load the drop in voltage between field terminals counterbalances the voltage of battery cell. On starting and heavy grades the current required causes a greater drop in voltage between the field terminals, thus diverting a charging current through

the cell. Similarly if the motor current required in service decreases, the field cell will discharge through the field winding. Thus, the field strength varies only within narrow limits and in such a system the coasting of the vehicle down grades and the braking or stopping of the vehicle may be employed to recharge the vehicle storage battery. With this system a motor is used in which both field and armature rotate; each being connected separately to the two driving wheels, no differential being required, and on account of the increased relative speed between field and armature a much lighter motor than the normal type, with stationary field and rotating armature, is required for any given service.

Efficiency of Transmission. — (*See also article on Gears and Gearing.*) Beaumont gives the following figures on the transmission efficiency of different types of automobile mechanisms.

Source of loss of power	Efficiency, per cent
One chain, and one and one-half pairs of bearings	89.5
One set of gears, two pairs of bearings	82.0
One set of gears, equivalent of two chains, three pairs of bearings.	74.0
Two sets of gears, four pairs of bearings	70.0

The following figures, given by F. Burgess (*Trans. Soc. Auto. Eng., 1912, Vol. 7, Part 2, p. 196*), are the results of an efficiency test of a straight type worm and worm gear for rear-axle drive of electric automobiles. The worm gear was made of phosphor bronze and had 39 teeth; the worm was made of case-hardened steel and had 4 threads.

R.p.m. of worm	Temperature of worm gear, °F.	Input, trans. dyn., h.p.	Output, brake h.p.	Efficiency, per cent
1393	74	1.64	1.03	61.2
1416	86	3.41	3.17	93.2
1370	90	5.48	5.13	93.7
1389	94	6.72	6.24	93.0
1400	108	9.43	8.5	90.2

Batteries. — There are three types of batteries in general use in electric vehicles, namely, the pasted lead, the "Iron-clad" lead, and the Edison. The performance characteristic (except life) of the first two are identical and are described in the article on *Batteries, Storage, Lead Type;* the characteristics of the Edison battery are described in the article on *Batteries, Storage, Alkaline Type.* The desirable characteristics of a battery for vehicle service are given in the article on *Batteries, Storage, Applications of.*

Number of Cells; Number of Plates. — Practice among electric automobile manufacturers is now practically standardized upon the installation of 42 or 44 cells of lead battery and 60 cells of Edison battery in both passenger and commercial classes of cars, as these numbers permit of charging from a 115-volt circuit with minimum loss in the rheostat. There are, however, in use many old designs of pleasure cars in which from 24 to 38 lead cells are used, and the small capacity delivery wagons in which 48 Edison cells are also employed. A 24-volt to 48-volt battery is used on industrial trucks. For a given watt-hour capacity a battery of a small number of cells is cheaper to buy and

maintain than one of a large number of cells, but is more expensive to charge from constant-potential d-c. mains on account of rheostat losses.

As pointed out above the rated current output of a vehicle battery for 4 hours should not be exceeded by the current required to drive the car at rated speed on hard, level asphalt (*see section above on Size of Motor*). Thus when the current under these normal conditions is known, the number of plates per cell can be determined from a knowledge of the discharge rates of the various types of plates which are under consideration. For A-type Edison batteries the normal (5-hr. rate) is 7.5 amp. per positive plate; conservative designers use this rate in applying batteries rather than the value of 9.5 amp. which could be obtained for 4 hours. The discharge rates per positive plate of 5¾ by 8⅝ in. for 4 hours and for the so-called "normal" times of discharge for the types of vehicle batteries made by the Electric Storage Battery Co. are given in the accompanying table.

Type of plate	"Normal" discharge		Amperes per positive plate discharged during 6 hr.
	Time, hours	Amperes per positive plate	
Exide	4	7	5.12
Hycap	5	5.5	4.8
Thin	6	4.25	4.13
Iron-clad	4.5	7	5.6

It will be noted that the above rule of installing a battery whose 4-hour discharge rate is approximately equal to the current required for running at rated speed under good street conditions will furnish a minimum practicable battery capacity, and that this capacity may have to be increased materially in order that the car may meet the necessary mileage requirements when operating on grades or poor roads. There is a tendency among electric car manufacturers to recommend battery equipment for given conditions which is much more liberal than they recommended a few years ago.

Construction of Battery Box.—In passenger cars a tight wood-lined compartment is usually supplied for carrying the battery. In commercial cars the battery compartment is also wood lined, and provision is commonly made for ventilation to facilitate charging in warm weather. The details of a typical underslung battery box are shown in Fig. 3. With this construction strips may be laid in the spaces between the floor boards in order to make the compartment entirely inclosed, as is frequently desirable in cold weather. A tight covering should be provided with an underslung battery box to prevent dirt and water falling upon the battery from the floor of the car.

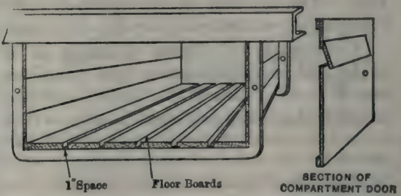


Fig. 3. Typical Underslung Battery Box

Controllers.—Controllers are usually of the drum type with two running positions, known as the "field-series" and the "field-parallel" positions. The motor field coils are arranged in two groups for connection either in series or in parallel.

The cells of the battery are connected permanently in series. In starting, either 2 or 3 resistance notches are used before the "field-series" notch is reached. A shunt on the fields is frequently used between the "field-series" and the "field-parallel" notches. For normal full-speed running the two groups of field coils are in parallel. An emergency speed is obtained by shunting a resistance around the fields when connected in parallel. It was formerly customary to split the battery into two groups which could be operated in series-parallel in order to obtain 4 economical running speeds, but trouble was experienced from the unequal discharge of the two halves of the battery. In the magnetic type of control which is used to some extent in passenger cars the changes in connection are affected by relays operated by an auxiliary electric circuit. Some forms of regenerative control have been tried with only moderate success, although in hilly localities regenerative control may be desirable on account of protecting the mechanical brakes.

Tires. — Tires for use on electric automobiles are generally more resilient than the standard types of tires commonly used on gasoline automobiles. "High efficiency" or special electric tires, both solid and pneumatic, have been developed by most of the tire manufacturers in response to a demand by the operators of electric cars for tires with low consumption of energy. Standard types of tires may consume as much as 100 per cent more energy than the special electric tires. Experience has shown that the smooth starting characteristic of an electric motor produces much less abrasion than the uneven acceleration with a gasoline motor, so that the rubber compounds in tires for electric cars are usually softer than the compounds in tires for gasoline cars and yet the lives of both types in point of distances traveled are approximately the same. Electric pneumatic tires are usually of the cord type; i.e., the "fabric" consists of parallel strands of cotton impregnated with rubber gum. Some types of solid electric tires, such as the Motz, have the sides undercut at intervals to allow for the "flow" of the rubber when under compression.

Carrying Capacities of Tires. — In general a tire of small transverse tread will consume less energy than a tire of the same compound with large tread when operating on a smooth, hard surface; also, a large wheel diameter gives less energy consumption than a small one as well as improving the riding qualities on uneven surfaces. The Society of Automobile Engineers have recommended the use of 32-, 36- and 40-in. tires as standard practice. (*See Trans. Soc. Auto. Eng., 1914.*) But in selecting tire equipment it should be remembered that there is a limit to the load which any size of tire will stand without permanent injury by disintegration. The maximum carrying capacities per wheel at present (1920) recommended by the B. F. Goodrich Co., are given in Table IV, as are also the recommended inflation pressures for pneumatic tires of the fabric type; the recommended inflation pressures for cord tires are approximately 80 per cent of the values in the table.

Examples of Commercial Car Design. — Table V gives data from the standard specifications of a number of the electric trucks.

PERFORMANCE; ENERGY CONSUMPTION. — The energy consumed by an electric automobile will depend very largely upon the care of the driver in coasting up to stops instead of braking directly from full speed; other important factors in determining the energy consumption are efficiencies of battery and driving mechanism, tire equipment, nature of roads and grades, number of stops and miles per day, etc. However, for a car which is operated under similar conditions day after day, the energy consumption per mile should be reasonably uniform, so that any considerable increase in energy consumption may indicate either careless driving or poorly adjusted mechanism, as for instance a dragging brake. Table VI gives a series of approximate values for

TABLE IV.—MAXIMUM CARRYING CAPACITY OF TIRES

Size of tire, inches	Maximum load, pounds per wheel				Recommended inflation pressure, lb. per sq. in.
	Solid		Pneumatic *		
	Single	Dual	Rear wheel	Front wheel	
2½	700	1400	....	....	..
3	1000	2000	375	450	45
3½	1300	2600	570	600	55
4	1700	3400	815	1200	65
5	2500	5000	1500	1700	85
6	3500	7000	....	....	..
7	4500	9000	....	....	..

* The allowable load per wheel for a given width of tire increases slightly with the wheel diameter. The sizes quoted in the table are 34 × 4½ in., 35 × 5 in., 36 × 6 in., and 38 × 7 in.

TABLE V.—DATA ON ELECTRIC TRUCKS

Rated load capacity, lb.	750	1000	2000	4000	7000	10,000
Speed, miles per hour	12	15	14	13	12	10
Miles per battery charge, approx.	45	45	45	45	40	35
Lead battery equipment:						
Number of cells	42	42	42	42	42	44
Number of pasted plates	11	11	13	17	23	27
Number of iron-clad plates	9	9	11	15	19	21
Edison battery equipment:						
Number of cells	48	60	60	60	60	60
Designation	A-4	A-5	A-6	A-8	A-10	A-12
Tires, front, in.	32 × 2½	36 × 3	36 × 3½	36 × 4	36 × 6	36 × 7
Tires, rear, in.	32 × 3	36 × 3	36 × 4	36 × 4D*	36 × 5D*	36 × 6D*
Weight, chassis and lead battery, lb.	2970	3325	4370	6000	8225	9300
Per cent weight on rear wheels †	50	50	50	50	50	50
Ampere rating of motor on 80 volts	20	32	32	45	62	62

* Dual.
† Chassis and battery

energy consumption per mile which are based upon a large number of reports upon the operation of electric cars in large cities; these reports were collected in connection with the Vehicle Research study conducted during 1912 to 1914 by the Electric Research Laboratory of the Massachusetts Institute of Technology. These figures are representative of experience with modern types of cars under average city conditions; the consumption for a given set of conditions may vary from the figures by as much as 20 per cent, either above or below.

TABLE VI. — ENERGY CONSUMPTION IN WATT-HOURS PER MILE, AT CHARGING BUS BARS

Type of car	Lead battery	Edison battery
2-passenger	300	400
1,000-pounds	550	750
2,000-pounds	650	900
4,000-pounds	830	1150
7,000-pounds	1100	....
10,000-pounds	1400	....

Garage Load Curves. — Fig. 4 shows a typical daily load curve for a public garage handling 75 electric passenger cars; Fig. 5 shows a similar typical daily curve for the private garage of a large department store (*see paper by E. E. Witherby, Proc. N. E. L. A., 1913*). The "off-peak" character of the electric vehicle load is indicated by these curves. In order to attract such off-peak load most light and power companies are now offering special rates for vehicle charging, frequently with the provision that no charging shall be done during the time of the power company's daily peak load.

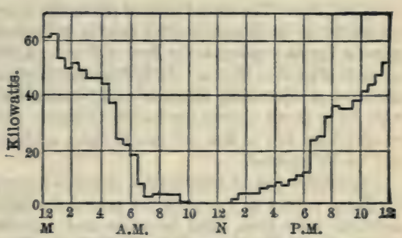


Fig. 4. Daily Load Curves of a Public Garage Handling Passenger Cars

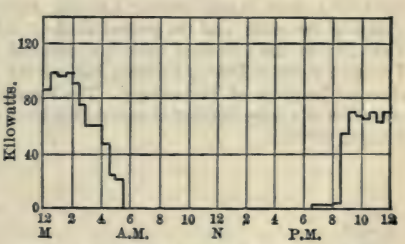


Fig. 5. Daily Load Curves of a Private Garage Handling Trucks

OPERATION.—In the operation of electric automobiles particular attention should be paid to the proper care of the storage batteries. The battery manufacturing companies issue complete instructions for the proper handling of their cells. The manufacturers of the battery, as well as the manufacturers of the car, are usually ready to confer with the operator of a car to the end that he may obtain satisfactory service from his machine. They should be consulted on the first indication of trouble in their respective portions of the equipment, should the cause of the trouble not be understood.

The following points upon the care and operation of electric cars should be observed:

1. A battery must always be charged with direct current and in the right direction.

2. Never bring an exposed flame near a battery while charging or immediately afterwards.

3. Do not allow the battery temperature to exceed 110° F.

4. Keep the cells filled to the proper level by adding distilled water only. Never put acid in an Edison battery under any circumstances.

5. Keep the outside of the cells free from foreign substances, both solid and liquid.

6. For boosting a lead battery during a specified short period, the maximum current rate I which may be used without reaching the gassing point is the quotient of the ampere-hours Q previously discharged (read from ampere-hour meter), divided by 1 plus the hours H available for boosting, viz., $I = \frac{Q}{1 + H}$.

7. The mechanism of a car should be inspected carefully at least once in two weeks.

8. A car should be entirely overhauled at least once each year, in order that worn parts may be located and replaced.

COSTS. — The initial cost of passenger electrics depends so largely upon the character of body and fittings that it is impossible to quote any but the most general figures; prices range from $2800 to $5000, the most popular cars selling at from $4000 to $5000. It is likewise impossible to quote definite figures on the cost of operating pleasure cars; prices for electricity for private charging vary from about 3 cents per kw-hr. upward; the cost of storage in public garages ranges from $15 to $40 per month, plus a charge for electricity at from 4 cents per kw-hr. upward.

The initial cost of commercial electrics ranges from about 60 cents per pound for large trucks to 75 cents per pound for light cars, based upon usual equipment of solid tires, lead batteries and express or stake bodies. (*For typical weights see Tables I and V.*) Special equipment such as winches or dumping bodies is additional. The operating expense will vary widely for different conditions, so that it is impossible to predetermine the total cost of operating a truck of a given rating without knowing the requirements of the service the nature of the roads, and the general method of handling and caring for the car. The data in the following table are deduced from estimates by Pender and Thomson given in *Vehicle Research Bulletin No. 3*, issued in 1913 by the Massachusetts Institute of Technology. The figures are based upon reports of the operation of a large number of cars during from 1 to 4 years in city trucking and delivery services.

TABLE VII.—COST OF OPERATING ELECTRIC TRUCKS

(Pre-war figures)

Rated load capacity, lb.	**1000**	**4000**	**7000**	**10,000**
Miles per year considered	10,500	9100	8850	8000
Cents per mile for:				
Tires, repairs and battery*	6.8	8.3	11.2	14.3
Electricity at 3 ¢ per kw-hr	1.5	2.2	3.0	3.6
Dollars per year for:				
Garage and lubricants	215	235	255	285
Driver and helper	1,000	1140	1210	1210
Depreciation, interest and insurance	380	464	532	685
Total annual expense, dollars	2,455	2794	3252	3610

* Pasted plate lead battery.

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AUTO-TRANSFORMERS AND COMPENSATORS. — (*See also Starters, Motor; Transformers.*) An auto-transformer or single-circuit transformer, also called a "compensator," consists of a transformer having the usual iron core but only one electrical circuit instead of two. This circuit is tapped at various points as shown in Fig. 1, and the primary and secondary circuits, while independent outside the transformer, unite in the same winding in the transformer. If an alternating voltage is impressed across the points *ab*, a magnetizing current will flow in the winding setting up an alternating flux which will link every turn and induce therein an alternating voltage. The voltage between any two taps, as *ac*, is proportional to the number of turns between the taps; thus any ratio of voltages may be obtained. If the secondary *ac* is loaded a current will flow in the primary and the primary and secondary currents will flow in the two parts of the winding as indicated in the figure.

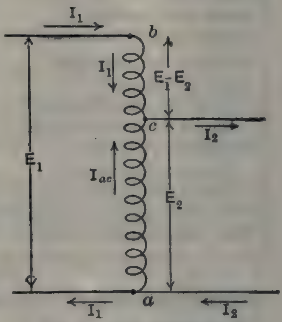

Fig. 1. Auto-transformer

VOLTAGE AND CURRENT RELATIONS. — Let N_1 be the total number of turns between *a* and *b*, N_2 the turns between *a* and *c*, E_1 the voltage across the terminals *a* and *b* and E_2 the voltage across the terminals *a* and *c*. Then, neglecting the resistance and reactance of the winding,

$$E_2 = \frac{N_2}{N_1} E_1.$$

Let I_1 be the current entering the terminal *b*, I_2 the current in the *external* circuit connecting *a* and *c*, and I_{ac} the current *in the transformer winding* between *a* and *c*. Then, neglecting the resistance and reactance of the winding and the magnetizing current,

$$I_2 = \frac{N_1}{N_2} I_1,$$

$$I_{ac} = I_2 - I_1 = \frac{N_1 - N_2}{N_2} I_1.$$

The current in the turns between *b* and *c* is

$$I_{bc} = I_1.$$

Auto-transformer Versus the Two-circuit Transformer.—For $N_2 = \frac{1}{2} N_1$ the current in the turns between *a* and *c* would be just equal, neglecting the exciting current, to the current in the turns from *b* to *c* (but opposite in direction). Consequently for a 2 to 1 transformation but one winding of an ordinary two-circuit transformer could be used, provided a tap was available at the middle point of this winding, and the rated output of the transformer could be obtained without the current in this winding exceeding its rated value. Since under these conditions there would be no current in the second winding, the heating of the transformer would be less than it would be were the transformer used as an ordinary two-circuit transformer, and therefore a greater output could be obtained without exceeding the nominal temperature rise.

In general, for the same power input into the connected load an auto-transformer requires but $\frac{E_1 - E_2}{E_1}$ of the copper required for a two-circuit transformer.

where E_1 is the high-tension and E_2 the low-tension voltage. The higher the ratio of transformation the less the saving in copper. There is also a serious objection to an auto-transformer of high ratio of transformation, in that an accidental ground on the high-tension lead, b in Fig. 1, would establish a high voltage between the low-tension leads and the ground, which may cause a dangerous shock to a person touching either low-tension lead or may cause other damage. This may be partially prevented by grounding the common point of the high- and low-tension circuits, the point a in Fig. 1, in which case an accidental ground on the high-tension side would produce a short-circuit, which would open the circuit breaker in the primary circuit, *provided the resistance between the two grounds is low and the circuit breaker operates properly.* These two provisions, however, may not always be realized even though reasonable care is taken, and it is therefore not considered good practice to use an auto-transformer of high ratio of transformation, except in special cases where economy of space is an important factor and danger from shock can be guarded against, e.g., on a-c. locomotives.

RATING OF AN AUTO-TRANSFORMER. — In commercial practice the rating of an auto-transformer in volt-amperes is taken equal to the difference between the high- and low-tension voltages multiplied by the rated current in that part of the winding, bc in Fig. 1, across which this voltage exists. The capacity of an auto-transformer for a given work bears to the capacity of a two-circuit transformer for the same purpose the ratio $\frac{E_1 - E_2}{E_1}$. For example, to step down the voltage of the supply mains from 500 to 400 volts and supply a load of 100 kv-a. would require an auto-transformer having a rating of $\frac{500 - 400}{500} \times 100 = 20$ kv-a. as against a two-circuit transformer having a rating of 100 kv-a. The weight, dimensions and cost of an auto-transformer are very nearly the same as for an ordinary two-circuit transformer having the same voltage and kv-a. rating.

APPLICATIONS. — Auto-transformers are used chiefly where the required change in voltage is small, e.g., for motor starters (*see Starters, Motor*), and for balancing the voltage between two or more circuits. The smaller the ratio of transformation the greater is the gain in cost and efficiency resulting from the use of an auto-transformer instead of a two-circuit transformer. Auto-transformers are also used to provide a neutral for the Edison three-wire system; see *Distribution Circuits*. In single-phase railway work single-phase auto-transformers are used on the locomotives for transforming the trolley voltage (from 3000 to 6000 volts) to the motor voltage (about 500). This is a rather high ratio of transformation for an auto-transformer, but as the saving in weight is very important and as one terminal is grounded, it has proven satisfactory for this service; see *Locomotives, Electric*.

Auto-transformers are frequently constructed three-phase (and occasionally two-phase). These are generally used for starting polyphase motors by providing a lower voltage for the starting period. They are also occasionally used in high-tension distribution. If the voltage applied to an a-c. motor is reduced to one-half, the motor starting current will be one-half and the volt-amperes one-quarter. Thus the starting current in the line will be one-quarter. A three-phase auto-transformer may be either Y or delta-connected. The Y connection is shown under *Motors, Polyphase Induction, Methods of Starting*. The delta connection cannot be made to give a balanced voltage less than half that of the higher voltage, as may be readily seen from an inspection of Fig. 2.

DESIGN. — The design of an auto-transformer is quite similar to that of a two-circuit transformer (q.v.) and the leakage reactance is calculated in the same way. For low-leakage reactance the primary and secondary coils must not be entirely separated, but each must be divided into sections and inter-

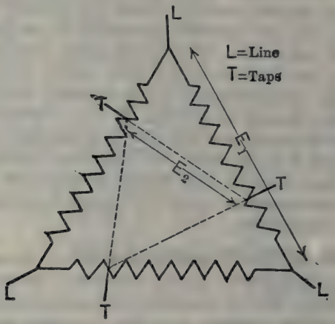

Fig. 2

mixed to as great an extent as possible. Having found the magnetizing current, core-loss, resistance and reactance in the usual manner, the calculations follow the same methods as for power transformers. In general, the efficiency at unity power factor is given by the equation

$$\text{Efficiency} = \frac{E_2 I_2}{E_2 I_2 + (r_1 - r_2)\, I_1^2 + r_2\, (I_2 - I_1)^2 + \text{core-loss}},$$

where r_1 is the total resistance of the winding measured between the high-tension terminals and r_2 the resistance measured between the low-tension terminals. The efficiency is better than that of a two-circuit transformer of the same kv-a. rating.

TESTS, SPECIFICATIONS, ETC. — See *Transformers*.

DIMENSIONS, WEIGHT AND COSTS. — For the same kv-a. rating the dimensions, weight and cost are very nearly the same for auto-transformers as for ordinary two-circuit transformers; see *Transformers*.

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BALANCES, CURRENT. — (*See also Ammeters; Electrodynamometers.*) The Kelvin balance consists of a system of two coils mounted one at each end of a rigid beam and free to move between two pairs of field coils. The balance may be used as an ammeter, voltmeter or wattmeter. It may be used for either a-c. or d-c. measurements, but for d-c. work it has been practically superseded by laboratory standard voltmeters. For a-c. work it is used only as a laboratory standard for calibrating other instruments.

A current balance when carefully constructed may be used for the absolute measurement of electric current, since the force of attraction between the coils can be calculated in terms of their dimensions and the current flowing. It has been successfully used to determine in absolute measure the electrochemical constant or Faraday (*see Electrochemistry, Principles of*).

Principle of Operation. — The principle of the current balance is similar to that of the electrodynamometer. The connections for current measurements are shown in Fig. 1. The current passes through all the coils in series and as a result of the action of the magnetic fields, the left-hand end of the beam is depressed and the right-hand end is elevated. The beam on which the middle coils are mounted has an index pointer and carries a weight that may be slid along a scale on the beam. With no current on and the sliding weight at zero on the scale, a counter weight on the swinging system is so adjusted that the index pointer on the system is opposite a fixed mark and the system is balanced.

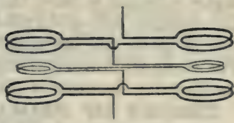

Fig. 1. Connections for Current Balances

Passing current through this system destroys the balance which is restored by sliding the weight along the beam. Since the torque exerted on the moving system is proportional to the square of the current, the current flowing will be proportional to the square root of the distance through which the weight has to be moved to balance the beam.

In the case of the wattmeter the current passes through the fixed coils, while the coils on the moving system serve as the potential coils. In this case the power is directly proportional to the distance through which the weight has to be moved to balance the system.

Range and Cost. — Kelvin balances are built in 5 standard sizes. The useful range and approximate cost of each instrument is given below (**pre-war prices**).

Type	Range, amperes	Approx. cost
Centi-ampere	0.025–1	$150
Deci-ampere	0.25–10	150
Deca-ampere	5–100	150
Hekto-ampere	30–600	150
Kilo-ampere	100–2500	250

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BATTERIES, PRIMARY. — (*See also Cells, Standard; Electrochemistry, Principles of.*) A primary battery or cell is a device for the *direct* transformation of chemical energy into electrical energy. It is becoming common practice, though not yet universal, to use the term "cell" for a single unit, and the term "battery" for a group of two or more units or cells.

For convenience of treatment primary batteries may be classified as wet batteries, dry batteries and standard cells. The latter are treated in the article on *Cells, Standard.* The wet battery at one time was largely used in laboratory testing, for telephones, bells and other devices requiring small amounts of energy. In recent years it has been largely supplanted by the dry battery and small storage battery cells (*see Batteries, Storage*). Dry cells, exclusive of the small flash light cells, are now (1921) used, in this country alone, to the extent of some hundred million a year, chiefly for gas-engine ignition and telephone work (*see Gas Engines* and *Telephone Instruments*).

Poles and Electrodes. — The pole or terminal of a battery which is at the *higher* potential is called the *positive* pole or terminal, the other pole being called the negative pole or terminal. The negative pole is the anode or *positive electrod* or plate and the positive pole is the cathode or *negative electrode* or plate. For example, in a copper-acid-zinc battery the copper is the positive pole but the negative electrode or plate.

THEORY OF PRIMARY CELLS. — The modern theory of primary cells is fully discussed in the article on *Electrochemistry, Principles of.* Only one or two points of practical moment will be mentioned here.

Electromotive Force. — A cell consists essentially of two metallic conductors or poles dipping into an electrolyte. Copper or carbon is commonly employed for the positive pole and zinc for the negative pole. The electrolyte may be sulphuric or nitric acid or sal-ammoniac, caustic soda or other salt. For a cell made of given materials the open-circuit e.m.f. is always the same provided the temperature, degree of concentration of the electrolyte and the purity of the materials are the same. The terminal e.m.f. or potential difference (p.d.) on closed circuit is always less than the open-circuit e.m.f. due (1) to the internal resistance of the battery, (2) to the polarization of the battery and (3) to the exhaustion of the battery.

Internal Resistance. — Let E be the initial open-circuit e.m.f. of the cell and V be the p.d. across its terminals, when it is supplying a given current I, then the "apparent" internal resistance of the cell is

$$r = \frac{E - V}{I}.$$

This is not the true resistance of the cell, for its net e.m.f. when a current I is being drawn from it is less than the initial open-circuit e.m.f. due to polarization (*see below*) and partial exhaustion. If, however, after measuring the p.d. on closed circuit the current is interrupted and the open-circuit e.m.f., say E', be measured *immediately* (i.e., before the polarized condition of the cell has had time to change) then the true internal resistance of the cell is

$$r' = \frac{E' - V}{I}.$$

Polarization. Depolarizers. — Polarization is the name applied to the changes produced in the relative concentrations of the electrolyte at the two poles of a cell or to the production at the poles of new chemical substances (such as hydrogen) as a result of the flow of current through it. A depolarizer is any substance which when placed in the electrolyte or on the poles of the cell will partially or wholly prevent these changes. Polarization always tends to reduce the effective e.m.f. of the cell. When a cell which has been polarized is

open-circuited, the relative changes in concentration gradually disappear, due to diffusion, and any new substances formed also tend to diffuse uniformly through the electrolyte, with the result that the open-circuit e.m.f. returns in time to nearly its original value, provided the active materials are not exhausted.

The chief cause of polarization in a cell is the formation of hydrogen gas at the positive pole or to the transfer of the metal from the negative pole to the positive pole. The depolarizer is usually an oxidizing agent which reduces the hydrogen or metal liberated at the positive pole to a form readily soluble in the electrolyte and thereby prevents its accumulation at the positive plate. This is the principle on which depends the depolarizing action of the various metallic oxides, such as manganese peroxide or cupric oxide. The same result may be obtained by surrounding the positive pole by a solution of a salt of itself which has the same acid radical as the electrolyte surrounding the negative pole, but which is less soluble, the two solutions being kept practically separated by a porous cup which renders the diffusion of one into the other very slow.

TYPICAL WET BATTERIES. — Numerous forms of wet batteries have been used; only some of the more common forms can be described here. The materials used and the e.m.f. developed and approximate range of internal resistance are listed in the following table.

TYPICAL WET BATTERIES

	Daniell	Gravity	Bunsen	Chromic acid	Edison-Lalande	Leclanché type
Positive pole....	Cu	Cu (b)	C	C	C	C
Negative pole....	Zn (a)	Zn (a)	Zn (a)	Zn (a)	Zn (a)	Zn
Electrolyte....	H_2SO_4 or $ZnSO_4$	$ZnSO_4$ or $MgSO_4$	H_2SO_4	H_2SO_4 or NaCl	KOH or NaOH	NH_4Cl
Depolarizer......	$CuSO_4$	$CuSO_4$	HNO_3	CrO_3	CuO	MnO_2
Separator........	Porous pot	None	Porous pot	With or without pot	None	With or without pot
E.m.f., volts.....	1.07 to 1.14	1	1.9–1.95	2	0.75	1.5
Resistance, ohms	0.3 to 30	0.1 to 6	Low	0.5 to 4	0.02 to 0.1	1 to 5

a. Amalgamated. b. In the Krüger cell copper-plated lead is used.

Daniell Cell (Fig. 1). — A typical form of this cell is shown in Fig. 1. Other forms, differing in certain details, are Muirheads cell, the Siemens & Halske Daniell cell, Minotto's cell (which may also be classed as a gravity cell, *see below*), etc. J is a glass or glazed earthenware jar containing a concentrated solution of copper sulphate, P is a porous pot containing dilute sulphuric acid (about 10 per cent by volume) or zinc sulphate solution or both, C is the positive copper pole and Z the negative zinc pole which is usually amalgamated. The chemical reaction which takes place may be represented by the formula

Fig. 1. Porous Pot Daniell Cell

$$Zn + CuSO_4 = Cu + ZnSO_4.$$

A current of 1 ampere for 1 hour deposits 1.186 grams of copper and liberates 1.219 grams of zinc. Hence

0.042 oz. copper is deposited per ampere-hour,
0.043 oz. zinc is used up per ampere-hour,
0.164 oz. copper sulphate crystals are used up per ampere-hour,
0.106 oz. zinc sulphate is formed per ampere-hour.

The latter when crystallized out will form 0.189 oz. zinc sulphate crystals. These figures do not include any loss of zinc due to local action, which may amount to 10 per cent or more.

E.M.F. of Daniell Cell. — The e.m.f. of a Daniell cell varies from about 1.07 volts to 1.14 volts, depending on the density of the copper sulphate solution and on the amount of zinc sulphate present in the dilute sulphuric acid. The cell has its highest e.m.f. at the start when the sulphate of copper solution is saturated and no sulphate of zinc has formed. Hence, in order that the e.m.f. shall remain more nearly constant, it is better to start with *both solutions saturated.* The resistance of the cell will be higher and its e.m.f. lower than when dilute sulphuric acid is used, but this lower value of about 1.10 volts will be maintained nearly constant while the cell is sending a current.

Internal Resistance of Daniell Cell. — This depends not only upon the dimensions of the cell but also upon the porosity of the pot or other separating medium. The type of cell shown in Fig. 1, having a pot 7 inches high which is quite porous, has an internal resistance of about 0.3 ohm. The resistance of the Siemens type of Daniell cell is from 10 to 15 ohms, and the resistance of Minotto's modification of the Daniell cell, in which a layer of sawdust or sand is used in place of the porous cup, may be as high as 30 ohms.

The resistance of a Daniell cell, like that of liquids generally, *diminishes* with *increase* of temperature.

Local Action. — Impurities in the zinc form with the zinc small short-circuited voltaic cells, resulting in a wasting away of the zinc without producing a current in the external circuit. This can be largely prevented by amalgamating the zinc, i.e., coating it with mercury, which is readily done by thoroughly cleaning the zinc by dipping it into dilute sulphuric acid and then rubbing mercury over its surface. In the Daniell cell metallic copper also forms in the pores of the porous cup where the zinc touches it; this can be prevented by covering with paraffin those portions of the cup which may come into contact with the zinc.

Care of Daniell Cells. — The type of cell shown in Fig. 1 must be taken to pieces when not in use. If it has to be put to one side for only an hour or two, it will be sufficient to lift the porous pot with the contained zinc rod bodily out of the cell, and to place it in another empty jar, or stand it in a dish while out of use.

Gravity Battery (Fig. 2). — The principle of this cell is the same as that of the Daniell cell, except that no porous pot is used, the copper sulphate and zinc sulphate being maintained separate by gravity. There are various modifications in the form of construction, known as the Meidinger cell, Calland cell, Kelvin tray battery, Krüger cell, etc. The gravity cell is still used in telegraph and telephone work.

The copper electrode is placed at the bottom of the cell, and is then covered with copper sulphate crystals. The zinc electrode is then put in place and the jar either filled with dilute zinc sulphate or with dilute sulphuric acid. When first set up the internal resistance is high, but if the cell is short-circuited for a considerable time the resistance is reduced due to the formation of zinc sulphate. To prevent evaporation the solution is covered with a layer of mineral oil.

Care of Gravity Battery. — The resistance of the cell increases rapidly with decrease of temperature; it should therefore be kept at a reasonably high temperature, say 70° F. A gravity cell must, of course, not be moved about, or if moved great care must be taken to avoid the two liquids being mixed together. To prevent the copper sulphate wandering to the zinc plate, it is well to allow the cell to send a weak current through an external circuit of considerable resistance even when the cell is not in ordinary use. The electrolyte should be renewed when the blue sulphate solution turns brown. The line of separation between the copper and zinc sulphates, or the "blue line," should be about halfway between the two electrodes.

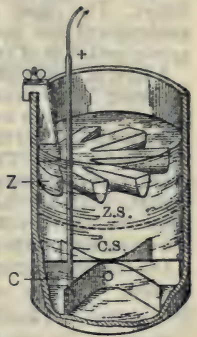

Fig. 2. Gravity Daniell Cell

Bunsen Cell. — In the Bunsen cell a zinc plate is placed in dilute sulphuric acid, as in the Daniell, but the copper plate is replaced by one of carbon and the copper sulphate solution by strong nitric acid, which is generally said to act as the depolarizer. The Grove cell differs from the Bunsen only in the use of platinum in place of carbon. Both cells give a high e.m.f., from 1.9 to 1.95 volts, and have low internal resistances, so they may be used for producing fairly large currents. When working the cells give off dark brown fumes of nitric peroxide, NO_2, and should be placed in the open air or under a chimney.

The chemical reaction which takes place may be represented by the formula $Zn + H_2SO_4 + 2\,HNO_3 = ZnSO_4 + 2\,H_2O + 2\,NO_2$.

Care of Bunsen Battery. — A Bunsen or Grove battery must be taken to pieces at the end of each day's use, since the mixing of the liquids through the walls of the very porous cup used to separate them would render the battery practically useless the next day. The porous pots should be placed in water after use, so that all the zinc sulphate solution may be dissolved out of the pores of the earthenware, for, otherwise, when the pots are dried the zinc sulphate solution will crystallize in the pores and cause the pots to fall to pieces.

Chromic Acid Cell (Fig. 3). — The chromic acid or potassium bichromate cell was devised by Poggendorff. In the original type of cell no porous pot was employed. In the Fuller cell, shown in Fig. 3, a porous pot is used. The depolarizer used is chromium peroxide, popularly called chromic acid, which may be purchased ready prepared, or may be formed by heating potassium or sodium bichromate with sulphuric acid (1 part by weight of the bichromate, 3 parts of acid and 9 parts of water). In the type of cell shown in the figure the wire connected to the zinc rod is well amalgamated or coated with gutta-percha to insulate it. In the porous pot containing the zinc, there is put a quantity of mercury to maintain the amalgamation, and either dilute sulphuric acid or a solution of common salt NaCl. The chromic acid solution is placed in the jar containing the carbon plate.

Fig. 3. Fuller's Mercury Bichromate Cell

The chemical reaction which takes place may be represented by the formula $3\,Zn + 2\,CrO_3 + 6\,H_2SO_4 = Cr_2(SO_4)_3 + 3\,ZnSO_4 + 6\,H_2O$.

This cell has an e.m.f. of about 2 volts, and is suitable for producing a fairly strong current for a short time. When much used the cell becomes saturated with the potassium and chromium sulphates, and a double salt, chrome alum, $K_2Cr_2(SO_4)_4$, crystallizes out and sticks so firmly to the bottom of the cell that it is somewhat difficult to remove.

Edison-Lalande Cell. — In this type of cell the positive pole is a plate of compressed cupric oxide (CuO), the surface of which is reduced to metallic copper. The cupric oxide acts as the depolarizer. The negative pole is amalgamated zinc and the electrolyte a strong solution (1 to 3 by weight) of caustic potash or of caustic soda.

A layer of heavy oil is poured over the solution to prevent evaporation and "creeping." No local action or polarization takes place in this cell; under normal conditions it is an easy matter to set it up to give any required number of ampere-hours, and to so proportion the constituents that they are all exhausted at the same time. This is a matter of considerable importance where closed-circuit working is employed, as in some systems of telegraphy and in alarm circuits. Although the e.m.f. of the Edison-Lalande cell is low, its resistance is also low, and the cell is capable of producing large currents. The cell may be left set up for months without deterioration.

The chemical reaction which takes place may be represented by the following formula

$$Zn + CuO + 2\,KOH = Cu + H_2O + K_2ZnO_2.$$

Leclanché Cells. — In the original form of this type of cell the positive pole was a plate of carbon embedded in a mixture of solid manganese peroxide and broken carbon contained in a porous pot. The electrolyte is sal ammoniac, NH_4Cl, and the negative pole zinc. The manganese peroxide acts as the depolarizer. The only object of the porous pot was to hold the carbon and manganese peroxide. In the later forms of this cell, such as the "agglomerate," Corsak, etc., the porous pot is dispensed with and a mixture of carbon and manganese peroxide are moulded together with a suitable binder, or the mixture of carbon and manganese peroxide is held together by a wrapping of canvas or sacking.

The chemical reaction which takes place may be represented by the formula

$$Zn + 2\,MnO_2 + 2\,NH_4Cl = Mn_2O_3 + H_2O + 2\,NH_3 + ZnCl_2.$$

If, however, too little sal ammoniac be present, zinc oxide or zinc oxychloride is formed instead of zinc chloride, and the solution becomes milky; hence when this happens, more sal ammoniac should be added.

The e.m.f. of a Leclanché cell is about 1.5 volts, but falls rapidly when the cell is used to send a strong current. It will, however, regain its value if the cell be left for some time unused, and it does not sensibly diminish when the cell is put to one side, even for some months. Hence, while the Leclanché cell is much inferior to the Daniell cell for the purpose of sending a steady current for an hour or two, it is much superior to the Daniell cell for producing intermittent currents at any time during the course of a year or more — for example, such currents as are employed for the ringing of electric bells, for house telephones, and for railway signaling.

Care of Leclanché Cells. — When the sal ammoniac becomes exhausted it should be thrown out and a new solution made. Three or 4 ounzes of sal ammoniac for a jar of ordinary size is required, the jar to be filled about one-third with water before putting in the electrodes.

DRY CELLS. — The modern dry cell may be looked upon as a modification of the Leclanché cell, the chief difference being that only enough water is added to the electrolyte to moisten an absorbent layer of pulp-board, blotting paper, cheese cloth or starch paste, this lining separating the positive and negative poles. The negative pole, which also serves as a container, is a hollow zinc cylinder; the bottom of this cylinder is also usually made of zinc. The

positive pole is a carbon rod, which may be either cylindrical or fluted. The absorbent layer above mentioned is placed next the zinc and is saturated with a solution of sal ammoniac and zinc chloride. The zinc chloride is necessary to reduce the rapid deterioration which would otherwise take place on open circuit. The space between this lining and the carbon electrode is filled with a mixture of granulated carbon and manganese peroxide, the latter being the depolarizer. The top of the cell is usually sealed with a pitch composition.

Desiccated Cells, sometimes called reserve cells, are usually constructed the same as the ordinary dry cell. The chief difference is that the cell is assembled without water, provision being made for filling the cells with water before they may be required for use. Desiccated cells will not deteriorate so long as they remain dry. It is necessary to fill them with water several hours before use, and 24 hours may be necessary before the cell will give its maximum current on short circuit.

E.M.F. of Dry Cells. — In new cells of practically all types the open-circuit e.m.f. is between 1.5 and 1.6 volts. The decrease in e.m.f. when the cell stands on open circuit is very slight, being only about 0.1 after the cell has stood many months. An open-circuit e.m.f. materially less than 1.5 volts is generally an indication of serious deterioration or of some other defect. The effect of temperature on the open-circuit e.m.f. is slight, amounting to only a few hundredths of a volt for all ordinary temperature ranges. Due to the relatively rapid polarization and increase of internal resistance with use, the average terminal voltage during the useful life of the cell is only about 1 volt.

Internal Resistance. — The internal resistance of a high-grade dry cell when new is usually less than 0.1 ohm, which may increase to 0.5 ohm within 9 or 12 months, even though the cell is not used during this time. The polarization of the cell in actual service causes a much greater decrease in the terminal e.m.f. than does the internal resistance drop, and therefore the internal resistance test is of little practical value.

Short-circuit Current. — Nine out of every ten users of dry cells consider the short-circuit current, i.e., the current produced through an ammeter having a relatively small resistance connected directly between the poles of the battery, as a direct measure of the value of the cell. There are other factors, however, which must be considered, such as the temperature of the cell, the service for which it is to be used, etc. According to D. L. Ordway (*Trans. Am. Electroch. Soc., 1910, Vol. 17, p. 346*) a standard 2.5 by 6-inch dry cell should give when new a short-circuit current (external resistance not over 0.01 ohm) of from 18 to 25 amperes; a cell giving a short-circuit current much above 25 or below 16 amperes should be looked upon with suspicion. A cell giving a short-circuit current much in excess of 25 amperes is liable to polarize rapidly, whereas if the short-circuit current is much under 16 amperes, it is probable that the cell has been made a long time or that cheap materials have been used.

The effect of temperature on the short-circuit current is pronounced. Between 10 and 80° C. the current increases about 1 ampere for each 10° increase in temperature. This effect is even more pronounced at very low temperatures. The short-circuit current returns to its normal value when the cell is restored to normal temperature.

Shelf-life. — The shelf-life of a dry-cell of the ordinary sizes is usually considered as the time in months that the cell may stand on open circuit without its short-circuit current falling below 10 amperes. This current is about half the short-circuit current when the cell is new, and represents a value which is probably lower than the minimum point at which a dealer could dispose of a

cell to the average consumer. The average shelf-life of high-grade cells is from 10 to 12 months, though the very best cells have a shelf-life of from 12 to 15 months. Many makes of cells have a shelf-life of only 8 to 10 months; and cells are on the market having a shelf-life of only 1 or 2 months.

The shelf-life is increased by storing the cells at a low temperature. Ordway gives the following results on standard dry cells kept in storage at the stated temperatures. These cells give initially a short-circuit current of about 20 amperes.

Temperature at which cells were stored, ° C....	0	25	50	75
Short-circuit current at 25° C. after 5 months....	18.1	17.4	0.5	0.4

Ampere-hour and Watt-hour Capacity. — The short-circuit current, however, is not a measure of the ampere-hours obtainable from the cell, as is indicated in the following table, taken from Ordway's paper. The ampere-hours given are those obtained from the various cells when discharged continuously through a resistance of 16 ohms until the closed-circuit voltage fell to 0.5 volt.

Brand of cell..............	A	D	G	J	M	P	V	X
Short-circuit current......	33.0	24.5	22.5	20.9	20.1	19.2	11.6	6.6
Ampere-hours..............	24.0	33.5	40.0	18.2	11.7	30.3	13.6	4.5

The letters are arbitrary designations.

Although the short-circuit current falls off with the age of a cell, even though the cell is not used, the ampere-hour capacity does not decrease in the same ratio. Ordway gives the following tests on samples from the same lot.

	Freshly made	9 months after manufacture
Short-circuit current....................	22.4	3.6
Ampere-hour capacity (through 2 ohms to 0.25 volt)........................	24.9	20.2

Effect of Rate of Discharge on Capacity. — Ordway gives the following results of tests on standard 2.5 by 6-inch cells, when the cells are discharged continuously through 2, 4, 8, 16, 24, 32 and 40 ohms respectively. By "end point in volts" is meant the terminal voltage per cell at the end of the stated number of hours.

HOURS OF CONTINUOUS SERVICE OF 2.5 BY 6-INCH DRY CELLS

End point in volts	Resistances used in ohms						
	2	4	8	16	24	32	40
1.2	4.3	10	39	142	260	414	549
1.0	9.3	35	94	296	548	889	1148
0.8	16.5	51	143	414	751	1078	1550
0.6	28.2	76	225	954	1240	1600	1763
0.4	55	207	648	1197	1711	2280	2040
0.2	160	450	882	1318	1914	2626	3140

WATT-HOURS FROM 2.5 BY 6-INCH DRY CELLS DISCHARGED CONTINUOUSLY

End point in volts	Resistances used in ohms						
	2	4	8	16	24	32	40
1.2	3.7	4.3	8.1	15.2	18.8	21.7	23.8
1.0	6.7	13.0	16.5	26.9	33.4	39.8	42.0
0.8	9.7	16.3	21.5	32.8	40.3	44.6	50.6
0.6	12.5	19.4	26.6	48.9	49.5	52.7	53.2
0.4	15.4	27.3	39.1	52.6	54.3	58.2	54.8
0.2	19.8	32.6	41.5	53.3	55.2	59.3	57.1

Capacity on Intermittent Service. — Neither the ampere-hour nor watt-hour capacity of a cell on *continuous* service is a measure of its capacity on intermittent service. What the user wishes to know is the actual number of hours of service that he can obtain from a cell when used to operate a definite piece of apparatus, which as a rule takes current only intermittently, e.g., for telephone or ignition service. From curves given in Ordway's paper for 3 cells connected in series and discharged through the stated resistances the following table has been made up.

HOURS OF INTERMITTENT SERVICE OF 2.5 BY 6-INCH DRY CELLS

(The hours given are the hours the cell is actually supplying current)

End point in volts, 3 cells	Continuous discharge			5 minutes each hour night and day			5 minutes each hour, 8 hours per day, 6 days per week		
	5 ohms	10 ohms	20 ohms	5 ohms	10 ohms	20 ohms	5 ohms	10 ohms	20 ohms
3.6	4	12	30	4	45	80	3	15	45
3.0	9	25	65	29	65	145	12	30	60
2.4	15	40	100	48	90	190	23	45	75
1.8	25	60	150	65	130	225	35	70	100
1.2	42	140	475	80	160	250	52	100	150
0.6	95	...	...	93	200	290	70	115	165

From the above table it is apparent (1) that the terminal voltage falls off more rapidly on continuous discharge than on intermittent discharge, counting hours of *actual discharge* only, but (2) that during the latter stages the terminal voltage falls off more rapidly for the intermittent discharge, this latter effect occurring earlier as the external resistance is increased. The second effect is probably due to the deterioration of the battery while standing on open circuit. It should be kept in mind that the length of time the cells were in service during the intermittent tests were respectively 12 and 40 times the period of actual discharge.

Proper Arrangement of Cells for Best Results. — The data given in the above tables may be used as a basis for the determination of the number and proper arrangement of cells for a given service. For example, if a certain piece of apparatus has a resistance of 8 ohms and cannot be operated at a voltage under 0.8, then 143 hours of service can be obtained from a single battery. If however 4 cells in parallel are used, the drain on each cell will be one-fourth as great as before, which is equivalent to discharging each cell through 32 ohms. Therefore the 4 cells will give 1078 hours of service, or 270 hours per cell. If the 4 cells were connected in series, then drain on each would be 4 times as great as for a single cell, which is equivalent to discharging each cell through 2 ohms, but each cell could discharge to 0.2 volt instead of 0.8. The 4 cells in series would then give 160 hours, or only 40 hours per cell. The parallel arrangement is therefore the best in this case.

For ignition service 6 cells in series are usually required in order to obtain a sufficiently high voltage. On heavy service of this kind two or more such series groups connected in parallel are usually more economical than a single group. For telephone service 3 cells in series are usually required in order to obtain a sufficiently high voltage. For such light service a single group of cells is more advantageous than two or more groups in parallel; any gain which might at first sight be effected is more than offset by the deterioration caused by local action on standing. This may be seen by a consideration of the data on intermittent service given above.

Tests of Dry Batteries. — The proper testing of dry batteries has been the subject of considerable discussion during the last few years. See *Trans. Am. Electrochem. Soc.*, 1909 to date. A preliminary report was submitted by a committee of this society in April, 1912. The following is a brief summary.

In measuring the terminal voltage of a dry battery a voltmeter having a fairly high resistance, 300 ohms or more, should be used. The ammeter and leads for measuring the short-circuit current should have a combined resistance of 0.01 ohm. The internal resistance test is not recommended.

Service Tests. — Dry batteries to be used in telephone work should be tested by discharging 3 cells, connected in series, through 20 ohms resistance for a period of 2 minutes, each hour, during 24 hours per day and 7 days per week, until the closed-circuit voltage of the battery at the end of a period of contact falls to 2.8 volts (0.93 volt per cell). Report the results as the number of days during which the closed-circuit voltage remains above the limiting value of 2.8 volts. This test has been modified and supplanted by the so-called A.T. and T. test, which is as follows: three cells connected in series are discharged through 20 ohms for 10 periods of 4 minutes each in 10 consecutive hours of 6 days per week. On the seventh day every other period is omitted. The end of the test is taken at 2.8 volts for the battery on closed circuit. The results are expressed as the number of days the test lasted.

For ignition service discharge 6 cells connected in series through 16 ohms resistance for 2 periods of 1 hour each per day, seven days per week, the periods being 11 hours apart. Determine at the end of every 12th period of closure the current which the 6 cells are capable of sending through a 0.5 ohm coil in series with an ammeter, the two being in parallel with the 16-ohm coil. The test is considered completed when the current through the 0.5 ohm coil at the end of a period of closure falls below 4 amperes. Report the results as the number of hours of actual discharge to this limiting value of the current.

For flash-light service discharge the battery through a resistance of 4 ohms for every cell in series, for a period of 5 minutes, once each day, until the closed circuit voltage at the end of a discharge period falls to 0.75 volt per cell in series. Report the result as the number of minutes of actual discharge until the voltage reaches this limiting value. At the present time (1921) the end point is gen-

erally taken as 0.50 volt per cell instead of 0.75 volt, because the modern lamps are usable to a lower voltage.

Full details for making these tests, together with a description of proper automatic arrangements for opening and closing the circuit at the designated times, will be found in the *Trans. Am. Electrochem. Soc.*, 1912, Vol. 21, p. 282. The timing device is an ordinary cheap clock provided with suitable contacts by means of which the hand closes the circuit.

Shelf-life Test. — Keep the cells open-circuited in a dry room at normal temperature. Determine the short-circuit current at the end of every 8 weeks. Report results as the number of months before the short-circuit current falls below 10 amperes.

SPECIFICATIONS FOR DRY BATTERIES. — See *Circular No. 79, Bureau of Standards*, April 25, 1919.

COST OF DRY BATTERIES (Pre-war prices). — Dry batteries of standard size (2.5 by 6 inches) cost at retail from 10 to 25 cents, depending upon the quality of the battery. When bought by the barrel, good dry batteries may be had for 10 cents apiece.

BIBLIOGRAPHY. — Cooper, W. R., *Primary Batteries*, Ayrton & Mather, *Practical Electricity*. For recent data on dry batteries see the *Trans. Am. Electrochem. Soc.*, particularly Ordway, D. L., *Some Characteristics of the Modern Dry Cell*, 1910, Vol. 17, p. 341, *Report of Committee on Dry Cell Tests*, 1912, Vol. 21, p. 275 (which contains references to numerous other articles). Helfrecht, A. J., *Depreciation in Small Dry Cells with Age*, Trans. Am. Electrochem. Soc., Sept. 26, 1919; *Electrical Characteristics and Testing of Dry Cells*, Bur. Stand., Circ. No. 79.

BATTERIES, STORAGE, ALKALINE TYPE. — (*See also Automobiles, Electric; Batteries, Storage, Applications of; Batteries, Storage, Lead Type.* The alkaline type of storage battery consists in its best-known form, i.e., the Edison battery, of an iron-nickel element immersed in dilute caustic-potash solution. The alkaline type of storage battery was first exploited commercially by Edison in 1904. At the present time (1921) it is being used extensively for the propulsion of electric vehicles, the operation of railroad block signals, the electric lighting of trains and in radio work.

THEORY OF ALKALINE STORAGE BATTERY. — Numerous active elements have been used in the alkaline type of storage battery. The elements in the Edison type of alkaline battery consist of nickel hydroxide for the active material of the positive plate, iron for the active material of the negative plate and a solution of potassium hydrate and lithium hydrate for the electrolyte.

The fundamental principle of the Edison storage battery is the oxidation and reduction of metals in an electrolyte which neither combines with nor dissolves either the metals or their oxides. Also, an electrolyte which, notwithstanding its decomposition by the action of the battery, is immediately re-formed in equal quantity, and is, therefore, a practically constant element without change of density or conductivity over long periods of time. Therefore, only a small quantity of such electrolyte is necessary, permitting a very close proximity of the plates. Furthermore, it is unnecessary to take hydrometer readings until about three hundred cycles of charge and discharge have been made; this is simply to determine when it is necessary to empty out the old solution and put in new. The active materials of the electrodes being insoluble in the electrolyte, no chemical deterioration takes place therefrom.

The chemical reactions in charging the Edison storage battery are: (1) the oxidation from a lower to a higher oxide of nickel in the positive plate, and (2) the reduction from ferrous oxide to metallic iron in the negative plate. The oxidation and reduction are performed by the oxygen and hydrogen set free at the respective poles by the electrolytic decomposition of water during the charge. The charging of the positive plate is, therefore, simply a process of increasing the proportion of oxygen to nickel. The proportions of nickel to oxygen in definite oxides of nickel are as follows:

	Atomic	Proportions	by Weight	
	Ni	O	Ni	O
NiO	1	1	1	.273
Ni_3O_4	1	1.33	1	.364
Ni_2O_3	1	1.5	1	.409
$Ni\ O_2$	1	2	1	.545

The relative amounts of oxygen necessary to oxidize nickelous oxide, or NiO, which is the oxide corresponding to the green nickel hydrate used in making the battery, to the various oxides are given in the three reactions:

$$(1)\quad 6\,NiO + 2\,O = 2\,Ni_3O_4$$
$$(2)\quad 6\,NiO + 3\,O = 3\,Ni_2O_3$$
$$(3)\quad 6\,NiO + 6\,O = 6\,NiO_2$$

The NiO_2 is capable of reacting with NiO according to the reaction

$$NiO_2 + NiO = Ni_2O_3.$$

Ni_3O_4 is considered as a combination of $NiO + Ni_2O_3 = Ni_3O_4$.

From a chemical standpoint a charged condition of the cell would, therefore, be represented in the positive plate by an atomic ratio of nickel to oxygen of at least 1 : 1.5 (or Ni_2O_3), depending on the charge. A discharged condition

would be represented by a ratio of 1 : 1.33 (Ni_3O_4) or lower, depending on the discharge.

The discharge of the cell is simply the reversal of the above reactions, the hydrogen reducing the higher oxides of nickel to lower oxides and the oxygen oxidizing the iron to ferrous oxide.

"Gassing." — In the alkaline type of cell, as in the lead type of cell, a certain amount of the water (H_2O) is broken up on charge by the electrolytic action of the charging current. The rate at which gas is evolved during a charge at the normal rate of current remains approximately constant during the first half of the charge, and increases rapidly during the latter portion of the charge. Of the total amount of gas evolved during a 7-hr. charge, about one-half is evolved during the first 5 hr. of the charge. As the oxygen and hydrogen thus liberated form an explosive mixture, provision should be made with a battery of any considerable size to carry the gases away to prevent their becoming a source of danger.

DESIGN. — The only alkaline type of battery in use at present (1921) in the United States is that made by the Edison Storage Battery Co. This battery is made up of a positive plate having as the active material a high nickel oxide, a negative plate having as the active material powdered iron, and an electrolyte consisting of dilute potassium hydrate solution.

Positive Plate. — The positive or nickel plate consists of perforated steel tubes, nickel plated, filled with alternate layers of nickel hydroxide and pure metallic nickel in thin flakes. The nickel is added to give the necessary conductivity to the active material.

The tube is formed from a perforated ribbon of steel, nickel plated, and has a spiral lapped seam. This tube, after being filled with active material, is reinforced with steel bands, which prevent the tube expanding away from and breaking contact with its contents. The tubes are flanged at both ends and held in contact with a steel supporting frame or grid made of cold-rolled steel, nickel plated.

Negative Plate. — The negative or iron plate consists of a grid of cold-rolled steel, nickel plated, holding a number of rectangular pockets filled with powdered iron oxide. These pockets are made up of very finely perforated steel, nickel plated. After the pockets are filled they are inserted in the grid and subjected to pressure between dies which corrugate the surface of the pockets and force them into contact with the grid.

After the plates have been prepared, as outlined above, the positive or nickel plate is further oxidized electrolytically and the nickel hydroxide converted to a high oxide of nickel, probably NiO_2. The negative or iron plate is reduced electrolytically, the powdered iron oxide being converted into metallic iron. See section above on *Theory* for an explanation of the chemical action.

Separation and Insulation. — After the plates are assembled into a complete element, narrow strips of hard rubber are inserted between the plates to insulate them from each other. The side insulator is provided with grooves and a step or projection at the bottom to insulate the complete element from the steel container. The step acts as a support to the element as well as insulating the plates from the bottom of the container. At the ends of the element, that is, between the outside negative plates and the container, are inserted smooth sheets of hard rubber.

Assembly. — The jar is made from cold-rolled sheet steel, nickel plated. The walls of the jar are corrugated to give the greatest amount of strength with minimum weight.

The cover is of sheet steel, nickel plated, provided with three mountings, two

being pockets for containing stuffing boxes about the terminal posts. The third mounting is an opening for filling the cell with electrolyte, and for the occasional (every four or five discharges) addition of distilled water to take the place of that which is lost during charge. This opening is provided with a hinged cap, on the under side of which is loosely hung a spherical segment of hard rubber. The latter, when the cap is closed, seats upon the circular opening beneath and is lifted by escaping gas, permitting its free egress. It closes the cell however to the admission of foreign gases or other material from without. The cap is opened only for filling and is held open or closed by a flat steel spring.

The plates are grouped on steel connector rods with steel spacing washers, all being attached to a steel terminal post. The terminal posts are insulated from the cover by means of hard and soft rubber washers and bushings.

The cells are assembled in wood trays, the positive and negative terminals of adjacent cells being connected together by copper connecting links. Each individual cell is insulated from the adjacent cell and from the containing wood tray.

RATING AND PERFORMANCE.—It is standard practice to rate an Edison battery in terms of the amperes it will give continuously for 5 hours. The rated capacity in ampere-hours is then 5 times this current rate. The normal rate of charge is the same as the normal rate of discharge and the time of normal charge is given as 7 hours. In the trade designations employed, e.g., A–6, the letter designates the size of the plate and the numeral the number of positive plates per cell, i.e., in the A–6 cell there are 6 positives; see below under *Dimensions, Weight and Cost.* The values of the rated current and output for each size of cell are given in the following table; the average voltage per cell during discharge is 1.2 volts. The G type is rated on a 3½ hour basis with a normal charging time of 4¾ hours. The normal current rate is 50 per cent higher than the A type of equivalent ampere-hour capacity.

RATED PERFORMANCE OF EDISON CELLS

Size of cell	Normal rate of charge and discharge, amperes	Rated capacity		Watt-hours per pound of cell for rated capacity	Average watts during discharge at normal rate
		Ampere-hours	Watt-hours		
B–2	7.5	37.5	48	12.2	9
B–4	15	75	90	13.0	18
B–6	22.5	112.5	135	13.7	27
A–4	30	150	180	13.3	36
A–5	37.5	187.5	225	13.4	45
A–6	45	225	270	14.1	54
A–8	60	300	360	13.1	72
A–10	75	375	450	13.2	90
A–12	90	450	540	13.2	108

The capacity of new cells increases for at least twenty cycles of charge and discharge. This betterment may be as much as 30 per cent above rating and comes from an improvement of conditions in the nickel electrode which is brought about by regular charging and discharging. It is expedited by overcharging. The capacity then decreases slowly with use until the electrolyte is renewed, which should be done before the capacity has fallen to the rated

value. The capacity again increases for a few charges and subsequently falls.

Voltage Characteristics. — The charge and discharge voltage characteristics of an Edison battery vary with the conditions, among which are temperature, condition of electrolyte, time since last discharge or charge, etc. Typical charge and discharge voltage curves at the normal rate are given in Fig. 1. It will be noted that the voltage rises rapidly at the beginning of charge, decreases somewhat during the second hour, and then rises gradually during the remainder of the charge. This hump in the voltage curve is characteristic of the Edison battery, and may lead to confusion as to battery's condition of charge. The value and duration of this hump may vary considerably even on successive charges of the same battery. The final charging voltage, with current on, is approximately 1.8 volts per cell, but ranges from 1.7 to 1.95 volts per cell. The rise in voltage near the end of charge is not sufficient to serve as a very satisfactory criterion of complete charge, but the Edison Storage Battery Co. recommends that if the extent of the previous discharge is unknown, the charge at the normal rate should be continued until the voltmeter reading has remained constant for 30 minutes at about 1.8 volts per cell.

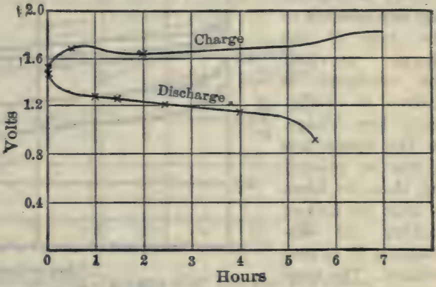

Fig. 1. Voltage Curves at Normal Rate

Effect of Temperature on Charging Voltage. — The normal average temperature of a battery on charge is between 90 and 100° F. The values of average and maximum charging voltage are higher for low temperatures of the electrolyte than for high temperatures. The range is indicated by the results of a series of tests given in the accompanying table.

Temperature, deg. Fahr.	Average voltage	Maximum voltage
35	1.88	1.94
55	1.81	1.92
75	1.76	1.88
95	1.70	1.84
115	1.67	1.77

Discharge Voltage. — On discharge the voltage performance of an Edison battery is more uniform than on charge, although during the early portion of the discharge the voltage is liable to vary considerably from the values indicated in Fig. 1. This depends chiefly on the length of time the cell has stood on open-circuit since the completion of the charge. The average discharge voltage is 1.2 volts per cell when discharged to 1.0 volt per cell (with current on) at the normal (5-hour) rate.

As with lead cells the open-circuit voltage is valueless as an indication of the state of charge. The open-circuit voltage in general will be about 20 per cent higher than the corresponding terminal voltage at normal rate of discharge.

Specific-gravity Characteristics. — The normal specific gravity of the electrolyte is about 1.200, and decreases slowly as the cell is used. (*See section below on Tests of Electrolyte.*) The specific gravity does not change materially on charge or discharge. A slight concentration has been noted on discharge, but the variation is too small to have commercial significance. The specific gravity decreases 0.002 with each 10° F. increase in temperature.

Variation of Capacity with Rate of Discharge. — The ampere-hour capacity of an Edison battery is practically independent of the rate of discharge, provided the discharge is carrried to a sufficiently low voltage. In this connection it may be pointed out that no harm is done an Edison battery by discharging it to zero voltage. The total ampere-hour outputs of a battery, when discharged to zero voltage at various current rates from one-third normal to four times normal, after a normal charge in each case, vary by only 2 per cent. As the low voltage part of a discharge cannot be considered useful, it is customary to consider the discharge completed for any rate of discharge when the voltage has reached a value differing from 0.9 volt per cell by an amount equal to the difference between the resistance drop in the cell corresponding to this rate and that corresponding to the resistance drop at the normal rate. Thus for a cell with an internal resistance (*for values see paragraph below on Internal Resistance*) of 0.003 ohm and a normal rate of 30 amperes, the terminating voltage for a discharge at 60 amperes would be 0.9 – 0.003 (60 – 30) = 0.81 volt. Fig. 2 shows a series of voltage curves for discharges at various rates following normal charges. The discharge has been discontinued in each case in accordance with the above rule. The variations of average voltage, ampere-hour output and time of discharge, with discharge rate are shown in Fig. 3, where values of these factors for the normal (5-hour) rate are each taken as 100 per cent.

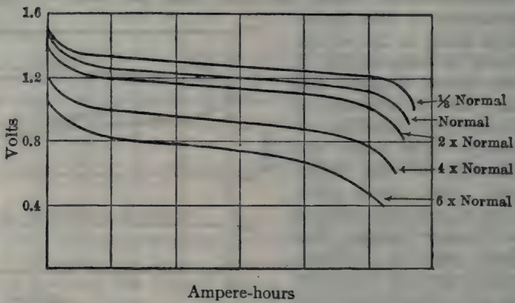

Fig. 2. Discharge at Various Rates

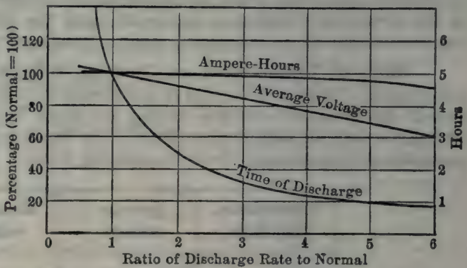

Fig. 3. Relation of Output Characteristics to Rate of Discharge

Variation of Capacity with Rate of Charge. — The energy contained in a newly charged Edison cell depends upon both the rate and length of charge. Fig. 4 gives the variation in the time of discharge at normal rate for various charge rates and periods for a particular battery. In this particular test when the cell was discharged at the normal 5-hour rate to 0.9 volt, after a 7-hour

charge at this same rate it gave this current for 5.9 instead of 5 hours. The curve for the normal rate of charge shows that the capacity for a 10-hour charge is 13.5 per cent and for a 14-hour charge 22.0 per cent greater than for a normal (7-hour) charge. The results of a series of charges of various lengths and subsequent discharges, both at the normal rate, are given in the accompanying table.

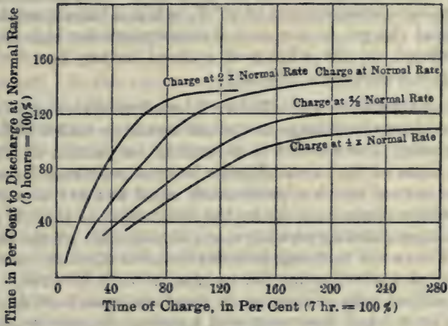

Fig. 4. Capacity at Normal Rate Discharge after Charging at Various Rates and Periods

Length of charge, hours	5.5	7.0	10.0
Length of discharge, hours	5.0	5.8	6.4
Average discharge voltage	1.19	1.202	1.203
Ampere-hour efficiency, per cent	91	83	64
Watt-hour efficiency, per cent	66	59	45

Variation of Efficiency with Length of Charge. — As shown by Fig. 4 the capacity of a cell depends upon the rate as well as upon the length of charge. For charge and discharge at the normal rate, the watt-hour efficiency is approximately 72 per cent for a short charge (i.e., one hour or less), it drops to about 58 per cent for a normal or 7-hour charge, and has smaller values for larger charges, being approximately 30 per cent for a 15-hour charge. The normal rate of charge has been chosen by the manufacturers such that for an output of approximately the rated number of ampere-hours at the normal rate the efficiency is higher for the normal rate of charge than for a rate which is considerably higher or lower than the normal.

Variation of Capacity with Temperature. — With the Edison battery, as with all alkaline batteries, there is a critical electrolyte temperature for each rate of discharge below which the capacity falls to a low value The higher the rate of discharge the higher is this critical temperature. If the electrolyte throughout the cell has been chilled below the critical point for a given rate of discharge, the full capacity can be obtained as soon as the cell is warmed above the critical range. A series of tests, reported by W. E. Holland (*Central Station, Nov., 1911*), showed that the temperature should be kept above 55° F. if a large current is to be required. In the operation of Edison batteries in electric automobiles in cold weather it is recommended that the batteries be given a warming charge shortly before the car is taken from the garage. See also next paragraph.

Rate of Cooling of Electrolyte. — The rate of cooling of the electrolyte depends upon the initial temperature of the battery, the circulation of air in

contact with the battery, the temperature of this air, and the rate at which current flows from the battery. Another series of tests reported by Holland (*Central Station, Nov., 1911*) showed that in an open box a battery which had been charged in a room temperature of approximately 65° F. could stand idle in still air at 8° F. for a period of about 4 hours before the electrolyte dropped to 55° F. When placed in a closed box it took about 13 hours for the battery to cool from an initial temperature of 90° F. (which a battery would have at the end of a normal charge) to the critical temperature for high discharge rates. These tests pointed out the efficacy of a closed battery box for keeping cells warm. (*See also Automobiles, Electric.*)

Internal Resistance. — The effective internal resistance of a cell determines the immediate change in voltage at the cell terminals with a sudden change in the discharge rate. For method of measuring see section on *Testing*, below. The value in ohms of the mean effective internal resistance for an A-type cell discharging at normal rate is approximately equal to 0.012 ohm divided by the number of positive plates as given by the designation of the cell size, and for a B-type cell is approximately equal to 0.024 ohm divided by the number of positive plates. For two or more cells in series the total effective internal resistance is the product of this value by the number of cells in series. The internal resistance of Edison cells is such as to cause an immediate drop in terminal voltage of between 7 and 8 per cent with a sudden increase in current equal to the normal rate. The virtual internal resistance increases slightly during the progress of a discharge.

Retention of Charge. — The rate of loss of charge of a battery during idleness varies with the temperature at which it stands. The loss is slight in the cold and becomes greater as the temperature increases. The variation for cells charged under normal conditions and discharged after resting for various periods in one case at normal temperature of 75° F., and in another case at 35° F. is shown in Fig. 5. Accordingly a cell which has not been used for 10 days following a normal charge may be expected to retain at least 85 per cent of its initial capacity. It will be noted that the *rate* of loss of capacity after the second day is practically the same at both temperatures.

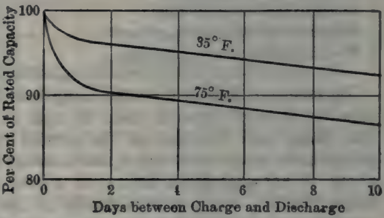

Fig. 5. Retention of Charge when Standing Idle

TESTING. — There are described below only those tests which are of interest in the commercial operation of the batteries. The methods of conducting several of the tests are the same as for the lead type of battery and are described more fully in the article on *Batteries, Storage, Lead Type*.

Tests of Electrolyte. — The normal specific gravity of the electrolyte at 80° F. is 1.200 as measured by hydrometer but may range from 1.160 to 1.230. Specific gravity is not necessarily a true indication of the suitability of the electrolyte, since harmful impurities may get into the electrolyte when watering or in other ways. The manufacturers recommend that the electrolyte be removed when the specific gravity has fallen to 1.160; this will usually be required after from 8 to 10 months of daily service. It is advisable to discharge the battery before renewing the electrolyte, and to give it a 12-hour charge, at the normal rate after renewing.

Test for Internal Resistance. — The internal resistance of a battery may be determined by momentarily opening the switch during discharge and noting the rise in voltage. This rise represents the *IR* drop in the battery due to its internal resistance and cell connections. Hence the value of the internal resistance at the given point of discharge will be the quotient of the volts rise divided by the current rate. This value divided by the number of cells in series will give the internal resistance per cell. Since on interrupting the current the voltage may rise for some time, due to change in the e.m.f., a better voltage reading may be obtained by momentarily reducing the current flow instead of opening the circuit. In this case the resistance is found by dividing the difference in voltage at the two rates by the change in current.

Capacity; Efficiency. — In testing for the capacity of an Edison battery at any given rate, the discharged battery should first be charged for 7 hours at the normal (5-hour) discharge rate. The battery is then discharged at the desired constant-current rate until the terminal voltage has reached the value as determined by the rule given in the paragraph above on *Variation of Capacity with Rate of Discharge.* The capacity in ampere-hours is the product of time of discharge in hours by the rate of discharge in amperes. The watt-hour efficiency of the battery is the ratio of the energy given out on the discharge to the energy put in on the charge. The term "volt efficiency" is used to express the relation of the average voltage of discharge to the average voltage of charge. For charge and discharge at the normal rate the value of the volt efficiency will be close to 72 per cent for any length of charge not extremely short or long. Thus 72 per cent is the limit of possible efficiency at the normal rate. The watt-hour efficiency on any normal rate test may be calculated accurately enough for all practical purposes by taking 72 per cent of the ampere-hour efficiency, i.e., by taking 72 per cent of the ratio of ampere-hours output to ampere-hours input.

Test Electrodes. — The best substance for a third electrode in analyzing voltage curves of alkaline batteries is a partially reduced oxide which will undergo reduction in the electrolyte without polarization, and which is insoluble in the electrolyte. Either cupric oxide or the high nickel oxide of a charged Edison positive fills these conditions for alkaline cells. For a discharge at normal rate the readings of the cell electrodes to a test electrode of cupric oxide are: iron, approximately 0.44 volt, nickel, 0.9 to 0.5 volt; for a test electrode of nickel oxide the readings are: iron, approximately 1.28 volts, nickel − 0.1 to − 0.4 volt.

INSTALLATION. — The A- and B-types of cells are usually assembled by the manufacturer in wooden trays holding from 2 to 12 cells each, according to requirements. There is an air space between cells for ventilation and insulation. The standard form of tray is made "bottomless," in order to prevent short-circuits between cells from the collection of dirt and moisture. If A-type cells are to be used where they will be pulled horizontally for filling, etc., trays with bottoms are used on account of the greater strength. Data on typical assemblies in bottomless trays are given in the following table:

Size of cell	B–2	A–4	A–6	A–12
Width of standard tray, in.	6 1/8	6 1/8	6 1/8	9
Over-all height (filler cap closed), in.	9 5/8	14 5/16	14 3/8	15 1/2
Length of trays, in.:				
2-cell tray	5	7 3/4	10	13 1/2
5-cell tray	10 3/4	17 1/8	22 3/4	32 1/4
8-cell tray	16 1/2	27 1/4	36 1/4	....
12-cell tray	24 3/8	40 1/8	....	....

For the extra high container which is sometimes furnished (designated by *H*) the over-all height for B-type is 2 in. greater and for A-type 3 in. greater than the figures given in the above table.

The over-all height with filler cap open is about 1½ in. greater than the height with filler cap closed.

If a battery is to be used for stationary service a clearance of 6 in. above cells should be allowed so as to permit the proper filling of the cells in place. In vehicles from which the battery is removed for filling the clearance need be only ¾ in.

OPERATION.—An Edison battery should never be operated in any manner except in accordance with the instructions received from the Edison Storage Battery Co. Disobeying these instructions may result in forfeiture of the guarantee with regard to the life of the cell. As the detailed instructions differ somewhat for various services, only general directions which apply to all services are noted below.

Charging. — The batteries are now usually shipped in a charged condition. The initial charge should continue for 12 hours at the normal rate. A similar overcharge should be given after 30 days of service, one after 60 days of service, and another after each renewal of electrolyte. The normal charging rates are given above in the section on *Rating and Performance;* the time of a normal charge is 7 hours. The boosting rate may be as high as desired provided the temperature does not exceed 115° F. in any of the cells. The best results are obtained when the temperature is kept between 75 and 95° F. Under average operating conditions the charge in ampere-hours necessary to replace a discharge is from 15 to 25 per cent greater than the discharge. The Edison Storage Battery Co. recommends that if an ampere-hour meter is used, it should be set to operate 20 per cent slow on charge.

Precautions as to Gases. — The battery should be well ventilated while charging, and no open flame or arcing contact should be allowed near the cells while charging or immediately afterward, as the evolved gases may be exploded.

Standing Idle. — An Edison battery may be allowed to stand idle in any state of discharge provided the level of the electrolyte is kept above the plates. After long idleness an overcharge may be required to bring the battery up to rated capacity.

Watering Cells.—The level of the electrolyte must be kept above the plates by adding distilled water from time to time. The frequency of adding water will depend upon the amount and rate of charging. The manufacturers of the battery supply an indicating filler which is of assistance in preventing slopping and over-filling.

Cleaning. — The trays and containers must be kept dry, and dirt or other foreign material must not be allowed to collect between or under cells. Dirt and dampness may cause leakage which may result in corrosion of the containers. The outside of the cans may be cleaned with a steam or air blast. If a container becomes leaky it should be returned to the manufacturer for repair.

DIMENSIONS, WEIGHT AND COST.—The Edison battery is made in two standard types, the plates of the A-type being approximately 4¾ by 9¼ in. and those of the B-type 4¾ by 4⅝ in. Plates of other dimensions can be obtained for special purposes. The sizes of cells are commonly designated by the type letter and a number which indicates the number of positive plates, as for instance, an A-6 cell has 6 positive and 7 negative plates. The several sizes of cells together with data on the dimensions and weight of each are given in the following table:

DIMENSIONS AND WEIGHTS OF EDISON CELLS

Size of cell	Over-all dimensions of cell, in inches			Weight in pounds	
	Length	Width	Height	Complete cell	Average per cell with trays and connectors
B-2	1.5	5.1	8.8	4.6	5.5
B-4	2.6	5.1	8.8	7.4	8.7
B-6	3.8	5.1	8.8	11.0	12.0
A-4	2.7	5.1	13.4	13.3	14.5
A-5	3.2	5.1	13.4	16.8	18.5
A-6	3.8	5.1	13.4	19.0	21.0
A-8	5.0	5.3	14.0	27.0	30.0
A-10	6.2	5.5	14.0	34.0	37.5
A-12	7.4	5.5	14.6	41.0	45.0

For services, such as the lighting of railroad cars, which require that the batteries work for long periods without the addition of water, the A-type plates are assembled in containers (designated by H) about 3 in. higher in each case than those indicated in the table, in order to obtain additional space above the plates for the electrolyte. The weight per cell is then about 15 per cent greater than the above.

Before the war the cost of Edison cells assembled in trays was approximately $0.92 per pound, including weight of trays and connectors. This is equivalent to a cost of approximately $410 per kilowatt at the normal (5-hour) rate of discharge or $105 per kilowatt at the one-hour rate of discharge. The manufacturers guarantee that these cells will show at least their rated capacity after being used a specified time, the usual guarantee period for A-type cells being at least 4 years and for B-type cells, at least 5 years.

BIBLIOGRAPHY.— Crocker and Arendt, *Storage Batteries;* Holland, W. E., *Effect of Low Temperature on the Alkaline Storage Battery,* Central Station, Nov., 1911; *The Edison Storage Battery, Its Conception and Method of Manufacture,* Com'l Car Jour., Jan., 1914; Turnock, L. C., *Preparation of the Active Materials and Electrolytes of Alkaline Accumulators,* Met. and Chem. Eng., 15, p. 259, Sept., 1916; Publications of the Edison Storage Battery Co., especially *The Edison Alkaline Storage Battery* (1919).

BATTERIES, STORAGE, APPLICATIONS OF. — (*See also Batteries, Storage, Alkaline Type; Batteries, Storage, Lead Type.*) The following list shows some of the more important applications of storage batteries; these are briefly treated in the following pages.

Central Stations:
- Emergency reserve — "Stand-by service."
- Load or voltage regulation.
- Taking peaks.
- Day load on small systems.
- Exciter reserve.
- Remote-control switch operation.

Isolated Plants:
- Mine hoists, steel mills and other heavy motor regulation; see Index.
- Carrying entire load during certain hours of light load.
- Load and voltage regulation in office buildings or hotels, where electric elevators are in service.
- Giving 24-hour service in residences.
- Operation of drawbridges.

Other uses are:
- Regulation of long feeders; see *Trolley Systems, Overhead.*
- Propulsion of pleasure cars, trucks, street cars, submarine boats, launches, industrial trucks and tractors, mine, and industrial locomotives, etc.
- Gas-engine ignition.
- Railway passenger-car lighting; see *Lighting of trains by Electricity.*
- Railway signaling, see *Signaling, Railway.*
- Telephone and telegraph (q.v.).
- Portable and small stationary lamps.
- Fire and burglar alarm (q.v.).
- Electroplating.
- Dental and other surgical work.
- Automobile starting, lighting and ignition; see *Ignition, Electric; Starting and Lighting Systems for Automobiles.*

Extent of Application in the United States. — The first application in America of storage batteries to central-station service was in 1886. In 1920, there were in service in the United States, for central-station "stand-by" work 193 storage batteries of the lead-lead acid type, having a combined capacity of approximately 177,000 kw-hr. at the one-hour rate of discharge. Most of these are of the Faure or Pasted type, some are of the Planté type, or a combination of Planté and Faure types.

There were in service in the United States in 1920 approximately 700,000 cells of the lead-lead acid type, operating telephones, lighting railway cars and operating signals.

At the end of 1920 there were in operation lead batteries for other uses approximately as follows:

For automobile, starting lighting and ignition 15,000,000 cells aggregating approximately 3,000,000 kw-hr.

For farm lighting plants 3,000,000 cells aggregating 1,000,000 kw-hr.

For electric passenger cars, street trucks, mine and industrial locomotives, industrial trucks and tractors, 1,166,000 cells aggregating 444,000 kw-hr.

STAND-BY BATTERIES IN CENTRAL LIGHTING STATIONS. — Formerly the storage battery in a central lighting station served a dual purpose: first, as a source of current in an emergency, and second, to take short peaks of load, thus cutting down the generating capacity in service and increasing the economy of plant operation. In certain cases the batteries are used in this manner to-day, but owing to the desire always to have the full battery reserve in case of emergency, the peak battery feature is being abandoned and the stand-by feature only is being retained.

This method is considered by some authorities to be even more conducive to economy of station operation than that in which the battery was also used on the peak, inasmuch as the presence of a fully charged battery behind the generating apparatus permits station operation at a much higher load factor than were the battery fully or partly discharged. Furthermore, as one of the important factors in the life of the battery is the amount of work it does, this life is greatly increased if the battery be used only for emergency service.

Connections for Battery and Its Auxiliaries, Stand-by Service. — Fig. 1 shows a general standard scheme for the operation of a battery on a 3-wire lighting system, either for peak work or stand-by emergency work. As will be noted in this scheme, the neutral of the battery is connected to the neutral of the system.

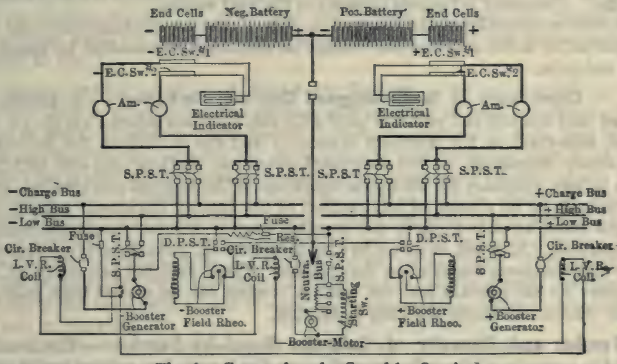

Fig. 1. Connections for Stand-by Service

Charging. — For stand-by emergency work it is standard practice to take the battery off the bus for charge and in most cases the battery is floated on only one bus. In this service the battery is kept fully charged and floating on the bus-bars continuously, except during charge. Except in case of an emergency discharge having been required from the battery, it will only be necessary to give the battery an occasional overcharge about once a week or once in two weeks in order to keep the active material in proper condition. Since this overcharge is given at the normal rate for only a few hours, once every week or every two weeks, a time for this overcharge can be selected when it will be safe to take the battery off the bus. It will also be noted that on charge, sufficient voltage is obtained for charging the complete battery by adding to the bus voltage the voltage of a simple charging booster. It is necessary with the 3-wire system to have two generators, one on each side of the system, in order to be able to charge the two halves of the battery at different rates in case the battery has become unbalanced on discharge. The diagram shows a simple shunt motor driving these two booster generators.

End-Cells and Switches for Stand-by Service. — The connections between the two sides of the battery and the outside bus-bars of the system are made through "end-cell switches," by means of which the number of cells on the system can be varied at will within certain limits. This scheme shows two end-cell switches in parallel on each side of the system, this number of switches being installed in this case in order to provide sufficient capacity for the discharge.

The number of end-cell switches necessary on each side of the system depends upon the current to be carried, upon the number of bus-bar voltages to be maintained from the battery simultaneously, and upon whether or not it is desired to keep the battery on the bus during charge. It is frequently required to float the battery on more than one bus. In such cases there must be one end-cell switch on each side of the system for each bus. If it is not desired to keep the battery on the bus during charge, and if the battery is to float on only one bus, one end-cell switch on each side of the system will be sufficient, provided the one end-cell switch has sufficient current-carrying capacity.

If, however, it is deemed advisable to keep the battery on the bus at all times, it will usually be necessary to have at least two end-cell switches on each side of the system and the number of end cells required will be greater than the number of end cells required where the battery can be taken off the bus for charge. This point is referred to again in the next section.

It will be noted from the diagram in Fig. 1 that on discharge the cells are connected in series and that the voltage on discharge is controlled by means of the end cells.

Importance of Well-designed End-cell Switches. — It should be noted that the end-cell switches used with this standard scheme are by far the most important of the auxiliary apparatus and the efficacy of the battery is absolutely dependent on the successful operation of these end-cell switches. The factors necessary for successful operation of the end-cell switches are, first, there must be no chance of interruption of the battery discharge; second, the switch must be able to carry the required current without dangerous heating or sparking. End-cell switches having a continuous current-carrying capacity of 10,000 amperes are now on the market. These switches can carry 40,000 amperes for six minutes and the contact brushes can be moved from point to point without injurious sparking or heating.

Number and Capacity of Cells for Stand-by Service. — The number and the capacity of the cells depends upon the lowest bus voltage maintained under normal operating conditions, the minimum allowable voltage at the end of discharge in case of emergency, and the load required to be carried during emergency.

If the battery can be taken off the bus for charge, the number of cells in the main battery will be equal to the lowest bus voltage (between outside bus bars) divided by 2.1. The number of cells in the entire battery will be equal to the lowest allowable voltage in emergency divided by the final voltage per cell at the rate of discharge required by the emergency. The difference between the number of cells in the main battery and the total number of cells will, of course, be the number of end cells. When the total number of cells in the battery is determined it will become necessary to put half the number of cells, including half the end cells, on each side of the system.

If it is deemed necessary to keep the battery on the bus at all times during charge and discharge, the total number of cells will be determined as indicated above, but the number of cells in the main battery will be equal to the bus voltage divided by 2.7, since this is the average cell voltage at the end of charge. In this case, at least two end-cell switches on each side of the system will be

required. On charge one end-cell switch on each side of the system is connected to the main bus and is set at such a position that the total voltage of the cells in circuit during charge equals the bus voltage. The other end-cell switch is set so as to include the cells being charged. That is, at the start of charge, the circuit from the charging booster to one end-cell switch will include all of the cells, and the circuit from the lighting bus to the other end-cell switch will only include sufficient cells to give the lighting-bus voltage. As the charge proceeds, the end-cell switch in the charging circuit will be shifted to cut out end cells as their charge is completed, and the end-cell switch connecting the battery with the lighting bus will be shifted to cut out cells as the voltage of the battery comes up, so that the voltage of the cells in circuit on the lighting bus can be held at the lighting-bus voltage.

It is seen that with this operation the end-cells included between the two end-cell switches will have to carry the charging current plus or minus any current transferred between the battery and the lighting bus. It is thus apparent that with this method of operation, unless especial care is taken to float the battery at the true lighting-bus voltage, the end cells will be worked unequally. It is also seen that, if it is desired to keep the battery on the bus at all times, the amount of end-cell copper is considerably increased and the cost of the battery installation is proportionately increased, for the number of end-cells required is greater than the number of end cells required when the battery can be taken off the bus for charge. On account of this increase in cost and the possibilities of unequal work on the end cells, it is desirable to install the simpler scheme shown in Fig. 1.

Determination of Size of Stand-by Battery. — As an example of the method used in determining the number and capacity of cells required for this class of work, suppose it were desired to install a battery to take care of an emergency of 20,000 amperes for 20 minutes at a normal bus voltage of 230 volts with a minimum allowable voltage during emergency of 225 volts. The 20-minute rate of discharge is 8 times the normal rate, as shown in Figs. 3 and 6 in the article on *Batteries, Storage, Lead Type*, and it will be noted from Fig. 6 that the final voltage per cell at the end of this rate is 1.5 volts. The total number of cells will be 225 divided by 1.5, or 150 cells. If it is desired to use standard practice, taking the battery off the bus for charge, the number of cells in the main battery will be equal to the normal bus voltage, 230, divided by 2.1, or 110 cells.

It will, therefore, be necessary, in this case, to install a battery having 150 cells, including 40 end cells, each cell having a capacity of 20,000 amperes for 20 minutes. This battery will be installed with 75 cells, including 20 end cells, on each side of the neutral. In standard stand-by practice the 20 end cells on each side of the neutral will be connected to the end-cell switches, with two cells per switchpoint for a few of the cells and with 3 or 4 cells per switchpoint for the remainder of the end cells. A certain number of two-cell connections will be required in order to take care of the variation in the bus voltage, as it may be necessary to cut in and out cells while the battery is floating. The rest of the equipment necessary should be in accordance with the diagram shown in Fig. 1.

LARGE REGULATING BATTERIES FOR RAILWAY WORK, ETC. — Often the load (particularly a railway load) on the power house is subject to frequent and violent fluctuations, causing severe strains on the generating apparatus in service and in many cases requiring the operation of a greater number of units than would be necessary to furnish a steady average load. This not only results in a lower load factor and consequent loss of economy, but it reduces the surplus capacity of the generating station by the extra

amount of apparatus required in operation. Another loss in economy is due to the rapid changes in the cut-off point of reciprocating engines, due to the violent fluctuations of load. Fig. 2 shows a simplified wiring diagram of the scheme generally used.

Carbon Regulator. — The carbon regulator shown in Fig. 2 consists of piles of carbon disks arranged in two groups on opposite sides of the fulcrum of a lever, the lever being subjected at one end to the tension of a spring and at the other end to the pull of a coil connected in series with the main bus-bar. Variations in the force applied to opposite ends of this lever will increase the pressure on one group of carbon disks and decrease the pressure on the other, thus varying their relative resistances. These piles are connected in series across the battery terminals, and the field winding of the booster exciter is connected between the middle point of the battery and a point in the circuit between the two groups of carbon disks. Variation in the resistances of these two piles, produced by variation of the pressure of the lever, will cause current to flow in one direction or the other through the field of the exciter, thus controlling the operation of the booster and compelling the battery to charge and discharge with small variations of current in the series coil.

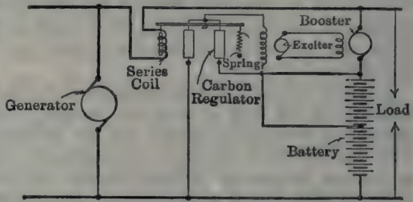

Fig. 2. Connections for Regulating Service

In this manner the load on the generator can be maintained constant within a few per cent on either side of the average value for which the apparatus has been adjusted. The load on the generator is determined by the tension of the spring, which may be adjusted to balance any average current in the series coil. If the average load on the line should exceed for a considerable period of time the generator load for which the spring is adjusted, the battery will be subjected to a sustained discharge which may in time exhaust its available capacity. If, on the other hand, the average load on the line falls off for a considerable period of time, the battery will be subjected to a sustained charge which may produce excessive and unnecessary overcharge. To follow these variations in average load, it is necessary to adjust the spring from time to time.

Automatic "Average" Adjustor. — In order to eliminate the necessity for such frequent adjustment by hand in cases where the average load is continually changing and where the generator is capable of handling these comparatively slow changes, the automatic average adjustor has been designed. This piece of apparatus consists of a small motor connected by a reducing gear to the drum which controls the tension of the carbon regulator spring. The armature of this motor is connected across the main bus-bars through a fixed resistance which maintains a practically constant current through the armature. The field of the motor is connected across the terminals of the booster.

Whenever the booster voltage is in the direction to discharge the battery, the motor field is energized and the motor revolves in the direction to increase the tension of the spring, thus gradually increasing the load on the generator and relieving the battery of its discharge until the booster voltage comes back to zero. If, on the other hand, the booster voltage is in the direction to charge the battery, the motor of the average adjustor will operate in the opposite direction, gradually relieving the tension on the spring and reducing the average load on the generator until the booster voltage is again brought back to zero

and the battery is again restored to its floating condition. This transfer of load between the battery and generator is brought about slowly so that the battery is still free to relieve the generator of all quick fluctuations of load. The battery, however, is relieved from a considerable amount of sustained peak charge and discharge.

Load-limiting Device. — To prevent the average adjustor from throwing load on the generator beyond its capacity, a load-limiting device has been designed. This consists of an electrolytic valve in series with the field of the motor. Ordinarily this valve is short circuited by a switch. When the drum which regulates the tension of the spring has traveled to a certain point corresponding to the maximum permissible load on the generator, a projection mounted on the shaft of this drum engages with the switch and opens it, whereupon the valve prevents current from passing through the motor field in a direction to cause a further increase in the spring tension. The valve will, however, permit the current to flow in the opposite direction so that as soon as the load has decreased sufficiently to cause the battery to charge, the motor will operate so as to relieve the tension on the spring and at the same time permit the switch to close, thus restoring the original adjustment.

Automatic Current Stop. — While the capacity of a storage battery to withstand an excessive momentary overload without injury is practically unlimited, the booster must be protected by an automatic circuit breaker. Whenever the setting of this circuit breaker is exceeded on heavy overload and the breaker opens, the entire maximum load is thrown on the generator. This usually results in opening the generator circuit breaker also and interrupting the supply of current to the line. To avoid this and permit the battery to furnish the maximum current which the booster can handle without danger of overloading the latter or opening the circuit, there has been designed an automatic current stop. This stop is arranged to automatically prevent the battery from discharging above a certain predetermined value, for which this stop has been adjusted. All load demand above this predetermined value is thrown upon the generator. The generator is thus subjected only to the excess of load above the maximum capacity of the battery booster, instead of having the entire load of the line suddenly thrown on it when the battery circuit breaker opens. The adjustment of this automatic current stop can be changed within certain limits to meet the requirements.

Batteries for Controlling Alternating-current Loads. — For controlling fluctuations of an alternating-current load a solenoid which is responsive to the energy component of the alternating current is substituted for the series coil shown in Fig. 2. The general operation of this scheme is the same as described above, except that it is, of course, necessary to discharge the battery on to the alternating-current bus through suitable alternating-current-direct-current transforming apparatus. There are in service at present several battery installations which regulate fluctuating alternating-current loads. Most of these installations are based on the carbon regulator principle, as outlined above.

Number of Cells and Capacity of Cells and Booster. — The number and capacity of cells required for regulating work depends upon the bus voltage and upon the nature of the fluctuating load. It is also frequently desirable to utilize a regulating battery for emergency and peak work, and the extent of the emergency and peak requirements will also be a factor in fixing the capacity of the battery. The number of cells installed will be equal to the bus voltage divided by 2.1, since this is the floating voltage per cell. A reference to Fig. 2 shows that during the regulation of load fluctuations all of the battery discharge passes through the armature of the booster generator. It is thus seen that it is necessary for the booster to generate sufficient voltage to make up for the drop

in battery voltage on discharge. This drop in battery voltage depends upon the capacity of the battery, and the amount and duration of the discharge. Before the booster can be properly designed it will be necessary to have complete data on these points.

Emergency Work. — For emergency work, where the battery is carrying the entire load without the assistance of any generating apparatus, it may or may not be necessary to keep the booster in circuit, depending upon whether or not it is desired to maintain a constant bus voltage. If it is not desired to maintain a constant bus voltage, a saving can be effected by cutting the booster out of circuit and allowing the battery to discharge directly on to the system.

Peak Work.— For peak work, where the battery must work in parallel with the generators, it will either be necessary to keep the booster in circuit in order to maintain the same voltage as the generator or else to give the generators a drooping characteristic so that the battery can discharge in parallel with the generators without the aid of the booster.

Current Capacity. — If it is desired to float the battery on the system at all times in order to regulate load fluctuations, it will be possible to float the battery about 75 per cent fully charged. If this battery must have a definite capacity always available for peak or emergency work, it is seen that the battery must be of sufficient capacity, when 75 per cent fully charged, to deliver the load called for by the emergency or peak conditions.

Determination of Size of Regulating Battery. — In all cases where a battery is to be considered for any one of the above conditions or any combination of them, it is recommended that the complete data summarized below be sent to the battery manufacturer with a request for a complete report covering the battery and accessories recommended to suit the conditions. The data necessary are as follows:

Number, type and capacity of boilers, if the plant is a steam plant.
Schedule of boiler operation, if the plant is a steam plant.
Number, type and rating of generators.
Schedule of generator operation.
Bus voltage maintained.
If the generators are of the alternating-current type, give frequency, number of phases, and the number and rating of any static transformers used.
If the station in question is a substation, describe the complete substation equipment along these same lines.
Give complete 24-hour load curve showing half-hour readings of the total load on the station bus bars. A graphic recording meter chart will be sufficient, if such an instrument is available.
If possible, obtain five-second readings on the total indicating ammeter or wattmeter, over a period of one-half hour during the time of maximum fluctuations.
If the battery is required to assist the generators on the peak, give the amount of load in kilowatts required from the battery and the length of time this load is required. It would be well to have an exact load curve showing the shape of this peak, if it is possible to obtain such a record.
If the battery is to be used for emergency, give the amount of the load and the time over which this emergency extends. In all cases state the exact nature of the load required, that is, whether it is a motor load, a lighting load, etc.

BATTERIES FOR REMOTE-CONTROL SWITCH OPERATION. — It is standard practice in central stations of any magnitude to install a storage battery of sufficient capacity to give ample insurance against interruption of

the switch-control circuit. It is the usual practice to utilize this battery, not only for the operation of the remote-control switches, but for the operation of the signal lamps and a sufficient number of emergency lamps throughout the station. In some stations the remote-control switches are operated from the exciter bus bars, but it is better practice to operate these switches together with their signal lamps and the emergency lamps from a separate source of power.

Control Battery Connections. — Fig. 3 shows a scheme which is more or less standard with the larger central-station companies. This figure shows a motor-generator set consisting of an induction motor driving a direct-current generator whose voltage can be varied from 110 volts up to 168 volts, the latter voltage being required to completely charge the 62 cells in series.

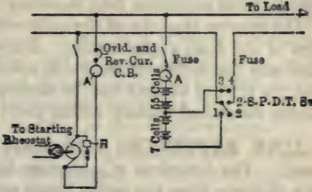

Fig. 3. Connections for Control Battery

Under normal operation the battery is floated on the 130-volt bus with the motor-generator set, the battery switches being thrown into positions 1 and 2. It will be noted that with this arrangement all 62 of the cells are floating on the bus. When it becomes necessary to charge the battery, the battery switches are thrown into positions 1 and 4, and the generator voltage is raised sufficiently to start the charge of the entire 62 cells. With this arrangement the voltage at the load bus bars will be the voltage of 55 cells. At the finish of charge the battery switches are thrown into positions 3 and 4. It will be noted that in these positions the group of 7 end cells is out of circuit, as they have received their complete charge, and the motor-generator set will be furnishing power to charge the 55 cells and to carry the load.

Size of Battery. — With this arrangement the voltage at the bus bars may vary from 145 volts to the minimum voltage of the battery on discharge. The number of cells for this scheme can be varied to suit the voltage conditions. The capacity of the battery is fixed by the maximum current demand, the total ampere-hour current demand over the period of estimated possible shutdown, and the minimum allowable voltage on the load bus bars. With these conditions known, it will be possible with the aid of trade catalogues showing the charge and discharge curves of the battery and the battery capacity at various rates, to select a battery to suit the conditions.

Control Battery on Exciter Bus. — If the oil-switch load is furnished from the exciter bus it will be necessary to float the emergency battery on the exciter bus, or to supply automatic means for throwing the battery on the exciter bus when the voltage of the exciter bus drops below a certain predetermined value. If this method is used, unless the battery is of sufficient capacity to carry the entire exciter load, it will be necessary to place a reverse-current circuit breaker between the battery and the exciter load, so that when the exciter bus "goes dead" the battery will be automatically cut off from the exciter bus and left on the remote-control switch bus.

EXCITER BATTERIES. — It is standard practice with the larger central-station companies to install a battery of sufficient capacity to carry the total maximum exciter load for from one hour to two hours continuously. The number of cells will depend upon the voltage of the exciter bus, which is usually 110 but is sometimes as high as 250 volts.

If it is desired to maintain constant the voltage on discharge, it will be necessary to utilize end cells by means of an end-cell switch. In this case the number of cells in the main battery will be equal to the bus voltage divided by 2.1. and

the total number of cells in the battery will be equal to the bus voltage divided by the voltage per cell at the end of discharge for the discharge rate used. The difference between the number of cells in the main battery and the number of cells in the whole battery will be the number of end cells. These end cells can be connected to the points on the end-cell switch singly or in pairs, depending upon the voltage regulation desired. If it is not desired to maintain a constant voltage on an exciter bus, the end cells will not be required. The total number of cells will be kept on the bus bars and as the voltage of the battery falls the field excitation of the generators can be maintained constant by changing the generator field rheostats.

The ampere capacity of the cells will be determined by the amount and duration of the load to be carried. Knowing the ampere capacity, the length of time and the minimum allowable voltage during the period, the number of cells and the type of battery can be determined by the use of the charge and discharge curve and battery rating given in trade catalogues.

LINE BATTERIES FOR RAILWAY WORK. — (*See also Trolley Systems.*) The functions of a storage battery floating on an electric railway line at a distance from the power house are as follows:

1. To improve the line voltage and displace a certain amount of feed wire.
2. To relieve the power house of fluctuations of load.
3. To keep the cars moving when the power supply is temporarily interrupted.
4. To supply power for the operation of a few cars or lights at night when the power house is shut down.

Location. — Before any calculations can be made, the location of the battery must be decided. Where there are no other determining factors, the battery would ordinarily be located about three-quarters of the distance from the power house to the end of the line, or, if the line is fed from both ends, midway between the feeding points. In some cases the location of the maximum momentary load, due, for example, to a severe grade or to the passing of two cars at a siding, will influence the location of the battery. In other cases the location of a suitable piece of property owned by the company, or the location of a car barn where attendance would be available, may be the determining factor.

Line Battery without Boosted Feeder. — In this class of line-battery installation the number of cells is determined by the average line voltage at the proposed battery site on the basis of approximately 2.1 volts per cell. Thus, if the average voltage at the battery site, as determined by voltage readings taken at frequent intervals, such as five or ten seconds, over a considerable length of time, should be found to be 500 volts, the number of cells suitable for this average voltage would be 238. If it is found that, owing to changes in load conditions at different hours of the day or on different days of the week, the average voltage varies considerably for prolonged periods of time, it may be necessary to provide means for changing the number of cells connected to the line. This may be accomplished by separating a group of cells at one end of the battery, and arranging suitable switches so that these cells may be connected either in series with the main battery when the average voltage is high or in parallel with an equal number of cells of the main battery when the average voltage is low It is found preferable to connect these end cells in parallel with a portion of the main battery, and thus keep them active rather than disconnect them from the system entirely.

Calculation of Size of Battery Required. — See article on *Trolley Systems, Overhead*, and *Bulletin No. 134* of The Electric Storage Battery Co.; also Chap. 44 of Lyndon's *Storage Battery Engineering*.

BATTERIES FOR SMALL CENTRAL STATIONS AND ISOLATED PLANTS. — The objects of a battery installation in small electric lighting and power plants may be any one or all of the following:

1. Twenty-four-hour service with but a few hours daily operation of engine.
2. Reduction in size of engine and dynamo otherwise required.
3. Improved voltage regulation.
4. A source of current during breakdown of machinery, or while repairs are being made.

Twenty-four-hour Service. — In this service the battery supplies the current during the hours when the engine is not running. As the capacity of a battery of normal size in many instances in this class of work, especially in residential service, is not generally exhausted in daily operation, it may usually be charged by running the engine and dynamo only two or three hours each day at the most convenient time. In summer, when the lighting hours are shorter, it may be sufficient to charge only once in two or three days.

Reduction in Size of Generator. — If the plant is so designed that the combined capacities of the dynamo and the battery are equal to the maximum load requirements, a smaller engine and dynamo may be installed than would be required if the dynamo alone had to carry the total maximum load. The battery will then be discharged in parallel with the dynamo, to assist it by taking a part of the total load on special occasions, as when the maximum number of lights is in use.

Improved Regulation. — Objectionable fluctuations of voltage and consequent flickering of lights, due, for example, to the operation of an electric elevator or other motor work, may be eliminated by floating the battery across the lighting bus.

Emergency. — In case of temporary derangement of the engine or dynamo, which may occur when the lighting service is most important, or in case repairs to the machinery are required, a battery will serve to tide over the break and avoid any interruption in the current supply.

Small Regulating Battery with Counter Cells. — Fig. 4 shows an ideal arrangement for small battery lighting plants where the capacity of the battery does not greatly exceed 300 ampere-hours at the normal rate. This figure shows 62 cells in the main battery with 8 counter cells, the whole being designed to operate on a 110-volt bus-bar. It will be noted that the battery is charged with two halves in parallel. During charge each half is placed across the bus-bars in series with a fixed resistance. At the beginning of charge the counter cells are all in circuit and assist the fixed resistances in reducing the bus voltage to the proper charging voltage. The counter cells are cut out of circuit as the battery voltage rises. On discharge, all of the cells in the main battery and the full number of counter cells are connected in series. As the voltage falls on discharge the counter cells are cut out of circuit, thus maintaining a steady voltage at the bus-bars. The number of cells and counter

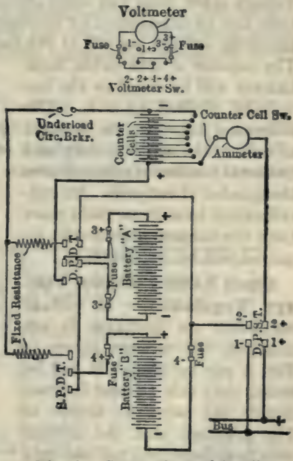

Fig. 4. Connections of Small Regulating Battery

cells shown in the figure are selected to maintain a steady voltage of 110 volts at the normal rate of discharge. The counter cells are composed of unformed battery plates or grids, and since they have very little active material they have practically no capacity. Each counter cell will, however, furnish an opposing voltage of approximately 2.3 volts; this counter voltage is practically constant over the range in current demand for an ordinary small station.

Small Regulating Battery with Charging Booster. — For larger-sized plants this method of operation is not economical owing to the loss in the charging rheostats and the loss in the counter cells. For a straight lighting plant where a battery of more than 400 ampere-hours capacity is required, it is usual to use the scheme shown in Fig. 5. It will be noted in this scheme that the cells in series are connected to the bus for discharge, the voltage being controlled by cutting in and out the end cells at one end of the series. On charge the required voltage to complete the charge is obtained by adding to the bus-bar voltage the voltage of a small booster generator.

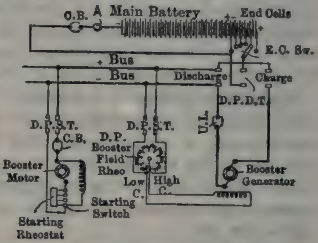

Fig. 5. Connections of Small Battery with Booster

Size of Battery. — For ordinary small plants which operate at a bus voltage of 110 the standard equipment consists of 64 cells, including 12 end cells. The capacity and type of the cells are fixed by the ampere-hour requirements and the minimum-voltage requirements on discharge. For any bus voltage other than 110 volts the complete number of cells, including the end cells, can be determined by dividing the minimum allowable bus voltage by the minimum voltage per cell at the end of discharge at the rate which will be used. The number of cells in the main battery will be equal to the bus voltage divided by 2.1. The difference between the number of cells in the main battery and the total number of cells will give the number of end cells. An end-cell switch having one point more than the number of end-cells required will be necessary.

Small Battery without Voltage Regulation. — If voltage regulation on discharge is not required, the scheme shown in Fig. 6 can be used. It will be noted that this scheme is practically the same as shown in Fig. 4 except that the counter cells have been omitted and only 56 cells are used.

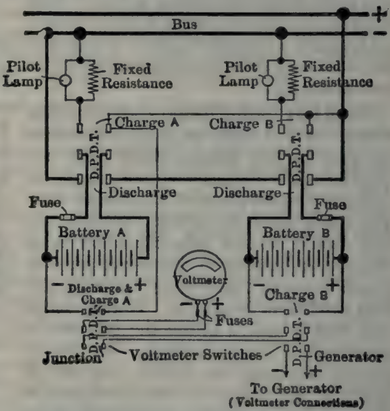

Fig. 6. Connections of Small Battery without Booster Cells

Small Battery on 3-Wire System. — In small central stations or isolated plants which utilize the 3-wire 110 to 220-volt system, the battery is arranged in two halves, one on each side of the neutral. A group of end cells on each side of the system and two booster generators direct con-

nected to one motor are used. This scheme of operation is shown in general in Fig. 1 which is described in detail above in the section on *Stand-by Batteries*.

Small Battery for Low-voltage Lighting — So-called Farm Lighting Plants. — Since the advent of the tungsten lamp 32-volt plants are growing very popular.

Numerous manufacturing concerns have gone extensively into this business and there are several concerns to-day marketing these plants by the thousands. One concern in America during 1920 marketed considerably more than 50,000 complete farm lighting plants.

All of these plants consist of a gas engine driving a 32–42-volt generator, a switchboard and a storage battery. The capacity of the low-voltage farm lighting plant varies from ½ kw. to 5 kw. Usually where the capacity required exceeds 5 kw., especially when the distance of transmission exceeds a few hundred feet, it is better to go to 110 volts, as the low-voltage plant is designed primarily for small loads and short distances of transmission.

There are various schemes of operation which are now successfully used, but they all include a storage battery. These schemes vary all the way from the one extreme, where only sufficient storage battery is installed to automatically start up the engine-generator unit when the load is switched on, to the other extreme where a storage battery of quite large capacity is used and all the load is usually taken from the storage battery, the engine generator set being run only at intervals (about once a week) to charge the battery.

Because of the multiplicity of schemes it will not be attempted here to describe any of them in detail and it is suggested that those interested obtain literature from numerous manufacturing concerns putting out these plants.

As an evidence of the growth in the use of storage batteries for this application there were in service at the end of 1920 in low-voltage farm lighting plants approximately three million storage battery cells aggregating a capacity of one million kilowatt-hours.

SMALL REGULATING BATTERIES FOR ELEVATORS, ETC. — Fig. 7 shows a scheme of load regulation which has been adopted as standard for situations where it is desirable to operate a lighting load and a fluctuating motor load from the same generator. This scheme is known as the "constant-current" scheme.

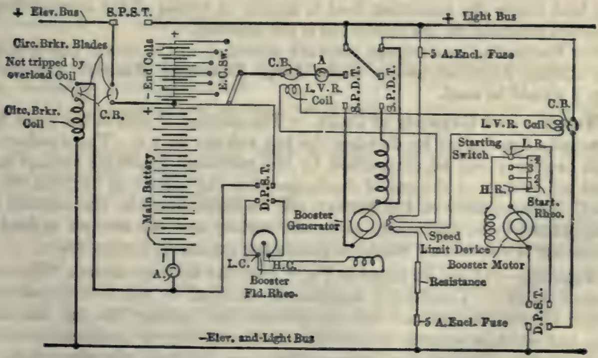

Fig. 7. Connections for "Constant-current" Regulation

Elevator Load Regulation. — This scheme finds its greatest use in office buildings, hotels, etc., where electric elevators are in service. The load on such a plant is a very fluctuating one on account of the heavy current required for starting the elevators. This starting current is generally about twice the elevator running current. Moreover, as the elevators are not in service continuously, the average current per elevator is usually much less than the running current per elevator. There is generally, therefore, a considerable difference between the average load on the station and the maximum momentary load. This difference varies with the number and type of elevators, the character of elevator service and the magnitude of the lighting load. If the fluctuations above and below the true average are a comparatively large percentage of the true average load, they are liable to cause fluctuations in the generator voltage with consequent flickering of lights. In addition to this objection, the generating capacity necessary without a battery must be increased and it is necessary to operate the whole plant at a poor load factor.

Constant-current Scheme. — Reference to Fig. 7 shows that the battery and the elevator circuit are in multiple, and that the booster is in series between the lighting bus and the elevator bus. The booster in the constant-current scheme, as the name would imply, is wound in such a manner that the current going through it is rendered constant. Therefore, the parallel system of battery and elevator circuit can only draw a constant current from the generator, and it follows that all of the fluctuations due to the elevator load must be taken by the battery. When the elevator current is greater or less than the average, the difference is given or absorbed by the battery. It is important to note that the battery voltage applied to the elevator motors is that of the battery alone and is therefore variable. Ordinarily this is not a disadvantage as the heavy load is caused by the starting of motors, in which case high voltage is not an object.

It will be noted that the scheme given in Fig. 7 shows end cells. These are used for regulating the voltage on the lights when the generator is shut down and the booster is out of circuit, the battery carrying the entire lighting and elevator load.

Other Uses of Small Regulating Batteries. — Electric-elevator service has been taken as an example, as this is the class of work usually encountered. This scheme may be used, however, for other fluctuating loads, such as are due to electric cranes, shop tramways, mine hoists, etc., and in general for any fluctuating load where the maximum load is greatly in excess of the average load and constant potential on the motor bus is not required.

For fluctuating loads where constant potential on the power bus is required, the scheme described above under *Large Regulating Batteries* and shown in Fig. 2 should be used.

Number and Capacity of Cells. — The number of cells in the main battery is usually determined by dividing the bus voltage by 2.1. The number of end cells necessary will be determined by the minimum allowable voltage and the final cell voltage at the end of discharge when the battery is carrying the entire load. The capacity of the battery will be determined by the amount of the fluctuations to be taken care of by the battery and the amount of load to be taken out of the battery when the generating apparatus is shut down.

Booster. — It will be noted in Fig. 7 that the diagram shows a differentially-wound booster. The desired regulation is obtained by means of the series field, which carries the constant current between the lighting bus and the elevator bus, and which is designed to produce a booster voltage in the direction to oppose this current. The relation between the series field and the shunt field is adjusted for any given average load.

A straight shunt booster with the field controlled by the carbon regulator as shown in Fig. 2 can be utilized with this constant-current scheme, if the solenoid is connected between the lighting bus and the elevator bus and if the carbon regulator actuated by this solenoid is utilized to control the field of the exciter as shown in Fig. 2. In all other respects this carbon regulator constant-current scheme operates according to the same general principle as the scheme for the differentially-wound booster.

It will be noted also that with the scheme shown in Fig. 2 the booster is in series with the battery and carries only the battery current, whereas in the constant-current scheme shown in Fig. 7 the booster carries the average current of the fluctuating load. The relative magnitude of the fluctuations as compared with the average will determine which booster will be least expensive and may, therefore, be a factor in deciding between the two schemes.

Two- and Three-wire Systems. — The scheme shown in Fig. 7 is for a two-wire system. This same general scheme is utilized for a three-wire system where the motor load is connected across the outside wires and the lights are balanced on each side of the neutral. Where the three-wire system is used, the general principles of the scheme are exactly the same as shown in Fig. 7, except that it will be necessary to have one booster generator and one set of end cells on each side of the system.

ELECTRIC-VEHICLE BATTERIES. — The growth in the use of storage batteries for electric vehicle propulsion has been enormous during the past few years. This is especially true in case of street trucks, industrial trucks and tractors and mining and industrial locomotives. The voltage and capacity of the battery necessary to operate any given vehicle is usually specified by the manufacturer of the vehicle, as the design of the vehicle determines the number of watt hours per car mile under certain definite conditions. Unless the vehicle user is a battery expert and a vehicle expert, it is not advisable that he specify either the number or capacity of cells to be used.

Desirable Characteristics of Vehicle Batteries. — The following are the chief characteristics of a good vehicle battery.

1. Low internal resistance, because of high current demand required for acceleration or for ascending grades.

2. Small capacity temperature coefficient, in order that the capacity of the battery shall not be unduly affected by chilling of the cells.

3. Voltage characteristic as nearly flat as possible under average conditions of discharge.

4. Construction of sufficient ruggedness to prevent breakage of parts in ordinary service.

5. Assembly such as to minimize possible labor and attention for flushing, cleaning, removing short-circuits, etc.

6. Ability to stand some abuse, as the average vehicle operator is not a battery expert.

The use of batteries for vehicle work is treated in greater detail in the article on *Automobiles, Electric.*

DRAWBRIDGE BATTERIES. — It is becoming standard practice to install storage batteries in connection with electrically-operated drawbridges, chiefly as an insurance against the interruption of power from the generating station. The voltage of drawbridge motors is usually 220 or 500. The number of cells and the capacity of the cells to meet the requirements will depend upon the maximum and minimum allowable voltages, and the total energy required from the battery before recharging.

Size of Drawbridge Battery. — Assume a 2000-ton bascule lift bridge with a 185-foot span. This bridge is to be operated with 220-volt motors. The time

required to open or close is 1½ minutes, or a total of 3 minutes for a complete opening and closing. Assume the following conditions: The battery will be required to open and close the bridge 40 times on one charge, the maximum allowable voltage during bridge operation will be 250 and the minimum allowable voltage 200; the average current demand over a complete cycle will be 100 amperes with a maximum momentary current demand of 320 amperes lasting a few seconds.

It is seen that the total energy in ampere hours required for the 40 complete openings and closings of the bridge would be equal to $\frac{3 \times 100 \times 40}{60} = 200$ ampere hours during a total period of $3 \times 40 = 120$ minutes or 2 hours. The battery, therefore, must have a capacity not less than 200 ampere hours at the 2-hour rate of discharge. Bearing in mind the voltage limits specified above, an inspection of Figs. 3 and 6 in the article on *Batteries, Storage, Lead Type,* will show that the battery capacity will be fixed by the voltage limits in this case and not by the ampere-hour requirements. In any case that may arise, if there is any doubt as to which requirement will fix the capacity of the battery, select a battery having the ampere-hour capacity required and then examine Fig. 6 to see if a battery having this ampere-hour capacity will meet the voltage requirements.

For a maximum allowable voltage of 250 and a floating voltage of 2.1 volts per cell, there would be required 120 cells. Referring to Fig. 6 and using the upper curves, for plates 10¾ inches square, we will assume that the battery must not discharge at a higher rate than 4 times the normal rate. At 4 times the normal rate the voltage drops almost immediately to 1.9 volts per cell. With 120 cells the voltage would drop to 228 volts. It is noted that the normal rate of this battery will be 80 amperes, and the average discharge of 100 amperes will be 1¼ times the normal rate. By interpolating a curve between the normal curve and four times the normal curve, it is seen that with an average discharge at this rate for 2 hours, the voltage per cell at the end of the two hours, with 100 amperes flowing, will be approximately 1.9 volts, or the voltage of the 120-cell battery will be 228 volts. If, now, the current were increased to 320 amperes (4 times the normal rate), and this current flow continued for an appreciable period of time, the voltage would drop further, by about 20 volts, leaving a voltage of 208 at the battery terminals. It is, therefore, evident that the voltage requirements will be met by a 120-cell battery having a capacity of 80 amperes at the normal rate.

A battery having a normal rate of 80 amperes could be discharged at 1¼ times the normal rate for something over 4 hours. It is, therefore, seen that the battery which we selected has more than ample ampere-hour capacity to meet the requirements, its capacity being fixed by the momentary current demand.

It usually will be found that the capacity of cells required for drawbridge operation is fixed by the maximum discharge rate and not by the ampere-hour capacity at the average rate.

Size of Charging Generator.— If a separate power plant is installed for the operation of the drawbridge, it will be well to install an engine-generator set having an available voltage range of from 220 to 325 volts. There may be required 2.7 volts per cell for charge, which with 120 cells would call for 324 volts. If the battery installed is of sufficient capacity to carry the bridge a complete day or more on one discharge, it is good practice to install a generator of only sufficient capacity to charge the battery at the normal rate and to operate the bridge at all times from the battery. In the case cited above the capacity of the generator would be approximately 80 amperes. If it is required ordinarily to supply the bridge from the generator and hold the battery for

emergencies, the generator must have sufficient capacity to carry the maximum current required by the bridge motors.

IGNITION BATTERIES FOR PORTABLE AND STATIONARY GAS ENGINES. — Probably the largest single application of storage batteries to-day is for the starting, lighting and ignition of gasoline automobiles. There are now in service in this use storage batteries of the lead-acid type numbering approximately 15,000,000 cells with an aggregate capacity of approximately 3,000,000 kw-hr. Batteries of from three to twelve cells (6 to 24 volts) are used for this purpose, but the common standard to-day is a 6-volt (3-cell) battery with a capacity of from 80 ampere hours to 150 ampere hours. For this purpose the essential requirement is a battery of as low internal resistance as can be designed compatible with a reasonable life.

For further details in regard to automobile starting and lighting batteries the reader is referred to the various manufacturers of automobiles and starting and lighting equipment and also to the bulletins and catalogs of the various storage battery manufacturers. See also *Ignition, Electric* and *Starting and Lighting Systems for Automobiles*.

For stationary engines of large capacity it is customary to use a voltage ranging from 90 to 110 volts. Glass-jar batteries of sufficient ampere-hour capacity to meet the requirements are installed in a cabinet in the main engine room. The number of cells in the main battery necessary to float on the system will be equal to the bus voltage divided by 2.1. The capacity of the battery will depend upon the number of ampere-hours required on one charge and the minimum allowable voltage during a discharge. These requirements being known, the correct number and type of cells can be determined from the characteristics of the batteries (*see Batteries, Storage, Lead Type and Alkaline Type*), in connection with trade catalogues.

Occasionally, there is installed in addition to the cells in the main battery a group of end cells which can be cut in on discharge by means of a single switch and thus hold up the voltage of the battery. The number of cells in this end-cell group usually varies from 3 to 8, depending upon the voltage requirements.

Usually a large ignition battery is charged by placing the two halves in parallel, each half being connected to the bus through a fixed resistance somewhat after the fashion explained above and indicated in Fig. 6.

BATTERIES FOR STEADY-VOLTAGE REQUIREMENTS. — It is frequently required to obtain a source of direct current with a steady voltage, as, for instance, in electroplating, instrument calibration, telephone work, laboratory work, etc. In such cases there should be chosen a battery that has more capacity at the normal rate than the ampere hours actually required before recharging, in order that the battery may be worked on the flat portion of the discharge curve.

For instance, to furnish 10 amperes continuously for 8 hours at about 20 volts, with a variation of 1 per cent between the beginning and the end of the discharge, the time of discharge must be limited to about 50 per cent of the total time. A cell having a capacity of 10 amperes for 16 hours should, therefore, be chosen.

BIBLIOGRAPHY. — See Bibliography in article on *Batteries, Storage, Lead Type*.

BATTERIES, STORAGE, LEAD TYPE. — (*See also Batteries, Storage, Applications of; Batteries, Storage, Alkaline Type; Electrochemistry, Principles of.*) The following is a brief outline of the contents of this article:

TYPES OF LEAD CELLS. — A secondary or storage cell is any voltaic couple which can be regenerated, after exhaustion, by passing a current through it in a direction opposite to the direction of current flow when the couple delivers energy to the external circuit. Three types of storage batteries are now in use in this country, the Planté type, the Faure type, and the alkaline or Edison type. The special features of the alkaline type are treated in a separate article, viz., *Batteries, Storage, Alkaline Type.*

Planté Type. — The principle of the storage battery was discovered in 1801 by Gautherot, a Frenchman. However, practically nothing further was done until 1860 when Gaston Planté constructed a storage cell consisting of two lead strips immersed in dilute sulphuric acid. Planté found that, by giving this cell a long charge by passing the current through in one direction, and then giving the cell a long charge in the opposite direction, and repeating this cycle many times, the capacity of the cell was considerably increased. Little improvement was made in the Planté cell until a number of years after Planté's discovery.

At present the Planté type of storage battery is made by preparing a pure lead plate with a large superficial area exposed and then oxidizing the surface electrolytically so that it is covered with lead peroxide. A plate formed thus is a positive plate. To form a negative plate the peroxide, after electrolytic formation, is reduced to metallic sponge lead by reversal of current. In this manner a thin layer of active material which is porous and which adheres firmly to the supporting lead plate is produced.

Use of Planté Type. — The Planté type is used chiefly where weight and space are of no great importance.

Faure or Pasted Plate. — In 1880 Camille A. Faure, in France, and Charles F. Brush, in America, simultaneously developed the "pasted" type of storage battery. In this type of storage battery the active material, on both the positive and negative plates, instead of being formed electrolytically, as was done by Planté, was applied, in the form of a paste, to a stiff lead-antimony alloy supporting grid. This type of plate is now commonly called the Faure type of plate, and it was the discovery of this type that gave the storage battery its first commercial impetus.

Use of the Faure or Pasted Plate Type. — The Faure type is used chiefly where it is desired to obtain the greatest possible capacity with a minimum of weight and space occupied. This type is used chiefly for vehicle propulsion and for central-station "stand-by service."

THEORY. — The electrochemical theories of the storage battery are quite complicated and the various authorities on these subjects do not agree on all details. In this outline no attempt will be made to cover the various theories in detail; only the simplest fundamental chemical actions will be described.

Positive and Negative Plates. — The terms positive and negative are employed throughout this article in accordance with engineering usage; that is, the positive plate is the one from which the current flows on discharge and the negative plate is the one into which current flows on discharge. In a lead battery the positive plate, on which the lead peroxide is formed, has a comparatively hard surface of a reddish-brown or chocolate color, while the negative plate, which carries the sponge lead, has a much softer surface of a grayish color.

Chemical Reactions. — The active elements of the lead-lead acid type of battery consist of lead peroxide (PbO_2) on the positive plate, sponge lead (Pb) on the negative plate and dilute sulphuric acid (H_2SO_4) for the electrolyte.

Whatever the secondary reactions may be, it is agreed that the final result on discharge is the formation of lead sulphate ($PbSO_4$) on both the positive and negative plates, the SO_4 radical of the sulphuric acid combining with the lead of both plates to form this compound, resulting in the formation of some water (H_2O), with a consequent decrease in the specific gravity of the electrolyte. On charge the electric current splits up the lead sulphate ($PbSO_4$), returning the SO_4 radical to the electrolyte, oxidizes the positive plate to its original condition of lead peroxide (PbO_2) and reduces the negative plate to its original condition of sponge lead (Pb). This action may be represented as follows:

$$\underset{+\text{Plate}}{PbO_2} + 2H_2SO_4 + \underset{-\text{Plate}}{Pb} = \underset{+\text{Plate}}{PbSO_4} + 2H_2O + \underset{-\text{Plate}}{PbSO_4}$$

This equation read from left to right is the equation of discharge; if read from right to left, it is the equation of charge. In practice, on charge, towards the end of charge some of the water (H_2O) is split up by the current into its component parts, hydrogen (H) and oxygen (O), the hydrogen being liberated at the negative plate and the oxygen at the positive plate. This occurs whenever the density of charging current is greater than can be utilized in decomposing the lead sulphate remaining in the plates.

DESIGN. — The capacity of a storage-battery plate depends not only upon the amount of active material but also upon the active surface exposed to the electrolyte. Plates should therefore be designed to expose a maximum amount of surface consistent with the strength of the supporting grid or lead base. In order that the active material of the plates can be acted upon by the electrolyte, it should be as porous as is consistent with its proper support by, and proper contact with, the grid or supporting lead base.

Sponge lead and lead peroxide possess little mechanical strength. Lead peroxide is a poor conductor. Since mechanical conditions require a certain amount of rigidity in the plates and since the current generated by the active materials must be carried away, a battery plate must necessarily consist of two parts, viz., the grid or supporting base and the active material which in the lead battery is finely-divided porous sponge lead or porous lead peroxide.

Lead Plates. — There are certain ideal requirements to be met by a lead storage-battery plate, among which may be mentioned:

1. The grid and active material should be so proportioned as to obtain, as far as possible, uniform current distribution over the entire surface.

2. The active material should be applied to, or formed on, the grid in such a manner that good electrical contact exists when the plate is new and this contact should be maintained during the life of the plate.

3. In pasted plates and pellet plates the grid should be of material that is not injuriously affected by the electrolyte.

4. The material of the grid should be such as to insure a minimum of local action between itself and the active material.

5. The surface of the active material exposed to the action of the electrolyte should be as great as is compatible with proper mechanical strength and a suitable provision for the natural expansion of the active material in use.

6. The plates should be so made and assembled as to allow the maximum possible diffusion of the electrolyte through and around the plates and through the containing vessel.

7. The grid or lead base should be of ample cross section to carry the current generated under working conditions without undue loss.

8. The lugs which collect the current from the grids should be of ample cross section, and when designed for heavy currents they should be so arranged as to distribute the current uniformly through the grid.

9. The negative, sponge-lead plate should be constructed so that the sponge lead retains its porous character and does not grow hard and dense in service and thus lose its capacity.

10. The lead and the electrolyte should be pure; otherwise secondary reactions will be set up, with a consequent decrease in efficiency.

Of the many lead plates on the market to-day, the most commonly used are the "pasted," Planté and composite or "pellet" plates. Cuts showing the construction of the various types of plates may be found in manufacturers' catalogues; the essential features of design are briefly discussed below.

Pasted Plates. — The pasted plate is usually made by applying to a hard lead-antimony grid a paste made of some oxide of lead, usually litharge (PbO) or red lead (Pb_3O_4), and some liquid and other substances. In the so-called "Iron-clad" battery made by the Electric Storage Battery Co., the active material is held in perforated hard rubber tubes. Various substances are used to mix with the lead oxide, the idea being to increase the hardness, porosity, toughness and conductivity. Some of the substances used by different manufacturers include anthrocine, glycerine, graphite, potassium silicate, asbestos, ammonium-sulphate, etc.

"Forming" of Pasted Plates. — After the grid is filled with the paste, the plate is dried. After being completely dried a number of plates are assembled in a forming bath of dilute sulphuric acid with dummy lead plates for the opposite electrode and the forming charge is given by passing the proper current through the voltaic couple thus formed. Positive plates are formed by connecting the plates to be formed as the anode; the current oxidizes the lead oxide further to lead peroxide (PbO_2). Negative plates are formed by passing the current in the opposite direction, reducing the lead oxide to sponge lead. After the forming charge the plates are dried and are ready for the market.

Planté Plates. — Of the various types of Planté plates, among those most commonly in use may be mentioned the central-web type; the cast-lead having no central web; and the composite or "pellet" type. The main idea in any of these methods of manufacture is to produce a large surface on which to form the active material.

Central-web Type. — In the central-web type there is a solid sheet or "web" of pure lead on which the ribs are formed. This web prevents the circulation of the electrolyte through the plate. The ribs are formed from the original lead plate, either by rolling, spinning or cutting. In the rolling and spinning processes the plate is formed from a lead blank by means of a number of steel disks placed side by side and separated by small spacers on a shaft. The lead blank is passed between two sets of disks by forward and backward movements. The disks gradually work deeper into the plate and squeeze up

lead into the spaces between the adjacent steel disks. In the cut type of plate the ribs are formed from the lead sheet by a tool, which at each stroke turns up one complete rib. The cutting edge works at an angle so that the finished ribs stand out from the surface. The ribs may incline upward from the central web and thus form pockets to hold the active material and prevent its falling away.

The disadvantage of the *web* type is the web itself. There is invariably a tendency towards unequal work on the two sides of the positive plate, this tendency being caused by difference in the plate spacing, unequal capacity of the negative plate on either side, inequality in the shape of the ribs, etc. Where the active material and the active surface of a plate are disposed in planes perpendicular to the face of the plate and extend *through* the plate, excessive action on one side simply works the plate a little further through from that side, the effect on the active material, however, being uniform throughout. With a plate provided with a central web, preventing such action, any inequality of work will charge or discharge one side more than the other, producing a tendency to buckle.

Cast-lead Type. — The type of plate which has no central web is made by casting pure soft lead in a mold, casting having the advantage of allowing for distributing metal in the plate without limitations in manufacturing process. The plate as it comes from the mold consists of a great number of short vertical ribs running entirely through the plate and bound together by transverse ribs to give strength to the plate. In this manner a large surface can be obtained and in a plate having no central web the electrolyte can circulate through the plate, and the active material will be uniformly worked throughout even though the amount of work on the two sides of the plate be unequal. The best-known form of this plate is the "Tudor" positive.

Composite or "Pellet" Type.— The composite or pellet-type plate is made by rolling up into pellets pure soft-lead corrugated ribbons. These pellets or buttons are then forced by pressure into circular openings in a grid composed of a hard-lead-antimony alloy. The openings in the grid are beveled towards the center so that when the pellets are formed the swelling action causes them to rivet tightly into place. The corrugations on the lead ribbon allow the electrolyte to circulate through the plate and thus expose the full surface of the closed lead spiral to the action of the electrolyte. It will be noted that in this type of plate additional mechanical strength is obtained from the supporting grid. The best-known form of this plate is the "Manchester" positive.

"Forming" of Planté Plates. — In all Planté positives, after the ribs or corrugations have been formed on the lead blank or the pellets have been placed in the hard grid, the plates are assembled in a sulphuric-acid bath containing some corrosive chemical, called a "forming agent," together with dummy lead plates. The forming agents used by various manufacturers are usually kept as trade secrets; the nature and method of using such agents determines largely the quality of the battery. To form the positive plates the dummies are connected as the cathodes, and a current is passed through the couple thus formed. The electrolytic action of this current causes lead peroxide (PbO_2) to be formed from the lead of the ribs. The strength and duration of the current produce the desired thickness of lead peroxide on the ribs or pellets.

Negative Planté plates are made from positive plates by electrolytically reducing the lead peroxide to sponge lead. In most types of Planté negatives, however, it is necessary to form them with an initial capacity considerably in excess of the capacity of the positive plate. This is due to the fact that in actual service the sponge lead shrinks and loses its spongy nature, thereby reducing its capacity, since insufficient surface is exposed to the action of the electrolyte.

"Permanizing" Process. — One of the large manufacturers of storage batteries has recently developed a process for making the Planté negative plate permanent. This method consists of injecting into the sponge lead a material which prevents the shrinking of the sponge lead and thus enables it to maintain its spongy character and its initial capacity. This permanizing process can be applied to the pasted negative plate as well as to the Planté negative plate. The most widely used form of the permanized negative plate is one in which the sponge lead, formed by reducing lead oxide electrolytically, is retained in boxes made by casting a soft-lead perforated sheet on a hard lead-antimony grid. This negative is known as the "Box negative."

Applications of the Various Types of Plates. — In stationary work requiring frequent discharges, where expert attention is available at all times, and where the conditions of operation are properly suited to its use, the all-lead Planté type of plate with through-and-through circulation is the plate that should be used. Where the battery is to receive little attention and where the conditions of service are variable and uncertain, the composite or "pellet" type of plate is the safest plate to use, on account of its rugged structure and its ability to stand great abuse. As stated above, for stand-by service and for vehicle service and other service where high capacity with a minimum weight and space is desired, the pasted type of plate is more desirable.

Electrolyte. — The electrolyte used with the lead type of battery is always a dilute solution of sulphuric acid. The specific gravity of the electrolyte, when the battery is fully charged, varies from about 1.210 for stationary batteries to 1.300 for automobile ignition batteries. These values have been adopted as standard by all the leading manufacturers.

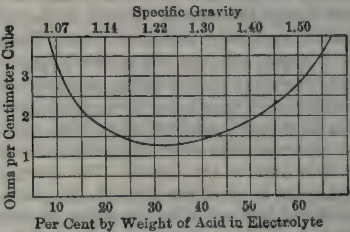

Fig. 1.

The proper specific gravity to use varies with the conditions. Fig. 1, showing the variation with specific gravity of the resistance of one cubic centimeter of electrolyte, shows that the resistance of dilute sulphuric acid is least at a specific gravity of from 1.224 to 1.240, this resistance increasing if the specific gravity be either increased or decreased. There are numerous other conditions which influence the selection of the proper specific gravity.

The curves in Fig. 2 show the specific gravity of various mixtures, both by weight and by volume, of one part of 1.840 specific gravity acid with from 1/10 to 7 parts of water. There is also a curve showing the percentage, by weight, of 1.840 specific gravity acid in mixtures of various specific gravities. These curves are approximately correct at 60° F. Unless a compensating hydrometer is used in determining the specific gravity, allowance must be made for temperature variation, on the basis of an increase of one point (i.e., one one-thousandth) in gravity for each 3 degrees Fahrenheit *decrease* in temperature, and vice versa; for instance, electrolyte that has a specific gravity of 1.210 at 70° F. will have a specific gravity of 1.213 at 61° F., and 1.207 at 79° F.

Impurities in Electrolyte. — The electrolyte should be free from organic substances, iron, chlorine, copper, arsenic, mercury, nitrates, acetates and the slightest possible trace of platinum. The various battery manufacturers issue exact specifications to the acid manufacturers, specifying the maximum amount of these injurious ingredients which the acid may contain. Electrolyte that is not approved by the company furnishing the battery should never be used.

Preparation of Electrolyte. — In preparing the electrolyte, sulphuric acid, approved by the battery manufacturer, should be diluted with sufficient pure distilled water to bring the mixture to the required specific gravity. The acid should be poured into the water; *never pour the water into the acid.* If the water is poured into the acid, the heat formed by the mixture is sufficient to cause sputtering and damage may ensue.

The sulphuric-acid manufacturing companies furnish electrolyte for battery work in such large quantities that they carry a stock of various standard mixtures. It will usually be found cheaper and more convenient to purchase the electrolyte ready-mixed than to purchase the concentrated sulphuric acid and prepare the mixture on the ground. The latter course, however, is sometimes adopted where the amount of acid used is considerable and where the item of freight saving is appreciable.

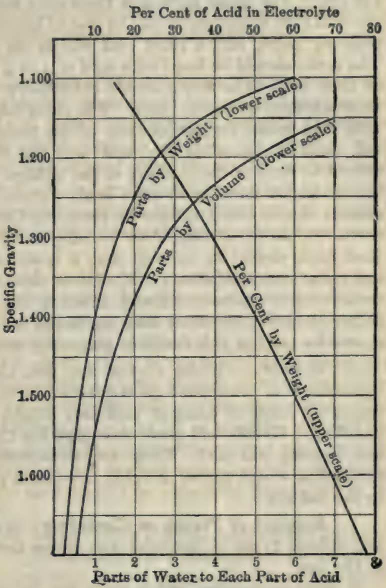

Fig. 2.

Containers for Lead Batteries. — The containing receptacles for holding the battery plates and electrolyte are usually rubber jars, glass jars or lead-lined wooden tanks.

Rubber Jars. — Hard-rubber jars are used exclusively in vehicle and portable batteries. In these batteries the plates are supported on ribs at the bottom of the jars. To reduce cleaning to a minimum, these ribs should be of sufficient height to leave ample space in the bottom of the jar for the reception of the active material which falls away from the plate. If the sediment in the bottom of the jars is allowed to reach the plates and short-circuit them, serious damage will result. The jars themselves should be made of the very best hard rubber which is not affected by sulphuric acid.

Glass Jars are ideal containers for small batteries when they are properly installed. The glass jar has the advantage that the plates and electrolyte can always be seen. The use of the glass jars is limited to the smaller sizes on account of their liability to break, due to strains left in the glass after annealing, temporary strains set up by unequal temperatures between the inside and outside of the jars, and to strains due to weight of the plates and electrolyte. Glass-jar manufacturers have, up to the present time, been unable to produce a thoroughly reliable jar larger than 21 by 13 by 18 inches.

Lead-lined Tanks. — In the larger sizes of batteries lead-lined tanks are standard. Tanks are generally made from specially selected yellow pine, dovetailed together without the use of nails or other metallic fastenings. The upper edges should be slightly beveled inward so that the moisture will drain into the tank. The bottoms under the linings should be drained and venti-

lated; these bottoms are usually constructed of slats across the tanks, separated by a small spacing to provide drainage. To further facilitate drainage under the lining, the upper surface of these slats should be grooved crosswise. Before the tanks are lined they should be coated inside and outside with two coats of acid-resisting paint, and a third coat should be added outside during installation. The tanks should be lined with lead of 3 to 4 pounds per square foot, depending on their size. The lining should extend over the upper edge of the tank and a short distance down the sides. The outer edge of the lining should be provided with drip points projecting clear of the tank, so that any drip from the lining will clear the wood of the tank and the tank supports. Especial attention should be paid to the seaming of the linings. All seams should be burned with the hydrogen flame with pure lead, without the use of any flux. The lower corners of the lining should be reënforced by puddling with lead. The upper corner should be reënforced by burning on an additional thickness of sheet lead. Each tank should be so built that it is self-supporting without the use of any braces or reënforcements. If this is done, any tank in the battery can be removed and replaced without affecting the remaining tanks in any way. A poorly constructed wood tank is bound to cause trouble. Special attention should be paid to this detail in preparing specifications.

Covers. — While covers are not absolutely necessary for stationary batteries, it is considered good practice to cover all cells. Covers should preferably be made of glass of sufficient weight to prevent excessive breakage in handling. Covers will more than pay for themselves in reducing evaporation and keeping out dirt. They are absolutely necessary in situations, such as steel mills, where the air is liable to contain particles of foreign matter injurious to the battery.

Support of Plates in Container. — As stated above, in the portable and vehicle types of batteries, the plates are supported on ribs at the bottom of the jar.

In stationary types, using glass jars or lead-lined tanks, it is preferable to support the elements from the top. In glass jars, plates are supported on the top edges of the jars by means of lugs cast or burned on to the plates. In lead-lined tanks it is, of course, necessary to insulate the plates from the lead lining. The plates are, therefore, supported in the tanks by vertical sheets of glass resting on the bottoms of the tanks. The bottom of the tank lining, under the glass sheets, should be heavily reënforced. These glass sheets should be ground top and bottom and the lower corners should be cut off at an angle of about 45°, to avoid injury to the lead lining during installation by sharp corners.

Separation and Separators. — Since the positive and negative groups, forming the complete element in a cell, are assembled with positive and negative plates alternating, it is necessary that some means be provided to prevent contact between adjacent positive and negative plates. Such contact would mean a short-circuit and might result in injury to the element. Various forms of separators have been used for keeping the plates apart. The most common practice to-day, and the one considered the best for stationary work, is to use wooden diaphragms. Each wooden diaphragm should consist of a thin sheet of porous wood mounted in slotted dowels which will allow free passage of the electrolyte between the plates. The wood should be specially treated to insure the absence of elements which in conjunction with the electrolyte might form acids injurious to the plates.

In vehicle batteries rubber separators are generally used in addition to the wood separator. That is, a rubber separator is placed on each side of each positive plate and one wood separator between each positive plate and the adjacent negative plate. In the so-called "iron-clad" battery, made by the

Electric Storage Battery Co., the active material of the positive plate is contained in rubber tubes, and therefore additional rubber separators are not necessary. Ignition batteries use wood separators only. Portable batteries use rubber only. Car-lighting batteries use wood only or rubber only. Yacht-lighting batteries use rubber only.

Sediment Space. — In all types of batteries, no matter what container is used, there should be a certain amount of free space between the bottoms of plates and the bottoms of jars. This space is designed to receive the sediment or active material which falls away from the plates. The amount of sediment space required will depend upon the service, and no fixed rule can be given to cover this point. If not prohibitive from space or expense limitations it would be well to have this sediment space great enough to take all of the active material which will be shed from the plates during their useful life.

Assembly of Parts. — All portable, automobile, car-lighting and other batteries that will be subject to jar or vibration should be assembled in the containing vessel in such a manner that the plates cannot move. They should be packed in as tightly as is possible with due consideration to the amount of electrolyte necessary and the avoidance of injury to the plates when being put into or taken out of the jars. Hard-rubber covers should be used and these covers should be so sealed as to avoid any splashing of the electrolyte. The crates containing the jars should be constructed, as far as possible, without the use of metallic fasteners and they should be thoroughly painted with acid-resisting paint. In portable batteries that will receive rough handling, if several jars are assembled in a single crate, they should be imbedded thoroughly in an acid-proof elastic compound.

Cells assembled in crates should be connected by burning together the adjacent positive and negative terminals. Cell connections should never be soldered. The main terminals of the series in one crate should be brought out and fastened in such a manner that these terminals will not be attacked by the acid.

The methods of assembling and connecting stationary types of cells are described below in the section on *Installation*.

METHODS OF TESTING. — The tests described below are intended to cover only those tests that are of interest in the commercial operation of batteries.

Test of Specific Gravity. — The specific gravity of the electrolyte is the most accurate guide as to the state of charge of a lead-type storage battery. Specific instructions furnished by the manufacturer of the battery are given, showing the range in specific gravity over a given amount of charge and discharge and the operator should be guided by the instructions of the manufacturer. The test of the specific gravity is made by means of a hydrometer having a suitable scale for the type of cell to be tested. In stationary types of batteries the hydrometer has a scale reading of 1150 to 1250 and is left floating in one cell. In all portable types of batteries, and ordinarily in vehicle- and car-lighting batteries, it is usually necessary to draw some of the electrolyte from the cell in order to test its specific gravity with the hydrometer, which should have a scale reading of 1150 to 1300. Hydrometer syringes, with hydrometer contained in a glass barrel, can be obtained on the market for this purpose. There is also on the market a regular acid-testing set consisting of a syringe for withdrawing the acid and a test tube into which the acid is poured from the syringe, in order that the hydrometer may be floated for reading.

Test for Impurities in Electrolyte. — The proper testing of electrolyte for impurities requires not only some knowledge of chemistry, but also experience in such work, in order to correctly interpret results; it is therefore inadvisable

for the ordinary user to attempt it. Furthermore, it is usually entirely unnecessary that he should, since the leading battery manufacturers make analyses for their customers free of charge. The harmful impurities most likely to be present are iron, hydrochloric acid, oxides of nitrogen, sulphurous acid, arsenic, organic matter and platinum. In order to insure freedom from the latter, it is advisable to specify acid which has not been concentrated in platinum.

Test of Internal Resistance. — The most accurate method of determining the internal resistance of a cell is to subject it to a discharge current of a certain amount, and after the voltage has become sufficiently constant to permit accurate reading with the discharge current flowing, the current is instantly interrupted and the rise of voltage noted. This rise of voltage divided by the current will give the internal resistance. If this test is made on a battery of more than one cell in series the result shows the internal resistance of the entire battery, including the cell connections.

Test of Capacity. — The capacity of a storage cell depends on various conditions, such as the temperature, the rate of discharge, the strength of the electrolyte, the character of service to which it has previously been subjected and the attention it has received. The "normal capacity" of a storage cell is usually expressed in ampere hours at the 8-hour rate at 70° F., down to a certain definite voltage per cell. For instance, when it is said that a cell has a capacity of 100 ampere hours, it is *usually* meant that this cell can be discharged at a rate of 12½ amperes continuously for 8 hours at 70° F., down to the limiting voltage specified by the battery manufacturer. In lead cells this limiting voltage is usually taken at 1.75 volts per cell. The "watt-hour" capacity of a battery is equal to the ampere-hour capacity multiplied by the average voltage during discharge.

The "available" ampere-hour and watt-hour capacity of a battery varies with the rate of discharge, the available capacity decreasing with increase of rate. The higher the rate of discharge the lower the limiting voltage at end of discharge. Therefore, to test the capacity at any given rate of discharge, the limiting voltage at that rate as specified by the manufacturer should be known.

Bearing in mind the above facts, to test the capacity of a storage battery at any given rate, the battery should be first charged at the normal rate, as specified by the manufacturer, until the voltage and gravity in all of the cells will rise no further and until all cells gas freely at both the positive and negative plates. The capacity of the battery, as a whole, will be limited by the capacity of the lowest cell and all cells should be as nearly as possible in a condition of full charge before the discharge is started to test the capacity of the battery. Individual cell readings should be taken of the voltage and gravity while the charging current is flowing.

When it is certain that the battery is fully charged, discharge the battery at a constant current at the desired rate, down to the limiting voltage at this rate. The ampere-hour capacity will be equal to the constant current in amperes, multiplied by the time of the discharge in hours; and the watt-hour capacity of the battery will be the ampere-hour capacity multiplied by the average voltage during discharge.

In order to be sure that the battery is in good condition it is well to take several complete discharges before making the final charge, preceding the final test discharge.

On each charge and discharge observations should be taken of the voltage, the current, the temperature and the specific gravity. The temperature of the room should be held as nearly constant as possible throughout the test. In order to facilitate the plotting of curves, the readings should be taken at intervals of time which are even factors of an hour.

Cadmium Test. — When a strip of cadmium is placed across the top of the plates in a cell, and properly insulated from them, there will be a difference in potential, as measured by a voltmeter, between the cadmium and the positive plates and between the cadmium and the negative plates; the sum or difference (depending on the relative directions of the two potential differences) between these two readings being equal to the internal voltage of the cell.

Such a test, when properly interpreted, is a valuable guide to the capacity and condition of the positive and negative plates separately. The results are, however, very apt to be misleading to one who has not had a wide experience in storage-battery work. See *Tech. Paper* No. 146, Bureau of Standards.

Test of Efficiency. — The ampere-hour efficiency of a battery is the ratio of the output on discharge in ampere-hours to the input on charge in ampere-hours; the watt-hour efficiency of a battery is the ratio of the output on discharge in watt-hours to the input on charge in watt-hours.

The efficiency of a battery depends upon numerous conditions, chief among which are the charge and discharge rates and the temperature.

It has been noted above that the *available* capacity of a battery depends upon the discharge rate. For instance, if a battery is discharged at the one-hour rate to the allowable final voltage limit, the energy taken out at this rate is approximately only fifty per cent of the energy that could be taken out at the normal rate. However, if, at the end of one hour's discharge at the one-hour rate, the rate of discharge be reduced to the normal rate, much of the available *normal* capacity of the battery can be obtained.

It is thus seen that any statement of the true efficiency of a battery should include the charge and discharge rates. It is also seen that in order to make a true efficiency test an average of several charges and discharges should be taken, and the battery should be given several discharges and charges before the actual test is begun. Failure to observe these conditions may result in figures that are misleading. The writer has seen test figures showing an efficiency of more than one hundred per cent; he has also seen figures showing efficiencies far below the actual efficiency.

To determine the efficiency of the battery, get all the cells in satisfactory condition by preliminary charge and discharge, as outlined above under the capacity test, and then take a preliminary discharge down to the final voltage specified by the manufacturer for this rate of discharge. The first test charge should then be made at the normal rate until the cells are fully charged. The first test discharge should then be taken at the specified rate down to the same final voltage used in the preliminary discharge. The watt-hour efficiency of the battery will then be the ratio of the total watt-hours discharged to the total watt-hours charged. If greater accuracy is desired, this cycle should be repeated several times (say, from four to six times) and the watt-hour efficiency will then be the ratio of the total watt-hours of all the discharges to the total watt-hours of all the charges.

RATING AND PERFORMANCE. — The capacity of a storage-battery plate of a given size varies somewhat with the conditions, chief among which are the specific gravity of electrolyte, the temperature of electrolyte, the amount of electrolyte, the original formation of the plate, the design of the grid, the age of the plate, the length of time between charge and discharge and the final voltage limit specified.

Rating of Stationary Batteries. — The rate of discharge of a storage battery is the number of amperes that it will supply continuously for 8 hours, for 3 hours or for 1 hour. The 8-hour rate is the so-called "normal" rating, the 3-hour rate is approximately twice the normal rating and the 1-hour rate is approximately 4 times the normal rating. The *capacity* of a storage battery is usually

stated in ampere hours. Since the capacity varies with the rate of discharge, it is necessary to specify the rate of discharge in stating the capacity in ampere-hours. If the rate is not specified, the normal rate is assumed.

Certain plates especially designed for high rates of discharge are rated in terms of the amperes they will supply for 6 hours, this rating being designated as "normal." The corresponding 1-hour rate is 4 times this 6-hour rate.

While the above is standard for ordinary batteries, plates for certain classes of service are designed to give the normal rate for from 7 to 7½ hours.

The capacity of a storage battery is sometimes expressed in kilowatts, but this is ambiguous unless the rate of discharge is specified.

Rating of Vehicle and Ignition Batteries. — It is standard practice to rate a vehicle battery in terms of the amperes it will give continuously for 4, 4½ or 5 hours, specifying the battery to have a capacity of so many ampere-hours at one of these rates. In most portable types of ignition batteries the batteries are rated at so many ampere-hours at the "service rate," i.e., the rate of discharge (amperes) corresponding to the service for which they are designed.

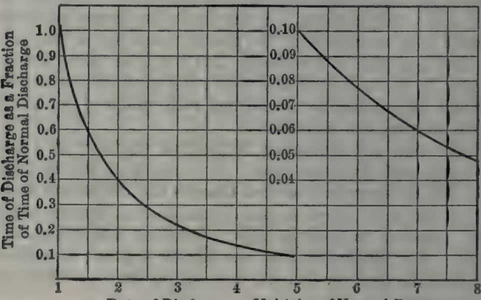

Fig. 3.

Variation of Rate of Discharge with Time of Discharge. — Fig. 3 shows an average capacity curve for a Planté plate, the time for a complete discharge at any rate being shown as a fractional part of the normal time and the rate of discharge for any given time being shown as a multiple of the normal rate.

From these curves it will be noted that a cell which will give its normal rate for 8 hours will give 4 times that rate for about 1⅛ hours; and a cell which will give 4 times the normal rate for 1 hour will give the normal rate for only 7 hours. These are approximately correct relations for the average Planté cell. Trade catalogues usually specify the normal rate for 8 hours and 4 times the normal rate for 1 hour. This is due to the fact that a cell which is regularly worked at the normal rate will tend to hold its 8-hour capacity, while a cell which is regularly worked at 4 times the normal rate will tend to lose some of its 8-hour capacity and finally give only 1 hour at 4 times the normal rate.

Variation of Capacity with Rate of Discharge. — It will be noted from the capacity curves that the available capacity in ampere-hours decreases with increase of rate of discharge. This is largely due to the time required for fresh electrolyte to penetrate into the pores of the active material of the plate. At the higher rates of discharge the diffusion of electrolyte is not sufficiently rapid to reach the more remote portions of the active material, and these portions are not fully available at these high discharge rates. As should be expected, if, after all of the available capacity at any given rate has been delivered

by the battery, the battery be allowed to rest or the rate of discharge be decreased, additional capacity can be obtained from the battery.

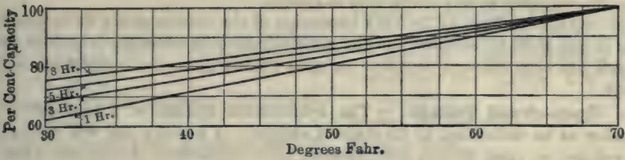

Fig. 4.

Variation of Capacity with Temperature. — Fig. 4 shows the variation of capacity with temperature at various rates of discharge. This change of capacity is temporary and the capacity will be restored to its original value when the temperature is restored to its original value.

Voltage and Specific-gravity Characteristics. — The charge and discharge voltage characteristics of a battery vary with the conditions. Fig. 5 shows typical average voltage and specific-gravity charging characteristics of a Planté type of cell designed for stationary service. It will be noted that the specific gravity increases gradually until all of the lead sulphate in the plates has been reduced, the (SO_4) radical being returned to the electrolyte to form sulphuric acid, which accounts for the steady increase in specific gravity and for the fact that the specific gravity will rise no further after the battery is completely charged.

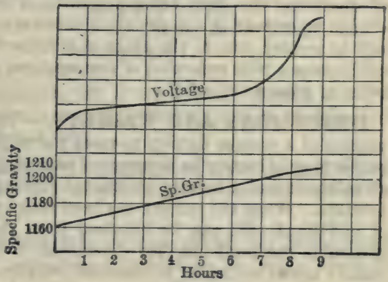

Fig. 5.

Final Charging Voltage (Fig. 5). — The scale of volts on the upper curve has been purposely omitted, but this curve shows the general shape of charging curve for any type of lead battery. The final charging voltage of any lead type of battery varies over a considerable range, according to the type of cell, age of plates, temperature of electrolyte, strength of electrolyte, etc. The final charging voltage may vary anywhere from 2.4 to 2.8 volts per cell. It will be noted that the voltage rises rapidly at the beginning and at the end of charge.

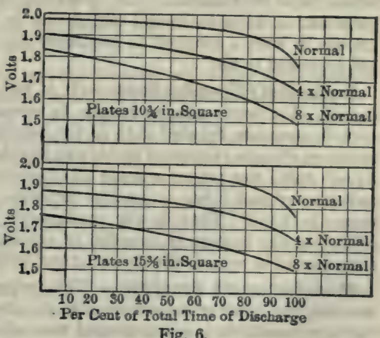

Fig. 6.

Discharge Voltage. — Fig. 6 shows discharge curves at the normal, four times the normal and eight times the normal rates, respectively, for two different sizes of Planté plates The upper set of curves are for plates 10¾ inches square and the lower set of curves are for plates 15⅝ inches square.

These curves, used in conjunction with the capacity curves shown in Fig. 3, will enable anyone to determine the approximate performance of any lead battery when the capacity at any given rate is known.

The voltage readings at the very beginning of the discharge are liable to vary more or less from those shown on the curves depending upon the length of time the cell has been allowed to stand on open circuit after the completion of the charge.

Open-circuit Voltage. — The open-circuit voltage of a lead cell is of no value as an indication of its state of charge. A healthy cell will regain its full voltage on open circuit, even though fully discharged, if sufficient time is allowed for depolarization.

Internal Resistance. — The true internal resistance of a cell may be determined by the method explained above in the paragraph on *Test of Internal Resistance*. This internal resistance determines the *immediate* change of voltage at the cell terminals with any sudden change of discharge rate. In the ordinary commercial types of lead cells at 70° F. the internal resistance is such as to cause an immediate drop in terminal voltage of between 5 per cent and 7 per cent, with a sudden increase of current equal to four times the normal rate of discharge, the drop being proportional to the change in current. If this change in current is not strictly momentary, the change in terminal voltage will be greater, due to the effect of polarization. The internal resistance of a cell increases with reduction of temperature; at 0 °F. the internal resistance is twice as great as at 70° F.

DIMENSIONS AND WEIGHTS. — In determining the dimensions and weights of a storage battery to meet the requirements, it is suggested that the proper type of battery be determined in accordance with the curves and examples given above and a battery be selected from trade catalogues to meet the specifications. When the battery has been thus chosen, the dimensions and weights of individual cells can be obtained from the trade catalogues. In order to determine the total installed dimensions and weights, it will be necessary to make a layout according to standard practice. The distances between adjacent cells and the dimensions of aisles are given below in the section on *Installation.*

SPECIFICATIONS, CONTRACTS AND PROPOSALS. — (*See also article on Specifications.*) In preparing specifications for a storage-battery plant and accessories, special care should be taken to avoid drawing up specifications in such a manner as needlessly to embarrass the manufacturers who are requested to bid.

Preliminary Specifications. — It is recommended that preliminary specifications of a very general nature be prepared. These specifications should contain complete detailed data of the conditions to be met, so that the manufacturers who are requested to bid can readily determine the number and capacity of cells, the proper scheme of operation, the correct design of booster, switchboard and other accessories recommended by them to meet the requirements in hand. That is, the preliminary specifications should state plainly just what the battery is intended to accomplish. These preliminary specifications should be sent to the manufacturer with a request for a complete proposal.

Manufacturer's Proposal. — This proposal should give detail specifications covering the apparatus recommended by the manufacturer. In order to obtain the best results, it is recommended that after these proposals are received, the prospective purchaser confer with the bidders that are to be considered, and that complete final specifications be prepared, covering all details of the necessary equipment. These final specifications can then be sent to the manufacturers for their final bids.

Final Specifications. — As a guide in preparing the final specifications the points enumerated below should be thoroughly covered:

Number of cells;
Number of plates in elements initially installed;
Size of plates;
Capacity of the elements initially installed;
Charging rate of the elements initially installed;
Size of the containing vessels expressed in the number of plates which the vessel can contain;
Separators and supports;
Electrolyte;
Insulation;
Assembling and lead burning;
Bus bars, both plain and reënforced;
End-cell switches;
All copper work and cables;
Switchboard in detail;
Detailed specifications for booster and exciter apparatus according to the A.I.E.E. Standardization Rules.
Foundation for booster and switchboard;
Testing instruments;
Battery room;
Erection;
Skilled and unskilled labor;
First charge;
Operation;
Test;
Freight, cartage;
Delivery;
General scheme of operation;
Temporary work;
Cutting of walls, etc.;
Access, storage and hoisting;
Acceptance.

If specifications are drawn, covering all of these points clearly and if the specifications are attached to, and form part of, the contract, no trouble should arise in the future from misunderstandings.

Contract. — The contract should cover the price, time of payments, guarantee, protection from patent litigation and insurance of the material during construction.

INSTALLATION AND ERECTION. — *Never install any battery without following explicitly the detailed instructions furnished by the manufacturer who made the battery.*

If the battery is of considerable capacity, detail drawings showing the method of installation should be obtained from the battery manufacturer. The leading manufacturers issue printed instructions and detail drawings describing the method of installation. Much time and money can be saved by obtaining these data from the battery manufacturers.

Spacing of Glass Jars. — The spacing between cells along rows varies from 1½ inches with the smaller types of multiple plate-glass cell batteries to 3½ inches for the largest size multiple plate-glass cell batteries. This space is the space from jar to jar. It is standard practice to place the jars on wood or glass sand trays filled with sand; the distance between the sand trays along the rows varies from ½ inch to 2⅛ inches, depending upon the size of the glass jars. As the outside dimensions of the sand trays are not usually given in trade catalogues, it would be well in figuring the length of the rows to use the spacing between glass jars as given above, for the outside dimensions of the glass jars are usually stated.

With glass-jar batteries the aisle space between the rows should never be less than 24 inches, and aisles of 36 inches are recommended. In the case of double rows or where a row is installed near a wall or partition, the distance between rows, or the distance between a row and the wall, should not be less than 6 inches.

All batteries should be installed in one tier, if space is available, though glass-jar batteries (and occasionally lead-lined wood-tank batteries) can be installed

in two or three tiers, if more space is not available. In installing batteries in more than one tier, sufficient headroom should be allowed in each tier to permit the removal of the elements from the jars without displacing the jars. The minimum headroom for a single-tier glass-jar battery varies from 4½ feet to 5 feet; for two tiers from 5 feet to 7½ feet and for three tiers from 7 feet to 12 feet, depending upon the size of the battery. In all cases the ceiling of the battery room should be of sufficient height to allow for the passage of a man without stooping.

Spacing of Lead-lined Tanks. — Lead-lined tank batteries should always be installed in one tier, if space is available. More than one tier should not be installed without consulting with the battery manufacturer. With lead-lined tanks the distance between tanks along rows is from 2 to 2¼ inches. The aisle space between rows should never be less than 30 inches and should always be sufficient to allow the tank to be pulled out into the aisle for repairs. Where space will not allow the tank to be pulled out its full length the aisle should have at least sufficient width to allow the tank to be stood on end in the aisle. In double rows or in rows next to the wall, at least 18 inches should be allowed between rows or between the tanks and the walls so that a man can get in for examination. The battery room should have sufficient headroom for a man to walk upright. With large-sized batteries where the amount of gases liberated on overcharge is considerable, sufficient headroom should be allowed for proper ventilation.

Dimensions of Containers. — For any given type of plate, either in glass jars or lead-lined wood tanks, there is one dimension for all jars or tanks which is practically constant, independent of the number of plates in the cell. This is the dimension parallel to the horizontal edge of the plates. This dimension is usually specified in trade catalogues as the "width." The other dimension is the dimension across the row and is usually specified in trade catalogues as the "length."

Dimensions of Battery Room. — Keeping in mind these definitions one can determine from trade catalogues the approximate inside dimensions of a battery room necessary to house the battery in question. Note, however, that any manufacturer will gladly furnish a sketch showing an ideal layout for a given battery and such a layout should always be obtained from the manufacturer, if there is sufficient time. If there is available space in a building already built, the battery manufacturer will gladly prepare a sketch showing the layout of any given battery in the available space.

Insulation of Cells. — Cells in stationary batteries, whether in glass jars or in wood tanks, must be insulated from the ground and from each other.

Glass jars are usually mounted in glass or wooden sand trays which rest on glass insulators. The glass insulators, in turn, rest on wooden racks or stringers, these racks or stringers being mounted on vitrified brick set on the battery room floor.

For lead-lined wood-tank cells the standard practice at present is to mount each tank on a sufficient number of oil insulators. These insulators consist of a glass insulator provided with a circular trough partly filled with oil, this oil surrounding a central section which supports the tank. Over the insulator is placed a lead-alloy cap, designed to exclude spray or other foreign matter from the oil. This cap rests on the central section of the glass insulator, there being no connection between the tank and the glass except at this section. The oil around this central section of glass is for the purpose of preventing a film of acid from collecting thereon; if, by any chance, acid or water should get into the trough, it would sink to the bottom, the oil would float on top, and perfect insulation would be maintained.

The glass insulator rests upon a heavy earthenware truncated cone, having three feet. Between this cone and the glass is a Y-shaped lead washer. Thus three-point support is obtained between the floor and the earthenware, and between the earthenware and the glass. This form of support is not necessary between the glass and the tank, because the wood and lead cap take up any surface inequalities.

Connections between Cells. — In glass-jar batteries of small size, elements in adjacent cells are usually bolted together. In the larger-sized glass-jar batteries and in all lead-lined wood-tank batteries the positive plates in one cell are burned to the negative plates in the adjacent cell through the medium of a lead bus bar. The bus bars at the ends of rows, and wherever current taps are made, are reënforced by a bar of copper embedded in the lead.

Battery-room Design. — Though no battery should be installed without specific instructions from the manufacturer, the following points should receive special attention.

Floor. — The floor of the battery room should be made acidproof and should be graded to drain. A wood floor should never be used, as it is bound to become acid soaked and eventually be destroyed. For small batteries a cement floor can be used, but, unless it is kept thoroughly washed, acid will eat holes in it. A glazed tile floor is preferable.

Ventilation. — All battery rooms should be well ventilated. With small batteries natural ventilation is usually sufficient; with large-capacity batteries, where the gassing during overcharge is considerable, artificial ventilation should be provided. Exhaust ventilation is preferable to compression ventilation.

Temperature. — The temperature of the battery room should be kept as near 70° F. as possible.

Exposed Metal. — The amount of exposed iron or metal work in the battery room should be reduced to a minimum on account of the action of acid fumes upon it. Any iron or metal work that must be in the battery room should be protected with an acid-resisting paint.

OPERATION. — *Never operate any battery in any manner except in accordance with the instructions of the battery manufacturer furnishing the battery.*

With all large battery plants there should be supplied by the manufacturer the proper testing instruments and detail instructions for the operation of the battery under the conditions obtaining. The blank forms furnished by the manufacturer should be filled in at regular intervals and mailed to the manufacturer for analysis and recommendations.

No general instructions for the proper operation of a battery are given in this article, since the different manufacturers differ in their opinions on certain points of operation.

Printed instructions for operating any type of battery can be obtained free of charge from the manufacturer.

Attention from Operator. — The successful operation of a battery plant does not require much time from the operator, but attention given the battery should be systematic and absolutely in accordance with the instructions furnished by the manufacturer. If the manufacturer's instructions are followed implicitly and systematically, it will be found that the battery can be kept in good condition with a small amount of labor and time. If the operator neglects to follow the regular instructions and does not appeal to the manufacturer until he gets into trouble, the usual result will be considerable loss in time and money.

Addition of Water. — Pure distilled water, or natural water that has been analyzed and approved by the battery manufacturer, should be added to the cells from time to time to replace evaporation. If there is not available a source

of pure, natural water, and if distilled water cannot be purchased cheaply, a distilling outfit will usually be found necessary with large sizes of batteries. Standard water stills, for this purpose, can be purchased on the market.

Addition of Acid. — *Acid should never be added to a battery except upon the recommendation of the battery manufacturer.* The manufacturer should specify the specific gravity and the proper amount of acid.

REPAIRS. — Very few repairs, other than battery plates, should be required. For replacing the plates in the larger sizes of batteries, a lead-burning outfit and the necessary lead-burning material will usually be required. No one but a skilled lead burner should attempt to burn in plates.

Smaller sizes of batteries, with bolted connections between adjacent cells, can be purchased from the manufacturer with each element assembled.

Leaky Tanks. — A leaky tank should be repaired immediately. No one but an expert sheet-lead burner should be allowed to repair a leaky lead-lined tank. To repair the leaky lead-lined tank it will be necessary to cut loose from the bus bars all of the plates in the leaky tank and remove the tank into the aisle. If it is necessary to keep the rest of the battery in commission while the repairs are being made, the cell or cells cut out should be jumped by a jumper of sufficient current-carrying capacity.

FIRST COST. — It is impossible to give exact figures for the total cost of installation of a complete battery plant of any size. An analysis of the first costs of a great many plants now in operation shows that the first cost per kilowatt for the entire battery installation, exclusive of building, varies from $80.20 per kilowatt to $250.00 per kilowatt. The kilowatt rating as used here and elsewhere in this article is an arbitrary rating determined by multiplying the one-hour discharge rate in amperes (four times the normal rate) by double the number of cells.

ANNUAL COST. — The total annual cost includes operation, maintenance, depreciation and interest.

Operating Cost. — The operating cost will consist largely of labor. All material except necessary water should be included under maintenance.

Maintenance and Depreciation. — The cost of maintenance of a battery varies with the conditions. Other things being equal, the life of a battery is dependent upon the work done. The maintenance cost of a battery varies from 4 per cent to 10 per cent per annum. When the battery is used only for strictly emergency service the maintenance cost may be less than 4 per cent.

These figures cover not only maintenance, but also what is commonly included under the term depreciation. The plate being the essential component of the battery, plate renewals, which are made from time to time, involve all improvements that have been made in design and construction and serve to bring the battery up to date. An increase of voltage or capacity to meet changed conditions may be effected by the addition of cells or by increasing the number of plates in each cell, without sacrifice of the original investment. In the opinion of the writer there is, therefore, little, if any, true depreciation in a battery installation which is properly maintained.

BIBLIOGRAPHY. — Lyndon, L., *Storage Battery Engineering;* Wade E. J., *Secondary Batteries.*

BEARINGS. — (*See also Friction; Lubricants and Lubrication; Shafting.*) The common types of shaft bearings may be classified as follows:

Plain Cylindrical Bearings. — The usual form of bearing is a metal cylinder of cast iron, brass, bronze or gun-metal, mounted in a pedestal, bracket or frame, which serves as a support for the shaft. The portion of the shaft within the bearing is called the journal. Plain bearings are usually lined with some soft metal, such as babbitt metal or other white metal. In a self-aligning bearing the seating of the bearing in the pedestal is made a portion of a sphere, so that the bush may automatically align itself with the shaft. Cylindrical bearings are sometimes provided with an oil bath within the pedestal just below the bearing proper; rings or chains running loosely over the journal through slots in the bearing and dipping into this bath supply a steady stream of oil to the rubbing surfaces.

T. D. Lynch (Amer. Soc. Test. Mat., June, 1913) gives the following data in regard to plain Babbitt bearings: (1) The constituent metals of the alloy (*see Alloys*) must be of superior grade. (2) A temperature of 500° C. or less has been found sufficient for satisfactory alloying (3) A pouring temperature of 460° C. has been found to give excellent results. (4) The shell should be heated to 100–150° C. before the bearing metal is poured in. (5) Babbitted bearings must not be jarred during the solidification of the metal. (6) A Brinell hardness of 23.5 for lead-base and 30 for tin-base babbitts have been found to give excellent results.

Roller Bearings. — In this type of bearing the journal is supported on rollers which are in turn supported by smaller shafts in smaller bearings. The rollers support the journal of the main shaft.

Step Bearing. — A step bearing is essentially a large pivot bearing used at the end of vertical shafts carrying a heavy load. The end of the shaft is provided with a steel plate or "step," which rests on a second plate or bearing. The bearing proper may be either of the ball or roller type, or may be a plane surface with one or more grooves into which oil can be forced under pressure. The stationary plate is sometimes supported on short helical springs and is held against rotation by dowel pins (*H. G. Reist, A.S.M.E. Journ. 40, p. 393, May, 1918*).

Thrust Bearings. — Thrust bearings are used on horizontal shafts when the shaft is subjected to a horizontal thrust. They are similar to step bearings for vertical shafts, except that a collar fitting around the shaft is used instead of a plate at the end of the shaft. Steamship shaft bearings have numerous collars on the shaft, with thrust blocks between them. Roller or ball bearings may be used instead of collars.

Ball Bearings. — A ball bearing consists essentially of a track of curved cross-section which is filled with a set of balls. For a discussion of the use of ball bearings in electric machinery see H. N. Turnbull (*Elec. Rev. and West Elec. 70, p. 1054, 1917*).

Pivot Bearings. — Various types of pivot bearings are used. The pivot may be either a flat surface, a pointed cone, a truncated cone, a hemisphere or a surface of special form, such as Shiele's "tractrix."

ALLOWABLE LOADS AND SPEEDS. — If the pressure on a bearing is too great the oil film between journal and bearing surface will be destroyed and the bearing will overheat and "seize." Various formulas have been developed for the allowable pressure in terms of the speed and type of bearing.

Safe Loads, Line Shaft and Mill Bearings. — F. W. Taylor (*Trans. A.S.M.E., 1905*), as the result of an investigation of line shaft and mill bearings

that were running near the limit of durability and heating, yet not dangerously heating, gives the formula

$$p = \frac{400}{v},$$

where p = pressure in pounds per square inch of projected area (i.e., product of diameter of the journal by its length) and v = peripheral velocity of journal in feet per second.

The formula is applicable to bearings in ordinary shop or mill use on shafting which is intended to run with the care and attention which such bearings usually receive, and gives the maximum or most severe duty to which it is safe to subject ordinary *chain* or *oiled* ball- and socket-bearings which are *babbitted*. It is not safe for ordinary shafting to use *cast-iron boxes*, with either sight feed, wick feed or grease-cup oiling, under as severe conditions as $p \times v = 200$.

Safe Loads, Miscellaneous Bearings. — Alford in *Bearings and their Lubrication*, gives the following allowable bearing pressures in pounds per square inch of projected area.

Kind of bearing and condition of operation	Allowable bearing pressure in lb. per sq. in. of projected area	
United States Naval Practice		
Main engine bearings	275– 400	For weight alone.
Main engine crank pin bearings	400– 500	For weight alone.
Steam turbine bearings	85	For weight alone.
Thrust bearings for torpedo boats	50	For weight alone.
Merchant Marine Practice		
Main engine bearings	400– 500	
Main engine crank pin bearings	400– 500	
High-speed Stationary Engine Practice		
Main bearings	60– 120	For dead load.
Main bearings	150– 250	For steam load.
Crank pin bearings, overhung crank	900–1500	
Crank pin bearings, center crank	400– 600	
Cross-head pin bearings	1000–1800	
Slow Speed Stationary Engine Practice		
Main bearings	80– 140	For dead load.
Main bearings	200– 400	For steam load.
Crank pin bearings	800–1300	
Cross-head pin bearings	1000–1500	
Gas Engine Practice		
Main bearings	500– 700	
Crank pin bearings	1500–1800	
Cross-head pin bearings	1500–2000	
Electrical Machinery Practice		
Generator and motor bearings	30– 80	
Main engine bearings, driving generators	40– 80	
Horizontal steam turbine bearings	60– 400	
Vertical steam turbine steps	200–1000	

Kind of bearing and condition of operation	Allowable bearing pressure in lb. per sq. in. of projected area
MISCELLANEOUS PRACTICE	
Bearings for slow speed and intermittent load as in punch presses, shears and the like	3000–4000
Main bearings of slow speed pumping engines	600
Heavy line-shaft bearings, bronze or babbitt lined	100–150
Light line-shaft bearings, cast-iron	15–25
Heavy slow-speed step bearings	2000
Drill-press thrust collars	325
Angular-thrust bearing for boring mill tables	75

Safe Loads, Roller Bearings. — The following table gives the safe load in pounds for Mossberg roller bearings (*Trans. A.S.M.E., 1905*). D = diameter of journal, in inches; d = diameter of roll, in inches; N = number of rolls; P = safe load on journals, in pounds. The rolls are enclosed in a bronze supporting cage.

D	d	N	P	D	d	N	P	D	d	N	P
2	1/4	20	3,500	6	11/16	24	50,000	15	1 3/8	28	255,000
2 1/2	5/16	22	7,000	7	13/16	22	70,000	18	1 3/8	32	325,000
3	3/8	22	13,000	8	7/8	22	90,000	20	1 1/2	34	400,000
4	7/16	24	24,000	9	1	24	115,000	24	1 1/2	38	576,000
5	9/16	24	37,000	12	1 1/4	26	175,000	..	...	..	....

Surface speed of journal from 0 to 50 feet per minute. Length of journal 1½ diameters. The rolls are made of tool steel not too high in carbon, and of spring temper. The journal or shaft should be made not above a medium spring temper. The box should be made of high-carbon steel and tempered as hard as possible.

Marks in his "Mechanical Engineers' Handbook" gives the following formulas for load capacities of roller bearings.

"For rollers such as are used on bridge turn tables and in similar places where the speeds of rotation are slow, the safe load in pounds is $P = cnld$, in which n = number of rollers in the bearing, l and d the length and diameter of each roller, in., $c = 360$ for hard cast-iron rollers running on hard cast-iron tracks, and $c = 850$ for hard steel rollers on hard steel tracks. Where $l > 5d$, smaller values of c should be used. In case conical rollers are used, d is the mean diameter.

"The Standard Roller Bearing Company determines load capacities of roller bearings by the formula $P = 130{,}000d^2 nl/3s$, in which P = load on bearing, lb.; d = diam. of roller, in.; n = number of rollers; l = length of each roller, in.; and s = circumferential speed of each roller, ft. per min. In bearings with conical rollers d is the diameter of the roller at its mid-length. The safe load per inch of length of a solid roller is taken at 2000 lb., with the assumption that one-third the number of rollers take the whole load on the journal.

"Another formula for the safe load on a roller bearing is $P = kld^2/(ND + 2000d)$ in which N = r.p.m. of the shaft; D = diam. of the sleeve or roller path, in.; and k = 1,200,000 to 2,000,000 for first class workmanship, hardened steel rollers with $l = d$, running on hardened ground surfaces; k = 400,000 for ordinary workmanship and soft steel rollers running on a soft steel shaft."

Safe Loads, Ball Bearings. — The following formula is given by Mr Henry Hess, 1910. See also paper by Mr. Hess in *Trans. A.S.M.E.*, 1907.

Let

W = total safe load on bearing in pounds,
n = number of balls,
d = diameter of balls in *eighths of an inch*.

Then for *radial* (cylindrical) bearings

$$W = Knd^2,$$

where K ranges from 0.9 to 9 depending upon the condition and type of bearing and the hardness of the balls, but for ordinary speeds is practically independent of the speed. (*See Kent's Mechanical Engineers' Pocket-Book.*)

For *thrust* bearings

$$W = \frac{K_1 nd^2}{\sqrt[3]{N}},$$

where N is the number of revolutions per minute, which may range from 3000 down to 1 revolution per minute, as for crane hooks and similar elements. For high quality ball K_1 ranges from 25 to 40 for a race having a cross section of radius 1.66 × (radius of balls). For unhardened steel, occasionally used for very large races, $K_1 = 0.5$.

In both types of bearings the balls must be carefully selected to make sure that all that are used in the same bearing do not vary among one another by more than 0.0001 inch. A ball that is more than that larger than its fellows will sustain more than its proportion of the load, and may therefore be overloaded and will in turn overload the races.

FRICTION OF BEARINGS. — The coefficient of friction of a well-lubricated plain cylindrical bearing is practically independent of the projected area of the journal, the pressure per unit area remaining constant, and in the case of plain bearings is also practically independent of the nature of the rubbing surfaces, provided these are smooth.

Friction of Plain Bearings. — The coefficient of friction of plain bearings however, does depend to a very great extent upon the following factors:

1. The method of lubrication.
2. The nature of the lubricant.
3. The temperature of the lubricant.
4. The peripheral velocity of the journal.
5. The pressure of the journal on the bearing surface.

1. The more perfectly the bearing is bathed in oil the less will be the coefficient of friction. The friction coefficient of a scantily lubricated bearing may be from 6 to 10 times the coefficient when an oil bath is used.

2. As a rule the lower the viscosity of an oil the less will be the coefficient of friction. There are other factors, however, which must be taken into account in selecting the proper lubricant for any service (*see article on Lubricants and Lubrication*).

3, 4 and 5. The variation of the friction coefficient with temperature, speed and pressure is shown by the curves in Fig. 1. The pressures are nominal

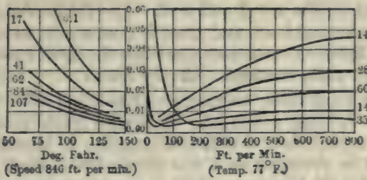

Fig. 1. Numbers at end of curves are pressures in lb. per sq. in.

pressures, i.e., pounds per square inch of projected area of journal. These curves are taken from a paper by Stribeck (*Zeit. Ver. Deutsch. Ing., 1902, Vol. 46s p. 1341*), and are test results on a Sellers bearing with oil rings; journal 13 inches long, 2.75 inches in diameter, lubricated with "gas motor oil." These curves are typical of ordinary bearings.

POWER LOST IN BEARINGS. — Let

f = coefficient of friction,
W = weight on journal or pivot in pounds,
r = radius in inches,
d = diameter in inches,
S = space in feet through which sliding takes place per minute.
r_1 = inner radius in inches,
r_2 = outer radius in inches,
n = number of revolutions per minute,
a = the half-angle of the cone, i.e., the angle of the slope with the axis.

Type of bearing	Friction torque in in.-lb.	Power loss ft.-lb. per min.
Flat surfaces	....	fWS
Shafts and journals	$\frac{1}{2}\ fWd$	$0.2618 fW\ dn$
Flat pivots	$\frac{2}{3} fWr$	$0.349 fWrn$
Collar-bearing	$\frac{2}{3} fW \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$	$0.349 fWn \frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$
Conical pivot	$\frac{2}{3} fWr \operatorname{cosec} a$	$0.349 fWrn \operatorname{cosec} a$
Conical journal	$\frac{2}{3} fWr \sec a$	$0.349 fWrn \sec a$
Truncated-cone pivot	$\frac{2}{3} fW \frac{r_2^3 - r_1^3}{r_2 \sin a}$	$0.349 fWn \frac{r_2^3 - r_1^3}{r_2 \sin a}$
Hemispherical pivot	fWr	$0.5236 fWrn$
Tractrix, or Schiele's "anti-friction" pivot	fWr	$0.5236 fWrn$

It should be noted that for peripheral speeds of 100 feet and over the coefficient of friction is approximately proportional to the square root of the peripheral velocity of the journal. These particular curves also indicate that the coefficient of friction is approximately inversely proportional to the square root of the pressure. Other experimenters, however, have found that this variation is inversely as the first power of the pressure.

Friction of Roller and Ball Bearings. — The friction coefficient of a well-made annular ball bearing is 0.001 and 0.002 of the total load on the journal

and is independent of the speed and load. The friction coefficient of a good roller bearing under normal loads and speed is from 0.0035 to 0.014; it rises very much if the load is light. It increases also when the speeds are very low, though not so much as with plain bearings. (*Henry Hess.*)

Lubrication is absolutely necessary with ball-and-roller bearings, although the contrary claim is often advanced. Under favorable conditions an almost imperceptible film is sufficient; a sufficient quantity to immerse half the lowest ball should always be provided as a rust preventive. Rust and grit must be kept out of ball-and-roller bearings. Acid or rancid lubricants are as destructive as rust. (*Henry Hess.*)

Comparison of Plain, Roller and Ball Bearings. — Thomas, Maurer and Kelso (*A.S.M.E. Journ. 36, p. 89, March, 1914*), give the following relative power consumption, based on tests of 20 bearings of each type:

Bearings	100 ft. per min.		300 ft. per min.	
	77° F.	100° F.	77° F.	100 F°.
Ball	1.0	1.0	1.0	1.0
Roller	2.2	2.5	2:7	3.0
Plain Babbitt	3.0	3.6	4.5	4.0

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BELTS AND BELTING. — (*See also Ropes and Rope Drive.*) Belts are usually made of tanned leather, though rubber belts are frequently employed, as well as various woven fabrics. Rubber belts are made of two or more layers of canvas connected together with a rubber composition, and then heated until the rubber vulcanizes. The "ply" of a rubber belt is the number of layers of canvas. Leather belts are referred to as single, light-double, medium-double, standard-double and 3-ply. Wire belts with paper driving surface have recently been developed in Germany (*Zeits. Vereines Deutsch. Ing. 63, p. 1057, Oct. 1919*).

Thickness of Belts. — The following figures are from a paper by Samuel Webber (*Am. Mach., May 11, 1909*). The thickness of leather belts, however, is variable, depending on the hide and process of manufacture.

BELT THICKNESS, INCHES

Leather		Rubber	
		30-oz. duck, new rubber vulcanized	
Single thickness	1/6	3-ply	0.18
Light-double	1/4	4-ply	0.24
Medium-double	5/16	5-ply	0.30
Standard-double	1/3	6-ply	0.35
3-ply	9/16	7-ply	0.40
		8-ply	0.45

Weight. — The average weight of leather per cubic inch is 1/30 pound (*Barth*). The average weight of rubber belting made of 30-ounce duck is about 0.045 pound per cubic inch.

Strength. — The strength of the solid leather in belts is from 2000 to 5000 pounds per square inch; at the lacings, even if well put together, only from 1000 to 1500. If riveted, the joint should have half the strength of the solid belt. Rubber belts have approximately the same tensile strength as leather belts. The working tension on the driving side is generally taken at not over one-third of the strength of the lacing, or from one-eighth to one-sixteenth of the strength of the solid belt.

Belt Tension. — Let

T_1 = actual, or "total," tension in pounds per square inch on driving side of belt,
T_2 = actual tension in pounds per square inch on slack side of belt,
T_0 = "initial" tension in belt, i.e., the tension in each side when the belt is at rest,
$T = T_1 - T_2$ = effective tension in pounds per square inch, i.e., T corresponds to the force actually driving the driven pulley,
T_c = the tension produced in the belt due to centrifugal action as it goes around the pulleys,
m = weight of belt per cubic inch in pounds,
f = coefficient of friction between belt and pulley,

n = ratio of arc of contact between belt and pulley to an arc of 180°, i.e., if the arc of contact is 180°, $n = 1$,

$C = 1 - \epsilon^{-nf\pi}$. The value of $\epsilon^{-nf\pi}$ may be found directly from the table in the article on *Exponential Functions*, putting $nf\pi = x$,

V = velocity of belt in feet per minute.

Then

$$T_c = \frac{mV^2}{9660}, \tag{1}$$

$$T = C(T_1 - T_c), \tag{2}$$

$$T_2 = T_1 - C(T_1 - T_c), \tag{3}$$

$$T_0 = \left(\frac{\sqrt{T_1} + \sqrt{T_2}}{2}\right)^2 \text{ approximately.*} \tag{4}$$

Barth recommends that for belts running at various speeds the initial tension T_0 should not be the same in all belts, but that the belts should be tightened initially to such a tension that under load the sum

$$A = T_1 + \tfrac{1}{2} T_2 \tag{5}$$

is a constant. Barth also states, basing his conclusion in part on the studies of the economics of belting made by F. W. Taylor (*Trans. A.S.M.E., Vol. 15, p. 204*), that the best values of A for greatest economy are the following:

	Machine belts	Counter-shaft belts
Maximum value of A, belt first put on	320	240
Minimum value of A, belt to be retightened	240	160

These values of A are for full-load conditions. The corresponding values of the various tensions are given below.

Belt Friction and Belt Slip. — The value of the factor C depends upon the coefficient of belt friction, which is subject to considerable variation, depending upon the condition of the belt and pulleys. Barth gives the following formula for f for leather belts on cast-iron pulleys

$$f = 0.54 - \frac{140}{500 + V}.$$

Barth also gives a formula for the belt slip which may be written

$$s = \frac{V_1 - V_2}{V} = \frac{320}{V}\left(\frac{1 + 0.0055\,V}{85 + 0.03\,V}\right),$$

where V_1 and V_2 are the peripheral velocities of the driving and driven pulleys respectively.

* See paper by C. G. Barth, *Trans. A.S.M.E.*, 1909, Vol. 31, p. 29. In this paper Barth also gives a more exact formula, and takes into consideration the difference in the relation between T_1, T_2 and T_0 for vertical and horizontal belts.

Power Loss Due to Belt Slip is proportional to the slip, i.e., if the slip is 0.03 the power lost is 3 per cent of the power transmitted by the belt. Under normal conditions the power loss due to belt slip is from 2 to 4 per cent. The power loss in mill shafting due to the friction of the shaft bearings ranges from 10 to 60 per cent, depending upon the type of bearing, belt tension, etc., See article on *Bearings*.

Power Transmitted by Belt. — Let

V = velocity of belt in feet per minute,
w = width of belt, in inches,
t = thickness of belt, in inches,
m = weight, in pounds, of 1 cubic inch of belt,
T_1 = actual tension in driving side of belt in pounds per square inch of belt cross-section,
s = slip of belt as a fraction of belt speed.

Then the horse-power transmitted by the belt is

$$P = \frac{wtCVT_1}{33,000}\left(1 - \frac{mV^2}{9660\,T_1}\right),$$

where C, defined above, depends on the belt friction and arc of contact between belt and pulley.

As a rough approximation C may be taken equal to 0.6, and the second term in the bracket zero, at least for low speeds, and the formula then becomes

$$P = \frac{wtVT_1}{55,000},$$

or the horse-power transmitted per square inch of belt cross-section is

$$p = \frac{VT_1}{55,000}.$$

The power lost in the transmission from one pulley to the other, excluding the loss due to shaft friction, is sP, the driving pulley absorbing approximately $(P + 0.5\,sP)$ horse-power and the driven pulley receiving $(P - 0.5\,sP)$ horse-power.

The following tables are derived from curves in the paper by Barth above referred to.

HORSE-POWER TRANSMITTED BY BELTS DRIVING MACHINES
(For $A = T_1 + 0.5\,T_2 = 240$)

Velocity, ft. per min.	500	1000	2000	3000	4000	5000	6000
Initial tension, T_0	124	120	121	128	136	144	152
Centrifugal tension T_c	0+	3	13	31	56	86	124
Difference, $T_0 - T_c$........	123	117	108	97	80	58	28
Tension on tight side, T_1...	210	212	211	207	198	187	173
Tension on slack side, T_2...	60	54	57	68	84	107	134
Effective pull, $T_1 - T_2$	150	158	154	139	114	80	39
Sum of tensions $T_1 + T_2$....	270	268	269	274	282	294	307
H.p. per sq. in. of section..	2.27	4.79	9.33	12.64	13.82	12.12	7.09

HORSE-POWER TRANSMITTED BY BELTS DRIVING COUNTER-SHAFTS

(For $A = T_1 + 0.5\ T_2 = 160$.)

Velocity of belt, ft. per min.	500	1000	2000	3000	4000	5000
Initial tension, T_0..........	82	81	83	89	96	102
Tension on tight side, T_1...	140	141	140	134	125	114
Tension on slack side, T_2...	40	38	41	53	69	92
Effective pull, $T_1 - T_2$.....	100	103	99	81	56	22
Sum of tensions	180	179	181	187	194	206
H.p. per sq. in. of section...	1.51	3.12	6.04	7.36	6.79	3.33

Proper Size of Belt to Transmit a Given Amount of Power. — Using the same notation as in the preceding paragraph, the proper cross-section of the belt in square inches is approximately

$$wt = \frac{55,000\, P}{VT_1},$$

or exactly

$$wt = \frac{33,000\, P}{CVT_1\left(1 - \dfrac{mV^2}{9660\, T_1}\right)}.$$

The various "handy formulas" for determining the proper cross-section of belt are all of the form of the approximate formula, and differ only in regard to the proper value to assign to the tension T_1. A working tension, that is the difference between the tensions on the tight and slack sides of the belt, of 45 pounds for a single (thickness) belt 1 inch wide is commonly used, giving a nominal tension of $6 \times 45 = 270$ pounds per square inch. Taking the coefficient C at 0.6, the actual tension in a belt designed in accordance with this rule is 450 pounds per square inch. The value of C varies through a wide range, say from 0.25 to 0.90, chiefly due to the variability of the coefficient of friction.

The following table, deduced from Barth's curves, may be considered as representative of modern practice.

CROSS-SECTION OF LEATHER BELT PER HORSE-POWER TRANSMITTED, IN SQUARE INCHES

Speed, ft. per min.	500	1000	2000	3000	4000	5000	6000
Machine belts.....................	0.44	0.209	0.107	0.079	0.072	0.082	0.141
Countershaft belts.................	0.662	0.320	0.166	0.136	0.147	0.300	.....

All belts should have a contact area with the smaller pulley of at least 165 degrees. If this is not possible to obtain under normal conditions an idler should be provided to increase the belt contact.

Pulley Face and Belt Speed. — The following table may be used for determining the pulley face and best belt speed for moderate-speed machines. For slow-speed machines the pulley sizes are the same as those used with moderate machines of the same frame dimensions.

PULLEY FACE AND BELT SPEED

Horse-power	Kilowatts	Belt width in inches	Pulley face in inches	Belt speed in feet per minute for moderate-speed machines
1 to under 5	1 to under 4	2	2½	2000
5 " 10	4 " 8	3	3½	2500
10 " 15	8 " 11	4	4½	2500
15 " 20	11 " 15	5	6	2500
20 " 25	15 " 19	6	7	3000
25 " 30	19 " 23	7	8	3000
30 " 40	23 " 30	8	9	3000
40 " 50	30 " 37	10	11	3000
50 " 60	37 " 45	12	13	3000
60 " 75	45 " 56	14	15	3000
75 " 100	56 " 75	16	17	3500
100 " 125	75 " 93	16	17	4000
125 " 150	93 " 112	18	19	4000
150 " 200	112 " 159	20	21	4500
200 " 250	159 " 186	22	23	5000
250 " 300	186 " 224	26	27	5000
300 " 350	224 " 261	30	31	5000
350 " 400	261 " 298	32	33	5000
400 " 450	298 " 336	36	37	5000
450 " 550	336 " 410	40	42	5000
550 " 650	410 " 485	44	46	5000
650 " 750	485 " 560	48	50	5000
750 " 850	560 " 634	52	54	5000
850 " 950	634 " 709	56	58	5000
950 " 1050	709 " 783	60	62	5000

Relations of Driving and Driven Pulleys. — The maximum ratio between the diameters of the driving and driven pulleys should not exceed 6 : 1. The ratio will determine the distance between the pulley centers, and good proportions are given in the accompanying table.

CARE AND OPERATION OF BELTS. — The following is taken from an article by F. W. Taylor (*Trans. A.S.M.E., Vol. 15, p. 204*). See also an excellent article by C. J. Morreson (*Eng. Mag. 51, p. 567, July, 1916*).

The best distance from center to center of shafts is from 20 to 25 feet.

Idler pulleys work most satisfactorily when located on the slack side of the belt about one-quarter way from the driving pulley.

Belts are more durable and work more satisfactorily made narrow and thick, rather than wide and thin.

It is safe and advisable to use a double belt on a pulley 12 inches diameter or larger; a triple belt on a pulley 20 inches diameter or larger; a quadruple belt on a pulley 30 inches diameter or larger. As belts increase in width they should also be made thicker.

Approximate Ratio	Minimum Distance Between Centers in Feet
2:1	8
3:1	10
4:1	12
5:1	15
6:1	20

The ends of the belt should be fastened together by splicing and cementing instead of lacing, wiring or using hooks or clamps of any kind.

Belts should be cleaned and greased every five to six months.

Belt-clamps having spring-balances between the two pairs of clamps should be used for weighing the tension of the belt accurately each time it is tightened.

Double leather belts, when treated with great care and run night and day at moderate speed, should last for 18 years (the tension being adjusted in accordance with Barth's rules).

In figuring the total expense of belting, and the manufacturing cost chargeable to this account, by far the largest item is the time lost on the machines while belts are being relaced and repaired.

The total stretch of leather belting exceeds 6 per cent of the original length.

The stretch during the first six months of the life of belts is 15 per cent of their entire stretch (the tension being adjusted in accordance with Barth's rules).

A double belt will stretch 0.81 per cent of its length before requiring to be tightened (the tension being adjusted in accordance with Barth's rules).

The most important consideration in making up tables and rules for the use and care of belting is how to secure the minimum of interruptions to manufacture from this source.

The average double belt when running night and day in a machine shop will cause an interruption to manufacture not oftener than once in sixteen months (the tension being adjusted in accordance with Barth's rules).

The oak-tanned and fulled belts showed themselves to be superior in all respects except the coefficient of friction to either the oak-tanned not fulled, the semi-rawhide, or rawhide with tanned face.

Belts of any width can be successfully shifted backward and forward on tight and loose pulleys. Belts running between 5000 and 6000 feet per minute and driving 300 h.p. are now being daily shifted on tight and loose pulleys to throw lines of shafting in and out of use.

The best form of belt-shifter for wide belts is a pair of rollers twice the width of belt, either of which can be pressed onto the flat surface of the belt on its slack side close to the driven pulley, the axis of the roller making an angle of 75° with the center line of the belt.

Dressings for Leather Belts. — We advise that no belt dressing should be used except when the belt becomes dry and husky, and in such instances we recommend the use of a dressing. Where this is not used beef tallow at blood-

warm temperature should be applied and then dried either by artificial heat or the sun. The addition of beeswax to the tallow will be of some service if the belts are used in wet or damp places. Our experience convinces us that resin should never be used on leather belting. (*Fayerweather and Ladew.*)

Some forms of belt dressing, the compositions of which have not been published, appear to have the property of increasing the coefficient of friction between the belt and the pulley, enabling a given power to be transmitted with a lower belt tension than with undressed belts. C. W. Evans (*Power, Dec., 1905*) gives a diagram, plotted from tests, which shows that three of these compositions gave increased transmission for a given tension, ranging from about 10 per cent for 90 pounds tension per inch of width to 100 per cent increase with 20 pounds tension.

Dressings for Rubber Belts. — Rubber belts will be improved, and their durability increased, by putting on with a painter's brush, and letting it dry, a composition made of equal parts of red lead, black lead, French yellow and litharge, mixed with boiled linseed oil and japan enough to make it dry quickly. The effect of this will be to produce a finely polished surface. If, from dust or other cause, the belt should slip, it should be lightly moistened on the side next the pulley with boiled linseed oil. (*From circulars of manufacturers.*)

Lacing of Belts. — In punching a belt for lacing, use an oval punch, the longer diameter of the punch being parallel with the sides of the belt. Punch two rows of holes in each end, placed zigzag. In a 3-inch belt there should be four holes in each end — two in each row. In a 6-inch belt, seven holes — four in the row nearest the end. A 10-inch belt should have nine holes. The edge of the holes should not come nearer than ¾ inch from the sides, nor ⅞ inch from the ends of the belt. The second row should be at least 1¾ inches from the end. On wide belts these distances should be even a little greater.

Begin to lace in the center of the belt and take care to keep the ends exactly in line, and to lace both sides with equal tightness. The lacing should not be crossed on the side of the belt that runs next the pulley. In taking up belts observe the same rules as in putting on new ones.

Setting a Belt on Quarter-twist. — A belt must run squarely on to the pulley. To connect with a belt two horizontal shafts at right angles with each other, say an engine-shaft near the floor with a line attached to the ceiling, will require a quarter-turn. First, ascertain the central point on the face of each pulley at the extremity of the horizontal diameter where the belt will leave the pulley, and then set that point on the driven pulley plumb over the corresponding point on the driver. This will cause the belt to run squarely on to each pulley, and it will leave at an angle greater or less, according to the size of the pulleys and their distance from each other.

In quarter-twist belts, in order that the belt may remain on the pulleys, the central plane on each pulley must pass through the point of delivery of the other pulley. This arrangement does not admit of reversed motion.

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BLOCKS AND TACKLE. — (*See also Ropes and Rope Drive; Chain and Chain Drive.*) A simple block used with rope tackle consists of one or mor pulleys mounted in a casing or shell. They may be single, double, triple, etc. depending upon the number of sheaves or pulleys. In chain blocks, instead o simple pulleys, spur or worm and wheel gearing (*see Gears and Gearing*) o differential pulleys are used. In the triplex block, made by the Yale and Town Co., the power is transmitted to the hoisting-chain wheel by means of a trai of spur gearing operated by the hand chain. In the duplex block the powe is transmitted through a worm wheel and screw.

Fig. 1 shows schematically a differential pulley. The two upper pulleys ar rigidly fastened together and rotate as one piece. The rims of these pulleys ar shaped to mesh with the links of the chain and prevent the latter from slipping. Let D_1 and D_2 be the respective diameters of the larger and smaller upper pulleys. Then if the diameter of the lower pulley is $(D_1 + D_2)/2$, the pull P' required to raise a load W, neglecting friction, is

$$P' = \frac{D_1 - D_2}{2D_1} \cdot W.$$

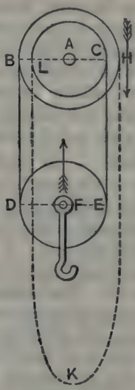

Fig. 1.

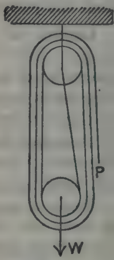

Fig. 2.

ACTUAL PULL REQUIRED; EFFICIENCY OF BLOCK AND TACKLE. — In the case of a simple block and tackle the pull P' required to raise a load W is, neglecting friction,

$$P' = \frac{W}{N},$$

where N is the number of lengths of rope shortened when the lower block rises (in Fig. 2 $N = 6$). The actual pull required is this theoretical pull P' divided by the efficiency expressed as a fraction or

$$P = \frac{100\,W}{Ne},$$

where e = the per cent efficiency. The distance in feet through which th pulling rope must be pulled to raise the load 1 foot is equal to the number (N of lengths of rope shortened.

In the case of a differential pulley the actual pull is

$$P = \frac{100\,(D_1 - D_2)\,W}{2D_1e},$$

where e is the per cent efficiency and the other symbols as above. The distanc through which the hand chain must be pulled to raise the load 1 foot is

$$\frac{2D_1}{D_1 - D_2}.$$

In any case, if the rope or chain pulls on the lower block at an angle, the block will be pulled out of the line drawn between the load and the upper block, and the effective pull will be less than the actual pull on the rope in the ratio of th cosine of the angle the pulling rope makes with the vertical, or line of action o the resistance, to unity.

Efficiency of Rope Blocks. — S. L. Wonson, *Eng. News*, June 11, 1903, gives the following:

Number of rope lengths shortened, N	Manila rope, 1¼ to 2 in.		¾-inch wire rope	
	Ratio of load to pull	Efficiency, per cent, e	Ratio of load to pull	Efficiency, per cent, e
2	1.91	96	....	..
3	2.64	88	2.73	91
4	3.30	83	3.47	87
5	3.84	77	4.11	82
6	4.33	72	4.70	78
7	4.72	67	5.20	74
8	5.08	64	5.68	71
9	5.37	60	6.08	68
10	....	..	6.46	65
12	....	..	7.08	59

Pull Required with Chain Blocks. — The following table is taken from a catalogue of the Yale and Towne Co.

Capacity in tons, (2000 lb.)	Pull in pounds required on hand-chain to lift full loads			Feet of hand-chain to be pulled by operator to lift load one foot high		
	Triplex	Duplex	Differential	Triplex	Duplex	Differential
¼	...	...	72	...	...	18
½	62	68	122	21	40	24
1	82	87	216	31	59	30
1½	110	94	246	35	80	36
2	120	115	308	42	93	42
3	114	132	557	69	126	38
4	124	142	...	84	155	..
5	110	145	...	126	195	..
6	130	145	...	126	252	..
8	135	160	...	168	310	..
10	140	160	...	210	390	..

For loads from 10 to 20 tons two hand-chains are provided.

SAFE LOAD. — Blocks are usually designed to carry a load as great as that of the rope or chain which fits the grooves in the sheaves (*see Ropes and Rope Drive; Chains and Chain Drive*).

BIBLIOGRAPHY. —See the works on *Machine Design* given in the Bibliography in article on *Bearings;* also circulars of Yale & Towne Mfg. Co., N. Y., and Boston & Lockport Block Co.

BLOWERS AND COMPRESSORS. — (*See also Draft, Mechanical; Fans.*) A blower is any kind of apparatus that is used for blowing or forcing air or other gas into or out of a room or other receptacle. When it is used to draw air from a room or vessel and discharge it into another vessel or into the external atmosphere it is commonly called an "exhauster." A fan is a blower in which motion is given to the air by means of thin blades or vanes (*see Fans*). A blowing engine is a large machine in which air is compressed in and discharged from a cylinder which is fitted with a reciprocating piston. When a blowing engine delivers air at high pressures, say 30 pounds per square inch and upwards, it is called an air compressor. Following is a brief description of the several kinds of blowers.

Displacement Blowers. — Into a closed tank containing air a stream of water is caused to flow, displacing the air and blowing it out through an opening provided for it. By using two tanks with suitable valves, the action may be made continuous.

Hydraulic Blowers. — A large tank is surmounted by a vertical pipe into the top of which water enters in such a way as to "entrain" or drag air along with it, which is separated from the water when it reaches the tank and is blown out through an air pipe while the water escapes through another opening. Tests of a compressor of this kind at Magog, P. Q., Canada, showed a capacity of from 967 to 1165 cubic feet of air per minute at a pressure of 53 pounds per square inch, and an efficiency of from 60 to 70 per cent. See paper by W. O. Webber, *Trans. A.S.M.E.*, Vol. 22, p. 599.

Jet Blowers. — A jet of water, compressed air, or steam flowing into a short pipe of a diameter greater than that of the jet will induce a current of air to flow through the pipe, which may thus be used as a blower or exhauster. The steam jet is in common use for blowing air into furnaces and gas producers, and is used as an exhauster to increase the draft of chimneys. It is usually very wasteful of steam, and should be used only in emergencies, when blowing machines are not available, or when the steam used to drive the jet is also used for other purposes, as for combining with the carbon of the fuel in gas producers.

Positive Rotary Blowers. — These are built like rotary pumps, with two rotating shafts, geared together so as to turn in opposite directions, each carrying an "impeller," a casting approximating a figure 8 in section; the curve of the two impellers is so shaped that they touch or nearly touch on lines parallel to the shafts during the whole period of rotation, and also nearly touch the casing which surrounds them. A blower thus built delivers a constant quantity of air at each revolution, differing in this respect from centrifugal fans which deliver a varying quantity according to the resistance the air meets at the outlet. The economical range of these blowers is between 8 ounces and 8 pounds pressure per square inch. For higher pressures then 8 pounds the blowing engine is more economical.

Blowing Engines. — A cylinder with a reciprocating piston when used for blowing air is called a blowing engine, whether it is driven by a steam engine, a water wheel or an electric motor. Blowing engines are commonly used to furnish air to blast furnaces and Bessemer converters, at pressures of from 4 to 30 pounds per square inch. They are simple in construction, but are usually large, heavy and costly relative to the quantity and pressure of air furnished by them, on account of their moderate speeds, and there is a tendency to supplant them by

High-speed Centrifugal Compressors. — These are built on the principle of steam turbines or high-pressure centrifugal pumps and are driven at very high

speeds, 1800 to 3500 r.p.m. The General Electric Co. makes a line of these compressors for pressures ranging from 1 to 3.25 pounds per square inch, and for delivering 800 to 28,000 cubic feet of free air per minute. By using several such compressors in series, one delivering into another, pressures of 100 pounds per square inch may be obtained.

POWER AND EFFICIENCY. — The useful work done by a blower is that equivalent to the isothermal compression of the air to the pressure in the receiver (or to the static plus the velocity pressure in the delivery main if the air were cooled to the atmospheric temperature) plus the work of delivering it against that pressure. If P_1 = absolute pressure at inlet in pounds per square foot; P_2 = pressure in the receiver; V_1 = volume of entering air at the pressure P_1, in cubic feet per minute, then useful work in foot-pounds per minute = $P_1 V_1 \log_e \frac{P_2}{P_1}$. The useful or "air horse-power" is this quantity divided by 33,000.

When P_2 is but slightly in excess of P_1 the work per minute is $(P_2 - P_1)V_1$ nearly, the error being less than 5 per cent when $P_2 = 1.1\ P_1$, and less than 1 per cent when $P_2 = 1.01\ P_1$.

When the air is compressed adiabatically, that is, without cooling during compression, the *useful* work is the same as in isothermal compression, but the actual work done in the cylinder (not including work lost by friction and leakage) is greater, or

$$3.463\, P_1 V_1 \left\{ \left(\frac{P_2}{P_1}\right)^{0.29} - 1 \right\}.$$

The efficiency of blowing machines (i.e., the ratio of the air horse-power to the horse-power required to drive the machine) ranges from 95 per cent in the case of a slow-moving engine with large inlet and outlet valves, down to almost zero in poorly designed rotating machines. Centrifugal compressors have an efficiency under normal load of from 40 to 60 per cent (*see Fans*). A 39 by 84-inch positive blower, made by the Connersville Blower Co., is reported to have given an efficiency ranging from 68.5 per cent at an air pressure of 0.5 pound per square inch to 86 per cent at a pressure of 3.5 pounds per square inch, the displacement of air in each case being approximately 18,000 cubic feet per minute. The efficiency of steam-jet blowers is very low.

SPECIFICATION FOR BLOWER. — The following memoranda are intended to assist in writing specifications. See also under ***Fans*** and article on *Specifications*.

Principal Characteristics and Conditions of Service.—Purpose of blowers.

Style and Description; Details of Construction. — Shall have horizontal, upward or downward discharge. Location of air outlet with reference to rotating parts. Hand or automatic closing of dampers when blower stops. How coupled to motor or engine, i.e., direct or belted. If direct driven, give details of coupler to disconnect blower from motor. Lubrication. Tools. Enclosure of moving parts to reduce danger.

Performance and Tests. — Cubic feet of air per minute. Air pressure, ounces or inches of water column at stated temperature.

ELECTRIC DRIVE OF COMPRESSORS. —(*See also Motors, Industrial Applications of.*) As compressors of the reciprocating type must be driven at a low speed, the synchronous motor is well adapted for driving them, chiefly because this motor operates normally at unity power factor and can be efficiently operated at leading power factors to counteract the lagging power factor

due to induction motors driving other apparatus on the system. Slow-speed induction motors must be built with a large number of poles and will then operate at very low power factors with a comparatively low efficiency. Other reasons favoring the synchronous motor drive for this service are the steady load and the new torque required in starting, as a by-pass is generally provided so that the compressor works only against atmospheric pressure when starting. The motor should be able to be started from the alternating-current side and should for this reason be provided with the usual "squirrel-cage" winding. There is usually sufficient flywheel effect in the machine itself to keep within a reasonable angle of displacement, so that no trouble is experienced from "hunting."

The centrifugal type of compressor operates at a comparatively high speed. For driving the smaller size compressors the squirrel cage induction motor is used, but for larger units where a low starting current is desirable the phase wound type of motor should preferably be selected. When design difficulties prevent the use of the induction motor for driving the larger size centrifugal compressors it has been the recent practice to use two-pole, 60-cycle synchronous motors built along the line of the turbo-alternator.

For direct-current installations compound-wound motors are generally used for driving reciprocating compressors and shunt-wound motors for the centrifugal type.

DIMENSIONS, WEIGHTS AND COSTS. — There is so great a variety of blowers and compressors that it is impossible in the space available to give a representative table of dimensions, weights and costs. The reader is referred to the manufacturers' catalogues and price lists. Representative manufacturers are the Ingersoll-Rand Co., the American Blower Co., the Sturtevant Co., and the Connersville Blower Co. (*See also Kent's Mechanical Engineers' Pocket-book.*)

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BOILERS, STEAM. — (*See also Chimneys; Condensers, Steam; Conveyors; Draft, Mechanical; Feed-water Heaters; Fuels; Pipes and Piping; Power Stations; Pumps; Separators, Steam; Steam.*) The simplest form of boiler is a closed metal cylinder partly filled with water, with a pipe for the introduction of feed water, another pipe for the escape of steam and means for maintaining a fire under the boiler. Modifications of this elementary form are made: (1) for the purpose of insuring that as large a fraction as possible of the heat energy generated by the burning fuel shall be absorbed by the water and as small a fraction as possible shall be lost in the escaping hot gases and by radiation; and (2) for the purpose of increasing the heating surface and therefore the evaporative capacity of the boiler, with as great an economy of space occupied and of cost of the metallic structure as possible, without at the same time endangering the safety of the boiler, or decreasing its durability, facility for cleaning or other desirable qualities.

DEFINITIONS. — Certain special terms used in reference to boilers are defined below.

Boiler Horse-power. — This term was originally introduced to indicate the size of boiler required for an average reciprocating engine, the boiler being said to have the same horse-power as that of the engine. Due to improvements in the design of steam engines a boiler of a given horse-power rating may be ample to supply an engine (e.g., a compound condensing engine) of 3 times the rating of the boiler. The American Society of Mechanical Engineers has adopted the following definition:

A boiler horse-power is equivalent to the evaporation of 34.5 pounds of water per hour from feed water at 212° F. to saturated steam at the same temperature.

Adopting Marks and Davis's figures for the properties of steam (*see Steam*) 34.5 pounds of steam from and at 212° F. is equivalent to 33,479 B.t.u. per hour, or to an evaporation of 30.018 pounds from 100° F. feed-water temperature into steam at 70 pounds gage pressure.

It is customary in the trade to consider 10 square feet of heating surface as equivalent to a boiler horse-power, for stationary boilers. The term boiler horse-power is not used in connection with locomotive or marine boilers, and there is a tendency to discontinue its use for stationary boilers, expressing their size in square feet of heating surface, and their evaporative capacity in pounds of water evaporated from and at 212° F. per hour.

Equivalent Evaporation. — Factor of Evaporation. — For the purpose of reducing the results of a boiler test made under certain conditions of feed-water temperature and steam pressure and quality to a common standard, it is customary to use the equivalent evaporation from and at 212° F. as that standard. The pounds evaporated under actual conditions are multiplied by a "factor of evaporation," F, which is determined by the formula

$$F = \frac{H - h}{970.4},$$

in which H = total B.t.u. above 32° F. in 1 pound of steam at the actual pressure and degree of superheat, and h = total B.t.u. above 32° F. of 1 pound of the feed water. The values of H and h are given in steam tables (*see Steam*). If the steam contains x per cent of moisture, use for H in the above formula

$$H = H'\left(1 - \frac{x}{100}\right) + \frac{h'x}{100},$$

where H' is the total heat of saturated steam at the given pressure and h' is the heat of the liquid at the given pressure, as given in steam tables.

Boiler Efficiency. — The efficiency of the boiler alone is usually defined as the ratio of the number of B.t.u. absorbed by the water per pound of combustible *actually* burnt, to the number of B.t.u. in one pound of the combustible. The over-all efficiency of the boiler and grate is the ratio of the B.t.u. absorbed by the water per pound of *coal as fired* to the number of B.t.u. in one pound of this coal. This over-all efficiency differs from the efficiency of the boiler alone according to the amount of coal lost through the grates.

"Economy" or Water Rate. — Boiler efficiency is also frequently expressed in terms of the number of pounds of water evaporated from and at 212° F. per pound of combustible in the coal actually burnt (this corresponds to the efficiency of the boiler alone), or per pound of coal as fired (this corresponds to the efficiency of boiler and grate). The number of pounds of water evaporated from and at 212° F. per pound of coal is frequently referred to as the "economy" of the boiler, or its "water rate."

The relation between efficiency and economy may be expressed as follows.

Let ϵ = boiler efficiency, in per cent,
B = B.t.u. per pound of coal,
w = economy, i.e., the equivalent number of pounds water evaporated per pound of coal.

Then

$$w = \frac{B\epsilon}{97{,}040}.$$

For example, if the coal contains 10,000 B.t.u. per pound and the boiler efficiency is 70 per cent, then the economy is $10{,}000 \times 70/97{,}040 = 7.2$ pounds water per pound coal.

CLASSIFICATION OF BOILERS. — Boilers may be classified as *externally* and *internally fired*, *water tube* and *fire tube*, *through tube* and *return tube*, *horizontal* and *vertical*. An *internally fired* boiler is one in which the furnace is built inside of the boiler, and its roof and sides form a part of the heating surface of the boiler; the locomotive and Scotch marine, the Lancashire and the vertical-tubular boiler are of this type. An *externally fired* boiler is one in which the furnace is separate from the boiler itself and is usually placed underneath it, but sometimes at the side of it in a structure lined with fire-brick. A *fire tube* boiler is one in which the hot gases pass through the tubes, whereas the water is contained in an external shell through which the tubes pass. A *water tube* boiler is one in which the water is contained in the tubes and the hot gases pass around them. A *through tube* boiler is one in which the hot gases pass directly through the tubes from the fire to the smoke flue, whereas in a *return tube* boiler the gases are caused to pass under the shell containing the tubes to the farther end, thence upward through the "back connection," or rear of the setting and return through the tubes to the "breeching" or flue connection to the chimney. The term "return tubular" is applied only to fire tube boilers.

APPLICATIONS OF VARIOUS TYPES. — Externally fired *return tube* boilers in sizes up to 200 horse-power and for pressures up to 150 pounds per square inch are very commonly used. Boilers of this type are comparatively cheap and do not require expensive setting. In large power stations externally-fired *water tube* boilers are almost invariably used in this country. These boilers are built in sizes up to 2300 horse-power and for all practicable pressures. Babcock and Wilcox, Heine, Sterling and Wickes boilers are of the water tube type. In the first two makes the tubes are slightly inclined, in the third they are quite steeply inclined, and in the last they are vertical.

Selection of Type of Boiler. — In selecting the type of boiler to be used in any particular case the following points should be taken into consideration: (1) steam pressure desired; (2) space available; (3) danger of explosion; (4) liability to minor troubles which cause temporary stoppage; (5) durability; (6) ease of making repairs; (7) facilities for cleaning and removing scale and for inspection; (8) water-storage capacity; (9) rapidity at which steam can be raised; (10) steadiness of water level; (11) dryness of steam; (12) cost, including setting.

The question of economy of fuel is not generally dependent on the type of boiler as the same economy may be obtained with all types provided each is so designed that (1) there is sufficient heating surface; (2) the fuel can be thoroughly burned in the furnace, so that no unburned gases are allowed to reach the heating surface, and (3) the path of the gases through the boiler is properly baffled so that they are not "short-circuited," leaving part of the heating surface out of the current.

DESIGN AND CONSTRUCTION. — The essential parts of the boiler in the case of a water tube boiler are the tubes, steam drums and mud drums, grates and setting. The tubes are usually 4 inches in diameter in large stationary boilers, and are usually more or less inclined. The steam drums are large cylindrical vessels which serve as a reservoir for the water and steam. The mud drums are arranged to collect a considerable part of the sediment formed by evaporation of the water. Suitable blowpipes are provided for draining off the water and for discharging the sediment and scale-forming material. The water level in the boiler is usually indicated by a gauge glass or try cocks, or both, connected either directly to the boiler shell or to a water column. Suitable manholes and handholes are provided to give access to the various parts of the boiler.

Heating Surface. — For maximum fuel economy with any kind of boiler it should be proportioned so that at least one square foot of heating surface should be given for every 3 pounds of water to be evaporated from and at 212° F. per hour. Still more liberal proportions are required if a portion of the heating surface has its efficiency reduced by: (1) tendency of the heated gases to short-circuit, that is, to select passages of least resistance and flow through them with high velocity, to the neglect of other passages; (2) deposition of soot from smoky fuel; (3) incrustation. If the heating surface is clean, and the heated gases pass over it uniformly, little if any increase in economy can be obtained by increasing the heating surface beyond the proportion of 1 square foot to every 3 pounds of water to be evaporated, and with all conditions favorable but little decrease of economy will take place if the proportion is 1 square foot to every 4 pounds evaporated; but in order to provide for driving of the boiler beyond its rated capacity, and for possible decrease of efficiency due to the causes above named, it is better to adopt 1 square foot to 3 pounds evaporation per hour as the minimum standard proportion.

For maximum commercial economy, taking into consideration not only the cost of fuel but also the interest, depreciation and taxes on the cost of the plant, a higher rate of driving may be required, even as high as 6 or 8 pounds per square foot of heating surface per hour at the time of maximum or peak load, if the high loads last only a few hours each day.

Measurement of Heating Surface. — The usual rule is to consider as heating surface all the surfaces that are surrounded by water on one side and by flame or heated gases on the other, using the external diameter for water tube boilers and the internal diameter for fire tube boilers.

Grates. — Three kinds of grates are employed, the stationary, the rocking or shaking, and the traveling grate. The grate bars are made of cast iron. Rock-

ing grates have the advantage of permitting clearing the fire-bed of ash and clinker without opening the fire door and require less manual labor than the stationary grate. A traveling grate is one form of a mechanical stoker (*see Stokers, Mechanical*).

Grate Surface. — The amount of grate surface required per horsepower and the proper ratio of heating surface to grate surface are extremely variable, depending chiefly upon the character of the coal and upon the rate of draft. With good coal, low in ash, approximately equal results may be obtained with large grate surface and light draft, and with small grate surface and strong draft, the total amount of coal burned per hour being the same in both cases. With good bituminous coal, like Pittsburgh, low in ash, the best results apparently are obtained with strong draft and high rates of combustion, provided the grate surfaces are cut down so that the total coal burned per hour is not too great for the capacity of the heating surface to absorb the heat produced.

With coals high in ash, especially if the ash is easily fusible, tending to choke the grates, large grate surface and a slow rate of combustion are required, unless means, such as shaking grates, are provided to get rid of the ash as fast as it is made.

The following table is adapted from a more extensive one given in Gebhardt's *Steam Power Plant Engineering.*

Nature of plant	Number of plants	Character of fuel	Average ratio of heating surface to grate surface
Central stations	10	Illinois screenings, 15 to 20% ash	65
Central stations	14	Bituminous	60
Central stations	1	Bituminous	31
Central stations	9	Anthracite	40
Mfg. plants	20	Anthracite	35
Office building	6	Bituminous	48

Calculation of Required Heating Surface and Grate Area. — From the data given above the proper grate area and heating surface may be calculated as follows:

Example 1. — Steam required per hour at the "peak of the load," 6000 pounds; 3000 pounds ordinarily. Steam per pound of the poorest coal under the given conditions of feed-water temperature and steam pressure at maximum rate of driving, 6 pounds. This corresponds to $6000/6 = 1000$ pounds of coal per hour. Maximum rate of combustion under worst conditions, 25 pounds of coal per square foot of grate per hour. The required grate surface is then $1000/25 = 40$ square feet. Heating surface for maximum economy at ordinary driving will be $3000/3 = 1000$ square feet, requiring $6000 \times 3/3000 = 6$ pounds of steam per square foot of heating surface per hour at highest rate. Ratio of heating to grate surface, $1000/40 = 25$ to 1.

Example 2. Let 3000 pounds of steam be required per hour uniformly. The coal is of good quality so that 9 pounds of water will be evaporated per pound

of coal when the rate of driving does not exceed 3 pounds of evaporation per square foot of heating surface per hour. The rate of combustion of this coal may be as high as 30 pounds per square foot of grate per hour, but to have more easy firing, 20 pounds is chosen. Then,

$$\begin{aligned} \text{Heating surface} &= 3000/3 = 1000 \text{ square feet;} \\ \text{Coal per hour} &= 3000/9 = 333 \text{ pounds;} \\ \text{Grate surface} &= 333/20 = 16.7 \text{ square feet;} \\ \text{Ratio of heating to grate surface} &= 1000/16.7 = 60 \text{ to } 1. \end{aligned}$$

Air Passages through Grates. — The usual practice is to make the air opening 30 to 50 per cent of the total grate area. With coal free from clinker much smaller openings may be used.

Flues and Other Gas Passages. — Rules are usually given making the area of gas passages bear a certain ratio to the area of the grate surface; thus a common rule for horizontal tubular boilers is to make the area over the bridge wall 1/7 of the grate surface, the flue area 1/8, and the chimney area 1/9.

For average conditions with anthracite coal and moderate draft, say a rate of combustion of 12 pounds of coal per square foot of grate per hour, and a ratio of heating to grate surface of 30 to 1, this rule is as good as any, but it is evident that if the draft were increased so as to cause a rate of combustion of 24 pounds, requiring the grate surface to be cut down to a ratio of 60 to 1, the areas of gas passages should not be reduced in proportion. The amount of coal burned per hour being the same under the changed conditions, and there being no reason why the gases should travel at a higher velocity, the actual areas of the passages should remain as before, but the ratio of the area to the grate surface would in that case be doubled.

Mr. Barrus states that the highest efficiency with anthracite coal is obtained when the tube area is 1/9 to 1/10 of the grate surface, and with bituminous coal when it is 1/6 to 1/7, for the conditions of medium rates of combustion, such as 10 to 12 pounds per square foot of grate per hour, and 12 square feet of heating surface allowed to the horse-power.

Boiler Settings. — The boiler proper is usually mounted in a brick setting with suitable iron buckstays. The setting also forms the walls of the furnace. Common red brick is usually employed for the setting except for those parts which are exposed to a sufficiently high temperature to require the use of fire brick. Sometimes a separate furnace, or "Dutch oven," directly in front of the boiler, is used.

OIL BURNERS AND FURNACES. — In burning liquid fuel in steam-boiler or other furnaces it is of the utmost importance that the fuel be completely atomized and that each minute particle be surrounded by sufficient air for its complete combustion. Failure in this results in the formation of smoke and soot. The following methods of injecting the oil and air into the furnace are used in connection with steam boilers:

(1) *Steam injection.* A fine jet of steam at high pressure is employed to inject both oil and air into the furnace. Furnaces fired by this method are very successful in burning oil without smoke, if the oil is not fed to the furnace at a rate faster than that for which the burner is designed. (2) *Air injection.* Air at about 40 pounds pressure per square inch is used to inject and atomize the oil. Burners of this type are very successful, and they give higher temperatures than steam injectors. The objection to them is the cost of the air-compressing machinery. (3) *Mechanical injectors.* The oil under pressure operates a revolving device in the nozzle of the burner which causes the oil to be delivered in a very fine spray. Some burners of this type have given excellent results. With these burners it is necessary to supply the oil under a high pres-

sure, say from 50 to 80 pounds per square inch and to heat it before reaching the burner to a temperature of from 120° to 180° F. in order to reduce its viscosity.

More important than the burner is the furnace. It must be of sufficient size to give ample room for complete combustion of the gases formed by decomposition of the oil before these gases are allowed to be chilled by contact with the heating surfaces of the boiler. For economy of oil the air supply should be regulated so that the excess of air is not more than 50 per cent above that theoretically required for complete combustion.

POWDERED FUEL BURNERS AND FURNACES.—In burning powdered fuel, it is necessary that the coal be dried to a moisture of 1 per cent or less. The coal must be ground to such fineness that 95 per cent will pass through a 100-mesh screen and 80 to 85 per cent through a 200-mesh screen. Reliable regulation of the coal and air supply seems to be the principal requirement of a good burner. John V. Cullney specifies the essentials as: (*a*) uniform feed; (*b*) proper mixture of coal and air; (*c*) proper control and ability to vary coal supply; (*d*) simplicity; (*e*) compactness. The life of furnaces using pulverized coal varies with the type of furnace. Information available at the present time indicates that the life of such furnaces is about the same as for furnaces using oil. The working temperatures of powdered fuel furnaces seem to range from 1800 to 3000° F.

Cost of Grinding and Handling, including Upkeep (Pre-war figures).—These items vary considerably with plants of different size and under different operating conditions. The cost of grinding, drying, handling, etc., per ton of fuel, as reported by four companies is as follows:

For Plant (*a*) about 45 cts. per ton, subdivided as follows:

Labor	15 cts.
Power	10
Repairs	14.5
Coal for drying	5.5
	45 cts.

For Plant (*b*), based on 200 tons in 24 hours, excluding overhead charges, interest, and depreciation.

Price of coal	Cost of grinding, handling, etc.
$1	17.6 cts.
2	21.8
3	23.9

For Plant (*c*), 140 tons per 24 hours in plant built 16 years ago.

	Cost per ton.	
Labor, operating	15.64 cts.	
Labor, repairs	1.70	
Supplies, fuel, power, etc.	14.71	
Oil and waste	1.25	
Repairs	4.54	
		37.84 cts.
Interest, depreciation and obsolescence		1.35
		39.19 cts.

For Plant (*d*), from 11.4 cts. per ton to 49.2 cts., depending upon the degree of pulverization, amount of coal consumed, etc.

SUPERHEATERS. — To obtain the advantages of superheated steam (*see Steam*), various forms of superheaters are employed. A superheater consists essentially of a number of small tubes, through which the steam is passed, arranged to be heated by the hot gases in the boiler or from some external source. In this country the superheater is nearly always placed within the boiler setting. Frequent cleaning of the exterior walls of the superheater tubes with steam is necessary to prevent the accumulation of soot. It is customary to provide for the flooding of the superheaters when the boiler is banked and when steam is being raised; this procedure may be undesirable when the feed water contains much scale-forming impurity, as superheater tubes are not readily cleaned internally.

PERFORMANCE OF BOILERS. — The number of boiler horse-power (i.e., the number of pounds of steam from and at 212° F. divided by 34.5) which can be obtained from a boiler depends upon the heating surface of the boiler, the grate area of the furnace, the quality of coal used, the method of firing, etc. Consequently the horse-power rating of a boiler is at best but an exceedingly rough indication of its capacity, and therefore the modern tendency is to rate a boiler in terms of the number of square feet of heating surface. If the horse-power rating is also given, this is usually taken arbitrarily for water tube boilers as the number of square feet of heating surface divided by 10; that is, 1 boiler horse-power is taken as equivalent to 10 square feet of heating surface. Some large modern boilers, however, are operated normally under conditions such that the boiler horse-power actually developed is twice the rating on this basis. Builders of fire tube boilers usually rate them on the basis of 11 to 12 square feet per boiler horse-power.

Boiler Capacity. — The capacity of a boiler may be expressed in terms of the boiler horse-power which may be obtained from it, or, more definitely, in terms of the number of pounds of water which can be evaporated in it. The capacity of a boiler, by which is meant the quantity of steam it will make in a given time, depends (1) upon the quantity of heat that can be generated in the furnace, and (2) upon the percentage of this heat that is absorbed by the boiler and not wasted in the chimney gases or by radiation.

The first essential of boiler capacity for a given heating surface is furnace capacity, and this depends chiefly upon three things, area of grate surface, quality of fuel and force of draft. It may also depend upon the kind of grate, as plain or shaking grate, or mechanical stoker; upon the roof of the furnace, whether formed of the heating surface of the boiler, or of fire brick: and upon the introduction of air above the fire in addition to that which passes through the grate. The second essential of boiler capacity is boiler efficiency. Other things being equal, anything that increases the efficiency of a boiler increases its capacity in the same ratio.

Factors Affecting the Efficiency of a Boiler. — Boiler efficiency depends: (1) on the quality of the coal (or other fuel) as regards its content of moisture and volatile matter (*see article on Fuels*); (2) on the completeness with which the coal is burned in the furnace, before the gases of combustion touch the heating surface; (3) on the combustion being effected with the least possible excess of air, above 20 per cent excess being necessary to avoid imperfect combustion; (4) on the cleanness of the heating surface, inside and out; (5) on the proper baffling of the gases, so as to avoid short-circuiting; (6) on the heat lost by radiation; and (7) on the extent of the heating surface relative to the quantity of heat generated, or to the rate of driving of the boiler, as measured by the heat absorbed in a given time per square foot of heating surface (*see section on Heating Surface, below*). In general the efficiency of a boiler may be represented approximately by the equation

$$\epsilon = \frac{T_1 - T_2}{T_1},$$

in which T_1 is the temperature of the fire and T_2 the temperature of the chimney gases. The temperature T_1 is proportional to the heat generated in the furnace, assuming the specific heat of the gases to be constant, and T_2 to the heat loss in the chimney. The loss by radiation is not considered in the formula, but this is generally not over 2 to 3 per cent when a boiler is driven at a normal rate.

Values of Boiler Efficiency. — The *highest* efficiency that is obtained with ordinary boilers and furnaces, fired by hand, is about 79 per cent with anthracite and 76 per cent with bituminous coal. As high as 81 per cent may possibly be obtained with bituminous coal with very large boilers, provided with mechanical stokers, and 82 per cent with oil fuel. With economizers, heating the feed water by the waste heat of the gases from the boiler, the combined efficiency of boiler and economizer may be as high as 90 per cent.

The necessary conditions for obtaining these high figures of efficiency are: (1) that the coal is of good quality, low in moisture; (2) that the coal is burned uniformly and with uniform conditions of air supply, so that the grate is not obstructed by clinker or imperfectly covered by coal; (3) that the rate of driving does not exceed 3 pounds evaporation per square foot of heating surface per hour; (4) that the air supply does not exceed 20 pounds per pound of carbon burned; (5) that the combustion of the gases is completed in the furnace, so that no unburned gases touch the heating surface of the boiler and become chilled below the point of ignition; (6) that the path of the gases through the boiler is so baffled as to cause them to traverse uniformly all parts of the heating surface; (7) that the heating surface is free from soot and dust on the outside and from scale or grease on the inside.

The conditions of high efficiency above described are obtained only in the best-managed plants, where the chimney gases are constantly analyzed as a check on the air supply (*see below under Tests of Boilers*). In the average plant in everyday practice the efficiency may be anywhere from 10 to 30 per cent below the best possible.

Boiler efficiencies are usually graded as follows: 50 to 60 per cent, poor; 60 to 70 per cent, fair; 70 to 75 per cent, good; over 75 per cent, excellent.

Effect of Rate of Evaporation on Efficiency. — On the basis of 10 square feet of heating surface per boiler horse-power (34.5 pounds of steam from and at 212° F.) approximately 3 pounds of steam are evaporated per square foot of heating surface when the boiler is developing its rating at normal steam pressure and temperature. If there is sufficient grate surface and draft the evaporation may be increased to 6 or in emergency to 15 pounds of steam per square foot of heating surface. The efficiency, however, falls off with increased rate of driving. The following figures are taken from a test by D. S. Jacobus (*Trans. A.S.M.E., 1911*) on a large Stirling boiler (23,650 square feet of heating surface).

Equivalent evaporation per square foot =	3.24	3.40	3.72	4.18	5.22	5.62	6.40	6.67	6.75	7.29
Over-all efficiency =	81.2	81.0	80.3	79.2	77.1	77.9	76.4	76.7	75.6	75.8

Still higher rates of driving and the corresponding efficiencies are given below, these figures being taken from a test of Babcock & Wilcox boilers designed for the United States warships "Cincinnati" and "Wyoming" (*Industrial Engineering, March, 1911*). The results are as follows:

Equivalent evaporation per square foot =	8.42	8.75	9.03	9.58	10.1	10.5	13.7	14.8
Over-all efficiency =	70.4	70.6	72.1	68.2	70.1	71.0	64.5	63.3

These efficiencies are all exceptionally high for the given rates of driving, but the relative efficiencies for the different rates are representative of good average practice.

Thickness of Fire and Efficiency. — Too thin a fire results in excess of air and too thick a fire in a deficiency, unless sufficient draft is provided by means of fans or tall chimneys (*see Draft, Mechanical*). There are so many factors to be considered that the proper thickness for maximum economy in any particular case can be determined only from actual test. Under ordinary conditions of hand-firing the thickness of fire for best efficiency ranges from 6 to 12 inches.

Air Supply and Efficiency. — The air supply for maximum efficiency, all other conditions remaining constant, is about 17 or 18 pounds per pound of carbon.

With an air supply lower than 17 pounds the efficiency is likely to fall off, on account of imperfect combustion. If the air supply is greater than 20 pounds the efficiency falls off very rapidly, the excess air supply causing a reduction of temperature in the furnace and an increase of heat carried into the chimney.

Heat Balance and Distribution of Losses. — By the "heat balance" is meant a statement accounting for ultimate distribution of the B.t.u. in the fuel supplied to the boiler. Such a statement is usually made up as follows. The figures given illustrate the ordinary range for the various items.

HEAT BALANCE PER POUND OF COMBUSTIBLE

Item	B.t.u.	Per cent
1. Heat absorbed by the boiler	......	80–50
2. Loss due to heating moisture in coal	......	0.2–2.0
3. Loss due to heating moisture formed by the burning of hydrogen	......	0.5–4.0
4. Loss due to heat carried away in the dry chimney gases	......	12–30
5. Loss due to incomplete combustion of carbon (CO in flue gas)	......	0.0–3.0
6. Loss due to		
a. Unconsumed hydrogen and hydrocarbons (smoke and soot)	......	0.0–1.5
b. To heating the moisture in the air	......	0.2–1.5
c. To carbon lost through grate	......	0.5–3.0
d. To radiation	......	1.0–3.0
e. Unaccounted for	......	1.0–10.0
Total	11,000 to 15,500	100

The five items under 6 are frequently given as a single item.

TESTING OF BOILERS. — The principal observations made during an "evaporation" test of a boiler are pounds of water evaporated, pounds of coal fired, steam pressure and feed-water temperature. Records of the water evaporated and the coal fired should be made for each hour of the test if possible. The steam pressures and feed-water temperatures are recorded every half-hour or oftener, and averaged for the whole test. All conditions should as nearly as possible be the same at the end as at the beginning of the test. When the furnaces are hand fired, tests of 8 to 10 hours duration are usually sufficient, but when mechanical stokers are used it is difficult to obtain an equal quantity and condition of the coal in the furnace at the beginning and end of the test, and a

large error in the results may be made if the test is much shorter than 24 hours. The average moisture in the coal should be determined and the ash and refuse should be weighed.

In all important tests the code of the American Society of Mechanical Engineers (new code to be published shortly, 1922) should be followed. The principal data and results given in this code are as follows:

PRINCIPAL DATA AND RESULTS OF BOILER TEST.

1.	Grate surface (width ...., length ....)	sq. ft.
2.	Total heating surface	sq. ft.
3.	Date	
4.	Duration	hr.
5.	Kind and size of coal	
6.	Steam pressure by gauge	lb.
7.	Temperature of feed water entering boiler	° F.
8.	Percentage of moisture in steam or number of degrees of superheating	per cent or ° F.
9.	Percentage of moisture in coal	per cent
10.	Dry coal consumed per hour	lb.
11.	Dry coal consumed per sq. ft of grate surface per hour	lb.
12.	Equivalent evaporation per hour from and at 212°	lb.
13.	Equivalent evaporation per hour from and at 212° per sq. ft. of heating surface	lb.
14.	Rated capacity per hour, from and at 212°	lb.
15.	Percentage of rated capacity developed	per cent
16.	Equivalent evaporation from and at 212° per lb. of dry coal	lb.
17.	Equivalent evaporation from and at 212° per lb. of combustible	lb.
18.	Calorific value of 1 lb. of dry coal by calorimeter.	B.t.u.
19.	Calorific value of 1 lb. of combustible by calorimeter	B.t.u.
20.	Efficiency of boiler, furnace and grate: $100 \times \left(\frac{\text{Item 16} \times 970.4}{\text{Item 18}}\right)$	per cent
21.	Efficiency of boiler and furnace: $100 \times \left(\frac{\text{Item 17} \times 970.4}{\text{Item 19}}\right)$	per cent

Flue Gas Analysis and Heat Balance. — See Kent's *Mechanical Engineers' Pocket-Book* and Gebhardt's *Steam Power Plant Engineering.*

SPECIFICATIONS FOR STEAM BOILERS. — As nearly all steam boilers are now made by large manufacturing concerns who have their own specifications as to details of construction, quality of material, etc., it is rarely the case that an engineer of a power plant or a consulting engineer is called on to make an original design of a boiler or to furnish detailed specifications for one. What is usually done is to call for bids on general specifications, naming the quality of coal to be used, the kind of furnace or stoker, the horse-power to be developed (34.5 pounds of water evaporated from and at 212° F. = 1 h.p.), when the boiler is driven at a rate not to exceed 3.5 pounds per square foot of heating surface per hour, evaporated from and at 212° F., and the overload capa-

city desired for higher rates of driving, or "the peak of the load." The bids received are then tabulated as to the details, such as heating surface, grate surface, size of combustion chamber, size of tubes, diameter of shell, quality of steel, style of riveting, factor of safety, space occupied, guarantees of capacity and economy, etc., and a selection made of the boiler that appears to be the most suitable for the location. Of two boilers that have the same heating surface the one that has the largest grate surface is the one that has the greatest overload capacity, if the draft is the same in both cases. Guarantees of capacity and economy may be disregarded when bids are for ordinary forms of boilers and furnaces, for the capacity and economy of two boilers of the same size of grate and heating surface will be practically the same whatever the type or form of the boiler, but they will vary greatly from causes independent of the boilers themselves, such as quality of coal, method of firing, kind of furnace, size of combustion chamber, etc.

OPERATION OF BOILERS. — To obtain the best results from boilers, considerable care must be paid to the firing and regulation of air supply.

Firing. — There are three common methods of hand firing: (1) the alternate in which fresh coal is fired on one side of the grate at a time, the combustion of volatile matter being assisted by the heated air passing through the other side; (2) the spread-firing, in which small amounts are spread thinly over the entire fire bed; and (3) the coking, in which a thick bed of fresh coal is fired directly in front of the door and allowed to coke, after which it is pushed back into the furnace. The best method for any particular kind of fuel, style of furnace and rate of driving should be determined by experiment.

Mechanical stokers (*see Stokers, Mechanical*) afford the following advantages: viz., the uniform feeding of coal, the close regulation of the air supply at all times, and consequently smokeless and efficient combustion with a wide range of loads, and a great reduction of labor cost in handling and firing fuel. In small plants with poor load factors the expense of installation is generally considered prohibitive, but in plants of 1500 kilowatts or more a considerable net annual saving can usually be obtained.

Air Supply. — (*See also Chimneys; Draft, Mechanical.*) The extent of the flue losses is determined by the amount of gas discharged, the temperature at which it is discharged, the amount of incompletely burned gas discharged and the amount of latent heat carried away in water vapor. The minimum economic amount of flue gas would obtain with the amount of air supplied to the fire exactly equal to the theoretical requirements of complete combustion. The admission of less air tends to the production of CO. More air dilutes the flue gas, increases the total amount of gas rejected and, in the same ratio, increases the amount of heat rejected, the stack temperature remaining the same. The theoretical air supply per pound of combustible (*see Fuels*) is approximately 12 pounds, but since the air and fuel cannot be mixed with perfect intimacy it is necessary to considerably increase this amount to prevent the formation of CO. See above under *Air Supply and Efficiency.*

CO_2 Recorders. — The per cent of CO_2 in the flue gas is an index of the air excess, provided combustion is complete. With perfect combustion and no excess air 20.9 per cent CO_2 should be obtained with pure carbon as fuel, or 18.6 per cent with a fuel composed of 95 parts C and 5 parts H. In practice more air must be supplied and 14 per cent CO_2 represents about the upper limit which should be attempted. 12 per cent CO_2 is good, 10 per cent fair and 8 per cent poor. Several types of CO_2 recorders have been devised. If properly maintained and interpreted they form a valuable check on the regulation of the air supply. Their indications, however, give no clue to the presence of CO, and are directly significant only when the formation of CO is prevented.

Flue-gas Temperature. — Because of the great heating surface which would be required it is not practicable to reduce flue-gas temperature to less than 75 degrees above the feed water. Economizers or flue-gas feed-water heaters (*see Feed-water Heaters*) assist in lowering the temperature of discharge but add considerably to the space required, increase the cost of installation and maintenance and increase the total draft required. Without economizers it is seldom economical to attempt to reduce the gas temperature below 450° F.

Smoke represents a minor element of loss *per se*, seldom exceeding 1 per cent, but it is generally an index of inefficient combustion. Smoke is usually due to a poorly regulated air supply, an imperfect mixture of air and fuel, or the too-rapid volatilization of hydrocarbons and their premature chilling in a half-burned state. The absence of smoke, however, is not a final index of efficiency, for the air supply may be excessive. See also article on *Smoke Prevention.*

Feed Water. — (*See Feed-water Heaters; Pumps and Pumping Engines.*)

Cleaning Tubes. — The tubes of a boiler should be kept clean, both internally and externally. For blowing the soot off the external surface the tubes of water tube boilers or off the internal surface of the tubes of fire tube boilers a steam jet blower is ordinarily used. For removing scale from the interior of water tubes different forms of scrapers or chippers are used, also rotary cutting devices driven at a high speed by steam, water or other power. For removing scale from the water side of fire tubes of horizontal tubular boilers the usual method is the operation of chipping and hammering by a man working inside of the boiler. Sometimes the scale is softened by the use of soda or other chemicals, before chipping, and a strong jet of water from a hose assists in detaching the scale. In locomotive practice it is often necessary to cut the tubes out of the boiler and run them through a cleaning machine, and then to weld a piece of each tube on one end to restore its original length, and to reset the tubes in the boiler.

DIMENSIONS OF BOILERS. — An ordinary fire tube boiler occupies a floor space of from 1.5 to 2.0 square feet per boiler horse-power (on the basis of 12 square feet of heating surface per horse-power). A water-tube boiler occupies from 0.5 to 1.0 square foot of floor space per boiler horse-power (on the basis of 10 square feet of heating surface per horse-power). With either type the height ranges from as low as 7 to as high as 30 feet depending not only on the horse-power but also on the style, whether horizontal or vertical. One boiler is sometimes placed directly on top of another, making a "double-deck" arrangement. For complete dimensions see the manufacturer's catalogues.

COST OF BOILERS (Pre-war figures). — The selling price of boilers per rated horse-power (10 square feet of heating surface = 1 horse-power) ranges from about \$8 to \$25, depending upon the size, the pressure they are to sustain, the style and design, etc. The lowest prices named are for large-sized ordinary horizontal tubular boilers (fire tube) for low pressures. For power plants of 1000 horse-power and over the price will range usually between \$10 and \$15 for pressures not over 150 pounds per square inch. For higher pressures and for boilers provided with superheaters the prices will be higher. For boilers of less than 100 horse-power the price per horse-power increases as the size decreases. The prices named are for the boilers with the usual fittings of grates, steam and water gauges, blow-off and stop valves, on board cars at the boiler works, and do not include the cost of erection nor of brickwork, flues or chimneys. (*See also Power Station.*)

The following approximate rules, in which P = boiler horse-power, may be used for determining the cost of boilers:

(1) Horizontal return tubular boilers.
Cost f.o.b. factory (\$) $= 180 + 6.4P$
Cost of setting (\$) $= 140 + 2P$.

(2) Horizontal water-tube boilers, 125 lb. pressure.
Cost f.o.b. factory (\$) $= 425 + 9P$.
Cost of setting (\$) $= 140 + 2P$.

(3) High grade water-tube boiler, 160 lb. pressure.
Cost f.o.b. factory (\$) $= 1000 + 10P$.

(4) High grade water-tube boiler, 200 lb. pressure.
Cost f.o.b. factory (\$) $= 1300 + 12P$.

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BONDS, RAILWAY TRACK. — (*See also Third Rail Systems; Trolley Systems; Wires and Cables.*) Rail bonds are electrical conductors for bridging the joints of rails. They consist either of a series of thin strips of annealed copper, or of one or more cables of copper wire, the ends of which are usually pressed or cast into solid copper terminals. Ribbon is more compact, but stranded wire is more flexible; the latter should always be used if space permits. Sometimes the termina s are made by upsetting the ends of a stranded conductor, so as to form a crude bond head. The terminals of the headed bond are then heated to a welding heat in furnaces, and pressed to the proper size and shape, the separate wires in the terminals becoming a solid mass of homogeneous copper. After being cleansed of all foreign substances the terminals are shaved to the proper size.

TYPES OF BONDS. — Bonds may be classified, according to the method of fastening them to the rail, as soldered bonds, brazed bonds and bonds applied by mechanical pressure.

Soldered Bonds (Figs. 1 and 2) usually consist of a series of thin strips of annealed copper with tinned terminals as shown in Figs. 1 and 2. They are

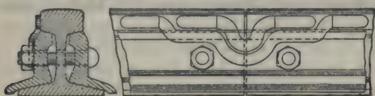

Fig. 1.

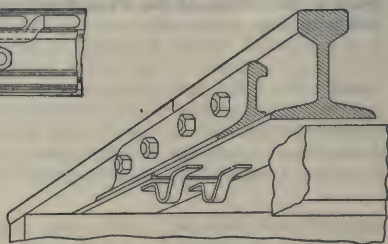

Fig. 2.

soldered direct to the head, foot or web of the rail. One or more bonds per joint may be used. A few companies employing well-trained workmen continue to install bonds of this type, but they have generally been abandoned.

Brazed Bonds resemble soldered bonds except that the terminals are enveloped in brass. They are brazed or welded to the rail by heat generated electrically in a carbon which constitutes one jaw of a clamp holding the bond against the rail. These are sometimes known as "electrically welded" bonds.

Expanded and Compressed Terminal Bonds. — Bonds fastened to the rail by mechanical pressure may be divided into two general classes, expanded terminal and compressed terminal bonds. Both kinds are called "stud terminal" bonds. The number of stud terminal bonds in use greatly exceeds that of any other kind, brazed bonds being second.

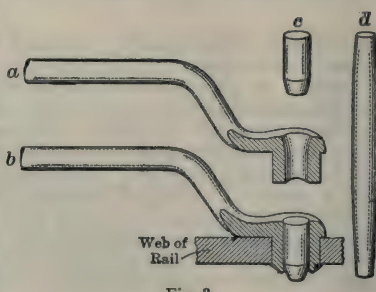

Fig. 3.

Pin-expanded Terminal Bonds (Fig. 3) have their heads drilled with an axial hole, through which a tapered steel pin *d* is driven, forcing the copper

outward and against the steel. This type of bond is fastened to the web of the rail.

Compressed Terminal Bonds (Figs. 4 and 5). — There are two kinds of compressed terminal bonds, in one of which direct pressure is applied at both ends of the head, and in the other, at one end only. The first type of bond is usually applied to the web of the rail by means of a heavy screw or hydraulic press (Fig. 8) which engages the bond head and causes it to compress longitudinally and expand laterally as the pressure is applied, bringing the copper into

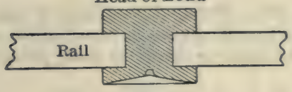

Fig. 4.

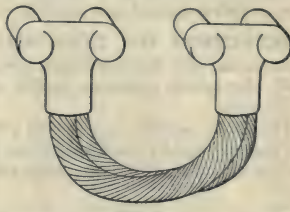

Fig. 5.

firm contact with the steel and spreading the projecting end of the terminal into a button-shaped rivet-head, as shown in Fig. 4. The second type of bond (Fig. 5) is applied only to the head of the rail, the terminal lugs being set in holes therein and expanded into contact by means of hammer blows.

Compressed Terminal Soldered Bond. — A type of bond which has been found very successful at the Detroit River Tunnel for third-rail work is a compressed terminal head bond soldered to the rail. The combination of mechanical adhesion and soldering has resulted in a bond of unusual durability. (*H. B. P. Wrenn.*)

Exposed Versus Concealed Bonds. — Whether soldered, brazed, expanded or compressed, bonds may be either exposed or concealed (Fig. 6) under the fish-plates. The former condition is preferable, if there is no likelihood of theft, as it permits inspection to be easily made. Where the bonds are exposed to theft, as, for example, on track rails unprotected by paving, concealed bonds are almost a necessity.

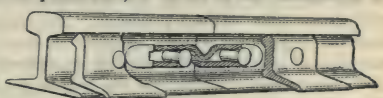

Fig. 6.

While concealed bonds are necessarily applied to the web of the rail, exposed bonds may be applied to the foot or head. Head bonds have the advantage of greater contact surface at the terminal studs, while foot bonds are less exposed to mechanical violence. Web bonds, unless concealed, have to be excessively long in order to span the fish-plates.

USES OF VARIOUS TYPES OF BONDS. — Bonds are used for track rails, third rails and girders of elevated and subway lines. Soldered bonds and compressed terminal head bonds find their best application in third-rail work, where good electrical contact is of greater importance than mechanical strength. Expanded terminal web bonds, especially of the concealed type with two stranded conductors, are regarded as the best for heavy track work, where mechanical strength and ease of installation are of the utmost importance.

SUBSTITUTES FOR BONDING. — Several efficient substitutes for bonding are now in use, such as electrical welding, cast welding and Thermit welding

Electrical Welding is performed by clamping an iron bar to the web of the rails, and bringing the bar and the neighboring part of the rail to a white heat by

means of an electric current. Another process accomplishes the welding by means of a carbon electrode. A modification of the latter process makes use of a composition bar which is clamped against the fish-plate. A soft steel rod placed in a groove between the rail and composition bar supplies the welding material.

Cast Welding is accomplished by setting a mold around the rail joint and pouring molten iron around it. The Thermit process is a modification of this, the iron being liberated at a white heat from a mixture of iron oxide and aluminum, which is ignited in a crucible.

SELECTION OF TYPE AND SIZE OF BOND. — Considerations determining the choice between concealed and exposed bonds are liability of theft, electrolytic corrosion, facility of inspection and injury to fish-plate bolts of old rails.

The choice between mechanical adhesion and soldering depends largely upon the importance of rapid installation, the mechanical stresses to be withstood in service, the type of labor available and the facilities for the use of drills, presses, etc.

Single vs. Double Bonding. — Joints are sometimes bonded with one and sometimes with two bonds. Double bonding has the advantages of less chance of complete failure and greater carrying capacity for a given cross-section of copper. It, however, has the disadvantages of being more expensive and giving uncertain results in testing.

Selection of Cross-sectional Area of Bond. — The cross-sectional area of a rail bond should, as a rule, be not greater than is necessary to keep its temperature at a safe working amount, unless greater area is required for mechanical strength. The resistance of the bonded joint is of secondary importance unless very high, because the resistance of the joints is usually a mere fraction of the total track-rail resistance (*see below*).

Carrying Capacity of Bonds. — The carrying capacity of rail bonds is not very well known as the manufacturers do not supply any data on this subject for their various products. The excellent heat conductivity of copper and the large heat storage capacity of steel rails tend to make the carrying capacity of bonds in cold weather considerably greater than that of free wire of the same size, especially if the bonds are short. In hot weather, however, the rails and consequently the bonds are likely to become hot from exposure to the sun's rays, thereby reducing the effective carrying capacity of the bonds.

Tests to determine the ultimate carrying capacity of soldered bonds indicated that a 500,000 circular mil bond of the type shown in Fig. 1 will carry 3500 amperes continuously without injury, but will melt off in 5 or 10 minutes at 10,000 amperes. On the basis of this test assuming the cooling surface per unit length of a bond to be proportional to the square root of its cross-section, the safe carrying capacity of a bond of A circular mils cross-section is

$$I = \frac{A}{5.6\sqrt[4]{A}}.$$

Or, the cross-section required for a given current is

$$A = 10\,I\sqrt[3]{I}.$$

Resistance of Bonds. — The total resistance of a bond is the sum of the resistances of the copper in the bond and the contact resistance between the body of the bond and the terminals and the contact resistance between the

terminals and the rail. The following table gives the resistance of a well-bonded joint at 75° F., including the body resistance and the two contact resistances at each end.

RESISTANCE OF BONDED JOINT IN MICROHMS (10^{-6} OHMS)

Size A. W. G. or B. & S. and C. M.	Diameter of terminals, inches	Length of bond in inches, between centers of terminals							
		9	12	15	18	24	30	36	42
500,000	1	28	33	39	44	55	65	76	87
400,000	1	32	39	46	52	66	80	93	106
300,000	1	38	46	55	63	80	97	114	131
250,000	7/8	44	54	65	75	95	116	136	157
0000	7/8	50	62	75	87	111	136	160	185
000	3/4	60	75	91	107	137	169	200	230
00	5/8	72	91	112	131	171	210	250	290
0	5/8	87	112	137	162	211	260	310	360

Bonding Efficiency and "Equivalent" Length of Joint. — The bonding efficiency* of a bonded rail system (having one bonded joint to each rail length) is the ratio of its conductance to that of an equal length of perfectly continuous rail without any breaks or joints. By "equivalent" length of a joint is meant the length of continuous rail having a resistance equal to that of the bonded joint. Let

B = bonding efficiency,
l = distance in feet measured along the rail between centers of bond terminals,
l' = equivalent length of bonded joint in feet,
L = length in feet of a full rail section,
r = resistance of rail per foot,
R_j = resistance of bonded joint.

Then

$$l' = \frac{R_j}{r},$$

$$B = \frac{1}{1 + \frac{l' - l}{L}}.$$

For values of r see article on *Rails*. The equivalent length of a properly bonded joint ranges from 1 foot to 6 feet, depending upon the size of rail, number and cross-section of bonds, length of bond, etc. (See also section on *Rebonding*, below.)

It is common practice to bond a joint with a bond having one-half the conductance of the rail. Hence for a 12-inch bond, and 30-foot rail section, assuming 9 inches between terminals measured along the rail, $l = 0.75$ and $l' = 2$, giving a bonding efficiency of 96 per cent.

Selection of Length of Bond. — The length of concealed bonds is necessarily determined by the dimensions of the rail and fish-plate, and by the bolt-hole drilling. The length should never be less than ten inches for single bonding

* The efficiency of individual bonds is sometimes stated as the ratio of the distance along the rail between centers of bond terminals to the equivalent length $= l/l'$.

and should generally be greater, it being usual practice to place the terminals between the first and second bolt holes. In double bonding it is customary to place one terminal of each bond between the first and second bolt holes of each rail, and the other terminals beyond the second bolt holes. It is not unusual to use concealed bonds 2 feet long for double-bonded rails on electrified steam roads. Exposed bonds are usually made as short as is consistent with the flexibility requisite to withstand vibration and other rail movements.

Tests by H. H. George (E.R.J., 1914, Vol. 44, p. 487) on No. 0000 stranded wire bonds showed the following relative number of vibrations to produce fracture.

Length of Bond inches	Relative number of vibrations
7	1.0
8	5.2
10	31.2
14	194.0

Effect of Vibration and Expansion of Rails. — The continual bending of a copper wire or ribbon will cause local hardening and crystallization of the metal, and eventually lead to fracture. It is, therefore, important to save track bonds from vibration as much as possible, and they should be initially designed to withstand the vibration they are likely to experience in use. The best way to accomplish this is to use long thin wires of soft annealed copper entirely free from surface imperfections.

According to the A. S. W. Co., Cat. No. 3, wires of from 0.040 to 0.045 inch diameter give the best general service for short bonds.

Concealed ribbon bonds, and to a less extent wire bonds, frequently break at the loop between rail bolts, first, because the operation of crimping slightly hardens the copper in the loop, and second, because the travel of the rails, due to their expansion and contraction, is wholly taken up in the loops, causing the copper to crystallize and break at this point. Of course this action is aggravated by any severe jarring or vibration of the joint.

SPECIFICATIONS. — (*See also article on Specifications and Contracts.*) Specifications for rail bonds should state the exact service conditions under which the bond is to be used, the style of adhesion desired, the part of the rail they are to be applied to, the style of conductor (ribbon, solid wire or cable), the cross-sectional area, the contact area of the stud, the formed length between centers of terminals, and the fish-plate and bolt layout, if the bonds are to be concealed.

INSTALLATION. — The foremost consideration in the installation of bonds is the cleanliness of the bonds and bond holes, or other adhesion surface. Unless this is secured the bonds will be electrically defective whatever their mechanical strength may be.

Soldered Bonds. — The rail surface is brightened by means of a carborundum or emery wheel, then tinned, using an acid flux. The bond is then clamped in place and the rail and bond heated by means of a blow-torch, to a temperature at which the solder will melt and cause the bond to adhere firmly to the rail.

Brazed Bonds. — The preliminary processes are the same as for soldered bonds except that a special clamp is used, the terminals of which are the electrodes of an electrical circuit, one being of copper and the other of carbon.

The surface of the rail being previously ground bright at the point where the weld is to be made, the brass-enveloped bond terminal is placed in position against the rail at this point, held there and pressed against the rail by the carbon

electrode, the copper electrode being in contact with the opposite side of the rail. The current, on passing from one electrode to the other, traverses the bond terminal and rail, the carbon becoming incandescent. The incandescent carbon pressing the copper against the rail quickly transmits sufficient heat at exactly the point where it is required, to produce the weld. To make a weld, the current is applied for a period of from 45 seconds to two minutes, depending on conditions.

Welding Outfit. — It is claimed by the manufacturers of one type of welding outfit that an average of over 100 bonds per day is readily installed by their car operating with four men, a bonder and three helpers. This car is about 6 feet, 10 inches long by 5 feet, 10 inches wide, and carries an 18-kilowatt rotary converter and transformer, with the necessary apparatus for their safe operation. To weld an average-size rail bond to the rail, an alternating current of about 2000 amperes at 5 volts is employed. On d-c. railways this is obtained by converting and transforming about 20 amperes at 500 volts taken from the trolley wire.

Pin-expanded Bonds. — The rail is drilled, usually through the web, with or without lubricant, using some form of drill especially adapted to this service.

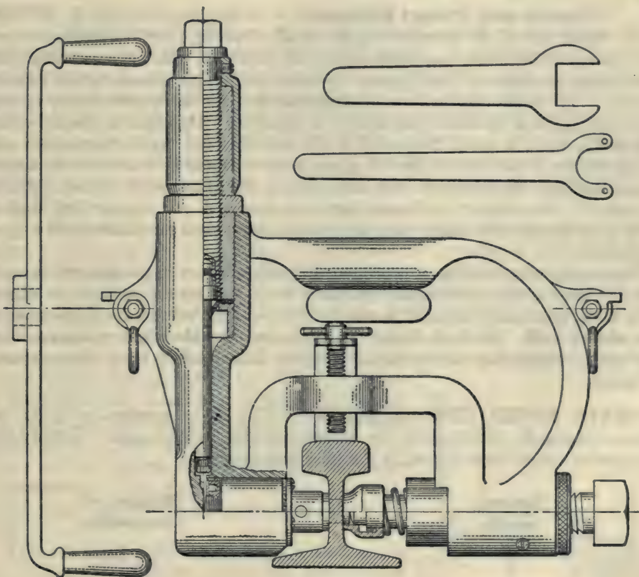

Fig. 8.

Drilling without lubricant has the advantage of giving a perfectly clean hole, and is now generally preferred. Holes are drilled to the same diameter as the bond studs, and rust or scale on the web of the rail is removed around the hole. The bond head is then inserted into the hole, and a long taper punch lubricated with grease is driven entirely through the terminal. Then a short drift pin is driven home, as shown in Fig. 3. The diameter of the hole is usually increased

about ⅛ inch by the expansion process. From 100 to 200 bonds may be installed with one taper punch before it is worn down to an appreciable extent.

This type of bond requires a smaller equipment in tools and materials than most other types and does not necessitate the use of any apparatus which obstructs the track and thereby endangers traffic.

Compressed-terminal Web and Foot Bonds. — The drilling and web cleaning having been performed as for a pin-expanded bond, and the bond heads inserted into their respective holes, a screw or hydraulic compressor, as shown in Fig. 8, is applied at both ends of the bond head, the conical point of the press fitting into the conical depression of the bond. Pressure is applied, either until a collar on the ram touches the rail, or until the head of the bond acquires the proper shape. Where no collar is used the point of the press (if of the screw type) sometimes cuts into the bond head; this may be avoided by placing a small amount of flake graphite mixed with oil in the depression of the bond head.

Compressed-terminal Head Bonds. — A four-spindle drill is used to drill four holes simultaneously in the rail heads. It is important to avoid drilling the holes too deep lest the copper should not touch bottom and therefore be unable to expand laterally. If, on the other hand, the hole is too shallow, expansion will occur too soon.

Pressure and Contact Resistance. — It has been found by P. M. Hall, P. C. Smith and C. B. Starbird (*St. Ry. Jour., Sept. 14, 1907*) that the pressure of a compressed-terminal rail bond against the steel rail, which gives a reasonably low-resistance value, is from 25,000 to 30,000 pounds per square inch of contact surface. To obtain this pressure a compressor giving a direct pressure of 25 tons per square inch of terminal steel section is required. This pressure being within the elastic limit of steel, the metal of the rail does not take a permanent set. Furthermore, if the contact surface of the bond terminal be increased, no appreciable decrease of resistance will occur unless the pressure is correspondingly increased. The contact resistance between annealed cast copper and steel is from 30 to 60 per cent higher than the resistance between annealed rolled copper and steel.

The copper of a bond head is hardened by the pressure it is subjected to, and, like the steel, is distorted within its elastic limit, causing the surfaces to adhere even if the pressure is reduced to one-third its original value, say 10,000 pounds per square inch. Between these two pressures, the electrical resistance does not vary. Expansion due to heat, therefore, has no effect upon the resistance of bonds.

TESTS AFTER INSTALLATION. — Out of 42 companies 15 do not inspect regularly, 6 test only by inspection, and melted snow. Twenty-five companies which inspect, test at the following intervals

Interval, months	Number of companies
3	2
6	10
12	12
18	1

Twenty-three of these companies use portable bond testers; four use autographic test cars.

Resistance Test. — The usual method of testing is to measure the drop of potential across the bonded joint and find simultaneously the length of continuous rail in which the same drop occurs, i.e., the "equivalent" length of the

bonded joint. Several ingenious instruments have been devised for making this comparison with ease and accuracy.

Differential Voltmeter Method. — The most accurate type consists of a differential voltmeter and three contact pieces for attachment to the rail. One winding of the differential voltmeter is connected to a pair of contact pieces *A* and *B*, which are placed in contact with the rail just over the centers of the bond terminals, see Fig. 9, and the other winding to a pair of contact pieces *B* and *C*, which span a variable length of continuous rail. The distance *BC* is varied by moving the contact *C* until the voltmeter shows no deflection. This indicates that the potential drop between *A* and *B* is equal to that between *B* and *C*, or in other words, that a length of continuous rail has been found which has the same resistance as the bonded joint. The tester should make sure that there is current in the rail while he is making the test.

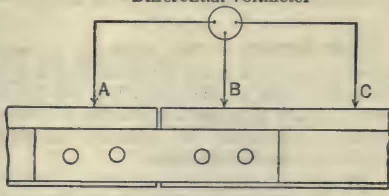

Fig. 9.

Conant Bond Tester. — R. W. Conant (Cambridge, Mass.) makes a bond tester in which the balancing is done by sound. A clockwork interrupter, an induction coil, and a telephone receiver is connected across a Wheatstone bridge (*see Bridges, for Electric Measurements*). The interrupted current causes sounds in the telephone receiver, which sounds diminish and disappear when the drop in the rail is balanced, by the bridge adjustment, to equal the drop across the joint. This type of apparatus has the advantage of being independent of the traction current in the rails, and is, therefore, constant in its sensitiveness.

Herrick Test Car. — A. B. Herrick has devised a recording instrument, which being placed on a car, is able to test the bonding of a railroad while running the car over it. The Herrick test car carries autographic recording apparatus, which measures by means of millivoltmeters, the potential drop across each joint, recording on sensitized paper by an electric spark, the scale of the record being usually about 1 inch to 60 feet of track. Wire-brush contacts made of steel of composition similar to that of the rail measure the drop across joints through which low-voltage current is circulated from a small motor generator set on the car. The car trucks are insulated from each other, current passing through the rail from one truck to the other, while the bond is under measurement. The apparatus can also automatically mark bad bonds by squirting paint on the rail.

The advantages of this method of testing are as follows: 1. Uniform impressed low-voltage current through joints while testing; 2. Heavy car passing over joint breaks any transient connection made by fish-plate; 3. Autographic record produced by the bonds themselves of their condition.

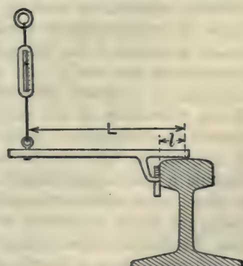

Fig. 10.

Mechanical Strength. — The mechanical adhesion of soldered bonds may be tested by means of a lever as shown in Fig. 10. It may be used as soon

as the terminals are cool. The operation of testing consists simply in submitting each bond terminal to a predetermined pull. A properly soldered bond should stand a shearing force of 1200 pounds per square inch of contact. Calling S this shearing force per square inch, A the square inch of contact, and P the pull, as registered on the balance, then

$$P = \frac{ASl}{L}.$$

REBONDING. — The resistance at which a joint should be rebonded depends upon how much potential drop is permissible in the tracks, and upon the relative cost of the energy loss and the cost of rebonding. The latter, in turn, depends upon the probable life of a new bond. If the energy loss is the primary consideration, the resistance at which rebonding becomes economical is given by the formula below.

Let

R = amount by which the resistance of the existing bond has increased over that of a new bond, ohms,

B = cost of rebonding once, dollars, allowing credit for scrap material,

I = root-mean-square current in the bond, amperes,

C = cost of an increment of energy, dollars per kw. hour,

T = expected life of the bond in days, i.e., the number of days in which its resistance is expected to increase R ohms, or in which it is to be removed for any reason whatsoever. This may be estimated from previous experience on the railway under consideration, the usual limitations being the life of the rails to which they are attached, corrosion by electrolysis, fracture due to crystallization and loosening of the terminals.

Then it is economical to rebond if R is equal to or greater than

$$\frac{B}{0.024\, I^2CT}.$$

It is usual to state the resistance of bonds in terms of the number of feet of rail having the same resistance. Let r be the resistance of the rail in ohms per foot; then the increased resistance R will be represented by an increased length, expressed in feet, equal to $\frac{R}{r}$.

It should also be noted that bonds in different parts of a railway system carry different amounts of current, those nearer the station bus carrying more than those at comparatively remote points. Hence, the economical resistance at which to rebond is less the nearer the bond is to the bus, a consideration that complicates the use of the above formula. Furthermore, as the economical resistance is inversely proportional to the square of the current, a slight error in the estimation of the root-mean-square current will lead to a considerable error in the resistance. Such errors are unavoidable, a circumstance that, taken in conjunction with the probable error in the estimated life, renders the above formula a mere approximation. It is, therefore, usual for railway companies to select some arbitrary total resistance at which to replace their bonds, this resistance being expressed in terms of equivalent length of rail. For urban lines this length varies from 3½ feet to 20 feet with an average of about 8 feet; for suburban lines, a length of 12 feet or more is common.

REPAIRS. — If a soldered bond becomes loose, it may often be resoldered, but defective bonds of other types are usually scrapped, unless the defect is of a very trivial nature. Pin-expanded bonds may, however, be reëxpanded into

larger holes, but with some loss in efficiency. Soldered bonds entail a comparatively large and increasing expense for repairs when applied to track rails, in spite of the fact that no failures may occur for several months after installation. Most bond failures are due to loose rail joints.

COSTS. — The cost of bonding is extremely variable, depending upon the type and size of bond, cost of labor, etc. The following cost data should therefore be used with caution.

COST OF BONDING

	Cost per joint		
	Labor	Material	Total
a. 2–500,000 C.M. bonds soldered to head of third rail..............	$0.66	$1.23	$1.89
b. 2–500,000 C.M. pin-expanded concealed bonds applied to track rail.	0.69	2.05	2.74
c. 2–400,000 C.M. compressed terminal concealed bonds applied to track rail.....................	....	....	1.90
d. 2–0000 bonds of same type as *c*......	0.50	1.00	1.50

Note.—In *a* and *b* the item for material includes inspection and the labor item includes foreman's salary; solder, acids and tools, not included.

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BRAKES AND BRAKING SYSTEMS. — (*See also Cars, Electric; Railways, Energy Requirements for.*) In order to stop a car or train a torque must be applied to the wheels in a direction opposite to the direction of motion of the car. This may be accomplished by applying a frictional retarding force to the wheel rims, by applying a retarding force to the axle by means of a magnetically operated friction clutch, by applying a reverse torque to the axles by operating the motors as generators, or by applying a frictional force to the rails directly by a "track brake." The method of applying a retarding frictional force to the wheel rims is the most general one and lends itself most readily to the system of manipulating the brakes by compressed air. This system has done much to improve the safety of travel on railroads. The other methods have all been tried and some have been put into practical operation, but only to meet special local conditions.

FRICTIONAL RESISTANCES IN BRAKE-SHOE SYSTEM. — The application of the usual brake-shoe system makes use of the frictional adhesion between the wheels and the track and between the brake shoes and the wheels. Both these quantities vary throughout a considerable range and it is therefore necessary to adjust the pressure between the various members so that it is possible to rely on a definite minimum value.

Adhesion between Wheel Rim and Rails. — The coefficient of adhesion between the wheel rims and rails varies from less than 15 per cent to over 30 per cent depending upon the condition of the track and the relative motion between the track and wheel rim. (*See Railways, Energy Requirements for.*) An adhesion of 15 per cent can usually be depended upon with normal track, and this can be increased to 25 per cent by the use of sand. But these values only obtain while the wheels are rolling on the track. If they begin to slide the coefficient decreases considerably. For this reason the braking effort must always be controlled so that the wheels do not slip.

Adhesion between Brake Shoe and Wheel Rim.— When the brakes are applied, the retarding force is applied below the center of gravity of the car body. The latter is therefore subjected to a couple and tends to press downward at the forward end and upward at the rear end, thus changing the distribution of weight on the axles and decreasing the adhesion on some axles or trucks. It is therefore not possible to figure on using for braking purposes the same weight per axle as exists at standstill. For this reason the brake-shoe adhesion must be less than the track adhesion. The coefficient of adhesion between the customary cast-iron brake shoe and the steel tire of the wheel varies with the speed, and decreases as the time of application increases. As the speed increases the coefficient drops off, being a maximum of from 30 to 25 per cent at speeds from 0 to 5 mi. per hr., 20 per cent at 20 mi. per hr., 14 per cent at 40 mi. per hr., and 7.5 per cent at 60 mi. per hr. Thus at high speeds a heavy pressure may be applied without stopping the wheels, while as the speed of the car diminishes the pressure on the brake shoes must be decreased in order to prevent gripping the wheels and causing them to slide on the track.

Effect of Angular Momentum. — In addition to overcoming the linear momentum of the cars the brakes must overcome the angular momentum of the gears and motor armatures. The effect of the latter is to introduce a tendency of the whole motor to rotate around the car axle and introduce additional strains on the gears and on the trucks. For this reason brake shoes hung between the wheels of a truck are better than those hung on the outside of the wheels.

HAND BRAKES are always provided on cars and locomotives whether power brakes are employed or not, as they are necessary to hold a car left out of service on a grade, because the air brakes will not hold a car standing idle for any

length of time. When a car is descending a very steep grade it is customary to set the hand brakes to hold the speed of the car and reserve the power brakes for emergency or for stopping the car. In hand braking equipment the "foundation" brakes (*see next paragraph*) are actuated through a drum or lever system which is hand operated.

POWER BRAKES. — The "foundation" brakes are that part of the brake equipment usually furnished separately from the power-braking equipment, and consist of the brake shoes, hangers, equalizers, levers, etc., back to the brake cylinder. To this is attached the desired form or make of power brake. Of the various forms of power brakes in use, viz., air, electric, regenerative and electro-pneumatic, the air-brake is the most generally used.

Air-Brakes. — In electric railway practice there are three systems of air-brakes in use, each of which is best suited to a definite type of service and has its particular field, as follows: (1) straight air-brake system for cars always operated singly; (2) emergency straight air-brake system for cars operated in trains of two or three but never more than three cars; (3) automatic air-brake system for cars operated in trains of any number of cars.

Straight Air-brake System. — The equipment for the straight air-brake system consists essentially of a motor-driven compressor, a reservoir, a brake cylinder, a motorman's valve, a train pipe, and the foundation brakes. The brakes are applied by direct pressure, that is by admitting air from the reservoir directly to the cylinder. The advantages of the system are that it is quick-acting and the braking effort is easily controlled to any value desired. Its disadvantage is that in trains a leak in the train pipe renders the brakes ineffective so that there is no means of applying the brakes on the trail-cars if the train should break apart. Sometimes instead of a compressor the motor-car carries a large reservoir which is charged at stations.

Emergency Straight Air-brake System. — This system involves a special valve and an extra pipe on each car so arranged that if the air pressure in the train pipe drops, due to cars breaking apart, this valve automatically turns the pressure of the reservoir into the brake cylinders and applies the brakes. The equipment of the motor car includes an extra pipe and each trail car has a reservoir, two pipes, automatic valve, brake cylinder and foundation brakes. The usual operation in service is like that of the straight air-brake.

Automatic Air-brake System. — This system involves the use of an auxiliary reservoir and a "triple valve" on each car. Whenever the air pressure in the train pipe is reduced, either intentionally or accidentally, this triple valve turns the air pressure of the auxiliary reservoir on each car into the brake cylinders. The brakes are applied by the motorman by opening the service pipe to the air by means of the engineer's valve. The engineer's valve has three positions, "off," "lap," and "on." By turning the handle to the "on" position the train pipe is opened to the atmosphere and the air continuously escapes. This applies the brakes with a continuously increasing pressure. When the desired pressure has been reached the handle is turned to the "lap" position, when the pressure of air in the train pipe and the pressure of the brakes on the wheels remains constant. By turning the handle to the "off" position pressure is restored in the train pipe, the brakes are released, and the reservoirs recharged.

Quick-action Triple Valve. — In any air-brake system an appreciable time elapses between the action of turning the engineer's valve and the actual application of the brakes. If there are many cars in a train the brakes may be applied on the leading cars sometime before they are applied on the rear cars and the result is that the rear cars bump into the forward cars, and may cause a derailment, particularly if this happens on a curve. To guard against this, the

"quick-action triple valve" is used on long trains. This consists of a special form of valve which causes a local intensification of the change in pressure in the train pipe on each car. The action is somewhat similar to a relay which is affected by slight changes in pressure and causes greater changes, and thus applies or releases the brakes more quickly.

Electromagnetic Brakes. — Any braking system making use of the electric current is not as reliable as the air system since if either the trolley comes off or the motors fail to pick up as generators the brakes are inoperative. Electromagnetic braking may be applied either to the axles or directly to the track.

Disc Brakes. — This system comprises two cast-iron discs mounted concentrically on the car axle. One is keyed to the axle and the other anchored to the truck frames. A coil is placed in one of these discs and when a current flows in the coil the two discs are attracted to each other by the magnetism and the friction between them retards the car. The current for the coils may be taken from the trolley or from the motors operating as generators. In the latter case the motors supply an additional retarding force. The braking effort is controlled by resistance in series with the magnet coils.

Track Brakes. — The principle of electromagnetic braking may also be applied to a track brake in which a magnet coil when energized draws a shoe against the rail, the pressure being regulated by a resistance in series with the magnet coil.

Regenerative Braking. — By supplying the field circuits of the motors with a current of proper value and direction by means of separate excitation they may be operated as generators and made to exert a strong retarding effort on the car axles. This separate excitation may be obtained by connecting the field windings of all the motors on a car in series with themselves and with a regulating resistance to the trolley, or by means of a motor generator set (on a locomotive) supplying a low potential of the proper value. The energy developed by the motors when acting as generators may be dissipated in rheostats or may be returned to the distributing system. In the latter case the voltage generated by the motors must be accurately controlled. Ordinarily, if the energy is returned to the line, the braking effort can only be obtained at speeds in the proximity of full speed. Very special means must be provided to make it possible to return energy to the line at low speeds.

This system has had its most elaborate application in the locomotives of the C. M. & St. P. Ry. of U. S. A., where experience over a considerable period of time shows a saving of approximately 15 per cent in the energy consumption with grades varying from 0.7 per cent to 2 per cent. Of greater importance than the saving of energy, is the saving in wear on wheel tires, brake shoes and track and the ease of control, as these features have reduced the number of derailments to a marked extent. In the early St. Paul locomotives, a special motor generator was provided to serve as an exciter. In the later locomotives, two of the main traction motors are used to supply the exciting currents for the rest of the motors when they act as generators.

Three-phase induction motors will provide regenerative braking at rated speed without auxiliary devices.

Electro-pneumatic Brakes. — This system has been proposed as an improvement over the quick-acting air-brake system. It involves controlling the application of the brakes on each car by means of an electromagnet receiving current from the locomotive cab. By energizing this circuit the motorman can set the air brakes on all cars practically simultaneously. Each car would be equipped with the usual brake cylinders and reservoir and merely the application of the brakes would be controlled electrically.

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BRICKS AND BRICK MASONRY. — (*See also Buildings, Allowable Unit Stresses in; Cement; Concrete.*) Brick construction may be made more ornamental than concrete, but is generally more expensive to place on account of the necessity of handling each brick as an individual unit. The proportion of building and engineering construction in which bricks are used, although still extensive, has been falling off in recent years with the increasing use of plain and reinforced concrete.

Kinds of Bricks. — Ordinary or clay bricks are made by moulding a rectangular block of nearly pure clay, or clay and clean sand, and subsequently burning it in a kiln for from one to two weeks. Pressed bricks are machine molded clay bricks used for facing work in high-class construction. Sand lime bricks are also used to some extent. They are made from sand cemented by lime and are hardened by being subjected to steam pressure at from 100 to 150 pounds per square inch for about one-half a day. Fire bricks are a special kind of clay bricks, which are made to stand high temperatures. The refractory properties of fire bricks depend chiefly upon the amount of silicon contained. The amount of iron oxide in fire bricks should not exceed 6 per cent. Vitrified bricks are very hard burned clay bricks which have been annealed by slow cooling; used in pavement work.

Physical Properties. — A common, practical test for a good brick is that it will give a clear, ringing sound when struck with a hammer. The color of a brick is not a reliable indication of its strength or properties. The water absorbed in a specified time by a number of bricks is sometimes considered to indicate their relative strengths, as water acts as a lubricant on the material and causes crushing to occur more readily than when dry, but there is considerable difference of opinion as to the value of this test.

The compressive strength of dry bricks ranges from 500 pounds per square inch for soft bricks to over 10,000 pounds per square inch for pressed bricks. The strength of the individual bricks, however, is of little importance except for comparing various kinds, as the strength of brick masonry is usually limited by the strength of the mortar used.

The specific gravity of bricks ranges from 1.6 to 2.6. See also paragraph below on *Brick Masonry* and article on *Weights of Materials*.

Several foreign countries have adopted legal standard sizes for brick, but the only standard sizes in the United States are those specified by the National Brick Mfg. Assoc. as follows:

Common brick........................	8¼ by 4 by 2¼ inches
Pressed brick........................	8⅜ by 4 by 2⅜ inches
Paving brick........................	8½ by 4 by 2½ inches

BRICK MASONRY. — Bricks are laid in lime or cement mortar, the mortar forming a cushion which fills the interstices and keeps out water. Joints, i.e., the spaces between adjacent bricks which are filled with mortar, are usually made from ¼ to ⅜ inch thick in outside walls, and from ⅜ to ½ inch thick in inner walls. In ordinary building work, where the weight of the wall itself is the greater part of the load, lime mortar is usually used because it is cheap. Where strength is required of brickwork, a rich Portland cement mortar should be used; see article on *Cement*. Tests upon the ultimate compressive strength of brick piers are quoted in detail in Johnson's *Materials of Construction*, and Baker's *Masonry Construction*. The values of loads allowed upon brick masonry in various localities range from 100 to 300 pounds per square inch; see *Buildings, Allowable Unit Stresses in*. A brick wall itself weighs from 100 to 145 pounds per cubic foot, depending somewhat upon the quality of the brick and the thickness of joints.

Measurement of Brickwork. — Bricks are usually sold by the thousand. Brickwork is commonly measured by the cubic yard in place, but the units of per thousand brick and per square yard of surface area are also used. The relation between the number of bricks used and the volume of finished brickwork depends upon the size of brick, thickness of joints and shape of the structure. For solid brick walls the number of bricks per cubic yard ranges from about 400 with ⅝-inch joints to about 500 with ¼-inch joints. An allowance of 3 or 4 per cent excess should be made in estimating to provide for waste and breakage.

Cost of Brickwork (Pre-war figures). — Hudson River common bricks cost prior to the war about $6.00 per thousand. Lime for mortar was quoted at $0.97 to $1.10 per 200-pound barrel. The total cost of brick masonry including labor and materials was from $6.00 to $15.00 per cubic yard, depending upon the locality, type of construction and quality of brick and mortar. An average figure for ordinary brick walls was from $8.00 to $10.00 per cubic yard.

BRIDGES FOR ELECTRICAL MEASUREMENTS. — (*See also Inductance and Inductive Reactance; Resistance and Conductance, Electric; Resistors, Standard; Wires and Cables, Insulated.*) Numerous arrangements of electric circuits, known under the general term of "bridges," are used for the comparison of unknown with known resistances, capacities and inductances. The fundamental principle of an electric bridge is the adjustment of the component circuits or arms of the bridge in such a manner that the drop in potential V in the arm formed by the circuit to be tested is to the drop of potential V_r in an arm having known constants as the drop of potential V_a in one "ratio" arm is to the drop V_b in the second "ratio" arm.

A bridge may be made up of separate resistance boxes (*see Resistors, Standard*), but where frequent tests are to be made it is more desirable to have all the resistances, keys, etc., mounted together in a single box. Complete bridges of this kind can be obtained from instrument manufacturers.

SLIDE-WIRE BRIDGE (Fig. 1). — This is the simplest form of bridge but is seldom used in commercial testing, as its range is limited. In principle it is the same as a Wheatstone bridge (*see below*), the difference being a structural one only, in that a wire is used for the "ratio" arms.

The slide wire, AB in Fig. 1, is generally a wire of uniform cross-section stretched along a meter scale which is divided into millimeters. All connections in the X and R circuits are made of heavy low-resistance material. A slider is provided for sliding over and making contact on the wire. When the bridge is balanced (i.e., no current through the galvanometer),

$$X = \frac{AR}{1000 - A},$$

where A is the distance in millimeters between the slider and the left-hand end of the bridge. The slide wire can be made uniform to 1/10 per cent and the error in observation may be even smaller than this. When using a ratio greater than 1 to 1 and when a low resistance is being measured, the errors are greater. It is possible when using an even ratio to make very exact comparisons of the standard and unknown resistances by reversing them and taking the mean of the two measurements.

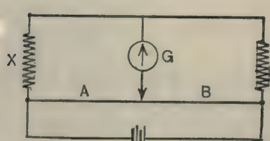

Fig. 1. Slide-wire Bridge

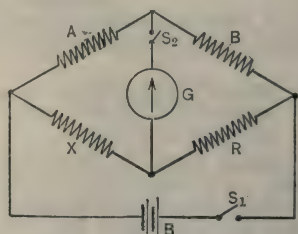

Fig. 2. Wheatstone Bridge

WHEATSTONE BRIDGES. — Fig. 2 shows the arrangement of resistances in the ordinary type of Wheatstone bridge used for comparing resistances. A and B represent two coils or sets of coils, usually referred to as the "ratio" coils, whose relative resistances must be known but whose actual resistance values are unnecessary; R is a standard variable resistance, or resistance box (*see Resistors, Standard*); X is the unknown resistance; G is a sensitive galvanometer connected across two points of the diamond and B is a battery connected across the other two points of the diamond.

For no deflection of the galvanometer when the switches S_1 and S_2 are closed, i.e., when the bridge is balanced,

$$X = \frac{A}{B} \cdot R,$$

where the letters represent the resistances of the four arms.

Construction. — Wheatstone bridges having the various parts mounted in one box may be divided into three classes: (1) portable bridges, in which not only the resistance coils and keys but also the battery and galvanometer are mounted in one box; (2) laboratory bridges, in which the coils and in some cases the keys are mounted in one box, but for which separate galvanometers and batteries must be provided; and (3) precision bridges, arranged in the same manner as laboratory bridges but made with greater care and capable of greater precision.

Since the construction of the resistance coils, blocks, plugs and other minor details of bridges is practically the same as for resistance boxes (*see Resistors, Standard*); only the arrangement of coils and connections will be described here.

Rheostat Arrangements. — The coils forming the rheostat (the variable resistance R in Fig. 2) may be arranged on the 1, 2, 3, 4 plan as shown in Fig. 3, on the 1, 2, 2, 5 plan, on the decade plan of Fig. 4, or on any other plan. (*See Resistors, Standard.*)

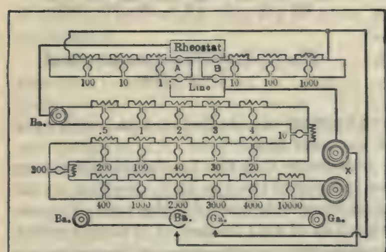

Fig. 3. Wheatstone Bridge (Post Office Type)

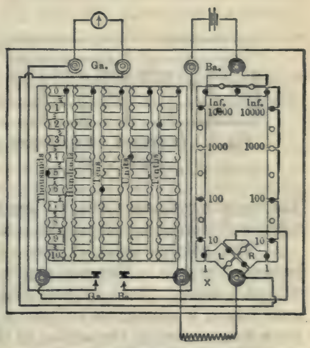

Fig. 4. Wheatstone Bridge (Anthony Type)

The advantage of the decade arrangement is that few plugs are required, and, therefore, there is less likelihood of error due to the contact resistance. A very good form of the decade type of rheostat is used in the Anthony type of Wheatstone bridge, shown in Fig. 4. In this bridge the coils may be joined in series or multiple or in any desired combination of series and multiple.

Plug and Dial Arrangements. — The resistances of the various arms of the bridge may be varied either by the insertion of plugs between the heavy metal terminals of the coils provided on the top of the box, or a dial arrangement similar to that employed on ordinary rheostats (but more carefully made to reduce the contact resistance) may be used.

Ratio Coils (Figs. 5 to 7). — The connections of the ratio arms in a common construction of dial bridge is shown in Fig. 5. The values of the coils are so chosen that the ratio of these coils is always that given by the stamping for various settings of the contact S. An advantage of this type of ratio coils is

that there is no error due to contact resistance in the ratio coils, as the contact is in series with the battery.

The arrangement of an improved form of ratio coils is shown diagrammatically in Fig. 6. This arrangement of ratio coils has the advantage over the old post-office arrangement shown in Fig. 7, in that there are but two plugs to operate and the coils can be checked by reversal.

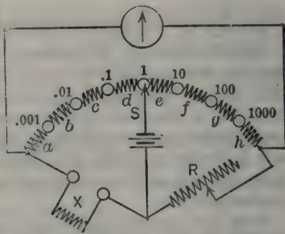

Fig. 5. Dial Type Ratio Coils

Precision of Measurements by Wheatstone Bridge. — The degree of accuracy with which a resistance can be measured by means of a Wheatstone bridge is determined by the following conditions:

1. The accuracy to which the resistances of the various coils is adjusted. The range of accuracy is from about 0.1 per cent for portable bridges to 0.02 per cent for precision bridges.
2. The relative value of the coil resistances and the contact resistances at plugs and terminals. A well-fitting, clean plug has a resistance of from 0.0001 to 0.00005 ohm; a poorly fitting or greasy plug a resistance of 0.01 ohm or higher.

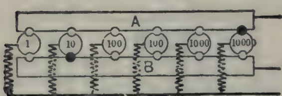

Fig. 6. Improved Arrangement of Ratio Coils

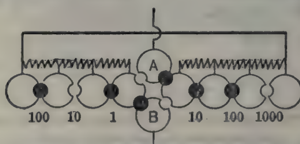

Fig. 7. Post-Office Arrangement of Ratio Coils

The contact resistance at a binding post between the binding post and wire, even when well clamped, ranges from 0.001 to 0.0001 ohm.

3. The relative value of the coil resistances and the insulation resistances between the coils. Dirt and grease on the top of the box may introduce a considerable error. Hard rubber, which is commonly used for the tops of resistance boxes, is also liable to have a thin film of acid formed on it when exposed for a long period to the action of the moisture and impurities in the air, which attack the sulphur in the rubber. Such a film will greatly reduce the insulation resistance between the lugs on top of the box.
4. The effect of changes of temperature in changing the resistances of the coils and in producing thermoelectric effects. The thermal e.m.f. in a copper-brass-copper circuit is about 2×10^{-6} volts per degree C. difference in temperature between the two junctions. Thermoelectric troubles can usually be avoided by reversing the connections of the battery to the bridge and taking the mean of the values of the resistances corresponding to the balance when the battery current flows through the bridge first in one direction and then in the other.
5. The maximum current that the coils will safely carry; this ranges from about 0.5 ampere for 1-ohm coils down to 0.005 ampere for a 5000-ohm coil.
6. The resistances of the galvanometer and battery, particularly the former. The battery resistance is usually small.
7. The relative resistances of the various arms of the bridge.
8. The sensitiveness of the galvanometer, i.e., the deflection per unit current.

Best Galvanometer Resistance. — Referring to Fig. 2, the best galvanometer resistance is

$$G = \frac{(A + X)(B + R)}{A + B + R + X},$$

where the letters designate the resistances of the various branches.

Best Location of Galvanometer. — Knowing the galvanometer and battery resistances (the galvanometer resistance is usually the larger), connect the one having the higher resistance so that it joins the junction of the two arms of the bridge having the highest resistances to the junction of the two arms having the lowest resistances.

Precautions in Making Measurements. — The following rules should be observed in using a Wheatstone bridge:

1. Do not employ a battery having an e.m.f. of over 5 volts; a lower value is desirable if sufficient sensitiveness can be obtained.
2. Always shunt the galvanometer during preliminary adjustments. The shunt circuit should be opened when the final balance is made.
3. See that all binding posts are screwed up tightly and all plugs firmly inserted. After withdrawing a plug those adjacent to it should be retightened.
4. For a preliminary balance use a 1 to 1 ratio.
5. In manipulating the keys be careful not to touch the metal work, as the heat and moisture of the hand is likely to set up appreciable electromotive forces.
6. Always close the battery switch first and then the galvanometer switch, to avoid momentary deflections of the galvanometer due to the transient e.m.f.'s set up while the currents are establishing themselves.
7. If the contact or lead resistances are appreciable relative to the resistances of the coils with which they are in series, these resistances should be separately determined.

Care of Bridge. — In order that a bridge or resistance box may remain in first-class condition, it must be carefully protected from dust and moisture by being covered when it is not in use. To insure high insulation the top must be kept clean, especially between the blocks. The plugs must be kept free from grit, grease or from contact with mercury. Grease may be removed by the use of a little benzol. Never use sand or emery paper for cleaning the plugs or holes, for the surfaces of the taper will be spoiled by this treatment. These materials work into the metal and cannot be removed, thus causing the plugs to "cut." If it becomes absolutely necessary to clean the plugs and sockets, a little of the very finest whiting may be employed. Extra holes of the proper taper bored in the brass blocks are convenient for holding the plugs when they are not in use, and also for attaching a movable terminal. Never apply undue force in inserting the plugs.

KELVIN BRIDGE. — The ordinary form of Wheatstone bridge is not suitable for measuring with accuracy a resistance of 0.1 ohm or less, due to the contact resistances introduced at the binding posts, this contact resistance being of the order of 0.0001 to 0.001 ohm. An arrangement of circuits to avoid these contact resistances devised by Lord Kelvin (*Wm. Thomson*) is shown in Fig. 8, and is known as the Kelvin or Thomson bridge. There will be no current in the galvanometer when

$$X = \frac{A}{B} \cdot R + \frac{bd}{a + b + d}\left(\frac{A}{B} - \frac{a}{b}\right).$$

If $\frac{a}{b}$ is made equal to $\frac{A}{B}$, then

$$X = \frac{A}{B} \cdot R.$$

It is also well to make the resistance of d extremely low in comparison with X or R, since the lower the resistance of d the less an error in $a/b = A/B$ will affect the accuracy of measurement.

In the above formulas X and R are the resistances between the arrowheads, which represent the "potential terminals." Such terminals are always provided on low-resistance standards (*see Resistors, Standard.*)

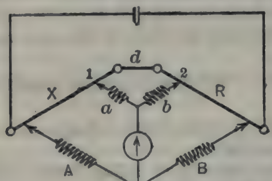

Fig. 8. Kelvin Bridge

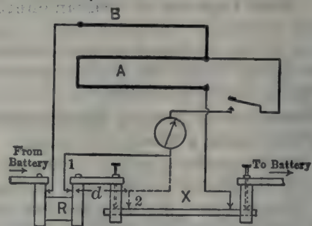

Fig. 9. Modification of the Kelvin Bridge

Simple Modification of the Kelvin Bridge. — When suitable resistances are not available for a and b, these resistances may be omitted and the galvanometer terminal be connected first at 1 and then at 2, and A adjusted for a balance in each case; let A_1 and A_2 be the corresponding values of A. Then

$$X = R\,\frac{A_1\,(B + A_2)}{B\,(B + A_1)}.$$

This method is very convenient for measuring the resistance X of a low-resistance shunt (*see Shunts*). The only standard resistances required are a low-resistance standard R and a resistance box with three terminals (or two separate resistance boxes). Fig. 9 shows such an arrangement.

HOOPES' CONDUCTIVITY BRIDGE. — This bridge is also a modification of the Kelvin bridge, designed for the rapid determination of the relative conductivity (*see Resistance and Conductance*) of samples of wire. It is extensively used in wire factories. A diagram of the connections is shown in Fig. 10.

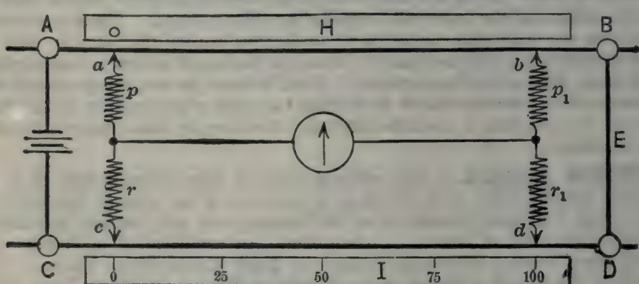

Fig. 10. Hoopes' Conductivity Bridge

The standard A–B and the unknown C–D are of the same metal; consequently if care be taken that they are at the same temperature, all corrections due to temperature are avoided. The arms p, r, p_1, r_1 are in the same case and are made of material of low temperature coefficient so that their relative values will not change. They are adjusted so that $p = r$ and $p_1 = r_1$; consequently at balance the resistance of c–d equals the resistance of a–b.

The sample $C-D$ is placed alongside a scale I divided into 100 parts, so that the graduations represent percentages of the total length of the scale. Accompanying the standard wire $A-B$ is a scale H, on which are laid off a number of points corresponding to the weights of the standard length (38 inches) of a range of sizes of sample wires.

To make a conductivity reading, the weight of the standard length of the sample $C-D$ is found to within an accuracy of $\frac{1}{20}$ per cent. The contact b is set at the point on scale H corresponding to this weight, the contacts a and c being at the zero points of their respective scales. After the case has been closed a sufficient length of time to allow both the standard and sample to assume the same temperature, the contact d is moved until the galvanometer shows no deflection; this will occur when the resistance between a and b is equal to that between c and d. The scale reading corresponding to the position of d for a balance is equal to the per cent conductivity.

The Hoopes' bridge is so designed that the standard wire with its scale is removable from the bridge and so that a single standard covers a range of sizes equal to 3 numbers of B. & S. gauge. Any number of standards can be supplied with a bridge, so that it can cover an extensive range of sizes and can also be used for wires of different materials. In order to keep the standard wire and the test wire at the same temperature the bridge is mounted in a metal-lined case and the scale read through a glass window in the case, the window being closed by a metal screen when readings are not being taken.

Resistance of Electrolytes. — The resistance of an electrolyte can be measured by using the Wheatstone bridge principle with an a-c. source substituted for the battery and an a-c. detector in place of the a-c. galvanometer. Alternating current must be used on account of the polarizing action of a direct current. A convenient source of alternating current is an induction coil with the secondary winding in place of the battery, as in an ordinary set-up. A telephone receiver makes a very good detector when an induction coil is used. The current from an induction coil gives a very clear and sharp sound in the telephone, making accurate balancing of the bridge quite simple. If a sinusoidal alternating current is used, a vibration galvanometer may be used to better advantage than a telephone. To measure merely the resistance of an electrolyte it is only necessary to place the electrolyte in a vessel provided with two electrodes, preferably of gold or platinum, and connect this vessel into one arm of a Wheatstone bridge and balance for alternating current. The other arms of the bridge are resistances which must be strictly non-inductive.

To determine the resistivity it is necessary to place the electrolyte in a vessel whose length and cross-section can be accurately measured. Such a vessel may be made from a glass tube, whose internal diameter is known, and in the two ends of which metal plugs are inserted. These plugs may be any convenient distance (say 20 cm.) apart. There should be several small holes in the side of the tube. The tube is then immersed in a vessel containing the electrolyte and the bridge balanced as before. The resistivity is then the product of the cross-sectional area of the tube and the resistance, divided by the length of the tube.

COSTS (Pre-war prices). — A good slide-wire bridge costs about \$20, a portable Wheatstone bridge from \$50 to \$125, depending upon the design, a laboratory Wheatstone bridge from \$25 to \$75 and a high-precision standard bridge from \$200 to \$400. A Hoopes' conductivity bridge with a single standard, covering 3 sizes of B. & S. gauge wire, costs about \$500; additional standards cost about \$50.

BIBLIOGRAPHY. — Laws, F. A., *Electrical Measurements;* Fleming, J. A., *Handbook for the Electrical Laboratory and Testing Room:* Gray, A., *Absolute Measurements in Electricity and Magnetism;* Northrup, E. F., *Methods of Measuring Electrical Resistance;* Bur. Stands., Sc. Pap. No. 288; *Catalogues of the Leeds & Northrup Co.*

BUILDINGS, ALLOWABLE UNIT STRESSES IN.—(*See also Brick; Cement; Concrete; Iron, Pig and Cast; Iron, Wrought; Steel; Structures, Simple; Timber.*) The following tables give the allowable unit stresses and loads in accordance with the building laws of the respective cities, as corrected to recent dates (*see Bibliography*).

TABLE I.—STEEL AND IRON

Loads in pounds per square inch

	New York	Chicago	Philadelphia	St. Louis	Boston
Compression:					
Rolled steel	16,000	14,000	16,250	14,000	20,000(*b*)
Cast steel	16,000	14,000	.....	14,000	....
Wrought-iron	....	10,000	12,500	10,000	....
Cast iron (in short blocks)	16,000	10,000	11,700	10,000	16,000(*b*)
Steel pins and shop or power driven rivets (bearing)	24,000	25,000	22,000	24,000	24,000
Steel field rivets, hand driven in (bearing)	16,000	.....	17,600	18,000	20,000
Wrought-iron pins and rivets (bearing)	....	.....	18,000	.....	....
Tension:					
Rolled steel	16,000	16,000	16,250	16,000	....
Cast steel	16,000	16,000	.....	16,000	....
Wrought iron	....	12,000	12,500	12,000	....
Cast iron	3,000	.....	.....	.....	....
Bending (extreme fiber stress):					
Rolled-steel beams	16,000	16,000	16,250	16,000	16,000(*c*)
Rolled-steel pins, rivets and bolts	20,000	25,000	.....	25,000	24,000
Riveted steel-beams	16,000(*a*)	16,000	16,250*	.....	16,000(*c*)
Riveted wrought-iron beams (net flange section)	....	.....	12,500	.....	....
Rolled wrought-iron beams	....	.....	12,500	12,000	....
Riveted wrought-iron pins	....	.....	.....	.....	....
Cast-iron compression side	16,000	10,000	17,500	10,000	10,000(*c*)
Cast iron, tension side	3,000	3,000	3,750	3,000	4,000
Shear:					
Steel, web plates	10,000	10,000	10,000	10,000	10,000
Steel, shop rivets and pins	12,000	12,000	11,000	12,000	10,000
Steel, field rivets	8,000	10,000	8,800	9,000	....
Wrought iron, web plates	....	.....	7,500	.....	....
Wrought-iron, shop rivets and pins	....	.....	9,000	.....	....
Wrought iron, field rivets	....	.....	7,200	.....	....
Cast iron	3,000	.....	.....	.....	2,000

* 14,500 for mild steel.

(*a*) Net flange section. (*b*) Bearing.

(*c*) Where top flange is stayed laterally at distance not greater than 20 times the width of flange $\left(\frac{L}{b} < 20\right)$. When this ratio is between 20 and 70 this formula applies up to $\frac{L}{b} = 70$. $f = 19{,}200 - 160\frac{L}{b}$. (*d*) $\frac{L}{b}$ not greater than 20.

TABLE II—TIMBER

Loads in pounds per square inch

	New York	Chicago	Philadelphia	St. Louis	Boston
Compression (short lengths):					
Oak, with grain	1400	900	......	1000	900
Oak, across grain	1000	500	......	500	500
Yellow pine, with grain	1600 *L*	1100 *L*	750 *L*	1100 *L*	1200 *D*
	1000 *S*	800 *S*		800 *S*	900 *So*
Yellow pine, across grain	1000 *L*	250	550 *L*	250 *L*	350 *D*
	800 *S*			250 *S*	250 *So*
White pine, with grain	1000	700	......	700	700
White pine, across grain	800	200	......	200	200
Spruce, with grain	1200	......	500	......	750
Spruce, across grain	800	......	300	......	200
Hemlock, with grain	800	500	350	500	......
Hemlock, across grain	800	150	250	150	......
Tension:					
Yellow pine	1200 *L*	1300 *L*	1800 *L*	1800 *L*	......
	900 *S*	1000 *S*	......	1000 *S*	......
White pine	700	800	......	800	......
Spruce	800	......	1250	......	1000
Oak	1200	1200	......	1800	......
Hemlock	600	600	1000	600	......
Bending (extreme fiber stress):					
Yellow pine	1600 *L*	1300 *L*	1600 *L*	1100 *L*	1600 *D*
	1000 *S*	1000 *S*	......	800 *S*	1200 *So*
White pine	1200	800	......	700	1000
Spruce	1200	......	1100	......	......
Oak	1200	1200	......	1000	1400
Hemlock	800	600	900	500	......
Shear:					
Yellow pine, with grain	150 *L*	130 *L*	100 *L*	150 *L*	150 *D*
	100 *S*	120 *S*	......	120 *S*	100 *So*
Yellow pine, across grain	1000 *L*	......	1125 *L*	......	......
	1000 *S*				
White pine, with grain	100	80	......	80	80
White pine, across grain	500	......	......	......	......
Spruce, with grain	100	......	75	......	100
Spruce, across grain	500	......	750	......	......
Oak, with grain	200	200	......	200	200
Oak, across grain	1000	......	......	......	......
Hemlock, with grain	100	60	62	60	......
Hemlock, across grain	600	......	625	......	......

F.S. = Factor of Safety. *L* = Longleaf Yellow Pine. *S* = Shortleaf Yellow Pine. *D* = Dense Grade. *So* = Sound Grade.

TABLE III.—STEEL AND IRON COLUMNS

Loads in pounds per square inch *

City	Medium steel	Wrought iron	Cast iron
New York....	$16{,}000-70\frac{L}{R}$ (*a*)	no values given	$9{,}000-40\frac{L}{R}$ (*b*)
Chicago......	$16{,}000-70\frac{L}{R}$ (*a*)	$12{,}000-60\frac{L}{R}$	$10{,}000-60\frac{L}{R}$ (*b*)
Philadelphia..	$\dfrac{16{,}250}{1+\dfrac{L^2}{11{,}000\,R^2}}$	$\dfrac{12{,}500}{1+\dfrac{L^2}{15{,}000\,R^2}}$	$\dfrac{11{,}700}{1+\dfrac{L^2}{400\,D^2}}$
St. Louis....	$16{,}000-70\frac{L}{R}$ (*a*) with maximum of 14,000	$12{,}000-60\frac{L}{R}$ with maximum of 10,000	$10{,}000-60\frac{L}{R}$ (*b*)
Boston.....	$20{,}000-100\frac{L}{R}$ (*c*) with maximum value of 12,000	....	$9{,}000-40\frac{L}{R}$ (*d*)

* All values obtained either by formulas or from tables shall be reduced for eccentric loading.

L = unsupported length in inches.
D = diam. or least side in inches.
R = least radius of gyration in inches.
S = stress in lb. per sq. in.

(*a*) $L/R >$ 120 not allowed.
(*b*) $L/R >$ 70 not allowed.
(*c*) $L/R >$ 160 not allowed.
(*d*) $L/R >$ 96 not allowed.

TABLE IV.—TIMBER COLUMNS

Loads in pounds per square inch

City	Yellow pine (b)	White pine	Oak
New York....	$1200-20\frac{L}{D}$ (*a*)	$900-17\frac{L}{D}$ (*a*)	$1200-20\frac{L}{D}$ (*a*) for white oak
Chicago......	$1100\left(1-\frac{L}{80\,D}\right)$	$700\left(1-\frac{L}{80\,D}\right)$	$900\left(1-\frac{L}{80\,D}\right)$
Philadelphia..	$750\left(1-\frac{L}{100\,D}\right)$	....	....
Boston (*a*)..	$1200-20\frac{L}{D}$ (*c*) with maximum value of 1000	$S=585$ for $\frac{L}{D}=10$; $S=230$ for $\frac{L}{D}=40$	$900-15\frac{L}{D}$ (*c*) with maximum values of 750
St. Louis.....	$1100\left(1-\frac{L}{80\,D}\right)$	$700\left(1-\frac{L}{80\,D}\right)$	$1000\left(1-\frac{L}{80\,D}\right)$

L = unsupported length in inches.
D = diam. or least side in inches.
S = stress in lb. per sq. in.

(*a*) $L/D >$ 30 not allowed.
(*b*) Longleaf.
(*c*) $L/D >$ 40 not allowed.

TABLE V.—LIVE LOADS ON FLOORS AND ROOFS

Pounds per square foot

Kind of building	New York	Chicago	Philadelphia	St. Louis	Boston
Dwellings, hotels....	40	40 to 50	70	50	50 to 100
Office buildings:					
First floor........	60	50	100	100	125
Above first floor...	60	50	100	60	75
Schools.............	75	75 to 100	....	75	50 to 100
Buildings for public assembly.........	100	100	120	100	100
Stores..............	120	100	120	150	125
Factories...........	120	100	150	150	125 to 250
Roofs:					
Pitch $<$ 20 degrees..	40	25 (*a*)	30	30 (*b*)	40 (*b*)
Pitch $>$ 20 degrees..	30 (*a*)	25 (*a*)	30	....	....

(*a*) Measured in horizontal plane. (*b*) Flat.

TABLE VI.—MASONRY AND BUILDING MATERIALS

Loads (compression) in pounds per square inch

Item	New York	Chicago	Philadelphia	St. Louis	Boston
Concrete (P), 1 : 2 : 4.....	500	400 M	208 (*b*)	500 (*d*)	450 (*e*)
		350 H			
Concrete (P), 1 : 2½ : 5 (*f*)	400	350 M	......	......	361 (*g*)
		300 H	......	......	....
Rubble stonework:					
Portland cement mortar	140	200 C	139 (*b*)	......	....
		100 NC	......	......	....
Lime mortar..........	no value given	120 C	70		
		60 NC			
Brickwork:					
Portland cement mortar, 1 : 3..........	250 (*c*)	250 (*a*)	209 (*b*)	300 (*d*) 250 (*d*)	278
Lime mortar, 1 : 4 ...	110 (*c*)	100	111 (*c*)	100 (*c*)	111 (*c*)
Granite	1000	......	......	......	1000 (*h*)
Limestone..............	700	......	......	......	556 (*h*)
Marble.................	600	......	......	......	556 (*h*)
Sandstone..............	400	......	......	......	417 (*h*)

P = Portland cement.
M = Machine mixed.
H = Hand mixed.
NC = Not coursed.
C = Coursed.

a = Pressed.
b = Kind of concrete not specified.
c = Mixture not specified.
d = Vitrified paving brick, 300: hard pressed brick, 250.
e = Mixture 1 : 6.
f = Mixture 1 : 2 : 5 for Chicago.
g = Mixture 1 : 7½.
h = Portland cement mortat 1 : 2 and not more than ½-inch joints.

TABLE VII.—BEARING CAPACITY OF SOILS

Tons per square foot

Item	New York	Chicago (a)	Philadelphia	St. Louis	Boston
Clay, soft	1	......	...	According to test; maximum not to exceed 2½ tons on "solid natural clay" unless satisfactory tests are made	2 (a)
Clay, hard, dry	4	2.25 (a)	3.5		5–6
Sand, firm and coarse	4	2.5 (a)	3.5		5
Sand, fine and dry, firm	3	2.5 (a)	...		4
Clay and sand, wet	2	1.5	...		4–5
Clay and gravel, well cemented	..	......	6		6

(a) In relatively thick beds protected against lateral displacement.

BIBLIOGRAPHY. — *The Building Code of The City of New York, with amendments to July 17, 1919; Revised Building Ordinances, Chicago; Amendments to May 26, 1920; Laws and Ordinances Relating to the Bureau of Building Inspection, Philadelphia, July 1, 1920; Building Laws of the City of St. Louis, 1917; The Building Law of the City of Boston, with amendments to Oct., 1919.*

BUS-BARS AND BUS-BAR STRUCTURES. — (*See also Circuit Breakers; Power Stations; Substations; Switches; Switchgear Equipment for Power Stations; Wires and Cables.*) Bus-bars, or "omnibus bars," as they were originally called, are the common circuits into which the various generators deliver their output and from which the different feeders draw their supply of power. In large-capacity, moderate-voltage a-c. plants the bus-bars are usually placed in structures and connected to the various generators, feeders, etc., by suitable wiring. The bus-bars and their connections, together with the bus-bar structures, form one of the most important parts of large plants, as the entire energy of the station is usually concentrated thereon.

BUS-BAR SYSTEMS. — Where there is only a single set of bus-bars either in d-c. or a-c. stations the connections are said to be arranged on the "single-throw" system; when the connections can be made to either of two sets of bus-bars the system is spoken of as "double-throw"; but if the connections can be made to both sets of bus-bars instead of only to either set, the system is spoken of as the "selector system." Occasionally three or more sets of bus-bars are used.

If there is only one set of bus-bars, but with switches provided for dividing it into two or more sections, it is spoken of as a "sectioned bus." Where there are two sets of these sectioned bus-bars connected together at the ends, the system forms a "ring bus." In many high-voltage plants having step-up transformers each generator normally connects to the low-tension side of its own transformers, but switches are provided so that any transformer or generator can connect to a bus; such a bus is spoken of as a "relay bus." Where a number of feeders connect to a bus which in turn connects to the main bus through a switch or breaker, such a bus is spoken of as a "group bus."

BUS-BAR MATERIAL. — Depending on the current and voltage, bus-bars may be made of wire, rod, tubing, cable or strap, either bare or insulated. Solid wire is seldom used for more than 200 amperes, rod is used for less than 1000 amperes, tubing for 300 to 600 amperes, cable up to 1000 amperes, while strap is used up to any capacity.

Strap for bus-bars, particularly for heavy current, possesses several advantages over other shapes, the chief one being the ease with which additional straps may be added and the excellent radiating surface secured. Straps of different sections are in use, a typical one being 3 inches × ⅛ inch. Where more than one strap is required, a space is left between adjacent bars making the so-called laminated bus. The usual spacing left with 3 inches × ⅛ inch bars is ⅜ inch. The connections from switches, circuit breakers, etc., to the bus are made of one or more similar straps suitably interleaved and clamped together.

With this construction and due to the large surface exposed in comparison to the section of copper used comparatively high current density may be employed for a small number of straps without exceeding a safe temperature rise. The exact amount of current to be carried for a given rise depends somewhat on local conditions, ventilation, etc., and whether the bus is being used for direct current, 25-cycle, or 60-cycle service, and the temperature rise is not the same for different parts of the bar. A typical test under average conditions, 60-cycle service, 25° rise, indicated that one bar would carry 650 amperes, two bars 1150, three 1500, four 1800, five 2000, six 2160, showing that due to skin effect, lack of ventilation, etc., the permissible current density falls off rather rapidly as the number of bars increases.

It is usually necessary to interleave the phases or to adopt special arrangements of the strap connections to balance the mutual induction, to secure proper cur-

rent distribution in the various straps for 60-cycle service to carry 3000 amperes or more without an excessive amount of copper.

Tubing for Small Currents and High Voltages. — For extremely high voltages with their correspondingly small currents, copper tubing for bus-bars and connections has many advantages over rods, cable or wire. These advantages are principally increased stiffness for the same amount of material, large and effective radiating surface, and the facility of making connections by flattening out the tubing at the point desired and bolting the tubing together at such points. On extremely high-voltage circuits tubing of approximately 1 inch outside diameter is not apt to be troubled by the brush discharge or corona effect that is sometimes noted with small wires or straps having sharp edges. In some plants iron piping is used for bus-bars and connections.

BUS-BAR STRUCTURES. — (*See also Power Stations; Substations; Switchgear.*) In large-capacity a-c. plants of 13,000 volts or less, with generators connected directly to the bus, the current that can be developed on a short-circuit is something enormous, and every precaution has to be taken to prevent trouble from spreading if it ever starts. For this reason it has become customary to employ masonry compartments and cellular construction for the oil circuit breakers and bus-bars. In higher-voltage plants open wiring possesses several decided advantages. The vertical walls and septums of the circuit-breaker and bus-bar structures are usually built of brick or concrete, and the horizontal shelves between the bus-bars are ordinarily made of concrete, sandstone, soapstone, slate or marble. These substances are named in the order of their increasing cost. In some instances the bus-bar structures have been made of asbestos lumber, transite or similar material.

Low tension bus-bars when not too heavy can be supported by the wall bushing for the lead. For heavier work, or where bushings are not used, the bus-bars are supported on porcelain pillars, petticoat insulators, and similar devices resting on the bus-bar shelf, or attached to the wall.

In the larger generating stations due to the mechanical stress caused by the magnetic effects of the tremendous short-circuit current resulting from the size and number of turbo generators represented in present day station practice, close attention must be given to the adequacy of the bus-bar supports. Various curves and formulæ have been deduced for the purpose of calculating the mechanical strain on bus-bar supports at the instant of short-circuit. A typical formula is the following:

$$F = .27\ K.V.A.^2 \text{ divided by } A \times V^2 \times Z^2 \quad \text{where}$$

F = Maximum force exerted in pounds per foot of bus.
$K.V.A$ = Normal rating of the station including all synchronous apparatus.
A = Distance between buses in inches.
Z = Impedance in per cent expressed in decimals to the point of short-circuit.
V = Line voltage.

In using this formula a typical example with 150,000 K.V.A. station capacity at 6600 volts, 8 per cent reactance, gives a maximum force on the bus-bars per foot of length, 735 pounds with 30 inch spacing between bars, 1470 pounds with 15 inch spacing between bars. With 4 feet between bus supports, each bus support would have to stand a strain of 2940 pounds if the buses are 30 inches on centers; 5880 pounds if the buses are 15 inches on centers. For heavy duty of this kind, multi-point supports are frequently used.

For supports for high tension bus-bars and connections it is customary to employ high-tension insulators of the pillar type, pin type, or suspension type, depending on the voltage.

The open system of wiring is preferable for any voltage higher than that for which generators can be conveniently wound. This opinion is based on the following reasons:

1. For the same kilovolt-ampere capacity back of an arc the current established is approximately inversely proportional to the voltage, and consequently the violence of the arc and its destructive effects are less on a high-voltage than on a low-voltage system.

2. The distance from wire to ground has to be greatly reduced from what could be obtained with open wiring in the same space, as the conductivity of the fireproof barriers is sufficiently good to permit large currents to flow with high voltages, in case an arc or a dead ground is established.

3. A more expensive building and more costly construction are usually needed for inclosed bus-bars and wiring than are required for open wiring.

4. Inspection and repairs are more difficult when bus-bars, wiring, disconnecting switches and similar appliances are boxed in masonry compartments, and the conductors are visible and accessible only by the removal of doors. Inspection will be more frequent and thorough and incipient trouble will be noticed far sooner with open wiring than with inclosed, as the station attendant can see everything in a walk of a few minutes, and will not have to remove many doors and visit two or three floors to examine the condition of the apparatus.

CONNECTIONS TO BUS-BARS. — Where the currents exceed 600 or 800 amperes, it is usual to employ laminated copper straps for connecting to bus-bars; for smaller currents, cable, wire, rod or tubing is used. Cable and, to a certain extent, bare wire are used for connections involving bends or long runs through conduits, but for straight runs or simple bends rod or tubing can be used. Tubing, though more costly than rod or wire, is stiffer for the same section, and can often be flattened out for making connections to studs, bars, etc., without the necessity of additional terminals.

BIBLIOGRAPHY. — Billhimer, F. M., *Current Capacity of Copper Bus-Bars*, El. Jour. 1918, Vol. 15, p. 94; Leonard S. G. and Riker C. R., *Repulsion between Bus-Bars*, El. Jour. 1917, Vol. 14, p. 491.

CALORIMETERS, FUEL. — (*See also Fuel.*) A fuel calorimeter is an instrument for determining the heating value of a fuel. Its essential features are a closed chamber in which a weighed sample of the fuel can be quickly and completely burned, a vessel of water surrounding this chamber, into which all the heat generated by the combustion is transferred, a delicate thermometer for measuring the rise of temperature of the water, means for igniting the fuel, and provisions for preventing loss of heat from the apparatus by radiation or by the escape of the gases and vapors produced by the combustion. The most approved form of the instrument is Mahler's modification of Berthelot's calorimeter. The combustion chamber is a strong cylindrical steel vessel enameled on the inside, called a "bomb," into which about 1 gram of powdered coal, contained in a small platinum dish, is placed. Oxygen under pressure of 20 to 25 atmospheres is introduced, and the coal is ignited by an electric spark and burned explosively. The bomb is set in a water pail of thin brass, which is heavily felted and surrounded by a double-walled vessel filled with water of the temperature of the room. A stirring apparatus is used in the pail to circulate the water around the bomb. The thermometer is finely graduated. Readings of the temperature are made and recorded every minute until the maximum is reached, and a few minutes afterward to obtain a correction for radiation. The weight of the water, together with the water equivalent of the bomb and pail, multiplied by the rise in temperature, corrected for radiation and other minor errors, gives the number of heat units generated by the combustion of the coal.

The Junker calorimeter is commonly used to determine the heating value of fuel gas. A measured volume of gas is burned with air or oxygen in a vessel which is surrounded with water, and the calculations are made in the same way as those for the Mahler calorimeter.

BIBLIOGRAPHY. — The Bureau of Standards (Washington) has published a comprehensive paper on *Combustion Calorimetry* (1915) in *Bulletin* 11, *No.* 2, p. 189, and a comprehensive monograph on *Industrial Gas Calorimetry*, (1914) Tech. Pap. No. 36. See also Bibliography under *Boilers, Steam.*

CALORIMETERS, STEAM. — (*See also Steam.*) For the purpose of determining the percentage of moisture in the steam, in a boiler or engine test, a steam calorimeter is used. Several forms of this instrument have been used but the most common is that of Professor Peabody, known as the throttling calorimeter. The action of this instrument depends upon the fact that the heat of saturated steam is greater the greater the pressure, and consequently if the pressure is reduced by throttling, the heat rendered available will convert the moisture into steam and in general produce more or less superheating.

A ½-inch pipe, closed at the end and perforated with several ⅛-inch holes in its walls, is inserted into the main steam pipe so that steam may enter these holes. The other end of the calorimeter pipe is throttled by an orifice 1/16 inch diameter through which the steam escapes into a chamber which has an outlet to the atmosphere. The temperature and pressure of the steam on each side of the orifice are observed. The steam in the chamber is superheated more or less, according to the amount of moisture contained in the sample drawn from the steam main.

The per cent of moisture in the steam is then

$$W = 100 \frac{H - h - K(T - t)}{L},$$

where H = total heat and L = the latent heat of saturated steam at the pressure of the steam in the main pipe; h = total heat of saturated steam at the pressure in the discharge chamber of the calorimeter (= 1150.4, corresponding to a pressure of 14.7, when this chamber opens directly to the atmosphere); K = specific heat of superheated steam (= 0.48 approximately); T = actual temperature in the discharge chamber; t = temperature of saturated steam at the pressure in the discharge chamber (= 212 when this chamber opens directly to the atmosphere). The above formula becomes

$$W = 100 \frac{H - 1150.4 - 0.48(T - 212)}{L},$$

when the discharge chamber opens directly to the atmosphere, and when the atmospheric pressure is 14.7 pounds per square inch.

When the steam is very moist, so as to reduce the superheating on the discharge side to 0°, the instrument fails, and a separating calorimeter, which is simply a small steam separator (q.v.), must be used between the throttling calorimeter and the steam pipe to collect the greater quantity of moisture. The moisture collected in the separator is then added to that determined by the calorimeter. The instrument must be thoroughly felted to reduce the error due to radiation. There is also usually a considerable error in obtaining steam of an average quality from the main pipe by the perforated tube.

BIBLIOGRAPHY. — Carpenter and Diederichs, *Experimental Engineering, Report of Committee on Power Tests, A.S.M.E.*

CAMBRIC, VARNISHED. — (*See also Insulating Materials, Wires and Cables, Insulated.*) Varnished muslin, variously known as varnished cloth or varnished cambric is an insulating material for cables, which consists of tapes of cotton fabric coated with insulating varnish, wound helically around the conductor with a thin layer of plastic compound between turns. This compound prevents the absorption of moisture, precludes air spaces and permits the layers of fabric to slide upon each other when the cable is bent. It is also used for machine coils without the plastic compound.

The fabric is prepared by coating it with a mixture whose principal constituents are boiled linseed oil, resin, gilsonite, and benzine. This mixture dries and the oil is oxidized by contact with air, leaving a hard smooth surface. Three such coats are usually applied. The plastic material between layers is usually a mixture of petrolatum and resin.

The varnished fabric usually has a thickness between seven and thirteen mils, the thinner cambric being used on smaller cables. There are usually between 65 and 70 threads to an inch.

The tensile strength of the varnished tape should be not less than 2000 pounds per square inch.

After the varnished cambric has been placed in hot petrolatum having a temperature of 150° C., removed from petrolatum and allowed to cool, the film of varnish should not become soft or tacky.

Specific Resistance. — The value of K in the formula $M = K \log \frac{D}{d}$ (See article on rubber) ranges from $K = 300$ to $K = 1000$, the usual value being between 400 and 600.

The resistance of varnished cambric is more affected by temperature changes than that of rubber or impregnated paper. The amount of variation differs in different makes and in different batches of the same make, so that the accompanying table should be considered as an approximation only.

Temperature		Per cent of resistance at 60° C
C.°	F.°	
15.5	60	100
18.3	65	61
21.1	70	37
24.0	75	24
26.7	80	15
29.4	85	11
32.2	90	8
35.0	95	6
37.8	100	5

Dielectric Strength. — The dielectric strength of varnished cambric depends greatly upon the size and shape of electrodes used for testing. Using the equipment described below, the puncture voltage per mil at 25° C. is betwen 800 and 1000 volts. This decreases rapidly as the temperature rises, being between 300 and 600 volts per mil at 75° C.

Farmer, Kennelly and Wiseman have shown that the apparent dielectric

strength of 0.33 mm. varnished cambric decreased more than 50 per cent in changing from very small discs to discs exceeding 15 inches in diameter.

Tests are most readily made between the rounded edge of a strip of brass 1/16 inch thick and 6 inches long, which constitutes the upper electrode, and a strip of brass about 3/8 inch wide and 8 inches long, which is placed directly below the strip of varnished cambric to be tested, and forms the lower electrode. The electrodes may be attached to a strip of wood about 16 inches long and 4 inches wide. Also the top electrode may be fastened to supports with revolving arms so that it can be thrown out of the way when the test has been completed. In order to prevent flash-overs from one electrode to another, a wide strip of varnished cambric is placed underneath the lower electrode and petrolatum smeared over this strip of varnished cambric before the strip to be tested is placed over the lower electrode. The test strip of varnished cambric is pressed firmly against the lower strip so that the petrolatum makes a seal between the two strips thus preventing flash-overs from the top electrode to the bottom one.

Power Factor. — The power factor of varnished cambric insulation in cables is much higher than that of impregnated paper. The following values are typical:

Temperature Deg. Cent.	Power factor, Per Cent
20	6.0
30	8.0
40	10.5
50	14.0
60	19.0
70	49.0
80	33.0
90	43.0
100	56.0

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CAPACITY AND CHARGING CURRENTS. — (*See also Alternating Currents; Condensers, Electric; Insulating Materials, Testing of; Transient Electric Phenomena and Oscillations; Transmission Lines; Wires and Cables, Insulated.*) In the section on *Capacity and Condensers*, in the article on *Electricity and Magnetism, Principles of*, are given the formulas for capacities in series and in parallel and for the energy stored in a charged condenser. For units and their interrelations see the articles *Units, Practical Electrical*, and *Units and Conversion Factors*. Commercial forms of condensers are described in the article on *Condensers, Electric*. The following is a brief table of contents of this article.

GENERAL RELATIONS AND DEFINITIONS. — Consider any number of conductors 0, 1, 2, 3, etc., either (1) at a great distance from all other conductors or (2) completely surrounded by a hollow conducting shell, the inside surface of which shell is to be considered as one of the conductors, say No. 0, of the system. The electrostatic condition of such a system of conductors is uninfluenced by any electrostatic effects produced outside the system; it may therefore be called an "electrostatically independent system."

Potential Coefficients (A). — Any conductor of an electrostatically independent system may be chosen as a conductor of reference; let this reference conductor be designated as conductor No. 0. Let v_{10}, v_{20}, v_{30}, etc., represent the potential drop from No. 1 to No. 0, from No. 2 to No. 0, from No. 3 to No. 0, etc., and let q_0, q_1, q_2, q_3, etc., represent the charges on No. 0, No. 1, No. 2, No. 3, etc. Then, if the relative positions of the various conductors and insulators in the field remain unaltered and the specific inductive capacities (*see Electricity and Magnetism, Principles of*) of the various insulating materials between the conductors are *constant* (not necessarily the same for each insulating material, however), the following relations hold for all values of the charges on and potential drops between conductors irrespective of how the conductors may be connected: *

$$\left.\begin{aligned} v_{10} &= A_{11}q_1 + A_{12}q_2 + A_{13}q_3 + \text{etc.}, \\ v_{20} &= A_{12}q_1 + A_{22}q_2 + A_{23}q_3 + \text{etc.}, \\ v_{30} &= A_{13}q_1 + A_{23}q_2 + A_{33}q_3 + \text{etc.}, \\ q_0 &= -(q_1 + q_2 + q_3 + \text{etc.}), \end{aligned}\right\} \qquad (1)$$

where the A's are all *constants* depending upon the distances apart of the con-

* By wires of small cross-section and length compared with the dimensions of the conductors.

ductors and the nature of the insulating medium between them. The coefficients A in these equations may be called the "potential coefficients" of the system of conductors. It should be noted particularly that *these coefficients are independent of how the conductors may be charged and of how they may be interconnected* (provided the connecting wires are small compared with surfaces of the conductors). In certain simple cases these coefficients A are readily calculated; see below.

Electrostatic Induction Coefficients (B). — The above equations may also be written

$$\left.\begin{aligned} q_1 &= B_{11}v_{10} + B_{12}v_{20} + B_{13}v_{30} + \text{etc.},\\ q_2 &= B_{12}v_{10} + B_{22}v_{20} + B_{23}v_{30} + \text{etc.},\\ q_3 &= B_{13}v_{10} + B_{23}v_{20} + B_{33}v_{30} + \text{etc.},\end{aligned}\right\} \qquad (2)$$

etc.

$$q_0 = -(q_1 + q_2 + q_3 + \text{etc.}),$$

where the B's are also constants and may be expressed directly in terms of the potential coefficients A by solving equations (1) for q_1, q_2, q_3, etc. The constants B are called the "electrostatic induction coefficients," and like the constants A are independent of how the conductors may be charged and of how they may be interconnected. The B's may be expressed directly in terms of the normal and grounded capacities of the various conductors; see below.

Normal Capacity of Two Conductors (C). — By the normal capacity between any two conductors is meant the capacity of the condenser formed by these two conductors when all the other conductors are connected to one another and to the conductor of reference, the two conductors of course being insulated therefrom. The normal capacity between any two conductors of a system, say Nos. 1 and 2, is, then, from equation (2),

$$C_{12} = \frac{B_{11}B_{22} - B_{12}^2}{B_{11} + B_{22} + 2\,B_{12}}. \qquad (2a)$$

When the arrangement of the conductors is perfectly symmetrical (as in a three-conductor cable), $B_{11} = B_{22}$ and the normal capacity between 1 and 2 is

$$C_{12} = \frac{1}{2}(B_{11} - B_{12}). \qquad (2b)$$

Grounded Capacity (C_g). — By the grounded capacity of any conductor of a system is meant the capacity of the condenser formed by this conductor as one "plate" and all the other conductors, including the conductor of reference, connected together as the other plate.* The grounded capacity of conductor No. 1, say, is then, from equation (2),

$$C_{1g} = B_{11}. \qquad (2c)$$

That is, the "electrostatic coefficient of self-induction" of any given conductor is the same as the grounded capacity of this conductor.

Capacity to Neutral (C_0). — In calculating the charging current, voltage drops, etc., in a single-phase or balanced three-phase transmission line it is sometimes convenient to consider the actual capacity between wires as made up of two capacities in series, each of twice the actual capacity between wires. This double capacity, viz.,

$$C_0 = 2\,C_{12}, \qquad (2d)$$

is called the capacity to neutral, since this capacity multiplied by the voltage to neutral, in either a single-phase or balanced three-phase line, gives the charge per wire, which charge is also in phase with the voltage to neutral. This prod-

* The term "grounded" arises from the fact that the conductor of reference is usually the ground.

uct, however, does not give the charge per wire when the system is unbalanced; the general equations (2) must then be used.

Charging Current and Capacity Susceptance. — The charging currents taken by the various conductors of a system are found by differentiating equations (2) with respect to time; this gives the instantaneous values of the charging currents. In the case of sine-wave voltages all of frequency f, the effective values and phase relations of the charging currents in each conductor, in terms of the voltage drops to the conductor of reference, *all expressed in vector notation* (*see Alternating Currents*), are as follows:

$$\left.\begin{aligned} I_1 &= j\,2\pi f\,(B_{11}V_{10} + B_{12}V_{20} + B_{13}V_{30} + \ldots\ldots\ldots\ldots) \\ I_2 &= j\,2\pi f\,(B_{12}V_{10} + B_{22}V_{20} + B_{23}V_{30} + \ldots\ldots\ldots\ldots) \\ I_3 &= j\,2\pi f\,(B_{13}V_{10} + B_{23}V_{20} + B_{33}V_{30} + \ldots\ldots\ldots\ldots) \\ &\text{etc.,} \\ I_0 &= -(I_1 + I_2 + I_3 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots) \end{aligned}\right\} \qquad (3)$$

Note that the quantities in the brackets are to be added *vectorially*.

In the case of a system of but two conductors, i.e., a simple condenser, these relations reduce to

$$I = j\,2\pi fCV, \qquad (3a)$$

when V is the voltage drop through the condenser, I the current in the direction of this drop, and C the capacity of the condenser. That is, the charging current of a simple condenser leads the voltage drop by 90° and is equal numerically to the product of this voltage by $2\pi fC$. The factor

$$b = 2\pi fC \qquad (3b)$$

is called the capacity susceptance * of the condenser. Capacity susceptance is expressed in mhos or micromhos, being of the same dimensions as conductance. Numerically, the charging current of a simple condenser may be then expressed as

$$I = bV. \qquad (3c)$$

When V is in volts and b in mhos, the current I is in amperes. For either a single-phase or balanced three-phase line the charging current per wire may also be expressed as $I = b_0V_0$, where $b_0 = 2\pi fC_0 = 2\,b$, and may be called the "capacity susceptance to neutral." When b_0 is in micromhos (see tables below) and V_0 is in volts to neutral, the charging current per wire is

$$I = 10^{-6}b_0V_0. \qquad (3d)$$

In a single-phase line $V_0 = V/2$ and in a balanced three-phase line $V_0 = V/\sqrt{3}$, when V in each case is the voltage between wires.

FORMULAS FOR THE CAPACITY OF SIMPLE CONDENSERS. — Let

K = specific inductive capacity of medium between conductors; medium assumed uniform; for air $K = 1$,
C = capacity of the condenser formed by the two conductors,
$C_0 = 2\,C$ = capacity to neutral.

Two Concentric Spheres. — Let r' = internal radius of outer sphere and r = external radius of inner sphere, both in centimeters, and R' and R the corresponding dimensions in inches.

$$C = \frac{Krr'}{r' - r} \quad \text{statfarads,}$$

$$= 2.822 \times 10^{-6}\frac{KRR'}{R' - R} \quad \text{microfarads.}$$

* The inductive susceptance of a condenser is $-2\pi fC$; see *Alternating Currents*.

Two Parallel Plates. — Unless the plates are large compared with their distance apart no simple formula can be deduced, since the electrostatic field at the edge of the plates is not uniform. Let S = surface, in square inches, of contact between the metal plate and dielectric (the dielectric sheet is usually larger than the metal sheet, hence S is generally the surface, one side only, of the metal plate); D = thickness of dielectric in inches, K = specific inductive capacity, and let s and d be the dimensions in centimeters corresponding to S and D respectively. Then for D small compared with S, the capacity of two parallel plates is

$$C = \frac{Ks}{4\pi d} \quad \text{statfarads}$$

$$= 2.246 \times 10^{-7} \frac{KS}{D} \quad \text{microfarads.}$$

Stack of Plates. — Let N = the number of *metal* plates in the stack; there will then be $N - 1$ effective dielectric sheets or $N - 1$ parallel plate condensers in series. Using the same notation as above, the capacity of a stack of N metal plates, *connected in series*, is

$$C = \frac{Ks}{4\pi d\,(N-1)} \quad \text{statfarads}$$

$$= 2.246 \times 10^{-7} \frac{KS}{D\,(N-1)} \quad \text{microfarads.}$$

Round Wire in Concentric* Sheath. — Dimensions as in Fig. 1; since the *ratio* only is involved, it is immaterial what units are used for D and d provided *both* are expressed in the *same* unit. For a length of cable long compared with its diameter,

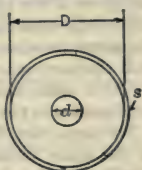


Fig. 1.

$$C = \frac{K}{2 \log_\epsilon \frac{D}{d}} \quad \text{statfarads per centimeter}$$

$$= \frac{7.354 \times 10^{-3} K}{\log_{10} \frac{D}{d}} \quad \text{microfarads per 1000 ft.}$$

Two Parallel Round† Wires. — Dimensions as in Fig. 2; since the *ratio* only is involved, it is immaterial what units are used for D and d provided *both* are expressed in the *same* unit. For a length of line large compared with the distance apart of the wires, the exact formula ‡ for the capacity *between wires* is

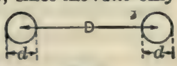


Fig. 2.

* When the wire is off center by a distance m (center of wire to center of sheath),

$$C = \frac{K}{2 \cosh^{-1}\alpha} \quad \text{statfarads per centimeter}$$

$$= \frac{16.93 \times 10^{-3} K}{\cosh^{-1}\alpha} \quad \text{microfarads per 1000 ft.,}$$

where
$$\alpha = \frac{D^2 + d^2 - 4m^2}{2Dd}.$$

† When the wires are far apart compared with the linear dimensions of their cross-section, the second group of formulas also applies approximately to wires of any shape of cross-section provided d is taken equal to the perimeter of the cross-section divided by π, i.e., equal to the " equivalent " diameter of the cross-section.

‡ Taking into account the non-uniform distribution of the charge on each wire; see *Pender and Osborne, Elec. World, 1910, Vol. 56, p. 667.*

$$C = \frac{K}{4 \cosh^{-1} \frac{D}{d}} \quad \text{statfarads per centimeter}$$

$$= \frac{8.467 \times 10^{-3} K}{\cosh^{-1} \frac{D}{d}} \quad \text{microfarads per 1000 feet.}$$

When D is greater than 10 d the following formulas for the capacity ***between wires*** may be used instead of the above with an error of less than 0.1 per cent:

$$\left.\begin{aligned} C &= \frac{K}{4 \log_e \frac{2D}{d}} && \text{statfarads per centimeter} \\ &= \frac{3.677 \times 10^{-3} K}{\log_{10} \frac{2D}{d}} && \text{microfarads per 1000 feet} \\ &= \frac{19.41 \times 10^{-3} K}{\log_{10} \frac{2D}{d}} && \text{microfarads per mile.} \end{aligned}\right\} \qquad (4)$$

The *capacity to neutral* in all cases is $C_0 = 2\,C$. Tables of capacity to neutral for various sizes of wires and various spacings, when separated by air ($K = 1$), are given in the tables below. Note that these tables and the above formulas are strictly applicable to ordinary overhead lines only when the distance from the wires to other conductors, particularly the earth, is large compared with their distance apart. However, the effect of the earth is usually small in most practical cases (see below), and the formulas and tables give a very fair approximation to the actual capacities.

The capacities of standard strands given in the following tables are calculated by the same formula as for smooth round wires using for the diameter d the diameter of the strand; see *Wires and Cables, Bare*. The values as thus calculated are therefore not exact, but the error is probably less than 3 per cent for all practical cases.

CAPACITY TO NEUTRAL* OF SMOOTH ROUND WIRES

Microfarads per 1000 FEET of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	0.01199	0.006608	0.005192	0.004618	0.004282	0.003884	0.003643	0.003477
000	0.4096	0.01099	0.006317	0.005013	0.004477	0.004161	0.003783	0.003555	0.003396
00	0.3648	0.01016	0.006055	0.004847	0.004344	0.004045	0.003688	0.003470	0.003319
0	0.3249	0.009458	0.005812	0.004692	0.004218	0.003936	0.003597	0.003390	0.003245
1	0.2893	0.008855	0.005587	0.004546	0.004100	0.003833	0.003511	0.003313	0.003174
2	0.2576	0.008332	0.005381	0.004408	0.003988	0.003735	0.003428	0.003239	0.003107
4	0.2043	0.007455	0.005010	0.004157	0.003781	0.003553	0.003274	0.003102	0.002980
6	0.1620	0.006753	0.004688	0.003933	0.003595	0.003388	0.003134	0.002975	0.002863
8	0.1285	0.006177	0.004406	0.003732	0.003426	0.003238	0.003005	0.002859	0.002755
10	0.1019	0.005693	0.004155	0.003551	0.003273	0.003100	0.002886	0.002751	0.002655
12	0.08081	0.005277	0.003931	0.003386	0.003132	0.002974	0.002776	0.002651	0.002562
14	0.06408	0.004921	0.003730	0.003235	0.003003	0.002858	0.002675	0.002558	0.002475
16	0.05082	0.004611	0.003549	0.003099	0.002885	0.002750	0.002580	0.002472	0.002394

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	0.003351	0.003171	0.003043	0.002947	0.002806	0.002706	0.002542	0.002436	0.002361
000	0.003276	0.003103	0.002981	0.002889	0.002753	0.002657	0.002498	0.002396	0.002323
00	0.003204	0.003039	0.002922	0.002833	0.002702	0.002610	0.002456	0.002358	0.002287
0	0.003135	0.002977	0.002864	0.002779	0.002653	0.002564	0.002416	0.002320	0.002251
1	0.003069	0.002917	0.002809	0.002727	0.002606	0.002520	0.002376	0.002284	0.002217
2	0.003006	0.002860	0.002756	0.002677	0.002560	0.002477	0.002338	0.002249	0.002184
4	0.002887	0.002752	0.002656	0.002582	0.002474	0.002396	0.002266	0.002182	0.002121
6	0.002777	0.002652	0.002563	0.002494	0.002392	0.002319	0.002197	0.002118	0.002061
8	0.002676	0.002559	0.002476	0.002412	0.002317	0.002248	0.002133	0.002059	0.002004
10	0.002581	0.002473	0.002395	0.002335	0.002245	0.002181	0.002073	0.002002	0.001951
12	0.002493	0.002392	0.002319	0.002262	0.002178	0.002118	0.002016	0.001949	0.001900
14	0.002411	0.002316	0.002247	0.002194	0.002115	0.002058	0.001961	0.001898	0.001852
16	0.002334	0.002245	0.002180	0.002130	0.002056	0.002002	0.001910	0.001850	0.001806

* The capacity *between* wires equals one-half the values given in this table.

CAPACITY TO NEUTRAL* OF SMOOTH ROUND WIRES

Microfarads per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	0.06332	0.03490	0.02741	0.02438	0.02261	0.02051	0.01924	0.01836
000	0.4096	0.05802	0.03336	0.02647	0.02364	0.02197	0.01998	0.01877	0.01793
00	0.3648	0.05366	0.03198	0.02559	0.02293	0.02136	0.01947	0.01832	0.01752
0	0.3249	0.04995	0.03069	0.02477	0.02227	0.02078	0.01899	0.01790	0.01713
1	0.2893	0.04676	0.02951	0.02400	0.02165	0.02024	0.01854	0.01749	0.01676
2	0.2576	0.04400	0.02842	0.02328	0.02106	0.01972	0.01810	0.01710	0.01640
4	0.2043	0.03937	0.02645	0.02195	0.01997	0.01876	0.01729	0.01638	0.01573
6	0.1620	0.03566	0.02475	0.02077	0.01898	0.01789	0.01655	0.01571	0.01512
8	0.1285	0.03262	0.02326	0.01971	0.01809	0.01710	0.01587	0.01510	0.01455
10	0.1019	0.03006	0.02194	0.01875	0.01728	0.01637	0.01524	0.01453	0.01402
12	0.08081	0.02787	0.02076	0.01788	0.01654	0.01570	0.01466	0.01400	0.01353
14	0.06408	0.02599	0.01970	0.01709	0.01586	0.01509	0.01412	0.01351	0.01307
16	0.05082	0.02434	0.01874	0.01636	0.01523	0.01452	0.01362	0.01305	0.01264

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	0.01769	0.01674	0.01607	0.01556	0.01482	0.01429	0.01342	0.01286	0.01246
000	0.01730	0.01639	0.01574	0.01525	0.01454	0.01403	0.01319	0.01265	0.01227
00	0.01692	0.01604	0.01543	0.01496	0.01427	0.01378	0.01297	0.01245	0.01207
0	0.01656	0.01572	0.01512	0.01467	0.01401	0.01354	0.01275	0.01225	0.01189
1	0.01621	0.01540	0.01483	0.01440	0.01376	0.01330	0.01255	0.01206	0.01171
2	0.01587	0.01510	0.01455	0.01413	0.01352	0.01308	0.01235	0.01187	0.01153
4	0.01525	0.01453	0.01402	0.01363	0.01306	0.01265	0.01196	0.01152	0.01120
6	0.01467	0.01400	0.01353	0.01317	0.01263	0.01225	0.01160	0.01118	0.01088
8	0.01413	0.01351	0.01307	0.01273	0.01223	0.01187	0.01126	0.01087	0.01058
10	0.01363	0.01306	0.01264	0.01233	0.01186	0.01152	0.01094	0.01057	0.01030
12	0.01316	0.01263	0.01224	0.01194	0.01150	0.01118	0.01064	0.01029	0.01003
14	0.01273	0.01223	0.01187	0.01159	0.01117	0.01087	0.01036	0.01002	0.009777
16	0.01232	0.01185	0.01151	0.01125	0.01085	0.01057	0.01008	0.009768	0.009536

* The capacity *between* wires equals one-half the values given in this table.

CAPACITY TO NEUTRAL* OF STANDARD STRANDS

Microfarads per 1000 FEET of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	0.0105	0.00725	0.00617	0.00558	0.00492	0.00454	0.00428
750,000	0.998	.....	0.00959	0.00683	0.00586	0.00533	0.00472	0.00437	0.00414
500,000	0.814	0.0254	0.00856	0.00630	0.00547	0.00501	0.00447	0.00415	0.00394
350,000	0.681	0.0181	0.00783	0.00591	0.00517	0.00476	0.00427	0.00398	0.00378
250,000	0.575	0.0147	0.00725	0.00558	0.00492	0.00454	0.00409	0.00383	0.00364
0 000	0.528	0.0135	0.00699	0.00542	0.00480	0.00444	0.00401	0.00376	0.00358
000	0.470	0.0122	0.00666	0.00523	0.00465	0.00431	0.00390	0.00366	0.00349
00	0.418	0.0112	0.00637	0.00504	0.00450	0.00418	0.00380	0.00357	0.00341
0	0.373	0.0103	0.00610	0.00488	0.00437	0.00407	0.00371	0.00349	0.00333
1	0.332	0.00958	0.00586	0.00472	0.00424	0.00396	0.00361	0.00341	0.00326
2	0.292	0.00891	0.00561	0.00456	0.00411	0.00384	0.00352	0.00332	0.00318
4	0.232	0.00790	0.00520	0.00429	0.00389	0.00365	0.00336	0.00318	0.00305
6	0.184	0.00712	0.00486	0.00405	0.00369	0.00348	0.00321	0.00304	0.00293

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.00410	0.00383	0.00365	0.00351	0.00331	0.00317	0.00295	0.00281	0.00271
750,000	0.00396	0.00371	0.00354	0.00341	0.00322	0.00309	0.00288	0.00274	0.00265
500,000	0.00378	0.00355	0.00339	0.00327	0.00310	0.00298	0.00278	0.00266	0.00257
350,000	0.00363	0.00342	0.00328	0.00316	0.00300	0.00289	0.00270	0.00258	0.00250
250,000	0.00351	0.00331	0.00317	0.00307	0.00292	0.00281	0.00263	0.00252	0.00244
0 000	0.00345	0.00326	0.00312	0.00302	0.00287	0.00277	0.00260	0.00249	0.00240
000	0.00337	0.00318	0.00306	0.00296	0.00282	0.00272	0.00255	0.00245	0.00237
00	0.00329	0.00312	0.00299	0.00290	0.00276	0.00267	0.00251	0.00240	0.00233
0	0.00322	0.00305	0.00293	0.00284	0.00271	0.00262	0.00247	0.00237	0.00229
1	0.00315	0.00299	0.00288	0.00279	0.00266	0.00257	0.00242	0.00233	0.00226
2	0.00308	0.00292	0.00281	0.00273	0.00261	0.00252	0.00238	0.00229	0.00222
4	0.00295	0.00281	0.00271	0.00263	0.00252	0.00244	0.00230	0.00222	0.00215
6	0.00284	0.00271	0.00261	0.00254	0.00244	0.00236	0.00223	0.00215	0.00209

* The capacity *between* conductors equals one-half the values given in this table.

CAPACITY TO NEUTRAL * OF STANDARD STRANDS

Microfarads per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	0.0554	0.0383	0.0325	0.0294	0.0260	0.0240	0.0226
750,000	0.998	.....	0.0506	0.0361	0.0309	0.0281	0.0249	0.0231	0.0218
500,000	0.814	0.134	0.0452	0.0333	0.0289	0.0264	0.0236	0.0219	0.0208
350,000	0.681	0.0955	0.0413	0.0312	0.0273	0.0251	0.0225	0.0210	0.0200
250,000	0.575	0.0776	0.0383	0.0295	0.0260	0.0240	0.0216	0.0202	0.0192
0 000	0.528	0.0713	0.0369	0.0286	0.0253	0.0234	0.0212	0.0198	0.0189
000	0.470	0.0644	0.0352	0.0276	0.0245	0.0227	0.0206	0.0193	0.0184
00	0.418	0.0590	0.0336	0.0266	0.0238	0.0221	0.0201	0.0189	0.0180
0	0.373	0.0544	0.0322	0.0258	0.0231	0.0214	0.0196	0.0184	0.0176
1	0.332	0.0506	0.0309	0.0249	0.0224	0.0209	0.0191	0.0180	0.0172
2	0.292	0.0470	0.0296	0.0241	0.0217	0.0203	0.0186	0.0175	0.0168
4	0.232	0.0417	0.0275	0.0227	0.0205	0.0193	0.0177	0.0168	0.0161
6	0.184	0.0376	0.0256	0.0214	0.0195	0.0184	0.0169	0.0161	0.0154

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.0216	0.0202	0.0193	0.0185	0.0175	0.0168	0.0156	0.0148	0.0143
750,000	0.0209	0.0196	0.0187	0.0180	0.0170	0.0163	0.0152	0.0145	0.0140
500,000	0.0200	0.0188	0.0179	0.0173	0.0164	0.0157	0.0147	0.0140	0.0135
350,000	0.0192	0.0181	0.0173	0.0167	0.0159	0.0153	0.0143	0.0136	0.0132
250,000	0.0185	0.0175	0.0168	0.0162	0.0154	0.0148	0.0139	0.0133	0.0129
0 000	0.0182	0.0172	0.0165	0.0160	0.0152	0.0146	0.0137	0.0131	0.0127
000	0.0178	0.0168	0.0161	0.0156	0.0149	0.0143	0.0135	0.0129	0.0125
00	0.0174	0.0165	0.0158	0.0153	0.0146	0.0141	0.0132	0.0127	0.0123
0	0.0170	0.0161	0.0155	0.0150	0.0143	0.0138	0.0130	0.0125	0.0121
1	0.0166	0.0158	0.0152	0.0147	0.0141	0.0136	0.0128	0.0123	0.0119
2	0.0162	0.0154	0.0149	0.0144	0.0138	0.0133	0.0126	0.0121	0.0117
4	0.0156	0.0148	0.0143	0.0139	0.0133	0.0129	0.0122	0.0117	0.0114
6	0.0150	0.0143	0.0138	0.0134	0.0129	0.0125	0.0118	0.0114	0.0111

* The capacity *between* conductors equals one-half the values given in this table.

25-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL*
SMOOTH ROUND WIRES

Charging current in amperes per mile = (susceptance from table) × *(volts to neutral)* × 10^{-6}

Micromhos per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0 4600	9.948	5.483	4.306	3.830	3.552	3.222	3.023	2.884
000	0.4096	9.115	5.241	4.158	3.714	3.451	3.139	2.949	2.817
00	0.3648	8.430	5.024	4.020	3.602	3.356	3.059	2.878	2.752
0	0.3249	7.847	4.821	3.891	3.499	3.265	2.983	2.812	2.691
1	0.2893	7.346	4.636	3.770	3.401	3.180	2.913	2.748	2.633
2	0.2576	6.912	4.465	3.657	3.309	3.098	2.844	2.686	2.576
4	0.2043	6.185	4.155	3.448	3.137	2.947	2.716	2.573	2.471
6	0.1620	5.602	3.888	3.263	2.982	2.811	2.600	2.468	2.375
8	0.1285	5.125	3.654	3.096	2.842	2.686	2.493	2.372	2.286
10	0.1019	4.722	3.447	2.946	2.715	2.572	2.394	2.283	2.203
12	0.08081	4.378	3.261	2.809	2.598	2.466	2.303	2.199	2.126
14	0.06408	4.083	3.095	2.685	2.492	2.371	2.218	2.122	2.053
16	0.05082	3.824	2.944	2.570	2.393	2.281	2.140	2.050	1.986

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	2.779	2.630	2.525	2.444	2.328	2.245	2.108	2.020	1.957
000	2.718	2.575	2.473	2.396	2.284	2.204	2.072	1.987	1.928
00	2.657	2.520	2.424	2.350	2.242	2.165	2.038	1.956	1.896
0	2.602	2.470	2.375	2.305	2.201	2.127	2.003	1.924	1.868
1	2.547	2.419	2.330	2.262	2.162	2.089	1.972	1.895	1.840
2	2.493	2.372	2.286	2.220	2.124	2.055	1.940	1.865	1.811
4	2.396	2.283	2.203	2.141	2.052	1.987	1.879	1.810	1.760
6	2.305	2.199	2.126	2.069	1.984	1.924	1.822	1.756	1.709
8	2.220	2.122	2.053	2.000	1.921	1.865	1.769	1.708	1.662
10	2.141	2.052	1.986	1.937	1.863	1.810	1.719	1.661	1.618
12	2.067	1.984	1.923	1.876	1.807	1.756	1.672	1.617	1.576
14	2.000	1.921	1.865	1.821	1.755	1.708	1.628	1.574	1.536
16	1.935	1.862	1.808	1.767	1.705	1.661	1.584	1.535	1.498

* The susceptance *between* wires equals one-half the values given in this table.

25-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL* STANDARD STRANDS

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) $\times 10^{-6}$

Approximate micromhos per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between conductors, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	8.70	6.01	5.10	4.62	4.08	3.77	3.55
750,000	0.998	.....	7.94	5.67	4.85	4.41	3.91	3.63	3.42
500,000	0.814	21.0	7.10	5.23	4.54	4.14	3.71	3.44	3.27
350,000	0.681	15.0	6.48	4.90	4.29	3.94	3.53	3.30	3.14
250,000	0.575	12.2	6.01	4.63	4.08	3.77	3.39	3.17	3.02
0 000	0.528	11.2	5.79	4.49	3.97	3.67	3.33	3.11	2.97
000	0.470	10.1	5.53	4.33	3.85	3.57	3.24	3.03	2.89
00	0.418	9.26	5.28	4.18	3.74	3.47	3.16	2.97	2.83
0	0.373	8.54	5.06	4.05	3.63	3.36	3.08	2.89	2.76
1	0.332	7.94	4.85	3.91	3.52	3.28	3.00	2.83	2.70
2	0.292	7.38	4.65	3.79	3.41	3.19	2.92	2.75	2.64
4	0.232	6.55	4.32	3.57	3.22	3.03	2.78	2.64	2.53
6	0.184	5.90	4.02	3.36	3.06	2.89	2.65	2.53	2.42

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	3.39	3.17	3.03	2.90	2.75	2.64	2.45	2.32	2.25
750,000	3.28	3.08	2.94	2.83	2.67	2.56	2.39	2.28	2.20
500,000	3.14	2.95	2.81	2.72	2.57	2.47	2.31	2.20	2.12
350,000	3.02	2.84	2.72	2.62	2.50	2.40	2.25	2.14	2.07
250,000	2.90	2.75	2.64	2.54	2.42	2.32	2.18	2.09	2.03
0 000	2.86	2.70	2.59	2.51	2.39	2.29	2.15	2.06	1.99
000	2.79	2.64	2.53	2.45	2.34	2.25	2.12	2.03	1.96
00	2.73	2.59	2.48	2.40	2.29	2.22	2.07	1.99	1.93
0	2.67	2.53	2.43	2.36	2.25	2.17	2.04	1.96	1.90
1	2.61	2.48	2.39	2.31	2.22	2.14	2.01	1.93	1.87
2	2.54	2.42	2.34	2.26	2.17	2.09	1.98	1.90	1.84
4	2.45	2.32	2.25	2.18	2.09	2.03	1.92	1.84	1.79
6	2.36	2.25	2.17	2.10	2.03	1.96	1.85	1.79	1.74

* The susceptance *between* conductors equals one-half the values given in this table.

60-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL *
SMOOTH ROUND WIRES

Charging current in amperes per mile = *(susceptance from table)* × *(volts to neutral)* × 10^{-6}

Micromhos per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
0000	0.4600	23.87	13.16	10.33	9.191	8.524	7.732	7.253	6.922
000	0.4096	21.87	12.58	9.979	8.912	8.283	7.532	7.076	6.760
00	0.3648	20.23	12.06	9.647	8.645	8.053	7.340	6.907	6.605
0	0.3249	18.83	11.57	9.338	8.396	7.834	7.159	6.748	6.458
1	0.2893	17.63	11.13	9.048	8.162	7.630	6.990	6.594	6.319
2	0.2576	16.59	10.71	8.777	7.940	7.434	6.824	6.447	6.183
4	0.2043	14.84	9.972	8.275	7.529	7.073	6.518	6.175	5.930
6	0.1620	13.44	9.331	7.830	7.155	6.745	6.239	5.923	5.700
8	0.1285	12.30	8.769	7.430	6.820	6.447	5.983	5.693	5.485
10	0.1019	11.33	8.271	7.069	6.515	6.171	5.745	5.478	5.286
12	0.08081	10.51	7.827	6.741	6.236	5.919	5.527	5.278	5.101
14	0.06408	9.798	7.427	6.443	5.979	5.689	5.323	5.093	4.927
16	0.05082	9.176	7.065	6.168	5.742	5.474	5.135	4.920	4.765

Size of wire, A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
0000	6.669	6.311	6.058	5.866	5.587	5.387	5.059	4.848	4.697
000	6.522	6.179	5.934	5.749	5.482	5.289	4.973	4.769	4.626
00	6.379	6.047	5.817	5.640	5.380	5.195	4.890	4.694	4.550
0	6.243	5.926	5.700	5.531	5.282	5.105	4.807	4.618	4.483
1	6.111	5.806	5.591	5.429	5.188	5.014	4.731	4.547	5.415
2	5.983	5.693	5.485	5.327	5.097	4.931	4.656	4.475	4.347
4	5.749	5.478	5.286	5.139	4.924	4.769	4.509	4.343	4.222
6	5.531	5.278	5.101	4.965	4.762	4.618	4.373	4.215	4.102
8	5.327	5.093	4.927	4.799	4.611	4.475	4.245	4.098	3.989
10	5.139	4.924	4.765	4.648	4.471	4.343	4.124	3.985	3.883
12	4.961	4.762	4.614	4.501	4.336	4.215	4.011	3.879	3.781
14	4.799	4.611	4.475	4.369	4.211	4.098	3.906	3.778	3.686
16	4.645	4.467	4.339	4.241	4.090	3.985	3.800	3.683	3.595

* The susceptance *between* wires equals one-half the values given in this table.

60-CYCLE CAPACITY SUSCEPTANCE TO NEUTRAL* STANDARD STRANDS

Charging current in amperes per mile = (susceptance from table) × (volts to neutral) × 10^{-6}

Approximate micromhos per MILE of each conductor of a single-phase or of a symmetrical three-phase line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches	Inches between cables, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.152	.....	20.9	14.4	12.3	11.1	9.80	9.05	8.52
750,000	0.998	.....	19.1	13.6	11.6	10.6	9.39	8.71	8.22
500,000	0.814	50.5	17.0	12.6	10.9	9.95	8.90	8.26	7.84
350,000	0.681	36.0	15.6	11.8	10.3	9.46	8.48	7.92	7.54
250,000	0.575	29.3	14.4	11.1	9.80	9.05	8.14	7.62	7.24
0 000	0.528	26.9	13.9	10.8	9.54	8.82	7.99	7.46	7.13
000	0.470	24.3	13.3	10.4	9.24	8.56	7.77	7.28	6.94
00	0.418	22.2	12.7	10.0	8.97	8.33	7.58	7.13	6.79
0	0.373	20.5	12.1	9.73	8.71	8.07	7.39	6.94	6.63
1	0.332	19.1	11.6	9.39	8.44	7.88	7.20	6.79	6.48
2	0.292	17.7	11.2	9.09	8.18	7.65	7.01	6.60	6.33
4	0.232	15.7	10.4	8.56	7.73	7.28	6.67	6.33	6.07
6	0.184	14.2	9.65	8.07	7.35	6.94	6.37	6.07	5.81

Size of cable, C.M. or A.W.G.	Feet between cables, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	8.14	7.62	7.28	6.97	6.60	6.33	5.88	5.58	5.39
750,000	7.88	7.39	7.05	6.79	6.41	6.15	5.73	5.47	5.28
500,000	7.54	7.09	6.75	6.52	6.18	5.92	5.54	5.28	5.09
350,000	7.24	6.82	6.52	6.30	5.99	5.77	5.39	5.13	4.98
250,000	6.97	6.60	6.33	6.11	5.81	5.58	5.24	5.01	4.86
0 000	6.86	6.48	6.22	6.03	5.73	5.50	5.16	4.94	4.79
000	6.71	6.33	6.07	5.88	5.62	5.39	5.09	4.86	4.71
00	6.56	6.22	5.96	5.77	5.50	5.32	4.98	4.79	4.64
0	6.40	6.07	5.84	5.66	5.39	5.20	4.90	4.71	4.56
1	6.26	5.96	5.73	5.54	5.32	5.13	4.83	4.64	4.49
2	6.11	5.81	5.62	5.43	5.20	5.01	4.75	4.56	4.41
4	5.88	5.58	5.39	5.24	5.01	4.86	4.60	4.41	4.30
6	5.66	5.39	5.20	5.05	4.86	4.71	4.45	4.30	4.18

* The susceptance *between* conductors equals one-half the values given in this table.

Single Round* Wire Parallel to the Ground. — Dimensions as in Fig. 3. The dotted circle represents the "image" of the wire in the plane. The capacity of the actual condenser formed by the wire and the earth (assumed equivalent to an infinite conducting plane parallel to the wire) is the same as the capacity to neutral of the fictitious condenser formed by the wire and its image, the distance between the two wires of this fictitious condenser being $D = 2H$. Hence using the approximate expressions, since the wire is practically always more than 10 times its diameter above the other, the capacity *between the wire and the earth* is

Fig. 3.

$$C = \frac{K}{2 \log_\epsilon \frac{4H}{d}} \quad \text{statfarads per centimeter}$$

$$= \frac{7.354 \times 10^{-3} K}{\log_{10} \frac{4H}{d}} \quad \text{microfarads per 1000 feet}$$

$$= \frac{38.83 \times 10^{-3} K}{\log_{10} \frac{4H}{d}} \quad \text{microfarads per mile.}$$

SYSTEMS OF THREE CONDUCTORS. — Three practical examples of an electrostatically independent system consisting of three conductors only are (1) three parallel overhead wires at a distance above the earth large compared to their distances apart, (2) two wires in a lead sheath and (3) two overhead wires and the earth. In the first case any one of the three wires may be considered as the conductor of reference; in the second case it is convenient to choose the sheath of the cable as the conductor of reference, whether grounded or not, and in the third case to choose the earth as the conductor of reference. The general equations (1) and (2) are applicable to either case. Designating the reference conductor as No. 3 instead of No. 0, and the other two conductors as No. 1 and No. 2 respectively, the general equations may be written

$$\left.\begin{aligned} v_{13} &= A_{11}q_1 + A_{12}q_2, & q_1 &= B_{11}v_{13} + B_{12}v_{23}, \\ v_{23} &= A_{12}q_1 + A_{22}q_2, & q_2 &= B_{12}v_{13} + B_{22}v_{23}, \\ & & q_3 &= -(q_1 + q_2). \end{aligned}\right\} \quad (5)$$

v_{13} and v_{23} indicate the potential drop from 1 to 3 and from 2 to 3 respectively. Solving the first set of equations for q_1 and q_2 gives

$$B_{11} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2}, \quad B_{22} = \frac{A_{11}}{A_{11}A_{22} - A_{12}^2}, \quad B_{12} = \frac{-A_{12}}{A_{11}A_{22} - A_{12}^2}. \quad (5a)$$

Possible Combinations of Three Conductors which may be used as a Condenser — "Part Capacities." — The following combinations are possible:

No. 1 against † No. 2 with No. 3 insulated,
No. 2 against No. 3 with No. 1 insulated,
No. 3 against No. 1 with No. 2 insulated,
No. 1 against No. 2 and No. 3 connected together,
No. 2 against No. 3 and No. 1 connected together,
No. 3 against No. 1 and No. 2 connected together.

* When the wire is at a height large compared with the linear dimensions of its cross-section these formulas also apply approximately to wires of any shape of cross-section provided d is taken equal to the perimeter of the cross-section divided by π.

† By "against" is here meant that the source of e.m.f. is connected between the two conductors designated, e.g., in the fourth case No. 1 is charged positively say, and an equal and opposite charge divides between 2 and 3.

The capacities corresponding to these various combinations may be called "part capacities," and may be designated by subscripts thus: C_{12} = capacity of No. 1 against No. 2 with No. 3 insulated (= normal capacity between 1 and 2), $C_{1(23)}$ = capacity of No. 1 against No. 2 and No. 3 connected together (= grounded capacity of No. 1), and so on for the others. The values of these part capacities can be deduced directly from equations (5), viz.,

$$\left.\begin{aligned} C_{12} &= \frac{1}{A_{11}+A_{22}-2A_{12}}, & C_{1(23)} &= B_{11} \\ C_{23} &= \frac{1}{A_{22}}, & C_{2(31)} &= B_{22} \\ C_{31} &= \frac{1}{A_{11}}, & C_{3(12)} &= -(B_{11}+B_{22}+2B_{12}) \end{aligned}\right\} \quad (6)$$

Three Overhead Parallel Wires, Effect of Earth Neglected. — Arrangement of wires and dimensions as in Fig. 4; distance apart of wires and diameters both in the *same* unit of length, but this unit may be either centimeters or inches. From equation (20c) in the article on *Electricity and Magnetism, Principles of,* the following closely approximate values of the A's (accurate to within less than 0.1 per cent when the distance apart of the wires is greater than 10 diameters) may be deduced, taking K as unity, since the dielectric in the case under consideration is air:

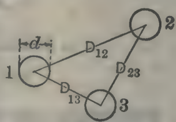


Fig. 4.

$$A_{11} = 2\log_\epsilon \frac{4D_{13}^2}{d_1d_3}, \quad A_{22} = 2\log_\epsilon \frac{4D_{23}^2}{d_2d_3}, \quad A_{12} = 2\log_\epsilon \frac{2D_{13}D_{23}}{d_3D_{12}}, \qquad (7)$$

all in c.g.s. electrostatic units for a length of one centimeter.*

From these relations the values of the coefficients B may be calculated for any arrangement of three parallel wires.

Equilateral Triangle Arrangement. — A common arrangement of the three wires of a three-phase transmission line is to place them so that their centers form the three vertices of an equilateral triangle. In this case $D_{12} = D_{23} = D_{13} = D$, say; and if all three wires are of the same diameter $d_1 = d_2 = d_3 = d$, say, then $A_{11} = A_{22} = 4\log_\epsilon \frac{2D}{d}$ and $A_{12} = 2\log_\epsilon \frac{2D}{d}$ in c.g.s. electrostatic units, and therefore, from equation (6) the normal capacity between any two of the wires with the third wire insulated is

$$C_{12} = \frac{1}{4\log_\epsilon \frac{2D}{d}} \quad \text{statfarads per centimeter,}$$

$$= \frac{3.677 \times 10^{-3}}{\log_{10} \frac{2D}{d}} \quad \text{microfarads per 1000 ft.,}$$

which is the same as the capacity between two parallel wires by themselves, see above.

Equilateral Triangle Arrangement with Balanced Three-phase Voltages. — For sine-wave voltages between the wires equal in effective value to V and differing in phase by 120 degrees, the above equations for the charges and

* To find the values of the B's in microfarads per 1000 feet change $2\log_e$ in these formulas to $136\log_{10}$ and apply equations (5a).

charging currents give the following relations between the *effective* values of the voltages, charges and charging currents, for each of the three wires:

$$Q = C_0 V_0, \qquad I = 2\pi f C_0 V_0,$$

where

$$V_0 = \frac{V}{\sqrt{3}} = \text{voltage to neutral},$$

$$C_0 = 2\, C_{12} = \text{capacity to neutral}.$$

When the C's are in microfarads the charge Q is in microcoulombs and the charging current in micro-amperes.

The charge on any particular wire is in phase with the voltage between that wire and the neutral (*see Alternating Currents*) and the charging current for that wire is 90 degrees ahead of the voltage drop from that wire to the neutral. For the same voltage V between wires in a single-phase system as in a three-phase balanced system, the charging current per wire in the three-phase system with the equilateral triangle arrangement of wires is $\frac{2}{\sqrt{3}} = 1.155$ times the charging current per wire in the single-phase system.

For any other arrangement of wires and for an unbalanced three-phase system, the general equations (5) and the general formulas for A_{11}, A_{22} and A_{12} given above must be used.

Two-conductor Cable. — Arrangement of wires and dimensions as shown in Fig. 5. The sheath forms the conductor of reference and when designated as conductor No. 3, general equations (5), (5a) and (6) apply directly to this case also. For the symmetrical arrangement of the wires shown in the figure, $A_{11} = A_{22}$, whence, dropping the first subscript,

$$B_1 = \frac{A_1}{A_1^2 - A_2^2}, \qquad B_2 = \frac{-A_2}{A_1^2 - A_2^2}.$$

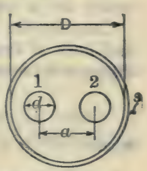


Fig. 5.

Russell (*Alternating Currents, Vol. 1*) gives the following values of A_1 and A_2, assuming a dielectric filling the space between the wires and the sheath to have the *same* * specific inductive capacity throughout:

$$A_1 = \frac{2}{K} \log_\epsilon \frac{D^2 - a^2}{Dd}, \qquad A_2 = \frac{2}{K} \log_\epsilon \frac{D^2 + a^2}{2\,Da}, \tag{8}$$

both in c.g.s. electrostatic units for a length of one centimeter. To reduce to practical units for a length of 1000 ft. change $2 \log_\epsilon$ to $136 \log_{10}$.

Normal Capacity of a Two-conductor Cable. — Substituting Russell's values for A_1 and A_2 in equation (6) gives

$$C_{12} = \frac{K}{4 \log_\epsilon \left(\frac{2\,a}{d} \cdot \frac{D^2 - a^2}{D^2 + a^2}\right)} \quad \text{statfarads per centimeter}$$

$$= \frac{3.677 \times 10^{-3} K}{\log_{10}\left(\frac{2\,a}{d} \cdot \frac{D^2 - a^2}{D^2 + a^2}\right)} \quad \text{microfarads per 1000 ft.}$$

Grounded Capacity of a Two-conductor Cable. — The capacity in this case, from equations (6), is

$$C_{1(23)} = B_{11} = \frac{A_1}{A_1^2 - A_2^2}.$$

* This is seldom realized in practice, since the dielectric is made up of insulation, fillers and braids having different specific inductive capacities; see *Wires and Cables, Insulated.*

By substituting Russell's values of the A's an approximate value for this capacity may be obtained; it will be greater than the normal capacity C_{12}.

Effect of the Earth on the Capacity of Two Overhead Wires. — As a fair approximation the earth may be considered as a conducting plane of infinite extent and the wires as parallel to this plane. Such a combination is electrostatically equivalent to two wires and their two "images" at a distance below the earth equal to the distance of the actual wires above the earth, see Fig. 6. Let q_1 and q_2 represent the charges on the actual wires and q_3 the actual charge on the earth, and V_{13} and V_{23} the potential differences between the two wires and the earth respectively (the p.d. between the two wires $v_{12} = v_{13} - v_{23}$); equations (5), (5a) and (6) then apply to this case also. The values of the A's in this case are (putting $K = 1$, since the dielectric is air)

$$A_{11} = 2 \log_e \frac{2 D_{1a}}{d_1}, \; A_{22} = 2 \log_e \frac{2 D_{2b}}{d_2}, \; A_{12} = 2 \log_e \frac{D_{1b}}{D_{12}}, \tag{9}$$

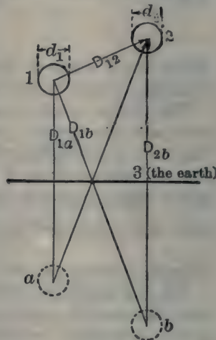


Fig. 6.

all in c.g.s. electrostatic units per centimeter length. To find the values of the B's directly in microfarads for a length of 1000 feet change $2 \log_e$ to $136 \log_{10}$.

Consider the special case where both wires are at the same height H, say, and both of the same diameter d; then from equation (9), putting D for the distance between their centers,

$$A_{11} = A_{22} = 2 \log_e \frac{4H}{d}, \quad A_{12} = 2 \log_e \frac{2H}{D'}, \tag{9a}$$

where

$$D' = \frac{D}{\sqrt{1 + \left(\frac{D}{2H}\right)^2}} = \text{"equivalent" distance apart.}$$

Then from equations (6)

$$C_{12} = \frac{1}{4 \log_e \frac{2D'}{d}} \quad \text{statfarads per centimeter.} \tag{10}$$

Comparing the formula for C_{12} with that for two wires by themselves, equation (4), it is evident that the effect of the earth in this case is to increase the capacity by an amount equal to the increase in capacity which would result from decreasing the distance between the wires from the actual distance D to the "equivalent" distance D'. For D small compared with the height H, which is usually the case, the equivalent distance D' is practically equal to D and the effect of the earth is therefore negligible.

Effect of the Earth When One Wire is Grounded. — Let No. 2 be grounded. Consider the same case as in the preceding paragraph, i.e., both wires at the same height H and of the same diameter d. From equation (5a)

$$B_{11} = B_{22} = 2C_{12}\left(\frac{A_{11}}{A_{11} + A_{12}}\right), \qquad B_{12} = -\,2C_{12}\left(\frac{A_{12}}{A_{11} + A_{12}}\right),$$

where C_{12} has the value given by equation (10) and the A's the values given by (9a). The charge taken by conductor No. 1 is then $B_{11}v$, the charge taken

by conductor No. 2 is $B_{12}v$, and the charge taken by the earth $(B_{11}+B_{12})v$, where v is the drop of potential from No. 1 to No. 2.

Effect of the Earth When Middle or Neutral Point of the System is Grounded. — In this case $v_{23} = -v_{13}, = v_0$, where v_0 = voltage to neutral. From equation (5) it is then evident that grounding the middle point of the system has no effect upon the charging current.

SYSTEMS OF FOUR OR MORE CONDUCTORS. — The method of calculating the capacities for a three-conductor system, described in detail above, may be readily extended to a system of any number of circuits, independent or otherwise, made up of any number of conductors.

Relations Between the *A's* and *B's* for a Four-conductor System. — For a four-conductor system the relations are as follows: put

$$M = A_{11}A_{22}A_{33} + 2\,A_{12}A_{13}A_{23} - (A_{11}A_{23}^2 + A_{22}A_{13}^2 + A_{33}A_{12}^2),$$

then

$$\left.\begin{array}{ll} B_{11} = \dfrac{A_{22}A_{33} - A_{23}^2}{M}, & B_{12} = -\dfrac{A_{12}A_{33} - A_{13}A_{23}}{M}, \\[2ex] B_{22} = \dfrac{A_{11}A_{33} - A_{13}^2}{M}, & B_{23} = -\dfrac{A_{23}A_{11} - A_{12}A_{13}}{M}, \\[2ex] B_{33} = \dfrac{A_{11}A_{22} - A_{12}^2}{M}, & B_{13} = -\dfrac{A_{13}A_{22} - A_{12}A_{23}}{M}, \end{array}\right\} \quad (11)$$

There are 32 part capacities in the general case of a four-conductor system but in cases of symmetry they reduce to a lesser number.

Three-conductor Cable. — Arrangement and dimensions as shown in Fig. 7. The sheath forms the conductor of reference; let it be designated as conductor No. 0. From symmetry $A_{11} = A_{22} = A_{33} = A_1$, say, and $A_{12} = A_{23} = A_{13} = A_2$, say. Equations (11) then reduce to

$$B_{11} = B_{22} = B_{33} = \frac{A_1 + A_2}{(A_1 - A_2)(A_1 + 2\,A_2)} = B_1, \text{ say,}$$

$$B_{12} = B_{23} = B_{13} = \frac{-A_2}{(A_1 - A_2)(A_1 + 2\,A_2)} = B_2, \text{ say,}$$

and the capacity between any pair of wires is

$$C_{12} = \tfrac{1}{2}(B_1 - B_2).$$

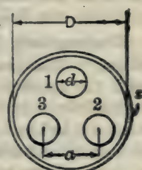


Fig. 7.

For sine-wave voltages equal in effective value and differing in phase by 120 degrees, the effective value of the charging current per wire is then, from equation (3),

$$I = 2\,\pi f\,(B_1 - B_2)\,V_0 = 2\,(2\,\pi f C_{12})\,V_0,$$

where $V_0 = \dfrac{V}{\sqrt{3}}$, the voltage V being the p.d. between wires. This current leads the voltage drop to neutral, V_0, by 90 degrees.

Calculated Value C_{12} for a Three-conductor Cable. — Russel (*Alternating Currents, Vol. I*) gives the following approximate values for A_1 and A_2, assuming the insulation fillers and braids to have the same specific inductive capacity (which is not usually the case),

$$A_1 = \frac{0.1360 \times 10^9}{K}\log_{10}\left(\frac{D^2 - 1.33\,a^2}{Dd}\right) \text{ and } A_2 = \frac{0.0680 \times 10^9}{K}\log_{10}\left(\frac{0.75\,D^6 - 1.78\,a^6}{a^2D^2(3\,D^2 - 4\,a^2)}\right)$$

in practical units for a length of 1000 feet. Using these values the normal capacity between wires is

$$C_{12} = \frac{3.677 \times 10^{-3} K}{\log_{10} \dfrac{2\,a\rho}{d}} \quad \text{microfarads per 1000 feet,}$$

where

$$\rho = \sqrt{\frac{(3\,D^2 - 4\,a^2)^3}{(3\,D^2)^3 - (4\,a^2)^3}}.$$

That is, the capacity is the same as that of two parallel wires by themselves but at a distance ρa between centers instead of the actual distance a.

Effect of the Earth on the Capacity of Three Overhead Wires. — For any kind of arrangement, as shown in Fig. 8 (the dotted circles are the "images" of the actual wires), the general expressions for the A's in c.g.s. electrostatic units * for a length of one centimeter are as follows:

$$A_{11} = 2 \log_\epsilon \frac{2\,D_{1a}}{d_1}, \quad A_{22} = 2 \log_\epsilon \frac{2\,D_{2b}}{d_2}, \quad A_{33} = 2 \log_\epsilon \frac{2\,D_{3c}}{d_3},$$

$$A_{12} = 2 \log_\epsilon \frac{D_{1b}}{D_{12}}, \quad A_{13} = 2 \log_\epsilon \frac{D_{1c}}{D_{13}}, \quad A_{23} = 2 \log_\epsilon \frac{D_{2c}}{D_{23}}.$$

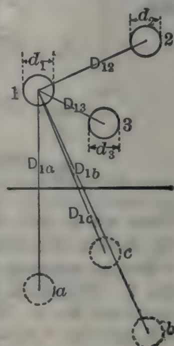


Fig. 8.

The values of the B's may then be calculated from equation (11) and the charging currents from equation (3). For any numerical case the calculations are tedious, but not difficult. Ordinarily the formulas given above neglecting the effect of the earth are sufficiently accurate for all practical purposes; compare with the effect of the earth on a two-wire line.

Electrostatic Induction From One Circuit to Another. — The general equations (1), (2) and (3) together with the method of calculation of the A's given in the last paragraph make it possible to calculate the induced voltages from one line to another, e.g., the voltages induced from a high-tension line to a telephone line. The method is straightforward, but tedious. The discussion given above indicates how such a problem may be attacked; space is not available for a detailed discussion here.

MEASUREMENT OF CAPACITY. — See the article on *Condensers, Electric.*

BIBLIOGRAPHY. — Russell, Alex., *Alternating Currents;* Ferguson, *Elements of Electrical Transmission;* Deutsch., W., *Graphical Method of Calculating the Capacity of Cylindrical Combinations*, Archiv f. Elektrotechnik, 2, p. 435, 1914, gives a method for calculating the capacity of parallel conductors of any form of cross-section whatever.

* To find from these A's the values of the B's in microfarads per 1000 feet change $2 \log_e$ in these formluas to $136 \log_{10}$ and apply equations (11).

CAR BARNS AND INSPECTION SHEDS. — (*See also Cars, Electric; Locomotives, Electric; Railways.*) In every electric railway system it is necessary to provide car barns and inspection sheds where the cars may be taken regularly for a systematic inspection. In some cases these barns are used for the storage of cars and in special cases are equipped as regular repair shops. Provision for inspection is necessary, provision for storage is optional, although it is customary to provide storage at least for those cars which are idle during the daytime. The expense of providing for the storage under cover of all the cars that are idle between midnight and 6 a.m. is of doubtful value, for the deterioration of the cars left in the open is likely to be less than the fixed charges on the increased capacity of the sheds.

Location. — The location of the car barn may be determined by several considerations. It may be located in the city on account of the convenience of the employees, the possibility of placing the general offices in the same building, and on account of the fact that the city service is an important part of the whole system. It may be located in the suburbs where real estate is cheap and ample space available. It may be located at a convenient point such as the center of gravity of the system, so that the dead mileage of the cars will be a minimum. Finally, it may be located adjacent to the power station, so that the repair shops of the two departments may be combined and thus become more efficient.

Construction. — Preparatory to designing a car house it is necessary to become acquainted with municipal and underwriters' regulations and conforming with these, to provide space for car storage, inspection, administration offices, line department, road department, car employees' lobby, car sign storage, sand drying and storage, salt storage, oil storage, wash-room and toilets.

The standard construction consists of a concrete foundation and brick walls, but many of the smaller railroads have mill frame buildings with 2-inch cement curtain walls. The floors are made of concrete with cement finish and sloped for drainage.

Illumination. — The illumination of a car barn should be both general and local. The general illumination should consist of incandescent lights, suspended from overhead between the tracks and spaced about a car length apart. These may be run five in series on the trolley circuit, although an ungrounded system is better. The intensity of illumination on the floor should be from 1 to 1.5 foot-candles or lumens per square foot. Frequent sockets for drop lights must also be provided for many portions of the car equipment are in dark corners.

Inspection Pits. — Inspection pits between the rails are usually 4 ft. 6 in. deep below the top of rails. It is important to have these pits thoroughly drained and well illuminated.

Tracks. — Tracks should be spaced not less than 11 feet between centers where there are no posts between tracks and not less than 13 feet where there are posts. The tracks at the entrance to the car house should be designed to permit the rapid movement of cars with the least interruption to traffic.

Fire Risks. — Attention should be given to the prevention of fire by the adoption of the following precautions:

1. Use of automatic sprinklers.
2. Provision of two sources of water supply such as mains and tank.
3. Division of building by fire-proof walls. The underwriters stipulate that no section shall contain cars to the value of over $200,000.
4. Avoiding proximity to inflammable buildings.

5. Having ample provision of water pails, sand pails, chemical extinguishers and short hoses, the last being usually 2 ½ inches in diameter and provided with 1⅛ inch nozzles.
6. Locating car house near a fire station.
7. Building tracks on a grade so that cars may be easily pushed out by hand.
8. Having numerous auxiliary fire alarms.
9. Consulting underwriters when planning.

Repair Shops. — It is usual to provide about 200 sq. ft. of repair shop floor per car owned. This area is usually divided about equally between departments, with, and those without car tracks. Where tracks are installed the usual spacing is from 15 to 16 feet between centers. Cars and car parts are usually lifted by travelling cranes, but in the smallest shops it is common practice to use jacks, hydraulic lifts, screw hoists and chain hoists. Transfer tables are preferred to ladder tracks for moving cars between departments.

Shop buildings should be surrounded with sufficient ground for storage of trucks, lumber, scrap material, etc.

BIBLIOGRAPHY. — Electric Railway Engineering Association, *Engineering Manual* and Proceedings.

CARS, ELECTRIC. — (*See also Control Systems for Railway Motors; Collectors, Current; Locomotives, Electric.*) The cars used on electric railways may be divided into two classes, viz., single-truck and double-truck. Each of these classes may be subdivided into types according as the cars are open or closed.

SINGLE-TRUCK CARS have a seating capacity as high as 32 passengers. Their characteristics are given in the accompanying table.

Item	Dimension
Over-all length	28 to 31 ft.
Rigid wheel base	5 to 7 ft.
Weight loaded and equipped	12 to 20 tons
Maximum speed	25 m.p.h
Maximum motor capacity, total	80 h.p.
Number of motors	2

They are only used in city service with frequent stops and usually have a cylinder-type controller at each end with hand brakes, although there are cases where these cars have been fitted with remote control and air brakes.

Considerable attention is being given at present to the subject of equipping these cars so that during the hours of heavy traffic they may be run in trains of two, controlled by one motorman. This causes less congestion in the streets for a given service and obviates the waste of operating throughout the day cars of large capacity designed to meet rush-hour conditions.

DOUBLE-TRUCK CARS.— The double-truck type of car has an over-all length of from 30 to 70 feet, a seating capacity of from 32 to 70 passengers and a maximum speed of from 40 to 70 m.p.h. One of these cars consists of a body mounted on two, four-wheel trucks. The car body under-frame is supported on a cross piece in the truck known as the bolster and is kept in position by a king-pin fitted into a king-pin center plate on the car body. The truck may swivel around this king-pin through quite an angle, guided by curved plates on each member.

Construction of Trucks. — The truck consists of axles, carried in journal boxes in side frames, transoms which connect the side frames, and the bolster which is carried by the transoms by means of springs (either spiral or elliptical) and a spring plate swung from the transoms. Coil springs are inserted between the journals and the side frames. The bolster may be of the "rigid," "floating" or "swinging" type depending upon the amount of play in the bolster. The rigid bolster is employed in locomotives only. The swinging bolster is most generally used for passenger cars, particularly in high-speed service, as it makes a much more easy riding construction. The three-axle truck is common in high-speed steam railroad practice, but the two-axle truck is generally used in electric railway practice as it allows more room for the motors. Four-wheel trucks are sometimes called "bogie" trucks or "swiveling" trucks to distinguish them from the "pony" or "radial" trucks having two wheels, which are used on steam locomotives.

Motor-car trucks have wheels of from 33 to 36 inches in diameter, and usually weigh from 10,000 to 15,000 pounds depending upon the size of the motors and body they are intended to carry. The rigid wheel base is from 6 to 8 feet. The problem of making a motor-driven truck guide properly is considerably more difficult than guiding the trucks of a car that is hauled, as there is a tendency for the motor-driven truck to get out of line and become cramped between the

rails. On this account and on account of the weight of the motors, trucks for a motor car must be stronger and heavier than for a trailer car.

Suspension of Motors. — The motors are usually hung between the axles on a single-truck car and between each axle and the transom on a double-truck car. In the "nose" suspension a lug is cast on the side of the motor frame away from the axle and this is attached to the transom with or without springs. The other side of the motor is carried by arms containing bearings which encircle the car axles. Thus about one-half of the weight of the motor is carried as a dead weight on the car axle, and the rest by the truck framing. In the "cradle" suspension both motors of a truck are carried by a cradle which is suspended from the car axles by springs. By this arrangement the entire weight of the motors is spring supported and carried by the axles, thus relieving the transom and side frames of the weight of the motors.

Maximum Traction Truck. — This is a special form of bogie truck with two axles having wheels of different diameters. The equalizing arrangements in this truck are such that about three-quarters of the total weight comes on the main axle which has the larger wheels and carries the motor, and the balance comes on the other axle which has small wheels. This arrangement gives a double-truck car with only two motors and yet provides that about three-quarters of the total weight shall be on driving axles, thus yielding "maximum traction." This type of truck is used on cars which are too long and heavy to use the two-axle rigid truck but which have such slow speed that only two motors are required. It is not adapted to high-speed work.

CAR BODIES may be classified as closed, convertible, semi-convertible and open.

Closed Cars.— The small closed type for city service is always of wood and has longitudinal seats. The general dimensions are given in the table below. Large closed cars with double trucks for interurban or rapid-transit service have both transverse and longitudinal seats, the proportion depending upon the proportion of long-haul traffic. The longitudinal seats give an arrangement permitting convenience of movement into and out of the car and allow more room for standing, but are not comfortable for long trips. Transverse seats are more comfortable for a long trip and for a service employing a high rate of acceleration.

Convertible Cars are constructed so that all the side panels of the body may be folded either up into the roof, or down to the floor leaving only the upright posts carrying the weight of the roof, thus converting a closed car into an open car at will.

Semi-convertible Cars. — A semi-convertible car, as its name implies, is arranged so that it may be partly opened in the summer time. It makes a very good car for high-speed interurban service because it is not advisable to operate open cars in high-speed service.

Open Cars usually have the transverse seats extending clear across the car. Access is obtained by steps running the length of the car on each side. They may be of the single- or double-truck type. They provide the maximum seating capacity for a given weight.

Cars for Rapid Handling of Traffic. — In large cities where the service is congested special means must be provided to aid and direct the rapid loading and unloading of cars. In a rapid transit service having stations this is accomplished by placing doors in the sides of the cars at the center and at or near the ends, and requiring all passengers to enter by the end doors and leave by the center doors. Both center and end doors are operated by compressed air and controlled by a guard at one end of the car. For street service the pre-payment or "pay as you enter" car is employed. This is built with extra long platforms

at each end. The passengers are required to deposit their fare in a box as they enter the car from the rear platform, where the conductor stands, and to leave by the front door and platform. The rear platform is made roomy to take care of congestion while the conductor is making change. Cars of this type recently installed in Chicago and Philadelphia are arranged for both entrance and exit of passengers at the front end, the conductor's station also being at this end.

The "One Man Safety Car" was introduced in 1914 to give more frequent service with decreased expense and in 1919, more cars of this type were built than of all other types combined. The principal data on a typical car are: Seats 32, single truck, length overall 28 feet, two 25 h.p. motors, wheel base 8 feet, wheels 24 inches, total weight equipped 7 tons, maximum speed 18 to 20 m.p.h., energy consumption 0.75 kw.-hr. per car mile. The car is equipped with air brakes, control system automatically interlocked with brakes and car doors and other special safety devices making it possible for one man to operate it with minimum possible chance of mistakes.

Construction of Car Bodies. — Most of the car bodies used on electric railways are constructed of wood, but for underground roads or for roads operating a high-class suburban service of multiple-unit trains the cars are preferably constructed of steel to reduce the dangers from collision and fire. This construction is demanded by consideration of safety, but it is directly contrary to the general tendency of the present day to reduce the weight of cars as much as possible. The proportion of weight of car to weight of load is very high and it entails a considerable waste of energy to propel this dead weight. The weight of car per passenger or seat capacity is a figure that should receive careful consideration and it should be kept at the minimum value compatible with strength and safety.

DIMENSIONS AND WEIGHTS OF TYPICAL EQUIPMENTS. — There are indicated in the following table the principle data for electric cars which are being operated in various typical services.

Service	City	City	City	City P. A. Y. E.	Suburban	Interurban	Subway	N. Y. C. R.R.	W. S. R.R.
Seating capacity	14	24	32	32	38	40	52	64	52
Number of trucks	1	1	1	2	2	2	2	2	2
Length over-all, ft.	22	30	32	41	38	40	50	60	49
Length of body, ft.	16	20	21	28	28	29	41	50	38
Number of motors	2	2	2	2	2	4	2	2	4
H.p. of each motor	40	40	40	70	70	50	200	200	75
Weight of body, lb.	6,000	9,530	7,500	15,800	15,670	19,200	34,300	55,000	35,110
Weight of trucks, lb.	4,600	4,800	4,500	10,400	10,600	16,000	22,500	14,940	23,000
Weight of electric equipment, lb.	5,990	6,962	7,062	9,500	8,350	16,542	21,510	25,460	21,680
Weight, total, lb.	16,590	21,292	19,062	35,700	34,620	51,742	78,310	95,400	79,790
Weight per passenger, lb.	1,185	887	594	1,118	911	1,293	1,506	1,490	1,534
Weight of load, lb.	2,240	3,640	4,760	4,760	5,600	5,889	7,560	9,240	7,560

BIBLIOGRAPHY. — Buck, A. M., *The Electric Railway*, New York, 1915; Burch, E. P., *Electric Traction for Railway Trains*, N. Y., 1911; Jackson, W., *Electric Car Maintenance*, New York, 1914; Master Car Builders Assn., *Car-Builders' Dictionary*, New York, 1916; *Electric Railway Journal*, Numerous places; *Electric Journal*, Oct. 1916, Oct. 1918.

CASTINGS, IRON AND STEEL. — These are made by pouring the molten material into sand or iron molds of the desired shape and allowing it to cool. Sand molds are generally used, and are made by packing molding sand about a wooden pattern having the shape of the desired casting and somewhat larger dimensions to allow for the contraction of the metal in cooling. The removal of the pattern leaves a hollow in the sand of the shape of the object to be cast. In order to have sufficient consistency the sand should either contain some clay and be somewhat moist, giving a "green sand" mold, or it should be mixed with a binding material giving a "dry sand mold."

Simple patterns may be made in one piece, but if the pattern be at all complicated it must be made in two or more pieces to permit withdrawal from the sand without disturbing the latter.

Steel castings are more difficult to make than iron castings, as they are more porous; thin steel sections should be avoided for such castings, as the steel cools too rapidly when thin.

In all castings sudden changes in the thickness of the metal should be avoided, as the unequal cooling of sections of different thicknesses causes temperature stresses and may crack the metal. This is particularly true in steel castings.

Care should be exercised in designing castings to make them such that the patterns can be readily removed from the sand molds.

Process of Making a Mold. — A wooden or iron box called a flask, consisting of sides only, is used in making the molds. This is divided into a bottom portion called the drag, and an upper portion called the cope; see Fig. 1. Intermediate portions, called checks, are used for complicated patterns.

A hole is made through the cope to convey the metal to the mold. Molds made of "dry sand" should be baked in an oven before pouring. The surface of the mold is frequently coated with "blacking."

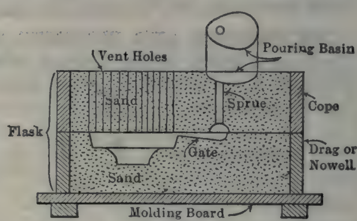


Fig. 1.

If holes are to be made in the casting they are formed by cores of hard-baked sand inserted after the pattern is removed and are held in place by studs of metal, called "chaplets," and by impressions in the mold made by projections on the pattern called "prints."

Cost of Iron and Steel Castings (Pre-war prices). — The cost of an iron casting, exclusive of the pattern, depends somewhat upon the intricacy of the pattern; representative costs in 1914 were for simple patterns of moderate size, 3 to 5 cents per pound; for large engine and dynamo castings 3 to 4.25 cents per pound; for small intricate castings 6 to 10 cents per pound. Malleable iron and steel castings about 50 to 100 per cent more.

BIBLIOGRAPHY. — See bibliographies in the articles on *Iron, Pig and Cast; Steel.*

CELLS, STANDARD. — (*See also Batteries, Primary; Electrochemistry, Principles of; Potentiometers.*) The need of a standard of electromotive force was recognized in the early days of the electrical industry, and the Daniell cell (*see Batteries, Primary*) was largely used for this purpose. This cell is readily prepared from ordinary chemicals, and is still used to some extent as a rough and ready standard. Such a cell, however, deteriorates quite rapidly, and has therefore been supplanted, for all accurate measurements, by other cells which possess to a greater degree the necessary characteristics of permanence and reproducibility. There are at present two types of standard cells in general use, the Clark and the Weston cells. The latter is the more generally used at the present time.

Pure Chemicals Required. — In order that the e.m.f. of any form of primary standard cell may accord with the stated value, great care must be exercised in the preparation of the materials, the processes for which have been carefully worked out by Kahle and later by Wolff and Waters (*Bull. Bureau of Standards, 1907, Vol. 4, p. 1*).

CLARK CELL. — In Fig. 1 is shown the Kahle's H-form of the Clark cell, which is one of the most satisfactory of the several forms in which the Clark cell is made. The container is of glass, which is mounted in a metal case with insulated binding posts connected to the two platinum terminals. The saturated solution of zinc sulphate forms the electrolyte, the paste of mercurous sulphate and zinc sulphate acts as a depolarizer, the mercury forms the positive pole and the zinc the negative pole. Detail directions for the preparation of the cell are given in the Bulletin of the Bureau of Standards, 1907, Vol. 4, p. 1.

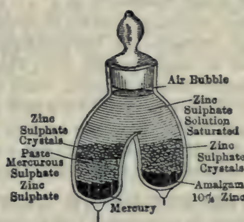


Fig. 1. Kahle's H-form of Clark Cell

Electromotive Force of Clark Cell. — The e.m.f. of the Clark cell, as determined at the time of the adoption of the international units (*see Units, Practical Electrical*), was given as 1.434 volts at 15° C. Subsequent investigations have shown that its true value is 1.4328 international volts at 15° C. At any other temperature t° C. its e.m.f. is

$$E_t = 1.4328\,[1 - 0.00077\,(t - 15)].$$

The chief objections to the Clark cell are its relatively high temperature coefficient and the length of time required for the e.m.f. to attain a constant value after a change in the temperature of the cell.

Carhart-Clark Cell. — To reduce the temperature coefficient and time lag of the saturated or "normal" Clark cell, Carhart devised a form of Clark cell in which the zinc sulphate solution was just saturated at 0° C. with no excess of crystals. The e.m.f. of this type of cell is approximately

$$E = 1.440\,[1 - 0.00039\,(t - 15)].$$

For accurate work, however, such cells must be calibrated, as the e.m.f. of individual cells may differ appreciably.

WESTON OR CADMIUM CELL. — In Fig. 2 is shown the construction of the "normal" Weston or cadmium cell. The container is of glass suitably mounted in a case provided with binding posts to which the platinum wires are attached. This cell differs from the Clark cell chiefly in the use of cadmium amalgam and cadmium sulphate in place of the zinc amalgam and zinc sulphate

for the negative pole and electrolyte respectively. The particular advantage in the use of cadmium instead of zinc arises from the low temperature coefficient and small temperature lag thus obtainable. It also has a longer life and polarizes less rapidly than the Clark cell.

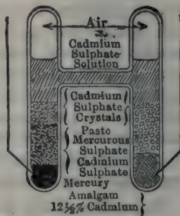


Fig. 2. Weston Cell

Electromotive Force of Weston Normal Cell. — The e.m.f. at 20° C. of the Weston normal cell (the word normal designating that the cadmium sulphate solution is saturated) is 1.01830 international volts. At any other temperature $t°$ C. its e.m.f. is

$$E_t = 1.01830 - 10^{-6}[40.6\,(t-20) + 0.95\,(t-20)^2 + 0.001\,(t-20)^3].$$

(See Bulletin Bureau of Standards, 1909, Vol. 5, p. 309.) The e.m.f.'s of normal cadmium cells as set up by various observers following the same specifications differ among each other by less than 1 part in 10,000.

Portable Cadmium Cell. — A portable form of cadmium cell is made, which differs from the normal cadmium cell chiefly in that the cadmium sulphate solution is just saturated at 4° C., and therefore does not contain an excess of cadmium sulphate crystals at ordinary temperatures. This type of cell is to be used between the temperatures of 4° and 40° C. The temperature coefficient is much smaller than that of the normal cell and is negligible for any ordinary measurements; each cell is accompanied by a certificate stating its e.m.f. The extreme variation among 145 cells of this type was found to be 0.0009 volt. This is at present the best form of secondary standard cell, being remarkably permanent. The e.m.f. is approximately 1.0186 international volts; the resistance, roughly, 200 ohms.

PRECAUTION IN USING STANDARD CELLS. — No appreciable current can be taken from a standard cell without alteration of its e.m.f., due to polarization. It is found that the change is not permanent, for the cell gradually recovers its original e.m.f.; of course the cell is unreliable until the recovery is complete. Consequently standard cells should be used only where their e.m.f. is opposed to an equal p.d., as in connection with potentiometers. They should always be protected by a key, which is closed only momentarily, and for preliminary adjustments a high resistance, several thousand ohms at least, should be in series with the cell. When an approximate balance has been obtained this resistance should be cut out and the final balance then made.

COSTS (Pre-war prices). — A normal Clark cell costs about $12, a normal cadmium cell $15, a portable Weston cell $15.

BIBLIOGRAPHY. — Laws, F. A., *Electrical Measurements;* Wolff and Waters, *Bull. Bur. of Stand.*, 1907, Vol. 4, p. 1; Wolff, *Bull. Bur. of Ltd.*, 1909, Vol. 5, p. 309.

CEMENT. — (*See also Concrete.*) The cements used in engineering construction are Portland, natural and Puzzolan (or slag) cements. Of these Portland cement is the most reliable and should always be used in reinforced concrete construction. The processes of manufacture and general characteristics of these cements are given in the following paragraphs, the descriptions being copied from the report of the Joint Committee as published in *Trans. Am. Soc. C. E.*, 1917.

PORTLAND CEMENT. — This is the finely pulverized product resulting from the calcination to incipient fusion of an intimate mixture of properly proportioned argillaceous and calcerous materials. Portland cement should be used in reinforced concrete construction and in any other construction that will be subject to shocks or vibrations or stresses other than direct compression.

Specifications. — The following specifications are taken by permission from the *Am. Soc. Test. Mat. Standards*, 1918.

The following limits shall not be exceeded:

Chemical Limits. — Loss on ignition, 4 per cent; insoluble residue, 0.85 per cent; sulfuric anhydride (SO_3), 2 per cent; magnesia (MgO), 5 per cent.

Specific Gravity. — The specific gravity of cement shall be not less than 3.10 (3.07 for white Portland cement). Should the test of cement as received fall below this requirement a second test may be made upon an ignited sample. The specific gravity test will not be made unless specifically ordered.

Fineness. — The residue on a standard No. 200 sieve shall not exceed 22 per cent by weight.

Soundness. — A pat of neat cement shall remain firm and hard, and show no signs of distortion, cracking, checking or disintegration in the steam test for soundness.

Time of Setting. — The cement shall not develop initial set in less than 45 minutes when the Vicat needle is used or 60 minutes when the Gillmore needle is used. Final set shall be attained within ten hours.

Tensile Strength. — The average tensile strength in pounds per square inch of not less than three standard mortar briquettes composed of one part cement and three parts standard sand, by *weight*, shall be equal to or higher than the following:

Age at test, days	Storage of briquettes	Tensile strength pound per sq. in.
7	1 day in moist air, 6 days in water....	200
28	1 day in moist air, 27 days in water...	300

The average tensile strength at standard mortar at 28 days shall be higher than the strength at seven days.

NATURAL CEMENT. — This is the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperautre only sufficient to drive off the carbonic acid gas. Natural cement does not develop its strength as quickly, nor is it as uniform in composition, as Portland cement. Natural cement may be used in massive masonry where weight rather than strength is the essential feature. Where economy is the governing factor, a comparison

may be made between the use of natural cement and a leaner mixture of Portland cement that will develop the same strength.

Specifications. — The following specifications are taken from the *Am. Soc. Test. Mat. Standards,* 1918 by permission.

Fineness. — The residue on a standard No. 100 sieve shall not exceed 10 per cent and on a standard No. 200 sieve shall not exceed 30 per cent by weight.

Soundness. — Pats of neat cement about 3 inches in diameter, ½ inch thick at center, tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat shall then be kept in air at normal temperature.

(*b*) Another pat shall be kept in water maintained as near 70° F. as practicable.

These pats shall be observed at intervals for at least 28 days, and, to satisfactorily pass the tests, shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

Time of Setting. — The cement shall not develop initial set in less than ten minutes, using the Vicat needle. Final set shall be attained in not less than 30 minutes nor more than 3 hours, using the Vicat needle.

Tensile Strength. — The minimum requirements for tensile strength for briquettes 1 sq. in. in cross-section shall be as follows; and the cement shall show no retrogression in strength within the periods specified:

Neat cement	Strength
24 hours in moist air	75 lb.
7 days (1 day in moist air, 6 days in water)	150 lb.
28 days (1 day in moist air, 27 days in water)	250 lb.

One part cement, three parts standard Ottawa sand	
7 days (1 day in moist air, 6 days in water)	50 lb.
28 days (1 day in moist air, 27 days in water)	125 lb.

PUZZOLAN OR SLAG CEMENT. — This is the finely pulverized product resulting from grinding a mechanical mixture of granulated basic blast-furnace slag and hydrated line. Puzzolan cement is not nearly as strong, uniform or reliable as Portland or natural cement, is not used extensively and never in important work; it should be used only for foundation work underground where it is not exposed to air or running water.

COST OF CEMENT. — As the cost of transportation is an important item in the cost of cement, the market price depends largely upon the place of sale. Quotations are given in the first number each month of *Engineering News-Record.*

BIBLIOGRAPHY. — *U. S. Government Specifications for Portland Cement,* Bur. Stand., Circ. No. 33, 1913. See also *Bibliography* in article on *Concrete.*

CHAINS AND CHAIN DRIVE. — Chains for hoisting and similar purposes may be made with straight open links, twisted links or with links with transverse studs to prevent the links from collapsing under heavy loads. These types of chains are known as straight-link chains, twisted chains and stud chains, respectively. The twisted chain is usually employed when the chain has to be wrapped around a smooth drum. Crane chains are straight-link chains carefully made to fit the corrugations of the winding drum.

Chains for transmitting power are of various forms. The more common types are block chains, roller chains and various makes of "silent" chains. In all these chains the links are made of several pieces of metal, the longitudinal pieces being riveted, bolted or screwed to the end pieces or studs. In the block chain, an example of which is the ordinary bicycle chain, the end pieces are solid blocks of metal; in the roller chain the end pieces form studs which carry small rollers. With block or roller chains the power is transmitted from or to the sprocket directly by the studs or rollers, whereas in the various types of "silent" chains, e.g., the Morse and Renold chains, the longitudinal pieces of the links are provided with lugs or fingers which mesh with the teeth of the sprocket wheel and thus pull it around.

Link belts are essentially power transmitting chains made by mounting a number of single chains side by side, so that they all move as a unit. Link belts are used both for power transmission and for belt conveyors.

DATA ON ORDINARY CHAINS (Penn. R.R. Specifications, 1920)

Description	Diam. of wire, inches	Width of links, inches	Length of 100 links, inches	Weight per foot, pounds	Breaking load, pounds	Safe working load, pounds
Twisted iron....	5/32	....	....	....	1,250	315
Twisted iron....	3/16	....	....	....	1,800	450
Straight-link....	1/4	1	100	0.70	3,400	850
Straight-link....	5/16	1 3/16	114	1.05	5,300	1,325
Straight-link....	3/8	1 3/8	125	1.55	7,700	1,925
Straight-link....	7/16	1 5/8	137	2.00	10,500	2,625
Straight-link....	1/2	1 13/16	150	2.55	13,700	3,425
Straight-link....	5/8	2 3/16	187	4.00	21,400	5,350
Straight-link....	3/4	2 5/8	217	5.75	30,700	7,675
Straight-link....	7/8	3	247	7.75	41,800	10,450
Straight-link....	1	3 3/8	280	9.75	54,700	13,675
Crane..........	3/8	1 1/4	113	1.65	8,200	2,050
Crane..........	7/16	1 1/2	125	2.10	11,200	2,800
Crane..........	1/2	1 5/8	144	2.70	14,600	3,650
Crane..........	5/8	2 1/8	175	4.25	22,800	5,700
Crane..........	3/4	2 1/2	200	6.00	32,900	8,225
Crane..........	7/8	3	235	8.00	44,700	11,175
Crane..........	1	3 3/8	262	10.00	58,400	14,600
Crane..........	1 1/8	3 7/8	300	13.00	74,000	18,500
Crane..........	1 1/4	4 1/4	350	16.00	91,400	22,850
Crane..........	1 1/2	5	412	23.00	131,500	32,875
Crane..........	1 3/4	5 7/8	475	30.00	177,300	44,325
Crane..........	2	6 3/4	575	40.00	233,900	58,475

Elongation of all sizes 12 per cent. Proof test one-half of breaking load.

SIZE, PITCH, STRENGTH AND WEIGHT OF CHAINS. — The size of an ordinary chain in which the links are made of wire or rods is specified by the diameter of the wire or rod. The pitch of a chain is the distance between the centers of successive links, usually expressed in inches. The proof test of a chain is the specified load it must carry without deformation, and is usually taken as one-half the breaking load. The ordinary safe load is usually taken as about two-thirds of the proof test, or one-third the breaking load.

The safe working load is dependent on the amount of rivet-bearing surface, speed, size of sprockets, etc., and ranges from 1/5 to 1/40 of the tensile strength.

STRENGTH OF ROLLER AND BLOCK "DIAMOND" CHAINS

Pitch, inches	Tensile strength, pounds	
	Roller	Block
½	1,200	....
⅝	1,200	....
¾	4,000	....
1	6,000	1200–2500
1¼	9,000	....
1½	12,000	5000
1¾	19,000	....
2	25,000	....

CHAIN DRIVE. — The advantages of chain drive are positive speed ratio, capability of transmitting large amounts of power at low speeds, not seriously affected by moisture, oil or grease, no stretch, and for short transmissions greater efficiency than leather, rubber or fiber belting. Roller chains should not as a rule be run at speeds of over 1000 feet per minute, and block chains not over 700 feet per minute.

Wherever possible, the distance between centers of shafts should permit of adjustment in order to regulate the sag of the chain. A chain should be adjusted, in proportion to its length, to show slack when running, care being taken to have it neither too tight nor too loose, as either condition is destructive.

The principal cause of trouble within the chain itself is elongation. It is the result of stretch of material or natural wear. Sudden jars or jolts beyond the limit of elasticity of the material will quickly render the chain useless. If for any reason a link elongates unduly it should be replaced at once, as one elongated link will eventually ruin the entire chain.

To minimize wear, chains should be kept well greased and protected from mud and grit, cleaned often, and when replaced put back so that they run in the same direction and same side up. A new chain should never be applied to a much-worn sprocket.

Sprockets. — Properly proportioned and machined sprockets are essential to successful chain gearing. For block chain these are obtained as follows:

Sprockets should be gauged to discover thick teeth and inaccurate diameters. A poor chain may operate on a good sprocket, but a bad sprocket will ruin a good chain. Sprockets of 12 to 60 teeth give best results. Fewer may be used, but cause undue elongation in the chain, wear the sprockets and consume too much power. Eight-tooth sprockets ruin almost every roller chain applied to them, and ten and eleven teeth are fitted only for medium and slow speeds with other conditions unusually favorable.

BIBLIOGRAPHY. — Kent's *Mechanical Engineers' Pocket-Book;* Hildage, H. T., *Chain Driving*, Mech. Eng. 30, p. 688, 1912 and *Transmission of Power by Chains*, Engineering, p. 99, 1914. See also circulars of Bradlee & Co., Phila., Link Belt Co., Phila., Yale & Towne Mfg. Co., N. Y., and other manufacturers. See also the works on *Machine Design* listed in the Bibliography at end of article on *Bearings*.

CHIMNEYS. — (*See also Draft, Mechanical.*) A chimney serves as a means for establishing a sufficient draft through a furnace to produce the combustion of the fuel. The amount of air *required* per hour depends upon the quantity of fuel burned per hour and the quality of fuel. The amount actually used depends also on the manner of firing, or the judgment and skill of the fireman. The *force required* to produce this draft in turn depends upon the quality of the fuel, the method of firing, the thickness of the fuel bed, the design of the furnace, the length and cross section of the gas passages, the height and cross section of the chimney, the location of the chimney with respect to neighboring buildings and hills, and the direction and velocity of the wind which may be blowing. The *force available* for producing the draft depends upon the height and cross section of the chimney and the difference in temperature of the hot gases in the chimney and the cold air outside. On account of the number of these variables and their great range of variation from time to time, it is practically impossible to deduce a rational formula for the size of the chimney required for the combustion of fuel at a given rate. The following approximate method for determining the height and cross section of a chimney, however, has been found satisfactory in practice. The formula connecting the height, cross section and rate of fuel consumption is an empirical one, deduced by the author from numerous actual cases, and first published in 1884. It has been extensively employed with satisfactory results by numerous engineers since that time.

INTENSITY OF DRAFT AND HEIGHT. — The difference in weight between the hot gases inside the chimney and the weight of an equal column of the external air is the cause of the draft produced by the chimney. The pressure due to this difference is called the "intensity" of the draft and is usually measured in inches of water column required to balance it. The measurement is made by a draft gauge, usually a U-tube partly filled with water, one leg connected by a pipe to the interior of the flue between the boiler and chimney and the other open to the external air. For finer and more accurate readings some form of multiplying gauge is used.

Let t_1 be the average temperature of the chimney gases, t_2 the temperature of the external air, both in degrees Fahrenheit, and let H be the height of the chimney in feet. Then the total intensity of draft produced by the chimney, expressed in inches of water column for normal atmospheric pressure, is

$$F = H\left(\frac{7.64}{460 + t_2} - \frac{8.00}{460 + t_1}\right).$$

For an atmospheric pressure of P inches of mercury, multiply the left-hand side by $\frac{P}{30}$.

The intensity of the draft and consequently the velocity and the volume of the hot gases will of course be greater the greater the temperature of the hot gases, but as the temperature increases the volume of a given weight increases, that is, their density decreases. When the temperature of the gases exceeds a certain value the increase in the velocity due to a further increase of temperature will be more than offset by the decrease in density. According to Rankine the temperature corresponding to the maximum value of the product of volume and density, that is the maximum weight of gases discharged, is, under ordinary conditions, 600° F., when the external temperature is 60°. Hence the total intensity of draft F_0 corresponding to maximum weight of discharged gases may be found approximately by substituting 600 in the above formula for t_1, and 60 for t_2. The formula then reduces to $F_0 = 0.007\ H$.

The actual intensity of the draft is usually less than that calculated from the

temperature taken at the bottom of the chimney, on account of the cooling of the gases as they ascend the chimney and from other causes. Taking the reduction at 20 per cent the formula expressing the relation of intensity of draft to height of chimney becomes $F_n = 0.0057\,H$ on the assumption of a temperature of 600° at the bottom of the chimney.

MINIMUM HEIGHT. — Hence when the intensity of the draft necessary to produce the necessary current of air through the breeching, boiler and furnace is known, the minimum height of chimney which will produce this draft is

$$H = 175\,F_n \text{ feet},$$

where F_n is the net intensity of draft in inches of water required. The following table gives the intensity of draft required under ordinary conditions.

DRAFT PRESSURE REQUIRED FOR COMBUSTION OF DIFFERENT FUELS

Kind of fuel	Total draft in inches of water	Kind of fuel	Total draft in inches of water
Straw	0.20	Slack, very small	0.7–1.1
Wood	0.30	Coal dust	0.8–1.1
Sawdust	0.35	Semi-anthracite coal	0.9–1.2
Peat, light	0.4	Mixture of breeze and slack	1.0–1.3
Peat, heavy	0.5	Anthracite, round	1.2–1.4
Sawdust mixed with small coal	0.6	Mixture of breeze and coal dust	1.2–1.5
Bituminous, run of mine	0.4–0.7	Anthracite slack	1.3–1.8

Thurston gives the following formulas for the intensity of draft and height of chimney in terms of the fuel consumption per square foot of grate per hour.

Let C = pounds of coal burnt per hour per square foot of grate. Then

for anthracite coal

$$F_n = 0.001875\,(C-1)^2 \quad \text{and} \quad H = \frac{(C-1)^2}{4};$$

for best Penn. or Welsh

$$F_n = 0.00148C^2 \quad \text{and} \quad H = \frac{C^2}{5};$$

for Pittsburgh or Illinois

$$F_n = 0.000833C^2 \quad \text{and} \quad H = \frac{C^2}{9}.$$

In any particular case the required draft may differ considerably from the figures given above. The design of a chimney should be left to an engineer of experience in this kind of work.

According to C. L. Hubbard (*Am. Elec., Mar., 1904*), the following heights have been found to give good results in plants of moderate size (500 horse-power or less), with sufficient draft to force the boilers 20 to 30 per cent above their rating:

With free-burning bituminous	75 feet
With anthracite of medium and large size	100 feet
With slow-burning bituminous	120 feet
With anthracite pea	130 feet
With anthracite buckwheat	150 feet
With anthracite slack	175 feet

For plants of 700 or 800 horse-power or over, the chimney should not be less than 150 feet high regardless of the kind of coal used. It is evident that the figures in the above table are subject to wide variations with different qualities of coal as regards caking and non-caking, clinkering and non-clinkering, etc., with different proportions of grate to heating surface and with different kinds of grates and furnaces.

On account of the elimination of the grate in oil-burning boilers and the injection of oil under pressure the height of a chimney for an oil-burning boiler is less than for a coal-burning boiler of the same capacity. According to Gebhardt a height of from 80 to 90 feet is sufficient to force oil-burning boilers to 50 per cent above their rating. In large oil-burning plants, however, the chimney is usually designed on the coal-burning basis, in order to permit the use of coal should this subsequently prove desirable.

CROSS-SECTION OF CHIMNEY. — All chimney formulas are based on the hypothesis that the capacity or theoretical coal consumption of a chimney varies directly as the area (or effective area) and the square root of the height.

Let C = coal consumption in pounds per hour.
A = area of actual cross-section of chimney in square feet.
H = height above the grates in feet.

Then the typical formula may be written:

$$C = KA\sqrt{H}$$

where K = a constant. Kent uses the formula

$$C = K'A'\sqrt{H}$$

where A' is the "effective" area, taken equal to $(A - 0.6\sqrt{A})$, and K' is a constant.

The value of the constant K, as given by different authorities, varies greatly, Toldt gives $K=5$; Prechtl, $K=6.4$; Molesworth, $K=9$; Ser, $K=9.3$; Hutton, $K=10$ to 16; Seaton and Rounthwaite, $K=12$; Henthorn, $K=16.6$; Brinckerhoff (average), $K=18.1$. Kent gives for K' the value 16.65. Toldt and Prechtl refer mainly to German metallurgical practice, Ser to general French practice, Molesworth and Hutton to English practice, Seaton and Rounthwaite to marine practice, Henthorn to American mill practice.

An average of 30 stacks of various sizes now doing good work gives $K=9.4$. An average of three notoriously overworked stacks gives $K=17.9$.

Ser's figure $K=9.3$, was obtained theoretically by allowing for twice the amount of air necessary for perfect combustion. By allowing an excess of one-half the amount necessary for perfect combustion, which result can readily be obtained by the use of automatic stokers, the constant $K=12$ will be obtained.

For preliminary calculations the above formula with $K=12$, gives practical results, but the chimney should be checked by comparison with known stacks of similar diameter and height for the final calculations.

The value of $K=12$ applies only to brick-lined stacks. In case an unlined iron or steel stack is being considered, the value of K may be increased to 14 or 15 and for small stacks 16 may be used.

STABILITY — THICKNESS OF WALLS. — All chimneys of any considerable size should consist of an outer stack of sufficient strength to give stability to the structure, and an inner stack or core, independent of the outer one, to protect the latter from the high temperature of the hot gases. This core sometimes extends up to a height of but 50 or 60 feet above the base of the chimney, but better practice is to extend it the full height or to 100 to 120 feet in chimneys over 120 feet high. The core is usually constructed of fire brick; the outer stack may be built of common brick, or radial brick (the Custodis chimney is an example of the latter) or of steel or of reinforced concrete. To determine whether a chimney is stable, treat it as a cantilever loaded throughout its length with a load equal to the total pressure (*see Elasticity and Strength; Wind Pressure*); at no section of the chimney should the stress in the walls exceed the safe working stress for the material of which it is constructed.

Brick Chimneys. — The base of a brick stack should rarely be less than one-tenth of the height. The allowable batter according to different authorities varies from 1 in 192 to 1 in 20 on each side, but the best practice lies between 1 in 30 and 1 in 40 for ordinary brick, with 1 in 60 to 1 in 80 for the Custodis or hollow-tile method of construction.

In brick chimneys practice varies as to the thickness of the walls. The linings are not exposed to wind pressure, and consequently can be much thinner than the outside wall. The usual practice is to make the steps about 50 feet high and 4 inches thick at the top up to a height of 150 feet.

For higher chimneys the lining should be 8 inches thick at the top. The outside walls for chimneys up to 150 feet high may be 8 inches thick at the top, with the steps about 50 feet high or the upper steps may be as high as 60 feet with 50 feet for the lower steps. For stacks built on the Custodis principle the top courses are from 8 to 13 inches thick, depending on the height. The thickness of the moulded brick is increased 2 inches every 5 meters, or about every 16½ feet.

All brick stacks should be topped with a waterproof cap, usually of cast iron, although in many cases it is made of stone or monolithic concrete.

Reinforced Concrete Chimneys began to come into extensive use in this country in 1901 and several hundred of them are now in use. Reinforced concrete chimneys built by the Weber Co., of Chicago consist of two parts, the lower double shell and the single shell above. The inside shell is usually 4 inches thick, while the thickness of the outside shell depends upon the height and varies from 6 to 12 inches. The single shell is from 4 to 10 inches thick. The height of the double shell depends upon the height of the chimney, nature and temperature of the gases, etc. The bending forces caused by wind pressure are taken up by vertical steel reinforcement; the resistance of the concrete itself against tension is not considered in calculation.

Steel Chimneys are largely used, especially for tall chimneys of iron-works, from 150 to 300 feet in height. The advantages claimed are: greater strength and safety; smaller space required; smaller cost, by 30 to 50 per cent, as compared with brick chimneys; avoidance of infiltration of air and consequent checking of the draft, common in brick chimneys. The best constructions for steel stacks include a number of vertical stiffeners riveted to the shell which support horizontal cast-iron or steel rings on which the linings are built. The vertical stiffeners are usually spaced about 5 feet apart, and the horizontal rings about 20 feet apart. By this method any section of the lining may be replaced without disturbing the other sections. The thickness of the metal at the top of the stack in such cases is usually ⅜ inch, increasing ⅛ inch

every 50 feet. Stacks in which the linings are not supported may be ¼ inch thick at the top increasing 1/16 inch every 30 feet. They are usually made cylindrical in shape, with a wide-curved flare for 10 to 25 feet at the bottom. A heavy cast-iron base-plate is provided, to which the chimney is riveted, and the plate is secured to a massive foundation by holding-down bolts. No guys are used.

Sheet Iron Smokestacks. — Guyed stacks of light sheet iron are frequently used for single boilers and even for quite large plants especially where the expected life of the plant is short. The smaller stacks are made up in lengths of about 20 feet of ⅛ inch steel connected by angle rings on the outside or the whole stack may be riveted in one piece on the ground and erected with a gin pole.

For these stacks the value of K in the general formula may be as high as 20 as they are usually connected directly to the boiler uptake and are exposed to high temperatures.

Such stacks deteriorate very rapidly and cannot be considered a desirable construction, but occasionally circumstances will require their use. Galvanized stranded wire cables form the best guys and the anchors may be concrete blocks for the larger sizes. For the smaller sizes the guys usually lead to the steel building structure.

WEIGHT OF SHEET-IRON SMOKESTACKS PER FOOT (Porter Mfg. Co.)

Diam., in.	Thickness, W.G.	Lb. per ft.	Diam. in.	Thickness, W.G.	Lb. per ft.	Diam., in.	Thickness, W.G.	Lb. per ft.
10	No. 16	7.20	26	No. 16	17.50	20	No. 14	18.33
12	"	8.66	28	"	18.75	22	"	20.00
14	"	9.58	30	"	20.00	24	"	21.66
16	"	11.68	10	No. 14	9.40	26	"	23.33
20	"	13.75	12	"	11.11	28	"	25.00
22	"	15.00	14	"	13.69	30	"	26.66
24	"	16.25	16	"	15.00	..	.....	.....

Evasé Stacks. — During the last few years a type of stack has been developed in Europe which offers marked advantages both as to cost and ease with which the draft may be controlled. The stack action is based on the injector principle, but the theory has not been well worked out as yet. The stack resembles a Venturi meter set up on end, the upper cone or diffuser enclosing an angle of 7°. These stacks are rarely over 60 or 70 feet in height and are usually applied to single boilers or batteries, in order that the control may be perfect. It is usually possible to attain an evaporation of about 2 lb. of water per square foot of boiler surface with the stack alone. For the higher ratings air is injected just below the Venturi throat, thereby inducing a higher rate of suction than the height of the stack would make. It is possible so to proportion the stack and blower capacities that a suction draft of 3 or 4 inches of water may be obtained, but this is usually unnecessary, as drafts of from 1 inch to 1½ inch will fulfill most of the requirements.

For empirical rules for design of Evasé stacks, see *Engineering of Power Plants*, Fernald and Orrok.

FOUNDATIONS. — Chimney foundations are as a rule constructed of concrete except where the nature of the soil necessitates the use of piles or a grillage of timber or steel. (For the bearing power of various soils see section on *Foundations* in article on *Power Stations*.) For masonry chimneys the foundation is designed to give the necessary support for the shaft without particular reference to its weight and shape. In steel and reinforced concrete chimneys the weight and shape of the foundation are important factors in securing the proper stability of the chimney, since the shaft is usually securely fastened to the foundation and the two form practically one mass.

SIZES OF FOUNDATIONS FOR HALF-LINED STEEL CHIMNEYS
(Selected from circular of Phila. Engineering Works.)

Diameter, clear, feet	3	4	5	6	7	9	11
Height, feet	100	100	150	150	150	150	150
Least diam. foundation, ft. and in.	15–9	16–4	20–4	21–10	22–7	23–8	24–8
Least depth foundation, feet	6	6	9	8	9	10	10
Height, feet	.....	125	200	200	250	275	300
Least diam. foundation, ft. and in.	.....	18–5	23–8	25	29–8	33–6	36
Least depth foundation, feet	.....	7	10	10	12	12	14

BREECHING. — The area of the flue or breeching leading from the boilers to the stack is usually made 20 per cent greater than that of the stack. When several boilers discharge into the same flue its section may be tapered and proportioned to the number of boilers. When two flues enter the stack on opposite sides at the same level a diaphragm is inserted between the two openings.

COSTS (Pre-war figures). — The following table gives the cost of brick chimneys:

COST OF BRICK CHIMNEYS

Approximate horse-power	Height, feet	Diameter flue, inches	Cost, erected
85	80	25	$598
135	90	30	786
200	100	35	1226
300	110	43	1492
400	120	51	1785
750	130	61	2528
1000	140	74	3060
1650	150	88	3750
2500	160	110	5095

The following approximate cost of radial brick chimneys (adapted from Gebhardt, *Steam Power Plant Engineering*) will serve to indicate the variation in cost with height and diameter:

COST OF RADIAL BRICK CHIMNEYS

Diameter in feet	Height in feet					
	75	125	150	175	200	250
4	$1400	$3500	.....	.....	.....	.....
6	2000	4300	.....	.....	.....	.....
8	2700	4700	$6200	$7100	$9,300	.....
10	3700	5100	7100	7900	10,500	$16,500
12	.....	.....	7800	9000	11,100	18,300
14	.....	.....	8300	9700	12,500	21,500
16	.....	.....	.....	.....	.....	24,300

The following are approximate costs of sheet-iron stacks:

COST OF GUYED IRON STACKS

Approximate horse-power	Height, feet	Diameter, inches	Cost, erected
25	40	16	$60
...	40	18	70
...	50	18	85
75	50	20	90
...	50	26	105
...	60	22	110
100	60	24	125
...	60	26	135
...	60	28	150
125	60	28	165
...	60	32	190
...	60	34	205
150	60	36	215
200	60	38	230
225	60	42	260
250	60	46	290
300	60	52	340
400	100	60	500

Brick and concrete stacks involve little expense for maintenance, 2 per cent being a liberal allowance for both maintenance and depreciation. Steel and sheet-iron stacks require occasional painting to prevent excessive corrosion.

BIBLIOGRAPHY. — Gebhardt, G. F., *Steam Power Plant Engineering;* Christie, W. W., *Chimney Design and Theory; Kent's Mechanical Engineers' Pocket-Book;* Kent, Wm., *Steam Boiler Economy;* Fernald and Orrok, *Engineering of Power Plants;* Menzin, A. L., *Proportioning Chimneys on a Gas Basis,* A. S.M.E. Trans., Vol. 37, p. 1065.

CIRCUIT BREAKERS. — (*See also Bus-Bars; Switchboards; Switches; Switchgear Equipment for Power Stations.*) A circuit breaker is a device to open an electric circuit which is normally held closed by the action of a latch, toggle or similar mechanism. This tendency to open is the feature that usually distinguishes a circuit breaker from a switch, as the latter, although used to open electric circuits, is without any latch or similar device and hence will remain in the closed position. This distinction between switches and circuit breakers is not always followed, as circuit breakers which open in oil are frequently spoken of as "oil switches."

TYPES OF CIRCUIT BREAKERS. — When a circuit breaker automatically opens the circuit as the result of certain abnormal conditions, such as overload, the circuit breaker is said to be "automatic," but if the breaker does not open unless tripped by the attendant it is spoken of as "non-automatic." An automatic breaker usually can be released by hand also. A breaker designed to be closed directly by the operator is spoken of as "direct controlled," while a breaker mounted at a distance is "remote controlled," The latter can be "manually operated" if worked through a system of bell cranks, levers or similar devices; "electrically operated" if worked by motor or solenoid; or "pneumatically operated" if controlled by compressed air.

Small circuit breakers, particularly of low voltage, are usually direct controlled, and large-capacity high-voltage breakers are usually remote controlled, some auxiliary source of power being employed. For such distant-controlled breakers automatic operation is secured through relays (*see Relays*). Relays are also frequently used with direct-controlled breakers.

Underload, Over-voltage, Under-voltage and Reverse Current Breakers. — The principal demand for circuit breakers is to have them open the circuit when the current reaches a certain predetermined value, and breakers are designed with this end in view. They are also built for underload conditions to open on minimum current, for over-voltage to open when the voltage exceeds a certain amount, for under-voltage to open when the voltage falls below a certain minimum value, for reversal when the current flows in the opposite direction through the breaker from that which was intended. It is, of course, possible to combine these various features of overload, underload, reversal, etc., in one and the same breaker. (*See Relays.*)

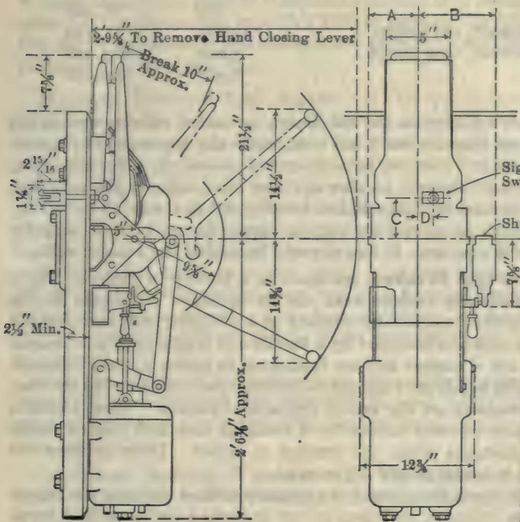
PRINCIPLE OF OPERATION. — The opening of a circuit breaker is usually secured by the releasing of a latch or the upsetting of a toggle joint that keeps the breaker closed. Various schemes have been adopted to secure the adjustment or "calibration" of the overload tripping device, so that it may be made to operate within reasonably wide limits, usually 80 to 160 per cent of the normal rating.

Breakers Opening in Air. — Fig. 1 shows the general arrangement of the magnetic circuit of a typical carbon-break circuit breaker, arranged for hand or solenoid operation, as used on heavy-capacity d-c. and low-voltage a-c. circuits. The current passes in at one stud across the contact brush and back through the other stud, thus forming a loop or turn in the electric circuit. A U-shaped iron frame is placed around the lower stud and the movable armature completes the magnetic circuit.

The current passing through the breaker magnetizes the iron circuit and tends to lift the movable armature. At a certain predetermined current the attraction overcomes the force of gravity, and the raising of the moving arm opens the latch or upsets the toggle which holds the breaker closed.

Two methods of securing the necessary range or calibration can be employed, one by varying the air gap between the stationary and moving iron, and the

other by giving the electromagnet more work to do by increasing the weight to be lifted. This latter method can employ either additional weights at a fixed



	Amperes	
	6000	8000
A	5⅝	6 7/16
B	8 1/16	8 13/16
C	5 11/16	5 15/16
D	1 9/16	2 1/16

Fig. 1. Carbon-break Circuit Breaker

distance from the fulcrum, or a fixed weight at a variable distance from the fulcrum.

For small currents the electrical circuit is carried more than once around the iron circuit, which results in a solenoid design with a movable plunger. With this scheme the tripping range is secured by adding weights to the solenoid or varying the air gap.

Oil Circuit Breakers (Figs. 2 to 6). — With high-voltage oil circuit breakers the tripping coil is usually a solenoid of a fairly large number of turns, which is connected in series with the main circuit or serves as the secondary of a current transformer whose primary is connected in the high-tension circuit. For polyphase circuits two or more tripping coils are used.

DESIGN. — For different classes of service various types of circuit breakers are used, and the functions of these types overlap more or less. The following table gives the approximate values of the maximum normal current and voltage for which these various types are used at present.

In connection with the carbon-break circuit breaker 24,000 amperes is the largest size in actual service, but larger breakers could be built if desired. The maximum d-c. voltage at present used for railway service in America, is 5000 volts but carbon breakers can be built for higher d-c. voltages if desired; they have been used up to 22,000 volts for a-c. service.

The fuse type has not been built for more than 100 amperes or 66,000 volts, but could be made for higher voltages if desired.

Type	Max. Current Amperes	Max. Vo ts.
Carbon Break	24,000	5,000
Fuse Type	100	66,000
Magnetic Blowout, old design	10,000	750
Magnetic Blowout, new design	3,000	3,000
Oil Break	6,000	220,000

The magnetic blowout type in the old design for street railway service has been built up to 10.000 amperes but it has been practically superseded by carbon breakers for this service. For d-c. railway service above 750 volts and up to 3000 volts the magnetic blowout breaker has been used owing to the very high speed that can be obtained and the efficient blowing out of the arc.

The oil break type is in service for current capacities up to 6000 amperes and for voltages up to 150,000. It has recently been built for 220,000 volts.

Air-break Carbon-type Breakers are made in a variety of designs; Fig. 1 shows a typical single-pole carbon-break circuit breaker. As shown in the figure, the circuit-breaker mechanism consists of a swinging arm carrying the main and the arcing contacts actuated by a handle and toggle-joint mechanism. The main contacts are of copper and are laminated to insure a perfect contact with the copper blocks which are fastened to the base and connected to the line. Above the copper contacts are the arcing contacts of carbon. These contacts consist of one or more carbons carried by a swinging arm and pressed firmly against stationary contacts when the breaker is closed. These carbons are mounted on a pivot to insure proper adjustment.

When the breaker opens the current is gradually shifted through the copper shunts to the carbon contacts, and thus no arc is formed until the final break takes place between the carbon contacts at the top. This arrangement, combined with the natural tendency of an arc to rise, prevents any injury to the breaker by the arc.

For large capacities strap connections are almost invariably used and to facilitate the employment of these strap connections the studs of large breakers are made of laminated construction.

Heavy capacity carbon-break circuit breakers are also made solenoid or motor-operated. With the solenoid design there is a definite end to the movement of the core that prevents damage to the mechanism from overtravel. The motor mechanism has a device that disconnects the worm gear and shuts down the motor when the breaker has closed and the toggle or latch has locked. This device is also so arranged that if an overload or short-circuit exists when the breaker is closed, the breaker will immediately trip out even though the control switch is held in the "close" position. This feature of being "nonclosable on overload" is one that is embodied in a large number of breakers of all kinds, and is a very valuable and useful one.

Switch and Breaker for Rotary Converters. — A combination motor-operated switch and circuit breaker has been supplied for various rotary converters and 600-volt feeder circuits supplying current for third-rail electrifications. One motor is provided for each combined circuit breaker and switch, and it has suitable clutches, shafts, operating rods and mechanism, so that in the act of closing the circuit breaker is first thrown in and the closing device is then disconnected before the switch is thrown in, so that in case of trouble the breaker can immediately trip out and open the circuit.

Air-break Fuse-type Breaker. — This type is a modification of the carbon-break type and is intended for moderate capacity circuits for voltages of 6000 to 60,000. This breaker consists essentially of a stationary and a movable arm

mounted on suitable insulators supporting the line connections. These arms are hinged together at one end. The free ends are held together against gravity and a strong spring by a piece of aluminum fuse wire that passes through a blow-out tube. When the current exceeds a certain amount, the fuse wire melts and the free end of the movable arm swings away from the stationary arm, the arc being ruptured in a blow-out tube.

Magnetic Blow-out Type Breaker. — This type of breaker is provided with auxiliary contacts that open in a strong magnetic field which blows out the arc formed at these auxiliary contacts. The main contacts are solid copper blocks bridged by a laminated copper brush. These open first when the breaker operates, leaving the final arc to be broken on the auxiliary contacts, which are protected by the magnetic blow-out device and which can be readily renewed at comparatively small expense. Although breakers of this type have been installed on the switchboards of power plants, they have been practically superseded by carbon breakers for this work. For such service as the protection of the circuits on a railway car, requiring a few hundred amperes at 600 volts, the magnetic blow-out principle is used to advantage, as the discharge vent from the blow-out compartment can be so set that the vapors from the arc will do no damage.

Since the advent of the high voltage d-c. railways operating at voltages of 1200–1500 or 2400–3000 the magnetic blowout effect has been utilized to great advantage in connection with the high speed breakers developed for that class of service.

Oil Circuit Breaker (Figs. 2 to 6). — The essential feature of this type of breaker is the opening of the circuit under oil and the smothering of the arc in a restricted space. These breakers are used almost to the exclusion of all other designs on a-c. circuits of large capacity and high voltage. Oscillograph tests show that the circuit is interrupted at the time of zero point of the alternating-current wave, and, therefore, there is little tendency to set up surges in the circuit. The rapid interruption of a direct current, however, is apt to set up surges, and therefore the oil-break design is not commonly used with d-c. service.

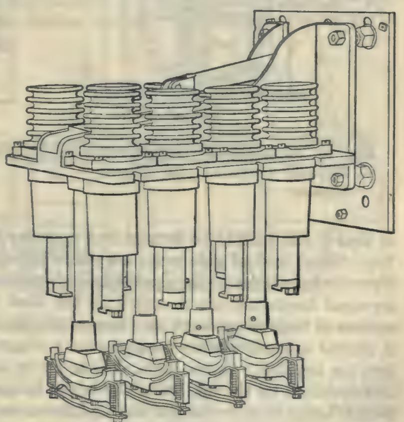


Fig. 2. 300-ampere, 15,000-volt, 4-pole Circuit Breaker

Oil-break circuit breakers are designed to meet various conditions of current, voltage and amount of power that they might be called upon to handle in case of a short-circuit. For moderate amounts of power where the size and cost of the breaker is to be kept to a minimum it is often possible to locate all the poles in one oil tank. For slightly larger amounts of power each pole is in a separate oil tank but all the poles are mounted on the same frame. For still greater amounts of power at moderate voltages each pole is in a separate tank and each tank in a separate compartment. For very high-voltage work each pole is in a

separate steel tank of such substantial construction as to be proof against any explosion due to the effects of a short-circuit.

Fig. 2 shows a 300-ampere, 15,000 volt, 4-pole breaker of a type that is built on various frames for various sizes up to 2000 amperes at 7500 volts, and 600 amperes at 25,000 volts. In this type of breaker each pole with its pair of contacts is in a separate tank with an insulating lining and all poles on the same frame. This breaker is readily arranged for manual operation or electrical operation by a solenoid and it can be mounted on a wall, framework, switchboard panel or on suitable supports of pipe or structural iron for open construction or for masonry compartments.

Fig. 3 shows a solenoid operated breaker of a type built on various frames for wall mounting, pipe mounting, or structure mounting, in capacities up to 2000 amperes at 15,000 volts and 1200 amperes at 25,000 volts. Each pole of this breaker is in a separate tank mounted on an independent frame and arranged

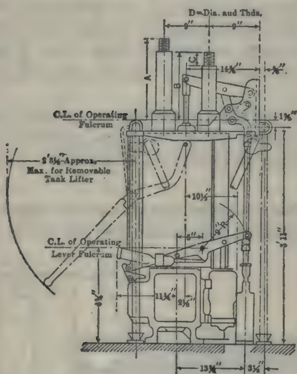


Fig. 3. 2000-ampere, 15,000-volt Solenoid Operated, Oil Circuit Breaker

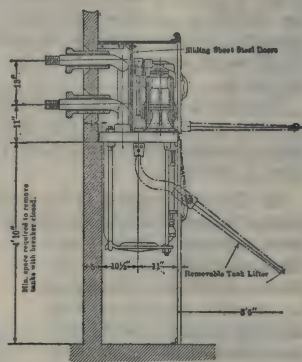


Fig. 4. Solenoid Operated, 15,000-volt, Oil Circuit Breaker

so that it can readily be located in a masonry compartment or on a framework. Two, three, or four pole units are made by connecting the mechanisms of two, three, or four poles to the same soleonid operating device. The mechanism and terminal insulators are mounted on the steel top of the breaker and a simple system of toggles operated by a powerful solenoid is used for closing the breaker. A second solenoid is used to disengage a trigger on the operating mechanism to trip the breaker. A two pole, double throw switch is mounted on the breaker and operated by the motion of the levers in opening or closing the breaker; this switch being used to control the signals on the switchboard.

Fig. 4 shows a solenoid operated back connected heavy capacity breaker built in capacities of 600 to 3000 amperes at 15,000 volts, guaranteed capable of rupturing 23,000 amperes per phase at 15,000 volts. Both terminals of each pole are in a single circular tank pressed out of ⅜ inch sheet steel, forming a seamless tank of great strength. An insulated lining is provided and each tank has a gauge glass for observing the height and condition of the oil. These breakers in service are placed in masonry structures and their design is based on securing the maximum rupturing capacity in a given space. A heavy steel casting carries the powerful operating mechanism as well as the contact rods, tanks, etc.,

making the breaker a self-contained unit. A modification of this breaker has the three poles independent, each pole being located in a separate masonry compartment.

Fig. 5 shows a 400 ampere 50,000 volt indoor frame mounted 3 pole solenoid operated oil circuit breaker. Each pole of the breaker is located in a welded steel tank with micarta lining. All of the operating mechanism is mounted in the tank top and provision is made for readily dropping any tanks to obtain access to the mechanism.

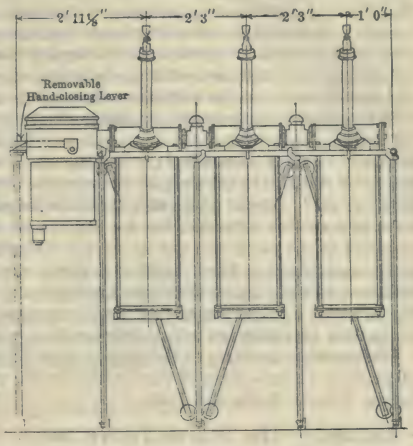


Fig. 5

This type of breaker is built both for indoor and for outdoor service, frame mounted, and floor mounted, for voltages from 25,000 to 73,000 frame mounted, and from 37,500 up to 155,000 volts for floor mounting. Designs are available up to 220,000 volts.

A three pole breaker is made up of three elements entirely independent of each other, except that they are connected by a single operating rod. The spacing of the poles, particularly on the floor mounting breaker, can readily be made to suit the station wiring. For manual operation the operating solenoid is replaced by a bell crank device. The condenser bushing leads that form the stationary terminals can readily be unclamped and removed through the cover. The series transformers for the operation of ammeters and relays are usually arranged to be clamped directly around the condenser bushing leads which form the single turn primary. This permits the use of a simple, compact and cheap form of series transformer.

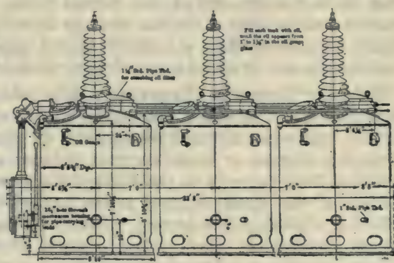


Fig. 6. Three-Pole Outdoor Breaker, 400-ampere 140,000 volts

Fig. 6 shows a 400 ampere, 140,000 volt outdoor floor mounting electrically operated breaker that is built in sizes from 50,000 up to 155,000 volts. This type of breaker is provided with cylindrical tanks having domed tops and bottoms, following out steam boiler practice and utilizing tanks that withstand pressures as high as 150 pounds per square inch. Designs are available up to 220.000 volts,

The breaker shown on Fig. 6 has main features corresponding closely with those of Fig. 5 previously described.

For outdoor service the condenser bushing leads of the breaker are covered with a series of porcelain insulators, and the space between the bushing and the insulators is filled with a moisture proof compound.

The operating mechanism is covered by a metallic hood and the pull rods connecting the poles of the breaker pass through pipes. The various joints are made thoroughly water proof. Access to the interior of the tank is secured by the removal of the mechanism cap or manhole cover which exposes the lever system and presents a sufficiently large opening to withdraw any necessary parts. The mechanism is so disposed that a terminal bushing complete with its contact details can be withdrawn without disturbing any other details, and the moving contact elements can be withdrawn through the mechanism cover.

Where circuit breakers are exposed to temperatures of 0° C. or below, ordinary oil becomes sufficiently viscous to seriously impair the efficiency of the circuit breaker. This is caused by the slowing up of the opening action due to the heavy oil and also to the sluggish action of the oil itself in flowing in and suppressing the arc. Special grade oil having low freezing characteristics is available to overcome this difficulty, but for very large units heating elements may be installed in the tanks and put in service during the continuance of dangerously low temperatures. These heating elements can easily be placed so as not to interfere with the action of the mechanism in any way. The cost of these heating elements and the amount of power consumed by them is less on very large units than the difference in cost between the low freezing oil and the ordinary oil.

High voltage breakers of the outdoor type have been in service for a number of years in various parts of the United States, and have also been supplied to Japan, France, Spain and Italy.

RATING — (*See below for Table of Sizes.*) The rating of a circuit breaker is usually given as the normal current which it will carry continuously with a temperature rise not exceeding 30° C. for carbon breakers and 50° C. for oil breakers, above the surrounding air. This rating has no direct relation to the maximum short-circuit current which the breaker will safely interrupt. (*See below under Ultimate Capacity.*) The tripping range of the breaker is usually from 80 to 160 per cent of this normal rating. The cross-section of the conductors and the area of contact surfaces depend so much upon the character of design, the cooling effect of bodies of oil and masses of metal, that no definite current density can be given as an average value for the various types of breakers.

Ultimate Breaking Capacity. — (*See table below.*) When a short circuit or ground occurs on a line, the rush of current thereby produced depends upon: (1) the total resistance and reactance of all parts of the circuit through which flows this current or other currents induced thereby, (2) the demagnetizing action of these currents on the fields of the generators producing them, and (3) the lag of this demagnetizing action behind the current producing it. The ability of a breaker to interrupt, without damage to itself, a current of a given magnitude depends upon various features of design, and it is evidently impossible to state how much synchronous or transforming apparatus may be safely connected directly to a breaker of a given rating, unless the conditions of operation are also fully specified.

In the table below the ultimate capacity of various breakers is given in terms of the amperes that they will actually rupture at the arc at their maximum rated voltage. They will rupture this current twice with a two minute interval between short circuits, and will be in condition to carry their full rated current, but may not be in condition to open up under short circuit without further attention.

For voltages less than the rated voltages the rupturing capacity in amperes can be increased usually more rapidly than the decrease in voltage, but a limit is reached when the short-circuit current amounts to approximately fifty times the normal current rating of the breaker.

Curves were published in the February 1918 Proceedings of the A.I.E.E. giving the short-circuit currents of typical American generators with various percentages

of reactance after the expiration of various lengths of time and these can be used to advantage in determining the short circuit to be expected in any system.

In determining the short-circuit characteristics of any particular system, the reactance of the generators and other devices has to be considered. On the basis of an average reactance of 8 per cent the momentary short-circuit current is roughly 12½ times normal and this usually dies down to about one-half this amount or 6¼ times normal by the time that any automatic breaker can open. At the expiration of a certain time that may be taken roughly as two seconds, the output of the system will be down to the continuous short-circuit basis of possibly two or three times normal rating.

The larger rupturing capacity breakers for 15,000 volts or less are almost invariably mounted in masonry compartments, usually with each pole in a separate cell and the leads carefully isolated from each other. The very high voltage breakers do not utilize the cellular construction and are very frequently arranged for outdoor installation.

DIMENSIONS, WEIGHTS AND COSTS. — Although the dimensions, weights costs etc. of breakers of various manufacturers naturally differ and are subject to change with the progress of designs, and while the same manufacturer frequently builds various grades for the same capacity the data that follows will give an idea of the range covered by the breakers of a single manufacturer.

CIRCUIT BREAKER DATA

Item	Amperes	Volts	Rupture, Amperes	Height, inches	Width, inches	Depth, ins.	Weight	Prices
1	400	750	......	24	4¾	14¼	130	$140
2	800	750	......	24	4¾	14¼	140	149
3	2000	750	......	24	4¾	14¼	150	227
4	8000	750	......	32	11⅞	21½	500	785
5	300	4,500	2,800	22¾	9⅛	16	185	100
6	300	15,000	1,100	31¼	15¾	18	220	235
7	2000	7,500	3,000	34	20½	25¼	660	1,060
8	300	7,500	2,900	26⅝	17¾	38 3/16	635	366
9	600	25,000	1,960	50⅝	20⅛	25¼	500	480
10	600	25,000	2,200	93	46½	30	1,040	1,300
11	600	25,000	5,350	120	54	29	1,760	1,700
12	600	15,000	18,000	87	54	31½	2,500	2,140
13	2400	15,000	23,000	99	74	31½	3,200	4,620
14	400	37,500	2,400	92	94	30	1,575	2,775
15	400	50,000	2,400	108	101	33	1,875	4,375
16	400	73,000	2,400	119	94	42	4,025	6,075
17	400	95,000	2,400	124	149	48	8,200	9,100
18	400	115,000	2,400	139⅜	172	58½	10,700	11,500
19	400	73,000	2,400	124	118	42	5,500	4,975
20	400	115,000	2,400	139⅜	174	58½	12,050	9,660
21	400	135,000	1,600	171	216	69½	26,600	14,875
22	400	115,000	3,750	157	201¾	69⅝	14,700	14,030
23	400	135,000	5,350	170	272	84¾	26,400	17,660

The information in the above table is based on apparatus manufactured by one of the largest American builders of switchgear and is practically correct at the date of its preparation (October, 1920). Changes are made from time to time in the design of equipment that may slightly modify this information.

The dimensions, height, breadth, and depth for carbon circuit breakers, items 1, 2, 3, and 4, refer to single pole switchboard mounting breakers; and the depth is given from the front of the panel, that is, the length of the studs projecting through the panel is neglected. For the switchboard mounting oil circuit breakers the dimensions are given from the back of the panel, that is, the length of the handle sticking through the board is neglected. All of the oil breakers are three pole, items 5, 6, and 7, being hand operated, the remainder being electrically operated. For the electrically operated breakers the dimensions include the operating solenoids. For cell mounting breakers the dimensions include the cells but the weight and prices are exclusive of cells, oil, current and potential transformers. For the high voltage breakers the dimensions are over the terminals and the width is based on the minimum spacing recommended for any particular voltage. The dimensions are in every case the maximum overall dimensions except as mentioned. The costs given are the approximate selling prices under date of October 1920, and are sufficiently accurate for the preparation of preliminary estimates, but for accurate work should be checked by actual quotations from a reliable builder.

Items 1 to 4 are single pole hand operated carbon circuit breakers, similar to Fig. 1.

Items 5 and 7 are hand operated oil circuit breakers with all contacts in the same tank and barriers between contacts.

Item 6 is an oil breaker with each pole in separate tanks but all tanks on the same frame.

Item 8 is an electrically operated breaker with each pole in a separate tank on a separate frame, and arranged for cell mounting, wall mounting or pipe mounting.

Item 9 is a three pole breaker with all poles in separate tanks, all tanks on a common frame arranged for wall mounting, pipe mounting, or cell mounting.

Items 10 and 11 are essentially cell mounting breakers with each pole in a separate cell, and items 12 and 13 are cell mounting breakers with circular tanks with each pole in a separate tank, all three tanks on the same frame.

Items 14, 15, and 16, are frame mounted solenoid operated indoor breakers.

Items 17 and 18 are solenoid operated floor mounting indoor breakers.

Item 19 is a solenoid operated frame mounting outdoor breaker.

Items 20 and 21 are solenoid operated floor mounting outdoor breakers with oval tanks.

Items 22 and 23 are solenoid operated outdoor breakers with cylindrical tanks.

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COLLECTORS, CURRENT. — (*See also Cars, Electric; Locomotives, Electric; Third-Rail Systems; Trolley Systems, Overhead; Trolley Systems, Underground.*) Current collectors for electric cars or locomotives are divided into three classes in accordance with the form of the working conductor from which they collect current, as follows:

(*a*) Overhead Trolley, which may be of the wheel, scraping bow or roller type.

(*b*) Third-Rail Shoes, which may be of the over-running or under-running type.

(*c*) Underground Conduit Plow.

WHEEL TROLLEY. — The wheel trolley consists of a grooved brass or copper wheel held in bearings in a prong called a "harp" at the end of a steel pole which is pressed upward by a system of springs and levers. The trolley wire is from 18 to 22 feet above the rails. The pole presses the wheel upward against the wire with a pressure of from 20 to 40 pounds, the higher pressure being used for the higher speeds. The current which a wheel trolley can collect is limited at the various speeds as indicated in the accompanying table. If greater currents are required, resort must be had to the third rail. The trolley wheel is not very satisfactory on cars in trains as each trolley pole requires the attention of a conductor to replace it at curves and switches, and moreover, it is difficult to manipulate the trolley pole from a platform between two cars.

Miles per hour	Amperes
5	1200
15	600
40	350
60	200

BOW TROLLEY. — This type of trolley is used to collect current from a high-voltage conductor because it is self-adjusting and needs no attention while the cars are running. At high voltages its current capacity is sufficient to supply power for a train of considerabie size but it would not have sufficient capacity to supply power for heavy trains at 600 volts. In America, bows are usually made of mild steel. In Europe aluminum is preferred but it has been found there that aluminum containing 6 per cent of copper, gives the best results. The bow may be held up either by simple springs in the same manner as the trolley pole or it may be mounted upon a pantagraph mechanism which is held up by springs and folded down by a compressed-air cylinder. The St. Paul 3000 volt locomotives have pantographs with two pans pressing upward with a pressure of about 50 pounds per pan and have successfully collected currents of 2000 amperes at low speeds. The pans are hinged to the pantograph proper in order to make better contact with the trolley wire. A special form of overhead construction must be used for the bow trolley as the bow would strike the downwardly projecting ears and the guy wires of the ordinary trolley construction. The roller trolley may be used in place of the bow on the same form of mechanism. The roller has a greater current capacity and causes less wear on the working conductor than the bow. The rollers used by the Butte, Anaconda and Pacific Ry., are steel tubes 5 inches in diameter and 2 feet long. They are each required to collect 750 amperes at 2400 volts.

THIRD-RAIL SHOES. — A third-rail shoe will collect currents as high as 2000 amperes at low speeds and 600 amperes at 60 miles per hour. As they are self-adjusting two or three may be placed on a locomotive or car just as well as one, so that there is practically no limit to the current that can be collected in this way. In fact it is always customary to put two on each side of each locomotive or car in order to prevent a cessation of current when passing

over breaks in the third rail due to switches or crossings. (See article on *Third Rails.*)

Over-running vs. Under-running Types. — The third rail for top contact or for over-running shoe is cheaper to install, to protect and to maintain and is most generally used. The under-contact rail is less liable to trouble from sleet, snow and ice and is therefore warranted where climatic conditions are bad.

UNDERGROUND CONDUIT PLOW. — The underground plow consists of an insulated steel plate hung from a movable structure on the car. On the two sides of this plate and thoroughly insulated from it are two shoes pressed outward from the plate by springs. These shoes press against the two working conductors which are usually steel Tees separated from each other by about 6 inches and supported on some form of ceramic insulator. The current is led from the shoes to the car body by flexible insulated conductors.

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COMPLEX QUANTITIES. — (*See also Vectors.*) The square root of a negative quantity is called an "imaginary" quantity, or a pure imaginary. A quantity consisting of the sum or difference of a real quantity and an imaginary quantity is called a "complex" quantity. For example, $\sqrt{-3}$ is a pure imaginary, and $2+\sqrt{-3}$ is a complex quantity. All the rules of ordinary algebra apply to pure imaginaries and complex quantities. For example, $\sqrt{-3}$ may be written $\sqrt{-1}\sqrt{3}$, and in general $\sqrt{-a}$, where a is a positive quantity, may be written $\sqrt{-1}\sqrt{a}$. The square root of minus one is called the imaginary unit and is usually represented by the symbol j (writers on pure mathematics use the symbol i), that is,

$$j=\sqrt{-1}.$$

Any complex quantity may then be written

$$a+jb,$$

where a and b are both real quantities.

Geometrical Representation of a Complex Quantity. — A positive real quantity may be represented by a line drawn in a given direction; a negative real quantity may be represented by a line drawn in the opposite direction. Multiplying a quantity by -1 then reverses its direction. Also, since multiplying a real quantity by $\sqrt{-1}$ twice is equivalent to multiplying it by -1, the operation of multiplying once by $\sqrt{-1}$ may be represented by turning the line representing the quantity through 90° in the positive direction of rotation. The positive direction of rotation is taken as the opposite direction to that in which the hands of a clock move. Hence, a complex quantity $a+jb$ may be represented by the line OP in the figure, where $OA=a$ and $AP=b$. The complex quantity $a+jb$ is then completely specified by a line of length $\sqrt{a^2+b^2}$ making an angle θ, with the axis of reference OX where $\tan\theta=\frac{b}{a}\cdot$

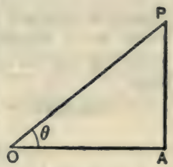

Fig. 1

The length $M=\sqrt{a^2+b^2}$ is called the magnitude of the complex quantity and the angle $\theta=\tan^{-1}\frac{b}{a}$ is called its angle. From the figure it is evident that the complex quantity $a+jb$ may also be written

$$a+jb=M(\cos\theta+j\sin\theta).$$

Expanding $\cos\theta$ and $\sin\theta$ into series (*see Series*) and adding, the resultant series obtained is the series for $\epsilon^{j\theta}$; hence

$$a+jb=M\epsilon^{j\theta}. \tag{1}$$

From the above definitions and equation (1) it is evident that complex numbers possess the following properties:

Addition of Two Complex Quantities. —

$$(a+jb)+(a_1+jb_1)=(a+a_1)+j(b+b_1).$$

Subtraction of Two Complex Quantities. —

$$(a+jb)-(a_1+jb_1)=(a-a_1)+j(b-b_1).$$

Multiplication of a Complex Quantity by a Complex Number. —

$$(a+jb)(a_1+jb_1) = aa_1 - bb_1 + j(ab_1 + a_1b)$$

or, putting $a+jb = M\epsilon^{j\theta}$ and $a_1+jb_1 = M_1\epsilon^{j\theta_1}$

where $M = \sqrt{a^2+b^2}, \quad M_1 = \sqrt{a_1^2+b_1^2},$

$$\tan\theta = \frac{b}{a},$$

and

$$\tan\theta_1 = \frac{b_1}{a_1}$$

we have $(a+jb)(a_1+jb_1) = M\epsilon^{j\theta}M_1\epsilon^{j\theta_1} = MM_1\epsilon^{j(\theta+\theta_1)}.$

Hence the product of two complex quantities is in general a complex quantity which has a magnitude equal to the product of the magnitudes of the two quantities and an angle equal to the sum of the angles of the two quantities.

Division of a Complex Quantity by a Complex Number. —

$$\frac{a+jb}{a_1+jb_1} = \frac{(a+jb)(a_1-jb_1)}{(a_1+jb_1)(a_1-jb_1)} = \frac{aa_1+bb_1-j(ab_1-a_1b)}{a_1^2+b_1^2}$$

or

$$\frac{a+jb}{a_1+jb_1} = \frac{M\epsilon^{j\theta}}{M_1\epsilon^{j\theta_1}} = \frac{M}{M_1}\epsilon^{j(\theta-\theta_1)}.$$

Hence the quotient of two complex quantities is in general a complex quantity which has a magnitude equal to the quotient of the magnitudes of the two quantities and an angle equal to the difference of the angles of the two quantities.

Example. — Suppose

$$I = \frac{10+j\,15}{3+j\,1};$$

then by the rule for division

$$I = 4.5 + j\,3.5,$$

that is, I contains a real part 4.5 and an imaginary part $j\,3.5$.

Equations Containing Complex Quantities. — Since a real quantity cannot be equal to an imaginary quantity it follows that any equation of the form

$$A + jB = A_1 + jB_1,$$

where A, B, A_1 and B_1 are all real quantities (which may, however, consist of any number of terms), is equivalent to the two equations

$$A = A_1$$

and

$$B = B_1.$$

Also, if one member of an equation reduces to the form $A+jB$, then the other member of this equation must likewise contain an equal real and an equal imaginary part.

CONCRETE. — (*See also Cement, Concrete, Reinforced.*) Concrete as used by engineers in construction is generally formed of an artificial mixture of Portland cement (*see Cement*) and an aggregate consisting of sand and gravel or broken stone. These ingredients are mixed with water either by hand or by machine mixers.

Light-weight Concrete.— During the Great War a light-weight concrete was developed for building concrete ships in the U. S. In this concrete the aggregate instead of being ordinary stone is made from calcined clay specially treated. This light-weight concrete weighs about 110 pounds per cubic foot and has a crushing strength of about 4000 pounds per square inch after 28 days ageing. It has to be made richer in cement than ordinary concrete and is consequently more costly. (*W. L. Scott, Engineering, 108*, p. 33, *July. 1919.*)

SPECIFICATIONS. — The quality of the materials can be determined only by careful tests, but for construction of minor importance and magnitude the following simple specifications should give good results.

Cement. — The cement shall be first-class American Portland cement of standard brand, guaranteed to conform to the Standard Specifications of the American Society for Testing Materials. (*See Cement.*) Such cement may be delivered either in bags or barrels but must be kept dry during storage.

Sand. — The sand shall be clean and coarse and free from dust, vegetable, loam or other organic matter, and other impurities, and should pass, when dry, a screen with holes $\frac{1}{4}$ of an inch in diameter.

Gravel. — The gravel shall consist of clean pebbles, and contain no foreign matter. It shall be screened over a ¼-inch mesh; the sand passing through the screen may be remixed in definite proportions with the gravel. If dirty or clayey, the gravel should be washed by a hose before mixing.

Broken Stone. — The broken stone shall consist of hard and durable stone such as trap, granite and limestone. Unless otherwise specified, all stone shall pass through a 2½-inch screen.

Water. — The water used for mixing shall be free from oil, acid and other injurious substances.

PROPORTION OF INGREDIENTS. — The amount of each ingredient is usually measured by volume. The concrete is designated by the proportion by volume of each of the ingredients in the following order: Cement, Sand, Stone or Gravel. For example, a 1 : 2 : 4 mixture is one consisting of one barrel of cement, two barrels of coarse sand, and four barrels of loose gravel or broken stone (standard cement barrels contain 3.8 cu. ft; four bags of packed cement may be considered as equal to one barrel).

The following proportions are somewhat generally adopted for different classes of work.

1:1:2 or 1:1½:3 for water tanks and standpipes carrying considerable pressure and required to be water-tight.

1:2:4 for arches, reinforced floors, beams, columns, engine foundations subject to vibration, sewers, and in general for structures subjected to bending stresses of some magnitude. A mixture as rich as this is also desirable where concrete is to be deposited under water since in such a case some of the cement may be washed away from the mixture.

1:2½:5 for bridge abutments and piers when laid in air, retaining walls and ordinary machine foundations.

1:3:6 for heavy walls, ordinary foundations, backing for stone masonry, etc.

Quantity of Ingredients.— The following rule devised by Wm. B. Fuller may be used for approximate determination of the qnantity of the various ingredients.

Let c = number of parts of cement,
s = number of parts of sand,
g = number of parts of gravel or broken stone. Then

$\frac{10.5}{c+s+g} = N =$ number of barrels of Portland cement required per cu. yd. of concrete,

$\frac{1.55}{c+s+g}s =$ number of cu. yd. of sand required for one cu. yd. of concrete,

$\frac{1.55}{c+s+g}g =$ number of cu. yd. of stone or gravel required for one cu. yd. of concrete.

For tables giving more accurate values see Taylor and Thompson, *Concrete, Plain and Reinforced.*

MIXING. — The ingredients consisting of sand and aggregate should be thoroughly mixed while dry until the mass is uniform in color and homogeneous. Mixing should preferably be done by machine. When necessary to mix by hand, the mixing should be done on a water-tight platform and the ingredients should be turned not less than six times. Enough water should be used to produce a mixture which will flow readily into its place in forms or elsewhere.

Concrete Mixing in Freezing Weather. — Reinforced concrete should not be mixed or deposited in freezing weather unless special precautions are taken to keep the materials free from ice, and to protect the concrete from freezing until it has been thoroughly hardened. To accomplish these results the aggregate may be heated, salt water used in mixing, and the concrete carefully covered after it has been deposited. Portland cement concrete which is to be deposited in large masses may be laid in freezing weather provided the surface appearance is not important, and provided that hardened surfaces be thoroughly cleaned from frost as well as dirt before a new surface is laid. Natural cement concrete should never be laid in freezing weather.

Concrete Shrinkage and Temperature Changes. — Cracks in cement may be caused by contraction in setting, or by temperature changes, as well as by excessive stress. To preserve a good appearance it is common to establish artificial cracks in long walls, sidewalks, etc. For spacing of such cracks see reference in *Bibliography*. The insertion of steel reinforcing bars even when not needed to carry stresses due to applied loads is an expedient often adopted to prevent the occurrence of such cracks. The amount of reinforcement to be used for this purpose should generally be not less than one-third of one per cent.

Effect of Sea Water and of Acids on Concrete. — The data available relating to the effect of sea water upon concrete indicate that there may be some chemical action resulting in a softening and disintegration of the surface; to offset this action as far as possible, concrete laid in sea water should be carefully proportioned and well mixed. There is also great danger from disintegration due to frost in cold climates in concrete lying between high water and low water. To protect against this the concrete may be protected by a surface of stone in the zone exposed to such action.

Thoroughly hardened concrete of good quality may be considered as resisting the action of acids and mineral oils as well as other building materials. Oils containing fatty acids may produce injurious effects by combining with the lime in the concrete, resulting in a disintegration of the latter.

Waterproofing Concrete. — Concrete may be made reasonably impervious to water under moderate pressures by using a rich mixture and by careful pro-

portioning and mixing. In the case of building foundations and other structures where leakage is not permissible it is usually advisable to protect the concrete by a separate coating applied either on the outside or inside of the wall or floor. The usual method of waterproofing the exterior is to construct a so-called membranous coating consisting of alternate layers of pitch and tarred felt. The inner surface of concrete basement walls and floors of many of the important buildings in New York City and other large American cities have been waterproofed by the use of a patented compound called "Hydrolithic Cement." Many so-called waterproofing powders, pastes, etc., are manufactured which are claimed to make concrete entirely water-tight if incorporated with it during mixing, but their value is problematical.

COMPRESSIVE STRENGTH AND WORKING STRESSES. — In the Joint Committee Report (*Trans. Am. Sec. C.E.*, 1917) the following table of compressive strength of thoroughly set concrete is given:

COMPRESSIVE STRENGTHS OF DIFFERENT MIXTURES OF CONCRETE

Tested in cylinders 8 inches in diameter and 16 inches long

Set 28 days under favorable conditions

In Pounds per Square Inch

Aggregate	1 : 3	1 : 4½	1 : 6	1 : 9	1 : 7½
Granite, trap rock........	3300	2800	2200	1800	1400
Gravel, hard limestone and hard sandstone.........	3000	2500	2000	1600	1300
Soft limestone and sandstone..................	2200	1800	1500	1200	1000
Cinders.................	800	700	600	500	400

The following recommendations are also made:

When compression is applied to a surface of concrete of at least twice the loaded area, a stress of 35 per cent of the compressive strength may be allowed in the area actually under load.

For concentric compression on a plain concrete pier, the length of which does not exceed four diameters, or on a column reinforced with longitudinal bars only, the length of which does not exceed twleve diameters, 22.5 per cent of the compressive strength may be allowed.

The extreme fiber stress of a beam, calculated on the assumption of a constant modulus of elasticity for concrete under working stresses may be allowed to reach 32.5 per cent of the compressive strength.

For reinforced structures see the full report of the Committee in the *Trans. A.S.C.E.*, 1917.

Modulus of Elasticity. — The value of the modulus of elasticity of concrete has a wide range, depending on the materials used, the age, the range of stresses between which it is considered, as well as other conditions. It is recommended that in computations for the position of the neutral axis and for the resisting moment of reinforced concrete beams and for the compression of concrete in columns it be assumed as:

(*a*) One-fifteenth of that of steel, when the strength of the concrete is taken as 2200 lb. per sq. in. or less.

(*b*) One-twelfth of that of steel, when the strength of the concrete is taken as greater than 2200 lb. per sq. in., or less than 2900 lb. per sq. in., and

(*c*) One-tenth of that of steel, when the strength of the concrete is taken as greater than 2900 lb. per sq. in.

COST OF CONCRETE (Pre-war prices). — The following table * gives approximate costs of concrete in place not including excavation forms, steel or miscellaneous items. The values include superintendence, overhead charges and general expense but not office expense nor profit.

Item	Cost per cubic yard	
	Range	Average
Mass concrete as in dams, piers, foundations, etc.:		
Labor only	$0.75 to $2.50	$1.25
Material and labor	3.00 to 9.00	5.50
Concrete in tunnels and conduits:		
Labor only	1.00 to 3.00	2.00
Material and labor	4.50 to 8.00	6.25
Concrete reservoirs and standpipes:		
Labor only	0.75 to 3.00	1.50
Material and labor	3.50 to 13.00	7.00
Concrete buildings:		
Labor only	0.75 to 4.00	1.50
Material and labor	4.50 to 9.00	6.50
Concrete bridges:		
Labor only	0.50 to 2.50	1.50
Material and labor	4.00 to 8.00	6.00
Concrete sewers:		
Labor only	0.75 to 1.75	1.50
Material and labor	3.50 to 8.00	6.00
Granolithic sidewalks:		
Labor only	1.00 to 3.00	1.75
Material and labor	6.25 to 9.00	7.00
Granolithic sidewalks:		
Labor only	2¢ to 4¢ per sq. ft.	2¾¢ per sq. ft.
Material and labor	5¢ to 14¢ per sq. ft.	10½¢ per sq. ft.

BIBLIOGRAPHY. — Taylor and Thompson, *Concrete, Plain and Reinforced;* Taylor and Thompson, *Concrete Costs; Handbuch für Eisenbetonbau;* Turneaure and Maurer, *Principles of Reinforced Concrete Construction;* Hool, *Reinforced Concrete Construction;* Hool and Johnson, *Concrete Engineers' Handbook.*

* Copied by permission from *Concrete Costs* (1910) by Taylor and Thompson, which should be consulted for detailed information.

CONCRETE, REINFORCED. — (*See also Cement; Concrete; Structures, Simple.*) Reinforced concrete beams, girders and slabs are made of Portland cement concrete with the addition of steel bars or steel mesh to resist tensile stresses. They are usually fixed at the ends by being built into columns, walls, or other girders and are continuous over intermediate supports; they are, therefore, subject to negative bending moments at the ends and at intermediate supports and positive bending moments between supports. It is common to use bottom reinforcement throughout the length and top reinforcement across the supports. Vertical or inclined reinforcement is also usually necessary to carry the diagonal tensile stresses occurring in the web. Fig. 1 shows the general character of a reinforced concrete beam and shows methods of making connections to column and floor beams.

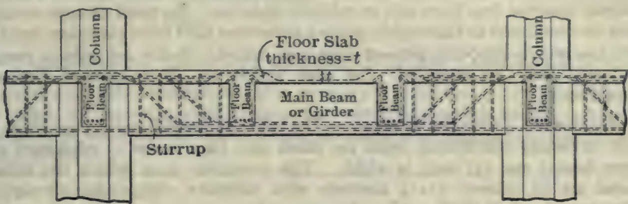

Fig. 1.

Character of Reinforcement. — The steel reinforcement may consist of round or square bars of which the diameter or side seldom exceeds 1½ inches. The square rods are sometimes twisted when cold, this type of bar being known as the Ransome bar. There are also various forms of corrugated or otherwise deformed bars on the market, some of which are shown in Fig. 2. Steel mesh is seldom used for ordinary beams, but is frequently employed in floor slabs. The steel reinforcement should preferably be of the same quality as that used for ordinary steel structures, although the use of a slightly less ductile material may be warranted in some cases where the steel is unlikely to be subjected to injurious shocks.

Fig. 2.

Proportions of Concrete; Forms. — The proportions of concrete for beams and other reinforced concrete structures subjected to flexure is commonly 1:2:4 (*see Concrete*). The forms or molds in which reinforced concrete beams are constructed are usually of wood bolted together, although steel forms are sometimes employed. In general, the size of the beam should be such that the forms can be manufactured from standard size boards. Forms should be watertight, and should be left in place until concrete has set sufficiently to carry the load to which it may be subjected at time of removal of forms.

Fireproofing. — In order to make reinforced concrete beams and columns fireproof the steel reinforcement must be protected by a reasonable amount of concrete. A common provision is that the metal in girder and columns be protected by a minimum of 2 inches of concrete; that the metal in beams be protected by a minimum of 1½ inches of concrete; and that the metal in floor slabs be protected by a minimum of 1 inch of concrete. It is also advisable that all corners shall be beveled or rounded.

Spacing of Reinforcement. — Lateral spacing should not be less than three diameters center to center, with a clear spacing of not less than one inch. The distance from the side of the beam to the center of the nearest bar should be not less than two diameters. More than two layers should not be used unless securely tied together by metal connections particularly at or near points where bars are bent.

Electrolysis and Corrosion in Reinforced Concrete. — The best experimental data available indicate that no danger from electrolysis need be anticipated to the steel reinforcement in carefully mixed and proportioned Portland cement concrete free from salt or calcium chloride. Portland cement concrete may be considered to thoroughly protect embedded steel from corrosion provided the concrete is properly proportioned and mixed. If the concrete is porous, corrosion may be expected. See also article on *Electrolysis of Grounded Structures.*

Reinforced Concrete Beams. — The table on the following page gives the safe uniformly distributed loads per lineal foot for a series of reinforced concrete beams one (1) inch in width. The constants upon which the table is based are as follows: 1:2:4 concrete with ultimate compressive strength of 2000 pounds at 28 days. Allowable stress in concrete = 650 and in steel = 16,000 lb. per sq. in. Modulus of elasticity of steel 15 times that of concrete. Area of steel = 0.77 per cent area of beam above center of reinforcement.

FORMULAS FOR REINFORCED CONCRETE STRUCTURES. — Detail formulas for design cannot be given here; see references in *Bibliography.*

Reinforced Concrete Slabs. — These resemble beams, are much used for floors of buildings, and usually form an integral part of the floor beams and girders as indicated in Fig. 1. They may be computed in the same manner as reinforced concrete beams (*see above*) provided the shears and moments due to the outer forces are known. For slabs supported on four sides the distribution of the load may be determined by the application of the following formula recommended by the Joint Committee (*see Concrete*).

$$r = \frac{l}{b} - 0.5,$$

in which r = proportion of load carried by the transverse reinforcement, l = length, and b = breadth of slab.

Using values above specified, each set of reinforcements is to be calculated in the same manner as slabs having supports on two sides only, but the total amount of reinforcement thus determined may be reduced 25 per cent, by gradually increasing the rod spacing from the third point to the edge of the slab.

If length of the slab exceeds one and five-tenths its width the entire load should be assumed as carried by transverse reinforcement.

Girderless floors or flat slabs are much used for factories and warehouses. In this type of construction the flat slab is supported directly on the columns which are constructed with capital of considerable size. For information regarding the methods of design, reference should be made to the Joint Committee Report, the Building Codes of Chicago and Boston, and the standard text-books.

Columns of Reinforced Concrete. — For such columns it is customary to limit to 15 the ratio of unsupported length to least width, and to consider the effective area as that within the protective coating (*see section on Fireproofing, above*), or for hooped columns or columns reinforced with structural shapes to the area within the hooping or structural shapes. The reinforcement may con-

sist either of longitudinal bars or of longitudinal bars connected by bands, hoops, or spirals, or of rigid structural forms. The following formulas may be used in design. Let A_c = cross-section of concrete, A_s = cross-section of steel, $A = A_c + A_s$ = total cross-section, n = ratio between modulus of elasticity of steel and concrete, usually 15, P = total safe load, f_c = allowable unit stress in concrete, f_s = allowable unit stress in steel, $p = A_s/A$. Then

$$P = f_c(A_c + n\,A_s) = f_cA[1 + (n - 1)p],$$

$$f_c = \frac{P}{A[1 + (n - 1)p]},$$

$$f_s = nf_c.$$

ALLOWABLE LOAD (POUNDS) ON REINFORCED CONCRETE BEAMS

Allowable uniformly distributed load in pounds per lineal foot, in excess of weight of beam, for end-supported rectangular reinforced concrete beams. ***Tabular values to be multiplied by width of beam in inches to get safe applied load** (dead + live + impact).*

Depth of beam, in.	Distance from top to center of reinforcement, in. = *d*	Area of steel, sq. in.	Moment of resistance, foot pounds	Span in feet							
				6	8	10	12	16	20	25	35
5	4.0	0.031	144	27	13	7	3	....	....	....	...
6	5.0	0.038	224	44	22	12	6	....	....	....	...
7	6.0	0.046	323	64	32	18	10	....	....	....	...
8	7.0	0.054	440	89	46	26	15	5	....	....	...
9	7.75	0.060	539	110	57	33	20	7	....	....	...
10	8.75	0.067	687	142	75	44	27	10	....	....	...
11	9.75	0.075	853	177	95	56	35	15	....	....	...
12	10.75	0.083	1,037	217	117	70	45	19	8	....	...
13	11.5	0.089	1,186	249	134	81	52	23	10	....	...
14	12.5	0.096	1,401	296	160	97	63	29	13	3	...
15	13.5	0.104	1,635	347	188	115	75	35	17	5	...
16	14.5	0.112	1,886	402	219	134	88	42	21	7	...
17	15.5	0.119	2,155	460	251	154	102	49	25	10	...
18	16.5	0.127	2,442	523	286	176	117	57	30	12	...
19	17.0	0.131	2,592	555	304	187	124	61	32	13	...
20	18.0	0.139	2,906	624	342	211	141	70	37	16	...
22	20.0	0.154	3,588	773	425	263	175	88	48	22	...
24	22.0	0.169	4,341	938	517	321	215	109	61	30	...
26	24.0	0.185	5,166	1119	618	385	259	133	75	38	6
28	26.0	0.200	6,063	....	728	455	307	159	91	48	10
30	28.0	0.216	7,032	....	847	531	359	187	109	58	14
36	33.5	0.258	10,070	....	1220	767	521	275	162	90	27
42	39.5	0.304	13,990	....	1704	1074	733	392	235	134	47
48	45.5	0.350	18,570	....	....	1435	981	528	320	187	70

COST. — See section on *Costs* in article on *Concrete*.

BIBLIOGRAPHY. — See *Bibliography* in article on *Concrete*.

CONDENSERS, ELECTRIC. — (*See also Capacity and Charging Current.*) Any two conductors separated from each other by a dielectric form an electric condenser. The capacity C of such a condenser is the quotient of the numerical value of charge Q on either conductor by the difference of potential V between the two conductors *when equal and opposite charges are given the two conductors*, i.e., $C = Q/V$. The usual way of giving the conductors equal and opposite charges is to connect them respectively to the two terminals of a battery or other source of electromotive force. Unless the conductors are close to each other relative to the linear dimensions of their surfaces the capacity is small. Large capacities are usually obtained by using flat plates separated from each other by a thin sheet of dielectric. Unless the two conductors are close to each other relative to their distances from other conductors their capacity is influenced by the presence of the other conductors.

Formulas for Capacity. — See the article on *Capacity and Charging Current.*

Condensers in Series and in Multiple. — See *Electricity and Magnetism, Principles of.*

Energy Stored in a Condenser. — See *Electricity and Magnetism, Principles of.*

Electric Absorption and Dielectric Hysteresis in Condensers. — When an ordinary condenser is charged by connecting it to the terminals of a source of constant e.m.f. the amount of charge which it takes depends upon the time during which the e.m.f. is applied, i.e., there is an apparent absorption of charge by the dielectric. Time is also required for the dielectric to give up this charge when the plates are short-circuited. Absorption is particularly pronounced in such heterogeneous substances as glass, paper, ordinary mica, etc. On account of this absorption both the capacity and apparent resistance of a condenser in general depend upon the time of application of the e.m.f., i.e., upon the time of electrification.

Dielectrics, when submitted to an alternating electric field, absorb a certain amount of energy over and above that corresponding to the direct current "ohmic" loss. This loss is made up of two elements, one due to the conversion of displacement current into the leakage current at the boundaries of electrically dissimilar elements of the insulation, and the other due to a property similar to magnetic hysteresis. The former depends upon the frequency but is not proportional thereto; the latter is directly proportional to the frequency.

TYPES OF CONDENSERS AND THEIR APPLICATION. — Condensers may be classified according to the nature of the dielectric used as: (1) air condensers; (2) mica condensers; (3) glass condensers; (4) paper condensers; and (5) electrolytic condensers.

Air Condensers. — Due to the low specific inductive capacity and relatively low dielectric strength of air, air condensers have very low capacity and are therefore seldom used except for standards of small capacity. For the latter purpose they are well suited, since their capacity is not appreciably affected by temperature, time of electrification, or frequency.

Mica Condensers. — These are built of alternate sheets of mica and metal foil, impregnated with paraffine. They are much used for standards of capacity and for telephone, radio and other work requiring condensers which have a small power factor, high insulation resistance and which remain constant. A good working thickness of mica is from 0.0015 to 0.002 inch. A single sheet of high grade mica 3 inches by 4 inches by 0.002 inch will give a capacity of about 0.004 microfarad and will withstand 1000 volts. In order that the insulation shall be

high (product of megohms by microfarads not less than 1000) and the absorption low, only sheets of the finest clear "ruby" mica should be used.

Fig. 1 shows the plan view and connection of a subdivided mica condenser. Since this arrangement permits of series, parallel, or series-parallel connection of the condensers a very large number of values of capacity may be obtained.

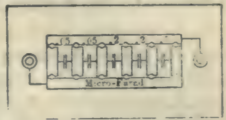

Fig. 1. Subdivided Standard Condenser

The power factor of well-made standard mica condensers ranges from about 0.1 per cent to about 1.75 per cent; see Grover, F. W., *Bulletin of the Bureau of Standards*, Vol. 3, No. 3.

Glass Condensers. — On account of its high dielectric strength and specific inductive capacity glass is particularly well suited for high-tension condensers, such as required for wireless telegraphy and the like. The Leyden jar is a common form of glass condenser. Moscicki's modification of the Leyden jar, shown in Fig. 2, consists of specially formed glass tubes closed at one end and coated inside with silver deposited chemically, the outer coating being sometimes applied in the same way and sometimes dispensed with altogether, the outer electrode in the latter case consisting of a mixture of glycerine and water in which the tubes are immersed.

Fig. 2. Moscicki Condenser

Mordey (*Jour. Inst. Elec. Eng., 1909, Vol. 43, p. 621*) reports a test on a Moscicki condenser consisting of 8 tubes having a total capacity of 0.03 microfarad, designed for 10,000-volt a-c. working. Each tube is 2 inches in diameter and 2 feet 9 inches long, or with connections 3 feet 2 inches long. The power factor of this condenser was approximately constant and equal to 1.0 per cent for frequencies from 40 to 60 cycles per second and for voltages from 5000 to 10,000 volts.

Condensers of this type are used to a considerable extent in Europe as a protective device on high-tension overhead transmission lines, as well as for wireless telegraphy. It has also been suggested by Mordey that they could be used economically for improving the power factor of highly-inductive loads, such as induction motors.

Paper Condensers. — Paper condensers for telephone and telegraph work are made either (1) by building up a stack of alternate sheets of tinfoil (0.0003 inch thick) and tissue paper, two sheets to the layer, each sheet about 0.001 inch thick, or (2) by rolling up alternate strips of foil or "foiled" paper and tissue paper, the roll after being dried and impregnated with paraffin or other wax being pressed into a cubical shape, and suitably mounted in airtight metal boxes. The "foiled" paper is made by depositing very fine flakes of tin on suitable tissue paper which is then run through a press which forces the flakes into contact with each other, resulting in a paper resembling the so-called silver paper used for wrapping tea but having a fairly high conductivity. According to G. F. Mansbridge (*see reference below*) this paper possesses two decided advantages over the ordinary metal foil, (1) it is much lighter, and (2) when punctured it is self-healing, the spark vaporizing the tin in the immediate vicinity of the puncture without destroying the paper itself. For a complete description of the method of making the paper and the condensers see Mansbridge, G. F., *Jour. Inst. Elec. Eng.*, 1908, Vol. 41, p. 535.

Paper condensers for telephone and telegraph work are usually built to withstand about 400 volts. The following weights and dimensions are taken from the British Post Office specifications for metal-cased telephone condensers

One-microfarad paper condensers designed for a working pressure of 1000 volts, and having the dimensions 8¾ by 6¼ by ½ inches and weighing 8 ounces each are made by the Western Electric Co.

DIMENSIONS AND WEIGHT OF PAPER CONDENSERS

British Post Office Specifications

Capacity, microfarads	Length, inches	Width, inches	Thickness, inches	Weight, ounces
0.5	4¾	2⅝	5/16	4
1.0	4¾	4¾	5/16	8
2.0	4¾	4¾	9/16	12
4.0	4¾	4¾	1⅛	20
10.0	4¾	4¾	2 11/16	46

The power factor of well-made paper condensers ranges from about 0.2 to 2 per cent, but unless care is taken in their manufacture the power factor may be considerably larger; values as high as 30 per cent have been obtained.

Paper condensers similar in construction to those used for telephone work are also used for power factor correction on distribution circuits up to 2300 volts. These condensers are usually made up of a number of units in parallel, each unit having a capacity of 5 kv-a. at 60 cycles. See *G. E. Bulletin, No. 49714 B, Feb, 1920*. Up to about 300 kv-a. the cost of a static condenser is less than that of a synchronous condenser (*see Motors, Synchronous*).

Electrolytic Condensers. — Electrolytic cells of the three following types have been used to a limited extent in Germany as condensers for telephone work: (1) acid cells, consisting of two small electrodes of platinum dipping into an acid solution, (2) sodium cells, in which the electrolyte is a solution of common salt, and (3) aluminum cells, in which the electrodes consist of aluminum and the electrolyte is some kind of basic solution. The capacity action of these cells arises from the formation of an extremely thin insulating layer at the anode of the cell which apparently has an enormously high specific inductive capacity. The anode and electrolyte therefore form two plates of a condenser having this thin layer of dielectric between them, forming a condenser of exceptionally large capacity per unit volume. The power factor of this condenser, however, is very high, due to the leakage current from anode to electrolyte, and consequently these cells are little used strictly as condensers, either in telephone or in power work. However, the aluminum cell is very extensively used as a lightning arrester, on power circuits, its capacity action combined with the self-healing property of the film making it an excellent device for this purpose (*see Lightning Protectors*).

TESTING OF CONDENSERS. — The three important properties of a condenser are its capacity, its insulation resistance and its power factor. The insulation resistance is determined by the use of direct current (*see Resistance and Conductance*). The power factor is measured by employing a source of alternating e.m.f. and a Wheatstone bridge and may be looked upon as a measure of the effective a-c. resistance. The power factor of a good mica condenser is practically constant over a wide range of frequency.

A great many arrangements of circuits have been devised for measuring the capacity of condensers, cables, etc. Only one or two of the simple methods

used in engineering practice can be described here. For a description and comparison of the various methods which have been used see Grover, F. W. *Bulletin of the Bureau of Standards*, Vol. 3, No. 3.

D-C. Versus A-C. Capacity Tests. — Due to the effect of electric absorption above described, the charge taken by a condenser depends upon the time of electrification; the discharge also depends on the time. Consequently, when the capacity is measured by any direct-current scheme a standard time of electrification and a standard method of determining the discharge must be chosen. The British Post Office call for the capacity to be measured by taking the instantaneous discharge (by ballistic galvanometer) after the condenser has been charged for a period of 10 seconds. As a rule the capacity measured by a-c. methods is less than that measured by d-c. methods, since the charge does not have time to soak in; for the same reason the a-c. capacity also decreases slightly with increase of frequency. Hence a-c. tests should be made at a standard frequency, preferably that of the circuit on which the condenser or cable is to be used. It is also desirable, when possible, to make the capacity test at the same voltage, both in value and wave form, as is to be used on the condenser.

Voltmeter-ammeter Test of Capacity. — Connect the condenser in series with an a-c. ammeter and impress on this circuit the selected a-c. voltage. Measure the voltage drop across the condenser by means of a high-resistance a-c. voltmeter. Let the current read by the ammeter (voltmeter circuit open) be I, the voltmeter reading V, the frequency f, then the capacity is

$$C = \frac{I}{2\pi f V},$$

provided the voltage is a pure sine wave and the power factor of the condenser is negligible. As neither of these conditions are usually fulfilled, such a test as a rule gives only a rough approximation to the true capacity.

Ballistic Galvanometer Method. — Fig. 3 shows the arrangement of circuits for testing the capacity of a cable. B is a battery giving the desired constant e.m.f., K a highly insulated key, G a ballistic galvanometer properly shunted (*see Galvanometers and Shunts*). The sheath of the cable is usually earthed. It is best to charge the core of the cable positively as shown. If a condenser is to be tested instead of a cable, the plate corresponding to the sheath may or may not be earthed, as desired. Care should be taken that there is no leakage of current directly from the battery into the galvanometer or into the cable.

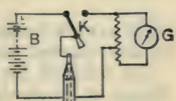

Fig. 3.

The procedure is as follows: Throw the key to the left for the desired length of time, note the position of the galvanometer needle (or spot of light), then throw the key to the right and note the first swing or maximum deflection of the spot of light. Call this D_x. Next, keeping the same battery, substitute for the cable a standard condenser of known capacity C and make a similar observation. Let the deflection in this case be D. Then the unknown capacity is

$$C_x = \frac{D_x}{D} C,$$

provided the damping of the galvanometer is the same in both cases.

While this method is satisfactory when applied to perfect condensers, it fails to a greater or less extent when applied to cables, especially if they are very long, or to any condenser having high absorption. With such condensers the discharge consists of two portions: a sudden rush when the key is first closed.

followed by a current gradually diminishing toward zero, due to the release of the "absorbed" charge. Both portions of the discharge are active in producing the deflection, hence the apparent capacity depends to a certain extent on the period of the galvanometer employed, as well as upon the time of electrification.

Wheatstone Bridge Method. — The following method gives a means of measuring both the capacity and power factor of a condenser, provided there is available a standard condenser whose capacity and power factor is known. The bridge is arranged as shown in Fig. 4. C is the standard condenser and C_x the condenser to be tested. All resistances must be non-inductive and adjustable. A is a source of alternating e.m.f. of pure sine-wave form and of frequency f. T is the detector; it may be either a telephone or, if the frequency be kept perfectly constant, a vibration galvanometer. For ordinary tests the telephone is the more convenient instrument; for good work with it the frequency should be high.

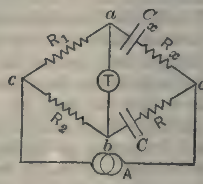

Fig. 4.

The balancing is effected as follows: with R_x and R both zero, adjust R_1 or R_2 until a minimum of sound is obtained; then adjust either R_x or R as the case may require, until the minimum is the best possible; then readjust R_1 or R_2, and so on, thus obtaining the perfect balance by successive adjustments.

$$C_x = \frac{R_2}{R_1} C,$$

$$\tan \phi_x = \tan \phi + 2\pi f\,(RC - R_x C_x),$$

where f is the frequency, ϕ_x is the angle whose *sine* is equal to the power factor of the condenser under test and ϕ is the angle whose *sine* is equal to the power factor of the standard condenser. The effective alternating-current conductance of the condenser under test is then

$$G_s = 2\pi f C_x \tan \phi_x,$$

which as a rule is many times the reciprocal of its insulation resistance as measured by direct-current methods (*see Resistance and Conductance*).

The value of the power factor for the standard is best determined at the Bureau of Standards in Washington; it is done by indirect reference to an air condenser.

Sources of Error. — The sources of error when the most refined measurements are to be made are the inductance or capacity of the various resistances, error in the ratio of R_1 and R_2, and electrostatic induction between the bridge and its surroundings. These sources of error are fully discussed by Grover above referred to.

COSTS OF CONDENSERS (Pre-war prices). — A good standard mica condenser of ⅓ microfarad, guaranteed to stand 300 volts maximum across its terminals costs about $25. An adjustable mica condenser, such as shown in Fig. 1, having a maximum capacity of 1 microfarad and suitable for voltages up to 300 volts costs about $60. Paper condensers, such as used on telephone circuits, cost about 70 cents per microfarad. A standard 1000 volt, 1 microfarad paper condenser can be had for about $3.75.

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Inst. Elec. Eng., 1908, Vol. 41, p. 535; Mordey, W. M., *Some Tests and Uses of Condensers*, Jour. Inst. Elec. Eng., 1909, Vol. 43, p. 618; Budd, A. D., *Tubular Electric Condensers* (Paper), Elec. W., 1910, Vol. 55, p. 748; Morse and Shuddemagen, *Properties of the Aluminum Electrode*, Proc. Amer. Acad. Sc., 1909, Vol. 47, p. 367; Brunet, P., *Capacities and Condensers*, Rev. Gen. d'El. 8, p. 237, Aug. 21, 1920; Taylor, W. B., *Static Condensers*, G. E. Review, 21, p. 567, 1918.

CONDENSERS, STEAM.— (*See also Cooling Systems for Power Stations; Power Stations; Pumps; Steam Engines; Turbines, Steam.*) The primary object of a condenser is to reduce the back pressure in the exhaust of a steam-engine or steam turbine, although in cases where the supply of suitable feed water is limited the recovery of the condensed steam is of equal importance. Theoretically the gain in the output of a given engine for the same steam input is proportional to the reduction in back pressure, but practically the gain is usually much less than this, depending upon the type of engine and conditions of operation.

CLASSIFICATION OF CONDENSERS. — Condensers are of two general types, jet condensers and surface condensers.

Jet Condensers. — In a jet condenser cooling water and the exhaust steam mingle together in a closed chamber, the water condensing the steam by direct contact. The cooling or injection water on entering the condenser is broken up into a fine spray or is spread out into a thin sheet. Jet condensers may be classified as follows:

Standard or Ordinary Jet Condensers, in which the cooling water, condensed steam and air are exhausted by a pump.

Barometric Condensers (Siphon Condensers), in which the cooling water and condensed steam are exhausted by a barometric column, the air being exhausted with the water and condensed steam or by means of pump. In certain types of barometric condensers the condensing water is forced into the condenser by atmospheric pressure (if the lift of the condensing water is not over 15 feet), and the condenser is then called a "siphon" condenser.

Ejector Condensers, in which the condensed steam and air are exhausted directly to the atmosphere by the momentum acquired by the cooling water and condensed steam as they pass through the condenser.

Rotary Condenser. — The best known form of this type of condenser is the Leblanc Condenser (made by the Westinghouse Machine Co.), which accomplishes the separate removal of water and air by means of a pair of relatively small turbine-type rotors on a common shaft in a single casing, which is integral with or attached directly to the lower portion of the condensing chamber. The condensing chamber itself is but little more than an enlargement of the exhaust pipe. The injection water is projected downwards through a spray nozzle, and the combined injection water and condensed steam flow downward to a centrifugal discharge pump under a head of 2 or 3 feet, which insures the filling of the pump. The space above the water level in the condensing chamber is occupied by water vapor plus the air which entered with the injection water and with the exhaust steam. This space communicates with the air-pump through a relatively small pipe.

The air-pump differs from pumps of the ejector type in that the vanes in traversing the discharge nozzle at high speed constitute a series of pistons, each one of which forces ahead of it a small pocket of air, the high velocity of which effectually prevents its return to the condenser. A small quantity of water is supplied to the suction side of the air-pump to assist in the performance of its functions. The power required for the pumps is said to approximate 2 to 3 per cent of the power generated by the main engine.

Surface Condensers. — Ordinary surface condensers consist of nests of small brass or copper tubes usually ¾ inch or 1 inch in diameter through which cooling water is forced and which are surrounded by an air-tight shell to which the exhaust steam is admitted. There may be one, two or more sets of tubes through which the steam passes in succession, the condenser being referred to

as a "single-flow," "double-flow" or "multi-flow" respectively. Steam is admitted at the top of the shell and the condensed steam drawn off at the bottom. Water enters the lower tubes and is discharged at the top, securing thus the advantages of the counter-flow principle. The rate of heat transmission depends largely upon the state of the tubes. Transmission is much retarded by a water film coating the exterior of the tubes and by interior coatings of scale, dirt or corrosion. In the so-called "dry-tube" type baffles are arranged to catch the drip from each set of pipes, drain it off at the side and so prevent it from falling on the pipes below.

Evaporative and Air-cooled Surface Condensers are sometimes employed when condensing water is scarce. Brief descriptions and references will be found in Gebhardt's *Steam Power Plant Engineering.*

CLASSIFICATION OF CONDENSER PUMPS. — (*See also Pumps and Pumping Engines.*) The following names are usually applied to the pumps used in connection with a condenser.

Injection or Circulating Pump, the pump used for injecting the cooling water into a condenser.

Wet-air Pump, the pump used for exhausting the air, condensed steam and hot water from a condenser when these are all exhausted together.

Dry-air Pump, the pump used for exhausting the air and water vapor only. A dry-air pump must be used when high vacua are required.

Hot-well Pump. — This name is applied to the wet-air pump when this pump exhausts to a hot-well. The hot-well is a well provided to hold the condensed steam, which is approximately at the same temperature as that of the exhaust steam in the condenser. If this hot water is to be used over again for boiler feed it is pumped from the hot-well to the boiler by the boiler feed pump (*see Boilers, Steam*).

CHOICE OF TYPE OF CONDENSER. — The chief advantage of the surface condenser over the jet condenser is that the condensed steam does not mingle with the cooling water and therefore the condensed steam may be used over again for boiler feed water, provided any oil which may be carried by the exhaust steam from the cylinders is eliminated before the steam reaches the condenser (*see Separators, Steam*). When there is plenty of cooling water readily handled a higher vacuum is usually obtained by means of a surface condenser than by means of a jet condenser. To offset these advantages, however, the surface condenser is much more expensive than a jet condenser, and requires as a rule a greater amount of cooling water.

Consequently the use of a surface condenser is in general justified only when the cost of obtaining suitable feed water is relatively high (*see also Feed-water Heaters and Purifiers*), and cooling water, unsuitable for feed water however, relatively cheap (*see also Cooling Towers, Ponds, etc*). Before finally selecting either type of condenser an estimate of the total annual cost, including interest, depreciation, maintenance and all operating charges should be made for each case.

According to Prof. Wickenden the following vacua, based on a 30-inch barometer, have been found most economical in general practice.

Piston engines	26 to 26.5 inches
Turbines at 20 per cent load-factor or less	27 to 27.5 inches
Reaction turbines at high load-factors	28 to 29.5 inches
Impulse turbines at high load-factors	28.5 to 29 inches

In summer months warm cooling water often renders these vacua impracticable. When cooling towers are required to conserve the water supply the economic

limits of vacuum are lower, due to the extra investment and the relatively warm state of the water. Jet condensers are cheaper to install and operate for vacua up to 27 inches. They require less water and give somewhat higher hot-well temperatures if closely regulated. For higher vacua the load on the dry-air pump becomes excessive, due to the air entrained with the cooling water. Surface condensers are then preferable and for extreme vacua they are indispensable.

DEGREE OF VACUUM AND BACK PRESSURE. — Let

V = degree of vacuum, i.e., the reading of vacuum gauge, in inches of mercury column,
B = reading of barometer.

Then the back pressure in pounds is

$$p = 0.491\,(B - V).$$

The degree of vacuum is usually referred to a barometric pressure of 30 inches. Calling V_0 the vacuum in inches of mercury corresponding to a barometric pressure of 30 inches, and B and V the observed barometric pressure and gauge reading respectively, then

$$V_0 = V + (30 - B).$$

(If V_0 is the vacuum in inches referred to 760 millimeters or 29.91 inches of mercury, substitute for 30 the number 29.91.)

Vacua from 20 to 29 inches are used in practice, corresponding to back pressures of from 4.5 to 0.5 pounds. The higher the vacuum the more cooling water required and the greater the cost of the air and circulating pumps.

CONDENSING WATER REQUIRED. — Let

T = temperature, in ° F., of dry saturated steam at the pressure corresponding to the desired degree of vacuum,
T_i = temperature, in ° F., of the injection water,
T_s = temperature, in ° F., of the condensed steam at bottom of condenser,
T_w = temperature, in ° F., of the discharge water (for a jet condenser $T_w = T_s$),
H = total heat (above 32° F.) of the exhaust steam, in B.t.u. per pound.

Then the weight of condensing water required per pound of steam is

$$W = \frac{H - T_s + 32}{T_w - T_i}.$$

In applying this formula the values of T_s and T_w must be determined and an allowance must be made for the regulation of the supply of condensing water.

	Jet condensers		Surface condensers	
	Parallel-current	Counter-current	Single and double flow	Multi-flow*
	Deg. F.	Deg. F.	Deg. F.	Deg. F.
T_s less than T by....................	10 to 15	5 to 10	5 to 10	0 to 5
T_w less than T by....................	10 to 15	5 to 10	10 to 20	0 to 10

* Cases have been reported where T_s and T_w are both *higher* than T by a few degrees. (*Proc. Inst. Nav. Arch., March., 1906.*) [Probably due to errors of gauges.]

Values of T_s and T_w. — Air is always present in exhaust steam and in the case of jet condensers a certain amount of air also enters the condenser with the cooling water. The effect of the air is to reduce the temperature of the steam (T_s) below that corresponding to saturated dry steam (T) at the same pressure. The amount of air present depends upon the type of condenser and also upon the amount of air in the cooling water in the case of a jet condenser.

The relations between T_s, T_w and T, as shown in the table on page 256, are found to hold in practice.

Allowance for Regulation of Injection Water. — It is usual to take for H in the above formula the value corresponding to dry saturated steam at the given pressure in the condenser (*see tables in article on Steam*), and then to increase the value of W so obtained by from 5 to 15 per cent to allow for imperfect regulation of the injection water and for the more or less unknown state of the steam as it enters the condenser.

Example. — A vacuum of 25.85 inches, referred to 29.92-inch barometer, is to be maintained in a surface condenser, with injection water at 60° F. The value of T from the steam tables is 126° F. and H = 1115 B.t.u. Take T_s = 120°, T_w = 115°, T_i = 70°; then

$$W = \frac{H - T_s + 32}{T_w - T_i} = \frac{1115 - 120 + 32}{115 - 70} = 22.8.$$

DIMENSIONS OF CONDENSERS. — There is so great a difference in the design of the various forms of jet condensers that it is impossible to give any average dimensions. See the catalogues of the makers; representative makes of the various types are listed above.

In the case of surface condensers the tube surface exposed to the steam varies inversely as the coefficient of heat transfer (i.e., the number of B.t.u. per hour per ° F. difference in temperature between the steam and water per square foot of cooling surface) and directly as the mean temperature difference between the water and the steam. Let

T_i = temperature, in ° F., of the injection water,
T_s = temperature, in ° F., of the exhaust steam at condenser pressure,
T_w = temperature, in ° F., of the discharge water,
Q = pounds of cooling water required per hour,
U = coefficient of heat transfer.

Then, according to Josse (*Power, Feb. 2, 1909*), the required cooling surface in square feet is

$$S = \frac{2.3\,Q}{U} \log_{10} \frac{T_s - T_i}{T_s - T_w}.$$

The value of U depends upon the metal used for the tubes, the condition of the external surface of the tubes and especially upon the condition of the internal surface of the tubes (scale and corrosion decreasing the value of U very markedly). and upon the velocity of the water, being greater the higher this velocity. U may be taken as 250 for water velocities of from 25 to 50 feet per minute and 375 for velocities of from 50 to 75 feet per minute. There is, however, considerable difference of opinion regarding the proper formula for S and the proper value to assign to U. (*See Hoefer, K., A.S.M.E., J. 41, p 962, Dec. 1919*).

In reciprocating engine plants an allowance of 2 square feet of tube surface per indicated horse-power is customary. In early steam-turbine installations an area of from 2 to 4 square feet per kilowatt was commonly employed. but subsequent improvements in condensers and the water rates of turbines have tended to reduce these areas. In the most modern plants

of large capacity condenser surfaces range from 1.2 to 2.5 square feet per kilowatt.

From the relations implied in the above formulas it is seen that the surface may be reduced by forcing large quantities of water at high velocities to pass the tubes; or, with a given surface, the vacuum may be heightened by increasing the flow of cooling water. However, the gain may be more than offset by the cost of pumping this water.

SIZE OF CONDENSER PUMPS. — (*See also articles on Blowers and Compressors; Pumps and Pumping Engines.*) The size of the injection or circulating pump required can be calculated directly from the quantity of injection water to be handled and the head against which it is to be pumped, including of course the friction head. Separate injection pumps for jet condensers are not usually required, as the head does not as a rule exceed that corresponding to the difference between atmospheric and condenser pressure. In the case of surface condensers the intake and discharge tunnels are usually at about the same level, and consequently the head is largely friction head. The friction head of a condenser is not readily calculated, but may be obtained from the manufacturer.

Wet-air Pumps. — The predetermination of the size of wet-air pumps is difficult, owing to the uncertainty in estimating the quantity of water vapor and air which they must handle. In the case of jet condensers the total weight of water (including the vapor) and air is equal to the combined weight of the exhaust steam and cooling water, whereas for a surface condenser only the condensed steam and air in it is handled by the wet-air pump. Hence the wet-air pump for a surface condenser is usually much smaller than for a jet condenser.

Formulas, partly analytical and partly empirical, for calculating the piston displacement of wet-air pumps will be found in Gebhardt's *Steam Power Plant Engineering*. For average practice the volume capacity or piston displacement is equal to the volume of *cooling water* multiplied by the following factors.

For jet condensers, single-acting pump	3
For jet condensers, double-acting pump	3.5

Piston speeds are usually about 50 feet per minute at full load.

For surface condensers the volume capacity is equal to the volume of the *condensed steam* multiplied by the following factors.

For reciprocating engines	10
For steam turbines	20

These figures are based on a study of some 200 installations (Gebhardt, 1910 ed.).

Dry-air Pump. — The required volume capacity is found in practice to be equal to the volume of the *condensed steam* multiplied by the following factors:

For vacua under 27 inches	20 to 30
For vacua 28 inches or over	50

In both cases the barometer is assumed as 30 inches. These figures are given by Gebhardt and are based on an investigation of some 50 installations.

POWER REQUIRED FOR CONDENSER PUMPS. — The power required for the circulating pump is calculated in the ordinary way from the weight of the water delivered and the head, including the friction head (*see article on Pumps and Pumping Engines*).

The power required for the wet-air and dry-air pumps can be accurately determined only when the proportion of air in the exhaust steam and also in the cooling water, in the case of a surface condenser, is known. Surface water under atmospheric pressure contains from 2 to 12 per cent, by volume, of air. To allow for leakage, a liberal factor of 20 per cent may be taken.

Power Required to Exhaust the Air.—Let

p_a = atmospheric pressure, pounds per square inch,
p_c = pressure in condenser, pounds per square inch,
T_i = initial temperature of injection water, in ° F.,
T_f = initial temperature of feed water, in ° F.,
T_c = temperature in condenser in ° F. (approximately the temperature of the exhaust steam),
W_w = weight of cooling water in pounds per *minute*,
W_s = weight of condensed steam in pounds per *minute*.

Then for a jet condenser the volume of air in cubic feet per minute at condenser pressure to be handled by the pump is

$$V_1 = \frac{(W_w + W_s)\, p_a\, (T_c + 460)}{310\, p_c\, (T_i + 460)},$$

and for a surface condenser

$$V_1 = \frac{W_s p_a\, (T_c + 460)}{310\, p_c\, (T_f + 460)},$$

assuming in both cases that there is 20 per cent air by volume in the water at atmospheric pressure. This percentage is probably high when the feed water is taken from a hot-well, since the heating of the water drives out the air. Also, the temperature T_c for a counter-current jet condenser and separate air pump is more nearly the temperature of the injection water than that of the steam.

The horse-power required to exhaust the air is then

$$P_a = \frac{p_c V_1}{66.2\, \epsilon} \left\{ \left(\frac{p_a}{p_c} \right)^{0.29} - 1 \right\},$$

where ϵ is the mechanical efficiency of the pump, which may range from 30 to 60 per cent. (*See article on Blowers and Compressors.*)

Power Required to Exhaust the Condensed Steam and Water.—The power required to exhaust the water and condensed steam from a jet condenser may be calculated in the same manner as for an ordinary pump (*see article on Pumps and Pumping Engines*), using the proper value of the head under which the pump operates. This will depend upon the arrangement of the condenser, location of the pumps and hot-well, etc.

If a wet-air pump only is used, then the total horse-power of this pump will be $P_a + P_w$, where P_w is the horse-power required to remove the condensed steam and water. If a dry-air pump is used in connection with the wet-air pump (which then exhausts the water only) the horse-power of the dry-air pump is P_a, and the horse-power of the wet-air pump P_w.

Per Cent of Total Available Energy Used by Condenser Pumps.—When steam-driven pumps are employed and the exhaust from these pumps is not utilized the steam consumption of the pumps is properly taken as a measure of the energy required for their operation: this may range from 5 to 20 per cent of the total steam generated by the plant. When the exhaust from the steam cylinders of the pumps is utilized for heating the feed water (*see Feed-water Heaters*), the *heat consumption* of the pumps is properly taken as a measure of the energy required for their operation; this may range from about 1 to 5 per cent of the total B.t.u. utilized by the main engines, for a large part of the heat in the steam used in the pumps is returned to the boiler via the feed water. When electrically-driven pumps are employed, the kilowatt-hour input to the motors driving the pumps is properly taken as a measure of the energy required for operating the pumps; this may range from about 2 to 8 per cent of the output of the main generators.

LOCATION AND ARRANGEMENT OF CONDENSERS. — Where there is only one engine installed the condenser is usually placed close to and just below the engine, so that all condensation may gravitate into it. In some large vertical steam turbine outfits the condenser (surface type) is at the base of the turbine and forms an integral part of the structure. When several engines or turbines are installed in a power house a separate condensing outfit may be used for each engine, or the so-called "central system" may be employed in which one condensing outfit serves several engines. This arrangement is particularly well adapted to plants in which the individual units are subjected to extreme variations in load, as in rolling mills.

Additional data on the location and arrangement of condensers will be found in the article on *Power Stations*.

COST OF CONDENSING EQUIPMENT (Pre-war figures). — An approximate formula that is sometimes used for determining the cost of jet condensers is,

$$\text{Cost in dollars} = 500 + 1.0 \times \text{h.p.},$$

Potter (*Power*, Dec. 30, 1913) gives:

Type	Capacity	Cost, dollars
Barometric (28-in. vac.)....	Up to 30,000 lb. steam per hour	1055 + 0.112 × (lb. steam cond. per hr.)
Jet (28-in. vac.)...........	Up to 30,000 lb. steam per hour	1176 + 0.1138 × (lb. steam cond. per hr.)
Surface (28-in. vac.).......	Up to 35,000 lb. steam per hour	1630 + 0.2038 × (lb. steam cond. per hr.)
Surface (26-in. vac.).......	Up to 30,000 lb. steam per hour	413 + 0.1015 × (lb. steam cond. per hr.)

Twenty-eight inch vacuum surface condensers with pumps cost from $1 to $1.80 per square foot of surface, depending on price of copper, or from $150 to $250 per 1,000 pounds of steam condensed per hour.

J. R. Bibbins (*Power, Jan, 1905*) gives a curve showing the relative cost of high-vacuum surface condensers compared with 26-inch-vacuum surface condensers. The following figures are taken from this curve.

Degree of vacuum, inches...........	26	26.5	27	27.5	28	28.5
Relative cost, per cent..............	100	111	123	137	158	205

Maintenance of Condensers. — The cost of maintenance of a surface condenser is subject to wide variations, depending upon the corrosion and deterioration of the condenser tubes, the exact cause of which is not often understood. With clean, fresh water, free from acid, the tubes of a condenser last indefinitely, but where the cooling water contains sulphur, as in drainage from coal mines, or sea water contaminated by sewage, such as harbor water, the deterioration is exceedingly rapid.

The maintenance of a jet condenser is much less affected by impurities in the water.

BIBLIOGRAPHY. — Gebhardt, *Steam Power Plant Engineering*, contains 62 pages on condensers and a full bibliography of the subject.

CONDUITS AND CONDUIT LINES, UNDERGROUND. — (*See also Distribution Lines; Transmission Lines; Wires and Cables, Insulated; Wiring of Buildings.*) The following is a brief table of contents of this article:

Underground cables are now almost universally installed in conduits made either of glazed tilc, wood or paper fiber. The pipes or conduits are laid so as to form a series of continuous ducts for a length of not over 400 feet, and are terminated in brick or concrete chambers, from which the cables are pulled into the ducts.

TERMINOLOGY. — While there is some confusion in the terminology of conduit lines, the best recent literature, except the spceifications of the American Electric Railway Engineering Association, sanctions the following definitions.

Conduit, a pipe or tube designed or used for containing electric wires or cables.

Duct, a passage or opening, designed or used for accommodating electric wires or cables.

Conduit Line, a group of installed conduits. (The expression *subway* is largely used for conduit line, but will not be used herein, as it is desired to avoid such expressions as "the subway subway," meaning the conduit lines of an underground railway.)

Splicing Chamber, a chamber built to give access to the ducts of a conduit line; frequently called a manhole. Properly, a manhole is the opening giving access to the splicing chamber.

Manhole, an opening giving access to an underground splicing chamber, from the surface of the ground; frequently, but improperly, used to designate the entire splicing chamber.

Service Box, a part of a splicing chamber with facilities for connecting distributing conductors to the mains. Service boxes are usually made of cast iron and set on the roof of the splicing chamber like a manhole casting.

USE OF CONDUIT LINES. — Conduit lines are used for the distribution of electrical energy wherever the unsightliness, danger or instability of pole lines prohibits the use of the latter. It is therefore in large cities that they have found their principal application. They are used for the transmission and distribution cables of lighting systems, power plants and railways, and for telephone and telegraph lines.

Since the invention of the Pupin loading-coil, the use of conduit lines for telephones has received considerable impetus. Underground telephone lines now extend from Boston to Washington, a distance of nearly 500 miles, creosoted pump-log being used for the conduit the greater part of the way. The most notable installation of conduit for trunk-line railroad electrification is that of

the New York Central which comprises over 1,600,000 duct feet, partly in tile conduit and partly in iron pipe.

Advisability of Double-conduit Line. — A conduit line of a large number of ducts should be avoided wherever possible. While a large line may be permissible for telephone work, it certainly is not desirable for light or power work. The entire output of a station or substation of considerable size should not be carried out through one conduit line, but should be divided between two or more lines kept well separated.

TYPES OF CONDUITS. — While tile conduit is used far more than any other kind, there are four other kinds in fairly extensive use; namely, fiber conduit, wrought-iron pipe, cement-lined iron and pump-log.

Tile Conduit. — Fig. 1 shows the tile conduits used in the Electric Zone of the N. Y. C. & H. R. R. R. near New York and Fig. 2, the standard of the

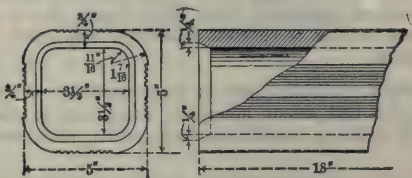


Fig. 1.

A.E.R.E.A. They represent what is probably the best practice up to date. The conduit shown in Fig. 1 weighs 16½ pounds per length of 18 inches. Four-

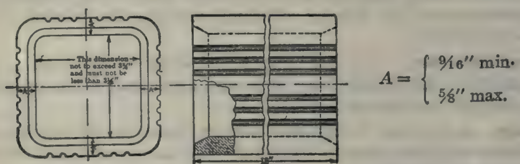


Fig. 2.

duct conduit is now comparatively little used; it weighs about 100 pounds per length of 3 feet.

Fiber Conduit consists of tubes, made by rolling paper saturated with asphalt or bituminous compound around a mandrel. Like iron pipe, which it resembles in appearance, lengths of fiber conduit are usually joined by screw and coupling although it is also made for socket, sleeve and drive joints. It is lighter and easier to handle than iron pipe with which it compares favorably in cost, but is more expensive than tile conduit. It is mechanically inferior to other types of conduit, and difficulty is often experienced in drawing heavy cable around bends without breaking the conduit. For some classes of work this disadvantage is entirely compensated for by superior gas and water tightness.

The dimensions of fiber conduits are given as follows by the Johns-Manville Company and substantially the same by the Fiber Conduit Company.

Socket joint				Sleeve joint			
Inside diameter, inches	Thickness of walls, inches	Approximate average weight per foot, pounds	Length of section, inches	Inside diameter, inches	Thickness of walls, inches	Approximate average weight per foot, pounds	Length of section, inches
1	¼	0.45	30	1	¼	0.45	30
1½	¼	0.75	60	1½	¼	0.75	60
2	¼	0.90	60	2	¼	0.90	60
2½	¼	1.05	60	2½	¼	1.05	60
3	¼	1.30	60	3	¼	1.30	60
3½	¼	1.60	60	3½	7/16	2.50	60
4	¼	1.85	60	4	½	3.20	60
Screw joint				Drive joint			
1½	5/16	0.85	60	2	¼	0.90	60
2	⅜	1.35	60	2½	¼	1.05	60
2½	⅜	1.70	60	3	¼	1.30	60
3	7/16	2.20	60	3½	¼	1.60	60
3½	7/16	2.50	60	4	¼	1.85	60
4	½	3.20	60				

Wrought-iron Pipe is used in city streets where the conduit line has to twist about sub-surface obstructions. It is more expensive than tile conduit and does not last as long on account of rusting. The usual sizes are 3-inch and 3½-inch pipe, 20 feet long, provided with threaded ends and couplings. (For weights and dimensions see article on *Pipes*.)

Cement-lined Iron Pipe consists of thin sheet-iron cylinders, like stovepipe, with a lining of hydraulic cement. When the conduit is old the iron usually rusts away without detriment to the construction.

Pump-log Conduit is used only on telephone lines, as its inflammability renders it dangerous for power work. It consists of wooden blocks with the

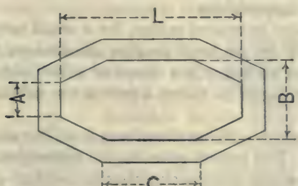


Fig. 3.

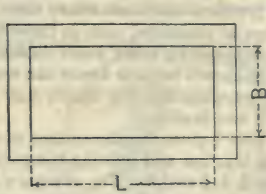


Fig. 4.

duct hole drilled out. A description of a typical pump-log installation is given below under Installation.

SPLICING CHAMBERS. — Splicing chambers, for straight runs, are usually built in the shapes shown in Figs. 3, 4, 5 and 6. That shown in Fig. 3 is usually regarded as ideal, as the excavation is a minimum, and the cables can

follow the walls very closely without being bent to a dangerously small radius, a frequent occurrence in chambers with square corners. The rectangular form shown in Fig. 4 is somewhat more common, but requires more excavation and more wall material than the first form, and is conducive to sharp bends in the

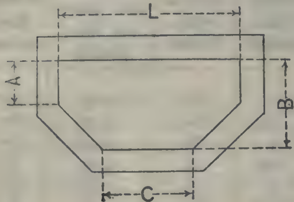


Fig. 5.

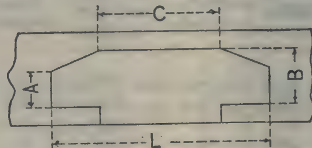


Fig. 6.

cable. The type shown in Fig. 5 is convenient where it is desired to have a wide chamber without occupying much space on one particular side of the duct line as, for example, where two duct lines run parallel and a short distance apart. In this case, economy may be obtained by the use of a common wall on the long side of the chamber.

Fig. 6 shows the side-wall type of chamber, which is entered from the side 'nstead of the top. This form is used in the side walls of tunnels or in retaining walls, and usually takes the form of a long niche in the wall. When built in tunnel walls not far from the surface of the ground, it is desirable to have them open from the street above as well as from the side, in order to avoid the necessity of having the use of a tunnel track when work is to be done in the chambers.

Material of Splicing Chambers. — Monolithic concrete is the cheapest and best material to use where a large number of identical chambers have to be built, as the same form can be used over and over again. Where such uniformity cannot be attained, as in streets congested with sub-surface construction, brick or concrete hollow tile are better and cheaper materials.

Dimensions of Splicing Chambers. — The height of large splicing chambers is usually determined by the height in which a man can stand upright and is seldom less than 6½ feet. The width is similarly influenced by the space required to work in, which is about 4 feet. The length depends upon the length of splice and the space required to curve the cable from the ducts to the supporting shelves or racks, considerations which make a length of 8 feet the practical minimum where there are large cables.

Special chambers, such as those for bare grounded cables, for a small number of telephone or other small cables, may be made of smaller dimensions than stated above, because a man can work on his knees in a chamber 4 feet high and 3½ feet wide, and the length may be reduced to about 4 feet if the cables are small and flexible.

Table I gives the dimensions of standard splicing chambers used by several large traction and lighting companies. The letters refer to the dimensions on Figs. 3, 4, 5 and 6. The average volume of a splicing chamber is about 16 cubic feet per splice, although a maximum of 40 and a minimum of 6 are sometimes found.

Spacing of Splicing Chambers. — It is found that the greatest length of cable that can be pulled through a glazed tile conduit without injuring the cable or requiring special apparatus is about 400 or 500 feet on a straight or slightly curved run. Hence at distances of 400 or 500 feet along a conduit line it is necessary to provide splicing chambers, where the separate lengths of cable may

TABLE I. — DIMENSIONS OF SPLICING CHAMBERS

Name of company	Fig. No.	A	B	C	Height inside	L	No. of splices
		Feet and inches					
Chicago Edison Co.......	3	2′ 3″	6′ 0″	3′ 3″	6′ 6″	7′ 6″	16
District Ry., London....	5	2 4	5 0	5 6	7 6	14 3	32
I. R. T. Subway, N. Y...	6	1 3	3 0	4 4	14 0	11 4	64
Long Island R.R.........	5	1 6	4 0	2 3	6 6	10 0	18
Long Island R.R.........	4	......	4 0	....	6 6	9 0	18
Long Island R.R.........	3	1 8	4 0	3 8	6 6	9 0	18
Manhattan Ry., N. Y. ..	4	......	6 0	....	6 6	8 0	16–48
New York Cent. R.R....	3	2 4	5 0	3 6	6 6	11 0	20
New York Cent. R.R....	5	2 6	5 0	3 6	6 6	11 0	20
New York Cent. R.R....	5	1 10	4 0	3 6	6 0	10 0	5
New York Cent. R.R....	6	1 10	3 0	3 6	6′ 6″–8′ 0″	8 6	20–32
Pennsylvania R.R.......	4	......	4 0	....	6 4	8 0	24
Philadelphia R.T........	5	1 8	3 6	curved	6 3	7 10	20
P. R.R. Tunnels.........	(Special shape)				4 9	6 0	

be pulled in and spliced. Local conditions and curves may prevent the full length being attained in all cases, but it is desirable to have no lengths greater than the standard, in order to avoid the necessity of keeping special long pieces of cable in stock.

Service Boxes. — Service boxes are usually iron castings similar in shape and size to an ordinary manhole casting, but provided with a completely waterproof inner cover which screws down on a gasket. The outer cover resembles that of a manhole. Inside the inner cover is a distribution board with copper bus bars, to which the main and feeder cables in the chamber connect through the bottom of the board.

Waterproofing and Drainage. — There is some diversity of opinion with regard to the value of waterproofing splicing chambers, although the modern tendency is to omit waterproofing and provide efficient drainage. Waterproofing is futile unless the duct lines leading to it are waterproofed. This is a very difficult and expensive process which seldom shows good results. Water will also enter chambers from the top and will be retained if the chamber is waterproof.

Every chamber should be provided with a sump into which the water can drain, Fig. 7. It is desirable to connect the sump to the sewer through a syphon and back-water valve. If this is not practicable the sump may be drained through the manhole by a hand pump.

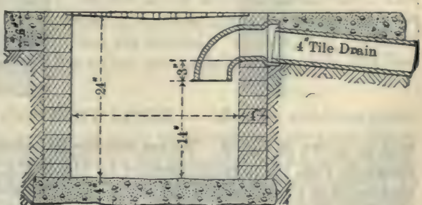


Fig. 7.

Where natural drainage cannot be secured, as, for example, where the chamber is below the sewer level, it is good practice to provide a special drain pipe to which all the chambers are connected, the drainage being towards a general sump pit, which is kept

dry by an automatic sump pump. The cost of such a system often compares favorably with that of waterproofing, besides yielding superior results.

The sump of a splicing chamber should be covered with an easily removable grating; wooden ones are often used on account of their property of floating when the chamber is flooded, thereby leaving the sump open for the pipe of a hand pump.

Manhole Castings. — Manhole castings are usually made with two covers (Fig. 8), the outer of which should be made strong enough to stand the weight of the heaviest vehicle, and the inner should be as light as consistent with the rough usage they receive in handling. Both covers should be provided with means to grip them with a hook-bar. Where power cables are used it is essential to provide ventilating holes. It is usual to lock the inner cover with a substantial brass padlock with protected keyhole.

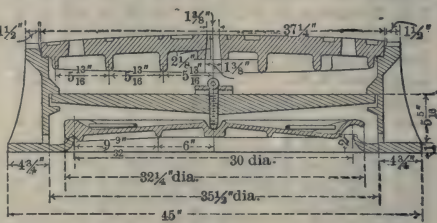


Fig. 8.

Cable Supports. — (*See also article on Wires and Cables, Insulated.*) Cables in splicing chambers are usually supported on iron brackets attached to the chamber walls. This construction is very satisfactory, especially if the power cables are properly wrapped with asbestos, but some engineers consider a shelf to be preferable, as it affords insulation and provides more protection between cables.

Concrete reinforced with expanded metal or wire cloth has proved satisfactory for shelves one inch thick, and is inexpensive. A good support for shelves in concrete splicing chambers is a pair of iron pins set in iron pipes, which are sunk in the chamber wall at a slight angle, so as to tilt the shelves toward the chamber

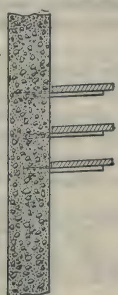


Fig. 9.

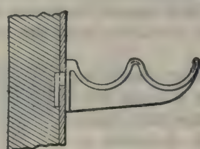


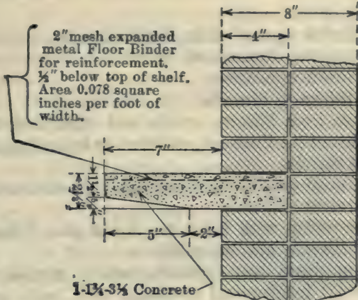
Fig. 10.

wall as shown in Fig. 9. Where splicing chambers are made of brick, the design shown in Fig. 11, is recommended by the A.E.R.E.A. Splicing chambers with shelves have to be made larger and more expensive than those with duplex racks (Fig. 10), if an equal number of cables has to be carried.

RODDING AND WIRING OF CONDUIT LINES. — The only test which it is usual to apply to conduit lines is rodding with a mandrel in order to ascertain whether the ducts are continuous and unobstructed.

The rods used for this purpose are of hickory about one inch in diameter

and three or four feet long and are fitted at the ends with steel couplings such as that shown in Fig. 12. The first rod is attached to a mandrel and pushed into the duct. Another rod is coupled to the first and the pair pushed further



Length of st'd shelf-5 Ft.

Fig. 11.

into the duct. By successively coupling other rods and pushing them into the duct, the mandrel is made to travel from one chamber to another. As soon as the mandrel emerges into the receiving chamber, the rods are pulled through and uncoupled. If an obstruction stops the mandrel an attempt is made to force it through by repeated blows, failing which, it becomes necessary to cut into the conduit line from the side.

Mandrels for Rodding. — Various types of mandrels are used for testing ducts, some hollow and smooth with sharp cutting edges, others fitted with numerous sharp projections and known as hedgehogs (Fig. 13).

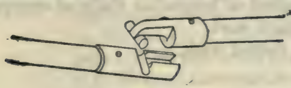


Fig. 12.

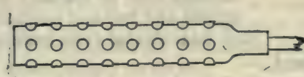


Fig. 13.

"Wiring" the Ducts. — It is usual to attach a galvanized steel wire (No. 10 or 12) to the last rod and leave the wire in the duct after the removal of the rods. The ducts may also be "wired" by the use of a conduit machine. This consists of a reel of steel tape and means for winding and unwinding the tape into the duct at the rate of about ten feet per second. This tape or "snake" having been pushed through the duct is pulled out again with the wire attached to its end.

SPECIFICATIONS FOR CONDUIT LINES. — (*See also article on Specifications.*) By far the greatest number of cable breakdowns occur from injury to the lead sheaths either by electrolysis or by abrasions during installation. Sharp projections in the ducts should therefore be carefully guarded against, as a cable pulled over such a projection will have a groove cut along its entire length and the effective thickness of the sheath will be thereby materially reduced. The specifications for inspection and rodding given below are typical of the best modern practice, but unfortunately duct inspection is usually so lax that often in a large installation scarcely a single duct conforms in every particular with the specifications. Engineers are now beginning to realize the practical importance of giving each duct a rigid inspection, and rejecting those which have any

roughness or irregularity. Scarcely less important is rigid inspection during installation, in order to assure the most perfect alignment.

Details of Construction. — The following items should be covered:

Conduits. —

Single-way, or four-way.

Holes circular or square with rounded corners.

Maximum outside dimensions.

Minimum inside dimensions.

Minimum thickness of walls.

Shall be free from blisters, cracks and other imperfections which, in the opinion of the Engineer, will tend to injure the cables to be accommodated therein.

Shall be of good quality tile, thoroughly glazed inside and outside.

Shall be straight and true.

Shall be provided with holes for dowel pins if ducts are four-way. Dowell pins may also be called for.

The sides of single duct conduits shall be combed with two (2) sets of three (3) longitudinal combings each, each combing to have a width of one-quarter (¼) inch and a depth of one-sixteenth (1/16) inch. Multiple-duct conduits shall be scored transversely near the ends.

Conduits shall be of stated length. It is usual to call for a certain percentage of shorter lengths (generally 1 per cent), in order to finish runs or stagger joints. The permissible variation from the specified length should be stated.

(If fiber conduit is to be used, substitute the corresponding requirements.)

Cement. —

Shall be of approved brand.

Briquettes made of neat cement and kept one day in air and six days in water shall show a stated tensile strength. A corresponding strength may be similarly required of briquettes made of one part cement and two and one-half parts sand.

Shall be protected from moisture during work.

Sand. —

Shall be clean and free from loam or salt.

Shall be sharp and of stated coarseness.

Stone. —

Shall be of crushed granite, lime stone, trap rock or other approved variety.

Shall pass through a sieve of stated mesh.

Brick. —

Shall be of good commercial hard-burned sewer brick, or other stated variety.

Concrete. —

Stated proportions of cement, sand and stone. (Usually 1 part of cement, 2 parts of sand and 4 parts of stone.)

Maximum size of aggregate. (Usually ¾ inch.)

Sand and cement shall be mixed dry and wetted with only sufficient water to make a stiff paste. The stone having been previously wetted shall be added while wet and thoroughly mixed until all the stones are covered with mortar. It shall then be deposited as rapidly as possible. Machine-mixed concrete will be accepted if made in a manner approved by the Engineer.

Cement Mortar. —

Stated proportions of cement and sand. (Usually 2½ to 1 of cement.)
Concrete mortar shall not be laid in freezing weather, and shall not be used after initial set has taken place.

Excavation. —

Shall always be of such depth as to leave a stated minimum distance between the top of the concrete over the conduits and the surface of the ground.

Ground on which conduits are laid shall be rammed solid before any concrete is laid.

Refilling Excavations. —

The best part of the material excavated shall be used.

Surplus material shall be carted away by the contractor (or will be carted away by the company).

Filling shall be thoroughly tamped and rolled, or flushed, as seems necessary to the Engineer, and shall be done in a manner to prevent, as far as possible, a settling of the earth after completion.

Obstructions. —

Obstructions encountered in the course of the work shall be overcome in a manner to be approved by the Engineer.

Laying Conduits. —

Shall be laid with ends square so as to leave a tight, well-fitting butt joint.

Joints shall be staggered horizontally and vertically.

Conduits shall be laid in a bed of cement mortar of about ¼-inch thickness.

Each joint shall be wrapped with two strips of burlap 6 inches wide and coated with neat cement mortar, the ends of the wrap to lap 4 inches. (It will insure more careful work if it be specified that the contractor shall supply rubber gloves to the men who lay the burlap.) Where conduits are laid on curves, the wraps shall be doubled if required to protect the openings between the ends of the conduits on the outside of the curve, and to exclude mortar from said openings. (This method of wrapping is not universal. If another method is to be used, corresponding details should be given.)

Conduits shall be laid with a mandrel of specified length and width and provided at the end with a rubber washer for wiping the joints.

Conduits shall be laid on a bed of concrete of stated depth, shall be covered at the top with a stated depth of concrete, and shall have a stated thickness of concrete on each side. Where the conduit line goes under railroad tracks, the concrete shall be suitably thickened and reinforced.

If conduits are four-way, they shall be laid with dowell pins at joints.

The alignment horizontally and vertically shall be satisfactory to the Engineer.

Drainage of Conduit Lines.— The grade of all conduit lines shall be such that water cannot stand in the ducts but shall drain into one or both splicing chambers.

Repairing. — All repairing shall be done in a manner satisfactory to the Engineer and the municipal authorities.

Extras. — No extras will be allowed the contractor for irregular work except

Rock blasting, if necessary.

Removal of pipes or other obstructions, if necessary.

Such work shall not be done without the consent of the Engineer. The contractor shall specify the cost of such work before it is begun.

Details of Terminals of Conduit Lines. — (Whether they go into power stations, etc.)

Splicing Chambers. — Shall be built according to plans supplied, unless local conditions interfere, in which case the suggested modifications shall be approved by the Engineer.

Chambers to be not more than a stated distance apart.

State any details pertaining to the design of the chamber and cable supports which may not be clear from the plans.

Tests. — The contractor shall notify the Engineer sufficiently in advance of the completion of the conduits to enable inspection at the factory to be arranged for.

Rodding, Cleaning and Wiring. — After the conduits are laid and the cement is sufficiently set, they shall be rodded and the contractor shall draw after such rods, wire brushes and a mandrel of specified dimensions. All mortar and other foreign matter shall be removed. If obstructions are found which cannot be removed by cleaners so as to pass the specified mandrel, the ducts shall be removed and relaid. Any expense incurred by such work shall be borne by the contractor. A galvanized wire of stated size shall be left in each conduit from splicing chamber to splicing chamber, and sufficient length shall be left at each end to permit it to be bent in order to prevent it from slipping into the duct.

Inspection. — During the process of construction, the Engineer reserves the right to inspect and the right to reject any and all parts which are not strictly in accordance with this specification.

INSTALLATION OF CONDUIT LINES. — The problems connected with the installation of conduits vary greatly with the local conditions. They involve the choice of trench excavation, drainage, removing or avoiding obstructions, concrete mixing and so on. The following details are gleaned from first-class examples of the various kinds of work described but should in no sense be regarded as standard.

Cross-country Conduit Lines. — The excavation for the conduit line of the American Telegraph and Telephone Company, between New York and Washington, was made partly by a trenching machine and partly by a trench plow. The trenching machine was of the Austin "caterpillar" type and dug a trench 18 inches wide and 3 feet deep at the rate of 3 feet of clean trench for each minute of actual working time. An engineer and two assistants were required to operate the machine, replacing, it is estimated, 50 laborers. Other sections were excavated by a trench plow drawn by two mules or horses, the furrow being made a few inches deeper at each pass, and the loose earth removed by shovelers. The conduit used in this installation was "pump-log" made of southern yellow pine 4½ inches square and 7 feet long with a circular duct 3 inches in diameter. These conduits were creosoted by the pressure process, with 15 pounds dead oil of coal tar per cubic foot of wood. Splicing chambers having concrete-block walls were built every 500 feet.

A wheel type trenching machine made by Pawling & Hornischfeger Co. has been successfully used by the Wisconsin Telephone Co. This machine has a cutting wheel 7½ feet in diameter, digging a trench 15, 18 or 21 inches wide and with a maximum depth of 5½ feet. The machine weighs 9 tons and is operated at a speed of 3½ feet per minute, by an operator and one helper.

Conduit Lines for Railroads are usually difficult to construct and operate for the following reasons:

(1) Owing to the right-of-way being usually on made ground, excessive quantities of concrete and reinforcement are required to make a reasonably strong duct construction.

(2) Owing to the width of the right-of-way being usually very restricted it is necessary to shore-up tracks in order to excavate close to them.

(3) Owing to the vibration caused by heavy trains, it is necessary to bury the conduits at a greater depth, first to avoid undue stress on the conduit, and second to avoid crystallization of the cable sheaths.

In order to have the conduits below the frost level, the depth of ballast must be neglected, as it has been found that with stone ballast on top of the ground, the frost penetrates the ground about as far as if the ballast were not there, unless the ballast is very dirty.

(4) There is considerable difficulty in obtaining best results from labor where there are continual interruptions from trains. On a busy section a duct construction gang engaged for ten hours can possibly work two full hours.

(5) Owing to the right-of-way being often quite low, in many cases alongside of rivers, duct construction is likely to be seriously impeded by the flooding of trenches.

(6) Where the right-of-way shows signs of settlement, as, for example, on marshy ground, continuous piling is necessary to support the ducts. This involves the use of the track for construction purposes for long periods, and thereby not only impedes, but also endangers traffic.

(7) Duct-line construction generally involves interference with signal and interlocking apparatus, thereby introducing danger and expense.

(8) Bridge abutments, bridges, culverts, and in fact all special right-of-way construction, present complicated problems which can be solved only at great expense.

Conduit Lines in Cities. — The obstructions due to sewer, water and gas pipes, car tracks and foreign conduit lines, also render conduit construction in big cities a complicated problem. Plans made in the office can seldom be followed in the field, as the municipal pipe plans are seldom reliable, and the supervision of an experienced civil engineer is needed to solve the numerous problems which constantly arise. Excavation is almost invariably performed by hand labor, and when the conduits have been laid and covered with concrete, it is usual to lay a plank over them in order that future excavators may not drive picks into them. Obstructions are often avoided by changing the grouping of the conduits.

Laying Conduits. — Conduits must be laid so that joints are mechanically strong and the ducts unobstructed. With this in view joints should be staggered horizontally and vertically, and each joint covered with a wrapping impervious to mortar in bulk. Formerly the wrapping was omitted and the conduits joined with stiff mortar, but this became too expensive when the labor unions insisted upon the employment of masons for this work.

Single-duct Conduits are laid with their joints wrapped in a lightweight fabric saturated with thin cement mortar. In the Pennsylvania Tunnels, canvas weighing 10 ounces per yard was used, and cut into six-inch strips. Cheese cloth doubled has also met with favor. When laying conduits it is necessary to have a long mandrel (Fig. 14) to remove loose cement from the ducts. This mandrel is usually provided with a rubber washer at the rear, and a hook-eye at the front end. The conduit layer is provided with a hook rod by means of which he draws the mandrel after him as he lays the conduit.

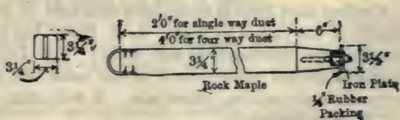
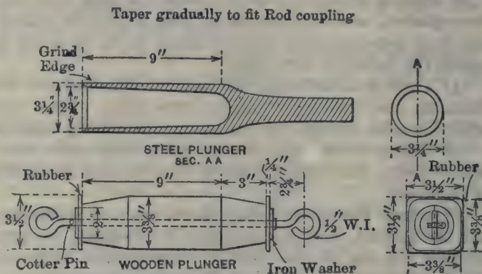


Fig. 14.

Four-duct Conduits are usually provided with holes for dowel pins by means of which the ducts are aligned. The joints are wrapped in burlap soaked in asphalt and afterwards painted with asphalt. No mandrel is used in this type of construction.

Concrete Covering. — However the conduits may be laid, the complete group is always inclosed in concrete to secure rigidity and protection, and the finished ducts are cleared of rubbish and obstructions by pushing a steel plunger through them by means of the rods described above. A steel plunger for this purpose is shown in Fig. 15, in which is also shown a wooden plunger with rubber washers, which should be drawn through the ducts to collect the loose particles left behind the steel plunger.



Steel Plunger, Steel Wire Flue Brush and Wooden Plunger to be passed through ducts in order named.

Fig. 15.

MAINTENANCE OF CONDUIT LINES. — The principal items of conduit-line maintenance are those relating to keeping the line clean and safe, namely, pumping out water, blowing out gas, removing mud, opening and closing manholes for the benefit of cable workers and inspecting the line to guard against theft and injury. Large systems have usually one or more wagons equipped with apparatus required for these purposes, and have men ready to go out with it upon emergency calls.

Removal of Water is usually the most important of operating troubles, especially where no drainage system is installed. When cable accidents occur, it is important to have the chambers accessible without delay and a portable pump is required. For this purpose a small gasoline or electric pump is useful, having a capacity of about 50 gallons per minute. Such a pump which has given good service is of the horizontal centrifugal type with horizontal discharge and costs about $200 complete with electric motor, hand priming pump and starting rheostat.

Ventilation does not occur naturally in conduit lines, because the cold air contained in them has no tendency to rise. Noxious gases therefore tend to accumulate in splicing chambers, endangering workers and making explosions possible. No permanent system of ventilation has proved successful, as it is found that pressure is maintained only at or near the blowing points. When it is necessary to blow out the chambers, it is therefore usual to employ a portable blower in conjunction with an air-tight false manhole cover.

Cooling duct lines is sometimes desirable in order to increase the carrying capacity of the cables in them. L. E. Imlay has accomplished this at Niagara by keeping the surrounding soil moist. The Duquesne Light Co. of Pittsburg installed several blowers each of 600 cubic feet per min. capacity and forced air into alternate splicing chambers. A reduction of temperature from 125° F. to 96° F. was obtained.

Protection from theft of cable is secured (1) by locking the inner manhole cover and having the lock combination periodically changed; (2) by patrolling

the line and (3) by using cable differing in some way from that used by neighboring companies, so that it can be easily identified if stolen.

Electrolysis is more fully treated in the article on *Electrolysis*. It should be noted that the prevention of electrolytic corrosion of cable sheaths depends more upon efficient drainage than anything else.

Maintenance of Conduit Lines Along Railroads present the following peculiar operating difficulties.

(1) Owing to the right-of-way being often on made ground, duct lines settle and crack, injuring the cables in them and preventing the removal and replacement of injured cables.

(2) Owing to the great depth of splicing chambers necessitated by railroad conditions, they are often full of water and cannot be cleared for repairs without pumping water out of as much of the system as is at the same level, a process which may take many hours to complete, possibly interfering with traffic during that time. Drainage is usually out of the question, owing to the absence of any kind of a drainage system below the surface system.

(3) The great depth of chambers requires the use of narrow chimneys connecting the chambers to the surface of the ground. This makes it almost impossible for employees to escape from chambers in case of trouble.

(4) Where improvements are made involving the raising of the right-of-way, as, for example, in eliminating grade crossings, ducts laid previous to the improvements become so deep that they are practically inaccessible for repairs and splicing chambers are correspondingly dangerous on account of their distance from the surface.

(5) The existence of water in low splicing chambers renders the cables particularly liable to electrolytic corrosion. This is a very serious matter where the grounded return is only a few feet away, as is almost invariably the case on a railroad. Electrolytic trouble cannot always be reduced by grounding the cable sheaths to the track rails, as such connections are seldom permissible where electric signals are used.

REPAIRS. — The principal repairs to conduit lines are those due to settlement and to damage done by adjacent building operations, such as the construction of sewers or railway tracks. It is sometimes desirable to replace the conduit line without disturbing the cables they contain. In such cases, the conduits are broken, the utmost care being taken to avoid injuring the cables. New conduits are then relaid on a firm foundation, after having been split longitudinally so as to fit over the cables. The whole construction is then rendered rigid by being inclosed in concrete.

COSTS. — The costs of conduit lines published from time to time are of merely local value, the labor of preparing a dry trench free from obstructions being an item whose variations are so great as to render insignificant the items of constant cost. Costs as low as 20c, and as high as $3.00 per duct foot, are recorded.

The items to be considered in making an estimate of the cost of a conduit line are the following, the quantities all being per foot of trench.

Labor: excavating trench; excavating for splicing chambers; boxing and bracing; removing obstacles, such as gas and water pipes; mixing and placing concrete; placing conduit; carting and dumping; repaving; superintendence, outside; office expense.

Material: conduit; manhole and service boxes; covers, shelves, etc.; sand, stone and cement; brick for splicing chambers (unless made of concrete); lumber for protecting top of ducts; lumber for bracing; incidentals.

Depreciation. — (*See also article on Depreciation.*) Conduit lines depreciate

very slowly from the effects of deterioration or obsolescence. There are few conduit lines in existence which have lost much of their value from age unless unforeseen circumstances have deprived them of some useful association upon which their value depended. The value of conduit lines often increases with age especially where they have been laid in growing districts, where the demand for duct space increases more rapidly than the supply.

BIBLIOGRAPHY. — American Electric Railway Engineering Association. *Specification for Electrical Conduit Construction* A.E.R.E.A. Handbook, 1919; Meyer E. B., *Underground Transmission and Distribution for Electric Light and Power*, N. Y. 1916. Reports of Underground Systems Committee of N.E.L.A.

CONTROL SYSTEMS FOR RAILWAY MOTORS. — (*See also Controllers; Motors; Railways, Electric Traction Systems for.*) The function of the control equipment is to regulate the speed and direction of the motors by certain definite systematic changes in connections. The speed of direct-current railway motors is controlled in two ways: (1) by connecting suitable resistances in series with the motors, which will reduce the voltage across the motors and thereby the current which they will take; (2) by changing the connection of the motors so that they will be connected at first in series, thereby applying half of the line voltage to each motor, and then in parallel across full line voltage. In most control equipments a combination of both methods is used. For the speed control of alternating-current railway motors an auto-transformer, or compensator, is used instead of the resistances.

The direction of rotation of the motors, d-c. or a-c., is changed by changing the direction of the current in either the fields or the armatures; it is customary to connect the terminals of each field coil to a reversing switch in order to accomplish this effect.

TERMINOLOGY. — The following terms are in general use.

Cylinder or Drum Control or Direct Control are names commonly applied to an equipment in which all the connections are made by contacts on a cylinder or drum which is manually operated by the motorman and located on the platform. This may, therefore, be called direct control.

Multiple-unit, Indirect, Remote Control or Train Control are names applied to an equipment in which the changes in connection of the main power circuit are made by switches called "contactors," usually located underneath the floor of the car, and controlled by electric circuits coming from a small master controller on the platform. There are two systems of multiple-unit control in use in this country, viz.,

Sprague-General Electric System. — In this system the contactors are closed by electromagnets which force a plunger against a spring, the latter normally holding the switch open.

Westinghouse "Unit Switch" System. — In this system the contactors are closed by compressed air, from the air-brake cylinders, the air valves at the switches being controlled electrically from the master controller.

Hand Control is a term applied to that method of control in which the motorman has it in his power to regulate the current to any value he pleases by moving the controller handle, the change in connections depending only upon the motion of the latter.

Automatic Control, as distinguished from hand control, is a type of control in which certain automatic devices prevent the motorman from causing the motors to take a current greater than a predetermined value. With this method of control the motors start with a definite current and as soon as the current has decreased to a specified value a change in the connections is automatically made. Thus the rate of acceleration and the current are kept practically uniform throughout the period of control. It is nearly always used in connection with multiple-unit control.

Rheostatic Control consists in connecting a resistance in series with the motor and short-circuiting consecutively parts of this resistance. It is seldom used at present, except on mining locomotives and for single motor operation.

Series-parallel Control, which is used on practically all railway equipments, includes the feature of connecting two motors and their resistances in series on the first step, then short-circuiting portions of the resistance consecu-

tively until all resistance is cut out, under which condition the motors will operate efficiently at approximately half speed. On the next step of the controller the two motors with resistance in series are connected in parallel and subjected to full line voltage. There are three methods of accomplishing the change from series to parallel.

Transition with Power Off. — In the so-called type L controller power is entirely cut off from both motors while the change in connection is being made. This was formerly used for large-size motors and locomotives but is not at present much used.

Transition with Series Resistance. — During the transition from series to parallel a resistance is placed in series with one motor and the other motor is first short-circuited, then disconnected from the main circuit, and finally, placed in parallel with the other motor. This method is in general use in equipments of small motors with the so-called type K controller.

Bridge Transition. — The so-called "bridge" method consists in grouping the motors and their resistances like the arms of a Wheatstone bridge, so that after the two motors are in full-series position the resistances may be placed in circuit again in parallel with the motors, without opening the circuit; the two motors are then connected in parallel with each other and each in series with its own resistance. This method is preferable to either of the other two in that both motors are in operation throughout the whole control period. There is no noticeable jerk and it is not necessary to open the circuit, which would cause flashing at the switches. It is used in certain forms of the K control and in most of the multiple-unit control equipments, particularly for motors of large capacities and for locomotives.

Series-parallel Control with Four-motor Equipment. — Whereas the three methods just described apply particularly to two-motor equipments, they are equally applicable to four-motor equipments by connecting two motors permanently in parallel and treating them as a unit.

TYPE K CONTROL. — The type of control as well as the construction of the controllers for ordinary single-car equipments has been practically standardized in this country, the large manufacturers supplying control equipments which are practically identical. This type of control is known as the type K. The various sizes of type K controllers are listed in the table below.

Where type K controllers are used for motors of large capacity they are sometimes adapted with a modification of the remote control by the addition of two electrically-operated main switches placed underneath the floor of the car, the function of which is to open the main power circuit every time it is necessary that it should be opened and thus remove all flashing and arcing from the controller. This expedient makes it possible to use a smaller controller for a given capacity of motors and obviates all danger to the passengers from fire and fright. The scheme is accomplished by substituting for the main power circuit on the controller an auxiliary circuit carrying only one or two amperes and every time this auxiliary circuit is opened in the main controller the main switches underneath the car open the power circuit. When the auxiliary circuit is closed the main switches close the main circuit. By means of an overload trip operated by a coil in the main circuit these switches are also used as circuit breakers and if the current taken by the car exceeds a certain value a relay opens the auxiliary circuit which in turn causes the main switch to open.

Capacity and Weight of Type K Controllers. — The more usual forms of type K controllers and the capacity in motors for which they are adapted are as follows:

TYPE K CONTROLLERS

Designation	Number of motors	Total h.p.	Number of points	Weight, pounds*
K-10-A	2	80	5 series 4 parallel	940
K-10-H	2	80	5 series 4 parallel	
K-11-A	2	120	5 series 4 parallel	1020
K-11-H	2	120	5 series 4 parallel	
K-12-A	4	120	5 series 4 parallel	1175
K-12-D	4	120	5 series 4 parallel	
K-28-B	4	160	5 series 5 parallel	1350
K-28-F	4	160	5 series 5 parallel	
K-34-D	2 or 4	360	6 series 4 parallel	2250
K-34-F	2 or 4	360	6 series 4 parallel	
K-35-G	2 or 4	240	5 series 3 parallel	1800
K-35-M	2 or 4	240	5 series 3 parallel	

* Weight includes cables, etc., but not motors.

The 10 H, 11 H, 12 D, 28 F, 34 F and 35 M have auxiliary contactors for opening the main circuit.

The K 34 and K 35 have "bridge" transition. All others short-circuit one motor during transition.

The letter "B" in the designation of a controller indicates that it has contacts added to it to make it possible to operate electric brakes by causing the motors to act as generators to energize electric brake shoes, either of the axle or rim type.

Method of Operation of Type K Control. — The principle of the type K control for small and moderate-size motors is shown in Fig. 1. The controller has an operating handle which moves the main cylinder and thereby changes the connections, and also a reversing handle which moves the reversing cylinder. The latter merely changes the direction of the current through the fields of all the motors with respect to the armature. These two handles or cylinders are interlocked so that the reversing handle can only be moved when the operating handle is in the off-position, thus preventing reversal with voltage on the motors.

One terminal of one of the motors is grounded throughout. The first three points are known as accelerating steps. As resistance is in circuit for each of these points the controller should not be left on any of them for a considerable

length of time, for there is a considerable power loss in the rheostats and they are not designed for continuous operation. The fourth step, full series, is an efficient "running point," giving about half normal speed. The next two steps are transition steps and are not marked as points on the controller as they must be passed over rapidly. During this period one terminal of the second motor is grounded, thus short-circuiting the motor which has one terminal grounded initially; the connection between the two motors is then opened; finally the two motors are connected in parallel but in series with a part of the rheostat. Points 5, 6, and 7 are accelerating steps with motors in parallel and resistance in series and are therefore not to be used continuously. At the last point the motors are in parallel, and all resistance is cut out; it is therefore an efficient high-speed running point. The two terminals of the armatures of each motor are led to fingers on the reversing cylinder as are also one terminal of each field and the two points of the main circuit to which the motors are connected. Thus, in reversing, the current is reversed in the armatures, but not in the fields.

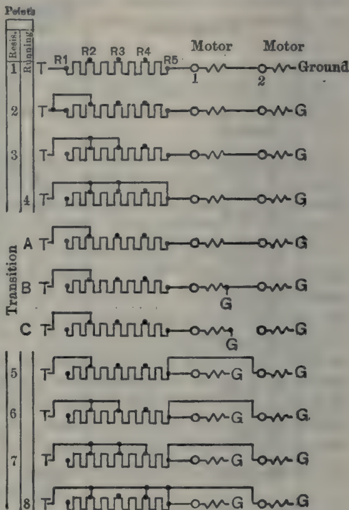


Fig. 1. Type K Control

In all controllers a magnetic blow-out and an "arc chute" are employed to interrupt the current quickly and direct it away from the contacts, in order to prevent short-circuiting other contacts. In the older types of controllers this was obtained from one large magnet coil and a large iron pole piece covering all the contacts. In the later forms each contact has an independent blow-out coil and small pole pieces. This gives a more powerful effect and more accurately directs the arc in the proper direction.

EFFICIENCY OF TWO- AND FOUR-MOTOR CONTROLLERS. — See *Railways, Energy Requirements for.*

REASONS FOR MULTIPLE-UNIT CONTROL. — When the total capacity of the motors on a car or locomotive exceeds 300 horse-power it is advisable, and when the capacity exceeds 400 horse-power it is necessary, to use the indirect or multiple-unit control, for the cylinder type of controllers required to handle the large currents become too bulky and dangerous to place on the platforms of passenger cars. The cylinder control is also inadequate when it is desired to control simultaneously the motors on the several cars in a train, which is necessary in order to obtain the high tractive effort necessary in high-speed service on elevated and underground railways.

For these two reasons the system of multiple-unit or train control was developed. As originally proposed by Sprague this consisted of a large cylinder controller on each car and each controller was actuated by a small motor instead of by hand. These small motors were controlled synchronously from a single point by means of auxiliary control circuits. With the growth in the capacity

of the motors this system became inadequate and was replaced by the systems now in use.

SPRAGUE-GENERAL ELECTRIC CONTROL. — There are two types in use, the type MK and the type MA. The chief difference in these two types is that the type MK is non-automatic whereas the type MA is provided

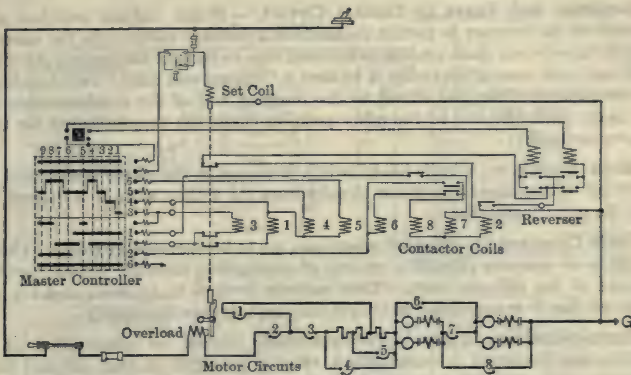


Fig. 2. Sprague-General Electric Type MK Control

with a current-limiting relay. Fig. 2 is a diagram of the type MK, showing the control circuits in light lines and the motor circuits in heavy lines.

The material included in the control equipment of a motor car consists of:

2 master controllers;
1 motor controller containing 8 or 10 contactors and a reverser;
3 master control switches;
1 main switch;
1 main fuse box;
1 set of rheostats;
Cables, Train Couplers.

Master Controller. — This is very similar to an ordinary railway drum-type controller but is much smaller, as the current carried by it is small. Each master controller is equipped with an operating handle, reversing handle, individual magnetic blow-outs and (optionally) a "deadman's handle," which automatically interrupts the current and applies the brakes when the motorman's hand is removed from the button located in the top of the handle.

Motor Controller. — This consists of an iron box lined with asbestos in which the several contactors and the reverser are placed, and is mounted under the car.

Contactors. — Each contactor consists of a powerful magnet operating an arm by means of a toggle joint against a spring pressure. This arm closes and opens the circuit in a strong magnetic field which acts as a blow-out. As one contactor can carry and break currents of several thousand amperes no extra circuit breakers are required. Each contactor is provided with interlocks in the form of relays so that contactor No. 2 cannot be operated until after No. 1 is closed, thus providing the proper sequence of operation under all conditions.

Reverser. — This is a switch with several circuits and contacts and is comparable to the reverser cylinder in an ordinary controller. These contacts are mounted on a rocker-arm actuated by two electromagnets, one for moving the switch to the forward and the other to the reverse position. The electromagnets receive their current from the master controller and are interlocked so that only one can be operated at a time.

Switches and Fuses in Control Circuit. — Motor cut-out switches are located on the reverser to permit cutting out a disabled motor. In the control circuit there is one main control switch and fuse to protect the control circuits, and near each master controller is located a "control and reset" switch, which in one position closes the circuit to the resetting coil of the overload relay in the main controller and in the other position closes the supply circuit for the master controller.

Main Switch. — A knife-blade switch is placed in the power circuit and is intended to disconnect the motor circuits from the trolley when it is desired to test the motor controller.

Train Couplers are provided where cars are to be operated in trains. These couplers or jumpers provide a means of connecting together similar control circuits of the different cars. They contain from 8 to 12 wires and are so designed that it is impossible to couple the cars together improperly.

Current-limiting Relay. — For automatic control, or acceleration at a predetermined current, a current-limit relay is provided on each car and this prevents each successive contactor from operating until the current in the motors has decreased to a predetermined value. On roads having a fairly level profile and operating with frequent stops this refinement is desirable, as it makes it possible for the motorman to accelerate the train every time at the maximum allowable rate and yet never exceed that rate except on a down grade. When this relay is provided the control equipment is known commercially as the type MA control.

WESTINGHOUSE "UNIT SWITCH" SYSTEM. — The Westinghouse multiple-unit control is known as "Unit Switch Control" and is designed either for the operation of several motor cars in a train or of single cars or locomotives, using either large currents or high voltage. There are four types of unit-switch control classified as follows:

Type H. L. — Hand-operated (non-automatic), using line voltage for the control circuits and having as many points on the controller dial as there are steps in the operation; also has a seperate reverser handle. One motor or group is short-circuited during transition from series to parallel.

Type A. L. — Automatically operated, using line voltage for the control circuits. The controller dial shows three points forward and three reverse, corresponding to switching, series running and parallel running, although there are more steps in the operation. Uses bridge transition from series to parallel.

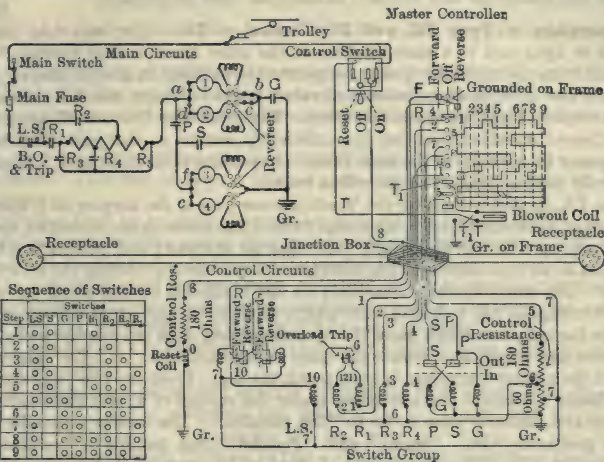
Type H. B. — Hand-operated, using current from a storage battery for the control circuits. Otherwise similar to the H. L.

Type A. B. — Automatically operated, using current from a storage battery for the control circuits. Otherwise similar to the A. L.

Operation of Type H. L. Control. — As typical of all of these classes the diagram in Fig. 3 shows the connections of the H. L. type for a four-motor equipment for either a car or a locomotive (*Cole, Electric Journal, Oct. 1912*).

As many as five motor cars per train may be operated by one man with this control system.

The main-circuit connections are made by means of a number of independent pneumatically-operated switches, known as unit switches, each provided with a strong magnetic blow-out and normally held open by a powerful spring. The overload trip that controls the opening of all switches in case of overload



Step	LS	S	G	P	R_1	R_2	R_3	R_4
1	o	o			o			
2	o	o				o		
3	o	o				o	o	
4	o	o				o		o
5	o	o			o	o	o	
	o	o	o			o	o	
6	o		o	o		o	o	
7	o		o	o		o		o
8	o		o	o	o	o		o
9	o		o	o	o	o	o	

Fig. 3. Westinghouse, Type H. L. Unit Switch Control

or short-circuit is actuated by the magnetic pull produced by one of the blow-out coils, which is in the main circuit. When a predetermined current value is exceeded, a plunger carrying two contact discs breaks the control circuit, which causes certain switches to open in the line and switch group. This trip can only be reset when the controller is in the off-position.

A pneumatically-operated reverser controls the direction of operation of the car. This reverser consists of two pistons similar to those attached to the unit switches, which serve to move the reverser drum to forward or reverse position. The air-brake system furnishes compressed air for operating the reverser and switches.

A multi-conductor train line extends the length of each car and is tapped off at the master controllers and at the unit switch group to form the circuits needed. This train line is made continuous throughout a number of cars by multi-conductor jumpers between cars fitted into receptacles on each car. The current for the electromagnets which operate the valves of the unit switches is taken from low-voltage taps on a resistance connected across line potential.

Operation of Type A. L. Control. — The A. L. control differs from the hand-operated control in that the acceleration of the train is automatic. The master controller has only three positions, switching, series running and parallel running. The "off" position is in the center. The unit switches are provided with interlocks which are electrically connected with the valve magnet in such a manner that the closing of one switch energizes the magnet of the next, thus producing automatic progression of the switches, under the direction of the limit switch. This limit switch controls the rate at which the resistance

is cut out of circuit so as to give uniform accelerating current. The limit switch consists of a solenoid operated by the current of one motor or a pair of motors. When this current exceeds a specified limit for which the switch is adjusted, the circuit is opened through a pair of contacts in the operating circuit. The circuit remains open so that no more unit switches can close until the accelerating current falls below the predetermined limit, when the control circuit is again closed and allows the unit switches to continue their progression.

Operation of Type PK and PC Control. — This type was first introduced in 1914 and is a simplification of the older Multiple Unit Systems. It uses air from the air brake system to operate the switches in groups thus requiring only two or three air cylinders and valves instead of many which is the cause of its simplicity and improved reliability. The two forms PK and PC are made by different manufacturers but are sufficiently similar so that cars with both forms may be operated in the same train. On account of its cheapness, it is quite generally used for single cars as well as trains.

The PK (manufactured by the Westinghouse Co.) is the simpler as it merely consists of a standard form K controller (which see) whose drum is rotated, notch by notch, by a pair of electrically controlled air cylinders and pistons to which is added an air operated main-line switch and a cylinder for rotating the reversing drum. The small Master Controllers on the platforms control small currents at low voltage in 7 or 9 control wires which admit or exhaust air at definite intervals to the cylinders which move the reversing drum, open or close the main-line switches and advance or return the main control cylinder. All the apparatus except the master controllers is underneath the car. From start to "full series" the change is gradual and automatic under the control of a "current-limiting" device, and similarly from series to "full parallel." The master controller has three positions "Off," "Series" and "Parallel" thus making it possible for equipments with different numbers of steps to be operated in the same train. It is possible to adapt old "K" controllers by merely adding the "PK Head" and main line switch to one of the old controllers and placing it underneath the car.

The "PC" Control (manufactured by the General Electric Co.) uses a similar combination of electrically controlled pneumatic cylinders to rotate, notch by notch, a main cam shaft and the cams on this shaft open and close spring actuated switches or "Contactors." These cam-operated switches cut out the starting resistance and change the motor connections from series to parallel. Cams also operate the main line switch while other cylinders actuate the reversing drum. The main line switches have suitable magnetic blow-out coils and serve as a circuit-breaker by means of a series trip coil.

The advantages claimed for both forms of this type of control are:

1. All power circuits and the weight of the main control apparatus are removed from the car platforms, i.e. the system is operated by remote control.
2. The main line current is always broken by large, suitably designed switches.
3. Multiple unit operation may be secured with less apparatus and therefore adapted to smaller equipments.
4. The operation of the switches being mechanical it is more positive and therefore the time element is definite.
5. Automatic or selective acceleration may be provided with less complexity.
6. A greater number of motor cars may be operated in multiple in one train. (16 motor cars in one train is claimed).
7. The current from the control circuits is so small that storage batteries may be used, thus making this control easily adaptable for operation on a-c. or high voltage d-c. systems.

SPECIFICATION FOR MULTIPLE UNIT CONTROL EQUIPMENT.—The following memoranda are intended to assist in writing specifications for a complete equipment for the electrical control for a multiple unit car or locomotive exclusive of motors and collecting shoes. See also article on *Specifications.*

Give a complete description of the service in which the equipment is to be used, including the maximum number and weight of cars or locomotives per train.

Similar motion of the master-controller handle shall always produce similar train motions.

A device shall be provided (this is optional) which will limit the rate at which the controller increases the motor voltage, and will assure even acceleration without surging, at the changes from series to series-parallel, and series-parallel to parallel, etc.

A relay shall be provided (this is optional) limiting the current to a specified maximum. (The use of such a relay is more usual on heavy, than on light, equipments.)

Provision shall be made so that acceleration can be arrested at any position by the master controller and so that the motor-circuit combinations shall never be beyond the position indicated by the master controller.

The controller shall automatically return to initial or open-circuit position when the general current supply fails, and when current is restored the control shall progress, as specified, to its former advanced position.

The reverser shall be interlocked so that it cannot be thrown when the motors are taking current.

The control apparatus shall operate satisfactory with a maximum line voltage of . . . . and a minimum line voltage of . . . . and shall never take a current exceeding . . . . amperes per car.

In the event of a train breaking in two, provision shall be made so that power shall be cut off from the detached rear portion without affecting the control of the front portion.

The master-controller handle shall be designed (this is optional) so that if the motorman releases it while operating a train, the power will be automatically cut off.

State whether controller is to be interlocked with air brakes.

Main circuit switch (for street cars only) shall be within easy reach of motorman at each end of car.

Each car or locomotive shall be provided with a control-circuit cut-out switch, which will enable the contactors on any car or locomotive to be disconnected from the control circuits.

Each car shall be provided with an automatic circuit breaker and devices for tripping it from any car of the train.

Number of motors	H.P. of each motor	Type of control	Weight of control equipment, pounds	Weight of each motor, pounds	Total weight of equipment, pounds	Cost of equipment, dollars
4	25	*K*	1200	1900	8,800	2400
4	50	*K*	2200	2450	12,000	3600
4	75	*K*	2200	3200	15,000	4800
4	75	Multi-unit	2800	3200	15,600	5100
2	125	Multi-unit	2700	4150	11,000	4000
2	200	Multi-unit	3200	6400	16,000	5600

CAPACITY, WEIGHT AND COST OF CONTROL EQUIPMENTS. — The preceding table gives the capacity and weight and the approximate costs of some typical control equipments.

CONTROL OF HIGH-VOLTAGE D-C. MOTORS. — The motors operating on systems of from 1200 to 1500 volts are usually designed to operate two in series, thus each receives normally 600 or 750 volts. However, the insulation of each motor must be designed to withstand the whole line potential, and each motor must be able to withstand momentarily the line voltage across its commutator, for if one motor slips the voltage will be unevenly divided between them. For operation on high voltage two motors in series are normally treated as a unit and the series and parallel connection made with these double units. The multiple-unit control is preferable with these high voltages and contactors or unit switches similar to those for 600 volts are used. To operate the control it is customary to supply a self-starting dynamotor (q. v.) which provides 600 volts for this purpose as well as for the lights and other auxiliary apparatus.

Provision for 600-volt Operation. — Since these equipments usually operate also over 600-volt sections of road, provision has to be made to change the connection of the dynamotor when the transfer is made. If the cars are to operate at reduced speed on the lower voltages, as is usually the case on entering the city districts, no change need be made in the motor connections. But if the cars must operate at 600 volts at high speed over an interurban section, then provision must be made to separate the pairs of motors so that all motors will be in parallel for full-speed operation on 600 volts. This requires a commutating switch with automatic protection in order to provide that it is always changed when the car passes from one section to another.

2400-volt Systems. — For locomotive work 1200-volt motors are constructed to operate two in series on 2400 volts. The control for such a locomotive is similar to the control for a 1200-volt equipment.

CONTROL OF A-C. COMMUTATOR MOTORS. — A transformer or compensator is always used to transform the line voltage (3000, 6000 or 11,000 volts) to a voltage suitable for the motors, which is usually from 400 to 500 volts for two motors in series. Taps on the low-voltage side of the compensator (or auto-transformer) provide the various voltages necessary to start and control the motors and thus there is no need of series-parallel control or rheostats, and the energy lost in the rheostats is obviated. A compensator or auto-transformer is usually preferred to a transformer as it is lighter for a given capacity. It is usually placed in oil in a tank suspended from the bottom of the car body. Fewer steps (5 or 6 in all) are required for the control of a-c. motors on account of the reactance of the circuits. To avoid open circuiting the connection to the motors in changing from tap to tap or short-circuiting the portion of the transformer between the taps a "preventive resistance" is connected in the circuit momentarily during the transition. A reverser is provided to reverse the connection of the series fields or exciting windings.

The control may be either of the cylinder or multiple-unit type. In the former type a standard controller may be adapted for the work. If the multiple-unit control is used the cores of all the magnets and contactors must be of laminated iron and a special design of magnet used on account of the difference in characteristics of a-c. and d-c. magnets. In some a-c. multiple-unit equipments a storage battery is used to supply current for the control circuit, in which case d-c. electromagnets may be used in the contactors.

Provision for D-C. Operation. — For operation on direct current as well as alternating current provision must be made to perform the following operations:

(1) cut the transformer out of circuit, (2) connect the motors for series-parallel control, (3) connect rheostats in circuit, (4) change the field connection of the motors, (5) change the connections of the compressor motors, (6) change the connections of the lighting circuits. All this is done by a "commutating switch" which is thrown over at the instant the change is made. This is so arranged that it can only be moved when the controller is at the off-position. The commutating switch is frequently operated automatically, so that when the car reaches a dead section of the trolley between the a-c. and d-c. sections, a no-voltage release throws everything to off-position and when the car reaches the new live section a "selector" coil moves the commutating switch to the proper position.

CONTROL OF THREE-PHASE INDUCTION MOTORS. — Three-phase induction motors for railway work may be controlled by three methods, viz: (1) Changeable pole windings, (2) concatenation of two motors, (3) variable resistance in secondary.

The first two methods were given a considerable trial by German manufacturers some years ago, and have been practically abandoned on account of their complications. In addition to their complications the variable resistance method must also be used with them to provide the smaller gradations of speed.

Variable-resistance Methods. — The secondaries of the motors have a definite winding and the terminals are brought to collector rings by means of which a three-phase starting resistance is connected into the circuit. The speed of the motors is controlled by varying this resistance. Under these conditions the operating characteristics of the motors are similar to those of a d-c. shunt motor. At all fractional speeds a considerable amount of energy is wasted in the rheostats and there is only one efficient running speed. For prolonged running at fractional speed the rheostats must have considerable heat-dissipating capacity. To reverse the direction of the motors a reverser is employed which reverses the connections of two of the three primary leads on each motor. Either the cylinder or multiple-unit control may be used. With induction motors it is desirable to provide a separate set of resistances for each motor to avoid the tendency of the motors to exchange current and "buck," which would occur if the driving wheels were not of exactly the same diameter and one set of resistances were used for all motors. With several induction motors on one car it is desirable to accurately maintain the same diameter of driving wheels on all axles in order to divide the load equally between the motors.

BIBLIOGRAPHY. — See bibliography under *Cars, Electric.* Axtell, C. J. *Type P. C. Control*, G. E. Rev. 1915, Vol. 18, p. 985; F. E. Case, *Type P. C. Control*, G. E. Rev. 1916, Vol. 19, p. 1015; Mardis, P. L. *Type PK Control.* El. Jour. 1917, Vol. 4, p. 436. G. E. Review, Vol. XVIII, 1915, p. 985, XIX, 1916, p. 1015. Elect. Journal, Oct. 1916, p. 462, Oct. 1917, p. 436.

CONTROLLERS. — (*See also Control Systems for Railway Motors; Regulators; Rheostats; Switchgear Equipment for Power Stations.*) Any device for regulating the current or voltage of an electric circuit may be called a controller. Various kinds of controllers have been given different names by users and manufacturers.

DEFINITIONS. — To avoid confusion the various names are used throughout this book as defined below.

Compensator or Induction Starter, an auto-transformer for supplying reduced voltage to the terminals of an induction motor during acceleration; see *Starters, Motor.*

Controller, a device which controls, in some predetermined manner, the operation of the apparatus to which it is connected; generally it controls the running speed of a motor. This article deals only with the latter type A controller frequently combines starting with running features; see *Starters, Motor.*

Regulator, any device for adjusting the voltage of a circuit. A compensator is a special form of regulator. The term regulator is sometimes reserved for devices which utilize inductive action for their control properti s; such a device will be designated specially as a potential regulator; see *Regulators.*

Resistor, the resistance portion of a controller or starter, through which flows the main current of the circuit whose voltage is controlled. A field rheostat is any device employing a variable resistance to control the voltage across the field windings of a generator or motor. The field rheostat of a d-c. motor indirectly controls the speed, because it controls the voltage and current of the field winding; see *Rheostats; Starters, Motor.*

Motorstarter, a controller designed for accelerating a motor to normal speed in one direction of rotation.

FUNCTIONS OF CONTROLLERS. — Controllers are designed to be used with motors of different kinds and to take care of the functions not incorporated in the motor design in order to enable the latter to operate under the specified conditions of load. The functions usually supplied by the controller are the following:

To limit the current during the acceleration of the motor.
To limit the torque during acceleration.
To change the direction of rotation of the motor.
To limit the load on the motor.
To disconnect the motor on failure of voltage.
To regulate the speed of rotation.
To start and stop the motor at fixed points on the cycle of operation, or at the limit of travel of the load.
To stop the motor.
To protect the operator from injury.

Not every controller has to embody all of these features in the same degree but these are the underlying points of controller design and they must be procurable when they are needed.

TYPES OF CONTROLLERS. — Controllers may be divided into the following classes: Face-plate, drum, drum-contactor, and magnetic contactor, controllers.

Face-plate Controller. — The simplest form of controller for starting and regulating the speed of d-c. motors is the face plate type shown diagrammati-

cally in Fig. 1, and intended for use with a variable speed motor connecting resistors in the armature and field circuit. With this type of controller the contacts are mounted on the face of a suitable base and a moving arm makes the connections to the armature and field circuits, the speed of the motor being changed by varying the field strength. The rheostat arm is made in two parts, the under part making contact with the segments R–1 to R–12 and with the contact ring E, while the top arm engages the upper row of round contacts. When starting the two arms are held together by a spring. The bottom arm is provided with a notched segment engaging the plunger forming part of the low voltage release magnet. The notched segment and pawl hold the arm in any operating position after the low voltage magnet is energized. To start the motor

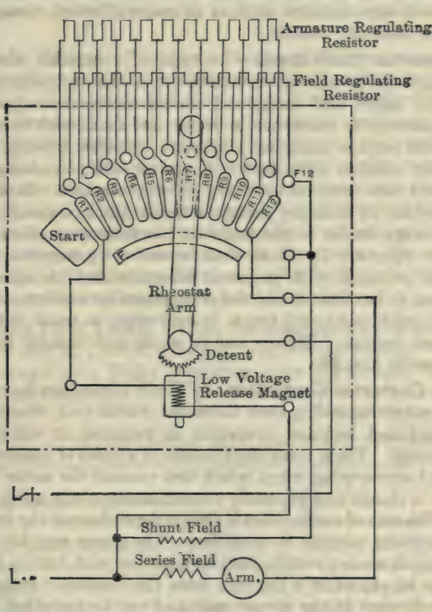


Fig. 1.

the contact arms are moved from the off position to contact R–1, and the connections can readily be traced from that point. The arms are gradually moved to the right eliminating successively each section of the armature resistor until the bottom arm makes contact with R–12. In this position the armature is connected directly across the lines and the segment E is disconnected from the rheostat arm. The shunt field circuit now is from the positive side of the line to the upper rheostat arm to the right-hand field contact F–12, thence to the field winding. This gives a motor speed due to full field strength. If it is desired to increase the speed of the motor the upper arm can be moved to the left across the field contacts to insert resistance gradually in the shunt field circuit, and thus within its range, give the increased speed desired, while the low voltage release magnet holds the lower arm on contact R–12. If the circuit is interrupted the low voltage release magnet will allow the lower arm to be carried to the off posi-

tion by means of a spring. This in turn picks up the upper arm and the two are moved quickly to the off position.

Drum Controller. — Another type of controller in common use is the drum controller. This is used with machine tools for varying the speed and reversing the direction of rotation of adjustable speed d-c. motors by means of armature and field resistance. On the larger sizes magnetic blowouts are used. The drum controller usually has two rows of contact fingers attached to the framework of the controller but insulated from it so as to be electrically separated from each other. Between these rows of fingers is mounted an insulated cylinder or drum which is revolved by the handle. On this drum are mounted copper segments of different lengths which engage the contact fingers. The length and location of these segments are such as to make different connections for each position of the controller handle.

Drum-Contactor Controllers. — A type of controller which, due to its wide application has found very extensive use, is the drum-contactor controller. It is used with series and compound wound motors for adjusting the motor speed by regulating the resistance in series and shunt with the motor armature or for crane hoist service requiring graduated dynamic braking while lowering. This controller consists of a series of spring closed contactors mounted on steel rods and a cam shaft for operating these contactors. In outward appearance it resembles an ordinary drum controller and in internal appearance and operation it is very similar to the magnetic contactor controllers, except that the contactors are operated by a cam shaft instead of by magnetic coils. The line contactors are provided with magnetic blowouts and the same type of rolling contact is used as has been so successful on magnetic contactor controllers. It has relatively quick make and break in operation, is small in size and is easy to operate and inspect. Repairs can be easily and cheaply made. Contacts have exceptionally long life.

Magnetic Contactor Controllers. — These controllers have many advantages which, notwithstanding their increased initial cost, recommend their use wherever rapid and frequent operation is required or where continuity of service is essential. These controllers consist of a series of magnetically operated contactors and accelerating relays which first connect the motor to the line with all resistance in circuit and gradually cut out resistance in series with the motor armature. The rate of acceleration is always dependent on the load of the motor. They are absolutely positive in operation. The motor may be completely controlled by a small master switch, field rheostat, or push button station, any one of which may be placed at a distance from the controller proper. The motor is always accelerated at the proper rate and can be provided with protection against overload and voltage failure. All safety features can easily be incorporated.

CONTROLLERS FOR INDUCTION MOTORS. — Controllers with squirrel cage secondaries usually operate by supplying the motors with various voltages obtained from transformer taps. If the motors are provided with wound secondaries the controllers frequently operate by varying the resistance in the secondary circuit. The resistors must then be designed to carry the running current of the motor continuously without overheating (*see Motors, Polyphase Induction; Starters, Motor*).

With a reversing motor, as in crane or rolling mill service, a similar method of control is used except that the controller is provided with two drums which are operated by a single handle. Primary reversing contacts are located on one drum so that the motor will run in one direction when the handle is turned to the right, say, and in the reverse direction when it is turned to the left. In going from the off position the first step connects the primary circuit of the motor

to the line and subsequent steps cut out the secondary resistance. In passing from full speed in one direction to full speed in the other the resistance is all cut into circuit before reversing. In the off-position the motor is entirely disconnected from the line.

Use of Contactors with A-C. Controllers. — For reversing mill or hoisting work using induction motors with wound secondaries many very ingenious and highly satisfactory installations have been put in service which use solenoid-operated magnet switches, or contactors, for the secondary and occasionally for the primary circuits. These are worked from a master controller or similar device, or are operated automatically by the positions of the rolls, hoist, etc. Automatic acceleration can be obtained in the same manner as with direct-current motors and various safeguards, such as dynamic breaking, can also be employed.

Automatic Control of Input to Flywheel Motor-generator Set. — Another application for contactor control with automatic features is with flywheel motor-generator sets which use a very heavy flywheel in connection with a d-c. generator and an a-c. motor with wound secondary. The power put into the flywheel or delivered up by it depends upon the variation in speed of the motor generator. By varying the resistance in the motor secondary this speed regulation can be secured. By the use of suitable relays the input to the motor and consequently the load on the a-c. system can be kept practically constant, while the output of the d-c. generator supplying power to a d-c. hoist or rolling-mill motor is undergoing wide fluctuations, the energy in the flywheel taking care of the difference between the constant input and the variable output. See also article on *Flywheels for Load Equalization*.

COST OF CONTROLLERS. — The following figures will serve as a rough indication of the cost of various sizes and types of controllers. Costs of controllers vary through wide limits depending upon the design, the amount of speed variation desired, etc.

COST OF CONTROLLERS FOR 220-VOLT MOTORS *

H.P. of Motor (approximate)	5	25	50	100
Face plate type speed adjusting	$38	$70	$100	$220
Drum type	64	100	158	...
Drum contractor	85	155	125	...
Magnetic contactor L.V.–O.L. Reversing	318	377	545	887
Reversing controllers for wound rotor induction motors	80	100	200	400

* Controllers for 110 and 550 volts cost relatively the same as for 220-volt motors.

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CONVERTERS, SYNCHRONOUS OR ROTARY. — (*See also Generators, Alternating-current; Generators, Direct-current; Motor Generators; Motors, Synchronous; Transformers; Substations, Railway.*) Since in general it is more economical to transmit electrical energy in the form of alternating currents and more convenient to utilize it in the form of direct currents, some means of converting from one form of electrical energy to the other is desirable. For this purpose synchronous converters and motor-generator sets (q.v.) are available. Synchronous converters are also called "rotary converters."

A synchronous converter is a machine very similar to a d-c. generator in which certain commutator segments, or the conductors connected to them, are connected to 2, 3, 4 or 6 collector rings as the case may be. When the movable member is caused to rotate, the voltage between any two collector rings is alternating. Such a machine, when driven by an engine or motor, may be operated as an a-c. generator or as a "double-current" generator giving alternating current from its collector rings and direct current from its commutator. If the collector rings are connected to a source of alternating currents the machine will run as a synchronous motor and direct current may be obtained from the brushes on the commutator; i.e., the machine, with but one set of windings acts simultaneously as an alternating-current motor and a direct-current generator. It has therefore the friction, core-loss and excitation loss of one machine instead of two, and since the motor and generator currents flow in the same winding and during at least the major part of each cycle are in opposite directions, they more or less balance each other and the armature RI^2 loss is much less than in either a motor or generator alone.

Synchronous Converter versus Motor Generator. — A converter is much more efficient and weighs and costs less than a motor-generator set of the same capacity. It also occupies less space. However, since only one winding is used, there is a definite relation between the e.m.f's. of the a-c. and d-c. terminals. The maximum value of the alternating wave bears a definite relation to the direct e.m.f. (*see below*). It is therefore necessary to supply the converter with a voltage of the same order as the direct voltage and this involves the use of transformers, if a high-voltage transmission line is used to supply the converter. Motors operating at voltages as high as 13,000 volts can be used in motor-generator sets.

Relative Efficiencies. — The efficiency of a converter is in the neighborhood of 93 per cent and of the transformers 97 per cent, thus the efficiency of the combination is about 90 per cent. The efficiency of a synchronous motor is in the neighborhood of 93 per cent and of a d-c. generator 92 per cent, thus the combination motor-generator set has an efficiency of 85.5 per cent. If the supply voltage is greater than 13,000 volts, transformers will also be needed for the motor-generator set and the net efficiency would then be 83 per cent.

Synchronous Converter versus Rectifiers. — A converter differs from a rectifier (q.v.), since the former gives a direct e.m.f. of constant and uniform value and the latter gives a pulsating unidirectional voltage and current. In the former the energy is stored in the form of magnetism for an instant whereas in the latter there is no magnetic field and no storage of energy. A rectifier will not work satisfactorily on an inductive d-c. circuit but a converter will.

APPLICATION OF CONVERTERS. — The most common application of synchronous converters is in electric railway work. The great majority of motors for electric traction are direct-current series type, operating at from 500 to 600 volts. The energy for these motors must be transmitted over long distances, which requires a high voltage a-c. transmission line and converters to link the d-c. distribution with the a-c. transmission. Converters have also

been developed for high voltage d-c. traction systems, these converters giving 1500 volts d-c. Two such converters may be connected in series for 3000 volt d-c. distribution (*Elect. J. 12, p. 154, Apr. 1915*). Synchronous converters are also used for lighting and power service and for electrolytic work.

TERMINOLOGY. — The following terms are used to describe certain characteristic features of the various kinds of synchronous converters.

Phases and Rings. — A single-phase converter has two collector rings and each ring is connected to the windings by as many equally spaced taps as there are pairs of poles. The taps for the two rings alternate at equal spaces. A single-phase converter is therefore a two-ring converter.

A three-phase converter has three rings and three equally spaced taps (one for each ring) for every pair of poles. A four-phase or quarter-phase converter has four rings and four taps for every pair of poles. A six-phase converter has six rings and six taps per pair of poles.

Shunt and Compound-wound Converters. — A converter may be shunt or compound wound, depending upon the service for which it is intended. The series winding is intended to make the converter take leading current when the load increases and thus increase the voltage at the a-c. terminals, but the ratio of the a-c. *terminal* voltage to the d-c. voltage remains unaltered.

Inverted Converter. — Sometimes a converter is operated to convert from d-c. to a-c. It is then called an "inverted converter." The machine will operate satisfactorily in this manner, but its speed depends upon the nature of the a-c. load. An inductive load in the a-c. circuit causes the armature to demagnetize the fields, with a resultant increase in speed. It is therefore dangerous to operate an inverted converter on an inductive load unless it is provided with a speed-limit device. This effect does not occur when the machine is operating as an a-c. motor, since its speed is fixed by the frequency of the supply circuit.

Cascade Converter or Motor Converter. — This is a combination of an induction motor and converter connected in series or concatenation (*see Motors, Induction*). The converter receives half the power in mechanical form from the shaft and half the power inductively, in the form of alternating current at half frequency, from the secondary of the induction motor. By this means the steadiness of a 30-cycle converter is obtained in a 60-cycle unit.

Split-pole Converter. — The "split-pole" or "regulating-pole" converter is designed to give a variable ratio of alternating e.m.f. to direct e.m.f., for operation in parallel with storage batteries and similar purposes. The field poles are divided into sections the excitation of which may be controlled independently. The effect of the split pole is primarily to produce a third harmonic in the flux distribution curve, which increases or decreases the *total* flux per pole and therefore the d-c. voltage, but does not change the a-c. counter e.m.f. between slip rings, since these rings are connected to the armature winding at points 120 electrical degrees apart. Split pole converters give a voltage variation of about 20 per cent.

Converter with Series Booster. — A synchronous converter with series booster is sometimes used for purposes similar to those of the regulating pole converter. It is merely a converter with a separately excited a-c. generator on the same shaft as the converter, and the armature of this generator is connected in series with the converter armature and the line. This generator acts as an a-c. booster and raises the line voltage.

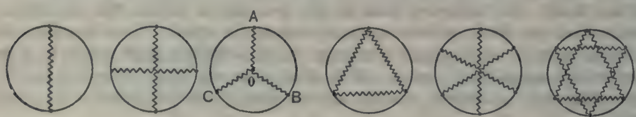
RATING AND PERFORMANCE. — The standards of the A.I.E.E. specify a maximum rise of 50 degrees C. on any part of the windings and

70 degrees on the commutator for continuous operation. Some manufacturers are more conservative in their continuous rating and give these figures for the temperature rise after operation for two hours at 25 per cent overload, occurring immediately after a continuous run. Twenty-five cycle converters will give momentarily an output of three times their normal rating without damage or injurious sparking at the commutator, but sixty cycle converters are more sensitive to overloads.

PRINCIPLES OF CONVERTER ACTION. — In this section are briefly treated those features of a synchronous converter, in which the converter differs in action from an a-c. or d-c. generator; see table on page 296, for a summary of the voltage, current and capacity relations.

Connections and Voltage Ratios. — The ratio of voltage on the a-c. side to that on the d-c. side depends upon the number of rings and type of connection employed.

Two-ring Converter. — The two collector rings are connected by taps to the same winding as the commutator; hence the alternating e.m.f. has the same value as the direct e.m.f. at the instant that the taps pass the brushes. As this is also the maximum value of the alternating e.m.f., the effective value (i.e., the value to be indicated by a voltmeter) will be 0.707 times the maximum or 0.707 times the direct e.m.f. Fig. 1 shows in a simple manner the connection of a single-phase converter. The external circle represents a two-pole arma-



Figs. 1 2 3 4 5 6

Transformer Connections and Vector Relation of Voltages in Synchronous Converter

ture winding and inside is shown the supply transformer connected to two taps diametrically opposite each other. The voltage across this transformer would be 0.707 E, where E is the voltage between the positive and negative brushes on the d-c. side.

Four-ring Converter. — If two additional collector rings are connected to conductors spaced half way between the former taps, there results the quarter-phase converter shown in Fig. 2. The voltage across each supply circuit or transformer is the same as before, but the two voltages will differ in phase by 90 degrees.

Three-ring Converter. — If three collector rings are connected to taps spaced 120 degrees apart the e.m.f. between adjacent collector rings will be the vector sum of AO and OB in Fig. 3. Since AO and OB each equal $0.5 \times 0.707\,E$ the voltage AB will be $\sqrt{3} \times 0.5 \times 0.707\,E = 0.612\,E$. Thus in a three-ring or three-phase converter the voltage between adjacent taps is 0.612 times the direct voltage and the connections of transformer are as in Figs. 3 and 4.

Six-ring Converter — Diametrical and Double Delta Connections. — A six-ring converter may be connected diametrical as in Fig. 5 in which case the voltage of each transformer will be $0.707 \times E$ and the result will be like the combination of three single-phase groups. A six-ring or six-phase converter

may also be connected "double delta" as shown in Fig. 6, which is similar to the combination of two groups of three-phase delta transformers. In both cases the voltage between adjacent taps of a six-phase converter is 0.355 E.

n-Ring Converter. — In general the voltage between taps of any converter having n equally spaced taps per pair of poles is

$$E_{ac} = \frac{E \sin \frac{\pi}{n}}{\sqrt{2}}.$$

Current Ratios. — To determine the ratio of the continuous current I to the alternating current I_3 per collector ring in a three-phase converter, for example, assume the d-c. output equal to the a-c. input with unity power factor; then

$$\sqrt{3}\, E_3 I_3 = EI,$$

and, from preceding paragraph,

$$E_3 = 0.612\, E,$$

where E_3 = a-c. voltage between lines; I_3 = alternating current per line; E = direct voltage; I = direct current. From these two relations

$$I_3 = 0.94\, I.$$

The ratios of currents for other converters are obtained similarly, and are given in the table on page 296.

In actual practice the current on the input side must be greater than that given by these relations, in order to supply the losses in the converter, and the alternating current will also vary inversely as the power factor, which is taken as unity in the table.

Resultant Coil-Current. — In any machine acting simultaneously as a motor and a generator the two currents must flow in opposite directions, and the current in any particular conductor will be the difference between the two. In any particular coil the direct current is constant in amount and direction from the instant the commutator segment connected to this coil passes the positive brush to the instant it passes the negative brush, and conversely from negative to positive brush. In any coil midway between the a-c. taps the alternating current is a maximum when this coil is half way between brushes (for unity power factor), and the current falls to zero as the coil reaches the interpolar position. Therefore, for the period of time that the direct current in a coil remains constant in amount and direction there is also in it a variable current changing from zero to a maximum and back to zero again. The net or resultant current will therefore have a wave shape and frequency somewhat as shown at R in Fig. 7. The heating in this particular coil will therefore be proportional to the product of the resistance of the coil by the square of this current, and it is readily seen that the power lost is less than that due to either the direct or alternating current alone.

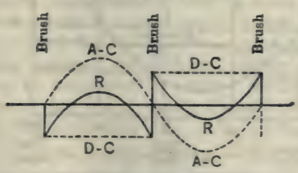


Fig. 7. Current in Coil Midway between Taps

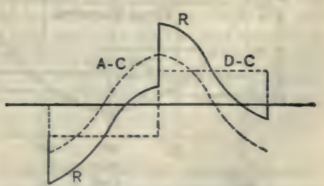
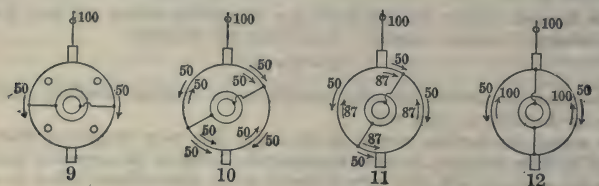


Fig. 8. Current in Coil at Tap in Two-ring Converter

A coil situated very near one of the a-c. taps will carry a direct current subject to the same law as before mentioned, but the alternating current in this coil is the same as that in the middle coil and has its maximum value when the middle coil is at the middle of the poles and not when this particular end coil is at the middle of the pole. The alternating current in this coil may therefore reach its maximum value soon after the coil has passed a brush, as shown in Fig. 8, and the resultant of the two currents is greater than that in the middle coil. The further a coil is situated from the middle coil of a group, the greater is the phase displacement of its alternating current, the greater is the value of the resultant current and the greater is the heating effect. Thus although the heating effect in a rotary converter winding is less than that of a direct-current machine having the same output it is different in each and every coil, is minimum in the central coil (when the power factor is unity) and is maximum at the coil nearest the tap, and the greater the angle between taps the greater is the total heating effect.

Distribution of Heat Losses in Armature Winding. — In Figs. 9 to 12 inclusive, representing a two-ring converter, the numbers outside the circles



Component Currents in a Two-ring Converter

represent the direct current in the winding (corresponding to 100 amperes in the external d-c. circuit and unity power factor on a-c. side) and the numbers inside the circles the instantaneous alternating current in the winding. It will be seen that the resultant current in a coil midway between taps, Figs. 9 and 12, never exceeds 50 amperes. A coil 30 degrees from the middle, Fig. 10, has a maximum current of 100 amperes. A coil 60 degrees from the middle, Fig. 11, has a maximum current of $50+87=137$ amperes, and a coil 90 degrees from the middle (tap coil) may have a current of $100+50=150$ amperes. The heating of the armature winding is therefore not uniformly distributed, though the conduction of heat from one part of the winding to another tends to equalize the temperature rise.

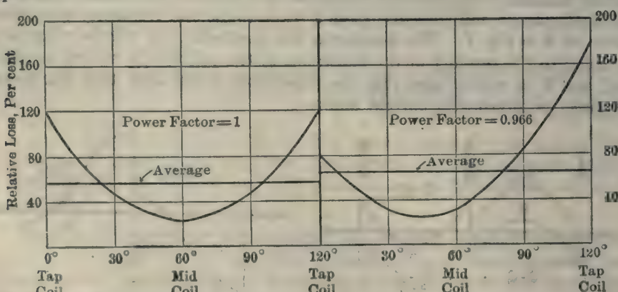


Fig. 13. Relative RI² in Individual Coils of Three-ring Converter

If the converter operates at a power factor different from unity, the position of minimum resultant current is no longer at the middle coil, but is moved one way with leading current and the opposite way with lagging current. Thus one end coil has improved heating conditions, and the middle coil and the other end coil have much worse heating conditions, the result being that the heating as a whole has increased and is more non-uniformly distributed than with unity power factor.

The power lost in each individual coil of a converter and the effect of the power factor on this loss is very well shown in Figs. 13 and 14, taken from a paper by J. E. Woodbridge (*A.I.E.E., 1908*). In Fig. 13 the curved line shows the relative RI^2 loss in a coil having any position throughout 120 degrees of one phase of a three-phase converter when the power factor is unity. The

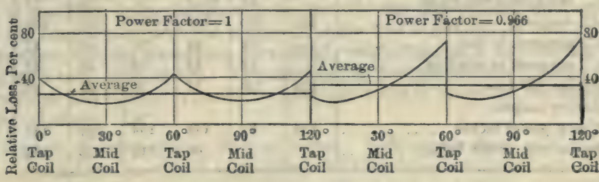


Fig. 14. Relative RI^2 in Individual Coils of Six-ring Converter

curve shows the ratio of the loss compared to the loss in the same machine acting as a d-c. generator of the same capacity. It will be noted that the middle coil has a loss of 22 per cent of that of the generator and the end coils 120 per cent, and that the average loss is 57 per cent. For a power factor of 0.966, representing a phase displacement of current of 15 degrees, the loss in individual coils ranges from a maximum value of 180 per cent at one tap to a minimum value of 22 per cent in the coil shifted 15 degrees to one side of the middle coil. The other end coil has a loss of a little over 80 per cent of the generator loss. The average value has been increased to 65 per cent.

In Fig. 14 the same ratios are shown for a six-phase converter, in which the winding of one phase is distributed over only 60 degrees. Consequently the conditions are better and the maximum loss due to low power factor is less.

Dependence of Output upon Number of Phases. — As a result of these conditions we have the following relations of the capacity of a given armature with various numbers and connections of taps, the capacity being based on an equal total amount of RI^2 loss.

It should be remembered that there are other losses besides coil losses in the converter, and that therefore practical figures are slightly different from those given in the table on page 296.

Field Excitation. — The variation of the field excitation of a synchronous converter has much the same effect as in the case of a synchronous motor (*see Motors, Synchronous*); that is, if its field is under-excited the armature will draw a lagging current which assists the field and sets up the necessary flux, whereas if the field is over-excited, the armature will draw a leading current which opposes the field and reduces the flux. By this means the converter may be made to take either a leading or lagging current. A leading current flowing over a line having inductance tends to raise the voltage at the receiving end of the line.

Line Compounding. — If there is sufficient leading current and sufficient inductance in the line, the voltage at the receiving end may be greater than at the sending end in spite of the resistance of the line. This is some-

VOLTAGE, CURRENT AND OUTPUT RATIOS

	D-C. generator	Converters					
		2-ring	3-ring	4-ring	6-ring diametrical	6-ring double delta	12-ring
D-C. volts	100	100	100	100	100	100	100
A-C. volts between lines	...	71	61.2	71	71	61.2	71
A-C. volts between rings	...	71	61.2	50	35	35	18
D-C. amperes	100	100	100	100	100	100	100
A-C. amperes in line	...	141	94	71	47	47	24
A-C. amperes in winding	...	71	55	50	47	47	45
Relative RI^2 loss	100	137	55	37	26	26	20
Relative output:							
Unity power factor	100	85	134	165	197	197	224
87 per cent power factor	...	...	99	115	129	129	135

times called "line compounding." The relation between the voltages at the two ends of the line may be expressed as follows:

Let

E = voltage to neutral at sending end;
V = voltage to neutral at receiving end;
I_1 = component of line current in phase with V, i.e., the power component of the line current;
I_2 = component of line current at 90 degrees to V, i.e., the *leading* reactive component of the line current;
r = resistance of one line;*
x = inductive reactance of one line;*

then, noting that I_2 is to be taken positive when leading,

$$E^2 = (V + rI_1 - xI_2)^2 + (xI_1 + rI_2)^2.$$

Use of Series Field. — In practice a series field is added to the converter and the shunt excitation is adjusted so that the armature current is lagging at no load. As the load increases the series field increases, the adjustment being such that at about ¾ load the proper excitation for unity power factor is given. Hence at all loads over ¾ the field will be over-excited, the current will be leading and the voltage will be raised or compounded.

DESIGN. — The design of a synchronous converter is very similar to the design of a d-c. generator (q.v.), except for certain special conditions due to the fact that the frequency is fixed by the frequency of the supply system, and that greater latitude is allowed in the choice of the nominal value of the d-c. armature reaction and copper density, because the real value of these quantities is the difference between their nominal d-c. values and their a-c. values.

* r and x should include respectively the resistance and reactance not only of the line wire, but also the transformers and reactance coils through which the line current passes.

Speed and Number of Poles. — The revolutions per minute and the number of poles are definitely related to the frequency of the supply circuit, in cycles per second, in accordance with the formula:

$$120 \times (\text{frequency}) = (\text{number of poles}) \times (\text{rev. per min.}).$$

The choice of the number of poles usually depends upon the commutator.

Commutator. — The design of the commutator is usually the limiting feature and therefore the first to be considered. This is particulary true of either high-frequency (60 cycle) machines or when the d-c. voltage is 600 or over. Three factors must be considered in the design of the commutator to secure successful commutation and life of the commutator, namely, peripheral speed, voltage between bars, thickness of bars. If the peripheral speed or the voltage between bars is too high commutation will be bad. If the commutator bars are too thin the commutator will not retain its shape and commutation will be bad. These three limiting factors are very closely related and in a high-voltage machine give very little choice. The diameter of the commutator depends upon the voltage and frequency, and its minimum value is given by the three following relations. Let

s = pitch of commutator bars in inches. This ranges from 0.15 in small machines to 0.20 in high-voltage or high-frequency machines, to 0.40 in liberally designed machines. These values include the width of bar and about 0.03 inch insulation between bars.
V = peripheral speed of commutator in feet per minute. This ranges from 3500 in liberally designed low-voltage, 25-cycle machines to 5000 in 60-cycle machines, and is extended to 6000 under compulsion.
e = average volts per bar = machine voltage divided by the number of bars between brush studs. Normal values are from 8 to 14. The maximum voltage between bars is about 1.57 times this.
f = frequency in cycles per second;
E = voltage between d-c. terminals;
p = number of poles;
n = total number of commutator bars;
d = diameter of commutator in inches.

Then

$$s = \frac{Ve}{10Ef}, \qquad n = \frac{pE}{e}, \qquad d = \frac{sn}{\pi}.$$

Armature Ampere Turns and Current Density. — Since the d-c. and a-c. armature reactions are opposed to each other the nominal value of the d-c. armature reaction ampere turns may be chosen much higher than in generators and values of from 4000 to 8000 ampere turns per pole are in common practice. The nominal or apparent value of the ampere conductors per inch of periphery varies from 500 to 900, and the apparent current density (d-c.) in the armature copper from 2500 to 5000 amperes per square inch.

Diameter of Armature. — The diameter of the armature per pole varies from 3.5 inches to 6 inches, and the diameter is usually from 6 inches to 8 inches greater than the diameter of the commutator, the difference depending upon the possibility of making a good mechanical construction of the end connections. The number of slots in the armature and the number of segments in the commutator must be a multiple of both the number of phases and the number of poles.

Armature Winding. — The winding of the armature is usually of the multiple drum type, although the series winding may be used The turns per pair of poles are divided by equally spaced taps (to the slip rings) into as many groups as there are phases. The number of turns in series between brushes is adjusted for the proper direct e.m.f. and a reasonable value of flux per pole. The alternating e.m.f. bears a definite relation to the direct e.m.f. depending upon the type of connection (*see above*), and also to some extent upon the shape and length of pole arc.

Flux Density in Air Gap. — The air gap or pole-face flux density usually has a value ranging from 40 to 60 kilolines per square inch as in generators.

Damping Copper in Pole Face. — In the pole face of every converter a squirrel-cage winding should be provided to assist in starting and to prevent hunting. The total cross-section of copper per pole ranges approximately from 1/10 to 1/5 of the armature copper per pole and the end rings must be of reasonable cross-section compared to the bars. The joints between bars and end rings must be carefully made.

Shunt and Series Fields. — The determination of the length of armature, length of commutator and the design of the field follow the same laws as the design of these parts for a d-c. generator. The proportioning of the series field results from the considerations given above under *Field Excitation*, the problem being that a certain amount of leading current is required at a given load and to make the armature take this leading current the field excitation must be increased by a certain number of ampere turns, from which the number of turns in the series field can be determined.

Equalizer Connection. — In large multi-polar machines it is customary to connect to a common ring all commutator bars which are at the same potential; these "equalizer connections" avoid local cross-currents in the armature from flowing through the brushes and causing bad commutation. These cross-currents are caused by unequal or uneven air gap.

Shaft, Bearings, Etc. — Since the transfer of energy is in the conductors themselves, there is no mechanical torque other than that to overcome friction and core-loss. Therefore the shaft, bearings and mechanical housing of a rotary converter are quite light and present no particular mechanical difficulties in mechanical design. For the same reason converters do not require very elaborate foundations. Machines of less than 1000 kilowatt capacity are usually supplied with a base and are complete in one piece, while larger sizes are supplied with foundation plates.

Kw. rating	500	300
Frequency	25	60
Core-loss	1.00	1.75
Armature RI^2	0.55	0.60
Shunt field RI^2	0.70	0.60
Brush RI^2	0.40	0.40
Bearing friction	0.55	1.50
Brush friction	0.30	0.65
Efficiency	96.50	94.50

Efficiency and Losses. — The efficiency and distribution of losses of a typical 25-cycle and 60-cycle converter are shown in the accompanying table. All values are in per cent of input at full load.

EXAMPLES OF DESIGN. — In the table on the next page are given design data on four representative converters of different capacities.

TESTING OF CONVERTERS. — The following are the usual tests made on converters to determine the efficiency, regulation, heating and to show any defects in construction.

1. Resistance of Armature, Shunt Field and Series Field. — The armature resistance is usually measured between points on the commutator diametrically opposite and the equivalent resistance calculated from this value by the equation

$$\text{Equiv. Res.} = \frac{4 \times (\text{diametrical resistance})}{(\text{number of poles})^2}.$$

SYNCHRONOUS CONVERTERS

Design Data

Item	Unit	1	2	3	4
Poles		4	6	6	6
Rating	kw.	500	500	100	300
Speed	rev. per min.	750	500	1000	1200
D-C. volts	volts	600	600	440	600
Frequency	cyc. per sec.	25	25	50	60
Number of phases		6	6	3	3
Armature reaction	amp. turns	15,000	6720	7400	3500
Nominal σ*	amp. cond.	1500	710	675	475
Flux density in gap	kilolines	58	52	23	53
Armature diameter	inches	25.6	36	21	28
Slots per pole		24	24	6⅝	21
Armature length	inches	11	17.25	7.1	10.5
Commutator diameter	inches	19.7	28	16.5	20
Number of segments		288	288	194	252
Periph. speed of commutator	feet per minute	3800	3650	4300	6280
Volts per bar	volts	8.3	12.5	13.6	14.3
Pitch segments	inches	0.17	0.306	0.266	0.25
Armature diameter per pole	inches	6.4	6	3.5	4.75
Nominal U†	amperes	6000	3600	2630	3400

* σ = ampere conductors (d-c.) per inch periphery.
† U = amperes (d-c.) per square inch.

To obtain the true RI^2 in a converter armature this equivalent resistance is multiplied by the square of the external direct current and by a constant as given in the accompanying table:

	Theoretical	Commercial
Single phase	1.39	1.47
Three phase	0.56	0.59
Quarter phase	0.37	0.39
Six phase	0.26	0.27

These values only hold if the converter is operating at unity power factor. For any other power factor the reactive component of the alternating current per phase must be found and the square of this component times the resistance per phase of the armature gives the additional RI^2 loss due to the lesser power factor.

2. **No-load Saturation Curve.** — The no-load saturation curve, as in a-c. generators and d-c. generators (q.v.) is usually plotted between commutator voltage and ampere turns of shunt field.

3. **Core-loss** as in a-c. and d-c. generators (q.v.).

4. **Phase Characteristic** at no load and at full load as in synchronous motors (q.v.).

5. **Synchronous Impedance** as in a-c. generators (q.v.).

6. **Starting Tests** to determine current and voltage necessary to start the converter on a-c., time to reach full speed, and voltage induced in field windings as in synchronous motors (q.v.).

7. **Heat Run.** — This may be made either with a resistance for load or two similar converters may be tested in parallel by the Hopkinson or "pump back" method; see *Transformers*. In addition to a source of d-c. power of the rated voltage of the converter to supply the losses, either a d-c. booster or an a-c. potential regulator is needed to adjust the load.

8. **Insulation Tests.** — See *Generators, Alternating-current,* and *Standardization Rules of the A.I.E.E.*

9. **Pulsation Test.** — A synchronous converter is very sensitive to any change in impressed voltage or frequency. A sudden change in either of these factors will cause a pulsation which will start the machine hunting, as discussed in the article on *Motors, Synchronous*. This hunting may increase until the machine falls out of step or flashes over. To determine whether a converter has a dangerous tendency of this kind, a test is made in which two similar machines are supplied with power from a common generator. Between each converter and the common connection a resistance is placed in each a-c. line, having a value that will give with full-load current a drop in voltage of 15 per cent of the rated voltage of the machine. Thus there is 15 per cent *RI* drop between each converter and the generator and 30 per cent between the two converters. The two machines are operated at no load and at rated voltage with the shunt fields adjusted for minimum input. The voltage across the commutators is observed for any periodic variation. Then the field of one machine at a time is varied from half normal to twice the normal value and any indications of periodic variations of the direct voltage noted. If the machines have a proper damper winding in the pole faces, they should not develop any dangerous hunting, even under the above unfavorable conditions.

SPECIFICATIONS FOR SYNCHRONOUS CONVERTER. — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Principal Characteristics and Conditions of Service. — Use to which converter is to be put, such as railway, lighting, motor driving or battery charging. Whether it is to convert from a-c. to d-c. or vice versa. Voltages and number of phases. Nominal rating in kilowatts or Institute rating in kilowatts. Frequency and speed.

Style and Description; Details of Construction. — Whether interpole; whether shunt or compound wound, or whether there is a split-pole field winding; whether rheostat is to be supplied for shunt field; if so, its characteristics. Proposed method of starting and whether starting apparatus is to be supplied. Whether a speed-limit device is required; if so, the shunt field rheostat shall have sufficient resistance to speed the machine for testing the speed-limit device, the latter being set at 15 per cent over rated speed. Whether an end play device (or oscillator) is desired and if so, what type or types are acceptable.

Work to be Done by Other Contractors. — Whether the synchronous converter contractor is to furnish and install the following: Main wiring, field

wiring, field-rheostat grids, dial plate and chains, starting panels, starting rheostat or motor generator, foundations.

Performance and Tests. — Temperature rises upon which nominal and Institute ratings are to be based. Overload capacity (*see Standardization Rules of the A.I.E.E.*). Commutation limits. Efficiency at 25, 50, 75, 100, 125 and 150 per cent nominal load. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation. Converters shall operate in parallel without "hunting" from no load to stated (say 200 per cent) overload, provided drop in high-tension lines due to resistance between any converter and any other synchronous apparatus in the system does not exceed a stated value (say 20 per cent), and provided that the phase variation does not exceed a stated value (say 2.5°). What voltage regulation is required, how it is to be obtained and whether it is to be hand or automatic. If the converter is shunt wound, state voltage-regulation requirements. What per cent reactance between a-c. bus and converter is required?

OPERATION OF CONVERTERS. — In the operation of synchronous converters the points mentioned below should receive special attention.

Transformer Connections. — The usual methods of connecting transformers to supply converters are shown diagrammatically in Figs. 1 to 6 and are discussed above in the section on *Voltage Ratios* and also in the article on *Transformer Connections*. Of the three-phase to six-phase connections the choice must be made with some care, as all connections are not equally good for each specific use of the converter.

Methods of Starting. — The several methods of starting converters are as follows:

Alternating-current Starting, which is the same as the starting of motors; see *Motors, Synchronous*.

Direct-current Starting. — The machine is started as a direct-current shunt motor and synchronized on the a-c. side after it is up to speed. This requires less power, but takes more time and more skill in order to synchronize. To secure maximum starting torque, the a-c. side should be disconnected from the transformers, which otherwise act as a shunt to a part of the starting current (*Newburg and Smith, Elec. J, 15, p. 24, Jan. 1918*).

Starting with Auxiliary Motor. — This involves the extra cost and extra continuous loss of the auxiliary induction motor. It is no more efficient than starting by direct current and requires the same amount of time. By the use of suitable relays the starting and synchronizing may be made entirely automatic (*Wensley, R. J., El. Ry. J. 53, p. 948, May 17, 1919*).

Combination Alternating- and Direct-current Starting. — The machine is started up with direct current, then disconnected from the d-c. mains and connected to a low-voltage tap of the a-c. supply and brought up to full speed. This method is more economical of power and time but requires more starting apparatus than either the a-c. or d-c. methods.

Field Break-up Switch. — All converters are supplied with a switch to open the field in several places to avoid the strain of the high potential induced in the field during starting and to reverse the direction of current in the field after the machine is up to speed in order to reverse the polarity in case it should not be right. This is usually a double-throw switch with several poles.

End-play Device. — In order to prevent the brushes from wearing grooves in the commutator and collector rings a device is mounted upon one end of the shaft to move the shaft end-wise back and forth periodically. In small ma-

chines this is a mechanical device consisting of a ball running between two warped surfaces. In large machines it consists of an electromagnet which periodically pulls out the armature a short distance. The magnetic pull of the main field poles pulls the armature back.

Speed-limiting Device. — In case a converter should be disconnected from the main a-c. generating circuit and still remain connected so that it would tend to operate from the d-c. side there is danger of its speed becoming dangerously high. To avoid this a centrifugal governor is placed on the shaft and arranged to electrically operate the main d-c. switches of the converter.

VOLTAGE REGULATION. — There are several methods of regulating the voltage delivered by the commutator of a converter.

Compound-wound Converter with External Reactance. — This method is automatic and will give about 10 per cent variation in voltage. It is standard practice for railway work; see also *Substations, Railway*.

Regulating or Split-pole Converter. — With this type of converter the voltage regulation is gradual and normally accomplished by hand, but by the addition of an automatic voltage regulator may be made automatic.

Shunt-wound Converter with Induction Regulator. — A large variation in voltage is possible but this method does not respond to quick changes. It is quite generally used in lighting work.

Shunt-wound Converter with Synchronous Booster. — A synchronous generator is carried on the same shaft as the converter and connected in series between the transformers and the collector rings of the converter. The method is good but expensive. G. A. Juhlin (*Inst. El. Eng. J. 55, p. 241, Apr. 1917*) gives a detailed discussion of the relative advantages of reactance and booster regulation.

Shunt-wound Converter and Taps on the Transformers. — The voltage ratio of the transformers may be varied. This is usually accomplished by connecting the line to different taps on the pirmaries, which involves opening the circuit or short-circuiting a portion of the transformer at each change.

WEIGHTS, SPEEDS AND COSTS: — The weights, speeds and costs of four commercial lines of converters are given in the following table. **The costs are based on the 1921 price level.** The first group is a line of 25-cycle converters without commutating poles, the second 25-cycle converters with commutating poles. The third and fourth groups refer to 60-cycle converters without and with commutating poles respectively. It will be noted that the smaller ratings are three-phase machines while the greater ratings are six-phase machines. The six-phase connection makes possible a saving in material at the expense of increased cost in manufacturing labor. The use of commutating poles also makes possible a considerable reduction in size of machine at the expense of increased labor, therefore all the higher capacity machines have commutating poles. The costs given are only approximate, as such figures vary greatly with commercial conditions. The machines listed are 600-volt converters for railway purposes. Machines for lower voltages would be somewhat less expensive.

WEIGHTS, SPEEDS AND COSTS

25 Cycle non-commutating pole type, 600 volts

Poles	Kw.	R.P.M.	Phases	Weight, pounds	Cost, dollars per kw. in 1921
4	200	750	3	18,000	16
4	300	750	3	20,000	12.50
4	500	750	6	24,000	11
6	500	500	6	27,000	13

25 Cycle, commutating pole type, 600 volts

4	500	750	6	15,000	11
4	1000	750	6	25,000	9
6	1500	500	6	37,000	8
8	2000	375	6	54,000	8
18	4000	167	6	180,000	11

60 Cycle, non-commutating pole type, 600 volts

6	200	1200	3	11,000	12.50
6	300	1200	3	13,000	12
8	500	900	6	18,000	11

60 Cycle, commutating pole type, 600 volts

6	500	1200	6	10,000	10
8	1000	900	6	23,000	12
16	2000	450	6	70,000	17

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CONVEYORS. — (*See also Power Stations; Telpherage.*) A conveyor is a mechanical device for the continuous handling of materials along a horizontal or inclined plane. There are four general types of conveyors, viz., the scraper or flight, the screw, the bucket and the belt.

THE FLIGHT CONVEYOR consists of a trough of any desired cross section through which are pulled a series of scrapers or "flights" attached to an endless chain. The improved forms of this type of conveyor have sliding shoes or rollers attached to the flights or to the chains, supported on runways.

Capacity. — The following table by S. B. Peck (*Trans. A.S.M.E., 1910*) gives the capacities of flight conveyors in tons of coal per hour when the conveyor is operated at a speed of 100 feet per minute.

CAPACITY OF FLIGHT CONVEYORS

Size of flight, in.	Horizontal				Inclined		
	Flights spaced			Lb. per flight	10°	20°	30°
	18 in.	18 in.	24 in.		24 in.	24 in.	24 in.
	Tons	Tons	Tons		Tons	Tons	Tons
4 by 10	33.75	30	22.5	15	18	14.25	10.5
4 " 12	42.75	38	28.5	19	24.5	18	13.5
5 " 12	51.75	46	34.5	23	28.5	22.5	16.5
5 " 15	69.75	62	46.5	31	40.5	31.5	22.5
6 " 18	.....	80	60	40	49	40.5	31.5
8 " 18	.....	120	90	60	72	57	48
8 " 20	.....	...	105	70	84	66.5	56
8 " 24	.....	...	135	90	120	96	72
10 " 24	.....	...	172.5	115	150	120	90
10 " 30	.....	...	220	147	184	146	116
10 " 36	.....	...	268	179	225	177	142
10 " 42	.....	...	315	210	264	210	167

Power Required. — The following formula gives approximately the horse power at the head wheel required to operate flight conveyors:

$$H.P. = (ATL + BWS) \div 1000.$$

T = tons of coal per hour; L = length of conveyor in feet, center to center; W = weight of chain, flights and shoes (both runs) in pounds; S = speed in feet per minute; A and B constants depending on angle of incline from horizontal and have the following values.

Angle, deg.	A	B	Angle, deg.	A	B	Angle, deg.	A	B
0	0.343	0.01	10	0.50	0.01	30	0.79	0.009
2	0.378	0.01	14	0.57	0.01	34	0.84	0.008
4	0.40	0.01	18	0.63	0.009	38	0.88	0.008
6	0.44	0.01	22	0.69	0.009	42	0.92	0.007
8	0.47	0.01	26	0.74	0.009	46	0.95	0.007

For suspended flight conveyors take B as 0.8 and for roller flights as 0.6 of the values given in the table.

Screw Conveyors. — Screw conveyors consist of a helical steel flight, either in one piece or in sections, mounted on a pipe or shaft, and running in a steel or wooden trough. These conveyors are made from 4 to 18 inches in diameter, and in sections from 8 to 12 feet long. The speed ranges from 20 to 60 r.p.m. and the capacity from 10 to 30 tons of coal per hour. It is not advisable to use this type of conveyor for coal, except the smaller sizes, as the flights are easily damaged by any foreign substance of unusual size or shape.

BUCKET CONVEYORS are of two types, having rigidly connected and pivoted buckets respectively. The buckets are carried by an endless chain driven by sprockets or pawls. Rigid buckets are used to convey coal and other materials over considerable distances when there is no intermediate point of discharge. They are built to carry as much as 2 tons of coal per minute.

Pivoted buckets may be used both as conveyors and elevators, and are particularly well adapted to the handling of coal and ashes in power plants. Their advantages are slow speed, silent operation, adaptability to change of direction without transfer, high efficiency and easy renewal of worn parts. Their disadvantages are danger of buckets sticking or upsetting and jamming in the supports, and the difficulty of preventing spill at the loading and turning points. Spilling in loading may be prevented by the use of special loading devices, by providing overlapping lips on the buckets or by placing small buckets between the main buckets to catch the spill.

Capacity. — Buckets are usually of 2-feet pitch, and range in width from 18 to 48 inches. They run at low speeds, usually not over 50 feet per minute, 40 feet per minute being the usual speed. At the latter speed the capacities when handling coal vary from 40 tons per hour for the 18-inch width to 120 tons per hour for the 48-inch width.

Power Required. — Prof. E. F. Miller gives the following formula for calculating the power required for a bucket conveyor making a rectangular circuit:

$$P = 0.004\,CL,$$

where P is the horse-power required, C is the capacity of the conveyor in tons of coal per hour, and L is the total lift in feet. This is an empirical formula deduced from numerous data on coal conveyors having capacities of from 20 to 50 tons and operating at speeds of from 40 to 55 feet per minute, and making lifts of from 40 to 80 feet. The power given by the above formula is for the conveyor loaded to its full capacity. The conveyor when running empty will require from 40 to 60 per cent of this. The smaller the conveyor the larger the percentage of power empty to power loaded.

BELT CONVEYORS. — Rubber and cotton belt conveyors are used for handling coal, ore, sand, gravel, etc., in all sizes. They combine a high carrying capacity with low power consumption. In the majority of cases the belt is troughed by means of idler pulleys set at an angle from the horizontal and placed at intervals along the length of the belt. Belt conveyors may be used for elevating materials up to about 23° incline. The belt may be run at any speed from 200 to 800 feet per minute, and may be from 12 inches to 60 inches in width. The most serious objection to belt conveyors is the lack of durability of the belts, their liability to destruction from accidental causes and the expense of their frequent renewals.

Link belts (*see Chains and Chain Drive*) are also used for conveying purposes.

Capacity. — The following table gives the capacity of the more common sizes of belt conveyors:

CAPACITY OF BELT CONVEYORS IN TONS OF COAL PER HOUR

Width of belt, in.	Velocity, feet per minute			Width of belt, in.	Velocity, feet per minute				
	300	350	400		300	350	400	450	500
12	34	..	...	20	96	112	128	...	...
14	47	..	...	24	139	162	186	210	...
16	62	72	82	30	218	254	290	326	...
18	78	91	104	36	315	368	420	472	520

For materials other than coal, the figures in the above table should be multiplied by the coefficients given in the table below:

Material	Coefficient	Material	Coefficient
Ashes (damp)	0.86	Earth	1.4
Cement	1.76	Sand	1.8
Clay	1.26	Stone (crushed)	2.0
Coke	0.60		

Power Required to drive a belt conveyor (*C. K. Baldwin, Trans. A.S.M.E., 1908*) depends on a great variety of conditions, as the spacing of idlers, type of drive, thickness of belt, etc. In figuring the power required, the belt should run no faster than is necessary to carry the desired load. If it should be necessary to increase the speed, the load should be increased in proportion and the power figured accordingly.

For level conveyors the horse-power required is

$$P = C \times T \times L \div 1000.$$

For inclined conveyors

$$P = (C \times T \times L \div 1000) + (T \times H \div 1000).$$

C = power constant from table below; T = load, tons per hour; L = length of conveyor, center to center, feet; H = vertical height material is lifted, feet; S = belt speed, feet per minute; B = width of belt, inches.

For each movable or fixed tripper add horse-power in column 3 of table. Add 20 per cent to horse-power for each conveyor under 50 feet long. Add 10 per cent to horse-power for each conveyor between 50 and 100 feet long. The formulas above do not include gear friction, should the conveyor be gear-driven.

	1	2	3	4	5
Width of belt, Inches	*C* for material weighing from 25 lb. to 75 lb. per cu. ft.	*C* for material weighing from 75 lb. to 125 lb. per cu. ft.	H.P. required for each movable or fixed tripper	Minimum plies of belt	Maximum plies of belt
12	0.234	0.147	½	3	4
14	0.226	0.143	½	3	4
16	0.220	0.140	¾	4	5
18	0.209	0.138	1	4	5
20	0.205	0.136	1¼	4	6
22	0.199	0.133	1½	5	6
24	0.195	0.131	1¾	5	7
26	0.187	0.127	2	5	7
28	0.175	0.121	2¼	5	8
30	0.167	0.117	2½	6	8
32	0.163	0.115	2¾	6	9
34	0.161	0.114	3	6	10
36	0.157	0.112	3¼	6	10

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COOLING SYSTEMS FOR POWER STATIONS. — When the supply of cooling water is limited or expensive, economy requires that it shall be artificially cooled and used continuously. This cooling is accomplished by natural or forced evaporation in open ponds, spray fountains, a series of artificial cascades or in cooling towers. The latter device is largely used.

COOLING TOWERS. — Cooling towers are usually made in the shape of large cylinders of sheet steel, filled with narrow boards or laths arranged in geometrical forms, or hollow tile, or wire network, so arranged that while the water, which is sprayed over them at the top, trickles down through the spaces it is met by an ascending air column. The air is furnished either by disk fans at the bottom or is drawn in by natural draft. In the latter case the tower is made very high, say 60 to 100 feet, so as to act like a chimney.

Make-up Water. — When used with jet condensers, the water produced by the condensation of the exhaust steam is sufficient to make up for the evaporation in the tower, and therefore only enough water for the boiler feed need be supplied continuously. In fact, the condensed steam is, as a rule, more than sufficient to make up for the loss due to evaporation, and there results a slight overflow, which carries with it the oil from the engine cylinders and tends to clean the system of the oil that would otherwise accumulate in the hot-well.

When a cooling tower is used with a surface condenser make-up water must be added to the cooling water, the amount ranging from 3 to 15 per cent, but in this case the boiler feed water can be used over and over again with but slight loss, provided the oil (there is none with turbines) is eliminated by suitable separators.

Reduction in Temperature Obtainable. — With a properly designed cooling tower the temperature of the hot water (and condensed steam in the case of a jet condenser) can be reduced 40 to 50° F. The water can usually be cooled to within 2 to 5 degrees of the wet bulb air temperature.

Power Required for Forced Draft in Tower. — The power required for the fan, when forced draft is used, averages 2 per cent of that developed by the main engines during the summer months, when maximum volume of air is required, and about 1 per cent during the winter months.

Cooling Ponds. — Where water for condensing is scarce it is found economical to store it in reservoirs where it is cooled by evaporation. Under the conditions prevailing in northeastern United States it has been found that a surface of 250 sq. ft. is sufficient to cool the condensing water required for one boiler horsepower (34.5 lb.) at 26-inch vacuum. In countries where the evaporating coefficient is higher, smaller surfaces may be used; see Ruggles, *Transactions A.S.M.E., Vol. 34, p. 561.*

Spray Ponds. — Where the area available for water storage is small recourse must be had to the spray system. Here the warm condensing water is sprayed into the air above the small pond; a portion evaporates, cooling the remaining portion, which falls to the tank to be used again. The sides of the tank are extended to prevent the spray from being carried away by the air currents. About 4 sq. ft. of surface are required for one boiler horsepower of steam condensed at 26-inch vacuum. It is even possible in small plants to have the spray pond on the roof of the power house. These ponds should not be used in crowded localities as the spray of water is likely to be a nuisance. The loss of water varies between the amount of boiler feed and twice that amount.

COSTS (Pre-war figures). — Cooling-tower costs vary greatly with climatic and operating conditions. When these are relatively favorable the cost may be kept as low as $2.50 per kilowatt for natural draft towers and $3.60 for fan draft.

Under ordinary central-station conditions the costs will probably average \$3.50 and \$5.00 per kilowatt respectively for the two types of towers. Assuming 27-inch vacuum, 70° air and 70 per cent humidity, forced-draft cooling towers cost about \$250 per 1000 lb. of steam condensed. For 24-inch vacuum the cost may be one-half this. Natural-draft towers cost about one-half to one-third as much as the forced-draft towers. As a general rule cooling towers do not pay when the pumping head exceeds 75 feet.

The following data on spray cooling ponds were given in the *Elec. World*, 66, p. 808, Oct. 9, 1915:

Steam condensed lb. per hr.	Circ. water, gal. per min.	Number of nozzles	Size of pond, ft.	Total cost, pond and equipment	
				Concrete basin	Clay basin
11,000	1320	35	50 × 128	\$3385	\$2105
20,000	2420	60	90 × 90	4825	3205
36,000	4320	110	112 × 120	7710	5010

BIBLIOGRAPHY. — Additional data on cooling towers, ponds and spray fountains and numerous references to original papers will be found in Gebhardt's *Steam Power Plant Engineering;* Fernald and Orrok, *Engineering of Power Plants;* Thomas, C. C., *The Cooling of Water for Power-plant Purposes*, A.S.M.E. Trans. Vol. 39, p. 625. See also Kent's *Mechanical Engineer's Pocket-Book.*

COPPER. — (*See also Electrochemical Processes, Industrial; Wires and Cables, Bare.*) The following discussion applies primarily to copper for electrical conductors. Copper mined in the vicinity of the Great Lakes is almost pure. That derived from the compound ores of the metal must be refined (usually electrolytically) to reduce it to the same degree of purity.

ROLLING-MILL PROCESSES. — The refined copper comes to the rod mill in bars weighing about 200 pounds each. These bars frequently have ridges along the sides, due to faults in castings, and the surface is often covered with a layer of oxide. They are heated in a furnace until sufficiently soft for rolling and are passed through a series of rolls diminishing in size until a rod of the proper diameter is obtained. The rod is then coiled up and immersed in a pickling liquid (10 per cent H_2SO_4) in order to dissolve the oxide formed during rolling. It is then washed and dipped in a fluid tallow mixture.

WIRE-MILL PROCESSES. — The rods having cooled are connected together by brazing and are drawn through a series of dies of decreasing diameter. The dies give the wire a dense hard exterior coating which increases its tenacity. As the strength obtainable is almost a direct factor of the work expended upon the wire, the smaller the size the greater the tensile strength per square inch, so that the strength of the wire is readily varied by changing the size of the rod and the number of dies.

Defects in Wire. — One of the most serious defects occurring to wire at this point is from ridged bars as above described. Ordinarily the bar will not be sufficiently heated to dissolve the copper oxide on the surface so that as the softened bar enters the first passes of the rolls, the ridges are lapped over, inclosing the oxide scale. The subsequent passes and the drawing through the dies obscure this flaw almost entirely, but it remains a serious menace to the toughness and the resistance to wear of the copper.

A second cause of trouble arises at the same point by overheating the copper in the softening furnace, in which case copper oxide is formed on the surface and quickly dissolves through the entire bar, thereby increasing the oxide content and tending toward the production of brittleness. Both of these dangers can be avoided by careful selection of the bars and by proper regulating of the temperature of the softening furnace.

As the production of the hard surface from drawing is at best a rather delicate operation, careless handling, uneven welding of the rods and unequal temperature of the wire while passing through the dies will all produce noticeable effects in the quality of the finished wire, so that care throughout the mill is absolutely necessary for the best results.

It, therefore, appears that the most efficient wire must possess not only high conductivity but the maximum torsion and tensile strength possible in commercial copper and that to obtain this it is first necessary to use high-grade copper and to prevent an excess of cuprous oxide entering it at any stage of the manufacture, and, secondly, to select as perfect bars as possible and to observe extreme care in every treatment through which they pass. (*Adapted from article by Carl F. Woods, Electric Railway Journal, 1909, Vol. 33, p. 195.*)

ANNEALING.— All wire when first drawn is more or less hard. It may be softened by annealing, i.e., by heating to a temperature between 250° to 400° C. The hardness is not affected by the rate of cooling. Annealed copper has a crystalline structure, whereas hard-drawn copper consists of grains elongated in the direction of drawing.

MECHANICAL PROPERTIES. — (*See Wires and Cables, Bare, for tables of tensile strength, etc., of various sizes of wire.*) The more important mechanical properties of copper are discussed in some detail below.

Tensile Strength and Elongation of Soft Annealed Copper. — The tensile strength of soft annealed copper is about 30,000 to 33,000 pounds per square inch with an elongation of about 25 per cent (in 10 inches) at the fracture. It has no true elastic limit, permanent elongation being produced by very small loads.

Tensile Strength and Elongation of Hard-drawn Copper. — (*See Wires and Cables, Bare, for tensile strength of various sizes of wire.*) Modern hard-drawn copper is equally affected by drawing throughout the section, no hard skin being produced. According to D. R. Pye, hard-drawn copper in wires up to $\frac{1}{2}$ inch diameter varies in tensile strength with the diameter according to a linear law of the form

$$T = 70,000 - 45,000\,D,$$

where T = tensile strength, pounds per square inch, and D = diameter of wire, inches.

The above constants agree approximately with the tables of the American Society for Testing Materials and represent values somewhat under those usually obtained.

The elongation at fracture is approximately represented by

$$E = 4\sqrt{D},$$

where E = per cent elongation at fracture.

Tests by G. C. Batson on 50-foot lengths of hard-drawn copper showed a tensile strength only ½ per cent less than that of 10-foot lengths, a fact which indicates that the material is very uniform.

Modulus of Elasticity of Hard-drawn Copper. — The modulus of elasticity of hard-drawn copper varies from 16×10^6 to 20×10^6, the higher values applying to small wires. The following tests by G. C. Batson are typical of commercial copper.

Diameter, in.	0.158	0.138	0.112	0.094	0.079	0.066	0.049
Modulus, lb. sq. in. units	17.7×10^6	17.9×10^6	17.5×10^6	17.7×10^6	17.1×10^6	19.5×10^6	19.2×10^6

An apparent modulus of 12×10^6 is often obtained from an initial test, due to straightening-out. (*See also Wires and Cables, Bare.*) The modulus decreases at temperatures above 150° C.

Elastic Limit of Hard-drawn Copper. — The true elastic limit of hard-drawn copper, or load at which permanent set begins, is not the same as the point where the strain begins to increase more rapidly than the stress. The latter point is usually between 30,000 and 35,000 pounds per square inch and the former, somewhat below (*see Wires and Cables, Bare*).

Compression Test. — Copper of good quality does not fracture under compression; it yields and flattens. According to Thurston the resistance to compression may be calculated, within the limits $e < \frac{1}{2}$, from the formula

$$C = 145,000\sqrt[3]{e}$$

where C = resistance in pounds per square inch of original area,
e = fractional compression.

Miscellaneous Mechanical Properties. — The torsional strength, shearing strength, hardness, resistance to impact and fatigue of copper are all discussed in Bureau of Standards Circular No. 73 on *Copper.*

Density. — The density of copper or, for all practical purposes, its specific gravity referred to water, is 8.89 at 20° C. This is the value which has been adopted as standard by the American Institute of Electrical Engineers, and most other authorities in the past. Recent measurements by the Bureau of Standards, the Calumet & Hecla Smelting Works, and the Reichsanstalt have indicated this as a mean.

CONDUCTIVITY AND RESISTIVITY. — *(See also Resistance and Conductance; Wires and Cables, Bare.)* F. A. Wolff and J. H. Dellinger give the resistivities of 89 samples of commercial copper from 14 important refiners and wire manufacturers in this and other countries. The mean for annealed wire is: Resistivity in ohms per meter-gram at 20° C. = 0.15292; per cent conductivity = 100.25. (Per cent conductivity is computed on the basis of 100 per cent conductivity corresponding to the standard resistivity of 0.15328 ohm per meter-gram at 20° C.) The mean result of data furnished by a large wire-manufacturing company, representing tests on more than 100,000,000 pounds of copper, is also given; for example, for annealed samples: Resistivity in ohms per meter-gram at 20° C. = 0.15263; per cent conductivity = 100.42.

Conductivity of Hard-drawn Copper. — The conductivity of hard-drawn No. 12 B. & S. wires was found to be less than the conductivity of annealed wires by a mean value of 2.7 per cent. The difference between the conductivity of annealed and hard-drawn wires increases as the diameter of the wire decreases The lowest resistivity and highest conductivity found for a hard-drawn wire were: Resistivity in ohms per meter-gram at 20° C. = 0.15386; per cent conductivity = 99.62; and for annealed wire were: resistivity in ohms per meter-gram at 20° C. = 0.15045; per cent conductivity = 101.88.

Effect of Bending on Conductivity. — Copper wire apparently decreases in conductivity when bent, but the greater part of this is caused by local changes in cross-section. This effect is negligible unless the wire is bent to a very small radius.

Effect of Melting. — Electrolytic copper which is drawn without having been melted has a higher conductivity than that which has been melted, J. H. Dellinger recording copper thus drawn as having a conductivity as high as 101.71 per cent after annealing at a dull-red heat. Lake copper likewise drawn without having been melted gave conductivities as high as 101.88 after annealing.

Temperature Coefficient of Resistivity. — It has been found by J. H. Dellinger (*Bulletin No. 147, Bureau of Standards, 1910, Vol. 7, No. 1*) that the temperature coefficient of copper is proportional to the conductivity instead of being virtually a constant, as hitherto assumed. This fact may be expressed by saying that the change of resistivity per degree C. of a sample of copper is 0.000597 ohm per meter-gram or 0.00681 micro-ohm per centimeter cube. (Dellinger's original figures were subsequently changed slightly; *see Circ. No. 31 of Bureau of Standards, 1912 ed.*) The 20° C. temperature coefficient of a sample of copper is found by multiplying the per cent conductivity by 0.00393 and dividing by 100. These rules apply only to copper furnished for electrical use and to the temperature range of 10° C. to 100° C. over which the temperature coefficient was found to be linear. The following table gives the temperature coefficients a_T in the formula:

$$R_t = R_T[1 + a_T(t - T)].$$

Ohms per meter-gram at 20° C.	Per cent conductivity	α_0	α_{15}	α_{20}	α_{25}	α_{30}
0.16134	95	0.00403	0.00380	0.00373	0.00367	0.00360
0.15966	96	0.00408	0.00385	0.00377	0.00370	0.00364
0.15802	97	0.00413	0.00389	0.00381	0.00374	0.00367
0.15753	97.3	0.00414	0.00390	0.00382	0.00375	0.00368
0.15640	98	0.00417	0.00393	0.00385	0.00378	0.00371
0.15482	99	0.00422	0.00397	0.00389	0.00382	0.00374
0.15328	**100**	0.00427	0.00401	**0.00393**	0.00385	0.00378
0.15176	101	0.00431	0.00405	0.00397	0.00389	0.00382

The boldface values in the table have been adopted as standard by the American Institute of Electrical Engineers.

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CORONA, ELECTRIC. — (*See also Electron Theory; Spark Gap; Transmission Lines.*) If a potential difference is established between smooth parallel wires, or between concentric cylinders, and gradually increased, a voltage is finally reached at which a hissing noise is heard. If it is dark, a pale glow, called the electric "corona," will be seen to surround the wires. When wattmeters are inserted in the line a loss is noticed to start at this critical voltage. There is also noted the characteristic odor of ozone, and the air around the wires becomes ionized. These phenomena are referred to as "corona effects."

The corona starts at the conductor surface, because the voltage stress per unit distance, or "voltage gradient," is highest there. The break-down or corona extends out to a point where the stress is below the break-down point of air. If the wires are close together, so that the stress is fairly uniform, the break-down immediately extends from conductor to conductor without corona, and the phenomenon is called a "spark-over." The spark and corona are exactly the same phenomenon. The corona may be considered as a spark-over from conductor to space, as distinguished from a spark-over from conductor to conductor. The spark-over does not extend to the other conductor when the separation is large compared to the wire diameter, because it reaches a point in space where the gradient is below the break-down gradient of air. Increasing the voltage after the corona point is reached causes the corona to extend until finally a spark occurs from conductor to conductor. The power loss increases very rapidly with increase in voltage above the critical point.

Corona is caused by either a-c. or d-c. voltages, and starts at approximately the same maximum stress. At the critical voltage, corona occurs only at the crest of the alternating wave. As the voltage is increased above the critical point, the loss extends over a greater portion of the wave. In the a-c. corona the eye sees a superposition of the corcna caused by the plus and minus half-cycles of the a-c. waves. If the effects of the half waves are viewed separately, it is noticed that a reddish haze surrounds the wire while it is negative, whereas the surface of the wire glows bluish-white while it is positive. Positive and negative d-c. corona have exactly the same appearance as the positive and negative corona of the correspodning a-c. half-wires. When a negative wire becomes oxidized, reddish tufts appear. If there are rough spots on the wire the corona starts at these points at a lower voltage. At points the positive corona extends out as a bluish-white spray; the negative corona appears as a red tuft.

Corona is also caused by transient voltages. Corona produced by voltages lasting less than a millionth of a second can be seen by the eye and distinction can be made between a positive and negative half-wave.

Corona Voltmeter. — For a given conductor arrangement and air density corona always starts at the same maximum voltage. Use has been made of this fact in the corona voltmeter (*Whitehead and Isshiki, J.A.I.E.E., May, 1920*) For this purpose, a wire in the center of a cylinder is generally used. In one form, the voltage is applied and the air density varied until the corona appears. The voltage is then readily calculated or read from a scale. A tapered wire has also been used. The starting point can be detected by the glow, the noise, the odor, or by the conducting ionized air.

Laws of Corona-Formation. — The laws of corona have been quite definitely worked out. The chief factors affecting corona formation are:

For a given spacing and air density the corona starting voltage is lowered by decreasing the wire diameter.

Increasing the spacing increases the corona starting voltage but the effect of changing the diameter is relatively much greater.

Decreasing barometric pressure decreases air density and decreases the corona starting voltage.

Increasing temperature decreases air density and decreases the corona starting voltage.

Dirt, water, etc., on the conductor surfaces lower the corona starting voltage by increasing the stress.

The *apparent strength* of air is not constant in irregular fields but greater at the surface of small conductors than large ones. The strength at the start of corona is always constant, however, at a distance from the conductor which is a known function of the radius. This corresponds to the strength in a uniform field of 76 kilovolts per inch (maximum value in case of a-c. voltage), which seems to mean that the actual strength of air is 76 kilovolts per inch. In order, however, that a finite thickness only may be brought to this stress in an irregular field, the stress at the conductor surface must be higher. At a given spacing the corona always starts at a lower voltage on small wires than large ones, because, although the *apparent strength* of air at the small conductor surface is greater, the stress produced by a given voltage is relatively much higher for the smaller wires.

Corona does not start at exactly the same voltages when the wires are alternately plus and minus. The difference is at most a few per cent and is greater for small wires than large ones.

For perfectly clean, smooth wires of uniform diameter, there is no loss until the *visual critical voltage* is reached, when the loss assumes a *definite value* and increases as the square of the difference between the applied voltage and the *disruptive critical voltage.* The *visual critical voltage* takes into account the *apparent strength* of the air, which is a function of the wire diameter; the *disruptive critical* voltage corresponds to the constant strength of air for uniform fields, which is about 76 kilovclts per inch, maximum value. The *visual critical voltage* is always higher than the *disruptive critical voltage. Due to irregularities there is always a loss below the visual critical voltage.* This loss follows the probability curve, because the loss occurs at chance rough spots, but on practical transmission lines the quadratic law is generally approximated even on this section of the curve. This is especially so for large conductors at low altitudes, where the difference between the visual and disruptive critical voltages is not great.

Corona loss increases with increasing frequency.

The formulas for calculating corona on transmission lines are given below.

CORONA ON TRANSMISSION LINES. — It is of great importance in the design of high voltage transmission lines to know the various factors that affect the corona formation and to be able to estimate accurately the starting voltage and loss for a given line.

The various characteristics may be calculated from the following formulæ:

Let e = r.m.s. value of the kilovolts to neutral $\left(= \frac{1}{\sqrt{3}}\right.$ kilovolts between wires for 3-phase; $=$ ½ kilovolts between wires for single-phase$\left.\right)$.

t = temperature in deg. F.,

b = barometric pressure, in inches,

$\delta = \frac{17.9b}{459+t}$ = specific gravity of air, referred to air at 77° F. and 29.9 inches barometric pressure,

r = radius of conductor, in inches,

$g_0 = 53.6$ = disruptive critical gradient, kilovolts per inch, r.m.s. value,

$g_v = g_0\delta\left(1 + \frac{0.189}{\sqrt{\delta r}}\right)$ = visual critical gradient, kilovolts per inch, r.m.s. value,

s = distance between conductor centers, in inches,

f = frequency, cycles per second.

Then the **disruptive critical voltage,** r.m.s. value of kilovolts to neutral, is

$$e_0 = 2.302 m_0 g_0 r\,\delta \log_{10}\left(\frac{s}{r}\right), \qquad (1)$$

where $m_0 = 1$ for polished wires, $= 0.98$ to 0.93 for roughened or weathered wires, $= 0.87$ to 0.83 for seven-strand cables.

The **visual critical voltage,** r.m.s. value of kilovolts to neutral, is

$$e_v = 2.302 m_v g_v r \log_{10}\left(\frac{s}{r}\right), \qquad (2)$$

where $m_v = 1$ for polished wires, $= 0.72$ for local corona all along cable, $= 0.82$ for decided corona all along cable.

The **power loss in fair weather,** in kilowatts per mile of single conductor, is

$$p = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{s}}(e - e_0)^2 \times 10^{-5}. \qquad (3)$$

The **power loss in stormy weather,** in kilovolts per mile of single conductor, is approximately,

$$p_s = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{s}}(e - 0.8e_0)^2 \times 10^{-5}. \qquad (3a)$$

Example. — To illustrate the use of the above formulas consider the case of a three-phase, 60-cycle line, No. 0, A.W.G. stranded cable (diameter 0.374 in.), spacing 120 inches, length of line 100 miles, temperature 100° F., barometer 28.85 inches.

Then $\quad \frac{s}{r} = \frac{120}{0.187} = 642,$

$$\log_{10}\left(\frac{s}{r}\right) = 2.81,$$

$$\sqrt{\frac{r}{s}} = \sqrt{\frac{0.187}{120}} = 0.0394,$$

$$\delta = \frac{17.9 \times 28.85}{459 + 100} = 0.925,$$

$$\sqrt{\delta r} = \sqrt{0.925 \times 0.187} = 0.426,$$

$$g_v = 53.6 \times 0.925\left(1 + \frac{0.189}{0.426}\right) = 71.6,$$

$$m_0 = 0.87.$$

The disruptive critical voltage is then,

$$e_0 = 2.302 \times 0.87 \times 53.6 \times 0.187 \times 0.925 \times 2.81,$$

$$= 52.0 \text{ kilovolts to neutral.}$$

The power loss is,

$$p = \frac{390}{0.925}(25 + 60) \times 0.0394(e - e_0)^2 \times 10^{-5}$$
$$= 0.014(e - e_0)^2 \text{ kw. per mile of conductor}$$
$$= 0.042(e - e_0)^2 \text{ kw. per mile of line.}$$

The conductor would begin to glow ($m_v = 0.72$) at

$$e_v = 2.302 \times 0.72 \times 71.6 \times 0.187 \times 2.81$$
$$= 62.6 \text{ kilovolts to neutral}$$
$$= \sqrt{3} \times 62.6 = 108.3 \text{ kilovolts between wires.}$$

The glow would be pronounced ($m_v = 0.82$) for

$$e_v = \frac{0.82}{0.72} \times 108.3 = 123.4 \text{ kilovolt between wires.}$$

The power loss per mile of line under storm conditions would be,

$$p_s = 0.042(e - 0.8 \times 52)^2$$
$$= 0.042(e - 41.6)^2 \text{ kw. per mile of line.}$$

This last calculation assumes a storm over the whole line at the same time; a condition that is most unlikely to occur. The storm loss will generally be less due to lower temperatures.

The loss on a transmission line will vary from day to day depending upon the temperature and weather conditions. The above losses are calculated for summer temperature. For winter the losses would be much lower.

Methods of Increasing Size of Conductors. — For equal conductivity an aluminum conductor has about a 25 per cent greater diameter than a copper conductor, and, therefore, approximately 25 per cent higher critical voltage.

The advantage of aluminum may be still further increased by the addition of a steel cable core. Such a cable has been in use for several years on a 165–154 kilovolt line in California.

A copper tube may also be used or a copper cable with a suitable core.

Another means of increasing the corona voltage for a given amount of metal is to arrange several conductors of the same potential close together.

Arrangement of Conductors. — In the above formulas it has been assumed for three-phase lines that the conductors are so arranged that they form the edges of an equilateral prism. When the conductors are not so arranged, but are placed symetrically in a plane, corona will start at a lower voltage on the center conductor than on the outside conductors. The actual critical voltage on the center conductor will be approximately 4 per cent lower, and on the outside conductors 6 per cent higher, than for the equilateral prism arrangement with the same spacing.

Voltage Variation Along Line. — In practice, due to the drop in voltage along the line, the corona loss will be different at different points on the line. This may be allowed for by calculating the loss per mile at various points, and plotting a curve with loss per mile as ordinates and length in miles as abscissa. The area of this curve then represents the total loss.

If an isolated system is operating near the critical corona voltage and one conductor becomes grounded, the corona loss will be quite high.

The Corona Limit of High Voltage Transmission Lines. — Safe and Economical Voltages. — It will generally be found that it is safe and economical to operate a line up to, but not above, the fair weather value of the disruptive critical voltage (that is, up to the value of e_0 given by equation 1). for average barometer and summer temperatures. This will give loss during storms, but as storms do not extend over the whole line at one time, it will generally not be serious. During cold weather the critical voltage will be higher and storm loss less.

Besides the loss of energy, corona loss may be undesirable from another standpoint, viz., since the loss occurs only on part of the wave, it may introduce harmonics if it is excessive.

Tables I and II give the values of the fair weather disruptive critical voltage, in kilovolts *between wires*, for various conductors at various spacings, at a barometric pressure of 29.9 inches = 760 mm. (standard pressure at sea level) and 77° F. = 25° C.

Table III gives the factors by which these voltages must be multiplied to give the corresponding disruptive critical voltage at various elevations. This correction factor is equal to the ratio of the barometric pressure at the given altitude to the barometric pressure at sea level.

TABLE I.—CORONA LIMIT OF VOLTAGE—SOLID WIRES

(Kilovolts between lines, three-phase, sea level, 25° C)

Size, A.W.G.	Diameter, inches	Diameter, cm.	Spacing in feet									
			3	4	5	6	8	10	12	14	16	20
			Spacing in centimeters									
			91	122	152	183	245	305	366	426	490	610
4	0.204	0.520	51	54	56	58	60	62	64	65	66	68
3	0.229	0.581	...	59	62	64	66	68	70	72	74	76
2	0.258	0.655	...	...	69	70	74	76	78	80	82	84
1	0.289	0.734	...	...	75	77	81	83	86	88	90	92
0	0.325	0.826	...	...	...	85	89	92	95	97	99	102
00	0.365	0.928	...	...	...	94	98	102	105	107	110	113
000	0.410	1.04	...	...	...	...	109	113	116	119	121	124
0000	0.460	1.17	...	...	...	...	120	125	128	131	134	138

Agreement of Calculated and Measured Losses on Actual Transmission Lines. — In most cases where losses have been measured on transmission lines the check is in close agreement with calculated values. The agreement is in fact within the limit of error of the measurements on commercial lines. Apparent discrepancies have generally been due to the fact that the measurements are often made at voltages below the *visual critical voltage*, when the loss depends upon irregularities, as discussed above. The loss between the *visual critical voltage* and the *disruptive critical* voltage closely follows the quadratic law for large conductors, but for new conductors, and especially for small

TABLE II.—CORONA LIMIT OF VOLTAGE–7–STRAND CABLES
(Kilovolts between lines, three-phase, sea level, 25° C.)

Size A.W.G. or cir. mils	Diameter inches	Diameter cm.	Spacings in feet									
			3	4	5	6	8	10	12	14	16	20
			Spacings in centimeters									
			91	122	152	183	245	305	366	426	490	610
4	0.230	0.584	...	56	58	60	62	64	66	68	69	71
3	0.261	0.663	...	62	65	67	70	72	74	76	77	80
2	0.290	0.736	...	...	71	73	76	79	81	83	85	87
1	0.330	0.839	...	...	79	81	85	88	91	93	95	97
0	0.374	0.950	...	...	...	90	95	98	102	104	108	109
00	0.420	1.07	...	...	...	98	104	108	111	114	117	121
000	0.470	1.19	...	...	...	...	114	118	121	124	127	132
0000	0.530	1.35	...	...	...	...	125	130	135	138	141	146
250000	0.590	1.50	...	...	...	...	138	144	149	152	156	161
300000	0.620	1.57	...	...	...	...	...	151	156	161	165	171
350000	0.679	1.73	...	...	...	...	...	161	166	170	175	180
400000	0.728	1.85	...	...	...	...	...	171	176	180	185	192
450000	0.770	1.96	...	...	...	...	...	178	184	190	194	200
500000	0.818	2.04	...	...	...	...	...	188	194	199	205	210
800000	1.034	2.62	...	...	...	...	...	...	234	241	244	256
1000000	1.152	2.92	...	...	...	...	...	...	256	264	270	281

TABLE III.—ALTITUDE CORRECTION FACTOR AT 25° C.

Altitude		Correction factor	Altitude		Correction factor
Feet	Meters		Feet	Meters	
0	0	1.00	5,000	1525	0.82
500	152	0.98	6,000	1830	0.79
1000	305	0.96	7,000	2135	0.77
1500	459	0.94	8,000	2440	0.74
2000	610	0.92	9,000	2745	0.71
2500	765	0.91	10,000	3050	0.68
3000	915	0.89	12,000	3660	0.63
4000	1220	0.86	14,000	4270	0.58

conductors at high altitudes, when the difference between e_0 and e_v is quite large, the loss will generally fall below the quadratic curve.

Special Calculations. — For small conductors the loss, in kilowatts per mile of conductor, is given by the expression,

$$p = \frac{390}{\delta}(f + 25)\sqrt{\frac{1 + \frac{0.93}{s} + 0.016}{s}}\,(e - e_d)^2 \times 10^{-5},$$

where $$g_d = g_0\delta\left[1 + \frac{0.189}{\sqrt{\delta r}}\,\frac{1}{(1 + 1480r^2)}\right]$$

and $$e_d = 2.302 m_0 g_d r \log_{10}\left(\frac{s}{r}\right).$$

BIBLIOGRAPHY. — Peek, *Dielectric Phenomena in High Voltage Engineering;* Lewis, *Some Corona Tests*, G. E. Review, May, 1920; also numerous articles by Peek, Whitehead, Ryan, and others in the *Trans. A.I.E.E.*, 1908 to date and in the *Jour. A.I.E.E.* to date.

COUPLINGS, DIRECT.—(*See also Belts and Belting.*) Couplings for connecting electrical apparatus together or to other machines can be divided into two distinct classes, namely, solid and flexible.

SOLID COUPLINGS (Fig. 1) are usually of the flanged type and consist of two steel castings rigidly bolted together. This type should be used where the two machines are mounted on a common iron base and where an exact alignment is possible. Large flanges should be provided for overcoming bending stresses which may occur when the couplings carry part of the load such as, for example, in three bearing sets. See also article on *Machine Tools, Electrical Operation of.*

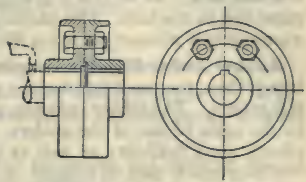

Fig. 1. Solid Flange Coupling

FLEXIBLE COUPLINGS are mainly used to connect machines which cannot be lined up properly or where there is fear that alignment cannot be maintained. Many different kinds of flexible couplings are in use, and they may be divided in two general classes, insulated and uninsulated. The insulated couplings are composed of castings separated by leather or rubber, as a flexible and insulating medium. The principal types of flexible couplings in use are the leather link, the laced belt, the rubber buffer and the mill-type coupling.

Leather-link Flexible Couplings (Fig. 2) consist of two iron castings connected together through leather punchings fastened at the ends by bolts to the alternate halves. For the smaller sizes these links are usually replaced by a single leather disk which connects both halves by means of bolts alternately fastened through opposite halves. The torque stresses are transmitted through these links or disks to the bolts which fasten them to the castings.

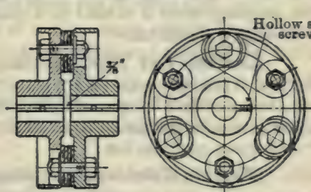

Fig. 2. Leather-link Flexible Coupling

In order to allow sufficient play for the heads of the bolts, alternate holes are bored to a large diameter, the bolts accurately fitting the other holes in the castings. By this means flexibility is obtained and a small amount of end or side play is permissible between the shafts of the two machines to which each half coupling is securely keyed. This type of coupling is recommended for shafts up to 3½ inches in diameter. For shafts between 3½ inches and 5 inches in diameter either this type or the leather-laced type may be used.

Leather-laced Flexible Couplings (Fig. 3) are recommended for shafts above 3½ inches in diameter on account of their structural advantages. They consist of two steel rings, an outer and an inner, with cast-iron hubs bolted to them. Slots are formed in these rings through which an endless leather belt is interwoven.

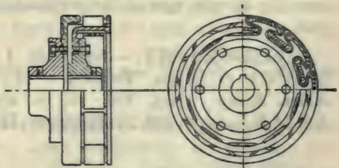

Fig. 3. Leather-laced Flexible Coupling

This construction not only gives great flexibility but, due to the two rings being concentric, it is not subjected to bending strains and therefore is espe-

cially adapted for transmitting a high torque commensurate with the strength of the belt and the size of the coupling employed.

As the outer ring of the coupling is only connected to the shaft of the machine through the coupling hub which is keyed to the shaft, machines using this form of coupling can be readily disconnected without unlacing or interfering with the coupling belt. To do this it is only necessary to remove the bolts, holding the outer ring to the hub. This partial disassembling also aids in the replacing of a worn-out belt without removing the coupling from the machine.

Rubber-buffer Flexible Coupling. — This coupling is made up of two cast-iron spiders, the small interlocking arms of which are separated by cylinders of soft rubber. The rubber cylinders are held in place by projecting plates screwed to the arms of one of the spiders. The construction of this type of coupling affords great flexibility as well as insulating qualities. This type of coupling was once much used on account of its high-insulating qualities but is now to a great extent being superseded by the two previous types.

Mill-type Flexible Coupling (Fig. 4). — Where the conditions are too severe for the belt or rubber of the flexible couplings described above, that is, where there is much grit or hot vapor present and where noise is not objectionable, the mill-type flexible coupling shown in Fig. 4 should be used. This is a rough sturdy coupling consisting of three steel castings, two of which are identical and called the "pods," while the center is called the "box." The most extensive use for this coupling has been in steel mills. Best results are obtained from this coupling when it is used on a constant load, as the noise under these conditions is at a minimum.

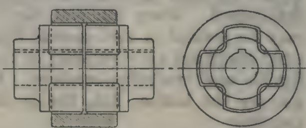

Fig. 4. Mill-type Flexible Coupling

Leather Key Type Flexible Couplings. — These are made in three distinct types, although all are constructed on the same fundamental principle, viz., two modified flanges, one driving the other by means of leather keys, the three types are known as the "single key," "two-piece multiple key," and the "three-piece multiple key."

Flexible Telescope Couplings. — This coupling is specially designed for use with motor driven Jordan engine in paper and pulp mills. On the Jordan side of the set it consists of a stout split cylinder on which are mounted four radial columns of leather washers securely bolted. The motor half is a split cylinder slotted to take the columns of the other half.

PROPER SIZE OF COUPLING. — The selection of the proper coupling should be determined by calculation and not by the method of choosing the bore corresponding to the shaft diameter. The coupling should have sufficient capacity to take care of the overload capacity of the motor or generator, as the case may be.

BIBLIOGRAPHY. — Smith, H. J., *Inertia Effects of Shaft Couplings*, Power, 1911, Vol. 33, p. 994; Bunnell, S. H., *Dimensions for Flanged Shaft Couplings*, Amer. Mach., 1903, Vol. 26, p. 1914; Ohnesorge, O., *Elastic Couplings*, A.S.M.E.J., 40, p. 718, 1918; Davey, H., *Flexible Couplings*, Eng., July 30, 1915.

CRANES. — (*See also Blocks and Tackle; Controllers; Hoists, Electric; Telpherage.*) A hoist is a machine for raising and lowering weights. A crane is a hoist with the added capacity of moving the load in a horizontal or lateral direction. That part of a crane carrying the hoist and movable with respect to the main structure of the crane is called the "trolley." Cranes are divided into two classes as to their motions, viz., rotary and rectilinear.

Rotary Cranes may be classified as follows:

Swing-cranes. — Having rotation, but no trolley motion.

Jib-cranes. — Having rotation, and a trolley traveling on the jib. When the jib or boom is movable, carrying a sheave on the end, the device is called a *derrick.*

Column-cranes. — Identical with the jib-cranes, but rotating around a fixed column (which usually supports a floor above).

Pillar-cranes. — Having rotation only; the pillar or column being supported entirely from the foundation.

Pillar Jib-cranes. — Identical with the last, except in having a jib and trolley motion.

Derrick-cranes. — Identical with jib-cranes, except that the head of the mast is held in position by guy-rods, instead of by attachment to a roof or ceiling.

Rotary cranes arranged so that they may be readily moved from place to place may be classified as:

Walking-cranes. — Consisting of a pillar or jib-crane mounted on wheels and arranged to travel longitudinally upon one or more rails.

Locomotive-cranes. — Consisting of a pillar-crane mounted on a truck, and provided with a steam-engine capable of propelling and rotating the crane, and of hoisting and lowering the load.

Rectilinear Cranes may be classified as follows:

Bridge-cranes. — Having a fixed bridge spanning an opening, and a trolley moving across the bridge.

Tram-cranes. — Consisting of a truck, or short bridge, traveling longitudinally on overhead rails, and without trolley motion.

Traveling-cranes. — Consisting of a bridge moving longitudinally on overhead tracks, and a trolley moving transversely on the bridge.

Gantries. — Consisting of an overhead bridge, carried at each end by a trestle traveling on longitudinal tracks on the ground, and having a trolley moving transversely on the bridge.

Rotary Bridge-cranes. — Combining rotary and rectilinear movements and consisting of a bridge pivoted at one end to a central pier or post, and supported at the other end on a circular track, provided with a trolley moving transversely on the bridge.

For descriptions of these several forms of cranes see Towne's *Treatise on Cranes.*

HAND-OPERATED TRAVELING CRANES. — The weight of a hand-operated traveling crane depends not only upon the capacity of the crane and the length of the span, but also upon the particular design. The following approximate formula will serve as a rough guide when actual data are not available:

$$W = KCL,$$

where W is the weight of the crane in tons, C the capacity of the crane in tons, L the length of the span in feet and K a constant ranging from about 0.01 for $L = 10$ feet to 0.012 for $L = 75$ feet.

ELECTRIC OVERHEAD TRAVELING CRANES. — Electric traveling cranes usually have 3 motors, one for the hoist, one for the trolley traveling on the bridge and one for moving the bridge. Cranes of over 15 tons capacity are usually provided with an auxiliary hoist of from 1/10 to 1/8 the capacity of the main hoist. Automatic brakes are used to sustain the load when lifted and to regulate the speed when lowering, the hoist motor usually having to *drive* the load down.

Speeds of Electric Traveling Cranes. — The standard speeds at which the various parts travel in the cranes manufactured by Pawling and Harnischfeger are as follows:

STANDARD SPEEDS OF ELECTRIC CRANES, IN FEET PER MINUTE
(Pawling & Harnischfeger)

Capacity, tons (2000 lb.)	Hoisting speed, ft. per min.	Bridge travel, ft. per min.	Auxiliary hoist	
			Capacity, tons	Speed, ft. per min.
5	25–100	300–450	..	......
10	20–75	300–450	3	30–75
25	10–40	250–350	3 10	50–125 25–60
40	9–30	250–350	5 10	40–100 25–60
50	8–30	200–300	5 10	40–100 25–60
75	6–25	200–250	15	20–50
125	5–15	200–250	25	20–50
150	5–15	200–250	25	20–50

Trolley travel speed from 100 to 150 feet per minute in all cases.

Weight and Dimensions of Electric Traveling Cranes. — Let

C = capacity of crane, in tons (2000 pounds),
W_B = weight of bridge alone, in tons,
W_T = weight of trolley alone, in tons,
W = total weight of crane, in tons.

Then

$$W_B = K_B CL, \qquad W_T = K_T C,$$
$$W = W_B + W_T = C(K_B L + K_T),$$

where K_B and K_T have the following approximate values (*see paper by S. S. Wales, Proc. Eng. Soc. Western Pa., 1902, Vol. 18, p. 146*).

L	K_B	C	K_T
25	0.012	1–25	0.3
50	0.012	25–75	0.4
75	0.013	75–150	0.5
100	0.015		

For example, for a 50-ton crane for a 60-foot span, the bridge alone would weigh $0.013 \times 50 \times 60 = 39$ tons, the trolley would weight $0.4 \times 50 = 20$ tons, and the total weight would be 59 tons, approximately. If the bridge is equipped with 2 wheels at each end, the maximum load on each wheel when the trolley, carrying full load, is at the end of the bridge would be approximately

$$\frac{39}{4} + \frac{20 + 50}{2} = 45 \text{ tons},$$

or if there are 4 wheels at each end, the maximum wheel load would be 22.5 tons.

The following table is taken from a catalogue of the Alliance Machine Co.

DIMENSIONS AND WHEEL LOADS OF ELECTRIC TRAVELING CRANES

60-foot span and 25-foot lift; wire-rope hoist

Capacity, tons (2000 lb.)	Distance from runway rail to highest point		Distance from center of rail to ends of crane	Wheel base of end truck		Maximum load per wheel; trolley at end of bridge
	Feet	Inches	Inches	Feet	Inches	Pounds
5	6	0	9	9	0	20,000
10	6	6	10	10	0	27,000
25	7	4	12	11	6	51,000
40	8	0	12	12	3	82,000
50	8	9	12	12	6	48,000*

* Has 8 track wheels on bridge.

Standard cranes are built in intermediate sizes, varying by 5 tons up to 40 tons.

Power Required. — The following is adapted from the paper by S. S. Wales above referred to. The frictional losses will of course depend to a certain extent upon the design of the crane, the use of rope or chain, etc., but the constants given below represent fair averages.

The same notation as in the previous section is used. In addition let

R_B = tractive effort in pounds per ton required to move the bridge,
R_T = tractive effort in pounds per ton required to move the trolley,
S_B = speed of bridge in feet per minute,
S_T = speed of trolley in feet per minute,
S_H = speed of hoist in feet per minute.

Then the power required, at shaft of bridge motor, to drive the bridge at speed S_B is

$$P_B = \frac{(C + W_B + W_T)\, R_B\, S_B}{33{,}000} \text{ horse-power.} \qquad (1)$$

The power required, at shaft of trolley motor, to drive the trolley at speed S_T is

$$P_T = \frac{(C + W_T)\, R_T\, S_T}{33{,}000} \text{ horse-power.} \qquad (2)$$

The power required, at shaft of hoist motor, to drive the hoist at speed S_H, assuming 60 per cent efficiency of gearing and hoisting tackle, is

$$P_H = \frac{CS_H}{10} \quad \text{horse-power.} \qquad (3)$$

For *constant-speed running*, Wales gives the following values for R_B and R_T, the former varying with L and the latter with C.

L	R_B	C	R_T
25	30	1–25	30
50	35	25–75	35
75	40	75–150	40
100	45	......	....

If the moving member is *accelerating* at the rate of a ft. per sec. per sec., to the value of R given in the above table should be added an amount

$$R_a = 64\,a.$$

The power as calculated by the above formulas is the power *output* of the motors; to obtain the power input divide by the motor efficiency, which ranges from 80 to 90 per cent.

Motor Equipment. — (*See also Motors, Industrial Applications of*). The bridge motor should have a 30 minute rating, 1.8 times the calculated power for constant full-load speed, the trolley motor 1.5 times the calculated power for full-load speed, whereas the rating of the hoist motor should be taken equal to the calculated power for constant full-load speed. This is a good approximation, but for accurate work the torque capacity of the bridge motor should be checked against the rate of acceleration required. A full discussion of the motor equipment of cranes is given by McLain and Jackson in the *Gen. Elec. Rev. 19, p. 506, June, 1916.*

Both direct-current and alternating-current motors are successfully used for crane work. The series direct-current motor has speed-torque characteristics especially well adapted for crane service. At light loads the speed increases and at heavier loads the lifting is slower and the raising effect consequently stronger. This characteristic of the series motor often results in considerable saving of time, due to the fact that the majority of loads in factories are light so that the increased speed of lift enables work to be performed in a shorter time. For a-c. motors a normal operating speed about 25 to 50 per cent higher than for d-c. motors is therefore generally selected and the slowing down for heavy work is accomplished by inserting resistance in the phase-wound rotor circuit.

Example. — Crane of 50-ton capacity, span 70 feet, bridge speed 200 feet per minute with full load, trolley speed 100 feet per minute with full load, hoist speed 15 feet per minute with full load. Then $C = 50$, $W_B = 45.5$, $W_T = 20$, $R_B = 40$, $R_T = 35$, and

$$P_B = \frac{115.5 \times 40 \times 200}{33{,}000} = 28 \text{ horse-power,}$$

and the motor required for the bridge would be $28 \times 1.5 = 42$ horse-power, or the nearest commercial size over 42, say 50 horse-power.

$$P_T = \frac{70 \times 35 \times 100}{33{,}000} = 7.43 \text{ horse-power,}$$

and the size motor required for the trolley would be $7.43 \times 1.25 = 9.28$ horse-power, or say 10 horse-power.

$$P_H = \frac{50 \times 15}{10} = 75 \text{ horse-power},$$

and a motor of this rating would be used for the hoist.

Control of Crane Motors. — The control equipments for crane motors should be of the regulating and reversible type. The starting and speed regulation of series motors is generally accomplished by inserting resistance in series with the armature and field, and with induction motors, resistance is inserted in the phase-wound secondary or rotor circuit. For cranes which do a large amount of lowering, dynamic control is becoming very generally used, and is very readily accomplished with direct-current motors. This also occasionally is used with a-c. motors of 200 h.p. and above, when direct current is available for excitation. On smaller a-c. motors a system of regenerative lowering in combination with solenoid load brakes is used.

Drum Versus Magnetic Controllers. — Both hand- and magnetic-control equipments are in general use. The former are satisfactory for small and medium-size motors, and consist simply of a drum-type controller with a set of separate resistances. For large-size motors too much physical effort would be required to move a controller of the necessary size and the magnetic control should then be selected. A magnetic-control equipment consists of a master controller, a contactor panel and the resistances. The contactor panel contains the contactors for cutting in or out the resistances, the interlocks and the current-limit relays for automatically controlling the sequence and rapidity with which the contactors operate. The master-controller handle can be thrown to the full-speed position quickly without causing an overload on the motor, since the current-limit relays automatically prevent the contactors from cutting out the resistances too rapidly.

Dynamic Control. — With dynamic control of d-c. motors the field is separately excited and the armature is connected to the line voltage with one section of a rheostat in series with it, and another section of the rheostat in parallel with it. This is accomplished by means of the controller, whose duty is to make each step on the lowering side keep the speed of the motor under control no matter whether the motor has to drive a light hook downward or hold back against a heavy load. In the former case the current which passes through the series rheostat also passes through the armature and produces the desired torque. The speed is controlled by varying the value of this rheostat just as in hoisting. If the motor has to act as a brake, the power generated in the armature is expended in the parallel rheostat and the speed is controlled by varying this rheostat. In actual practice the two rheostats are varied simultaneously so that if a certain point on the controller tends to cause the motor to drive a load at a high speed it will also cause the motor to hold back at a high speed. The practical result is that an operator always has his load under control, and does not have to worry about dropping his load. On the lowering side of the controller, a motor holds back against its load even when power fails, because the motor then acts as a self-excited series generator, and consequently the solenoid brake alone is not depended on to prevent the load from falling.

In the case of a phase-wound induction motor it is necessary to excite one portion of the primary winding with direct current in order to generate a voltage in the secondary winding. A rheostat is then connected to the secondary windings of the motor and the speed can be controlled as with d-c. motors. As direct current is necessary for excitation dynamic braking is not very often used with a-c. crane motors.

Solenoid brakes are also provided for holding the load when the hoist motor is stopped. If dynamic braking is not provided, mechanical brakes must be provided in addition to the solenoid brakes to assist in holding the load when the motor is stopped and for regulating the speed when lowering.

Solenoid brakes are, as a rule, applied by gravity when the armature current is shut off and released by the solenoid which is energized when the controller is thrown on the first notch. The solenoids of direct-current motors are generally connected in series with the motor circuit, although with polyphase induction motors they are connected directly across one phase thus receiving the full line voltage.

All crane motors should also be protected by safety-limit switches arranged to cut off the supply current when the limit of motion is reached and thus automatically provide against accidents from over-travel.

SPECIFICATION FOR ELECTRIC CRANE. — The following memoranda are intended to assist in writing specifications. See also *Specifications*.

Principal Characteristics and Conditions of Service. — Span between crane rails and load to be lifted on main hoist and on auxiliary hoists, if any. Characteristics of current, a-c. or d-c. (phases, cycles and voltage).

Style and Description of Apparatus. — Structural work of open-hearth steel according to some standard steel specifications. Maximum stresses allowable. Whether hand operation is to be provided for. Distance of hook from crane rails, vertically and horizontally, when at extreme limits of its motion. Details of brake. Description and characteristics of motors to be supplied. Whether or not a foot-walk is to be supplied. Whether trolley is to be above bridge or submerged. Hoisting drum to have bronze bearings. Track wheels shall have chilled treads, ground true, with double flanges, and shall be keyed to their axles. Axle bearings shall be bronze. Details of lubrication and bearings. Location of carriage. Details of control. All gearing (except when shrouded) shall be cut, etc. Wiring details. Controller details.

Dimensions. — Supply a diagram of clearances.

Work Done by Other Contractors. — Track rail and weight thereof.

Performance and Tests. — Speeds of main travel, lateral travel and hoists. Time to get up speed or acceleration rates.

COST OF CRANES (Pre-war figures). — The cost of cranes depends to so great an extent upon the design, motor equipment, etc., that no reliable average figures can be given. As a rough approximation the cost of a hand-operated traveling crane may be taken as from about 5 to 7 cents per pound of total crane weight and the cost of an electric traveling crane as from about 10 to 13 cents per pound of total crane weight.

BIBLIOGRAPHY. — Anton Böttcher, A., *Cranes*, trans. by *A. Tolhausen;* Hess, H. D., *Machine Design, Hoists, Cranes;* Broughton, H. H., *Electric Cranes;* Wilda, Herman, *Cranes and Hoists;* Brown, R. B., *Designing Electric Cranes and Brakes*, Machinery's Reference Series No. 47; Hill, C. W., *Electric Crane Construction;* Circulars of Morgan Engineering Co., Alliance, O., Pawling and Harnischfeger, Milwaukee, Wis.; Yale and Towne Mfg. Co.

CROSS ARMS. — (*See also Poles for Overhead Lines; Insulator Pins; Insulators.*) Cross arms are usually of wood though sometimes of iron. "Buck arms" or "reverse arms" are cross arms attached to a pole at right angles to the principal arms, and are used for taking off wires at right angles to the line, either at the junction of intersecting lines or at services. "Double arms" are pairs of cross arms attached to opposite sides of a pole so as to act as one compound arm. Double arms are used to increase the strength of an arm and to permit the use of two pins and two insulators for supporting a single wire where additional strength is required.

Cross arms are used principally for supporting pins, insulators and wires, though lightning arresters, transformers, switches and other miscellaneous appliances are often mounted on them, usually for the purpose of keeping the pole free of incumbrances so that it will be more easy to climb.

Alley Arms. — When city distribution lines are located in alleys it is common to locate poles next to the property line. Where it is not permissible to let arms overhang private property special arms may be used which extend on one side of the pole only. These must be well braced and should not be used for dead ending wires. A better construction is obtained by locating two poles on opposite sides of the alley and putting special cross arms across the alley between them.

FORCES ON AND STRESSES IN CROSS ARMS. — The forces which a cross arm resists are:

(a) Vertical forces due to weight of pins, insulators, wires (with sleet) and accidental loads due to linemen standing on arms, etc.

(b) Transverse horizontal forces due to wind pressure on the wires (with sleet) at right angles to the line.

(c) Longitudinal horizontal forces due to the pull of the wires where the pull is unbalanced. Unbalanced pull is usually due to an angle in the line, the ending of the wire at the arm, a change in the size of wire at arm and an unequal tension in the spans on the two sides of the arm.

The principal internal stresses produced in an arm from these forces are:

(1) A bending force in a vertical plane due to vertical forces (a).

(2) A bending force in a horizontal plane due to horizontal forces (c).

(3) A twisting force about the longest axis of the arm due to the "pin leverage" of the horizontal forces (c). The pin leverage is the distance from the center of the wire to the axis of the arm.

Of these stresses the most destructive is probably the twisting stress which tends to split the arm in a vertical plane through the pin holes and along the grain of the wood. On this account the pin and insulator should be no taller than necessary, the pin should extend completely through the arm to give the best distribution of bearing pressure and, where stress is heavy, the arm may be strengthened by two "strengthening bolts" (machine bolts with nut and washers) put horizontally through the arm one on each side of the pin, one being near the top and the other near the bottom of the arm.

The vertical and horizontal bending stresses are of some importance and may be computed by the usual beam formula (*see Structures, Simple*). Data of tests on strength of cross arms for these stresses are given below.

Strength Tests of Cross Arms. — (*Forest Service, Cir. No. 204.*) Tests made on 3¾ in. by 4¼ in. by 6 ft., 6-pin air-dried cross arms with vertical load distributed equally at each pin hole gave the results at top of next page.

The tests showed that for ordinary use the strength, for vertical loads of 6-pin arms, need not be considered in calculations of line construction, except in the rare case of abrupt change of the grade of the line.

Kind of wood	Av'ge max. load in lb.
Longleaf pine, 75 per cent heart..........	10,180
Longleaf pine, 100 per cent heart.........	9,780
Shortleaf pine..........................	9,260
Longleaf pine, 50 per cent heart..........	8,980
Shortleaf pine, creosoted................	7,650
Douglas fir.............................	7,590
White cedar.............................	5,200

DIMENSIONS AND RATING. — Cross arms are rated in terms of the number of pins they are designed to carry, 2-pin arms being the minimum and 10-pin arms the maximum ordinarily used.

Cross-section. — When arms are made directly from the original logs they are sometimes dimensioned in even inches, i.e., 4 in. by 5 in., 5 in. by 7 in., etc. In most cases they are made from lumber in stock and when finished are a quarter of an inch or more under the even sizes. Several "standard" dimensions have been adopted for distribution arms. In the accompanying table Standard I is that given by Miller (*American Telephone Practice*) and found in various catalogues of dealers in electrical supplies. It is the standard of the American Electric Railway Engineering Association for light service cross arms. Standard II is that given in Report of Committee on Overhead Line Construction. of the National Electric Light Association, and Standard III is that of the American Electric Railway Engineering Association for heavy service cross arms

Size of arms	Width, inches	Depth, inches
Standard I....	3¼	4¼
Standard II....	3½	4½
Standard III....	3¾	4¾

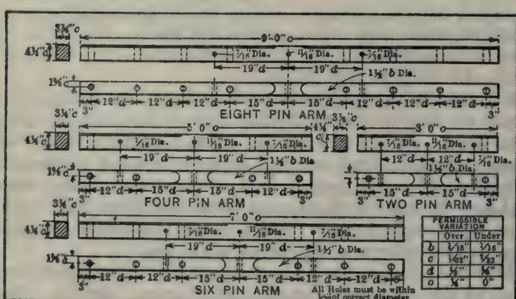

Fig. 1. Standard I Cross-Arms

Distance between Pole Pins. — It is desirable to have the space between pole pins (i.e., the two pins on opposite sides of the pole) as large as possible to give a clear space for climbing the pole. It usually varies from 16 in. to 30 in.

Distance between Other Pins. — This usually ranges from 10 in. to 21 in. Under ordinary circumstances 12 in. has been found sufficient to prevent wires on adjacent pins swinging in contact in the middle of the span. Where small wires (such as No. 10 telephone wires) are used on long spans (150 ft. or more) contact is to be expected occasionally where 12 in. spacing is used.

Distance from End Pin to End of Arm. — This is usually 3 or 4 in., which is about the minimum that can be used without the arm splitting due to pin leverage.

In order that a line may have a neat systematic appearance it is desirable that the pin spacing be the same for all arms, whether 2 pin, 4 pin, etc. In the tabulations below the arms included in each standard (defined above under *Dimensions and Rating*) have been grouped on this principle which shows that Standard I really consists of one regular group and 5 odd arms while Standard II consists of 2 regular and 1 odd arm. Referring to Standards I and III it will be noted that the length of arm is always a multiple of one foot.

STANDARD DISTANCES BETWEEN PINS

Standard	Group	Number of pins	Length of arm	End pin to end of arm	Between pole pins	Between other pins
			Ft. In.	In.	In.	In.
I	Regular	4	4 0	4	16	12
	Regular	6	6 0	4	16	12
	Regular	8	8 0	4	16	12
	Regular	10	10 0	4	16	12
	Odd	2	3 0	4	28	..
	Odd	4	5 0	4	18	17
	Odd	4	6 0	4	22	21
	Odd	4	6 0	4	24	20*
	Odd	6	8 0	4	18	17½
	Odd	8	10 0	4	17½	15¾
II	Regular	4	5 7	4	30	14½
	Regular	6	8 0	4	30	14½
	Odd	8	9 2	4	30	12
III	Regular	2	3 0	4	28	..
	Regular	4	5 0	4	28	12
	Regular	6	7 0	4	28	12
	Regular	8	9 0	4	28	12
	Regular	10	11 0	4	28	12

* Alternative found in some catalogues.

Pin Holes. — 1½ in. diameter is standard for wood pins. N.E.L.A. specifications require that holes shall be tested with steel gauges taking one of the nominal size without forcing but not taking one 1/32 in. larger.

Bolt Holes. — For fastening the arm to the pole there is usually one 11/16 o. ⅝-in. bolt hole at the middle of the arm. For fastening cross-arm braces there are two 7/16 or ⅜-in. bolt holes located 38 in. apart (Standard II) or 40–42 in. apart (Standard III).

High-tension Wooden Cross Arms.—These are not standardized but are specially designed and made as required. They differ from distribution arms in being wider and deeper with larger pin holes and wider pin spacing. The

following tabulation will give data on some arms that have been used for different voltages.

Voltage	Number of pins	Width of arm	Depth of arm	Length of arm	Spacing of pins	Pin hole diameter
		Inches	Inches	Inches	Inches	Inches
25,000	2	6	6	64	48	2¼
55,000	2	5	7	88	72	2½

Special high-tension arms are usually unnecessary for circuits of 5500, 6600, 11,000 and 13,200 volts. Standard 2200-volt distribution arms may be used, the wire spacing being increased by not using certain pins. Thus a standard 2200-volt 6-pin arm may be used for a 11,000-volt 4-pin arm by leaving the middle pin holes vacant. For higher voltages the strength of arm and size of pin hole is usually insufficient.

SPECIFICATIONS. — (*See article on Specifications.*) The points to be covered in a set of specifications are:

Material. — Usually pine or fir, sometimes western hemlock. (*See Poles for Overhead Lines.*)

Quality of Material. — Usually should be "straight grained, free from knots (or free from large, loose or unsound knots), sapwood, pitch pockets, shakes, checks, loose heart, rot, worm holes or other defects." The strictness of the specifications, or of the inspection, must, however, be governed in these respects by local conditions. In some localities a quality of material can be obtained cheaply, and should be required, which would be unreasonable to expect where wood is expensive.

Wood should be dried before manufacture. (When arms are dried after drilling, the pin holes become elliptical, i.e., smaller across the grain, due to shrinkage of the wood and the pins do not fit well.)

Dimensions, Pin holes, etc., as noted in preceding section.

COST. — The cost of cross arms is subject to considerable variation and depends largely upon the locality in which they are purchased. The following figures are rough approximations and are exclusive of freight.

APPROXIMATE COST OF CROSS ARMS

Cents per linear foot

Kind	Washington fir, F.O.B., Seattle	Longleaf pine, "75 per cent heart," F.O.B., New Orleans	Commercial yellow pine F.O.B., New York
3¼ by 4¼ inches.....	5	6	5
3½ by 4½ inches.....	7	10	...
3¾ by 4¾ inches.....	8	...	...

In car load lots about 25 per cent less than above.

BIBLIOGRAPHY. — American Electric Railway Engineering Assn. *Engineering Manual*, and Committee Reports; Kapper, F., *Overhead Transmission Lines and Distribution Circuits.* New York, 1915; National Electric-Light Assn. Committee Reports; Lundquist, R. A., *Transmission Line Construction*, N. Y., 1912; U. S. Forest Service Circulars.

DAMS. — (*See also Hydraulics; Hydrology; Power Stations; Structures, Simple*). A dam is a structure built to interrupt a stream's flow and raise the level of the water, thereby impounding water which may be used for power development, water supply, navigation, irrigation or numerous other purposes.

Coffer-dam. — A dam built for the purpose of holding back water from an area, which would otherwise be flooded, and making it accessible for construction purposes, is called a coffer-dam. It is generally a temporary affair.

Diversion Dam. — This name has been applied to dams built for the purpose of diverting the whole or a portion of a stream into a side channel or pipe line by which it is conducted to one or more places where it is used for power or other purposes.

Storage Dam. — A storage dam is built for the purpose of storing flood waters for use during periods of low stream flow. In many power developments a storage dam is essential to continuous output. Storage dams are sometimes used as diversion dams.

Wing Dam. — In swiftly flowing streams having steep slopes, a diversion dam is sometimes built out from one shore diagonally upstream toward the other, but not to it. This special form of diversion dam is known as a wing dam.

Classification with respect to Materials and Design. — According to the materials used and the design, dams may be classified as:

Wooden dams	Masonry dams
Earth dams	Solid gravity
Rock dams	Hollow gravity
	Arched

By a **gravity dam** is meant one in which the force of gravity is relied upon for resisting the thrust and overturning moment of the water. Earth and loose rock dams, timber cribs and most masonry dams are of this type.

By an **arched dam** is meant one which in plan forms an arch, convex to the up-stream side. The ends of the arch terminate in the valley walls, which thus act as abutments to the arch so formed.

Often a gravity dam has been built in the form of an arch, using the possible arch action as a factor of safety, and depending wholly on the gravity action to resist the thrust of the water. Many of the highest dams in existence are so constructed and are generally in localities where the stream's valley is relatively narrow and deep.

LOCATION AND HEIGHT OF DAMS. — For power developments the location of the dam depends largely on the topography and geological formation of the site. The total fall in a stream to be utilized for power may be developed in whole by a dam or in part by a dam and in part by a canal, flume or pipe to the power-house. Thus the location and height of the dam is fixed by economic considerations to develop the whole head at the least cost, consistent with safety and reasonable operation and maintenance expense. The cost of lands to be overflowed has considerable influence.

WOODEN DAMS. — Wooden dams are made in a variety of forms. The most common form of wooden dams consists of a crib of timbers filled with broken rock and covered on the top and up-stream face with wooden sheeting.

EARTH DAMS. — A typical earth dam is indicated in Fig. 1. No dam is absolutely watertight. The successful construction of a dam of any type embodies the condition that leakage or seepage will not be sufficient to endanger

the structure or its foundation and that an excessive amount of impounded water will not be wasted.

An earth dam and its foundation must be made sufficiently tight that the seepage of water through it will not be great enough to move the minute particles of which it is composed.

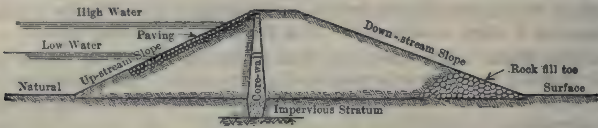

Fig. 1. Typical Earth Section

Materials and Details. — The materials used in the construction of earth dams are loam, sand, gravel and clay of various proportions. The dam is approximately trapezoidal in section, having a top width and side slopes which are determined by the materials used. The best of the available materials are used to provide a selected impervious part. The impervious part or core may be placed near the upstream face; but, as impervious materials are relatively unstable when saturated, the core is usually placed in the center of the dam and supported on each side by the more stable pervious materials. The downstream toe is often built of loose rock or otherwise well drained to prevent slipping when saturated by water seeping through the dam.

Cores of masonry, clay puddle or other materials are sometimes built in the center of the dam, extending into the foundation to an impervious stratum. Timber cores above the water gradient are not sufficiently durable for a permanent structure. Masonry core walls usually have a top width of from 2 to 4 feet with side batters of ½ to ¾ inch per foot down to the foundation, but, if well reinforced may be much thinner. There is a divergence of opinion among engineers regarding the practical use of masonry corewalls for earth dams. However, it has not been shown that they impair the safety of the dam and they are often imperative to guard against burrowing animals.

The downstream slope of the dam should be protected from rain scour by sodding or seeding it. For long slopes, provision is often made for draining off the rain water by building berms, a few feet wide every 30 feet or so on the slope, containing gutters for lateral drainage.

Riprap, hand laid paving or masonry is used on the upstream face to protect it from the action of waves and ice. Solid paving should be laid on a layer of well-drained broken stone or gravel to prevent internal pressure on the paving when the pond is drawn down rapidly.

Dimensions. — The top width of the dam is often determined by requirements for a roadway. Where not so fixed it may be made approximately equal to one-fifth the height of the dam plus five feet. The top of the dam must be built sufficiently high to prevent overtopping by waves during extreme high water.

Simple rules cannot be given governing the side slopes of earth dams. From the nature of the materials available, this angle of repose when placed in the dam must be determined by tests or practical experience. The upstream face is continually saturated and the downstream face is more or less damp, rendering the materials less stable than when in a dry embankment. High dams are usually given flatter slopes than low dams and often the lower part has flatter slopes than the upper part. Low dams often have up and downstream slopes

of 2 to 1. The Ticitus Dam (New York State) is 110 feet high, has an upstream slope of 2.4 to 1 and a downstream slope of 2.5 to 1 with a 30 foot top width. The Gatun Dam (Panama) is 115 feet high, has an upstream slope of 4 to 1 and a downstream slope of 16 to 1 with a top width of 100 feet. It is thus seen that the variation in practice is very great, depending upon the nature of the materials and the methods of placing them in the dam.

Foundations. — In preparing the foundation for earth dams the surface soil is stripped to a depth sufficient to reach sound material, and all loose stones, stumps, roots, etc., removed. If the foundation is rock, the surface is cleaned to aid in making a good bond with the earth. The foundation should be examined for perviousness. If deemed necessary, one or more trenches should be dug to an impervious stratum and refilled with impervious material. Between an earth dam and a rock foundation is a plane of weakness which is usually guarded against, in the absence of a masonry corewall, by building several low walls of masonry across the site to resist seepage of water along the rock surface.

Construction of Rolled Earth Dams. — The material for rolled earth dams is placed in layers 6 inches to 12 inches thick and well rolled with a grooved roller or otherwise compacted while damp. Sprinkling may be necessary during the compacting process. It is often provided that each finished layer shall be harrowed and dampened to provide a bond with the next layer above.

Hydraulic Fill Earth Dams. — Hydraulic fill earth dams are constructed by sluicing the materials into place. If available material is found at an elevation higher than the crest of the dam and other conditions at the site are favorable, the materials are washed out of the borrow pits by a stream of water under heavy pressure, collected by the flow of the water and transported by gravity through sluices to the site. The materials are discharged from the sluices near the edges of the dam where the coarser particles, possessing the greatest angle of repose, remain. The balance of the material is washed by the water towards the center of the dam, the particles depositing automatically and progressively according to size until only the finest particles reach the center of the dam. Here a pond of water is allowed to remain through which the fine materials settle to form an impervious core.

When the materials are not located at an elevation sufficiently high to permit of sluicing to the site, they are transported by cars, carts or other means to the edges of the dam and sluiced into place as above described by streams of water.

The greatest difficulty in the construction of hydraulic fill dams is to obtain a core which is composed of sufficiently fine materials to make a tight dam and still possess adequate stability for a firm structure. If the material in the core is too fine to drain properly as the construction progresses, it will exert virtually a liquid pressure on the outside embankments which may be sufficient to cause them to slip. The degree of fineness in the core can be regulated by adjusting the elevation of the pool at the center of the dam, the deeper the pool the finer the size of particles deposited in the core.

ROCK DAMS. — In localities where earth is not available, broken rock has been used in the construction of dams. The rock fragments are of a size that can be most conviently quarried and handled. Such dams are made tight by a masonry corewall at the middle of the dam or on the upstream face. The natural angle of repose of loose rock is about 1 to 1 but the side slopes of rock dams should not be steeper than 1½ to 1 unless the face is hand laid to a considerable thickness and in any case not steeper than 1¼ to 1.

MASONRY CONCRETE DAMS. — During the latter part of the last century, rubble masonry was used extensively for the construction of dams; but in recent years this type has been practically superseded by plain and reinforced concrete. In massive structures cyclopean concrete is generally used, consisting of plain concrete containing a liberal percentage of large stones or "plums." The usual mix for masonry dams consists of 1 : 3 : 6 cyclopean concrete for solid gravity and large arch dams; 1 : 2 : 4 to 1 : 2½ : 5 reinforced concrete for thin arch dams; 1 : 2 : 4 reinforced concrete for decks and struts of hollow dams and 1 : 2½ : 5 to 1 : 3 : 6 reinforced concrete for buttresses of hollow dams. Special provision for water tightness must be made, particularly for solid gravity dams where water pressure in horizontal joints increases the overturning moment to be resisted.

Foundations for Masonry Dams. — Most masonry dams over 30 feet high require solid rock foundations. From this all soil and disintegrated rock is removed and the surface scrubbed clean. All cracks and fissures at the site and for some distance above it must be located and grouted. Often, in poor rock, a longitudinal trench is taken out and refilled with masonry to form an impervious barrier to the passage of water. This is placed at the upstream face or heel of the dam (see Fig. 3).

For dams on earth foundations, a masonry or sheet piling cut-off is built into the foundation at the heel of the dam extending down to a stratum of impervious material. Where this is impractical, undermining by seepage may be prevented by providing a path of enforced percolation of sufficient length. This varies from 5 to 18 times the head on the dam according to the materials of the foundations. The proper length of path may be obtained by one or more cut-offs, by a downstream apron, by an upstream apron or by a combination of these. The length of the path of percolation, as affecting uplift and erosive force is the length of the actual plane of contact between the structure and the earth including aprons and both sides of all cut-offs, providing the cut-offs are not closer together than twice their depth. The foundations below the dam must be adequately protected from wash by proper paving.

Dimensions and Cross-sections of Solid Gravity Dams. — Typical high solid gravity non-overflow and spillway dams are indicated in Figs. 2 and 3.

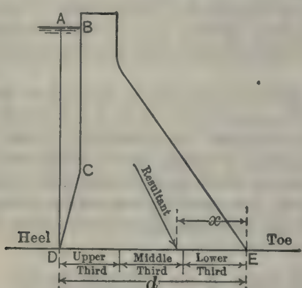

Fig. 2. Typical Solid Gravity Non-Overflow Masonry Section

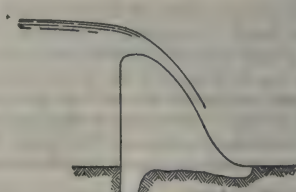

Fig. 3. Typical Solid Gravity Spillway Masonry Section

The top width of non-overflow dams is generally made from 10 per cent to 14 per cent of the height of the dam and is placed at or a few feet above high water sur-

face. For spillway dams the top is made approximately parabolic to conform to the natural path of the water as it leaves the crest. If the descending sheet falls free of the face of the dam, a partial vacuum will occur behind it and the atmospheric pressure on the upstream side will become effective as an overturning force. The curve or "bucket" at the downstream side of the base is for the purpose of deflecting the water and protecting the foundation. This may be omitted for low dams on hard rock.

The proportions of the section below the top are possible of mathematical determination once the forces acting on it are known. A common method is the "cut and try" by which a profile is first assumed and then investigated with regard to the conditions of stability previously given. It is then adjusted and again tested until it results in a satisfactory section. It is customary to start at the top and design the dam progressively towards the base.

A series of imaginary horizontal joints are arbitarily assumed. When the design has been tested for stability at the highest joint and has been properly adjusted to conform to the conditions of stability, the section of the dam is fixed and is not influenced by the design of the lower part of the dam. The design above the next lower joint is then tested and adjusted by changing the slope of the up- or down-stream faces or both between that and the preceding joint. The section to this joint is then fixed and other joints treated in like manner. This method of design applies to all types of gravity dams. See section below on *Forces Acting on Dams.*

Hollow Concrete Dams. — A hollow concrete dam consists of an inclined decking of reinforced slabs or arches resting on a series of parallel buttresses

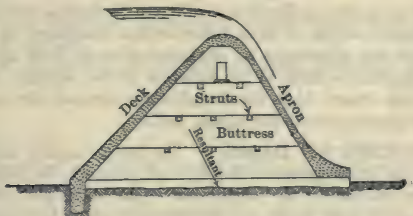

Fig. 4. Typical Hollow Gravity Spillway Section

which have their footings on the foundations. Fig. 4 shows a section of such a dam designed for a spillway. With a low dam and rock foundation, the downstream decking or apron may be omitted and the face made vertical.

Stability against overturning and sliding is obtained by providing a considerable batter to the upstream face, thereby obtaining a large vertical component of head water pressure to add to the weight of the structure.

Arched Concrete Dams. — A typical arch dam is indicated in Fig. 5. A dam of this type consists of a single masonry arch spanning the valley and is adaptable to narrow sites having steep rocky sides.

Arch dams are usually treated as sections of simple cylinders and the thickness at any elevation determined by the equation

$$t = \frac{pr}{s},$$

where t = the required thickness in feet,
p = the horizontal load on the dam in pounds per square foot,
r = the radius of the dam in feet,
s = the allowed unit stress in pounds per square foot.

As the arch is fixed at the base and at each end, the actual stresses are influenced by expansion and contraction due to changes in temperature and moisture

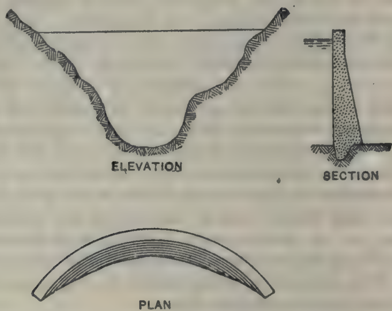

Fig. 5. Typical Arch Dam

contents, contraction due to setting of the concrete and the application of the loading, variations in span and radius at different elevations and other considerations. Theoretical analysis of these influences are considered by most engineers to be impractical and allowances for deviation of stress from that indicated by the foregoing equation is made in the adoption of low working stresses. These stresses should not exceed ⅛ to 1/12 the ultimate stress of the masonry or from about 30,000 to 40,000 pounds per square foot.

FORCES ACTING ON DAMS. — The forces usually considered in the design of dams are water pressure, earth or silt pressure, ice pressure, weight of the dam and the reaction of the foundation. One linear foot is usually considered in the design of solid dams and the distance center to center of buttresses in hollow dams.

Water Pressure. — The weight of water per cubic foot is usually taken equal to 62.5 pounds.

It facilitates computations if the horizontal and vertical components of the water pressure are considered separately instead of the resultant pressure.

When the water surface is at or above the top of the dam, the total horizontal pressure, in pounds is,

$$P = 62.5\frac{(h_2^2 - h_1^2)l}{2},$$

where h_2 and h_1 are the vertical distances in feet of the bottom and the top of the dam respectively below water surface, and l is the length in feet of dam considered in the design. The location of the resultant horizontal pressure above the base of the dam is

$$x = \frac{3h_1h + h^2}{6h_1 + 3h},$$

where h is the height of the dam.

If the water surface is below the top of the dam, the total horizontal pressure is

$$P = 62.5\frac{h_3{}^2 l}{2},$$

where h_3 is the depth of water in feet. The location of the resultant above the base of the dam is

$$x = \frac{h_3}{3}.$$

For vertical faces the vertical component of water pressure is zero; but for inclined faces the vertical component is the actual weight of water $A\ B\ C\ D$ (Fig. 2), directly above the face. The resultant vertical water pressure passes through the center of gravity of $A\ B\ C\ D$.

The uplift pressure per square foot of the base of solid dams, due to water leaking through the dam, is,

$$p = 62.5\ ch_v,$$

where c is the percentage of the area assumed to be subjected to uplift. The head, h_v, is considered to vary uniformly from headwater pressure at the heel to tail water pressure at the toe. The value of c cannot be unity unless the dam is on earth foundations. For rock it has been assumed variously from zero to 0.66 according to the nature of the foundation.

Total uplift pressure is,

$$P = 62.5cdl\frac{h'_v + h''_v}{2},$$

where h'_v and h''_v are head and tail water pressures respectively, d is the width and l the length of the base.

The distance of the resultant uplift pressure from the heel of the dam is,

$$x = \frac{2h'' + h'}{3(h'' + h')}d.$$

Uplift is also considered as existing in any horizontal building joint as well as at the base. Drains are sometimes placed in dams to reduce possible head water uplift pressure. Head water uplift pressure is never considered to exist to an appreciable extent in hollow dams. Uplift from tail water is always present.

Earth Pressure. — Submerged earth or silt deposited against dams exerts a pressure on the structure in addition to the water pressure. The amount and location of this pressure can be determined by the ordinary methods for retaining walls if the weight and angle of repose when submerged are used. An approximate method for determining silt pressure is to assume it a liquid weighing 62.5 pounds. It will then exert a pressure equivalent to additional water pressure of the same depth.

Ice Pressure. — Much uncertainty exists in the determination of ice pressure on dams. Pressures amounting to 47,000 pounds per lineal foot of dam have been used in the design of important dams in cold climates. The configuration of the gorge above the dam has considerable influence on the possible ice thrust. The reader is referred to the late C. L. Harrison's paper on this subject (*see Bibliography*).

Weight of the Dam. — The weight of masonry varies from 130 to 165 pounds per cubic foot. The weight most commonly adopted for concrete dams is 140 to 150 pounds per cubic foot.

Reaction of the Foundation. — The resultant of all forces acting on the dam, including the weight of the dam, must be balanced by an equal and opposite reaction of the foundation.

Requirements for Stability. — A dam may fail by sliding, by overturning or by crushing of the materials in the dam or the foundations. This may occur at the base or at any horizontal plane above the base. Sliding has been known to occur on clay-filled seams below the base after the foundation near the dam has become eroded by the flowing water. A dam must therefore be investigated for stability at various elevations, considering for each elevation only those forces acting above it as though the base of the dam or a horizontal building joint actually existed at that elevation. The following are the fundamental conditions for the design of gravity dams.

(*a*) At any elevation the resistance to sliding shall be greater than the horizontal component of all forces acting on the dam above that elevation. Adhesion of the dam at any horizontal joint or at the base is not considered except as an additional factor of safety. The angle that the resultant of all forces makes with a vertical line must therefore not exceed the angle of repose of masonry on masonry or masonry on the material composing the foundation, as indicated by laboratory tests of smooth specimens. The natural or artificial roughness of rock foundations and projecting plums in horizontal joints provide the necessary margin of safety. For dams on good rock the tangent of the angle of inclination of the resultant force with the vertical should not exceed 0.75. For dams on earth foundations the tangent should not exceed 0.1 to 0.2, depending upon the materials, or the dam should be well anchored to deep cut-off walls or piling.

(*b*) To prevent overturning, the resultant of all forces above any joint must pass inside the extremity of the joint. To provide a margin of safety and to prevent the existence of tension the resultant must intersect the joint at a distance from the extremity equal to at least one-third the width of the joint. In other words, the resultant must fall within the "middle third" of the joint, and this applies to the condition of full or empty reservoirs, as tension is always objectionable. It can be shown that, if the resultant falls within either outside third of the joint, tension will exist at the opposite side of the dam.

(*c*) The compressive stresses in any part of the dam must not exceed safe working values. The unit stress on any joint is assumed to vary as a straight line from the heel to the toe. If the resultant force intersects the joint at its exact center, the unit stress will be uniform over the joint. If the resultant intersects at any other point, the stress will be a maximum at the edge of the base nearest the point of intersection and a minimum at the other end.

In Fig. 2, let

p' = the unit *vertical* stress at E;
p'' = the unit *vertical* stress at D;
x = the distance from the resultant to point E;
d = the width, DE, of the joint;
l = the breadth of the joint;
w = the vertical component of all forces on the dam above the joint including uplift at the joint;
p'_v = the unit uplift at point E;
p''_v = the unit uplift at point D.

Let these quantities be expressed in feet or pounds per square foot. Then, if the resultant intersects the joint within the middle third and the joint is rectangular,

$$p' = \frac{2w}{ld}\left(2 - \frac{3x}{d}\right) + p_0',$$

and

$$p'' = \frac{2w}{ld}\left(\frac{3x}{d} - 1\right) + p_0''.$$

The maximum *inclined* stress in the dam may be twice the value given by these equations for *vertical* stresses. Therefore due allowance must be made in the selection of working *vertical* stresses or the maximum inclined stresses determined by extended calculations.

Vertical pressures should not exceed 1/21 of the ultimate strength of the masonry, or from about 16,000 to 22,000 pounds per square foot.

APPENDAGES TO DAMS. — The more common appendages to dams are the following:

Sluice Gates. — As their name implies, these are large gates placed in the dam for the control of the water level, or for relieving the dam in times of flood. A common type of gate is the so-called Tainter gate, which is slightly curved and revolves about its axis of curvature. A later improvement on this is the Hall gate, which permits of finer regulation.

Flash Boards. — These are light boards, or their equivalent, placed on the crest of the spillway held in position by light rods or frames. Their object is to temporarily increase the height of the dam during the dry season so as to conserve the water which would otherwise be wasted over the spillway at times when the wheels were not running. They are made light not only to facilitate handling, but also that they may give way quickly in emergency before a dangerous head could be created by sudden flood.

Log Runs. — On any stream where the driving of logs is practiced, it is generally a requirement that provision be made in a dam for the sluicing of logs. This is accomplished by a chute or trough placed generally near the spillway, with its upper end slightly below the pond level and closed by a sluice gate. To this the logs are guided by log booms fastened to the dam and cribs along the shore.

Fish-ways. — A fish-way is a gradual incline connecting the water below the dam with the pond above. A constant stream of water passes through it with a velocity against which the fish may readily swim. This is sometimes attained by using baffles across a portion of the stream, leaving a passageway at the side. The slope of a fish-way should not be greater than 1 in 4 and its ends should be well beneath low water level. Fish-ways are generally more or less covered in order to protect the fish from interference, but should never be made dark, as they would not be used by the fish. Fish-ways are generally required by State law on all natural water ways.

COST OF DAMS (**Pre-war figures**). — It is difficult to give items of cost for dam construction, due to the great influence of local conditions. As earth embankments should not be attempted in places where material is not available near the site, the figures for unit cost may be fairly well approximated. Excavation in earth will cost from 20 to 30 cents per cubic yard; embankment 25 to 40 cents; puddle cores 45 to 70 cents; hand-laid paving $2.00 to $3.00; rip-rap $1.50 to $2.00; sodding 20 to 30 cents per square yard. Figures for masonry dams depend largely on the ease with which the materials may be brought to the site. The following figures serve to show approximately the unit costs under conditions normally favorable. Rock excavation $1.00 to $2.00; rubble

masonry laid in natural cement $3.00 to $5.00; same laid in Portland cement $4.00 to $6.00; concrete masonry, Portland cement, $5.00 to $7.00; reinforced concrete $7.00 to $11.00; rock-faced stone masonry $10.00 to $15.00; dimension stone masonry $15.00 upward.

BIBLIOGRAPHY. — Some of the more important books treating of dams are the following: Creager, *Engineering for Masonry Dams;* Smith, *Construction of Masonry Dams;* Wegmann, *Design and Construction of Dams;* Morrison and Brodie, *Masonry Dam Design;* Parker, *The Control of Water;* Bligh, *The Practical Design of Irrigation Works;* Etcheverry, *Irrigation Practice and Engineering*, Vol. III; Thomas and Watt, *Improvement of Rivers;* Mead, *Water Power Engineering.*

DECIMAL EQUIVALENTS. — The following table will be found useful in converting common fractions into decimals.

8ths	16ths	32nds	64ths	Decimal	8ths	16ths	32nds	64ths	Decimal	8ths	16ths	32nds	64ths	Decimal
			1	0.015625			11	22	0.34375				43	0.671875
		1	2	0.03125				23	0.359375		11	22	44	0.6875
			3	0.046875	3	6	12	24	0.375				45	0.703125
	1	2	4	0.0625				25	0.390625			23	46	0.71875
			5	0.078125			13	26	0.40625				47	0.734375
		3	6	0.09375				27	0.421875	6	12	24	48	0.75
			7	0.109375		7	14	28	0.4375				49	0.765625
1	2	4	8	0.125				29	0.453125			25	50	0.78125
			9	0.140625			15	30	0.46875				51	0.796875
		5	10	0.15625				31	0.484375		13	26	52	0.8125
			11	0.171875	4	8	16	32	0.5				53	0.828125
	3	6	12	0.1875				33	0.515625			27	54	0.84375
			13	0.203125			17	34	0.53125				55	0.859375
		7	14	0.21875				35	0.546875	7	14	28	56	0.875
			15	0.234375		9	18	36	0.5625				57	0.890625
2	4	8	16	0.25				37	0.578125			29	58	0.90625
			17	0.265625			19	38	0.59375				59	0.921875
		9	18	0.28125				39	0.609375		15	30	60	0.9375
			19	0.296875	5	10	20	40	0.625				61	0.953125
	5	10	20	0.3125				41	0.640625			31	62	0.96875
			21	0.328125			21	42	0.65625				63	0.984375

DEMAND METERS. — (*See also Ammeters; Ampere-hour Meters; Watt-hour Meters; Wattmeters.*) For some time past various attempts have been made to inaugurate systems of charging for electric energy which would be more equitable to both the consumer and the central station than a flat kilowatt or kilowatt-hour rate. The so-called "maximum demand system" has found favor with many, and may find a much more general application.

The system is based on the fundamental assumption that the charge to any consumer should be made up of two parts; one part fixed by the maximum power in watts demanded by the consumer at any time during a certain definite period, and another part fixed by the total energy in kilowatt-hours used during the same period. The object of the demand system of charging is the improvement of the load factor of the station and the more equitable distribution of its fixed charges among its customers.

Demand metering cannot be regarded as an exact science, because of the large number of variables involved and the various ways in which they enter the problem. These include the capacity of generators, lines and transformers, and the diversity factor, load factor, and power factor of the system. No practical meter can be devised to take care of all these variables, consequently charges based on the use of demand meters in addition to the usual energy meters must necessarily be somewhat arbitrary.

Requirements of a Demand Meter. — The kilowatt-hours supplied to the consumer are readily measured by a watthour meter (q.v.); to record the maximum power (kilowatts) taken by the consumer, various forms of "maximum demand meters," usually called simply "demand meters," have been devised. In the case of a practically constant d-c. voltage at the consumer's premises, a device which measures the maximum current is as satisfactory as one which measures maximum power, but when the voltage or power factor (in case of an a-c. system) varies, the maximum current indicator is not suitable.

In any case the device should be one in which the demand measured is not the instantaneous peak of the load demanded by the consumer, but is the average of the power demanded over an appreciable time interval, for the maximum demand recorded should not be influenced by short-circuits, excessive current flow in starting motors, or by any abnormal consumption of energy that covers too short a time to have any real effect on the capacity which must be provided by the central station to take care of the demand.

The time interval over which the demand should be taken should theoretically differ with the character of the installation and the relation of the maximum power demanded to the maximum capacity of the central station. In relatively large consumers' installations the time should be carefully chosen with reference to the time that the central station can endure an overload successfully. Hydroelectric plants have a very definite power limit which is reached with maximum gate opening, while steam-power plants have considerable thermal inertia and their boilers and generating apparatus can usually carry relatively large overloads for a considerable time. These considerations require much shorter time intervals for large users of hydroelectric power than would be necessary in steam plants of corresponding magnitude.

Demand intervals of 15 minutes, 30 minutes, and 1 hour are those in general use at the present time. For very special installations, intervals as short as 1 minute and as long as 3 hours have been used. The general tendency is toward the use of a single interval for a given system, and toward the use of intervals larger than those formerly used.

On account of the desirability of improving the load factor, the off-peak system has been developed, which gives a preferential rate to customers utiliz-

ing current during the times of low load on the central station. Demand meters giving time of day as well as demand are used for this class of metering.

The general subject of demand meters has received considerable attention during the past five years and many devices have been developed and placed on the market. The whole question of charging for electric service, including the necessary devices to be used, is still in a transition state, and is complicated by the inclusion of the power factor and balance of the customer's load as elements in determining the cost of supply.

Classification of Demand Meters. — Demand meters may be classified into (1) curve-drawing instruments, giving the load-time curve of the installation; (2) integrated-demand meters, consisting of an integrating meter combined with a device which registers the energy consumption from time to time in such a way as to indicate or record the maximum demand; (3) lagged-demand meters, which require a certain time interval for the indication to reach the value corresponding to the load. Under each of these types there are a number of further distinctions, such as length of demand interval, whether the demand intervals begin at specified times of the day or may be so chosen as to include the maximum average load during any time interval of the given length, and whether the maximum demand is simply indicated without reference to time or is recorded so that the time of its occurrence may be determined. Some of the types of demand meters in commercial use are briefly described below.

Curve-drawing Instruments as Demand Indicators. — (*See also Wattmeters.*) The most complete knowledge of the conditions attending the consumer's use of energy may theoretically be obtained by using a curve-drawing wattmeter, with which the average demand for any time interval may be found and also the time at which it occurred. This statement, however, requires considerable modification in practice. Unless the paper speed is fairly high, the lines will run together so as to make it difficult or impossible to determine the demand with any approach to accuracy. Curve-drawing wattmeters are too expensive in first cost for any but comparatively large installations, and one of the simpler and cheaper devices described below is ordinarily used. The indications of a curve-drawing wattmeter are difficult of comprehension for the ordinary customer who knows nothing of integration. Such wattmeters are used largely by central stations to accumulate demand data. Curve-drawing ammeters may be used on the assumption of constant voltage and power factor, but do not give as accurate information as wattmeters.

Integrated-Demand Meters Indicating Maximum Demand Only. — The Type M–4 demand meter of the General Electric Company consists essentially of a demand-registering and a timing mechanism mechanically connected and mounted within the same case. The demand-registering element is electrically operated by means of a contact in the register of the watthour meter in conjunction with which the demand meter is used. The registering element consists of a train of gears which drive a demand pointer forward over a dial. This train is actuated by a ratchet and pawl mechanism operated by an electromagnet which receives an impulse every time a certain amount of energy is registered by the watthour meter. The speed of advance of this pointer is therefore proportional to the power. In order that the meter may show the demand for a given time interval, it is necessary that the pointer-driving mechanism shall be reset to zero position at the end of each time interval, leaving the pointer itself in the highest position reached. This resetting is accomplished by a mechanism controlled by a constant-speed motor similar in construction and operation to the motor element of an induction watthour meter. This demand meter is for use on a-c. circuits, and is arranged for intervals of 15, 30, or 60 minutes. The Type M–5 demand meter for d-c. circuits is similar to the pre-

ceding, but has an 8-day keywind clock to give the time interval. The Type M–6 demand meter differs from the M–5 in having a small d-c. motor to keep the clock movement wound. The latest development of this series is the M–7 demand meter register, Fig. 1. This replaces the standard register of either an

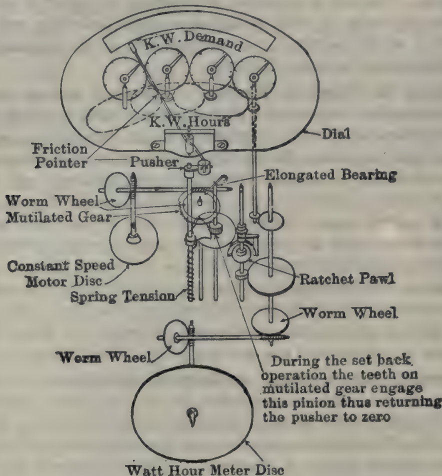

Fig. 1. Type M–7 Demand Meter Register.

I–14 or D–6 watthour meter. In addition to the usual four kilowatt-hour dials, it contains a demand dial. The principle of operation is the same as that of the other demand meters of the Type M series, but the timing device is a small disk-type constant speed induction motor built into the register. In the M–7 the demand registering mechanism is thus mechanically connected to the watthour meter, instead of electrically as in the preceding Type M demand meters. When new I–14 or D–6 watthour meters are supplied with this form of demand register they are called respectively Type IM–14 or DM–6 watthour demand meters.

The Type RO watthour demand meter of the Westinghouse Company is a combination of induction watthour meter, induction wattmeter, and an escapement mechanism. The auxiliary disk of the wattmeter is located in the same air gap as the watthour meter disk, and tends to move against the torque of a spring. This tendency is checked by an escapement controlled by the rotation of the watthour meter disk. The auxiliary disk thus advances step by step at a speed proportional to the load until the driving torque is equal to the spring torque. Starting from zero, the time required to reach equilibrium on any constant load is a constant, since the deflection and the rate of deflection vary in direct proportion. The auxiliary disk drives a demand pointer through a dog, and a ratchet holds it in the highest point reached, until released by hand. A second ratchet allows the auxiliary disk to fall back to the position of equilibrium with the spring tension whenever the load drops, but prevents it from advanceing except as controlled by the escapement.

Integrated-Demand Meters With Arbitrary Time Interval, Recording all Demands and Time of Occurrence. — The earliest instrument of this

class was the printometer, later made by the General Electric Company as the Type P demand meter. This instrument is designed to print on a paper tape at regular time intervals the time of record and the consumption of energy up to that time, the energy consumption being that registered by a watthour meter connected to the circuit. The instrument contains a set of cyclometer type-wheels which are moved forward at a rate proportional to the rate of energy consumption in the circuit. This is accomplished by a solenoid, the energizing circuit of which is closed through a contact wheel fixed to one of the spindles of the gear train of the watthour meter. The reading of the cyclometer is printed on a paper tape (see Fig. 2) by a rubber platen and a copying ribbon, the rubber platen being actuated by a solenoid which is energized at regular time intervals by means of a contact making clock. The time is also printed opposite each

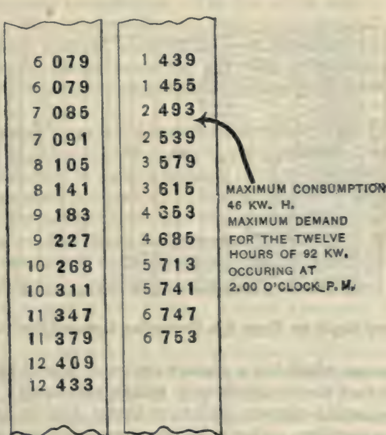

Fig. 2. Record Made by Type P Demand Meter.

cyclometer reading. Thus, the difference between consecutive records on the tape is proportional to the energy consumption in watt hours in the circuit during a definite time interval. The average demand over a period of time, corresponding to the interval between successive operations of the printing solenoid, is obtained by dividing the watthours of energy consumption during a time interval by the length of the interval.

The Type RA recording-demand watthour meter of the Westinghouse Company consists of a polyphase watthour meter combined with a mechanism for obtaining a permanent graphic record of the integrated demand over successive predetermined time intervals. The total energy consumption is shown in the usual way by the register of the watthour meter. The time interval is controlled by a clock which requires winding once a month. A separate spring mechanism advances the record paper. The pen is moved across the paper by the gear train of the watthour meter, and at the end of each time interval a release mechanism frees it from the gear train and allows a weight to move it back to zero where it is again meshed with the gear train to repeat its advance during the next time interval. Just before the release occurs, the record paper is advanced slightly (see Fig. 3) so that the pen makes a distinct record of the maximum travel, which is a measure of the integrated demand. The time of

occurence is shown by the time figures printed on the paper. This meter is made with time intervals of 15, 30, or 60 minutes. It is a self-contained structure, with the metering and demand-recording mechanisms mechanically connected.

The Piek demand instrument, which was the forerunner of the Type RA demand meter, is very similar to the latter, but uses the same clock to establish the time intervals and to drive the chart.

The Type G demand meter of the General Electric Company is used in connection with a watthour meter, and gives a graphic record on a circular chart of the demands integrated over definite time intervals, with the time of day and day of week of their occurrence. It consists essentially of a registering element and a timing element. The registering element is electrically connected

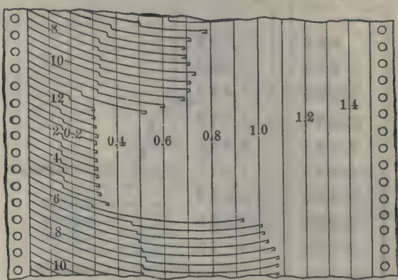

Fig. 3. Record Made by Type RA Recording Demand Watthour Meter.

to the watthour meter, which has a contact device on one of the spindles of the register. This contact device alternately makes and breaks the circuit of a solenoid in the registering element each time that a definite amount of energy is registered by the watthour meter. An armature operates a ratchet and pawl mechanism to advance the stylus by a small step for each operation of the contacts. At the end of each time interval a cam mechanism operated by the timing device allows the stylus to return to the zero position, where it is re-engaged ready to integrate the energy used in the following interval. The timing mechanism rotates the chart continuously, giving a record in the form of a saw-tooth polar curve. The chart gives a record for one week. The time interval is either 15, 30, or 60 minutes.

Integrated-Demand Meters Recording Time and Equal Amounts of Energy, Non-Arbitrary Time Interval. — The Ingalls demand recorder is operated in connection with a watthour meter. It contains a clock which uniformly advances a tape upon which a mark is made by a printing solenoid energized through contact in the meter register. Each mark corresponds to a predetermined amount of energy. The number of marks in a given time interval is proportional to the average power.

Thermal-Storage Lagged-Demand Meters. — These meters all show maximum demand only, and the indication depends on the duration of the load as related to the "demand interval" of the meter, as well as on the previous history of the load. The demand interval of this type of meter is usually taken as the time required for the indication to reach 90 per cent of the full value of a steady load which is thrown suddenly on it. In principle, all such meters are differential thermometers.

The Wright demand indicator, manufactured by the General Electric Company, is a differential air thermometer having one of the bulbs surrounded by a heating element. The expansion of the air in this bulb drives a liquid over into a graduated tube, giving a measure of the ampere demand. It is suitable for use on constant-voltage circuits, either direct current or alternating current of constant power factor. It leaves no record of the duration of the maximum demand nor of the time at which it takes place. After the indicator has been reset there is no original record of previous maximum demands.

The Type H–2 demand indicator of the General Electric Company is an ampere-demand instrument also based on the differential thermometer principle, but using a pair of strips of thermostatic metal. One of these is in proximity to a heating element traversed by the load current, while the other nominally follows the room temperature. The two strips tend to move in opposite directions, and with no current flowing through the heating device the pointer remains at zero in spite of changes in room temperature. Current flowing through the heating device heats the adjoining strip and moves the pointer over the scale. The pointer remains at the highest point reached until manually reset.

The Westinghouse Type RH demand meter employs the principle of the hot-wire wattmeter, and thus takes account of watt demand. It contains two heating elements which tend to move a demand pointer in opposite directions. The electrical arrangements are such that the difference in the heating is proportional to the power in the circuit, and the thermal capacity of the heating elements makes the reading depend also upon the storage of heat in them. The demand pointer remains at the highest point reached until manually reset.

Overdamped-Wattmeter Lagged-Demand Meter. — The Type W polyphase demand indicator of the General Electric Company consists essentially of a polyphase watthour meter with a system of springs opposing the rotation of the disk and thus converting it into a polyphase wattmeter. A large number of damping magnets are used, giving heavy overdamping and preventing the meter from recording demands of short duration. The rotation of the disk against the opposing springs moves a demand pointer forward, and this pointer remains at the highest point until manually reset. This type of demand meter is now but little used.

Demand Devices Taking Account of Power Factor. — The increasing appreciation of the effect of low power factor on the cost of energy supply is beginning to show in the development of demand devices taking power factor into account. For example, the Westinghouse Company has made an instrument consisting of two Type RA watthour demand meters in one case with their charts interlocked and driven by the same clock. One of these meters is connected to record power demand and the other records reactive-component demand. The same object is also attained by the General Electric Co. with its Type P demand meter used in connection with a watthour meter and a reactive voltampere-hour meter.

COSTS. — The following figures are approximate and are intended as a rough guide.

Type	Description	Price
Integrated-Demand Meters, maximum demand only:		
Types M-4 and M-5 .... Type M-6..............	15, 30, or 60 min. interval....	$22 to 25
Type M-7 Demand Meter Register...............	....	17 to 19
Type IM-14..............	2-wire or 3-wire single-phase..	25 to 47
Type DM-6..............	3-wire polyphase............	49 to 73
Type RO.................	2-wire or 3-wire single-phase..	29 to 44
Type RO.................	3-wire polyphase............	57 to 87
Integrated-Demand Meters, all demands and time:		
Types P-2 and PS-2.......	....	118
Type RA (Polyphase)......	15 30, or 60 min. interval....	115 to 123
Type G-2	15, 30, or 60 min. interval....	65
Lagged-Demand Meters:		
Type H-2 Demand Indicator..................	Maximum amperes 2 to 25..	6
Type RH Demand Meter...	5 to 50 amps., 110 volts; used with instrument transformers for higher capacities...	22 to 42

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See also Section X of the *Code for Electricity Meters*, prepared by the Electrical Testing Laboratories for the Joint Meter Committee of the Association of Edison Illuminating Companies and of the National Electric Light Association. This section is given in the 1920 Report of the Committee on Meters of the National Electric Light Association.

DEPRECIATION. — Depreciation is loss of value due to expiration of useful life. It is brought about by wear and tear and the action of the elements which eventually make replacement necessary. *Service value or serviceability* may or may not suffer diminution as depreciation accrues. Depreciation does not properly include loss of value due to accidental damage, to obsolescence, to inadequacy or to price changes. Replacements due to accidents such as fire may be covered by insurance. If not, they should be charged directly to operation or amortized over a reasonable period. Replacements due to obsolescence or inadequacy should be financed from the savings resulting from the use of the more modern or adequate equipment. As for market or price changes, it is impossible to show property on the books at its value under constantly fluctuating prices. The accounting problem involved in making replacements in kind at changed price levels is discussed below. Depreciation as specifically defined * is accrued liability to replace and it involves four problems which may be described as follows:

First, *physical*, to determine the amount of the liability at any time and the rate of its accrual.
Second, *financial*, to make provision for meeting the liability.
Third, *accounting*, to keep the proper records.
Fourth, *management or regulation*, to determine the effect of the accrued and accruing liability on the economic position of the enterprise whose assets are under consideration.

THE PHYSICAL ASPECT. — A piece of property whose useful life has ended has suffered depreciation to the extent of the difference between its value when new and its scrap value. This difference is called the wearing value. Various assumptions may be made as to the distribution of the depreciation over the life of the property, but the usual practice is to assume it uniform. Then the annual depreciation is equal to the wearing value divided by the life in years. The accrued depreciation at any time is equal to the product of the annual depreciation and the age in years.

Since the extent and rate of depreciation must be known while the property is still in service, if proper provision is to be made for replacement, the life used in determining the annual depreciation must be a prediction. Such a prediction can be made only on the basis of experience, and is a subject for the best engineering judgment, since many factors, such as severity of use, quality of material and of workmanship, and climatic conditions, must be considered. Tables of lives of various classes of property have been prepared, but should be used with discretion, since they are unreliable, for the following reasons among others. First, many of the renewals considered by the compilers to be due to depreciation were due to obsolescence or inadequacy. Second, improvements in the arts are continually increasing the durability of many classes of equipment. Third, replacements are at times postponed past the time when true economy would dictate replacement because of the financial situation of the owning company, and as a result, the experience recorded becomes an unreliable guide. In the bibliography appended reference is made to tables of lives which may be used as a guide to judgment.

Frequently it is desired to create a reserve which will provide for replacing property made up of parts having different lives whose independent replace-

* The term "depreciation" has through use by courts and regulatory commissions come to have the specific and somewhat narrow construction defined above. In general usage, the word "depreciation" denotes decrease in value from any cause, but discussion is here confined to the more technical use of the term.

ments are to be charged to depreciation, and it is desired to place a life on the piece of property as a whole. This is properly done by dividing the sum of the wearing values of the parts by the sum of their annual depreciations, the quotient being the composite life. The weighted average life of the parts, the weighting figures being the wearing values of the respective parts, cannot be used in lieu of the composite life defined above. To be correct, the average life must be further weighted by the relative frequencies of renewal of the respective parts. The test of a composite life is that the wearing value of the property which it covers shall, when divided by it, give the annual depreciation for the same property. This the first method assures, but the second does not.

The scrap, or salvage value of a piece of property is a subject for engineering estimate. The expense involved in preparing for market and marketing the scrap is a deduction from the scrap value but the cost of removing the unit to be replaced is a charge to operation.

In certain types of property, life can better be measured in service units than in time units. However, in the case of a large and varied property, time, being of general application, forms the most satisfactory basis for computing depreciation.

As the operation of a property continues, the engineers responsible for determining depreciation must constantly check up and revise their opinions as to the lives of various units, basing their conclusions on the frequency of renewals, due to depreciation, as proved in practice. The rate of accrual of depreciation should be refigured from time to time on the basis of these studies. At intervals an estimate should be made of the depreciation accrued on the property. In making this estimate it must be understood that accrued depreciation or accrued liability to replace is a physical fact, independent of the assets accrued to meet the liability as reflected by the balance sheet, and the estimate must be a quantitative statement of that physical fact, as nearly as it can be determined.

THE FINANCIAL ASPECT. — As depreciation and the consequent replacement liability accrues, it is necessary to build up assets to offset it. To this end a portion of the revenue derived from the operation of the property should be retained by the corporation for that purpose. Building up assets to offset the depreciation liability gives assurance that the property will be maintained in its full usefulness and earning power in so far as these are governed by its physical condition. In public utility mortgages, it is frequently provided that a certain percentage of the gross operating revenue shall be expended or reserved for the maintenance of the property. Such provisions have been thought to be particularly necessary where, (*a*) operation cannot be discontinued, (*b*) property cannot be removed to another location, (*c*) scrap or recovery value of physical property is so small a proportion of original cost as to be of negligible importance to the creditor. These reasons, while controlling with the creditor, do not prove the value of this method of measuring maintenance and depreciation requirements.

Building up assets offsetting accruing replacement liability serves to maintain the equity of the stockholders as well as to protect the physical condition and the earning power of the creditor's security. If the funds required to take care of replacement are paid out in dividends a partial liquidation takes place, which may or may not be sound business, but in any event the fact should be clearly understood by all concerned, and clearly reflected by the books of account.

In the case of a property made up of many parts having different lives, at no time, after the initial construction, will all parts be new at once. Therefore, the assets reserved to offset depreciation will never all be needed for financing replacements. From this it may be argued that it is necessary to reserve only enough funds to meet such abnormal amounts of replacement expenditures as

occasionally occur. Under this plan no distinction would be made between maintenance and depreciation, so much each month being set aside out of income, the amount being revised from time to time as found necessary, and all costs maintaining the property paid out of the reserve. If this plan is followed it must be recognized that the company's assets have been allowed to decrease and that the depreciated value of the property, plus the reserve, is less than its original cost. This consideration is particularly pertinent in the case of a regulated utility.

In reserving funds for effecting replacement due to depreciation various methods are used for determining the amount to be periodically set aside. Sometimes it is made a fixed percentage of the gross receipts. Sometimes it is a uniform amount arbitrarily fixed. These methods have only simplicity to recommend them. The correct system is to make the reservations in accordance with the accrual of depreciation as determined by the engineers. Two methods have been generally used. One is the straight line method in which the set-up for each piece of property is equal to the wearing value of that piece of property divided by its life. The other is the sinking-fund method in which the set-up for each piece of property is of such amount that, when the piece of property has reached the end of its life, the sum of the set-ups made on its account shall, with the compound interest accumulated on those set-ups, equal the wearing value. In the second case the reserve fund must be credited periodically with interest upon accumulated balances at the rate used in computing the set-ups. This interest may be charged either against income or surplus. The latter method is well adapted to the case of regulated utilities, as will be discussed later.

The assets set aside or retained to offset depreciation may be handled in various ways. Specific assets, such as bank deposits and interest-bearing securities, may be designated as a depreciation reserve fund, and kept equal to the depreciation liability as shown on the books, or the offsetting assets may be allowed to remain a part of the general assets and used in the business. The latter method is usually preferred, since a going concern can, as a general thing, derive a larger return from capital used in the business than in that invested outside. However, care must be exercised that there is always a sufficiency of quick assets to meet any sudden demand. In theory, when expected replacements become necessary, they may be financed by borrowing money on the security of the property in which the reserve against depreciation has been invested. This, however, is not generally practicable due to the provisions of prior liens.

The Accounting Aspect. — It is highly important that the accounting treatment of depreciation reflect accurately both the lessening of remaining life of physical plant and the financial plan by which provision is being made for replacement. The accounting processes involved are primarily: (*a*) Current credits to depreciation reserve either by charges to income or appropriation of surplus.* (*b*) The crediting of interest to the reserve where current reservations are calculated on the sinking fund basis. (*c*) The segregation of assets against the reserve liability or funding. (*d*) The auditing of charges against the reserve for the purpose of determining their propriety in the light of the basis on which credits to the reserve have been made.

There is not always a clear distinction between maintenance charges to be included in current operating expenses, and replacement charges to be made

* In the case of a company which has not from the beginning made provision for replacement of physical property, and in which considerable depreciation has already accrued, the initial charge in establishing such a reserve, if it is to cover depreciation accrued, must be against surplus.

against the reserve. The final test must always be whether or not the reserve has been built up from credits calculated to cover charges such as the one in question. In a business where assets are progressively exhausted, as in the case of a mine, the reserve account may be dispensed with by making the depreciation credits direct to property. This practice is not proper in the case of a continuing enterprise.

Expenditures which serve to increase the investment in a property are chargeable to construction. When a piece of property is replaced by another which by reason of greater size, superior quality, or higher price level costs more than the original piece, the procedure is as follows, assuming that an amount equal to the wearing value of the original property has been accrued in the depreciation reserve. The amount realized from scrap or salvage is credited to the depreciation reserve to bring the amount in the reserve up to the full original cost of the item to be replaced. The original cost is then written out of property and plant account and depreciation reserve concurrently reduced. The whole of the cost of the replacing unit is then added to property and plant account, the result being to increase property and plant account by the excess cost of the new property. In case the new property costs less than the old, there will be a net credit to property and plant. This is sound accounting, since the property account should represent the actual cost of the existing property at all times.

In case the piece of property is replaced for reasons other than depreciation, the depreciation accrued should be charged against the reserve, and the remainder of the cost of replacement cared for in other ways. For example, if a car estimated to last 30 years is accidentally destroyed at the end of 10 years, only a part of its cost will have been accumulated in the reserve. The balance of the amount at which the car is carried in the property account must either be appropriated from surplus, charged to operation, or set up as a suspense item, and amortized over an appropriate period. The latter method would be used ordinarily only in cases where large amounts are involved. The proper division of charges in such cases involves engineering judgment.

Reference has been made to periodic engineering estimates of the extent of accrued depreciation. These estimates should be used to check the balance in

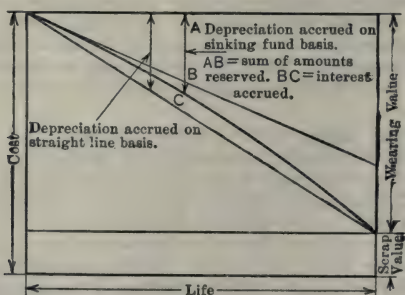

Fig. 1.

the depreciation reserve. If the reserve has been set up on the straight line basis, and the accrued depreciation estimated on the same basis, the two should be in general agreement. If the reserve has been set up on the sinking fund basis, it should be somewhat smaller than the depreciation estimated to be accrued on the straight line basis since in the case of any given piece of property, the reserve

accruals including interest will be more rapid in the latter part of its life than in the earlier part. (See Fig. 1.) If the reserve is found to be too large or too small by a substantial amount, the amounts currently reserved should be decreased or increased with the object of bringing the balance in the reserve back into line. Usual practice does not provide for accumulating an amount in the reserve in excess of original cost less salvage, therefore cost of removing the unit to be replaced must be charged to operation.

The foregoing comments on the accounting aspects of depreciation further define the engineering problems involved, rather than accounting procedure.

The Management or Regulatory Aspect. — In the case of property used in unregulated competitive business, the value at any time is the cost at current prices of a property which can be used for the same purpose, less the accrued depreciation, the latter being computed as the product of present cost less salvage, and the percentage of expired life. It is necessary to deduct the depreciation in order to obtain a measure of the service which can still be extracted from the property. It is necessary to know the rate of accrual of depreciation in order that the management may know whether or not the business is presently profitable and to know the accrued depreciation in order that the balance sheet may show the true condition of the property.

In the case of regulated utilities, frequently, though inaccurately, described as non-competitive, value is fixed by a regulatory body subject to judicial review. No final rule of general application has been enunciated, though various elements of value have been specified and defined. It is generally agreed, however, that, in any event, the rates allowed must provide for current operating expenses and reserve for depreciation, before they can provide for return on the value fixed. Since the function of the depreciation reserve is to provide for replacement at original cost, (increases or decreases in cost being cared for through capital account) "original cost," not "present value," is considered here. As the depreciation reserve accumulates and is invested either in extensions to the utility's plant, or independent thereof, it furnishes a source of income aside from that earned by the original property.

If the reserve has been set up on a straight line basis, the original cost of the property, plus the assets which offset the reserve, less depreciation accrued, equals the original investment. If the offsetting assets are invested so as to earn a return equal to that allowed on utility property, it is fair to fix the earnings from utility operation so as to yield a return on the property at its depreciated value, but if this is done the earnings of the reserve are a part of the corporate revenues and should go to the owners. But it is seldom possible to re-invest so advantageously and still fulfill the essential requirement of having the assets readily avaliable for financing replacements. If a return is allowed on the undepreciated cost of the property, then the income from the invested reserve should be considered part of that return and the balance collected in rates. If, however, the depreciation reserve is set up on a sinking fund basis, the earnings of the reserve are needed as a part of the reserve, and are not available as return. Then the company must be allowed in rates a return on the undepreciated cost of its property. As the income from the reserve appears in corporate income, the interest payments to the reserve should be made out of corporate income. Since normally a part of the reserve will be invested in utility property, and part in quick assets, the interest rate used in calculating credits to the reserve must be a compromise between the return allowed on utility property and that obtainable from quick assets.

In cases where the depreciation reserve is less than the accrued replacement liability, to the extent that returns to investors have been excessive, it may properly be considered that partial liquidation has occurred. Where returns

have not been excessive and rates have been insufficient to permit full reservation to cover replacement liability, equitable consideration must be given to that fact.

Calculations. — The following notes are illustrative of the calculations necessary to determine the proper amounts to be currently reserved for the purpose of financing replacements.

Let C = cost of property,
S = scrap value,
n = life in years,
r = annual interest rate,
A = annual reserve credit,
M = monthly reserve credit.

Then, on the *straight-line* basis:

$$A = \frac{C-S}{n},$$

$$M = \frac{A}{12} = \frac{C-S}{12n}.$$

On the *sinking fund* basis:

Assume first that the reserve credit is made annually and interest is credited annually, then:

Case I.—If interest *is* allowed each year on that year's reserve credit, *i.e.*, set-up made at beginning of year:

$$A = (C-S)\frac{r}{(1+r)^{(n+1)} - (1+r)}.$$

Case II.—If interest *is not* allowed each year on that year's reserve credit, *i.e.*, set-up made at end of year:

$$A = (C-S)\frac{r}{(1+r)^n - 1}.$$

Assume next that the reserve credit is made monthly and interest is credited monthly, then:

Case III.—If interest *is* allowed each month on that month's reserve credit, *i.e.*, set-up made at beginning of month:

$$M = (C-S)\frac{\frac{r}{12}}{\left(1+\frac{r}{12}\right)^{(12n+1)} - \left(1+\frac{r}{12}\right)}.$$

Case IV.—If interest *is not* allowed each month on that month's reserve credit, *i.e.*, set-up made at end of month:

$$M = (C-S)\frac{\frac{r}{12}}{\left(1+\frac{r}{12}\right)^{12n} - 1}.$$

Illustrative Example: Cost of property \$1000. Net scrap value \$200. Life, 10 years. Interest rate on depreciation reserve 4%.

On the straight-line basis:

$$A = \frac{1000-200}{10} = \$80,$$

$$M = \frac{80}{12} = \$6.67.$$

On the *sinking fund* basis:

Case I.

$$A = (1000 - 200)\frac{.04}{(1.04)^{11} - 1.04} = \$64.07.$$

Case II.

$$A = (1000 - 200)\frac{.04}{(1.04)^{10} - 1} = \$66.63.$$

Case III.

$$M = (1000 - 200)\frac{.00\frac{1}{3}}{(1.00\frac{1}{3})^{121} - 1.00\frac{1}{3}} = \$5.41.$$

Case IV.

$$M = (1000 - 200)\frac{.00\frac{1}{3}}{(1.00\frac{1}{3})^{120} - 1} = \$5.43.$$

In large enterprises it is the usual accounting practice to make credits to the reserve each month. It will be seen that the difference between the monthly reserve credits determined under Case III and Case IV are less, in proportion, than the probable error in the life assigned to the property.

Composite Life. — A piece of property is made up of two parts. The first part has a wearing value of $2000 and lasts 20 years; the second part has a wearing value of $1000 and lasts 10 years. Depreciation reserve is credited on the straight-line basis:

W. V.	Life	Annual Set-up
2,000	20	100
1,000	10	100
3,000		200

The composite life is 3000 divided by 200 = 15 years. If life is weighted by wearing value, it averages as follows:

W. V.	Life	Product
2,000	20	40,000
1,000	10	10,000
3,000		50,000

The weighted average is 50,000 ÷ 3000 = 16⅔ years. Dividing the wearing value by this gives 300 ÷ 16⅔ = $180 for the annual reserve credit which is insufficient by $20. If frequency of rewewal is introduced we have the following:

W. V.	Relative Frequency of Renewal	Product	Life	Product
2,000	1	2,000	20	40,000
1,000	2	2,000	10	20,000
		4,000		60,000

The weighted average is 60,000 ÷ 4000 = 15 years, which is correct.

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TABLES OF LIVES OF PHYSICAL PROPERTY AND ANNUAL PER CENT DEPRECIATION.

Grunsky, *Valuation, Depreciation, and the Rate Base*, p. 274; Proceedings American Electric Railway Engineering Association, 1912, p. 560; Floy, *Valuation of Public Utility Properties*, p. 188; Foster, *Engineering Valuation of Public Utilities and Factories*, p. 194; Gorsuch, W. S., in M. Merriman's *American Civil Engineers' Handbook*, p. 269 of 1920 Edition; Wyer, *Regulation, Valuation and Depreciation*, p. 243.

Numerous Commission decisions contain data on life of physical property, those most quoted being the decisions of the California and Wisconsin Railroad Commissions.

DERIVATIVES. — (*See also Equations, Differential; Integrals; Maxima and Minima; Series, Mathematical*).

Differentials. — Let y be a function of x, i.e., a quantity which varies continuously with variations of x. If x be increased or decreased by the smallest amount conceivable, y will change by a correspondingly small amount. Such small variations are called "differentials," and are symbolized thus: dy and dx, where d is not a quantity but a symbol meaning "the smallest conceivable value of" y or x.

Derivative or Differential Coefficient. — The ratio $\frac{dy}{dx}$ is the rate of change of y with regard to x and is called the "derivative" or "differential coefficient" of y with respect to x.

Geometrical Meaning of a Derivative. — If a curve be plotted between x and y for any equation $y = f(x)$, and a tangent drawn to the curve at any point, Fig. 1, then the tangent of the angle θ between this tangent and the axis of x is equal to $\frac{dy}{dx}$.

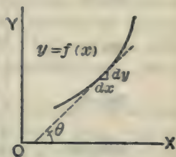

Fig. 1

Differentiation. — Differentiation is the process of obtaining the derivative of a function. If $y = f(x)$

$$\frac{dy}{dx} = \frac{df(x)}{dx}$$

$$= \frac{f(x+dx) - f(x)}{dx}.$$

Example. — If

$$y = ax^2$$

$$\frac{dy}{dx} = \frac{a(x+dx)^2 - ax^2}{dx}$$

$$= \frac{a\,dx\,(2\,x + dx)}{dx}$$

$$= a\,(2\,x + dx).$$

As dx is the smallest conceivable value of x, it is so small compared to x that it may, with the smallest conceivable error, be neglected. Hence,

$$\frac{dy}{dx} = 2\,ax.$$

Formulas for Differentiation. — (u, v, x and z are variables; a is a constant.)

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{d}{dx}(au) = a\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

When x is itself a function of some other variable z,

$$\frac{du}{dx} = \frac{du}{dz} \cdot \frac{dz}{dx}.$$

TABLE OF DERIVATIVES

Function $f(x)$	Derivative $\frac{d}{dx} f(x)$	Function $f(x)$	Derivative $\frac{d}{dx} f(x)$
x^n	nx^{n-1}	$\tan^{-1} ax$	$\frac{a}{1 + (ax)^2}$
$\sin ax$	$a \cos ax$	$\sinh^{-1} ax$	$\frac{a}{\sqrt{1 + (ax)^2}}$
$\cos ax$	$- a \sin ax$	$\cosh^{-1} ax$	$\frac{a}{\sqrt{(ax)^2 - 1}}$
$\tan ax$	$\frac{a}{\cos^2 ax}$	$\tanh^{-1} ax$	$\frac{a}{1 - (ax)^2}$
$\sinh ax$	$a \cosh ax$	$\log_a x$	$\frac{1}{x} \log_a \epsilon$
$\cosh ax$	$a \sinh ax$	$\log_\epsilon x$	$\frac{1}{x}$
$\tanh ax$	$\frac{a}{\cosh^2 ax}$	$\log_{10} x$	$\frac{0.4342944819}{x}$
$\sin^{-1} ax$	$\frac{a}{\sqrt{1 - (ax)^2}}$	a^x	$a^x \log_\epsilon a$
$\cos^{-1} ax$	$-\frac{a}{\sqrt{1 - (ax)^2}}$	ϵ^x	ϵ^x

Note: See also Table of Integrals in article on *Integrals* noting that the column there headed "Function" is the derivative of the column headed "Integral."

Second Derivative. — The derivative of a derivative, i.e., $\frac{d}{dx}\left(\frac{dy}{dx}\right)$, is usually written $\frac{d^2y}{dx^2}$. Similarly $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$ is called the third derivative and is written $\frac{d^3y}{dx^3}$, and so on.

DISPATCHING OF TRAINS BY TELEPHONE. — (*See also articles on Telephony.*) Up to 1907 train dispatching was done almost entirely by the telegraph. In that year the telephone began to replace the telegraph for this purpose. At the beginning of 1920, 83 railroads in the United States, operating 120,000 miles of line, had abandoned telegraphic train dispatching in favor of the telephonic method.

Telephone train dispatching has many advantages over the former method. The dispatcher's work consists in the main of gathering information and issuing orders for train movements. These operations are much quicker by the use of the telephone and are no less accurate. Special training in the use of the apparatus is not as necessary as when telegraphy is employed. All the agencies are brought into closer personal relations, with the result that there is better co-operation and better discipline. The dispatcher can call and speak at the same time, which is impossible by telegraph.

It now is possible to operate dispatching lines as long and with as many stations as railway conditions may permit. The length of line and number of stations are not limited by the nature of the electrical equipment. Lines of 300 miles length are in operation, and 65 stations on one line are in successful use. More than 65 stations could be operated successfully if desired.

Dispatching circuits are multi-station lines, like the telegraph-dispatching lines they displace. Broadly speaking, they are selective party lines, but are equipped with systems of apparatus which allow the placing of many more stations on a line than is possible with any other party-line system so far developed.

Several train-dispatching systems are on the market, the principal ones being known as the Gill System, the Western Electric System, the Cummings-Wray System, and the Kellogg System. These are in general similar in that the dispatcher has means of calling any station selectively. In some systems he can call several stations at once by setting keys for them in advance and operating a common calling key.

Each way station is equipped with a device called a selector. These selectors can be operated only by the dispatcher. They work on the step-by-step principle and the whole object of the device is to close a contact at the station called and thus to ring a bell. No provision is made for cutting the way-station telephone set on and off the line, as secrecy is not aimed at. Any way station may communicate with the dispatcher at will by lifting the telephone receiver. The dispatcher wears a head telephone constantly and so listens upon the line at all times.

Train orders are written as given and received and are read back to the dispatcher by each station, the dispatcher underscoring each word once for each correct repetition from a way station.

BIBLIOGRAPHY. — See Bibliography in article on *Telephone Instruments and Circuits.*

DISTRIBUTION LINES. — (*See also Conduits and Conduit Lines; Distribution and Transmission Systems; Transmission Lines; Wires and Cables; Wiring of Buildings.*) Two types of construction are employed, overhead and underground. Overhead construction has the following advantages (1) lower first cost, (2) easier to repair, (3) easier to change, (4) adaptability to higher voltages; while underground construction has the advantages (5) less unsightly, (6) less dangerous to the public, (7) less subject to damage by external agencies. The majority of circuits in use are overhead. Underground circuits are principally used in the central portions of the larger cities. When underground circuits are used a large part of the construction consists of a composite of underground and overhead construction.

MATERIALS FOR LINE CONSTRUCTION. — The principal elements for wood-pole construction are the conducting wires, insulation on the wires, insulators, tie wires, pins, cross arms, cross-arm braces, bolts, poles, guy wires, guy anchors. For underground construction, see *Wires and Cables, Insulated,* and *Conduits and Conduit Lines.*

Wires for Overhead Construction.— The conducting wires are usually of copper, sometimes of aluminum. Iron is not used as it costs more than copper for equal conductivity. Aluminum is used to a much less extent than copper; see articles on *Aluminum* and on *Copper.* Soft-annealed copper is usually employed for ordinary distribution work, as it has the greatest conductivity and is the easiest to work with. Medium hard-drawn and hard-drawn wires are used in lines where great mechanical strength is necessary.

For overhead city distribution copper wires ranging in size from No. 6 B. & S. (or A. W. G.) to 500,000 circular mils are used; see *Wires and Cables.* Sizes larger than No. 0000 B. & S. are usually stranded. Conductors as small as No. 14 B. & S. have been used for overhead work but are too small, as they are frequently broken by wind and sleet.

Standard Sizes of Wire. — Of the gage numbers between No. 6 B. & S. and 500,000 circular mils (*see Gages, Wire*), some are but little used because (1) the difference between consecutive sizes is less than is found necessary in usual practice and (2) the difference is too small to be readily detected by the ordinary lineman or stock keeper. In practice the number of sizes to be used and carried in stock has usually been reduced by omitting certain sizes; No. 5, 3 and 1 are nearly always omitted; practice regarding the omission of larger sizes varies. The most commonly used are No. 6, 4, 2, 0, 0000 B & S. and 500,000 circular mils.

Wires for Underground Lines. — See *Wires and Cables, Insulated.*

Covering on Overhead Wires. — Overhead conductors for city distribution are covered with weatherproof braid. Two grades are recognized, "double braid" and "triple braid," according to the number of coverings, though the actual thicknesses are not the same for different makes. Double braid is considered suitable for use on voltages of 600 or less, and triple braid for voltages up to 2500 constant potential and on series-arc circuits of all voltages. To prevent the possibility of double-braid wire being used for voltages over 600 and to reduce the number of kinds of wire used, it is good practice to have all No. 0000 wires and smaller covered with triple braid. Ordinarily cables larger than No. 0000 are not used on voltages above 600 and may therefore be double braided.

Object of Covering on Overhead Wires. — The covering on overhead conductors is solely for the purpose of limiting the short-circuit current due to an accidental cross or grounding. The normal insulation of the line is

maintained by the insulators alone (*see below*); any reinforcement obtained from the insulation on the conductors is neglected in practice. While weatherproof braid is an imperfect insulator, it serves to eliminate the greater proportion of the short-circuits and arcs which would occur, due to momentary contact, were bare wires used. Rubber and other more perfect insulators are not used because of expense and the impossibility of maintaining perfect insulation, due to weakness, against mechanical injury and deterioration.

Bare wire is usually used on circuits operating at 10,000 volts and over to avoid giving a false sense of security. For voltages between 2500 and 10,000 weatherproof wire is often used, though the protection afforded against short-circuits is doubtful.

Extra Cost and Weight due to Weatherproof Braids. — In estimating the cost of a distribution line, the additional weight and cost of the weatherproof braids should be taken into account. The following is a rough comparison of bare, double-braided and triple-braided copper wire, the cost being based on copper at 15 cents per pound.

COMPARISON OF BARE AND WEATHERPROOF COPPER WIRES

Conductor	Relative weights			Relative Costs		
	Bare	Double braid	Triple Braid	Bare	Double Braid	Triple Braid
No. 6 B. & S., solid	100	123	137	100	127	141
500,000 cir. mil cable	100	112	117	100	115	121

The cost and weight of weatherproof insulation for aluminum wires are greater than for copper of the same conductance on account of the larger cross-section of conductor required.

Insulators for Overhead Lines. — City distribution circuits are ordinarily carried on double-petticoat deep-groove glass (D.P.D.G.) insulators; see article on *Insulators for Overhead Lines*.

Tie Wire. — The conductor is attached to the insulator by a tie wire of the same material as the conductor, though soft wire is usually employed even for hard-drawn conductors; the tie wire is either bare or insulated to correspond to the conductor. The size of tie wire is often the same as that of the conductor; for small wires it is merely a piece of conductor. With large conductors it may be as much as three sizes smaller. See also the section on *Installation* in the article on *Wires and Cables, Bare.*

Pins. — See article on *Insulator Pins*.

Cross Arms. — See article on *Cross Arms*.

Poles. — See article on *Poles for Overhead Lines.*

DESIGN OF CITY DISTRIBUTION LINES. — City distributing systems should be designed so that service may be given to any building in the city and ultimately to every building present and future. On certain streets pole lines may be omitted without defeating this object. The arrangement of lines which will serve scattered customers with the least number of poles will usually contain many poles which should not be used if the ultimate arrange-

ment were immediately constructed. Preliminary studies and designs should be made, first, of arrangements suitable for servicing the initial expected customers, second, of arrangements for ultimately servicing customers on every lot in the city, third, of a plan of extension by which the initial arrangement can be extended to the ultimate with the minimum expense in changing lines and services.

The Pole Line. — The pole lines perform two functions: (1) of carrying feeders from the station to the mains and (2) of carrying the mains supplying services to buildings immediately adjacent. In the old cities of the eastern part of the United States where the streets are crooked general rules for systematic line work cannot be followed far. In the newer cities of the West and South the streets are laid out at regular intervals and at right angles, dividing the city into rectangular blocks of equal size. In such cases the following rules should be followed:

(1) A pole line should continue on the same side of the street throughout its entire length and disconnected lines built in the same street should be on the same side, so that they may be connected when desired without crossing the street.

(2) The spacing between poles should be an exact divisor of the length of a block (including cross street), giving a uniform number of poles per block.

(3) Whenever the line crosses a street where there is, or may be, an intersecting line there should be a corner pole on the proper side of the intersecting street for making a junction.

These rules logically lead to the use of corresponding sides of all parallel streets.

When each block contains lots fronting on all four surrounding streets the servicing of every lot in the city would require poles on all streets, though on the streets in at least one direction the lines may be discontinuous. Often the lots all front on the streets in one direction, which are laid out to be principal streets, in which case no service lines are necessary in cross streets, though at intervals connecting lines are necessary.

Trunk Lines. — Where the location of power house or substation is fixed, trunk lines must be laid out from such point, but often a study of possible trunk-line arrangements made before the location of the power house is fixed will show that other locations are more advantageous. If distributing station (power house or substation) is centrally located, there should be at least four main trunk lines (of poles) from it, say north, east, south and west. A short distance from the station they should be divided into branches, then subdivided into smaller branches and finally merge into the service lines. The trunk lines should be laid out: (1) on back streets where the large poles, numerous and heavy wires and heavy guying will not be conspicuous, (2) on side streets or streets little built up so that interruption to service due to fire in adjacent buildings will be infrequent, (3) on streets where there are few trees, (4) on streets where there are no jogs or offsets to weaken the line and require heavy guys, (5) on streets which lead directly to the section supplied, penetrate its center and intersect the maximum number of service lines. Even when the station location is excellently chosen these desirable conditions will have to be compromised to a serious degree.

Street and Alley Service Lines. —With the symmetrical arrangement of lots described above there is often an alley through each block parallel with the principal streets. Under these conditions there is therefore a choice of two methods of laying out the lines: (1) run the lines on the principal streets servicing the houses from the front and (2) run the lines in the alleys servicing

the houses from the rear. The disadvantages of the first are unsightliness of the poles and wires, difficulty of avoiding or of trimming shade trees; the disadvantages of the second are discontinuity of alleys, proximity of buildings (inflammable barns and out buildings in residence districts and of windows and fire escapes in business districts) and lack of established grade.

Composite Distribution. — The most expensive and difficult part of an underground system is that for taking the current from the conduit to the customer's building. This includes underground secondary mains, transformers, handholes, service pipes and wires. Where it is necessary to remove poles from streets but not to remove overhead wires from private property the pole lines are replaced by conduits which contain primary feeders and mains only. From the conduit in the street a small branch runs into the interior of each block where the ducts (usually iron pipes) come to the surface at the foot of a terminal pole up which the cables run. At the top the cables connect to overhead wires. The interior of each block contains a complete overhead distribution system of poles, transformers, secondary mains and services. Sometimes the servicing is done from a single centrally located pole and at others there may be a pole line of several spans length running longitudinally through the block. The most serious disadvantage of this method is that the poles and wires frequently have to be on private property, where no permanent rights may be obtained. Being dependent on concessions from one or more customers, the company does not have the independent position that a public service company should in order to serve all equally and fairly. The poles and circuits are therefore often put up of insufficient size without proper guying.

The composite system has the advantage of the overhead system in less unsightliness, less accessibility of public to high-voltage wires and less trouble from trees. In thickly built-up blocks its use results in a tangle of wires over roofs and along walls which may be as dangerous in case of fires or high winds as an overhead system. The composite system is used (1) in small cities where the load density is small and (2) in large cities in an annular district between overhead and underground construction; in either case it is usually an intermediate step from overhead to underground construction.

Crossing of Waterways; Submarine Cables. — Where a distributing system is divided by a navigable waterway the connection is usually made by submarine cables laid on the surface of the bottom, or better, below the surface in trenches dredged for the purpose. Cables may be single or multiple conductors, the latter being usually used for alternating currents to avoid reactance due to wire armor of cable. Two conductor cables may conveniently be of concentric type to give a true circular exterior. In laying cables it is desirable to keep them approximately parallel. Where one crosses under another it may be impossible to remove it. Each cable should be in a single length without joints, and the length should be made as short as possible, as repairs are very difficult and often impossible. At ends unimportant cables may be brought up a pole and connected to overhead wires; important cables should land in a cable house with suitable provision for disconnecting the cable or for transferring the overhead circuit to a spare cable in case of trouble, and also with lightning arresters. Submarine cables are weak links in a distribution system and may sometimes be avoided by crossing channels at sufficient height to clear the masts of ships.

Calculation of Size of Wires. — The size to be used depends upon the voltage drop which should be permitted, considering the probable growth of the load. The following table of per cent voltage drop in the various lines is representative of ordinary practice.

	For light
	Per cent
House wiring	2
Service wires	2
Secondary mains	5
Transformers	2
Primary mains	5
Primary feeders	10

The drop in the feeders is usually compensated for by raising the voltage at the substation or power station or by using voltage regulators; see *Distribution and Transmission Systems.*

Formulas for calculating the size of wire for a given length of line, given load and given distribution of load are given in the articles on *Transmission Lines* and *Wiring of Buildings.* Due to the uncertainty regarding the probable increase of load, a close calculation of the size of wire is seldom made, the engineer relying largely on his experience and judgment, making only a rough calculation as a check. For overhead lines 500,000 circular mils is usually the largest size used on account of the difficulty of supporting a larger wire; greater conductance is obtained by installing parallel circuits. For underground lines No. 0000 B. & S. three-conductor cable is the largest size that can be conveniently drawn into the ducts; 1,000,000 c.m. is the usual maximum size of single-conductor cable used.

Effect of Diversity Factor. — It should be noted that in a distribution circuit the maximum load on a feeder is less than the sum of the maximum loads on the mains which it feeds, these in turn are less than the sum of the maximum loads on the transformers connected to these mains, and so on. Therefore, whenever a circuit divides or subdivides, the aggregate sectional areas should ordinarily be greater after division than before. The total drop in voltage from power house or feeding point to a customer's lamp or motor is also usually less than the sum of the maximum drops in the parts of the circuit which are in series (such as house wiring, services, secondary mians, etc.) because these component drops do not have their maximum value simultaneously.

Stresses on Poles. — An overhead line is a framed structure. The poles are struts resisting the weight of wires, including sleet on the conductors, etc., insulators, cross arms and transformers and the downward pull of the guy wires. The horizontal pull of the line wires is balanced by the horizontal component of the pull of the guy wires, which transmit it to the ground. In addition to being struts the poles resist certain bending stresses, but these should be but a small part of the normal horizontal tension of the wires. The principal normal bending stresses are of two classes: (1) constant stresses, due to the unbalanced pull of service and other wires which do not exert sufficient force to require guys, and (2) variable stresses, due to the force of the wind at right angles to the line on both the poles and the wires. Poles also resist a twisting force where the tension on one side of an arm is greater than on the other, due to a difference in the number or weight of wires ending on the two sides. The poles, wires and guys should be so disposed that the bending and twisting forces on the poles are insignificant. The calculation of the strains produced in a pole is given in the article on *Poles for Overhead Lines.*

Arrangement of Wires on Poles.— The arrangement of wires is governed by mechanical, electrical and practical considerations. For mechanical reasons it is desirable that:

1. The largest wires be on the lowest cross arm, in order to reduce the bending stress on the pole to a minimum.
2. The largest wires be on the pins nearest the pole, in order to reduce the bending stress on the cross arm to a minimum.
3. The wires be arranged symmetrically on the two sides of the pole, especially those which end at the pole, in order to reduce the twisting stress on the pole to a minimum

For electrical reasons (which, however, are of minor importance) it is desirable that:

4. The wires of any one circuit be as close together as practicable (on adjacent pins), in order to reduce the self-inductance of the circuit.
5. The wires of a three-phase circuit be arranged to form the edges of an equilateral prism and the wires of a two-phase circuit be arranged to form the edges of a square prism, in order to render the inductances and capacities of the wires respectively equal.
6. The wires of different circuits be placed as far apart as practicable, in order to reduce their mutual inductance.

For practical reasons it is desirable that:

7. The highest voltage wires be on the top cross arms and on the pins farthest from the pole, in order to reduce the danger of accident to linemen.
8. The mains, which have the greatest number of taps, be on the lowest cross arms, in order to avoid danger of accidental crossing (with contact) of the wires.
9. The arrangement be systematic throughout; this is absolutely essential for safe and economical operation.

As it is impossible to meet all of these conditions the actual arrangements used are compromises and are governed by the relative importance attached to the several desirable conditions. In some cases the electrical requirements (4) to (6) have been considered of most importance, resulting, for example, in an arrangement subordinated to the idea that the three wires of a three-phase circuit must be arranged exactly in an equilateral triangle. It appears, however, that these electrical requirements are really the least important of the considerations and that in most practical cases can be entirely neglected.

Transpositions. — Due to the effect of mutual electromagnetic induction (*see Inductance*) an alternating or varying current flowing in one circuit will induce a voltage, and therefore a current, in any parallel circuit; and due to the effect of mutual electrostatic induction (*see Capacity*) an alternating or varying voltage in one circuit will induce currents in a parallel circuit, even though there is no metallic connection between the two circuits. It is possible, however, by properly transposing equal alternate lengths of the wires forming the two sides of each of the parallel circuits, to neutralize these effects. Fig. 1 shows diagrammatically a scheme of transposition whereby a two-wire circuit can be protected from both electromagnetic and electrostatic induction from a parallel circuit and vice versa, provided neither circuit is grounded. The more frequent the transpositions the more thoroughly are the effects due to inequality in the spacing of wires and poles eliminated. Transpositions cannot be made effective in eliminating inductive effects when either circuit is grounded, otherwise than at the neutral point. See also *Transmission Lines*.

Fig. 1. Transposition of Two Two-wire Circuits

Methods of calculating the induced voltage and induced currents in one line due to currents and voltage in a neighboring line are indicated in the articles on *Inductance* and *Capacity*.

Disturbances in Telephone Circuits due to neighboring power circuits are the most common results of electromagnetic and electrostatic induction. When the power circuit is grounded and carries either (1) fluctuating direct currents (e.g., a d-c. railway) or (2) alternating currents it may produce a noise in the telephone receiver, even though both the power circuit and telephone line are transposed. See also *Telephone Lines*.

Trees. — Trees constitute the most serious obstacle to the proper planning, construction and operation of overhead lines. The principal methods of meeting this difficulty are (1) avoiding them, (2) going over them, (3) going under them, (4) going through them. In most cases a combination of these methods is used. Where trees are a serious factor it is necessary to examine the route of every line in detail and the size and location of the trees may become the determining feature of the whole design. In such cases nearly all rules of good construction and systematic arrangements are violated in the interests of expediency.

The methods by which trees may be avoided are: using alleys instead of streets, or vice versa, choosing streets without trees for important lines, taking side of street with fewest trees, and finally the very bad arrangement of crossing the street back and forth to avoid the trees either individually or in groups.

The plan of going over the trees is the proper one in the case of all small trees and is perfectly satisfactory until the trees grow up and touch the wires. It is therefore only a temporary method, especially where the trees are of tall, quick growing varieties. In going over small trees it is well to have poles tall enough to allow for wires clearing after several years growth. It is usually impracticable to go over large full-grown trees because of cost of poles, unsightliness of very tall poles, and the difficulty of properly guying them to resist wind and the unbalanced pull of wires, which is magnified by the great leverage. It is also difficult or impracticable to take off service wires over the tops of tall trees.

In the case of very large trees it is sometimes practicable to take the wires under the trees on short poles. Where the wires pass the trunks it may be necessary to spread them or necessary to pass between large branches, with insulators fastened to the trees to maintain clearance. While such a line may be kept fairly clear under normal conditions, there is apt to be trouble during storms from branches falling on the wires or from limbs bent by wind or snow touching them.

With trees of moderate size it is usually necessary to take the wires through the trees among the leaves and small branches. Such wires are a constant source of trouble and expense. The branches and leaves should be trimmed from around the wires as much as possible, including not only those in contact with the wires but such as will be brought into contact by wind or which will grow into contact during the season. In addition it is usual to protect the wires in the worse places by tree insulation consisting of split tubes of wood or bamboo.

Pole Transformers. — The primary (1100- or 2200-volt) mains are usually run to transformers mounted on poles and the voltage there stepped down to the lamp or motor voltage (110 or 220 volts), and secondary mains run from the transformer to the buildings in the immediate vicinity. These transformers usually range in size from ½ to 50 kw., but transformers of ¼ kw. were formerly common and transformers larger than 50 kw. are sometimes used in factory districts where the unsightliness of the supporting structure (several poles framed

together) does not have to be considered. Very small transformers are objectionable because of their poor regulation, lower reliability and the cost of frequently changing them as the load increases.

Use of Single and Polyphase Transformers. — Transformers are usually of the single-phase type, two being used for motors on two-phase circuits and two for small motors and three for large motors on three-phase circuits. Three-phase transformers are also used for motors on three-phase circuits, and have the advantage of reducing the amount of wiring on the poles. Single-phase transformers have the advantage of being interchangeable between the single-phase lighting and polyphase power circuits where the same voltage is used.

Voltage Ratios of Pole Transformers. — Lighting transformers were early standardized with voltage ratios based on multiples or submultiples of 10 : 1, that is, 1000 and 2000 volts primary to 50, 100 or 200 volts secondary, or 1040 and 2080 primary to 52, 104 and 208 volts secondary. It was found that on account of the voltage drop in the transformers and secondary mains, motors wound for this voltage did not operate well. Instead of remedying this difficulty by winding the motors for a lower voltage or by raising the voltage on the generators and lamps, power transformers were introduced with windings based on a ratio of 9 : 1 thus giving with $^{1040}/_{2080}$ volts primary a secondary voltage of approximately $^{115}/_{230}$. It was soon found that many complications followed from the use of transformers of the two ratios, so that companies dropped one or the other ratio and made their transformers interchangeable. Of the two ratios the 10 : 1 ratio is preferable, because it is the more extensively used, gives a higher voltage for primary distribution and agrees better with the general principles according to which voltages have been standardized.

Service Wires. — The service wires are those which connect the house wiring with the main on the street. Usually these wires extend in a single span from the nearest pole to the house. At the house they are fastened to insulators similar to those used on the line and mounted on brackets attached to the house. These brackets are often the ordinary wooden bracket used in line work, though the special iron brackets made for the purpose are neater and more secure. The bracket should take the strain off the service span, so that where the service passes through the wall it will not be under the strain of the span. The wires should pass through the wall in porcelain bushings and should have a drip loop between the bracket and bushing so that water will not follow the wire into the building. Where service wires do not conveniently reach the house at point of entrance, they are carried along the wall to such point, being supported at intervals by insulators on brackets. See also *Wiring of Buildings*.

At the pole the service wires are sometimes attached directly to the mains that supply them; while this is the easiest method it has the disadvantage that the strain in the service wire will come on the mains, which may also be injured by the attaching and detaching of numerous service wires. When the service wires do not slope upward or downward at a considerable angle they are liable to become crossed with the main of opposite polarity. Since the general direction of the service wire is at right angles to the main, the service wire should naturally originate on a cross arm at right angles to the arm carrying the mains. In good service work consequently a cross arm is attached to the pole below the main, and all the service wires to both sides of the street run from this. One tap to each of the wires constituting the main can then be used for a number of service wires.

Front and Rear Servicing. — When houses are serviced from a pole line on the street the service wires usually enter the front of the house while if serviced from alley they enter the rear. Service wires can be run around detached houses from front to rear on brackets on the wall, but this is unsightly, expensive and increases the service wire drop. Service entrances can be changed by changing the inside wiring, but the expense is usually heavy and the damage to decoration of rooms sometimes makes it prohibitive. In the wiring of a building the error is frequently made of bringing out services without regard to location of supply lines.

Attic and Basement Servicing. — The service-wire entrance and the center of distribution in the house is generally in the attic for houses serviced overhead and in basement for those serviced underground. Buildings arranged for underground service are sometimes supplied from overhead lines by taking service wires down poles and under sidewalks in iron pipes.

In changing an existing system from a street to an alley distribution, or from overhead to underground service, the center of distribution in the house must be correspondingly changed, or a connection run from the new service entrance to old center of distribution large enough to carry the current without unduly increasing the drop.

Sectionalizing of Distribution Circuits, Fuses and Cut-Outs. — An overhead alternating-current system supplied from a single bus is sectionalized: (1) at the switchboard in power station or substation into circuits having no external interconnection by knife switches or oil switches with fuses or automatic trip; (2) at the poles where long branches leave, by transformer cut-outs, either fused or solid, or by pole-type oil switches; (3) at transformer primaries by fused cut-outs. Switches and fuses are used but sparingly in the primary mains, and in a small compact circuit none are necessary. No switches or fuses are ordinarily used on the secondary side of transformers or on secondary mains. Occasionally transformers have been provided with switches in secondary side so that part of the transformers on a network could be cut out during times of light load to save core-loss. Such sectionalizing has been little used, as the small savings have not justified the complication, care and hazard to service due to mistakes.

When several transformers feed the same secondary main, the opening of a primary cut-out, either by blowing of fuse or by hand, does not make the transformer primary dead, as it is still alive from the secondary side. In case the fuse of one transformer on a secondary network blows the additional load thrown on an adjacent transformer may cause the fuse of that to blow, and all transformers to go out in succession. Under these conditions the fuses cannot be replaced in one transformer at a time. To avoid the interruption to service involved in leaving the fuses out until a time of day when one transformer can carry the whole load temporarily, or in having the whole primary circuit out while the fuses are replaced, it is sometimes possible to have all the transformers on a single secondary network on the same branch of the primary main, this branch being sectionalized from the rest of the primary circuit by a switch.

Lightning Protection. — (*See also article on Lightning Protectors.*) Practically all disturbances from lightning enter a system through the overhead distributing circuits. The most serious effects are not to the distributing circuit itself but to the switchboard and machinery in the station. The effects on the circuit consist of splitting of poles, puncturing of insulators, puncturing of transformers, blowing of transformer fuses. There may also be damage to meters or appliances on the premises of consumers. The protection principally used includes (1) lightning arresters and choke coils in the station (prin-

cipally to protect station apparatus), (2) lightning arresters at intervals on the lines, (3) ground wires over the lines, (4) lightning rods on poles and (5) grounding of the circuit.

Use of Lightning Arresters. — On alternating circuits lightning arresters are used at intervals on the primary but not on the secondary circuits. The amount of line which a lightning arrester will protect is less the more severe the lightning discharge. It is impracticable, and probably impossible, to space the arresters close enough together to absolutely protect a circuit. Theoretically the number of arresters used should be such that the sum of the loss due to the damage by lightning and the expense of providing and maintaining arresters to avoid damage is a minimum. The effects of lightning are too variable and the money loss due to interruption of service is too indeterminate to admit of correct distribution of arresters being determined by calculation. In practice it has been found that when no arresters are used many lightning discharges of considerable intensity do no great amount of damage, also that where the arresters are used much of the damage done is to the arresters themselves, and that their failure is a cause of a considerable number of interruptions to service. It is usually well to begin by using arresters sparingly on the lines and putting additional arresters on if found necessary. The maximum number of arresters would be reached when one was provided for each transformer bank.

The importance of lightning protection is greatest in a composite system, and in most cases underground or submarine cables should be protected by lightning arresters wherever they connect to exposed overhead circuits.

Ground wires are principally used over transmission lines but may be used to advantage over city distribution wires in exposed places. Where adjoining buildings and trees are higher than the pole line, these foreign objects answer the same purpose and little additional screening effect will be obtained from a special ground wire.

RECORDS OF CIRCUITS. — Overhead line construction is constantly changing due to erection, moving and removal of poles, extension of mains, connection and disconnection of service, erection and changing of location and size of transformers. The ease with which changes may be made in the lines necessitates a system of records in such form that any details can be changed at frequent intervals without making a completely new record. The great amount of complicated detail subject to frequent change makes it impracticable to record every feature. In compiling a system of records it is therefore important to determine: (1) what features should be recorded and what omitted, (2) a method of recording information which will be easily corrected. Written records and maps are both used, though for most purposes the latter gives a clearer, more comprehensive and useful record.

Maps of circuits have usually become useless, soon after they have been prepared either because obsolete from lack of correction or unintelligible because of successive erasures and interlineations. The following method of keeping map records has been found to give good results: (1) a map of the circuits is prepared on tracing cloth, giving the circuits on the date of preparation; (2) a blue print is taken from this tracing and all changes marked in pencil or crayon on this print, the print and not the tracing being used for correcting and reference; the tracing is not subject to wear or tear; (3) before the blue print becomes illegible from correction or wear the accumulated corrections are made on the tracing and a new up-to-date blue print substituted, and the process repeated. The corrections on the blue print can be made by line foremen or other unskillful persons, at the time that the line changes are made, while those on the tracing should be made by a draftsman who will make them neatly

and with minimum damage to the cloth. If no colored inks are used on parts subject to change, such a tracing will last a long time and will only have to be redrawn when changes have become so numerous or radical as to amount to a rebuilding of the circuits.

COST OF DISTRIBUTION CIRCUITS. — So many varying items enter into the cost of a complete distribution system that it is impossible to give comprehensive figures of general application. A cost of from $150 to $250 per kilowatt of maximum load at the distributing switchboard may be taken as an indication of the magnitude of the total cost of an overhead distribution system for both light and power, including all outside construction from the power station or the substation to the customer's service inlet, including meters.

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DISTRIBUTION AND TRANSMISSION SYSTEMS. — (*See also Alternating Currents; Conduits and Conduit Lines; Distribution Lines; Electricity and Magnetism, Principles of; Ground Connections; Grounding of Electric Circuits; Insulators for Overhead Lines; Insulator Pins; Poles for Overhead Lines; Power Stations; Substations; Transmission Lines; Trolley Systems; Wiring of Buildings; Wires and Cables.*)

This article treats of the following subjects:

The design and construction of pole lines and underground circuits are treated in detail in the articles on *Distribution Lines, Conduits and Conduit Lines, Transmission Lines, and Wiring of Buildings.* Requirements regarding the grounding of circuits and methods of making ground connections are treated in the articles on *Grounding of Electric Circuits* and *Ground Connections.*

Circuits designed for transmitting relatively large amounts of power from one fixed point to another are called transmission lines, while those for delivering small amounts at numerous points are called distribution circuits. Transmission lines usually have no or few branches, while it is characteristic of distribution circuits to have many branches. The various systems of transmitting and distributing electric energy for light and power may be classified under two general heads, viz., constant current or series and constant potential or multiple systems, and each of these may be subdivided into direct-current systems and alternating-current systems.

CONSTANT CURRENT OR SERIES SYSTEM. — The lamps or other devices are connected in series and the current through them is kept constant, the voltages varying automatically to increase or decrease the energy delivered. It is the principal system used in the United States for street lighting, but is now little used for any other purpose. At one time such circuits were extensively used for commercial arc lighting in stores, but the series arc became obsolete for commercial use when the arc lamp was perfected for use in multiple circuits. The value of the current used for street lighting ranges from 1.75 to 9.6 amp., depending upon the type of lamp used.

Source of Current. — Direct current for series circuits is usually obtained from d-c. series generators (*see Generators, Direct-Current*) or from an a-c. circuit by rectifying the alternating current from a constant-current transformer (*see Rectifiers*); adjustable resistance in series in a constant potential d-c. circuit is sometimes used. Alternating current is usually obtained from a constant-current transformer on a constant potential a-c. circuit; other methods sometimes used are an adjustable resistance or reactance in series on constant potential a-c. circuits and by automatic regulating reactance coil on a constant potential a-c. circuit.

Cut-Outs, By-Passes and Transformers. — Switching out of lamps on series circuits is accomplished by short-circuiting them; this leaves the lamps charged to the potential of that point in the circuit. To make them safe to handle "absolute cut-outs" are used which also disconnect both conductors to the lamp, leaving the circuit closed.

To avoid the excessively high voltage which would occur when the circuit opened, due to series incandescent lamps burning out, an automatic by-pass is provided in multiple with the lamps, sometimes consisting of a piece of paper which punctures on a moderate rise of voltage, or of a choke coil which takes but little current at normal lamp voltage. Sockets for series incandescent lamps are arranged to automatically close the circuit when the lamp is withdrawn.

Transformers, one at each lamp, are sometimes used to insulate the secondary circuit containing the lamp from the primary or for reducing the current for low-current lamps used in series with lamps taking higher current.

Advantages and Disadvantages of Series Lighting. — Constant current or series systems have the advantage that low-voltage lamps may be used on high-voltage circuits without the expense, losses or complication of transformation. They have the disadvantage that the lamps are dangerous to handle, the efficiency is low at light loads and it is impracticable to distribute any large amount of power on a single circuit. In the series system the current and consequently the loss in the conductor is the same irrespective of the load, while in the multiple system, the current is proportional to the load, and the watts lost in the line therefore vary as the square of the load and the per cent loss directly as the load. For this reason the series system is not an economical one where the load varies and averages much below full load, which is the case with most commercial loads. For street lighting where all the lamps are turned on and off at once the efficiency at partial loads is of no importance. In constant-current systems the resistance of the circuit does not affect the uniformity of the voltage on the lamps at different parts of the circuit, while with constant-potential systems it does. For a scattering load of lamps, such as street lamps, a uniform light can therefore be obtained from the several lamps, with a much smaller weight of copper and fewer wires by using the series than by using the multiple system.

Thury System. — In Europe the constant-current or series system, with direct current of extra high voltage, has been used for long-distance power transmission under the name "Thury System." The current is obtained by connecting several generators in series, and is utilized by a number of motors also in series. The advantages are simple switchboards, no transformers and minimum strain on line insulators, the latter due first to the fact that in direct current the effective voltage is as high as the maximum voltage, and second, that in a constant-current system the working voltage only remains at its maximum value during the short period when the load is also a maximum. Among the disadvantages are the necessity for insulating frames of generators and motors, need of speed governors on motors, necessity of converting the current by moving machinery in every case where it is used for lighting and in most cases for power. The Thury System is not used in the United States.

CONSTANT-POTENTIAL OR MULTIPLE SYSTEM. — In this system, which is the one principally used for electrical distribution, the voltage between conductors is kept as constant as practicable and the current varies as the load changes. The degree to which the voltage approximates constancy throughout the system is called the "regulation" of the system.

Use of Direct or Alternating Current. — Either direct or alternating current can be used equally well for certain purposes, principally those for which the

heating effect of the current is used, including the lighting of carbon and tungsten incandescent lamps, cooking and heating. For certain purposes, where the current effects chemical or physical changes, such as charging storage batteries and electroplating, direct current is essential; for other purposes, such as operating arc lamps and tantalum incandescent lamps, it is better; whereas for other purposes, such as for Nernst lamps, the alternating current is better. For motive power, the direct current is most favorable where acceleration, variable speed and adjustable speed are desirable, whereas alternating current gives best results where uniform unvarying speed is desired. While the fields of the two kinds overlap to such a large extent that either kind can be used for general distribution, it is found that it is often more advantageous to use both. For low-voltage, underground distribution, direct current is advantageous because the heavy currents can best be carried on single conductor cables; no subway transformers and no small, high tension fuses are required.

Alternating current may be supplied either directly from the generators in the power station or from the secondaries of transformers in substations. Direct current may be supplied either directly from direct-current generators or from converter substations (*see Substations, Railway*) supplied with high-voltage alternating current.

Light and Power Circuits. — For a load consisting of both electric lamps and electric motors the same circuit may be used throughout for both classes of service or entirely or partially independent circuits may be employed. The use of the same circuit throughout for light and power service has the advantages of reduced number of wires, transformers and meters, reduced weight of wire, and reduced capacity of transformers and meters, whereas the use of separate circuits for light and power has the advantage of requiring less capacity of feeder regulators and, sometimes, of less weight of wire for the same perfection of regulation on the lighting circuits and also simplifies the problem of balancing the phases in polyphase distribution (*see below*). In the business districts of many cities the Edison three-wire direct-current system is used, in which case light and power are usually supplied from the same circuit, the motors as a rule being connected to the outside wires. In business districts where the lighting is done by alternating current the lighting is frequently on single-phase circuits and the power on separate polyphase circuits or on 500-volt direct-current circuits. In residential districts, where the lighting is done by single-phase alternating current, small motors are usually put on same services as the lights; whereas larger motors (from 1 to 5 and in some instances as large as 15 horse-power) are put on separate meters, services and transformers, but the same primaries are used for both services. In factory districts when the power is the predominating load and is supplied by polyphase circuits the incidental lighting is sometimes taken off the same services, but usually there are separate meters and transformers. There are a large number of cases where the same polyphase primaries are used for power and light with separate transformers and secondaries; in these cases the lighting transformers are usually distributed as equally as convenient between the several phases, in order to balance the load, though occasionally they are all connected to a single phase. In a few cases the same secondaries have been used for power and light usually on the four-wire three-phase plan.

Circuits with Branches. — There are two methods of providing proper distribution circuits for an extensive area, known respectively as the "tree system" and the feeder and main system. The former is the most extensively used but is not susceptible of providing as good voltage regulation as the latter.

Tree System; Interconnected Networks. — The simplest method of constant-potential distribution is to run one set of wires through the middle of

the district to be served, running branch wires from them wherever convenient for reaching loads at the sides. At the beginning, the single set of wires is large enough to carry the whole current, but the size is diminished as the current branches off. In this system adjacent branches will often have widely different voltages, due to unequal drop because of varying loads. If wires are run connecting the out-lying ends, currents will flow and partially equalize the differences in voltage. This plan, if carried out systematically, develops into a network, usually composed of two sets of parallel mains crossing at right angles and connected at each point of crossing.

Feeder and Main System. — It is practicable to keep the voltage on the customers' electric lamps within 2 to 5 per cent of being constant, even when it is necessary to raise the voltage of the generators from 10 to 20 per cent, from no load to full load, by arranging the distribution circuits on the feeder and main plan. The area to be served is first divided into compact sections; each small enough to be fed from a central point with a voltage drop within the desired percentage. A system of wires called "mains" are run from this central point, called the "feeding point," or "distribution center," to the customers. The mains form a little circuit originating at the feeding point instead of at the power house; they may be arranged on either the tree or interconnected network plan. A set of wires called "feeders" is run from the power house to the feeding point. As no current is taken off at intermediate points, these wires are not tapered and any desired drop of voltage may occur in the feeders without affecting the lamps, provided the power-house voltage is raised proportionally.

Two-wire System. — The simplest multiple system is the two-wire system, where all devices are connected directly in multiple. This system is used very extensively for direct-current light and power, from isolated plants, for power circuits (usually 500 volts) and railway circuits from central stations. It is also used for single-phase alternating-current distribution for both primary and secondary circuits.

Edison Three-wire System. — The three-wire system, Fig. 1, is obtained by replacing the out-going wire of one two-wire system and the return wire of a second two-wire system by a single wire, called the "neutral." The voltage between the outside wires is then double the voltage between the neutral and either outside wire. For example, 110-volt lamps may be connected between the neutral and either outside wire, and 220-volt motors may be connected between the two outside wires, and both the lamps and motors be supplied with their rated voltage. The neutral wire carries a current which depends only upon the *difference* in the loads on the two sides of the system and their distribution. As a rule the neutral of a three-wire main is made equal in cross-section to each outside wire. With perfectly balanced load the three-wire system with all three wires of the same size results in a saving in copper of 62.5 per cent as compared with a two-wire system supplying the same load at the same regulation.

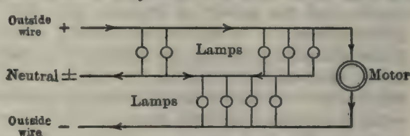

Fig. 1. Three-wire System

The three-wire system is used very extensively for direct-current light and power distribution from central stations, also for large isolated plants and for alternating current for lighting on the secondary circuit. Three-wire systems are usually 110 volts on each side of neutral, or 220 volts between outsides,

though there are several systems using 220 volts on each side and 440 volts between outsides.

The Edison three-wire distribution system, as used in the business sections of large cities, consists of a set of interconnected three-wire mains supplied by two-wire or three-wire feeders from one or more power houses or substations. The different feeders feed into the same set of mains at different points. At the power house or substation end the feeders are often all supplied from a common bus, though where there is a great difference in the length of the feeders there are sometimes two busses which are run in multiple when the load is light but separated when it is heavy, the short feeders on one called the "low bus," and the long feeders on the other called the "high bus," because of the relative voltages. As the feeders from the several substations connect to interconnected mains, the bus voltage in each is raised or lowered in accordance with the drop in its own feeders. All mains which connect feeding points supplied from a given bus are made large enough to allow large equalizing currents to flow through them, without requiring any great drop in the mains, in order to equalize the voltages at these points.

ALTERNATING-CURRENT SYSTEMS. — The simplest form of alternating-current system is the single-phase system, for which the connections are exactly the same as for the direct-current systems. Both two-wire and three-wire circuits are used, the former for primary circuits and small secondary circuits and branches, and the latter for large secondary circuits. The principal use of single-phase circuits is for electric lighting and auxiliary uses, such as heating or cooking, and fan motors. For delivering large amounts of power a two-phase or three-phase system is generally used.

Two-phase Systems. — In this system there are two single-phase currents having a difference in phase of 90°, or a quarter of a cycle. These currents may be distributed on the three-wire, four-wire or five-wire system; see also under *Transformer Connections* in the article on *Transformers*.

Three-wire Two-phase Systems. — Each single-phase current has a separate outgoing wire but unites in a common return wire. Each two-phase motor has two circuits, each connected between an outside wire and the return wire. The voltage between the two outside wires is 41 per cent greater than between outside wire and return wire and the current in the return wire is 41 per cent greater than in each outside wire. This is an unsymmetrical system and has the disadvantage that even a balanced load will cause a distortion and unbalancing of the delivered voltage of the two phases because of the unsymmetrical drop in the common return wire.

Four-wire Two-phase System. — Each of the two single-phase circuits has a complete, independent two-wire circuit. There are two variations, first where the circuits are insulated from each other, in which case a cross between either wire of one circuit with either wire of the other will change the voltage stress to ground, but will not affect the delivered voltage or cause a short circuit; second, when the neutrals of the two circuits are connected. In the latter case, from each wire of one circuit to either wire of the other circuit, the voltage is 71 per cent of the voltage between wires of the same phase. The four-wire system with insulated phases is probably the most extensively used of the two-phase systems.

Five-wire Two-phase System. — This is a modification of the four-wire system, with interconnected neutral in which the common neutral is extended as a fifth wire. Lamps may be connected from each of the four wires to the neutral. The five-wire system may be considered as two three-wire single-phase systems, one for each phase, with a common neutral wire.

Three-phase Systems. — In this system there are three single-phase alternating currents with a phase difference of 120°, or of one-third of a cycle. These currents may be distributed on the three-, four- or six-wire systems.

Three-wire Three-phase System. — Each single-phase current has a separate outgoing wire; the three return currents neutralize so that no return wire is required. The three wires are necessarily interconnected, the voltages are usually the same between any two and the currents equal in each of the three conductors, provided the loads on the three phases are equal, i.e., provided the load is balanced. When equally loaded the voltage drops in the three conductors are equal and symmetrical. This is the most extensively used of the three-phase systems.

Four-wire Three-phase System. — This is a modification of the three-wire system in which a neutral wire is extended as a fourth wire. Lamps or transformers may be connected from each of the three wires to the neutral, which carries only the unbalanced current, due to the differences in loading of the three phases. The voltage between the three outside wires is 73 per cent greater than from each outside wire to neutral.

Six-wire Three-phase System. — If to a three-wire, three-phase system, three wires are added, one with voltage midway between that of each pair of outside wires, lamps may be divided into six groups, between the three outside wires and the three adjacent middle wires. The result is the same as though there were three single-phase three-wire circuits, one for each phase, with the six outside wires combined in pairs giving three common outside wires in place of the three pairs. When connected in this way, the three middle wires cease to be neutrals, as between the three there is a three-phase voltage equal to one-half of that between the three outside wires.

Six-phase System. — This is used for circuits in the interior of substations (q.v.), such as from transformers to rotary converters, but is not used for distribution.

Monocyclic System. — This is a variety of unsymmetrical polyphase system now obsolete. It may be regarded as a species of two-phase system with voltage on one phase one-fourth of that on the other and with the two wires of the larger phase acting as a common return for the small phase.

Star or Y, and Mesh or Δ Connections. — See article on *Alternating Currents*.

Transformer Connections and Phase Transformations. — See article on *Transformers*.

Transformations of Systems. — Any electric system can be transformed to any other electric system by the use of rotating machinery; see *Motor-Generators* and *Converters, Synchronous*. This method is used chiefly for changing from alternating to direct current and for changing the frequency of alternating current. Conversion from alternating to direct current is also accomplished by means of mercury arc rectifiers, particularly for series d-c. arc lighting and for charging storage batteries; see *Rectifiers*. Conversion from any alternating voltage to any other alternating voltage at the same frequency is accomplished by means of transformers (q.v.).

Frequencies in Use in the United States. — At present the two frequencies in most general use and adopted as standards in most new work are:

60 cycles per second or 7200 alternations per minute, used by the majority of companies operating alternating-current lighting systems.

25 cycles per second or 3000 alternations, generally used for alternating-current railway or power work or where the alternating current is to be converted into direct current before final use.

In addition to the above other frequencies ranging from 140 to 25 cycles per second are in use as follows:

140 cycles or 16,800 alternations. Formerly standard frequency of Ft. Wayne electrical works, now obsolete for new apparatus but still used in many small towns.

133⅓ cycles or 16,000 alternations. Formerly a standard frequency of Westinghouse E. & M. Co., and Stanley Co., now obsolete for new apparatus but still used in many small towns.

125 cycles or 15,000 alternations. Formerly a standard frequency of General Electric Co., now obsolete for new apparatus but still used in many small towns.

50 cycles or 6000 alternations. Used extensively in several large systems in Southern California.

40 cycles or 4800 alternations. A compromise frequency at one time standard with the General Electric Co. used on several large systems in that part of New York state around Albany and including the General Electric Co.'s system at Schenectady. Also used by certain mills in New England and elsewhere.

35 cycles or 4200 alternations. Used on the large system of the T.C.R.T. Co., in Minneapolis and St. Paul.

33⅓ cycles or 4000 alternations. Used by the P.G.E. Co., in Portland, Oregon.

16⅔ cycles or 2000 alternations and 15 cycles or 1800 alternations have been proposed for use on single-phase electric railway work and it has even been suggested that they be standardized for that purpose but at the present time very little progress has been made toward their introduction.

Choice of Frequency. — For new light and power systems, the choice of frequency depends on (1) the frequency of existing systems in same or adjoining territory, and (2) the proportion of load which will be direct current. It is generally advantageous to have a new system of the same frequency as that of existing systems in the same or adjoining territory, so that a physical connection can be made between them or load transferred from one to the other without the use of frequency changers. The choice of frequency now (1914) is in most cases confined to the two standard frequencies, 60 and 25 cycles. Aside from the question of matching adjacent systems, the choice depends principally on the proportion of current to be converted into direct current. Where all load is direct current, 25 cycles has the advantage; where all load is alternating-current lighting, 60 cycles is best.

Use of Two Frequencies. — When the bulk of the load is direct current, but a small though important part is alternating-current lighting, two frequencies, 25 and 60 cycles, are sometimes used. Sometimes the two frequencies are generated by separate prime movers, though sometimes all current is generated at 25 cycles and the 60-cycle current obtained from frequency changers (*see Motor Generators*).

RELATIVE VOLTAGES AND WEIGHTS OF COPPER REQUIRED FOR VARIOUS SYSTEMS. — The comparison of the various systems with respect to the weight of copper required, as tabulated below, is based upon the assumptions (1) that the energy delivered, (2) that the energy loss and (3) that the maximum voltage strain on insulation between any wire and ground are respectively *the same* for all systems.

When the neutral or middle point of the system is grounded, then the maximum voltage strain on the insulation to ground is the maximum instantaneous value of the voltage between any outside wire and neutral. In the case of an alternating sine wave of voltage the maximum instantaneous value is equal to $\sqrt{2}$ times the effective value determined by a voltmeter. When the neutral is not grounded then an accidental ground on any leg will throw full line voltage across the insulation between any other leg and ground.

On this basis of comparison, assuming a sine wave of voltage and 100 per cent power factor for the alternating-current systems, the relative voltages between outside wires and the relative weights of copper are as follows:

RELATIVE VOLTAGES AND RELATIVE WEIGHTS OF COPPER

System*	Relative voltages between outers		Relative weights of copper*	
	Grounded neutral	Insulated neutral	Grounded neutral	Insulated neutral
	Per cent	Per cent	Per cent	Per cent
Direct-current (2-wire)	100	50	100	400
Single-phase (2-wire)	71	35.5	200	800
Two-phase (4-wire)	71	35.5	200	800
Three-phase (3-wire)	61	35.5	200	600

* Neutral wire not included; when neutral is added increase the figures given in the ratio of weight of neutral to combined weight of the outside wires for the system in question. For example, the addition of a neutral wire equal in size to either outside wire to a d-c. 2-wire system with grounded neutral gives a relative weight of copper of $100 \times 1.5 = 150$ per cent.

Effect of Power Factor. — To correct the above tabulation for alternating-current systems in cases where the power factor is less than unity divide the relative weight of copper given by the square of the power factor (i.e., for a single-phase two-wire system with insulated neutral and 70 per cent power factor the weight of copper will be $800/0.70^2 = 1630$).

VOLTAGES FOR D-C. AND A-C. SYSTEMS. — Unless otherwise specified the "voltage" of a polyphase system refers to the potential difference between adjacent outside wires; see *Alternating Currents* for the relations between the various voltages.

A considerable number of voltages are standard or semi-standard. In general they may be considered as derived from a lamp voltage of 50 or 100 multiplied by the factors 2, 3, 5 and 10, and increased by 4, 5, 10, 15 and 20 per cent for various reasons, principally to allow for line loss. In addition to this system of voltages there is another system which arises from the use of Y and Δ connections on the three-phase system; these voltages are related to the above by the factor $\sqrt{3} = 1.73$, either as a multiplier or a divisor. For example, the voltage between the outside wires of a four-wire three-phase system supplying 115-volt lamps connected between each outside wire and neutral is $\sqrt{3} \times 115 = 198$ volts.

Voltages Commonly Used. — The voltages most used are:

Direct current:

100 to 120 volts	Lighting, small power and field excitation.
200 to 240 volts	Power, lighting and field excitation.
500 to 600 volts	Power, electric urban railways.
1200 to 1500 volts	Interurban electric railways.
2400 to 3000 volts	Electric trunk lines.

Alternating current, secondary distribution:

100 to 120 volts	Lighting, small power.
200 to 240 volts	Power.
400 to 480 volts	Power.
500 to 600 volts	Power.

Alternating current, primary distribution:

1000 to 1200 volts	Lighting (obsolete).
2000 to 2400 volts	Lighting and power.

Alternating current, intermediate distribution:

5000 to 6000 volts	Generating station to substation.
6000 to 7200 volts	Generating station to substation.
10000 to 12000 volts	Generating station to substation.
12000 to 13200 volts	Generating station to substation.

Alternating current, high tension transmission:

20000 to 24000 volts	Generating station to substation.
30000 to 36000 volts	Generating station to substation.
40000 to 48000 volts	Generating station to substation.
50000 to 60000 volts	Generating station to substation.
60000 to 72000 volts	Generating station to substation.
100000 to 110000 volts	Generating station to substation.

CIRCUITS FOR TYPICAL CITY DISTRIBUTION SYSTEM. — For a city covering a space ten miles square, with current generated at one point, a typical distribution would consist of the following lines: —

(1) From generating station to substations; alternating current, 60 cycles, three-phase, 11,000 volts.

(2) From lighting substations in business center; direct current, 110–220 volts, three-wire.

(3) From lighting substations in residence district; alternating current, 60 cycles, one-phase, 2200 volts for house lighting, and alternating current, 60 cycles, one-phase, 6.6 amperes series circuits for street lighting.

(4) From power substation in factory district; alternating current, 60 cycles, three-phase, 2200 volts.

(5) From railway substation; direct current, 550 volts.

(6) From secondaries of lighting transformers; alternating current, 110–220 volts, three-wire for large transformers and alternating current, 110 volts, two-wire for small transformers.

(7) From secondaries of power transformer; alternating current, 220 volts, three-phase.

EFFICIENCY OF DISTRIBUTION. — The ratio of the energy registered by the customers' meters, or which would be so registered if all customers had meters, to the energy supplied by the generator to the bus-bar in the generating station, may be called the over-all efficiency of distribution. The losses may be divided into several kinds: line loss, transformer loss, converter loss, meter loss and error, leakage and unaccounted for. For some purposes it is useful to consider eacn loss from two points of view: (1) as a loss of energy, and (2) as a loss of power at full load; the first may be called the energy loss and the second the capacity loss. The corresponding efficiencies are usually designated as all-day or energy efficiency and the full-load or capacity efficiency respectively.

Fixed and Variable Losses. — The total energy loss consists of two components, (1) a fixed loss independent of load, including the core-loss of transformers, the core-loss, excitation, friction and windage of rotating converting

apparatus, the loss in the shunt coils of meters, the copper loss in constant current circuits, and the loss in the arc of mercury arc rectifiers for constant current circuits; (2) a variable loss proportional to the square of the current, including the copper loss of constant potential circuits and of transformers, the armature copper loss of rotating converting apparatus. The effect of the fixed loss on the per cent efficiency depends on the load factor of the load, while that of the variable loss depends both on the load factor and the shape of the load curve.

Representative Losses. — For a lighting system, the full-load losses in primary feeders, primary mains, transformers, secondary mains, services and meters may be expected to be as much as 17.5 per cent of the power generated and the daily energy loss 33.3 per cent of the energy generated, giving 82.5 per cent capacity efficiency and 66.7 per cent energy efficiency respectively.

Effect of Nature of Load. — In making estimates of the efficiencies of particular systems, the effect of the following items should be considered: (1) relation of transformer capacity to maximum loads, (2) load factor, (3) shape of load curve, (4) power factor, (5) diversity factor.

REGULATION. — An ideal constant potential distribution would have one uniform, unvarying voltage, and would be said to have perfect regulation. The greater the variation from such constancy the poorer the regulation. The regulation is usually specified in per cent variation, either "above or below" a standard mentioned. The standard is usually either the nominal voltage desired or the actual average voltage obtained.

Evil Effects of Poor Regulation. — The evil effects of high voltage are short life of electric lamps, excessive speed of direct-current motors, excessive exciting current of induction motors and burning out of motors and other devices; on the other hand, low voltage greatly diminishes both the candle-power and efficiency of electric lamps, decreases the maximum power of motors and increases the current which a motor will take for a fixed horse-power output. As electric lamps are much more sensitive to change of voltage than motors, separate circuits are often used for lighting and power, the former having devices for regulation which are omitted from the latter.

The following figures give roughly the quantitative effect of voltage variation between 5 per cent below and 5 per cent above normal:

Each per cent decrease in voltage decreases candle-power of carbon incandescent lamps	5 Per Cent
decreases torque of induction motor	2 Per Cent
Each per cent increase in voltage decreases the life of carbon incandescent lamps*	13 Per Cent
increases the magnetizing current of induction motors	2 Per Cent

Ordinary Limits of Regulation. — Roughly, the maximum voltage variation at the lamps on a lighting system should never exceed 5 per cent, i.e., the regulation above or below normal should never be greater than 2.5 per cent, and should be as much less as is economically feasible; the voltage variation on power systems is usually 10 per cent (5 per cent above or below normal) and is sometimes considerably more.

Calculation of Regulation. — In order to calculate the variation in voltage at any receiving device or group of such devices from no load to full load, the voltage at the generating or substation or feeding point being assumed constant, the impedance drops in all parts of the distribution system must be calculated

* Average for 5 per cent increase in voltage; the first per cent increase in voltage decreases the life 18 per cent, the fifth per cent 8 per cent.

and properly combined. The various parts of the system to be considered are the house wiring, service wires (or leads from street mains to the house), secondary mains, transformers, primary mains, primary feeders. See *Distribution Lines; Transmission Lines; Wiring of Buildings.*

In making such calculations account should be taken of the fact that the loads or currents in the several parts of the system seldom have their maximum values at the same time. The maximum drop in house wiring will not occur simultaneously in all houses nor will the maximum service drop occur together on all services, etc. Furthermore, the maximum house wiring drop for a given house may not occur at the same time as the maximum drop on the secondary mains, transformer or primary mains to which it is connected.

Effect of Line Reactance. — The regulation of an alternating-current system of unity power factor will be poorer than that of a direct-current system of the same copper efficiency (see tabulation above) because of additional drop due to line reactance.

Effect of Lagging Power Factor. — A lagging power factor usually makes the regulation of an alternating-current system worse than it would be for unity power factor and therefore much worse than for a direct-current system of the same copper efficiency.

Effect of Leading Power Factor. — A leading power factor usually makes the regulation better than it would be for unity power factor and may give even better regulation than can be obtained from a direct-current system of the same copper efficiency.

Effect of Currents in Neutral. — Any current in the neutral wire of a balanced system produces a drop which tends to unbalance the voltage of the two sides. In the case of a three-wire direct-current or single-phase system *one per cent drop in neutral produces two per cent difference in the voltages on the two sides.* Voltage drop in the neutral therefore affects more seriously the regulation than an equal amount in the outside wires. If the currents in the neutral all have the same direction, say toward the station, the voltage drops in the various parts of it will be cumulative, and though the drop in each section may be small and no current may actually reach the station, the aggregate effect may be serious. The individual loads on the two sides of the neutral should be connected so that the unavoidable neutral currents flow alternately in each direction, thus causing drops in alternate directions thereby neutralizing each other over the total length of the circuit.

Feeder Neutral in Three-wire Systems. — When the "feeder and main" system is used with a three-wire system, two-wire feeders should be used only on the outside wires, and only a single "feeder" neutral be used. That is, with respect to the neutral, the "tree" system gives better regulation for the same weight of copper than the "feeder and main" system, but with respect to the outside wires the latter system gives the better results. This is a point which has not always been recognized in laying out three-wire systems.

VOLTAGE CONTROL. — (*See also Controllers.*) The more common methods of controlling the voltage at feeding points when the feeder and main system of distribution is used (*see above*) are the single bus system, the high and low bus system, and feeder regulators.

Single Bus System. — (*See also article on Bus-Bars.*) All the feeders may be connected to a single bus and the bus voltage be raised as load increases so as to compensate for average drop of all the feeders on it. This method gives excessive voltage on such feeders as are comparatively short and low voltage on those that are long. Usually maximum voltage is carried in the evening

during the lighting peak and lower voltage during day giving very poor regulation on power circuits having maximum load during the day and low load at night. This method is used on most small direct- and alternating-current distribution systems.

High and Low Bus System. — The bus may consist of two parts which may be separated, and operated at different voltages, feeders of greater average drop connected to the "high" (voltage) bus and the others to the "low" (voltage) bus. The voltage of each bus is raised or lowered in accordance with average drop of feeders on it as in the single bus system. This method is used extensively on the Edison three-wire direct-current systems, as noted above. The unequal drop of the several feeders on same bus are equalized by heavy interconnection through mains.

Feeder Regulators. — A feeder may have a separate regulator adjustable to compensate for its own drop. This method is sometimes used on direct-current railway feeders and very extensively on alternating-current lighting feeders. See article on *Regulators*. A feeder or other distribution circuit which contains a voltage regulator is frequently referred to as a "boosted circuit." Boosted circuits are not usually interconnected but may be if the boosting is of the same nature in each.

Control of Polyphase Systems. — In a polyphase system the load may be as equally divided among the phases as possible and the voltage regulated with respect to any one phase taken as representing the average of all; or the lighting load may be connected on a single phase and the voltage regulated for this phase alone. The former method is more common, as it permits of full output of all the phases of the generator being used and gives a more equal voltage on the several phases of polyphase motors.

Transformers as Outside Boosters. — An auto-transformer or an ordinary transformer may be connected to a feeder at any point and used as a "booster" to raise or lower the voltage. In the case of an ordinary transformer the primary and secondary are connected in series thus converting the transformer into an auto-transformer; see article on *Auto-Transformers*. Such boosters are not adjustable and have a bad effect on the regulation as they give excessive voltage on the boosted part of line at light load.

To cut such a booster out of service without taking it off the circuit the primary coil must be open circuited and the secondary short circuited. *Caution:* the main circuit must be opened before cutting out booster, because if the primary is opened while current is flowing in the secondary the booster becomes a step-up transformer and may give a dangerously high voltage on the primary. If on the other hand the secondary is short circuited first, a destructive short-circuit current may flow through it and the primary, and when the primary is opened a dangerous arc will form.

Use of Lamps of Different Voltage. — Attempts have been made to compensate for the difference in voltage at the various points of a network by using lamps of different rated voltage, high-voltage lamps being used near the point of supply and low-voltage lamps at the ends of line. This plan has not been successful because the difference in voltage between any two points is not constant. Frequently some of the high-voltage lamps are used by mistake where low-voltage lamps are intended; the results are then poorer than when no difference is made in lamp voltage. While lamps of two or more voltages may be used temporarily as an expedient when regulation is very bad, it is better practice to have all lamps on a system of a single uniform voltage.

Use of Transformers of Different Ratio. — Transformers are also made with taps so that a uniform secondary voltage can be obtained with a varying primary voltage. The plan is not a good one, because if the difference in ratio

is correct for uniform voltage at full load it gives unequal voltages at light load. It also has the very serious disadvantage that the haphazard changing of transformer ratios by ignorant linemen to compensate for dim light, due perhaps to lamps already blackened by excessive voltage, makes it impossible to carry out any systematic plan for securing the best average regulation for the system as a whole.

BALANCING OF LOAD. — In three-wire direct-current or single-phase systems and in all polyphase systems supplying single-phase load it is necessary to approximately balance the load between the two sides or the several phases as the case may be. Unbalanced load has two bad effects: (1) it loads the two sides or phases of the system unequally, making it impossible to get full output out of the lightly loaded side or phase without overloading the other; (2) it makes the regulation of the system worse by causing high voltage on the lightly loaded side or phase and low voltage on the other.

The first difficulty is not serious, because through the conduction of heat from the loaded to the unloaded coils of transformers and generators the machine capacity is not reduced in proportion to the unbalance; for moderate unbalancing, say up to 10 per cent greater load on one side or phase than on the other, it is doubtful if any appreciable effect could be discovered. As the total load usually consists of a great number of small parts, a very little foresight in dividing the load in the first place between the sides or phases, and in suitably distributing subsequently connected load, will give a balance good enough for all practical purposes. When polyphase alternators were first installed for supplying existing single-phase lighting circuits, it was supposed to be important to have a close balance of load between the phases, and early switchboards were therefore provided with transfer switches for throwing single-phase feeders from phase to phase. It was later found that instead of being necessary to transfer the load from phase to phase, following the diurnal or annual variations of load, that the circuits could stay on the same phase indefinitely and that transfer switches were unnecessary and undesirable. The time when rebalancing of this kind is necessary is usually when new circuits are established, at which time the changes of connections are best made by changing the taps to the bus-bars.

Motor-generator Balancer. — On direct-current three-wire systems the difference in current on the two sides may be balanced without taking the neutral current to the generator by a balancer consisting of two similar machines mechanically coupled and electrically connected in series. Each machine is wound for the voltage of one side of the system (110 volts for a $^{110}/_{220}$-volt system); the common connection between the machines is connected to the neutral and the other two terminals to the two outsides, see Fig. 2 (the field windings are omitted for clearness). The unit acts as a motor generator (q.v.), whichever machine happens to be on the light side being the motor for the time being and the other the generator. By strengthening the field of the one on the side where the voltage is low and weakening that of the other, so as to keep the same total voltage, the voltages on the two sides as well as the currents may be balanced if necessary.

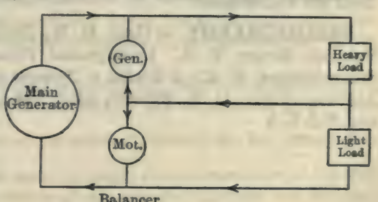

Fig. 2. Motor-generator Balancer

The output of the balancer generator and the input of the balancer motor are practically equal to each other, neglecting the losses in the machines, and each is equal to

$$\frac{P_1 - P_2}{2},$$

where P_1 is the load on the heavily loaded side of the system and P_2 the load on the lightly loaded side. For example, if the load on one side is 110 kw. and on the other side 90 kw., the load on each unit of the balancer is 10 kw.

It would, however, be unsafe to use a balancer as small as such calculations for the normal unbalancing would indicate as correct, because in case of a short-circuit on one side of the system, or of loss of a large amount of load on one side, due say to the blowing of fuses, the balancer would be dangerously overloaded. This may not only destroy the balancer, but may burn out many lamps and motors on one side due to excessive voltage. In small systems, say up to 100-kw. total capacity, the balancer should be able to operate momentarily with any loading up to the capacity of the main generator, and in large systems should have capacity sufficient to burn off a short-circuit on one side of the system without creating an unduly high voltage on the lightly loaded side.

Dynamotor as a Balancer. — A smaller and cheaper balancer is obtained by using a dynamotor (q.v.), which is a single machine with two windings on one armature and two commutators. The armatures are connected in series and balance the currents, but as there is only one field the voltages cannot be balanced.

Alternating-current Balancing Coil. — On alternating-current single-phase three-wire systems it is usual to obtain the neutral from the middle point of the coil of the transformer supplying the current. It can, however, be obtained at any point of the two-wire circuit without going back to the transformer by using a balance coil. A balance coil is a transformer with two similar windings connected in series across the circuit and with the neutral wire connected to the common point between the windings. Whichever coil is on the lightly loaded side acts as the primary and the other as the secondary, thereby balancing the circuit by transferring one-half of the difference in load from the heavily loaded side to the lightly loaded side. Such coils, which are essentially auto-transformers with a 2:1 ratio, may be used in various ways: to obtain the neutral for an unbalanced three-wire circuit from a two-wire circuit; to supply a 110-volt load from a 220-volt circuit; to supply a large 110-volt load from a 110/220-volt circuit without connecting to the neutral. In practice balance coils are not much used as the neutral can more cheaply be obtained from the transformer.

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DRAFT, MECHANICAL. — (*See also Blowers and Compressors; Chimneys; Fans; Pipes and Piping.*) Any system producing draft either by steam jets or by blowers is called a mechanical draft system. The relative advantages of mechanical draft as compared with chimney draft are: (1) The supply of air may be more readily controlled; (2) a fan permits a lower temperature of flue gases, which increases the boiler efficiency; (3) the boiler may be forced to high overloads in case of emergency; (4) the draft is uninfluenced by climatic conditions; (5) the system is low in first cost. The disadvantages of mechanical draft are: (1) danger of breakdown; (2) cost of maintenance and operation; (3) a stack or other means must be provided to carry off the flue gases wherever their discharge at low levels constitutes a public nuisance. Many plants with tall stacks are provided with mechanical-draft apparatus as a means of forcing the boilers in case of heavy overloads. Mechanical draft is also frequently used in connection with fuel economizers.

Steam Jets for producing artificial draft are simple, inexpensive and easily applied, but are very uneconomical, since they require from 6 to 11 per cent of the total steam generated (A. J. Whitham, *Trans. A.S.M.E.*, Vol. 17).

Forced Draft. — A blower system creating an excess of pressure beneath the fire is known as a forced-draft system. A single fan is commonly employed to supply a number of boilers through a system of ducts beneath the floor. The air is sometimes taken from a chamber built around the breeching, thus utilizing a part of the heat energy in the exhaust gases.

Induced Draft. — In the induced-draft system the fan is placed between the breeching and the stack. All leakage of air is inward, avoiding inconvenience when the doors are opened. This system is frequently used in plants which have high peak loads, being ordinarily installed in connection with fuel economizers. A by-pass directly from breeching to stack is usually provided for use when mechanical draft is not required or in case of accident. A double system of fans is also sometimes provided as a further insurance against accidental interruption.

POWER REQUIRED — SIZE OF FANS. — The power and size of fans required to operate either a forced-draft or induced-draft system may be roughly calculated by the method given in the article on fans. The static pressure may be roughly approximated from the data given in the article on chimneys, the figures there given for loss of draft being taken as the static pressure to be overcome. The quantity of air required per boiler horse power depends upon the quality of the coal, upon the care taken in firing and upon the efficiency of the boiler and grate (*see Boilers*).

Let R be the number of pounds of coal required per boiler-horse-power hour, W the pounds of air required per pound of coal, P the boiler horse power and $w = \frac{1.33\,B}{460 + t}$ = pounds per cubic foot of gas passing through the fan, where B is the barometric pressure in inches of mercury and t the temperature of the gas in degrees Fahrenheit; then the volume of gas passing through the fan per minute is $Q = \frac{RWP}{60\,w}$. This formula and the formula for w are strictly applicable only to the case of a forced-draft fan, but the two formulas may also be used with but a small error for an induced-draft fan, taking for t in the latter case the temperature of the discharged products of combustion (about 500° F., when no economizer is used and about 300° F. with an economizer). The size of fan and the power required to operate it will be greater for induced draft than for forced draft, since in the former case the volume of gas passing through

the fan is greater. According to Gebhardt the power required to operate a forced-draft system for a plant of 1000 boiler horse power or more, will range from 1 to 5 per cent of the plant capacity.

COSTS (**Pre-war figures**). — According to Gebhardt the cost of a forced-draft fan, engine and stub stack for a plant of 1000 boiler horse power or more will approximate 20 to 30 per cent of the cost of an equivalent brick chimney. The cost of a single induced fan, engine, stack, etc., will approximate 40 to 50 per cent of the cost of an equivalent brick chimney, and the double-fan outfit will be 50 to 60 per cent of that of an equivalent brick chimney. Two per cent is a liberal allowance for maintenance and depreciation of a brick or concrete chimney, while for a mechanical-draft system an allowance of from 4 to 10 per cent should be made. It costs nothing to operate a chimney, while from 1 to 5 per cent of the cost of operation of a boiler plant is chargeable to a mechanical-draft system.

BIBLIOGRAPHY. — Gebhardt, *Steam Power Plant Engineering; Mechanical Draft*, published by B. F. Sturtevant Co., Boston.

DREDGES, ELECTRICALLY OPERATED. — (*See also Motors, Industrial Applications of.*) Electrically operated dredges are now used very generally, especially in connection with gold mining. This drive has indisputably proved its advantages over steam drive from the standpoint of both operation and economy, as individual motors can be applied to the various units of the dredging machinery, and a larger percentage of the power input can thereby be directly applied in useful work.

The standard form of dredge is of the continuous-chain, close-connected bucket type, ranging in capacity from 3 to 16 cubic feet and with a digging depth of from 40 to 60 feet. As the power is mostly obtained by transmission from some hydroelectric power system or central generating station, alternating current is generally used for driving the dredge machinery. For a modern gold-mining dredge this consists of the digger or bucket line, the winch, the high- and low-pressure pumps, the priming pump, the screen and the stacker.

Bucket Line. — The bucket line is supported on a sheet-steel or girder structure of massive construction, so as to resist the heavy strains while in operation, especially when striking rock. The speed varies from 50 feet (with from 16 to 25 buckets) to 75 feet (with from 35 to 50 buckets) per minute, depending upon the condition of the ground.

For operation and control of the digger a variable-speed induction motor is used. This is located on the lower deck and belted to the driving pulley, which is generally situated in the rear of the pilot house on the upper deck. The duty imposed upon this motor is severe, as it must operate under conditions calling for power varying from 75 per cent overload down to 25 per cent of its rated capacity. A drum-type controller for forward and reverse operation is provided, including the necessary resistance for continuous operation on any notch of the controller from one-half to full speed. The maximum starting torque is required and obtained at about the fourth point of the controller, thus leaving three points on which to bring the motor up to half speed, at which time nearly full rated torque is required. As a result of these conditions the ordinary motor designed for intermittent service cannot be successfully applied.

The raising and lowering of the bucket-line ladder is generally accomplished by a friction clutch which can be connected to the digger motor. For the larger size dredges, however, a separate motor is often provided to perform this duty.

Winch. — To keep the dredge in place and to move it about or hold it against the bank when digging, head lines are used, which are controlled from the forward end and operated by a six- or eight-drum winch driven by a variable-speed motor. The winch motor, though of smaller capacity, is of the same staunch construction as the digger motor and is equipped with a suitable controller and resistance to permit its continuous operation from one-half to full speed. It has been found advisable to equip the motors for this service with solenoid brakes, by means of which the motor can be brought to a standstill almost instantly. It is then ready for the reverse operation without the usual reversing of the motor through the controller, which is not only bad practice but may result in a burn-out due to the heavy strain on the windings.

Pumps. — The high- and low-pressure pumps for supplying water to the screens and sluices are generally connected to individual motors. Constant-speed squirrel-cage motors of compact construction and large overload capacity, with a speed of from 600 to 900 r.p.m., are usually installed for this work.

To prevent the filling up of the basin, in which the dredge floats, when digging in shallow water, it is sometimes found necessary to install a sand pump,

which carries the fine tailings from the sluice boxes to the top of the rock pile by way of the stacker.

Priming Pump. — A priming pump is required for priming the large pumps, or for supplying water on the tables during the "clean-up." It generally consists of a small centrifugal pump driven by a high-speed direct-connected squirrel-cage induction motor.

Screen. — Either a shaking or revolving screen is used to separate the gravel from the clay and permit the fine particles containing the gold to pass through on to the gold tables and sluices below. For this service a constant-speed motor is recommended, which can be placed on the upper deck and belted down to the driving pulley of the screen.

Stacker. — After screening the large rocks are carried on a belt conveyor to the end of the stacker and deposited on the spoil in the rear of the dredge. For operating this conveyor a constant speed motor is installed at the extreme end of the stacker, where it can be readily housed.

Monitor. — When a dredge is working in a high bank of hard material actual experience has demonstrated that it is advisable to lower the water level in the pit and cut the bank down with a "Giant," water being furnished by a three- or four-stage, high-pressure centrifugal pump driven by a phase-wound induction motor. The Giant is mounted on the bow of the boat, and as the dredge swings from one side of the cut to the other, the stream of water is thrown against the bank about two feet above the water level. This undermines the bank, causing the material to cave into the pond where it is brought up by the bucket line. The Giant is also used in localities where the water in the pond cannot be raised high enough to bring the bucket line near the top when the dredge is stepping ahead.

Power Required. — The sizes of motors required for driving the machinery of some dredges of different capacities are given in the following table. The proper speeds will depend on whether the motors are to be belted or direct connected to the machines which they are to drive.

SIZES OF MOTORS FOR DREDGES

Machinery to be driven	3 cu. ft. dredge, 40-ft. digging depth		5 cu. ft. dredge, 40-ft. digging depth		7 cu. ft. dredge, 40-ft. digging depth		13½ cu. ft. dredge, 40-50 ft. digging depth		16 cu. ft. dredge, 55-ft. digging depth	
	H.P.	R.p.m.	H.P.	R.p.m.	H.P.	R.p.m.	H.P.	R.p.m.	H.P.	R.p.m.
Bucket line	75	720	100	720	150	600	300	514	400	360
Winch	15	720	25	720	25	720	35	600	50	600
High-pressure pump	40	900	50	900	50	900	150	720	150	600
Low-pressure pump	25	600	25	600	25	600	75	600	100	600
Priming pump	7½	1200	7½	1200	7½	1200	25	1200	35	1200
Screen	25	900	25	900	25	600	75	600	150	600
Stacker	20	900	20	900	25	600	50	600	50	600

Voltage of Motors. — The power for operating a dredge is generally transmitted from some existing power system, and the connection between the line and the dredge is, as a rule, made by means of armored cable carried on floats. When this transmission can be economically carried out at a voltage of 2200, it is customary to provide the motors for this voltage, thus eliminating the expense of installing step-down transformers. When the transmission voltage is above 2200 volts, it is desirable to step it down to 440 volts and use motors for this voltage. When step-down transformers are required, their capacity is usually taken as two-thirds of the total horse-power load, allowing one kilowatt for each horse-power.

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DYNAMOTORS. — (*See also Converters, Synchronous; Motor Generators; Transformers.*) A dynamotor is a direct-current device combining both motor and generator action in one magnetic field. It has an armature having two separate windings and two separate commutators, one at each end of the armature. Either winding may be used as the motor winding, and the other as the generator winding. Such a machine performs the same function in a direct-current circuit that a power transformer does in an alternating-current circuit, i.e., serves as a means of transforming high-voltage direct current into low-voltage direct current, or vice versa.

Performance Characteristics. — The device corresponds to two machines in which there is only one core-loss, one friction loss, one excitation loss, but two losses due to RI^2 in the two armature circuits. It is therefore more efficient than a motor-generator set, but less efficient than one machine. Since the currents in the two windings flow in opposite directions their resultant magnetic effect is zero. The machine has therefore no armature reaction (except for the slight amount due to the current to overcome the losses in the machine). It is not subject to the troubles of field distortion and bad commutation that occur in either motors or generators. It is impossible to compound a dynamotor, since any increase in field strength intended to increase the voltage of the generator would decrease the speed of the motor by the same amount and no change would result. The ratio of the two voltages is therefore fixed by the number of turns, and only varies from this by the loss due to RI drop in both windings. These two drops are additive, and therefore the regulation of such a machine is not very good.

Applications. — Dynamotors are used largely to give large currents to start other motors, or to give low voltages or a fractional voltage in a multi-voltage system for speed control. The motor of the combination may be wound for the line voltage and the generator for any fraction of the line voltage. Thus the combination supplies a large current at a low voltage, which will give a good starting torque in motors connected to it with a reasonable consumption of power. They are used as equalizers or "balancers" in three or multi-wire circuits, but are not as desirable for this work as motor-generator sets with compound-wound machines, on account of their poor regulation. They are also used to supply a low voltage for such purposes as telephone and telegraph systems and the low voltage and large currents for electrolytic work.

Dynamotors are also used in radio work in connection with vacuum tube detectors and amplifiers, in order to obtain from a battery of 4 to 10 volts a voltage of 300 for operating the tube.

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ELECTRICITY AND MAGNETISM, PRINCIPLES OF. — In this article are given definitions of the various electric and magnetic quantities, together with a brief statement of the fundamental principles in accord with which all electric and magnetic phenomena take place.

The following is a brief table of contents of this article:

UNITS AND QUANTITATIVE LAWS. — Three different systems of units are in use in terms of which electric and magnetic quantities are expressed. These systems are known as the c.g.s. electrostatic system, the c.g.s. electromagnetic system and the practical system (*see also Units, Practical Electric*). The various units in the practical system have been given short names which are in common use, e.g., ampere, volt, coulomb, etc. For the sake of brevity the corresponding units of the electrostatic system will be designated by these same names with the prefix "stat," and the corresponding units in the c.g.s. electromagnetic system will be designated by these same names with the prefix "ab," the latter prefix arising from the term "absolute" sometimes applied to this system.

The use of the electrostatic and electromagnetic systems of units arises from the manner in which certain of the fundamental quantitative relations, or "laws," were originally formulated. The practical system is related to the electromagnetic system, by even multiples of ten times the units in the latter, e.g., 1 volt $= 10^8$ abvolts, 1 ampere $= 10^{-1}$ abamperes, 1 ohm $= 10^9$ abohms.

The quantitative relations given below between *electric* quantities are independent of the system of units employed provided *all the quantities involved are expressed in the same system of units.* (*See Units and Conversion Factors.*) For some of the *magnetic* quantities, certain additional practical units have been introduced which are not related to the c.g.s. electromagnetic units by multiples of 10; consequently, to avoid confusion it is best to reduce all quantities to electromagnetic units before applying any of the formulas given.

WORKING HYPOTHESIS REGARDING THE NATURE OF ELECTRICITY. — The various facts of experience described as electric and magnetic phenomena can best be coördinated by assuming at the outset the existence of electricity as an actual entity, possessing the following properties:

1. Electricity exists in two forms, which for convenience may be called "positive" and "negative" electricity. These names arise from the fact that the two kinds of electricity produce certain effects which are just the opposite of each other, and, therefore, under certain conditions the external effect of one kind of electricity may completely neutralize the effect of the other kind.

A given quantity of electricity is called an "electric charge."

2. All matter contains electricity of both kinds, even when in the neutral or "uncharged" state. An uncharged body contains in every elementary portion (or molecule) equal quantities of positive and negative electricity, so thoroughly

"mixed" that no external effects are produced, unless the two kinds of electricity are constrained to take up a definite position or to move in a definite manner.

3. A body or portion of a body can be charged with electricity of either kind either by adding to it electricity of that kind or by taking away from it electricity of the other kind. In general only the *surface* of a body can be charged, i.e., the excess or deficit of charge exists only at the surface of separation between this body and some other body, the interior elements of the body containing, as in the neutral state, equal amounts of electricity of opposite kinds.

4. It is impossible to charge a body with electricity of one sign without producing on the same or on some other body an equal charge of the opposite sign.

5. Forces can be produced on the electricity in a body by various external agents, without necessarily producing a force on the body as a whole. When such forces are produced the two kinds of electricity are either displaced relatively to each other or are constrained to move through the body in opposite directions. This displacement or motion is always opposed by forces set up within the body itself. The opposing forces are of two kinds, (1) a force analogous to the opposing force set up in an elastic body when it is stretched or compressed, and (2) a force analogous to the resisting force due to mechanical friction produced when one body moves over another.

6. When the electricity in a body is acted upon by an external force, but cannot escape from that body to another, this force is transmitted to the body itself.

7. Electricity possesses inertia, just as ordinary matter possesses inertia; i.e., a force is required to change the motion of a charge of electricity just as a force is required to change the motion of a portion of matter. The "effective" inertia of a given quantity of electricity depends upon the path over which it is caused to move and upon the nature of the surrounding bodies (*see Electron Theory*).

Unit of Charge or Quantity of Electricity. — The unit of electric charge or quantity of electricity in the practical system of units is called the coulomb. For the relation between the coulomb, statcoulomb, abcoulomb, and other units of quantity see *Units and Conversion Factors*.

ELECTRIC FIELDS OF FORCE. — In any portion of a substance in which the electricity is acted upon by a force tending to move it, there is said to be an "electric field of force," or briefly an "electric field." An electric field is also said to exist in any region of free space where a charge, if placed there, would have a force exerted upon it tending to move it.

Intensity of an Electric Field (F). — The "intensity" of an electric field at any point is defined as the force per unit positive charge exerted on a charge at this point by the agent or agents producing the field i.e., by the agent or agents tending to move the charge. The direction of the field intensity, or the direction of the field, is defined as the direction of the force acting on a *positive* charge at this point. A positive charge then moves or tends to move in the direction of the field and a negative charge moves or tends to move in the opposite direction.

Units of Electric Field Intensity. — The unit of electric field intensity has not been given any special name, but since it is of the same nature as electromotive force per unit distance, the intensity at any point may be conveniently expressed as so many volts, abvolts or statvolts *per centimeter* or *per inch;* see *Units and Conversion Factors*. Hence electric field intensity is frequently called the potential, or voltage, "gradient."

Lines of Electric Force. — Lines of Electric Intensity. — A line drawn in an electric field in such a manner that its direction at each point coincides

with the direction of the field at that point is called a "line of electric force." A line of force is usually a curved line, though in certain special cases it may be straight. Any number of such lines may be drawn in an electric field, but no two of these lines can intersect. The density of these lines, i.e., the number drawn through unit area perpendicular to their direction, may be chosen arbitrarily to represent the value of the field intensity at this area, and when so drawn are preferably called "lines of electric intensity," as distinguished from flux lines and stream lines defined below. The term "lines of force," however, is frequently used to designate any one of these three sets of lines, but this loose use of the term is liable at times to lead to much confusion. The term "line of electric force" will be used in this article to designate merely the *direction* of the field at any point; in any statement involving the *density* of these lines the proper one of the other terms will be employed.

Electric Equipotential Surfaces. — A surface drawn in an electric field in such a manner that it is perpendicular at each point to the line of force through that point is called an "electric equipotential surface." The electric intensity has no component along such a surface, and therefore no work is required to move a charge from one point to another over any path in such a surface.

ELECTROMOTIVE FORCE (E). — The work done by the field intensity F in moving unit positive charge around any *closed* path or circuit (Fig. 1) in an electric field is defined as the "electromotive force," abbreviated "e.m.f.," acting around this path. Electromotive force is not a force in the mechanical sense but is *work per unit charge*. The relation between electromotive force E and field intensity F is analogous to that between work and mechanical force viz.,

$$E = \int (F \cos \theta)\, dl, \qquad (1)$$

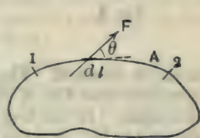

Fig. 1.

where dl represents an elementary length of the path, $(F \cos \theta)$ the *component* of the field intensity along dl and $\int$ represents the integral around this closed path. When the field intensity has the same value F at every point of a path and coincides in direction with the path at every point, the electromotive force acting around the closed loop is

$$E = Fl \qquad (1a)$$

where l is the total length of the path.

Units of Electromotive Force. — The practical unit of electromotive force is the volt; see *Units, Practical Electrical*. See also *Units and Conversion Factors* for the other units and their interrelations.

Sources of Electromotive Force. — The primary sources of e.m.f. are (1) two dissimilar bodies in contact (*see Batteries; Electrochemistry, Principles of*) and (2) a varying magnetic flux linking the circuit (*see the articles on Generators; Transformers, etc.*). The e.m.f. is said to be "located" at the surface of contact in the case of a "contact" e.m.f., or in that portion of the circuit which is linked by the varying magnetic flux in the case of an "induced" e.m.f., for experience shows that the value of the work integral, equation (1), around any circuit passing through a given surface of contact or linking a given varying flux is always the same, irrespective of the shape of the circuit and of the nature of substance through which the path passes, provided there are no other sources of e.m.f. in this path. Contact e.m.f.'s are usually extremely

small (of the order of 0.001 volt) except in the case of metallic substances in contact with electrolytes, in which case they may amount to several volts, see *Batteries; Electrochemistry, Principles of.* E.m.f.'s of 500,000 volts or more can be produced by a varying magnetic flux.

Direction of an Electromotive Force. — The direction of the e.m.f. *in* any portion of a circuit is taken as the direction in which a positive charge would be forced *around* a circuit containing only this one source of e.m.f. A closed circuit may contain several sources of e.m.f.; in this case the resultant e.m.f. acting around in the circuit is the *algebraic* sum of all these e.m.f's, the e.m.f.'s acting around the circuit in one direction being taken as positive and those acting around the circuit in the opposite direction being taken as negative. Those e.m.f.'s which act in the opposite direction to the resultant e.m.f. are called "back" or "counter" e.m.f.'s.

A convenient symbol for a source of constant e.m.f. is shown in Fig. 2; the long light line represents the positive terminal and the short heavy line the

Fig. 2.

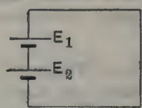

Fig. 3.

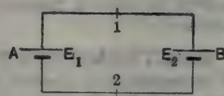

Fig. 4.

negative terminal. Fig. 3 shows two e.m.f.'s acting around a circuit in the same direction, and Fig. 4 shows two e.m.f.'s acting around the circuit in opposite directions. In the first case the resultant e.m.f. is $E_1 + E_2$ and the two e.m.f.'s are said to be "in series," and in the second case the resultant e.m.f. acting in the right-handed direction around the circuit is $E_1 - E_2$ and the two e.m.f.'s are said to be "in opposition." When E_2 is less than E_1 then E_2 is a back or counter e.m.f.

DIFFERENCE OF ELECTRIC POTENTIAL (V). — Consider a portion of a path A between any two points 1 and 2 in an electric field (Fig. 1). The total work done on unit charge, *by the e.m.f.'s external to this portion of the path* when unit charge moves from 1 to 2, is called the "drop of electric potential" from 1 to 2. The term "difference" of electric potential is also commonly used to designate this quantity; the term "drop" is preferable since it signifies the direction of the difference. Electric potential drop is of the same nature as electromotive force and is expressed in the same units. It is frequently abbreviated "p.d." Potential drop may be due either (1) to a "back" electromotive force, analogous to the back pressure of a pump, or (2) to an opposing force analogous to that due to the resistance of a pipe to the flow of water.

Let the path from 1 to 2 be in the direction of the line of force from 1 to 2, let E_{12} be an e.m.f. whose source is between 1 and 2 and whose direction is from 1 to 2, then the general expression for the potential drop from 1 to 2 is

$$V_{12} = \int_1^2 F\,dl - E_{12}. \tag{2}$$

The first term represents the "resistance" drop and $-E_{12} = E_{21}$ represents the "back" pressure. If the field intensity along the path from 1 to 2 is negligible, then $V_{12} = -E_{12}$; hence an e.m.f. in a given direction is equivalent to a negative drop of potential, i.e., is equivalent to an actual *rise* of potential. Comparing

(2) with equation (1) it is evident that the total drop of potential around a *closed* circuit is always zero. This relation is conveniently expressed by the formula

$$\Sigma V = 0 \qquad (3)$$

which holds for every closed path or circuit. This is one way of stating Kirchhoff's second "law."

Voltage and Voltage Gradient. — The term "voltage" is commonly used for either an e.m.f. or a potential difference, particularly when these quantities are expressed in volts. Since from equation (2) the field intensity F is equal in volts to the drop in voltage per unit length, the electric field intensity is also frequently called the voltage "gradient."

Positive and Negative Terminals. — That terminal of a device which is at the higher potential is called the positive terminal, the other terminal being called the negative terminal. The drop of potential is *always* from the positive to the negative terminal, irrespective of the direction of flow of electricity through the device.

FLOW OF ELECTRICITY. — Whenever an electric field is set up in a substance by any means whatever a displacement of the electricity in that substance always takes place, the nature of the displacement depending upon the nature of the substance. In every case the positive electricity within the substance is displaced in the direction of the field intensity and the negative electricity in the opposite direction, until an opposing force of some kind is set up which just balances the forces due to the impressed field.

Conductors and Insulators or Dielectrics. — A substance in which a *constant* electric field is always accompanied by a *continuous* displacement or *flow* of electricity through the substance is called an "electric conductor;" when there is no flow of electricity through a conductor the field within the conductor is *always* zero. Every substance is a conductor of electricity, at least to a slight extent, but some substances are far better conductors than others. A substance in which a constant electric field produces only a relatively small continuous flow of electricity is called an "electric insulator" or "dielectric." A *perfect* dielectric may be defined as a substance through which it is impossible to set up a continuous flow of electricity; no such substance exists, but very poor conductors are approximately such. Free space may be considered a perfect dielectric, but free space differs from a substance in that it does not contain electricity.

Metals are the best conductors, carbon and most moist substances are fair conductors; dry non-metallic bodies such as air and other gases (at normal pressure), porcelain, glass, rubber, and dry paper are very good insulators.

A wire covered with an insulating substance or supported on insulating substances is said to be "insulated," even though its ends are connected to a source of e.m.f.

Electric Current (I). — The displacement of the electricity within a substance cannot be measured directly, but only in terms of some effect produced thereby. Two effects which always occur when a displacement of electricity is produced are (1) a magnetic field is established around the path along which the displacement takes place (but disappears when the electricity comes to rest) and (2) heat is developed in the path of the displacement. The magnetic field produced by a displacement or flow of electricity is usually taken as the measure of the *rate* of flow, i.e., of the quantity of electricity displaced per unit time through a surface perpendicular to the direction of the displacement. This rate of flow is called the "intensity" of the electric current, or simply the "electric current," and is usually represented by the symbol I.

Direction of the Electric Current. — A flow of positive electricity in one direction is equivalent magnetically to a flow of an equal amount of negative electricity in the opposite direction; hence the total flow along a given path is the sum of the positive electricity displaced per unit time in one direction past a point in this path plus the negative electricity displaced per unit time past this point in the opposite direction. The "direction" of the electric current is taken as the direction in which the positive electricity is displaced, and is, therefore, the same as the direction of the field intensity.

Current due to Varying Electric Field. — When the electric field in any substance is *varying*, the total magnetic effect produced is found to depend not only upon the rate of displacement of the electricity within the substance, but also upon the *rate of change* of the electric field. In fact, a magnetic field is produced around a path in *free space* along which the intensity of the electric field is varying, see *Waves, Electromagnetic.* In dealing with varying electric fields it is found convenient to consider the variation of the field intensity as equivalent to an actual flow of electricity, and to take as the total *equivalent* electric current the flow of electricity which would produce the same magnetic field as that actually observed; the *actual* flow of electricity is then in general less than this equivalent current, but the difference is negligible except in substances which are good insulators.

Units of Electric Current. — The practical unit of electric current is the ampere; see *Units, Practical Electrical.* For the relation between this unit and the statampere and abampere see *Units and Conversion Factors.*

Continuity of an Electric Current. — When a varying electric field is considered as equivalent to an electric current, it is found that the total equivalent current coming up to any point or surface in any network of circuits, no matter how complicated, is *always* equal to the total current leaving that point or surface, irrespective of the nature of the substances through which the currents are established and irrespective of whether the currents are constant or are varying. For example, in Fig. 5,

$$I_1 + I_2 = I_3 + I_4 + I_5. \qquad (4)$$

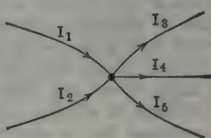

Fig. 5.

Or, calling the currents coming up to any point positive, and the currents leaving that point negative, the *algebraic* sum of all the currents at any point in a network of circuits is always zero. This fundamental principle is conveniently expressed by the formula

$$\Sigma I = 0 \qquad (5)$$

which holds at every junction point both for variable and for continuous currents. This is Kirchhoff's first "law."

Stream Lines of Electric Current. — Current Density (σ). — As a consequence of the continuity of an electric current, the total current in any substance of any size or shape may be looked upon as made up of a number of small streams of electricity flowing side by side, the strength (quantity of electricity per second) of each stream being constant throughout its length. If the cross-section of each stream at any point is so chosen that each stream represents unit current (unit quantity per second), then the number of these streams crossing unit area of a surface perpendicular to their direction will be equal to the current per unit area of this surface, or to the "current density" at this surface. Each such stream may be represented graphically by a line coinciding with its axis; such lines are called "stream lines." When the stream lines are drawn as described their direction at any point gives the direction of

the current at this point and the number of these lines per unit area perpendicular to their direction is equal to the current density at this point.

In the case of an insulated wire the stream lines are parallel to the axis of the wire, except in the immediate vicinity of its ends; this non-uniformity at the ends in the case of a long wire is negligible. The stream lines in an ordinary wire may also, in most practical cases, be considered as all coinciding, and the wire may, therefore, be treated as a geometrical line as regards *external* effects. However, in the case of a short rod or strip (such as an ammeter shunt) connected in the circuit by wires attached to its ends, the stream lines are not in general parallel but diverge from one terminal and converge toward the other.

Continuous, Direct, Pulsating, Alternating and Oscillatory Currents.— A "continuous" electric current is defined as a current which does not vary with time. A "direct" current is a current which is always in the same direction but may vary or pulsate in value. The term "direct current" is ordinarily used to designate either a continuous current or a current which varies or pulsates only by an inappreciable amount, such as the current from a battery or direct-current generator (q.v.). A "pulsating" current is a direct current which pulsates by an appreciable amount, such as the current from a rectifier (q.v.). An "alternating" current is a current which reverses in direction, being first positive and then negative, but alternates between *constant* maximum positive and negative values; see article on *Alternating Currents*. An "oscillatory" current is a current which reverses in direction, oscillating between positive and negative values which either decrease or increase with time.

Conduction Current and Displacement Current. — Experience shows that when an electric field is established in any substance, the total equivalent electric current set up depends (1) upon the value of the field intensity, (2) upon the rate of change of the field intensity, and (3) upon the nature of the substance in which the field is established. The current density σ at any point at any instant may in general be expressed by the relation

$$\sigma = \gamma F + \frac{1}{4\pi}\frac{d}{dt}(kF), \tag{6}$$

where F is the field intensity, γ and k are coefficients depending upon the chemical nature and physical condition of the substance at the point in question, and $\frac{d}{dt}$ means the rate of change with respect to time, the factor 4π arising from the historical definition of the quantity k, see below. The total current may then be considered as the sum of two components having respectively the densities

$$\sigma_1 = \gamma F \qquad \text{and} \qquad \sigma_2 = \frac{1}{4\pi}\frac{d}{dt}(kF). \tag{6a}$$

The first of these components (σ_1) is called the "conduction" current, and the second (σ_2) is commonly called the "displacement" current. The term "displacement" current, however, is somewhat a misnomer, for both components of the total current are probably due, in part at least, to a displacement of electricity; whereas a displacement current is set up by a varying electric field in a *free space*, although there is probably no electricity in free space to be displaced. The conduction current is the only appreciable component in substances usually classed as conductors, and the displacement current is appreciable only in substances ordinarily classed as dielectrics. The conduction current in a dielectric is usually small, though measurable; it is frequently called the "leakage" current. When the electric field in a dielectric is rapidly varying the displacement current may be many times greater than the conduction or leakage current through the dielectric.

Conductivity (γ) **and Resistivity** (ρ). — The quotient of the density σ_1 of the conduction current by the field intensity F, i.e., the coefficient γ in the expression $\sigma_1 = \gamma F$ is called the "conductivity" of the substance at the point in question. Since in an ordinary conductor the displacement current is inappreciable, the conductivity of an ordinary conductor is also equal to the density of the total current divided by the field intensity, i.e., for a conductor

$$\sigma = \gamma F, \tag{7}$$

where σ represents the density of the total current. Experience shows that for a given conductor at constant temperature (and also at constant pressure in case of a gas) this coefficient γ is a constant irrespective of the strength, distribution or time variation of the current. The value of γ for a dielectric, however, is not in general a constant but depends upon the time variation of the field intensity; see below under *Capacity and Condensers.*

The above relation between σ and F may also be written

$$F = \rho\sigma, \tag{7a}$$

where ρ is the reciprocal of the conductivity γ. The constant ρ is called the "resistivity" or "specific resistance" of the substance. Values of γ and ρ for various conductors are given in the article on *Resistance and Conductance*, and also the methods of measuring these quantities. The values of γ and ρ for some of the more common insulating materials are given in the article on *Insulating Materials, Properties of.* For the units of conductivity and resistivity see *Units and Conversion Factors* and also the next two paragraphs.

Conductance (g) **and Resistance** (r). — To cause a *continuous* current I to flow through a given conductor in which there is no source of e.m.f. a difference of potential must be established by some external agent between the ends of the conductor. Let V be the potential drop established through the conductor, from any point 1 to any other point 2, then the quotient

$$g = \frac{I}{V} \tag{8}$$

is defined as the "conductance*" from 1 to 2, and the quotient

$$r = \frac{V}{I} \tag{8a}$$

is defined as the "resistance*" from 1 to 2. Conductance is the reciprocal of resistance and vice versa. The practical unit of resistance is called the ohm, and the practical unit of conductance is called the mho; see *Units and Conversion Factors.*

It should be carefully noted that the definitions expressed by equations (8) and (8a) hold only when there is no e.m.f. in the portion of the circuit under consideration; this condition is realized only when the current remains constant in value (i.e., a *continuous* current) and the conductor is of uniform material and at constant temperature throughout. Also, the definition is meaningless unless the same current flows through each cross-section of the conductor and the drop of potential is the same between all points in the two end surfaces; i.e., the end surfaces must be equipotential surfaces.

Factors Upon Which Conductance and Resistance Depend. — The resistance or conductance of a given portion of a conductor included between

* Frequently called the "ohmic" or "d-c." conductance and resistance respectively to distinguish these quantities from the "effective" or "a-c." conductance and resistance; see article on *Resistance and Conductance.*

two equipotential surfaces and bounded laterally by a surface through which no stream line passes, depends upon (*a*) the conductivity of the conductor (*b*) the dimensions of this portion of the conductor, and (*c*) the distribution of the stream lines over each cross-section perpendicular to them. Consider the case of a straight wire or bar over the cross-section of which the current is uniformly distributed. Let l be the length of the bar, A its cross-section and ρ its resistivity. Then from equation (2) the drop of potential along the length l is $V = Fl$; from equation (7a) $F = \rho\sigma$; therefore $V = \rho\sigma l$. The total current $I = \sigma A$; whence the resistance and conductance are

$$r = \rho \frac{l}{A} \quad \text{and} \quad g = \gamma \frac{A}{l}. \tag{9}$$

Hence, the conductivity of a substance is equal to the conductance, and the resistivity is equal to the resistance, of a unit cube of this substance when the current through this cube is parallel to four edges of the cube and is uniformly distributed over the section at right angles to these four edges. Hence conductivity is frequently expressed as mhos per centimeter cube or inch cube; see *Resistance and Conductance; Units and Conversion Factors.*

Equation (9) is applicable only to a conductor in which the stream lines are uniformly distributed and parallel to each other, such as in insulated wires carrying continuous or comparatively slowly varying currents. When the stream lines are not parallel to each other, let dl represent an elementary length along any stream line and σ the current density at dl; then taking the integral along the stream line from one end surface to the other, the resistance is

$$r = \frac{\rho \int \sigma \, dl}{I} \tag{9a}$$

where I is the total current. The calculation of the value of σ at each point along the line from one surface to the other is difficult except in certain special cases of symmetry; see however, equation (22) below.

Ohm's Law and Resistance Drop. — From the relations noted above it is evident that, in a conductor (*a*) which remains at constant temperature, (*b*) in which there is no internal e.m.f., and (*c*) in which the distribution of stream lines remains unaltered, *the quotient of the voltage by the current is a constant.* This relation is known as Ohm's Law; it holds only when the three conditions specified above are fulfilled.

The above relation may also be written $V = rI$ which is merely another form of equation (2); i.e., if r is the resistance between any two points 1 and 2 in an electric circuit in which the current is I, then the integral of the electric field intensity along the path from 1 to 2 is equal to rI. This drop of potential due solely to resistance between 1 and 2 is called the "resistance drop" from 1 to 2, and is always in the direction of the current. When there is an e.m.f. in the circuit between 1 and 2 and the distribution of current remains the same,* the total drop from 1 to 2, from equation (2), is

$$V_{12} = rI_{12} - E_{12}, \tag{10}$$

the subscripts indicating the directions of the various quantities. Equation (10) is sometimes referred to as the "modified" Ohm's law.

* In general, the distribution of current depends upon the time variation of the voltage; see article on *Skin Effect.*

Terminal and Impressed Voltages. — The application of this equation is best shown by considering a simple circuit containing a generator and a motor or other receiving device, Fig. 6. Let I be the current in this circuit, let R_g be the internal resistance of the generator, R_m the internal resistance of the motor and R_l the total resistance of the "line" or connecting wires, let E_g be the e.m.f. developed by the generator and E_m the back e.m.f. developed by the motor, and let V_g be the terminal voltage of the generator and V_m the terminal voltage of the motor. Through the generator the current flows from the − to the + terminal, the e.m.f. is from the − to the + terminal, and the drop of potential is from the + to the − terminal. Hence

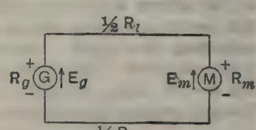

Fig. 6.

$$V_g = E_g - R_g I, \tag{10a}$$

i.e., the terminal voltage is less than the generator e.m.f. by an amount equal to the resistance drop in the armature. Through the motor the current flows from the + to the − terminal, the e.m.f. is from the − to the + terminal, and the drop of potential is from the + to the − terminal. Hence

$$V_m = E_m + R_m I, \tag{10b}$$

i.e., the terminal voltage is greater than the back e.m.f. of the motor by an amount equal to the resistance drop through the motor.

The terminal voltage of the motor is less than that of the generator by an amount equal to the resistance drop in the line, i.e.,

$$V_m = V_g - R_l I. \tag{10c}$$

The expression "impressed" electromotive force is also used to designate the rise of potential from the negative to the positive terminal of any receiving device, whether it be a motor, bank of lamps, or any "straight" resistance, i.e., a resistance containing no source of electromotive force.

Series Circuits. — When several conductors are connected end to end so that the *same current* flows through each of them (Fig. 7), they are said to be connected "in series." Let I_{12} be the current in each conductor in the direction from 1 to 2, let R', R'', R''', etc., be the resistances of the various conductors and E'_{12}, E''_{12}, E'''_{12}, etc., be the e.m.f.'s in the circuit between 1 and 2 in the direction from 1 to 2. Then the potential drop from 1 to 2 is

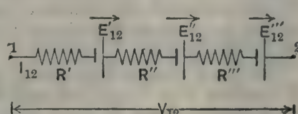

Fig. 7.

$$V_{12} = R'I_{12} - E'_{12} + R''I_{12} - E''_{12} + R'''I_{12} - E'''_{12}, \text{ etc.}$$

Therefore, the resistances between the points 1 and 2 are equivalent to a single resistance

$$R = R' + R'' + R''', \text{ etc.,} \tag{11}$$

and the e.m.f.'s between the points 1 and 2 are equivalent to a single e.m.f.

$$E_{12} = E'_{12} + E''_{12} + E'''_{12}, \text{ etc.} \tag{11a}$$

The equivalent conductance of several conductances G', G'', G''', etc., in series *when there are no e.m.f.'s in the path*, is G where

$$\frac{1}{G}=\frac{1}{G'}+\frac{1}{G''}+\frac{1}{G'''}+\dots \tag{11b}$$

Parallel Circuits. — When several conductors are connected to two common junction points so that the *same potential drop* is established through each (Fig. 8), they are said to be "in parallel." Let the currents, e.m.f.'s and resistances be as designated in the figure. Then from equation (5)

$$I_{12}=I'_{12}+I''_{12}+I'''_{12}+\text{etc.}$$

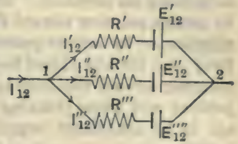

Fig. 8.

and from equation (10a)

$$V_{12}=R'I'_{12}-E'_{12}=R''I''_{12}-E''_{12}=R'''I'''_{12}-E'''_{12}=\text{etc.},$$

from which relations the currents in the individual branches may be calculated.

When there are no e.m.f.'s in the various branches, the combined resistance of the several branches from 1 to 2 is R where

$$\frac{1}{R}=\frac{1}{R'}+\frac{1}{R''}+\frac{1}{R'''}+\dots, \tag{12}$$

and the combined conductance is

$$G=G'+G''+G'''+\dots, \tag{12a}$$

where G_1, G_2, G_3, etc., are the individual conductances.

Two Parallel Circuits. — In the special case of two conductors in parallel, and no e.m.f. in either, the combined resistance is

$$R=\frac{R_1R_2}{R_1+R_2}. \tag{12b}$$

Series-parallel Circuits. — When a circuit is made up of several conductors some of which are in series and some in parallel, it is called a "series-parallel" circuit. The total resistance or conductance of such a circuit can be calculated from the constants of the several branches by applying successively the formulas for series and for parallel circuits.

Kirchhoff's Network Laws. — The relations given above for conductors in series and in parallel are special cases of two general laws already stated, namely:

1. The algebraic sum of the currents coming up to any junction in a network of conductors is always zero (equation 5).
2. The algebraic sum of the potential drops around any closed loop in a network of conductors is always zero (equation 3).

These two laws are known as Kirchhoff's Laws, from the name of the scientist who first clearly enunciated them. By making use of them one can always predetermine (*a*) the current in each branch of a network when the resistance of each branch and the e.m.f. in each branch are known, or (*b*) the e.m.f. in each branch when the current in each branch and the resistance of each branch are known.

It should be carefully borne in mind in applying these laws that a current leaving a point is equivalent to a negative current *entering* that point, and that

an e.m.f. in any chosen direction is equivalent to a *rise* of potential in that direction. In working out any problem concerning a network of circuits it is convenient to make a diagram of the network and to place on each branch in this diagram a number or symbol to represent the value of the current in this branch and an arrow or subscripts to indicate the direction of the current represented by this number or symbol, and wherever there is an e.m.f. to place a number or symbol to represent the value of this e.m.f. and an arrow or subscripts to indicate its direction. Then at any junction point those currents represented by arrows pointing toward the point are to be considered positive (say) and those represented by arrows pointing away from the point are to be considered negative; and for any closed loop those currents and e.m.f.'s represented by arrows pointing around the loop in the clockwise direction (say) are to be considered positive and those pointing around the loop in the counter-clockwise direction are to be considered negative. With this understanding, these laws may be written

$$\Sigma I = 0 \text{ at every point,} \tag{13a}$$

$$\Sigma E = \Sigma rI \text{ for every closed loop,} \tag{13b}$$

where I, r and E represent the current, the resistance and the e.m.f. respectively in each branch of the loop, and the symbol Σ indicates the algebraic sum of the quantities following it.

These equations enable one to write down a set of simultaneous equations for the given network, but it will be found that at least one of the current equations may be derived directly from the other current equations, and that at least one of the potential equations may be derived from the other potential equations. That is, the number of independent equations of each form will be one less than the number which it is possible to write down. It should also be noted that it is frequently unnecessary to write down formally all the possible independent equations; many of the simpler problems can be solved by writing down two independent expressions for the potential drop between each pair of points and equating these two expressions.*

When the currents and e.m.f.'s are *varying*, the first law applies to the total current (conduction and displacement current), but the second law in the form given in (13b) applies only to closed loops of conduction current; in both cases the instantaneous values of the currents and e.m.f.'s must be taken when the quantities are varying.

ELECTRIC ENERGY AND POWER. — From the definition of potential drop it follows that the total work done by the external agents in forcing Q units of electricity from any point 1 in a circuit to any point 2 is $W = VQ$. From the definition of electric current the quantity of electricity carried from 1 to 2 when the current I is established from 1 to 2 is $Q = It$, where t is the time during which the current exists. Hence when a current I is established through any device for a time t by an impressed voltage V, the energy input to this device is

$$W = VIt \tag{14}$$

and the power input, i.e., energy input per unit time, is

$$P = VI. \tag{15}$$

When V and I are expressed in volts and amperes respectively the power input is in watts; if t is in seconds, the energy input is in joules or watt-seconds. When V and I are in statvolts and statamperes respectively, or in abvolts and abam-

* The application of these laws to the solution of complicated network problems is given in detail in Del Mar's *Electric Power Conductors*, New York, 1913.

peres respectively, the power input is in ergs per second, if t is in seconds the energy input is in ergs. See *Units and Conversion Factors*.

Applying the above relations to the simple circuit shown in Fig. 6 containing a generator and a motor (armature windings only are considered), the power *input* to the motor armature is

$$P_i = E_m I + R_m I^2, \tag{15a}$$

the power *output* of the generator armature is

$$P_0 = E_g I - R_g I^2 \tag{15b}$$

and the power lost in the line is

$$P_l = R_l I^2.$$

The term $R_g I^2$ represents the power lost in heating the armature winding of the generator due to its resistance, and the term $R_m I^2$ represents the power dissipated as heat in the armature winding of the motor. The net electric input to the generator armature is $E_g I$ and the gross mechanical output of the motor armature is $E_m I$. The gross mechanical input to the generator is greater than $E_g I$ and the net mechanical output of the motor is less than $E_m I$ by an amount equal to the friction and "core-loss" in the respective machines; see *Generators and Motors*.

Joule's Law. — That portion of the power input to any device which is equal to the product of the resistance of the conductors forming the winding of the device by the square of the current through this winding is always converted into heat. That is, when a current I flows through a resistance r heat is always "dissipated" in this resistance, and the rate of dissipation is

$$P_h = rI^2. \tag{15c}$$

This experimental fact is known as "Joule's Law." This law applies directly only to continuous or non-varying currents. The relation $P_h = rI^2$ is, however, used as the basis for defining the "effective" resistance of a conductor to an alternating current (q.v.).

DIELECTRIC FLUX (ψ) AND DIELECTRIC FLUX DENSITY (D). — As noted above the displacement current through a dielectric at any point depends upon the rate of change of the electric field intensity and upon the nature of the dielectric, and the density of this displacement current at any point may be expressed by the relation

$$\sigma_2 = \frac{1}{4\pi}\frac{d}{dt}(kF), \tag{16}$$

where F is the field intensity and k a coefficient depending upon the nature of the dielectric. The coefficient k in this expression is the so-called dielectric coefficient or dielectric constant.

The quantity kF, whose rate of change is equal to 4π times the density of the displacement current, is called the "dielectric flux density," and may be represented by the symbol D. Then

$$D = kF. \tag{17}$$

The direction of the dielectric flux density is arbitrarily chosen to be the same as that of the electric field intensity F. Through any surface of area A at each point of which the dielectric flux density has a constant value D and is perpendicular to that surface, there is said to exist a "dielectric flux" equal to DA. The total dielectric flux through a surface may be represented by the symbol ψ.

In general, the total dielectric flux through any surface perpendicular at each point to the direction of the field intensity at that point (i.e., through an equipotential surface) is

$$\psi = \int D\,ds, \tag{18}$$

where ds represents any elementary area of this surface and D the flux density per unit area at ds, and $\int$ represents the sum of all the products $D\,ds$ for that surface. The total displacement current through this surface is then

$$i = \frac{1}{4\pi}\frac{d\psi}{dt}. \tag{19}$$

Lines of Dielectric Flux. — The electrostatic flux through any surface may be represented by lines drawn in the same direction as the lines of electric intensity, but of such a density that *their number per unit area* perpendicular to their direction at any point *is equal* to the *dielectric flux density* at this point. The number of these lines cutting any surface is then equal to the total dielectric flux through this surface. The ratio of the number of flux lines through any surface to the number of lines of electric intensity through that surface is equal to the dielectric coefficient of the substance in which the field exists.

ELECTRIC CHARGE (Q) AND DIELECTRIC FLUX (ψ). — *Within* any substance of uniform structure throughout the dielectric flux lines are continuous lines, i.e., the number of these lines coming up to one side of a surface *within* such a substance is equal to the number of these lines leaving the other side of that surface. Experience shows that it is impossible to produce an appreciable dielectric flux in those substances ordinarily classed as conductors; *hence dielectric flux lines cannot pass through a good conductor*, but terminate at its surface. Every dielectric is a conductor to at least a slight extent, and on account of this fact the dielectric flux lines coming up through one dielectric to the surface of contact between this dielectric and another do not all pass through the second dielectric, but some of them terminate at this surface.

Experience shows that to establish an electric field in the dielectric around a conductor, electricity must be *conducted through the conductor* to the surface of contact between the conductor and the dielectric. For example, consider a good conductor in contact with a perfect dielectric, Fig. 9; a momentary conduction current must flow through the conductor along the stream lines of the conduction current, represented by the dotted lines. While the field is being established (and therefore varying) a displacement current is set up in the dielectric requiring an equal conduction current in the conductor, and consequently $\frac{1}{4\pi}$ times the rate of change of the dielectric flux (ψ) established in the dielectric must be equal to the conduction current (i) flowing up to this surface through the conductor, i.e.,

Fig. 9.

$$\frac{1}{4\pi}\frac{d\psi}{dt} = i,$$

or

$$\psi = 4\pi \int i\,dt = 4\pi Q, \tag{20}$$

where Q is the quantity of electricity (current multiplied by time) conducted through the conductor to this surface.

This relation is a general one, viz., the total dielectric flux from any area A in the surface of a *conductor* is equal to 4π times the total charge on this area. Hence every flux line originates at a positively charged conducting surface and terminates at a negatively charged conducting surface, 4π of these lines connecting each unit positive charge to each unit negative charge.

The quantity of electricity conducted through a conductor when a momentary current is established through it can be readily measured by means of a ballistic galvanometer (*see Galvanometers*) and consequently the dielectric flux (equal to $4\pi Q$) may be readily determined.

Units of Dielectric Flux. — Dielectric flux may be expressed in the same unit as electric charge, viz., coulombs, statcoulcombs or abcoulombs; see *Units and Conversion Factors.*

Surface Density of Charge (σ_c) and Dielectric Flux Density (D). — When there is no current in a conductor there can be no electric field within it, see equation (7), and therefore *the surface of a conductor in which no current is flowing is always an equipotential surface.* Hence the lines of electrostatic intensity, in the surrounding dielectric, and therefore the dielectric flux lines also, must leave or enter this surface in a direction perpendicular to it. The dielectric flux density in the dielectric just outside a conducting surface in which there is no electric current is perpendicular to this surface and is

$$D = 4\pi\sigma_c, \tag{20a}$$

where σ_c is the charge per unit area of the surface at this point, or the "surface density" of the charge.

Dielectric Flux Density Due to a Number of Charged Conductors. — It can be shown that when any number of charged conductors are surrounded by a *uniform* dielectric, the dielectric flux density at any point in the field may be expressed by considering each elementary surface having a charge q as producing at any point P at a distance r from q a flux density equal to q/r^2, in the direction of the line from q to P when q is positive and in the direction of the line from P to q when q is negative. The total flux density at P due to all the charges is then the *vector summation*

$$D = \sum \frac{q}{r^2}. \tag{20b}$$

Dielectric Flux Density at any Point due to a Uniformly Charged Wire of Circular Cross Section and Infinite Length. — Let q be the charge per unit length, K the dielectric coefficient of the surrounding dielectric, assumed uniform, and r the perpendicular distance from the center of the wire to any point P, then the dielectric flux density at P due to this wire is

$$D = \frac{2q}{r} \tag{20c}$$

The resultant dielectric flux density at any point due to any number of *uniformly* charged wires is the vector sum of the flux densities due to each separately. However, when two or more wires are close together the distribution of charge on them is not uniform,* but when the wires are more than 10

* The distribution can be calculated readily in the case of two parallel wires; see Pender and Osborne, *Elec. World, 1910, Vol. 56, p. 667.*

diameters apart the error introduced by the assumption of uniform charge is practically negligible.

ELECTROSTATIC CAPACITY (C) AND CONDENSERS. — To establish a given dielectric flux ψ through a given dielectric a certain difference of electric potential is always required. Consider any portion of an electric field (Fig. 10) between the two equipotential surfaces S and S_1 bounded laterally by a surface tangent at each point to the flux line through that point. Let V be the drop of potential from S to S_1, and let ψ be the dielectric flux through this region. When there is no source of e.m.f. between S and S_1 the quotient

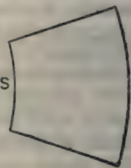

Fig. 10.

$$C = \frac{\psi}{4\pi V} \tag{21}$$

is defined as the "electrostatic capacity" of this portion of the field. Compare with the definition of conductance, equation (8).

When the equipotential surfaces S and S_1 are the surfaces of two conductors, the two conductors and the dielectric between them are said to form an "electric condenser." Practical forms of condensers are described in the article on *Condensers, Electric*. When all the flux lines from one conductor end on the second conductor (e.g., when they are given equal and opposite charges by connecting them respectively to the two terminals of a source of e.m.f.), then the flux from one to the other is equal to $4\pi Q$ where Q is the numerical value of the total charge on either conductor. The capacity of the condenser may then be written

$$C = \frac{Q}{V}. \tag{21a}$$

When there are several charged conductors in the field the total flux from one conductor does not in general all end on another single conductor, but, some of the flux lines from No. 1, say, may run to No. 2, some to No. 3, etc. Let ψ_{12} be *that portion* of the flux from any conductor 1 which ends on any other conductor 2, and let V_{12} be the drop of potential from 1 to 2, then the capacity between conductor No. 1 and conductor No. 2 is

$$C_{12} = \frac{\psi_{12}}{4\pi V_{12}}. \tag{21b}$$

Or, calling Q_{12} that portion of the charge on No. 1 which is balanced by an equal and opposite charge on No. 2, the capacity between 1 and 2 is

$$C_{12} = \frac{Q_{12}}{V_{12}}. \tag{21c}$$

Units of Capacity. — The unit of capacity in the practical system of units is the farad (*see Units, Practical Electric*), but as this is a very large unit, a unit equal to one-millionth of a farad, called the microfarad, is usually employed. The c.g.s. electrostatic unit may be called the statfarad and the c.g.s. electromagnetic unit the abfarad. See *Units and Conversion Factors*.

Factors Upon Which Capacity Depends. — The capacity of a given portion of a dielectric depends upon (a) the dielectric coefficient k of the dielectric, (b) the length of the dielectric flux lines through it, (c) the cross-section of the dielectric at right angles to the flux lines, and (d) upon the distribution of the flux lines over this cross-section. Compare with electric conductance. In

general, the capacity of any portion of a dielectric bounded laterally by flux lines and at the ends by equipotential surfaces (Fig. 10) can be expressed by the formula

$$C = \frac{k\psi}{4\pi \int D\,dl}, \tag{21d}$$

where k is the dielectric coefficient, ψ the total dielectric flux through the given portion of dielectric, dl any elementary length along one of the flux lines and D the dielectric flux density at this point, the integral being taken along the flux line from one end surface to the other. When the end surfaces are conductors charged with $+Q$ and $-Q$ units respectively, then

$$C = \frac{KQ}{\int D\,dl}. \tag{21e}$$

By the application of this formula the capacity of various practical forms of condensers may be calculated; see the article on *Capacity and Charging Current, Calculation of*. It should be noted that the capacity of a condenser depends upon the *distribution* of the dielectric flux (k being assumed constant), but not upon the absolute value of the flux; i.e., for a given dielectric and given distribution of flux the capacity is a constant. In general, when any conductor or dielectric of a different specific inductive capacity is placed in the electric field set up by the charged plates of a condenser, the distribution of the flux, and therefore the capacity of the condenser, is altered.

Relation Between Conductance and Capacity. — Comparing equations (9a) and (21d), it is apparent that when the dielectric flux lines and the current stream lines have the same distribution in any given region, the ratio of the conductance of this region to the capacity of this region is $4\pi\gamma/k$ where k is the dielectric coefficient and γ the conductivity of the material in this region. Hence the formulas for the capacity and conductance of the dielectric between the plates of any shape or size of condenser differ only by a constant coefficient. That is, if C is the capacity of any condenser, then

$$g = \frac{4\pi\gamma}{k} \cdot C \tag{22}$$

is the conductance of the dielectric between its plates. Values of C for various cases are given in the article on *Capacity and Charging Current.*

Charge and Discharge of a Condenser. — To charge a condenser a difference of electric potential must be established between its plates. This may be done, as noted above, by connecting the two plates of the condenser respectively to the two terminals of any source of e.m.f., see Fig. 11. If the dielectric has a very high resistance and the source of e.m.f. has a constant value E, the current set up in this circuit will continue only until a difference of potential equal to E has been established across the two plates of the condenser, or until a charge equal to CE has been transferred from the "negative" to the "positive" plate of the condenser. The establishment of the electric flux through the dielectric of the condenser may be looked upon as setting up in the dielectric itself an opposing force analogous to the opposing force set up in a spring when it is compressed. When the opposing force just balances the impressed force a steady state is attained, just as the compressing of a spring ceases

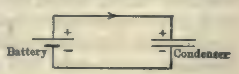

Fig. 11.

when the force producing the compression is just balanced by the opposing force due to the elasticity of the spring.

When a condenser has thus been charged, the wires connecting it to the source of e.m.f. may be removed and the condenser remains charged for a length of time depending upon the resistance of the dielectric separating the plates; the higher this resistance the longer the time that the condenser remains charged. The plates may also be moved apart and they still retain their charges, one plate a positive charge and the other a negative charge, but the distribution of these charges on the plates will in general become altered. Experience shows that a mechanical force is required to separate the charged plates irrespective of whether or not they are connected to the source of e.m.f.

When the two charged plates are "short-circuited" by a wire, as shown in Fig. 12, a momentary current is established through the wire and the electric field between the plates and the charges disappear. The quantity of electricity discharged through the wires is equal to the quantity of charge originally on either plate. A charged condenser, therefore, acts like a source of e.m.f., the direction of this e.m.f. *around* the circuit containing the condenser being in the direction *through* the condenser from its negative to its positive plate. A condenser when it is being charged may also be looked upon as producing a back e.m.f., that is, an e.m.f. opposing the e.m.f. which charges it. When a condenser is considered from this point of view only the *conducting* portion of the circuit is to be considered in applying Kirchhoff's Laws. When the condenser has an appreciable leakage its resistance must be considered to be in *parallel* with its e.m.f.

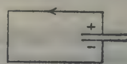

Fig. 12.

Charging or Capacity Current and Leakage Current. — The displacement current through the dielectric of a condenser is frequently called the "charging" or "capacity" current. The conduction current through the dielectric is called the "leakage" current. Let C be the capacity of the condenser, g the conductance of the dielectric, and v the voltage across the condenser, then the total current through the condenser is

$$i = gv + C\frac{dv}{dt}, \tag{23}$$

where $\frac{dv}{dt}$ represents the rate of change of v with time. The component gv of this current is the leakage current and the component $C\frac{dv}{dt}$ is the charging or capacity current.

Capacities in Series. — When several capacities* are connected end to end so that the *same dielectric flux* passes through each of them, they are said to be "in series." The total capacity of any number of individual capacities C_1, C_2, C_3, etc., connected in series is C where

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{24}$$

Compare with conductances in series, equation (11b).

Capacities in Parallel. — When several capacities* are connected between the same pair of equipotential surfaces so that the *same potential drop* is established through each, they are said to be "in parallel." *When there are no e.m.f.'s*

* Either condensers, or dielectrics of different kinds, sizes or shapes in contact along equipotential surfaces.

in any of the circuits between the two equipotential surfaces the total equivalent capacity of any number of capacities C_1, C_2, C_3, etc., connected in parallel is

$$C = C_1 + C_2 + C_3 \dots \qquad (25)$$

Compare with conductances in parallel, equation (12a).

SPECIFIC INDUCTIVE CAPACITY (K) **AND DIELECTRIC COEFFICIENT** (k). — From equation (21d) it is evident that when the capacity of a given condenser is measured (1) when a dielectric A is between the plates and (2) when some other dielectric B is between these plates, then the ratio of the two capacities is the same as the ratio of the dielectric coefficients of the two dielectrics. The "specific inductive" capacity of any dielectric is defined as the ratio of the capacity of a condenser having this substance as its dielectric to the capacity of the same condenser when air forms the dielectric between the plates. The specific inductive capacity is, therefore, independent of the system of units employed.

The c.g.s. electrostatic system of units is based on the arbitrary choice of unity as the dielectric coefficient of air; hence, in the c.g.s. electrostatic system of units the specific inductive capacity and the dielectric coefficient are numerically equal. In the c.g.s. electromagnetic system of units the dielectric coefficient of air is not unity but $\frac{1}{9 \times 10^{20}}$ (i.e., the reciprocal of the square of the velocity of light in air). In the practical system of units, when the centimeter is taken as the unit of length, the dielectric coefficient of air is $\frac{1}{9 \times 10^{11}}$. Hence, calling K the specific inductive capacity of any dielectric referred to air as unity, and k its dielectric coefficient, then in the

c.g.s. electrostatic system $\quad k = K$,

c.g.s. electromagnetic system $\quad k = \frac{K}{9 \times 10^{20}}$,

Practical system (cm. as unit of length) $\quad k = \frac{K}{9 \times 10^{11}}$.

Values of the specific inductive capacity of various insulating materials are given in the article on *Insulating Materials, Properties of.*

Electric Absorption and Residual Charge. — The value of the dielectric coefficient k of a given dielectric is not strictly a constant unless the dielectric is perfectly homogeneous. In the case of such nonhomogeneous substances as glass, mica, rubber, paper, cloth, etc., the dielectric coefficient is found to depend upon the time of electrification, i.e., upon the length of time that the voltage is applied, its value increasing with the time of electrification (*see Insulating Materials, Testing of*). This phenomenon is sometimes described as "electric absorption," the idea being that the charge from the plates of the condenser soaks into the dielectric, for an increase in the dielectric coefficient for a given impressed voltage means a greater quantity of electricity conducted to the plates. This idea is also in accord with the experimental fact that when such a condenser is discharged by short-circuiting it with a wire, Fig. 12, the wire then being removed, a "residual" charge appears on the plates after a lapse of a few seconds.

Dielectric Hysteresis. — A phenomenon closely associated with electric absorption is the fact that when the electric field in a heterogeneous dielectric is caused to vary rapidly an amount of heat is dissipated in the dielectric greatly in excess of that which can be accounted for in terms of its leakage resistance as determined by continuous-current measurements. This may be due in part to

an actual increase in the resistance of the dielectric with the speed of variation of the field, or may be due to a phenomenon analogous to magnetic hysteresis, i.e., to a lag of the dielectric flux density behind the electric field intensity (*see Magnetic Materials, Properties of*). Whatever may be the cause of this extra loss of power for rapidly varying fields, it is generally described as the loss due to "dielectric hysteresis." The heat developed is in many cases quite appreciable. See also the article on *Condensers, Electric.*

DIELECTRIC STRENGTH. — ELECTRIC SPARK AND ELECTRIC CORONA. — Experience shows that when an electric field is established in a dielectric and the intensity of this field is increased, a point is reached at which the dielectric loses its insulating property and becomes a conductor. This condition of affairs is usually accompanied by a spark which burns a hole through the dielectric, i.e., the dielectric is "punctured." Under other conditions the breakdown may not be permanent, but may result in the acquisition of a high conductivity by the dielectric only while the voltage gradient is maintained above the critical value, the dielectric regaining its insulating property when the field is reduced below this critical value. This latter condition is usually described as the formation of an electric "corona" in the dielectric; in the case of air the formation of corona manifests itself by a bluish light in the air around the conductors between which the field is established. Whether the breakdown produced by a given voltage is of the nature of a puncture or results in the formation of a corona depends chiefly upon the *distribution* of the dielectric flux produced in the dielectric (*see Corona, Electric*).

The critical field intensity or voltage gradient at which breakdown occurs is called the "dielectric strength" of the dielectric. The dielectric strength depends upon the nature of the dielectric, its value for the various dielectrics ordinarily employed in practice depending decidedly upon their chemical and physical nature (*see Insulating Materials, Properties of*). It is also found that for a given dielectric the critical voltage *gradient* at which breakdown occurs depends in general upon (*a*) the distribution of the dielectric flux just prior to breakdown and (*b*) upon the thickness of the dielectric. It is naturally to be expected that the *voltage* (total potential difference) required to produce a breakdown would depend upon the distribution of the dielectric flux and the thickness of the dielectric, for the voltage gradient at all voltages depends upon these factors (*see above under Capacity*), but there is as yet no satisfactory explanation of the dependence of the *critical gradient* upon these factors.

In fact, but little is known regarding the nature of an electric breakdown, and even the values of the dielectric strength are known only approximately in most instances, for in many of the tests made to determine its value the distribution of the electric flux was not known. The values of the dielectric strength given in the article on *Insulating Materials, Properties of*, must, therefore, be considered only as rough approximations except for conditions identical with those under which the tests were made.

ELECTROSTATIC ENERGY. — From the general relations expressed by equations (15) and (23) it is evident that when the potential difference between the plates of a condenser is increased from 0 to V the energy input is

$$\int_0^t gv^2\,dt + \int_0^V Cv\,dv.$$

The energy represented by the first term on the right-hand side of this equation is dissipated as heat in the dielectric, but the energy represented by the second term, which, when C is constant, may be written

$$W = \tfrac{1}{2}CV^2, \tag{26}$$

does not represent a dissipation of heat; this is a fact of experience. Moreover, when the condenser is discharged, by short-circuiting its plates with a wire, this same amount of energy $\frac{1}{2} CV^2$ is transferred to the wire. Hence, the energy represented by $\frac{1}{2} CV^2$ is said to be stored in the condenser, or preferably in the *dielectric* of the condenser, for the electric force F exists only in the dielectric. This stored energy is called the "electrostatic" energy. It is analogous to the energy stored in a spring when the latter is compressed or stretched. Equation (26) may also be written

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{8\pi}\psi V \cdot \tag{26a}$$

The electrostatic energy *per unit volume* of an electric field may be written

$$w = \frac{DF}{8\pi} = \frac{kF^2}{8\pi} = \frac{D^2}{8\pi k}, \tag{26b}$$

where k is the dielectric coefficient, F the electric field intensity or potential drop per unit distance, and D the dielectric flux density or flux per unit area perpendicular to the direction of the drop.

It should be noted that equations (26) to (26b) are based on the assumption that k is a constant, independent of the value of F. When this condition does not hold, the energy required to establish the field is $\int_0^V Cv\,dv$, the evaluation of which depends upon the relation between C and v.

MECHANICAL FORCES IN AN ELECTROSTATIC FIELD. — Experience shows that all bodies (conductors or insulators) in an electric field exert in general mutual mechanical forces upon one another tending to produce such a relative motion as will *decrease* the energy of the field. Let f be the component of the force tending to move any body in the field in a given direction and let dW be the *increase* in the energy of the field due to displacing the body a distance dx in this direction, then

$$f = -\frac{dW}{dx}, \tag{27}$$

provided this displacement does not cause a change in the existing electric charges in the field. As a consequence of this general relation it can be shown that every charged surface exerts a force of repulsion on every other surface charged with electricity of the same sign, and a force of attraction on every surface charged with electricity of the opposite sign.

In the special case of the two conductors forming a condenser the force of attraction exerted by one conductor on the other is

$$f = -\frac{V^2}{2}\frac{dC}{dx}, \tag{27a}$$

where V is the p.d. across the condenser, C the capacity of the condenser, and dC represents the increase in the capacity of the condenser when one conductor moves a distance dx away from the other. This relation results from the substitution of (26a) in (27). For example, the capacity of a parallel plate condenser is approximately,

$$C = \frac{kA}{4\pi x},$$

where k is the dielectric coefficient, A the area of the smaller plate, and x the distance between the plates. Hence the force of *attraction* is

$$f = \frac{V^2kA}{8\pi x^2}.$$

Principle of the Electrometer. — This relation suggests a method of measuring potential difference, for, by transposing,

$$V = x\sqrt{\frac{8\pi f}{kA}}.$$

If the force acting on the upper plate is measured by means of a balance, and if A and x are also measured, and the dielectric between them is air ($k = 1$ in the electrostatic system, by definition), then all the data necessary for the calculation of V is at hand. The above formula for V is approximate only, since the capacity formula is approximate, due to the assumption of a uniform flux density in the dielectric between the plates. As a matter of fact the flux density near the edges of the plate is not uniform, but it is possible to correct for this non-uniformity; see article on *Electrometers*.

MAGNETS AND MAGNETIC SUBSTANCES. — A magnet may be defined as any body which possesses the property of attracting pieces of iron or steel* and which when freely suspended takes up a definite position with respect to the geographical meridian. A magnetic substance is any body which acquires this property when it is placed near a magnet or near a conductor carrying an electric current. A body which is given this property is said to be "magnetized." A magnetic needle is a magnetized needle of iron or steel; the north seeking end of such a needle is called its north pole and the south seeking end its south pole. When such a needle is freely† suspended near a magnet or a conductor carrying an electric current a couple is bound to be exerted upon it which causes it to take up a definite direction. The needle is said to "point" in the direction of a line drawn through it from its south to its north pole.

MAGNETIC FIELD OF FORCE. — Any region in which a magnetic substance (e.g., a piece of soft iron), when placed therein, becomes magnetized is said to be a "magnetic field." A magnetic field exists in and around every magnetized substance and around every stream line of electric current. The direction of the magnetic field at any point P is arbitrarily chosen as the direction in which a small magnetic needle point would point when placed at P without disturbing appreciably the existing conditions.

Magnetic Flux (ϕ). — Consider a small closed turn of wire, Fig. 13, placed in a magnetic field with its plane perpendicular to the direction of the field. Experience shows that when such a turn of wire is removed from the field in any manner whatever (the coil remaining short-circuited on itself or forming part of a closed circuit), or when the magnetic field is caused to disappear in any manner whatever, a momentary electromotive force is set up or "induced" in this coil, which in turn causes a momentary electric current to flow through the coil. This e.m.f. exists only while the coil is *moving across* the field or while the field through the coil is *varying*.

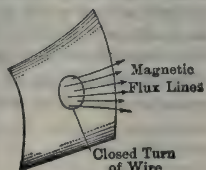

Fig. 13.

* With a force in excess of the gravitational force, which latter is extremely small.

† A needle is said to be freely suspended when there is no controlling force exerted upon it through its suspension tending to make it take up any definite position.

The time integral of the induced e.m.f. when the coil is removed entirely from the magnetic field is taken as the measure of the "magnetic flux" existing through the coil when in its original position. That is, calling e the e.m.f. induced in the coil at any instant due to its motion through the field, and t the time during which the e.m.f. exists in the coil, then the magnetic flux through the coil when in its original position is

$$\phi = \int_0^t e\,dt. \tag{28}$$

This quantity is readily measured by means of a ballistic galvanometer; see *Magnetic Testing.*

Units of Magnetic Flux. — The unit of magnetic flux in the c.g.s. electromagnetic system is frequently called a "maxwell" or simply a "line." The unit of flux in the practical system of units is sometimes called a "weber"; see *Units and Conversion Factors.*

Magnetic Flux Density (B). — Experience shows that the magnetic flux through any closed loop, such as the turn of wire described above, depends upon the area inclosed by this loop. The magnetic flux per unit area through any surface perpendicular to the direction of the field is defined as the "magnetic flux density" at this surface, and is usually represented by the symbol B. By the flux density at any *point* is meant the flux density at an infinitely small surface drawn perpendicular to the field at this point. The direction of the magnetic flux density at any point is the same as that in which a magnetic needle would point if placed at this point; i.e., the direction of the flux density and the direction of the magnetic field are the same. When the flux density has the same value B at every point of a surface of area A and is perpendicular to this surface, then the total flux through this surface is

$$\phi = BA. \tag{29}$$

The total magnetic flux across any surface S may in general be expressed mathematically by the surface integral

$$\phi = \int (B \cos \alpha)\,ds, \tag{29a}$$

where ds represents any elementary area of this surface and $(B \cos \alpha)$ the component of the flux density perpendicular to ds.

Units of Magnetic Flux Density. — The unit of magnetic flux density in the c.g.s. electromagnetic system is called the "gauss"; no name has been given to the corresponding practical unit.

Magnetic Flux Lines. — Continuity of Magnetic Flux. — Magnetic flux can be represented by lines drawn in the field in such a direction that their direction coincides at each point with the direction of the field at that point, and of such a number that their density at each point (number per unit area perpendicular to their direction) is equal to the magnetic flux density at that point. Such lines are called "magnetic flux lines." Experience shows that lines thus drawn in a magnetic field always form *closed* loops, i.e., a magnetic flux line has no ends. As a consequence of this fact the total magnetic flux coming up to any surface in a magnetic field is always equal to the total flux leaving that surface. (Compare with stream lines of electric current.) When the flux

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leaving a surface is considered as equivalent to a negative flux coming up to that surface, this relation may be expressed by the formula

$$\Sigma\phi = 0 \tag{30}$$

at every surface, the summation being an algebraic one. This is analogous to Kirchhoff's first law for electric circuits.

Magnetic Fields Due to Electric Currents. — Experience shows that every stream line of electric current is always accompanied by a magnetic field the flux lines of which *link* the stream line of current. That is, the flux lines thread the loops formed by the stream lines and the stream lines thread the loops formed by the flux lines; see Fig. 14.

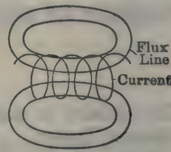

Fig. 14.

Right-handed Screw Law. — The direction of the current flowing around any electric circuit and the direction in which the flux lines due to that current thread this circuit are related to each other in the same manner as the direction of motion of a point on the edge of the head of a right-handed screw placed at the center of the circuit and the direction of advance of the screw. Or, if one faces the electric circuit looking in the direction of the flux lines threading it, the current producing these lines is in the clockwise direction around the circuit. The relative direction of the current and its magnetic flux may be briefly described by saying that the current is in the right-handed screw direction with respect to the flux which it produces.

MAGNETICALLY INDUCED ELECTROMOTIVE FORCE. — The measure of magnetic flux is based on the experimental fact that whenever the magnetic field threading an electric circuit changes, an electromotive force is induced in that circuit. When the circuit is formed by a single turn of wire this induced e.m.f. is, from the definition above, equal to the rate of change of this flux with respect to time, i.e., $e = \frac{d\phi}{dt}$. When the circuit is in the form of a coil each turn of which links the flux, the e.m.f. induced in *each* turn is equal to $\frac{d\phi}{dt}$ where ϕ is the flux which links that particular turn. When each turn links the *same number of flux lines*, then the total induced e.m.f. in a coil of N turns is

$$e = N\frac{d\phi}{dt}. \tag{31}$$

When the change in flux is due to a motion of a circuit or part of a circuit through a magnetic field the induced e.m.f. in any conductor is also equal to the number of flux lines which *cut across* this conductor.

Magnetic Linkages (λ). — The condition that each turn be linked by the same flux ϕ is seldom the case; some of the flux lines link only part of the turns, see Fig. 14. In general, the total e.m.f. is

$$e = \frac{d}{dt}(\phi_1 + \phi_2 + \ldots \phi_n),$$

where ϕ_1, ϕ_2, etc., represent the fluxes linking the various turns. The sum $(\phi_1 + \phi_2 + \ldots \phi_n)$ may be called the total number of "magnetic linkages" and may be conveniently represented by the symbol λ, viz.,

$$\lambda = \phi_1 + \phi_2 + \ldots \phi_n,$$

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and the total induced e.m.f. may then be written

$$e = \frac{d\lambda}{dt}. \tag{31a}$$

When all the N turns link the same flux, ϕ, then $\lambda = N\phi$.

Direction of the Induced E.M.F. — The direction of this induced e.m.f. around the circuit is found to be in the *left-handed* screw direction with respect to the *increase* of flux; viz., if one faces the circuit looking in the direction of the *increase* of flux, the induced e.m.f. is in the counter-clockwise direction. The current which would be set up by this e.m.f., however, would produce a flux linking the circuit in the right-handed screw direction. Hence a change in the magnetic flux through an electric circuit always sets up an e.m.f. which tends to produce a current around this circuit in such a direction as to set up an *opposing* flux. This fact may be expressed mathematically by writing a minus sign before $\frac{d\phi}{dt}$ in equation (31), i.e., by putting

$$e = -N\frac{d\phi}{dt}. \tag{31b}$$

The value of $\left(-N\frac{d\phi}{dt}\right)$ is then the e.m.f. induced in the circuit in the right-handed-screw direction with respect to the increase of flux. Or stated in other words $\left(-N\frac{d\phi}{dt}\right)$ represents the *rise* of electric potential and $N\frac{d\phi}{dt}$ represents the *drop* of potential around the circuit in the right-handed screw direction with respect to the increase of flux.

WORK DONE BY A VARYING MAGNETIC FLUX. — Consider a coil A (Fig. 15) of N turns of wire, and let each of these N turns be linked by a flux ϕ due to some external agent, e.g., another coil B in which an electric current is flowing. Let the flux ϕ through A due to B be increasing at any instant at the rate $\frac{d\phi}{dt}$ in the *left-handed* screw direction with respect to the current I in A at this instant. Then there is induced in A at this instant an e.m.f. in the direction of I equal to $e = N\frac{d\phi}{dt}$ and, therefore, the electric power developed in A at this instant is $ei = NI\frac{d\phi}{dt}$. This power is transmitted to the coil A as a result of the varying flux through it; hence the power

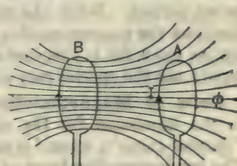

Fig. 15.

$$p = NI\frac{d\phi}{dt} \tag{32}$$

may be looked upon as the magnetic power input, this power being converted within the coil into electric power.

Magnetic Displacement Current. — The varying flux established by B through A may be looked upon as the means whereby this energy is transferred through the magnetic field, just as an electric current may be looked upon as the means whereby energy is transferred from one point to another in an

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electric circuit. Hence the varying flux may be looked upon as a "magnetic displacement current."

Magnetomotive Force ($\mathfrak{F}$). — The above expression for the rate of transfer of energy by a varying magnetic field may be compared with the rate of transfer of energy, or power input $P = EI$, corresponding to an electric current I flowing in a circuit in which there is *back* electromotive force E. The strength of the "magnetic displacement current" may be chosen arbitrarily* as

$$\text{Magnetic displacement current} = \frac{1}{4\pi}\frac{d\phi}{dt},$$

just as the strength of the electric displacement current is taken as $\frac{1}{4\pi}\frac{d\psi}{dt}$, where ψ is the total electrostatic flux. Then the quantity corresponding to the electromotive force must be chosen numerically equal to

$$\mathfrak{F} = 4\pi NI \tag{33}$$

and may be called the "magnetomotive force," abbreviated "m.m.f." The magnetic power input may then be written

$$p = \mathfrak{F} \times (\text{Magnetic displacement current}).$$

where $\mathfrak{F}$ is the *back* magnetomotive force.

Direction of the Magnetomotive Force. — The closed path of the magnetic flux lines through a magnetic field is called a "magnetic circuit," just as the path of the stream lines of an electric current is called an electric circuit. In general, a magnetomotive force is produced in every magnetic circuit wherever it is linked by an electric circuit, equal in numerical value to $4\pi NI$, where N is the number of times the current I links this circuit. The magnetomotive force is taken as positive when the current links the flux lines in the right-handed screw direction, for then an increase in the flux corresponds to a magnetic power output. When the current links the flux lines in the left-handed-screw direction the magnetomotive force is taken as negative, i.e., a "back" magnetomotive force, for in this case an increase in flux corresponds to a magnetic power input; see the special case considered above.

Units of Magnetomotive Force. — Ampere-Turns. — The product NI in the expression for the magnetomotive force, equation (33), is called the "current-turns"; when I is expressed in amperes it is called the "ampere-turns." The magnetomotive force as above defined differs from this product only by a constant numerical factor; hence the ampere-turns of a coil may be taken as a measure of the magnetomotive force produced by it. This unit of magnetomotive force, namely, one ampere-turn, is the unit commonly employed in the practical calculation of magnetic circuits. The c.g.s. electromagnetic unit is called the gilbert; 1 ampere-turn = 1.2566 gilberts. See *Units and Conversion Factors.*

MAGNETIZING FORCE OR MAGNETIC FIELD INTENSITY (H). — Experience shows that the magnetic flux density produced at any point by a given magnetomotive force depends (*a*) upon the position of the point with respect to the source of the m.m.f. and (*b*) upon the nature of the substances through which this m.m.f. produces the magnetic flux. Compare with the electric current density or dielectric flux density produced at a given point by

* The factor 4π arises from the manner in which the conceptions of the magnetic field were originally developed.

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a given electromotive force. These facts lead to the conception of the flux density at any point in a magnetic field as being due to a "magnetizing force" H at that point, this magnetizing force H depending solely upon the *magnetomotive forces* producing the field and the *distribution of the flux lines*, as distinguished from the flux density B which depends not only upon these two items but also upon the *nature of the medium at the point in question.*

From analogy with the relation between electromotive force and electric field intensity, the magnetizing force (also called the "magnetic field intensity") at successive points along any closed path in a *magnetic* field may be defined by the relation that its line integral around such a path is equal to the total magnetomotive force acting around this path, viz.,

$$4\pi NI = \int (H \cos \theta) dl, \tag{34}$$

where dl represents any elementary length of this path, see Fig. 16, $(H \cos \theta)$ the value of the component of the magnetizing force at dl in the direction of dl, and NI the total number of current turns linked by the path. Experience shows that such a definition leads to a simple means of expressing in a quantitative manner the interrelations of a number of experimental facts. Magnetizing force may also be expressed as the force in dynes which would act on a "unit magnetic pole," see p. 412.

When the path coincides in direction with the magnetizing force at each point, equation (34) may be written

$$4\pi NI = \int H dl. \tag{34a}$$

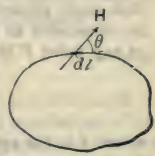

Fig. 16.

For the application of this formula see the articles on *Magnetic Properties of Iron; Generators; Motors.*

Direction of the Magnetizing Force. — Experience shows that except for points inside a permanent magnet the magnetizing force H and the flux density B are always in the same direction. For points inside a permanent magnet the direction of the magnetic field intensity H, due solely to the magnet itself, is opposite to the direction of the flux lines, i.e., a permanent magnet produces a "demagnetizing force" on itself.

Lines of Magnetizing Force. — The magnetizing force at any point in a magnetic field may be represented by lines drawn in the field in such a direction that their direction coincides with the direction of the magnetizing force at each point, and of such a number per unit area perpendicular to their direction that their density at each point gives the value of the magnetizing force at that point. Such lines are called "lines of magnetizing force" or "lines of magnetic field intensity." In general the lines of magnetizing force and the magnetic flux lines coincide in direction (except within the substance of permanent magnets), *but their densities are different.* Only in non-magnetic substances do the flux lines and lines of magnetizing force coincide. The simple expression "lines of force" is frequently used to designate either the flux lines or the lines of magnetic intensity, but it is evident that such a loose use of this term is liable to lead to confusion when speaking of the magnetic field within a magnetic substance.

Units of Magnetizing Force. — Magnetizing force is of the nature of magnetomotive force per unit length, just as electric field intensity is of the nature

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of electric potential difference per unit length. Hence the c.g.s. electromagnetic unit of magnetizing force may be called the "gilbert per centimeter"; compare with volts per centimeter. When the magnetomotive force is expressed in ampere-turns, the magnetizing force is expressed in ampere-turns per centimeter or per inch; see *Units and Conversion Factors*.

Values of the Magnetizing Force in a Uniform Medium. — When the medium surrounding the stream lines of an electric current is of a *uniform magnetic nature throughout*, the magnetizing force at any point may be calculated from the shape and distribution of the stream lines of the current, irrespective of whether the medium be non-magnetic or highly magnetic, e.g., iron. See *Units and Conversion Factors* for numerical multipliers to change them into practical units.

Magnetizing Force at any Point Due to an Element of a Current-Stream Line (Fig. 17). — Consider any closed stream line of electric current and let the surrounding *medium be uniform* in its magnetic properties throughout the region in which the magnetic field produced by this stream line exists. It can be shown that each elementary length dl of this stream line may be considered as contributing to the magnetizing force H at any point P in this region an amount

$$dH = \frac{(I \sin\theta)\, dl}{x^2}, \qquad (35)$$

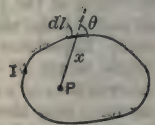

Fig. 17.

where I is the current flowing along this stream line, x the distance from P to dx, and θ the angle between x and dl. The direction of dH is perpendicular to the plane determined by x and dl. The total magnetizing force at P is then the *vector sum* or *vector integral* of dH for all the elementary lengths into which the stream line is divided.

Magnetizing Force Due to a Straight Wire (Fig. 18). — Applying equation (35) to the case of a straight wire of circular cross-section carrying a current I, the magnetizing force at any point P due to a length l of this wire is

$$H = \frac{I}{x}(\sin\theta_1 + \sin\theta_2), \qquad (35a)$$

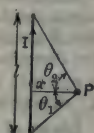

Fig. 18.

where x is the perpendicular distance from P to the wire and θ_1 and θ_2 the angles designated in Fig. 18.

When the wire is very long compared with x, this becomes

$$H = \frac{2I}{x}. \qquad (35b)$$

This formula also holds approximately for any point outside a wire of any shaped cross-section, provided x is large compared with the maximum diameter of this section. For a point *inside a long wire of circular cross-section* of radius a the magnetizing force is also given by (35b) when I is taken to represent *that part of the current inside the circle through P* concentric with the axis of the wire. When the current density is uniform over the cross-section, as is usually the case (*see, however, the article on Skin Effect*), the magnetizing force inside the wire is

$$H_i = \frac{2xI}{a^2}. \qquad (35c)$$

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Magnetizing Force on the Axis of a Circular Coil of N Turns.— Let I be the current, r the mean radius of the coil, and x the distance of the point from the center of the circle; then

$$H = \frac{2\pi NIr^2}{(r^2 + x^2)^{3/2}} \tag{35d}$$

Magnetizing Force due to a Solenoid.—A solenoid is a helical coil of wire, each turn having the same radius. Let N = total number of turns, I = current in abamperes, r = mean radius of the helix in centimeters, l = length of helix in centimeters. Then at any point on the axis of the helix (inside or outside) at a distance of x centimeters from its center, the magnetizing force in gilberts per centimeter is

$$H = \frac{2\pi NI}{l}\left[\frac{0.5\,l + x}{\sqrt{r^2 + (0.5\,l + x)^2}} + \frac{0.5\,l - x}{\sqrt{r^2 + (0.5\,l - x)^2}}\right] \tag{35e}$$

This formula holds only when the thickness of the winding is small compared with the mean radius r. When l is large compared with r this reduces to

$$H = \frac{4\pi NI}{l}. \tag{35f}$$

For all points inside the solenoid (whether on the axis or not) at a distance from the ends large compared with r, that is, inside the central portion of a long solenoid, the field is uniform over the cross-section of the solenoid and its value is given by (35f). An exact formula for field intensity at any point inside a solenoid is given by O. Billieux in *Rev. Gen. d'El.* 6, *p. 827, Dec. 13, 1919.*

Magnetizing Force Inside a Toroid (Fig. 19).—A toroid is a cylinder bent into the form of a closed ring, making a shape like a doughnut. When such a ring is *uniformly* wound with an insulated wire so that the turns of the wire are close together and cover the entire surface of the toroid, the magnetic field is confined entirely within the space inclosed by these turns, and therefore, when the core on which the wire is wound is of uniform magnetic material throughout, both the lines of magnetizing force and the flux lines must be circles concentric with the hole in the "doughnut." The magnetizing force will have the *same* value at every point on the circumference of any *one* of these circles, and, therefore, from equation (34a) the value of H at any point P within the core is

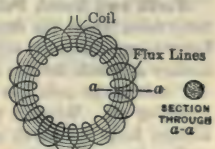

Fig. 19.

$$H = \frac{4\pi NI}{l}, \tag{35g}$$

where N is the total number of turns on the core, I the current in each turn and l the length of the circumference through P. Unless the hole in the "doughnut" has a large radius compared with the radius of the cross-section of the core H will not be uniform over this section, since l for the various points in the cross-section will differ considerably.

It should be noted that the value of H is independent of the material of the core provided only that the core be of uniform material throughout. That is,

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equation (35g) applies to an iron core as well as to an air or wood core, provided the iron is uniform throughout and there is no air gap across the path of the flux lines. Even a mechanically perfect contact between two pieces of iron of the same kind, however, is sufficient to vitiate the above formula.

MAGNETIC PERMEABILITY (μ). — PARAMAGNETIC AND DIAMAGNETIC SUBSTANCES. — The quotient of the magnetic flux density B at any point by the magnetizing force H at that point is defined as the "magnetic permeability" μ of the substance as medium at that point, viz.,

$$\mu = \frac{B}{H}. \tag{36}$$

Compare with the definition of dielectric coefficient, equation (17.) The c.g.s. electromagnetic system of units is based on the arbitrary assumption of unity as the value of the magnetic permeability of air. Any substance which has a magnetic permeability greater than that of air is called a "paramagnetic substance," and any substance which has a permeability less than that of air is called a "diamagnetic substance." The only substances which are strongly paramagnetic, i.e., which have a permeability considerably greater than that of air, are iron, steel, nickel and cobalt, and certain alloys of non-magnetic elements. The only substance which is appreciably diamagnetic is bismuth, which has a permeability of about 0.9998. All other elements are practically non-magnetic, i.e., their permeabilities differ from unity by less than 1 per cent.

The permeability of a given sample of any highly magnetic substance is not a constant, but depends upon the value of the magnetizing force; see the curves in the article on *Magnetic Properties of Iron.* The permeability also depends very largely upon the previous heat treatment and the exact composition of the material, and also upon its previous magnetic history; these relations are also discussed in detail in that article. The methods of measuring permeability are described in the article on *Magnetic Testing.*

North and South Poles. — That portion of the surface of any magnetized body from which the flux lines pass out into the air (or into any substance of lower permeability) is said to be a north magnetic pole, and that portion of the surface at which the flux lines enter the body is said to be a south magnetic pole. A "unit north pole" is a pole from which 4π flux lines emerge into the surrounding air. When a magnetic needle is placed near the surface of a magnetized body its north seeking end points away from this surface when this surface is a north pole and toward the surface when this surface is a south pole.

Difference of Magnetic Potential (U). — Consider any two points 1 and 2 in a magnetic field (Fig. 20) and let the path between them from 1 to 2 pass through an electric circuit producing a magnetomotive force in the direction from 1 to 2, then from analogy with the electric circuit, equation (2), the expression

$$U_{12} = \int_1^2 (H\cos\theta)\,dl - \mathfrak{F}_{12} \tag{37}$$

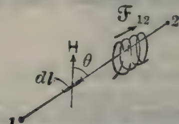

Fig. 20.

is called the "drop of magnetic potential" from 1 to 2. From the definition of magnetizing force, equation (34), it follows that around any *closed* circuit the drop of magnetic potential is always zero. A magnetomotive force $\mathfrak{F}_{12}$ is, therefore, equivalent to a rise of magnetic potential from 1 to 2.

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When there is no source of m.m.f. between 1 and 2 and the path coincides with a line of magnetizing force, the drop of magnetic potential is

$$U_{12} = \int_1^2 H\, dl. \tag{37a}$$

Magnetic potential difference is of the same nature as magnetomotive force and may, therefore, be expressed in the same units, viz., gilberts or ampere turns.

Magnetic Equipotential Surfaces. — A surface drawn in a magnetic field in such a manner that this surface is perpendicular at each point to the magnetizing force at this point (i.e., to the line of magnetizing force through this point) is called a "magnetic equipotential surface," compare with *electric equipotential surface*.

Magnetic Reluctance ($\mathcal{R}$). — To establish a magnetic flux ϕ through a given portion of a substance which is *not itself linked by a source of m.m.,f.* a difference of magnetic potential must always be established between the end surfaces of this substance. Let U be the magnetic potential drop established from one surface to the other, then the quotient

$$\mathcal{R} = \frac{U}{\phi} \tag{38}$$

is defined as the magnetic reluctance of the given portion of the substance. Compare with electric resistance. The c.g.s. electromagnetic unit of reluctance is called the oersted. It should be noted that the above definition is meaningless except when applied to a portion of a substance of which the end surfaces are magnetic equipotential surfaces and through every cross-section of which the same flux passes.

Factors upon Which Reluctance Depends. — The magnetic reluctance of a given portion of a substance included between two equipotential surfaces and bounded laterally by a surface through which no flux line passes depends upon 1. the magnetic permeability of the substance, 2. the dimensions of this portion of the substance and 3. upon the distribution of the flux lines over each cross-section perpendicular to them. The relations are identical with those which determine the electrical resistance of a conductor, the magnetic permeability taking the place of the electric conductivity. For example, *for a straight bar of constant cross-section A and length l, through which the flux lines are straight, parallel and uniformly distributed, the reluctance is*

$$\mathcal{R} = \frac{l}{\mu A}. \tag{38a}$$

Note particularly that this formula for reluctance is applicable only under the special conditions just stated. Formulas for other cases are much more complex; see Douglas, J. F. H., *Reluctance of Irregular Magnetic Fields*, A.I.E.E., Proc., 34, May, 1915.

Magnetic reluctance is not a constant quantity even for a given material and given flux distribution, unless this material is non-magnetic. For all highly magnetic materials μ depends upon the magnetizing force and therefore also upon the flux density. It should also be noted that the magnetic reluctance does not represent a "resistance" in the sense of something which causes a dissipation of energy.

Magnetic Permeance ($\mathcal{P}$). — The reciprocal of magnetic reluctance is called "magnetic permeance." The permeance of a straight bar under the conditions specified above is

$$\mathcal{P} = \frac{\mu A}{l} \tag{39}$$

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The permeability of a substance is, therefore, equal to the permeance of a unit cube of this substance when the flux through the cube is parallel to four edges of the cube and is uniformly distributed over the section at right angles to these four edges.

Magnetic permeance is analogous to electric conductance, except that it is not a factor which affects the *dissipation* of energy in a substance. It does, however, enter into the expression for the energy *stored* in a magnetic field in the same way that the electrostatic capacity of a dielectric is a determining factor in the expression for the energy stored in the electric field.

Kirchhoff's Laws for the Magnetic Circuit. — As already noted, equation (30), the total magnetic flux coming up to any surface in a magnetic field is always zero, provided a flux leaving a surface is considered as a *negative* flux coming up to that surface. This fact may be represented by the formula

$$\Sigma\phi = 0 \tag{40}$$

for every surface in the field. Similarly, from the definition of magnetic potential drop, it follows that the *total* magnetic potential drop around any closed circuit is zero, or that the total magnetomotive force acting around any closed circuit is equal to the sum of the reluctance drops around that circuit, which may be represented by the formula

$$\Sigma\mathfrak{F} = \Sigma\mathcal{R}\phi. \tag{40a}$$

These two equations are identical in form with those representing Kirchhoff's laws for the electric circuit, equation (13). They are, however, not so easy to use for practical calculations, for the magnetic flux is not confined to approximately geometrical lines like the currents in a network of insulated wires, but in general fills all space surrounding the coils which establish the magnetomotive forces; also, when there is iron or other magnetic material in the circuit, the permeability depends on the flux density and the previous history of the iron. (The distribution of magnetic flux in and around an iron circuit is analogous to the distribution of current in and around an uninsulated mass of copper of the same shape as the iron circuit immersed in a liquid having a conductivity about equal to that of carbon.) Only in the special case of a uniformly wound circular ring or toroid are the lines of induction confined *entirely* to an iron circuit; in general a certain number also exist in the air and in whatever other substances are in the vicinity of the iron circuit.

SELF AND MUTUAL INDUCTION. — When the current in a given electric circuit varies with time the magnetic flux accompanying this current also varies with time, and, since this flux links the current which produces it, an e.m.f. is induced in each turn of the circuit equal to the rate at which the flux through this turn is varying, and in such a direction as to *oppose* the change in the current. That is, an *increasing* electric current is always accompanied by a *back* e.m.f. due to the increase in the magnetic flux which accompanies this increase of current. Or, viewed from another point of view, an increase in the velocity of a stream of electricity develops a back pressure, just as when the velocity of a stream of water is increasing it develops a back pressure or "velocity head" due to the inertia of the water. The magnetic field accompanying an electric current may, therefore, be looked upon as a result of its "electromagnetic inertia," or "electromagnetic mass"; see also the article on *Electron Theory*.

Again, when an electric current *decreases* its accompanying flux decreases and an e.m.f. is set up in the circuit tending to oppose this decrease, i.e., tending

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to keep the current from decreasing. This is again analogous to the effect of inertia in the case of ordinary matter.

Coefficient of Self-Induction or Inductance (L). — In general, the coefficient L by which the rate of change of the current $\left(\frac{di}{dt}\right)$ in any circuit must be multiplied to give the self-induced electromotive force e is called the "coefficient of self-induction" or simply the "inductance" of the electric circuit. In general, then, the self-induced e.m.f. in an electric circuit is

$$e = L \frac{di}{dt}, \tag{41}$$

where L is the inductance of the electric circuit and $\frac{di}{dt}$ represents the change in the current per unit of time. Since $e = \frac{d\lambda}{dt}$, see equation (31a), where λ is the number of magnetic linkages between the electric circuit and the flux established by the current i, the inductance may also be defined by the relation

$$L = \frac{d\lambda}{di}. \tag{42}$$

That is, the *inductance is equal to the increase in the number of linkages per unit increase in the current.* When the permeability of the magnetic circuit is *constant* the inductance is also a constant equal to the linkages per unit current. When every flux line is linked by every stream line of electric current, and the permeability of the entire magnetic circuit is constant, then

$$L = \frac{4\pi N^2}{\mathcal{R}}, \tag{42a}$$

where N is the number of turns forming the electric circuit and $\mathcal{R}$ is the reluctance of the complete magnetic circuit.

Formulas for the inductance of various electric circuits are given in the article on *Inductance and Inductive Reactance.*

Units of Inductance. — The practical unit of inductance is called the henry; for the relation between the henry and the abhenry and millihenry see *Units and Conversion Factors.*

Coefficient of Mutual Induction. — Mutual Inductance (M). — In general, the coefficient M_{ab} by which the rate of change of the current $\left(\frac{di_a}{dt}\right)$ in a circuit A must be multiplied to give the electromotive force e induced by this current in another circuit B, is called the "coefficient of mutual induction" or simply the "mutual inductance" between A and B; see Fig. 15. In general then, the e.m.f. induced in any circuit B by a varying current i_a in any other circuit A, is

$$e_b = M_{ab} \frac{di_a}{dt}, \tag{43}$$

where M_{ab} is the mutual inductance between A and B.

It can be shown from the principle of the conservation of energy that the mutual inductance of a circuit A with respect to a second circuit B must be equal

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to the mutual inductance of B with respect to A, that is, $M_{ab} = M_{ba}$. Whence, the e.m.f. induced in A when the current in B increases by an amount di_b is

$$e_a = M_{ab}\frac{di_b}{dt}.$$

Since $e_b = \frac{d\lambda_{ab}}{dt}$, where λ_{ab} is the number of linkages between the circuit B and the flux through B due to the current i_a, the mutual inductance may also be defined by the relation

$$M_{ab} = \frac{d\lambda_{ab}}{di_a}. \tag{44}$$

That is, the *mutual inductance between two circuits A and B is equal to the increase in the number of magnetic linkages of the circuit B per unit increase of the current in A*, and vice versa. When the permeability of the magnetic circuit is *constant*, the mutual inductance is also a constant equal to the linkages of B per unit current in A, and vice versa. When every flux line linking *both* A and B is linked by every turn in A and every turn in B, then

$$M_{ab} = \frac{4\pi N_a N_b}{\mathcal{R}_{ab}}, \tag{44a}$$

where N_a and N_b are the number of turns forming the circuits A and B respectively and $\mathcal{R}_{ab}$ is the reluctance of that *part* of the magnetic circuit through which the flux from A to B passes when there is current *in one coil only*.

The units of mutual inductance are the same as those of self-inductance; see, *Units and Conversion Factors*.

Formulas for mutual inductance for a few special cases are given in the article on *Inductance, and Inductive Reactance*.

Instantaneous Potential Drop Through a Coil. — Consider a coil of wire which has a resistance r and an inductance L. Then when this coil contains no other source of e.m.f. than its own self-induced e.m.f., the expression for the instantaneous potential drop through the coil is

$$v = ri + L\frac{di}{dt}, \tag{45}$$

where i is the instantaneous value of the current and $\frac{di}{dt}$ the increase in this current per unit of time. Compare with the equation

$$i = gv + C\frac{dv}{dt},$$

for the instantaneous current through a condenser, where g is the conductance of the dielectric in the condenser, v the potential drop across it, and $\frac{dv}{dt}$ the increase in the potential drop per unit of time.

When there is another coil near the first coil, and the two coils have resistances r_1 and r_2 and self-inductances L_1 and L_2 respectively, and a mutual inductance M, then the potential drops through them are respectively

$$\left.\begin{aligned} v_1 &= r_1 i_1 + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \\ v_2 &= r_2 i_2 + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \end{aligned}\right\} \tag{46}$$

where i_1 and i_2 are the currents in the two coils in the *same* direction.

Effective or "Total" Inductance. — In certain cases of symmetry, when the algebraic sum of all the currents in the field is zero, the voltage drop along any *portion* of a circuit may be represented by an expression of the form

$$v = ri + L\frac{di}{dt},$$

although there may be several electric circuits in the vicinity in which electric currents are flowing. An example of this is the case of the wires of a three-phase transmission line arranged so that they form the three edges of an equilateral prism; see *Inductance and Inductive Reactance.* The coefficient L in this case is called the "effective self-inductance" of this particular portion of the circuit; it really takes into account not only the self-induction of the particular part of the circuit under consideration but also the mutual induction of the rest of the circuit or circuits with respect to this particular portion. Similarly, when the voltage drop in any portion of a circuit involves not only the current i_1 in this circuit but also a current i_2 in some part of this circuit or in some other circuit, the coefficient M_{12} in the expression

$$v_1 = r_1i_1 + L_1\frac{di_1}{dt} + M_{12}\frac{di_2}{dt}$$

is called the effective mutual inductance of the part 2 with respect to 1 although L_1 may also be due in part to circuit 2.

Leakage Inductance. — In discussing the action of a transformer (q.v.), which is merely two electric circuits linking the same iron core, it is more convenient to deal with the *resultant* flux due to the currents in *both* electric circuits or windings instead of considering the fluxes due to the two windings separately. Referring to Fig. 21, let ϕ_r represent that portion of the resultant flux due to the currents i_1 and i_2 in the two windings 1 and 2, and let ϕ_1 represent that part of the total flux which links 1 only and ϕ_2 that part of the total flux which links 2 only. Let λ_1 be the linkages between ϕ_1 and circuit 1 and λ_2 be the linkages between ϕ_2 and circuit 2. Then

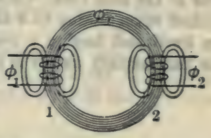

Fig. 21.

$$L_1' = \frac{\lambda_1}{i_1} \quad\text{and}\quad L_2' = \frac{\lambda_2}{i_2},$$

are called the "leakage inductances" of the two windings respectively; the reluctance of the paths of ϕ_1 and ϕ_2 are practically constant since the air portion of these paths forms a greater part of the reluctance in each case.

Let i_1 be the current in winding 1 and i_2 be the current in winding 2 in the *opposite* direction to i_1 (this is the actual relation in a transformer during most of the time) and let e_1 be the impressed e.m.f. across the terminals of the first or primary winding and e_2 the terminal e.m.f. at the terminals of the second or secondary winding when the current i_2 is flowing. Then

$$\left.\begin{aligned} e_1 &= r_1i_1 + L_1'\frac{di_1}{dt} + N_1\frac{d\phi_r}{dt} \\ e_2 &= N_2\frac{d\phi_r}{dt} - r_2i_2 - L_2'\frac{di_2}{dt} \end{aligned}\right\} \qquad (47)$$

where r_1 and r_2 are the resistances of the two windings and N_1 and N_2 are the number of turns in the two windings respectively.

Comparing equation (47) with (46) and noting that $e_1 = v_1$ and $e_2 = -v_2$, and i_2 in (47) is taken in the opposite direction from i_2 in (46), it may be shown that*

$$L_1' = L_1 - \frac{N_1}{N_2} M,$$

$$L_2' = L_2 - \frac{N_2}{N_1} M.$$

Whence the leakage inductance of each winding is very much less than the total self-inductance of that winding.

Fundamental Equations of the Transformer. — Comparing equations (47) and (46) it may also be seen that*

$$\phi_r = \frac{M}{N_2}\left(i_1 - \frac{N_2}{N_1} i_2\right).$$

Put $i_2' = \frac{N_2}{N_1} i_2$, that is, i_2' is the current which would be produced in the secondary if it had the same number of turns as the primary. The difference

$$i_m = i_1 - i_2'$$

is called the "magnetizing current," and the quantity $L_m = \frac{M}{N_2}$ may be called the magnetizing inductance. Then

$$\phi_r = L_m i_m.$$

In the above deduction no account is taken of the eddy-current and hysteresis loss in the iron core; these losses (the eddy-current loss in particular) may be looked upon as due to a third current i_3 flowing in a single turn short-circuited on itself and having a resistance r_3 and negligible inductance. The equation for this tertiary circuit is then

$$0 = \frac{d\phi_r}{dt} - r_3 i_3.$$

Whence the corresponding current in the primary winding is $i_3' = \frac{i_3}{N_1}$, which may also be written

$$i_3' = gN_1 \frac{d\phi_r}{dt},$$

where $g = \frac{1}{r_3 N_1^2}$ is called the "leakage conductance" of the transformer. The total primary current is then

$$i_1 = i_2' + i_m + i_3'.$$

From equation (47) and the relations given in the last two paragraphs the complete theory of the transformer may be developed.

ENERGY OF THE MAGNETIC FIELD. — Energy is required to establish a flow of electricity just as energy is required to set a column of water in motion, this "energy of motion" of electricity being analogous to the kinetic energy of a moving body. This energy of motion is most conveniently ex-

* In this equation i_1 and i_2 are taken in the opposite direction.

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pressed in terms of the magnetic field which accompanies the flow of electricity or electric current; the mathematical expression for it may be put into various forms.

Magnetic Energy of Single Electric Circuit, Permeability Constant. — For example, consider a single electric circuit and the magnetic field which is established around this circuit when the current in it increases from zero to a value I. If at any instant the current has the value i and number of magnetic linkages between this current and its own flux at this instant is λ, then when the current increases by di the linkages increase by an amount $d\lambda$ in the right-handed screw direction with respect to the current and therefore the *electric output* of the circuit during this change is, from equation (31a), $id\lambda$, which is also equal to the *magnetic* power *input* into the magnetic circuit which it links. Hence the total energy input into the magnetic circuit or magnetic field is

$$W = \int_0^I i d\lambda = \int_0^I L i di,$$

since by definition $d\lambda = L\,di$, see equation (42). When the permeability is constant L is also constant, whence for constant permeability

$$W = \frac{1}{2} LI^2. \tag{48}$$

Equation may also be written

$$W = \frac{1}{2} \lambda I = \frac{1}{8\pi} \frac{\mathfrak{F}^2}{\mathcal{R}} = \frac{1}{8\pi} \mathcal{R}\phi^2 \tag{48a}$$

where $\mathfrak{F}$ is the magnetomotive force ($= 4\pi NI$ when the coil has N turns in a concentrated winding), $\mathcal{R}$ is the reluctance of the magnetic circuit and ϕ the total magnetic flux.

Since the impressed m.m.f. per unit length of a magnetic flux line is equal to the magnetizing force H, and the flux per unit area perpendicular to this flux line is equal to the flux density B, the *energy per unit volume* of the magnetic field is

$$w = \frac{HB}{8\pi} = \frac{\mu H^2}{8\pi} = \frac{B^2}{8\pi\mu}. \tag{48b}$$

These various formulas should be compared with the corresponding formulas, equation (26), for electrostatic energy.

Magnetic Energy of Two or More Electric Circuits, Permeability Constant. — It can also be readily shown that the total energy required to establish currents I_1, I_2, etc. in several electric circuits linking one or more magnetic circuits of *constant* reluctance is

$$\begin{aligned} W = {} & \tfrac{1}{2} L_1 I_1^2 + \tfrac{1}{2} L_2 I_2^2 + \tfrac{1}{2} L_3 I_3^2 + \dots \\ & + M_{12} I_1 I_2 + M_{13} I_1 I_3 + M_{23} I_2 I_3 + \dots \end{aligned} \tag{49}$$

where the L's and M's represent the self and mutual inductances respectively. This may also be written

$$W = \frac{1}{2} \Sigma \phi I, \tag{49a}$$

where the summation is an *algebraic* one and includes every complete *turn* of the electric circuit, I being the current in this turn and ϕ the magnetic flux linking this turn in the *right-handed* screw direction with respect to the current.

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The energy per unit volume at any point in the magnetic field due to any number of electric circuits is represented by equation (48b), where H and B are taken as the resultant magnetizing force and flux density respectively at this point.

When the permeability is not constant the *energy transferred to unit volume* of the magnetic field due to any number of currents is

$$w = \frac{1}{4\pi}\int_0^B H\,dB \qquad (50)$$

provided the magnetizing force H and the flux density B are either in the same or directly opposite directions, as, for example, in the case of a uniformly magnetized iron toroid. To integrate this expression requires a knowledge of the relation between B and H. Note also that, due to the phenomenon of magnetic hysteresis, part of the energy required to establish a magnetic field in iron or other magnetic substance is dissipated as heat and is not recoverable when the field disappears; therefore, only part of the energy represented by this formula is "stored" in the field in a recoverable form. See article on *Magnetic Properties of Iron.*

MECHANICAL FORCES IN THE MAGNETIC FIELD. (*See also article on Electromagnets.*) — Experience shows that all bodies in which an electric current exists, and all bodies in which a magnetic flux exists, exert in general mutual mechanical forces upon one another tending to produce such a relative motion of these bodies as will *increase* the energy of the magnetic field. Let f be the component of the force tending to move any body in the field in a given direction and let dW be the increase in the energy of the field due to displacing the body a distance dx in this direction, then

$$f = \frac{dW}{dx}, \qquad (51)$$

provided this displacement does not alter the existing magnetomotive forces in the field.

Similarly, calling T the component of the torque tending to turn any body in the field about a given axis, and dW the *increase* in the magnetic energy due to the turning of the body through an angle of $d\alpha$ radians about this axis, then

$$T = \frac{dW}{d\alpha}, \qquad (51a)$$

provided this displacement does not alter the existing magnetomotive forces in the field.

Equations (51) and (51a) also give the actual force and torque respectively during a change in position which *does* cause a change in the magnetomotive forces in the field, provided dW is taken to represent the net increase in the energy of the field. This force and torque may differ greatly from the steady state force and torque; see article on *Electromagnets.*

Force Produced by a Magnetic Field on a Coil Carrying a Current. — From equation (32) the energy *output* of an electric circuit, when the magnetic flux threading it in the right-handed screw direction with respect to the current in it increases by an amount $d\phi$, is $dW = NI\,d\phi$, where N is the number of turns linked by this increase in flux and I is the current in each turn. This is the energy *input* into the magnetic field. Whence if an increase in flux $d\phi$ is

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produced through the coil when it moves a distance dx, the force acting on the coil in the direction of dx is

$$f = NI\frac{d\phi}{dx}; \tag{52}$$

$d\phi$ represents the increase in the flux linking the coil in the *right-handed* screw direction with respect to the current in it or the number of flux lines which cut the coil as the result of its motion. Similarly, when the coil is so mounted that it can move only about a fixed axis, then the value of the torque tending to turn it about this axis is

$$T = NI\frac{d\phi}{d\alpha}, \tag{52a}$$

where $d\phi$ represents the increase in the flux linking the coil in the right-handed screw direction with respect to the current in it when the coil turns through an angle α (in radians).

From these relations it follows that a coil carrying an electric current, when in the magnetic field due to any other agent (current or permanent magnet), always tends to take up that position in which it will embrace the maximum possible flux linking the coil in the right-handed screw direction with respect to the current in it. This accounts for the attraction of two parallel coils when they carry currents in the same direction, and the repulsion of two such coils when they carry currents in opposite directions. This principle is useful in determining the direction of motion of the moving element in such devices as the electric motor, galvanometer, current balance, electrodynamometer (q.v.).

Torque on the Coil When its Plane is Parallel to the Magnetic Field. — When the coil is placed with its plane parallel to the flux lines due to some other agent (e.g., a permanent magnet or another coil carrying a current), the flux linking the coil due to this agent is zero; see Fig. 22. Let the two circles represent sections of the two sides of the coil, its plane being perpendicular to the page, and let the dot in the left-hand circle indicate that the current is up through this side of the coil and the cross in the other circle that it is down through the other side. Let B be the flux density of the field constant for each point along the flux line since the flux lines are parallel, and let A be the area of the coil and let $\phi = BA$, that is, ϕ represents the total flux which *would be produced* through the coil by a uniform field of flux density B at *right angles* to it. Then the torque on the coil when its plane is *parallel* to the field is

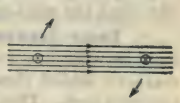

Fig. 22.

$$T = N\phi I. \tag{53}$$

This relation is useful in calculating the torque on the moving element of a galvanometer, ammeter, electrodynamometer, wattmeter, etc.

Average Torque on a Coil Rotating in a Magnetic Field. — Consider a coil which is rotating with an angular velocity ω about a fixed axis in a magnetic field due to some other agent (e.g., an armature coil rotating in the magnetic field produced by the current in the field coils). Let the current in this coil be constant and in the same direction with respect to the coil while the coil turns from the position in which it embraces the maximum flux ϕ in the left-handed screw direction to the position (a half revolution in a 2-pole machine) when it embraces this same maximum flux ϕ in the right-handed

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screw direction. The total change in the flux while the coil turns through this angle, π radians in the case of a 2-pole machine, is $2\,\phi$, whence the average torque turning the coil through this half revolution is

$$T = \frac{2\,N}{\pi} I\phi \cdot \tag{54}$$

That is, the average torque is proportional to the product of the current by the total flux per pole. When a commutator is provided to change the direction of the current every half turn, then the torque is in the same direction for a complete turn. See *Motors*.

When the coil is mounted in a slotted iron core which rotates with the coil as in an ordinary motor, this torque is exerted partly on the wire and partly on the teeth of the core.

Force on a Wire in a Magnetic Field. — Consider a wire of length l forming part of a closed circuit, Fig. 23. Let B be the value of the flux density at the wire, I be the current in the direction indicated, and let the lines representing the flux be perpendicular to the wire in the direction from the eye to the page. When this wire moves a distance dx to the left the flux threading the closed loop formed by the circuit is increased by an amount $d\phi = Bldx$, whence the force acting on the wire is, from equation (51),

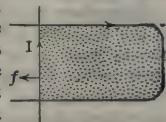

Fig. 23.

$$f = BlI. \tag{55}$$

Left-hand Rule. — The relative directions of this force, the flux density B and the current I may be conveniently determined by pointing the forefinger of the left hand in the direction of the flux and the middle finger in the direction of the current (I), then if the thumb is held perpendicular to these two fingers it will point in the direction in which the force tends to move the wire. Compare with the right-handed rule for e.m.f.

Force between Two Current-Carrying Conductors. — The mutual force between any two electric circuits carrying currents i_1 and i_2, due to the magnetic field of these currents, may be readily found by combining equations (49) and (51). This gives for the component of this force in any direction the value

$$f_x = i_1 i_2 \frac{dM}{dx} \tag{56}$$

where dM is the increase in the mutual inductance between the two circuits when one circuit is displaced a distance dx with respect to the other, the distance dx being measured in the direction in which it is wished to find the component of the force. In this formula i_1 and i_2 are both to be considered as positive when they link the mutual flux in the same direction. From this relation follows the well known fact that two coils, or conductors, carrying currents which are in the *same* direction *attract* each other, since when one coil is moved *toward* the other their mutual inductance *increases*.

The mutual inductance of two long parallel wires when these are far apart relative to their diameters is

$$M = 2\,l \log_e \frac{2\,l}{D},$$

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where D is their distance apart and l their length, both in centimeters (*see article on Inductance and Inductive Reactance*). Hence, from equation (56) the force exerted by one of two parallel wires on the other is

$$f = \frac{2\, i_1 i_2}{D} \quad \text{dynes per centimeter} \tag{56a}$$

and is at right angles to the wire and in the plane formed by two. This formula is also approximately true for parallel conductors of any cross-section, provided they are far apart relative to the greatest dimension of their cross-section. When this condition does not hold, e.g., in the case of parallel bus-bars close to each other, the formula for the force is much more complex (*see Dwight, H. B., Repulsion between Strap Conductors, Elec. World, 70, p. 522, Sept. 15, 1917*).

Forces on Magnetic Bodies in a Magnetic Field. — In general the reluctance of a magnetic field to the flux set up by a given magnetomotive force depends upon the relative positions of the various magnetic bodies in the field with respect to one another and with respect to the electric circuit producing this m.m.f. When any magnetic body in the field is displaced the total reluctance will in general be changed due chiefly to the change in the dimensions of the air portion of the circuit. From equations (48a) and (51) it can be shown that the force acting on any magnetic body in the field is in the direction of the flux lines threading it and has the value

$$f = -\frac{1}{8\,\pi}\phi^2 \frac{d\mathcal{R}}{dx}, \tag{57}$$

provided the magnetomotive force remains constant; where ϕ represents the total flux threading the body and $d\mathcal{R}$ represents the *increase* in the reluctance of the magnetic circuit corresponding to a displacement dx of the body in the direction of the flux lines. The minus sign in this formula indicates that the force is always in the direction in which a motion of the body would *decrease* the reluctance of the circuit. In deducing this expression it is assumed that the permeability of each body in the field is constant. It can also be shown that to a close approximation the same formula holds for actual magnetic bodies, for which the permeability is not a constant.

The above relation accounts for the attraction of one magnet for another when their unlike poles are nearer each other than their like poles, and the repulsion of two magnets when their like poles are nearer than their unlike poles. It also accounts for the attraction of iron or other paramagnetic substance by *either* pole of a magnet or by either "face" of an electric circuit, and the repulsion of a diamagnetic substance by either pole of a magnet or either face of an electric circuit; see article on *Electromagnets*.

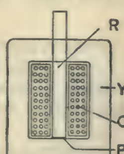

Fig. 24.

As a special application of equation (56) consider the electromagnet shown in Fig. 24, which is one form of "permeameter" (q.v.). R is a rod of iron the flat end P of which makes contact with the yoke Y. This rod passes through a hole in the top of the yoke. C is a magnetizing coil. The flux lines through the rod are uniformly distributed and pass perpendicularly into the yoke at the joint P. Let A be the area of the end of the rod and let B be the flux density at this area. When the rod is raised a distance dx, so that an air gap of length dx is formed at P, an increase in the reluctance of the magnetic circuit is produced due to

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the formation of an air gap at P, and a decrease in reluctance is produced by the shortening of the iron part of the circuit, i.e., by the reluctance of a length dx of the rod. Due to the high permeability of the iron compared with that of the air, the decrease in the reluctance of the iron part of the circuit may be neglected in comparison with the increase in reluctance due to the formation of the air gap. The net increase in reluctance may then be taken as the reluctance of this air gap of length dx, that is $d\mathcal{R} = dx/A$, since the permeability of air is unity. Whence, from equation (56), the force required to raise the rod is

$$f = \frac{1}{8\pi}\frac{\phi^2}{A} = \frac{B^2A}{8\pi}. \tag{56a}$$

That is, the tractive force per unit area between the rod and the yoke is proportional to the square of the flux density in the rod.

HYPOTHESIS REGARDING THE NATURE OF MAGNETISM. — The peculiar properties possessed by a magnetic needle or other magnet may be accounted for by assuming that at least some of the molecular charges in a magnetic substance have an orbital motion without friction, forming a kind of "molecular solar system." The magnetization of such a substance is then due to the setting of the molecular currents in planes more or less parallel to the plane of the magnetizing current. In the case of a permanent magnet these molecular currents retain their parallel or "polarized" condition even when the magnetizing current is removed. For example, when a needle is magnetized by a current in a coil placed around it, the molecular currents are set at right angles to the axis of the needle. A magnetic needle placed in a magnetic field, therefore, tends to set itself so that each molecular current embraces maximum flux and hence "points" in the direction of this flux.

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ELECTROCHEMICAL PROCESSES, INDUSTRIAL. — (*See also Electrochemistry, Principles of; Furnaces, Electric.*) The principal industrial electrochemical processes are the following :

Electroplating,
Galvanoplasty, including electrotyping,
Electrolytic refining of metals,
Electrolytic winning of metals,
Electrolytic oxidation and reduction,
Electrolysis of sodium and potassium in chloride solutions,
Electrolysis of water,
Electric-furnace processes (*see* ***Furnaces, Electric***),
Production of ozone.

Some of the more important of these various processes are briefly described below.

ELECTROPLATING. — Electroplating consists in covering a conducting surface (usually metallic) with a thin, smooth, compact, well-adhering layer of metal, by depositing this metal electrolytically from an aqueous solution of one of its salts. The anode of the electroplating vat consists of rods or plates of the same metal as that of the salt in solution and is connected to the positive terminal of the source of electricity. The object to be plated forms the cathode, and is connected to the negative terminal. The anode dissolves approximately to the same extent that the cathode gains, so that the amount of the metal ions in the bath remains nearly constant.

Suspension of Objects to be Plated. — The cathodes are always suspended in the bath between two rows of anodes, so that they will be plated uniformly on both sides. When the cathode is of irregular shape, or very large, it must be turned frequently during the plating in order to get a uniform deposit. The cathodes and anodes are suspended by copper wires from horizontal metallic tubes, the ends of which rest on the edge of the plating vat. The metallic tubes are permanently connected to the source of the electricity, so that as soon as the cathodes are suspended in the bath, electrolysis begins. Small objects, such as tacks, pins and screws, are suspended in the vat in a wire basket, which is, of course, plated simultaneously. To get a uniform plated surface the objects should be well shaken during electrolysis. The anodes are removed from the bath only when they are nearly used up and have to be replaced.

Construction of Vats. — Large plating vats are made of wood lined with some specially prepared substance resembling pitch, or with lead. Small tanks for silver or gold plating are usually porcelain lined.

Voltage and Current Density. — Electroplating tanks are always connected in parallel, so that they will be electrically independent of each other.

Low-voltage generators of from 5 to 6 volts are, therefore, used in plating, and each tank must be connected directly to the generator through a regulating rheostat in order to regulate the voltage.

The proper current density in any plating process is that density at which a good deposit is formed. This may vary within certain limits for a given solution and temperature; it is a function of the temperature and the nature of the solution.

Washing and Pickling. — In order to make the metal adhere well to the surface to be plated, the surface must be smooth and perfectly clean. It is first polished, and the grease is then removed by dipping it into a hot alkaline bath containing 10 per cent, by weight, of sodium carbonate or sodium hydrate.

After washing off the alkali, the object is dipped into a bath called a "pickle," the purpose of which is to remove any oxide that may have been produced by the alkali, and to give a bright surface. The pickle is then washed off with water and the object is suspended immediately in the electroplating vat.

Pickling Solutions. — The pickle varies with the nature of the metal treated. Cast iron and wrought iron are pickled in a solution consisting of 15 parts, by weight, of water to 1 part of concentrated sulphuric acid. A suitable pickle for zinc is dilute sulphuric or hydrochloric acid. Copper, brass, bronze and German silver are pickled first in a bath consisting of 200 parts, by weight, of nitric acid of specific gravity 1.33, 1 part of common salt and 1 part of lampblack. The lampblack is intended to form some nitrous acid from the nitric acid. The object is then washed in boiling water and is immersed in a "bright dipping bath," to give a bright surface. This bath consists of 75 parts, by weight, of nitric acid of specific gravity 1.38, 100 parts of concentrated sulphuric acid and 1 part of common salt.

After the plating is finished, the object is removed from the plating bath, washed in hot water and placed in warm sawdust to dry.

Plating by Dipping. — A thin film of metal may be deposited on a metal by dipping it into a solution of a salt of a metal which is electronegative (*see Electrochemistry, Principles of*) with respect to the metal to be plated, e.g., by dipping iron into a copper-sulphate solution. A small amount of iron dissolves and an equivalent amount of copper is deposited on the remaining iron. Of course, only a thin film can be produced in this way, for, as soon as the iron is covered with copper, the action ceases. No external electric current is needed in this process.

Plating by Contact. — When the metal to be plated is electronegative with respect to the metal to be deposited on it, the electro-deposition can be obtained by connecting the former to a zinc rod. In this case, the solution must be of a complex salt, in order to reduce the deposition of the metal in solution on the zinc itself. For example, silver may be deposited on copper by connecting a zinc rod or plate to the copper object by a wire and dipping both into a potassium-silver-cyanide solution; the zinc dissolves and silver is deposited on the copper, while some silver is also deposited on the zinc. No external current is needed in this process.

Nickel Plating.— The solution ordinarily used consists of 50 parts, by weight, of the double nickel-ammonium sulphate, $NiSO_4 \cdot (NH_4)_2SO_4 \cdot 6\ H_2O$ with from 25 to 50 parts of ammonium sulphate to 1000 parts of water. The anodes are nickel. The solution is made acid enough to redden litmus slightly, either by addition of a small amount of sulphuric acid or one-half per cent of citric acid. The proper current density on the cathode is about 0.6 ampere per square decimeter (5.5 amperes per square foot) of exposed surface, which requires about 2 volts. The surface should be perceptibly coated with nickel in two or three minutes, and a few bubbles of hydrogen are liberated continuously. If the current is too weak, the surface becomes discolored, and if too strong hydrogen is evolved more rapidly and the surface turns dark.

Iron is sometimes copper plated before it is nickel plated, but this is not necessary, for nickel adheres to iron perfectly well if the surface has been properly cleaned.

A nickel-chloride solution gives good results in plating any metal except iron. Iron always eventually rusts if plated in a chloride bath.

Copper Plating. — The metals on which copper is usually plated, such as zinc, iron and tin, are more electropositive than copper. On dipping any of these metals into an acid copper-sulphate bath, they would become covered with a layer of copper, which in some cases is spongy and does not adhere

well. In order to reduce the velocity with which this reaction takes place a solution of the double cyanide of copper and potassium, $KCu(CN)_2$, is used. This can be made by dissolving cuprous cyanide in potassium cyanide to form a 3 to 8 per cent solution,* with an excess of 0.2 per cent of potassium cyanide. This bath is generally heated to 50° to 60° C. The proper current density at the cathode is about 0.5 ampere per square decimeter (4.6 amperes per square foot) which requires about 3 volts at room temperature. For further details, see Circular of the Bureau of Standards, No. 52.

Surfaces that have already received a thin coating of copper in a cyanide bath are sometimes thickened in an acid copper-sulphate bath. The cyanide must be washed off on transferring to the sulphate bath. A sulphate bath may be made by dissolving 150 grams of copper sulphate $CuSO_4 \cdot 5\ H_2O$ and 50 grams of concentrated sulphuric acid in 1 liter of water. The proper current density is about 0.7 ampere per square decimeter (6.5 amperes per square foot) which requires less than 1 volt.

Small springs are very much weakened by copper plating in a cyanide bath, and are very likely to break while suspended, slightly stretched, in the bath during plating. This may be due to absorbed hydrogen. See *Met. and Chem. Eng.*, Vol. 16, p. 83, 1917; *Trans. Am. Electrochem. Soc.*, Vol. 32, p. 247, 1917; and Vol. 33, p. 169, 1918.

Zinc Plating. — Electrolytically deposited zinc is of a dull color and is not as pleasing in appearance as layers obtained by dipping in melted zinc, but electrolytic zinc has been shown to protect iron better for a given thickness of deposit than a coating made from melted zinc. (*Burgess, Electrochem. and Met. Ind., Vol. 7, p. 17, 1905.*) A suitable solution consists of 200 grams of zinc sulphate, $ZnSO_4 \cdot 7\ H_2O$, 40 grams of sodium sulphate, $Na_2SO_4 \cdot 10H_2O$ and 10 grams of zinc chloride per liter, slightly acidified with sulphuric acid. The current density in the cathode is from 0.5 to 2 amperes per square decimeter (4.6 to 18 amperes per square foot) which requires from about 1 to 2.5 volts. Zinc anodes are used. A little more zinc is dissolved than is deposited, due to the free acid. The acid must be replaced as it is used up. The resistance may be reduced by warming the bath to 40° or 45° C.

Brass Plating. — If an acid solution of zinc and of copper sulphates were electrolyzed, only copper would be deposited. In a cyanide solution of zinc and copper, however, these metals are deposited simultaneously in the form of an alloy. (*Spitzer. Zeit. f. Electrochemie, Vol. 11, p. 367, 1905.*) The copper is deposited more easily than the zinc, so that at a low current density, 0.1 ampere per square decimeter (0.93 ampere per square foot), only a small amount of zinc is deposited, but at 0.3 ampere per square decimeter (2.8 amperes per square foot), the deposit contains only 80 per cent of copper. Increasing the current density changes the composition of the deposit only slightly.

A suitable bath for brass plating is made by substituting zinc cyanide for half of the copper cyanide in the solution given above for plating. Brass anodes are used. Other brass baths that have been found to give good deposits are the following:

1 liter water,
14 grams sodium carbonate, dried,
20 grams sodium sulphate, dried,
20 grams double cyanide of potassium and copper,
20 grams monosodium sulphate,
20 grams double cyanide of potassium and zinc,
1 gram potassium cyanide,
2 grams ammonium chloride.

*An *n* per cent solution of a substance contains *n* parts, by weight, of that substance in 100 parts, by weight, of the solution.

With the electrodes 15 centimeters apart, current density 0.3 ampere per square decimeter (2.78 amperes per square foot) about 3 volts are required.

1 liter water,
15 grams double cyanide of potassium and copper, crystallized,
16.5 grams double cyanide of potassium and zinc,
25.0 grams sodium sulphite,
2.0 grams potassium cyanide, 98 per cent.

Current density 0.3 ampere per square decimeter (2.78 amperes per square foot) requiring 3 volts when the electrodes are 10 centimeters apart. (*Schlötter, Galvanostegie, 1. Teil, p. 238, 1910.*)

Silver Plating. — The double cyanide of potassium and silver is universally used for silver plating on account of the smooth deposit obtained with this solution. The deposit from a nitrate comes down in the form of isolated crystals, which do not cover the surface completely. The solution contains from 1 to 5 per cent silver as potassium-silver cyanide, $KAg(CN)_2$, with 0.5 per cent of free potassium cyanide. Too much or too little free cyanide gives a bad color to the deposit. A good silver plating bath may be made up as follows:

20 grams potassium silver cyanide,
10–12 grams potassium cyanide, 99 per cent,
1 liter water.

The current density is 0.3 ampere per square decimeter, at about 1 volt. (*Schlötter, Galvanostegie, 1. Teil. p. 149, 1910.*) The anodes are silver.

Silver is deposited only on a surface of copper or copper alloy. Other metals must be copper plated before silvering. In order to make the silver adhere well, the copper surface must be amalgamated by dipping the cleaned surface in a "quicking bath," consisting of a solution of 30 grams of potassium-mercury cyanide, $KHg(CN)_2$, and 30 grams of potassium cyanide in 1 liter of water. On removal from the quicking bath, articles are washed and placed immediately in the silvering bath.

Gold Plating. — The solution used in gold plating contains from 0.35 to 1 per cent of gold as the double cyanide of gold and potassium, $KAu(CN)_2$, with twice as much free potassium cyanide. The current density on the cathode is about 0.2 ampere per square decimeter (1.9 amperes per square foot), which requires about 1.5 volts. The anodes are pure gold. The solution may be used hot or cold. The deposit from a hot solution is more dense, more uniform and of a richer color. The color of the gold deposit may be influenced by simultaneously depositing some other metal. Green gilding may be obtained by adding a little silver cyanide to the bath, until the desired tint is obtained. The solution should be cold. To give the deposit a red tint, a little copper cyanide is added to the solution.

Other Plating Processes. — Plating with the following metals is sometimes carried out: platinum, tin, lead, iron, antimony, and arsenic.

Plating on Aluminum frequently does not wear well, on account of the difficulty in getting a perfectly clean aluminum surface on which to plate. This is due to the rapidity with which a thin invisible film of oxide or hydroxide forms on aluminum when exposed to the air or to any solution. One method of overcoming this difficulty is to immerse the aluminum in a solution of potassium hydrate until hydrogen is evolved, and then dip without previous washing in a potassium-silver-cyanide solution. The aluminum is immediately covered with a layer of silver. It is still better to amalgamate by dipping into a 0.5 per cent solution of mercuric chloride immediately after treating with hydrate. The chloride is rinsed off, and the object again treated with potassium hydrate and

then immediately suspended in the silvering bath. (*Langbein, Electrodeposition of Metals, 4th ed., p. 409, 1902.*)

Burgess (*Electrochem. Ind., Vol. 2, p. 85, 1904*) recommends cleaning first with dilute hydrofluoric acid, then with a mixture of 100 parts of sulphuric acid and 75 parts of nitric acid, both concentrated. After rinsing with water the surface is immediately plated with zinc, as this metal is found to adhere better than many others. Starting with the zinc surface, other metals are readily deposited.

GALVANOPLASTY. — Galvanoplasty is the art of reproducing by electrolysis articles of various kinds, or of making finished products, such as set-up type, copper tubes, etc.

Electrotyping. — The object is to make a copper plate which shall be an exact duplicate of type which has been set up ready for printing. First an impression of the type is made in wax, which is then covered with a thin layer of graphite, by dusting the fine powder over the wax surface with soft brushes. A thin layer of copper is then formed by sprinkling the surface with iron filings and pouring over the surface a solution of copper sulphate. The iron goes into solution, depositing copper on itself and on the graphite. The wax plate is then washed in water and suspended in an acid bath of copper sulphate, where copper is deposited electrolytically until a thin sheet that can be stripped from the wax has been formed. After removing the copper sheet from the wax plate, a melted lead-antimony alloy is poured on its reverse side, making a plate approximately 0.5 inch thick. This is then used for printing in place of the original type. The economy of this procedure comes in the saving of wear on the type, and the relatively small amount of type which has to be kept in stock.

The current density ranges from 1 to 2 amperes per square decimeter (0.9 to 1.8 amperes per square foot) and the volts per cell from 0.75 to 1.5.

Copper tubes are made by the Elmore process by depositing copper on a conducting cylinder which rotates in an acid copper-sulphate bath. The surface must be conducting, but the copper must not stick so firmly that the cylinder cannot be slipped out of the tube when finished. In order to keep the outer surface of the tube smooth, it is frequently polished during the deposition of copper. Copper sheets may be made by making tubes of large diameter and cutting them open.

Metallic foil may be made by the electrolytic deposition of a thin metallic layer on a surface from which it can be removed.

Parabolic mirrors are made by depositing copper electrolytically on a parabolic glass surface that has been silvered, and separating the metal from the glass by warming. (*Cowper-Cowles, Electrolytisches Verfahren zur Herstellung parabolischer Spiegel, 1904.*)

ELECTROLYTIC REFINING. — The method of refining metals electrolytically is as follows. The impure metal is made the anode of an electrolytic bath, the electrolyte of which is, at the start, a solution of a pure salt of the metal to be refined, and the cathode is a sheet of the refined metal. On passing the current, metal dissolves, along with certain of the impurities. The impurities which are electronegative with respect to the principal metal dissolve; those electropositive with respect to the principal metal remain adhering to the anode. When the latter finally drop from the anode they may be dissolved by the free acid, in which case they would be precipitated again on coming in contact with the anode or the cathode. Some metals are precipitated as an insoluble salt as soon as they dissolve, and are thus removed from the further action of the current.

When it goes into solution, the principal metal is, therefore, separated from those metals which are electronegative with respect to it. When it is deposited

on the cathode, it is separated from those metals which are electropositive with respect to it. The bath thus becomes contaminated with certain of the impurities in the anode, and these would eventually be deposited on the cathode if the bath is not purified from time to time.

Anode Mud. — The metals which do not dissolve drop to the bottom of the tank forming the "anode mud." It is from this mud that the platinum, gold and silver are recovered in copper refining. For methods of working up anode mud, see Kern, *Met. and Chem. Eng.*, Vol. 9, p. 417, 1911.

Copper Refining. — The object in refining copper electrolytically is to obtain as pure copper as possible for electric conductors and to obtain the precious metals contained in the crude copper. A representative composition of crude copper anodes for American refineries is the following:

Copper	98–99.5 per cent.
Silver	0–300 oz. per ton.
Gold	0–40 oz. per ton.
Arsenic	0–2 per cent.

The refined copper is about 99.95 per cent copper. The electrolyte is a solution of $CuSO_4$ and H_2SO_4 containing from 4 to 10 per cent of free acid and from 12 to 20 per cent of $CuSO_4$. The current density ranges from 0.43 to 4.8 amperes per square decimeter (4 to 45 amperes per square foot) of cathode surface and the volts per cell range from 0.1 to 0.3. (*Ulke, Die Elektrolytische Raffination des Kupfers, p. 42, 1904.*) The electrolyte circulates slowly from tank to tank. The cathodes, called *starting sheets*, are thin sheets of refined copper.

Multiple and Series Systems of Arranging the Electrodes. — The tanks for holding the solution and electrodes are made of wood and are frequently lined with lead. There are two methods of arranging the electrodes. In the "multiple system" all the cathodes of one tank are connected, and likewise the anodes, the cathodes having an anode on each side, as shown in Fig. 1.

Fig. 1. Multiple System

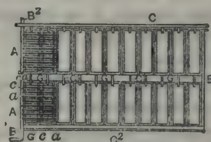

Fig. 2. Walker Multiple System

Fig. 3. Series System

The most economical method as regards the use of copper of connecting tanks in the multiple system with each other is that devised by Walker (*U. S. Pat. 687, 800, 1901*). In this system the current flows from tank to tank without being collected in a single bus bar in this passage. The *cathodes* of the first tank rest on a conducting bar G (Fig. 2) on which the *anodes* of the second tank also rest. The figure shows two series of tanks with a leading in bus bar B, bus bar B' connecting the two series, and leading out bus bar B^2. A more recent improvement is to have the cathodes of a tank make direct contact with the anodes of the adjacent tank.

In the Hayden or "series system," the electrodes are arranged as shown in Fig. 3. The impure copper plates are suspended in the solution at equal distances apart, only the end ones being connected to the dynamo. The current enters at the electrode A, which dissolves, and on flowing through the tank passes through the intermediate plates. Pure copper is deposited on the sides of the plates facing A, and dissolved from the opposite surfaces facing B. Some

conducting preparation is painted over the sides of the intermediate electrodes facing *A*, so that the copper deposited can be separated from the impure copper. When a certain amount of the impure copper has been dissolved, the electrodes are removed, the pure copper is separated from the impure, and the latter is melted and cast into new electrodes. This method is in use at the large plant of the Nichols Copper Company, Laurel Hill, New York, and at the Baltimore Copper Smelting & Rolling Co., Canton, Md. All others use the multiple system.

Comparison of the Two Systems. — The energy required in the series system is about 70 per cent of that required in the multiple system. Contacts give more trouble in the multiple than in the series system. The electrodes in the series system cost more to prepare than in the multiple system. The investment in a series plant of given capacity is less than in a multiple plant. Since lead-lined tanks cannot be used in the series system, the maintenance expense of a series plant is greater than that of a multiple plant.

Nickel Refining. — Nickel may be refined by using a weakly acid solution of nickel chloride or nickel sulphate. To get thick deposits the solution must be heated to from 50° to 90° C. Nickel and copper may be separated by the David H. Brown process, in which nickel is separated from an alloy of nickel and copper by depositing the nickel electrolytically. Very pure nickel required for special purposes is refined electrolytically by the Orford Copper Company, by a secret process. Foerster obtained good results with a current density of 0.5 to 2.5 amperes per square decimeter (4.5 to 23 amperes per square foot) between 50° C. and 90° C. The neutral nickel-sulphate solution contained 30 grams of nickel per liter. Günther obtained good deposits at 60° to 65° C., with 4 to 5 amperes per square decimeter (37 to 45 amperes per square foot) at 3.5 to 4 volts, with nickel sulphate baths.

Silver Refining. — Two cases arise, (1) the separation of silver from copper in an alloy consisting mainly of these two metals, and (2) the separation of silver from relatively small amounts of gold and platinum.

Dietzel Process. — One method of separating silver and copper is the Dietzel process. This consists in dissolving both of the metals as anode in a weakly acid solution of copper nitrate. The solution is then transferred to another vessel and the silver is precipitated by metallic copper, following which the copper is deposited electrolytically.

The current density is about 1.5 amperes per square decimeter (14 amperes per square foot) of cathode surface, and the voltage per cell from 2.5 to 3.5 volts.

Moebius Process. — Silver is separated from small amounts of other metals by the Moebius process. In this process the anodes consist of the impure silver, the cathodes of thin sheet silver, and the electrolyte is a slightly acid dilute silver-nitrate solution. In the earlier type of apparatus the cathodes were stationary; in the later type the cathode is a rotating sheet of silver. In both the pure silver is scraped off as crystals.

The current density is about 3 amperes per square decimeter (28 amperes per square foot) of cathode surface, and the voltage per cell from 1.4 to 1.5 volts Another type is the Balbach-Thum cell (*Met. and Chem. Eng., Vol. 9, p. 444, 1911*).

Gold refining is carried out in a slightly acid solution of gold chloride. Gold anodes do not dissolve in a solution of gold chloride $AuCl_3$, or of chloroauric acid, $HAuCl_4$, but the chlorine liberated comes off in the gaseous form. If some free alkali chloride or hydrochloric acid is present, the gold dissolves. The resulting gold is never less than 999.8 fine, and frequently is 1000 fine.

The baths contain about 3 per cent of free hydrochloric acid with 30 to 40 grams of gold as chloroauric acid per liter. The bath is heated to 60° or 70° C.,

and the current density is about 10 amperes per square decimeter (93 amperes per square foot) at about 1 volt.

Lead refining is carried out by the Betts process. The electrolyte is a solution of lead fluosilicate ($PbSiF_6$) containing 60 to 70 grams of lead and 120 to 130 grams of SiF_6 per liter, and 0.1 per cent of glue. The object in refining lead is to recover the copper, antimony, bismuth, gold and silver. Tin cannot be separated from the lead electrolytically, on account of its proximity to lead in the electrolytic series.

The current density ranges from 1.3 to 1.7 amperes per square decimeter (12 to 16 amperes per square foot) of cathode surface, and the voltage per cell from 0.30 to 0.38 volt.

According to Duisberg (*Journ. of Ind. and Eng. Chem. for 1912, p. 752*) iron is now refined electrolytically by the firm Langbein-Pfanhauser & Co., Leipzig. It has valuable magnetic properties, and is used in electromotors. The electrolysis is carried out at 100° to 120° C. See also Guillet, *J. Iron and Steel Inst.* Vol. 90, pp. 66–80, 1914, and Story, *Tr. Am. Electrochem. Soc.*, Vol. 29, pp. 357–367, 1916.

Refining of Other Metals. — A number of other metals may be refined electrolytically, such as mercury, tin, bismuth, and antimony, but these are relatively unimportant.

ELECTROLYTIC WINNING OF METALS. — The attempts that have been made to win metals directly from their ores or from matter are based on the same principle as that underlying metal refining. These attempts have not been successful until recently, however, on account of the large amount of impurity that gets into the bath and on account of mechanical difficulties. The best known of such attempts, are the following.

Marchese Process. — In the Marchese process it was attempted to obtain pure copper from a matte containing principally copper, lead, iron and sulphur, by electrolyzing this matte as an anode in a sulphate bath. The process seemed a success at first, but finally failed.

Siemens and Halske Process. — The Siemens and Halske process for extracting copper consists in electrolyzing a solution of iron sulphate and copper sulphate in a cell in which the anode and cathode are separated by a linen diaphragm. The anode was carbon; the cathode pure copper. In the cathode compartment the solution loses copper by deposition on the cathode. The solution then circulates to the anode compartment, where the ferrous sulphate is oxidized to ferric sulphate. See below, under *Electrolytic Oxidation*. From here the solution circulates to another vat where fresh ore, consisting of copper pyrites ($Cu_2SFe_2S_3$) that has been roasted so as to change the iron sulphide to oxide, is treated. The ferric sulphate dissolves the cuprous sulphide as follows:

$$Cu_2S + 2\ Fe_2(SO_4)_3 = 2\ CuSO_4 + 4\ FeSO_4 + S.$$

This solution, enriched in copper, is then conveyed to the cathode compartment, and the cycle is completed. This process also failed when tried on a commercial scale.

Hoepfner Process. — In the Hoepfner process the copper ore, consisting of copper sulphide, is dissolved in a cupric chloride solution containing a relatively large amount of sodium chloride. The reaction is as follows:

$$Cu_2S + 2\ CuCl_2 = 4CuCl + S.$$

The cuprous chloride, insoluble in water, is held in solution by the sodium chloride. The anode and cathode of the electrolytic cell are separated by a diaphragm. The solution first circulates to the cathode compartment, where part

of the copper is deposited, and then to the anode, where the cuprous chloride is oxidized to cupric chloride. The solution from the anode then circulates to fresh ore.

Copper Hydrometallurgy at Chuquicamata. — A process is now being perfected by the Chile Exploration Company for winning copper directly from the ore of the Chuquicamata copper mine, in Chile. The ore is dissolved in dilute sulphuric acid in leaching tanks. The solution is then conducted to the electrolyzing tanks where the copper is deposited, using insoluble anodes and copper cathode starting sheets. Originally fused magnetite anodes were used, but these were later replaced by *duriron* (ferro-silicon containing 12 per cent silicon). The electrolytic refinery will have a capacity of 335,000 pounds of copper per day. (*E. A. Cappelen Smith, Met. and Chem. Eng., 1914, Vol. 12, pp. 291–294.*

Salom Process. — Lead was at one time extracted from galena (PbS) by the Salom process, consisting in electrolyzing the powdered galena as cathode in sulphuric acid. The hydrogen deposited on the galena combines with the sulphur, forming hydrogen sulphide and leaving the lead as lead sponge. This process was given up, partly at least, on account of the poisonous action of the hydrogen sulphide. (*Trans Am. Electroch. Soc. Vol. 1, p. 87, 1902; Vol. 4, p. 101, 1903.*)

Electrolytic Zinc. — Within recent years a large quantity of high grade electrolytic zinc has been produced in the United States and Canada, by a process similar to that used for copper at Chuquicamata. Sulphate solutions are used, with lead anodes and aluminum cathodes, from which the zinc is stripped in thin sheets. The solution must be free from impurities in order to get good deposits. See Hansen, *Bull Am. Inst. Mining Eng.*, Vol. 135, p. 615, 1919; T. French, *Trans. Am. Electroch. Soc.*, Vol. 32, pp. 321–328, 1917; *Met. and Chem. Eng.*, Vol. 18, pp. 549–550, 1918.

Detinning Tin Scrap and Old Cans. — The two principal methods of recovering tin from tin scrap and old tin cans are due to Goldschmidt. The first consists in electrolytically dissolving the tin and depositing it in the metallic state; the second method, which however is not electrolytic, converts the tin into tin tetrachloride by treating with dry chlorine.

In the electrolytic method the solution consists of a sodium hydrate solution, containing a certain amount of carbonate, absorbed from the air, and sodium stannate. The best concentrations are: 10 to 12 per cent alkalinity, not over 7 per cent free alkali; carbonate not over 3 per cent; stannate not over 5 per cent. Temperature, 60° to 70° C. About one-tenth of the solution must be replaced every week.

The tin scrap is suspended in the solution in baskets (39 by 35 by 18 inches) of perforated sheet iron, containing 100 to 130 pounds of scrap. Old cans are cleaned, compressed and then cut up. The cathodes are sheet iron 39 by 35 inches. The tin dissolves from the anode and is deposited as spongy tin on the cathode. The scrap is left in the bath for three hours. The spongy tin is compressed by a hydraulic press into cylinders weighing six pounds; these are subsequently melted in a furnace with sealed tubes. This sponge contains 50 per cent tin and 50 per cent ash. The ash is subsequently reduced with carbon in an open-hearth furnace, yielding 70 per cent of the tin.

Very careful working is necessary to remove the tin so completely that the iron can be sold to open-hearth plants. Even then the iron contains 0.1 to 0.5 per cent tin.

The cathode current density is never over 0.75 ampere per square decimeter (7 amperes per square foot), at 2 to 3 volts per cell. (*Electrochem. and Met. Ind., Vol. 7, p. 79, 1909; Met. and Chem. Eng., Vol. 10, p. 202, 1912.*)

ELECTROLYTIC OXIDATION AND REDUCTION is frequently carried out on substances in solution. One of the advantages of the electrolytic method is that no other substance has to be added to the solution. In carrying out an electrolytic oxidation or reduction a porous diaphragm is used to separate the anode and cathode compartments. The substance to be reduced is placed in the cathode compartment, where the whole or a part of the hydrogen that would be evolved by the current while in the nascent state, acts on the substance in question. The intensity of the reduction may be varied by varying the potential difference between the solution and the cathode. This is accomplished either by increasing the current density on the cathode, or by making the cathode of different metals on which the overvoltage (*see Electrochemistry, Principles of*) is different. It frequently happens that the metal composing the cathode has a marked accelerating effect on the velocity of the reduction. Thus the reduction may be much more complete on one metal than on another, though the potential difference between the metals and the solution is the same in both cases. The same considerations apply to oxidation on the anode.

Typical Reduction and Oxidation Processes. — Examples where electrolytic reduction has been found useful are: in the manufacture of white lead by corroding a lead anode in a suitable solvent, e.g., the Luckow process; in the preparation of the lower salts of vanadium, molybdenum and titanium. Examples where electrolytic oxidation has been employed are: in the oxidation of the lower cerium sulphate, $Ce(SO_4)$, to $Ce(SO_4)_2$, the latter being useful as an oxidizing agent; in oxydizing potassium manganate, K_2MnO_4, to permanganate, $KMnO_4$; in the oxidation of potassium ferrocyanide, $K_4Fe(CN)_6$, to the ferricyanide, $K_3Fe(CN)_6$; and of chromium sulphate to a chromate.

Sodium permanganate has been made on a semi-commercial scale by electrolyzing ferro-manganese anodes in sodium carbonate or hydrate solutions (*Trans. Am. Electrochem. Soc., Vol. 35, pp. 371–384, 1919*). Potassium permanganate has been made in the same way (*Chem. and Met. Eng., Vol. 21, pp. 680–681, 1919*).

ELECTROLYSIS OF SODIUM OR POTASSIUM CHLORIDE. — The electrolysis of sodium or potassium chloride may yield several different products, depending on the kind of cell employed. In the following discussion the reactions for sodium choride are given; these reactions also apply to potassium chloride when K is substituted for Na.

Production of Electrolytic Bleach. — If there is no diaphragm and the solution is kept cool, the chlorine and hydrate react on each other to form principally sodium hypochlorite or "electrolytic bleach," according to the equation

$$2\,NaOH + Cl_2 = NaOCl + NaCl + H_2O.$$

As soon as hypochlorite is formed, it begins to change to chlorate, but at first not as rapidly as it is formed. As the concentration of the hypochlorite increases, a larger and larger proportion changes to chlorate, until finally the concentration of the hypochlorite reaches a limit, which depends on the temperature, original salt concentration, current density, and other factors. The hypochlorite then changes to chlorate as rapidly as it is formed.

The reactions by which hypochlorite changes to chlorate are two; (1) the action of the discharged ClO ion on water:

$$6\,ClO + 3\,H_2O = 2\,HClO_3 + 4\,HCl + 1\tfrac{1}{2}\,O_2,$$

and (2) in a slightly acid solution:

$$2\,HOCl + NaOCl = NaClO_3 + 2\,HCl.$$

If the solution is warmed to about 50° C. the conditions are so changed that the limiting concentration of hypochlorite is much lower than when cold. Sodium and potassium hypochlorites are used only in solution, but the chlorates are crystallized out.

By further electrolysis of a cooled sodium (or potassium) chlorate solution, perchlorate, $NaClO_4$, is produced by the action of the liberated ClO_3 ion on water:

$$2\ ClO_3 + H_2O = HClO_4 + HClO_3$$

and

$$HClO_4 + NaOH = NaClO_4 + H_2O.$$

In most of the cells used for producing hypochlorite, a vessel is divided into a number of narrow compartments by bipolar or intermediate electrodes, consisting of carbon plates (Hass and Oettel) or platinum-iridium wire wound on glass plates (the Kellner cell). The solution is cooled by circulation. There is no essential difference between a hypochlorite and a chlorate cell, except that the latter is used at a higher temperature. (*See numerous volumes in the Engelhardt Monographs.*)

Production of Chlorine and Caustic. — If the anode and cathode are separated by a diaphragm of some kind, sodium hydrate is produced at the cathode by the action of the alkali metal on water:

$$2\ Na + 2\ H_2O = 2\ NaOH + H_2,$$

while at the anode chlorine is set free, which may be used to make bleaching powder.

Various devices are used to separate the hydrate and chlorine in cells intended for these products. In the McDonald cell (*Electrochem. Ind., Vol. 1, page 387, 1903*) and others a porous diaphragm is used. In the Hargreaves-Bird cell (*U. S. Pat. 655,343, 1900; 596,157, 1897*), the porous diaphragm is supported by a heavy copper gauge cathode, which is wetted only by the solution percolating through the diaphragm. In the Townsend cell (*Electrochem. and Met. Ind., Vol. 5, page 209, 1907*) the cathode compartment is filled with kerosene oil. During electrolysis the solution in the anode compartment percolates through the porous diaphragm and the perforated cathode, and on coming in contact with the oil forms drops and sinks to the bottom of the cathode compartment where it is collected. In the "bell process" the anode is inside an inverted nonconducting, nonporous bell, and the cathode is a conducting ring outside. Two important diaphragm cells are the Allen-Moore cell and the Nelson cell (*Trans. Am. Electrochem. Soc., Vol. 35, pp. 239–249, 1919*).

In the Castner cell the compartments are separated by a mercury diaphragm, acting as an intermediate electrode; and in the Solvay and the Whiting cells a mercury cathode is used, which, when charged with sodium in the electrolytic cell, is decomposed by water in a different compartment. The sodium amalgam reacts with water like metallic sodium. (*See the Engelhardt Monographs.*)

ELECTROLYSIS OF WATER. — The electrolysis of water is carried out on a commercial scale for the production of hydrogen and oxygen. The electrolyte is usually a 15-per cent sodium hydrate solution. A diaphragm of canvas is used to separate the hydrogen and oxygen in the Flamand cell, now in use by the International Oxygen Company at Newark, N. J. The vessel containing the electrolyte is cast iron and acts as the cathode; the anode is steel. In the Siemens and Halske apparatus the gases are separated by a metallic gauze while bubbling to the surface of the solution; the Garuti and

Pompeili apparatus is based on a similar principle. (*See Viktor Engelhardt, Die Elektrolyse des Wassers, 1902.*)

ELECTROTHERMAL PROCESSES. — Some of the more important industrial electrothermal processes and products and the various types of electric furnaces are described in the article on *Furnaces, Electric.*

OZONE (O_3) is made by the silent discharge of electricity through dry air. The electrodes are water cooled, and protected to prevent the passage of sparks. From 8000 to 50,000 volts alternating are applied, this voltage being obtained from a small step-up transformer. For the Siemens and Halske ozonizer the concentration of the ozone is about 2 or 3 grams per cubic meter of the air which passes through it, and the yield is from 18 to 37 grams per kilowatt hour. (*Electrochem. Ind., Vol. 2, p. 67, 1904.*) Milk has been sterilized by high tension discharges; one process being the Shelmerdine process (*Electrical Times, Vol. 45, pp. 502–505, 1914*).

BIBLIOGRAPHY. — In addition to the references given above, the following books should be consulted for further information regarding industrial electrochemical processes:

Arens, *Handbuch der Elektrochemie;* Askenasy, editor, *Einführung in die Technische Elektrochemie, Vol. 1, Elektrothermie;* Blount, *Practical Electrochemistry;* Engelhardt, editor *Monographien über angewandte Elektrochemie;* Foerster, *Elektrochemie Wässeriger Lösungen;* Haber, *Grundriss der Technischen Elektrochemie;* Pring, *Some Electrochemical Centres;* Richards, *Metallurgical Calculations;* Rodenhauser and Schoenawa, *Electric Furnaces in the Iron and Steel Industry* (translated by C. H. Von Baur); Billiter, *Die elektrochemischen Verfahren der chemischen Gross-Industrie;* Thompson, *Applied Electrochemistry;* Allmand, *Applied Electrochemistry.* See also the volumes of the *Metallurgical and Chemical Engineering; Transactions of the American Electrochemical Society; Zeitschrift für Elektrochemie; Transactions of the Faraday Society.* See also Bibliography in *Electrochemistry, Principles of.*

ELECTROCHEMISTRY, PRINCIPLES OF. —(*See also Electricity and Magnetism, Principles of; Elements, Chemical; Electrochemical Processes, Industrial; Furnaces, Electric.*) Electrochemistry is the science which deals with the phenomena resulting from the direct transformation of electrical into chemical energy or the converse transformation of chemical into electrical energy. By general usage the term, especially as applied to industrial processes, has come to include also those thermochemical phenomena which occur at temperatures produced in electric furnaces, although in such processes electrical energy frequently plays no other rôle than that of generating heat.

The following is a brief table of contents of this article:

DEFINITIONS. — The following terms are commonly used:

Element. — An element is a substance which cannot be decomposed into simpler substances by any known means of chemical analysis. (*See Table of Electrochemical Equivalents below, and article on Elements, Chemical.*)

Atoms and Atomic Weights. — An atom is the smallest mass of an element that can enter into chemical combination with another element or itself. The atomic weight of an element is a number which is proportional to the smallest known combining mass of the element, that of oxygen being chosen as the standard and taken as 16. The relative proportions, by mass, of the elements forming any substance are simple multiples of these numbers. For table of the atomic weights of the more common elements see below, and for a complete table see article on *Elements, Chemical.* On the atomic theory the atomic weight is proportional to the mass of the atom.

Molecules and Molecular Weight. — A molecule is the smallest mass of a substance which can exist and preserve its chemical properties. The molecular weight of an element or of a chemical compound is equal to the sum of the atomic weights of the atoms contained in the molecule, and may readily be calculated from the atomic weights when the molecular symbol of the compound is known. Thus the molecular weight of oxygen, O_2, is $2 \times 16.00 = 32.00$ as there are two atoms in the molecule. The molecular weight of silver nitrate, $AgNO_3$, is $107.88 + 14.01 + 3 \times 16.00 = 169.89$.

Radical. — A radical is a combination of two or more elements which persists as a group in chemical reactions, e.g., the SO_4 in H_2SO_4 is a radical, as exemplified by the reaction $H_2SO_4 + Zn = ZnSO_4 + H_2$.

Formula Weight. — The formula weight of an atom, radical or molecule is the sum of the atomic weights of the elements of which it is formed. For example, the formula weight of an oxygen atom, O, is 16; the formula weight

of an oxygen molecule, O_2, is 32; the formula weight of the radical NO_3 is $14.01 + 3 \times 16.00 = 62.01$.

Valency. — An adequate consideration of valency would involve a discussion beyond the limits of this article. As a simple definition, the valency of an acid or acid radical may be taken as the number of replaceable hydrogen atoms which the acid molecule contains, the valency of the hydrogen atom being always equal to unity. Thus hydrochloric acid, HCl, and nitric acid, $H(NO_3)$, are univalent acids and the elements H and Cl and the radical NO_3 are univalent; sulphuric acid, $H_2(SO_4)$, is a bivalent acid and SO_4 a bivalent radical; $H_3(PO_4)$ is a trivalent acid and PO_4 a trivalent radical, etc. Similarly, in a salt like sodium chloride, NaCl, sodium is a univalent metal, since one atom of sodium replaces one atom of hydrogen in the corresponding acid. But in barium chloride, $BaCl_2$, barium is a bivalent metal, as it replaces the hydrogen of two molecules of hydrochloric acid to form the salt. To completely describe the valency of a salt it is therefore necessary to stipulate the valency of the basic and acid constituents; thus sodium chloride, NaCl, is a uni-univalent salt; sodium sulphate, Na_2SO_4, is a uni-bivalent salt, the basic constituent being univalent and the acid constituent bivalent. Barium sulphate, $BaSO_4$, on the other hand, is a bi-bivalent salt. The valency of a number of elements varies according to the compound of which the element forms a part, e.g., copper is bivalent in cupric salts like $CuSO_4$ but univalent in cuprous salts as CuCl; iron is bivalent or trivalent according as it is a constituent of a ferrous or ferric salt respectively.

Bonds or Affinities. — An atom or radical is sometimes said to possess a number of "bonds" or "affinities" equal to its valency in the compound of which it forms a part.

Equivalent Weight — Chemical Equivalent. — The equivalent weight of an atom, ion or radical is defined as its formula weight divided by its valency. The equivalent weight of an element or compound is also referred to as its "chemical equivalent." The equivalent weight of an acid, base or salt is its molecular formula weight divided by the highest valency of either of its ions. As hydrogen exhibits no other valency than unity its equivalent weight and atomic weight are identical. Similarly, the equivalent weight and molecular weight of hydrochloric acid, HCl, are each 36.47. Copper in copper sulphate, $Cu(SO_4)$, and in cupric chloride, $CuCl_2$, has a valency of 2, hence equivalent weights of these salts are one half their molecular weights respectively.

In computations of electrochemical reactions it is frequently convenient to take as the unit of mass (or weight) of a substance, a mass in grams equal to the number expressing the atomic weight, ionic weight, molecular weight or equivalent weight of the substance. The following names have been given these units.

Gram-Atom or Gram-atomic Weight. — A number of grams of an element equal to its atomic weight, e.g., one gram-atom of oxygen is 16 grams.

Gram-Molecule or Mol. — A number of grams of a substance equal to its molecular weight, e.g., one mol of silver nitrate, $AgNO_3$, is 169.89 grams.

Gram-Equivalent. — A number of grams of an atom, radical or molecule equal to its equivalent weight, i.e., to its formula weight divided by its valency. For example, a gram-equivalent of sulphuric acid is $98.09/2 = 49.04$ grams. A gram-equivalent is usually represented by the chemical formula divided by the valency, e.g., a gram-equivalent of sulphuric acid is denoted by $\frac{1}{2} H_2SO_4$.

Electrolyte — Electrolysis. — When an electric current is passed through certain substances a chemical reaction takes place at the places where the current enters or leaves the substance. Such substances are called "electro-

lytes;" if the current effects a decomposition or chemical change in the electrolyte the process is called "electrolysis."

Electrodes — Anode and Cathode. — Electrodes are the conductors by which the current enters and leaves the electrolyte. The electrode at which the current enters the electrolyte is called the "anode," and that by which the current leaves the electrolyte is called the "cathode." The anode is therefore that electrode which is connected to the positive terminal of the generator; the cathode that connected to the negative terminal.

Ions — Anions and Cations. — The constituents of an electrolyte which are *primarily* liberated or deposited from an electrolyte at the electrodes are called "ions." The term also applies to the constituents of an electrolyte which conduct the current. (*See sections below on Dissociation Theory and Theory of Electrolytic Conduction.*) The ions liberated at the anode are called "anions," while those liberated at the cathode are called "cations." For example, when a current enters and leaves an aqueous solution of copper sulphate, $CuSO_4$, at platinum electrodes, the $CuSO_4$ is decomposed, the copper Cu being liberated from the solution and deposited on the cathode, and the SO_4 being liberated at the anode where it reacts with the water to form oxygen gas and sulphuric acid. The copper in the sulphate solution is therefore a cation and the SO_4 an anion. In general the metal atoms in salts and bases and the replaceable hydrogen atom in acids are cations, while the acid radicals (e.g., SO_4) and the basic radicals (e.g. OH in KOH) are anions.

The terms anion and cation are also used to designate specifically the negative and positive constituents of an electrolyte. For example, in dilute aqueous solution, a molecule of hydrochloric acid, HCl, is looked upon as consisting of one hydrogen cation and one chlorine anion; sulphuric acid, H_2SO_4, of two hydrogen cations and one SO_4 anion. There is much evidence to indicate that a negative charge of fixed amount is associated with each anion and a positive charge (or deficit of negative electricity) of equal amount is associated with each cation, irrespective of the chemical nature of the ion, except that the charges carried by any ion are directly proportional to its valency. The charge carried by each individual univalent ion as determined by Millikan, is $4.774 \pm 0.005 \times 10^{-10}$ electrostatic units, the charge carried by every bivalent ion is twice this amount, etc., $4.774 \pm 0.005 \times 10^{-10}$ electrostatic units of negative electricity, being the charge of an electron. Hence on the basis of the electron theory, a univalent anion as Cl has associated with it one free electron, a divalent anion as SO_4 two free electrons, while a univalent cation as H has a deficit of one electron, a divalent cation as Cu has a deficit of two electrons, etc. See *Electron Theory*.

To distinguish cations and anions from neutral atoms or molecules, small plus or minus signs are often written over the symbol, the number of such signs being equal to the valance of the ion. Thus

$\overset{+}{H}$, $\overset{+}{Na}$, $\overset{++}{Cu}$, $\overset{+++}{Fe}$ (ferric iron) represent cations; $\overset{-}{Cl}$, $\overset{-}{NO_3}$, $\overset{--}{SO_4}$, $\overset{---}{PO_4}$ anions, etc.

Ionic Weight. — The formula weight of an ion is called its ionic weight.

Gram-Ion. — A number of grams of an ion equal to its ionic weight, e.g., one gram-ion of hydrogen is 1.008 grams, and one gram-ion of SO_4 is 96.07 grams.

Electrochemical Equivalent. — The electrochemical equivalent of an ion is the mass in grams of the ion liberated or deposited by one coulomb of electricity.

Electrochemical Constant or "Faraday." — The electrochemical constant, denoted by F, and called the "Faraday," is the number of coulombs required to liberate one gram-equivalent of any ion. It is a constant for all ions, and its value, as adopted by the International Congress of Applied Chemistry in 1903, is $F = 96{,}540$ coulombs. (*See also page 452.*)

Solvent, Solute and Solution. — When a substance A "dissolves" in another substance B, the substance A which dissolves is called the "solute," and the substance B is called the "solvent." The two substances together form a "solution."

Osmotic Pressure. — The osmotic pressure of a solution may be regarded as the force in virtue of which a solute tends to diffuse from regions of higher to regions of lower concentration. It can be measured by separating the solution and solvent by means of a diaphragm which is permeable to the solvent but not to the solute. (See section below on *Theory of Solutions*.)

Specific Conductance (κ). — The specific conductance of a solution is the reciprocal of its specific resistance, i.e., it is the conductance of a column of liquid one centimeter long and one square centimeter cross section. It is expressed in reciprocal ohms, or mhos, and is denoted by κ.

Concentration (c). — The concentration (c) of a solution is the quantity of the solute contained in unit volume of the solution. Thus it may be expressed as grams per cubic centimeter or grams per liter, etc.

Equivalent Concentration (η). — The equivalent concentration of a solution is the number of gram-equivalents of solute contained in one cubic centimeter of the solution; it is denoted by η. Calling w the equivalent weight of the solute, $\eta = c/w$.

Dilution (ϕ). — The dilution of a solution is the reciprocal of its concentration, i.e., it is the number of cubic centimeters of solution in which one gram-equivalent of solute is dissolved. It is denoted by ϕ. Hence, $\phi = 1/\eta$ and $\eta = 1/\phi$.

Normal Solution. — A normal solution is a solution containing one gram-equivalent of solute per liter. For such a solution $\eta = 10^{-3}$ or $\phi = 1000$.

Equivalent Conductance (Λ). — The equivalent conductance of a solution at the dilution ϕ is the conductance which a volume (in cu. cm.) of the solution containing one gram-equivalent of the solute would have, if placed between parallel plate electrodes one centimeter apart. It is denoted by Λ_ϕ or Λ_η, according as the dilution or concentration of the solution is given. Hence $\Lambda_\phi = \phi\kappa$, or $\Lambda_\eta = \kappa/\eta$.

The dilution or concentration of a solution should always be stated in connection with its equivalent conductance, otherwise the expression is indefinite.

Heat of Reaction. — The heat of reaction is the heat energy given out or absorbed in a chemical reaction. It is usually expressed in calories; see *Heat and Heat Effects*. Ostwald recommends the use of a calorie equal to 100 times the mean gram-calorie for expressing thermochemical data and this will be adopted in the present discussion. It will be denoted in this article by "cal."

Exothermic Reactions. — Reactions in which heat is given out to the surrounding bodies.

Endothermic Reactions. — Reactions in which heat is absorbed from the surrounding bodies.

NOTATION. — The notation used in this article is, in most instances, that proposed by the Bunsen Gesellschaft and adopted by the Fifth International Congress of Applied Chemistry in Berlin, 1903. For convenience of reference the symbols most frequently used are tabulated below.

κ = specific conductance in reciprocal ohms;
η = concentration, expressed as gram-equivalents of solute per cubic centimeter solution;
ϕ = dilution, expressed as cubic centimeters of solution per gram-equivalent solute;

Λ = equivalent conductance;
Λ_∞ = equivalent conductance at infinite dilution;
γ = degree of ionization;
E = electrical potential difference;
I = current strength;
R = resistance [W adopted by International Congress]
e = single potential difference, taken positive in direction of rise of potential;*
e_h = potential difference measured against a normal hydrogen electrode;
e_c = potential difference measured against a normal calomel electrode;
R = gas constant per mol
F = the Faraday = 96,540 coulombs per gram-equivalent
T = absolute temperature in degrees centigrade.
t = centigrade temperature [Θ adopted by International Congress]

CHEMICAL EQUATIONS. — A chemical equation, such, for example, as that representing the solution of zinc in a dilute hydrochloric acid solution (Aq = water),

$$Zn + 2\,HClAq = ZnCl_2Aq + H_2,$$

is to be interpreted as follows: one gram-atom (65.37 grams) of zinc reacting with 2 mols ($2 \times 36.47 = 72.94$ grams) of hydrochloric acid in aqueous solution forms one mol ($65.37 + 2 \times 35.46 = 136.3$ grams) of zinc chloride in solution and one mol ($2 \times 1.008 = 2.016$ grams) of hydrogen gas. Since atomic weights are relative numbers, any unit of weight, as the pound or kilogram, may be substituted for gram in this statement.

If the reaction is intended to express not only the chemical change which takes place, but also the energy change involved, the "heat of the reaction" must be included, thus:

$$Zn + 2\,HClAq = ZnCl_2Aq + H_2 + 342 \text{ cal.}$$

This thermochemical or energy equation signifies that the intrinsic energy represented by the initial system, consisting of 65.37 grams of zinc and 72.94 grams of hydrochloric acid dissolved in enough water to form a dilute solution, is 342 calories (1 gram of water from 0° to 100° C.) greater than that represented by the final system, consisting of 136.3 grams of zinc chloride dissolved in water and 2.016 grams of hydrogen gas; in other words when the substances in the initial system react and form the substances in the final system, an amount of heat energy is evolved equal to 342 cals. When energy is given up by a system to the surroundings (exothermic reactions), as in the above illustration, it is regarded as positive. If a reaction is accompanied by an absorption of heat energy (endothermic reactions), i.e., if the system takes up heat and thereby tends to cool the surroundings, the sign of the heat of the reaction is taken as negative.

FARADAY'S LAWS. — The first quantitative relations between the magnitude of an electric current and its chemical effect were discovered by Faraday in 1834 and are known as Faraday's Laws. They may be stated as follows:

First Law. — The quantity of an electrolyte decomposed by an electric current is directly proportional to the total quantity of electricity which passes through the electrolytic cell; or, the rate of chemical decomposition is directly proportional to the current. The amount of decomposition is independent of the voltage and intensity of the current, size of electrodes and concentration of electrolyte, so long as the total *quantity* of electricity flowing through the

* This notation is employed to harmonize the notation in this article with that used elsewhere in this book. The Berlin Congress recommends the use of the symbol ϵ for a potential difference taken positive in the direction of the *drop* of potential. See also Bancroft, *Trans. Am. Elec. Chem. Soc.* XXXIII, 79, 1918; *Committee Reports on Signs of Potentials*, ibid. XXXIII, 85, 1918.

circuit remains constant. (These factors, however, do affect the *ultimate products* of an electrolysis.)

Second Law. — A given quantity of electricity always decomposes *equivalent* weights of different electrolytes irrespective of their nature. For example, if three electrolytic cells containing aqueous solutions of silver nitrate $AgNO_3$, copper sulphate $CuSO_4$, and cuprous chloride $CuCl$ (dissolved in sodium chloride), respectively, be connected in series and the same quantity of electricity passed through each, it will be found if a grams of silver are deposited in the first cell, b grams of copper in the second and c grams of copper in the third, that the following proportion holds between these weights: $a : b : c = \frac{107.88}{1} : \frac{63.57}{2} : \frac{63.57}{1}$, where 107.88 and 63.57 are the atomic weights of silver and copper respectively. In copper sulphate the valency of copper is 2, while in cuprous chloride its valency is 1; silver is always univalent and hence the weights a, b, c are proportional not to the atomic weights but to the *equivalent weights* of the metals in the compounds from which they are electrolyzed. It follows from this that a given current will deposit in a given time twice the weight of metal from a salt in which it is combined with a valency of 1, as from a salt in which it exists with a valency of 2. Thus, the same current deposits in a given time twice the weight of copper from a cuprous as from a cupric salt solution. In general, if the ion in a compound has a valency n, the quantity of electricity required to liberate it will be n times as great as that required to liberate the same ion from a compound in which it is univalent.

It is to be noted, since the numbers expressing the atomic weights of the elements, and hence their equivalent weights, are purely relative, that the relative weights of substances decomposed by a given quantity of electricity may be expressed in grams, kilograms, pounds or any other unit of weight. Thus, in the above illustration the quantity of electricity which will deposit $\frac{63.57}{2} = 31.79$ pounds of copper from a copper sulphate solution will deposit 107.88 pounds of silver.

Faraday's Laws are the expression of the results of direct experiment and have been tested to the limit of precision with which physical and chemical measurements are capable of being carried out at the present time. All evidence indicates that they hold rigidly, i.e., that they are exact laws of nature. In cases where exceptions have seemed to exist these have been shown to arise from secondary causes. See Kraus, *Trans. Am. Elec. Chem. Soc., 21, 119, 1912* for special cases of electronic and ionic conduction.

Value of the Electrochemical Constant or Faraday. — The determination of the number of coulombs required to deposit one gram-equivalent of an ion, i.e., the constant connecting the quantity of electricity and the mass of a substance liberated, involves two distinct investigations; first, the measurement of the number of coulombs which pass through a given electrolytic cell, and second, the determination of the amount of the chemical decomposition. The importance of the constant has enlisted the skill of a number of the ablest physicists, and its value is still being investigated at the present time.

The various methods which have been employed vary primarily in the means adopted for measuring the current. A primary instrument the constants of which can be computed from its dimensions, is essential. Tangent galvanometers and various forms of current balances or dynamometers have been constructed for this purpose. The ultimate precision of the measurements with these instruments is thus referred back to instrumental constants and to the

accuracy with which the horizontal component of the earth's field, or the value of the acceleration due to gravity, is known at the place where the research is carried out, respectively.

The amount of silver deposited from a silver nitrate solution has been adopted almost universally for the chemical part of the research. The amount of silver deposited depends to a slight extent upon whether or not the solution at the anode is allowed to diffuse to the cathode. The following résumé of the determinations of the mass of silver in milligrams deposited by one coulomb is given (*Guthe, Bulletin of the Bureau of Standards, 1905, Vol. 1, p. 363*).

TABLE I. — ELECTROCHEMICAL EQUIVALENT OF SILVER IN MILLIGRAMS PER COULOMB

Observer	Year	Found	Electrochemical equivalent corrected by		
			Richards and Heimrod	Van Dijk	Guthe
		mg.	mg.	mg.	mg.
Mascart	1884	1.1156	......	1.1155	1.1153
Fr. and W. Kohlrausch	1884	1.1183	1.1173	1.1182	1.1177
Rayleigh and Sidgwick	1884	1.1179	1.1175	1.1178	1.1176
Gray	1886	1.1183	......	......	1.1178
Koepsel	1887	1.1174	......	......	1.1169
Pellat and Potier	1890	1.1192	......	1.1191	1.1189
Patterson and Guthe	1898	1.1192	1.1175	1.1180	1.1177
Pellat and Leduc	1903	1.1195	......	1.1192	1.1190
Van Dijk and Kunst	1904	1.1182	......	1.1180	1.1178
					Mean 1.1176

If the atomic weight of silver be taken as 107.88 (latest corrected value) the value of the Faraday from the average of Guthe's corrected values is $F = \frac{107.88}{0.0011176} = 96{,}530$ coulombs. This value agrees very closely with the value $F = 96{,}540$ adopted by the International Congress of Applied Chemistry in 1903 and based on the atomic weight of silver = 107.93, and on the definition of the legal ampere as that current which in one second deposits 0.001118 gram of silver. (*See Report of International Electrical Congress, 1893; Definition legalized by the U. S. Government in 1894.*) In view of the above agreement it seems advisable to adhere to the value 96,540 until by international agreement another value shall be adopted. For a complete résumé of recent work on the value of Faraday the *Bull. Bureau of Standards*, *13*, 479, 1916, should be consulted. The Bureau of standards gives 96,503 as the most probable value, and recommends 96,500 as the best round value.

ELECTROCHEMICAL EQUIVALENTS OF THE ELEMENTS. — From the definitions of the electrochemical equivalent and the Faraday, it follows that the electrochemical equivalent of any ion, expressed in grams per coulomb (one ampere per second) is equal to

$$\frac{\text{gram-equivalent of the ion}}{96{,}540}.$$

Table II contains the values of the electrochemical equivalents of the more common elements which may be found useful in certain calculations. The

TABLE II. — ELECTROCHEMICAL EQUIVALENTS OF THE MORE IMPORTANT ELEMENTS

(Based on atomic weights of 1921, and $F = 96,540$ coulombs.)

Element	Symbol	Atomic weight	Valence	Milligrams deposited by one ampere in one second	Grams deposited by one ampere in one hour
Aluminum	Al	27.1	3	0.0935	0.3366
Antimony	Sb	120.2	3	0.4152	1.495
Arsenic	As	74.96	3	0.2589	0.9319
Barium	Ba	137.37	2	0.7115	2.562
Bismuth	Bi	208.0	4	0.5387	1.939
Bromine	Br	79.92	1	0.8279	2.981
Cadmium	Cd	112.40	2	0.5821	2.095
Calcium	Ca	40.07	2	0.2076	0.7472
Carbon	C	12.005	4	....	....
Cerium	Ce	140.25	3	0.4843	1.744
Chlorine	Cl	35.46	1	0.3673	1.322
Chromium	Cr	52.0	2	0.2694	0.9696
Chromium	Cr	52.0	3	0.1795	0.6462
Cobalt	Co	58.97	2	0.3054	1.099
Cobalt	Co	58.97	3	0.2036	0.7331
Copper	Cu	63.57	1	0.6586	2.371
Copper	Cu	63.57	2	0.3293	1.186
Fluorine	Fl	19.0	1	0.1968	0.7086
Gold	Au	197.2	1	2.043	7.353
Gold	Au	197.2	3	0.6810	2.451
Hydrogen	H	1.008	1	0.01043	0.03758
Iodine	I	126.92	1	1.313	4.733
Iron	Fe	55.84	2	0.2894	1.042
Iron	Fe	55.84	3	0.1929	0.6946
Lead	Pb	207.10	2	1.073	3.863
Lithium	Li	6.94	1	0.0725	0.261
Magnesium	Mg	24.32	2	0.1260	0.4534
Manganese	Mn	54.93	2	0.2845	1.024
Manganese	Mn	54.93	3	0.1897	0.6827
Mercury	Hg	200.6	1	2.077	7.479
Mercury	Hg	200.6	2	1.039	3.740
Nickel	Ni	58.68	2	0.3039	1.095
Nickel	Ni	58.68	3	0.2026	0.7290
Oxygen	O	16.00	2	0.08287	0.2984
Platinum	Pt	195.2	2	1.011	3.640
Platinum	Pt	195.2	4	0.5055	1.820
Potassium	K	39.10	1	0.4051	1.458

TABLE II. — *Continued*

Element	Symbol	Atomic weight	Valence	Milligrams deposited by one ampere in one second	Grams deposited by one ampere in one hour
Silver	Ag	107.88	1	1.118	4.025
Sodium	Na	23.00	1	0.2382	0.8576
Strontium	Sr	87.63	2	0.4539	1.634
Sulphur	S	32.06	..	......	......
Thallium	Tl	204.0	1	2.113	7.607
Thallium	Tl	204.0	2	1.057	3.804
Tin	Sn	118.7	4	0.3076	1.107
Titanium	Ti	48.1	4	0.1245	0.448
Tungsten	W	184.0	2	0.9550	3.431
Zinc	Zn	65.37	2	0.3386	1.219

electrochemical equivalent of any radical may be readily calculated from its formula; for example, the electrochemical equivalent of SO_4 is

$$(32.07 + 4 \times 16)/2 \times 96{,}540 = 0.0004975 \text{ gram or } 0.4975 \text{ milligram.}$$

CONDUCTIVITY OF ELECTROLYTES—Solutions. — (*See also article on Resistance and Conductance.*) The specific conductance of an electrolytic solution varies between wide limits. It depends upon the nature of the solute and of the solvent, on the temperature and on the concentration of the solution. The effect of these factors is discussed below. For numerical values see Kohlrausch and Holborn, *Leitvermögen der Electrolyte;* Landolt-Börnstein, *Tabellen; Tables Annuelles Internationales de Constantes.*

Fused Electrolytes. — Most inorganic salts conduct electrolytically when heated to a sufficiently high temperature to cause them to pass into the liquid state. For such electrolytes Faraday's, Ohm's, and Joule's Laws hold as they do in the case of solutions. The conductance increases in general from 1 to 1½ per cent degree centigrade. An exhaustive résumé of all matters relating to fused electrolytes may be found in Lorenz's *Electrolyze der Geschmolzene Salze.* On account of their enormous concentration (100 per cent) the specific conductance of fused salts is generally very high. This, together with the difficulty of obtaining a non-conducting chemically inert vessel of suitable shape to contain the salt and the difficulty of regulating high temperatures to a fraction of a degree, makes the measurement of the conductivity of fused electrolytes far more difficult than in the case of solutions. Fused silica and natural quartz crystals have been successfully used for conductivity cells. See Goodwin and Mailey, *Physical Review,* Vol. 25, p. 469, 1907; Vol. 26, p. 28, 1908; Goodwin and Kalmus, *Physical Review,* Vol. 28, p. 1, 1909; Arndt, *Zeit. für Electrochem.* Vol. 12, p. 337, 1906; Vol. 13, p. 509, 1907; Vol. 14, p. 662, 1908; Foot and Martin, *Am. Chem. J.* 41, 451, 1909; *Trans. Am. Elec. Chem. Soc.*, 21, 105, 1912; Landolt-Börnstein *Tabellen.*

VOLTAIC AND ELECTROLYTIC CELLS. — When a cell formed by two electrodes and one or more electrolytes gives out electric energy the cell is called a "voltaic" cell. All ordinary chemical batteries are voltaic cells. If across

the terminals of a cell an external electromotive force, greater than the electromotive force developed within the cell, is impressed, then a current will flow through the cell in the opposite direction, and the cell will absorb electric energy. A cell which absorbs electric energy is usually referred to as an "electrolytic cell." The cells used in electrolytic refining and similar processes are electrolytic cells. An electrolytic cell may or may not have an electromotive force on open circuit, but may develop an electromotive force in virtue of the chemical changes or changes in concentration which take place due to the current forced through it by some external source. A voltaic cell becomes an electrolytic cell when the electromotive force impressed across its terminals exceeds (and opposes) its own electromotive force.

POLARIZATION. — When an electric current passes through either a voltaic or an electrolytic cell there is, in general, developed at the electrodes of the cell as a result of the chemical actions and changes in concentration which take place, a *back* or *counter-electromotive* force, in addition to its open-circuit electromotive force (if any), and an *increase in the resistance* of the cell, in addition to the change in resistance due to the heating effect of the current. Both of these effects cause a decrease in the strength of the current through the cell. These two phenomena are said to be due to "polarization" and "transition resistance" respectively.

Even in the case where the electrolyte, as a whole, suffers no change in concentration, as in the electrolysis of copper sulphate between copper electrodes, a counter e.m.f. of polarization is produced in consequence of the difference in concentration of the copper ions in the neighborhood of the anode and cathode. Vigorous stirring of the electrolyte tends to reduce this concentration difference and the resulting polarization, but, except for very low current densities, it cannot be completely eliminated. It is for this reason, in refining processes, that energy is required to transfer the metal from anode to cathode in addition to that necessary to overcome the internal resistance of the cell.

A transition resistance results from the formation of a film of poorly conducting material over the electrode and may or may not be present according to the character of the electrolysis.

Measurement of E.M.F. of Polarization and Transition Resistance. — As both transition resistance and polarization tend to cut down the current flowing through the cell, they cannot be distinguished by this effect alone. The transition resistance may be determined by measuring the ohmic resistance of the cell before and after the passage of the current by the usual alternating-current method. The existence of an e.m.f. of polarization in a cell which normally has no open-circuit e.m.f. may be qualitatively demonstrated by short-circuiting the cell through a galvanometer immediately after breaking the primary circuit; if the cell be polarized, a current which diminishes to zero will flow through it in the reverse direction. A voltmeter, or, better, an electrometer connected across the electrolytic cell, will indicate the polarization voltage the instant after the current is broken. This voltage will diminish as the polarization disappears, the rate depending upon the current through the voltmeter and the rate of diffusion of the electrolytic products causing the polarization. Owing to the rapidity with which the e.m.f. of polarization falls off after the exciting cause is removed, special precautions must be observed in its measurement. One of the best methods is to connect up the circuit containing the applied e.m.f., and electrolytic cell with a tuning-fork interrupter and electrometer so that at the instant the battery current is broken at each vibration of the fork, the circuit containing the cell and electrometer or voltmeter is closed, and vice versa. By this arrangement, the e.m.f. of polarization is measured during the fraction of a second that the battery circuit is open,

and before it has had time to sensibly diminish. The polarizing current may also be regarded as practically constant.

It is frequently of importance to know, not the polarization of the cell as a whole, but the polarization at each electrode. This is obtained by measuring the drop in potential at each electrode against a normal hydrogen or calomel electrode while the impressed e.m.f. is acting on the electrolytic cell, or immediately after the current through the cell is interrupted.

REVERSIBLE ELECTRODES — DEPOLARIZERS. — If the chemical and thermal actions which take place at a given electrode when a given quantity of electricity passes in one direction through a cell are exactly the reverse of the actions which take place when the same quantity of electricity passes through it in the opposite direction, the electrode and the solution with which it is in contact are said to form a "reversible electrode." Such an electrode does not polarize provided the concentration of the solution is kept constant.

There are two types of reversible metal-liquid electrodes, namely:

Electrodes of the First Type, consisting of a metal in contact with a solution of one of its own salts, e.g., copper in copper sulphate, zinc in zinc sulphate, etc. The two electrodes of a Daniell cell are of this type.

Electrodes of the Second Type, consisting of a metal in contact with a solution containing one of its difficultly soluble salts and a second soluble salt of some other metal, having the same anion. The difficultly soluble salt, called the "depolarizer," must be present in excess as solid. Such an electrode is mercury in contact with a solution of zinc sulphate containing mercurous sulphate in excess as solid; this is one of the electrodes of the Clark cell.

Reversible Gas Electrode. — An electrode consisting of "platinum black" saturated with an atmosphere of hydrogen gas and dipping partially into a solution containing hydrogen ions is also a reversible electrode. This is a special form of electrode of the first type in which a gas by being occluded in platinum is made to play the rôle of a metal.

Other "gas electrodes" e.g., chlorine, may be similarly prepared.

CONTACT POTENTIALS; ELECTROMOTIVE FORCE. — The electromotive force of a voltaic cell, the back-electromotive force of an electrolytic cell, and the electromotive force of a thermocouple, are all due to differences of potential which always exist at the junction of dissimilar substances. These single potential differences are called "contact potentials," the word "difference" being understood. For a general discussion of this subject see Langmuir, *Trans. Am. Elec. Chem. Soc.*, 29, 125, 1916.

Metal-metal Potentials. — The potential at the junction of two dissimilar metals may be measured either by the Peltier effect, i.e., by the heat developed or absorbed at the junction when a known quantity of electricity is forced across it, or by the thermoelectric force set up in a circuit formed by the two metals in question when the two junctions are kept at different temperatures. In the former case the potential may be calculated from the expression $e = H/nF$ when H is the Peltier heat developed or absorbed at the junction by the passage of nF coulombs. In the latter case $e = T\frac{de}{dT}$, where $\frac{de}{dT}$ is the thermoelectric coefficient of the junction at the absolute temperature T. Both methods give results which prove that the potential difference thus arising is of the order of magnitude of a few millivolts, except in the unusual combination of antimony and bismuth where it amounts to about 0.03 volt. Table III taken from an article by Caswell, *Physical Review, 33, 401, 1911*, contains the values of the metal-metal potentials for the more common metals against copper at ordinary

temperatures. The columns headed *A* give the values as determined by the first method, columns headed *B* give the values as determined by the second method. The plus sign (+) indicates that at a junction formed by the given metal and copper the given metal is at the higher potential, the minus sign (−) that it is at a lower potential than the copper.

TABLE III. — CONTACT POTENTIALS BETWEEN COPPER AND OTHER METALS, IN MILLIVOLTS

(Temperature 15–25° C.)

Metal	LeRoux	Jahn		Edlund		Caswell		Other Observers
Against Copper	*A*	*A*	*B*	*A*	*B*	*A*	*B*	*A*
Antimony...	− 5.64	......	......	......	......	......	......	−3.06 Lecher
Iron........	− 2.93	−3.68	−3.07	− 2.96	− 3.08	......	......	......
Cadmium...	− 0.53	−0.72	−0.72	+ 0.16	+ 0.21	......	......	......
Zinc........	− 0.45	−0.68	−0.41	− 0.01	− 0.02	......	......	......
Copper.....	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
Silver.......	......	−0.48	−0.58	+ 0.03	+ 0.04	+ 0.03	+ 0.06	......
Gold.......	......	......	......	+ 0.33	+ 0.50	......	......	......
Lead.......	......	......	......	+ 0.50	+ 0.57	......	......	......
Tin........	......	......	......	+ 0.56	+ 0.82	......	......	......
Aluminium..	......	......	......	+ 0.70	+ 0.89	+ 0.70	+ 0.89	......
Platinum....	......	+0.37	+0.38	+ 1.02	+ 1.23	+ 0.85	+ 0.66	......
Palladium...	......	......	......	+ 2.17	+ 2.43	......	......	......
Nickel......	......	+5.07	+5.44	......	......	+ 6.0	+ 6.35	+6.75 Barker +8.04 Cermak
Bismuth....	+22.3	......	......	+17.7	+17.6	+16.1	+16.0	......

The contact potential difference between any two of these metals is the *algebraic* difference between the potentials of the two metals against copper; e.g., the contact potential difference between iron and platinum according to Jahn's measurements (*A*) is $+ 0.37 - (- 3.68) = 4.05$ millivolts.

The contact potential difference between any two metals is a function of the temperature, the sign of the potential for a given pair of metals sometimes reversing as the temperature rises. Although of importance in thermopiles, pyrometry and radiometry, metal-metal potentials are of little significance in considerations of the voltaic cell.

Metal-gas Potentials. — The contact potential differences between two metals should be carefully distinguished from potential differences having their origin in the so-called "Volta effect," i.e., potential differences which are manifested when two dissimilar metals in metallic contact are separated by an air or other gaseous gap. The two phenomena are quite distinct, there being no known relation between them.

	Volts
Zinc — air — lead	0.210
Lead — air — tin..............	0.069
Tin — air — iron..............	0.313
Iron — air — copper..........	0.146
Copper — air — platinum......	0.238
Platinum — air — carbon......	0.113

Values of the potential difference across the air gap between combinations of some of the more common metals are given in the table at the bottom of the preceding page. There is a fall of potential from the zinc to the lead through the air. This table is taken from data by Ayrton and Perry, *Phil. Trans., Vol. 171, 1880.*

Liquid-liquid Potentials. — When two dissimilar electrolytic solutions are brought into contact the phenomenon of diffusion takes place between them until a homogeneous mixture results. At the same time a difference of potential is produced between the solutions, the magnitude of which depends upon the velocity of migration and the relative concentration of the ions taking part in the diffusion, the charge which they carry, the absolute temperature and the gas constant. Liquid junction potentials like metal-metal potentials are, in general, of small magnitude. Thus the potential difference between two solutions of potassium chloride, one solution having 10 times the concentration of the other, is only 0.0004 volt, and diminishes as the ratio of the concentrations approaches unity, being, in fact, proportional to the logarithm of this ratio. Potassium chloride is often used as an intermediate electrolyte when it is desired to reduce the liquid-liquid potentials of a voltaic combination to a minimum because of the small potential to which it gives rise. For further details of eliminating liquid-liquid contact potentials, see Ostwald-Luther's *Physico-Chemische Messungen*, pages 448–449.

Liquid potentials are greatest between acid or between alkali solutions. Thus, between two hydrochloric acid solutions whose concentrations are in the ratio 10 to 1 at a temperature of 18° C., there is a potential difference of about 0.038 volt; between two sodium-hydrate solutions under the same conditions there is a potential difference of 0.034 volt, these voltages also varying as the logarithm of the concentration ratio. For formulas, see page 482.

Metal-liquid Potentials. — From what has been said regarding the magnitude of metal-metal and liquid-liquid potentials, it follows by the process of elimination that the main seat of the electromotive force of a voltaic cell must reside at the junctions between metals and liquids. The *absolute* value of the potential difference between a metal and the solution with which it is in contact can be found only on certain assumptions regarding the nature of surface tension; this point is again referred to below. For practical purposes the value of this potential is expressed as the e.m.f. of a cell constructed with two half elements, one of which is the given metal and solution, and the other some form of "normal" or reference electrode. Two normal electrodes are in common use, namely, the calomel electrode and the hydrogen electrode.

Normal Calomel Electrode. — The normal calomel electrode consists of mercury in contact with a normal solution of potassium chloride saturated with mercurous chloride, the latter being present in excess as a solid.

Normal Hydrogen Electrode. — The normal hydrogen electrode consists of a strip of platinum coated with a thin deposit of platinum black and saturated with hydrogen gas at atmospheric pressure. The electrode is mounted partially surrounded by hydrogen gas and partially dipping into an acid solution of such a concentration that it contains 1 gram-equivalent of hydrogen ion per liter. A sulphuric-acid solution containing 2 gram-equivalents per liter sufficiently fulfills this condition. Hydrogen gas is allowed to bubble through the solution continuously.

Following the recommendation of the International Congress of Applied Chemistry in 1903, all metal-liquid potentials should hereafter be given as directly measured against either the normal calomel or the hydrogen electrode, uncorrected for liquid junction potentials.

From a normal hydrogen electrode to a normal calomel electrode there is

a rise of potential through the electrolyte of 0.283 volt, hence, if from a hydrogen electrode to any metal electrode the rise of potential is e_h, and if from a calomel electrode to any metal electrode the rise of potential is e_c, then

$$e_h = e_c + 0.283 \text{ volt.}$$

The difference between any two metal-liquid potentials remains the same irrespective of the normal electrode to which they are referred.

Specific Electrode Potentials. — On the assumption that the equivalent conductance ratio is a true measure of the degree of ionization of an electrolyte, and that the gas laws hold for solutions up to normal concentration, it is possible to calculate from electrode potential measurements against a normal electrode what the potential difference would be between an element and a solution containing ions of that element at a concentration of one gram formula weight per liter. Such potential differences (measured directly or calculated) are called specific electrode potentials. Much experimental work remains to be done before these fundamental constants are known with precision. For only a few metals are there known at present better than + 0.01 volt. The following table contains values for some of the more typical electrochemical reactions. These values are all referred to the hydrogen electrode as standard. The parenthesis ( ) signifies one electronic change which is subtracted from or added to an atom or ion. For a more complete table see the Monograph of Abegg, Auerbach and Luther, *Abhandlungen Deutsch. Bunsen Gesellschaft*, Vol. 2, No. 5, (1911) and No. 8 (1915), or Washburn's *Principle of Physical Chemistry*, p. 203.

TABLE IV. — SPECIFIC ELECTRODE POTENTIALS

Elements (electrodes) in equilibrium with aqueous solutions containing one gram formula weight of the indicated ions per liter. The electrode assumes a charge against the solution of the sign and magnitude indicated.

Electrochemical reaction	Volts	Electrochemical reaction	Volts
$Li - (-) = Li^{+}$	−3.03	$2H^{+} + 2(-) = H_2$	0.0
$K - (-) = K^{+}$	−2.93	$Cu^{++} + 2(-) = Cu$	+0.34
$Na - (-) = Na^{+}$	−2.72	$O_2 + 2H_2O + 4(-) = 4OH^{-}$	+0.41
$Mg - 2(-) = Mg^{++}$	−1.55	$I_2 + 2(-) = 2I^{-}$	+0.62
$Zn - 2(-) = Zn^{++}$	−0.76	$Fe^{+++} + (-) = Fe^{++}$	+0.74
$Fe - 2(-) = Fe^{++}$	−0.43	$Ag^{+} + (-) = Ag$	+0.80
$Cd - 2(-) = Cd^{++}$	−0.40	$Hg^{+} + (-) = Hg$	+0.80
$Co - 2(-) = Co^{++}$	−0.29	$Br_2 + 2(-) = 2Br^{-}$	+1.10
$Ni - 2(-) = Ni^{++}$	−0.22	$Cl_2 + 2(-) = 2Cl^{-}$	+1.35
$Pb - 2(-) = Pb^{++}$	−0.12	$Au^{+} + (-) = Au$	+1.5
$Sn - 2(-) = Sn^{++}$	−0.10	$F_2 + 2(-) = 2F^{-}$	+1.9
$Fe - 3(-) = Fe^{+++}$	−0.04		
$H_2 - 2(-) = 2H^{+}$	∓0.0		

Variation of Electrode Potentials with Concentration. — According to Nernst's theory of electrode potentials (*see below*) the *change* in the value of these potentials with concentration may be calculated by the following formula

$$e_1 - e_2 = \frac{0.000198\, T}{n} \log_{10} \frac{c_1}{c_2},$$

where e_1 is the potential at concentration c_1, and e_2 the potential at concentration c_2, T is the absolute temperature, and n the valence of the metal in the solution.

This formula is also justified by experiment. For example, the electrode potential of silver in a normal silver-nitrate solution at 25° C. is 0.80 referred to the normal hydrogen electrode. If the concentration of the silver ions be reduced to 1/1,000,000 its value in a normal solution, the electrode potential of silver will become

$$e_2 = 0.80 - \frac{0.000198 \times 298}{1} \log_{10}\left(\frac{1}{10^{-6}}\right)$$

$$= 0.80 - 0.0590 \times 6 \times 0.45 \text{ volt.}$$

Electrochemical Series — Nobility of the Elements. — The elements, as arranged in Table IV, constitute the "Electrochemical Series." Those elements for which e_h is negative are said to be less "noble" than hydrogen, while those for which e_h is positive are said to be more "noble" than hydrogen. The alkali metals having the greatest tendency to form ions in water stand at one end, while the "noble" metals, such as gold, platinum, palladium, having but a very slight tendency to form ions, are at the other end. The halogens which go into solution as negative instead of positive ions stand at the lower end of the series. Other series have been given which differ from the above in that the elements are not compared under similar conditions in regard to the concentration of the solution with which they are in contact. Changing the electrolyte, e.g., to potassium cyanide, alters not only the numerical values of the potentials, but may completely change the order in which certain elements occur in the series.

Any metal if placed in a solution of a salt of a metal standing below it in the series will tend to replace it; thus, zinc precipitates iron, copper, silver, etc., from their solutions but will not displace the alkali metals from solutions of their salts. Any metal standing above hydrogen will tend to displace it from an acid solution with evolution of hydrogen. Metals below hydrogen in the series will not dissolve in acid by a simple replacement of hydrogen. From a chemical standpoint the adoption of the hydrogen electrode as a standard has the advantage over the calomel standard that it divides the metals into two groups according to their behavior towards acids.

The approximate electromotive force of a battery consisting of two metals dipping into normal solutions of their respective salts may also be computed at once from the table by taking the difference of the two corresponding electrode potentials. Thus a zinc-copper cell should have an electromotive force equal approximately to $E_{zn-cu} = (e_h)_{zn} - (e_h)_{cu} = -0.76 - 0.34 = 1.10$, the rise of potential being from the zinc to the copper in the cell. As a matter of fact, this combination, the Daniell cell, has an electromotive force of approximately 1.10 volts. Other conclusions to be drawn from the table will be pointed out below.

ELECTROMOTIVE FORCE AND HEAT OF REACTION. — The electromotive force of a reversible cell maintained at constant temperature may be readily calculated from the first and second law of thermodynamics (*see Thermodynamics*). By a "reversible" cell is meant a cell which does not polarize and which operates under conditions such that changes which take place within the cell constitute a thermodynamically reversible process. (*See Thermodynamics, Principles of.*) The discussion given below also applies, with a rough degree of approximation, to most commercial cells, which, as a rule, are not strictly reversible.

Gibbs-Helmholtz Equation. — In the case of a reversible cell, such, for example, as the Daniell cell,* shown in Fig. 1, the relation between the heat of reaction (H) and the electromotive force (E) of the cell is calculated as follows.

* The special construction of this cell renders it reversible.

From Faraday's Laws, the quantity of electricity which must flow through the cell in order to deposit or liberate one gram-ion at either electrode is nF coulombs, where n is the valency of the ion of highest valency involved in the chemical reaction and F is the electrochemical constant (*see above*). Hence, the external work done by the cell (when there is no other external work than electrical work) is nFE joules, where E is the e.m.f. of the cell. If the cell is kept at the same temperature (T degrees, absolute scale) as the surrounding bodies, then the external work nFE done by the cell is the maximum external work it can do at this temperature T (*see Thermodynamics, Principles of*). Hence, from equation (8) of the article on thermodynamics,

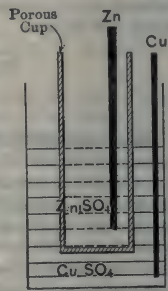

Fig. 1.

$$nFE = H' + nFT\frac{dE}{dT},$$

or

$$E = \frac{H'}{nF} + T\frac{dE}{dT},$$

where H' = the heat of reaction, in joules per gram-ion, corresponding to the chemical change which takes place.* H' gives the change in intrinsic energy corresponding to the chemical changes which take place within the cell. This can be found by an independent calorimetric measurement, by causing the reaction to take place under such conditions (*constant volume*) that heat is the only form of energy produced.

Temperature Coefficient of Electromotive Force. — From the above equation it follows that the energy developed by the cell is equal to the heat energy of the chemical reaction taking place within it plus a certain other quantity of energy equal to $nFT\,dE/dT$. This may be either positive or negative, according as the temperature coefficient of electromotive force dE/dT is plus or minus, i.e., according as the electromotive force of the cell increases or decreases with the temperature. If the electromotive force on open circuit increases with the temperature of the cell, then on closed circuit the cell will tend to cool, i.e., its temperature will fall, and its electromotive force will likewise decrease, the energy $nFT\,dE/dT$ being derived from the heat energy within the cell (or surroundings). If the electromotive force on open circuit decreases with increasing temperature of the cell, then on closed circuit the cell will tend to heat up, i.e., its temperature will rise and its electromotive force will decrease. A cell for which dE/dT is other than zero can work at constant temperature only by absorbing heat from or giving out heat to some surrounding body.

Only for the unique conditions that $dE/dT = 0$, or that $T = 0$, i.e., when the electromotive force is independent of the temperature or the cell works at temperatures approaching absolute zero, is $E = H'/nF$. By a curious chance it happens that the first of these conditions practically obtains in the case of the Daniell cell considered above. The electromotive force of the cell is nearly independent of the temperature. Its observed value is $E = 1.10$ volts, while its value computed by the above equation, putting $dE/dT = 0$, is 1.09 volts.

Thomson's Rule. — In computing the electromotive force of a battery from heats of reaction it is usually necessary to content oneself with a first

* Since 1 large mean calorie (Ostwald calorie) = 418.6 joules and $F = 96{,}540$, this may be written, putting H = heat of reaction in large mean calories,

$$E = \frac{H}{2.035\,n} + T\frac{dE}{dT}.$$

approximation and neglect the second term in the above equation, since at present few data are available from which the value of dE/dT can, a priori, be obtained. The approximate equation

$$E = \frac{H'}{nF} = \frac{H}{230.5\, n}$$

is known as Thomson's rule. (H' is in joules, H in large mean calories.)

This rule also applies approximately to most commercial cells, although these are not strictly reversible. Of course, the heat of reaction used must be that corresponding to the actual chemical changes which take place in the cell.

Nernst's Thermodynamic Theorem. — Nernst, by assuming that the values of the intrinsic energy and free energy of a system not only become equal at the absolute zero but also that they approach equality asymptotically at this point, has been able to express the electromotive force (E) of a cell explicitly in terms of the thermal properties of the substances taking place in the reaction. So far as the new formula has been tested a satisfactory agreement has been found between theory and experiment. The principles involved are of fundamental importance and are fully discussed in Nernst's *Silliman Lectures, Applications of Thermodynamics to Chemistry*. See also Nernst's *Theoretical Chemistry*.

Concentration Cells. — All commercial primary cells (*see Batteries, Primary*) are chemical cells, i.e., they derive the major part of their energy from the chemical reactions occurring within them. There is also a class of cells in which the electrical energy developed by the cells at constant temperature is derived from the heat energy of the surroundings, i.e., cells in which $H = 0$, and, therefore,

$$E = T\frac{dE}{dT}.$$

Integration of this equation shows that for such cells the electromotive force is proportional to the absolute temperature. Such cells are very simple of construction and are called concentration cells. They are of considerable importance from a theoretical point of view, and their electromotive force can be readily and exactly computed by an application of Nernst's formula for electrode potentials (*see above*). A pure concentration cell can be formed of any chemical or physical system in which the various parts tend to diffuse into each other without the evolution or absorption of heat, by constructing a cell in which the equalization of concentration, be it gaseous or liquid, is effected electrically. Numerous cases are discussed in Le Blanc's *Electrochemistry*, English translation, page 184, 1907 ed. If the two solutions forming a concentration cell when mixed have an appreciable "heat of dilution" the general Gibbs-Helmholtz formula and not the above special case is applicable.

Carbon Generator. — The problem of converting the energy liberated in the combustion of carbon to carbon dioxide directly into electric energy has not yet been practically solved. The reaction, $C + O_2 = CO_2$, liberates a very large quantity of heat, namely, 961 large calories per gram-atom of carbon consumed. Of this, only about 10 per cent is converted into electrical energy through the agency of a steam engine and dynamo. The difficulties encountered in devising a commercial carbon generator are the following: (1) the velocity of the above reaction is small, except at high temperatures; (2) it is difficult to utilize a gas as an electrode, except through the agency of a conducting medium which occludes it, such as platinum black; (3) a satisfactory high-temperature electrolyte which will not deteriorate in the presence of CO_2, is difficult to obtain; and (4) the slight tendency of carbon to form ions.

ELECTROLYTIC DECOMPOSITION. — Faraday's laws enable one to determine the amount of the substances primarily liberated or deposited at the electrodes of any electrolytic cell, but they do not enable one to predict the nature of the secondary chemical actions which may take place. (*See section on Secondary Reactions.*) The factors which determine what products will be formed in any given case are numerous. The chemical process occurring at the cathode is always of the nature of a reduction; at the anode, of an oxidation, these terms being used in their most general sense. It may be that the electrolyte, taken as a whole, undergoes no change, as is the case, for example, of a copper-sulphate solution when electrolyzed between copper electrodes. Here the chemical reaction at the cathode is the exact reverse of that at the anode. Generally, however, this is not the case; different chemical products are usually formed at the two electrodes, and appear either in the form of gas or as a solid precipitate, or remain dissolved in the electrolyte.

Decomposition Voltage of Electrolytes. — When a gradually increasing difference of potential is applied to the electrodes of an electrolytic cell, electrolysis does not in general begin at once but only after a certain minimum electromotive force is reached, even though the cell has no open-circuit e.m.f. It should be noted that this condition does not apply to the case when the electrodes are such that the electrolyte as a whole undergoes no change as a result of electrolysis. Thus, in the electrolysis of a zinc-sulphate solution between zinc electrodes an exceedingly small electromotive force is sufficient to maintain a steady current through it indefinitely (assuming that concentration changes at the electrodes are completely eliminated by stirring). Although, in this case, electrolysis starts at once, the electrolyte as a whole is not decomposed, for as much zinc dissolves at the anode as is deposited at the cathode. If a platinum anode be substituted for the zinc anode so that as a result of electrolysis zinc is separated at the cathode and oxygen gas is liberated at the anode, an e.m.f. of 2.35 volts is required to start the electrolysis. Again, if the electrolyte is an aqueous solution of sulphuric acid it requires a minimum voltage of 1.67 volts to liberate oxygen and hydrogen gas against smooth platinum electrodes. The minimum voltage necessary to decompose a compound electrolytically is called its "decomposition voltage." It is influenced by a variety of factors and conditions which will now be considered.

Measurement of Decomposition Voltage. — Decomposition voltages have been experimentally determined in several ways. Certain investigators have taken the value necessary to produce visible electrolysis as the criterion; others have followed the polarization at the electrodes until it became constant and assumed this value as that at which electrolysis begins. Results obtained by the so-called ammeter-voltmeter method must be accepted with caution. In this method the value of the electromotive force applied to the cell is gradually increased from zero and plotted as abscissas and the corresponding currents through the cell plotted as ordinates. The resulting curves have the general form shown in Fig. 2.

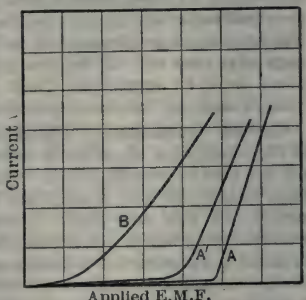

Fig. 2.

The slope of these current-voltage curves depends upon the internal resistance of the cell, the size and the distance apart of electrodes. The critical voltage at which the current suddenly increases in value is, however, quite sharply

defined with some electrolytes, e.g., silver nitrate between platinum electrodes, and other salts from which a metal is deposited. Below the critical voltage very little or no current passes through the cell and no visible decomposition at the electrodes is apparent. When the point A is reached, visible electrolysis begins. The sharpness of the bend in the curve and its course below A depend in large measure upon the sensitiveness of the galvanometer used for detecting the current, the size of electrodes and the tendency of the products of the electrolysis to go back into solution.

Residual Currents. — With a sensitive reflecting galvanometer, a steady deflection may be observed for days without any apparent electrolysis when an e.m.f. of a few tenths of a volt (far below the "decomposition voltage") is applied to platinum electrodes dipping into acidulated water. It might seem from this that Faraday's law is here violated. These slight currents are to be explained, however, by a diffusion of the products liberated slowly and in minute quantities at the electrodes, back into the solution. Currents produced in this manner are called "residual currents" and play an important rôle in the electrolysis of fused salts and probably also in conduction in solid electrolytes. The larger the residual current the less sharply marked is the decomposition point, see A'. For many fused salts the current-voltage curve is of the form shown in B, Fig. 2. It would be difficult to say from this curve at what voltage decomposition actually began.

Table V gives the experimentally determined decomposition voltages of some of the more common acids, bases and salts.

TABLE V. — DECOMPOSITION VOLTAGES OF NORMAL SOLUTIONS
(*From Le Blanc's Text-book of Electrochemistry, p. 289.*)

Solution	Voltage	Solution	Voltage
Acids:		Bases:	
Dextrotartaric	1.62	Ammonium hydrate	1.74
Dichloracetic	1.66	½ n. diethylamine	1.68
Hydrazoic	1.29	¼ n. methylamine	1.75
Hydriodic	0.52	Potassium hydrate	1.67
Hydrobromic	0.94	Sodium hydrate	1.69
Hydrochloric	1.31	⅛ n. tetramethyl ammonium hydrate	1.74
Malonic	1.69		
Monochloracetic	1.72	Salts:	
Nitric	1.69	$AgNO_3$	0.70
Oxalic	0.95	$CdCl_2$	1.88
Perchloric	1.65	$Cd(NO_3)_2$	1.98
Phosphoric	1.70	$CdSO_4$	2.03
Pyrotartaric	1.57	$CoCl_2$	1.78
Sulphuric	1.67	$CoSO_4$	1.92
Trichloracetic	1.51	$NiCl_2$	1.85
		$NiSo_4$	2.09
		$Pb(NO_3)_2$	1.52
		$ZnBr_2$	1.80
		$ZnSO_4$	2.35

Calculation of Decomposition Voltage from Heat of Reaction. — If the electrolytic process is reversible in the thermodynamic sense, then Gibbs-Helmholtz's equation (*see above*) is directly applicable. Since however the temperature coefficient of the maximum work (or free energy) is known in but

a few instances, one usually employs Thomson's rule, which gives a fair approximation in the case of most commercial processes, even though these processes are not strictly reversible. That is, calling H the heat of formation of the compound which is decomposed from the substances which are liberated at the electrodes (i.e., H is the heat of the reverse reaction corresponding to the complete chemical actions which take place in the cell), the decomposition voltage is

$$E = \frac{H}{230.5\,n},$$

where n = the valency of the ion of highest valency involved in the reaction, and H is expressed in large mean calories per gram-molecule.

Tables giving the voltages required to liberate a number of elements and radicals from various electrolytes computed by Thomson's rule from heats of formation in dilute solutions are given by J. W. Richards in *Jour. Franklin Institute*, 1906.

Example. — In the formation of one mol of solid lead chloride from metallic lead and chlorine gas 828 large mean calories are liberated. The valency of lead is 2; of chlorine, 1. As solid lead chloride does not conduct electricity appreciably at ordinary temperatures, to decompose it electrolytically, it must be either dissolved or fused. If a saturated solution containing an excess of solid lead chloride be electrolyzed, metallic lead will be deposited at the cathode, chlorine gas liberated at the anode, and an equivalent amount of solid lead chloride will pass into solution, the net result then being a decomposition of solid lead chloride into its elements lead and chlorine. Under these conditions the decomposition voltage by Thomson's rule is

$$E = \frac{828}{230.5 \times 2} = 1.79 \text{ volts},$$

as against 1.612 volts by actual measurement.

It should be pointed out that Nernst has recently applied his new heat theorem (*see above*) to the calculation of the e.m.f. of cells similar to the above, with very satisfactory results. Thus the computed value of the e.m.f. in the above case from thermal data by the formula developed by Nernst is $E = 1.594$ volts, in excellent agreement with the observed value. See Nernst's *Theoretical Chemistry.*

Calculation of Decomposition Voltage from Electrode Potentials. — Le Blanc was the first to measure by means of a normal electrode the single potential drop at the cathode when various salts were electrolyzed and found that the cathodic drop in potential necessary to discharge a given cation is numerically equal to the electrode potential which would be established if the separated metal were substituted for the actual cathode. For example, to deposit metallic zinc from a normal zinc-sulphate solution it is necessary to impress a potential from solution to electrode as measured against a normal hydrogen electrode of at least 0.76 volt, to overcome the natural rise of potential from the zinc electrode to the solution (as measured by the normal hydrogen electrode) of this amount. See table page 460.

Before continuous electrolysis can begin, however, not only must the applied e.m.f. be sufficient to establish the necessary cathodic drop e_c, but in addition it must also raise the anode potential to a value e_a, corresponding to the electrode potential of the anion which is liberated. If, as is usually the case, several different anions are present in the electrolyte that one will be liberated first which can part with its negative charge with the least expenditure of energy, i.e., at the lowest voltage. It frequently happens that e_c is established long

before e_a is reached, in which case Le Blanc found that e_c remains constant while the impressed voltage E (and hence e_a) is increased up to the decomposition point. The minimum e.m.f. at which decomposition of a given electrolyte begins may therefore be written $E = e_c + e_a$ where e_c and e_a are the cathodic and anodic polarization values produced by that cation and anion respectively which gives up its charge most readily. In an electrolyte containing several different anions and cations all take part in the *conduction* of a current through the solution, but only specific ions are effective in electrolysis, i.e., in giving up their charges to the electrodes.

Overvoltage. — When a gas is evolved at either electrode the above relations are usually more complicated owing to the phenomenon known as overvoltage. Thus in the electrolysis of an acid between polished platinum or other metal electrodes, hydrogen gas is evolved, to liberate which the cathode potential must be raised to a value in excess of that required to liberate hydrogen at a reversible (platinum black) electrode.

This excess of voltage which is necessary to liberate a gas at any electrode over that required to liberate the same gas against a *reversible* gas electrode is called "overvoltage." Its value depends upon the character of the material of the electrode against which the gas is set free and upon the nature of the gas. It is particularly large in the case of hydrogen. In Table VI are given values of the overvoltage for hydrogen as determined by Caspari when this gas is *visibly* set free. The magnitude of the effect is closely related to the power of the metal surface to occlude the gas in question. Thus, platinum and (to a less extent) iron absorb hydrogen readily and hence these metals exhibit slight overvoltage phenomena. Zinc and mercury, on the other hand, have little or no tendency to absorb hydrogen and hence this gas is liberated with great difficulty against cathodes of these metals, with correspondingly high overvoltages.

For discussion of overvoltage phenomenon, see Möller, *Ann. d. Physik*, 27, 566, 1908; Crabtree, *Trans. Faraday Soc.*, Vol. 9, 125, 1913; Bennett and Thompson, *Proc. Am. Elec. Chem. Soc.*, 29, 269, 1916; MacInnes, *Am. Chem. J.* XLI, 194, 1919.

TABLE VI. — OVERVOLTAGE OF HYDROGEN FOR VISIBLE ELECTROLYSIS.*

Electrode	Overvoltage in volts	Electrode	Overvoltage in volts
Pt, platinized	0.005	Cu	0.23
Au	0.02	Cd	0.48
Fe	0.08	Sn	0.53
Pt, polished	0.09	Pb	0.64
Ag	0.15	Zn	0.70
Ni	0.21	Hg	0.78

* *Zeit. für Phys. Chem.*, Vol. 30, p. 89, 1899.

Electrolytic Separation of Metals. — The reason that two or more metals can be electrolytically separated from each other by a regulation of voltage follows from the preceding discussion. Consider a solution containing two salts, say silver and copper nitrates, each of normal concentration If a gradually increasing e.m.f. be applied to inert electrodes dipping into this solution, electrolysis will begin when a voltage is reached sufficient to set free

simultaneously any anions and cations present in the solution. Referring to Table IV, it will be seen that copper stands 0.80 − 0.34 = 0.46 volt above silver in the electrochemical series, and hence, the latter metal can be deposited from the solution with e.m.f. nearly half a volt less than copper. No copper ions can separate until the applied e.m.f. has been increased 0.46 volt above that necessary to first separate the silver. The decomposition e.m.f. necessary will, of course, depend on the nature and concentration of the anions present. As electrolysis proceeds, the solution becomes weaker and weaker with respect to silver ions and the voltage necessary to separate them from the solution increases. If the electrolysis is continued until the silver remaining in solution has a concentration only 1/1,000,000 that at the start (the limit of analytical determinations), the change in voltage will be approximately 0.35 volt (*see section above on Variation of Electrode Potentials with Concentration*). This is still insufficient to bring the applied e.m.f. up to a value sufficient to permit the copper to deposit, and hence, silver and copper may be completely, for all practical purposes, separated from each other.

It is to be noted that as the electrolysis proceeds and the silver is removed from the solution a greater and greater percentage of the current is *conducted* by the copper. The solution in the neighborhood of the cathode becomes continually weaker with respect to silver ions. In a quiet electrolysis the silver ions at the cathode are replenished by diffusion from the interior of the electrolyte. This process may be greatly accelerated by violently stirring the solution, as, for example, by the use of a rapidly rotating electrode. This permits good deposits being obtained with higher current density and greatly reduces the time required for the separation.

Secondary Reactions. — In the preceding sections the principles underlying *primary* electrolysis have been explained. The ultimate products which are produced at the anode and cathode are often quite different from the simple ions or ionic complexes liberated by the action of the current. In general, the primary process of discharge of ions at cathode and anode is followed by a secondary reaction of one of the following types:

1. The discharged ions form molecules, e.g.:

At cathode $2\,H = H_2$ gas,
At cathode $x\ Cu = Cu_x$ solid,
At anode $2\,Cl = Cl_2$ gas.

This may occur at both anode and cathode.

2. The discharged ions react with each other or break up, e.g :

At anode $2\,OH = H_2O + \frac{1}{2}\,O_2$,
At anode $\begin{matrix} COO \\ | \\ COO \end{matrix} = 2\,CO_2$,
At anode $2\,CH_3COO = 2\,CO_2 + C_2H_6$.

This is very characteristic at the anode, particularly of organic anions.

3. The discharged ions react with the solvent, e.g.:

At cathode $Na + H_2O = NaOH + \frac{1}{2}\,H_2$,
At anode $SO_4 + H_2O = H_2SO_4 + \frac{1}{2}\,O_2$.

4. The discharged ions react with the solute, e.g.:

At cathode $8\,H + 2\,H_2SO_4 = H_2SO_3 + S + 5\,H_2O$,
At cathode $K + KAg(CN)_2 = Ag + 2\,KCN$.

5. Discharged ions react with the electrode, e.g.:

At cathode $4\,H + PbO_2 = Pb + 2\,H_2O$,
At anode $2\,Cl + Cu = CuCl_2$.

Which of the above reactions is most likely to occur in any given case depends to some extent upon, (a) current density; (b) temperature; (c) concentration of the solution.

In general, it may be said that at the cathode a reduction takes place, while at the anode oxidation (in its general sense) occurs. The intensity of these reactions depends upon the electrode potential at which the reaction takes place. This can be varied by the use of different electrode materials (thereby increasing the overvoltage). The efficiency of certain secondary reactions has also been found to depend upon the nature of the electrode (catalytic action). The efficacy of electrolytic reduction and oxidation in organic chemistry where the intensity of the reaction must be carefully regulated likewise rests on the above factors.

Passivity. — A metal is said to assume the "passive" state when it comports itself towards acids like a noble metal, i.e., becomes insoluble. Iron affords a very striking example of this phenomenon, although it is exhibited by other metals as well. Thus, if dipped in strong nitric acid, or if anodically polarized, it becomes passive. Several explanations have been advanced to account for the phenomenon, one being that it is due to an invisible film of oxide on the surface, and another that the surface assumes a condition in virtue of which the velocity of reaction between it and the surrounding solution is infinitely slow. For résumé of theories see *Trans. Am. Elec. Chem. Soc.*, *29*, 217, 1916.

Alternating-current Electrolysis. — The decomposition of an electrolyte, and the solution of metal electrodes by electrolysis, can be effected under certain conditions by an alternating current as well as by a direct current. The chief determining factors are the velocity with which the chemical reaction takes place and the periodicity of the alternating current.

THEORY OF SOLUTIONS. — To understand the theory of the various phenomena described above, a knowledge of the theory of solutions is necessary.

Osmotic Pressure. — The present theory is based upon Van't Hoff's thermodynamic and experimental investigations published in 1887. When a substance is brought into contact with a second substance in which it is more or less soluble, it dissolves in virtue of a force (the nature of which is imperfectly understood), which is called its "solution tension" or "solution pressure." If the solute is present in excess, the process of solution continues until the solution becomes saturated, i.e., until equilibrium is established between the solute and solution, such that the solution pressure of the former is just balanced by an opposing pressure created within the solution which tends to cause the dissolved substance to separate out. This pressure is called the osmotic pressure of the solution. Under these conditions there are just as many molecules of solute passing into solution as separate out of the solution during any interval of time. A clearer conception of the osmotic pressure of a solution, as above defined, may be obtained from the following considerations.

Semi-permeable Membranes. — If a solution be placed at the bottom of a cylinder and carefully covered with a layer of pure solvent, the process of diffusion will begin at once and continue until a homogeneous dilute solution is formed. If the experiment be repeated with the single modification, that the pure solvent is separated from the solution by a piston consisting of a semi-permeable diaphragm which will permit the solvent to pass freely through it in either direction, but which will not allow the solute to pass through, the process of diffusion will take place as before, provided the piston is movable. Thus the piston will be forced back by the osmotic pressure of the solution while the solvent passes freely through it thereby diluting the solution. Membranes possessing the property of semi-permeability are not only theoretically con-

ceivable but may actually be prepared. Such a membrane is formed by the precipitate of copper ferrocyanide deposited within the walls of an unglazed porous cell when copper sulphate and potassium ferrocyanide solutions are allowed to diffuse into the cell from within and without respectively. This membrane is permeable to water, but impermeable to sugar and many other organic and inorganic substances, such as the copper and potassium salts from which it is formed. Most membranes, in fact, possess the property of semi-permeability to a limited degree.

Measurement of Osmotic Pressure. — To prevent the piston being forced up by the diffusing solution, a downward pressure P must be applied to it equal to that exerted upon it from below by the solution. The magnitude of P is, therefore, a measure of the osmotic pressure of the solution. Owing to experimental difficulties, it is impracticable to measure osmotic pressures in precisely the manner just described, but by slightly modifying the arrangement of the apparatus it may readily be done. Thus, if the solution is contained in a closed vessel, A, Fig. 3, the walls of which are made semi-permeable by depositing a membrane of copper ferrocyanide within the pores of the cell, and this be immersed in the solvent B, the osmotic pressure will be exerted as before against the membrane from within. If the latter could stretch like a rubber balloon, it would do so as the solution became diluted; being, however, restrained by the material of the walls of the cell on and in which it is deposited from doing this, dilution of the solution can take place only by the solvent passing through the fixed membrane into the solution cell. This it will do unless the cell A is hermetically sealed. If an open manometer M of small bore be inserted in the top of A, the solvent as it passes into the cell will cause the solution to rise gradually in M until the hydrostatic pressure thus produced prevents further entrance of solvent. When equilibrium is established, the hydrostatic pressure gives a measure of the osmotic pressure of the solution then existing in the cell. As the entrance of an appreciable amount of solvent into the cell reduces the concentration of the solution, the pressure thus measured is less than that of the original solution; hence for quantitative work an open manometer is replaced by a closed mercury manometer.

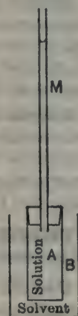

Fig. 3.

Laws of Osmotic Pressure in Non-electrolytic Solutions. — By an apparatus similar in principle to that just described the following laws have been verified. They hold for dilute aqueous solutions of many organic substances, such as sugar, and for many solutions in organic solvents, all of which possess the common property of being non-electrolytes.

1. The osmotic pressure of a solution is directly proportional to its concentration (and therefore inversely proportional to the volume in which a given mass of the solute is dissolved).

2. It is directly proportional to the absolute temperature.

3. It is independent of the nature of the solute, being a function solely of the number of mols of substance dissolved in unit volume of solution.

4. The numerical magnitude of the osmotic pressure p of one mol of any substance dissolved in a volume v of solution at the absolute temperature T is identical with the gaseous pressure exerted by one mol of a perfect gas at the temperature T and occupying the same volume v.

The gas laws are therefore not only directly applicable to solutions, but the numerical value of the gas constant $\overline{R}$ is the same for each. Thus the combined laws of Boyle and Gay-Lussac for gases have the same form for solutions, namely

$$pv = \overline{R}T,$$

where p = the osmotic pressure, v = the volume of the solution containing one mol of solute, and T = the absolute temperature.

The accompanying table gives the numerical value of $\bar{R}$ when the various quantities are expressed in the units designated. In heat units

$$\bar{R} = 1.985 \frac{\text{calories}}{\text{deg. cent.}}.$$

Abs. pressure p	Volume v	Abs. temp. T	Gas constant $\bar{R}$
Atmospheres	Liters	Deg. cent.	0.0821
Grams per sq. cm.	Cu. cm.	Deg. cent.	84,800
Dynes per sq. cm.	Cu. cm.	Deg. cent.	0.83161×10^8

When applied to concentrated solutions of non-electrolytes the above laws suffer deviations analogous to those which they exhibit when applied to highly compressed gases.

Osmotic Pressure in Electrolytic Solutions. — The first of the above laws for non-electrolytic solutions applies to electrolytic solutions only within narrow limits, while the third and fourth laws fail absolutely. The osmotic pressure of this class of solutions is much greater than the equation $pv = \bar{R}T$ would lead one to expect. Thus, a 0.1 normal sodium chloride solution has an osmotic pressure 1.9 times as great as that which it should have on the basis of the above formula. A potassium sulphate solution containing 0.1 gram-equivalent per liter has an osmotic pressure 2.3 times as great as that calculated. To bring these results within the scope of the gas-law formula, Van't Hoff proposed the empirical equation $pv = i\bar{R}T$, where i is a constant having the value unity for non-electrolytes and a value greater than unity for electrolytes.

THE DISSOCIATION THEORY. — Arrhenius, in 1887, gave the first satisfactory explanation of the anomalous behavior of electrolytes, which show not only abnormally high osmotic pressures, but also abnormally high boiling points and abnormally low vapor pressures and freezing points. On thermodynamic principles these phenomena can all be shown to be proportional to the total number of molecular complexes per unit volume and entirely independent of their chemical constitution. Abnormally high osmotic pressures may therefore be explained on the assumption that each molecule, when dissolved, exists in a more or less dissociated condition, i.e., at any given instant there are in the solution more discrete particles or complexes than there would be if each molecule remained continuously intact. The ions are not supposed to be permanently separated, but a continuous process of dissociation and combination is assumed to be going on all the time. If the further assumption be made that the dissolved substance is electrolytically active only during that fraction of time while it exists in a dissociated or ionized state, in other words, that dissociated molecules are alone capable of being acted upon by electric forces, the connection between electrolytes and solutions showing deviations from the gas laws is explained.

On the assumptions of this theory, sodium chloride, NaCl, in aqueous solution is partially dissociated or ionized according to the equation, $NaCl \rightleftarrows \overset{+}{Na} + \overset{-}{Cl}$, and therefore at any instant there are present in the solution not only sodium-chloride molecules, but also positively charged sodium ions and an equal number of negatively charged chlorine ions, the number of which at any instant depends upon the average fraction of a second during which the molecules are dissociated or "ionized." If the salt were "completely, i.e., 100 per cent ionized" there would be no molecules, as such, present in the solution, but instead twice the number of ions as there were molecules originally dissolved. Hence the osmotic pressure of such a solution would be twice as great as that calculated

from the equation $pv = \bar{R}T$; in equation $pv = i\bar{R}T$, i would be equal to 2. Again, if a given volume of solution contains one mol of potassium sulphate, K_2SO_4, which is 75 per cent ionized according to the equation $K_2SO_4 \rightleftarrows 2\overset{+}{K} + \overset{-}{SO_4}$, then at any instant there will be $1 - 0.75 = 0.25$ mol of K_2SO_4 molecules, 0.75 mol of $\overset{-}{SO_4}$ ions, and $2 \times 0.75 = 1.50$ mol of $\overset{+}{K}$ ions present in the solution, or $0.25 + 0.75 + 1.50 = 2.5$ molecular complexes for every mol of sulphate dissolved; hence $i = 2.5$.

Although the nature of the ions resulting from the ionization of acids, bases and salts is now known to be more complicated in many cases than assumed in such simple ionization reactions as the above, owing to their hydration or solvation with molecules of the solvent, still the above hypothesis of Arrhenius has proved the most fruitful conception which has been introduced into chemistry during the last thirty-five years and it remains to-day, modified in the light of recent research, the best working hypothesis with which to interpret electrolytic phenomena in aqueous solutions. It is a natural sequel to the free ion theory of Clausius which in turn displaced the old chain theory of Grotthuss.

THEORY OF ELECTROLYTIC CONDUCTION. — The external effects of a current flowing through an electrolyte cannot be distinguished from those produced by a current of the same strength conducted by a metal. Thus the magnetic effect of a current flowing through a helical glass tube filled with electrolyte is the same as that produced by the same current flowing through an equivalent circuit of an equal number of turns of copper wire. A current may be induced in a closed ring of electrolyte just as in a ring of metal. Ohm's Law and Joule's Law hold for conduction in electrolytes as well as in metals. The mechanism of the conduction in the two cases, however, is very different, as shown by the phenomena produced at the junction of two conductors in one of which the conduction is metallic or "electronic" and in the other of which it is electrolytic or "ionic."

Migration of Ions. — Electrical conduction, both metallic and electrolytic, is believed to be a convection phenomenon. In the case of electrolytic conduction there is much evidence to indicate that the convection consists in the transport of electrically charged ions through the electrolyte, both with and against the direction in which the current is assumed to flow. Simultaneous convection of anions and cations in opposite directions may not only be made visible to the eye but their velocities may also be directly measured. On the assumption that the ions are the carriers of charges, the motion of which constitutes the current, the "velocity of migration" can also be calculated from conductivity measurements and the experimentally determined "transport ratios" (*see below*).

Velocity of Migration. — In Table VII are given the results of direct measurements of the migration velocity of various ions and also the calculated velocities, both expressed in centimeters per second and for a potential gradient of one volt per centimeter.

The velocities are directly proportional to the potential gradient. They all increase rapidly with the temperature. They vary but slightly with the concentration of the solution if the concentration is not too great, and for dilute solutions may be regarded as practically constant.

From the table it is seen that hydrogen migrates with the maximum velocity. Hydroxyl $\overset{-}{OH}$ has the next greatest velocity. The high velocity of the hydrogen ion explains the fact that aqueous solutions of the mineral acids are as a class the best electrolytic conductors, and the high velocity of the hydroxyl ion accounts for the relatively good conductivity of the inorganic bases.

As the velocity with which an ion migrates under the action of a given force depends upon the friction which it has to overcome in moving through the solution, an intimate relation should be expected between it and the rate of diffusion and fluidity (reciprocal of viscosity) of the solution. The viscosity of a solution diminishes or its fluidity increases rapidly with an increase in temperature (from one to two per cent per degree) and experiment shows that the migration velocity increases at approximately the same rate. The increased velocity of migration of the ions with rising temperature is the cause of the high temperature coefficients of electrolytes.

TABLE VII. — VELOCITIES OF MIGRATION OF IONS

Ion	Velocities in cm. per sec. per volt per cm.		
	0.1 gram-equivalent per liter, 18° C.		Infinite dilution, 18° C.
	Observed	Calculated	Calculated
$\overset{+}{H}$	0.0026	0.0028	0.003263
$\overset{-}{OH}$	....	....	0.001802
$\overset{-}{Cl}$	....	....	0.000677
$\overset{+}{K}$	....	....	0.000669
$\overset{+}{NH_4}$	....	....	0.000663
$\overset{-}{NO_3}$	....	....	0.000639
$\overset{-}{ClO_3}$	....	....	0.000570
$\overset{+}{Ag}$	....	....	0.000562
$\overset{-}{SO_4}$	0.00045	0.00049	....
$\overset{-}{Cr_2O_7}$	0.00047	0.00047	....
$\overset{+}{Ag}$	0.00049	0.00046	....
$\overset{+}{Ba}$	0.00039	0.00037	....
$\overset{+}{Ca}$	0.00035	0.00029	....

Transport Ratio. — When a solution is subjected to electrolysis a change in concentration is found to take place in the immediate vicinity of the anode and cathode, but practically no change takes place in the middle portion of the solution, provided means are employed to prevent convection and diffusion from the electrodes. Suppose that the solution about the anode and cathode respectively is analyzed after a known current has passed for a known time.

Let

a = number of gram-equivalents of the *solute* which disappear from the region about the anode.

c = number of gram-equivalents of the *solute* which disappear from the region about the cathode.

then $a + c =$ total number of gram-equivalents of the solute which is decomposed.

The ratio $n_a = c/(a + c)$ is called the transport ratio for the anion and the ratio $n_c = 1 - n_a = a/(a + c)$ is called the transport ratio for the cation. These ratios are also frequently referred to as "Hittorf's transference numbers." For a given electrolyte n_a is proportional to the number of gram-equivalents of the anions which migrate through the solution from the cathode towards the anode and n_c to the number of gram-equivalents of the cations transported from the anode towards the cathode.

On the assumption that the ions are the sole carriers of the charges the motion of which constitutes the electric current, the ratio of the current carried by the anions or cations to the total current is the same as the above transport ratio for these ions, since the charge carried by one gram-equivalent of every ion is constant = 96,540 coulombs.

TABLE VIII. — TRANSPORT RATIOS n_a OF ANIONS IN AQUEOUS SOLUTIONS AT ABOUT 18° C.

(Values in parenthesis are somewhat uncertain. From Le Blanc's *Electrochemistry*.)

Gram-equivalents per liter / Liters per gram-equivalent	0.01 / 100	0.05 / 20	0.2 / 5	1 / 1	2 / 0.5	5 / 0.2
K{Cl, Br, I}, NH_4Cl	0.506	0.506	.....	.....	.....	.....
NaBr, NaCl	0.604	0.604	.....	.....	.....	.....
LiCl	0.670	0.680	0.697	.....	.....	.....
KNO_3	.....	.....	0.496	0.487	0.479	.....
$NaNO_3$	.....	.....	0.614	(0.611)	(0.608)	0.585
$AgNO_3$	0.528	0.528	0.527	0.501	0.476	.....
$KC_2H_3O_2$	.....	.....	0.33	(0.331)	(0.332)	0.335
$NaC_2H_3O_2$	.....	.....	(0.43)	(0.425)	0.421	.....
KOH	.....	.....	0.736	(0.740)	.....	.....
NaOH	.....	(0.81)	(0.82)	0.825	.....	.....
LiOH	.....	.....	0.85	(0.873)	.....	.....
HCl	0.167	0.165	.....	.....	.....	.....
HNO_3	0.170	0.170	0.170	.....	.....	.....
½ $BaCl_2$	0.553	.....	.....	.....	.....	.....
½ $CaCl_2$	(0.58)	(0.61)	(0.66)	0.686	(0.700)	0.737
½ $MgCl_2$	.....	(0.63)	0.68	0.709	(0.729)	0.776
½ $CdCl_2$	0.570	0.570	(0.65)	(0.72)	0.745	0.865
½ CdI_2	0.558	0.606	0.86	.....	.....	.....
½ $Ba(NO_3)_2$ at 25°	.....	0.544	0.545	.....	.....	.....
½ K_2CO_3	.....	(0.39)	(0.41)	0.434	0.413	(0.380)
½ Na_2CO_3	.....	(0.52)	(0.53)	0.548	(0.542)	.....
½ K_2SO_4	0.505	0.512	.....	.....	.....	.....
½ Na_2SO_4	0.610	0.624	.....	.....	.....	.....
½ $CdSO_4$	0.616	0.635	0.672	0.746	.....	.....
½ $MgSO_4$	0.620	0.633	(0.66)	0.74	(0.76)	.....
½ $CuSO_4$	.....	0.625	0.657	.....	.....	.....
½ H_2SO_4	0.176	0.175	.....	.....	.....	.....

Note. — The ½ before the various bivalent electrolytes indicates that 1 gram-equivalent = ½ mol.

Table VIII contains the generally accepted values of the transport ratios for the more common ions. These numbers vary but slightly with the temperature. They tend to approach the value 0.5 with increasing temperature, i.e., as the temperature rises both ions of a given electrolyte tend to conduct equal per cents of the current. With increasing dilution the transport ratios of good electrolytes approach constant limiting values; these are practically reached in solutions whose concentration is 0.01 gram-equivalent per liter.

Relation between Velocity of Migration and Transport Ratios. — On the assumption that the ions are the carriers of the charges the motion of which constitutes the electric current, and on the further assumption that the moving ions do not carry along with them different quantities of the solvent, the ratio of the velocity u_a of the anions to the velocity u_c of the cations in any solution must be equal to the ratio of the transport ratios $n_a = c/(a+c)$ and $n_c = a/(a+c)$ of the anions and cations respectively, i.e.,

$$\frac{u_a}{u_c} = \frac{n_a}{n_c} = \frac{c}{a}.$$

The transport ratios n_a and n_c may be determined experimentally as described above and by the use of this relation the *relative* velocities of migration may be compared with the ratio calculated from the direct determination of the velocities u_a and u_c. See Washburn's *Principles of Physical Chemistry.*

Hydration or Solvation of Ions. — The above relation between the velocities of migration and the transport ratios has been found to hold only approximately in a number of instances. The probable explanation of this lack of agreement between theory and experiment is that the changes in concentration at the two electrodes are due not only to the migration velocities of the ions, but also to the fact that the ions carry along with them molecules of the solvent, the amount of solvent carried by the anions and cations being different. This "hydration" or "solvation" of the ions has recently been proved by direct experiment (*Washburn, Zeit. für Phys. Chem., Vol. 66, p. 513, 1909*), by adding to the solution an indifferent substance upon which the electric current produces no migratory effect (e.g., sugar or raffinose). See also Lewis, J., *Am. Chem. Soc., Vol. 32, p. 862, 1910.*

Conductance of Electrolytes. — (*See also article on Resistance and Conductance.*) While no simple relations have been found to hold between the *specific* conductances of different electrolytic solutions, very important results follow at once from a comparison of their *equivalent* conductances.

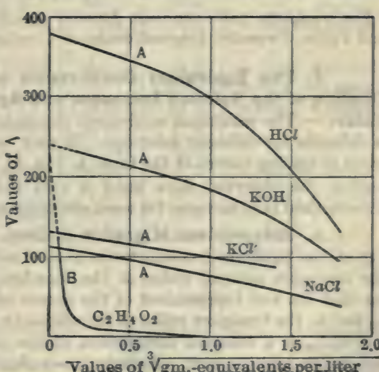

Fig. 4.

Table IX shows the relation between the equivalent conductance Λ and the concentration m in gram-equivalents per liter for some typical aqueous solutions. In Fig. 4 certain of these data are shown graphically. From a study of these results the following important conclusions may be drawn.

TABLE IX. — EQUIVALENT CONDUCTANCE Λ OF TYPICAL ELECTROLYTES DISSOLVED IN DIFFERENT QUANTITIES OF WATER, AT 18° C.

Concentration in gm.-equivalents per liter = $m = 1000\,\eta$	Dilution in liters per gm.-equivalent = $v = \phi'/1000$	KCl	NaCl	KNO_3	$AgNO_3$	½ $CuSO_4$	½ H_2SO_4	HCl	CH_3COOH	KOH	NH_3
0.0001	10,000	129.07	108.10	125.50	115.01	109.95	......	(378)	107	......	(66)
0.0002	5,000	128.77	107.82	125.18	114.56	107.90	......	(378)	80	......	53
0.0005	2,000	128.11	107.18	124.44	113.88	103.56	(368)	377	57	......	38.0
0.001	1,000	127.34	106.49	123.65	113.14	98.56	361	376	41	(234)	28.0
0.002	500	126.31	105.55	122.60	112.07	91.94	351	375	30.2	(233)	20.6
0.005	200	124.41	103.78	120.47	110.03	80.98	330	373	20.0	230	13.2
0.01	100	122.43	101.95	118.19	107.80	71.74	308	369	14.3	228	9.6
0.02	50	119.96	99.62	115.21	......	62.40	286	366	10.4	225	7.1
0.05	20	115.75	95.71	109.86	99.50	51.16	253	358	6.48	219	4.6
0.1	10	112.03	92.02	104.79	94.33	43.85	225	351	4.60	213	3.3
0.2	5	107.96	87.73	98.74	......	37.66	214	342	3.24	206	2.30
0.5	2	102.41	80.94	89.24	77.5	......	205	327	2.01	197	1.35
1	1	98.27	74.35	80.46	67.6	25.77	198	301	1.32	184	0.89
2	0.5	92.6	64.8	69.4	......	......	183.0	254	0.80	160.8	0.532
3	0.33	88.3	56.5	(61.3)	......	......	166.8	215.0	0.54	140.6	0.364
5	0.2	......	42.7	......	......	......	135.0	152.2	0.285	105.8	0.202

Note. — The ½ before $CuSO_4$ and H_2SO_4 indicates 1 gram-equivalent = ½ mol.

For complete tables consult *Landolt and Börnstein, Kohlrausch and Holborn,* and *Tables Annuelles Internationales de Constantes.*

1. The Equivalent Conductance of a Solution Increases as the Dilution of the Solution Increases and Approaches a Limiting Maximum Value. — The maximum equivalent conductance, usually denoted by Λ_0 or Λ_∞, is different for different solutions. It may be found graphically for good electrolytes (giving curves of the form A, Fig. 4), by extrapolating the curve representing the observed data until it cuts the ordinate of zero concentration. This method fails however for weak electrolytes such as acetic acid, see Curve B.

2. Independent Migration of Ions. Ionic Mobilities. — The product of the equivalent conductance of any solution at infinite dilution Λ_∞ and the transport ratio of either of the ions forming the electrolyte is a constant for that ion and independent of the nature of the electrolyte. That is, calling n_a and n_c the transport ratios of the anions and cations respectively, the product $l_a = n_a\Lambda_\infty$ is a constant irrespective of the nature of the electrolyte of which this anion forms a part; similarly $l_c = n_c\Lambda_\infty$ is a constant for the cation irrespective of the nature of the electrolyte of which it forms a part. For example, the product $n_c\Lambda_\infty$ for the potassium ion is a constant whether the solution be one of potassium chloride, potassium bromide, potassium sulphate, etc. These constants l_a and l_c are called the "mobilities" or "ionic conductivities" of the ions. This relation was discovered by F. Kohlrausch in 1876 and stated as the law of the independent migration of ions.

Since, by definition, $n_a + n_c = 1$, it follows that

$$\Lambda_\infty = l_a + l_c.$$

In other words the maximum equivalent conductivity of a solution (at infinite dilution) is equal to the sum of the mobilities of the anions and cations which form the electrolyte; or, the mobility of an ion depends solely upon the nature of the ion and not upon the nature of the substance of which it originally formed a part.

From this relation the mobility of any ion may be determined directly from the equivalent conductance when the mobility of the ion with which it is associated is known.

The values of the mobilities of the more common ions at 18° C. and at infinite dilution are given in Table X. From this table the equivalent conductance of any given electrolyte at infinite dilution may be readily calculated by the formula $\Lambda_\infty = l_a + l_c$. Thus, for silver nitrate, $AgNO_3$, at 18° C. $\Lambda_\infty = l_{Ag} + l_{NO_3} = 54.3 + 61.7 = 116.0$.

TABLE X. — MOBILITIES OF TYPICAL IONS AT INFINITE DILUTION AND 18° C.

(Values at t° may be computed by the formula $l_t = l_{18}\,[1 + \alpha\,(t - 18) + \beta\,(t - 18)^2]$) (Kohlrausch)

Anions	l_{18}	α	β	Cations	l_{18}	α	β
F	46.6	0.0232	0.000094	Li	33.4	0.0261	0.000142
Cl	65.5	0.0215	0.000067	Na	43.5	0.0245	0.000116
Br	67.0	......	....	K	64.6	0.0220	0.000075
I	66.5	0.0206	0.000052	Rb	67.5	0.0217	0.000069
SCN	56.6	......	....	Cs	68.0	......	....
ClO_3	55.0	0.0207	0.000054	NH_4	64.0	0.0223	0.000079
BrO_3	46.0	......	....	Tl	66.0	......	....
IO_3	33.9	0.0233	0.000096	Ag	54.3	0.0231	0.000093
ClO_4	64.0	......	....	H	315.0	0.0154	−0.000033
IO_4	48.0	......	....	½ Ba	55.0	0.0239	0.000106
NO_3	61.7	0.0203	0.000047	½ Sr	51.0	......	....
MnO_4	53.4	......	....	½ Ca	51.0	......	....
OH	174.0	0.0179	0.00008	½ Mg	45.0	0.0255	0.000132
CHO_2	47.0	......	....	½ Zn	46.0	0.0256	0.000133
$C_2H_3O_2$	35.0	0.0236	0.000101	½ Cd	46.0	......	....
½ SO_4	68.0	0.0226	0.000084	½ Cu	46.0	0.0240	0.000107
½ Cr_2O_7	72.0	......	....	½ Pb	61.0	0.0244	0.000114
½ CO_3	60.0	0.0269	0.000155				
½ $(COO)_2$	63.0	......	....				

The mobilities at any other dilution ϕ may also be determined in the same way from a measurement of the conductivity Λ_ϕ at this dilution and the corresponding transport ratios n_a and n_c, but the product $n_a\Lambda_\phi$ or $n_c\Lambda_\phi$ for ions in fairly concentrated solutions is not independent of the nature of the electrolyte.

Interpretation of the Laws of Electrolytic Conduction on the Dissociation Theory. — The interpretation of the above experimental facts in the

light of the dissociation theory is simple. On this theory, the value of Λ_ϕ is determined by three factors, any one of which, if zero, will make $\Lambda_\phi = 0$. These are:

1. The fraction of the total number of molecules in one gram-equivalent of the solute which are dissociated at any instant. This fraction, called the "degree of ionization" or "ionization coefficient," is usually denoted by γ_ϕ, the subscript ϕ indicating the dilution to which it refers.

2. The average velocities with which the ions migrate while free. Let u_c and u_a be these velocities, in centimeters per second, for the cations and anions respectively, for a potential gradient of one volt per centimeter.

3. The charge carried by the ions. This is fixed, since one equivalent weight of every ion carries with it $F = 96{,}540$ coulombs.

If a volume ϕ of the solution containing one gram-equiv. of the solute be imagined placed between two parallel plate electrodes 1 cm. apart, and an e.m.f. of one volt be applied, the quantity of electricity conducted across the electrolyte in one second, i.e., the current, will be equal to the equivalent conductance Λ_ϕ (definition of Λ_ϕ). The current is also equal to the total quantity of electricity conducted by the cations plus that conducted by the anions; hence

$$\Lambda_\phi = \gamma_\phi F u_a + \gamma_\phi F u_c = \gamma_\phi F\,(u_a + u_c).$$

The experimental fact that for many electrolytes at infinite dilution Λ_ϕ is equal to the sum of two constants l_a and l_c (the mobilities of the ions) which depend only on the nature of the ions, indicates that the velocity of migration u_a or u_c of any given ion depends only upon the nature of the ion and not upon the electrolyte of which it forms a part, and that the degree of ionization at infinite dilution is the same for all such solutions; there is much evidence to indicate that this is practically 100 per cent (*see below*).

Degree of Ionization. — The fact that Λ_ϕ increases with the dilution may be due either to the velocities u_a and u_c or to γ_ϕ increasing with the dilution, or to both causes combined; F is a constant. In concentrated solutions both the velocities and the degree of ionization undoubtedly decrease as the concentration increases. For dilute solutions, however, there is considerable evidence to show that the frictional resistance opposed to the motion of the ions changes but slightly with increasing dilution, and hence the values of the migration velocities in such solutions may be regarded as nearly constant. If this be assumed, and also that the *nature* of the ions remains unchanged, i.e., that u_c and u_a at the dilution ϕ are practically the same as at infinite dilution, $\phi = \infty$, then the observed increase in Λ_ϕ with the dilution must be due to an increase in γ_ϕ. On these assumptions, the solute becomes more and more ionized as the dilution increases, and at infinitely great dilution all the molecules of the solute are continuously dissociated or ionized, i.e., as ϕ approaches infinity, γ approaches unity.

Hence for $\phi = \infty$, $\Lambda_\infty = F\,(u_c + u_a)$, and therefore

$$\frac{\Lambda_\phi}{\Lambda_\infty} = \frac{\gamma_\phi F\,(u_c + u_a)}{F\,(u_c + u_a)} = \gamma_\phi.$$

This relation affords the simplest way of determining the coefficient of ionization γ_ϕ of an electrolyte, namely, finding the ratio of its equivalent conductance at dilution ϕ to its maximum conductance at infinite dilution. The two assumptions mentioned above should not be lost sight of when applying this formula. In general, the viscosity of the solution must be corrected for except at great dilutions. See Washburn and Clark, *Jour. Am. Chem. Soc.*, *38*, *1916*.

The values of γ_ϕ computed by the above simple formula for a number of typical electrolytes are given in Table XI.

TABLE XI. — IONIZATION COEFFICIENTS OF TYPICAL ELECTROLYTES AT VARIOUS DILUTIONS COMPUTED FROM RATIO $\frac{\Lambda_\phi}{\Lambda_\infty} = \gamma_\phi$

(Kohlrausch and Holborn's data)

Liters per gm.-equivalent $= v = \frac{1}{1000}\phi$	Gm.-equivalents per liter $= m = 1000\,\eta$	NaCl	HCl	KOH	$\frac{1}{2}CuSO_4$	NH_4OH	$C_2H_4O_2$
1	1	0.675	0.784	0.770	0.217	0.0037	0.0036
10	0.1	0.839	0.914	0.891	0.379	0.0139	0.0131
100	0.01	0.933	0.964	0.954	0.608	0.0403	0.0406
1000	0.001	0.978	0.982	0.979	0.856	0.118	0.117

Note.—The ½ before the $CuSO_4$ indicates that 1 gram-equivalent of $H_2SO_4 = \frac{1}{2}$ mol. of H_2SO_4.

Calculation of Velocity of Migration of Ions. — From a knowledge of the mobility of an ion its actual velocity of migration may be easily computed on the assumptions of the dissociation theory. Thus for aqueous solutions at infinite dilution, in which the ionization may be assumed complete, $l_c = Fu_c$ and $l_a = Fu_a$; hence $u_c = \frac{l_c}{F}$ and $u_a = \frac{l_a}{F}$. From Table X we have for the sodium ion $l_{Na} = 43.5$ and for the chlorine ion $l_{Cl} = 65.5$; therefore the velocity in centimeters per second per volt per centimeter with which these ions migrate at 18° C. is $\frac{43.5}{96,540} = 0.000450 \frac{\text{cm.}}{\text{sec.}}$ and $\frac{65.5}{96,540} = 0.000677 \frac{\text{cm.}}{\text{sec.}}$ respectively. For other dilutions the degree of ionization must be taken into account. The values given in Table VII are obtained in this way. For the relation of velocity of migration to diffusion phenomena see Nernst's *Theoretical Chemistry.*

Effect of Concentration on Degree of Ionization. — It has been pointed out that the concentration of a solution has relatively little effect on the velocity with which the ions migrate, except at high concentrations. Hence the large increase of the equivalent conductance with the dilution ϕ, see *Fig. 4, and Table IX*, must be due to a change in the degree of ionization. It can be shown on theoretical grounds that the ionization coefficient γ of a binary electrolyte should vary with the dilution ϕ or concentration η according to the equation

$$\frac{\gamma^2}{\phi(1-\gamma)} = K \quad \text{or} \quad \frac{\eta\gamma^2}{1-\gamma} = K,$$

in which K is a constant called "the equilibrium constant of the ionization reaction," for the given temperature and pressure. This formula, known as "Ostwald's Dilution Law," holds exactly only for weak electrolytes, i.e., those which are slightly ionized like the organic acids, and which give rise to dilution curves of the type *B, Fig. 4.* For good electrolytes, giving curves of the type *A*, the formula fails completely. A wholly satisfactory explanation for this discrepancy has not yet been given. (*See discussion in address by James*

Walker, British Association Meeting, 1911. Also G. N. Lewis, Zeit. f. physik. Chem., 70, 216, 1910.) Instead of obeying the above law, curves of the type A satisfy an exponential equation of the form

$$\frac{(\eta\gamma)^n}{\eta(1-\gamma)} = \text{constant},$$

in which n is a constant depending upon the electrolyte and having a value between 1.40 and 1.56. Other "dilution laws" have also been proposed by Kohlrausch, Van't Hoff, Rudolphi, Kraus, Bray, and Bates.

Effect of Temperature on Ionization. — The per cent to which a substance is ionized in solution may increase or decrease with the temperature. The sign and magnitude of the effect depend upon whether the ionization reaction is accompanied by an absorption or evolution of heat. Substances which dissociate with evolution of heat become less ionized with increasing temperature; substances which dissociate with absorption of heat become more ionized with rising temperature. Highly dissociated substances like sodium chloride, hydrochloric acid, etc., belong to the former class. Water which is very slightly ionized belongs to the latter class. In general a variation of temperature from 0° to 100° C. produces a change in ionization of only a few per cent. Recent investigations by Noyes carried out at temperatures up to 360° C. have confirmed these predictions of theory. The effect of temperature on γ may be computed through its effect on K by means of Van't Hoff's thermodynamic relation

$$\frac{d \log K}{dt} = \frac{Q}{\bar{R}T^2},$$

where K is the above equilibrium constant of the ionization reaction, Q the heat of the corresponding reaction, $\bar{R}$ the gas constant and T the absolute temperature to which the values of K and Q refer. For derivation of this equation see Nernst's *Theoretical Chemistry*.

Negative Temperature Coefficients of Electrical Conductivity. — The fact that certain electrolytes, e.g., a phosphoric-acid solution, may have a negative temperature coefficient is readily explained in terms of the above relations. If the increase in the velocity of migration of the ions with rising temperature is more than offset by a diminution in the average number of free ions, resulting from a decrease in ionization, the conductivity of the solution will diminish. By combining solutions having positive and negative temperature coefficients in suitable proportions, electrolytes have been prepared which have nearly a zero temperature coefficient over quite a range of temperature. The following mixture, proposed by Manganini, has this property: 121 grams mannite, 41 grams boracic acid, 0.06 gram potassium chloride dissolved in sufficient water to make one liter. Its specific conductance at 18° C. is $\kappa = 0.00097$. Such a solution is well adapted for a liquid resistance just as manganine wire is adapted for resistance coils.

Conductivity of Fused Electrolytes. — In Table XII are given the specific conductance (κ) and the equivalent conductance (Λ) of several typical fused salts at their respective melting points; also for comparison the equivalent conductance of these salts in infinitely dilute aqueous solutions at 18° C. It will be seen that the values of the specific conductance (κ) of the salts at their respective melting points are much greater than the values of the specific conductance of the same salts in normal aqueous solutions. On the other hand, if the volume ϕ occupied by one gram-equivalent of the salts in the fused state is determined by specific gravity measurements and the equivalent conductance Λ computed ($\Lambda = \phi\kappa$), it will be seen that these values are less

than the maximum equivalent conductance Λ_∞ of the same salt in aqueous solutions. Thus the conducting power of one equivalent of fused sodium nitrate at 305° C. is less than when the same weight of salt is dissolved in one or more liters of water.

TABLE XII. — SPECIFIC AND EQUIVALENT CONDUCTANCE OF TYPICAL FUSED SALTS AT THEIR MELTING POINTS

Salt	Temperature, melting point, °C.	Specific conductance, κ	Equivalent conductance, Λ	Λ_∞ at 18° C. in aqueous solution
$NaNO_3$	305	0.9510	42.15	105.2
KNO_3	334	0.6225	33.59	126.3
$LiNO_3$	250	0.7886	30.36	95.1
$AgNO_3$	218	0.6815	29.22	116.0
$AgClO_3$	215	0.3676	18.09	109.3

If electrical conduction in fusions is wholly electrolytic it is probable that it is determined by the same three factors which determine the conductance of solutions, and that the equivalent conductance may be expressed by an equation of the general form

$$\Lambda = \gamma F\,(u_c + u_a).$$

As yet, however, all attempts to measure the velocity with which ions migrate in fused salts have led to no satisfactory or conclusive results. Hittorf's transference ratios cannot be determined as in solutions, since in a pure fused salt concentration differences do not occur at the electrodes. Only indirect estimates of the value of γ have been possible. The evidence thus far obtained, however, points to a high rather than to a low state of ionization in molten salts. For a résumé of the present status of this question see Goodwin, *Trans. Am. Electrochem. Soc., 1912.*

The equivalent conductance of fused salts increases in general from 1 to 1½ per cent per degree centigrade. As the increase has been found in a number of cases to be almost identical with the increase in the fluidity of one equivalent weight of the fused mass, it is probable that the velocity of migration of the ions increases in this proportion. The ionization of fused salts is certainly not a result of temperature, as in the case of gases; there is evidence, in fact, that the ionization of some fused salts may decrease with increasing temperature as it does in their aqueous solutions.

THEORY OF CONTACT POTENTIALS. — In the case of liquid-liquid junctions and metal-liquid junctions a satisfactory theory, based on the dissociation theory and the theory of solutions, has been worked out.

Theory of Liquid-liquid Potentials. — Nernst (*Zeit. für Phys. Chem., 4, page 129, 1889*) was the first to show that in the case of electrolytes a difference of potential necessarily arises between two solutions in contact which gradually diminishes, until the solutions are completely mixed. The general theory, worked out by Planck (*Ann. der Physik, Vol. 40, page 561, 1890*) is complicated. It leads, however, to simple formulæ in the following special cases.

Case I. — The junction is between two dilute solutions of the *same solute at different concentrations* c_1 and c_2, both ions of the solute having the valency n:

for example, c_1-HCl — c_2-HCl or c_1-$ZnSO_4$ — c_2-$ZnSO_4$. The potential rise between solutions 1 and 2 is given by the equation

$$e = \frac{\bar{R}T}{nF}\frac{l_c - l_a}{l_c + l_a}\log_e\frac{c_1}{c_2}$$
$$= \frac{0.000198}{n}T\frac{l_c - l_a}{l_c + l_a}\log_{10}\frac{c_1}{c_2}\text{ volts,}$$

where $\bar{R}$ = the gas constant; T = absolute temperature; F = the Faraday = 96,540 coulombs; n = valence of ions; l_c and l_a the mobilities of the cations and anions, respectively; and c_1 and c_2 the concentrations (or, more exactly, the corresponding osmotic pressures) of the ions in the two solutions respectively.

It follows from this formula that the potential at a liquid junction approaches zero if the electrolyte is composed of two ions which migrate with nearly equal velocities, and increases as the difference in their velocities increases. Thus $\overset{+}{K}$ and $\overset{-}{Cl}$ ions have nearly identical mobilities, and hence between any two solutions of potassium chloride there exists only a very small potential difference. For a concentration ratio of 10 to 1 the potential difference at 18° C. is only 0.0004 volt. This theory also explains why liquid-liquid potentials are greatest between acids or between alkali solutions, for hydrogen and hydroxyl ions possess relatively great mobilities.

Case II. — The liquid junction is between solutions of two *different solutes*, each containing two ions of the same valence n and at the *same concentration*. Here again the process of diffusion gives rise to potential difference between solution 1 and solution 2, the value of which may be computed by the formula

$$e = \frac{\bar{R}T}{nF}\log_e\frac{l_{c_2} + l_{a_1}}{l_{c_1} + l_{a_2}}$$
$$= \frac{0.000198\,T}{n}\log_{10}\frac{l_{c_2} + l_{a_1}}{l_{c_1} + l_{a_2}}\text{ volts,}$$

where l_{c_1}, l_{a_1} and l_{c_2}, l_{a_2} are the mobilities or migration velocities of the cations and anions in the two solutions respectively. The above formulæ as well as the general formula for mixed electrolytes have been satisfactorily verified by experiment.

For most recent work see *Trans. Faraday Soc. Vol. 8, p. 86, 1912.*

Theory of Metal-liquid Potentials. — By extending the conception of the process of solution of solids in liquids to the case of metals, which pass into solution only as positively charged ions, Nernst (*Zeit. f. Phys. Chem., 4, 129, 1889*) developed an osmotic theory of metal-liquid potentials. This applies to all kinds of electrodes which are reversible in the thermodynamic sense, or which from an experimental standpoint are non-polarizable. For all such reversible electrodes Nernst has shown that the potential from metal to liquid may be expressed by the relation

$$e = \frac{\bar{R}T}{nF}\log_e\frac{P_M}{p}$$
$$= \frac{0.000198}{n}T\log_{10}\frac{P_M}{p}\text{ volts,}$$

where again $\bar{R}$ is the gas constant, T the absolute temperature, n the valence of the metal ions, F the Faraday, p the partial osmotic pressure of the metal ions present in the solution and P_M an integration constant characteristic for each metal, which depends upon the temperature and solvent, but is independent of the nature of the solute.

Electrolytic Solution Pressure. — The constant P_M in the above formula is called the "electrolytic solution pressure of the metal M." Since $e = 0$ when $P_M = p$, the electrolytic solution pressure P_M is sometimes regarded as equal to the osmotic pressure of the metal ions which would be just sufficient to prevent the metal electrode from dissolving or passing into the ionic form, in which case there would be no development of a potential difference at the metal-liquid junction due to this cause. Hence the numerical value of P_M for a given metal and solvent may be considered as a measure of the tendency for the metal to dissolve in the solvent, i.e., to pass from the metallic to the ionic form. Thus the alkali and alkali-earth metals, which pass readily into the form of ions, have a very high electrolytic solution pressure, while the noble metals like gold, silver and platinum have a very low solution pressure. From this point of view the electrolytic solution pressure of a metal may be taken as a measure of its place in the electrochemical series. The only method thus far proposed for obtaining the value of P_M is by computation from Nernst's formula for electrode potentials given above. The value to be assigned to e depends upon the experimental determination of the "absolute" difference in potential (*see next paragraph*) between the electrode and the solution with which it is in contact. At present there is some question as to whether the actual difference of potential between the electrode and solution may not be partially due to the phenomenon of selective absorption of the ions, which is not taken into account in the above formula. For this reason the values of P_M frequently computed from the measured electrode potentials are open to question. See Le Blanc's *Lehrbuch der Elektrochemie, pages 230–235;* Lehfeld, *Phil. Mag. 48, 430, 1899.*

Absolute Electrode Potentials. — On the assumption that the potential difference between a metal and a solution is zero when the surface tension of the metal is a maximum and that this condition is fulfilled in "dropping electrodes," or may be obtained in a properly adjusted Lippmann capillary electrometer, the absolute potential rise from the solution to the mercury in the normal calomel electrode has been found to be $e = 0.5600 + 0.0006(t - 18° \text{ C.})$ volt. It is in terms of this value for the normal calomel electrode that "absolute" electrode potentials have been expressed. On account of the uncertainty attaching to the interpretation of this value it is better for the present to adopt the arbitrary value zero as the potential drop at the calomel electrode.

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ELECTRODYNAMOMETERS. — (*See also Ammeters; Balances, Current; Wattmeters.*) One of the most reliable and accurate laboratory instruments for current and power measurements, particularly a-c. measurements, is the electrodynamometer.

This instrument depends upon the action of one circuit carrying current upon another carrying the same current (*see Electricity and Magnetism, Principles of*). The working parts of the instrument consist of two coils, one fixed and the other movable.

SIEMENS ELECTRODYNAMOMETER. — The simplest form of electrodynamometer is that devised by Siemens, shown diagrammatically in Fig. 1. The coils are arranged at right angles to one another as shown, the heavy lines representing the fixed coil and the light lines representing the movable coil. The force of attraction or repulsion is proportional to the current in the fixed coil times the current in the moving coil, and consequently if the same current flows in the two coils, the deflection is proportional to the *square* of the current. This force is balanced by the torsion of a spring. By turning the torsion head at the top of the instrument the coils are kept at right angles. The torsional force is proportional to the angle D through which the head is turned; hence the current is

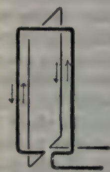

Fig. 1. Siemens Dynamometer

$$I = K\sqrt{D},$$

where K is a constant which is obtained by sending a known current through the instrument. When arranged with separate binding posts for the current or fixed coil and for the potential or swinging coil, as shown in Fig. 2, the instrument may be used as a wattmeter for either direct-current or alternating-current power measurements. The power in watts is then

$$P = K^2R_1D,$$

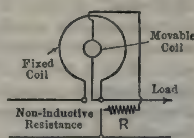

Fig. 2. Dynamometer Wattmeter

where K has the same value as before, and R_1 is the total resistance of the potential circuit, including the coil and external resistance R. This power includes the loss in the potential circuit, which is V^2/R_1, where V is the impressed voltage; this correction is usually small.

Uses of Siemens Electrodynamometer. — The instrument may be calibrated on direct current and may be used to measure either direct or alternating current or power, since when properly designed its readings are independent of frequency and wave form. High-grade standard wattmeters suitable for the precise calibration of commercial indicating wattmeter or watt-hour meters on alternating current and non-unity power-factor circuits are made on the electrodynamometer principle (*see Wattmeters*).

Design of Siemens Electrodynamometer. — The movable coil is usually suspended by a silk thread. Attached to the coil, at the point of suspension, is a spiral spring, the other end of which is attached to a movable collar. This collar carries a pointer which moves over a scale and denotes the angle of torsion of the spring. A second pointer is rigidly attached to the movable coil and registers with a fixed mark on the scale, when the fixed and movable coils are at right angles. The current is led in and out of the movable coil through mercury cups.

Dynamometers and wattmeters for use with alternating current should have as little metal used in their construction as is practical, as eddy currents will be set up in the metal parts that will influence the coil system and change the con-

stant of the instrument. The case and supporting frames should therefore be of insulating material.

Range of Siemens Electrodynamometer. — Commercial instruments of this type for measuring currents are not suitable for measuring currents of less than 0.02 ampere. Precision wattmeters (q.v.) built on the same principle may be used to measure power of the order of 1,000,000 watts on unity power factor.

Sources of Error. — The interaction of the earth's magnetic field and the current in the movable coil may be appreciable when the instrument is used for direct-current measurement unless the movable coil is perpendicular to the earth's field. This action may be detected by sending a direct current through the *movable coil only;* any deflection will be due to the earth's field.

When the instrument is used to measure alternating currents the earth's field has no effect. However, eddy currents induced in the various parts of the windings or case, or in neighboring conductors, may produce a disturbing influence. This may be tested by sending an alternating current through the *movable coil only*, the other coil being open; any deflection will be due to eddy currents. The "zero" position of the pointer should also be such that there is no mutual induction between the two coils when in the "zero" position. This can be tested by short-circuiting one coil and sending an alternating current through the other; any deflection will be due to mutual induction.

The instrument can be used for measuring the power input to circuits of low power factor only when the self induction of each coil and of the external so-called "non-inductive" resistance (*see Fig. 2*) is negligible (*see Fleming, Handbook of the Electrical Laboratory and Testing Room*).

Degree of Precision. — When properly calibrated and properly used the readings of a good Siemens electrodynamometer may be relied upon as accurate to within 1/10 per cent for a full-scale deflection. The smaller the deflection the less the percentage accuracy.

Rowland Electrodynamometer. — This instrument is identical in principle with a Siemens electrodynamometer, except that instead of noting the angle through which a torsion head must be turned, the moving coil is allowed to deflect and the deflection is noted by means of a telescope and scale. Its chief application is for measuring small alternating currents and small amounts of power in alternating-current circuits. It is seldom used except in special investigations requiring a high degree of precision or for the measurement of alternating quantities of small magnitude; e.g., alternating currents of the order of 0.1 ampere or power of from 0.01 to 1 watt. Its uses are fully described in Rowland's "Physical Papers," Johns Hopkins Press, Baltimore, 1902.

Irwin Astatic Electrodynamometer. — This instrument has two fixed circular coaxial coils of the same diameter through which the current circulates in opposite directions, producing a field which is directed radially outward. The movable element is suspended by a wire or silk fibre and consists of two semicircular coils mounted on a thin disc of mica, the current in each being in opposite directions. Any effect which may be due to a non-uniform stray field is reduced by the fact that the movable coils are close together.

COSTS (Pre-war prices). — A simple Siemens electrodynamometer suitable for measuring currents of from 0.025 to 5 amperes costs about $75. A Rowland electrodynamometer with shunt box costs about $450. An Irwin astatic galvanometer costs (in 1920) about $150.

BIBLIOGRAPHY. — Laws, F. A., *Electrical Measurements;* Fleming, J. A., *Handbook of the Electrical Laboratory and Testing Room;* Gerard, E., *Mesures Électrique.*

ELECTROLYSIS OF UNDERGROUND STRUCTURES. — (*See also Electrochemical Processes, Industrial; Electrochemistry, Principles of.*) The following discussion will be limited to the electrolysis of underground structures, that is, to the corrosion of metal work in the earth, due to the action of stray currents from the grounded rails of electric railroads.

The track rails of electric railways practically always form part of the electric circuit feeding the cars. As these rails are either laid in the ground or on ballast on the ground, and as the ground is a partial conductor, some of the current will leave the rails and flow in the ground. If pipes, lead-sheathed cables or structural foundations happen to be in the ground near the railway, part of the stray current from the rails will enter these underground structures and flow along them, leaving them at some other point to return to the station bus. If the current is direct, the underground structure from which current leaves to enter the earth, becomes the anode of an electrolytic cell, the earth containing the electrolyte. This anode corrodes, due to the liberation of acids from the salts contained in the earth. Occasionally the cathode, or underground structure into which current enters from the earth, is liable to corrosion by secondary causes. In the case of railways using alternating current for distribution, the stray currents are usually a large proportion of the total current, but the electrolytic effects are negligible. (See J. L. R. Hayden, *Trans. A.I.E.E., Vol. 26, p. 221.*)

In the case of direct-current railways, it is therefore desirable to confine the current to the track rails as far as practicable. This may be done by maintaining the rail bonds in a high degree of efficiency and by keeping down the voltage drop in the rails by means of insulated negative feeders, as described by G. I. Rhodes, *Trans. A.I.E.E., Vol. 26, p. 247, 1907.*

While such means will assist materially in mitigating electrolysis troubles, it is often necessary to take special steps to protect underground structures without relying upon the railway companies to confine their currents to the rails.

MINIMUM VOLTAGE TO PRODUCE ELECTROLYSIS. — The conduction of current through moist ground is almost entirely electrolytic, ordinary conduction being almost negligible. Electrolytic corrosion occurs whenever current flows from metal into the ground, regardless of whatever difference of potential may exist between different parts of the circuit. In order that current may flow, however, it is in general necessary that the difference of potential between anode and cathode exceeds the algebraic sum of the e.m.f.'s of combination and separation of the compounds involved in the electrolytic process. An iron anode in a soil containing iron salts in solution will be attacked when the e.m.f. is infinitesimal.

CORROSION EFFICIENCY AND PASSIVITY. — When an electric current is passed through an electrolyte, the mass of the anode dissolved may be calculated theoretically by Faraday's law (see index). In practice this amount may be exceeded, or the reverse. The term "coefficient of corrosion" or "corrosion efficiency" is used to designate the ratio of the actual to the theoretical mass of the anode dissolved. The corrosion efficiency is usually less than 100 per cent owing to the occurrence of secondary reactions. Thus the corrosion of iron, which primarily occurs by the formation of ferrous salts, should theoretically occur at the rate of 1.045 grams per ampere-hour, but in practice the rate may be anywhere from zero to forty per cent more than the theoretical value. In the case of iron in soil, without protection, the corrosion efficiency is practically independent of the temperature over a range from 0° C. to 40° C. It is also independent of the kind of iron, whether wrought iron, steel or

machined cast iron. The efficiency is greater the lower the current density, varying from 20 to 140 per cent for a range of current densities between 5.0 and 0.05 milliamperes per square centimeter. It is greater with increasing moisture content up to saturation of the soil. Data on steel encased in concrete are given below, under "Protection of Steel by Concrete."

DISTRIBUTION OF EARTH CURRENTS. — The distribution of stray currents in the earth depends not only upon the resistance of the earth but also upon the resistance and distribution of the conducting structures in the earth.

Earth Resistance. — As the earth conducts electrolytically, its specific resistance can be calculated from the concentration of the salt solutions it contains. Such calculations are, however, generally useless, as, even with moderate specific conductance, the extent of the current path in the earth is so great that its resistance is very low, except near the electrodes. The effect of the electrodes is to concentrate the current streams into limited areas, thereby increasing the effective resistance of the earth. It is, therefore, obvious that the resistance of the earth depends more upon the electrode area than anything else.

The resistance of the earth is low when the electrodes consist of considerable lengths of track rails. This is particularly noticeable in the case of single phase lines, due to the skin effect and inductance of the rails. Thus J. Dalziel and J. Sayers (Inst. Civil Engineers, London, 1909) found that, on the Midland Railway, the current did not continue along the rails for any considerable length, but within a few hundred yards of the car sank gradually into the earth to re-enter the ballast at a very short distance from the power station. H. F. Parshall found that on a line eight miles long, the earth-return current was 60 per cent of the total. Probably the most exact data of this kind are those due to A. W. Copley (*Trans. A.I.E.E., Vol. 27, p. 1171, 1908*) who stated that on the New York, New Haven & Hartford Railroad, before the adoption of the three-wire system, the percentage of current in the earth was 25 per cent on the four-track sections and 60 per cent on the single-track sections.

Electrolytic Zone. — When a drop of potential occurs in a track rail, the current spreads out from it into the soil in much the same way that the leakage lines of magnetic force from a long electromagnet spread out into the air; that is, almost the entire leakage current is confined to a limited zone of the earth, the extent of this zone depending upon a complicated function of both the potential gradient and the total drop in the rails themselves. The stream lines of the current will tend to crowd into a pipe or cable sheath (on account of its low resistance) if such conductor is within this zone, but this tendency rapidly diminishes as the distance between the conductor and the rails increases. Compare with the tendency of an iron bar to become magnetized when placed in the leakage field of an electromagnet; the induced magnetization rapidly diminishes as the distance of the bar from the magnet is increased. Conductors at a comparatively short distance from the rails are, therefore, not liable to electrolysis unless the difference of potential between them and the rails is large or the leakage path of the current from the rails to the conductors is of low resistance. Thus in England, where the drop in the grounded return is limited to a total of seven volts, Messrs. Cunliffe found experimentally that the electrolytic zone is confined to within three feet of the track. The voltage drop under this rule is taken as the mean between the average and the momentary maximum values during a 15 to 30 minute period of a schedule run, at the time of maximum traffic, exclusive of exceptional occasions.

STRUCTURES AFFECTED BY CORROSION. — Stray currents can set up corrosion of the rails, rail spikes, cross-bonds, steel columns, etc., through which they escape into the earth, and they can set up corrosion of gas and water pipes, cable sheaths, building structures, etc., which they may enter and leave in their course through the earth. The protection of the former class of materials is entirely the concern of the railroad company which originates the earth currents; not so with the latter class, which is often of great concern to the municipalities or public-service corporations which own them. Electrolytic surveys should, therefore, be made with a clear understanding of which class of property is under suspicion of danger. As a rule, electrolysis is most destructive to the grounded metal of the railroads; foreign pipes, cable sheaths, etc., being affected in comparatively rare instances. Of all properties foreign to the railroads, the thin sheaths of telephone cables are most susceptible to electrolytic corrosion.

Destruction of Concrete by Electrolysis.— Some experimenters have found that where iron imbedded in concrete becomes an anode, not only is the iron corroded but the concrete also is cracked. This action is due to the internal stress set up by the increase of volume which the iron suffers when it changes to an oxide or salt. The current has no direct effect upon the concrete at the anode even when sufficiently strong to liberate chlorine from the brine used to impregnate it. (C. E. Magnusson, *Trans. A.I.E.E., June, 1911.*) At the cathode, however, the concrete becomes softened and loses its bond with the electrode. See papers by E. B. Rosa, B. McCallum and A. S. Peters, Nat. Assn. of Cement Users, 1912, and abstracted in *Eng. News, Vol. 68, p. 1162, 1912.* No action, however, occurs on concrete through which current flows, except at the electrodes.

ELECTROLYTIC SURVEYS. — An electrolytic survey is an investigation made to determine the condition of grounded metallic structures and the soil in which they are imbedded and of their electrical conditions with the view of ascertaining what conditions tending to produce damage, may exist.

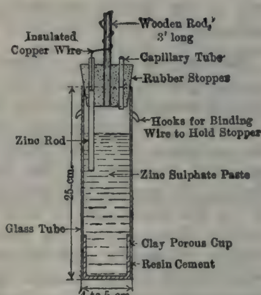

Fig. 1.

Potential Readings. — Random readings between rails and pipes give no quantitative information about electrolytic conditions. As a matter of fact, a difference of potential between a pipe and rail at any point is often greater the less the stray currents at that point. To illustrate this, we may assume a pipe to be outside the electrolytic zone and, therefore, safe from electrolytic corrosion. If, at any point, the rail is at the same potential as the pipe, the potential of the rail at every other point will differ from the pipe. Also, if the part of the rail which is at the potential of the pipe happens to be at some distance from the power station, the pipe will be positive to the rail at all points between the earthed point and the power station. Hence the fact that the pipe is positive to the rail does not indicate that it is subject to electrolysis. A more important illustration is the case of a pipe carrying little current itself but connected to some distant pipe which is carrying considerable current. In such a case, a pipe may be highly electropositive to the rails and yet be quite innocent of electrolytic tendencies. These illustrations are merely specific cases of the

general principle that potential readings are useless unless taken with reference to the resistance of the earth between rails and conductors, which in turn depends upon the direction and distribution of the current stream lines. The potential difference between the structure tested and earth affords more complete information than can be secured from any other practicable class of observations. For this purpose, it is necessary to use a non-polarizable electrode such as that shown in Fig. 1. The zinc sulphate paste shown therein is made by adding saturated zinc sulphate solution to fine zinc sulphate crystals until the mixture has attained a semi-fluid condition. The Bureau of Standards is developing improved types of electrodes.

Measurement of Leakage Current in Grounded Return System.— The columns and foundations of elevated structures and subways, rail spikes, bare negative cables and bonds are the parts most likely to be attacked electrolytically.

In the case of the elevated railways of New York City, after about nine years' operation with the steel structure a part of the return system, it was found necessary to remove all metallic connections in the Borough of Manhattan between the track rails and structure and to install negative feeder cables to compensate for the conductivity thereby sacrificed.

The corrosion of rail spikes, while happily rare, is a matter of such grave concern to the railroads and public that it should be carefully watched for. It occurs when ties are old and water-logged or when improperly treated with preservative. Timber is ordinarily classed with the non-conductors, because, when dry and well-seasoned, it has a high dielectric strength and practically infinite resistance. When green or moist, however, it becomes an electric conductor of comparatively low resistance. The resistance along the grain is much less than across it, and porous woods, such as oak, are better conductors than the non-porous woods such as pine. The conductivity of wood is due primarily to the presence in its pores of electrolytes formed from the salts found in natural timber, from preservatives and from salts originating from coal fumes or ashes. The flow of current, from the rails and spikes into the ties, fills the pores of the wood with iron salts, which add to the electrolytic conductivity and permit the leakage of more current. The effect is, therefore, cumulative, the leakage current increasing until the pores of the wood are completely saturated with electrolyte. Cases have been known where spikes were pitted more than half way through and where the rail flanges were badly corroded. Ties creosoted by the hollow-cell process, which leaves the fibers empty, are particularly likely to acquire high electrolytic conductivity. Red oak treated with zinc chloride is also a bad tie from the electrolytic point of view.

Columns of elevated railroads, subways, passenger stations, etc., are best tested for electrolytic trouble by means of a sensitive galvanometer used in the following way. Iron clamps with pointed tips are fastened to the column at points four or five feet apart, care being taken that the points penetrate the paint and make metallic contact with the steel. Wires are run from these clamps to a galvanometer and the deflection noted. The galvanometer having been calibrated, this gives the drop of potential in the column. The cross-sectional area of the column being known, its resistance may be calculated from the known resistivity of steel (usually 11 times that of copper). The potential drop, divided by the resistance, gives the flow of current in the column. Knowing the direction of flow, its amount and the efficiency of corrosion, the actual damage being done by electrolysis may be calculated as a definite weight of metal per annum. This method is being pursued with great success on the Electric Zone of the New York Central Railroad, the galvanometer employed

being a Queen & Co.'s E-8010 with tube E-8011, a calibration resistance and a tripod. A deflection of one scale division is equivalent to about 0.000003 volt. Considering an average column with a resistance of 0.000,004 ohm for a 4-foot length, a deflection of one scale division corresponds to three-fourths of an ampere. Where the columns are to be encased in concrete, permanent testing terminals should be provided, preferably in the form of small iron pipes screwed into the steel and ending flush with the concrete.

Measurement of Leakage Current in Pipes, Cable Sheaths, etc. — It is not uncommon to find potential readings taken between different systems of pipes, without regard to the location of the connections, the results being recorded as differences of potential between those systems of pipes. The potential of a pipe system, however, may vary from point to point, and consequently such readings have no significance unless the points between which the potential difference is measured are specified. The significance of potential readings between specific points is that they afford an indication of the potential gradient normal to the tracks and thereby help to determine the electrolytic zone.

Making use of all possible connections to the pipe, the potential difference between these points and the anode end of the grounded system should be determined as described below and the limits of the "electrolytic zone" ascertained by noting where the potential gradient in the pipe becomes negligible. For this purpose, water hydrants and water pipes constitute the best connection points.

Potential readings between points on the same pipe line or cable sheath are sometimes made the basis of potential curves showing the drop in the pipe or sheath. Such potential curves may be very significant when taken from cable sheaths, but are of little use for pipes on account of the variable joint resistance of the latter. Thus in case of a cable sheath of uniform resistance along the entire length tested, where the potential curve is flat, the sheath carries no current; when it is a straight sloping line, the sheath carries current without giving current to or taking current from the earth; where it changes its slope, current is either entering or leaving the sheath.

Hering's Method. — The current in a pipe may also be measured by the following method. (*C. Hering, Trans. A.I.E.E., June, 1912.*) The fundamental principle is as follows. Let *P*, Fig. 2, be a part of an underground pipe which has been uncovered and through which an unknown current *I* is flowing. Let *D* be a sensitive galvanometer, millivoltmeter or any other suitable form of detector; there should preferably be no variable resistance like an unbonded pipe joint between the two contact points. Let *A* be an ammeter, *B* a few battery cells and *R* an adjustable resistance; the shunt circuit containing them is connected as shown, anywhere outside of the points of application of the voltage detector, the farther away the better; they may even be on the other side of a joint.

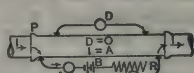

Fig. 2.

To find the current flowing in the pipe adjust the resistance *R* until *D* reads zero; then the current due to the battery *B* will be exactly equal and opposite to the current in the pipe. Hence the reading of the ammeter *A* gives the current in the pipe.

If *D* is a galvanometer with proportionate deflections, instead of a mere detector, then by taking a deflection immediately after the shunt circuit has been opened a reading proportionate to the drop of voltage for that current will be obtained. The instrument *D* is thereby calibrated to read the pipe currents directly and can be used for this purpose thereafter; the test with the battery current is therefore merely of the nature of a preliminary calibration, and need be carried out only once for each station.

Instead of attempting to adjust the current in the shunt to bring the voltage

D to zero, it is often more convenient to use a regular measuring instrument for D instead of a mere zero detector, and to pass a definite current through the shunt, say 10, 50 or 100 amperes, reading the two deflections of D when this current is on and when it is off; this had best be repeated several times. The difference between these two readings then corresponds to the current in the pipe. The best current to use is that which will reduce the original deflection as much as possible. By thus using the difference between a large and a small deflection the errors due to a loose zero, which are so common with highly sensitive instruments, are reduced.

Methods for overcoming fluctuating currents are given by Hering in the paper cited above.

Having thus calibrated the voltmeter V at each of two neighboring stations, the currents which enter or leave the pipe between them may be determined, with the fluctuating currents, by taking the readings of the two instruments simultaneously by means of visual or telephonic signals, preferably at times when the currents are momentarily steady.

Measurement of Current Leaving Pipe at Any Point. — The strength of the current at the point where it is leaving the sheath may be determined by the following test, which involves the use of a temporary bond in the form of a stout copper cable, electrically connected through an ammeter with both sheath and rail. Let

V = total drop of potential in the rails from *their* anodic center to the point where the temporary bond is connected.

By "anodic center of the rails" is meant the center of gravity of the leakage current *leaving* the rails.

v = difference of potential between the sheath and rail at the bonding point prior to the insertion of the bond.

v_1 = same, after the insertion of a bond which carries I amperes. The current in the bond is read from the ammeter.

Then the earth current, i, after the insertion of the bond will be

$$i = I \cdot \frac{k}{1 - k},$$

where

$$k = \frac{v_1}{v} \cdot \frac{V - v}{V - v_1}.$$

The earth current from the pipe, without bonding, would be

$$\frac{V - v}{V - v_1}(I + i).$$

For example, suppose the total drop in the rails to be 100 volts and the differences of potential between the pipe and the rails at the bonding point to be five volts before the bond is inserted, and one volt after. Suppose the current in the bond is found to be 50 amperes.

Then

$$V = 100. \qquad v = 5. \qquad v_1 = 1. \qquad I = 50.$$

$$k = \frac{1}{5} \cdot \frac{100 - 5}{100 - 1} = 0.19.$$

$$i = 50 \cdot \frac{0.19}{1 - 0.19} = 12 \text{ amperes, approximately.}$$

Without bonding, the current would be

$$= \frac{100 - 5}{100 - 1}(50 + 12) = 59 \text{ amperes approximately.}$$

The above formula is based upon the following assumptions:

(1) That the drop in the rails V is not affected by the insertion of the bond.

(2) That the area over which the current from the pip or sheath enters the earth is short. For proof of formula, see *Elect. World, 1910, Vol. 35, p. 407.*

MITIGATION OF ELECTROLYSIS.—Where stray currents are unavoidable, electrolysis may be mitigated by the use of drainage bonds, insulation or electric shielding.

Protection from Electrolysis by Drainage Bonds. — Bonds are not only useful for testing purposes; they may be applied permanently to reduce the earth currents. There are certain effects, however, which may render their use inadvisable.

1. If one piping system is connected to the rails or bus, a difference of potential will be established between it and all other underground metallic structures and it will, therefore, attract current from the latter and expose them to electrolytic danger. A bonded piping system thus becomes a part of the trolley return circuit and the owner may become a party to whatever damage may occur in the other structures.
2. A considerable current in a gas pipe is a serious fire hazard and in a lead cable sheath is a menace to continuity of service.
3. Electrolysis is promoted at all imperfect joints and connections.
4. In the case of power cables, the sheath currents may be great enough to add unduly to the heating of the cables.

In spite of these objections bonds are very largely used to protect cable sheaths from corrosion. To be most effective, the sheath should be connected to the negative bus by an insulated cable and the bus itself should be connected to the track rails by insulated cables only.

Protection of Steel by Concrete. — The conductivity of concrete depends upon its porosity and wetness. Tests have shown that when wet the specific resistance may be as low as 20 or 30 ohms per yard cube and when dry, as great as 2000 ohms. When the potential gradient in the concrete around an anode exceeds about 2 volts per centimeter, the temperature rises and the corrosion of iron is usually so rapid as to cause the concrete to crack open. At ordinary temperatures, the efficiency of corrosion of iron in concrete is low, but if the

Temperature, degrees, Centigrade	Corrosion efficiency, Per cent
10	2.0
20	2.5
30	3.0
40	3.5
50	5.0
60	11.5
70	22.5
80	37.5
90	44.5
100	47.0

temperature is high, due either to the high current density or external causes, the efficiency of corrosion may become quite high, as shown by the preceding table compiled from *Tech. Paper No. 18 of the Bureau of Standards.* For any fixed temperature the amount of corrosion for a given number of ampere-hours is independent of the current strength.

Passivity at ordinary temperatures is due principally to the drying of the film of anode rust by endosmose (*see Endosmose*) and to a less extent to the concentration of calcium hydrate near the cathode, where it becomes converted into chalk, which fills the pores of the concrete. Both of these actions have the effect of increasing the resistance from anode to cathode, but the effect of the former action, which is the principal one while the current flows, does not last long after the stoppage of current. (W. A. Del Mar and D. C. Woodbury, *Elec. World, 1917, Vol. 70, p. 916*).

Briefly stated, concrete affords good protection to steel work if the current density is not high enough to excessively heat the concrete.

Salt should never be used in concrete if there is the slightest probability of action by electric currents, since the addition of even a fraction of one per cent of chlorine is sufficient to increase the rate of corrosion a hundred fold.

Protection of Steel by Paint. — Experiments by M. Toch, C. E. Magnusson, G. H. Smith and others have shown that unpainted steel imbedded in concrete can be electrolytically corroded at the anode, but that a good insulating plant applied to the steel prevents such corrosion. Acid-proof paints with tar or asphalt base such as are commonly used to protect steel imbedded in concrete are usually effective. A typical paint of this kind has the following composition, 16 parts coal-tar paint, 4 parts Portland cement, 3 parts kerosene.

When the P. D. between cathode and ground does not exceed 5 volts the corrosive efficiency of an anode, so protected, is usually less than 1 per cent.

Protection of Sheaths by Tape or Braid. — H. W. Fisher found that electrolysis is not prevented by covering the lead with a weatherproof tape or braid saturated with insulating compound. With such coverings the electrolytic action is apt to be concentrated in spots and thus eat through the lead more quickly. For the same reason, lead-covered cables, laid in wooden boxes filled with pitch or bitumen, deteriorate rapidly under electrolytic action.

The Laclede Gas Company of St. Louis (*Elec. World, Vol. 57, p. 1103, 1911*) however, has had favorable experience with tape protection. Their wrought-iron pipe, in sizes from 3 to ¾-inch, was coated with a tar and pitch mixture, heated and thinned sufficiently to flow easily, and onto this a 4-inch paper ribbon was wrapped spirally, its edges overlapping. This paper covering was then tar painted and again wrapped with paper, the process being repeated until four successive coats were applied. Pieces of pipe thus insulated were placed in the ground under the most distinctive conditions of electrolysis, along with other lengths not so treated. After being taken up at the end of two years the unprotected pipes were badly pitted and almost consumed, while the insulated piping was practically the same as when laid. Although no test has yet been made carrying the insulated pipe to total disintegration, it is believed that pipe so treated will have its life at least doubled, and if this is true an expenditure for insulation equal to that of the cost of the bare pipe is justified. Only service runs are being so treated, the cast-iron mains being less subject to corrosion and electrolysis than the service pipes. The tar and paper coating is very hard when cooled, and the pipe lengths need to be handled with no more care than bare pipe (J. L. Fitzhugh).

Protection by Insulated Joints. — In a number of installations, flow of stray current on metallic pipe lines has been prevented by the use of a sufficient

number of insulating joints. It is found that where a pipe line is laid with every joint insulated, the line has such a high electrical resistance that no measurable current flows, although considerable potential gradient may exist in the earth parallel to the pipe line. Insulating joints in cable sheaths are not found to afford an effective means of preventing electrolysis.

Protection by Shielding. — Underground structures have been protected from electrolysis by connecting to the structure an auxiliary metallic conductor located so as to cause the current to flow to earth from the auxiliary conductor. The shielding conductor must be so placed as to prevent the current from leaving the structure to be protected, or at least, to cause its magnitude to be greatly reduced.

Protection by Booster. — It is often proposed to render grounded metal work electronegative to the rail return by means of a booster, but such proposals are seldom carried into execution on account of the expense they involve.

A scheme adopted with success at Karlsruhe, Germany, involves the use of a booster which does not have to carry the main current and which is therefore comparatively small. At places where there is danger for gas pipes or water pipes, electrodes are placed in the earth in the neighborhood of the pipes and these electrodes are connected to the positive pole of a low-voltage generator or storage battery, while the pipe to be protected is itself connected with the negative pole. In this way electric current is forced to enter the pipe from the earth so that anodic destruction of the pipe is impossible. The power consumption is said to be insignificant. (Geppert & Liese, *Elek., Kraftb. Bahnen*, Feb. 14, 1912.)

Boosters are used to prevent the pitting of condenser tubes, especially where salt water is used for cooling. An example of such an installation is in the Power Station of the Long Island Railroad on the East River, New York.

BIBLIOGRAPHY. — A very complete bibliography of the subject up to 1908 is given in a paper by W. H. Gee, *Electrolytic Corrosion*, Jour. Inst. El. Eng., 1908, Vol. 41, p. 425. In addition to these references and those given in the text, the following more recent papers should be consulted:

Report of the American Committee on Electrolysis, 1921. (This contains a great deal of valuable information on the subject, including a description of European practice and protective legislation. It also contains a bibliography of contributions which are regarded by the Committee as having permanent value.)

Bureau of Standards Technologic Papers, 15, 18, 25, 26, 27, 28, 32, 52, 54, 55, 62, 63, 72, and 75. (These papers cover almost every phase of the subject.)

ELECTROMAGNET WINDINGS. — (*See also Electricity and Magnetism, Principles of; Electromagnets, Lifting and Plunger; Generators; Motors; Transformers.*) Any coil or winding carrying an electric current forms an electromagnet; the term electromagnet is also generally used for such a coil, particularly when it has a magnetic core, whether it is actually carrying a current or not. The winding of most electromagnets is in the form of a cylindrical helix of one or more layers; such a winding is called a solenoid. This winding is usually built up on a bobbin, which may be fixed rigidly to a magnetic core, or the core or part of it may be movable, in which case the electromagnet is called a plunger electromagnet. The electromagnet may also be built upon a horseshoe shaped core, with a movable yoke or "armature" which is attracted to the core when a current is established in the winding. See *Electromagnets, Lifting and Plunger*.

DESIGN AND CONSTRUCTION OF WINDINGS — The problem is to construct a coil which, for a given voltage across its terminals, will produce a given number of ampere-turns, and which will not overheat. Usually one or more of the dimensions of the coil are also fixed by the conditions under which the coil is to be used. The following factors must be taken into account in designing a winding:

Insulation from Core; Design of Bobbin. — When the core is fixed, two washers of hard rubber or vulcanized fiber are forced on at either end of the core. The core is then insulated with a wrapping of paper, mica or oiled linen, and is then ready to be wound. When the core is movable, the two end washers are forced on to the ends of a brass tube or a tube made of the same material as the washers. When a metallic tube is used the washers are sometimes made of the same metal. In the case of a quick-acting plunger magnet a metallic bobbin, if used, should be slotted, in order to avoid eddy currents; this also applies to all forms of alternating-current electromagnets.

Insulation of Wires; Insulation Between Layers; Baking. — The wire may be insulated with a cotton, silk or asbestos wind or by a coating of enamel. In high-voltage solenoids the various layers of wire are insulated from each other by paper, mica or oiled linen wrappings, or the entire winding is divided into several sections separated by vertical washers of insulating material. The wound coil may be further insulated by dipping it into an insulating varnish in a vacuum or at atmospheric pressure, after which it is either air-dried or baked.

Aluminum wires and ribbons are used extensively by some manufacturers, especially abroad. In this case the layer of oxide on the wire is the only insulation which is used, but inasmuch as this insulation cannot be destroyed by heat, the coils can be run at much higher temperatures, which is of special value for lifting magnets and similar devices. The oxide layer will stand a potential stress of about 0.5 volt. See article on *Use of Naked Aluminum Wire in Electromagnets*, by H. F. Stratton, Elec. Wld., 1912, Vol. 60, p. 400.

Temperature Rise of Winding; Watts per Square Inch. — The rise of temperature of the winding will depend primarily upon the average rate at which heat is developed by the electric current and the amount of exposed surface from which this heat can be radiated; the temperature rise will also depend upon the depth of winding, the circulation of the air, etc. The hottest spot in the winding should never reach a higher temperature than 105° C. When the interior of the winding is at 105° C. the temperature of the external surface, as measured by a thermometer, will usually be much less (15° C. less), as will also the average temperature measured by the change of resistance method.

As a rough approximation a solenoid winding should be so designed that the average power developed will not exceed 0.5 watt per square inch of radiating surface for an open winding, and will not exceed 0.7 watt per square inch of radiating surface for an iron-clad solenoid. In figuring the radiating surface of an open winding, the surface of the hole through the solenoid is not included, and the end surfaces are included only when the solenoid is short. By the radiating surface in the case of an iron-clad solenoid is meant the surface of the winding which is in contact with the iron. A radiation of 0.5 watt per square inch and 0.7 watt per square inch for an open and an iron-clad winding respectively corresponds roughly to an average temperature rise of approximately 60° C.; for other rates of radiation the temperature rise will be approximately proportional to the watts per square inch radiated.

For short-time service, i.e., when the solenoid is energized only for short intervals with long intervals between the applications of power, the thermal capacity of the solenoid will permit of a greater dissipation of energy in the winding without overheating it.

Space Factors; Round Versus Square Wire; Layer Versus Haphazard Windings. — By the space factor of a winding is meant the ratio of the space occupied by the conductors to the total space occupied by the conductors, the insulation on the conductors and the voids between conductors. The space factor for strips and square wires is greater than for round wires, but strip and square wires are not extensively used in small sizes because of the increased amount of insulation required for a given section of conductor, and because of the tendency of such wires to twist in winding so that they lie upon their corners instead of upon their faces. However, for conductors of larger section than No. 10 B. & S. gauge, square wire is often used.

In winding wires larger than No. 18 B. & S. it always pays to wind them carefully in smooth layers ("layer" wound), but for smaller sizes used for open solenoids (as distinguished from iron-clad solenoids) the gain in space factor does not as a rule warrant this care and the wires are wound in a more or less haphazard fashion ("haphazard" wound). For iron-clad solenoids, however, a layer winding is always used, for economy of material requires that the winding space be kept as small as possible. The dotted curves A and B in Fig. 1 for haphazard windings are taken from an article by F. A. Willard (*Elec. Wld., 1906, Vol. 47, p. 823*).

Round wires are sometimes so wound that the wires of one layer lie in the hollows between the wires of the layer underneath; the wires in this case are said to be embedded. Objections to this procedure, however, are that each layer must be started from the same end and the insulation on the wires becomes tightly compressed and therefore is less effective; in most instances the extra labor and the diminution of insulating quality offset the small gain in space factor, which seldom exceeds 3 per cent.

The space factor s for a layer winding of round wire without embedding, and making no allowance for extra insulation between layers is

$$s = \frac{\pi}{4}\left(\frac{d}{d + 2t}\right)^2, \tag{1}$$

where d is the diameter of the conductor and t the thickness of insulation. Values of s for various sizes of wire and various thickness of insulation are given by the curves in Fig. 1. The "over-all" space factor, including the allowance for the space occupied by the extra insulation, if any, between layers, is equal to the value of s from these curves multiplied by $(1 - e)$, where e is the ratio of the space occupied by this extra insulation to the total winding space.

Thickness of Insulation. — Magnet wire is usually referred to as "single covered," "double covered" and "triple covered," depending upon the number of layers of insulating threads wrapped around it. Different manufacturers

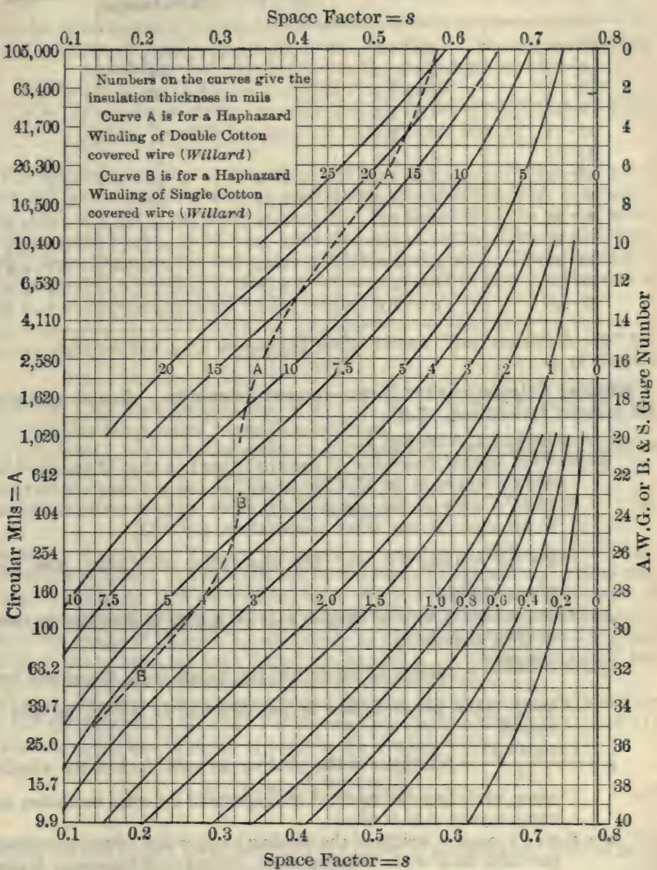

Fig. 1. Space Factor Curves

use different thicknesses for these layers, with the result that the thickness of the insulation on a "double cotton covered" wire of a given size or gauge number depends upon the manufacturer of the wire. In the following table is given the range in insulation thickness, taken from the catalogues of several large manufacturers.

INSULATION THICKNESS IN MILS

Size of wire A.W.G. or B. & S.	Single* cotton covered	Single* silk covered	Asbestos covered (Deltabeston)	Enamel covered
0000 – 00	4.5 – 7.5	....	....	....
0 – 3	4.5 – 7.5	....	18	....
4 – 7	4.5 – 7.5	....	16	....
8 – 10	4.5 – 7.5	....	14	....
11 – 12	2.5 – 5.0	....	12	....
13 – 15	2.5 – 5.0	....	10	1.5
16 – 19	2.5 – 5.0	1 – 2	10	1.1
20	2.0 – 4.0	1 – 2	10	1.1
21 – 23	2.0 – 4.0	1 – 2	....	1.0 – 1.0
24 – 28	2.0 – 4.0	1 – 2	....	0.8 – 1.1
29 – 33	2.0 – 4.0	1 – 2	....	0.7 – 1.0
34 – 36	2.0 – 4.0	1 – 2	....	0.4 – 0.7
37 – 40	2.0 – 4.0	1 – 2	....	0.3 – 0.6

* "For double covered" multiply these thicknesses by 2, and for triple covered multiply by 3.

Winding Calculations for Direct-current Solenoids. — Round solid wires are assumed throughout. Let

s = space factor, from Fig. 1.

k = ratio of specific resistance of conductor used to that of standard annealed copper at 20° C. For copper of 100 per cent conductivity at 20° C., $k = 1$; for copper of any other per cent conductivity, say C per cent, at any other temperature, say $t°$ C.,

$k = \frac{100}{C} + 0.004\,(t - 20)$. See also *Wires and Cables, Bare,* and *Wires, Resistor.*

A = cross section of wire in circular mils (= square of diameter in thousandths of an inch).

$n = \frac{1{,}270{,}000\, s}{A}$ = number of conductors per square inch, the square inch being taken perpendicular to the direction in which the wire is wound.

$\rho = \frac{1{,}100{,}000\, sk}{A^2}$ = resistance of the winding per cubic inch of the winding space, excluding the space, if any, occupied by extra insulation between layers; ρ is in ohms.

$w = 0.271\, s + 0.040$ = weight of the winding (copper and cotton insulation) per cubic inch of the winding space, exclusive of the space, if any, occupied by extra insulation between layers; w is in pounds.*

E = impressed volts.

(NI) = ampere-turns, where N is the total number of turns and I the current in amperes.

* This formula also holds approximately for other kinds of insulation and for most alloy resistance wires. Calling g_c the specific gravity of the conductor and g_i the specific gravity of the insulation, the exact formula is

$$w = 0.0362\,(g_c - g_i)\, s + 0.0284\, g_i.$$

Underhill gives g_i as 1.6 for asbestos, 1.4 for cotton, and 1.0 for silk.

p = allowable watts per square inch of radiating surface (may be taken approximately as 0.5 for open and 0.7 for iron-clad solenoids, see above).

S = radiating surface, in square inches, calculated as described above under *Temperature Rise of Winding*.

l = mean length of turn in inches (see Fig. 2 for a simple solenoid).

T = depth of winding space in inches (see Fig. 2), excluding the space occupied by the extra insulation, if any, between layers.

L = length of winding space in inches (see Fig. 2).

$V = LlT$ = volume of winding space in cubic inches, excluding space occupied by the extra insulation, if any, between layers.

The problem is usually to find the size of wire and necessary winding space for a coil which will give, without overheating, a required number of ampere-turns (NI) at a given impressed voltage E. The diameter of the core or spool is also usually known, or at least must not exceed certain fairly well-defined limits, depending upon the service for which it is to be used. The procedure is then to assume a reasonable value for the mean length of turn l. The size of wire is then immediately fixed by the relation

$$A = \frac{kl\,(NI)}{1.16\,E}. \tag{2}$$

From Fig. 1 the corresponding size of wire (A. W. G. or B. & S. gage) and the corresponding space factor s may then be found.

The volume V and radiating surface S of the coil must then satisfy the relation

$$\frac{LTS}{l} = \frac{k\,(NI)^2}{1{,}470{,}000\,ps}. \tag{3}$$

The length L and depth T of the winding space must be so chosen that this relation will be satisfied; as a rule this can be done only by cut and try. Note that changing the depth T of the winding space will also change the mean length of turn l, unless the diameter of the core or spool is so altered as to keep l constant. The cross section of the wire varies directly as l, as shown by equation (2), and therefore the value of the space factor s to be used in (3) will depend upon l, but only to a slight extent except in the case of very small wires.

Having determined the cross section of the wire (A), the mean length of turn (l) and the dimensions (L and T) of the winding space so that both (2) and (3) are satisfied, the total number of turns in the winding will be

$$N = nLT = \frac{1{,}270{,}000\,sLT}{A}, \tag{4}$$

and the current is then equal to the given number of ampere-turns divided by N and the total length of wire is equal to Nl.

The total resistance R and total weight W (including insulation) of the wire may then be found directly from a wire table, or may be calculated from the formulas

$$R = \rho V = \frac{klN}{1.16\,A} = \frac{1{,}100{,}000\,skLTl}{A^2}, \tag{5}$$

$$W = wV = (0.271\,s + 0.040)\,LTl. \tag{6}$$

See note at bottom of preceding page regarding w.

Calculation of Open Solenoid of Circular Cross Section. — In the case of a coil wound on a spool of diameter D, see Fig. 2, the mean length of turn is $l = \pi (D + T)$. Assuming that the outside cylindrical surface of the winding is the only radiating surface, which is only approximately true, as pointed out above, $S = \pi (D + 2T)L$. Whence, putting

$$Q = \frac{k(NI)^2}{1,470,000\ ps}, \qquad (7)$$

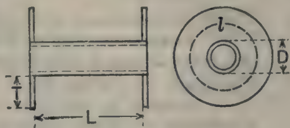

Fig. 2.

the required length of coil for a given diameter of core D and depth of winding T is

$$L = \sqrt{\frac{(D+T)Q}{(D+2T)T}}. \qquad (8)$$

When L is given instead of T, then the required depth of winding is

$$T = \frac{1}{4}\left[\left(\frac{Q}{L^2} - D\right) + \sqrt{\left(\frac{Q}{L^2} - D\right)^2 + \frac{8QD}{L^2}}\right]. \qquad (9)$$

In either case, the number of turns, total resistance, weight, etc., are found as described above for the general case.

Example. — Required to design an open solenoid 10 inches long and having an internal diameter of 1.5 inches, to give 12,000 ampere-turns at 110 volts, the heat developed not to exceed 0.5 watt per square inch of radiating surface (taken as the outside cylindrical surface of the coil). Assume a mean length of turn equal to 10 inches, then from (2) the required cross section of the wire, assuming copper at 70° C., is $A = 1130$ cir. mils. From Fig. 1 the space factor is then $s = 0.62$, assuming single cotton-covered wire with 2-mil insulation wound in layers. From (7) the value of Q is then $Q = 380$, whence from (9) the required depth of winding is $T = 2.36$ inches. The actual mean length of turn is then $l = 12.1$ inches which substituted in (2) gives for the cross section of the wire required, $A = 1368$. This wire section gives a space factor, see Fig. 1, of $s = 0.63$, which agrees practically with the value $s = 0.62$ used above. The nearest commercial size of wire is No. 19 B. & S., having a cross section of 1288 cir. mils. If this size of wire is used the actual ampere-turns will be $NI = 11,300$. From (4) the total number of turns will then be $N = 14,700$, and from (5) the total resistance will be $R = 143$ ohms. The current is then $I = 0.77$ amperes. The total weight of the winding is then from (6),

$$W = 60 \text{ pounds.}$$

BIBLIOGRAPHY. — Hedges, G. L., *Solenoid and Electromagnet Windings*, A.I.E.E., Proc. 34, Nov., 1915; George and Pender, *Calculation of Electromagnet Windings*, El. World, 65, p. 529, Feb. 27, 1915; see also *Bibliography* in article on *Electromagnets, Lifting and Plunger*.

ELECTROMAGNETS, LIFTING AND PLUNGER. — (*See also Electricity and Magnetism, Principles of; Electromagnet Windings.*) A solenoid provided with a movable magnetic core, or with a fixed core and movable "armature," serves as a very convenient means of causing an electric current to produce a direct mechanical pull This principle is utilized in various forms of lifting magnets, relays (q.v.), contactors (*see Switches*), electric brakes, clutches, etc. In the paragraphs following are given the formulas required in calculating the pull of various kinds of electromagnets in terms of their dimensions and ampere-turns, and also a brief statement of the applications of the various types.

Approximation of Formulas; Leakage Factor. — In applying the formulas given below, it should be noted that in general the effect of magnetic leakage is neglected. The leakage factor varies so greatly with the different forms of electromagnets that it is impossible to go into this matter in detail in the limited space available for this article. The designer, in making an allowance for leakage, has to rely chiefly upon his previous experience with other magnets of the same general form and dimensions. Merely as a rough guide to the designer who has not had this experience, it may be stated that for magnets of reasonable shape and dimensions, the formulas for pull given below may be relied upon to give the actual pull with an error of less than ± 10 per cent, the actual pull usually being less than the calculated pull. Under certain conditions the agreement between the actual pull and calculated pull may be much closer than the difference just stated.

SIMPLE SOLENOID AND PLUNGER. — The simplest type of electromagnet is a simple solenoid, as shown in Fig. 1, consisting of a cylindrical coil of circular or rectangular section and an iron plunger which fits into the inside of this coil. When current passes through the coil the plunger is attracted.

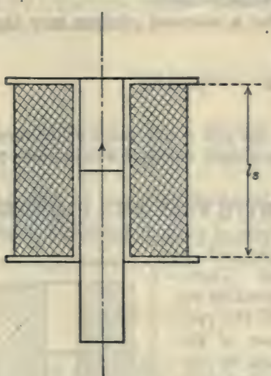

Fig. 1.

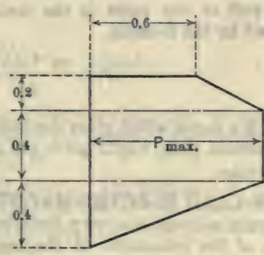

Fig. 2.

Variation of Pull of Simple Solenoid During the Stroke. — The pull of the simple solenoid varies between approximately zero when the plunger is at the lower end of the coil and a maximum which is nearly constant over approximately 40 per cent of the length of the coil. The maximum is reached when the plunger has entered the coil a distance of approximately 40 per cent. When the plunger has entered the coil 80 per cent of its length, the pull de-

creases again, reaching a value of about 0.6 of the maximum pull when the plunger is even with the top of the coil. The approximate pull variation is shown in the diagram, Fig. 2; of course the actual variation is a smooth curve, such as shown by curve A in Fig. 6.

Calculation of Pull of Simple Solenoid. — The calculation of the exact pull of such a magnet is very complicated, being dependent not only upon the ampere-turns, but also upon the shape of the coil, the size and length of the plunger and the induction in same. For practical purposes the maximum pull, when the plunger is at least as long as the coil, can be represented by the formula

$$P_1 = \frac{cANI}{l_s} \qquad \text{pounds,} \tag{1}$$

where c = the pull in pounds per square inch per ampere-turn per inch of coil length,
A = the area of the cross-section of plunger, in square inches,
I = the current in the coil, in amperes,
N = the number of turns in the coil,
l_s = the length of the coil, in inches.

It has been found by the author that c varies between 9×10^{-3} and 10.5×10^{-3} when the plunger is *magnetically saturated*. For practical purposes sufficiently close results for a *saturated* plunger are obtained if c is taken equal to 10^{-2}. The formula shows that the maximum pull for a *saturated* plunger is directly proportional to the current, which makes this type of magnet especially suitable for relays and instruments which should be very sensitive. When the flux density in the plunger is well below the knee of the B–H curve, the factor c varies almost directly as the ampere-turns, and the maximum pull, therefore, varies approximately as the square of the ampere-turns.

The pull at any point in the stroke for a *saturated* plunger may then be expressed by the formula

$$P_1 = \frac{10^{-2} ANI}{l_s} \cdot k \qquad \text{pounds,} \tag{2}$$

where k is a factor which gives the ratio of the pull at any point of the stroke to the maximum pull; Fig. 2 gives the approximate value of k at various points of the stroke.

IRON-CLAD ELECTROMAGNET WITH FLAT-END PLUNGER. — The simple solenoid is a very inefficient type of magnet, because a large percentage of the reluctance of the magnetic path is found in the long air path outside the coil. Therefore, the plunger magnet is modified by putting an iron return circuit around the outside of the coil, thus reducing considerably the reluctance of the path and increasing the work which can be obtained with a certain amount of power, and with a certain expenditure of energy in the coil. Fig. 3 shows this type of "iron-clad" magnet. In its highest position the plunger strikes against the frame, so that in reality the magnet represented is a special form of magnet with stop, which is described below. If it is desired to have the pull decrease towards the end of the stroke, a hole similar to the one on the lower end is drilled in the upper end of the frame, and the plunger permitted to protrude through it.

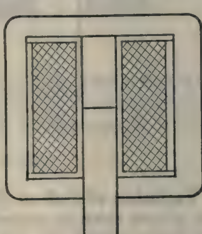

Fig. 3.

Use of Stop to Increase Pull. — The total pull on a plunger depends on the total number of flux lines which pass through it. For a given number of ampere-turns the total number of flux lines may be increased by decreasing the reluctance of the magnetic circuit. This reluctance may be still further reduced by using a plug or stop in the upper part of the solenoid of such a length that the air-gap is central to the coil for the maximum travel which is required. This form is shown in Fig. 4.

Pull of Stop on Plunger; "Air-gap Pull." — The pull P_2 between the stop and plunger may be expressed by the formula

$$P_2 = \frac{B^2 A}{72 \times 10^6} \quad \text{pounds,} \qquad (3)$$

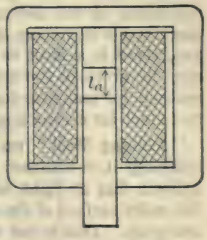

Fig. 4.

where B is the flux density in the air-gap* perpendicular to the end surface of the plunger, in lines (maxwells) per square inch, and A the area of the end of the plunger, in square inches (*see Electricity and Magnetism, Principles of*). The value of B may be calculated from the ampere-turns of the coil and the dimensions of the magnetic circuit in the same manner as the flux due to the field coils of a generator is calculated; see *Generators, Direct-current*. Or, neglecting leakage and the reluctance of the iron part of the path, B may be calculated approximately from the formula

$$B = \frac{3.19\, NI}{l_a} \quad \text{lines per sq. in.,} \qquad (4)$$

where NI is the total ampere-turns of the coil and l_a is the length of the air-gap in inches. Combining (3) and (4) gives

$$P_2 = 1.4 \times 10^{-7} A \left(\frac{NI}{l_a}\right)^2 \quad \text{pounds.} \qquad (5)$$

This pull of the stop on the plunger is usually referred to as the "air-gap pull." The actual variation of this air-gap pull with the length of the air-gap, for a particular magnet, is shown in curve B, Fig. 6.

Total Pull of Iron-clad Solenoid with Stop. — The total pull of the iron-clad solenoid with stop may be looked upon as due to two components, the solenoid effect P_1 given by equation (2) and the pull P_2 between stop and plunger given by equation (5); hence the total pull is

$$P = 10^{-2} ANI \left(\frac{k}{l_s} + \frac{1.4 \times 10^{-5}\, NI}{l_a^2}\right) \quad \text{pounds.} \qquad (6)$$

When the air-gap is short and at the center of the solenoid $k = 1$. The variation of this total pull with length of air-gap for a particular electromagnet is shown in curve C, Fig. 6.

IRON-CLAD SOLENOID WITH CONED PLUNGER. — It will be noted that for a given air-gap pull P_2, the ampere-turns required are directly

* Strictly, B in this formula is the actual air-gap flux density less the flux density which would be produced by the solenoid were there no iron whatever in its magnetic circuit. This correction, however, is smaller than the probable error in calculating B and may therefore be neglected.

proportional to the length of the air-gap. If, therefore, for the same stroke the length of the path for the magnetic lines through the air-gap can be reduced, the pull for the same ampere-turns will be increased, or if the pull remains constant the ampere-turns to obtain it can be decreased. This is accomplished by coning the end of the plunger, as shown in Fig. 5. For such a magnet the stroke should not be much in excess of the plunger diameter, as the leakage increases considerably with longer strokes, and this defeats the object of the coning. As the induction in the iron plunger is greater than the induction in the air-gap, there is a limit to the practicable value which may be given to the cone angle; this limit is about reached for cast iron for a cone angle of 28° and for cast steel for a cone angle of 19 degrees.

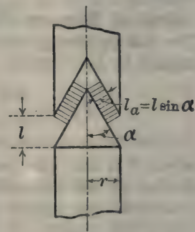

Fig. 5.

"Air-gap Pull" on Coned Plunger. — In the following it is assumed for simplicity's sake that a uniform flux at right angle to and distributed over the entire surface of the cones passes between plunger and plug. This is not strictly correct, especially for long strokes and the pull calculation is therefore only approximately correct.

Let

l = length of stroke, in inches,
A = total cross-section of plunger, in square inches, $= \pi r^2$ where r is the radius of the plunger, see Fig. 5,
α = angle of the cone, in degrees, see Fig. 5,
NI = total ampere-turns of coil.

Then the flux density in the plunger is

$$B_i = \frac{3.19\, NI}{l \sin^2 \alpha} \quad \text{lines per sq. in.} \tag{7}$$

and the air-gap pull in the direction of the stroke is

$$P_2 = 1.4 \times 10^{-7} A \left(\frac{NI}{l \sin \alpha}\right)^2 \quad \text{pounds.} \tag{8}$$

The solenoid pull, as found by experiment, is practically the same as for a flat-end plunger, equation (1). Whence the total pull on the coned plunger is

$$P = 10^{-2} A NI \left(\frac{1}{l_s} + \frac{1.4 \times 10^{-5} NI}{l^2 \sin^2 \alpha}\right) \quad \text{pounds,} \tag{9}$$

where l_s is the length of the solenoid winding assuming the gap at the center of the solenoid ($k = 1$).

Note that $l \sin \alpha = l_a$ is the "effective" length of the air-gap, i.e., the length perpendicular to the surface of the cone. Hence, comparing with equation (6), it is seen that the pull on a coned plunger for the same effective air-gap is the same as on a flat-end plunger, but the length of the stroke is increased in the ratio of $\frac{1}{\sin \alpha}$. Comparing equation (7) with equation (4), it is seen that this advantage is gained by increasing the flux density in the plunger by the square of $\frac{1}{\sin \alpha}$. For small air-gaps, therefore, the coned plunger becomes saturated much more quickly for the same current than does the flat-end plunger,

and the leakage and the reluctance of the iron part of the path produces an appreciable effect, causing the pull to become almost constant for small air-gaps instead of increasing as shown by the approximate equation (9). This effect is clearly shown by the curves D and E in Fig. 6.

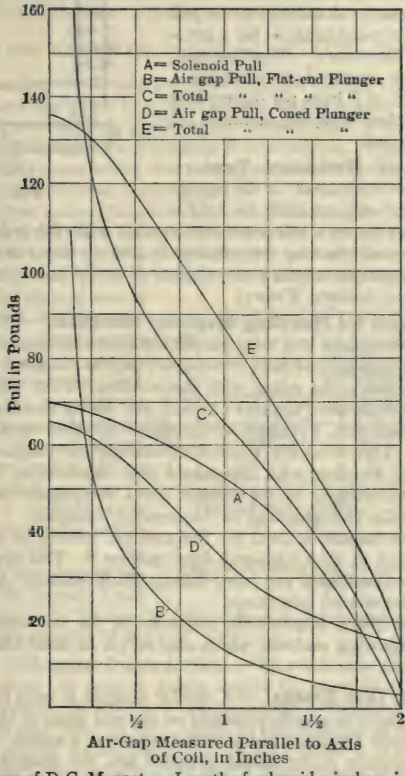

Fig. 6. Pull Curves of D-C. Magnet.—Length of solenoid 3 inches; internal diameter of solenoid 1¾ inches; external diameter 2¾ inches; diameter of plunger 1¼ inches; angle of cone 20°; stop projects 1 inch inside coil; number of turns 300; current 57 amperes.

HORSESHOE-TYPE ELECTROMAGNETS. — Another type of electromagnet which is used quite extensively is the horseshoe-type electromagnet, shown in its simplest form in Fig. 7. This magnet is not suitable for very long air-gaps, because the leakage increases very rapidly with increasing distance between armature and poles. The total pull on the "armature" K of such a magnet, neglecting the leakage, may be expressed approximately by the relation

$$P = 2.8 \times 10^{-7} A \left(\frac{NI}{2\,l_a + l'}\right)^2 \quad \text{pounds,} \tag{10}$$

where A = cross-section of each pole, in square inches, NI = total ampere-turns, l_a = length of each air-gap in inches, and l' = length of air-gap equivalent to the

reluctance of the iron. This last depends on the permeability and dimensions of the iron part of the circuit; calling l_i the mean length of this iron circuit and μ its permeability, and assuming the mean cross-section of this path to be the same as the cross-section A of each pole, then $l' = l_i/\mu$. A more exact calculation of the pull may be made by calculating, as for a generator field (*see Generators, Direct-current*), the ampere-turns required to establish a given flux density of B lines per square inch in the gap, and then applying equation (3) to determine the pull.

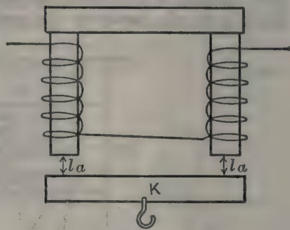

Fig. 7.

Applications of Horseshoe Type. — This type of electromagnet is frequently used where it is only desired to hold a weight after the armature has come into contact with the poles. A modified form of this magnet whereby the winding is put on the yoke connecting the two poles is used extensively as a no-voltage release on hand starters for direct-current motors (*see Starters, Motors*).

Lifting Magnets for Handling Magnetic Materials. — Various forms of horseshoe electromagnets are used for lifting magnetic material. For lifting long pieces, such as pipes and rails, the magnet may be designed with a relatively large distance between the poles, with the winding on the yoke; this type is usually referred to as the "bi-polar" type. For lifting shorter pieces, such as plates, slabs, billets, etc., a magnet with a relatively short distance between the poles is better. This is readily secured by making the magnet with a central pole carrying the winding, with the second pole surrounding the winding and central pole. Accordingly as the magnet has a rectangular or circular cross-section it is called a "rectangular" or "concentric" magnet.

The concentric magnet is used for the greatest variety of purposes, and is frequently referred to as a "general duty magnet." This type of magnet is especially suited for lifting pig iron, billets, skull-crackers, scrap, and other magnetic material of high reluctance.

In all types of lifting magnets the voids inside the coil winding are usually filled with impregnating material which also serves to make the magnet waterproof.

Magnetic Disc Brakes. — A similar magnet is used for magnetic disc brakes. The magnet is usually mounted on the end plate of the motor and attracts a disc-shaped armature which operates against a spring. When the disc is under the influence of the spring pressure and the magnet deënergized, it presses against a series of stationary and movable discs, the latter being connected with the motor shaft, and causes friction between these parts, which tends to retard and stop the motor. The movement of the armature on these disc brakes is very small, being of the order of ¼ inch.

Magnetic Clutches. — Of similar construction are magnetic clutches. In this case the magnet is mounted on the end of the shaft of the driving machine, whereas the armature, which consists of a circular ring, mounted to a hub by means of a somewhat flexible connection, is connected with the driven member. When the magnet is energized the disc is attracted and takes part in the rotation, thereby driving the second shaft. A pair of collector rings are provided to convey the current from the stationary wires to the rotating magnet. In this case also the movement of the armature is very slight. In order to make the brake release quickly a non-magnetic material is often provided between the magnet and the armature which prevents sticking, due to residual magnetism.

SPEED OF MOVEMENT OF PLUNGER. — In order to obtain quick action, the flux, upon closing the circuit, should reach its full value in as short a time as possible. The flux being a function of the current, the speed depends upon the rapidity with which the current reaches its full value. The time required for the current to reach its full value depends upon the quotient of the inductance L divided by the "effective" resistance R of the circuit. The larger this ratio $\frac{L}{R}$, which is called the "time constant" of the circuit (*see Transient Electric Phenomena*), the longer the time required for the current to reach its full value. The inductance L is proportional to the square of the number of turns in the solenoid winding and inversely proportional to the total reluctance of the circuit. The effective resistance R depends not only upon the d-c. or ohmic resistance of the winding, but also upon the eddy currents and hysteresis loss set up when the current is changing.

In Fig. 8 is given current-time curve showing the change of the current during the switching-in period of a direct-current magnet. It will be seen that, after the closure of the circuit, the current first rises to a certain value which corresponds to a flux just sufficient to cause a movement of the armature and lift the plunger. As the plunger moves the flux increases, thereby causing a counter electromotive force, which tends to reduce the current. This counter e.m.f. depends upon the speed of the plunger. In the case shown the current drops off continuously until the plunger strikes against the stop, at which moment it has a value of approximately one-third of the value which started the motion of the plunger. After the plunger has come to rest, the current again increases and gradually reaches the value which is dependent upon the terminal voltage and the resistance of the coil.

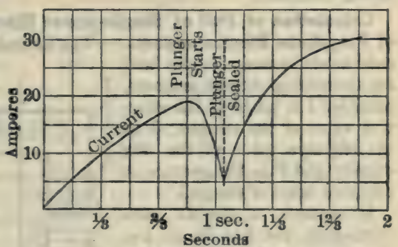

Fig. 8. Current-time Characteristics of D-C. Electromagnet

Methods of Obtaining Quick Action. — To reduce the eddy-current effect, the cross-section of the magnet frame should be as small as consistent with other considerations. Where very quick action is required it is sometimes advisable to slot this frame at right angles to the direction of the eddy currents or laminate it similar to transformers. In such cases it is also advisable to eliminate the brass tube on which the coil is very frequently wound and which acts as a guide for the plunger, or to slot this brass tube parallel to its axis. It is also advisable to slot or laminate the plunger.

Another method of obtaining quick action is to impress at the start a high voltage on the coil and insert resistance into the circuit of the coil as the plunger rises, in order to protect the magnet from overheating. This reduces the ampere-turns at the end of the stroke, which is permissible in most cases, because usually the pull of the magnet increases very rapidly toward the end of the stroke, as is indicated by the formulas given above.

ALTERNATING-CURRENT ELECTROMAGNETS.—Electromagnets for producing a mechanical pull may also be designed to operate on alternating current. The flux in an alternating-current magnet passes through zero twice per cycle. The pull, which varies with the square of the current, therefore becomes zero twice every cycle, and it can be shown that it also varies

according to a sine curve when the current is sinusoidal. The average effective pull is one-half of the maximum pull. Whenever the pull is less than the load, there is a tendency for the plunger to move away from the stop, and this causes rattling or humming of the magnet. This humming may be overcome in different ways, one method being to use a "shading coil," described below.

Polyphase Electromagnets. — It can be shown that if three magnets are used and each is supplied with current from one phase of a three-phase source, or if two magnets are used and each is supplied with current from one phase of a two-phase source, then the resultant pull will be constant at any moment, and if the plungers are rigidly connected there will then be no chattering. The most common form of three-phase magnet is shown in Fig. 9. This consists of a core having three poles and a plunger of similar construction. Over each pole there is wound a coil which is supplied from one of the three phases of the circuit. There are various modifications of the polyphase magnet, but their general principle is the same. In calculating the total pull, the pull of each pole is figured separately and the several pulls, which are equal to one another, are combined vectorially, since they differ in time phase (*see Alternating Currents*).

Calculation of Pull of Single-phase Electromagnets, or of One Phase of Polyphase Electromagnets. — The formulas for pull given above for direct-current electromagnets also hold for alternating-current magnets, provided I is taken as the effective value of the current, B as the effective value of the flux density and P as the average or effective value of the pull.

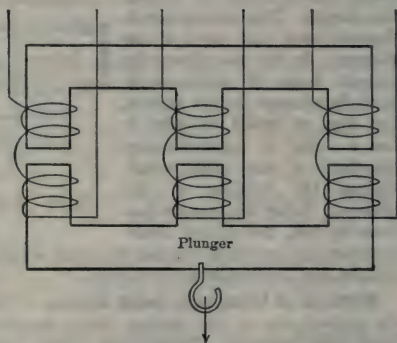

Fig. 9.

In contrast to the d-c. electromagnet, the flux in an a-c. electromagnet for a given impressed e.m.f. is approximately constant irrespective of the length of the air-gap. This is due to the fact that the opposition to the flow of current through the winding is due almost entirely to the back e.m.f., due to the alternation of the flux, and only to a very small extent to the resistance of the winding, the action in this respect being similar to that of a transformer (q.v.) or induction motor (*see Motors*). The back e.m.f. being practically equal to the impressed e.m.f., the flux producing this back e.m.f. is also proportional to the impressed e.m.f., and is therefore practically constant when the impressed e.m.f. is constant.

Since the flux remains practically constant, the current I, as may be seen from equation (4), must vary approximately proportionally to the length of the air-gap l_a; this proportionality holds only approximately, since equation (4) neglects the magnetic leakage and the reluctance of the iron part of the magnetic circuit. The actual variation of the current with the length of the air-gap for a particular a-c. electromagnet is shown in Fig. 10.

Using the same notation as used above for d-c. electromagnets and in addition putting E = effective value of impressed voltage per phase, and f = frequency of impressed voltage in cycles per second, the current taken by each phase of

the magnet, neglecting the iron losses, leakage and reluctance of the iron part of the magnetic circuit, will be

$$I = \frac{10^7 E l_a}{2 f N A^2} \quad \text{amperes.} \tag{11}$$

A more accurate calculation of the current, taking into account the losses and the magnetic leakage, may be effected by the method used for calculating the current in a transformer (*see Transformers*).

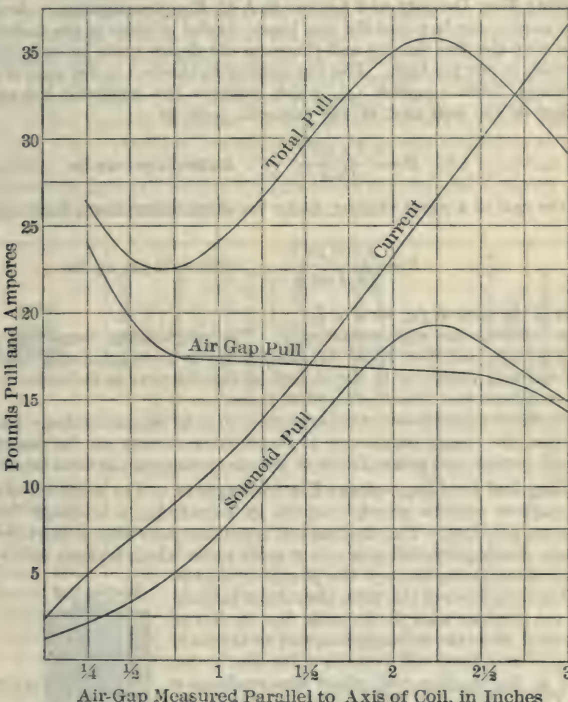

Fig. 10. Pull and Current Curves of A-C. Magnet.—Length of solenoid 5 inches; internal diameter of solenoid 3 inches; external diameter 4½ inches; diameter of plunger 2⅜ inches; stop projects 1½ inches inside of coil; turns 144; voltage 220; frequency 60 cycles per second.

Substitution of the value of the current given by (11) in equation (6) for the total pull on a plunger with a flat end gives the approximate formula

$$P = \frac{10^5 E}{2 f N}\left(\frac{k l_a}{l_s} + \frac{72\,E}{f N A}\right) \quad \text{pounds.} \tag{12}$$

This equation also applies to a coned plunger, when l_a is taken equal to the length of the air-gap perpendicular to the face of the cone.

Fig. 10 shows the pull curve of an alternating-current magnet with flat-end

stop and plunger. It will be noted that the current is roughly proportional to the air-gap. The solenoid pull reaches a maximum at an air-gap of about 2¼ inches and then falls again approximately proportional to the air-gap; the air-gap pull is constant over a wide range of the travel. The result is a total pull curve which has a maximum at approximately 2¼ inches and which drops until an air-gap of approximately ¾ inch is reached, and from there on it increases again, due to the increase of the air-gap pull, the latter being caused by the diminution of the leakage flux as the air-gap decreases.

Limiting Flux Density and Losses in A-C. Electromagnets.—Attention must be paid to the fact that the iron losses, similar to those of the transformer increase with the flux density and therefore the design must be such that the flux density is not too high. The flux density in the iron in the case of a flat-end plunger, from equation (4), which neglects the magnetic leakage and reluctance of the iron part of the magnetic path, is

$$B = \frac{1.6 \times 10^7 E}{fNA} \quad \text{maxwells per sq. in.} \qquad (13)$$

and in the case of a coned plunger, under the same assumptions, from equation (7), is

$$B = \frac{1.6 \times 10^7 E}{fNA \sin \alpha} \quad \text{maxwells per sq. in.} \qquad (14)$$

where α is the cone angle, see Fig. 5.

These relations are approximate only. The magnetizing component of the exciting current, and from it the flux density can be more accurately calculated by the methods employed in the design of transformers or induction motors. See *Transformers* and *Motors, Polyphase Induction*.

The iron and copper losses can also be calculated by similar methods, and from these losses the energy component of the exciting current can be determined. The total current and power factor of the electromagnet can then be deduced.

Shading Coil for Single-phase Electromagnets.—The humming of single-phase magnets may be greatly reduced by introducing a so-called "shading coil" in the pole face. This shading coil is nothing more than a short-circuited secondary winding, consisting of one or more turns, which encloses only part of the total flux passing through the plunger. Due to the leakage reactance of this turn, the current induced in it is out of phase with the inducing flux, so that at the moment when the inducing flux, due to the main winding, is zero, there still remains a flux due to the current in the shading coil, which flux produces a pull. The result is that the combined pull from the main flux and the shading coil flux never becomes zero. Fig. 11 shows the arrangement of the shading coil. This coil is mounted in the plunger or in the plug close to the pole face, in order to reduce the length of the path for the magnetic lines which are interlinked with the shading coil. Naturally the shading coil has no effect with long air-gaps, and it is therefore imperative that a good magnetic contact be obtained when the plunger is in the sealed position to get the greatest possible effect of the shading coil.

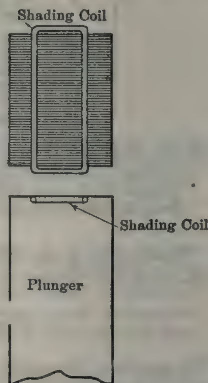

Fig. 11. Shading Coil.

Incidentally the shading coil also increases considerably the maximum pull for a given impressed e.m.f., as the maximum pull is also due to the com-

bined main and local flux. As an example, a plunger magnet without shading coil, which gave a minimum pull of zero and a maximum pull of 28 pounds, had, after the introduction of the shading coil, a minimum pull of 18 pounds and a maximum pull of 143 pounds.

COSTS, WEIGHTS AND DIMENSIONS OF ELECTROMAGNETS. — Due to the great variety in the designs, it is not possible to give unit costs of electromagnets. The subject of the most economical magnet design has been discussed by Wilkander (*Trans. A.I.E.E., 1911, Vol. 30, p. 2019*), but unfortunately the most economical design will usually be found not to be suitable for practical purposes, because it results in a magnet which is too long compared to its diameter, and which usually cannot be incorporated in the machine with which it is to be used. Therefore, magnets as they are found in practical application deviate greatly from the most economical design. Also for the same energy output (usually expressed as inch-pounds or foot-pounds) there is as much as a 1 to 3 variation, depending upon the service conditions as to speed of operation, stroke, etc., which they have to meet. The following table of costs (**pre-war figures**), weights and dimensions of some typical electro magnets is given merely as a rough guide.

Use for which designed	Over-all length, inches	Over-all diameter, inches	Total weight of active iron and copper, pounds	Length of stroke, inches	Pull at beginning of stroke, pounds	Voltage	Watts input at end of stroke	Power factor at end of stroke	Factory cost, 1914
1. Relay, D.C.	3¼	2⅛	2.9	⅛	0.15	110–500	9	....	$5.10
2. Relay, A.C.	3¼	2⅛	2.9	⅛	0.10	110–550	15	0.39	6.95
3. Lifting, D.C.	5	3	8	1	8	110–550	50	....	6.40
4. Lifting, D.C.	11	7	80	3	20	110–500	210	....	33.00
5. Lifting, A.C.	5½	4	10	⅞	7	110–550	30	0.30	8.15
6. Lifting, A.C.	14	6¼	90	2½	50	110–550	400	0.31	32.00
7. Brake, D.C.	5¼	8½	72	⅛	160	110–220	50	....	21.00
8. Brake, D.C.	6	15	320	5/32	1260	110–220	210	....	39.00
9. Clutch, D.C.	6½	10	82	⅛	60	110–220	52	....	19.00
10. Clutch, D.C.	7	18	210	⅛	400	110–220	115	....	35.00

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ELECTROMETERS. — (*See also Electrodynamometers; Voltmeters: Wattmeters.*) An electrometer is primarily an instrument for measuring potential differences, but under certain conditions may also be used as a wattmeter; in the latter case it is usually called an electrostatic wattmeter. The deflection of the instrument is due to the attraction or repulsion of electrostatic charges. It may be used for either direct- or alternating-current measurements.

There are a great variety of forms of electrometers. On account of the care required in its use, the ordinary quadrant electrometer is seldom employed except for laboratory purposes, but various modifications of the electrometer provided with pointer and scale so as to be direct reading are used commercially for high voltage measurements. Such instruments are known as electrostatic voltmeters; for description see article on *Voltmeters*.

Electrometer versus Galvanometer or Electrodynamometer. — The advantage of the electrometer over the galvanometer or electrodynamometer is that it takes no current when used for constant (d-c.) voltage measurements. Due to its electrostatic capacity, however, it does take a certain amount of charge which should always be allowed for if the capacity of the electrometer is appreciable compared with any other capacity which affects its reading. Also, when used for a-c. measurements, the charging current taken by the electrometer should be allowed for, if this charging current is appreciable. This charging current is usually considerably less than would be taken by an electrodynamometer of the same degree of sensitiveness.

KELVIN QUADRANT ELECTROMETER (Fig. 1). — Two brass quadrants a and a' are connected together and two quadrants b and b' are connected together and these respective pairs are well insulated from one another. The "needle" n (a light aluminum vane) is suspended by a silk, silvered quartz or other fiber and insulated from the quadrants.

A light aluminum vane v is suspended from the needle by a fine conducting wire and dips into a conducting solution, usually 60 per cent sulphuric acid, which serves as a means of connecting the needle to any external source of p.d. or to the ground. If a conducting suspension is used this may be employed to connect the needle to the external source of potential. The sulphuric acid also keeps the air dry in the case containing the quadrants, and the motion of the vane in the acid damps the vibrations of the needle. The suspension carries a mirror m by means of which the deflection of the needle may be read.

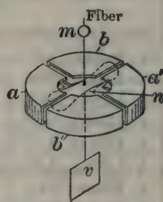

Fig. 1. Quadrant Electrometer

Formulas for Quadrant Electrometer. — In order to obtain a straight line calibration curve the needle must be properly shaped and must be suspended midway (vertically) between the two quadrants and must have a symmetrical zero position with respect to the two quadrants. Under these conditions when the two pairs of quadrants are charged to potentials V_a and V_b and the needle to the potential V_n above any fixed potential, say that of the ground, the angular deflection is

$$D = \frac{1}{K}(V_a - V_b)\left(V_n - \frac{V_a + V_b}{2}\right),$$

where K is a factor, which is practically a constant. The value and constancy of the factor K should be determined by calibration.

There are three ways of using the instrument, as indicated in the diagram,

Fig. 2. E is a known voltage, V the p.d. to be measured, a and b the two pairs of quadrants and n the needle.

Case I. $V = E \pm \sqrt{E^2 - 2KD}$. (*See footnote.*)

Case II. $V = \dfrac{KD}{E}$

Case III. $V = \sqrt{2KD}$.

Cases I and II are applicable to measurements of constant (d-c.) potentials only. Case III may be used for the measurement of either alternating or direct potentials. In the case of alternating potentials V is the *effective* value.

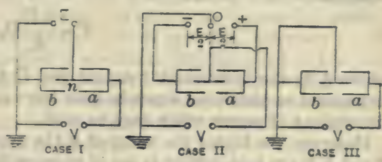

Fig. 2.

Range of Quadrant Electrometers. — The range of a quadrant electrometer when used as in Case I or II depends upon the maximum voltage E which can be impressed between the two pairs of quadrants and between the quadrants and the needle, and also upon the fineness and material of the suspension fiber. Case III is not applicable to the measurement of very low voltages. Electrometers can be purchased suitable for measuring direct voltages as low as about 2 volts, and as high as 50,000 volts, and for alternating voltages as low as about 2 volts and as high as 25,000 volts.

Extension of Range by Use of Auxiliary Condensers (Fig. 3). — By connecting the p.d. to be measured across two condensers in series and measuring the voltage across only one of them, the instrument may be used for the measurement of high voltages of practically any magnitude. The connections are as shown in Fig. 3. If the condensers have no leakage, then

$$V = V_1 \frac{C_1 + C_2 + c}{C_2},$$

where V_1 is the voltage read by the electrometer, C_1 and C_2 the capacities of the two condensers and c the capacity of the condenser formed by the electrometer quadrants and needle. If C_1 and C_2 are large compared with c, then c may be neglected, but the larger C_1 and C_2 the greater will be the charge (and therefore the charging current, in case of an a-c. measurement) taken by the measuring circuit.

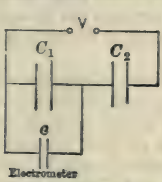

Fig. 3. Electrometer Range Extended by Condenser

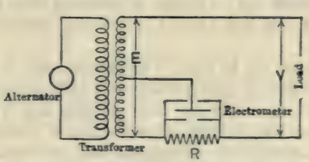

Fig. 4. Electrostatic Wattmeter

Quadrant Electrometer as Electrostatic Wattmeter (Fig. 4). — The quadrant electrometer may be used to measure with a fair degree of precision

* Use the − sign if V is less than E, the + sign if V is greater than E, and call the deflection positive in every case.

very small amounts of power, of the order of 1 watt, when the voltage giving this power is 5000 volts or more. It therefore serves as a very convenient means of measuring the power loss in small samples of insulating materials at high voltages. See paper by E. H. Rayner, *Jour. Inst. Elec. Eng.*, 1912, Vol. 49, p. 3.

The connections used by Rayner are shown in Fig. 4. R is a non-inductive resistance. The needle of the electrometer is connected to the middle point of the high-tension winding of the transformer. *On the assumption that the charging current of the electrometer and the difference in phase between V and E may be neglected,* the power supplied to the load is

$$P = \frac{2K}{R} D,$$

where K is the instrument constant and D the deflection. The range of the instrument as thus used depends upon the value of R, the higher R the smaller the amount of power which may be read.

However, the higher the value of R the greater the phase difference between E and V if the load current is out of phase with V, and therefore for small power measurements when the load has a low power factor *e.g.*, when it is a condenser, an allowance must be made for this difference in phase angle. Also, when the charging current of the electrometer is comparable in magnitude with the load current a correction must also be applied on this account. These corrections are discussed in detail in Rayner's paper.

COST (Pre-war prices).—An ordinary Kelvin quadrant electrometer suitable for measuring potentials of from 400 to 1300 volts costs approximately $150.

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ELECTRON THEORY. — (*See also Electricity and Magnetism, Principles of; Electrochemistry, Principles of.*) The electron is the smallest known separate charge of negative electricity. The basic assumption of the electron theory is that the atoms of all substances contain one or more electrons. The atoms, normally in a neutral electrical condition, become positively charged by the loss of one or more electrons. *This assumption that in every body, whether charged or uncharged, there exists electricity of both signs distributed not continuously but as discrete particles*, was first introduced,* long before the discovery of the separate existence of the electron, to account for certain discrepancies found in applying the electromagnetic theory of light to the optical properties of material media.

In studying the results of Crookes on the cathode rays J. J. Thomson was led to the conclusion that in the conduction of electricity through gases at low pressure the negative current, or cathode ray, is carried by extremely small particles or corpuscles, now called electrons. The work of Thomson, Wilson and others led to the first announcement of the separate existence of the electron together with the values of its negative charge and of its mass.

Properties of the Negative Electron. — The results of numerous experiments justify the following assumptions regarding the electron.

1. The charge carried by a single electron is the smallest possible charge which can exist in nature and no charge of electricity can be produced which is not an integral multiple of this charge. The value of the charge carried by a single electron is approximately

$$e = 4.9 \times 10^{-10} \quad \text{c.g.s electrostatic units}$$

($= 1.6 \times 10^{-20}$ c.g.s. electromagnetic units). This charge is also equal to that carried by the hydrogen atom in electrolysis. (There is also considerable evidence indicating that the electron *is* electricity pure and simple, having no mass in the ordinary sense; *see below*.)

2. The "effective" mass of an electron may be defined as the force required to give it unit acceleration.† All experiments indicate that the "effective" mass of an electron, as thus defined, is approximately

$$m = 8.9 \times 10^{-28} \quad \text{grams,}$$

provided the electron is not moving with a velocity greater than one-tenth the velocity of light; for velocities greater than one-tenth that of light the "effective" mass of the electron increases with increase in velocity (*see below*). The mass 8.9×10^{-28} is about one-seventeen hundredth (1/1700) that of a hydrogen atom.

Unless otherwise stated, wherever the expression "mass of an electron" is used in this article, it is to be understod that this *effective* mass is meant.

3. An electron having a charge e and moving with velocity v produces the same magnetic field as an elementary length ds of a conduction current of strength i, where $ev = ids$. (*See Electricity and Magnetism, Principles of.*)

4. Every neutral atom of matter contains at least one electron which is held in position by forces analogous to elastic forces, that is, an electron may oscillate within the atom, or may be displaced by an impressed electrostatic field. The electrons are always of the same nature irrespective of the substance in which they exist.

5. An electron may be forced from the atom by the influence (mutual repulsion) of a "free" electron moving at a high velocity in its immediate vicinity.

* H. A. Lorentz: Proceedings of the Amsterdam Academy, 1878.

† This is, $f = ma$, where f is the force, a the acceleration, and $m = \frac{f}{a}$ is the "effective" mass.

This action is usually spoken of as a bombardment, or collision, although it is not necessary to assume that the free electron actually hits the atom.

6. When an electron is expelled from an atom the atom manifests the properties of a positively charged body.

7. In every substance there exists in addition to the electrons within the atoms a certain number of "free" electrons, i.e., electrons which can move freely in the inter-atomic spaces; these free electrons may also pass from one substance to another. Under certain conditions, e.g., in a gas at ordinary pressures, a free electron may attach to itself one or more atoms or molecules.

Properties of the Positive Particle. — Experiment also justifies the following assumptions regarding the positively charged particles:

1. The smallest positive charge that it has been found possible to produce is numerically equal to that of an electron, viz., 4.9×10^{-10} c.g.s. electrostatic units.

2. The smallest possible positive charge is always associated with a particle having a mass of the same order of magnitude as that of an atom, this mass being never less than that of a hydrogen atom.

3. The positively charged particle may be looked upon as an atom from which one or more electrons have been expelled; its mass therefore depends upon the nature of the atom. A "free" positively charged particle may also attach to itself one or more neutral atoms or molecules.

Relation between Electrons and Ions. — The term "ion" is usually reserved to designate any charged body having a mass of the order of magnitude of that of a molecule or atom. In this sense an electron is not an ion, but the positively charged particle is an ion. If, however, the electron becomes attached to an atom or molecule, then this combination forms an ion. In the case of the discharge of electricity through gases at low pressure, the electron, although it is not attached to an atom or molecule, is sometimes called an ion. See also *Electrochemistry, Principles of.*

APPLICATIONS OF THE ELECTRON THEORY. — All of the foregoing conclusions have received ample experimental verification. The phenomena which have presented the most important evidence are the cathode rays, the condensation of water drops on free electrons, the emission of electrons from metals at high temperatures, and from certain metals under the influence of ultra violet light, and the properties of radio-active substances. From this mass of evidence it is now certain that the atoms of all substances contain one or more electrons; that many of the common properties of matter are linked with the numbers and arrangements of the electrons in the atoms; that the neutral atoms may lose or gain electrons and so become charged ions; that the electron may be separated from an atom and its independent existence studied; and that in all substances there are always present a certain number of free electrons in motion among the molecules and atoms due to their mutual collisions in accordance with the kinetic theory of matter; it appears probable that this theory governs also the motion of the free electron.

The electron theory has led to simple explanations of many of the electrical and optical properties of matter, and the study of the properties of electrons to a number of useful applications in the field of electrical engineering. Some of the more striking results in both fields are briefly described in the following paragraphs.

Electron Emission from Hot Bodies (Thermionic Discharge.) — (*See also Radio Communication.*) When a metal is heated to a high temperature in a high vacuum it gives off electrons freely. This fact was first studied by Richardson. Langmuir by improvements in methods of obtaining high vacua

has carried the investigation much further, clearing up many disputed questions of theory and leading to practical applications of far-reaching value. The amount of the electron emission is subject to the temperature of the metal or cathode, and to the electric field at its surface, as due for example to a neighboring anode.

The theory assumes that in accordance with the kinetic theory of matter the electrons within the metal are in a constant state of motion, and that of those nearest the surface a certain proportion escape, a few at high, the greater number at lower velocities, in accordance with Maxwell's Distribution Law. When an electron leaves the surface the metal becomes positively charged, thus putting a retarding force on the electron and tending to make it return The distance traversed by the electron is greater the greater its initial velocity, and in the presence of a neighboring neutral or positively charged surface it may not return at all. Obviously the higher the temperature of the heated metal or cathode the greater the velocities of the electrons leaving it, and the greater the volume of electron discharge. The following law due to Richardson has been corroborated by Langmuir and others

$$i = a\sqrt{T}\epsilon^{-\frac{b}{T}},$$

in which i is the current emitted per unit area from the cathode, T its absolute temperature, ϵ the base of the natural system of logarithms, and a and b are constants. For tungsten i is 0.0042 ampere at 2000° C. and 31.7 amperes at 3000° C. according to Langmuir. The value of the current given by the foregoing equation is known as the "saturation current," and it is independent of the potential of the anode.

When, by increasing the temperature of the cathode and the voltage of the anode, it is attempted to increase the current passing, a condition is reached in which the current is independent of the temperature and increases with the three-halves power of the voltage. The value of the current for a cylindrical filament of radius r in a coaxial cylinder is:

$$i = \frac{2\sqrt{2}}{9}\sqrt{\frac{e}{m}}\frac{V^{3/2}}{r},$$

where V is the difference of potential and $\frac{e}{m}$ is the ratio of the charge to the mass of the electron.

This new condition is due to the negative field at the surface of the cathode caused by the volume of negative charge existing in the stream of electrons in the space between anode and cathode; this is known as the "space-charge effect." The space charge drives back into the cathode all electrons leaving it except just so many as will balance the field of the positive electrode. Thus the current is a function of the potential of the anode and independent of the temperature of the cathode.

The volt-ampere characteristic of the thermionic tube is thus regulated by three general properties of the tube. In the first region, best observed at low-cathode temperatures, the current is low, the space-charge effect is absent, and the current is determined by the initial velocities of the electrons emitted by the cathode. This region is largely obscured for the hotter cathodes commonly used, and in this case the volt-ampere characteristic is determined at the lower voltages by the space-charge effect. At higher voltages the third region is reached in which the saturation current is reached, dependent on the temperature but independent of the voltage.

The two-electrode tube conducts in only one direction and is an excellent rectifying device for small alternating currents. This is the principle of the

Fleming valve and kenotron. If a third electrode, in mesh or grid form, be placed between the cathode and anode, very small potentials on the grid will cause wide variations in the potential gradient at the surface of the cathode, and so cause wide changes in the value of the electron discharge. This device has proved of immense value as a relay for magnifying radio signals and telephonic speech, as exemplified in the audion and thermionic amplifier. The same principle is used in larger tubes for generating sustained oscillating currents for radio transmission, as exemplified in the pliotron. At present both rectifying and generating tubes are limited to small current capacities (see *Radio Communication*) owing to difficulties of tube construction.

Cathode Rays. — When an electric discharge is passing in a vacuum tube, at a certain value of the pressure of the gas contained in the tube it is noticed that a greenish phosphorescence occurs on the walls of the tube. By placing solid bodies in the tube it is found that the phosphorescence is due to something proceeding in straight lines perpendicularly from the surface of the cathode. The name "cathode rays" has been given to this agent which produces the phosphorescence. These rays are deflected by both electric and magnetic fields and communicate a negative electric charge to an insulated conductor. These "rays" consist of a rapidly moving stream of electrons. The amount of the deflection of the rays, under the action of magnetic and electrostatic fields, can be calculated on the assumptions of the electron theory, and a comparison of the calculated with the observed deflections shows the two to be in close agreement.

The cathode rays give rise to X-rays when they strike a target or other obstacle. The use of a hot cathode with a focusing negative shield for regulating the velocities of emission has led to great improvements in X-ray tubes and also to further knowledge of the X-rays (see *X-rays*). In this way it is possible to obtain X-rays of single values of frequency and these by interference methods have led to important evidence as to the molecular structure of matter.

Kanalstrahlen. — If the cathode of a tube producing cathode rays is perforated, faint luminous streaks are seen to proceed through these perforations in a direction opposite to that of the cathode rays. The name "kanalstrahlen" has been given to this phenomenon (the German name is very generally retained in English). These rays are also deflected by both electric and magnetic fields but to a lesser extent and in relatively the opposite direction from the deflection of cathode rays, and communicate a *positive* charge to an insulated conductor.

In terms of the electron theory these rays are positively charged *ions*. Calculations from actual measurements, making this assumption, show that the charge carried by each of these ions is numerically equal to that of an electron, and that the mass of each ion is never less than the mass of a hydrogen atom, but may be greater, depending upon the nature of the gas and the pressure in the tube. That is, the cathode particle is an electron traveling in one direction and the kanalstrahlen particle is what is left of the atom which has lost an electron, traveling in the opposite direction.

Electrical and Thermal Conductivity. — One of the fundamental assumptions of the electron theory is that in conductors there are many electrons which are free to move about among the molecules besides those electrons having fixed positions of equilibrium. It is likely that these electrons are not always the same ones; the molecules are probably continually gaining and losing electrons. It is reasonable to assume that the more free electrons (on the average) in a body the better electrical conductor it is. According to the kinetic theory of matter, the molecules of all substances not at the absolute zero of temperature are in a state of ceaseless agitation; if the assumption is made that the *free* electrons share in the motion of the molecules, then the more free

electrons there are in a metal the more rapidly will kinetic energy be communicated from any molecule to neighboring ones, that is, the better conductor of heat the metal will be.

In order to deal quantitatively with the matter some assumption as to numerical values must be made; the one that has been made is that the mean kinetic energy of the electrons is equal to the mean kinetic energy of the molecules. Using the above assumption and also making one as to the relation between the number of electrons per unit volume and the temperature, it has been found possible to explain: 1. Joule's law for the production of heat in a conductor carrying an electric current; 2. the Wiedemann–Franz law ,which states that for pure metals the ratio of the thermal to the electrical conductivity is independent of the nature of the metal and is proportional to the absolute temperature; 3. the Peltier and Thomson effects. The theory also throws much light on the nature of the Hall effect.

Electrolysis. — Faraday's laws of electrolysis (*see Electrochemistry, Principles of*) are readily explicable in terms of the electron theory. It is only necessary to assume that each positive univalent ion in the solution is an atom from which one electron has been expelled, each positive bivalent ion is an atom from which two electrons have been expelled, etc., and that each negative univalent ion is a neutral atom to which one electron has attached itself, each negative bivalent ion is a neutral atom to which two electrons have attached themselves, etc.

Magnetic Properties. — The magnetic properties of bodies may be explained in terms of the electron theory by assuming that the electrons in the molecules of magnetic substances revolve in orbits within the molecule, thus making each molecule produce a magnetic field similar to that produced by a single turn of wire carrying an electric current.

Ionization of a Gas. — Gases at or near atmospheric pressure are normally very good insulators, i.e., have an extremely high insulation resistance. However, a gas may be rendered a fairly good conductor in a number of ways, viz., by passing X-rays through it, letting ultra-violet light fall upon a metal plate immersed in it, subjecting it to the influence of the rays given off by radium, passing it close to a flame or a hot piece of metal or by establishing in it a brush discharge or corona (q.v.). A gas rendered conducting in any of these ways is said to be "ionized."

In terms of the electron theory the conductivity thus given to a gas is attributed to the breaking up of some of the molecules of the gas into positive and negative ions. To account for the observed facts it is necessary to assume that, at pressures of 20 cm. or over, by barometer, the electrons and positive ions thus formed immediately attract molecules to themselves just as an electrified rod attracts dust particles. Since the force of attraction is independent of the sign of the charge both the positively and negatively charged particles thus produced would act in very much the same manner. In gases at low pressures, however, the electrons, or the positive ion, would rarely come within attracting distance of a molecule and even if it did collect molecules about itself a *collision* with a molecule would probably reduce it to its simplest form at once. A collision in gases at high pressure probably has the same effect but the ion very soon reforms owing to the greater number of molecules in its vicinity.

Ionization by Collision. — The electron theory assumes that at least a few free ions (positively charged particles or electrons with one or more molecules attached) exist in every substance. The fewer these free ions, the better the insulating properties of the substance. Under the action of an electric field these ions are given a velocity which depends upon their mass and upon the distance which they can travel before colliding with other molecules

or ions. In the case of a gas the "mean free path" is relatively large, being greater the less the density.

The ions present in a gas may therefore acquire, under the action of an electric field, a very considerable velocity. When one of these rapidly moving ions comes very close to a molecule of the gas it exercises a large force of repulsion on the electrons in the molecule and may thereby expel one or more electrons from the atom. The electric field therefore causes a large increase in the number of ions present in the gas and consequently the conductivity of the gas is largely increased.

Townsend has explained spark-over in terms of this theory. So long as the current is due only to the free electrons originally present and those caused by collision the current for a given voltage cannot continue indefinitely. But, if the voltage is high enough to give ionizing velocities to the positive ions also, a cumulative generation of both positive ions and negative electrons occurs, giving an indefinite increase of current, i.e., a spark-over.

Efforts have also been made to explain the arc, but these are not completely satisfying. The hot cathode is essential for the arc, and its temperature is probably due to the bombardment of the positive ions.

Theory of the Corona. — (*See also article on Corona.*) The electric field is strongest at the surface of a transmission wire, and stronger the smaller its radius. Thus with increasing voltage, ionization sets in first at the surface. This results in conductivity of the surrounding air or gas, causing an increase in the effective diameter of the wire to a point where the decreasing electric field is just balanced by the electric strength of the air. The law of corona formation on a clean round wire in air is, very closely:

$$E = 33\,\delta\left(1 + \frac{0.26}{\sqrt{\delta r}}\right),$$

where E is the surface electric intensity in kilovolts per cm., δ the relative density of the air, and r the radius of the wire in cm.

Townsend has given an explanation of the above law in terms of the theory of ionization by collision. Corona is a spark phenomenon. The influence of the radius of the conductor is satisfactorily shown, but the explanation of the influence of the density of the air depends on experiments made at very low gas pressures and is not so clear. The law of the power loss between transmission lines, due to corona, has not been explained.

The above law of corona formation is obeyed with great constancy and accuracy when clean round wires are used. This fact, together with the simple form of the law has led to the suggestion of the use of corona as a standard and a means of measurement of high voltage (*Whitehead*).

Optical Phenomena. — A number of otherwise obscure optical phenomena are satisfactorily explained in terms of the ionization theory, on the assumption that in general there exist in a given molecule one or more electrical systems of electrons capable of vibrating at different frequencies. In this way it is possible to explain the change of refractive index with the wave length, selective absorption, emission and reflection, the Zeeman effect, the Faraday effect, and other optical phenomena.

MASS OF THE ELECTRON. — Since to account for the various observed facts noted above it is necessary to assume that a moving electron produces a magnetic field (which assumption is in accord with the experimental fact that a moving charged body of finite dimensions produces a magnetic field), it follows that the total energy of motion of an electron is

$$W = \tfrac{1}{2} m_0 v^2 + W_e,$$

where the term $\frac{1}{2} m_0 v^2$ represents the energy which would be required to give the electron a velocity v *if it had no electric charge whatever*, and W_e is the energy stored in the magnetic field established by the motion of the charge carried by the electron. This expression may also be written

$$\tfrac{1}{2}(m_0 + m_e)v^2,$$

where $m_e = 2\,W_e/v^2$. The quantity m_0 is the ordinary or mechanical mass of the electron; the quantity m_e is called the "electromagnetic mass" of the electron. The latter, however, is more analogous to the coefficient of self-induction than it is to mechanical mass, since it is part of the expression for the energy of the *magnetic field* established by the moving electron.

The total "effective" mass of the electron is then

$$m = m_0 + m_e.$$

Since it is the total "effective" mass that enters into all calculations of $\frac{e}{m}$ from experimental data, it is impossible by direct experiment to determine m_0 and m_e separately. However, m_e can be calculated in terms of the charge e and the velocity, provided certain assumptions are made regarding the distribution of charge in the electron. Several different formulas have been proposed, based on different assumptions, regarding the distribution of the charge. All these contain a *constant* term independent of the velocity, and *this constant term, within the limits of experimental error, is found to be equal to the total mass m calculated from experimental determinations of the ratio* $\frac{e}{m}$ *and the charge e*, provided the velocity of the electron is less than one-tenth that of light (see next paragraph). Hence the mechanical mass m_0 of the electron is negligible, within the limits of experimental error, compared with the electromagnetic mass m_e.

The above considerations are the basis of the assumption, now generally made by physicists, that the mass of the electron is *entirely* electromagnetic i.e., that the electron is electricity pure and simple. In fact, some physicists go farther and try to explain all mass in terms of electricity. A satisfactory theory of this kind, however, cannot be developed until the properties of the positively charged particle are better understood. See J. J. Thomson, *The Corpuscular Theory of Matter.*

Variation of the Mass of the Electron with Velocity. — When the velocity of the electron approaches that of light both theory and experiment indicate that its electromagnetic mass increases. The formula which best satisfies the experimental facts is that deduced by Lorentz, viz.,

$$m_e = \frac{8.9 \times 10^{-28}}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}$$

where v is the velocity of the electron and V the velocity of light. For $\frac{v}{V}$ less than 0.1 this formula gives a value for m_e constant to within an error of 1 per cent. Since the mechanical mass of the electron (if there is such) is negligible compared with m_e, the above formula also represents, within the limits of experimental error, the variation of the total effective mass with velocity.

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of Matter; gives an account of the structure of the atom and other matters of a similar nature. Rutherford, E., *Radioactivity;* detailed information on radioactive processes; J. T. Townsend, *Ionization of Gases by Collision;* the original statement of the theory of ionization by collision; later papers on this subject by the same author; recent work in this field by Paul D. Foote and others in the publications of the National Bureau of Standards. An excellent review of the earlier work on hot cathode tubes is found in Thomson's *Conduction of Electricity through Gases*. The recent work by Langmuir and associates is published in the Physical Review, (Vol. 2, p. 450, 1913, and later numbers) and several other prominent scientific journals. The most recent work (1920) on the structure of the atom and kindred topics in the electron theory is A. Sommerfield's *Structure of the Atom*.

ELECTROSTATIC PRECIPITATION OF SUSPENDED PARTICLES. — (*See also Electricity and Magnetism, Principles of.*) The removal and collection of finely divided solid or liquid particles carried in suspension in air or other gases may be accomplished by the application of a high potential unidirectional electrical current to such gases.

The Cottrell Precipitation Processes embodying this principle have been applied with successful results in a large number of installations by the Research Corporation of New York City and by the Western Precipitation Company of Los Angeles, California.

Principle. — The essential elements of an electrical precipitator are two sets of electrodes. One set, known as the discharge electrodes, are of such form as to facilitate an electric discharge from their surface, as for instance a wire or a light chain, or a strip of metal having relatively sharp edges, while the other set, known as collecting electrodes, are of such shape as to prevent as far as possible any discharge from their surface, as for instance, a flat plate or a pipe with a smooth interior surface. These electrodes are so arranged in the precipitator that the different types oppose each other and between them a silent or glow discharge is maintained by supplying to the discharge electrode electrical energy of a unidirectional character and at a relatively high voltage.

In practice the collecting electrodes are grounded for reasons of convenience and safety. A typical form of precipitator, for example, consists of a grounded pipe along the central axis of which is placed a wire connected to a source of unidirectional high voltage direct power.

The air or other gas carrying the suspended liquid or solid particles which are to be removed is passed between the discharge and collecting electrodes. During passage the particles of suspended matter are charged and are driven away from the discharge electrodes and over to the collecting electrodes upon which they are deposited. The air or other gases are unaffected and pass on out of the precipitating chamber.

It has been found that the most effective precipitating action is obtained when the potential difference between the electrodes is sufficient to produce a glow known as the corona on the discharge electrodes (*see Corona*), and when the discharge electrodes are charged negatively rather than positively.

The voltage employed in a precipitator depends upon the size and type of discharge electrode used, the gap distance between the discharge and collecting electrodes, and the temperature and other characteristics of the gas being treated. There is a definite relation between the velocity of the gases through the precipitator and the length of the path between the electrodes. In other words, the particles must be under the influence of the electric field for a suitable period of time in order to secure satisfactory removal of these particles from the gases. The length of this period depends to a considerable extent upon the character of the particles to be precipitated. If the particles are fluffy in character, light, dry and very finely divided, a longer time will be required than if they are relatively heavy or coarse or of a liquid or sticky nature.

In practice it is common to use for collecting electrodes pipes ranging from 6 to 12 inches in diameter and from 6 to 15 feet in length, and gas velocities from 6 to 15 feet per second. If the collected material is liquid in nature it will trickle down the surface of the collecting electrodes and will drop into a suitable container provided for the purpose beneath the precipitator. In such cases the precipitator is self cleansing. If the collected material is dry or at all sticky in nature it is often necessary to dislodge it from the collecting electrodes by mechanical means. Usually the cleaning is accomplished by striking or rapping the collecting electrode plates, whereupon the material falls into hoppers placed beneath the precipitator. In some cases it is also necessary to clean the dis-

charge electrodes occasionally, and when such a condition is anticipated provision is also made for rapping the framework which supports the high tension or discharge electrodes.

The power consumption of an electrical precipitator is governed by the number of electrodes and their length, type and size, as well as by the characteristics of the gas to be cleaned, as regards temperature, conductivity, etc. The power consumption is in no cases excessive and is usually quite small. The usual consumption of energy is from 1 to 2 kilowatt-hours per 100,000 cubic feet of gas.

Typical Installations. — The Cottrell Processes are in use for the removal of suspended particles from the gases given off by sintering machines, reverberatory furnaces, lead and copper converters and blast furnaces, drying, roasting, and calcining kilns, electric furnaces, and gas producers. They are also in use for cleaning the hot gases from furnaces burning pyrites or other sulphur-bearing materials in sulphuric acid manufacturing plants, and for the removal of acid mist from the gases given off by sulphuric acid concentrators, nitrating towers, pickling vats, etc. They have been applied also for cleaning gases from iron blast furnaces and the recovery from such gases of potash, manganese and other valuable materials.

At Garfield, Utah, 200,000 cubic feet of gas per minute coming from copper converters are treated for the recovery of material containing lead, silver, bismuth, etc.

At Duluth, Minnesota, 5000 cubic feet per minute of producer gas is cleaned in a Cottrell precipitator having twenty-five collecting electrode tubes each 10 inches in diameter and 12 feet in length, the suspended particles of tar in the gases being completely removed so that the gas is readily available for gas engine use.

At the plant of the Davison Chemical Company at Baltimore, Maryland, 30,000 cubic feet of gas per minute coming from sulphuric acid concentrators are treated by the Cottrell processes, with the recovery of over 20 tons per day of 40° Bé. sulphuric acid.

At the Washoe smelter of the Anaconda Copper Company, at Anaconda, Montana, Cottrell precipitators are being installed to handle all of the gases going to the main smelter stack, amounting to some 3,000,000 cubic feet per minute. The Cottrell Processes will also be used at this plant for the purification and recovery of arsenic from the flue dust recovered in the main precipitation installation.

At the plant of the Security Cement and Lime Company at Hagerstown, Maryland, the gases from four cement kilns are treated, recovering approximately 20 tons per day of dust containing from 8 to 10 per cent water-soluble K_2O.

At the plant of the International Smelting Company at Miami, Arizona, two Cottrell installations are in service collecting and recovering suspended particles from the gases from four Wedge roasters used as driers and from three copper converters.

BIBLIOGRAPHY. — A few recent articles describing the Cottrell Process and its applications are: Schmidt, W. A., *Trans. Am. Inst. Chem. Eng. Vol.* 8, p. 35, 1915; Cottrell, F. G., *Eng. and Min. J.*, Vol. 101, p. 385, 1916; Bradley, *Met. and Chem. Eng.*, Vol. 13, p. 911, 1915; Aldrich, C. H., *Trans. Am. Electroch. Soc.*, 28, p. 119, 1915; Egbert, H. D., *The Blast Furnace and Steel Plant*, 6, p. 39, 1918; Landolt, P.E., *Chem. and Met. Eng.*, Vol. 25, 1921, contains fairly complete bibliography up to Sept., 1921; *G. E. Rev.*, 1921.

ELEMENTS, CHEMICAL. — (*See also Electrochemistry, Principles of; Weights of Materials; Heat and Heat Effects.*) An element is a substance which cannot be further decomposed into simpler substances by any known means of chemical analysis. At the present time about 100 elements (including the radioactive elements) have been discovered. Recent investigations in radioactivity have proved that substances now regarded as stable elements may be undergoing a process of transformation, the rate of which is, however, too slow to be detected by any known means. In the case of other highly radioactive elements the rate of spontaneous transformation is relatively great; thus it has been shown that helium is an element resulting from the disintegration of radium and the unstable elements formed in the process of its breaking up. The "life" of these unstable elements varies from a few minutes to many years.

Atomic Weights. — Elements always combine in definite proportions (or simple multiples of these proportions). It is possible to assign to each element a number such that the relative proportions (by mass) in which two or more elements combine are simple multiples of these numbers. The minimum combining mass or weight of oxygen has been chosen as 16, for certain practical reasons, and in terms of this standard the smallest combining weights of the other elements are determined by experiment. The numbers thus obtained are called "atomic masses" or more commonly "atomic weights." Formerly the atomic weight of hydrogen was taken as unity, in which case the atomic weight of oxygen equalled 15.98. On the atomic theory the atomic weight is proportional to the mass of the atom. The values of the atomic weights of the elements, as revised to 1921 by the International Committee on Atomic Weights, are given in the following table.

The Periodic Law. — The most important relation between the atomic weights was discovered by Mendelejeff and Lothar Meyer and is known as the Periodic Law. This states that "*the properties of the chemical elements are periodic functions of their atomic weights.*" Thus the elements may be arranged according to ascending values of their atomic weights in groups or families in each of which the chemical and physical properties of the elements are related. The existence and properties of several elements have been predicted by the aid of this law before their discovery.

Atomic Numbers. — Investigations on the X-ray spectra of the elements have shown that *atomic numbers* may be assigned to them which are of more fundamental significance than their atomic weights. These numbers are determined by the number of electrons in the atom; the frequency of the lines emitted by the X-ray spectrum of an element is determined by them. See papers by Moseley, Soddy, Rutherford and Harkins.

INTERNATIONAL ATOMIC WEIGHTS, 1921

Element	Symbol	Atomic weight	Element	Symbol	Atomic weight
Aluminium	Al	27.1	Molybdenum	Mo	96.0
Antimony	Sb	120.2	Neodymium	Nd	144.3
Argon	A	39.9	Neon	Ne	20.2
Arsenic	As	74.96	Nickel	Ni	58.68
Barium	Ba	137.37	Nitron (radium emanation)	Nt	222.4
Bismuth	Bi	208.0	Nitrogen	N	14.008
Boron	B	10.9	Osmium	Os	190.9
Bromine	Br	79.92	Oxygen	O	16.00
Cadmium	Cd	112.40	Palladium	Pd	106.7
Caesium	Cs	132.81	Phosphorus	P	31.04
Calcium	Ca	40.07	Platinum	Pt	195.2
Carbon	C	12.005	Potassium	K	39.10
Cerium	Ce	140.25	Praseodymium	Pr	140.9
Chlorine	Cl	35.46	Radium	Ra	226.0
Chromium	Cr	52.0	Rhodium	Rh	102.9
Cobalt	Co	58.97	Rubidium	Rb	85.45
Columbium	Cb	93.1	Ruthenium	Ru	101.7
Copper	Cu	63.57	Samarium	Sa	150.4
Dysprosium	Dy	162.5	Scandium	Sc	45.1
Erbium	Er	167.7	Selenium	Se	79.2
Europium	Eu	152.0	Silicon	Si	28.3
Fluorine	F	19.0	Silver	Ag	107.88
Gadolinium	Gd	157.3	Sodium	Na	23.00
Gallium	Ga	70.1	Strontium	Sr	87.63
Germanium	Ge	72.5	Sulphur	S	32.06
Glucinum	Gl	9.1	Tantalum	Ta	181.5
Gold	Au	197.2	Tellurium	Te	127.5
Helium	He	4.00	Terbium	Tb	159.2
Holmium	Ho	163.5	Thallium	Tl	204.0
Hydrogen	H	1.008	Thorium	Th	232.15
Indium	In	114.8	Thulium	Tm	168.5
Iodine	I	126.92	Tin	Sn	118.7
Iridium	Ir	193.1	Titanium	Ti	48.1
Iron	Fe	55.84	Tungsten	W	184.0
Krypton	Kr	82.92	Uranium	U	238.2
Lanthanum	La	139.0	Vanadium	V	51.0
Lead	Pb	207.20	Xenon	Xe	130.2
Lithium	Li	6.94	Ytterbium (Neoytterbium)	Yb	173.5
Lutecium	Lu	175.0	Yttrium	Yt	89.33
Magnesium	Mg	24.32	Zinc	Zn	65.37
Manganese	Mn	54.93	Zirconium	Zr	90.6
Mercury	Hg	200.6			

ELEVATORS, ELECTRIC. — (*See also Hoists; Motors, Industrial Applications of.*) Elevators may be divided in two general classes, freight and passenger. The former may be divided into (1) slow-speed elevators (75 feet per minute or less), (2) infrequent duty elevators and (3) high-speed (250 feet per minute or less) material-handling elevators provided with accurate means for landing so as to accommodate wheeled vehicles. The latter may be divided into local service (450 feet per minute and less), which requires rapid rates of acceleration and accurate landing; and high-speed express (600 feet per minute and more) which requires smooth acceleration and efficient landing. These express elevators sometimes must be operated as local elevators for a few floors at the top of the building and are often of the two-speed type.

Power Required. — The horse-power of an elevator motor depends on the load to be raised, the speed of travel and the elevator efficiency, that is, the combined efficiency of the motor, the gear and the drive. It is customary to counterbalance the weight of the cage, and if it is under or overbalanced this amount must be added or deducted from the live load to get the actual load.

$$\text{Horse-power} = \frac{(\text{Unbalanced load in lb.}) \times (\text{Speed of elevator in ft. per min.})}{33{,}000 \times (\text{Efficiency})}$$

An efficiency of 0.50 is generally used in problems of this kind.

As the load is intermittent the motors are generally rated on the basis of 55° C. rise, with full load for one-half hour. The starting torque should be from 2 to 2½ times full-load torque.

Direct-current and Alternating-current Systems are both in general use for elevator service. On a-c. systems the induction motor, either of the squirrel-cage or wound-rotor type, is used. These motors may be either single-speed, or multi-speed, depending upon the kind of service.

For Low-speed Freight Elevators, single-speed motors of either type should be used; the compound wound direct-current motor whose series field is cut out when it is fully up to speed, or the single-speed squirrel-cage induction motor, or the single-speed slip-ring induction motor. Squirrel-cage induction motors are sometimes not permitted on power systems, which lack capacity, but they are usually permitted up to 25 h.p.

For High-speed Material-handling Elevators, which are intended to move wheeled vehicles, it is necessary to provide an extremely positive and slow speed for accurate landing, and for this purpose the adjustable-speed direct-current motor equipped with dynamic braking control is suited.

Local Service Passenger Elevators for 450 feet per minute and less require a running speed of about 150 feet per minute and require extremely rapid acceleration and deceleration. For this purpose the single speed, a-c. or d-c. motor is available up to about 150 feet per minute. Between 150 and 450 feet per minute the adjustable speed motor in either a-c. or d-c. is necessary. For 450 feet per minute with direct current, the direct-coupled motor is sometimes used, but for alternating current the geared motor is necessary in all speeds.

For High-speed Express Passenger Elevators, the direct-coupled, direct-current motor is used exclusively at present. This motor is direct-coupled to a 36-inch drum and runs at 60 revolutions per minute. Its speed is controlled by shunt field down to 45 revolutions per minute and below this by dynamic braking down to about 15 revolutions per minute. Sometimes this motor is controlled by connecting its armature to a multi-voltage supply system of 62–125–187 and 250 volts.

Elevator Controllers must fulfill the most exacting service requirements

They must insure a smooth acceleration, afford positive speed control under widely varying loads, bring the elevator quickly, but smoothly, to a stop, or from high to low speeds, regardless of load. Accurate stops at the landings must be made and devices should be provided to protect against slack cable. overtravel, overloads, failure of voltage, etc. Controllers may be of the full-magnet type, the semi-magnet type or the mechanically operated type. High-speed passenger elevators running from about 100 to 600 feet per minute are practically always installed with full-magnet controllers.

Direct-current Control Equipment. — With a direct-current equipment the car is started by moving the car switch or car-controller handle directly to the full on position. This releases the solenoid brake, after which the motor starts with full field strength and with resistance in the armature circuit. After the motor has started, the contactors on the contactor panel automatically cut out the dynamic brake resistance, the armature resistance and the series field (if used), when the motor reaches its normal speed. Current relays limit the current taken during the acceleration to a predetermined value. An overload circuit breaker is provided which opens the circuit, while running, in case an abnormal current would tend to flow. This overload circuit breaker is generally interlocked with the car switch in such a way that should the motor be stopped by the action of the circuit breaker relay, this can be reset from the car by merely moving the control lever to the off position. The control equipment is generally arranged to give two speeds — one, a range of from rest up to the normal speed of the motor, secured by cutting out the armature resistance step by step, and a higher range, up to twice normal speed, secured by inserting resistance in the shunt field of the motor.

In stopping, the controller handle can be brought back to the second point, giving half speed, and then to the first point, which introduces the resistance in series with the armature and shunts the armature with the dynamic brake resistance which is relatively low. In this way it is possible to give a slow landing speed of about 25 per cent of the normal speed and which does not vary very much with the load in the car. Moving the control handle to the off position stops the car. It causes the motor to be disconnected from the circuit at the same time applying the solenoid brake, which is actuated by a spring or weight. The coil of the dynamic brake is connected to the motor armature so that this circuit is kept closed in stopping until the motor comes practically to rest, and the braking effect of the current generated in this local circuit is available to assist the mechanical brake.

Limit switches actuated by the elevator winding drum are provided to stop the elevator at predetermined limits of travel. In addition to these, overtravel switches are also ordinarily provided in the elevator hatchway as an extra precaution. These switches are arranged to open the control circuit on overtravel of the car. The first-named switches are generally connected so that after the car has been automatically stopped at either limit of travel, it is possible for the operator to start the car in the reverse direction only. When the hatchway-limit switches open, it is, however, generally impossible for the operator to reverse the car without first going to the elevator machine, thus insuring attention to the cause which made it necessary for the hatchway-limit switch to operate the car.

An auxiliary switch which will automatically stop the motor in case the cable slackens from any cause, such as the breaking of the cable or the catching of the cage, is also generally provided. It is mounted on one side of the winding drum and operated by a cross arm extending across the drum. Should the cable become slack the movement of the cross arm will serve to trip the switch, opening the control system and stopping the elevator. The slack-cable switch cannot

be automatically reset, but after having tripped out must be reset by hand thus also obliging the operator to go to the source of trouble before the elevator can be again operated.

Alternating-current Control Equipment. — As with the direct-current system there are also many types of controllers made for the alternating-current system. The best types are those known as the full-magnet controllers, which are automatic in operation and controlled from a car switch in the same manner as the direct-current type. All controllers should be protected by a relay which disconnects the motor from the line when the phase rotation in the supply circuit is, for accidental or other reasons, reversed.

Single-speed alternating-current motors are preferably not to be used for speeds above 130 feet per minute. For higher elevator speeds the two-speed motor is used, and the elevator speed corresponding to the slowest motor speed should not be over about 150 or 200 feet per minute.

A complete alternating-current control equipment generally comprises a contactor panel, a car switch, one or more limit switches, a slack cable switch and a solenoid brake. The solenoid brake is designed to be connected directly to the motor terminals, and when the circuit to the motor is closed the solenoid is energized and the brake retracted. Upon the opening of the main-line circuit, whether this is done intentionally in operating the elevator or is the result of accident, the solenoid is deënergized and the brake is instantly applied. The brake should be designed so as to permit of a gradual rather than an instantaneous braking effect, so as to avoid jarring the car by stopping it too suddenly.

The acceleration is accomplished by a number of alternating-current contactors. These are so connected as to cut a section out of each of the three phases of the rotor circuit simultaneously, and also to change the pole connections of the motor, the rate of acceleration or retardation being governed automatically by a number of time limit relays.

Energy Consumption. — In all elevators, particularly in those electrically operated, the use of power increases in proportion to the number of starts and stops. This is a natural result of the use of the electric motor, in which the largest part of the energy is required for the process of acceleration, the next largest for electrical and mechanical retardation, and a lesser amount for the actual running speed, with a small proportion devoted to the continuous excitation of the field. It therefore follows that in this form of elevator, which is now so widely used, the main part of the consumption of power is directly related to the number of stops and starts which result from the number of persons carried. The diversities in results which have been reported from time to time as to the consumption of current by electrical elevators may therefore be attributed to variations in traffic conditions. The results of a series of tests made upon the operation of a one-to-one traction elevator in a ten-story building as reported by R. P. Bolton (*A.S.M.E., 1910*) are given in the table on the following page.

SPECIFICATIONS FOR ELECTRIC ELEVATORS. — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Performance. — Where elevator is to be situated. Purpose, i.e., whether for passenger or freight service. Characteristics of current supply. Length of run and number of stops. Full running speed and number of feet traveled from rest to full speed. Noiselessness outside of spaces occupied by machinery. Maximum kilowatt-hours per car mile. (This is often supplied by bidder for a given schedule.)

Machinery. — Type of hoisting machinery. Who supplies supports and framing for machines. Details of drum and shaft materials, and minimum diameter.

ENERGY CONSUMED BY AN ELECTRIC ELEVATOR AT VARIOUS LOADS AND STOPS

Machine: One-to-one Electric Traction.
Total Weight of Car: 3956 pounds; Overbalance: 1060 pounds.
Capacity: 2500 pounds at a speed of 500 feet per minute.

Stops at floors No.												
Up	1 and 9							1, 5 and 9				
Down	9 and 1							9, 5 and 1				
Distance..	101 ft. 6 in. one way or per trip = 52 trips per car mile											
Load, lb...	Operator	666	1060	1360	2010	2360	2660	Operator	666	1060	2010	2660
Kw-hr. per car mile..	2.345	2.075	1.945	1.87	2.15	2.495	3.22	3.09	2.855	2.52	2.915	3.85

Stops at floors No.								
Up	1, 3, 5, 7 and 9				1, 2, 3, 4, 5, 6, 7, 8 and 9			
Down	9, 7, 5, 3 and 1				9, 8, 7, 6, 5, 4, 3, 2 and 1			
Distance...........	101 ft. 6 in. one way or per trip = 52 trips per car mile.							
Load, lb............	Operator	666	1060	2010	Operator	666	1060	2010
Kw-hr. per car mile	4.91	4.185	3.975	4.25	7.285	6.75	6.7	7.425

(Drums are usually of gray cast iron and shaft of forged open-hearth steel). Worm and gear details. The worm shall be of open-hearth steel and the gear of phosphor bronze. Motor details (*see articles on Motors.*) Method of taking up thrust of shaft and drum. Details of brake.

Safety Devices.—(a) Centrifugal governor: When the speed exceeds a stated speed, the car shall be gradually retarded. (b) Machine governor: When speed of car reaches a certain amount, greater than that at which the centrifugal governor acts, and less than that required to throw the safety clutches on the car, the circuit breaker shall open. (c) Limit stops, upper and lower: When the car approaches the limits, the controller circuit shall be automatically opened, and the brake applied. (d) Circuit-breaker details.

Cables.—Description, diameter and minimum number. The number of cables shall be sufficient to prevent any slipping of the cables on the drum when the car is operated under maximum load and contract speed.

Switchboard and Wiring. — Details of switchboard. Details of controller cable, which should be extra flexible and insulated with the best quality rubber.

Shaft Equipment. — Description of shaftway inclosures, with drawings.

Elevator pit; general description; whether elevator contractor will find it ready, for use and whether any machinery may be installed below the pit. Penthouse; general description; whether elevator contractor will find it ready for use, and whether any machinery may be installed outside or above penthouse inclosure. Beams for support of sheaves, guides, governors, bumpers, etc. Description and drawings. Whether to be supplied by elevator contractor. Buffers under elevator, description. Counterweights, description; the material is usually grey cast iron. Shaftway fittings, including supports, oil buffers, grating for overhead sheaves, material of sheaves (usually tough grey cast iron), bearing boxes, drip pans, guide rails. The guide rails are usually steel tees of a stated weight per foot. The details of their attachment to the shaft structure should be specified. Doors, details.

Car. — Details of construction, including statement of sides where door or doors are required. Safety clutch, details. Slack-cable device, details. Counterbalance chains shall be attached to bottom of car to compensate for weight of elevator cables.

Cage. — Details of construction, fireproofing, floor details, including finish of floors at edges, folding gate, lighting, emergency door, etc. There shall be no strain on the cage when the safety devices operate. Operating equipment, including controller, emergency switch, lighting switch, safety lever, annunciator in connection with signal system, etc.

Control System.— Control mechanism, description. Number of speeds by controller. Car to stop upon release of lever. Controller shall be out of way of passengers, yet allow operator to keep one hand on the gate while the other is on the switch. Provision shall be made to operate the car from the switchboard when not being operated from within. If the operator reverses while the car is running, the reversal of direction shall be gradual and without shock to passengers. The following operations shall be accomplished by controller: (a) Make and break circuit and reverse motor without destructible arcing at contacts or flashing at brushes. (b) Give easy acceleration of car independently of operator; stopping or starting without shock or jar. (c) Lift brake, cut out starting resistance quietly and operate fields by successive steps in starting. (d) Apply brake and cut in special field resistance in series with armature in stopping in order to attain a retardation of the car prior to the pressure of the brake shoes upon the brake pulley. (e) Other control will be considered.

Signaling and Indicator System.—Details of signaling system. Signals shall indicate correctly the position of the car whatever the motions of the latter. The indicator system shall give the operator in the car a clear signal showing what floor to stop at and whether passengers want to go up or down.

General Matters. — Details of test. Instruction by contractor in use of equipment. During the operation of the cars in the shaft and before its final completion, the contractor shall install on the cars a 6-inch gong, which will ring automatically at each floor up and down the shaft as a warning signal. Lubrication details. Painting details. Tools to be supplied.

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ENDOSMOSE OR ELECTRIC OSMOSIS. — (*See also Electrochemistry, Principles of.*) If a current is passed along a film of water on a solid insulating surface, the water will be attracted to the negative electrode. Similarly an electric current flowing through a porous partition, or capillary opening, filled with conducting liquid, causes a flow of the liquid through the partition or opening. This phenomenon is known as endosmose or electric osmosis. While various physicists have offered explanations of this phenomenon, none have received general acceptance.

The quantity of a given liquid which is carried through a given porous diaphragm in a definite time varies directly with the current strength and is independent of the area and thickness of the diaphragm. The quantity varies with the nature of the solution, being greater with liquids of high specific resistance and high specific capacity. The direction of flow is generally from positive to negative electrode, but under certain conditions the flow may be in the opposite direction.

Applications of Endosmose. — Endosmose may be utilized to remove water from wet substances. Thus if a wooden box be equipped with perforated metallic plates at opposite ends and be filled with wet turf and current circulated through the turf from one end to the other, water will ooze out of the perforations of the cathode plate. Endosmose is utilized in tanning processes to accelerate the passage of tanning fluids through the hides, and efforts are being made to utilize it for drying peat, extracting sap and similar purposes.

Endosmose of Negative Feeders. — If electric conductors covered with a saturated braid or a number of such braids be made the cathode of a water bath and say 110 volts applied between anode and cathode, the braid will blister in a few minutes and the blisters will finally burst, grounding the conductor to the water. The same action goes on at lower voltages at a proportionately lower rate, but with equal certainty. No action is observable if the wire be made the anode instead of the cathode. The same phenomenon is said to be observable with rubber insulation, but the writer has tried several samples of high-grade American compounds made with 30 per cent of Para rubber and obtained no signs of endosmose after subjecting them for a week to a pressure of 110 volts. C. H. Wordingham (*Elec. Engineer, London, Vol. 26, 1900, p. 93*) found, however, that certain makes of rubber insulation are porous and water may enter and eventually destroy the whole of the rubber on the negative conductors. For this reason insulation of low specific resistance should never be used on negative feeders. For the same reason it is not practicable to maintain a negative contact rail on electric railways, below earth potential, as the insulators soon become saturated with moisture and thereby become conductors. Thus S. B. Fortenbaugh, relating the experience of the Tube Railways in London, says:

1. That the difference of potential between the positive conductor and earth is always normally considerably greater than the potential existing between the negative conductor and earth.

2. That a reversal of polarity is always instantly accompanied by a considerable increase in the normal leakage current between the positive and negative conductor.

3. The insulation of the negative conductor to earth cannot be indefinitely maintained.

Effect of Endosmose on Insulation Measurements. — When testing wires having insulation of low specific resistance, endosmose may further lower the resistance if the wire is negative to the water bath.

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EQUATIONS. — (*See also Complex Numbers; Errors of Observation; Logarithms; Vectors.*) The subjects treated in this article are

QUADRATIC EQUATION. — The solution of

$$ax^2 + bx + c = 0$$

is
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a}\left[1 \pm \sqrt{1 - \frac{4ac}{b^2}}\right]$$

CUBIC EQUATION. — The solution of the general cubic equation of the form

$$ax^3 + 3bx^2 + 3cx + d = 0 \qquad (1)$$

is obtained as follows: Put

$$y = ax + b$$
$$H = b^2 - ac$$
$$G = a^2d - 3abc + 2b^3$$

then (1) becomes

$$y^3 - 3Hy + G = 0. \qquad (2)$$

The nature of the roots of this equation depends upon the algebraic sign of H and the relative numerical magnitudes of G and H.

Case I. H positive and G numerically less than $2H\sqrt{H}$. (G may be either positive or negative.) Put

$$\phi = \frac{1}{3}\sin^{-1}\left[\frac{G}{2H\sqrt{H}}\right]$$

then the three roots of (2) are

$$y_1 = 2\sqrt{H}\sin\phi$$
$$y_2 = 2\sqrt{H}\sin(\phi + 120) \qquad (3)$$
$$y_3 = 2\sqrt{H}\sin(\phi - 120)$$

and all three roots are real.

Case II. H positive and G numerically greater than $2H\sqrt{H}$. Put

$$u = \frac{1}{3}\cosh^{-1}\left[\frac{\pm G}{2H\sqrt{H}}\right]$$

the plus sign to be used when G is positive, the negative sign when G is negative thus making $\cosh 3u$ positive in either case.

Then the three roots of (2) are

$$y_1 = \mp 2\sqrt{H}\cosh u$$
$$y_2 = \pm\sqrt{H}\cosh u + \sqrt{-3H}\sinh u \qquad (4)$$
$$y_3 = \pm\sqrt{H}\cosh u - \sqrt{-3H}\sinh u$$

the upper signs to be used when G is positive, the lower when G is negative. The first root in this case is real, the other two being complex quantities. See articles on *Complex Quantities* and *Hyperbolic Functions.*

Case III. H negative and G any value. Put

$$u = \frac{1}{3} \sinh^{-1} \left[\frac{\mp G}{2 H \sqrt{-H}} \right]$$

the negative sign to be used when G is positive, the positive sign when G is negative, thus making $\sinh 3u$ positive in either case. Then the three roots of (2) are:

$$\begin{aligned} y_1 &= \mp 2 \sqrt{-H} \sinh u \\ y_2 &= \pm \sqrt{-H} \sinh u + \sqrt{3H} \cosh u \\ y_3 &= \pm \sqrt{-H} \sinh u - \sqrt{3H} \cosh u \end{aligned} \qquad (5)$$

the upper sign to be used when G is positive, the lower sign when G is negative. The first root in this case is real, the other two complex quantities.

Note that in each case the sum of the three roots is zero. Also, in the special case when $G = 2H\sqrt{H}$, the angle ϕ is 30°, and therefore from (3) the three roots are $\sqrt{H}$, $\sqrt{H}$ and $-2\sqrt{H}$; this also follows from (4).

Examples. — *Case I.*

$$y^3 - 6y + 2 = 0.$$

$H = 2$ and is positive and $G = 2$ and is less than $2H\sqrt{H}$.

$$\phi = \frac{1}{3} \sin^{-1} \left[\frac{2}{4\sqrt{2}} \right] = 6.90°$$

whence $y_1 = +0.3399$; $y_2 = 2.2618$; $y_3 = -2.6018$.

Case II.

$$y^3 - 3y + 4 = 0.$$

$H = 1$ and is positive and $G = 4$ and is greater than $2H\sqrt{H}$.

$$u = \frac{1}{3} \cosh^{-1} 2 = 0.439$$

whence $y_1 = -2.196$; $y_2 = 1.098 + 0.785\sqrt{-1}$; $y_3 = 1.098 - 0.785\sqrt{-1}$.

Case III.

$$y^3 + 6y + 2 = 0.$$

In this case, $H = -2$ and $G = 2$

$$\begin{aligned} u &= \frac{1}{3} \sinh^{-1} \frac{-2}{-4\sqrt{2}} = 0.1155 \\ y_1 &= -0.3275 \\ y_2 &= 0.1638 + 2.467\sqrt{-1} \\ y_3 &= 0.1638 - 2.467\sqrt{-1} \end{aligned}$$

SIMULTANEOUS EQUATIONS. — Given n independent equations in n unknowns, these n equations fix the values of each of the n unknowns. To solve such a set of simultaneous equations in x, y and z, say, solve each of

the three equations for x in terms of y and z. Equating these three values for x gives two equations in y and z. Solving each of these two equations for y in terms of z and equating these two values of y gives a single equation in z. The solution of this last equation then gives the value of z. Then substitute this value of z in either of the equations in y and z, and solve for y. Then substitute these values of y and z in any one of the original equations and solve for x.

Determinants. — In the case of linear simultaneous equations (i.e., when x, y and z occur only to the first power), the equations may be solved by determinants. This method is a considerable time saver when the number of unknowns is greater than three, but when the number of unknowns is three or less the straight substitution method is preferable.

The determinant of a set of simultaneous equations is formed by writing the equations one below the other with the same unknown in the same relative position in each. The block of numbers forming the coefficients of the unknowns is called the determinant. For example, the determinant of the equations

$$\begin{aligned} w + x + y + z &= 6 \\ w + y + 3z &= 4 \\ w + 2x + 3y &= 1 \\ w + 3x + z &= 3 \end{aligned}$$

is

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 0 & 1 \end{vmatrix}$$

The values of any one of the unknowns, say y, is found by writing a second determinant, D_y, exactly like the determinant D, except that the constants forming the right-hand members of these equations are substituted for the coefficients of y in the determinant, that is

$$D_y = \begin{vmatrix} 1 & 1 & 6 & 1 \\ 1 & 0 & 4 & 3 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{vmatrix}$$

Then $$y = \frac{D_y}{D}$$

and similarly for the other unknowns.

The value of any determinant is found by making use of the following rules:

(1) If a determinant has two equal rows or columns, it is equal to zero.

(2) To any row or column one may add or subtract any number of times any other row or column without altering the value of the determinant.

(3) To multiply any row or column by a number is the same as multiplying the determinant by that number.

(4) If all the terms in a row or column except one are zero, the determinant reduces to one of a lower order which may be obtained by striking out the row and column which intersect at the term in question, and multiplying the whole by that term, changing the sign of this term, however, if it is removed by an odd number of terms from the principal diagonal. The principal diagonal is

the line of terms beginning at the upper left-hand corner and ending at the lower right-hand corner. Thus,

$$\begin{vmatrix} 1 & 2 & 8 & 5 \\ 3 & 4 & 6 & 9 \\ 0 & 3 & 0 & 0 \\ 6 & 7 & 4 & 3 \end{vmatrix} = -3 \begin{vmatrix} 1 & 8 & 5 \\ 3 & 6 & 9 \\ 6 & 4 & 3 \end{vmatrix}$$

the principal diagonal being that with the figures 1, 4, 0 and 3. It is immaterial whether the distance from the diagonal is counted along a row or a column.

(5) The value of a determinant of the second order is

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

The reduction of determinants is effected by altering the terms according to the above rules until a row or column is obtained in which all terms but one are zero. This enables a reduction of order to be effected in accordance with rule 4. Reductions are continued until one of the second order is obtained.

Example. — By rule (2), the determinant D given above may be written as follows, the top row being subtracted from each of the others.

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 2 & -1 & 0 \end{vmatrix}$$

By rule (4) this reduces to

$$D = \begin{vmatrix} -1 & 0 & 2 \\ 1 & 2 & -1 \\ 2 & -1 & 0 \end{vmatrix}$$

By rule (3) this may be written

$$D = \frac{1}{2 \times 2} \begin{vmatrix} -2 & 0 & 4 \\ 2 & 4 & -2 \\ 2 & -1 & 0 \end{vmatrix}$$

By rule (2), adding the top row to each of the others, this reduces to

$$D = \frac{1}{2 \times 2} \begin{vmatrix} -2 & 0 & 4 \\ 0 & 4 & 2 \\ 0 & -1 & 4 \end{vmatrix}$$

which by rule (4) reduces to

$$D = \frac{-2}{2 \times 2} \begin{vmatrix} 4 & 2 \\ -1 & 4 \end{vmatrix}$$

whence by rule (5) $\quad D = -\frac{1}{2}(4 \times 4 + 1 \times 2) = -9.$

Similarly, $\quad D_y = 21$

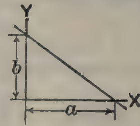
and therefore $\quad y = -\frac{21}{9} = -2.333.$

Similarly, $\quad w = 13.33, \ x = -2.667, \ z = -2.333.$

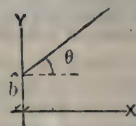
EQUATIONS OF COMMON CURVES.

Straight Line. —

$$\frac{x}{a} + \frac{y}{b} = 1$$

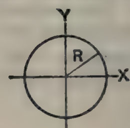


or $$y = x \tan \theta + b.$$



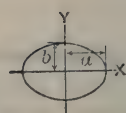
Circle. —

$$x^2 + y^2 = R^2.$$



Ellipse. —

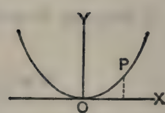
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$



Parabola (Vertical). —

$$y = kx^2$$

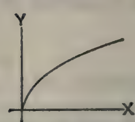
where k is a constant.



Parabola (Horizontal). —

$$y = k\sqrt{x}$$

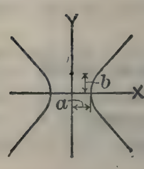
where k is a constant.



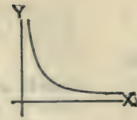
Hyperbola. —

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ (Horizontal).}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \text{ (Vertical).}$$



Rectangular or Equilateral Hyperbola. —

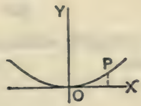


$$y = \frac{k}{x}$$

where k is a constant.

Catenary. —

$$y = \frac{1}{k}\left[\cosh kx - 1\right]$$



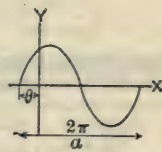
where k is a constant The length of arc from O to P is

$$\lambda = \frac{1}{k}\sinh(kx).$$

See tables in article on *Hyperbolic Functions*.

Sinusoid. —

$$y = A\sin(ax + \theta).$$



DIFFERENTIAL EQUATIONS. — Differential equations of the following forms are met with in the theory of alternating and transient currents.

The following notation is used: $\epsilon = 2.7183\ldots$ = base of natural system of logarithms; x, y, z are variables. A, ϕ, γ, and θ are integration constants. Other letters represent known constants.

$$\frac{dy}{dx} = ay. \tag{1}$$

Solution: $$y = A\epsilon^{ax}.$$

$$\frac{dy}{dx} + ay = 0. \tag{2}$$

Solution: $$y = A\epsilon^{-ax}.$$

$$\frac{dy}{dx} + ay = b.$$

Solution: $$y = \frac{b}{a}\left[1 - A\epsilon^{-ax}\right]. \tag{3}$$

$$\frac{d^2y}{dx^2} = -a^2y. \tag{4}$$

Solution: $$y = A\sin(ax + \phi).$$

$$\frac{d^2y}{dx^2} = a^2y. \tag{5}$$

Solution: $$y = A\sinh(ax + \phi).$$

$$\frac{d^2y}{dx^2} + 2u\frac{dy}{dx} + (u^2 + a^2)\,y = 0. \tag{6}$$

Solution:

Case I. a^2 positive:— $y = A\epsilon^{-ux}\sin(ax + \phi)$.
Case II. a^2 negative:— $y = A\epsilon^{-ux}\sinh(ax + \phi)$.
Case III. $a^2 = 0$:— $y = A\,(x + \phi)\,\epsilon^{-ux}$.

$$\frac{d^2y}{dx^2} + 2u\frac{dy}{dx} + (u^2 + a^2)\,y = B\sin(\omega x + \theta). \tag{7}$$

The complete solution of this equation consists of the solution of No. 6 plus the term

$$\left(\frac{B\sin\delta}{2u\omega}\right)\sin(\omega x + \theta - \delta), \tag{a}$$

where

$$\delta = \tan^{-1}\frac{2u\omega}{a^2 + u^2 - \omega^2}\cdot$$

For each additional sine term added to the right-hand member of the equation, there will be a corresponding term of the same form as (**a**) in the solution.

$$\frac{d^ny}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \ldots a_1\frac{dy}{dx} + a_0y = B\sin(\omega x + \theta). \tag{8}$$

Solution:

$$y = A_1\epsilon^{m_1x} + A_2\epsilon^{m_2x} + \ldots A_n\epsilon^{m_nx} + KB\sin(\omega x + \theta + \delta),$$

where m_1, m_2, etc., are the n roots of the equation

$$m^n + a_{n-1}m^{n-1} + \ldots a_1m + a_0 = 0,$$

and K and δ are found by substituting the $KB\sin(\omega x + \theta + \delta)$ by itself in the given differential equation and equating the coefficients of $\sin(\omega x + \theta)$ and $\cos(\omega x + \theta)$ respectively on the two sides of the resulting equation. When the second member of the differential equation is a constant, B, the sine term in the solution becomes simply $\frac{B}{a_0}\cdot$

Note that all of the preceding equations are merely special cases of the general equation (8).

$$\frac{d^2y}{dx^2} + 2u\frac{dy}{dx} + (u^2 - q^2)\,y = \frac{1}{c^2}\frac{d^2y}{dz^2}\cdot \tag{9}$$

The complete solution of this equation contains an infinite number of terms of the form

$$y = \epsilon^{-(u-s)x}\left[A_1\epsilon^{mz}\sin(\omega x + nz + \phi_1) + A_2\epsilon^{-mz}\sin(\omega x - nz + \phi_2)\right], \tag{a}$$

where A_1, ϕ_1, A_2, ϕ_2 and two of the four constants ω, s, m and n are integration constants (fixed by the terminal conditions). The values of m and n in terms of ω and s are

$$m = c\sqrt{ab}\cos\frac{\eta + \epsilon}{2},$$

$$n = c\sqrt{ab}\sin\frac{\eta + \epsilon}{2},$$

where

$$a = \sqrt{(s+q)^2+\omega^2}, \qquad \epsilon = \tan^{-1}\left(\frac{\omega}{s+q}\right),$$

$$b = \sqrt{(s-q)^2+\omega^2}, \qquad \eta = \tan^{-1}\left(\frac{\omega}{s-q}\right).$$

The values of ω and s in terms of m and n are

$$\omega = \frac{\sqrt{FG}}{c}\cos\frac{\alpha+\beta}{2},$$

$$s = \frac{\sqrt{FG}}{c}\sin\frac{\alpha+\beta}{2},$$

where

$$F = \sqrt{(n+cq)^2+m^2}, \qquad \alpha = \tan^{-1}\left(\frac{m}{n+cq}\right),$$

$$G = \sqrt{(n-cq)^2+m^2}, \qquad \beta = \tan^{-1}\left(\frac{m}{n-cq}\right).$$

The solution of equation (9) may also be written as a series of terms of the form

$$y = M\epsilon^{-(u-s)x}\sin(\omega x+\phi+\mu), \tag{b}$$

where

$$M = \frac{A}{\sqrt{2}}\sqrt{\cosh 2(mz+\gamma)+\cos 2(nz+\theta)},$$

$$\tan\mu = \tanh(mz+\gamma)\tan(nz+\theta),$$

where A, ϕ, γ and θ are integration constants, and the relations between the other constants ω, s, m and n are the same as above.

In the special case when $q = 0$, the solution of equation (9) is

$$y = \epsilon^{-ux}[f_1(\omega x+nz)+f_2(\omega x-nz)], \tag{c}$$

where f_1 and f_2 are any two arbitrary functions and ω and n are connected by the relation

$$\frac{\omega}{n} = \frac{1}{c}.$$

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx} = 4a^2y. \tag{10}$$

Solution is an infinite series:

$$y = A\left[1+(ax)^2+\frac{(ax)^4}{2^2}+\frac{(ax)^6}{(3!)^2}+\frac{(ax)^8}{(4!)^2}+\ldots\right]. \tag{a}$$

This series is absolutely convergent for all values of x, but for $ax > 1$ the following approximate series is more convenient and sufficiently accurate.

$$y = \frac{B\epsilon^{2ax}}{\sqrt{2ax}}\left[1+\frac{1}{16ax}+\frac{3^2}{2(16ax)^2}+\frac{3^2.5^2}{3!(16ax)^3}+\frac{3^2.5^2.7^2}{4!(16ax)^4}\right]. \tag{b}$$

ERRORS OF OBSERVATION. — When a quantity is measured with all possible accuracy many times in succession, the numbers expressing the results are found to differ by amounts, which, although generally small, are occasionally considerable in comparison with the quantity measured. Though these differences may be decreased by improved methods, better instruments or greater skill, they can never be entirely removed. They are known as the errors of observation. The following formulas, which are derived from the Theory of Least Squares, apply to such errors and not to errors which can be eliminated by correcting mistakes of the observer or defects of instruments or methods of observation. *That is, they apply only to errors which may be either positive or negative, the chance of a positive error occurring being exactly the same as the chance of a negative error occurring.*

Weighted Observations. — Sometimes in spite of the care with which observations are taken, there are reasons for believing that some observations are better than others. In this case the observations are given different "weights" or numbers expressing their relative practical worth. A weighted observation is an observation multiplied by its weight.

PROBABLE VALUE OF SEVERAL OBSERVATIONS. — The most probable value of a quantity which is observed directly several times with equal care is the arithmetical mean of the measurements.

Example. — The length of a room (in feet) is measured ten times with the following results:

20.05	20.06	19.98	19.99	20.00
20.07	20.01	20.05	19.95	19.98

The arithmetical mean of these lengths is 20.01 feet, which is accordingly the probable length of the room.

The most probable value of a quantity which is observed directly several times, but where the observations have different weights, is equal to the sum of the weighted observations divided by the sum of the weights.

Example. — In the above example if the fourth and fifth observations are twice as reliable as the others, and the seventh three times as reliable, the most probable value would be the sum of 20.05, 20.06, 19.98, (19.99 × 2), (20.00 × 2), 20.07, (20.01 × 3), 20.05, 19.95 and 19.98, divided by 14, which is equal to 20.01, which happens in this instance to agree with the unweighted average.

PROBABLE ERROR OF ANY ONE OF SEVERAL OBSERVATIONS. — The probable error of a number of direct observations made with equal care is found by the following formula:

$$r = 0.6745\sqrt{\frac{\Sigma v^2}{n-1}},$$

where

n = number of observations.
r = probable error of a single observation.
v = residual found by subtracting the arithmetical mean from each measurement.

Example.—In the example cited above, $n = 10$. The values of v in the order of the observations given above are as follows:

+0.04	+0.05	−0.03	−0.02	−0.01
+0.06	0	+0.04	−0.06	−0.03

The sum of the squares of these quantities is 0.0152. The probable error in any one of the ten observations is

$$r = 0.6745\sqrt{\frac{0.0152}{9}} = 0.0277.$$

The probable error of each of a number of direct observations, where the observations have different weight, is found by the following formula, in which p represents the weight of an observation.

$$r_1 = 0.6745\sqrt{\frac{\Sigma pv^2}{n-1}}.$$

Example. — In the example cited above for weighted observations, the values of v are the same as the values of v in the preceding example.

The values of pv^2 are as follows:

0.0016	0.0025	0.0009	0.0008	0.0002
0.0036	0	0.0016	0.0036	0.0009

and their sum is 0.0157.

Hence the probable error of a single observation

$$r_1 = 0.6745\sqrt{\frac{0.0157}{9}} = 0.0282.$$

PROBABLE ERROR OF THE ARITHMETICAL MEAN. — If

r = probable error of a single observation,
n = number of observations,
r_0 = probable error of the arithmetical mean,

$r_0 = \dfrac{r}{\sqrt{n}}$ for observations of equal weight, or

$r_0 = \dfrac{r_1}{\sqrt{\Sigma p}}$ for unequal weight.

Example. — In the example cited above for observations of equal weight, $r = 0.0277$ and $n = 10$, and $r_0 = 0.0277/\sqrt{10} = 0.0088$.

In the example cited above for observations of unequal weight, $r_1 = 0.0282$ and $\Sigma\ p = 14$, and $r_0 = 0.0282/\sqrt{14} = 0.0075$.

It should be noted that the probable error of the mean decreases inversely as the square root of the number of observations.

Example. — The probable error in making a single reading of a watt-hour meter on an electric locomotive is 5 per cent, after all constant sources of error have been eliminated. If 25 locomotives are used, what will be the probable error in the total energy consumption obtained, by adding the watt-hour meter readings? Answer, one per cent.

$$r_0 = \frac{5}{\sqrt{25}} = 1.$$

PROBABLE ERROR IN A RESULT CALCULATED FROM THE MEANS OF SEVERAL OBSERVED QUANTITIES. — Let Z = a sum or difference of several independent quantities.

Let r_1, r_2, r_3, etc., be the probable errors in these quantities. Then the probable error of Z is equal to

$$\sqrt{r_1^2 + r_2^2 + r_3^2 + \text{etc}}$$

Let $Z = Az$, where z is an observed quantity, and A, a known number. Let be the probable error in z. Then the probable error in Z is Ar.

Let Z be the product of two independently observed quantities z_1 and z_2 whose probable errors are r_1 and r_2 respectively. Then the error in Z is equal to

$$\sqrt{z_1^2 r_2^2 + z_2^2 r_1^2}.$$

Let Z be any function of the independently observed quantities z_1, z_2, z_3, etc., whose probable errors are r_1, r_2, r_3, etc. Then the probable error in Z is equal to

$$\sqrt{\left(\frac{dZ}{dz_1}\right)^2 r_1^2 + \left(\frac{dZ}{dz_2}\right)^2 r_2^2 + \left(\frac{dZ}{dz_3}\right)^2 r_3^2 + \text{etc.}}$$

METHOD OF LEAST SQUARES. — A set of simultaneous equations containing more equations than unknowns cannot have any exact solution unless the superfluous equations are deducible from the others. They may, however, have a most probable solution which is such that the sum of the errors remaining after corrections for the deduced values of the unknowns shall be a minimum. This result is obtained as shown in the following example:

$$\begin{aligned} s - t + 2u &= 3, \\ 3s + 2t - 5u &= 5, \\ 4s + t + 4u &= 21, \\ -s + 3t + 3u &= 14. \end{aligned}$$

The coefficients of s are 1, 3, 4 and -1. Multiplying by these coefficients:

$$\begin{aligned} s - t + 2u &= 3, \\ 9s + 6t - 15u &= 15, \\ 16s + 4t + 16u &= 84, \\ s - 3t - 3u &= -14. \end{aligned}$$

Add these equations and obtain $27s + 6t = 88$.

The coefficients of t are -1, 2, 1 and 3. Multiply the original equations by these coefficients.

$$\begin{aligned} -s + t - 2u &= -3, \\ 6s + 4t - 10u &= 10, \\ 4s + t + 4u &= 21, \\ -3s + 9t + 9u &= 42. \end{aligned}$$

Add these equations and obtain $6s + 15t + u = 70$.

The coefficients of u are 2, -5, 4 and 3. Multiply the original equations by these coefficients.

$$\begin{aligned} 2s - 2t + 4u &= 6, \\ -15s - 10t + 25u &= -25, \\ 16s + 4t + 16u &= 84, \\ -3s + 9t + 9u &= 42. \end{aligned}$$

Add these equations and obtain $t + 54u = 107$.

The equations

$$\begin{aligned} 27s + 6t &= 88, \\ 6s + 15t + u &= 70, \\ t + 54u &= 107, \end{aligned}$$

are then solved in the ordinary way, the solution being

$$s = 2.47, \quad t = 3.56, \quad \text{and} \quad u = 1.92.$$

BIBLIOGRAPHY. — The subject of error of observation is very fully treated in *A Text Book on the Method of Least Squares*, by Mansfield Merriman.

EXPONENTIAL FUNCTIONS. — (*See also Roots and Powers.*) When the relation between any variable y and another variable x is such that x occurs as an exponent of one or more terms, y is said to be an exponential function of x. Of particular importance in connection with electric circuits are the exponential functions ϵ^x and ϵ^{-x}, where ϵ is the base of the natural logarithms (*see Logarithms*).

ϵ^x may be obtained directly from the table of common logarithms (*see Logarithms*) from the relation that ϵ^x is equal to the number whose logarithm to the base 10 is $0.4343\,x$ (or more exactly $0.4342945\,x$), which may be written symbolically:

$$\epsilon^x = \log_{10}^{-1}(0.4343\,x).$$

Similarly,

$$\epsilon^{-x} = \frac{1}{\log_{10}^{-1}(0.4343\,x)}.$$

Example. — Find the value of ϵ^{10}. We have $0.4343 \times 10 = 4.343$, whence $\epsilon^{10} = 22{,}050$.

For convenience, however, the following table giving the value of ϵ^x and ϵ^{-x} from $x = 0$ to $x = 6$ is given. For larger values of x use the above relations. Also note that for $x < 0.1$ (*see Series*).

$$\epsilon^x = 1 + x \text{ with an error of less than 0.47 per cent,}$$
$$\epsilon^{-x} = 1 - x \text{ with an error of less than 0.54 per cent,}$$
$$\epsilon^x = 1 + x + \frac{x^2}{2} \text{ with an error of less than 0.016 per cent,}$$
$$\epsilon^{-x} = 1 - x + \frac{x^2}{2} \text{ with an error of less than 0.018 per cent.}$$

EXPONENTIAL FUNCTIONS ϵ^x AND ϵ^{-x} **0.00 - 0.89**

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	ϵ^x	1.0000	1.0101	1.0202	1.0305	1.0408	1.0513	1.0618	1.0725	1.0833	1.0942
	ϵ^{-x}	1.0000	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
0.1	ϵ^x	1.1052	1.1163	1.1275	1.1388	1.1503	1.1618	1.1735	1.1853	1.1972	1.2093
	ϵ^{-x}	0.9048	0.8958	0.8869	0.8781	0.8694	0.8607	0.8521	0.8437	0.8353	0.8270
0.2	ϵ^x	1.2214	1.2337	1.2461	1.2586	1.2712	1.2840	1.2969	1.3100	1.3231	1.3364
	ϵ^{-x}	0.8187	0.8106	0.8025	0.7945	0.7866	0.7788	0.7711	0.7634	0.7558	0.7483
0.3	ϵ^x	1.3499	1.3634	1.3771	1.3910	1.4049	1.4191	1.4333	1.4477	1.4623	1.4770
	ϵ^{-x}	0.7408	0.7334	0.7261	0.7189	0.7118	0.7047	0.6977	0.6907	0.6839	0.6771
0.4	ϵ^x	1.4918	1.5068	1.5220	1.5373	1.5527	1.5683	1.5841	1.6000	1.6161	1.6323
	ϵ^{-x}	0.6703	0.6637	0.6570	0.6505	0.6440	0.6376	0.6313	0.6250	0.6188	0.6126
0.5	ϵ^x	1.6487	1.6653	1.6820	1.6989	1.7160	1.7333	1.7507	1.7683	1.7860	1.8040
	ϵ^{-x}	0.6065	0.6005	0.5945	0.5886	0.5827	0.5769	0.5712	0.5655	0.5599	0.5543
0.6	ϵ^x	1.8221	1.8404	1.8589	1.8776	1.8965	1.9155	1.9348	1.9542	1.9739	1.9939
	ϵ^{-x}	0.5488	0.5434	0.5379	0.5326	0.5273	0.5220	0.5169	0.5117	0.5066	0.5017
0.7	ϵ^x	2.0138	2.0340	2.0544	2.0751	2.0959	2.1170	2.1383	2.1598	2.1815	2.2034
	ϵ^{-x}	0.4966	0.4916	0.4868	0.4819	0.4771	0.4724	0.4677	0.4630	0.4584	0.4538
0.8	ϵ^x	2.2255	2.2479	2.2705	2.2933	2.3164	2.3396	2.3632	2.3869	2.4109	2.4351
	ϵ^{-x}	0.4493	0.4449	0.4404	0.4360	0.4317	0.4274	0.4232	0.4190	0.4148	0.4107

EXPONENTIAL FUNCTIONS ϵ^x AND ϵ^{-x} 0.90–3.09

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.9	ϵ^x	2.4596	2.4843	2.5093	2.5345	2.5600	2.5857	2.6117	2.6379	2.6645	2.6912
	ϵ^{-x}	0.4066	0.4025	0.3985	0.3946	0.3906	0.3867	0.3829	0.3791	0.3753	0.3716
1.0	ϵ^x	2.7183	2.7456	2.7732	2.8011	2.8292	2.8577	2.8864	2.9154	2.9447	2.9743
	ϵ^{-x}	0.3679	0.3642	0.3606	0.3570	0.3535	0.3499	0.3465	0.3430	0.3396	0.3362
1.1	ϵ^x	3.0042	3.0344	3.0649	3.0957	3.1268	3.1582	3.1899	3.2220	3.2544	3.2871
	ϵ^{-x}	0.3329	0.3296	0.3263	0.3230	0.3198	0.3166	0.3135	0.3104	0.3073	0.3042
1.2	ϵ^x	3.3201	3.3535	3.3872	3.4212	3.4556	3.4903	3.5254	3.5609	3.5966	3.6328
	ϵ^{-x}	0.3012	0.2982	0.2952	0.2923	0.2894	0.2865	0.2837	0.2808	0.2780	0.2753
1.3	ϵ^x	3.6693	3.7062	3.7434	3.7810	3.8190	3.8574	3.8962	3.9354	3.9749	4.0149
	ϵ^{-x}	0.2725	0.2698	0.2671	0.2645	0.2618	0.2592	0.2567	0.2541	0.2516	0.2491
1.4	ϵ^x	4.0552	4.0960	4.1371	4.1787	4.2207	4.2631	4.3060	4.3492	4.3929	4.4371
	ϵ^{-x}	0.2466	0.2441	0.2417	0.2393	0.2369	0.2346	0.2322	0.2299	0.2276	0.2254
1.5	ϵ^x	4.4817	4.5267	4.5722	4.6182	4.6646	4.7115	4.7588	4.8066	4.8550	4.9037
	ϵ^{-x}	0.2231	0.2209	0.2187	0.2165	0.2144	0.2122	0.2101	0.2080	0.2060	0.2039
1.6	ϵ^x	4.9530	5.0028	5.0531	5.1039	5.1552	5.2070	5.2593	5.3122	5.3656	5.4195
	ϵ^{-x}	0.2019	0.1999	0.1979	0.1959	0.1940	0.1920	0.1901	0.1882	0.1864	0.1845
1.7	ϵ^x	5.4739	5.5290	5.5845	5.6407	5.6973	5.7546	5.8124	5.8709	5.9299	5.9895
	ϵ^{-x}	0.1827	0.1809	0.1791	0.1773	0.1755	0.1738	0.1720	0.1703	0.1686	0.1670
1.8	ϵ^x	6.0496	6.1104	6.1719	6.2339	6.2965	6.3598	6.4237	6.4883	6.5535	6.6194
	ϵ^{-x}	0.1653	0.1637	0.1620	0.1604	0.1588	0.1572	0.1557	0.1541	0.1526	0.1511
1.9	ϵ^x	6.6859	6.7531	6.8210	6.8895	6.9588	7.0287	7.0993	7.1707	7.2427	7.3155
	ϵ^{-x}	0.1496	0.1481	0.1466	0.1451	0.1437	0.1423	0.1409	0.1395	0.1381	0.1367
2.0	ϵ^x	7.3891	7.4633	7.5383	7.6141	7.6906	7.7679	7.8460	7.9248	8.0045	8.0849
	ϵ^{-x}	0.1353	0.1340	0.1327	0.1313	0.1300	0.1287	0.1275	0.1262	0.1249	0.1237
2.1	ϵ^x	8.1662	8.2482	8.3311	8.4149	8.4994	8.5849	8.6711	8.7583	8.8463	8.9352
	ϵ^{-x}	0.1225	0.1212	0.1200	0.1188	0.1177	0.1165	0.1153	0.1142	0.1130	0.1119
2.2	ϵ^x	9.0250	9.1157	9.2073	9.2999	9.3933	9.4877	9.5831	9.6794	9.7767	9.8749
	ϵ^{-x}	0.1108	0.1097	0.1086	0.1075	0.1065	0.1054	0.1044	0.1033	0.1023	0.1013
2.3	ϵ^x	9.9742	10.074	10.176	10.278	10.381	10.486	10.591	10.697	10.805	10.913
	ϵ^{-x}	0.1003	0.0993	0.0983	0.0973	0.0963	0.0954	0.0944	0.0935	0.0926	0.0916
2.4	ϵ^x	11.023	11.134	11.246	11.359	11.473	11.588	11.705	11.822	11.941	12.061
	ϵ^{-x}	0.0907	0.0898	0.0889	0.0880	0.0872	0.0863	0.0854	0.0846	0.0837	0.0829
2.5	ϵ^x	12.182	12.305	12.429	12.554	12.680	12.807	12.936	13.066	13.197	13.330
	ϵ^{-x}	0.0821	0.0813	0.0805	0.0797	0.0789	0.0781	0.0773	0.0765	0.0758	0.0750
2.6	ϵ^x	13.464	13.599	13.736	13.874	14.013	14.154	14.296	14.440	14.585	14.732
	ϵ^{-x}	0.0743	0.0735	0.0728	0.0721	0.0714	0.0707	0.0699	0.0693	0.0686	0.0679
2.7	ϵ^x	14.880	15.029	15.180	15.333	15.487	15.643	15.800	15.959	16.119	16.281
	ϵ^{-x}	0.0672	0.0665	0.0659	0.0652	0.0646	0.0639	0.0633	0.0627	0.0620	0.0614
2.8	ϵ^x	16.445	16.610	16.777	16.945	17.116	17.288	17.462	17.637	17.814	17.993
	ϵ^{-x}	0.0608	0.0602	0.0596	0.0590	0.0584	0.0578	0.0573	0.0567	0.0561	0.0556
2.9	ϵ^x	18.174	18.357	18.541	18.728	18.916	19.106	19.298	19.492	19.688	19.886
	ϵ^{-x}	0.0550	0.0545	0.0539	0.0534	0.0529	0.0523	0.0518	0.0513	0.0508	0.0503
3.0	ϵ^x	20.086	20.287	20.491	20.697	20.905	21.115	21.328	21.542	21.758	21.977
	ϵ^{-x}	0.0498	0.0493	0.0488	0.0483	0.0478	0.0474	0.0469	0.0464	0.0460	0.0455

EXPONENTIAL FUNCTIONS ϵ^{x} AND ϵ^{-x} 3.10–5.29

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3.1	ϵ^{x}	22.198	22.421	22.646	22.874	23.104	23.336	23.571	23.807	24.047	24.288
	ϵ^{-x}	0.0450	0.0446	0.0442	0.0437	0.0433	0.0429	0.0424	0.0420	0.0416	0.0412
3.2	ϵ^{x}	24.533	24.779	25.028	25.280	25.534	25.790	26.050	26.311	26.576	26.843
	ϵ^{-x}	0.0408	0.0404	0.0400	0.0396	0.0392	0.0388	0.0384	0.0380	0.0376	0.0373
3.3	ϵ^{x}	27.113	27.385	27.660	27.938	28.219	28.503	28.789	29.079	29.371	29.666
	ϵ^{-x}	0.0369	0.0365	0.0362	0.0358	0.0354	0.0351	0.0347	0.0344	0.0340	0.0337
3.4	ϵ^{x}	29.964	30.265	30.569	30.877	31.187	31.500	31.817	32.137	32.460	32.786
	ϵ^{-x}	0.0334	0.0330	0.0327	0.0324	0.0321	0.0317	0.0314	0.0311	0.0308	0.0305
3.5	ϵ^{x}	33.115	33.448	33.784	34.124	34.467	34.813	35.163	35.517	35.874	36.234
	ϵ^{-x}	0.0302	0.0299	0.0296	0.0293	0.0290	0.0287	0.0284	0.0282	0.0279	0.0276
3.6	ϵ^{x}	36.598	36.966	37.338	37.713	38.092	38.475	38.861	39.252	39.646	40.045
	ϵ^{-x}	0.0273	0.0271	0.0268	0.0265	0.0263	0.0260	0.0257	0.0255	0.0252	0.0250
3.7	ϵ^{x}	40.447	40.854	41.264	41.679	42.098	42.521	42.948	43.380	43.816	44.256
	ϵ^{-x}	0.0247	0.0245	0.0242	0.0240	0.0238	0.0235	0.0233	0.0231	0.0228	0.0226
3.8	ϵ^{x}	44.701	45.150	45.604	46.063	46.525	46.993	47.465	47.942	48.424	48.911
	ϵ^{-x}	0.0224	0.0221	0.0219	0.0217	0.0215	0.0213	0.0211	0.0209	0.0207	0.0204
3.9	ϵ^{x}	49.402	49.899	50.400	50.907	51.419	51.935	52.457	52.985	53.517	54.055
	ϵ^{-x}	0.0202	0.0200	0.0198	0.0196	0.0195	0.0193	0.0191	0.0189	0.0187	0.0185
4.0	ϵ^{x}	54.598	55.147	55.701	56.261	56.826	57.397	57.974	58.557	59.145	59.740
	ϵ^{-x}	0.0183	0.0181	0.0180	0.0178	0.0176	0.0174	0.0172	0.0171	0.0169	0.0167
4.1	ϵ^{x}	60.340	60.947	61.559	62.178	62.803	63.434	64.072	64.715	65.366	66.023
	ϵ^{-x}	0.0166	0.0164	0.0162	0.0161	0.0159	0.0158	0.0156	0.0155	0.0153	0.0151
4.2	ϵ^{x}	66.686	67.357	68.033	68.717	69.408	70.105	70.810	71.522	72.240	72.966
	ϵ^{-x}	0.0150	0.0148	0.0147	0.0146	0.0144	0.0143	0.0141	0.0140	0.0138	0.0137
4.3	ϵ^{x}	73.700	74.440	75.189	75.944	76.708	77.478	78.257	79.044	79.838	80.640
	ϵ^{-x}	0.0136	0.0134	0.0133	0.0132	0.0130	0.0129	0.0128	0.0127	0.0125	0.0124
4.4	ϵ^{x}	81.451	82.269	83.096	83.931	84.775	85.627	86.488	87.357	88.235	89.121
	ϵ^{-x}	0.0123	0.0122	0.0120	0.0119	0.0118	0.0117	0.0116	0.0114	0.0113	0.0112
4.5	ϵ^{x}	90.017	90.922	91.836	92.759	93.691	94.632	95.583	96.544	97.514	98.494
	ϵ^{-x}	0.0111	0.0110	0.0109	0.0108	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102
4.6	ϵ^{x}	99.484	100.48	101.49	102.51	103.54	104.58	105.64	106.70	107.77	108.85
	ϵ^{-x}	0.0101	0.0100	0.0099	0.0098	0.0097	0.0096	0.0095	0.0094	0.0093	0.0092
4.7	ϵ^{x}	109.95	111.05	112.17	113.30	114.43	115.58	116.75	117.92	119.10	120.30
	ϵ^{-x}	0.0091	0.0090	0.0089	0.0088	0.0087	0.0087	0.0086	0.0085	0.0084	0.0083
4.8	ϵ^{x}	121.51	122.73	123.97	125.21	126.47	127.74	129.02	130.32	131.63	132.95
	ϵ^{-x}	0.0082	0.0081	0.0081	0.0080	0.0079	0.0078	0.0078	0.0077	0.0076	0.0075
4.9	ϵ^{x}	134.29	135.64	137.00	138.38	139.77	141.17	142.59	144.03	145.47	146.94
	ϵ^{-x}	0.0074	0.0074	0.0073	0.0072	0.0072	0.0071	0.0070	0.0069	0.0069	0.0068
5.0	ϵ^{x}	148.41	149.90	151.41	152.93	154.47	156.02	157.59	159.17	160.77	162.39
	ϵ^{-x}	0.0067	0.0067	0.0066	0.0065	0.0065	0.0064	0.0063	0.0063	0.0062	0.0062
5.1	ϵ^{x}	164.02	165.67	167.34	169.02	170.72	172.43	174.16	175.91	177.68	179.47
	ϵ^{-x}	0.0061	0.0060	0.0060	0.0059	0.0059	0.0058	0.0057	0.0057	0.0056	0.0056
5.2	ϵ^{x}	181.27	183.09	184.93	186.79	188.67	190.57	192.48	194.42	196.37	198.34
	ϵ^{-x}	0.0055	0.0055	0.0054	0.0054	0.0053	0.0052	0.0052	0.0051	0.0051	0.0050

EXPONENTIAL FUNCTIONS ϵ^x AND ϵ^{-x} 5.30–5.99

x	Function	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.3	ϵ^x	200.34	202.35	204.38	206.44	208.51	210.61	212.72	214.86	217.02	219.20
	ϵ^{-x}	0.0050	0.0049	0.0049	0.0048	0.0048	0.0047	0.0047	0.0047	0.0046	0.0046
5.4	ϵ^x	221.41	223.63	225.88	228.15	230.44	232.76	235.10	237.46	239.85	242.26
	ϵ^{-x}	0.0045	0.0045	0.0044	0.0044	0.0043	0.0043	0.0043	0.0042	0.0042	0.0041
5.5	ϵ^x	244.69	247.15	249.64	252.14	254.68	257.24	259.82	262.43	265.07	267.74
	ϵ^{-x}	0.0041	0.0040	0.0040	0.0040	0.0039	0.0039	0.0038	0.0038	0.0038	0.0037
5.6	ϵ^x	270.43	273.14	275.89	278.66	281.46	284.29	287.15	290.03	292.95	295.89
	ϵ^{-x}	0.0037	0.0037	0.0036	0.0036	0.0036	0.0035	0.0035	0.0034	0.0034	0.0034
5.7	ϵ^x	298.87	301.87	304.90	307.97	311.06	314.19	317.35	320.54	323.76	327.01
	ϵ^{-x}	0.0033	0.0033	0.0033	0.0032	0.0032	0.0032	0.0032	0.0031	0.0031	0.0031
5.8	ϵ^x	330.30	333.62	336.97	340.36	343.78	347.23	350.72	354.25	357.81	361.41
	ϵ^{-x}	0.0030	0.0030	0.0030	0.0029	0.0029	0.0029	0.0029	0.0028	0.0028	0.0028
5.9	ϵ^x	365.04	368.71	372.41	376.15	379.93	383.75	387.61	391.51	395.44	399.41
	ϵ^{-x}	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026	0.0025	0.0025

A more complete table is given in the *Smithsonian Mathematical Tables, Hyperbolic Functions* (*No. 1871*), by G. F. Becker and C. E. Van Orstrand, Washington, 1909. Semi-exponentials, i.e., values of $\frac{1}{2}\epsilon^x$ and $\log_{10} \frac{1}{2}\epsilon^x$, are given in *Tables of Complex Hyperbolic and Circular Functions*, by A. E. Kennelly, Cambridge, Mass., 1914.

FACTORIALS. — The multiple product represented by

$$n\,(n-1)\,(n-2)\;.\;.\;.\;3 \times 2 \times 1$$

is called "n factorial," and is represented either by the symbol $n!$ or $\underline{n|}$. The following table gives the value of $n!$ up to $n = 10$.

$1! = 1.$	$6! = 720.$
$2! = 2.$	$7! = 5040.$
$3! = 6.$	$8! = 40320.$
$4! = 24.$	$9! = 362880.$
$5! = 120.$	$10! = 3628800.$

FANS. — (*See also Blowers and Compressors; Draft, Mechanical.*) For ventilating and drying purposes and for producing mechanical draft, two kinds of fans are used, the disk fan and the centrifugal fan.

Disk Fans. — A windmill reversed, that is, driven by power applied to the shaft, may be used to blow air in large volume at low pressure, and it is then called a disk fan. Such fans are in common use for blowing air into or out of a room for the purpose of ventilation. Usually they consist simply of a rapidly rotating shaft carrying four or more nearly flat blades slightly inclined to the plane of rotation. Sometimes these blades are given various curved forms for the purpose of increasing the efficiency of the fan.

Centrifugal Fans. — The older form of centrifugal fan is that of a paddle wheel, a rotating shaft carrying from four to twelve radial arms, to which are attached flat blades in axial planes. In modified forms the blades are bent backward or forward, with reference to the direction of rotation, from an axial plane. Very large centrifugal fans up to 25 feet diameter and 10 feet width have been built for ventilation of mines, and in many cases without an external casing, but usually a spiral casing is used, with a tangential discharge outlet, with two "eyes," or circular inlets, concentric with the shaft, one on each side of the casing. In a recent modification of the centrifugal fan known as the "multivane" or "Sirocco" fan, the number of blades is increased to from 30 to 80; the blades or vanes are made relatively very long, axially, and narrow, radially, the ratio of length to width being as much as 9 to 1; they are made of curved form, the outer edge tipped forward in the direction of rotation and the diameter of the eyes in the casing and the radial distance of the vanes from the shaft are greatly increased. The pressure produced by centrifugal fans varies practically as the square of the speed of the tips of the blades and with the forms of the blades, and reaches a maximum commercially of about 1 pound per square inch for ordinary types of centrifugal or multivane fans. Much higher pressures have recently been obtained from special forms of fans built on the principles of steam turbines (*see Blowers and Compressors*).

POWER REQUIRED. — In the article on *Blowers and Compressors* is given the formula for "useful" work done by a blowing machine. Since an ordinary fan seldom produces a pressure of more than 1 per cent of normal atmospheric pressure, the change in the volume and static pressure of the air as it passes through a fan is negligibly small in comparison with the initial volume and pressure, and the formula for the theoretical horse power reduces to

$$P = \frac{1}{33{,}000}\left[\frac{dQ^3}{2g(60A)^2} + 144\,p_sQ\right],$$

where

p_s = the excess of static pressure at discharge over the initial static pressure, in pounds per square inch,

Q = cubic feet of air displaced per minute,

$d = \dfrac{1.33\,B}{460 + t}$ = weight of one cubic foot of air at the barometric pressure B (inches of mercury) and at $t°$ F.,

A = cross section of full outlet opening in square feet,

g = acceleration due to gravity in feet per second (= 32.2),

$p_v = \dfrac{d}{2 \times 144\,g}\left(\dfrac{Q}{60A}\right)^2$ = pressure in pounds per square inch necessary to give an average linear velocity $V_a = \dfrac{Q}{A}$ feet per minute to the air.

This pressure p_v is called the "velocity" pressure.

Useful or "Air Horse Power" of the fan may then be written

$$P = \frac{(p_v + p_s)\,Q}{229}.$$

The total pressure $p_v + p_s$ developed by the fan is called the "impact" or "dynamic" pressure. The pressures developed by a fan are usually expressed in inches of water column; hence putting H_v and H_s for the corresponding inches of water column required to balance p_v and p_s, the above formulas become

$$H_v = 0.832 \times 10^{-6} \frac{d\,Q^2}{A^2},$$

$$P = \frac{(H_v + H_s)\,Q}{6360}.$$

When the discharge outlet is restricted (e.g., by partially closing the discharge valve), the cross section A refers to the area of the discharge pipe between the restriction and the fan, not to the cross section of the restricted opening.

Actual Horse Power. — The actual horse power required to drive the fan is equal to the value given by the last formula divided by the efficiency. The fan efficiency depends not only upon the design of the fan, but also upon the relative values of the velocity and static pressures. The maximum efficiency of a centrifugal fan ranges from 40 to 70 per cent. The maximum efficiency of disk fans usually ranges from 30 to 40 per cent. The table below, taken from curves published by F. R. Still of the American Blower Co., in the *Jour. Wes. Soc. Eng.*, 1902, is fairly representative of the performance of ordinary steel-plate centrifugal fans. For a given type of fan and given ratio of velocity pressure within the casing on the outlet side to the static pressure, there is a fixed relation between the linear velocity $V_a = \frac{Q}{A}$ of the air within the casing on the outlet side and the peripheral velocity V_f of the fan; the ratio $\frac{V_a}{V_f}$, as calculated from Still's curves, is also given in the table below.

$\frac{H_v}{H_s}$	Efficiency	$\frac{V_a}{V_f}$	$\frac{H_v}{H_s}$	Efficiency	$\frac{V_a}{V_f}$
0.00	0.00	0.00	0.4	0.38	0.45
0.05	0.30	0.22	0.5	0.37	0.47
0.10	0.43	0.30	1.0	0.32	0.51
0.15	0.43	0.35	1.5	0.30	0.53
0.20	0.42	0.39	2.0	0.28	0.54
0.30	0.41	0.42	∞	0.22	0.58

More recent fans of the multivane or "Sirocco" type, when properly proportioned, have given efficiencies of 60 per cent and upwards.

THE VOLUME OF AIR DISCHARGED per minute is $Q = V_a A$ where V_a is the linear velocity of the air in feet per minute. When the speed is kept constant and the outlet pipe is partially closed, the linear velocity of the air through the full cross section A of the outlet decreases and the power required

to drive the fan decreases. A fan running at constant speed requires maximum power when the discharge outlet is wide open; the volume discharge is also a maximum under these conditions.

For an ordinary steel-plate fan running at a given speed the maximum volume of air per minute which it can deliver is roughly $\frac{1}{3}DWV_f = \frac{\pi}{3}D^2WN$, where D is the diameter of the fan in feet, W its width (at periphery) in feet, V_f the peripheral velocity of the fan in feet per minute and N the velocity in revolutions per minute.

The capacity of disk fans working against very low resistances is directly proportional to the area of the circle inclosing the fan and to the number of revolutions per minute, and varies with the form and inclination of the vanes. An ordinary type of propeller fan of diameter D feet will deliver $Q = KD^3N$ cubic feet per minute, where K ranges from 0.4 to 0.7 (depending on the design of the fan) when the resistance is small, the delivery decreasing rapidly as the resistance is increased.

DIMENSIONS AND SPEED. — The full discharge outlet A of an ordinary steel plate fan is usually about two-thirds the product of the width W by the diameter D of the fan, that is $A = \frac{2}{3}DW$. The width of the fan ranges from one-third to one-half the diameter. Using the latter figure, we have $D = \sqrt{3A}$. From this formula and the table given above a rough estimate of the dimensions of a fan for a given service may be made. For example, 20,000 cubic feet of air per minute at 60° F. is to be discharged against a static head of 1 inch of water; what must be the approximate dimensions and speed of the fan and what will be the approximate horse power required? The size of fan and the horse power required will depend upon the linear velocity of the air through the fan, the greater this velocity the smaller the fan and the greater the horse power. Using the symbols as defined above we have $Q = 20{,}000$, $H_s = 1$, $d = 0.0766$. A reasonable value of the linear velocity V_a is 2000 feet per minute. Then $A = 10$, $D = 5.48$, $W = 2.74$, $H_v = 0.255$, $\frac{H_v}{H_s} = 0.255$, $\frac{V_a}{V_f} = 0.41$, $V_f = 4880$, $N = 284$, efficiency $= 0.415$, H.P. $= 9.6$. That is, the diameter of the fan should be approximately 5 feet 6 inches, its width 2 feet 9 inches, its speed 285 r.p.m., and 9.6 horse power will be required to operate it. If V_a is taken as 4000 feet per minute, then the required diameter will be approximately 3 feet 10 inches, its width 1 foot 11 inches, its speed 650 r.p.m., and 20 horse power will be required to operate it.

The above method of arriving at the size of fan is extremely rough at best; for more accurate data the reader is referred to manufacturers' catalogues.

SPECIFICATIONS FOR FANS. — The following memoranda are intended to assist in writing specifications. See also article on *Blowers* and article on *Specifications*.

General description of fan and service for which it is to be used. Rating, cubic feet of air per minute, under specified conditions of test. Whether exhaust or pressure. How supported. Whether fixed or oscillating. Details of motor, i.e., whether a-c. or d-c., frequency, phases and voltage.

MOTOR DRIVE. — (*See also Motors, Industrial Applications of.*) The load on a fan motor increases as the speed rises and unless some sort of shuttering is provided the motor will be fully loaded when it comes up to speed. Shuttering is essential and the starting conditions should be carefully considered when a synchronous motor is intended for driving the fan. As the torque required to start a fan is usually small, a motor with low-resistance starting winding may be used without requiring a very heavy current. To keep the

current down, however, a low tap on the starting compensator must be used and an intermediate step may be very desirable.

Fig. 1 shows the starting-torque curve of a shuttered centrifugal fan from rest to full speed, and also a speed-torque and a speed-current curve of a synchronous motor with a squirrel-cage starting winding of fairly low resistance plotted for 50, 70 and 100 per cent of normal voltage. If the motor is started from the 50 per cent tap of the compensator it will bring the fan to about 80 per cent of full speed. If the motor is then thrown over to the full-line voltage it may exert an excessive torque, and the current, as shown by the current curve, will rise to a very high value. If an intermediate tap is provided — say a 70 per cent tap — the torque exerted will be less but still enough to bring the motor to about 95 per cent of synchronous speed and the current will be much less. Application of the field would then pull the machine into step after which it could be thrown over to the line with a very little rush of current. For a variable-speed service the synchronous motor is of course not suitable and a varying-speed brush-shifting motor or a phase-wound induction motor must be used.

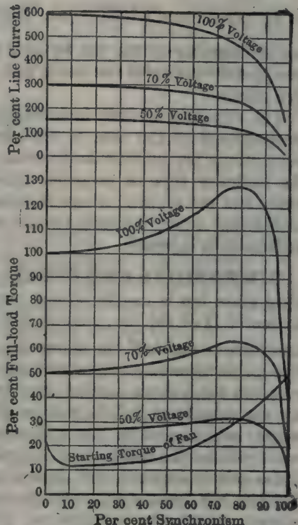


Fig. 1. Curves of Synchronous Motor with Low-resistance Starting Winding

For direct-current installations shunt motors are preferred, the speed adjustment being accomplished either by field control, armature control or a combination of the two. Field control is generally used where the motor operates for long intervals at speeds between 50 and 100 per cent of maximum speed; the combination control is used if the motor operates for long intervals at speeds between 75 and 100 per cent of maximum speed, and is occasionally required to operate at speeds below 75 per cent; and armature control is used advantageously for motors operating at long intervals near full speed, but where occasionally a lower speed is required. Ventilating fans offer one of the best applications for armature control because the torque and current decreases with the cube of the speed.

With very large mine fans, having a high inertia, the time required for acceleration is comparatively long — one minute or more — and this must be considered when the starting resistances are selected. Starting resistances for motors driving such large fans should therefore as a rule be of a larger size than would ordinarily be the case.

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FEED-WATER HEATERS AND PURIFIERS. — (*See also Boilers; Power Stations, Steam Electric.*) Impurities in the water supplied to a boiler may cause: (1) the formation of scale on the heating surface of the boiler, resulting in a decrease of boiler efficiency and overheating and consequent weakening of the tubes and plates; (2) the corrosion and consequent weakening of the tubes and plates; (3) an increase in the amount of suspended moisture carried over by the steam from boiler to engine. Sulphates of lime and magnesia, soluble salts of silica, iron and aluminum and suspended matter are the chief scale-forming impurities. Acids, organic matter, magnesium chloride and sulphate cause corrosion, while priming is induced by the presence of organic matter, sodium carbonate and other alkalies.

By the introduction into the feed water of various chemicals, as, for example, carbonate of soda, the scale-forming impurities may be changed into harmless substances by the reaction between the impurities and the "boiler compounds" introduced. Again, the adhesion of the scale to the heating surfaces may be prevented by the introduction into the feed water or boiler of such substances as kerosene oil. Suspended matter, such as sand, mud, insoluble organic matter, etc., may be eliminated by mechanical filtration or by allowing the water to stand in settling tanks. Organic impurities usually float on top of the water when the boiler is making steam and may be blown out through a "surface blow-out." Precipitated matter may be ejected by frequent blowing off before it has time to form a crust. Scale, when it has once formed, can be removed only by cleaning the tubes with some form of scraper. Very hard sulphate-of-lime scale may be softened so as to be more easily scraped, by dissolving a considerable quantity, say 50 to 100 pounds, of caustic soda in the boiler and slowly boiling it at atmospheric pressure for from 12 to 24 hours. Carbonates of lime and magnesia, two of the chief scale-forming impurities, may be almost completely precipitated by raising the temperature of the water to 290° F. (= boiling point for a pressure of about 60 pounds absolute).

SAVING DUE TO PREHEATING FEED WATER. — Although the heating and subsequent filtering or settling of the feed water results in the elimination of certain of the impurities, the primary object of preheating the feed water is to reduce the coal consumption. The heat required to raise the temperature of 1 pound of feed water 11° F. is approximately 1 per cent of the total heat required to convert the water into saturated steam at ordinary pressure. Hence, for every 11° F. the temperature of the feed water is raised by the application of heat which would otherwise be wasted, a saving of approximately 1 per cent will be effected in the amount of coal required. Let H = total heat of the steam at the given pressure, h_0 = heat of the water entering the heater, h = heat of the water leaving the heater, then the saving, expressed as a percentage of the heat that would be required by a boiler without a superheater, is

$$S = 100 \frac{h - h_0}{H - h_0}.$$

The values of H, h and h_0 may be taken from steam tables (*see Steam*). This formula also gives the saving in fuel, provided the boiler efficiency is the same with or without the feed-water heater. However, if the boiler is overdriven, the installation of a feed-water heater will effect a greater saving than that given by the formula, since when the work to be done by the boiler is reduced the boiler efficiency will also increase.

TYPES OF FEED-WATER HEATERS. — The heat used for preheating the feed water may be derived: (1) from the exhaust steam; (2) from the flue

gases, in which case the heater is usually called a "fuel economizer"; (3) from live steam taken directly from the boilers, in which case the primary function of the heater is to purify the feed water, and it is therefore usually called a "live-steam purifier." The heat may be transmitted from the steam to the water either (1) by allowing the steam to mingle with the water and give up its heat by condensation, or (2) by passing the water and steam through separate chambers arranged so that the heat is conducted to the water through the walls of the chambers. These two types of heaters are referred to respectively as "open" and "closed" heaters. Closed heaters may be either of the water-tube or steam-tube type; in the former the steam surrounds a set of tubes through which the water is passed, while in the latter the steam is passed through a set of tubes surrounded by water. A closed heater in which the steam pressure is less than atmospheric is called a "vacuum" or "primary" heater; such heaters are frequently used in the exhaust of condensing engines. "Atmospheric" or "secondary" heaters are those in which the steam pressure is that corresponding to the back pressure of the engines or pumps. A heater in which the steam pressure is the same as that in the boiler is called a "pressure" heater.

Open Heaters. — The essential parts of an open heater are: (1) a shell containing (2) a set of trays or pans to catch the scale-forming elements precipitated from the water; (3) a filter to take out suspended impurities; and (4) an oil separator to extract the oil from the steam before it enters the superheater.

Dimensions. — C. L. Hubbard (*Practical Engineer, Jan. 1, 1909*) gives the following:

Exhaust heaters should be proportioned according to the quality of the water to be used, the size being increased with the amount of mud or scale-producing properties which the water contains, regardless of the quantity of water to be heated. The general proportions of an open heater will depend somewhat upon the arrangement of the trays or pans, but an approximation of the size of shell for a cylindrical heater is as follows: $A = H \div aL$; $L = H \div aA$; in which A = sectional area of shell in square feet; L = length of shell in linear feet; H = total weight of water to be heated per hour divided by the weight of steam used per horse power per hour by the engine; a = 2.15 for very muddy water, 6.0 for slightly muddy water, and 8.0 for clear water.

The pan or tray surface varies according to the quality of the water, both as regards the amount of mud and the scale-making ingredients. The surface in square feet for each 1000 pounds of water heated per hour may be taken as follows, for the vertical and horizontal types respectively:

Very bad water	8.5 and 9.1
Medium muddy water	6 and 6.5
Clear and little scale	2 and 2.2

The space between the pans is made not less than 0.1 of the width for rectangular and 0.25 of the diameter for round pans. Under ordinary circumstances it is not customary to use more than six pans in a tier, in order to obtain a low velocity over each pan. The size of the storage or settling chamber in the horizontal type varies from 0.25 to 0.4 of the volume of the shell, depending on the quality of the water; 0.33 is about the average. In the case of vertical heaters, this varies from 0.4 to 0.6 of the volume of the shell. Filters occupy from 10 to 15 per cent of the volume of the shell in the horizontal type and from 15 to 20 per cent in the vertical.

Temperature Elevation of Feed Water. — Let H = total heat of steam in B.t.u. above 32° F. at the pressure of the steam in the heater; I =

initial temperature of the water entering the heater; F = final temperature of the water leaving the heater; K = ratio of weight of exhaust steam condensed per hour in the heater to weight of the feed water per hour; then, neglecting loss due to radiation,

$$F = \frac{I + K(H + 32)}{1 + K}.$$

Closed Heaters. — (*H. L. Hepburn, Power, April, 1902.*) Let W = pounds feed water per hour; A = sq. ft. of heating surface between steam and water; T_s = temperature of the steam; I = initial temperature of the water; F = final temperature of the water; U = B.t.u. transmitted per sq. ft. per hr. per deg. difference of temperature between the steam and the water; then

$$A = \frac{2.30\,W}{U} \log_{10} \frac{T_s - I}{T_s - F}.$$

The value of U varies widely according to the condition of the surface and with the velocity of the water, and also depends upon whether the heater is of the water-tube or steam-tube type. Gebhardt gives the following values of U: for multi-flow water-tube heaters, 250 for plain copper or brass tubes and 300 for corrugated tubes with a water velocity of 50 feet per minute; for single-flow water-tube heaters, 175 for plain brass with a water velocity of 12.5 feet per minute; for coil water-tube heaters, 300 for copper tubes with a water velocity of 150 feet per minute; for steam-tube heaters, 120 for iron tubes.

Economizers. — Economizers for boiler plants are usually made of vertical cast-iron tubes contained in a long rectangular chamber of brickwork. The feed water enters the bank of tubes at one end, while the hot gases enter the chamber at the other end and travel in the opposite direction to the water. The tubes are made of cast iron because it is more non-corrosive than wrought iron or steel when exposed to gases of combustion at low temperatures. An automatic scraping device is usually provided for the purpose of removing dust from the outer surface of the tubes.

Economizers are of value in plants operating with steady load in which little exhaust steam is available. The annual maintenance usually amounts to 10 or more per cent of the original cost.

The amount of saving of fuel that may be made by an economizer varies greatly according to the conditions of operation. With a given quantity of chimney gases to be passed through it, its economy will be greater (1) the higher the temperature of these gases; (2) the lower the temperature of the water fed into it; and (3) the greater the amount of its heating surface.

The maximum saving of fuel which may be made by the use of an economizer when attached to boilers that are working with reasonable economy is about 15 per cent. If the boilers are not working with fair economy on account of being overdriven, then the saving made by the addition of an economizer may be much greater. Barrus reports the following results of economizer tests:

Heating surface, boiler, square feet	**1,894**	**1,058**	**5,592**	**3,126**
Heating surface, economizer, square feet	1,600	1,920	1,290	1,600
Temperature of gases leaving boiler, degrees F.	376	361	403	435
Temperature of gases leaving economizer, degrees F.	231	254	299	279
Temperature of feed water entering economizer, degrees F.	95	79	111	84
Temperature of feed water entering boiler, degrees F.	175	145	169	196
Increased evaporation produced by economizer, %	10.5	7	9.3	12.8

and W. R. Roney (*Transactions A.S.M.E., Vol. 15*), reports:

Plant tested	1	2	3	4	5	6	7	8	9
Gases entering economizer, degrees F	610	505	550	522	505	465	490	495	595
Gases leaving economizer, degrees F	340	212	205	320	320	250	290	190	299
Water entering economizer, degrees F	110	84	185	155	190	180	165	155	130
Water leaving economizer, degrees F	287	276	305	300	300	295	280	320	311
Gain in temperature of water, degrees F	117	192	120	145	110	115	115	165	181
Fuel, saving, per cent	16.7	17.1	11.7	13.8	10.7	11.2	11.0	15.5	16.8

Temperature Elevation of Feed Water. — Let A = square feet of economizer heating surface per boiler horse power, T_1 = temperature (Fahrenheit) of flue gases entering economizer (ranges from about 450 to 700° F.); I = temperature of feed water entering economizer; F = temperature of feed water leaving economizer; W = pounds feed water per boiler horse power per hour; G = pounds flue gas per pound coal; C = pounds coal per boiler horsepower hour; S = specific heat of flue gases; U = B.t.u. transmitted per sq. ft. per hr. per deg. difference of temperature between flue gas and water; then

$$F - I = \frac{A\,(T_1 - I)}{\dfrac{W}{U} + \dfrac{(W + CGS)\,A}{2\,CGS}}.$$

A varies from 3.5 to 5, and U from 2.25 to 3.3, depending upon the conditions of operation. Putting $W = 30$, $S = 0.2$ and $U = 3.3$, the above formula reduces to

$$F - I = \frac{A\,(T_1 - I)}{9.1 + \dfrac{(5W + GC)\,A}{2\,GC}},$$

which is the formula advocated by the Green Economizer Co.

Rating and Cost (Pre-war cost figures). — The ordinary closed heater has from ⅓ to ½ square feet of heating surface for each boiler horsepower, and costs from 75 cents to $1 per horsepower. Economizers are sometimes rated by tubes, the usual area per tube being 15 square feet. From 3.5 to 5 square feet are usually installed per boiler horsepower. Economizers cost approximately $1.25 per square foot of heating surface, this figure including erection and brick setting.

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FIRE-ALARM TELEGRAPH. — *(See also Telegraph Instruments and Apparatus; Telegraph Systems; Wiring of Buildings for Miscellaneous Devices.)* The object of fire-alarm telegraph systems is to notify the fire-fighting forces of a community promptly of the existence and location of a fire, and also to afford a means of communication between the various branches of the fire-fighting organization whether at a fire or in quarters.

The simplest form of fire-alarm system is shown in Fig. 1, where several fire alarm boxes are arranged in series in the alarm or "box" circuit extending from the fire-alarm headquarters throughout the district covered by the boxes. These boxes are provided with a clockwork mechanism, so that when the box is started or "pulled" by the person sending the alarm, a break wheel, carrying notches

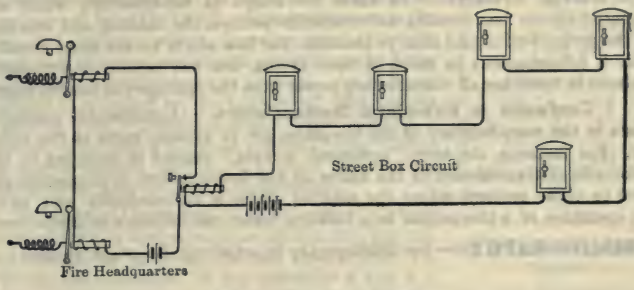

Fig. 1. Fire-Alarm System

corresponding to the code number of the box, is caused to revolve, thus making and breaking the circuit in accordance with the code. These makes and breaks cause the operation of the relay at headquarters and by means of a local circuit the gong sounds the alarm at headquarters. As many gongs as may be desired may be operated by the same relay, these being placed in the engine-house, men's quarters, or wherever else it is desired to give the alarm.

Types of Boxes. — The placing of a number of boxes on the same circuit gives rise to the possibility of confusion, due to two or more boxes being pulled at once. This is called "interference." A box which makes no provision against this confusion is called an "interfering box." A common type of box, termed the "non-interfering box," is so arranged that if two or more boxes on the same circuit are pulled at once the signal of the first box to be started will be transmitted without interference from the others, but the signals of the others will not be transmitted. In a later development, and one that is largely used, the arrangement is such that if any number of boxes up to four on the same circuit are simultaneously pulled, no interference and no loss of signals will be entailed, each box securing possession of the line in succession, the deferred boxes waiting their turn, so that the signals of all the boxes will be transmitted without confusion. These are called "non-interference succession boxes."

Gong Circuits. — In fire departments which have a number of engine and other apparatus houses, it is customary to establish a single fire-alarm headquarters, at which all box circuits center. From this headquarters other circuits, termed "gong circuits," extend to the apparatus houses. An alarm received over a box circuit may be transmitted to the gong circuits either manually or automatically.

Manual Systems. — In manual systems the box alarm is received at headquarters and then is set up on a transmitter, which, when started, automatically repeats the alarm over the desired gong circuits. The manual part of this operation may consist in placing a disc, notched on its periphery to accord with the desired box number, in the transmitter, and then starting the mechanism, the notches on the disc effecting the desired makes and breaks in the gong circuits. This method is employed in the fire-alarm system of the Borough of Manhattan, New York.

Another manual method of transmission is to set up the desired box number on a dial transmitter, which, when set in motion, transmits the alarm to the gong circuits. Still another method is to actually re-transmit the box numbers manually by means of a Morse key.

Automatic Systems. — In these the relays at headquarters, which are operated by the box circuits, automatically perform the making and breaking of the gong circuits, and thus re-transmit the box alarm to the gong circuits. This is advantageous in point of time saving, but has certain objectionable features in removing all discretionary power from the operators at headquarters.

Confusion in Fire-alarm Nomenclature. — Considerable confusion exists in the nomenclature of the fire-alarm art in various parts of the country. For instance, the term "automatic system," instead of applying to the automatic re-transmission of the alarm at fire headquarters, is frequently applied to those systems where the original alarm is automatically sent, as by the operation of a thermostat in a building under the influence of undue heat.

BIBLIOGRAPHY. — See *Bibliography* in article on *Telegraph Systems*.

FLUXMETER. — (*See also Galvanometers.*) The Grassot fluxmeter is essentially a dead-beat galvanometer, the suspension fiber of which is designed to exert no appreciable torsional force on the moving system when the latter is displaced.

Construction. — Fig. 1 is a diagram showing its construction. The coil C swings in the uniform air gap between the pole pieces NS of a permanent magnet and the soft iron core A. The system is supported at the top by a single cocoon fiber, the upper end of this fiber being attached to a flat spiral spring R, to minimize the effect of shocks. The torsional force exerted by such a fiber is extremely small. S and S_1 are very thin silver strips, serving to lead the current to and from the coil C. These strips are in the form of springs, which, however, are extremely weak and therefore exert but a small theoretically inappreciable torsional force on the coil. An index or pointer is attached to the instrument, this pointer swinging over a calibrated scale. The fluxmeter is also provided with a mirror in addition to the pointer so that it may be used in conjunction with a lamp and scale or with a telescope and scale.

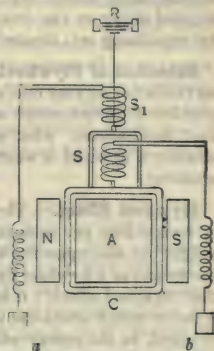

Fig. 1. Grassot Fluxmeter System

Principle of Operation. — When a given quantity of electricity is discharged through the moving coil, for example by changing the flux threading an exploring coil connected to the terminals a and b, a force is exerted upon the coil tending to deflect it. The only opposing forces (neglecting the small and theoretically inappreciable torsional forces) are the mechanical friction to motion and the back e.m.f. induced in the coil due to its motion through the field of the permanent magnet. The latter is proportional at any instant to the velocity of the coil, and the frictional force is also approximately proportional to this velocity. If both forces are directly proportional to the velocity, when a given quantity of electricity is discharged through the circuit, the coil comes to rest at a definite point, depending only upon its initial or "zero" position and the total quantity discharged through it. As the quantity of electricity discharged through an exploring coil, when the magnetic flux threading it is changed, depends solely upon the change in flux and the resistance of the coil and the rest of the circuit in series with it, the instrument may be calibrated to read directly the change in the flux density produced in the region occupied by the coil. In practice the friction is not exactly proportional to the velocity and the fiber and leading in springs usually exert an appreciable force on the coil. The instrument therefore has not proved altogether satisfactory.

Applications. — In motor or in dynamo work an exploring coil, consisting of one or more turns of wire, may be fixed or wound in position and the change in flux of induction observed on exciting the field magnet. Even in the largest work where some minutes may be taken to reach the limit of magnetization, no error is thus introduced. The fluxmeter can also be used for the measurement of magnetic field strength, pole strength and the distribution of magnetism in bar magnets. It is also adapted to the measurement of permeability (q.v.) and hysteresis (q.v.).

COST (Pre-war figures). — A direct reading fluxmeter with one exploring coil costs about $75. Additional exploring coils cost from $8 to $10 each.

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FLYWHEELS FOR LOAD EQUALIZATION. — (*See also Hoists; Motors, Industrial Application of; Steel Mills, Electric Drive of.*) There are two methods in general use for equalizing a fluctuating load, the storage battery and the flywheel. The former is mostly used in connection with a central service where the peaks are of considerable duration and for which service the flywheel is entirely unsuitable. For short fluctuating loads, such as steel-mill service, mine hoists, etc., the flywheel is particularly suitable, and it may be applied either to the driving motor direct or to an intermediate motor-generator equalizing set, depending on the operating conditions.

Selection of Flywheel. — The problem of selecting a flywheel for a given service is not an easy one, and each case must be treated separately. The general problem is to determine what effect a flywheel will have on smoothing out the load fluctuations; what effect it will have on the motor and supply system; whether a flywheel is warranted; and what size flywheel will result in maximum economy.

Flywheel Effect. — The effectiveness of a flywheel, rotating at a given number of revolutions per minute, in equalizing the load, depends not only upon its weight but also upon the square of its radius of gyration (q.v.). The product of the weight of a flywheel by the square of its radius of gyration is commonly called the "flywheel effect"; it is proportional to the moment of inertia of the flywheel. A factor which limits the usefulness of a given flywheel is the allowable variation in speed, as the energy which a flywheel is able to give up is proportional to the difference between the squares of the initial and final speeds. Another point to be considered is the fact that a flywheel is not an inexhaustible source of energy. The time during which it can supply a certain number of horse-power is limited, and it can, with a given drop of speed, only supply a given number of horse-power seconds. A certain flywheel, for example, may easily take care of a peak amounting to 100 per cent overload for one second, but it may be entirely inadequate to handle a 50 per cent overload lasting for five seconds.

Induction Motor with Flywheel. — Let the motor at any instant be developing a torque T, and let the opposing torque of the load at this instant be T_1. Then $T = T_1 + \frac{I}{32.2}\frac{dw}{dt}$, where I is the moment of inertia of all the revolving parts and w their angular velocity. In the case of an induction motor the torque T is practically proportional to the slip (*see Motors, Induction*). By making use of this relation and the above equation the following working formula is readily deduced for the change in the torque developed by an induction motor for an interval during which the opposing torque of the load remains constant. Let

T_0 = torque, in pound-feet, developed by motor at a given instant.
T = torque, in pound-feet, developed by motor t *seconds later.*
T_1 = opposing torque, in pound-feet, due to the load, *assumed constant.*
R = ratio of torque (in pound-feet) to slip (expressed as a decimal fraction) for the given motor, obtained from characteristic curves of the motor.
N = synchronous speed of motor, in r.p.m.
k = radius of gyration of flywheel, in feet (the inertia of the other rotating parts is usually negligible in comparison with that of flywheel).
W = weight of flywheel in pounds.

Calculate the constant

$$A = \frac{308\,R}{k^2WN}.$$

Then the torque developed by the motor t seconds from the time that its torque was T_0 is

$$T = T_1 - \frac{T_1 - T_0}{\epsilon^{At}},$$

where ϵ is the base of the natural system of logarithms (*see Exponential Functions*).

This equation shows that if the load torque suddenly increases from the value T_0 to a new constant value T_1, then the torque developed by the motor does not change immediately to T_1, but builds up to this value at a rate depending upon the value of the constant A. The smaller A the more slowly does the torque of the motor change; if A is sufficiently small and the load represented by the torque T_1 is of short duration, then the torque of the motor may increase only a relatively small amount before the load drops back to its original value. The constant A can be made small: (1) by having a flywheel with a large flywheel effect; (2) by using a high-speed motor, or (3) by designing the motor so that for a given torque the slip is large, i.e., with high secondary resistance.

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FREQUENCY CHANGERS.— (*See also Generators; Motor-Generators; Motors.*) Frequency changers are motor-generator sets in which both machines are designed for operation with alternating current, but for two different frequencies, thus a 25-cycle motor may drive a 60-cycle generator. They are used for three general classes of work which may be typified as follows:

(a) To supply 60-cycle current for lighting from a 25-cycle power circuit. The 60-cycle current is much more satisfactory for lighting than is the 25-cycle current.

(b) To supply 25- or 15-cycle current for single phase railway from a 50- or 60-cycle power system.

(c) To tie together two large power stations which started independently with different frequencies and as a result of growth and changed economic conditions or ownership, find it desirable to be able to exchange energy either for purposes of evening up the demand or to meet the emergency of an accident to the generators of one of them. Most of the large power stations in a given neighborhood, even when owned by different corporations, are tied together to help each other out in case of trouble in one of them. For this type of service it is desirable that the frequency changers be "reversible," that is, either machine may be the driving motor.

While several types have been proposed and discussed on paper, only two types have attained, or are likely to attain, any great commercial importance. These are the "Induction-synchronous" and the "Synchronous-synchronous" types. The former is not ordinarily reversible while the latter is reversible.

INDUCTION-SYNCHRONOUS TYPE. — An induction motor direct connected to a synchronous alternating-current generator (either single, two or three-phase at either end or any combination thereof) may be designed, by choosing a suitable relation between the number of poles on each machine, to receive power at any frequency and deliver power at any other frequency. Thus a 10-pole motor operating on a 25-cycle circuit driving a 24-pole generator at 300 r.p.m. will deliver approximately 60 cycles. This combination has the disadvantage that an increasing load causes decreasing speed and therefore decreasing frequency in the load circuit. This can be taken care of in some cases by choosing a ratio of poles which will give a nominal ratio of 25 to 62.5 cycles for instance, and by added resistance in the secondary of the induction motor, regulate the speed at all loads so that the generator gives 60 cycles. Where the secondary system or load is an independent circuit without other generators there is no objection to allowing the frequency to vary slightly from the nominal value.

If there are to be two frequency changers in parallel supplying the load the adjustment of the division of load may be accomplished either by suitably designing the resistances of the secondary of the two induction motors so that they have the same slip, or by having an adjustable resistance connected in these secondary circuits of the motors. The great advantage of the induction-synchronous type is that, due to the characteristics of the induction motor, a variation of one or two per cent in the frequency of either of the two systems will not cause a serious overload on the set, as is the case with the synchronous-synchronous type.

SYNCHRONOUS-SYNCHRONOUS TYPE. — This consists of a synchronous motor driving a synchronous alternating-current generator. The constant speed characteristic of a synchronous motor makes it particularly suited as the driving motor of a frequency changer, as this insures a constant ratio of frequencies irrespective of the load on the set.

Such sets are more generally used than any other type, particularly for link-

ing together two systems whose frequencies are fixed by the presence of other generators. However, any considerable variation in the frequency of one system will place a considerable overload on the set and may cause it to pull out of step. Consequently, it is advisable that the capacity of the set be not too small in proportion to the capacity of the system. The synchronous motor usually has an amortisseur or squirrel-cage winding in its pole faces in order to assure good starting ability and to reduce hunting (*see Motors, Synchronous*). This type of set is reversible, that is, will transfer energy either way between the systems which it ties together. The reversal of the flow of energy is accomplished by moving one of the stationary armatures in its cradle as described under parallel operation below.

In order that a synchronous-synchronous set may operate properly in parallel and divide the load proportionately with other synchronous apparatus on the two systems to which the motor and generator are respectively connected, the number of poles of both motor and generator must be carefully chosen. Thus to change from 25 to 60 cycles per second, the highest speed which allows an exact transformation of frequency is 300 rev. per min., and this requires 10 poles on the 25-cycle machine and 24 poles on the 60-cycle machine.

In order to build less expensive sets it is frequently the custom to use a speed of 750 rev. per min. with a 4-pole machine on the 25-cycle circuit and a 10-pole machine which will give 62.5 cycles for the other circuit. All the other machines on the latter circuit must then operate at 62.5 cycles. Other combinations of poles and speeds for various combinations of frequencies can be readily calculated by means of the usual relation between poles, speed and frequency; see *Generators, Alternating-current*, and attached tables. High-speed sets with few poles are desirable because they are smaller and less expensive for a given capacity. Each motor-generator set must be provided with direct current for field excitation, either from a direct-connected exciter or from a special exciter circuit.

Division of Load on Synchronous Sets. — The parallel operation of two or more sets of this type involves certain complicated considerations. In the first place the division of the load depends not only upon the voltage regulation of the individual machines constituting a set, but upon the mechanical position of the armatures on the shaft. Two sets built apparently the same may not divide the load equally because of inaccuracy in the placing of the key-ways, etc. To avoid this trouble it is customary to mount the stationary member of one machine of each set movable in a cradle, so that the angular phase position of this member (usually the stationary armature) with respect to the base, may be adjusted. The two sets to be operated in parallel are loaded, and the stationary armature of one machine is rotated in the cradle until the load on the two machines divides properly.

Synchronizing Frequency Changers. — The synchronizing of such sets is difficult, and several complicated conditions must be satisfied. The theory is quite involved and is discussed at length in a paper by J. B. Taylor on *Parallel Operation, Trans. A.I.E.E., Vol. 25, p. 113*, from which the attached table is taken. The essential requirement is that it is necessary to synchronize not only the motor with its supply circuit, but also the generator with its load circuit. Thus when the motor has been synchronized properly, it may be found that the generator is 180 degrees out of phase with the load circuit. To bring the generator into phase it is necessary to cause the motor to "slip" one or more poles successively until the generator comes into proper phase; this can be done by reversing the field of the motor one or more times depending upon the number of poles on the generator and on the motor. The necessity for more than one reversal is due to the fact that with a large number of poles on both motor and

generator there are only a few combinations in which the poles of the motor and the generator of one set match up with the poles of motor and generator of another set. Thus with a 10-pole motor and 24-pole generator (the most common arrangement) when the motor slips one pole or 180 electrical degrees of its circuit, the generator slips $^{24}/_{10} \times 180$ or 432 electrical degrees of its circuit. It is necessary to make the motor slip 5 poles or 5×180 degrees to make the generator slip the first even multiple of 360, that is $5 \times 432 = 2160$ or 6×360 degrees. In this case it might be necessary to make the motor slip 4 poles before getting the right combination. Five reversals would reproduce the original conditions and, after that, more reversals would repeat the cycle of possible combinations. The table shows the possible combinations for various commonly used sets.

In practice, this difficulty is overcome by the use of a special synchronizing indicator (*see Synchronizers*) having two hands on the same dial; one hand shows the phase relation of each member of the set with respect to its particular external circuit. It is necessary to synchronize the set when both hands are not only stationary but point to the zero position. If only the motor were properly synchronized it would be found that both hands were stationary, the motor hand pointing to zero but the generator hand pointing to some other position.

POSSIBLE COMBINATIONS FOR SYNCHRONOUS-SYNCHRONOUS FREQUENCY CHANGERS

Cycles		Poles		Speed	Kw.	Chances of correct Synch.	Field reversing switches	
Motor	Gen.	Motor	Gen.	R. P. M.	Capacity		Motor	Gen.
25	60	10	24	300	Max.	1 in 5	Yes	Yes
25	62.5	4	10	750	600	1 in 2	....	Yes
25	62.5	8	20	375	Max.	1 in 2	....	Yes
40	60	4	6	1200	150	1 in 2	....	Yes
40	60	8	12	600	3000	1 in 2	....	Yes
40	60	12	18	400	Max.	1 in 2	....	Yes
25	50	4	8	750	600	Certain	....	....
25	50	8	16	375	Max.	Certain	....	....
33.3	60	10	18	400	Max.	1 in 10	Yes	Yes
30	60	6	12	600	1000	Certain	....	....
30	60	8	16	450	Max.	Certain	....	....

Explanation of Table. — The first two columns show the frequencies of the two circuits to be tied together, the third and fourth columns, the number of poles on motor and generator respectively, the fifth column, the speed of the set. The sixth column, the maximum capacity in kw. in which such sets are available in commercial practice ("max." means as large as desired). The seventh column states the chances of the set coming in right the first time, and indicates the maximum number of trials that might be necessary before the right combination is found; thus in the first line, four trials might have to be made to secure proper synchronizing. The last two columns tell whether it is necessary to have field reversing switches on each member. These are used to make the motor slip a pole and to reverse the polarity of the generator. The use

of reversing switches on both members of a set reduces the number of trials necessary to get proper synchronizing.

Synchronous-synchronous sets are started by means of a squirrel-cage winding in the field poles which makes the motor start (at a reduced voltage) as an induction motor and pull into step without synchronizing.

By providing ample capacity in the motor, particularly in field copper, these sets may be used for power-factor correction of the motor circuit by over-excitation of the motor.

The efficiency of these sets is quite high, but is of course the product of the efficiencies of the two machines. The machines may be wound for high voltage, thus eliminating the need of transformers for voltages of 13,000 or less. Two-bearing enclosed sets are quite popular in order to economize in space and in friction loss.

INDUCTION-TYPE FREQUENCY CHANGERS. — When the stator of an induction motor is excited from a supply circuit having a frequency of f_1 cycles per second and the rotor of this motor is driven by another motor in the *opposite* direction to that in which it would rotate due to the currents in its stator winding, the frequency of the current induced in the rotor winding is

$$f_2 = \frac{N_0 + N}{N_0} \cdot f_1,$$

where N_0 = synchronous speed of the motor and N = actual speed at which its rotor is driven. This combination of two motors (the driven motor really acts also as a generator) may therefore be used as a frequency changer.

Neglecting the losses in the driven motor, the electrical input into its stator is

$$\frac{f_1}{f_2} \times \text{(output of set at frequency } f_2\text{)}$$

and the mechanical output of the driving motor is

$$\frac{f_2 - f_1}{f_2} \times \text{(output of set at frequency } f_2\text{)}.$$

While this combination is less expensive than the usual motor-generator set, it has the disadvantage of poor regulation, as every change in the potential of the supply circuit is transmitted to the receiving circuit; it is therefore but seldom used.

STATIONARY FREQUENCY CHANGERS. — See article on *Radio Communication*.

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FREQUENCY INDICATORS. — The frequency of the current supplied by a generator may be determined directly from its speed, N revolutions per minute, and number of poles p, from the formula

$$f = \frac{Np}{120}.$$

The same formula also applies to a synchronous motor, N and p being the speed and number of poles of the motor. The speed may be measured by a revolution counter and stop watch or by means of a tachometer, or speed indicator. The latter may be calibrated to read the frequency directly for a given generator or motor.

By the term Frequency Indicator or Meter is to be understood an instrument. the indications of which are determined directly by the frequency of the circuit connected to its windings, as distinguished from an electrical Tachometer or Speed Indicator which generally consists of a D.C. Voltmeter operated by a permanent field generator, the scale being calibrated to read "cycles." The permanent field generator, or so-called magneto, is directly driven from the shaft of the A.C. generator, the frequency of which it is desired to measure in this way.

INDICATING FREQUENCY METERS. — Indicating frequency meters are made in a variety of types, the most common being (1) the moving-vane or moving-coil type, (2) the induction type, and (3) those operating on the principle of the mechanical resonance of an iron reed acted upon by the magnetic field produced by a current from the given source.

Other types, less common, consist of a synchronous motor with an attachment for indicating its speed on a scale which is calibrated in frequency.

A wave meter is a type of frequency meter used for measuring radio frequencies which are much higher in value than the frequencies used in power work. For description, see *Wave Meters*.

Moving-vane Type of Frequency Indicator. — A common form of frequency indicator of this type consists of a moving vane, with pointer attached, so mounted that it is acted upon by two coils set at right angles to each other. One coil is connected in series with a non-inductive resistance and the other in series with a comparatively large inductance. The vane tends to set itself in the direction of the resultant field due to the currents in two coils, there being no controlling spring, and consequently for a fixed ratio of these currents the vane takes up a definite position irrespective of the values of these currents. The two circuits are so connected to the source of supply that when the voltage across the mains varies, the voltages across the two circuits of the instrument vary proportionally to each other, and therefore the position of the vane and pointer is unaffected by voltage variations. An increase in the frequency, however, decreases the current through the inductive circuit relative to that through the non-inductive circuit, and consequently causes a deflection of the vane and pointer; similarly a decrease in the frequency increases the current through the inductive circuit, producing a deflection in the opposite direction. The resistance and inductance of each circuit is so adjusted that for the standard frequency, e.g., 60 cycles, the pointer stands in the middle of the scale. A given instrument is suitable for a range of frequency generally from about 25 per cent below, to 25 per cent above normal frequency.

Effect of Wave Form. — Such instruments are affected to a very small extent by the wave form of the voltage on the mains to which they are connected.

This is readily understood by considering that wave forms differing from a pure sine are the equivalent to a superimposed higher frequency harmonic which affects the frequency meter in the same way as would a higher frequency of the

same effective value. Numerous expedients have been devised to overcome this error by various combinations of resistive and inductive circuits, and by providing a small adjustable resistance in series with the inductance, this resistance being set to correspond with the wave form on which the instrument is to be used.

Moving-coil Type of Frequency Indicator. — This type is similar to the moving-vane type except that two moving coils, rigidly fastened together, constitute the moving element and the stationary element is a single coil.

Resonant-Circuit Frequency Indicator. — A form of instrument of this type, in which the sensitiveness is greatly increased, is described by *Pratt and Price, Trans. A.I.E.E., 1912, Vol. 31, p. 1595*. The two moving coils or armatures are connected in series with suitable resistances, inductances and condensers, so adjusted that the two circuits are nearly in electrical resonance (*see Alternating Currents*) with the impressed frequency. A 6-inch deflection of the pointer over the scale of the instrument is readily obtained for a change of frequency from 55 to 65 cycles per second, and by changing the constants of the circuits a 6-inch deflection can also be obtained for a change of frequency from 60 to 61 cycles per second. This instrument shows only a trace of variation due to wave form, voltage or temperature

Induction-type Frequency Indicator (see Fig. 1) — In these instruments a disk *G*, is subjected to torque in opposite directions by two small shaded pole motors *C*, the construction being similar to that of an induction voltmeter. The circuit *I*, of one motor is inductive and that in the other is practically noninductive, *H* in figure. The disk or rotating member is provided with one edge eccentric, so that the torque developed on the disk varies with its position relative to the electromagnets. The disk then finds a position in which thetorques are balanced, which position will depend on the relative currents in the two electromagnets. The current in the inductive element depends upon the frequency as in the types described above, and consequently the deflection of the disk is a measure of the frequency.

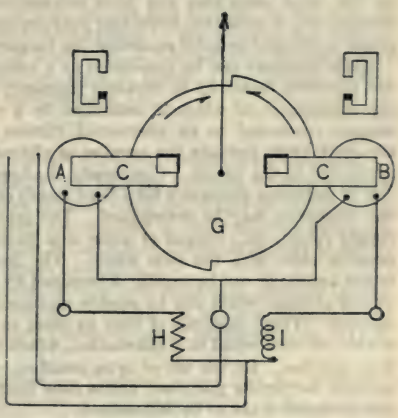

Fig. 1.

It will be understood from the above that the moving vane, moving arm, and induction type frequency meters do not operate with a control force such as a spring or gravity as do voltmeters, etc. The introduction of such a mechanical control force would render the instruments sensitive to changes of voltage and temperatures as these would vary the electrical torque without changing the mechanical countertorque, thus producing an error. Therefore, all these frequency meters have movements and pointers which will remain at rest at any position they may be placed when not connected to the circuit. When coils are energized, the movement assumes a position at which the differentially-acting coils are balanced.

Vibrating-reed Type of Frequency Indicator. — In these instruments a series of vibrating reeds is provided, each tuned to vibrate at a frequency corresponding to a certain number of cycles per second, or sometimes the tuning is done for differences as small as ¼ cycle per second or less. Some arrangement is provided by means of which the frequency of the alternating circuit is communicated to these vibrating reeds. In one arrangement all of the reeds are mounted on a common support which is vibrated by a single electromagnet through which the alternating current whose frequency is to be measured passes. In another arrangement the reeds are attached to a rigid support but are constructed of magnetic materials and acted on by one or more electromagnets directly. These instruments are quite accurate in service if the voltage is not varied beyond that for which the instruments are intended, in which case the wrong reed may be made responsive.

The frequency of modern power circuits varies but little and the rate of variation is slow. Thus the pointers of the usual frequency meters are generally either at rest or moving very slowly, introducing a personal element of doubt on the part of the observer as to operativeness. The argument in favor of the vibrating reed frequency meter is that the indicator is in motion constantly, showing that the instrument is operating. On the other hand, frequency meters indicating by means of pointers and graduated dials in the same way as voltmeters and ammeters, are generally preferred, being more easily read.

Miscellaneous Indicators. — Graphic recording frequency meters operate on the same principles and have movements similar to those of moving vane, moving coil, or induction-indicating instruments, to which are added clock-driven paper charts and pens for recording the frequency variations at different times.

Cycle counters can be considered as a variety of frequency meters although not intended primarily to measure frequency. They consist of a clock train, the escapement of which is controlled by a vibrator oscillated by an electromagnet connected to an alternating current circuit. The dials operated by the clock train indicate the number of alternations elapsed or occurring while connected to the circuit. Conversely, if allowed to be connected to an a-c. circuit of unknown frequency for a fixed length of time, as for example, 10 seconds, the frequency may be determined. The principal use of cycle counters is for timing relays, etc., instead of using stop watches, as much greater accuracy is possible than with the stop watch. The frequency of the circuit should be either known or can be readily determined by operating the cycle counter for a period of time which can be readily measured by ordinary means, such as 10 seconds. Then, intervals of time as short as 1 cycle can be readily measured within an error equal to 1 alternation, which is obviously an impossibility with stop watches. By the use of cycle counters, chronographs or oscillographs are avoided for timing purposes, unless time intervals within an error of less than one alternation are to be determined.

COST OF FREQUENCY INDICATORS. — Switchboard frequency meters of the moving-vane induction or moving-coil type (not tuned), cost from $45 to $65. The tuned, or electrical resonance, switchboard type cost from $75 to $85. Vibrating-reed frequency indicators for switchboard service cost from $35 to $95; portable indicators of this type cost from $110 to $125; duty included in both cases. The various types are made for standard frequencies of 25, 40 and 60 cycles per second and for circuits of 110 or 220 volts. For higher voltages potential transformers should be used.

BIBLIOGRAPHY. — Jansky, C. M., *Electrical Meters*, N. Y., 1913; Pratt and Rice, *Resonant Circuit Frequency Indicator*, Trans. A.I.E.E., 1912, Vol. 31, p. 1595; Bulletins of Manufacturers.

FRICTION. — (*See also Automobiles, Electric; Bearings; Belts and Belting; Brakes and Braking Systems; Friction Drive; Gears and Gearing; Hydraulics, Principles of; Lubricants and Lubrication; Pipes and Piping; Railways, Energy Requirements for; Ropes and Rope Drive; Valves; also under name of machine in question.*) Friction is the tangential force set up whenever an external force is applied to a body tending to move it, or actually moving it, over the surface of another. There are two distinct types of friction, namely, "sliding friction" and "rolling friction." Again, the force required to start a body sliding or rolling over the surface of another body (in addition to the force required to overcome the inertia of the first body) is in general greater than the force required to keep it sliding or rolling at slow speed after it is once started. The force required to start a body is called the "static friction" or the "friction of rest" and the force required to keep it in motion is called the "kinetic friction" or the "friction of motion." There is evidence to indicate that there is no abrupt change in the value of the friction from rest to motion, but that the change is a continuous one, varying rapidly with the speed at low speeds. At first the friction decreases with the speed, at moderate speeds it is nearly constant, and at higher speeds increases rapidly with the speed. Air friction must also be taken into account at very high speeds.

COEFFICIENT OF SLIDING FRICTION (f) AND ANGLE OF REPOSE (θ). — The coefficient of sliding friction is defined as the ratio of the force required to move a body along a horizontal plane surface to the normal component of the force pressing the body against this surface. If the body is resting on an inclined plane and the force normal to this plane is due only to the weight of the body, the angle of inclination of this plane to the horizontal required to start the body is called the angle of repose. The tangent of this angle is equal to the coefficient of static friction, i.e.,

$$f = \tan \theta.$$

The friction of a bearing is a special case of sliding friction (*see Bearings*).

Factors Affecting the Coefficient of Sliding Friction. — For a given normal pressure (force per unit area perpendicular to surface of contact) the coefficient of friction is approximately independent of the area of contact, all other conditions being the same, except in the case of fibrous materials, in which case the coefficient increases with extent of surface.

The coefficient of friction, however, is as a rule materially affected by the pressure, speed, degree of smoothness of the surfaces in contact, the condition of these surfaces (whether dry, moist, greasy or oily), temperature in case of lubricated surfaces, etc.

Friction of Fluids Against Solids. — Thurston states that for all fluids, whether liquid or gaseous, the resistance is: (1) independent of the pressure between the masses in contact; (2) directly proportional to the area of rubbing-surface; (3) proportional to the square of the relative velocity at moderate and high speeds, and to the velocity nearly at low speeds; (4) independent of the nature of the surfaces of the solid against which the stream may flow, but dependent to some extent upon their degree of roughness; (5) proportional to the density of the fluid, and related in some way to its viscosity (*see Pipes and Piping; Hydraulics, Principles of*).

Friction of Lubricated Surfaces approximates more closely the laws of fluid friction the more thoroughly the surface is lubricated (*see also Bearings; Lubricants and Lubrication*).

Values of the Coefficient of Sliding Friction. — Due to the numerous factors which affect the coefficient of friction, the values of this coefficient given

by various authorities are found to differ widely. The table below will serve to indicate the order of magnitude of the coefficient in the cases stated, but it should be kept in mind that these values are only rough approximations (*see also Belts and Belting, Brakes and Braking Systems; Ropes and Rope Drive*).

COEFFICIENT OF STATIC FRICTION BETWEEN DRY, SMOOTH SURFACES

Materials	Pressure, lb. per sq. in.	*f*	Authority
Wrought iron on wrought iron	187–560	0.25–0.41	Rennie.
Wrought iron on cast iron	187–672	0.28–0.38	Rennie.
Steel on cast iron	187–672	0.30–0.40	Rennie.
Brass on cast iron	187–784	0.21–0.23	Rennie.
Yellow pine on yellow pine	100–1500	0.25–0.32	Messiter and Hanson.
Spruce on spruce	100–800	0.13–0.53	Messiter and Hanson.
Metals on metals	....	0.15–0.25	Rankine.

COEFFICIENT OF STATIC FRICTION AND ANGLES OF REPOSE OF BUILDING MATERIALS

Materials	Angle of repose, degrees	*f*	Authority
Dry masonry and brickwork	31–35	0.6–0.7	Rankine.
Masonry and brickwork with damp mortar	36.5	0.74	"
Masonry on dry clay	27	0.51	"
Masonry on moist clay	18.25	0.33	"
Timber on stone	22	0.4	"
Timber on timber	11.3–26.5	0.2–0.5	"
Timber on metals	11.3–31	0.2–0.6	"
Iron on stone	16.7–35	0.3–0.7	"
Earth on earth	14–45	0.25–1.0	"

COEFFICIENT OF KINETIC FRICTION (*Rankine*)

Materials	Dry surface	Wet surface	Soapy surface	Oily or greasy surface
Wood on wood	0.25–0.5	....	0.04–0.2	....
Metals on oak	0.5–0.6	0.24–0.26	0.2	....
Metals on elm	0.2–0.25	....	....	....
Metals on metals	0.15–0.2	0.3	....	0.03–0.08
Hemp on oak	0.53	0.33	....	....
Leather on oak	0.27–0.38	....	....	....
Leather on metals	0.56	0.36	0.15–0.23	....
Bronze on lignum vitæ	....	0.05	....	....

Power Lost Due to Sliding Friction. — Let

f = coefficient of friction,
W = total force acting normal to surface of contact,
v = velocity in feet per second,

then the power lost is

$$fWv \text{ foot-pounds per second} = \frac{fWv}{550} \text{ h.p.}$$

COEFFICIENT OF ROLLING FRICTION. — Let

F = resisting force in pounds tangent to circumference of wheel,
r = radius of wheel in feet,
W = load on wheel in pounds,

then the coefficient of rolling friction (f) is defined by the relation

$$F = \frac{fW}{r}.$$

Note that the value of f depends upon the unit in which r is expressed. If r is expressed in inches instead of feet f will be 12 times as great.

Factors Affecting the Coefficient of Rolling Friction. — Rolling friction is a consequence of the irregularities of form and the roughness of surface of bodies rolling one over the other. Its laws are not yet definitely established in consequence of the uncertainty which exists in experiment as to how much of the resistance is due to roughness of surface, how much to original and permanent irregularity of form, and how much to distortion under the load. See also Schultz, B. B., *Theory of Resistance to Rolling of a Hard Body Over a Plastic Surface*, A.S.M.E. J., 37, pp. 478 and 555, Aug. and Sept., 1915.

Values of Rolling Friction. — The following are some reported values of rolling friction, when r is expressed in feet,

Lignum-vitæ roller on oak track	0.0016
Elm roller on oak track	0.0027
Car wheel on iron or steel rail	0.0015–0.002
Steel-tired wagon wheel on soft soil	0.065
Steel-tired wagon wheel on smooth hard road	0.02
Steel-tired wagon wheel on wood	0.0185
Steel-tired wagon wheel on asphalt	0.012

Power Lost Due to Rolling Friction. — Let

f = coefficient of friction, in feet,
W = total vertical load on wheel, in pounds,
r = radius of wheel, in feet,
n = number of revolutions of wheel per second,
v = linear speed in feet per second,

then the power lost in friction is

$$\frac{fWv}{r} = 6.28\, fWn \text{ foot-pounds per second}$$

$$= \frac{fWv}{550\, r} = \frac{fWn}{87.5} \text{ h.p.}$$

BIBLIOGRAPHY. — Numerous references will be found in *Kent's Mechanical Engineers' Pocket-Book.*

FRICTION DRIVE. — (*Adapted from paper by W. F. M. Goss, Trans. A.S.M.E., 1907.*) A friction drive consists of a fibrous or somewhat yielding driving wheel working in rolling contact with a metallic driven wheel. Such a drive may consist of a pair of plain cylinder wheels mounted upon parallel shafts, or a pair of beveled wheels, or of any other arrangement which will serve in the transmission of motion by rolling contact.

Suitable fibrous materials for the driving wheel are straw fiber, leather fiber, tarred fiber, sulphite fiber, or leather; suitable materials for the driven wheel are iron, aluminum and type metal. See also P. L. Weston, *Magnetic Steel-band Drive*, El. Rev. 81, p. 383, Oct. 19, 1917.

Crushing Strength and Safe Load for Fiber Wheels. — The crushing strength of each fibrous material, as determined by finding the load under which the wheel failed before 15,000 revolutions had been made, is given in the following table, together with the safe working load, taken as one-third the crushing load.

Material	Crushing load, lb. per in. width	Safe load, lb. per in. width
Straw fiber........	750	250
Leather fiber......	1200	400
Tarred fiber.......	1200	400
Sulphite fiber......	700	233
Leather...........	750	250

Coefficient of Friction. — The coefficients of friction between the various fibrous materials and the three metals are given in the table below, these being maximum values, corresponding to a slip of about 2 per cent. The friction at constant slip was found to be practically independent of the pressure between the limits of 150 and 400 pounds per inch width of face in contact.

Horse-power Transmitted. — Goss gives the following formula for the maximum horse-power which can be safely transmitted by a friction drive

$$P = \frac{\pi d}{12} \times \frac{WPN \times 0.6f}{33{,}000} = kdWN,$$

in which d = diameter of driving wheel in inches, W = width of face in inches, P = safe working pressure in pounds per inch of width, N = revolutions per minute, f = coefficient of friction, 0.6 a factor for the decrease of the coefficient in service and for the loss in journal friction, k a coefficient including P, f and the numerical constants.

COEFFICIENTS OF FRICTION AND HORSE-POWER OF FRICTION DRIVES

Surface of Driving Pulley	On iron		On aluminum		On type metal	
	f	k	f	k	f	k
Straw fiber..........	0.255	0.00030	0.273	0.00033	0.186	0.00022
Leather fiber.........	0.309	0.00059	0.297	0.00057	0.183	0.00035
Tarred fiber..........	0.150	0.00029	0.183	0.00035	0.165	0.00031
Sulphite fiber.........	0.330	0.00037	0.318	0.00035	0.309	0.00034
Leather..............	0.135	0.00016	0.216	0.00026	0.246	0.00029

BIBLIOGRAPHY. — Additional data on friction drive will be found in Kent's *Mechanical Engineers' Pocket-Book.*

FUELS. — (*See also Boilers; Calorimeters, Fuel; Gas; Gas Producers; Power Stations.*) Commercial fuels are wood, peat, coal, charcoal, coke, petroleum and gas. These fuels in the raw state all contain carbon and hydrogen as the heat-producing elements, together with oxygen, nitrogen, sulphur, earthy matter and moisture, which are undesirable and detract from the value of the fuel. Sulphur in the form of sulphide of iron, which frequently exists in coal, tends to cause spontaneous combustion.

Wood is rarely used for the production of large amounts of energy. Peat, which is formed in bogs or marshes by the partial decomposition or destructive distillation of vegetable materials, is unsuitable for fuel until dried. Although extensively used in Europe for both heating and power purposes, its use in this country has hardly gone beyond the experimental stage. Charcoal is the solid material left after evaporating the major portion of the volatile ingredients of wood or peat, or other vegetable matter. Coke is similarly the solid material left after evaporating the volatile ingredients of coal. Coke is of dark gray color, with slightly metallic luster, porous, brittle and hard. It is hygroscopic, i.e., absorbs and retains moisture when exposed to the air. One pound of coal yields from 0.35 to 0.90 pound of dry coke, depending on the kind of coal from which it is made. Coke is used chiefly in blast furnaces and foundries; its high cost prevents its use for the production of power.

In power plants, the fuels used are coal, oil and gas. Coal and oil will be treated in detail below; for a discussion of gas see the article on *Gas*.

DEFINITIONS. — When a fuel is heated to red heat in a non-oxidizing atmosphere, the carbon in the solid residue is called "fixed carbon." The hydrocarbons and other gaseous compounds which distil off are called "volatile matter." When a fuel burns or oxidizes, the solid mineral matter left is the ash. Fuels in the raw state also contain a certain amount of water or "moisture." The fixed carbon and volatile matter together are called the "combustible" though the nitrogen and oxygen in the volatile matter are not actually combustible. The determination of the fixed carbon, volatile matter, moisture and ash in a fuel is called the "proximate analysis." The determination of the moisture and ash of the fuel and the constituent elements of the combustible, i.e., the carbon, hydrogen, oxygen, nitrogen and sulphur, is called the "ultimate analysis." The "heating" or "calorific" value of a fuel is the number of units of heat energy developed as the result of the complete combustion of a unit weight (or mass) of the fuel. In this country and in England, the heat energy is expressed in British thermal units, abbreviated B.t.u. (1 B.t.u. = 777.5 ft.-lb. = 0.2928 watt-hour = 3.927×10^{-4} hp.-hr.) and the unit of weight is 1 pound. The heating or calorific value should be expressed as so many B.t.u. per pound of *combustible*, although it is sometimes given in B.t.u. per pound of *dry fuel* or per pound of *fuel as fired* (including ash and moisture).

MOISTURE IN FUELS. — The analyses of the solid fuels given above are of perfectly dried fuels. Wood when freshly felled contains on an average about 40 per cent of moisture, varying with different species. After eight to twelve months drying in the open air, the moisture is reduced to 20 or 25 per cent. If dried in an oven to greater dryness, it will, on exposure to the atmosphere, absorb moisture again, and the percentage it will then contain will vary with the dryness or dampness of the air; that is, wood is hygroscopic.

The moisture in coal may be surface moisture, received from rain while being transported or in storage, and which may be dried out on exposure to the atmosphere, or hygroscopic moisture, contained inside of the lumps, which cannot be

dried out without subjecting the coal for some time to a temperature considerably above 212° F. The anthracite, the semi-bituminous coals and the bituminous coals of the Appalachian coal field seldom contain hygroscopic moisture in excess of 1 or 2 per cent, but it is a characteristic of the western bituminous coals and lignites that the hygroscopic moisture is much higher; thus in some Illinois coals it is as high as 14 per cent. When this moisture is dried out, the coal will again absorb it on being exposed to the atmosphere. Some lignites contain as much as 40 per cent moisture.

ULTIMATE ANALYSES OF VARIOUS FUELS. — Typical ultimate analyses of various fuels together with the percentage of ash are given below; the analyses are for fuels from which all moisture has been expelled.

	C	H	O	N	S	Ash
Wood	50	6	41	1	..	2
Peat	59	6	30	1	..	4
Lignite	69	5	16	1	1	8
Bituminous coal	76	5	10	1	1	7
Semi-bituminous coal	86.5	4.5	3	1	0.5	4.5
Semi-anthracite	78.5	3.5	2	1	2	12
Anthracite	77	2.5	2	1	1	16.5
Charcoal	82	2	..	1	..	15
Coke	89	..	..	..	1	10
Petroleum (Texas)	84.5	11	3	..	1.5	..
Natural gas	70	25	1	4	..	..

In the progression from wood to anthracite, the chief change in chemical constitution, as shown in the above table, is a decrease in the oxygen from 41 per cent to 2 per cent. The hydrogen also decreases, but less rapidly. In all the varieties of coal, the ash is exceedingly variable, ranging from as low as 2 per cent up to 25 per cent or more. Coals high in ash are usually also high in sulphur. The ash and sulphur may be removed to a considerable extent by crushing and washing.

AIR REQUIRED FOR COMBUSTION. — The theoretical weight of air A required for complete combustion of 1 lb. of fuel may be determined from the formula,

$$A = 0.115\,C + 0.346\left(H - \frac{O}{8}\right) + 0.043\,S,$$

where C, H, O and S are the percentages (by weight) of carbon, hydrogen, oxygen and sulphur in the fuel. This gives about 12 lb. of air per pound of combustible. In the best boiler practice, the weight of air required is from 16 to 20 lb. per pound of coal, but in actual practice, the air supplied varies between much wider limits. See article on *Boilers*.

COAL. — **The proximate analysis** of coal is made by separating a sample by successive heatings at different temperatures, into moisture, volatile matter, fixed carbon and ash. For the determination of moisture, a rather large sample, say 2 ounces or 60 grams, should be taken, crushed to about ¼-inch size, and heated for two hours to 140° C. (284° F.) or until the coal ceases to decrease in weight. If a finely crushed small sample is taken, much of the moisture may be lost by air-drying while crushing and weighing. For the

determination of the other constituents, a sample of about 1 gram of finely crushed coal is taken, dried for an hour at 105° C. (221° F.) and weighed, then heated at a red heat in a covered crucible until all the volatile matter is driven off, weighed, heated with a blast lamp to a white heat, the cover being off the crucible, until all the carbon is burned away, leaving the ash, which is weighed. The analysis should be reported as in the example below, the percentages being percentages of weight.

	Moist coal	Dry coal	Combustible
Moisture	10.0	.....	.....
Volatile matter	30.0	33.33	57.14
Fixed carbon	40.0	44.45	42.86
Ash	20.0	22.22	.....
	100.0	100.0	100.0

Method of Sampling. — Proper sampling of coal is difficult. So much depends upon it that it must be properly done. For instructions in this field, see Bulletins of the Bureau of Mines and Reports of Committee D-5, A.S.T.M. (1916).

Classification of Coal. — The percentages of fixed carbon and volatile matter in the combustible furnish a means of classifying different kinds of coal. Kent's classification is as follows:

	Fixed carbon	Volatile matter	Heating value per lb. of combustible, B.t.u.
Anthracite	97 to 90	3 to 10	14,600 to 15,000
Semi-anthracite	90 to 85	10 to 15	14,700 to 15,500
Semi-bituminous	85 to 75	15 to 25	15,500 to 16,000
Bituminous, Eastern	75 to 60	25 to 40	14,800 to 15,500
Bituminous, Western	65 to 50	35 to 50	13,500 to 14,800
Lignite	Under 50	Over 50	11,000 to 13,500

The U. S. Geological Survey classifies coals into six groups, as follows: (1) anthracite; (2) semi-anthracite; (3) semi-bituminous; (4) bituminous; (5) sub-bituminous, or black lignite; and (6) lignite.

Anthracite is hard, shiny, burns with little or no smoke, is slow to ignite, burns slowly and breaks into small pieces when rapidly heated. Anthracite is crushed at the mine and the lumps separated into different sizes by passing them over screens or parallel bars.

Semi-anthracite is similar to anthracite, but is less hard, less shiny and burns more rapidly. It can usually be distinguished by its tendency to soil the hands, while true anthracite does not.

Semi-bituminous coal is softer than semi-anthracite. The combustible portion of semi-bituminous coals is very uniform in composition. The volatile matter is usually from 18 to 22 per cent of the combustible matter. Such fuels

are usually low in moisture, ash and sulphur and rank among the best steaming coals in the world.

The ash content runs from 3 to 8 per cent and for the higher grades, their heating value is in the neighborhood of 14,500 B.t.u. or better, per pound of dry coal.

Bituminous coal is also a soft coal. It is distinguished by high percentage of volatile matter, which causes it to give off dense volumes of smoke when heated. It requires careful firing and furnaces especially adapted for it to prevent smoke. There is a wide variation in its physical properties.

Sub-bituminous coal is commonly known as "black lignite." It is generally black and shining, closely resembling bituminous coal, but it "weathers" or disintegrates more rapidly on exposure and lacks the prismatic structure of bituminous coal. Its calorific value is generally less than that of bituminous coal.

Lignite, or "brown coal," is intermediate between coal and peat. It usually has a woody structure and is distinctly brown in color, even on a fresh fracture. It carries a higher percentage of moisture than any other class of coals, its mine samples showing from 30 to 40 per cent of moisture. It is fragile and rapidly splits into fine pieces upon exposure to air.

The following analyses of representative coals of the six classes are given by Prof. N. W. Lord:

Class 1. Anthracite Culm. Penn.
Class 2. Semi-anthracite. Arkansas.
Class 3. Semi-bituminous. W. Va.
Class 4(*a*). Bituminous coking. Connellsville, Pa.
Class 4(*b*). Bituminous non-coking. Hocking Valley, Ohio.
Class 5. Sub-bituminous. Wyoming, black lignite.
Class 6. Lignite. Texas.

COMPOSITION OF ILLUSTRATIVE COALS—CARLOAD SAMPLES

Proximate Analysis of "Air-dried" Sample							
Class	**1**	**2**	**3**	**4a**	**4b**	**5**	**6**
Moisture	2.08	1.28	0.65	0.97	7.55	8.68	9.88
Vol. comb.	7.27	12.82	18.80	29.09	34.03	41.31	36.17
Fixed carbon	74.32	73.69	75.92	60.85	52.57	46.49	43.65
Ash	16.33	12.21	4.63	9.09	5.85	3.52	10.30
Loss on air-drying	3.40	1.10	1.10	4.20	Undet.	11.30	23.50
Ultimate Analysis of Coal Dried at 105° C.							
Hydrogen	2.63	3.63	4.54	4.57	5.06	5.31	4.47
Carbon	76.86	78.32	86.47	77.10	75.82	73.31	64.84
Oxygen	2.27	2.25	2.68	6.67	10.47	15.72	16.52
Nitrogen	0.82	1.41	1.08	1.58	1.50	1.21	1.30
Sulphur	0.78	2.03	0.57	0.90	0.82	0.60	1.44
Ash	16.64	12.36	4.66	9.18	6.33	3.85	11.43

COMPOSITION OF ILLUSTRATIVE COALS — *Continued.*

Results Calculated to an Ash and Moisture Free Basis							
Class	1	2	3	4a	4b	5	6
Volatile comb.	8.91	14.82	19.85	32.34	39.30	47.05	45.31
Fixed carbon	91.09	85.18	80.15	67.66	60.70	52.95	54.69
Ultimate Analysis							
Hydrogen	3.16	4.14	4.76	5.03	5.41	5.50	5.05
Carbon	92.20	89.36	90.70	84.89	80.93	76.35	73.21
Oxygen	2.72	2.57	2.81	7.34	11.18	16.28	18.65
Nitrogen	0.98	1.61	1.13	1.74	1.61	1.25	1.47
Sulphur	0.94	2.32	0.60	1.00	0.87	0.62	1.62
Calorific Value in B.t.u. per pound of Combustible by Dulong's Formula							
Air-dried coal	12,472	13,406	15,190	13,951	12,510	11,620	10,288
Combustible	15,286	15,496	16,037	15,511	14,446	13,235	12,889

Heating Value of Coal. — The heating value of coal depends on its percentage of total combustible and on the heating value per pound of that combustible. The latter differs in different districts and bears a relation to the percentage of volatile matter as shown by the above table.

For coals in which the volatile matter is less than 40 per cent of the combustible, the heating value per pound of combustible may be approximately determined (within about 2 per cent) by means of the following table. When the volatile matter is in excess of 40 per cent, the figures in the table may have a plus or minus error of as much as 4 per cent, since the coals high in volatile matter differ greatly in their content of oxygen.

Volatile matter, per cent of combustible	Heating value, B.t.u. per pound of combustible	Volatile matter, per cent of combustible	Heating value, B.t.u. per pound of combustible	Volatile matter, per cent of combustible	Heating value, B.t.u. per pound of combustible
3	14,940	28	15,660	43	14,220
6	15,210	32	15,480	45	13,860
10	15,480	37	15,120	47	13,320
13	15,660	40	14,760	49	12,420
20	15,840				

The heating value per pound of combustible of all the semi-bituminous coals, containing from 15 to 25 per cent of volatile matter in the combustible, is within 1½ per cent of 15,750 B.t.u. per pound. This is within the limits of error of sampling, of chemical analysis or of calorimetric determination.

The heating value of any coal may also be calculated from the ultimate analysis by means of Dulong's formula,

$$\text{B.t.u. per pound} = 146\,C + 620\left(H - \frac{O}{8}\right) + 40\,S,$$

in which C, H, O and S are respectively the percentages by weight of carbon hydrogen, oxygen and sulphur. The probable error of this formula is not over 2 per cent for any coal containing less than 40 per cent volatile matter in the combustible. For coals higher in volatile matter, the error is sometimes larger. When the percentages are expressed as percentages of the various elements in the *combustible*, the formula gives the B.t.u. per pound of combustible; when the percentages are percentages of the various elements in a given weight of **coal,** the formula gives the B.t.u. per pound of coal, which is, of course, less than the B.t.u. per pound of combustible in proportion to the per cent of moisture and ash present. In general, if K be the per cent of combustible in a given sample, and H the heating value per pound of combustible, the heating value per pound of coal is $\frac{KH}{100}$.

For the direct determination of the heating value of coal or other solid fuel, a bomb calorimeter, in which the fuel is burned in an atmosphere of compressed oxygen, is the most accurate instrument. (*See Calorimeters, Fuel.*)

Average Heating Values. — The following figures give the average commercial heating values of the principal power plant fuels, other than oil and gas.

	B.t.u. per pound
Anthracite	11,500–14,000
Semi-anthracite	13,000
Semi-bituminous coal	14,000–15,000
Bituminous	11,000–14,000
Lignite (as fired)	8,300
Lignite (dry)	11,300
Coke	12,000
Charcoal	11,600–13,500
Peat (as fired)	4,000– 8,000
Peat (dry)	5,000–10,000
Wood (dry)	6,600– 9,900
Bagasse (45 to 55 per cent water)	3,000– 3,500
Tan bark (65 per cent water)	2,700
Tan bark (dry)	6,000– 9,500
Straw	5,100– 6,700

The Commercial Value of a Coal is not always in direct proportion to its heating value. Excessive moisture causes a reduction in its temperature of combustion, which reduction, in steam boiler practice, decreases the efficiency; excessive ash tends to obstruct the draft and thus cause imperfect combustion. High percentages of volatile matter tend to cause smoke and soot, and require the use of special furnaces.

Specifications for Coal. — For the reasons just stated, contracts for the purchase of coal on specifications of quality should penalize excess of moisture, ash and volatile matter above certain stated percentages. For a full discussion of purchase of coal by specification and specifications used by the Government, see latest Bulletins of the U. S. Bureau of Mines relating to this subject. Kent's specifications are as follows:

Anthracite and Semi-anthracite. — The standard is a coal containing 5 per cent volatile matter, not over 2 per cent moisture and not over 10 per cent ash. A premium of 0.5 per cent on the price will be given for each per cent of volatile matter above 5 per cent up to and including 15 per cent, and a reduction of 2 per cent on the price will be made for each 1 per cent of moisture and ash above the standard.

Semi-bituminous and Eastern Bituminous. — The standard is a semi-bituminous coal containing not over 20 per cent volatile matter, 2 per cent moisture and 6 per cent ash. A reduction of 1 per cent in the price will be made for each 1 per cent of volatile matter in excess of 25 per cent, and of 2 per cent for each 1 per cent of ash and moisture in excess of the standard.

Western Coals. — For western coals in which the volatile matter differs greatly in its percentage of oxygen, the above specification based on proximate analysis may not be sufficiently accurate, and it is well to introduce the heating value, as determined either by a calorimeter or by calculation from the ultimate analysis.

The standard is a coal containing not over 6 per cent moisture and 10 per cent ash in an air-dried sample, and having a heating value of 14,500 B.t.u. per pound of pure coal (coal free from moisture and ash). For lower heating value per pound of pure coal, the price shall be reduced proportionally, and for every 1 per cent increase in ash or moisture above the specified figures, 2 per cent on the price shall be deducted.

Space Required for Storage. — The space occupied by a ton of coal depends both upon the quality of the coal and the size of the lumps. A ton of 2240 pounds of anthracite of pea size, or smaller, occupies a space of from 36 to 45 cubic feet. A ton or 2240 pounds of bituminous coal occupies a space of from 40 to 50 cubic feet. In estimating the space required for storage, 45 cubic feet per ton (2240 pounds) is usually assumed.

Weathering of Coal. — Anthracite coal when exposed to the weather undergoes practically no change except the oxidation of the sulphur content, which is small. Bituminous coal contains a larger percentage of sulphur, the oxidation of which, if present in sufficient amount, may develop sufficient heat to cause spontaneous combustion. Some lignites are rapidly disintegrated when exposed to the air. Experiments on carload lots of Illinois coal (*F. W. Wheeler, Trans. A.S.M.E., 1908*) showed that the screenings and 3-inch nut coal lost 1.3 per cent of its heating value in one month, and 2 per cent in six months. Pillar coal in the mine, exposed underground twenty-two to twenty-seven years, showed only 3 per cent less heating value than the fresh face coal from the same mine.

Cost of Coal. — The cost of coal delivered to any power plant depends not only upon the quality of the coal and the size of the lumps, but also upon the cost of transportation, the railroad or water facilities for delivering the coal, and upon various conditions affecting the cost of mining. In making an estimate involving the cost of coal, one should obtain quotations from the dealers for the specific locality and time under consideration.

LIQUID FUEL. — (*See also Boilers; Gas Engines.*) Crude petroleum and various distillates of petroleum, such as gasoline, kerosene, etc., are largely used as fuel, the extent of their use in any locality depending chiefly on their relative cost as compared with coal. In Texas, California, Russia and other places near to oil wells and where coal is relatively expensive, petroleum has largely replaced coal as a fuel for steam boilers. Gasoline, kerosene and heavier oils are also extensively used in various types of internal combustion engines. Crude petroleum is composed chiefly of hydrocarbons, which distil at

different temperatures, the lightest vapors being driven off as low as 113° F., and heavier vapors and oils at temperatures rising to 600°, above which waxes and residuum are formed. The crude oil contains small percentage of water, sulphur and oxygen as impurities. The specific gravity, weight and heating value of California oil are given as follows by J. N. LeConte (*Jour. A.S.M.E., Aug., 1911*):

CALIFORNIA OIL

Degree Baumé	Specific gravity	Weight per barrel, pounds	B.t.u. per pound	B.t.u. per barrel (42 gallons)
10	1.000	350	18,280	6,398,600
12	0.986	345	18,400	6,349,800
14	0.972	340	18,520	6,302,400
16	0.959	336	18,640	6,256,500
18	0.946	331	18,760	6,211,400
20	0.933	327	18,880	6,167,900
22	0.921	322	19,000	6,126,000
24	0.909	318	19,120	6,084,000
25	0.903	316	19,180	6,063,800

Oils from other sources have different densities and heating values, thus Lima, O., crude is reported to have a sp. gr. of 0.792; Beaumont (Texas) oil, sp. gr. 0.92, B.t.u. per pound 19,060; Pennsylvania heavy crude, sp. gr. 0.886, B.t.u. 20,736; Caucasian light crude, sp. gr. 0.884, B.t.u. 22,027. California oil, six lots, used in a boiler test at Redondo, Cal., contained moisture 1.82 to 2.70 per cent; sulphur 2.17 to 2.607; B.t.u. per pound 17,717 to 17,966.

The following table shows the relative heating values of crude petroleum and coal, based on oil of sp. gr. 0.885; B.t.u. per pound 20,000; 1 barrel, 42 gallons = 310 pounds.

Coal B.t.u. per pound	1 pound oil = pounds coal	1 barrel oil = pounds coal	1 ton (2240 pounds) coal = barrels oil
10,000	2	620	3.61
11,000	1.818	564	3.97
12,000	1.667	517	4.33
13,000	1.538	477	4.69
14,000	1.429	443	5.05
15,000	1.333	413	5.42

Advantages of Liquid Fuel. —

1. Reduction in number of firemen in proportion of 5 or 6 to 1.
2. Easy lighting of fires and more regular supply of heat.
3. Fires readily regulated to suit demand for steam, and can be promptly extinguished.
4. Small proportion of refuse and its easy disposal.
5. Storage tanks can be located to best advantage, while coal bins must be near the boilers.

6. No sparks; no dust; no loss by banking.

Disadvantages of Liquid Fuel. —

1. Fire risk. Use prohibited by some city ordinances.
2. Offensive odor. Use prohibited by some cities.
3. Vapor forms explosive mixture with air.
4. Supply limited.
5. Burners make objectionable roaring noise.
6. Heating surface apt to become coated with residue.
7. Tendency of the oil to creep by valves and leak.
8. Necessity for auxiliary apparatus to start oil fire or maintain it or both.

Boiler Efficiency with Oil Fuel. — Although boiler efficiencies as high as 82 per cent or above are reported with oil-burning furnaces, the average is probably nearer 72 per cent, i.e., the efficiency with oil is about 2 per cent higher than with coal.

Gasoline, Kerosene and Alcohol. — The following is taken from the *Smithsonian Physical Tables* (1914):

Fuel	Spec. grav. at 15° C.	Cal. per gram	B.t.u. per lb.
Petroleum ether..........	0.684–0.694	12,210–12,220	21,980–22,000
Gasoline.................	0.710–0.730	11,100–11,400	19,980–20,520
Kerosene................	0.790–0.800	11,000–11,200	19,800–20,160
Alcohol with 7–9 per cent water and denaturing material................	0.8196–0.8202	6,440– 6,470	11,590–11,650

GASEOUS FUEL. — See articles on *Gas and Gas Engines.*

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FURNACES, ELECTRIC, AND ELECTRIC FURNACE PRODUCTS. — (*See also Electrochemical Processes, Industrial.*) Electric furnaces may be classified as follows:

Arc furnaces.
Induction furnaces.
Resistance furnaces.
  a. Current conducted by the materials heated:
    1. With electrolysis.
    2. Without electrolysis.
  b. Current conducted by a special resistor.

In the arc furnace the heat is produced by an electric arc (*see Arc, Electric*) usually between carbon electrodes. The induction furnace is essentially a static transformer (*see Transformers*) with the low-tension "winding" formed by the material to be heated. In the resistance furnace the current is supplied to the material to be heated (i.e., to the furnace "charge"), or to the special resistor, by connecting the charge or resistor directly to the source of the current supply. The heat developed in both the induction and resistance furnaces arises from the passage of the current through the resistance offered by the charge or special resistor.

The arc furnace may be considered a resistance furnace in which the resistor is a gas. Since the resistance of a gas at atmospheric pressure is greater than that of any solid resistor of the same dimensions as the arc, the amount of heat that can be produced in a small space will be greatest with an arc furnace.

TEMPERATURE AND DISTRIBUTION OF HEAT. — The advantage in electric heating is that a higher temperature can be produced than by using fuel, that the heat is produced inside the furnace where it is needed, and that the heat can be easily and accurately regulated. In the arc the hottest part of the positive carbon is estimated to be between 3900° C. and 4000° C. absolute. (*Waidner and Burgess, Bull. Bureau Stds., Vol. 1, p. 123, 1905.*) The temperature of the arc itself increases with the current. (*Kayser, Handbuch der Spektroscopie, Vol. 1, pp. 154–160, 1900.*)

The electric energy delivered to the furnace as heat is used as follows: (1) to heat the charge to the desired temperature, which involves heating up the furnace walls, if cold at the start. This energy is equal to the mass of the charge times the temperature rise times the specific heat. If the charge is melted or vaporized in the furnace, additional heat must also be supplied. This item may be reduced by delivering the charge to the furnace already hot as in steel refining. (2) To supply the energy needed for the reaction; the energy so required cannot be reduced. (3) To supply the loss due to conduction and radiation through the walls and electrodes, and the heat carried off by hot gases. A part of the heat carried off by hot gases may be recovered in heating water in boilers, so that it is not a complete loss.

MAXIMUM SIZE OF ELECTRIC FURNACES. — The largest workable capacity of an open-arc single hearth with a compact bundle of electrodes has been found in practice to be from 2500 to 3000 kilowatts at from 30,000 to 40,000 amperes and from 75 to 90 volts. These large sizes are always used with the three-phase system, so that the maximum total power absorption of a furnace is from 7500 to 9000 kilowatts. In carbide furnaces double three-phase furnaces are used with six electrodes, in place of three, in the same shaft, and the power absorbed is from 15,000 to 18,000 kilowatts. Great progress has been made in electric-furnace construction, by closing the furnace at the top and by having special means of feeding in the charge, thereby avoiding the dust nuisance and protecting the workmen from the heat, as well as distributing the

charge more uniformly. (*Taussig, VII Int. Cong. App. Chem., Sec. 10, p. 24, 1910; Trans. Faraday Soc., Vol. 5, p. 254, 1909; VIII Int. Cong. App. Chem., Vol. 21, p. 105, 1912.*)

DESIGN OF FURNACE WALLS. — To reduce the conduction of heat through the walls and electrodes to a minimum amount, these must be properly designed. The heat flow through the walls of three different shapes may be computed by the formulas below. In all cases

k = thermal conductivity of the walls, at the mean temperature $(t_1 + t_2)/2$. This coefficient k may be expressed in any convenient unit, e.g., gram calories per centimeter cube per second per ° C., or watts per inch cube per ° C. See article on *Heat and Thermal Properties* for values.

H = total heat conducted per second through the walls.

t_2 = temperature of the inside surface of the wall.

t_1 = temperature of the outside surface of the wall.

Hollow Rectangular Parallelopiped. —

$$H = \left(\frac{A}{\vartheta} + 0.54\,\Sigma l + 0.15\,n\vartheta\right)k(t_2 - t_1),$$

where A = the area of the six inner surfaces, ϑ = thickness of wall, Σl = the total length of all the inner edges, n = the number of corners. This applies where all three inner dimensions are greater than $\frac{1}{5}\vartheta$. (Langmuir, Adams, and Meikle, *Trans. Am. Electrochem. Soc., Vol. 14, p. 53, 1914.*)

Hollow Sphere. —

$$H = \frac{\pi kDd\,(t_2 - t_1)}{l},$$

where D = outside diameter of sphere, d = inside diameter, l = thickness of wall, all in the same unit.

Hollow Cylinder. —

$$H = \frac{2\pi kL\,(t_2 - t_1)}{2.3\log_{10}\dfrac{D}{d}} + \frac{\pi kDd\,(t_2 - t_1)}{2l},$$

where L = mean height of inner and outer walls, D = outside diameter, d = inside diameter, and l = thickness of top and bottom walls, all in the same unit. The first term gives the flow of heat through the cylindrical walls, the second the flow of heat through the top and bottom. (Hering, *Trans. Am. Electroch. Soc., Vol. 14, p. 215, 1908*)

Linings and Composite Walls. — The most refractory substances do not have the lowest thermal conductivities. Consequently, it is advantageous to use a highly refractory substance only for the inner part or lining of the walls, using only such a thickness that the drop in temperature will be sufficient to permit of a less refractory substance of a lower conductivity being used for the next layer. "Graded" walls of several layers may be employed. See Ray and Kreisinger, *Flow of Heat through Furnace Walls*, Bull. No. 8, Dept. of the Interior, 1912.

Refractories for Furnace Walls. — The most refractory substance is carbon, which, however, is a good heat conductor. Some of the products of the electric furnace, as silicon carbide and siloxicon (a substance containing varying amounts of carbon, silicon and oxygen) stand next to carbon as refractories, and do not have such high heat conductivities. (*See* Fitzgerald, *Electrochem. Ind., Vol. 2, p. 439, 1904.*) The numerical values of heat conductivities of refractories at high temperatures are known for only a few substances and then only approximately. See article on *Heat and Thermal Properties;* also article by Hering in *Met. and Chem. Eng.*, Vol. 9, p. 625, 1911, and *the table below for graphite and carbon.* For electrical conductivity see article on *Resistance and Conductance.*

DESIGN OF ELECTRODES FOR ELECTRIC FURNACES. — The electrodes of an electric furnace should be so designed that the energy will be carried into the furnace with a minimum energy loss. The loss due to the electrical resistance is directly proportional to the electrode's length; this should therefore be made as short as convenient. The loss due to electrical resistance will be smaller the greater the cross-section of the electrode, but the heat loss from the furnace through the electrode will be directly proportional to the cross-section. It is therefore possible to find a cross-section of a given material which will give a minimum total loss for a given length. The cross-section that would give the minimum loss on certain assumptions, only approximately true, is found from the equation

$$S = 0.346\, LI \sqrt{\frac{r}{k(t_2 - t_1)}},$$

and the loss itself, watts, is

$$h = 2.89\, I \sqrt{kr\,(t_2 - t_1)},$$

where S = cross-section of electrode, L = its length, I = the current in amperes carried by the electrode, t_2 and t_1 the temperatures, in ° C., of the hot and cold ends of the electrode respectively, r = its mean electrical resistivity in ohms per unit cube and k = its mean heat conductivity for the mean temperature $(t_1 + t_2)/2$. If S and L are in centimeters, r and k must be per cm.^{3}; if S and L are in inches, r and k must be per in.^{3}.

The values of k and r for carbon and graphite are not known accurately at high temperatures. The following values have been computed from measurements of Hering (*Trans. Am. Electroch. Soc., Vol. 17, p. 166, 1910*).

Material	Temperature °C.		Thermal cond. g-cal. per cm.^{3} per °C. per sec.	Electrical resistivity, ohms per cm.^{3}
	Hot end	Cold end		
Carbon	300	40	0.0891	0.00422
	701	50	0.124	0.00381
	902	60	0.130	0.00377
Graphite	355	66	0.399	0.000837
	516	70	0.325	0.000827
	707	87	0.309	0.000802

SMALL LABORATORY FURNACES. — A great variety of electric furnaces have been devised. A few typical laboratory furnaces will be described in this section. Some industrial furnaces are described in the next section:

Moissan's Furnace (Fig. 1). — Moissan's work was carried out in a furnace consisting of two horizontal electrodes, mounted so that the distance between the two ends could be adjusted longitudinally by a screw thread. An arc was formed between these electrodes in a cavity formed by some refractory material, such as lime. The substance to be heated was placed in a crucible under the arc as shown in Fig. 1. When the substance was not to be exposed to the gases of the arc, a furnace was made with a carbon tube passing through it at right angles to the electrodes and immediately below the arc. The substance to be heated was placed inside this tube.

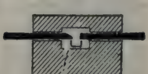


Fig. 1.

Borchers' Furnace (Fig. 2). — A type of furnace due to Borchers consists in a carbon rod placed between two larger electrodes. The charge is either packed around the small rod or placed under it. This type of furnace is convenient where a temperature below that of the arc is desired.

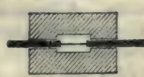


Fig. 2.

Héroult Furnace (Fig. 3). — A furnace that takes its name from Héroult consists in a crucible with one or more electrodes connected together above the crucible. The crucible is represented in Fig. 3 packed in carbon in an iron container. Graphite crucibles may be turned out from graphite electrodes. The charge in the crucible is usually melted by forming an arc between the crucible and the electrode above with an adjustable resistance in series with the arc. After the charge has melted the electrode may be partly immersed in the bath. In case it is desired to melt a substance in a crucible without the use of an arc, a smaller piece of carbon may be placed between the crucible and the electrode as in Borchers' furnace. If after the substance has been melted it is desired to pass the current through the bath itself, as for example, in case a salt is to be electrolyzed, the upper electrode may be raised, the thin rod removed with a pair of tongs and the electrode then lowered into the bath. The salt will not solidify during this operation.

Fig. 3.

Arsem Furnace (Fig. 4). — It frequently happens in the laboratory that it is desired to heat a substance to a high temperature in a vacuum or in some pure gas, such as hydrogen or nitrogen. A very convenient furnace for this purpose has been designed by Arsem (*Trans. Am. Electroch. Soc., Vol. 9, p. 153, 1906*), and has been extensively employed in research work. This furnace may be obtained from the General Electric Company in more than one size. A vertical section is shown in Fig. 4. It consists of a chamber *A* and cover *B* made of a gun-metal casting turned true at the joint. A lead gasket *C*, 1/16 inch thick, forms an air-tight joint when the cover is fastened down by the cap-screws *D*. The tube *J* through which the air is removed from the furnace is soldered into the cover. The window *E* is a disk of clear white mica 0.005 inch thick clamped between lead washers *F*.

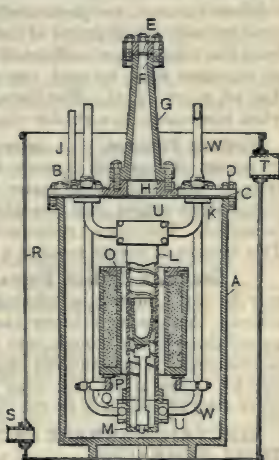


Fig. 4.

The electrodes *W* are brass tubing which are insulated from the cover. The clamps *UU* for holding the heater are copper. The heater *L* is a helix, usually of graphite, which is made by boring out a graphite electrode and cutting it along a spiral as shown. Metallic heaters may also be used. The lower end of the heater rests in the graphite cup which also holds the crucible support insulated from it by a lava ring. The screen for preventing radiation is a double-walled cylindrical box of Acheson graphite filled with graphite powder.

The water jacket *R* is a galvanized-iron tank provided with an inlet *S* and an outlet *T*. In a vacuum in the small size of furnace 9 to 10 kilowatts produce a temperature of 2500° C.

High-pressure Furnaces. — The Arsem furnace may be of course used with the internal pressure above an atmosphere, but it is not designed for high pressures. A furnace for working up to 200 atmospheres has been designed by Hutton and Petavel. (*Phil. Trans., Series A, Vol. 207, p. 421, 1908, and Electrochem. and Met. Ind., Vol. 6, p. 97.*) For a modified form see *Pring and Fairlie,* (*VIII Int. Cong. App. Chem., Vol. 21, p. 79, 1912*). For a furnace for spectroscopic work on gases at pressures up to 200 atmospheres, see *King, A. S., Astrophysical Journal, Vol. 28, p. 300, 1908.*

Heræus Furnace (Fig. 5). — A very useful type of furnace is due to the firm of Heræus. This consists in a tube wound in its middle portion with an electrical resistor. The ends of the tube are cooled by the air, or may be cooled by a coil of copper pipe through which water flows. The tube may be of some non-conducting substance, such as porcelain, in which case a ribbon of metal may be wound directly on the tube. Furnace tubes with grooves for winding with wire are now made by the Norton Company of Worcester, Mass., from fused alumina. These, however, are porous and cannot be used for a vacuum furnace. Glazed German porcelain may be heated up to 1180° C. and a vacuum maintained. At temperatures higher than this the glazing melts and air leaks into the tube. A nickel tube may be used for higher temperatures.

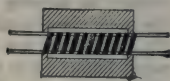


Fig. 5.

The metallic winding for carrying the current may be of platinum, nickel, nichrome, tungsten, molybdenum or any other suitable high-resistance material (*see Wires, Resistance*).

These furnaces are particularly useful when the temperature is to be held constant over a long period of time. For most purposes a constant current will keep the temperature sufficiently constant. For greater constancy some kind of regulator must be used (*see Bodenstein and Kranendieck, Z. f. Elektroch., Vol. 18, p. 417, 1912*). The Heræus type of furnace is usually more satisfactory when homemade, as a greater choice of materials is then possible.

Hoskins Furnace. — A convenient furnace for heating rather large crucibles or masses of material is made by the Hoskins Manufacturing Company of Detroit, Michigan. The heater consists of two rows of narrow, thin carbon plates which extend over two sides of the cavity which receives the substance to be heated. The contact resistance between the plates may be varied by pressure. This furnace of course requires a very large current at a low voltage.

Granular Carbon Furnace (Fig. 6). — A carbon or graphite crucible may be easily heated by placing it in a trough surrounded by granular gas carbon; this conducts better than coke. The current is passed through the trough, into which it is conducted by carbon rods. Clay crucibles should not be used in this way for temperatures over 1000° C., as at a high temperature they are attacked by the carbon.

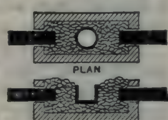


Fig. 6.

ELECTRIC-FURNACE PRODUCTS AND INDUSTRIAL FURNACES. — Some typical industrial furnaces and their products are described below.

Calcium Carbide. — Willson and Horry Furnaces. Calcium carbide was first made on a large scale by Willson in 1892 at Spray, N. C., by heating lime and carbon in an arc furnace. The reaction that takes place is

$$CaO + 3\,C = CaC_2 + CO.$$

The furnace consisted of a carbon plate 3 by 2.5 feet with a carbon electrode suspended above it. The whole was surrounded by brick walls. Furnaces similar to this were at first used at Niagara Falls, with the lower electrodes

mounted on a car which was removed when filled with an ingot of carbide. Later the Horry rotary furnace was used (*U. S. Pat. 656,156*). The Carbide Company keeps the style of their present furnace secret. In Europe the Willson type of furnace is used. Carbide may be either formed in an ingot in the crucible of the furnace and removed solid, or it may be drawn off in the liquid state. When an ingot is formed it has been found better to have the current flow between two electrodes suspended over the crucible, in place of having the crucible form one electrode. (*Conrad, Electrochem. and Met. Ind., Vol. 6, p. 397, 1908.*) The purity of the carbide is in the neighborhood of 80 per cent. The yield of 80 per cent carbide is about 5 kilograms per 24 kilowatt hours.

Carborundum. — Acheson Process. — Silicon carbide or carborundum was first made on a large scale by Acheson. It is produced from quartz and carbon when these substances are heated in an electric-resistance furnace, according to the reaction

$$SiO_2 + 3\,C = CSi + 2\,CO.$$

The furnace has a granular carbon core around which the charge, consisting of quartz, carbon, sawdust, and sodium chloride, is packed. The latest furnaces are 9.15 meters long by 3.67 meters wide, and absorb 1600 kilowatts. The current is 20,000 amperes (*Min. Industry, Vol. 16, p. 155, 1907; Vol. 17, p. 112, 1908*). From measurements on a 750-kilowatt furnace it was found that the carbide is formed at 1840° C. and decomposes when heated above 2240° C. (*Saunders, Trans. Am. Electroch. Soc., Vol. 21, p. 425, 1912.*) The yield is about one kilogram of crystallized carbide for 8.5 kilowatt hours. Silicon carbide is used as an abrasive, as furnace linings and as a substitute for ferro-silicon in the manufacture of steel. (*Fitzgerald, Carborundum, Vol. 13, in the Engelhardt Monographien über angewandte Elektrochemie.*) For description of Norton Company's plant see Fitzgerald, *Met. and Chem. Eng., Vol. 10, pp. 519–521* (*1912*).

Silundum is the trade name for silicon carbide made by exposing hot carbon rods to silicon vapor. These rods are used for electric heating. (*J. Eng. Chem., Vol. 7, pp. 565–571, 1915.*)

Siloxicon is a product of the incomplete reduction of silica, and may be represented by the formula SiCO, though compounds with varying proportions of these elements are found. Siloxicon is used for crucible linings. It is made by heating silica and an insufficient quantity of carbon for complete reduction of the silica.

Silox is a spongy substance of about the same composition as siloxicon, made by the General Electric Company, by heating silica and carbon in a closed arc furnace and condensing the distillate in a large chamber. It is used for heat insulating.

Silicon is made in an arc furnace from coke and sand. At the high temperature produced the silica is completely reduced and the melted metal is drawn off in amounts weighing from 600 to 800 pounds. It varies from 90 to 97 per cent in purity (*F. J. Tone, Electrochem. and Met. Ind., Vol. 7, p. 192, 1900; Min. Industry, Vol. 17, p. 768, 1908*).

Graphite. — Berthelot (*Ann. de Chem. et de phys., Series 4, Vol. 19, p. 393, 1870*) defines graphite as that allotropic form of carbon which when oxidized at low temperature with powerful oxidizing agents (potassium chlorate and nitric acid) gives graphite oxide. Arsem suggests as a definition for graphite that it is that allotropic form of carbon whose density lies betewen 2.25 and 2.26. (*Trans. Am. Electrock. Soc., Vol. 20, p. 105, 1911.*)

Graphite is made by heating carbon, containing a small amount of impurity, to a high temperature in an electric resistance furnace. Anthracite coal is graphitized in bulk; electrodes and crucibles are also graphitized after moulding.

Acheson's theory of the formation of graphite is that the carbon first forms a carbide, which decomposes at a higher temperature, leaving the carbon in the form of graphite. This theory is not confirmed by Arsem's experiments. (*See Fitzgerald, Künstlicher Graphit, Vol. 15 of the Engelhardt Monographien über angewandte Electrochemie.*)

Carbon Bisulphide is made in a specially designed furnace by heating together sulphur and carbon. Most of the disagreeable features encountered in the manufacture of this substance are thus avoided. (*Taylor, Trans. Am. Electroch. Soc., Vol. 1, p. 115, 1902; Vol. 2, p. 185, 1902.*)

Phosphorus is a substance the production of which the use of the electric furnace has much simplified. It is made in the Readman-Parker furnace according to Wöhler's process:

$$Ca_3(PO_4)_2 + 3\ SiO_2 + 5\ C = 3\ CaSiO_3 + 5\ CO + 2\ P.$$

(*Min. Industry, Vol. 6, p. 537, 1897; Vol. 7, p. 557, 1898.*)

Alundum is the trade name of fused aluminum oxide, which is made by the Norton Company of Worcester. Aluminum oxide is fused in an arc furnace. (*Min. Industry, Vol. 19, p. 28, 1910.*) Fused alumina is used as an abrasive, as a refractory substance for furnace linings, and porous crucibles of this substance are used in analytical laboratories.

Aluminum. — Hall and Héroult Processes. — Aluminum is now produced by the electrolysis of a solution of alumina in fused cryolite ($AlF_3.3\ NaF$) to which other fluorides, such as those of aluminum and of sodium, are added in some factories. The aluminum sinks to the bottom of the crucible and is drawn off. This process was discovered nearly simultaneously by C. M. Hall and Héroult. The heat developed by the current in passing through the solution is sufficient to keep the bath melted. The cathode is an iron trough lined with carbon, and the anode consists of a number of carbon rods suspended over the crucible. The Aluminum Company of America uses as anode for one crucible 48 carbon rods 3 inches in diameter and 15 inches long. The electromotive force applied to each crucible is 5.5 volts; the current is 10,000 amperes. The yield is 1.75 pounds of aluminum per horse-power day. For further information see *Min. Industry, Vols. 6, 14, 15, 17, 20.*

Sodium, Potassium. — Castner Process. — Sodium and potassium are obtained by the electrolysis of their fused hydrates, usually in the cell designed by Castner (*U. S. Pat. 453,030, filed 1890; see also Becker, Die Elektromettallurgie der Alkalimettale*). At Holcomb's Rock, Va., sodium is made by the electrolysis of fused sodium chloride. (*Mineral Ind., Vol. 19, p. 614, 1910.*)

Calcium is made by the electrolysis of fused calcium chloride, to which calcium fluoride is added to lower its melting point. Calcium is made at Holcomb's Rock, Va., probably in a cell devised by Seward and Von Kügelgen (*U. S. Pat. 880,760, 1908; Min. Industry, Vol. 16, p. 131, 1907; Vol. 17, p. 99, 1908*); and abroad by the method of Rathenau (*Z. f. Elektroch., Vol. 10, p. 508, 1907*). In the Ratheneau method the cathode is an iron rod which just touches the surface of the melted calcium chloride. The calcium solidifies when deposited on the cathode by electrolysis. As the calcium grows the rod is withdrawn so that a rod of calcium is produced. (*Trans. Am. Electrochem. Soc., Apr., 1920.*)

Zinc may be made by the electrolysis of fused zinc chloride. In the Swinburne–Ashcroft process sodium chloride is added to zinc chloride in such quantity that the resulting mixture contains 28 per cent zinc. The cell is a brick-lined, sheet-iron vessel. The anode is carbon, the cathode, melted zinc. Each vat takes 4.5 volts, with a cathode current density of 400 amperes per sq. ft. The temperature of the fused salt is 450° C. (*Electroch. and Met. Ind., Vol. 3, p. 65, 1905.*)

Magnesium is made by the electrolysis of fused carnallite ($MgCl_2.KCl.6H_2O$) and floats to the surface of the salt, which is, of course, anhydrous when melted. Instead of carnallite fused magnesium chloride may be used, which is dried in the presence of ammonium chloride ($1NH_4Cl$ to $1MgCl_2$) to prevent decomposition. The ammonium chloride is then removed by volatilization and condensed.

Cerium may be made by electrolyzing fused cerium chloride ($CeCl_3$). Its iron alloy sparks when filed and is used in gas lighters.

ELECTRIC FURNACES IN METALLURGY. — Under special local conditions, where iron ore is plentiful, where coke is expensive and where power is cheap, the electric reduction of iron ore is carried out commercially, as at Trolhätten and Domnarfvet, Sweden, and at Héroult, California. The furnaces have a shaft resembling a blast furnace, with a crucible at the base into which the electrodes project in a slanting position. (*For detailed accounts, see volumes of the Met. and Chem. Eng.*) More recently it has been found better to have the electrodes vertical. (*Taussig, VIII Int. Congress App. Chem., Vol. 21, p. 105, 1912.*)

Electric tin smelting has been tried on a commercial scale with apparent success (*Met. and Chem. Eng., Vol. 9, p. 453, 1911*) as well as the smelting of copper and nickel (*Met. and Chem. Eng., Vol. 11, p. 22, 1913*), and zinc (*see volumes of the Met. and Chem. Eng.*). The use of electric furnaces in steel refining and in the production of ferro-alloys is much more extensive for in this case power does not need to be so cheap as in the reduction of iron ore.

Steel Refining. — Usually the steel which is refined in electric furnaces is taken from Bessemer converters or open-hearth furnaces and poured directly into the electric furnace. The advantages of the electric furnaces are (*Walker, Met. and Chem. Eng., Vol. 10, p. 371, 1912*):

1. Complete removal of oxygen,
2. Absence of oxides caused by additions, such as silicon manganese,
3. Production of electric steel ingots of 8 tons and less that are practically free from segregation,
4. Reduction of sulphur to 0.005 per cent if desired,
5. Reduction of phosphorus to 0.005 per cent as in the basic open-hearth process, but with complete removal of oxygen.

The number of electric steel refining furnaces in use was increased greatly by the war.

The recent progress in electric steel refining consists in an improvement in existing methods and in a reduction of costs. While in 1911 it was considered good practice to melt and refine steel scrap in six hours at 750 kilowatt hours per ton, the same operation is now carried out in four hours with 600 kilowatt hours. (*Héroult, VIII Int. Cong. of App. Chem., Vol. 21, p. 59, 1912.*)

Some of the types of furnaces used in steel refining are the following:

Stassano Steel Furnace (Fig. 7). — This furnace consists of a closed chamber with three electrodes connected to a three-phase system above the slag. The furnace rotates so as to stir all of the metal. This furnace is used in Italy, Odessa, and Newcastle-on-Tyne (*Met. and Chem. Eng., Vol. 10, p. 66, 1912.*)

Héroult Steel Furnace (Fig. 8). — This furnace consists of a crucible lined with refractory material. Carbon electrodes project into it through the roof. An arc is formed where the current passes from each electrode into the slag. The power is regulated by an electrical automatic regulator which moves the electrodes up and down as required.

Girod Steel Furnace. — This furnace consists of a crucible with several soft-steel rods projecting through the base. These form one electrode; the other

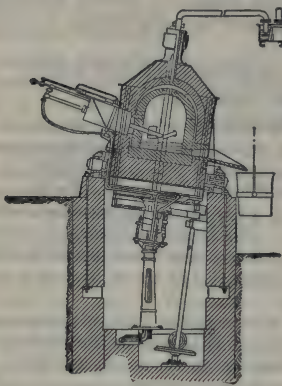


Fig. 7.

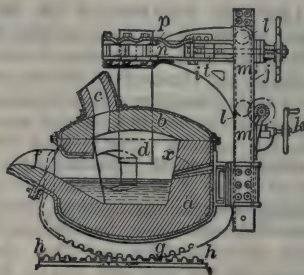


Fig. 8.

is one or more carbon rods suspended from above. The steel electrodes of course melt several inches below the surface of the refractory lining of the crucible. The furnaces are sold by C. W. Leavitt and Co., 30 Church St., New York, from whom the following data have been obtained.

These furnaces require from 65 to 70 volts at frequencies from 25 to 50 cycles. The power factor is about 80 per cent and the duration of heat when the metal is charged in the melted state is 1½ to 2½ hours. The number of electrodes are from 1 to 3, according to the capacity of the furnace. For a furnace of about 12 tons capacity, the maximum power required is 1200 kilowatts measured at the terminals of the electric generator; the energy consumption when the metal is charged cold is 900 kilowatt hours per ton of steel; with a melted charge, from 150 to 250 kilowatt hours. The electrodes are so designed that the current does not exceed 5 amperes per square inch of cross-section. The number of consecutive heats possible without repairs is:

	Linings	Cover	Electrodes
With cold charge....................	30 to 40	20 to 30	10
With melted charge..................	60 to 90	40 to 50	20

To handle the furnace 6 persons are necessary: a melter, 2 assistant melters, 2 workmen and a boy.

Other Arc Furnaces more or less similar to the Héroult furnace are the Keller furnace (*Trans. Am. Electroch. Soc., Vol. 15, p. 96, 1909*), the Nathusius furnace (*Met. and Chem. Eng., Vol. 10, p. 227, 1912*) and the Snyder furnace (*Trans. Am. Electroch. Soc., Vol. 28, pp. 221–230, 1915*).

Induction Furnaces. — Induction furnaces are transformers in which a melted ring of steel is the secondary. The Kjellin furnace, Fig. 9, consists of a single deep ring of metal. It has only a small area of contact between the metal and slag, and the slag is not easily heated. The use of this furnace is therefore

restricted in its application (*Kjellin, Trans. Am. Electroch. Soc., Vol. 15, p. 175, 1909*).

A modified form of induction furnace is the Röchling-Rodenhauser furnace. This furnace has two annular rings combined in the form of a figure 8. The central portion carries the currents induced in both circuits, as well as a current from electrodes supplied by extra secondary coils. This current passes through the lining of the furnace, which has sufficient conductivity when hot.

These furnaces, as well as another modification, known as the Frick furnace, can be obtained from Siemens and Halske, represented in this country by Dr. G. K. Frank, 80 West St., New York. Another design of induction furnace is due to Hiorth (*Trans. Am. Electroch. Soc., Vol. 20, p. 293, 1911*).

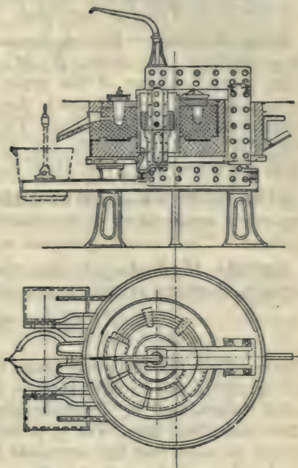


Fig. 9.

Pinch Effect in Induction Furnaces. — The magnitude of the current which can be sent through a trough of melted metal is limited by the so-called pinch effect (*Trans. Am. Electroch. Soc., Vol. 11, p. 329, 1907*). On account of the attraction of the current elements for each other, a compressing force is exerted on the metal which causes a decrease in the cross-section at some point. If the current is too great the metal may be entirely separated and the circuit broken.

Northrup-Ajax Furnace. — This is an induction furnace designed for operation at high frequency, 20,000 cycles per second or more. No iron core is required, the primary winding merely surrounding the crucible which contains the substance to be melted. The crucible itself or the metal which it contains serves as the secondary. The high frequency is obtained from the discharge of a condenser through the primary, the capacity of the condenser and the inductance of the primary coil being so chosen as to give the desired frequency. (*See Transient Electric Phenomena.*) The condenser is charged through a step-up transformer to about 8000 volts and is short-circuited through the primary coil by a spark-gap. The condenser is made up in units of 0.07 microfarad capacity, each unit being 12 + 16 × 13 inches. One unit is required for each 1.5 kilowatts in-put to the furnace. The efficiency of this furnace is from 50 to 60 per cent. Temperatures exceeding 1600° C. are readily obtained. This furnace is manufactured by the Pyroelectric Instrument Co., Trenton, N. J. (*Chem. and Meth. Eng., Vol. 19, p. 155, 1918.*)

Ferro-Alloys. — Ferro-alloys were originally made from iron ore, the oxide of other metal, carbon, and flux, but on account of impurities, scrap iron and steel shavings are now used in place of iron ore (*Met. and Chem. Eng., Vol. 8, p. 133, 1910*). Arc and resistance furnaces similar to those used for steel refining are used. For detailed information see Keeney, *Bull. Am. Inst. Mining Eng. No. 140, pp. 1321–1373, 1918.*

Ferro-Silicon is the most important of the ferro-alloys. It is made from iron, quartzite, and carbon. At the Keller-Leleux works at Livet it has been found practicable to turn out 20 tons of 30 per cent ferro-silicon with 4000 horse-

power during a day. (*See preceding reference.*) For an account of the uses of the other ferro-alloys, see *Electrochem. Ind., Vol. 1, p. 583, 1903; Electrochem. and Met. Ind., Vol. 4, p. 247, 1906.*

Brass Melting. — Electric furnaces are now used to a considerable extent for brass melting. (*St. John., H. M., Electric Brass Melting, Elec. J.*, 16, 373, *Sept.*, 1919.) Some of the types of furnaces used are the Booth rotating furnace (*Trans. Am. Electroch. Soc., Vol. 33, p. 247, 1918*); the Rennerfelt furnace (*Met. and Chem. Eng., Vol. 12, p. 275, 1914; Trans. Am. Electroch. Soc., Vol. 29, p. 497, 1916; Vol. 31, p. 87, 1917*); and the Helberger furnace (*Met. and Chem. Eng., Vol. 12, p. 644, 1914*).

FIXATION OF ATMOSPHERIC NITROGEN. — One of the most important applications of the electric furnace is the fixation of atmospheric nitrogen. The various processes employed are described below.

Carbide Method. — In this method calcium carbide is heated in pure nitrogen, forming calcium cyanamide according to the reversible reaction

$$CaC_2 + N_2 \rightleftarrows CaCN_2 + C.$$

The carbide is heated in iron drums by a thin carbon conductor running through the center of the drum. Heat is evolved by the reaction. The product is called "nitro-lime" or "lime-nitrogen," and contains 12 to 15 per cent nitrogen. (*Met. and Chem. Eng., Vol. 5, p. 78, 1907.*) It is used directly as a fertilizer, or may be converted into ammonia by superheated steam. The yield in nitrogen by the carbide method is about 51.6 grams per kilowatt hour, including the manufacture of the carbide.

Direct-Oxidation Method. — In this method the nitrogen and oxygen in air are caused to combine in a high-voltage arc according to the reversible reaction

$$N_2 + O_2 \rightleftarrows 2\,NO.$$

On cooling the NO is further oxidized to nitric dioxide. The nitric dioxide on treatment with water gives nitric and nitrous acids:

$$2\,NO_2 + H_2O = HNO_3 + HNO_2.$$

Only about 1 to 3 per cent of the air treated is oxidized. The yield in the Birkeland-Eyde furnace is 12.7 grams of nitrogen fixed per kilowatt hour. (*Trans. Faraday Soc., Vol. 2, p. 98, 1906.*) Furnaces of three different designs are now in operation for oxidizing nitrogen: that of Birkeland and Eyde, that of Schönherr, and that of H. and G. Pauling.

Direct Synthesis of Ammonia. — In this method, due to Haber, ammonia is formed directly from a mixture of nitrogen and hydrogen by passing over a catalyzer between 500° C. and 700° C. at 200 atmospheres, according to the reversible reaction

$$\tfrac{1}{2}\,N_2 + \tfrac{3}{2}\,H_2 \rightleftharpoons NH_3.$$

(*Zeit. f. Elektroch., Vol. 16, p. 244, 1910; Vol. 19, p. 53, 1913.*) The efficiency of this process has not been made public.

Bucher Process. — This process was discovered in 1846 by Lewis Thomson (Bucher, *Am. Inst. Chem. Eng., Vol. 9, p. 335, 1916*) and is represented by the following reaction:

$$Na_2CO_3 + 5C + N_2 = 2NaCN + 3CO$$

which requires metallic iron as catalyzer. The temperature is between 900° and 1050° C. Many attempts have been made to use this process, but at present

there is no plant in operation. A plant was built at Saltville, Va., to make cyanide for war requirements. The furnaces were 8-inch iron tubes heated externally by hot gases. The reacting mixture is finely ground, briquetted hot with water, dried, and then treated with nitrogen. See also Ferguson and Manning, *J. Ind. and Eng. Chem., Vol. 11, p. 946, 1919;* Posnjak and Merwin, *J. Washington Acad. of Sci. Vol. 9, p. 28, 1919.*

Serpek Process. — In this process aluminum nitride is made by heating aluminum oxide, carbon, and nitrogen together. On heating the aluminum nitride with water, ammonia and aluminum hydrate are formed. The product obtained from the furnace is said to contain 20 to 24 per cent of nitrogen, and the power required per unit of nitrogen is said to be only one-half of that used in the calcium-carbide method. (*Bull. de la Soc. ind. de Mulhouse, Vol. 79, p., 39. 1909. U. S. Pat. 996,032, 1911.*)

BIBLIOGRAPHY. — Borchers, *Electric Smelting and Refining;* Kershaw, *Electrometallurgy;* Macmillan and Cooper, *Electrometallurgy;* Alfred Stansfield, *The Electric Furnace;* Rodenhauser, Schoenawa and Vom Baur, *Electric Furnaces in the Iron and Steel Industry;* Department of the Interior Bulletins: No. 77, Lyon, Keeney and Cullen, *The Electric Furnace in Metallurgical Work*, 1916; No. 67, Lyon and Keeney, *Electric Furnaces for Making Iron and Steel*, 1916; No. 81, Lyon and Keeney, *The Smelting of Copper Ores in the Electric Furnace*, 1915. See also references in text above, and bibliography in article on *Electrochemical Processes, Industrial.*

FUSES. — (*See also Circuit Breakers; Wiring of Buildings.*) Metal strips or wires which open electric circuits by melting or fusing when the current reaches a predetermined value are called fuses. They were used in the earliest electric plants to furnish automatic protection for feeder and generator circuits. Their use is now confined largely to low-voltage distribution circuits and to the protection of small pieces of apparatus such as motors and small-capacity transformers.

All fuses have an inherent time-element feature due to the fact that the current must heat the fuse metal up to its melting temperature. This time lag varies with the size and type of fuse, the large ones in general taking longer to reach their fusing temperature than the smaller sizes, due to their thermal capacities. There are three designs of fuses in general use, namely, the open or link, the expulsion, and the enclosed or cartridge types. The choice of type depends upon the service for which the fuse is intended.

RATING OF A FUSE.—The rating of a fuse depends somewhat on its type and general design. Fuses will, as a rule, carry their normal rated current indefinitely but will blow at a certain overload varying from 10 per cent in an enclosed fuse to about 80 per cent in an open fuse if the overload continues a sufficient length of time. For greater overloads the length of time required for blowing diminishes rapidly.

OPEN OR LINK FUSES were the original type of automatic protection. The early ones were small copper wires and had the drawback of forming copper globules and of possessing a high fusion temperature. In order to reduce the temperature of the molten metal there were employed alloys of low fusing points made of lead, tin, or other metals. These fuses were soft and were easily damaged when tightening up the contact nuts. The next step was to use alloy fuses with copper tips, and these are still used to some extent.

As the price of aluminum was reduced this material was used largely for fuses as it has a high conductivity (thus reducing the amount of metal to be fused), a fairly low melting point and almost complete vaporization of the metal fused. By using wide strips of aluminum cut to form two or more bridges, fairly reliable open fuses can be made up to 1200 amperes capacity.

Any metal strip which is exposed to drafts is apt to be very erratic in its behavior as a fuse. To protect link fuses from drafts, and also to remove the danger from molten metal which may be thrown at the time of blowing, it is desirable to install them in porcelain fuse boxes.

EXPULSION FUSES consist essentially of open fuse wires or strips placed in a holder, so designed that the expulsion of the gases formed by the melting of the fuse blows out the arc. The earliest designs of this type comprised a removable fuse holder of lignum vitæ or similar tough, close-grained wood, equipped with terminals which fitted into suitable blocks. Later types have the fuse placed in a fiber tube and arranged to blow out through one end like a bomb.

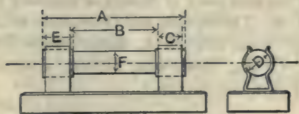
The fuses used for the protection of distributing transformers mounted on poles or houses are usually of the expulsion type. For such outdoor service a combination fuse and disconnecting switch is frequently used. Modifications of the expulsion-type fuse were formerly used to advantage in railway service, but in recent car equipments circuit breakers have practically replaced fuses and cut-outs.

INCLOSED OR CARTRIDGE FUSE.—This type of fuse consists essentially of a fusible wire, strip or sets of wires and strips inclosed within a tube or cartridge usually of fiber. The tube is filled with a material to exclude the

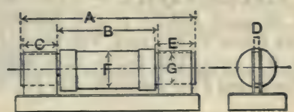
air and to facilitate the opening of the circuit when the fuse blows by absorbing the gases formed and chilling out the arc. Suitable terminals are provided so that the fuse may be mounted in a fuse block.

National Electrical Code Fuses. — When inclosed fuses were first put on the market each manufacturer developed his own spacings and designs of terminals, so that there was no uniformity and the fuse of one manufacturer could not be used in the fuse holder of another manufacturer. To avoid this confusion the representatives of the fuse builders and the National Board of Fire Underwriters finally adopted the standard dimensions and types of contacts given in the accompanying table. Up to 60 amperes ferrule type contacts (i.e., cylindrical metal ends) are used and from 61 to 600 amperes knife-blade contacts are employed. One set of dimensions are used for fuses up to 250 volts and another for fuses up to 600 volts. Fuses that correspond to the accepted dimensions and that meet other requirements agreed on are known as National Electrical Code (N. E. C.) fuses and are interchangeable. Fuses are made for higher voltages than 600 volts and larger currents then 600 amperes, but they have not been accepted by the National Board of Fire Underwriters.

DIMENSIONS OF N. E. C. CARTRIDGE FUSES



Style of Terminal for Cartridge Fuses
0-60 Amperes



Style of Terminal for Cartridge Fuses
61-600 Amperes

Voltage	Rated capacity	Form	A Length over terminals	B Distance between contact clips	C Width of contact clips	D Diameter of ferrules or thickness of terminal blades	E Min. length of ferrules or of terminal blades outside of tube	F Diameter of tube	G Width of terminal blades
	Amp.		Inches	Inches	Inches	Inches	Inches	Inches	Inches
0–250	0–30	Form 1	2	1	½	9/16	½	½	...
	31–60	Form 1	3	1¾	⅝	13/16	⅝	¾	...
	61–100	Form 2	5⅞	4	⅞	⅛	1	1	¾
	101–200	Form 2	7⅛	4½	1¼	3/16	1⅜	1½	1⅛
	201–400	Form 2	8⅝	5	1¾	¼	1⅞	2	1⅝
	401–600	Form 2	10⅜	6	2⅛	¼	2¼	2½	2
251–600	0–30	Form 1	5	4	½	13/16	½	¾	...
	31–60	Form 1	5½	4¼	⅝	11/16	⅝	1	...
	61–100	Form 2	7⅞	6	⅞	⅛	1	1¼	¾
	101–200	Form 2	9⅝	7	1¼	3/16	1⅜	1¾	1⅛
	201–400	Form 2	11⅝	8	1¾	¼	1⅞	2½	1⅝
	401–600	Form 2	13⅜	9	2⅛	¼	2¼	3	2

Cost of N. E. C. Fuses. — In moderate quantities of standard packages the price of 30-ampere 250-volt fuses is about 21 cents each, and of 400-ampere 600-volt fuses about $4.65 each. The others fall between these limits.

Plug Fuses. — A plug fuse is a fuse mounted in a plug of the standard Edison-lamp base dimensions. They are in common use on lighting and power circuits up to 30 amperes capacity. The outer face of the plug usually has a mica covering which is discolored by the blowing of the fuse. A brass cap is also sometimes used as a cover.

RENEWABLE FUSES. — The standard enclosed fuse when it has once blown has usually to be returned to its manufacturer for refilling. Various fuses have been designed that fit in the standard N. E. C. fuse holders and arranged so that the fuse elements can be readily renewed. These have now been accepted by the National Board of Fire Underwriters.

For high voltage work fuses have been made using carbon tetrachloride, or some similar liquid to extinguish the arc when the fuse blows.

SPECIFICATIONS FOR FUSES. — The following memoranda are intended to assist in writing specifications. See also articles on *Specifications*.

Open or inclosed. Wire or ribbon. Current to blow fuse in a specified time To blow without violence at a stated current. Voltage of circuit above ground.

BIBLIOGRAPHY. — Arthur, W., *Characteristics of Standard Enclosed Fuses*, E. W. 1917, Vol. 69, p. 456; Dillard, A. E., *The Design and Operation of Horn-Gap Fuses*, E. W., 1915, Vol. 66, p. 1305; Somerville, A. A., *Electric Fuse Testing*, E. W., 1913, Vol. 61, p. 144; Squier, C. W., *Equipment Defects, Fuses, Fuse Blocks and Fuse Boxes*, E. Ry. Jr., 1914, Vol. 44, p. 219.

GAGES, SHEET-METAL. — (*See also Copper; Gages, Wire; Iron, Wrought; Steel.*) There are two principal gages for sheet iron and steel used in the United States, one established by Act of Congress in 1893 and the other recommended by a joint committee of the American Society of Mechanical Engineers and the American Railway Master Mechanics Association.

U. S. Standard Gage. — The former gage, which is known as the U. S. Standard Gage, is based upon the fact that a cubic foot of iron weighs 480 pounds. The scale of this gage has been arranged so that each gage number represents a certain number of ounces in weight, or an equal number of 640ths of an inch in thickness. This gage is used for determining duties and taxes levied on sheet and plate iron and steel, and in its application a variation of 2½ per cent either way is allowable.

Decimal Gage. — The latter gage, which is known as the Decimal Gage, is not based upon the weight but upon the thickness of the metal. Each number of the gage is the number of thousandths of an inch of thickness. This gage has been adopted recently by the Association of American Steel Manufacturers, the American Railway Master Mechanics' Association, and by most of the principal railroads of the United States, Canada and Mexico. The decimal system of gaging was recommended by the American Institute of Mining Engineers in 1877 and by the American Society of Mechanical Engineers in 1895.

Brown and Sharpe Gage. — Copper and brass plates are rated by the Brown and Sharpe gage. The plate thicknesses on this gage correspond to the diameters on the American Wire Gage. (*See Gages, Wire.*)

The following tables give the dimensions and weights of sheet metal according to gages described above.

U. S. STANDARD GAGE FOR SHEET AND PLATE, IRON AND STEEL

No. of gage	Approximate thickness in decimal parts of an inch	Approximate thickness in millimeters	Weight per square foot in pounds avoirdupois	Weight per square meter in kilograms
0000000	0.5000	12.7000	20.00	97.65
000000	0.4687	11.9062	18.75	91.55
00000	0.4375	11.1125	17.50	85.44
0000	0.4062	10.3187	16.25	79.33
000	0.3750	9.5250	15.00	73.24
00	0.3437	8.7312	13.75	67.13
0	0.3125	7.9375	12.50	61.03
1	0.2812	7.1437	11.25	54.93
2	0.2656	6.7469	10.62	51.88
3	0.2500	6.3500	10.00	48.82
4	0.2344	5.9531	9.375	45.77
5	0.2187	5.5562	8.750	42.72
6	0.2031	5.1594	8.125	39.67
7	0.1875	4.7625	7.500	36.62
8	0.1719	4.3656	6.875	33.57
9	0.1562	3.9687	6.250	30.52
10	0.1406	3.5719	5.625	27.46
11	0.1250	3.1750	5.000	24.41
12	0.1094	2.7781	4.375	21.36
13	0.0937	2.3812	3.750	18.31
14	0.07812	1.9844	3.125	15.26
15	0.07031	1.7859	2.812	13.73
16	0.06250	1.5875	2.500	12.21
17	0.05625	1.4287	2.250	10.99
18	0.05000	1.2700	2.000	9.765
19	0.04375	1.1112	1.750	8.544
20	0.03750	0.9525	1.500	7.324
21	0.03437	0.8731	1.375	6.713
22	0.03125	0.7937	1.250	6.103
23	0.02812	0.7144	1.125	5.490
24	0.02500	0.6350	1.000	4.882
25	0.02187	0.5556	0.875	4.272
26	0.01875	0.4762	0.750	3.662
27	0.01719	0.4366	0.687	3.357
28	0.01562	0.3969	0.625	3.052
29	0.01406	0.3572	0.5625	2.746
30	0.01250	0.3175	0.5000	2.441
31	0.01094	0.2778	0.4375	2.136
32	0.01016	0.2580	0.4062	1.983
33	0.009375	0.2381	0.3750	1.831
34	0.008594	0.2183	0.3437	1.678
36	0.007031	0.1786	0.2812	1.373
38	0.006250	0.1587	0.2500	1.221

STANDARD DECIMAL GAGE FOR SHEET AND PLATE, IRON AND STEEL

Standard decimai gage in inches	Approximate thickness in millimeters	Weights per square foot in pounds avoirdupois	
		Iron, Basis: 480 pounds per cubic foot	Steel, Basis: 489.6 pounds per cubic foot
0.002	0.0508	0.08	0.0816
0.004	0.1016	0.16	0.1632
0.006	0.1524	0.24	0.2448
0.008	0.2032	0.32	0.3264
0.010	0.2540	0.40	0.4080
0.012	0.3048	0.48	0.4896
0.014	0.3556	0.56	0.5712
0.016	0.4064	0.64	0.6528
0.018	0.4572	0.72	0.7344
0.020	0.5080	0.80	0.8160
0.022	0.5588	0.88	0.8976
0.025	0.6350	1.00	1.0200
0.028	0.7112	1.12	1.1424
0.032	0.8128	1.28	1.3056
0.036	0.9144	1.44	1.4688
0.040	1.0160	1.60	1.6320
0.045	1.1430	1.80	1.8360
0.050	1.2700	2.00	2.0400
0.055	1.3970	2.20	2.2440
0.060	1.5240	2.40	2.4480
0.065	1.6510	2.60	2.6520
0.070	1.7780	2.80	2.8560
0.075	1.9050	3.00	3.0600
0.080	2.0320	3.20	3.2640
0.085	2.1590	3.40	3.4680
0.090	2.2860	3.60	3.6720
0.095	2.4130	3.80	3.8760
0.100	2.5400	4.00	4.0800
0.110	2.7940	4.40	4.4880
0.125	3.1750	5.00	5.1000
0.135	3.4290	5.40	5.5080
0.150	3.8100	6.00	6.1200
0.165	4.1910	6.60	6.7320
0.180	4.5720	7.20	7.3440
0.200	5.0800	8.00	8.1600
0.220	5.5880	8.80	8.9760
0.240	6.0960	9.60	9.7920
0.250	6.3500	10.00	10.2000

BROWN AND SHARPE GAGE FOR COPPER AND BRASS PLATES

B. & S. Gage No.	Thickness, inches	Weight, pounds per sq. ft.	
		Copper	Brass
0000	0.4600	21.30	20.20
000	0.4096	18.90	18.20
00	0.3648	16.90	16.00
0	0.3249	15.00	14.30
1	0.2893	13.40	12.70
2	0.2576	11.90	11.30
3	0.2294	10.60	10.10
4	0.2043	9.45	8.96
5	0.1819	8.41	7.98
6	0.1620	7.49	7.11
7	0.1443	6.67	6.33
8	0.1285	5.94	5.64
9	0.1144	5.29	5.02
10	0.1019	4.71	4.47
11	0.09074	4.20	3.98
12	0.08081	3.74	3.54
13	0.07196	3.33	3.16
14	0.06408	2.96	2.81
15	0.05707	2.64	2.50
16	0.05082	2.35	2.23
17	0.04526	2.09	1.99
18	0.04030	1.86	1.77
19	0.03589	1.66	1.57
20	0.03196	1.48	1.40
21	0.02846	1.32	1.25
22	0.02535	1.17	1.11
23	0.02257	1.04	0.99
24	0.02010	0.93	0.88
25	0.01790	0.83	0.79
26	0.05194	0.74	0.70
27	0.01420	0.66	0.62
28	0.02164	0.58	0.55
29	0.01126	0.52	0.49
30	0.01003	0.46	0.44
31	0.008928	0.41	0.39
32	0.007950	0.37	0.35
33	0.007080	0.33	0.31
34	0.006304	0.29	0.28
36	0.005000	0.23	0.22
38	0.003965	0.18	0.17
40	0.003145	0.15	0.14

* The specific gravity of copper plate is taken as 8.89 and of brass plate as 8.45.

BIBLIOGRAPHY.—*Report of Committee on Standard Thickness Gage for Metals*, Trans. A.S.M.E., 1895, Vol. 16, p. 641; *Report of Committee on Standard Gages*, Proc. Am. Ry. Master Mech. Assn., 1895, p. 149.

GAGES, WIRE. — (*See also Gages, Sheet Metal.*) The sizes of wires having a diameter less than ½ inch are usually stated in terms of certain arbitrary scales called "gages." The size or gage number of a solid wire refers to the cross-section of the wire perpendicular to its length; the size or gage number of a stranded wire refers to the total cross-section of the constituent wires, irrespective of the pitch of the spiraling. Larger wires are usually described in terms of their area expressed in circular mils. A circular mil is the area of a circle 1 mil in diameter, and the area of any circle in circular mils is equal to the square of its diameter in mils. See *Units and Conversion Factors*.

There are a number of wire gages in use, the principal ones being the following.

AMERICAN OR BROWN AND SHARPE WIRE GAGE. — This gage is the one commonly used in the United States for copper, aluminum and resistance wires. The gage is designated by either of the abbreviations A. W. G. or B. & S.

Basis of the A. W. G. or B. & S. Gage. — The diameter of wires having successive numbers on this gage are in the ratio of $\sqrt[39]{92}$ (= 1.1229 approx.) to 1, and the No. 36 wire has a diameter of 5 mils. No. 35 A. W. G., therefore, has a diameter of $5 \times 1.1229 = 5.61$ mils and so on until No. 0000 is reached, having a diameter of 460 mils.

The ratio $\sqrt[39]{92}$ is approximately equal to $\sqrt[6]{2}$, which is 1.1225. This circumstance makes it possible to have a group of wires of regular gage size with an aggregate area approximately equal to that of another regular gage size.

The following approximate relations are also useful:

An increase of 1 in the number increases the resistance 25 per cent.
An increase of 2 in the number increases the resistance 60 per cent.
An increase of 3 in the number increases the resistance 100 per cent.
An increase of 10 in the number increases the resistance 10 times.

A No. 10 A. W. G. copper wire has the following approximate characteristics:

Ohms per 1000 feet	1
Circular mils area	10,000
Weight, pounds per 1000 feet	32

A No. 10 A. W. G. aluminum wire has the following approximate characteristics:

Ohms per 1000 feet	1.6
Circular mils area	10,000
Weight, pounds per 1000 feet	9.5

Remembering these rules it is easy to find the approximate size, resistance, area, or weight of any size wire. For example, a No. 12 A.W. G. copper wire has a resistance of 1 plus 60 per cent = 1.6 ohms per 1000 feet approximately. Its area, being inversely as its resistance, is $\frac{10,000}{1.6} = 6250$ circular mils; its diameter is therefore $\sqrt{6250} = 79$ mils and its weight, $\frac{32}{1.6} = 20$ pounds per 1000 feet.

U. S. STEEL WIRE GAGE. — This gage, known also as the "Washburn and Moen," "Roebling," "American Steel and Wire Co.'s" gage, is the one usually employed in the United States for steel and iron wire. It is frequently abbreviated "S. W. G.," but to avoid confusion with the British Standard Wire Gage (*see below*) it should be abbreviated "Stl. W. G." or "A. (steel) W.G."

BIRMINGHAM (OR STUBS') WIRE GAGE. — This gage is still used in the United States for some purposes, e.g., to designate the size of brass wire, and is also employed to a limited extent in Great Britain. It is usually abbreviated "B. W. G." It is sometimes referred to as the "Stubs'" gage, but it should not be confused with the Stubs' Steel Wire Gage.

BRITISH STANDARD WIRE GAGE. — This gage, usually called simply the "Standard Wire Gage," and abbreviated "S. W. G.," is also known as the "New British Standard" (abbreviated "N. B. S."), the English Legal Standard, or the Imperial Wire Gage, and is the legal standard of Great Britain for all wires, as fixed by order in Council, August 23, 1883. It was constructed by modifying the Birmingham Wire Gage, so that the differences between successive diameters were the same for short ranges, i.e., so that a graph representing the diameters consists of a series of a few straight lines.

EDISON WIRE GAGE. — The size of a wire on this gage is equal to its cross-sectional area in circular mils divided by 1000. For example, a solid wire 0.2 inch in diameter has the number $(200)^2/1000 = 40$. This gage is now rarely used.

OTHER GAGES. — In addition wire sizes are sometimes specified in terms of the "Old English Wire Gage," known also as the "London Gage," and the "Stubs' Steel Wire Gage."

COMPARISON OF WIRE GAGES. — A comparison of the different gages, in terms of the diameters (in mils or thousandths of an inch) of solid wires corresponding to the various numbers, is given below. The cross-section in circular mils is the square of the diameter in mils.

COMPARISON OF WIRE GAGES

Diameters in Mils

(*Bureau of Standards, Circular No. 31*)

Gage No.	American wire gage (B. & S.)	Steel wire gage	Birmingham wire gage (Stubs')	Old English wire gage (London)	Stubs' steel wire gage	(British) Standard wire gage	Gage No.
7–0	.....	490.0	...	......	...	500	7–0
6–0	.....	461.5	....	......	...	464	6–0
5–0	.....	430.5	...	......	...	432	5–0
4–0	460	393.8	454	454	...	400	4–0
3–0	410	362.5	425	425	...	372	3–0
2–0	365	331.0	380	380	...	348	2–0
0	325	306.5	340	340	...	324	0
1	289	283.0	300	300	227	300	1
2	258	262.5	284	284	219	276	2
3	229	243.7	259	259	212	252	3
4	204	225.3	238	238	207	232	4
5	182	207.0	220	220	204	212	5
6	162	192.0	203	203	201	192	6
7	144	177.0	180	180	199	176	7
8	128	162.0	165	165	197	160	8
9	114	148.3	148	148	194	144	9

COMPARISON OF WIRE GAGES — *Continued*

Diameter in Mils

(*Bureau of Standards, Circular No. 31*)

Gage No.	American wire gage (B. & S.)	Steel wire gage	Birmingham wire gage (Stubs')	Old English wire gage (London)	Stubs' steel wire gage	(British) Standard wire gage	Gage No.
10	102	135.0	134	134	191	128	10
11	91	120.5	120	120	188	116	11
12	81	105.5	109	109	185	104	12
13	72	91.5	95	95	182	92	13
14	64	80.0	83	83	180	80	14
15	57	72.0	72	72	178	72	15
16	51	62.5	65	65	175	64	16
17	45	54.0	58	58	172	56	17
18	40	47.5	49	49	168	48	18
19	36	41.0	42	40	164	40	19
20	32	34.8	35	35	161	36	20
21	28.5	31.7	32	31.5	157	32	21
22	25.3	28.6	28	29.5	155	28	22
23	22.6	25.8	25	27.0	153	24	23
24	20.1	23.0	22	25.0	151	22	24
25	17.9	20.4	20	23.0	148	20	25
26	15.9	18.1	18	20.5	146	18	26
27	14.2	17.3	16	18.75	143	16.4	27
28	12.6	16.2	14	16.50	139	14.8	28
29	11.3	15.0	13	15.50	134	13.6	29
30	10.0	14.0	12	13.75	127	12.4	30
31	8.9	13.2	10	12.25	120	11.6	31
32	8.0	12.8	9	11.25	115	10.8	32
33	7.1	11.8	8	10.25	112	10.0	33
34	6.3	10.4	7	9.50	110	9.2	34
35	5.6	9.5	5	9.00	108	8.4	35
36	5.0	9.0	4	7.50	106	7.6	36
37	4.5	8.5	...	6.50	103	6.8	37
38	4.0	8.0	...	5.75	101	6.0	38
39	3.5	7.5	...	5.00	99	5.2	39
40	3.1	7.0	...	4.50	97	4.8	40
41	.....	6.6	...	......	95	4.4	41
42	.....	6.2	...	......	92	4.0	42
43	.....	6.0	...	......	88	3.6	43
44	.....	5.8	...	......	85	3.2	44
45	.....	5.5	...	......	81	2.8	45
46	.....	5.2	...	......	79	2.4	46
47	.....	5.0	...	......	77	2.0	47
48	.....	4.8	...	......	75	1.6	48
49	.....	4.6	...	......	72	1.2	49
50	.....	4.4	...	......	69	1.0	50

GALVANIZING FOR IRON OR STEEL, Specification for Acceptance Test. — These specifications give in detail the test to be applied to galvanized material, as recommended by the Electric Railway Engineering Association, the National Electric Light Association, etc. All specimens shall be capable of withstanding these tests.

(a) **Coating.** — The galvanizing shall consist of a continuous coating of pure zinc of uniform thickness, and so applied that it adheres firmly to the surface of the iron or steel. The finished product shall be smooth.

(b) **Cleaning.** — The samples shall be cleaned before testing, first with carbona, benzine or turpentine, and cotton waste (not with a brush), and then thoroughly rinsed in clean water and wiped dry with clean cotton waste.

The sample shall be clean and dry before each immersion in the solution.

(c) **Solution.** — The standard solution of copper sulphate shall consist of commercial copper sulphate crystals dissolved in cold water, about in the proportion of 36 parts, by weight, of crystals to 100 parts, by weight, of water. The solution shall be neutralized by the addition of an excess of chemically pure cupric oxide (CuO). The presence of an excess of cupric oxide will be shown by the sediment of this reagent at the bottom of the containing vessel.

The neutralized solution shall be filtered before using by passing through filter paper. The filtered solution shall have a specific gravity of 1.186 at 65° F. (reading the scale at the level of the solution) at the beginning of each test. In case the filtered solution is high in specific gravity, clean water shall be added to reduce the specific gravity to 1.186 at 65° F. In case the filtered solution is low in specific gravity, filtered solution of a higher specific gravity shall be added to make the specific gravity 1.186 at 65° F.

As soon as the stronger solution is taken from the vessel containing the unfiltered neutralized stock solution, additional crystals and water must be added to the stock solution. An excess of cupric oxide shall always be kept in the unfiltered stock solution.

(d) **Quantity of Solution.** — Wire samples shall be tested in a glass jar of at least two (2) inches inside diameter. The jar without the wire samples shall be filled with standard solution to a depth of at least four (4) inches. Hardware samples shall be tested in a glass or earthenware jar containing at least one-half (½) pint of standard solution for each hardware sample. Solution shall not be used for more than one series of four immersions.

(e) **Samples.** — Not more than seven wires shall be simultaneously immersed, and not more than one sample of galvanized material other than wire shall be immersed in the specified quantity of solution.

The samples shall not be grouped or twisted together, but shall be well separated so as to permit the action of the solution to be uniform upon all immersed portions of the samples.

(f) **Test.** — Clean and dry samples shall be immersed in the required quantity of standard solution in accordance with the following cycle of immersions.

The temperature of the solution shall be maintained between 62° and 68° F. at all times during the following test.

First. Immerse for one minute, wash and wipe dry.
Second. Immerse for one minute, wash and wipe dry.
Third. Immerse for one minute, wash and wipe dry.
Fourth. Immerse for one minute, wash and wipe dry.

After each immersion the samples shall be immediately washed in clean water having a temperature between 62° and 68° F., and wiped dry with cotton waste.

In the case of No. 14 galvanized iron or steel wire, the time of the fourth immersion shall be reduced to one-half minute.

(g) **Rejection.** — If after the test described in section "f" there should be a bright metallic copper deposit upon the samples, the lot represented by the samples shall be rejected.

Copper deposits on zinc or within one inch of the cut end shall not be considered causes for rejection.

In the case of a failure of only one wire in a group of seven wires immersed together, or if there is a reasonable doubt as to the copper deposit, two check tests shall be made on these seven wires and the lot reported in accordance with the majority of the sets of tests.

GALVANOMETERS. — (*See also Ammeters; Electrodynamometers; Fluxmeter; Voltmeters.*) The essential parts of a galvanometer are a permanent magnet (electromagnet in Einthoven String Galvanometer, see below) and a coil or wire designed to carry an electric current, the magnet and coil being so mounted that the passage of the current through the coil causes a deflection of either the coil or the magnet, as a result of the mechanical force with which the current and magnet act on each other. See *Electricity and Magnetism, Principles of.* Ordinary direct-current ammeters and voltmeters are operated on the same principle, but the term galvanometer is usually reserved for instruments designed for measuring relatively small currents, the deflection being read either by means of a telescope and scale or by means of a lamp and scale. In portable test set galvanometers the deflection of a pointer is read.

Moving-needle versus Moving-coil Galvanometers. — Galvanometers in which the moving element is the permanent magnet, in the form of a light needle, are called needle galvanometers, whereas those in which the moving element is the coil are called moving-coil galvanometers. The former type of galvanometer is required when extreme sensitiveness is desired, but cannot as a rule be used in a commercial laboratory, on account of the disturbing influences of the magnetic fields due to the lighting and power circuits in the laboratory and in neighboring buildings. Railway circuits, even though a mile or more away, may produce a disturbing field sufficient to render impossible the use of a sensitive needle galvanometer. For ordinary laboratory work, however, the moving-coil galvanometer can be made sufficiently sensitive, and since the permanent magnets used to produce the controlling field can be made very strong, the effect of stray fields can be rendered relatively negligible.

Astatic Galvanometer. — The disturbing effect of stray magnetic fields on a needle galvanometer can be partially prevented by the use of two sets of stationary coils and two needles, or the equivalent, the two needles being so arranged that any external field will produce opposing torques, and the coils so connected that their torques are additive. The Broca galvanometer, Fig. 1, described below, is a modern type of astatic instrument.

Damping and Logarithmic Decrement. — Due to the mechanical friction to motion and the induced currents set up in the adjacent metal or coils of the instrument as a result of the motion of the moving element, successive swings of the moving element when it is once set in motion decrease in amplitude. This effect is known as damping. Successive swings from the zero or equilibrium position decrease approximately in geometric ratio. Hence the ratio of the logarithm of one swing to the logarithm of the next swing, to the other side of the zero, is approximately a constant. This constant, which is a measure of the damping, is called the logarithmic decrement of the instrument.

Ballistic Galvanometer. — A ballistic galvanometer is one in which the moving element has a relatively long period and small damping. It can be shown that when a quantity of electricity Q is discharged through such a galvanometer in an interval of time *very short* compared with the period of the galvanometer, then the first throw or swing of the moving element is approximately proportional to the quantity of electricity Q. Such an instrument therefore serves as a very convenient means of measuring the capacity of a condenser (*see Capacity and Charging Current; Condensers, Electric*) as well as for measuring the quantity of electricity discharged through a circuit by transient induced electromotive forces (*see Magnetic Testing*).

Effect of External Resistance on Damping. — Since the damping of a galvanometer is due in part to the currents induced in the coils of the instrument by the motion of the moving element, and since these currents depend upon

the total resistance in series with these coils, it follows that the damping depends upon the constants of the external circuit. In particular, a low-resistance shunt around the galvanometer terminals may increase the damping very considerably. Consequently, in any measurements involving the comparison of initial throws or swings of the moving elements, care must be taken that the same external resistance is kept in the external circuit, or that a universal shunt (*see Shunts*) connected to give a high multiplying power (100 or more) be used.

Dead-beat Galvanometers. — When the damping of a galvanometer is very large, the free motion of the moving element ceases to be periodic, and for a constant current sent through the coils the moving element swings out to its new position and comes to rest without oscillation. Such an instrument is said to be dead-beat. This dead-beat feature is very desirable when steady deflections are to be observed; it may be obtained mechanically by mounting a light vane of mica on the moving element, or by placing a solid piece of metal close to the moving element so that the motion of the latter will induce currents in the metal.

Aperiodic Galvanometer. — When the damping is *just sufficient* to render the instrument dead-beat, the galvanometer is said to be aperiodic.

Alternating-current Galvanometers. — The ordinary type of galvanometer cannot be used for measuring alternating currents, since the twisting moment depends upon the *direction* of the current. An ordinary galvanometer may, however, be used indirectly by connecting in series with it a thermo-couple, the latter being heated by a coil carrying the alternating current to be measured; this is the principle of Duddell's thermo-galvanometer. The "twisted-strip" galvanometer in which the deflection is caused by the untwisting of a strip, due to the heating action of the current, has also been used as an alternating-current galvanometer. These instruments, however, are both decidedly inferior to the Einthoven "vibration" galvanometer (Fig. 3) described below.

Tangent and Sine Galvanometers are of historical interest only, and need not be described here.

Sensitiveness of Galvanometers. — The sensitiveness or deflectional constant of a galvanometer may be defined as the scale deflection in millimeters produced by a current of one micro-ampere passing through the galvanometer coil, the scale being at one meter distance from the mirror; this is called the *current sensitivity*. If the instrument has a scale attached, the current sensitivity is the number of smallest scale divisions per micro-ampere. It may also be stated in terms of the potential difference in mico-volts, which must be applied to the terminals of the galvanometer to produce unit deflection, or as the deflection in millimeters, at meter distance of scale, per mico-volt on the terminals; this is called the *voltage*, or *e.m.f.*, *sensitivity*. The sensitiveness of a galvanometer must be accompanied by good *zero-keeping quality*.

The megohm sensitivity of a galvanometer is the number of megohms which must be inserted in a circuit containing an e.m.f. of 1 volt in order to obtain a galvanometer deflection of 1 mm., the scale being at a distance of 1 meter from the mirror. This is the same, numerically, as the current sensitivity.

TYPICAL MODERN GALVANOMETERS. — Only a few typical forms of modern galvanometers can be described here.

The Broca Galvanometer. — This galvanometer is one of the best of the moving-needle type. It owes its sensitiveness and freedom from disturbance by external magnetic fields to the unique construction of its system. This system, shown in Fig. 1, consists of two vertical steel wires placed side by side and magnetized to have consequent poles at their centers.

This construction makes a system that is very astatic and though magnets of

considerable size are used, they are placed so close together that the system has a very small moment of inertia. A small mirror is located below the magnet system and below this an aluminum damping vane that moves in a chamber, the size of which can be varied, thus changing the damping. The whole system is suspended by a quartz fiber which serves as a control for the system, although the main control is obtained by a movable permanent magnet on the base of the instrument. The stationary coils are made interchangeable and removable and vary in resistance from 10 ohms to 1000 ohms per pair.

Either lamp and scale or telescope and scale can be used to read the deflections of this galvanometer. In the latter case the scale must be brilliantly illuminated, owing to the small size of the mirror.

The resistance of this galvanometer can be varied by connecting the fixed coils in parallel or series or by substituting coils of different resistances. To vary the damping two rods on either side of the instrument case are pushed in

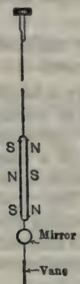


Fig. 1. Broca Galvanometer System

Fig. 2. D'Arsonval Galvanometer

or pulled out as required, to increase or decrease the size of the damping chamber. The period is changed by changing the position of the control magnet in reference to the system.

In the following table is given the sensitiveness of the instrument for various periods and resistances.

Resistance of coils in series in ohms	Period in seconds	Deflection in mm. at 1 m.	
		For 1 micro-ampere	For 1 micro-volt
10.9	8	218	19.9
107	12	1190	11.1
935	11	2450	2.62

D'Arsonval Galvanometers. — For the great majority of electrical measurements requiring galvanometers the D'Arsonval types, of which there are many forms, are the most satisfactory. Fig. 2 shows a typical form of construction, suitable either for mounting on the wall or on a tripod. This galvanometer has for its frame an iron casting which also serves as the permanent magnet and to hold the upper suspension tube. In the wall type this magnet casting is attached to a backboard holding the telescope and scale and the soft-iron core.

The coil is wound with copper wire, free from iron and suspended from above by a wire or strip of phosphor bronze, silver or steel, which also serves as one terminal of the coil. The other terminal is in the form of a spiral of phosphor bronze or silver strip brought off from the bottom of the coil. The coil carries a light mirror and the entire moving system is visible through a large window in a metal plate that covers the front of the magnet casting, this front plate being removable. The deflection is read by a telescope and scale, the scale curved so that the scale reading will be closely proportional to the deflection.

The following table gives the approximate sensitiveness of galvanometers of this type for different coil resistance and period.

MILLIMETERS PER MICRO-AMPERE

Periods in seconds	Galvanometer resistance			
	50 ohms	300 ohms	500 ohms	1000 ohms
7	50	150	200	300
10	100	300	450	600
12	150	500	650	800

D'Arsonval galvanometers of a slightly different form, having a sensitiveness 50 per cent greater than the above, are on the market. Portable galvanometers, or ammeters of high sensitiveness (0.5 to 10 micro-amperes per smallest scale division) are also used in connection with thermo-couples and for measuring insulation resistance.

Einthoven String Galvanometer. — The Einthoven string galvanometer, a diagrammatic view of which is shown in Fig. 3, is essentially a moving-coil galvanometer, though the conductor which moves is but a single filament. *NS* are the poles of a powerful electromagnet, and the fine conducting "string" *AB* stretched in the narrow air gap between the poles forms the moving element. When a current is passed through this conducting string, it deflects at right angles to the field of the electromagnet as indicated by the solid arrows. This deflection is observed by means of a microscope or by projecting an enlarged image of the string on a screen by means of a projection lantern, or the deflection may be recorded on a photographic plate.

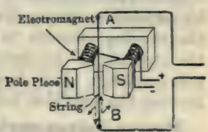


Fig. 3. Einthoven String Galvanometer

The electromagnet is relatively large and the pole faces are so shaped that an exceedingly high flux density is secured in the narrow air gap in which the string is stretched. The magnetic circuit is always fully saturated so that small variations in the exciting current do not affect the flux density in the air gap. The string, which may be an extremely fine silver wire or a silvered quartz fiber, is held under tension, which is adjustable by means of a micrometer screw. The resistance may vary from 5 ohms to 10,000 ohms, depending on the material of the string. The period depends on the tension on the string and may be reduced to less than 0.01 second or may be increased to 8 or 10 seconds with corresponding increase in sensibility.

The following table gives the characteristics of some Einthoven string galvanometers built by the Cambridge Scientific Instrument Company, when used on direct current.

CHARACTERISTICS OF EINTHOVEN STRING GALVANOMETERS

Material of string	Diameter of string	Resistance in ohms	Period in seconds	Magnifying power of microscope	Deflection* in mm.	
					Per micro-amp.	Per micro-volt
Silver wire.......	0.02 mm.	4.7	......	500	4.4	0.94
	0.002 mm.	20,000	......	500	62,500	3.13
Silvered quartz fiber........	0.003 mm.	6,600	......	650	333,000	50.5
	0.002 mm.	5,800	0.008	750	30	0.005
	0.002 mm.	3,890	0.005	750	9.7	0.003

* These are apparent deflections as observed through the microscope. For example, in the case of the galvanometer having the constants given in the third line, the actual deflection is $333000/650 = 512$ mm. per micro-ampere. An apparent deflection of about ⅓ mm. can be detected by the eye, consequently with this particular galvanometer a current of $10^{-6}/3 \times 333000 = 10^{-12}$ amperes, approximately, may be detected.

Vibration Galvanometer. — The Einthoven galvanometer may also be used as a vibration galvanometer for detecting an alternating current. If an alternating current is sent through the string and the tension on the latter is adjusted so that the free period of the string is the same as the period of the current, then the string will be set in vibration, the amplitude of the vibration depending upon the strength of the current. Consequently when the instrument is thus tuned, it may be used as a detector of extremely minute alternating currents. When thus used the instrument is capable of detecting an alternating current of 10^{-12} amperes, effective value.

A moving coil galvanometer may also be used as a vibration galvanometer by providing means for adjusting the free period of the moving element to resonance with the period of the current to be measured. Fig. 4 shows such a galvanometer, the free period being adjusted by moving the movable bridge up or down.

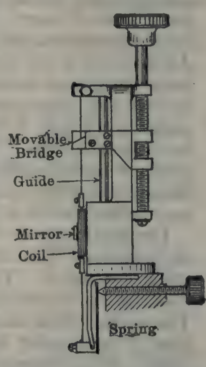


Fig. 4. Vibration Galvanometer.

Marine Galvanometers. — Any galvanometer that will give deflections that are not affected by the rolling of a ship and consequently can be used on shipboard may be called a marine galvanometer. A common form of marine galvonmeter is of the D'Arsonval type with the coil held by suspensions, both above and below, that are stretched under considerable tension.

The moving system of the galvanometer is carefully balanced, so that a 30-degree inclination will not cause a deflection of more than 3 mm. with the scale at 1 meter distance. It has a very stable zero and a sensitivity of 10 to 20 millimeters per micro-ampere with a 3-second period.

USE AND CALIBRATION OF GALVANOMETERS. — Galvanometers, other than ballistic, are used in engineering work primarily as current *detectors* in various bridge (q.v.) and potentiometer (q.v.) measurements, and

therefore careful calibration is usually unnecessary. In order to determine the degree of precision of such measurements, however, the sensitiveness of the galvanometer must be known. This may be determined by connecting in series with a battery of known e.m.f. a fairly high resistance, and connecting the terminals of galvanometer across a known small fraction of this resistance. If the galvanometer resistance is known or previously measured, the current through the latter may then be calculated by Kirchhoff's laws (*see Electricity and Magnetism, Principles of*), and consequently the millimeters deflection per micro-ampere determined. If the sensitiveness of the galvanometer is too great for any particular measurement, it may be reduced by properly shunting it (*see Shunts*).

Calibration of Ballistic Galvanometer. — The ballistic galvanometer is frequently used for capacity measurements and for the determination of the magnetization curves of samples of iron or steel (*see Magnetic Testing*). One method of calibration is to charge a standard condenser to a known voltage, and discharge it through the galvanometer, noting the deflection, and repeating the test for several known voltages. As the capacity of ordinary standard condensers is dependent upon the time of charging (*see Condensers, Electric*), this method is not very accurate.

Use of Standard Solenoid. — A better method is to use a standard solenoid. Such a solenoid should have a length about 50 times its diameter. A standard used in a number of experiments conducted under the auspices of the Standardization Committee of the American Society for Testing Materials is made of 1520 turns of No. 18 A.W.G. double-cotton-covered copper wire carefully wound in a single layer upon a cylindrical core of red fiber approximately 75 inches long and 1.5 inches in diameter. (A convenient way of winding such a coil uniformly is to cut a shallow thread in the core of such a pitch that the wire when wound in the groove between the threads will lie up snug.) A secondary coil of 2000 turns of No. 36 A.W.G. double-silk-covered wire was wound in 2 layers over a length of 6 inches, at the middle of this coil.

Connect the primary coil of this solenoid in series with an adjustable resistance, reversing switch and source of constant e.m.f. Connect the secondary coil in series with the galvanometer. Let

n_1 = the number of turns *per inch length* of the primary coil,
I_1 = the strength of the current in amperes in the primary coil,
N_2 = the number of turns in the secondary coil,
R = the total resistance in ohms of the secondary coil, galvanometer and any extra resistance which may be in the secondary circuit,
A = the area in square inches of the mean cross-section of the primary coil.

Then if the current I in the primary coil is reversed a quantity of electricity

$$Q = \frac{2.54 \times 8\pi}{10^9} \frac{n_1 I_1 A N_2}{R} \quad \text{coulombs}$$

$$= 0.0638 \frac{n_1 I_1 A N_2}{R} \quad \text{microcoulombs}$$

is discharged through the galvanometer. Hence, by calculating Q and noting the galvanometer swings when currents of various strengths are reversed in the primary coil, a curve can be plotted giving the quantity of electricity corresponding to a swing of any value; such a curve will be approximately a straight line.

COSTS (Pre-war figures). — A Broca galvanometer costs from $40 to $50 depending upon its sensitiveness. A D'Arsonval galvanometer costs from $18 to $100, including telescope and scale. An Einthoven galvanometer costs from

$400 to $500. Vibration galvanometer (type shown in Fig. 4) costs (1921) about $110.

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GAS. — (*See also Internal Combustion Engines; Gas Producers.*) The word gas in its most general sense means a substance which is capable of indefinite expansion. The word is also used in a restricted sense to mean any mixture of gases suitable for illumination or for fuel. It is in this sense that the word will be used in this article.

Gas is usually measured in thousands of cubic feet, at 60° F. and at a pressure of 30 inches of mercury. The term "permanent" or "fixed" gas is used to designate a gas which will not precipitate any of its constituents when cooled.

KINDS OF GAS. — In addition to natural gas, which exists in limited quantities, in restricted localities (in this country, chiefly in western Pennsylvania, West Virginia, Ohio, Indiana, Kansas, Texas and California), there are four types of manufactured gas. These various types all contain the same constituent gases, but in different proportions, depending upon the process of manufacture and the fuel from which they are made. Gas may be made from practically any kind of solid or liquid fuel, but coal is the fuel usually employed.

Coal-Gas. — This name is given to the mixture of permanent gases resulting from the distillation in closed retorts of the volatile matter in coal. The heavy hydrocarbons and various impurities are extracted by cooling and "washing" the gas and passing it through purifiers containing hydrated lime or sesquioxide of iron. The residue left in the retorts is coke; the condensed hydrocarbons form coal-tar; ammonia, sulphur compounds and cyanogen are absorbed in the washers and purifiers; all these by-products are of value.

Coal-gas is used chiefly for illumination, but in this country carburetted water-gas (see below) is more extensively employed for this purpose. Coal-gas is seldom used for power purposes on account of its high manufacturing cost as compared with producer-gas (see below), which, though unsuitable for illumination, makes a very satisfactory fuel for gas engines.

Water-Gas. — This name is given to the mixture of permanent gases resulting from the reactions which take place when steam is passed through a body of coal, coke or charcoal heated to redness or beyond, the hot gas being condensed and purified in much the same manner as coal-gas. The chief reaction in the formation of this gas is the decomposition of the steam by the heated carbon, forming hydrogen and carbon monoxide, the reaction being $C + H_2O = CO + 2\ H$. This reaction is a cooling process, hence the production of water-gas is intermittent, consisting of two stages: (1) the "blowing up" or heating the fuel in the producer by blowing air into it, producing CO_2 by the reaction $C + 2\ O = CO_2$, and (2) the production of water-gas by blowing steam through the fuel, the supply of air having been shut off. Water-gas is generally made from anthracite coal and contains not only hydrogen and carbon monoxide, but also small amounts of oxygen, nitrogen and light hydrocarbons. Pure water-gas is not suitable for illumination, since it is deficient in the heavy hydrocarbons which render coal-gas a good illuminant. This defect is readily supplied by adding naphtha vapor to the water-gas as it comes from the producer and superheating the mixture thus formed; the naphtha vapor and the superheated steam in the gas react to form permanent gases of the hydrocarbon group. Water-gas thus heated is called "carburetted water-gas" and forms an excellent illuminant. In this country, by far the greater proportion of illuminating gas is carburetted water-gas. Water-gas, pure or carburetted, is seldom used for power purposes, on account of its high cost.

Siemens Gas. — This name is given to the mixture of permanent gases resulting from the chemical reactions which take place when air is blown or drawn into a deep bed of coal. At the bottom of the bed, where the air is in excess, CO_2 is formed and this is reduced on passing through the upper part of

the bed to CO by the reaction $CO_2 + C = 2\ CO$. The resulting gas also contains hydrogen, resulting from the reaction between the moisture in the coal and the hot carbon, this reaction being the same as in the production of water-gas. The nitrogen of the air is also present in the gas, forming more than half its volume, thereby making a very dilute or "lean" gas. This process has been superseded by the producer-gas process described below.

Producer Gas. — This name is given to the mixture of permanent gases resulting from the chemical reactions which take place when both steam and air are blown through a bed of hot coals. The gas is essentially a mixture of water-gas and Siemens gas. The process is a continuous one, enough air being supplied with the steam to maintain the producer bed at the proper temperature, thus eliminating the "blowing up" operation in the water-gas process; also, due to the presence of the steam and the reactions which take place between it and the carbon, the proportion of hydrogen is increased and the proportion of nitrogen decreased, thus making a gas "richer" than Siemens gas. The exact composition of producer gas depends upon the kind of fuel used and the type of producer, of which there are numerous forms (*see Gas Producers*). The gas made in any particular form of producer is frequently called by the name or type of the producer, for example, "Mond gas," "Dowson gas," etc. Producer gas is used in power plants employing large gas engines; it is also used for firing ceramic kilns and metallurgical furnaces, and, to a limited extent, for firing steam boilers.

The thoroughness with which the gas must be cleaned depends upon the purpose for which it is to be used. For gas-engine work, it is particularly important that no tar should be present. On this account, anthracite coal is frequently used for the production of the gas for this purpose, in spite of the relatively high cost of such coal.

On the basis of the average figures from both the Bureau of Mines Testing Station and commercial plants, the following gas yield and heating value may be expected per ton (2000 pounds) of fuel as fired.

Fuel	Cubic feet of gas per ton of fuel as fired	B.t.u. per cubic foot of gas	B.t.u. in gas per ton of fuel as fired
Charcoal	160,000	135	21,600,000
Anthracite	140,000	135	18,900,000
Bituminous coal	135,000	150	20,200,000
Lignite	72,000	155	11,100,000
Peat	60,000	175	10,500,000

Other Kinds of Gas. — "Oil-gas" is a mixture of gases resulting from the vaporization of crude oil and superheating the vapor; it is used to improve the illuminating qualities of water-gas. "Coke-oven gas" is the gas formed as a by-product in coke ovens. Its properties are intermediate between those of coal-gas and Siemens gas; the composition of the gas, however, varies with the kind of coal and with the time, changing from the beginning to the end of the process. "Oil-water gas" is produced by the chemical reactions between steam and oil heated to a high temperature in closed retorts. The cost of manufacture is low where oil is cheap. "Blast-furnace gas" is a waste product of blast furnaces, and is a mixture of CO, CO_2 and N, the latter being the largest constituent. The relative proportions of CO and CO_2 vary with the conditions of the furnace. An average analysis of blast-furnace gas cooled to atmospheric

temperature is 25 per cent CO, 10 per cent CO_2, 65 per cent N. Coke-oven gas, oil-water gas and blast-furnace gas are all suitable for use in gas engines.

CALORIFIC OR HEATING VALUE. — The calorific or heating value (*see Heat and Thermal Properties*) of a given quantity of gas is the heat energy developed by its complete combustion. In this country, the calorific value of a gas is usually expressed in B.t.u. per cubic foot, the volume being measured at 60° F. and at a pressure of 30 inches of mercury. The total heating value of the gas produced from a given quantity of coal is less than that of the coal (or other fuel) from which it is made. In the case of coal-gas, the fixed carbon in the coal is not "gasified," and, therefore, its calorific value contributes nothing to that of the gas. In the case of water-gas and producer-gas, the carbon in the coal is converted into carbon monoxide, but the heating value of carbon monoxide is less than that of the carbon in it, since this carbon is already partially oxidized. Heat energy is also required to distil the hydrocarbons in the coal and in addition there are losses due to radiation. The calorific value of producer-gas made from a given quantity of coal varies from 55 to 85 per cent of the calorific value of the fuel from which it is made, depending upon the character of the fuel, the size of the producer and the manner of operating it.

EFFECTIVE OR NET HEATING VALUE. — The discharge from a gas engine is at a temperature considerably greater than the boiling point of water, and, consequently, the latent heat of condensation of the water vapor in the products of combustion is not available for the production of power, whereas in a calorimetric determination of the heating value of a gas this latent heat of condensation is included. The difference between the heating value, as determined by calorimetric measurement and the latent heat of condensation of the water vapor in the products of combustion, is called the effective or net heating value of the gas. The net (or low) heating value of producer-gas is usually about 5 per cent lower than the gross (or high) heating value. The high heating value has been generally adopted in this country in reporting tests, etc.

COMPARISON OF DIFFERENT KINDS OF GAS. — The following table shows what may be considered the average composition (percentage by volume), weight and calorific value of the different types of gases used for illumination and for fuel. It should be noted, however, that the characteristics of any particular gas may vary considerably from the figures given in the table, depending upon the quality of the coal used and the size and type of producer.

COMPARISONS OF DIFFERENT KINDS OF GASES

Item	Natural gas	Coal-gas	Water-gas	Up-draft producer gas: Anthracite	Bituminous	Lignite	Peat	Wood	Oil	Blast-furnace gas
CO	0.50	6.0	45.0	22.7	18.28	18.72	21.00	13.6	10.2	23.0
H	2.18	46.0	45.0	15.5	12.90	13.74	18.50	4.0	10.6	2.0
CH_4	92.6	40.0	2.0	0.0	3.12	3.44	2.20	8.0	6.1	2.0
C_2H_4	0.31	4.0	.....	0.0	0.18	0.17	0.40	0.0	3.8	0.0
CO_2	0.26	0.5	4.0	5.5	9.84	10.55	12.40	12.9	6.1	12.0
N	3.61	1.5	2.0	56.0	55.64	53.22	45.50	61.7	63.2	58.0
O	0.34	0.5	0.5	0.3	0.04	0.16	0.00	0.0	0.0	0.0
Average B.t.u. per cu. ft.	1000	730	320	135	150	155	175	135	215	90

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GAS LIGHTING. — (*See also Gas; Gas Producers; Illumination, Laws of.*) Illuminating gases are rated in terms of "calorific value," expressed in B.t.u.'s per cubic foot, and in "illuminating power," or the candle-power of a standard luminous flame burner, in its most efficient state, consuming 5 cubic feet of the given gas per hour. Measurements of volume are reduced to standard conditions of temperature and pressure, viz., 60° F., and a 30-inch barometric height. Gas pressures are sometimes stated in atmospheres and in pounds per square inch, but more commonly by the inches of water manometer column which the gas sustains against atmospheric pressure (*see Units and Conversion Factors*). The supply pressure ranges from 1.5 to 4 inches in low-pressure systems and from 40 to 60 inches in high-pressure systems. Most of the gas systems in the United States are of the low-pressure type, but high-pressure systems are common in Europe. The chief commercial varieties of gas and their more important properties are as follows:

Coal Gas, produced by distillation from bituminous coal, consists chiefly of hydrogen, methane and richer hydrocarbons. It varies greatly in quality with the grade of coal and the stage of distillation. Values of 16 candle-power and 600 B.t.u.'s are representative of the usual commercial supply. The use of coal gas is diminishing.

Water Gas, produced by the reaction of steam and incandescent coal or coke, consists chiefly of hydrogen and carbon monoxide, and is usually enriched with oil gas to raise its calorific and luminous value. The properties of water gas supplied in different cities range from 18 to 22 candle-power and from 580 to 700 B.t.u.'s per cubic foot. Many plants supply a mixture of coal gas and water gas.

Oil Gas is a mixture of rich hydrocarbon gases produced by "cracking" petroleum oils by heat. Its candle-power is from 40 to 60 and its calorific value 1200 B.t.u.'s or more. On account of its richness oil gas is often diluted with air or mixed with water gas for commercial delivery.

Pintsch Gas is a type of oil gas stored under a pressure of 150 pounds per square inch in steel tanks for self-contained and portable systems of lighting. It is rated at approximately 900 B.t.u. and 36 candle-power. An essential feature of the Pintsch system is an automatic reducing valve which supplies the gas at a constant low pressure adapted to flame and mantle illuminants.

Acetylene, produced by the reaction of water and calcium carbide, is a simple rich gas with a rating of about 1470 B.t.u.'s and 240 candle-power. For use in portable and self-contained systems acetylene is dissolved in acetone under a pressure of about 150 pounds per square inch. The usual container has a storage capacity of 10,000 volumes at this pressure.

Gasoline-air Gas is a mixture of air with gasoline vapor produced by forced evaporation. The generator commonly used comprises a blower operated by weights or water power, a carburretor and a mixing chamber. In the most widely used type of carburretor air is forced through sheets of cotton cloth which dip in gasoline and a rich mixture is produced. In the mixer the gas is diluted with air to a mixture containing from 2 to 5 gallons of gasoline to 1000 cubic feet, which is well above the explosive limit. Self-contained lamps in which the fuel mixture is locally generated by heat from the lamp have a wide use in street lighting. The luminous value of this gas in flames is very low, but it is well adapted to incandescent burners.

Natural Gas consists chiefly of methane, has a calorific value of about 1000 B.t.u.'s and little luminous value in open flames. It is excellently adapted to incandescent burners

PERFORMANCE OF FLAME ILLUMINANTS. — Light from a flame is due to minute incandescent carbon particles liberated from the gas by heat. The highest efficiency is obtained when the migration of these particles is slow; hence the pressure at the flame should not exceed two inches of water and the flame should be set but little above the smoking point. The horizontal light distribution from flames is sensibly uniform. The spherical reduction factor ranges from 85 to 92.5 per cent, depending on the occlusion of light by the burner. The batswing and fishtail burners used with ordinary gas should give fully 90 per cent of the rated illuminating power of the gas when properly adjusted. Acetylene is commonly burned in duplex burners with air holes giving slight pre-aeration. The usual one-foot acetylene burner gives 40 candle-power.

INCANDESCENT MANTLES AND BURNERS. — The incandescent or Welsbach mantle is the ashy residue of a woven structure of vegetable fibre, such as cotton, ramie or artificial silk, impregnated with oxides of cerium and thorium, with traces of other rare earths. The strength of mantle varies greatly with the material and weave of the fibrous base. Its luminous properties depend on the purity and proportions of the oxides used. Mantles are graded in quality chiefly by strength to resist shock, but the better grades are greatly superior in the maintenance of their candle-power.

The incandescent burner is made in two general types, upright and inverted. Each has a Bunsen tube with adjustable gas cock and air inlet. The entraining action and flame projection of the upright type is fairly self-regulating for a wide range of gas pressure and consumption. The inverted type admits of a narrow range of adjustment. The conditions required to obtain the highest efficiency from mantle burners are : (*a*) all gas and air ports must be clear and adjusted to give the proper air-gas mixture and best projection pressure at the flame; (*b*) the flame should burn at the highest possible temperature and the mantle should be completely immersed in the hottest or outer zone of the flame.

Performance of Mantle Burners. — Candle-power and efficiency ratings are based on lamps fitted with new mantles adjusted to the most efficient conditions, this adjustment being, however, of such a character that any one coming into casual contact with gas lamps is quite competent to make it. The average performance of mantle burners in service is always somewhat below the rated values and depends to a marked degree on maintenance conditions. Service deterioration is due to (*a*) the fouling of the burner with dust, (*b*) the improper adjustment of air and gas ports, (*c*) the shrinkage of mantles and loss of their active materials, and (*d*) the collection of dirt on glassware. Low-grade mantles show very rapid deterioration, often amounting to 40 per cent in 500 hours. R. F. Pierce (*Trans. Ill. Eng. Soc., Vol. VII, p. 686*) reports from reliable tests under most unfavorable conditions, in an atmosphere charged with iron oxide dust and so infested with insects that some of the burner tubes became entirely clogged the inherent deterioration in candle-power of well-adjusted mantle lamps of the best grade to be as follows: Duration of test, 1000 hours; loss due to mantle alone, 2.5 per cent; loss due to burner alone, 2.5 per cent; loss due to dirt on glassware, 10 per cent; total reduction in candle-power, 15 per cent. Tests of commercial installations receiving skilled maintenance show an average performance from 15 to 25 per cent below the rated candle-power and efficiency. Without skilled maintenance the average performance is generally from 30 to 50 per cent below rated candle-power and efficiency. (*Proc. Nat. Elec. Light Assn., 1911, Vol. I, p. 809.*)

It is important to note that the principal sources of deterioration in gas lamp candle power, as a general thing, are analogous to those operating in electric

lamps and that deterioration from dust and varying conditions of energy supply are of the most importance and are substantially of equal weight with the two forms of illuminants.

In an investigation of gas and electric lamps under actual service conditions, the performance of the gas lamps was found to be within 10 per cent of laboratory rating and in some installations the performance was even somewhat above laboratory rating. On the other hand, some of the electrical installations fell as far as 30 per cent below laboratory rating, due principally to blackened bulbs or to excessive voltage drop in the circuits.

There is a general tendency in comparing gas and electric installations, to disregard the influences which tend to produce discrepancies in the performance of electric lamps and to lay too much emphasis on the discrepancies exhibited by gas lamps. For all practical purposes and under reasonable good service conditions, laboratory tests offer the best and most practical methods of comparison between the two illuminants. Where good gas service conditions prevail, the average performance of gas installations is not likely to be as low as from 30 to 50 per cent below rated candle-power, the figures mentioned above in connection with systems where skilled maintenance is not avialable.

The efficiency of incandescent gas lamps varies with the pressure and the calorific value of the gas. At a given pressure the lumen-hours per cubic foot of gas are closely proportional to the calorific value of the gas. The pressure variations common to most gas systems have little effect on lamp efficiency, as the candle-power and rate of gas consumption are about equally affected.

PERFORMANCE OF GAS ILLUMINANTS. — The curves of Figs. 1 to 3 show the light distribution of the gas illuminants in general use.

PERFORMANCE OF GAS ILLUMINANTS

Illuminant	Glassware	Gas conditions			Light production			
		C.P.	Pressure, in.	Cu. ft. per hour	L.H.C.P.	M.S.C.P.	Lumens	Lumen-hours per cu. ft.
1-Mantle upright...	Clear chimney......	21.5	2.5	5.12	90.5	96.3	1210	236
" "	Opal dome..........	21.5	2.5	5.12	130.2	85.7	1078	211
" "	Opal globe..........	23.6	2.0	3.8	71.1	65.3	821	216
4-Mantle upright...	Alabaster globe.....	23.3	2.0	18.8	263.6	259.5	3261	173
" "	Clear globe, opal reflector..........	23.3	2.0	18.8	332.2	279.4	3511	187
1-Mantle inverted..	Ground glass ball...	21.9	2.5	3.31	68.8	54.7	688	208
" "	Green flashed reflector............	21.9	2.5	3.31	91.5	47.4	596	180
" "	Satin holophane reflector............	21.7	2.5	3.31	71.4	46.6	586	177
" "	Alba reflector.......	18.13	2.00	3.65	65	46.1	579	159
4-Mantle inverted..	Alabaster globe.....	21.9	2.5	13.03	207.5	150.8	1895	146
" "	Clear globe.........	21.9	2.5	13.03	273.6	174.3	2187	168
5-Mantle inverted..	Clear globe and enamelled ref.........	23.0	2.5	17.07	516	266	3343	196

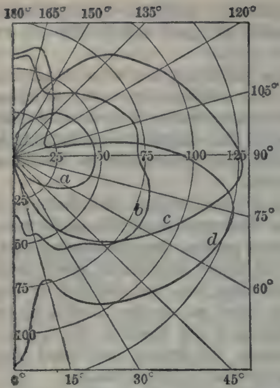

Fig. 1. Light Distribution of Upright, Single-mantle Gas Lamps

a. Junior type, mica chimney, 1.66 cu. ft. per hour
b. Standard type in opal globe, 3.8 cu. ft. per hour
c. Standard type in clear chimney, 4.66 cu. ft. per hour
d. Standard type with opal dome, 4.66 cu. ft. per hour

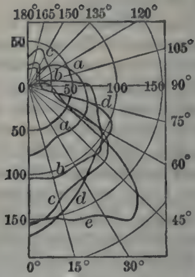

Fig. 3. Light Distribution of Single-mantle Inverted Gas Lamps

a. In roughed glass globe, 3.31 cu. ft. per hour
b. With flat opal reflector, 3.31 cu. ft. per hour
c. With opal dome reflector, 3.31 cu. ft. per hour
d. With enamelled cone reflector, 3.31 cu. ft. per hour
e. With holophane reflector No. 6321, 3.65 cu. ft. per hour
All tests with water gas of 20.5 to 22 c.p.

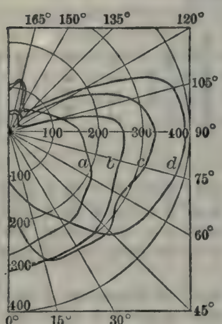

Fig. 2. Light Distribution of Gas Arcs

a. 4-mantle inverted, alabaster globe, 13.03 cu. ft. per hour
b. 4-mantle inverted, clear globe, 13.03 cu. ft. per hour
c. 4-mantle upright, alabaster globe, opal reflector, 18.8 cu. ft. per hour
d. 4-mantle upright, clear globe, opal reflector, 18.8 cu. ft. per hour

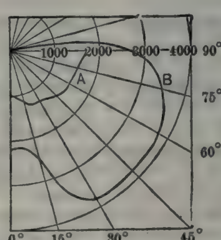

Fig. 4. Light Distribution of High-pressure Inverted Gas Lamps

A. 1-mantle, 31.6 cu. ft. per hour
B. 3-mantle, 70.3 cu. ft. per hour at 72.1 inches-pressure

The preceding table gives performance data of low-pressure gas illuminants under average conditions of test. Under ideal laboratory test conditions, the results would probably be somewhat better.

It should be noted that in column 7 in this table, values of mean spherical candle power are given for various types of gas lamps equipped with reflectors or globes. It may tend to mislead certain readers to try to interpret these values in terms of complete units, because of the difficulty of arriving at accurate conclusions concerning the effectiveness of the reflectors and accessories. It is, perhaps, safer to list mean spherical candle power values in reference to lamps unequipped with reflectors or other accessories, and care should therefore be exercised in the use of this table to take into account that the lamps are equipped with auxiliaries.

High-pressure Gas Systems. — The amount of air entrained in the mantle burner at ordinary low pressures is insufficient to produce the highest flame temperature and mantle efficiency. Three methods of gas supply are designed to improve this condition, viz., (*a*) to provide a gas pressure at the lamps of from 40 to 60 inches of water, (*b*) to provide gas at ordinary pressure and compressed air from a separate system of piping and (*c*) to supply both gas and air under pressure. The results produced by the three systems are approximately on a par. Results of the operation in Germany of inverted high-pressure lamps as cited by Wrightington (*Lectures on Illuminating Engineering, Johns Hopkins University, 1911, Vol. 2, p. 872*) may be summarized as follows:

Mantle life, 110 hours. Globe life, 2230 hours. Energy for gas compression, 1 horse-power-hour for 1400 cubic feet of gas. Consumption of 3-burner lamp, 65 cubic feet per hour. Costs per lamp-hour of 3-burner inverted lamp: compression, 0.31 cent; maintenance and renewals, 0.58 cent; gas at 61 cents per 1000 cubic feet, 3.98 cents; total per lamp-hour, 4.87 cents; total per lamp-year of 3675 hours burning, $178.97.

The light output of the high-pressure inverted single-mantle lamp is approximately 1500 mean lower hemispherical candle-power for a gas consumption of from 25 to 30 cubic feet per hour. The 3-mantle inverted lamp gives approximately 3500 m.l.h.c.p. at 70 cubic feet per hour. These efficiencies can be sustained only by the most careful attention to maintenance conditions. Fig. 4 shows typical light distribution curves of high-pressure lamps.

Methods of Automatic Ignition. — The automatic ignition of gas lamps may be accomplished by the following methods:

(*a*) A pilot flame consuming from 0.05 to 0.10 cubic foot per hour is fitted to each burner. As the main cock is opened a momentary jet of gas is supplied to the pilot tube, causing the ignition of the main burners. On some remote-controlled street-lighting systems a momentary extra pressure is applied as the gas is turned on to produce this jet action at the local burners.

(*b*) A high-tension spark is produced in the gas issuing at each burner by means of an induction coil. The primary circuit contains a battery, ignition switch and vibrating interrupter. The secondary coil is in series with the several spark terminals and each end of the wire is grounded. Good insulation of the secondary wiring is essential.

(*c*) The mantle may be provided with a small spot of sponge platinum which is raised to incandescence by catalytic action due to the rapid absorption of gas. Such self-lighting mantles are apt to be short-lived.

A clock-controlled ignition cock is frequently employed with show-window lamps and street lamps operated during definite time intervals on a flat rate system of charges.

COST OF GAS LIGHTING (Pre-war figures). — The following table shows the approximate cost of operation of various gas lamps. The assumed

cost of gas is $1.00 per thousand cubic feet and the assumed average light output as 80 per cent of the nominal ratings with clear glassware. These costs may be taken in reference to service conditions, including the effects of dirt.

Type of lamp	Cost per 1000 lamp-hours			Cost per 100,000 lumen-hours
	Renewals and maintenance	Gas	Total	
1-Mantle upright..........	$0.60	$5.00	$5.60	$0.65
1-Mantle miniature........	0.40	2.00	2.40	0.70
1-Mantle inverted.........	0.50	3.30	3.80	0.63
4-Mantle upright..........	1.65	18.80	20.45	0.74
4-Mantle inverted.........	1.65	13.00	14.65	0.80

Gas vs. Electricity. — For comparison with the above it may be noted that under initial laboratory performance conditions for given assumed lumen per watt ratings the operating costs of 100-watt tungsten lamps with energy at 5 cents and 10 cents per kw-hr. are respectively $0.75 and $1.40 per 100,000 lumen-hours. Corresponding costs for 250-watt lamps are $0.61 and $1.15 respectively. The advantages of electric lighting are chiefly convenience, flexibility in the size and location of illuminants. The chief disadvantages of electric lighting are danger of shock from abnormal voltages, the fire risk of faulty wiring and the greater likelihood of service interruption. The color of high-grade incandescent mantle lamps is a closer approach to daylight than that of the tungsten lamp, but is considered by many as less pleasing.

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GAS PRODUCERS. — *(See also Gas; Internal Combustion Engines; Power Stations, Gas-Electric.)* The name gas producer is usually applied to the apparatus used for making and purifying producer-gas (*see Gas*). The name, however, is sometimes limited to the apparatus in which the heating of the fuel takes place, and again it is used to designate the apparatus used in the manufacture of any kind of gas. In this article, the name gas producer will be used as a general term for all the apparatus used in making and purifying producer-gas, including the generator in which the heating of the fuel takes place, the evaporator or boiler for producing the necessary steam, and various filters and scrubbers.

The Generator. — The simplest form of generator is merely a cylindrical structure of fire-brick, provided with a grate, a roof and an outlet for the gas. A bed of coal from three to five feet thick is maintained in it and air and steam are forced through this bed. The oxygen of the air combines with some of the fixed carbon in the coal, forming carbon monoxide, the steam reacts with the heated carbon, forming hydrogen and carbon monoxide, and the hydrocarbons in the coal are driven off by the heat. In addition to adding combustible gases to the gas by its decomposition, the steam diminishes the formation of clinker, reduces the temperature of the escaping gases and makes a gas lower in nitrogen, and, therefore, of higher heating value, than would be produced if air alone were used. For facilitating the removal of ashes, revolving grates are used, or else the grate is dispensed with and the bottom of the producer rests in a shallow pit filled with water, from which the ashes are removed from time to time. The coal is usually fed by filling a hopper set in the roof of the producer, and opening its bottom so as to drop the coal, but sometimes special forms of rotating hoppers or feeders are used which distribute the coal evenly over the bed. To avoid choking of the producer by the " caking " of some varieties of soft coal, stirring apparatus is sometimes used in the upper part of the producer to break up the cake.

Evaporator. — The evaporator is essentially a steam boiler, the water being heated by causing the hot gases from the generator to circulate around the pipes or other container holding the water. The steam thus formed is fed into the generator with the proper proportion of air.

Filters and Scrubbers. — In order to remove the dust, tar and other harmful ingredients, the gas is passed through various devices designed to absorb or precipitate these substances. The filter consists of some porous material, such as shavings, excelsior or sawdust, in a suitable retainer. Liquid " scrubbers " are for the purpose of bringing the gas into intimate contact with water, which absorbs or precipitates the various impurities. This is accomplished by causing the gas to bubble through water, by passing the gas through a chamber into which water is sprayed, or by passing the gas over surfaces continually kept moist by a film of water. The spray type of scrubber is usually a large cylindrical shell filled with coke, the latter aiding in absorbing the tar from the gas as it passes through. Rotating scrubbers are also employed, which may be designed either to mix thoroughly the gas and water, or to drive out the impurities by centrifugal force. Rotating scrubbers are particularly effective in removing tar, a large amount of which is present in gas made from bituminous coal.

TYPES OF PRODUCERS. — The various forms of gas producers, of which there are a great number, differ chiefly in details of construction and in the arrangement of the various parts of the apparatus. Two distinct processes of making producer gas are in use—the up-draft process and the down-draft process. For commercial purposes these processes are applied by different manufacturers in different ways resulting in the following four general types of producers:

(*a*) Up-draft suction producers
(*b*) Up-draft pressure producers
(*c*) Down-draft producers
(*d*) Double-zone producers.

In the suction and down-draft types, the pressure in the gas generator is below that of the atmosphere. This is also true in at least one of the zones of the double-zone producer. In these types, the "suction" or draft through the producer may be produced by an exhauster or directly by the suction of the gas engines which they supply.

In the pressure producer the air and steam are forced through the fuel-bed under a light positive pressure (usually equivalent to from 2 inches to 8 inches of water in the generator space above the fuel-bed).

The double-zone producer is, as its name implies, a combination of the up-draft and down-draft principles. Two incandescent zones are maintained and the gas is withdrawn at the center or waistline of the producer.

UTILIZATION OF BY-PRODUCTS. MOND PROCESS. — Ammonia is about the only by-product of producer gas which has enough commercial value to justify the additional expense required to save it. This saving is accomplished by the Mond process, which is the one in most general use. An excess of steam is injected with the air, lowering the temperature and thereby preventing the destruction of the ammonia in the coal. (The proportion of hydrogen and carbon dioxide in the gas is also increased and the proportion of nitrogen decreased.) The gas, after passing through a mechanical scrubber, is caused to pass through a tower where it is thoroughly washed with dilute sulphuric acid. The ammonia in the gas combines with the sulphuric acid, forming ammonium sulphate. This acid solution of ammonium sulphate circulates through the tower over and over again until it reaches a given degree of saturation; then a certain amount of it is by-passed out of the system and fresh acid is added. In this system, the heat energy lost by the gas in cooling is also used to preheat the air which is used in generating the gas, effecting a considerable increase in the economy of the process. From a ton of coal containing not less than 1.3 per cent nitrogen, it is possible to procure 80 or 90 pounds of sulphate of ammonia and from 140,000 to 160,000 cu. ft. of 140 B.t.u. gas.

RATING AND PERFORMANCE. — A gas producer is usually rated in horse-power, the rating being the number of horse-power which can be developed by gas engines supplied from the producer when the latter is gasifying coal at a maximum rate. Such a rating is, of course, indefinite, as the power developed by the engines depends upon their design and the percentage of load which they are carrying, and the rate of gasification depends upon the quality of coal and the care taken in firing. The following data are the results of a series of tests on bituminous coal and lignites made by the U. S. Geological Survey at St. Louis on a 250-horse-power Taylor producer and a three-cylinder vertical Westinghouse 235-horse-power gas engine. These tests are summarized in a paper by R. H. Fernald in the *Jour. West. Soc. Eng., 1907, Vol. 12, p. 551.* The table on page 625 is derived from the curves there given.

All items in this table affected by load factor are given only for loads ranging from 90 to 100 per cent of full load (235 brake horse-power). The weight of coal per horse-power-hour does not include the coal required to produce the steam injected into the fuel bed. The efficiency is based on calorimetric measurements, no deduction being made for the latent heat of condensation of the water vapor in the products of combustion of the gas.

DATA ON 250 H.-P. GAS PRODUCER

B.t.u. per lb. of fuel as fired	9000	10,000	11,000	12,000	13,000	14,000	15,000
Lb. of fuel per brake hp.-hr	1.9	1.7	1.5	1.3	1.1	1.0	0.95
Lb. fuel required per hr. per sq. ft. of fuel bed to develop rated load	11.6	10.1	8.9	7.8	7.0	6.2	5.5
Cu. ft. of gas per lb. of fuel as fired	38	45	52	57	65	74	87
Lb. steam per lb. of fuel as fired	0.30	0.35	0.40	0.45	0.50	0.55	0.60
Efficiency—Per cent of B.t.u. in fuel in gas produced	66	69	70	72	75	80	85

Efficiencies. — The following are average efficiencies of conversion and scrubbing (i.e. percentage of B.t.u. in fuel available in cold, clean gas) for producer-gas power plants:

Up-draft plants, per cent	70
Down-draft plants, per cent	76
Double-zone plants, per cent	73

For other than normal rating the efficiencies may be taken on the following basis:

Per cent rated capacity	Per cent of efficiency of full rating
100	100
75	97
50	92
25	80

Dimensions — F. C. Tryon (*Power, Dec. 1, 1908*) gives the following rules for determing roughly the dimensions of the various parts of a producer plant:

1. Fuel-bed cross-section 0.125 square foot per horse-power capacity. This gives the formula

$$d = \frac{\sqrt{P}}{2.5},$$

where d is the diameter of fuel bed in feet and P the horse-power capacity.

2. For sizes smaller than 100 horse-power, the walls of the generator should be 9 inches thick; for larger sizes, 12 inches thick. Hence, for sizes smaller than 100 horse-power, the external diameter of the generator in feet is $d_0 = d + 1.5$ and for larger sizes $d_0 = d + 2$.

3. Height of generator approximately twice its internal diameter; for sizes under 100 horse-power the height is usually greater than twice the internal diameter while for larger sizes the height is usually slightly less than twice.

4. Diameter of wet scrubber three-fourths of the internal diameter of generator

5. Height of wet scrubber one and one-half times the height of generator.

6. Diameter of dry scrubbers, of which there should be two, equal to internal diameter of generator.

7. Height of each scrubber 3 to 4 feet for plants ranging from 25 to 200 horse-power.

COSTS (Pre-war figures). — The following approximate formulas for costs of producer-plant equipments have been derived from a large number of actual installations. They include the purchase price of generators, evaporators and the necessary scrubbers and filters, but do not include erection, buildings or foundations; P is the horse-power rating of the producer.

Up to 250 horse-power.......... $\$500 + 10 \times P$
250 to 1000 horse-power... $20P - \$2000$

The cost of such plants erected including producer equipment, gas engine and foundations averages from $80 to $110 per rated horse-power, exclusive of buildings, for plants up to 250 horsepower. Plants from 1000 to 5000 horse-power average about $70 per horse-power, including buildings.

Gas holders.......................... $\$1000 + \$0.26 \times P$.

Detailed manufacturers' estimates of investment cost and cost of operation of twenty-six producer plants are given in the paper by Mr. Fernald in the *Jour. Soc. Wes. Eng., 1907, Vol. 12, p.551.*

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GEARS AND GEARING. — The following terms are commonly employed.

Gear Wheel or Gear. — A wheel with teeth cut in its periphery, designed to mesh with the teeth of a similar wheel or with a screw or worm.

Pitch Circle or Pitch Line and Pitch Diameter. — Referring to Fig. 1, the two tangent circles A and B with their centers on the axes of the two gears, are called the pitch circles or pitch lines of the two gears. The diameter of the pitch circle is called the pitch diameter.

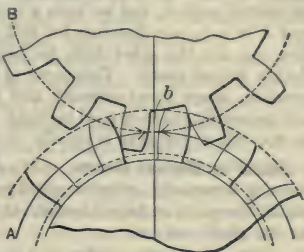

Fig. 1. Spur Gear

Backlash. — Referring to Fig. 1, the distance b, which is the difference in the width of the tooth space and the width of the tooth, measured along the circumference of the pitch circle, is called the backlash.

Pitch of a Gear. — The *circular pitch* of a gear is the distance in inches, measured along the *arc* of the pitch circle, between the center lines of two successive teeth; it is also

$$\text{Circular pitch} = \frac{\pi \times (\text{diameter})}{\text{Number of teeth}}.$$

The *chordal pitch* is the distance in inches, measured along the *chord* of the pitch circle between the center lines of two successive teeth; let N be the number of teeth, D the diameter of pitch circle, then

$$\text{Chordal pitch} = D \sin\left(\frac{180°}{N}\right).$$

The *diametrical pitch* is the quotient of the number of teeth by the diameter of the pitch circle.

Pinion. — The smaller wheel of a gearing consisting of two gear wheels is called the pinion; e.g., the gear wheel on the shaft of a railway motor is called the pinion. The name pinion is also used for the smaller of two gears mounted on the same shaft.

Sprocket. — A gear wheel driven by a chain.

Gear Ratio or Velocity Ratio. — The ratio of the number of teeth in the larger wheel of a gearing to the number of teeth in the smaller wheel; this is also equal to "velocity ratio," or the number of revolutions of the smaller wheel corresponding to one revolution of the larger wheel.

Spur Gear. — A gear wheel in which the external surfaces of the teeth lie on a cylinder and the center planes of the teeth pass through the axis of the gear — the common type of gear wheel.

Bevel Gears. — A gear wheel in which the external surfaces of the teeth lie on a cone, so that the gear may mesh with another gear having its axis inclined to the axis of the first.

Stepped Gears. — Two gears of the same pitch and diameter mounted side by side on the same shaft will act as a single gear. If one gear is keyed on the shaft so that the teeth of the two wheels are not in line, but the teeth of one wheel slightly in advance of the other, the two gears form a stepped gear. If mated with a similar stepped gear on a parallel shaft the number of teeth in

contact will be twice as great as in an ordinary gear, which will increase the strength of the gear and its smoothness of action.

Twisted Teeth. — If a great number of very thin gears were placed together, one slightly in advance of the other, they would still act as a stepped gear. Continuing the subdivision until the thickness of each separate gear is infinitesimal, the faces of the teeth instead of being in steps take the form of a spiral or twisted surface, and we have a twisted gear. The twist may take any shape, and if it is in one direction for half the width of the gear and in the opposite direction for the other half, we have what is known as the herring-bone or double helical tooth. This form of tooth is much used in heavy rolling-mill practice, where great strength and resistance to shocks are necessary. They are frequently made of steel castings. The angle of the tooth with a line parallel to the axis of the gear is usually 30 degrees.

Spiral or Helical Gears. — If a twisted gear has a uniform twist it becomes what is commonly called a spiral gear (properly a helical gear). The line in which the pitch surface intersects the face of the tooth is part of a helix drawn on the pitch surface.

Pitch Angle. — The pitch angle of a helical gear is the angle between the center line of the tooth and a line perpendicular to the axis of the gear.

Screw or Worm. — A spiral wheel may be made with only one helical tooth wrapped around the cylinder several times, in which it becomes a screw or worm. If it has two or three teeth so wrapped, it is a double- or triple-threaded screw or worm.

Worm-Gearing. — When the axes of two spiral gears are at right angles and a wheel of one, two or three threads works with a larger wheel of many threads, it becomes a worm gear, or endless screw, the smaller wheel or driver being called the worm, and the larger, or driven wheel, the worm wheel. With this arrangement a high velocity ratio may be obtained with a single pair of wheels. For a one-threaded wheel the velocity ratio is equal to the number of teeth in the worm wheel. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm.

Differential Gearing. — Various forms of differential gears are used. The object is to devise a motion such that the velocity ratio is equal to the ratio of the number of teeth in one gear wheel to the *difference* between the number of teeth in this gear wheel and the number of teeth in the other wheel. In one form of differential gearing one wheel meshes with a set of teeth on the inside of a second wheel. The second wheel is so arranged that it cannot turn about its own axis but its center is caused to move in a circle concentric with the first wheel.

But for the limitation that the difference between the wheels must not be too small, the possible ratio of speed might be increased almost indefinitely, and one pair of differential gears made to do the service of a whole train of wheels. If the problem is properly worked out with bevel gears this limitation may be completely set aside, and external and internal bevel gears, differing by but a single tooth if need be, made to mesh perfectly with each other.

DESIGN OF GEARS. — In order that the teeth of wheels and pinions may run together smoothly and with a constant relative velocity, it is necessary that their working faces shall be formed of certain curves called odontoids. The essential property of these curves is that when two teeth are in contact the common normal to the tooth curves at their point of contact must pass through the pitch point, or point of contact of the two pitch circles. Two such curves are in common use — the cycloid and the involute.

The design of gears is treated in detail in Kent's *Mechanical Engineers' Pocket-Book* and in Halsey's *Handbook for Machine Designers*.

Materials and Construction.* — Gears are usually made of cast iron or steel. To obtain the most satisfactory results the teeth should be machined. Rawhide pinions, cambric-cloth pinions, and composite gears, consisting of alternate sheets of rawhide or fiber and steel or bronze, are also used where smooth and quiet running are necessary. Of this latter form of gearing the cloth pinions have the advantages of being strong, not liable to damage from cold or hot oil, unaffected by atmospheric changes, and vermin proof.

A product of heavy duck bonded with bakelite, known as "micarta," is coming into extensive use for gears where quietness of running is desirable. Lunch and Talley (*Elec. J. 13, p. 368, Aug., 1916*) give the following as the physical properties of this material: Tensile strength parallel to laminations, 10,000 pounds per square inch; compression strength perpendicular/parallel to laminations, 35,000/17,000 pounds per square inch; transverse strength, maximum fiber stress perpendicular or parallel to laminations, 17,000 pounds per square inch.; coefficient of expansion per inch per degree centigrade, 0.00002 inch parallel to, and 0.000085 inch perpendicular to laminations; specific gravity, 1.4; weight per cubic inch, 0.05 pound; oil absorption practically zero; water absorption (50 hours immersion at 21° C.) 0.25 to 2 per cent by weight, depending upon relative amount of edge exposed.

POWER TRANSMITTED BY GEARING. — The strength of gear-teeth and the horse-power that may be transmitted by them depend upon so many variable and uncertain factors that the formulas and rules given by different writers show a wide variation. The pitches indicated in the accompanying table are recommended * as representing safely conservative practice and, although somewhat smaller teeth may afford sufficient strength in some cases, these standards should be adhered to with as few exceptions as possible.

Maximum horse-power transmitted	Diametrical pitch	Maximum horse-power transmitted	Diametrical pitch
1/6	12	10	4
1/4	10	15	3
1/2	10	20	3
3/4	10	25	3
1	8	50	2 1/2
2	8	60	2 1/2
3	6	75	2
5	6	100	2
7 1/2	5	125	1 1/2
		150	1 1/2

EFFICIENCY OF GEARING. — An extensive series of experiments on the efficiency of gearing, chiefly worm and spiral gearing, is described by Wilfred Lewis in *Trans. A.S.M.E.*, Vol. 7, p. 273. The average results are shown in a diagram, from which the approximate average figures in the table on the following page are taken.

* By D. B. Rushmore.

The experiments showed the advantage of spur gearing over all other kinds in both durability and efficiency. The variation from the mean results rarely exceeded 5 per cent in either direction, so long as no cutting occurred, but the variation became much greater and very irregular as soon as cutting began. The loss of power varies with the speed, the pressure, the temperature and the condition of the surfaces.

The excessive friction of worm and spiral gearing is largely due to the end thrust on the collars of the shaft. This may be considerably reduced by roller bearings for the collars. When two worms with opposite spirals run in two spiral worm gears that also work with each other, and the pressure on one gear is opposite that on the other, there is no thrust on the shaft. A low efficiency for a worm gear means more than the loss of power, since the power which is lost reappears as heat and may cause the rapid destruction of the worm.

PER CENT EFFICIENCY OF SPUR, SPIRAL AND WORM GEARING

Gearing	Pitch angle, degrees	Velocity at pitch line in feet per min.				
		3	10	40	100	200
Spur pinion	..	90	93.5	97	98	98.5
Spiral pinion	45	81	87	93	95.5	96.5
Spiral pinion	30	75	81.5	89	93	94.5
Spiral pinion	20	67	75	84.5	90	92
Spiral pinion	15	61	70	80.5	87	90
Spiral pinion or worm	10	51	61.5	74	82	86
Spiral pinion or worm	7	43	53	72	76.5	81.5
Spiral pinion or worm	5	34	43	60	70	76.5

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GENERATORS, ALTERNATING-CURRENT. — (*See also Alternating Currents; Electricity and Magnetism, Principles of; Generators, Direct-Current; Motors; Standardization Rules.*) In all dynamo-electric machines of the usual forms (excepting the homopolar) the electromotive force induced in any one conductor or turn varies in value and alternates in direction. If the terminals of any coil or group of coils are brought out to an external circuit by means of revolving contacts, such as slip rings, an alternating current will flow in the circuit. In order to obtain a direct or continuous current it is necessary to add a commutator or rectifier. Thus the most simple and elementary form of electric generator is an alternating-current generator and the direct-current generator is a special form adapted by the addition of a commutator to give a continuous or unidirectional current.

CLASSIFICATION. — Alternating-current generators may be classified according to several different distinguishing characteristics. The following classification considers these characteristics in the order of their prominence.

Synchronous Generators. — In the synchronous type the action of inducing the electromotive force results from the relative motion of the armature conductors and a constant magnetic field produced by exciting coils in which a continuous or direct current flows. The frequency of the alternating electromotive force depends directly on the number of field poles and the angular velocity of the revolving part.

Induction Generators. — In the induction type the magnetic field is of the rotating polyphase type and is produced by polyphase alternating currents flowing in the same windings with the load current. The mechanical construction is usually that of an induction motor with a short-circuited polyphase winding on the revolving member. The frequency depends upon the characteristics of the external circuit. In practice the frequency is determined or "set" by a synchronous machine, either generator or motor, in the external circuit. This "frequency setter" is necessary to supply the exciting current of the induction generator; that is, by means of the frequency setter the power factor of the total load is adjusted to equal the inherent power factor of the generator.

Advantages and Disadvantages of Induction Type. — A minority of the units of a station may be of the induction type with advantage as they will cause less disastrous effects in case of a short circuit on the system. Their instantaneous short-circuit current is less. A certain number of the units must be of the synchronous type to "set" the frequency and supply the exciting current for the induction machines. Consequently, if there is a large proportion of induction machines the synchronous machines operate at a poor power factor unless there is much capacity effect in the load system, such as synchronous motors or line capacity.

Revolving-field and Revolving-armature Types. — In the revolving-field or revolving-armature type there are two, or some multiple of two, poles, each pole having its own coil for the direct-current excitation, and the flux in each pole is to all intents and purposes constant in value. The relative movement of the poles and conductors causes the variation in flux interlinkages both in direction and intensity.

The early machines were constructed with an armature revolving inside a stationary field. As sizes and voltages increased it was found that a more effective use of the material could be obtained by making the armature the external member, and that the insulation of the high-voltage member was better preserved if that member was kept stationary.

At the present time all generators of large capacity (500 kilowatts and greater) and for high voltages (600 volts and up) are made of the revolving-field type.

Inductor Type. — In the inductor type the direct-current excitation is concentrated in one (usually stationary) coil and the variation in magnetic flux is

obtained by revolving a spider with bare projecting poles which alters the reluctance of the magnetic path. Thus the flux-threading any particular armature coil is always in the same direction, but varies in intensity or quality.

Comparison of the Inductor Type with Revolving Field or Revolving Armature Type. — There are no windings or insulation on the moving member of the inductor type, which is therefore adapted to high speeds. The greater variation in the reluctance of the main magnetic path and the consequent variation in magnetic densities in the field structure of the inductor causes excessive eddy currents, hence the construction of this type has been abandoned for machines of large power. It is still used for radio work however. See *Radio Communication*.

Single Phase, Two Phase or Quarter Phase and Three Phase. — The single-phase generator is usually about 30 per cent heavier and more costly than a polyphase generator of the same rating. By changing the internal armature connections a polyphase machine may frequently be reconnected to be a three-, two- or single-phase machine.

As transmission by three-phase currents is more economical of copper than by two-phase or single-phase currents, all power transmission lines are three-phase. Consequently unless there is some local condition requiring single or two phase, the three-phase generator is preferable.

Two-phase and three-phase generators of the same capacity and voltage strain are of practically the same dimensions, weight and cost.

Trade Terms. — It has become almost universally the custom to classify an alternating-current generator as *ASB*, *AQB* and *ATB;* the *A* standing for alternator, *S* for single phase, *Q* for quarter phase, *T* for three phase and *B* for revolving field.

Separately-excited and Self-excited Types. — For years it was attempted to develop a satisfactory and simple self-excited alternator and some were very sucessful but not simple. The object was to obtain a constant voltage on the load by means of automatic self-excitation. However, with the perfection of the automatic voltage regulator the need for automatic self-regulation ceased, and there is now very little demand for self-excited alternators.

METHODS OF RATING. — All a-c. generators are rated in kilo-volt-amperes (kv-a.) and unless otherwise specifically stated are rated at that kv-a. which they will give continuously with a rise in temperature not exceeding certain values depending upon the character of the insulating materials used; see *Standardization Rules A.I.E.E.*

Machines, or those parts of machines, insulated with mica, asbestos or similar heat resisting material are allowed a rise in temperature of 70° C. as measured by thermometer, or 75° if measured by resistance. This includes most turbo-alternators, both armature and field, and the fields of many other alternators.

Machines, or those parts of machines insulated with varnished cambric or impregnated cotton or linen, are allowed a rise of 50° C. as measured by thermometer, or 55° if measured by resistance. This includes the armatures of most slow and moderate speed machines.

Most machines have their fields designed sufficiently liberally to enable them to give their rated kv-a. at 80 per cent power factor with rated voltage, and this is stated in the specifications. It should be noted that the heating of the *armature* depends upon the kv-a. and not upon the kilowatt load. The lower the power factor of the load connected to the alternator, the greater the heating of the *field coils* for the same kv-a. output.

VOLTAGE. — Alternators are now built to generate voltages up to 13,000 and 15,000 volts between lines, either single phase or polyphase. Above that

voltage the extra cost of the insulation and the danger of damage from discharges cause it to be less expensive to install transformers with a lower voltage alternator. Some engineers consider the limit of economical voltage of generators to be even lower than 13,000 volts.

FREQUENCY. — The frequency depends upon the speed of rotation and the number of poles. If the speed of rotation of the revolving part is given in revolutions per minute, the frequency is

$$f = \frac{\text{r.p.m.}}{60} \times \frac{\text{number of poles}}{2}.$$

In the early alternators it was found much more economical to run at high speeds, and thus high frequencies such as 133 and 125 cycles per second, were customary; but, as systems increased in size and complexity, electrical difficulties arose as a result of these high frequencies and the tendency has been to reduce the frequency till now we have 60, 50, 40 and 25 cycles per second as usual frequencies, of which 25 and 60 are standard in this country. In Europe 50 cycles is used instead of 60. Of late there has been an attempt to have 15 or 16 cycles standardized for railway work.

A frequency of 25 cycles is preferable where there is a very long transmission line, on account of the lower inductive voltage drop, and where there is much synchronous machinery, such as synchronous motors and rotary converters, as these are more stable and better adapted to parallel operation at low frequencies.

A higher frequency (60 cycles in U. S. A., and 50 in Europe) is preferable for electric lighting, as the light is steadier. The higher frequency is also preferable where many transformers are used, as these are cheaper and more efficient at the higher frequencies.

PHASE AND LINE VOLTAGES AND CURRENTS. — For moderate voltages, up to 3000 between lines, the method of connection is decided by such details, as which will give a convenient number for the conductors per slot for the required voltage, but for higher voltages the Y connection is usually chosen as it causes a lesser strain on the machine insulation for a given voltage between lines. See also p. 666.

Single Phase. — In a single-phase generator the voltage per phase is the same as the voltage between lines and the current per phase is the same as the current per line, the product of voltage and current giving the volt-ampere rating of the machine.

Two Phase or Quarter Phase. — In a properly designed machine of this type each phase supplies half the rating, thus the voltage and current per phase in the machine are respectively the same as the voltage and current per phase on the line. The current is equal to $1000 \times (\text{kv-a.}) \div 2 \times$ volts per phase.

Three Phase. — Machines of this type may be connected either Y or Δ. In a Y-connected machine the current per phase is the same as the current in each line and is equal to $1000 \times (\text{kv-a.}) \div \sqrt{3} \times$ (volts between lines), while the voltage per phase is (volts between lines) $\div \sqrt{3}$.

In a Δ-connected machine the line current is equal to $1000 \times (\text{kv-a.}) \div \sqrt{3} \times$ (volts between lines) and the current per phase is equal to (line current) $\div \sqrt{3}$, while the voltage per phase is equal to the voltage between lines.

DESIGN OF SALIENT POLE GENERATORS. — The procedure in designing an alternating-current generaotr for a given power output, voltage and frequency to fulfill given requirements regarding regulation, efficiency, etc., is partly analytical and partly empirical. The data of four specific designs are

given below in the section on *Examples of Design and Performance.* The method of procedure is to lay out a preliminary design from rough calculations, calculate the performance of this design, modify the preliminary design where this calculation indicates, recalculate the performances, etc., until a design is arrived at which meets the given requirements.

Definitions. — The following terminology is used in the discussion of the design of generators:

Pole Arc. — The arc subtended by one pole face, measured along the periphery of the armature; in the following discussion it will be expressed in inches.

Pole Pitch. — The arc measured along the periphery of the armature from the center of one field pole (N-pole, say) to the center of the next field pole (S-pole); in the following discussion it will be expressed in inches.

Ampere Conductors per Inch of Armature Periphery. — The product of the number of conductors per inch measured along the periphery of the armature by the effective amperes flowing in each conductor.

Slot Pitch. — The distance in inches measured along the armature periphery between the centers of two adjacent slots. The slot pitch is equal to the pole pitch divided by the number of slots per pole.

Coil Pitch. — The number of slots spanned by a coil; that is, if a coil has one side in slot number 1, and the other side in slot number 7, the pitch of the coil is $7 - 1 = 6$. A coil which spans a distance exactly equal to the pole pitch is said to have a "full pitch," but if it spans a lesser distance it is said to have a "fractional pitch." For example, if there are six slots per pole and a coil has one side in slot number 1, and the other side in slot number 5, it is said to have a fractional pitch of $\frac{4}{6}$ or $\frac{2}{3}$.

Leakage Factor.—The ratio of the flux per pole which enters the armature core to the total flux (including the leakage flux) which would be produced by the field winding.

Preliminary Calculation of Main Dimensions. — Let

P_0 = output in kilovolt amperes (kv-a),
ρ = ratio; pole arc divided by pole pitch,
B = average magnetic flux density per square inch in air gap,
σ = ampere-conductors per inch of periphery,
V = peripheral velocity in feet per minute at gap,
D = diameter of armature in inches,
L = length of armature along shaft in inches,
N = revolutions per minute,
p = number of poles,
f = frequency in cycles per second,
$A = D \div p$.

Then the following relation holds for either a single-phase or a polyphase machine, other than turbo-alternators:

$$P_0 = \frac{\rho \sigma B V D L k_2 k_3}{144 \times 10^9}, \tag{1}$$

Five of the six design constants in the right-hand member of this equation are subject to choice; the equation then fixes the sixth constant. The choice of the values of the various constants should be based upon modern practice, an idea of which is given below in the following paragraphs. For values of the constants k_2 and k_3 see the section below on *Predetermination of Performance.*

For turbo-generators see section below on *Design of Turbo-alternators.*

Ratio of Pole Arc to Pole Pitch (ρ) is governed by two antagonistic phenomena; for the sake of small magnetic leakage and good form factor of

e.m.f. wave a low value is desired; for the sake of low reluctance to the main flux and economy of material a high value is desired. The usual values are ranged from 0.6 to 0.7.

Magnetic Flux Density (B) is limited by the exciting ampere turns required to produce it, length of gap and sometimes by the heating resulting from core loss. Usual values B are given in the accompanying table.

Cycles	Lines per sq. in.
25	44,000 to 58,000
60	32,000 to 48,000
125	30,000 to 40,000

Ampere-Conductors per Inch of Periphery (σ) is limited by the heating resulting from high current densities. If ventilation is good, insulation thin, or if the slots are very deep, as in armatures of large diameters, the values of σ may be high. Usual values of σ for continuous rating and for a rise in temperature of 40° C. are:

Diameter of armature, in.	Less than 500 volts	500 to 2000 volts	2000 to 7000 volts	Above 7000
0 to 20	250 to 350	200	...	...
20 to 30	350 to 600	400	300	250
30 up	600 to 1000	700	500	400

Naturally, with special means of ventilation, σ may be much greater than the values here given, as, for example, in turbo-alternators with forced ventilation.

Peripheral Velocity (V) is altogether a matter of mechanical design and values may be found from 2000 feet per minute to 10,000. Usual values are from 3000 to 6000.

Diameter and Length of Armature. — The diameter of armature (D) is related to the number of revolutions per minute (N) by the formula

$$D = \frac{12\,V}{\pi N}\,.$$

Hence, for a given value of N, the diameter D is fixed when V is chosen. This value of D together with the chosen values of ρ, B, σ and V then fixes the value of L by equation (1).

If, however, the value of N is not given, reasonable values for D and L may be found by assuming that, in accordance with normal and economical conditions, L is approximately equal to the pole pitch. This gives, putting A = diameter divided by number of poles, $L = \pi A$, and equation (1) reduces to

$$P_0 = \frac{\pi\rho B\sigma VAD}{144 \times 10^9}\,.$$

This fixes D when ρ, B, σ, V and A are chosen. The value of D must, of course, be adjusted to the frequency and an even number of poles (*see next paragraph*). Usual values of the diameter per pole are given in the accompanying table.

Cycles	A = diameter per pole, in.
25	5 to 12
60	2.5 to 5
125	1.75

Number of Poles (p) **and Revolutions per Minute** (N). — The

number of field poles is related to revolutions per minute N and the frequency f by the formula

$$f = \frac{Np}{120}, \quad \text{or} \quad p = \frac{120f}{N}.$$

Hence when the frequency and speed are both given, the number of poles is fixed. When N is not given, but L is assumed approximately equal to πA as above, the number of poles is found, when D has been determined, from the relation

$$p = \frac{D}{A}.$$

A value of A must of course be chosen such that p is an even number. The speed N is then obtained from the relation

$$N = \frac{120f}{p}.$$

Armature Windings. — While there are many forms of armature windings which may be used, there are practically only two forms that are in general use in this country. These are the "Chain Winding" and the "Lap Winding."

Chain Winding (Fig. 1). — This winding is characterized by having a number of coils equal to half the number of slots; that is, there is only one side of a coil in each slot. The coils may be either form wound and insulated, in which case slots with open faces are required; or they may be wound by hand in place, in which case partly or entirely closed slots may be used.

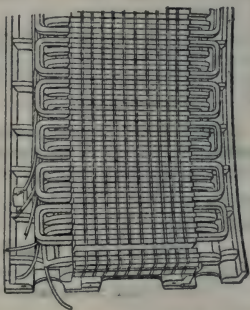

Fig. 1. Chain Winding

There are at least two kinds of coils in each machine. These are characterized by the shape of their end connections, as these end connections lie some in one plane and some in another. If there is more than one slot per pole per phase, there must be four different shapes of coils, having two different pitches; the coils of any one phase per pole are placed concentrically. The chain winding is most generally used in high-voltage machines, 2000 volts or higher.

Lap Winding (Fig. 2). — This winding contains a much larger number of coils, all of the same form and size. There may be two, four, six, etc., sides of coils in each slot, the coils being placed side by side two coils deep. This winding is similar to the multiple-drum winding of a direct-current armature. With two coil sides per slot the slots must be open, but with more than two coil sides per slot the slots may be partly closed. This type of winding is satisfactory for low voltages and very convenient if there are a large number of slots per pole.

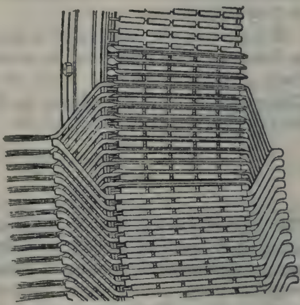

Fig. 2. Lap Winding

The coils are connected in groups of 2, 3, etc., per pole per phase, depending upon the slots per pole per phase. Successive groups or poles of the same phase are connected in series or multiple by "pole connections."

In both forms of winding each coil may contain from one to many turns in series.

Turns in Series per Phase (S). — Let

σ = ampere-conductors per inch of periphery,
D = diameter of armature at gap,
Q = number of phases,
I = effective amperes per phase at rated load,
S = turns in series per phase.

Then, for a uniformly distributed winding, such as is used in two- or three-phase machines, and all conductors of each phase in series.

$$S = \frac{\pi\sigma D}{2QI}.$$

If the conductors are arranged in 2, 3, or more parallel paths, the factor 2 in the denominator should be changed to 4, 6, etc., respectively. For single-phase machines *see special treatment* below.

Number of Slots. Pitch of Slots (s). — The number of slots per pole in the armature of a two-phase generator may be any multiple of 2, and in a three-phase generator any multiple of 3. The pitch of the slots for any assumed number of slots per pole is

$$s = \frac{\pi D}{p \times (\text{slots per pole})},$$

where D is the diameter of the armature at the gap and p the number of poles.

Number of Conductors per Slot is $\sigma s/I$, where σ is the number of ampere-conductors per inch of periphery, s the slot pitch, and I the effective full-load amperes per phase.

Size of Conductors. — The size of each conductor is a compromise between a size which will give a reasonable current density in the conductor, and a size which will properly fill a reasonable slot (*see discussion of magnetic circuit below*). The current density in the copper conductor should be from 2500 amperes per square inch in small machines to 2000 in medium and moderate voltage and 1600 in high-voltage machines.

The conductors are arranged in the slots so that the slots are usually about four times as deep as they are wide. The size of the slots should be such as to allow a space for insulation over and above the space occupied by the conductors and such cotton covering as they may have. The space required for this extra insulation in straight slots is as follows:

Voltage	Coil sides per slot	Insulation, inches	
		Vertical	Horizontal
500	2	0.350	0.150
2,000	2	0.430	0.180
3,000	2	0.50	0.30
5,000	1	0.75	0.375
6,600	1	0.835	0.50
13,000	1	1.125	0.75

For example, the necessary vertical depth of slot may be found in a machine insulated for 2000 volts with 2 coil sides per slot by multiplying the over-all

diameter of cotton-covered wire, or depth of bar, by the number of conductors in a vertical row and adding 0.43 to the product.

Slot Factor. — The ratio of cross section of copper in a slot to cross section of slot is known as the slot factor. Normal values of the slot factor for a 1000-kilowatt machine are given in the accompanying table. For smaller machines the slot factor is slightly less and for larger machines slightly greater.

Voltage	Slot factor
500	0.5
1,000	0.46
2,000	0.42
4,000	0.35
6,000	0.33
10,000	0.30

Preliminary Dimensions of Magnetic Circuit. — The diameter of the armature and the axial length of the pole face for the preliminary design are fixed by the above calculations. There remain to be determined the air gap, the dimensions of the armature teeth, the length and radial depth of the armature iron, the dimensions of the field poles, and the dimensions of the field yoke. Preliminary values for these quantities may be arrived at as indicated below.

Air Gap. — The minimum clearance between armature core and field poles in modern generators is given in the accompanying table. The average length of the air gap, due to the chamfer on the poles, is about 25 per cent greater.

Armature Diameter, in.	Minimum air gap, in.
40	0.125
80	0.160
120	0.200
160	0.312
200	0.440
240	0.500

Armature Teeth. — The dimensions of the armature teeth depend upon the number and dimensions of the slots, which in turn must be such as to contain the necessary conductors. Another important limitation to the width of the teeth, however, is the value of the magnetic flux density therein. Let s = pitch of slots, a = pole arc, g = average radial depth of air gap. Then the effective number of teeth under each pole is $(a + 2g) \div s$. The effective width of the tooth is its width at a section one-third the distance from its minimum width towards its maximum width. The effective axial length of the teeth is approximately equal to 0.9 of the axial length of the pole face L determined above. From these data the effective cross section of the teeth perpendicular to the flux lines is determined, and this divided into the total flux per pole ϕ gives the average flux density in the teeth. The value of ϕ from the preliminary calculations above is $\pi \rho LDB \div p$, where ρ = ratio of pole arc to pole pitch, D = diameter of armature at gap, B = average magnetic flux density in gap, and p = number of poles. The average flux density in the teeth in modern alternating-current generators ranges from 90,000 to 110,000 lines per square inch.

Armature Core. — The axial length of the armature core is usually slightly greater than the axial length of the pole face. A fair allowance is twice the radial depth of the air gap. The core is usually built up of sheet-steel punchings, 14 mils thick, the punchings having received previously a thin coat of varnish. Ventilating ducts from 3/8 to 1/2 inch in width (measured along the shaft) are usually provided, these ducts dividing the core into sections from 2.5 to 3 inches thick (measured along the shaft). The effective axial length of the armature iron is therefore the total length of the armature core less the space occupied by the air ducts and insulation between punchings. This latter is about 10 per cent of the net length after deducting the space occupied by the air ducts.

One-half the total flux per pole passes through each section of the armature

core. The flux density in the armature core of modern alternating-current generators ranges from 50,000 to 70,000 lines per square inch. Let B_a = the value of the flux density chosen, l = effective axial length of armature iron, ϕ = flux per pole. Then the necessary radial depth of the armature core is $\phi \div 2 B_a l$.

Field Poles. — The axial length of the pole face f is about 0.5 in. less than L, the length of armature. The pole arc a has already been determined. The radial length of the magnet core (c in Fig. 3, see below) and the width of the magnet core (b in Fig. 3) have yet to be determined. The width of the magnet core is usually from 65 to 75 per cent of the pole arc (see Fig. 3), and the radial length in modern machines from 50 to 150 per cent of the width. The dimensions chosen must be such that the core will accommodate the field spool, the approximate size of which is determined as follows:

The number of ampere turns required per pole to force the flux through the air gap is: $0.313\,\phi g \div af$, where ϕ = flux per pole; g = average radial depth of gap; a = pole arc, and f = axial length of pole face. The ampere turns required for the rest of the magnetic circuit is about 40 per cent of this for most 25-cycle machines and 20 per cent of this for most 60-cycle machines. Hence the total net ampere turns required to produce the flux ϕ is approximately $0.44\,\phi g \div af$ for 25-cycle machines, and $0.38\,\phi g \div af$ for 60-cycle machines. The actual ampere turns required on each field spool at full load, however, will be greater than this, on account of the armature reaction, by an amount depending upon the armature ampere turns per pole and the power factor. For preliminary calculations of the field winding space, however, it is sufficient to increase the net ampere turns as above calculated by about 50 per cent, giving as the approximate total number of ampere turns per pole $F = 0.66\,\phi g/aL$ for 25 cycles and $F = 0.57\,\phi g \div aL$ for 60 cycles.

It is usual to allow a current density of 1000 amperes per square inch in the field copper. The cross section of the copper in the winding is then $F \div 1000$. The total cross section of the winding including the cotton insulation and interstices will be 40 per cent greater than the cross section of the copper when wire is used, and 15 per cent greater when strip is employed. The field pole may then be laid out to accommodate this winding, which is usually wound on a spool, allowing 3/8 inch for the insulating collar at each end, and 0.25 inch for the thickness of the spool and protecting material.

In deciding upon the width of the core, the flux density in the core should also be considered. This should not exceed 100,000 lines per square inch in a laminated steel core. Let ϕ = useful flux per pole, ν = leakage factor, f = axial length of core, b = width of core, then the flux density is $B_c = \nu\phi \div bf$. The value of ν in modern generators ranges from 1.1 to 1.5.

Field Yoke. — The diameter of the armature and the radial length of air gap and field magnet cores fixes the internal diameter of the field yoke (or external diameter in case of revolving-field type). The radial depth of the yoke, which is usually of cast iron or cast steel, is determined by the flux density desired. Let ϕ = useful flux per pole, ν = leakage factor, e = axial length of yoke, and B_y the flux density in the yoke, then the radial depth is $\nu\phi \div 2 B_y e$. In modern machines B_y ranges from 25,000 to 30,000 lines per square inch in cast-iron yokes, and from 70,000 to 80,000 lines per square inch in cast-steel yokes.

Field Winding. — It is unnecessary to attempt an accurate calculation of the field winding until a design of the armature and magnetic circuit has been adopted which will give the required regulation. The method of calculating the regulation and determining the number of field ampere turns is given in the next section.

After the excitation in ampere turns per pole required for various loads and power factors desired (according to the specifications or the service required of the generator) has been determined, that condition of operation which requires the greatest number of ampere-turns F is selected. A field winding which will give this excitation and a margin for safety, with the exciter voltage specified, is then designed. Using this value of F and a current density of 1000 amperes per square inch, the cross section of the field winding is $1.4\,F \div 1000$ when made of insulated wire, or $1.15\,F \div 1000$ when made of insulated strip. The length of the coil will be $c - 0.75$ (see Fig. 3), allowing 3/8 inch at each end for the collar and insulation. Therefore the depth of winding will be for wire $\delta = 1.4\,F \div 1000\,(c - 0.75)$ + thickness of spool on which the wire is wound, or $\delta = 1.15\,F \div 1000\,(c - 0.75)$ + thickness of spool. The spool is usually 0.25 inch thick.

The radiating surface of the coil is then

$$A = 2\,c\,(f + b + 4\,\delta).$$

Resistance and Temperature Rise of Field Winding. — The rise in temperature per watt radiated per square inch of surface of a field coil depends on the peripheral speed of the revolving field or revolving armature. The accompanying table gives the temperature rise in degrees centigrade per watt radiated per square inch. For other rates of radiation the temperature rise is proportional. Let w = watts radiated per square inch for the assumed temperature rise (usually 45° C.), A = area of radiating surface, then the allowable loss per pole is wA, and the field current is then $I_f = wA \div v$, where v = volts per pole. The volts per pole should be taken as 80 per cent of the exciter voltage divided by the number of poles (all field spools in series), allowing 20 per cent for emergencies (low speed, etc.). The number of field turns per pole is then $t = F \div I_f + 0.5$, the half turn being added for convenience in connecting up. The proper resistance (r) of the field winding is then $wA \div I_f^2$, and the necessary cross section (q) of the conductor in square inches is

	Peripheral speed, ft. per min.	Temp. rise °C. per watt per sq. in.
Revolving field	1,000	45
	2,500	40
	5,000	25
	10,000	15
Revolving armature	1,000	80
	2,500	60
	5,000	45
	10,000	30

$$q = \frac{0.0186\,t\,(f + b + 2\,\delta)}{12{,}000\,r}.$$

(The resistance of 1000 feet of copper 1 square inch in cross section at 60° C. is 0.0093 ohm and $2\,(f + b + 2\,\delta)$ is the mean length of turn.) From this cross section the nearest size of wire may be chosen and r, w and volts per pole calculated by reversing the above procedure.

The cross section of the winding should now be recalculated, and the necessary adjustments made to make it fit the winding space available on the field poles.

Bearings, Shafts, Bedplates, etc. — Alternating-current generators may be belt-driven or direct-connected and the latter type may have either a horizontal or vertical shaft.

Belt-driven generators are only used in the smaller sizes up to 500 kilowatts. The smallest sizes usually have two bearings and an overhanging pulley. The larger sizes frequently have three bearings and the weight of the pulley and pull of the belt are taken by two of the bearings.

Generators with horizontal shafts for direct connection to water wheels are

usually built with two bearings and a coupling and are complete in themselves, i.e., they have bedplates, shafts and bearings.

Generators with horizontal shafts for direct connection to steam engines are usually built without bedplates, shafts or bearings. The generator frame is supported on the foundation by a suitable sole plate. The engine bearing serves for one of the generator bearings. The revolving field is carried by an extension of the engine shaft. The shaft and bearings are usually furnished by the engine builder and the revolving member of the generator is pressed on the shaft at the factory where the generator is built.

Vertical shafts are sometimes used with water-wheel-driven generators, and in this case the weight of the revolving part of the generator may be balanced by the upward thrust of the water jet.

In some machines of the revolving-field type the magnetic yoke is made extremely large to serve as a flywheel. Such machines are known as flywheel generators.

For the purpose of preparing the foundations for the generator, the manufacturer usually supplies in advance a template showing the shape and size of bedplate, number, size and spacing of bolt holes.

PREDETERMINATION OF PERFORMANCE OF SALIENT-POLE GENERATORS. — From the above calculations a preliminary drawing to scale of the machine may be laid out. The next step is to calculate its performance, i.e., predetermine what will be the regulation, the efficiency and the temperature rise in the various parts.

The calculations for a quarter-phase or a three-phase machine are very similar and are given together in the paragraphs immediately following. The special features of a single-phase machine are described below. Examples of specific designs and the tested performance are also given below.

Magnetization Curve. — The first step is the calculation of the magnetization curve, i.e., a curve showing the relation between the voltage per phase at no load (at normal speed) and the field ampere turns. To do this the useful flux per pole corresponding to any given value of the no-load voltage is first calculated, and then the field ampere turns per pole required to produce this flux are determined. A sufficient number of points to give a magnetization curve up to 20 per cent above rated voltage should be calculated.

Useful Flux (i.e., the flux cut by the armature conductors.) — Let

E = a given value of the terminal voltage per phase with zero armature current,

f = frequency in cycles per second,

S = number of turns in series per phase,

k_1 = a constant, depending on the shape of the pole shoe, which may be called the "pole shoe constant,"

k_2 = a constant, depending on the distribution of the armature winding, which may be called the "winding-distribution constant,"

k_3 = a constant, depending on the pitch of the armature coils, which may be called the "pitch constant."

Then the useful flux per pole entering the armature is

$$\phi = \frac{E \times 10^8}{4.44\, k_1 k_2 k_3 f S}.$$

Pole Shoe Constant (k_1). — This constant is proportional to the form factor of the flux distribution around the periphery of the armature. In all modern machines the pole faces are so shaped that this distribution is practically a sine wave, and therefore $k_1 = 1$. This is usually done by making the air gap at the pole tips greater than at the center of the pole.

One method of doing this is to make the outline of the pole face a portion of a circle of such a radius (less than the radius of the armature in revolving field machines) that the gap at the tips is twice the gap at the center.

For very accurate predeterminations the distribution of the flux is carefully calculated. (*See S. P. Thompson, Dynamo Electric Machinery, Vol. II, p. 206; C. A. Adams, Trans. A.I.E.E., Vol. 33.*)

Winding Distribution Constant (k_2). — This constant allows for the fact that if there is more than one slot per pole per phase, the conductors in the various slots of one phase under a pole do not generate e.m.f.'s of exactly the same phase. Since the slots pass under the pole consecutively the e.m.f.'s of the conductors reach a maximum consecutively. These e.m.f.'s must therefore be combined vectorially and not merely added together. k_2 is different in single-phase, two-phase and three-phase machines.

The following table gives the values of k_2 for uniformly distributed windings with equally spaced slots.

Slots per pole	Value of k_2		
	1 phase	2 phase	3 phase
1	1.000	.....	.....
2	0.707	1.000	.....
3	0.666	.....	1.000
4	0.65	0.925	.....
6	0.64	0.912	0.966
8	0.64	0.905	.....
9	0.64	.....	.....
12	0.64	0.90	0.960
18	0.64	0.90	0.960
24	0.635	0.90	0.958
∞	0.632	0.90	0.958

These constants apply to a winding uniformly distributed around the armature periphery. This is always the condition in a two- or three-phase machine. A single-phase machine usually has its working winding irregularly distributed. so the above constants are only of theoretical interest. (*See special treatment of single-phase machines below.*)

Pitch Constant (k_3). — It is sometimes desirable to use coils having a fractional pitch, particularly in machines of large pole pitch, in order to save copper and I^2R loss, and also in any machine to give a particularly good wave shape. When this is done each turn connects in series two conductors generating e.m.f.'s which are not in phase and their resultant is therefore not as great as their arithmetic sum. The constant k_3 is the ratio of this resultant or vector sum to the arithmetic sum of the two e.m.f.'s. The relation between k_3 and winding pitch expressed as a percentage of the pole pitch is given in the accompanying table.

Per cent pitch	k_3
100	1.00
83	0.97
80	0.95
75	0.93
67	0.87
50	0.71

Leakage Factor (ν). — The leakage factor, i.e., the ratio of the total flux produced by the field to the flux which enters the armature,

may be determined by calculating the permeance of the path of the armature flux and the permeance of the various leakage paths. The sum of the permeances of the main and leakage paths, divided by the permeance of the main path, is then the value of ν.

Referring to Fig. 3, the average radial depth of the air gap is approximately 1.25 g, and the reluctance of the gap is therefore 1.25 g/af. The reluctance to the useful flux of the iron part of the magnetic circuit is about 20 per cent of that of the gap in 60-cycle machines, and 40 per cent in 25-cycle machines. Hence the approximate value of the permeance of the path of the main flux is

$$P_0 = \frac{af}{1.5\,g} \text{ for 60 cycles, and } P_0 = \frac{af}{1.75\,g} \text{ for 25 cycles.}$$

The flux emanating from or entering each side of a pole has a path l_1 inches long and cf square inches in cross section. The permeance of this path to the plane midway between a pair of poles is $2\,cf/l_1$. There are two of these paths in multiple, one in each direction, from opposite sides of each pole. The total permeance of this path per pole is therefore $4\,cf/l_1$. If a uniform m.m.f. acted on this path at all points the flux would be proportional to this permeance, but since the m.m.f. varies from 0 at the yoke to the full m.m.f. per pole at the pole shoe, the average m.m.f. is one-half the m.m.f. per pole. The leakage flux through this path is therefore proportional to the m.m.f. per pole and to one-half this permeance. Hence the "effective permeance" of this path is

$$P_1 = \frac{2\,cf}{l_1}.$$

The same reasoning applies to the flux which leaks out from the end surface bc (see Fig. 3) of the poles, giving as the effective permeance of this path

$$P_2 = \frac{2\,bc}{l_1 + b/2} = \frac{2\,bc}{l_2}.$$

The leakage betwe n the pole tips is due to the total m.m.f. per pole, and the permeance of this path is therefore

$$P_3 = \frac{4\,c_2 f}{l_3}.$$

The leakage from the faces of the poles at the chamfer is also due to the total m.m.f., and the permeance of this path is

$$P_4 = \frac{4\,c_3 f}{l_4}.$$

The leakage factor is then

$$\nu = 1 + \frac{P_1 + P_2 + P_3 + P_4}{P_0}. \tag{3}$$

Ampere Turns. — The ampere turns required to produce the useful flux ϕ may now be calculated by the following systematized procedure. The symbols are: ϕ = useful flux per pole; ν = leakage factor; T = pole pitch; l = effective length of armature iron; s = pitch of slots at gap; D = diameter of armature at gap; D_1 = outside diameter of armature iron. Other dimensions as shown in Fig. 3. All dimensions are in incl.es.

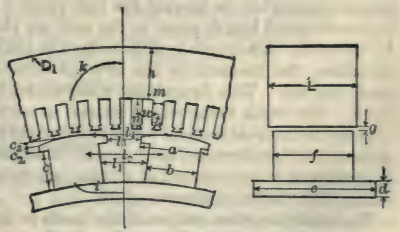

Fig. 3. Dimensions of Magnetic Circuit

Prepare a table like the following:

Part	Flux	Cross section, A	Flux density $=$ flux$/A$	Ampere turns per inch, m *	Length of path, λ	Total ampere turns $= m\lambda$
Field yoke....	$0.5\ v\phi$	de	......	......	i	......
Field pole.....	$v\phi$	$0.95\ bf$	......	......	c	......
Air gap........	ϕ	af	......	......	See below	......
Arm. teeth....	ϕ	See below	......	......	n	......
Arm. core......	$0.5\ \phi$	hl	......	......	k	......

* The value of m is found from the B–H curves in the article on *Magnetic Properties* when the flux density has been calculated.

Field Yoke. — The field yoke carries only half as much flux as the pole piece, as the flux divides at this point. The material is usually cast iron or cast steel. Find the magnetic density as indicated and refer to the proper magnetization curve to find the ampere-turns magnetizing force per inch for this density (see Fig. 4). The length of path is one-half the distance from the center of one pole to the center of the adjacent pole and is shown by i in Fig. 3. Find the total ampere turns as indicated.

Field Pole or Magnet Core. — This carries all the flux. The material is usually sheet steel of high permeability, but sometimes solid steel (formerly). The factor 0.95 is used for laminated steel poles; for solid poles this factor is of course unity. The length of this path is usually taken as c, the length of the space for the field spool. This is not strictly accurate, but the density in the pole shoe (c_2) is so low that the excitation required for this part is negligible.

Air Gap. — All the flux in the pole piece does not cross the gap, as some leaks across the interpolar space. The value of the flux in the gap is taken the same as the useful flux in the armature. For the area of gap section the cross section (af) of the pole shoe is taken. The length of the path in the air gap is taken as the mean of the gap length. As the maximum length (at the pole tips) is usually made twice the minimum, and the outline of the pole face is made the arc of a circle, the average length is usually 1.25 times the minimum. The ampere turns per inch are $0.313 \times$ (density per square inch).

The total ampere turns calculated as above indicated is sometimes multiplied by a constant (0.9 to 1.1), to allow for the spreading of the flux into the slots.

Armature Teeth. — The flux is confined at any one time to a certain portion of the teeth per pole known as "teeth under one pole." Since the flux spreads somewhat on leaving the pole piece it is logical to assume that it takes up a peripheral length equal to the pole arc plus twice the length of the air gap. If, therefore, this length is divided by the pitch of teeth at the periphery of the armature, the quotient is the average number of teeth carrying flux at any given instant. This figure may quite properly contain a fractional number of teeth. The teeth being wedge shaped or sectors of a circle, that cross section (not the mean cross section) which will give the average excitation must be chosen, since saturation increases more rapidly than the cross section decreases. A good approximate value is found at a point one-third the distance from the minimum width towards the maximum width.

The effective cross section of the teeth is then equal to this width multiplied by the product of the effective length of the core by the number of the "teeth

under one pole." The effective length of core is the net length of iron in the core after deducting the space occupied by air ducts and insulation between sheets of steel. This latter is usually 10 per cent of the measurable length of iron. The effective length (l) = 0.9 × (total length of armature core less space occupied by ducts).

Armature Core. — The flux divides again in the armature core, one-half the useful flux being in each section of the armature core. The core is made of annealed sheet punchings. The cross section of core is equal to the radial depth of core back of the slots multiplied by the effective length of iron in the core. The length of path is a little greater than one-half the pitch of poles at this radius, and is as shown at (k), Fig. 3.

The total ampere turns per pole as thus calculated, corresponding to the chosen value of the voltage per phase give one point on the magnetization curve. As noted above, a sufficient number of points should be calculated in the same manner to enable one to extend the saturation curve up to about 120 per cent of rated voltage per phase.

Armature Resistance. — The length of wire in the armature winding is estimated from the mean length of one turn and the number of turns. Let L = total length of armature core in inches; γ = per cent pitch of coils; D = diameter of armature at gap; p = number of poles; S = number of turns in series per phase; a = cross section of conductor* (wire or strip); m = number of parallel paths per phase; k = 10 for low-voltage machines, and 12 for high-voltage machines (k allows for the curves in the ends of the coils). Then the mean length of turn is

$$l = 2L + \frac{k\gamma D}{p},$$

and the resistance *per phase* is

$$R = \frac{0.0093\, lS}{12{,}000\, am},$$

where 0.0093 is the resistance at 60° C. of 1000 feet of conductor having a cross section of 1 square inch. 60° C. is the approximate temperature of the armature conductors at full load.

This is the resistance per phase to a direct current. The alternating-current resistance, due to eddy currents and hysteresis, is about 15 per cent greater; see paragraph on *Load Loss*, below.

In a single-phase or two-phase machine this resistance is the same as the resistance between terminals. In a Y-connected three-phase machine the resistance between terminals is twice the resistance per phase. In a Δ-connected three-phase machine the resistance between terminals is two-thirds of the resistance per phase.

Armature Leakage Reactance. — The load current in flowing through the armature conductors sets up a local magnetic flux which interlinks with the armature conductors, producing inductance. This inductance causes a loss of voltage and a "dephasing" effect, or lag of current behind the e.m.f.

The inductance L of any circuit, expressed in henries, is

$$L = 1.016\pi S^2 P \times 10^{-8},$$

where S = number of turns in series,

P = permeance of flux path in inches.

* If pressed cable is used, the cross section of the copper in the cable is approximately 82 per cent of the cross section of the cable, exclusive of insulation. The length of each of the wires forming the cable is about 7 per cent greater than the length of the cable.

The corresponding reactance is $2\pi fL$, where f is the frequency in cycles per second.

There are several paths for this so-called armature leakage flux, each path surrounding one or more slots, namely, the path around each slot, the path around a group of slots of one phase, the path around the end connections. The effect of the flux of one phase on the conductors of another (mutual inductance) must also be considered.

The slots lying under a pole have a path of greater permeance than those lying opposite the interpolar space or between poles, as the former are more completely surrounded by iron.

With the exception of the path around the end connections, the permeance of each path is readily calculated by the same process as used above in calculating the leakage factor. The path around the end connections is so complex and the permeance so small compared with that in the core proper, that it is convenient and sufficiently accurate to add a percentage for this flux. This is done by allowing for every inch of the projecting length of the end connections one-tenth as much flux or inductance as for an inch of the embedded portion of the conductors.

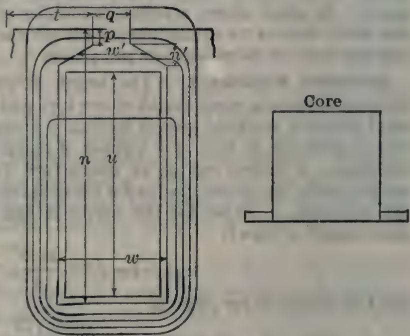

FIG. 4. Leakage Flux

In calculating the permeance of the various paths only the length of the path in air is considered, as the reluctance of the path in iron is so small compared to the path in air as to be negligible.

Let l = effective length of armature core; l' = length of end connections (one end); P_1 = the total permeance of all the leakage paths for a slot under a pole piece; P_2 = the total permeance of all the leakage paths for a slot midway between two poles, and the other quantities as in Fig. 4. Then

$$P_1 = \left(\frac{u}{3w} + \frac{n'}{w'} + \frac{p}{q} + \frac{t}{2g}\right)\left(l + 0.1\,l'\right)$$

$$P_2 = \left(\frac{u}{3w} + \frac{n'}{w'} + \frac{p}{q} + \frac{t}{t+q}\right)\left(l + 0.1\,l'\right)$$

In addition to the above symbols, let f = frequency in cycles per second; p' = the slots per pole per phase; s' = slots in series per phase; c = effective conductors per slot = $2 \times$ (turns per phase)/slots per phase; k_3 = "pitch constant" (*see above*). Then the effective reactance in ohms per phase corresponding to the permeance P_1 is

$$x_1 = 20.1 f p' c^2 s' k_3 P_1 \times 10^{-8},$$

and corresponding to the permeance P_2 is

$$x_2 = 20.1 f p' c^2 s' k_3 P_2 \times 10^{-8}.$$

For reactance of turbo-generators see p. 658.

Reactance Drop.— For preliminary or approximate calculations it is better and more conservative to use the value found for the inductance of the slots "under the poles," as this gives a greater reactance and voltage loss, and, as the poles usually cover about two-thirds of the armature periphery, this is

nearer correct. That is, the armature reactance drop per phase is taken as Ix_1, where I is the armature current per phase.

For more accurate calculations both values must be used, and the power component and reactive component of the current considered separately. Let θ be the power-factor angle and I the armature current per phase. Then the reactive drop per phase is

$$IX = I\sqrt{(x_1 \cos\theta)^2 + (x_2 \sin\theta)^2},$$

which is therefore equivalent to taking for the effective armature reactance

$$X = \sqrt{(x_1 \cos\theta)^2 + (x_2 \sin\theta)^2}.$$

Armature Reaction. — When current flows through the armature the armature winding becomes the seat of a magnetomotive force which reacts on the field m.m.f. and either distorts or diminishes the useful flux. If the current in the armature is in phase with the generated e.m.f. it causes a "cross magnetizing" force acting along an axis passing midway between any pair of poles.

As the phase of the current in the armature changes, the direction of the magnetizing force due to the armature m.m.f. shifts, and a component is introduced either opposed to the field magnetizing force (for lagging current) or assisting the field magnetizing force (for leading current). In a polyphase generator the magnetizing force due to the armature m.m.f. for a given armature current is constant in magnitude and has a fixed direction with respect to the field magnetizing force, depending on the phase of the current.

If the field iron had no polar projections nor interpolar spaces, the direction of the armature magnetizing force would be such that the angle between this magnetizing force and a line perpendicular to the field magnetizing force would be equal to the phase angle between the induced voltage and current.

With the usual type of alternator having interpolar spaces the reluctance of the path offered to the armature m.m.f. is intentionally much greater than that offered to the field m.m.f. The result of this is that the effect of armature reaction is minimized. The non-uniformity of the field iron, however, changes the relative directions of the two magnetizing forces and renders accurate calculations difficult. As a rule, however, the approximation resulting from the assumption of uniform distribution of field iron is sufficiently accurate. The error introduced by this assumption is on the safe side, since the armature reaction as thus calculated is greater than its actual value.

Armature-Reaction Ampere Turns per Pole. — Let S = number of turns in series per phase; I = effective value of armature current per phase; p = number of poles, and k_3 = pitch constant of the winding (*see above under Magnetization Curve*). Then the armature ampere turns per pole effective in producing armature reaction are

$$\frac{\sqrt{2}\,k_2 k_3 S I}{p}, \quad \text{for two-phase machine,}$$

$$\frac{1.5\sqrt{2}\,k_2 k_3 S I}{p}, \quad \text{for three-phase machine.}$$

As noted above the armature reaction for a given effective value of the current is constant in magnitude. The armature reaction in single-phase generators is given in the discussion of these machines below.

The armature-reaction ampere turns of different machines at full rated load is greater for a high than for low pole pitch, and is less for high frequencies than for low frequencies. The permissible armature-reaction ampere turns depend, of course, upon the desired regulation.

A reasonable value for armature reaction would be between 1500 and 5000 ampere turns per pole for a 25 cycle machine and between 1000 and 2000 for a 60 cycle machine.

Synchronous Reactance. — The effect of the armature leakage reactance and armature reaction upon the terminal voltage of a generator at given field excitation are of like character, since the voltage induced in the armature due to each of these causes is in quadrature with the current. The "synchronous reactance" of a generator is the equivalent reactance which would produce the same effect as the armature leakage reactance and armature reaction combined. The synchronous reactance may be predetermined by finding the excitation ampere-turns (from magnetization curve) corresponding to a voltage equal to the drop due to leakage reactance and adding to these ampere turns the armature reaction ampere turns. The voltage from the magnetization curve corresponding to this sum divided by the armature current per phase gives the synchronous reactance per phase. In the calculation of regulation by the "magnetomotive force method" (see below), however, it is more convenient to express the synchronous reactance in terms of the excitation ampere turns to overcome it.

Regulation. — The regulation of a generator machine is defined as the ratio of the difference in terminal voltage at no load and at full load to the full-load voltage, the field excitation being kept constant at its full-load value. Expressed as a percentage the regulation is $100(V_0 - V) \div V$ where V is the full-load terminal voltage and V_0 the no-load terminal voltage at full-load field excitation. Two different methods have been employed for the calculation of regulation, one known as the "magnetomotive-force" method, and the other as the "electromotive-force" method. Both are approximations, the first giving a value lower than the actual value, and the second a value higher than the actual value. More accurate methods are given in the *Standardization Rules of the A.I.E.E.*, which see.

Magnetomotive-force Method. — One way in which this method has been applied is the following: Let V = terminal voltage per phase at full load; R = alternating-current resistance per phase in ohms (= 1.15 times direct-current resistance); I = full load amperes per phase; $\cos\theta$ = power factor of the load; X = leakage reactance per phase in ohms. Calculate

$$V' = \sqrt{(V\cos\theta + RI)^2 + (V\sin\theta + XI)^2} \quad \text{and} \quad \theta' = \tan^{-1}\left[\frac{V\sin\theta + XI}{V\cos\theta + RI}\right]$$

taking θ' positive for I lagging behind V'. From magnetization curve find m = excitation ampere turns corresponding to V' and let n = armature reaction ampere turns per pole for current I (see p. 635). The total field ampere-turns is then

$$F = \sqrt{m^2 + n^2 + 2\,mn\sin\theta'}.$$

Let V_0 = voltage per phase from the magnetization curve corresponding to this excitation F; then the regulation is

$$100\,\frac{V_0 - V}{V}.$$

The difference between V' and V and between θ' and θ is usually quite small, and in preliminary calculations m may be taken as the excitation ampere turns corresponding to V, and θ' may be taken equal to θ.

Electromotive-force Method. — Let V = terminal voltage per phase at full load; R = alternating-current resistance per phase in ohms (see preceding paragraph); XI = volts per phase from magnetization curve correspond-

ing to the field excitation required to send full-load current through armature on synchronous impedance test; I = full-load amperes per phase; $\cos\theta$ = power factor. Then the voltage per phase E at no load, corresponding to the full-load excitation, is the vector sum of V, RI and XI, or

$$E = \sqrt{(V\cos\theta + RI)^2 + (V\sin\theta + XI)^2}.$$

The regulation is, as before, 100 $(E - V)/V$, and the full-load field ampere turns F is taken from the point corresponding to E on the magnetization curve.

Losses. — The losses in a synchronous alternating-current generator or motor are:

(*a*) Friction, bearing and windage.
(*b*) Excitation or field copper loss.
(*c*) Core loss.
(*d*) Armature copper loss.
(*e*) Load loss.

Of these the first three are approximately constant for various loads, but the last two vary as the square of the load current.

Friction and Windage depend in magnitude upon the details of the physical or mechanical construction, the amount of induced ventilation and speed. It is impossible to give a method of predetermining this quantity which will apply to various designs and makes. Each manufacturer has an empirical formula for each line of machines. This loss ranges from 2.5 per cent in 100 kv-a machines to 0.5 per cent in 10,000 kv-a. machines. These values are for complete machines with their own bearings (two in number). Some machines are designed with only one bearing, the other bearing being a part of the prime mover (hydraulic or steam) in which case the friction chargeable to the generator is less. Some machines with devices to produce ventilation, such as fan blades attached to the revolving part, have greater friction losses.

Core Loss. — The core loss is made up of hysteresis and eddy-current losses. These losses are principally in the armature core and teeth, but if proper care is not taken there may be a considerable loss in the frame of the machine and the pole shoes.

It is a simple matter to calculate the magnitude of these losses in the armature core proper, as the frequency and flux density are definite in this part, but the losses in the teeth are due not only to the fundamental frequency and main flux, but also to pulsations due to the passage of pole tips past the teeth and the leakage flux of the armature. The total core loss is the sum of the hysteresis and eddy-current losses. See *Magnetic Properties of Iron and Other Metals* for curves of hysteresis and eddy-current losses and formulas for their calculation.

Excitation Loss. — The calculation of the field current and of the resistance per pole of the field winding is given above in the section on *Field Winding*. The total power required for excitation will be the product of this resistance, the square of the current and the number of poles.

If the machine is separately excited, which is usually the case, the losses in the field rheostat are, by convention of the Am. Inst. of E. E., not chargeable against the generator.

Armature Copper Loss. — The calculation of the direct-current resistance per phase of the armature winding is given above in the section on *Armature Resistance*. The total armature copper loss is equal to the number of phases times this direct-current resistance times the square of the current per phase.

Load Loss. — When a current flows in the armature conductors a local

flux is set up which will cause eddy currents in these conductors, if they are not well subdivided, and in the surrounding iron, as well as a hysteresis loss in the surrounding iron. The loss due to this load flux is called the "load loss."

The load loss is a function of the leakage flux and the subdivision of the conductors. A large number of turns of fine wire will involve a very small loss. If the conductors must be large, they may be made of stranded cable pressed to shape. It is almost impossible to calculate this loss, and very difficult to measure it. A rough method is to assume the resistance per phase increased by 15 per cent, as this loss, like the true copper loss, is proportional to the square of the armature current. The total amount of the loss is less than 1 per cent of the input in well-constructed machines.

Efficiency. — Let P = total output in kilowatts; R_a = resistance per phase of armature; I_a = armature amperes per phase; R_f = resistance per pole of field winding; I_f = field current; q = number of phases; p = number of poles; C = total core loss in kilowatts; and F = friction and windage loss in kilowatts. Then the per-cent efficiency is

$$\frac{100\,P}{P + C + F + (1.15\,R_a q I^2_a + p R_f I^2_f) \times 10^{-3}}.$$

This assumes the load loss equivalent to increasing the armature resistance (as calculated or measured by direct current) by 15 per cent. If the load loss is determined from the short-circuit core loss the formula for efficiency is

$$\frac{100\,P}{P + C + F + L + (R_a q I^2_a + p R_f I^2_f) \times 10^{-3}},$$

where L is the load loss in kilowatts.

The efficiency is a maximum for that load at which the constant losses are equal to the variable losses.

Customary values for the efficiency at full load and each of the losses at full load for various sizes of generators are given in the following table. These values are merely indications and vary with the frequency, voltage, speed, power factor, etc. A 60-cycle low-voltage machine will be likely to have a better efficiency than a machine of the same rating for 25 cycles or high voltage.

EFFICIENCY AND LOSSES, USUAL VALUES

Rating, kv-a.	Efficiency, per cent	Friction, per cent	Excitation, per cent	Core, per cent	Armature,* per cent
100	91	2.5	2.5	2.4	1.6
500	94	1.4	1.5	2.2	1.2
1,000	95	0.9	0.9	2.1	1.0
2,000	96	0.6	0.7	1.8	0.9
3,000	96.5	0.6	0.6	1.7	0.8
5,000	97	0.55	0.4	1.6	0.5
10,000	97.2	0.5	0.35	1.5	0.45

* Copper and load loss.

Heating. — The rise in temperature of the field coils is ascertained as an incidental step in the calculation of the field winding as explained above.

The rise in temperature of the armature is best determined by a method given by Arnold (*see Wechselstromteknik, Vol. IV, p. 141*). In this calculation the losses in the projecting portions of the end windings are assumed to be

radiated by the end windings while the core loss and copper loss in that portion of the winding embedded in the slots are radiated by the surface of the armature core. Let L = length of armature iron in inches; λ = mean length of armature turn in inches (*see Armature Resistance, below*); R_a = armature resistance per phase in ohms; I_a = armature amperes per phase; D_1 = outside diameter of armature punchings in inches; D = inside diameter of armature punchings; z = number of ventilating ducts; q = number of phases; t = rise in temperature in degrees centigrade per watt radiated per square inch of surface. Then the watts to be radiated are

$$P = \text{core loss} + \frac{2\,LqR_aI^2_a}{\lambda},$$

and the radiating surface, including only one-half the area in the air ducts, as the sides of the air ducts are not as effective as the rest of the surface, is

$$A = \pi L(D_1 + D) + \frac{\pi}{4}(D_1^2 - D^2)(2 + z).$$

The rise in temperature by resistance in degrees centigrade is then

$$T = \frac{Pt}{A}.$$

The value of t for a well-ventilated stationary armature ranges from 30 to 40, depending upon the thickness of coil insulation, and the peripheral speed of the field. For a stationary field and revolving armature the value of t given above for the rise in a revolving field may be used.

Checking Calculations. — Substitution in the following formula gives an excellent check on the above calculations:

$$\frac{DL_f}{Q} = \frac{KA \times 10^9}{fB_gna},$$

where D = diameter of rotor at gap in inches; L_f = length of pole face parallel to shaft; Q = total kv-a. of generator; f = frequency in cycle per second; n = armature-reaction ampere turns; a = pole arc in inches; A = diameter per pole in inches; B_g = flux density, lines per sq. in. in air gap; and K = 22.5 for single-phase and 15.9 for two-phase or three-phase generators.

SINGLE-PHASE GENERATOR. — Single-phase generators do not make as effective use of the material as do polyphase generators, for the reason that the armature winding can occupy effectively only about one-half of the peripheral surface of the armature; if it occupies more than this there will be voltages generated in the windings which are so out of phase with each other that the resultant voltage is only from 63 to 70 per cent of the sum of all the voltages generated.

Therefore, if a polyphase generator is used as a single-phase machine with the same magnetic densities and the same copper densities, the output will be much less on account of the lesser voltage available. It is, therefore, customary to overload the magnetic elements of the machine somewhat to raise the voltage and thus reduce the overload on the copper which would be necessary to get the desired output. However, if both the iron and copper densities are increased until the machine gives as much output single phase as it is intended to give polyphase, there will be an increased heating. Thus, for the same heating, obtained by a readjustment of the iron and copper losses, a machine of a given first cost will give about 75 per cent as much output single phase as may be obtained polyphase.

In addition to the disadvantage of a single-phase generator that it cannot

make use of all the periphery of the armature, it also labors under the disadvantage that its armature reaction is pulsating instead of constant, and this introduces an additional loss in the form of eddy currents.

By using two phases in series of a three-phase machine, or one phase of a two-phase machine, a fairly good single-phase machine is obtained. By arranging the winding slightly differently the same number and arrangement of inductors (or coil sides) can be connected up to give a simpler winding requiring no crossings of coils, that is, a winding "in one plane."

The formula for the calculation of the e.m.f. of a single-phase generator is $E = 4.44\ k_2 k_3 f S \phi\ 10^{-8}$, where the symbols, with the exception of k_2, have the same significance as in the formula for polyphase machines, p. 629. The value of k_2 depends upon the portion of the armature periphery (including the teeth between slots) occupied by the main winding. Let A = this fraction of the armature surface, then the corresponding values of k_2 are given in the accompanying table. The value of A corresponding to a uniformly distributed winding is unity.

A	k_2
1	0.63
3/4	0.785
2/3	0.83
1/2	0.90
1/3	0.956

The average value of the armature-reaction ampere turns per pole of a single-phase machine is SI/p, where S = the number of turns, I = the armature current per phase, and p = the number of poles, but this quantity pulsates between 0 and twice the above value. The evil effects of this pulsating may be reduced by employing a short-circuited winding having its axis at 90 degrees to the main field winding. A squirrel cage or "amortisseur" winding in the pole faces is frequently employed for this purpose.

Due to this pulsating action the load losses are greater and this should be taken into account in calculating the efficiency and regulation by considering the effective armature resistance as 1.15 to 1.5 times the direct-current resistance.

The leakage reactance of a single-phase machine pulsates between a value equal to that obtained by the formula above in the paragraph on *Armature Leakage Reactance*, and a value two-thirds as great. Satisfactory results are obtained by multiplying the value obtained from the formula by 0.85 for the effective single-phase value. In this case the whole winding is considered as one phase.

DESIGN OF TURBO-GENERATORS. — In turbine-driven generators the steam turbine is usually built by the same company that builds the generator, and the two machines are practically one unit. The steam turbine has developed to such an extent that for a given capacity in power it weighs less than a reciprocating engine, occupies much less space, costs less and is more economical of steam and fuel. Good economy in steam turbines, however, is obtained only with high angular velocities. This has made it difficult to apply the turbine to useful purposes. The electric generator has shown itself to be the most suitable device to absorb the power of the turbine and make it available in subdivided form and at convenient speeds for general application. However, it required many years of experience to develop an electric generator which would operate successfully at the high angular velocities suitable for direct connection to the steam turbine. The principal difficulty was the design of a construction which would withstand the enormous stresses in the revolving member which resulted from the centrifugal force. Thus the design of the machine as a whole is largely a question of the type and construction of the rotor, which is the field.

Normal Weights and Speeds. — Most turbo-generators have much more copper on the fields and less on the armature than do slow-speed machines, although the total amount of copper is not far different in the two classes. The turbo-generator has much less magnetic iron on account of its high speed. It is quite normal in machines of 5000 kv-a. capacity to operate at 3600 r.p.m. and machines of 20,000 kv-a. at 1800 r.p.m. To do this involves the use of peripheral speeds of from 15,000 to 27,000 feet per minute. At these high speeds the amount of material per kilowatt is much reduced. Increasing the speed ten-fold reduces the weight of material to about one-quarter. This low weight is somewhat offset by the higher cost of construction necessary to withstand the enormous centrifugal forces. Most of the machines, even the large sizes, have two or four poles, or at most six poles. This involves the use of a pole pitch of from 30 to 40 inches in 60-cycle machines, and 60 to 90 in 25-cycle machines. Since such large powers are concentrated in such small bulk, a great deal of energy in the form of losses must be dissipated in a small space. Thus special means of ventilation must be provided, such as numerous air ducts, fan blades on the rotor, or a separate blower and a supply of clean, cool air.

Construction of Rotor. — The design of a turbo-generator differs from that of the salient-pole type described above, because the revolving field of the turbo-generator consists of groups of coils placed in slots and distributed concentrically about the pole centers or cores, which resemble large teeth. The center of a pole occupies about one-third of the pole pitch and has no slots; see Fig. 5. The distributed field winding gives a peaked shape to the flux wave in the air gap, which is made to approach a sine form as near as practicable as

Fig. 5. Rotor Construction

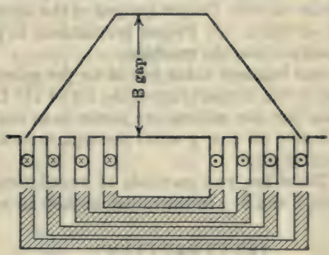

Fig. 6. Field Distribution

shown in Fig. 6, which shows a developed cross-section of one pole. Thus the pole arc is the same as the pole pitch, but the average flux density is approximately 0.63 times the maximum.

The air gap is usually very large in order to provide a path for the large volume of ventilating air required. On account of the high peripheral speeds customary the diameter per pole is high.

The centrifugal force in the revolving member is from 1000 to 1500 pounds for every pound of material near the periphery. A method for determining the centrifugal force in every part is given in *Electric Machine Design*, by Gray, and in *High-speed Dynamo-electric Machinery*, by Hobart and Ellis.

The rotor must be of substantial and rigid construction, so that the critical speed of vibration is above the normal speed of operation. There are several methods of construction, prominent among which are:

a. A solid steel forging turned to shape with radial slots containing a distributed field winding. The shaft is in one piece with the field core; see Fig. 5.

b. A built-up structure consisting of steel discs about three inches thick held

between the two end-plates by through-bolts; the radial or parallel slots being milled in the assembled structure.

c. Steel laminations with radial slots, assembled on a forged steel shaft.

d. Laminated pole pieces attached to a spider by keys. (These are no longer being built although there are a large number in service.)

In these revolving fields the length is about the same as the diameter, and in large machines the length is greater than the diameter. The air gap is from one to two inches. A uniform air gap is necessary to prevent noise and strains.

Construction of Stator. — Since the pole pitch is large, a large number of slots per pole is used (12 to 24) and this gives a low armature self inductance and high short-circuit current. Due to the long pole pitch the end connections are long and subjected to considerable mechanical forces, as a result of the leakage flux. They must, therefore, be held securely in place by nonmagnetic supports. For armature reaction in ampere-turns per pole values of from 8000 to 20,000 are common, and the field winding is usually of a capacity of three times the armature ampere-turns. The regulation is poor (20 to 30 per cent). This defect is readily overcome by the use of automatic voltage regulators (see *Regulators*). The current density in the copper (2000 amperes per square inch) is lower than in slow speed machines, while the constant "ampere conductors per inch periphery" has high values, viz., 600 to 1500.

Provision for Ventilation. — The ventilation of these machines is a problem similar to that of air-blast transformers. Thus there must be provided: (*a*) sufficient air to carry off the heat generated with a reasonable rise in temperature of the machine and the air; (*b*) Ample duct capacity to prevent too high velocity of the air; (*c*) Proper spacing of the ducts so that there is no part very far from an air duct; (*d*) Proper precautions to prevent the air from carrying dirt and moisture into the machine.

One hundred cubic feet of air per minute at 20° C. will carry off 1 kw. with a rise in the temperature of the air of 18° C. Velocities of the air of from 5000 to 6000 feet per minute are common in the machine proper. The surface cooled will give off 4 or 5 watts per square inch with a rise of 35 to 45° C. above the cooling air.

There are two methods of ventilation, the "radial" and the "axial"; see Figs. 7 and 8. In the radial method air enters the air gap at both ends and flows

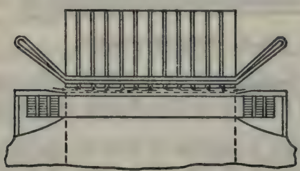

Fig. 7. Radial Ventilation

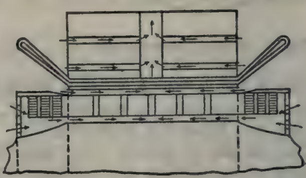

Fig. 8. Axial Ventilation

out radially through air ducts ½ inch wide) at right angles to the shaft. These air ducts are placed in the stator every two or three inches. In the axial method the stator core contains many holes parallel to the shaft through which the air passes, usually entering at both ends and passing out through large radial ducts at the middle of the core. Generally both methods are employed in one machine, and whichever method predominates determines the classification. The larger the outside diameter of the machine, the more appropriate is the axial method, because of the depth of core. The machine is totally enclosed in either case and

the air is taken in at a definite entrance and expelled at a definite exit, usually drawn from outside the power-house and expelled outside also. An excellent discussion of ventilation is given by Lamme in the *Trans. A.I.E.E., Jan., 1913.* See also Knowlton, *G. E. Review, May, 1916;* Newbury, *Trans. A.I.E.E., Nov., 1919;* Lamme and Newbury, *Trans. A.I.E.E., Nov., 1916.*

Suitable Speed and Number of Poles. — For 25-cycle machines, 2 poles and 1500 r.p.m. are used for all sizes including 35,000 kv-a. For 60 cycles, 2 poles and 3600 r.p.m. are used up to 6000 kv-a. 4 poles and 1800 up to 30,000 kv-a. and 6 poles for larger sizes. Table I gives representative values of dimensional constants.

TABLE I.—GENERAL DIMENSIONS AND DESIGN CONSTANTS OF TURBO-GENERATORS

	1	2	3	4	5	6	7
Frequency	25	25	25	60	60	60	60
Poles	2	2	2	2	2	4	6
Kv-a.	2,000	5,000	20,000	2,000	5,000	20,000	30,000
R.P.M	1,500	1,500	1,500	3,600	3,600	1,800	1,200
Diam., rotor, in.	38	38	61	23	25	50	75
Length rotor, in.	35	56	58	46	60	75	94
Stator slots	48	48	72	30	36	72	108
Depth of slots, in.	4.5	5	7	3	4	6	8
Peripheral velocity, ft. per min.	15,000	15,000	24,000	22,000	24,000	24,000	24,000
σ, amp.-cond. per in.	600	900	1,300	600	900	1,300	1,500
n, arm. reaction amp.-turns	11,000	16,000	37,000	6,600	11,000	16,000	15,000
Flux per pole, mega-lines	58	100	175	40	60	83	112
Avg. gap density, kilolines per sq. in.	28	30	32	24	26	28	30
Air gap length, in.	1.	1.25	2.5	0.75	1.	1.5	1.5
Rotor slots	36	36	48	24	24	48	72
Depth of slots, in.	5.	5.5	7.	3.5	4.5	6.	8
Weight of generator, lb.	50,000	100,000	300,000	34,000	62,000	250,000	430,000

Preliminary Calculation of Main Dimensions. — Equation (1), page 634, applies, but customary values of some of the constants are quite different for reasons stated above.

ρ is usually unity with distributed windings.

B, the average density in air gap varies from 25,000 lines per square inch in 60-cycle machines to 32,000 in 25-cycle machines. Maximum density is 1.57 times the average.

V, the peripheral velocity, varies from 15,000 feet per minute in small, low frequency machines to 27,000 in large machines, figured at the surface of the revolving field.

σ, the ampere conductors of the armature per inch periphery of the field, must be figured on the same diameter as *V* in order for the equation to check. σ has values as in Table I.

D = outer diameter of field, in inches.
L = total length of field parallel to shaft, in inches.
k_2 and k_3 the same as on page 634.

Connection of armature winding may be single-phase, two-phase, three-phase Y, or three-phase delta. For all three-phase machines for more than 6600 volts, the Y-connection is usually employed, for the reasons given on page 642.

Number of Turns in Series per Phase (S) is found as on page 637 for a preliminary value.

Armature Reaction Ampere Turns (n) per pole is found as per formula on page 647, and is an important criterion of design in turbo-generators, running very close to the values given in Table I.

Number of Slots and Conductors per Slot are determined by trial from the turns in series by the relation

$$2\,QS = cs,$$

where Q is the number of phases, S the turn in series, c the conductor per slot, and s the total number of slots. With values of $2\,QS$ set, values of $c = 0.5$, 1, 2, 4 and any other even number are tried until the combination which will most nearly satisfy the equation is found. s must be divisible by the number of poles and the number of phases, in order to make a conventional winding. Usual values for s are given in Table I.

Size of Armature Conductor. — For preliminary purposes, a current density of 2000 amp. per square inch may be assumed, which divided into the current per phase gives the cross-section of one "effective" conductor in square inches. By "effective conductor" is meant the conductor which carries the whole current per phase. On account of the large currents in large machines it is frequently advisable to divide the winding into multiple paths, two paths in multiple for instance, then each conductor carries half the phase current and is called half an effective conductor. To reduce eddy currents and facilitate bending, the conductors are usually made up of from 4 to 30 strands of rectangular wire laid parallel and insulated from each other. This makes it possible to build up a conductor of convenient shape, but necessarily reduces the slot factor.

Size of Armature Slots is determined, first, by the space necessary for conductors and insulation, and, second, by the depth which is best suited to the diameter of armature. The slot factor is low for machines of this size for three reasons: (1) high voltage is customary, (2) a considerable part of the slot is left vacant to serve as a passage for ventilating air, (3) the winding is placed deep in the slot to give sufficient reactance to limit the instantaneous short-circuit current. A typical slot would be 6 inches deep by 1 inch wide with the winding and wedge occupying only 5 inches of the depth. Customary slot factors are given in Table II, usual depths in Table I and space allowed for insulation and air in Table II. The dimensions given for insulation include all the insulation on both sides of the coil, that is, an allowance of 0.30 inch means 0.15 inch on each side. The area of the slot is equal to the cross-section of the copper divided by the slot factor and the width of slot is equal to the cross-section divided by a suitable depth as judged from values in Table I.

The Armature Winding is usually a double layer barrel with as many coils as slots and one, two or three turns per coil. The coils usually have a pitch two-thirds that of a pole in order to shorten the end connections, save copper and space, and reduce the copper loss. It is of the lap winding type shown in Fig. 2. There may be as many circuits in multiple in each phase as there

TABLE II.—SLOT FACTOR AND SLOT INSULATION

Machine voltage	Slot Factor	Insulation, inches	
		Depth	Width
2,000	0.35	1.25	0.17
4,000	0.35	1.50	0.25
6,000	0.33	1.50	0.32
10,000	0.30	2.00	0.46
13,000	0.27	2.00	0.55

are poles, in order to reduce the amount of current per circuit. Each turn is usually given two twists so that it lies near the top of the slot on one side and near the bottom on the other side.

Armature Loss. — The probable armature resistance is figured at this point to be sure that the loss in the armature does not exceed that which may be dissipated. The resistance is figured as on page 645, except that the straight portion of each coil extends 1 or 2 inches beyond the core, adding 7 to 8 inches to the mean length of turn. It is customary to figure the resistance of armature at 100° C., at which temperature the resistance of 1000 feet of copper of 1 square inch section is .0107 ohm. The total I^2R loss should be from 0.2 per cent to 0.3 per cent in moderate size machines; less in large machines. See Table IV.

The Total Useful Flux in Armature is found as given on page 641, and has very high values as shown in Table I. This is because the output per pole is so great. The leakage coefficient runs quite uniformly from 1.1 to 1.15 and is low because of the large pole pitch and distributed field winding.

The Magnetic Circuit is adjusted so that the magnetic densities are about as given in Table III. In the stator and rotor teeth and in the air gap the density is not uniform, so that the maximum density (in the center of a field pole) is about 1.57 times the average density (see *Induction Motors, Polyphase*). For a preliminary design this ratio may be assumed.

TABLE III.—USUAL MAXIMUM MAGNETIC DENSITIES

Part	25 cycles	60 cycles
Armature core	90,000	80,000
Armature teeth	100,000	85,000
Air gap	50,000	45,000
Rotor teeth	100,000	100,000
Rotor core	90,000	90,000

Rotor Slots and Construction. — The rotor teeth in the final analysis must be very carefully studied to determine if there is sufficient material at the root to withstand the centrifugal force due to the masses of the tooth and the material in one slot. For the preliminary lay-out a number approximating

those given in Table I is selected, say 36 for a 2-pole machine, and these are spaced as if there were 54 slots, but two groups of 9 each are omitted at symmetrically placed positions, thus forming the pole centers. This will give 18 slots per pole, in which will be placed 9 coils per pole, each of a different pitch and all concentric with the pole center.

For each diameter there is a certain number and depth of slots which gives the best proportion between amount of copper and strength of tooth and this may be judged from Table I. The width of a slot is made approximately 0.5 to 0.66 of the pitch on the circle at the bottom of the slots.

The Length of Air Gap is much influenced by its function in ventilation. For the electrical characteristic of the machine it is found desirable to have the length such that the ampere-turns excitation expended in the gap at no load and normal voltage are about equal to the armature reaction at rated load.

No Load Magnetization Curve. — This is calculated as for induction motors (q.v.) on account of the distributed field winding and the peak wave of flux. The average densities are figured on a basis of full pitch pole arc and all stator teeth per pole carrying flux. The maximum density in gap, stator teeth and rotor teeth is obtained by multiplying the average density by 1.57 for a sine wave. With saturation in the teeth (over 110,000) or with specially wide pole centers, this constant becomes 1.5 or 1.4 respectively.

On account of the long air gap (long compared to slot openings and duct widths) the effect of slots in contracting the flux in the gap is not marked, so that the density in the air gap may be taken the same as at the surface of the rotor. Investigation shows that the increase in density in the air gap due to the slots is only from 2 to 4 per cent and this is balanced by figuring the density at the surface of rotor instead of at the mean gap diameter. The section of rotor teeth is figured at a point one-third the distance from the bottom of the slots and the total section includes the one large center tooth and several smaller teeth.

The average length of the path of the magnetic flux in the two cores is considerably less than one-half the pole pitch on account of the distributed flux. It is usually about 100 to 120 magnetic degrees.

The materials used are generally silicon steel for the stator and forged steel for the rotor. The tendency of the flux to pass through the slots instead of the teeth is not so great in these machines as in other types because, in the rotor, the flux can spread out with saturation, and in the stator, because the densities are not very high. A magnetization curve between no load volts between terminals and ampere-turns per pole is usually plotted up to a voltage 120 per cent of rated voltage.

Armature Leakage Reactance. — The formula used for slow-speed alternators will not apply because of the large number of slots per pole and long end connections and the formula used for induction motors is not suited because of the long air gap.

Arnold (*Vol. IV, p. 8*) gives a formula and proof which has been found to apply very well to present day turbo-generators. Referring to Fig. 4, all dimensions in inches, let P_s be the effective leakage flux per ampere-conductor in the slot, then

$$P_s = 3.2L\left(\frac{u}{3w} + \frac{n'}{w'} + \frac{p}{q}\right),$$

where L = total length of core in inches.

Let P_t be the leakage flux per ampere-conductor from tooth tip to tooth tip, set up by the belt of s_1 slots per pole per phase, then

$$P_t = L\left(2.36 \log_{10} \frac{\pi z}{2q} + A\right),$$

where z = pitch of teeth at face and "A" has values as follows:

s_1 =	1	2	3	4	5	6	7	8	9	12	15
A =	0	.3	.6	1	1.7	2.1	2.7	3.2	3.8	4.8	5.5

Let P_c be the leakage flux per ampere-conductor around the end connections, set up by and interlinking the whole belt of s_1 coils, then

$$P_c = l_c\left(1.18\, s_1 \log_{10} \frac{2\, l_c}{U_c}\right),$$

where l_c = length of end connection at one end = $\left(\frac{10\,\gamma D}{2p}\right)$ as in the calculation of mean length of turn.

U_c = perimeter of a belt of coils = $u + 2s_1w$ in a double layer winding (see Fig. 4).

Then the total reactance in ohms per phase is

$$x = 2\,\pi f p s_1\, c^2\, k_2 k_3 (P_s + P_t + P_c) 10^{-8},$$

where f = frequency;

p = number of poles;

c = effective conductors per slot = $\frac{2\,S}{p s_1}$.

The reactance drop in volts is "x" times the current per phase, and this drop divided by the rated voltage is the per cent reactance, or per cent reactance drop, which is usually from 8 to 16 per cent.

Excitation at Rated Kv-a. — This is figured as on p. 648, Magnetomotive Force Method, giving conservative values, or as recommended in the Standardization Rules of the A. I. E. E. The excitation required in ampere turns per pole is found for full kv-a. output at 100 per cent and 80 per cent power factor.

Design of Field Coils. — These are usually designed to give the excitation required at 80 per cent power factor with about 80 per cent of rated exciter voltage, that is at 200 volts d-c. impressed across the field circuit terminals.

The suitable cross-section (sq. in.) of field conductor is found by the relation

$$a = \frac{0.0115 \times (mlt) \times F \times p}{E_x \times 12{,}000},$$

where $(m\,l\,t)$ = mean length of field turn (ins.) figured as for the armature turn except as to pitch diameter (at center of slots) and as to pitch of end connections, usually 67 per cent, and

F = required ampere-turns *per pole*;

p = number of poles;

E_x = Assumed voltage across field;

0.0115 = resistance per square inch per 1000 feet of copper at 125° C., the temperature commonly assumed for field copper.

This section, a, is definite for the conditions assumed and is independent of the number of turns. The number of turns depends upon the allowable loss or heating. The more turns (more copper) the less the loss.

The working field current I_f is found (a) by assuming 1500 amp. per square inch in this section of conductor for the excitation for 100 per cent power factor or (b) by assuming a definite loss for excitation and dividing this loss by the assumed voltage on the field. The number of field turns per pole is then $t = F \div I_f$ and these turns are distributed equally in the available slots. It should be noted that each turn lies in two slots. The field slot factor has values from 0.4 to 0.6, the lower value applying where much space must be devoted to ventilating air. Field coils made up with mica and asbestos insulation operate successfully with a specific loss of 0.90 watt per square inch of coil surface, giving a rise of about 50° C.

Efficiency and Losses. — The losses are friction, excitation, core-loss, armature I^2R and load loss. The last two are usually combined and referred to as the "armature copper loss." The principal friction is that due to windage and is a function of the ventilation. The excitation loss is treated above. The core-loss is best calculated on the basis of the loss per cubic inch at the given maximum densities as discussed in the article on *Magnetic Properties*. The material used is usually 14 mils thick, having a hysteresis coefficient $\eta = 0.001$ and eddy current coefficient $\epsilon = 0.00007$. In turbo-alternators the core-loss by test usually comes out very close to the calculated value, thereby differing from some other machines in which a factor greater than unity must be used for practical results.

The armature copper loss is greater than the true ohmic loss by from 25 to 50 per cent on account of the load losses.

TABLE IV. — USUAL EFFICIENCIES AND LOSSES

Kv-a	Per cent efficiency	Per cent friction	Per cent excitation	Per cent core-loss	Per cent armature
2,000	96	1.50	0.70	1.20	0.60
5,000	97	1.20	0.45	1.00	0.35
10,000	97.5	1.00	0.35	0.90	2.5
20,000	98	0.80	0.25	0.77	0.18
30,000	98.2	0.70	0.20	0.75	0.15

Heating of Armature and Field may be estimated by means of the data given under the section above on *Ventilation;* 100 cubic feet of air per minute is provided for each kilowatt of loss. The rise in the armature coils is about 100° C. per watt per square inch of coil surface and in the field coils about 110° C. per watt per square inch.

Synchronous Impedance is very high on these machines and the steady short-circuit current correspondingly small. It is frequently the case that with excitation for rated voltage and no load, the short-circuit current is less than the rated load current. Thus voltage regulation by external means is absolutely necessary, the inherent regulation being from 20 per cent to 30 per cent at unity power factor and rated load.

TESTS OF ALTERNATING-CURRENT GENERATORS. — (*See also Standardization Rules of the A.I.E.E.*) The principal tests are

Magnetization or saturation test;
Core-loss and friction tests;
Synchronous-impedance test;
Load-loss test;
Resistance measurements;
Heat runs;
Insulation tests.

Examples of test results are given below; typical test curves are given in Fig. 9.

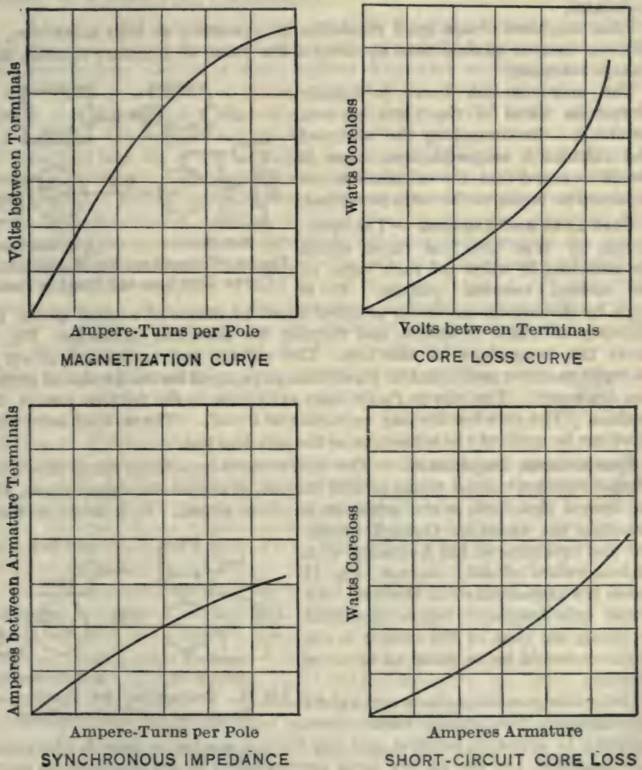

Fig. 9. Typical Test Curves

Magnetization Curve. — The magnetization curve, popularly named the no-load saturation curve," shows the relation between the no-load voltage and the current in the field. Fig. 10 shows the connections for making this test. On account of the existence of the hysteresis loop (*see article on Magnetic Properties of Iron*), it is necessary, in running this test, to increase the field current gradually from point to point and never reduce the value at any step until the highest excitation has been obtained. The curve differs in shape from the magnetization curve of a closed sample of iron because the magnetic circuit of the alternator contains a considerable air gap, the magnetization curve of which is a straight line. Therefore, if the saturation curve of a machine contains a portion which is very straight, the indications are that the air gap is of considerable magnitude. If the machine is operated at very high magnetic densities, this is indicated by the fact that the point corresponding to rated voltage is found at a point on the curve where it is nearly horizontal.

Some machines obtain good regulation by operating at high saturation, as this is a cheaper method than by using a low value of armature reaction and leakage reactance.

The magnetization curve is usually plotted in terms of the volts between terminals. In comparing the observed and calculated magnetization curve it should be noted that the calculations are expressed in terms of the volts per phase.

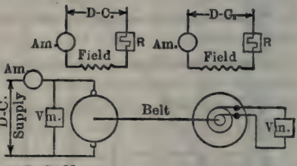

Fig. 10. Connections for Magnetization Curve, Core Loss and Friction Tests

Core Loss and Friction. — The open-circuit, or true core loss curve shows the core loss in watts for each value of the no-load terminal voltage. It is made by driving the generator at rated speed by means of a small motor, the efficiency of which is known, and varying the generator excitation. Fig. 10 shows the connections for this test. The voltage and mechanical power P (= input to motor multiplied by its efficiency) required for each value of excitation are noted. The power P_0 for zero excitation is the friction loss in the machine. The core loss for any excitation is $P - P_0$. The no-load saturation curve can be made at the same time as the core-loss test.

Synchronous Impedance. — The synchronous-impedance curve shows the relation between various values of field current, or excitation ampere turns, and the current that flows in the armature on short circuit. It is made by short circuiting the armature through ammeters and operating at full frequency with various values of field current. Fig. 11 shows the connections for this test. Of course only fractional values of normal excitation are used, or the current in the armature would be so great as to cause damage.

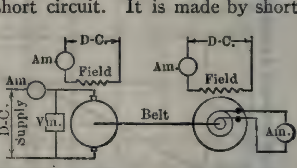

Fig. 11. Connections for Synchronous Impedance and Load-loss Tests

The synchronous-impedance curve gives approximately the ampere turns corresponding to armature reaction and the leakage reactance drop in the armature combined, i.e., the ampere turns corresponding to the synchronous reactance. The approximation arises from the effect of the armature resistance and the low saturation or magnetization of the magnetic circuit under the short-circuit conditions.

Stray Load-losses. — These include iron losses, and eddy-current losses in the copper, due to fluxes varying with load and also to saturation.

The following is quoted from the *Standardization Rules of the A. I. E. E.:* Stray load-losses shall be determined by operating the machine on short circuit and at rated-load current. This, after deductiug the windage and friction and I^2R loss, gives the stray load-loss for polyphase generators and motors. These losses in single-phase machines are large; but the Institute is not yet prepared to specify a method for measuring them.

Resistance Measurements must be made when all parts of the machine are at some known temperature. They are also made after the heat run, when the machine is hot, to check the temperature as measured by thermometers (*see article on Resistance and Conductance*).

Field Resistance is measured by the simple voltmeter-ammeter method (*see article on Resistance and Conductance*).

Armature Resistance. — The armature resistance between terminals is also measured by the simple voltmeter-ammeter method, by connecting two of the terminals to a sorce of direct current (the other terminal being free), the rotor (field or armature of course being at rest). In the case of a single-phase or two-phase machine the resistance as thus measured is the resistance per phase. In the case of a three-phase machine, let R_t = the resistance as thus measured and R_p = the resistance per phase. Then for a Y-connected armature $R_p = R_t \div 2$, and for a Δ-connected armature $R_p = 3R_t \div 2$. If the connection is not known, the calculation of the resistance drop and copper loss may be figured correctly assuming it either Y or Δ connected, provided the resistance per phase and current per phase are both calculated on the same assumption regarding the connection. In the case of a two-phase machine the resistance between terminals A and B and between B and C (B being the common terminal) should each be measured, and the average taken. In the case of a three-phase machine the resistance between A and B, B and C, and A and C should be measured and the average taken.

Calculation of Regulation and Efficiency from Tests. — From the results of the preceding observations, the regulation and efficiency of the machine at various loads may be calculated by the methods given above, in the discussion of *Predetermination of Performance,* using the test data instead of the calculated quantities. The synchronous-reactance ampere turns may be taken as equal to the synchronous-impedance ampere turns, since the resistance drop in the armature on short circuit and reduced excitation is seldom over 10 per cent of the synchronous-reactance drop, and as the two are in quadrature the synchronous-impedance drop differs from the synchronous reactance drop by less than one per cent. In using the magnetization curve and the synchronous-impedance curve one must keep in mind whether they are plotted in terms of voltage and current per phase or in terms of volts between lines and line current.

Full-load Saturation Curve. — This curve shows the relation between the terminal voltage and field current with full-load current in the armature. It may be plotted from tests or calculated from the magnetization curve and the synchronous impedance in the same manner as the regulation is calculated.

Armature Leakage Reactance. — There are three methods by which the leakage reactance may be determined:

(1) **Synchronous-impedance Method.** — This gives only approximate results but is very generally used as a synchronous-impedance test is made on every generator. Let n be the field ampere turns per pole required to force full-load current through the short-circuited armature. Let a be the calculated

armature-reaction ampere turns per pole (see above), with rated current. Then $n - a$ is the excitation necessary to induce the leakage reactance voltage (IX) in the armature. From the open-circuit saturation curve find the voltage corresponding to $(n - a)$ ampere turns. Reducing to volts per phase, if necessary, and dividing by the current per phase, the result is the leakage reactance X per phase in ohms.

(2) **Full-load Characteristic Method.** — This method is also approximate. Let the excitation in ampere turns per pole required to give rated voltage at full non-inductive load be q. Let the ampere turns to give rated voltage without load be m. Then $n = \sqrt{q^2 - m^2}$ is taken as the synchronous-reactance ampere turns, and the leakage reactance is calculated as described in the preceding paragraph.

(3) **Inductance-measurement Method.** — This method gives exact values, but is not very often employed. From an external source of proper frequency, full-load current is sent through the armature of the machine to be tested, with the fields excited to the normal value but not rotating. The voltage drop across the armature terminals is measured for several different positions of the coils with respect to the poles, ranging through an arc of about one-half the pole pitch. Let Z = the voltage per phase divided by the current, R = the resistance per phase corrected for load loss (i.e., the effective resistance as determined from the short-circuit core-loss curve), then the leakage reactance per phase is $X = \sqrt{Z^2 - R^2}$. This reactance will vary with the position of the coils and the maximum and minimum may be found by making this calculation for the different positions of the coils with respect to the fields.

Heat runs are made at rated load and various other specified loads. The parts in which the rise in temperature is of interest are:

Armature core surface;
Armature core ventilating ducts;
Armature conductors;
Collector rings;
Both pole tips;
Field winding;
Bearings;
Frame;
Room.

The temperature of the field and armature winding should be measured both by thermometers and by the resistance method. In taking the temperature of a hot surface by thermometer a small pad of waste should be placed over the bulb after the thermometer is put in place. The pad should not be too large or it will prevent the normal radiation from the surface.

With large machines it is inconvenient and expensive to test under full-load conditions. For determining the heating without actually developing the full power of the machine there are several methods available.

Reversed-field Method. — The field circuit may be tapped and the full-load field current sent through a portion of the field coils in a direction opposed to that for normal operation. For example, in a 24-pole machine the current in 8 of the field coils may be directed in such a manner that they are reversed with respect to the remaining 16. The armature winding will therefore generate a voltage due to the 8 poles that are not neutralized and therefore of approximately one-third rated value. If the armature is short circuited a current will flow due to this reduced voltage, and this current can be made to approximate closely the full-load value by making a proper division of the poles. If the machine is operated under these conditions the friction, excitation, core

loss and armature copper loss will be very nearly equal to their value under normal operating conditions, yet to drive the machine only an amount of power approximately equal to the sum of the losses is required.

Synchronous-motor Loading. — If two machines are available they may be connected up as a generator and synchronous motor and the field excitation of the motor adjusted so that a current of full-load value, but having a large reactive component, will flow in the armature, the power factor being nearly zero. The power required to drive the generator at full-load excitation and at full-load current will then be small.

Direct-current Loading. — Full-load losses may be simulated on a three-phase generator by connecting the three phases in delta and leaving the delta open at one point, to which a direct-current ammeter and a source of direct current may be connected in series. The delta is first closed through an alternating-current ammeter and the triple-frequency current in the delta (*see Alternating Currents*) is measured. The direct-current ammeter and source of direct current are then connected in series with the delta and the direct current increased until the sum of the squares of the local current of triple frequency and the direct current is equal to the square of the rated current per phase.

Open-circuit Short-circuit Method. — Another ingenious method, advocated by Hobart, consists in alternating open-circuit and short-circuit tests, each under exaggerated conditions, so that at the end of each hour the total watt hours lost in each part are equal to the watt hours that would be lost in normal operation. For example, let the core and field copper loss be 4 kilowatts, and armature copper loss 1 kilowatt. If, now, the machine is operated for 20 minutes with armature short circuited and the excitation adjusted to give 3 kilowatts copper loss, and for 40 minutes on open circuit with excitation adjusted to give 6 kilowatts core and field copper loss, then the loss per hour in the armature copper is 1 kw.- hr., and in the field winding and core 4 kw.- hr., the same as under normal conditions. For an exact division of the time between the open-circuit and short-circuit conditions, the core and field copper loss during the short-circuit run must be taken into consideration. After several hours of this relaying the machine will have reached a temperature corresponding to full-load operation.

Insulation Tests. — The insulation of a machine is usually tested by applying a given voltage between the conductors and the part of the machine from which they are insulated. An insulation resistance test is also made.

Voltage Tests. — During manufacture the coils are tested separately, and after the machine is assembled a voltage test between the completed windings and frame is made. After the heat run, while the machine is still warm, a third test of the insulation between windings and frame is made.

For this third test an alternating voltage is applied between armature and frame, according to the *Standardization Rules of the A.I.E.E.* (q.v.)

The voltage should be increased gradually and the final value should be applied for one minute. In this test the machine acts as a condenser and therefore care should be taken that the frequency is not so high or the inductance of the supply circuit so great as to set up resonance. It is advisable to have considerable resistance in the supply circuit.

To prevent an uneven distribution of voltage, all the terminals of the winding should be connected together by fine wires and one terminal of the high-potential circuit connected to the common connection. The high potential should be measured by a spark gap connected in shunt to the machine.

Insulation Resistance. — The insulation resistance is measured by connecting one terminal of a 500-volt direct-current supply to the windings of the ma-

chine through a voltmeter with a 500 scale, and connecting the other 500-volt terminal to the frame. If the resistance of the voltmeter is R_v ohms and the voltmeter deflection x, the insulation resistance is $R = \frac{R_v}{x}(500 - x)$.

EXAMPLES OF DESIGN AND PERFORMANCE. — In the tables on pp. 667 and 668 will be found the essential data both of the mechanical and electrical features of four representative alternators. The list of items will be found useful as a guide in collecting data on various machines. Performance data are deduced from tests.

OPERATION. — In the operation of an alternating-current generator the following factors should be considered:

Phase Connections and Grounding. — When a third harmonic is present in the e.m.f. wave of a three-phase generator, the triple frequency e.m.f.'s in the three phases (or windings) are additive when the three phases are connected in Δ, that is, the Δ forms a short circuit to the third harmonic e.m.f.'s, and a large triple frequency current may therefore be set up in the windings irrespective of the load on the machine. On this account, large three-phase generators are usually Y-connected, since with this connection the third harmonic e.m.f.'s between any two terminals of the machine neutralize each other. However, when two or more Y-connected machines are operated in parallel with their neutrals grounded, a triple frequency cross-current of considerable magnitude may be set up between the machines, unless the wave form of the e.m.f.'s of the various generators are exactly the same, which is practically never the case. To prevent such cross-currents with Y-connected machines with grounded neutral, it is the usual practice to ground the neutral of but one generator at a time. Provision must of course be made to shift this ground connection from one machine to any other, so that a ground connection can always be maintained irrespective of which machine or group of machines may be running.

The advantages and disadvantages of grounding the neutral are discussed in the article on *Grounding of Electric Circuits*.

Division of Load between Alternators in Parallel. — The division of load between two or more alternators operating in parallel cannot be changed by altering their field excitation, as is the case with direct-current generators. Changes in the load taken by any alternator of a group can be effected only by admitting more or less steam to the driving engine or turbine (or water to a water-wheel). In order that the various alternators shall share the combined load properly, it is therefore necessary that the governors on the several prime movers give the same speed-load characteristics.

Although the field excitation has no effect on the distribution of the load among the alternators, it does affect the power factor of the load delivered by each machine. The excitation of each alternator should be so adjusted that it delivers its load at the same power factor as the others.

Starting a Single Generator. — Before a generator is started up its bearings must be inspected and cleaned and filled with oil if necessary. The machine is then brought up to the proper speed and the bearings again inspected to see that the oil-rings are running properly. The exciters or excitation circuit is then put in readiness and the rheostat in the alternator field circuit adjusted for maximum resistance. Before exciting the field the armature insulation must be thoroughly dry. If it is not the armature is short-circuited through an ammeter and run for several hours at a partial excitation to give about rated current in the short-circuited armature. When the insulation is thoroughly dry the short-circuit is removed and the excitation adjusted to give rated voltage at the armature terminals with correct speed. To shut down the machine the load is first

MECHANICAL DATA ON TYPICAL A-C. GENERATORS

Dimensions in Inches, Weights in Pounds

Type	1 *ATB*	2 *ATB*	3 *ATB*	4 *ATB*
Poles	4	8	54	48
Kv-a. rating	6250	2500	2500	5000
R.p.m.	1800	375	133	150
Voltage between terminals	4000	6600	2300	4000
Frequency	60	25	60	60
Connection	Y	*Y*	*Y*	*Y*
Armature diam. at face	40	92	170	192
Armature diam. at back	64	120	180	204
Armature, total length	52	34	20	30
Armature air ducts				
Number	24	7	8	11
Width	0.5	½	½	½
Slots, number	72	144	648	432
Slots, dimensions	3.06×0.75	2⅝×0.9	2⅛×0.4	2⅛×⅝
Conductor size	0.16×0.08	0.75×0.17	0.75×3/16	0.6×0.15
Conductors, no. in mult.	39	1	2	4
Conductors, no. per slot	78	4	2	4
Pitch of connection	55	⅔	⅞	1
Air gap, minimum length	0.81	0.5	0.375	0.313
Air gap, average length	0.81	0.625	0.47	0.415
Field pole arc	30	23	6	8
Field pole, length along shaft	52	33.5	19.5	20
Magnet core, width	8.66	15	4	5.5
Magnet core, radial depth	4.25	8	7	9
Spool, no. turns	90	96.5	33.5	49.5
Spool, size of conductor	(.625×.19)	2¼×0.06	1.5×0.16	1⅛×0.14
Yoke, length	....	36	24	40
Yoke, radial depth	....	4	4	5
Total weight	69,000	140,000	170,000	300,000

ELECTRICAL DATA ON TYPICAL A-C. GENERATORS

Percentages are all in terms of rated or full-load values

		1 *ATB*	2 *ATB*	3 *ATB*	4 *ATB*
Rating	kv-a.	6250	2500	2500	5000
Rating	kw.	6250	2000	2500	5000
Rated volts per phase	volts	2310	3820	1350	2300
Full-load current per phase	amperes	902	219	628	723
Flux per pole	maxwells	48.4	43.3×10^6	5.6×10^6	12.6×10^6
Arm. res. per phase at 25° C.	ohms	0.006	0.072	0.018	0.14
Field resistance at 25° C.	ohms	0.327	0.43	0.37	0.84
Excitation amp. turns, no load	amp. turns	14,600	12,770	7230	7700
Excitation amp. turns, full load	amp. turns	18,000	13,470	7830	8280

ELECTRICAL DATA ON TYPICAL A-C. GENERATORS — *Continued*

Percentages are all in terms of rated or full-load values

		1 *ATB*	2 *ATB*	3 *ATB*	4 *ATB*
Friction loss	kw.	48	29	14	25
Core loss at rated volts	kw.	75	40	43	125
Syn. imp., volts between term.	volts	...	3070	1000	1700
Syn. imp., total amp. turns	amp. turns	11,430	5300	2800	3070
Short-circuit core loss, full-load current	kw.	29	3.4	5.3	19.2
Leakage reactance drop	per cent	17	23	7.8	7.3
Regulation at full load	per cent	18	16.7	5.8	4.75
at power factor of	per cent	100	80	100	100
Friction loss	per cent	0.54	1.4	0.54	0.48
Core loss	per cent	1.38	1.9	1.66	2.4
Field copper loss	per cent	0.44	0.65	0.77	0.45
Armature copper loss	per cent	0.29	0.52	0.90	0.46
Load loss	per cent	0.15	....	0.20	0.12
Efficiency at full load	per cent	97	95.5	96.0	96.1
Rise in temp. by thermometer:					
Armature	°C.	40	35	45	40
Field	°C.	46	35	45	40

removed by opening the circuit breaker; then the field rheostat is turned to maximum resistance as is also the rheostat in the exciter field if there is an individual exciter. Then the field circuit is opened.

Paralleling of Generators. — Before connecting a generator to bus bars to which one or more other generators are connected, the following conditions must be satisfied:

1. The frequency of the generator must be the same as that of the bus bars.
2. The frequency of the generator, and therefore its speed, must be constant for an appreciable interval of time.
3. The voltage of the generator must be the same as the voltage of the bus bars.
4. The generator and bus-bar voltage must be in phase.

If the two machines have not the same frequency or if the frequency is not constant, a condition will occur intermittently in which the two voltages are 180° apart or the two machines are in series on a short circuit, and a dangerous current will flow. If the voltages are not equal, a large "wattless" or reactive current may flow, and if the two voltages are not in phase, a large power current will flow which will cause a mechanical shock. To indicate when these conditions are fulfilled any one of several "synchronizing" devices may be employed, as described in the article on *Synchronizers and Synchroscopes.*

Synchronizing with Lamps. — The simplest method of synchronizing small machines is to use incandescent lamps as shown in Fig. 12. In Fig. 12A, the connections are such that the lamps remain dark when the above conditions are satisfied, while in Fig. 12B they will remain bright under these conditions. If the frequencies are wrong the lamps will flicker (the slower the flicker the nearer the two frequencies). If the voltages are wrong the lamps in 12A will

glow slightly but steadily. Transformers should be used with the lamps in case of high-voltage machines.

Hunting. — Unless the angular velocities of two machines which are connected in parallel remain the same, either both constant or both varying together, a cross current will flow, due to the phase displacement between them. This current will tend to drag ahead the machine which is lagging, but due to the inertia of the rotating parts the machine which is at first lagging will "overreach" and become leading, and under certain conditions a cumulative see-saw action will be set up. When this takes place the machines are said to "hunt." The value of this current is proportional to the short-circuit current of the machines and to the angular displacement expressed in electrical degrees. In a machine having a large number of poles (40), a very small variation in angular displacement of the prime mover may cause a considerable (20-fold) phase displacement in electrical degrees.

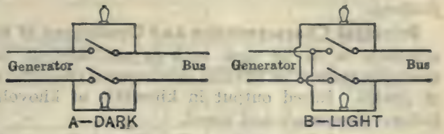

Fig. 12. Connections of Lamps for Synchronizing.

To prevent hunting it is necessary to have:

1. A prime mover giving reasonably constant tangential effort and angular velocity. In general, the maximum variation in angular displacement between any machine and the bus bars should not exceed 2.5 electrical degrees.

To secure this condition the machines shou d have constants such that they are not especially sensitive as pendulums to the strokes or impulses of the particular engines used.

2. A governor which is not too sensitive to slight and sudden variations in load, i.e., a damped governor.

3. A low-resistance drop (usually not over 10 per cent) in the connections between the machines. This refers particularly to groups of machines in power houses miles apart.

4. Short-circuited or "Amortisseur" windings on the fields of the machines. Currents induced in these windings cause them to act as electrica' brakes.

Short Circuits. — An alternator when suddenly short circuited will deliver for an instant a current many times as great as will flow after conditions have become constant. This is due to the fact that it takes a finite period of time for the increased armature reaction to weaken the magnetic field. During this short period much damage may be done to circuit breakers, etc. This is especially true of large turbo-alternators, as these, due to their construction, have a very large transient short-circuit current.

Use of External Reactance. — It is therefore frequently the case that reactance or choke coils are connected in circuit with these machines to prevent a dangerous current flowing in case of sudden short circuit. See *Reactance Coils*.

Induction Generators on Short Circuit. — Induction generators are free from this fault, since they lose their excitation almost immediately on short circuit, and are therefore becoming popular in large central stations.

Use of Imbedded Thermometers. — In large generators thermocouples or resistance thermometers (*see Pyrometers*) are sometimes embedded in the estimated hottest spot of the winding, and connected to a suitable indicating device to show at all times the maximum temperature of the machine. See also *Standardization Rules of the A.I.E.E.*

SPECIFICATION FOR A-C. GENERATOR. — The following memoranda are intended to assist in writing specifications. See also article on *Specifications.*

Principal Characteristics and Conditions of Service. — Service for which generator is to be used, such as a-c. lighting, single-phase railway service, operating railway or lighting synchronous converters, etc. Voltage and number of phases. Rated output in kilowatts or kilovolt amperes at stated power factor. Frequency and speed.

Style and Description; Details of Construction. — Type of generator, revolving field, induction type, etc. Details of speed, governing of prime mover, and how generator is connected to prime mover, e.g., direct or belt-driven by steam turbine, reciprocating engine, water wheel, water turbine, gas engine, oil engine, etc., with vertical or horizontal shaft. Whether compensated. Whether exciter is to be supplied; if so, its characteristics. If field rheostat is to be supplied, its characteristics, including the effect upon the generator voltage of each step and of all steps; whether to be controlled by hand or automatically. Restriction of excitation current and voltage and requirements respecting carrying capacity of slip rings. Accessibility of armature. Whether embedded thermometer coils shall be furnished. Windings shall be clamped securely to prevent any vibration of overhanging parts. Mechanical protection of armature conductors if exposed. If belt driven, specify pulley details; whether bed plate is desired.

Work to be Done by Other Contractors.—Whether Contractor is to furnish and install the following. Main wiring, field wiring, field rheostat grids, dial plate and chains. Point of division between engine and generator contracts.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*) Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load. High-potential tests of insulation. Requirements regarding effects of moisture upon insulation. Requirements for parallel operation, i.e., whether the machine is to operate in parallel with similar machines or different ones. It is usual to specify that the terminal voltage shall vary according to a sine law. Regulation with 100 per cent power factor and normal speed; the load may be varied from zero to 150 per cent of rated load without causing more than a stated variation of voltage. the exciter field being kept constant.

DIMENSIONS, WEIGHT AND COST (1922 prices). —While generators vary widely in their specific weights and costs, that is weight per kv-a. rating, and cost per kv-a. rating, they may be divided into classes in which these characteristics are fairly definite.

The conditions primarily affecting the specific weight are: method of rating, speed, frequency, voltage and size. In addition there are the peripheral speed and mechanical construction which cannot be easily classified. The method of rating is fundamental. For purposes of comparison the rating may be taken as the output in kv-a. which each machine will give continuously with a rise in temperature not exceeding 50° C. On this basis the specific weight decreases as the speed, frequency or capacity increases, and increases with increase of voltage. The cost per pound decreases as the frequency decreases, and as the capacity increases. The cost per pound increases as the voltage and peripheral speed increase.

Alternating-current generators may be divided into three classes according to their speeds and purposes:

High speed, as turbine-driven generators.

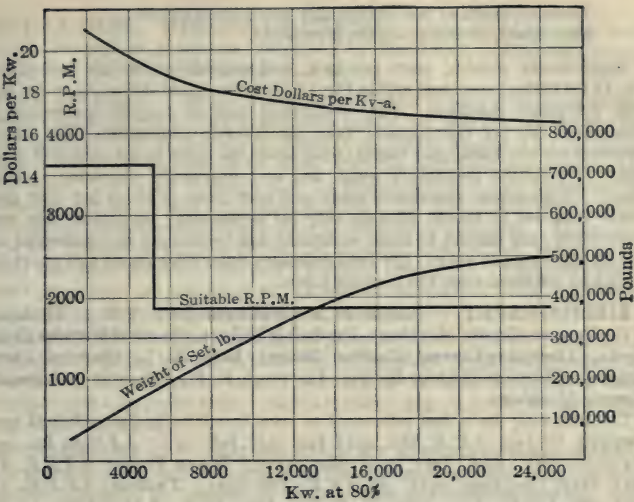

Fig. 13. Cost, Weight and Suitable Speed for 60-cycle Turbo-generator Sets, Including Turbine.

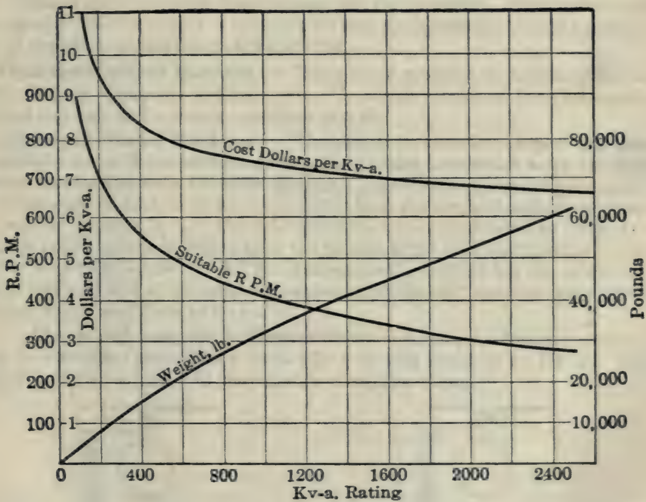

Fig. 14. Cost, Weight and Suitable Speed of Water-wheel driven 60-cycle A-C Generator.

Medium speed, as belt-driven and water-wheel-driven.
Slow speed, as engine-driven generators.

Approximate weights, costs per kv-a. and suitable speeds are indicated in Fig. 13 for turbo-generators and in Fig. 14 for water-wheel-driven 60-cycle, 2300-volt, polyphase machines. The weight and cost of 25-cycle generators are from 10 to 20 per cent greater than for 60-cycle generators. The cost of machines for less than 2300 volts is practically the same as for 2300-volt equipment. Two-phase generators weigh and cost practically the same as three-phase. Single-phase generators weigh and cost from 25 to 30 per cent more. These data are of course suitable only for preliminary estimates; the costs, particularly, are subject to large variations due to changes in commercial conditions from year to year. For final estimates exact dimensions and quotations should be obtained from the manufacturers.

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GENERATORS, DIRECT-CURRENT. — (*See also Alternating Currents; Electricity and Magnetism, Principles of; Generators, Alternating-Current; Motors, Direct-Current; Standardization Rules.*)

APPLICATIONS. — Direct-current, or, as it is sometimes called, continuous-current apparatus was in very general use before the alternating-current type was introduced, and wherever the distance to which energy is to be transmitted is not a factor, there is no doubt that the direct-current system is to be preferred. However, if energy is to be transmitted over long distances the alternating-current system with its transformer has unequivocal advantages. The result of these two conditions is that a compromise is adopted. The majority of the generating stations provide alternating currents and many of the motors use direct currents provided by rotary converters. Thus the direct-current generators are becoming of less relative importance while the direct-current motors maintain a very important position.

DEFINITIONS. — The fundamental principle involved in the construction and operation of any type of dynamo-electric machine is the production of an electromotive force in one or more conductors by the relative motion of these conductors and a magnetic field. Such a machine may, as a rule, be used either as a generator or motor.

Shunt Machine. — A shunt machine, either generator or motor, is one in which the entire field excitation is derived from a circuit of many turns and high resistance connected in "shunt" or multiple with the armature circuit. The characteristic of a shunt machine is poor regulation; that is, the voltage of a shunt generator decreases as the load increases. This is so marked that some shunt generators may be short-circuited, their terminal voltage dropping to zero, without resultant harm.

Series Machines. — A series machine, generator or motor, is one in which the entire armature current flows through the field winding. Series generators are usually built to supply a constant current to an external circuit irrespective of the effective resistance of that circuit.

Separately Excited Machine. — This type of machine is sometimes used in large stations where the exciting current is readily obtained from separately excited bus-bars and a voltage regulator is used.

Compound-wound Machine. — This type of machine has on each field pole in addition to its shunt winding a few turns of thick wire which carry the load current and are known as the series winding. This winding causes the excitation to increase as the load increases and tends to keep the terminal voltage constant or even to increase it. If the field windings are proportioned to cause the voltage at full load to be higher than the voltage at no load the machine is said to be "over-compounded." A "flat compounded" machine has the same voltage at full load and at no load; an "under compounded" machine has a lower voltage at full load than at no load.

Short- and Long-shunt Connections. — A compound-wound machine may be connected short shunt as in Fig. 1 or long shunt as in Fig. 2. The choice is merely a matter of convenience of station wiring.

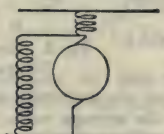

Fig. 1. Short-shunt Connections

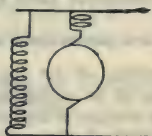

Fig. 2. Long-shunt Connections

Commutating Pole or Interpole Machines. — These machines have small auxiliary poles alternately placed with respect to the main poles and excited by a few turns in series with the load. The effect of these poles is to improve the operation of the machine in the matter of commutation; see below.

Bi-polar Machines. — Small continuous-current machines are usually of the bi-polar or two-pole type with a more or less inclosed frame of cylindrical shape.

Multi-polar Machines. — Large direct-current machines are of the multi-polar type, that is, have a large number of radial pole pieces.

Belt-driven and Direct-connected Types. — Direct-current generators may be of either the "belt-driven" or the "direct-connected" type, as determined by the method of connecting to the driving unit. Belt-driven generators are characterized by a higher angular velocity than direct-connected.

RATINGS. — It has been common practice (up to 1914) to rate direct-current machines on the basis of the output in kilowatts which they will give continuously with a maximum rise in temperature by thermometer of 50° C. above the surrounding air at 25° C. See however the recent recommendations of the A.I.E.E. in the article on *Standardization Rules.*

VOLTAGE. — The standard voltages for which continuous-current machines are built are:

80 volts for use on shipboard.
110–125 volts for lighting purposes and incidental small power motors, fan motors, cooking utensils.
220 volts for three-wire systems with lighting and power combined.
500–600 volts for power alone and particularly for railway service.
1200–2400 volts for special railway service and heavy traction.
2000–6000 volts for series-arc-light circuits fed by series machines.

ELECTROMOTIVE FORCE INDUCED IN ARMATURE. — Let

Z = total number of armature conductors
p = number of field poles.
m = number of parallel conducting paths between the positive and negative brush sets; that is $\frac{Z}{m}$ is the number of armature conductors in series between positive and negative brush sets. (*See below under Armature Windings.*)
ϕ = total useful magnetic flux per pole.
N = revolutions of armature per minute.
f = frequency in cycles per second.

If a coil of wire be revolved about an axis in a magnetic field, as shown in Fig. 3, each length of conductor, or side of the coil, will pass entirely around the armature in $\frac{60}{N}$ seconds. The time taken for a conductor to pass through the magnetic field under each pole, or from a to b, is $\frac{60}{Np}$. Hence the average value of the electromotive force induced in each armature conductor as it passes under each pole is $\frac{Np\phi}{60 \times 10^8}$ volts, since a cutting of 10^8 lines per second gives one volt. Since in an actual machine the conductors are uniformly distributed around the surface of the armature, this is also the average voltage per conductor in each of the conductors between a and b at any instant. Since

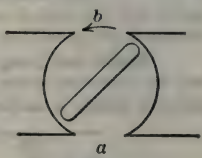

Fig. 3. Elementary Generator

there are $\frac{Z}{m}$ conductors in series between the positive and negative brush sets, the average value of the total electromotive force between the brushes, when a suitable commutator (see below) is provided, is

$$E = \frac{p\phi ZN}{60 \times 10^8\, m} \text{ volts.}$$

The electromotive force in each conductor alternates (i.e., passes through a complete cycle of positive and negative values) with a frequency of $f = \frac{pN}{120}$ cycles per second. Hence the formula for the average e.m.f. between brushes is

$$E = \frac{2\, f\phi Z}{10^8\, m} \text{ volts.}$$

The number of turns (S) in series between the positive and negative brushes is $Z/2\,m$, whence E may also be expressed as

$$E = 4 f\phi S \times 10^{-8} \text{ volts.}$$

Commutator. — A continuous electromotive force can be obtained from the machine if provision is made for reversing the connection from each conductor to the external circuit at the same time that the direction of e.m.f. induced in the conductor reverses. This is accomplished by the commutator. Taps leading from the front connections of the armature windings are connected to the segments of the commutator, so that as the segments come alternately under positive and negative brushes the current delivered to the circuit is always in the same direction.

ARMATURE WINDINGS. — Though there are a large variety of types of armature windings the practical man and even the designing engineer seldom meets types other than (1) the multiple-drum or lap winding and (2) the two-circuit series drum or wave winding. The other types may be used in a few special and exceptional machines but a designer may work for many years without finding any necessity for using them.

Multiple-drum or Lap Winding. — This type of winding is very common in direct-current machines, rotary converters, induction motors and is somewhat used in alternating-current generators. Its chief advantage is that it affords a very free choice in the number of coils and slots, and is very simple to lay out and connect up. Its disadvantage is that it is not easily adapted to high voltages. Its chief characteristic is that there are always as many cir-

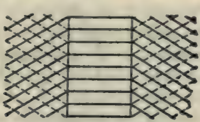

Fig. 4. Lap Winding

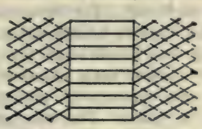

Fig. 5. Wave Winding

cuits in multiple and as many studs of brushes as there are poles. The distinguishing feature in appearance is the direction of bending of the end connections, as represented by Fig. 4. The characteristic form of the coils is shown in Fig. 7.

Conditions for Lap Winding. — The customary simplex multiple-drum winding fulfills the condition expressed by the formula

$$C = pz = sb,$$

where

C = total number of coil-sides or bars,
p = number of poles,
z = average pitch* $= \dfrac{\text{front pitch} + \text{back pitch}}{2}$,
s = total number of slots,
b = number of coil-sides or bars per slot.

The total number of sides (C) is double the number of coils, as each coil has two sides. C must be a multiple of the number of slots (s) and of the number of poles (p).

The average pitch (z) must be an even number so that the front and back pitches may be different and odd ($z - 1$) and ($z + 1$), respectively. There are always two layers of coil-sides, top and bottom. One side of each coil lies in the top layer in one slot and in the bottom layer in another slot. The actual front and back pitches of a coil are always odd because they are made from a coil-side in the top layer (odd numbers) to one in the lower layer (even numbers) as in Fig. 6. Here the pitch is $14 - 1 = 13$.

Fig. 6. Pitch of Connection


Fig. 7. Lap Winding

In order that the winding should progress continuously the front pitch, or pitch of commutator connections, must differ by one coil (or two coil-sides) from the back pitch, or actual pitch of coils. The actual spread of a formed coil (Fig. 7) is the same front and back and is equal to the "back pitch."

The total number of coils $Q = C/2$ is equal to the number of commutator segments and must be a multiple of the number of slots. In general if Q is a common multiple of the number of poles and the number of slots, a multiple-drum winding is possible. It is sometimes desirable that the number of slots should not be a multiple of the number of poles. In order to group the coils in poly-coils z should be a multiple of the coil-sides per slot.

Series-drum or Wave Windings, Simplex.—This type of winding is used in direct-current armatures where the ordinary multiple drum would give either too low a voltage or require too many turns of fine wire in each coil. Its advantage is that it gives an armature that is better balanced magnetically and that only two brush studs are necessary. A greater number of studs may, however, be used. There are always two circuits in multiple, regardless of the number of poles. The characteristic reverse bends of its face conductors are shown in Fig. 5. The characteristic form of the coils is represented in Fig. 8.

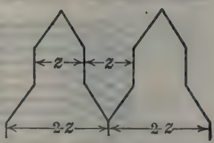

Fig. 8. Wave Winding

Conditions for Wave Winding.—The conditions to be fulfilled in laying out a series-drum winding are expressed in the formula

$$C = pz \pm 2 = sb,$$

* Let the coil-sides be numbered consecutively from slot to slot, and let a point travel through the conductors in the order in which they are connected: if this point starting at conductor No. 1, say, passes through the point or commutator connection to conductor No. 8 and thus through the back connection to No. 3, then the front pitch is $8 - 1 = 7$ and the back pitch is $8 - 3 = 5$, and the average pitch is 6.

where

C = total number of coil-sides or bars,
p = number of poles,
z = average pitch of end connections,
s = total number of slots,
b = number of coil-sides or bars per slot.

The total number of coil-sides (C) is double the number of coils and must also be a multiple of the number of slots. In the formula $+ 2$ is preferable to $- 2$, as the positive sign gives shorter end connections.

The average pitch z may be odd or even. If it is odd the front and back pitches are both equal to z. If it is even the front and back pitches must be $(z - 1)$ and $(z + 1)$. For a wire winding the back pitch must be one greater than a multiple of the coil-sides per slot b in order to fit the coils into the slots in groups.

The total number of slots (s) is very much restricted and is intimately connected with the number of poles unless the expedient of using a dead coil is used (*see below*).

The bars or coil-sides per slot b must be even and cannot be 4 or 8 in a four-pole machine and cannot be 6 or 12 in a six-pole machine, unless a dead coil is used.

Use of Dead Coils. — By leaving one of the coils dead or out of circuit, a greater choice of slots and conductors is available. Four coil-sides per slot for 4 poles and 6 coil-sides for six poles may then be employed. In general $s \times b$ may be any number divisible by the number of poles. The formula is then

$$C = pz - 2 = sb - 2.$$

Two bars or one coil are not connected in at all and the number of commutator segments is equal to the number of active coils or one less than the total coils. The total number of active coils is $C/2$.

Usual Arrangements for 4- and 6-pole machines with no dead coil are:

Four Poles.—For $b = 6$, then z may be 25, 31, 37, etc., and usual values fulfilling all conditions are $s = 17, 21, 25$, etc. For $b = 10$, then z may be 47, 57, 67, etc., and s may be 19, 23, 27, 31, etc.

Six Poles.—For $b = 4$, then z may be 17, 21, 25, etc., and s may be 26, 32, 38, etc.

Multiplex Wave Windings. — It sometimes becomes desirable to provide a winding which has more than two circuits in multiple but not as many as the number of poles. In such a case a multiplex wave winding would be selected. These are windings in which the circuit passes completely around the armature more than once. If to do this we use two entirely separate electric circuits we have a duplex doubly reëntrant winding, denoted by the symbol O O. If, on the other hand, the winding closes on itself after passing twice around the armature we have a duplex singly reëntrant winding, denoted by the symbol ◎. These types of windings are sometimes used when it is desired to have a number of circuits in parallel different from the number of poles. Thus we may have a six-pole machine with four circuits in multiple which with a given number of inductors would give a greater voltage than a multiple-drum winding and lesser voltage than a series drum. This would be a duplex winding.

Conditions for Multiplex Winding. — The conditions are imposed by the formula

$$c = pz \pm 2m = sb,$$

where tne symbols have the same meaning as before and m is the number of

multiple windings and $2m$ the number of circuits in multiple. If m and z are prime to each other we have a singly reëntrant winding. The greatest common factor of m and z gives the number of times the winding reënters or in other words the number of independent windings.

The multiplicity and complexity may be carried to a very extreme limit. (*See under Armature Windings in any standard textbook.*) These multiplex windings are hardly ever used in the United States and England and only occasionally in Germany and France.

ARMATURE REACTION (OR ARMATURE INTERFERENCE). — The armature reaction of a d-c. machine has a very important influence on the commutation and regulation of both generators and motors. It is the effect of the magnetomotive force of the current in the armature conductors on the magnetic field set up by the field coils.

Separate Fluxes by Field and Armature Currents. — When current flows in the field coils and no current flows in the armature, a flux is set up following a path directly across the armature from pole to pole as in Fig. 9. On the other hand when current flows in the armature and there is no current in the field a flux is set up in the armature across the axis of the poles, as in Fig. 10.

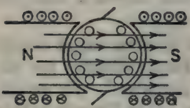

Fig. 9. Field Flux

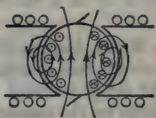

Fig. 10. Armature Flux

Fig. 11. Distorted Flux

Resultant Flux. — As a result of this action the flux is shifted around so that one tip of each pole has its density increased and one tip has the density decreased as in Fig. 11. If the brushes are at the geometrical neutral they are no longer on an axis at right angles to the flux; that is, they are no longer at the neutral point with respect to the resultant flux. With the brushes in this position the coil underneath a brush is cutting flux and generating a voltage, which is short-circuited by the brush and causes sparking. The brushes must, therefore, be moved a small angle in the direction of rotation in a generator (in the opposite direction in a motor) until they are on the neutral axis. The shift of the brushes aggravates the conditions, but nevertheless, unless the armature strength is too great, a position can be found in which a brush short-circuits a coil that is in the real neutral position and is inactive.

Relations Shown Vectorially. — In Fig. 12 F_1 and A_1 are the m.m.f.'s of the field and armature with the brushes on the geometrical or apparent neutral axis

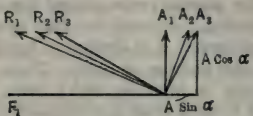

Fig. 12. Vector Relations of M.M.F.'s

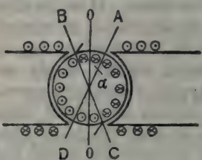

Fig. 13. Demagnetizing and Cross-magnetizing Turns

R_1 is the resultant of these. The brushes which are in line with A_1 are not at right angles to R_1. If the brushes are moved to an axis at right angles to R_1 as at A_2, then the resultant becomes R_2 and the desired conditions have not been

attained. By moving the brushes still further to A_3 giving the resultant R_3 we are able to get R_3 and A_3 at right angles to each other. The stronger the armature flux as compared to the field flux the greater will be the angle through which the brushes must be moved.

When the brushes are moved through an angle α to the position A_3 there is one component of the armature m.m.f., $A \sin \alpha$, directly opposed to the field m.m.f. Another component, $A \cos \alpha$, is at right angles to the field m.m.f. This is better understood by a reference to Fig. 13.

Demagnetizing Action and Cross-magnetizing Action. — Let the movement of the brush from A_1 to A_3 be represented in Fig. 13 by the movement from O to B, or through the angle α. Then the conductors in the angle $2\,\alpha$, between A and B and between C and D, carry currents whose m.m.f. directly opposes the magnetic strength of the field coils; these are known as back or demagnetizing conductors. The remainder of the armature conductors, in A–C and B–D, carry currents which give a m.m.f. at right angles to the field and cause a distortion of the flux; these are known as cross-magnetizing conductors.

It should be noted that while all the conductors in the sections A–B and C–D subtending an angle $4\,\alpha$ constitute back ampere-turns, these are opposed to the strength of two poles. When we come to our quantitative treatment in which our unit of design is a pole we make use of the turns of $2\,\alpha$ as the back ampere-turns per pole.

Effect of Interpoles on Armature Reaction.* — The demagnetizing effect of armature reaction may be almost entirely overcome by placing the brushes in the neutral axis O—O. To prevent sparking under these conditions interpoles must be used. The use of interpoles does not in itself prevent armature reaction, but makes possible sparkless commutation when the brushes are set in such a position as to reduce armature demagnetization to a minimum.

ARMATURE INDUCTANCE. — Each coil of the armature carries a current flowing in one direction as it travels from a positive brush to a negative brush and in the opposite direction as the coil travels from a negative to a positive brush. Thus the direction of the current in each coil reverses during the time the commutator bars to which the coil is connected pass under the brush. To reverse the current in any circuit it is necessary to take out the energy stored up in the form of magnetic flux linked with the circuit, and put back an equal amount represented by a flux in the opposite direction. In doing this there is induced in the circuit a voltage which is proportional to the time rate of change of the flux and which opposes any change in the value of the current. This voltage is the e.m.f. of self-inductance of the coil.

Methods of Improving Commutation. — To minimize the voltage of self-inductance it is necessary to keep the number of ampere-turns in each coil as low as possible and to cause the reversal to take place as slowly as possible. It is possible to neutralize the voltage of self-inductance by introducing an opposing voltage, which is accomplished in practical machines by giving the brushes a forward lead or by using interpoles.

Forward Lead of Brushes. — The brushes are moved in the direction of rotation in a generator (opposite direction in a motor) away from the true neutral until the coil undergoing reversal is moving in a flux coming from an adjacent pole tip of such a density that it induces by rotation in that particular coil a voltage that opposes and neutralizes the voltage of self-inductance.

Effect of Interpoles on Commutation. — Interpoles or commutating poles are placed between the main poles over the neutral space. These poles

* See *Motors, Direct-current.*

are excited by the load current until they give a flux of the proper direction and value to induce the desired neutralizing voltage. (*See also above under Armature Reaction.*)

Increasing Resistance in Path of Short-circuit Current also aids commutation. This changes the time constant of this circuit, that is, some of the stored energy of the magnetism is dissipated in I^2R loss in this resistance instead of in sparking at the brush. The usual method of accomplishing this is to use carbon brushes which have a higher resistance of contact than metal brushes. Another method is to introduce a high resistance in the connection between the winding and the commutator segment.

DESIGN. — The steps in the systematic procedure in the design of a d-c. machine of given voltage and power rating are given below. Simplex windings are assumed.

1. Statement of problem.
2. Suitable speed.
3. Number of poles.
4. Diameter and length of armature.
5. Length of air gap.
6. Number and size of slots.
7. Total flux, preliminary.
8. Number of turns on armature, preliminary.
9. Armature reaction, ampere-turns.
10. Number of conductors per slot.
11. Form of armature winding.
12. Size of armature conductors.
13. Exact size of slots.
14. Armature resistance and heating.
15. Design of commutator and brushes.
16. Reactance voltage.
17. Exact value of flux.
18. Cross-section of magnetic path; usual flux densities.
19. Excitation; calculation of magnetic circuit.
20. Stability factor.
21. Armature reaction or interference.
22. Field winding, series, shunt.
23. Interpoles or commutating poles.

The performance of the machine should then be calculated from the preliminary design to determine whether the design meets the imposed conditions and such modifications as are necessary should then be made. The steps in calculating the performance are the calculations of:

24. Core-loss.
25. Friction.
26. Excitation loss.
27. Armature circuit loss.
28. Efficiency.
29. Regulation.
30. Heating.

Symbols. — The following notation is employed uniformly throughout this article; other symbols, and the same symbols with primes or subscripts, are defined in the paragraphs in which they are used.

(AR) = armature reaction ampere-turns per pole.
B = flux density in air gap, lines per sq. inch.
b = number of coil sides per slot.

F = total field ampere-turns per pole.
c = number of conductors per slot.
D = outside diameter of armature, in inches.
E = terminal armature voltage.
e = reactance voltage of short-circuited coil.
$f = \frac{pN}{120}$ = frequency of voltage induced in armature.
I = full-load line current, in amperes.
L = length of armature, in inches.
m = number of parallel paths between brushes.
N = revolutions per minute.
P = power output in kilowatts in case of generator; input in case of motor.
p = number of poles.
$\mathcal{P}$ = permeance of local magnetic path of short-circuited coil.
q = cross-section of one armature conductor, in square inches.
R_a = effective resistance of armature between brushes, in ohms.
$S = \frac{Z}{2\,m}$ = number of turns in series between brushes.
s = total number of slots.
V = peripheral velocity of armature in feet per minute.
V_c = peripheral velocity of commutator in feet per minute.
C = total number of coil-sides.
$z = \frac{C}{p} = \frac{bs}{p}$ = average pitch of end connections.
ν = leakage coefficient.
$\rho = \frac{\text{pole arc}}{\text{pole pitch}}$; pole pitch is the arc from pole center to center of adjacent pole, measured at air gap; pole arc is the arc covered by the pole face.
σ = ampere-conductors per inch of armature periphery.
ϕ = total useful flux per pole.

1. Statement of Problem. — The rating in kilowatts or horse-power and the voltage are always given. The current and speed may or may not be given. The line current at full load is found as follows:

For a generator $\qquad I = \frac{1000\,P}{E}$.

For a motor $\qquad I = \frac{746 \times (\text{Horse-power})}{E \times (\text{Efficiency})}$.

The proper efficiency to assume may be taken from the table of usual values given below in section 28.

2. Suitable Speed (N). — Machines are classified in commercial practice as high speed, moderate speed and low speed. The proper speed for a machine of a given size for each class is given in the curves in the section below on *Cost, Weight, and Speed.*

3. Number of Poles (p) depends upon the size, speed and character of the machine. There is, however, much variation depending upon specific conditions. Reasonable arrangements are indicated in the first table on page 682.

4. Diameter and Length of Armature are related by the following formula:

$$D^2L = \frac{610\,P \times 10^9}{\sigma\rho\,BN}.$$

Kw. rating	Number of poles		
	High speed	Moderate speed	Low speed
0– 10	2	4	4
10– 50	4	4	4
50–100	4	4–6	6
100–300	6	6	8
300–500	6	8	10
500 and greater	..	..	12 and more

Usual Values of the Constants in the formula at bottom of page 681 are:

Magnetic Density in Air Gap (B) has values from 40,000 lines per square inch in small machines, 55,000 in medium and 70, 000 in large machines. In general, the density in the air gap is taken the same as the density at the pole face.

Ampere-Conductors per Inch of Periphery, or the Specific Loading (σ) is the product of the total conductors around the armature times the current in each, divided by the periphery of the armature. The usual values for σ are given in accompanying table.

Kw. rating	σ-amp.-cond. per inch of periphery	
	For continuous rating	For intermittent rating
0–150	250–400	700
150–400	400–600	1000
400 and greater	600–900	1200

Ratio of Pole Arc to Pole Pitch (ρ) varies from 0.6 to 0.85 with an average value of 0.7.

Determination of D and L. — Having found D^2L the two factors may be separated by two methods. Assuming a square pole face (a desirable proportion for economy of armature and field copper), then

$$L = \frac{\pi\rho D}{p},$$

and substituting in D^2L we can solve for D.

Another method of determining separate values for D and L is by a consideration of the peripheral speed. The peripheral speed is fairly uniform in d-c. machines, having values of from 3000 to 5000 feet per minute. Assume an average value near 4000. The diameter of armature is given by the formula

$$D = \frac{(\text{Peripheral speed}) \times 12}{\pi N}.$$

5. Length of Air Gap in inches for a given diameter has a value given approximately by the table in the next paragraph. This gives a good value for preliminary work but it may be found advisable to change it later in the design.

6. Number and Size of Armature Slots. — A rough approximation to practice gives the number of slots as four times the diameter in inches. General practice is shown in the following table.

D=arm. diam., in.	Air gap, in.	No. of slots	Depth of slots, in.
5	0.08	25– 40	0.6
10	0.10	30– 60	0.75
15	0.12	40– 80	1.00
20	0.125	60–100	1.25
30	0.15	80–150	1.50
50	0.19	100–200	1.75
100	0.25	150–300	2.50
150	0.35	200–400	3.25

7. Total Flux in the Armature (ϕ) is fixed by the assumed gap density B and the length L and diameter D of the armature:

$$\phi = \frac{\pi \rho LDB}{p}.$$

8. Number of Turns in Series (S) between brushes is

$$S = \frac{E \times 10^8}{4 f \phi}.$$

9. Armature Reaction Ampere-Turns per Pole are

$$(AR) = \frac{IS}{p}.$$

The armature reaction ampere-turns per pole (AR) should not exceed certain values and should properly check as explained in the accompanying table.

Kw. rating	Armature reactive ampere turns per pole (AR)
0–100	1500–3500
100–300	3500–6000
300–500	6000–8000

If the (AR) comes out higher than advisable it is reduced by increasing the value of the flux by increasing either B, L or D.

10. Number of Conductors per Slot (c). — If s is the total number of slots on the armature, the proper number of conductors per slot (c) must fulfill two conditions which are sometimes antagonistic and therefore the solution is a compromise.

For a multiple-drum winding:

$$c = \frac{\pi \sigma p D}{Is} \quad \text{and} \quad c = \frac{2\,pS}{s}.$$

For a series-drum winding:

$$c = \frac{2\,\pi \sigma D}{Is} \quad \text{and} \quad c = \frac{4\,S}{s}.$$

c must be an even number and give a number of conductors which can be conveniently arranged in a slot having a depth of from 3 to 4 times its width. If c comes out very large for a multiple winding we then choose a series winding.

11. Form of Armature Winding (for general discussion of forms of windings see above).

Multiple-drum Winding. — If the number of conductors per slot (c) indicates the desirability of a multiple-drum winding, we choose an arrangement having an even number of coil sides per slot and assume the conductors per slot

at a value the nearest multiple of the coil sides per slot, thus grouping the conductors into coils.

Series-drum Winding.—If a series-drum winding is indicated:

For a four-pole machine choose a number of conductors per slot (c) divisible by 6 and a number of slots (s) in the series 29, 33, 37, etc., endeavoring to obtain that value of $\frac{sc}{4}$ that comes nearest to our preliminary value of S, the number of turns in series between brushes. Or for a four-pole machine choose a number of conductors divisible by 10 and a number of slots in the series 27, 31, 35, that gives a value of $\frac{sc}{4}$ the nearest the preliminary value of S.

For a six-pole machine choose a value of c divisible by 4 and a number of slots (s) in the series 26, 32, 38, that gives a value of $\frac{sc}{4}$ nearest the desired value of S.

The preceding values of s give the best winding arrangement but there are other combinations giving unequal front and back pitches and longer connections.

Revised Value of Turns in Series (S).—Having chosen arbitrarily values for s and c to fit the form of winding desired we must revise our preliminary assumption for the turns in series, S, to accord with the actual winding. Thus for a multiple drum $S = \frac{sc}{2p}$, and for a series drum $S = \frac{sc}{4}$.

12. **Size of Armature Conductors.**—The current in each conductor is equal to the line current I divided by the number of poles in a multiple-drum winding, and to $I/2$ in a series-drum winding of the simple type. The cross-section of each conductor is equal to the current per conductor divided by the allowable amperes per square inch, which varies from 3000 in small machines and 2000 in intermediate size machines to 1500 in large machines.

13. **Exact Size of Slots, Slot Factor.**—The total number of slots is definitely set by the style of winding. The maximum allowable depth depends upon the diameter of the armature. The width of each slot should be from 0.4 to 0.6 of the pitch of the slots, which is the quotient of the periphery of the armature divided by the number of slots. The ratio of the total cross-section of copper in a slot to the cross-section of the slot is known as the slot factor and has more or less consistent values.

Kw. rating	Slot factor	
	For 125 volts	For 500 volts
0– 10	0.28	0.20
10– 50	0.35	0.25
50– 100	0.40	0.30
100– 400	0.50	0.40
400–1000	0.60	0.50

Type of slot	No. of bars or coil-sides per slot	Allowance for coil and slot insulation, in.	
		Width	Depth.
Open	2	0.09	0.30
Open	4	0.14	0.35
Open	6	0.15	0.35
Partly closed	4	0.15	0.40

The actual dimensions of the slots are found by adding to the width and depth of the cotton covered conductors, suitably arranged, an allowance for the coil and slot insulation. The allowances in the preceding table are for voltages of 500 and less.

14. Armature Resistance and Heating. — The mean length of turn is obtained from a drawing or is roughly estimated as

$$l = 2L + \frac{10D}{p}.$$

The equivalent resistance at 60° C. of the armature from brush to brush is

$$R_a = \frac{0.0093\,Sl}{12{,}000\,qm}.$$

The voltage drop in the armature is R_aI.

The loss in armature copper is R_aI^2.

The rise in temperature of the copper in ° C. is given approximately for preliminary purposes by

$$T_c = \frac{100\,R_aI^2}{(a+b)\,ls},$$

where

a = width of coils in slot, in inches.
b = depth of coils in slot, in inches.

15. Design of Commutator and Brushes. — The diameter of the commutator must be less than the diameter of the armature. The peripheral speed should be about 3000 feet per minute and should not exceed 4000 unless a special construction is used. The number of segments should be such that the average volts per bar should be less than 15.

$$\text{Volts per bar} = \frac{(\text{Rated voltage}) \times (\text{Poles})}{\text{Number of segments}}.$$

To avoid a weak construction the pitch of segments should not be less than 0.20 inch. Of this amount about 30 mils is occupied by insulation.

Dimensions of Brushes. — The width of the brushes is limited by the number of commutator segments which it is permissible to cover and the width of these segments. The number of segments covered is limited by the reactance voltage, as explained below. In general, the width of the brushes is from 2 to 3 times the pitch of segments. The area of the brush surface is such as to allow from 40 to 50 amperes per square inch of brush contact surface at full load. This surface is distributed equally over a number of studs equal to one-half the number of poles with a multiple-wound armature and may be all on one stud or disposed at pleasure with a series-wound armature.

Total Length of Commutator. — The total length of brushes per stud or the active length of the commutator is

$$l_c = \frac{2 \times (\text{Total surface of positive brushes})}{(\text{Width of brushes}) \times (\text{No. of studs})}.$$

The total length of commutator is greater than this by about 0.5 inch to allow for clearances between brushes and at the ends. It is also customary to allow an extra space at the end of the commutator for insulation to prevent creepage. Commutators in general have a gross superficial area of from 0.67 to 1.0 square inch per ampere of total current.

Resistance of Brushes and Brush Contact is a variable quantity. Some designers allow 2 volts total drop in positive and negative brushes under all conditions of load.

Another method is to use the values for the resistance per square inch of each brush and brush contact as given in the accompanying table.

Kind of brush	Resistance of each brush, ohm				
	20 amp. per sq. in.	40 amp. per sq. in.	60 amp. per sq. in.	80 amp. per sq. in.	100 amp. per sq. in.
Hard..........	0.051	0.03	0.023	0.018	0.015
Medium........	0.037	0.021	0.015	0.012	0.01
Soft...........	0.024	0.014	0.011	0.009	0.008

Friction of Brushes on Commutator.—The value of this loss is

$$\text{Friction in watts} = \frac{(\text{No. of brushes}) \times (\text{Pressure per brush}) \times V_c \times k}{45},$$

where

V_c = peripheral speed of commutator, feet per minute.
k = coefficient of friction = 0.2 to 0.3.

The usual value of pressure on the brushes is from 1.5 to 2 pounds per square inch of contact surface.

Rise in Temperature of Commutator is

$$T = t \times \frac{(\text{Brush } I^2R) + (\text{Brush friction})}{\text{Surface of commutator}}$$

where t has the value indicated in the accompanying table: t is the temperature rise in °C. for a total commutator loss of 1 watt per sq. in.

Peripheral speed of commutator, ft. per min.	Value of t.
2000	15
3000	12
4000	10

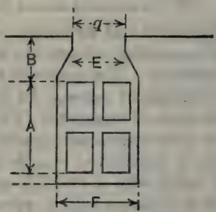

Fig. 14. Slot Diagram

16. Reactance Voltage.—The armature leakage flux per ampere-conductor may be resolved into three parts for convenience in analysis. These three fluxes are: P_s flux inside the slot itself, P_t flux from tooth tip to tooth tip, and P_c flux about the end connections.

Referring to the dimensions in Fig. 14 these fluxes are given by the expressions:

$$P_s = 3.2\, l \left(\frac{A}{3F} + \frac{B}{E}\right)$$

$$P_t = 2.35\, l \log_{10} \left(\frac{\pi m}{2q}\right)$$

$$P_c = 1.17\, l_c \left(\log_{10} \frac{2\, l_c}{U_c}\right),$$

where l = effective length of iron core,
m = interpolar distance between pole tips,
q = width of slot opening at face,
l_c = length of end connection at one end,
U_c = perimeter of one coil of end connections,
and all dimensions are in inches.

Also let t = turns in series between adjacent commutator segments,
= turns per coil in a multiple winding,
= $p/2$ times turns per coil in a series winding,

$$C = kt\frac{\text{(brush thickness)}}{\text{(segment pitch)}},$$

k = 2 for a full pitch winding and approaches unity as the pitch decreases,

then

$$L = Ct\,(2P_s + 2P_t + P_c)10^{-8} \quad \text{henries.}$$

Frequency of commutation,

$$f_c = \frac{12\, V_c}{120\ \text{(brush thickness)}}.$$

V_c = peripheral speed of commutator, feet per minute

$$I_c = \text{amperes per circuit in armature} = \frac{(I\ \text{line})}{\text{no. of mult. cir.}}.$$

Then assuming that the current in the short-circuited coil varies according to the sine law, the effective, or r.m.s., value of the reactance voltage is

$$e_x = 2\pi f_c L I_c \text{ volts.}$$

17. Exact Value of Flux Leakage Coefficient (v). — The exact number of turns was determined under *Form of Armature Winding*, section 11, above. The useful flux in the armature must then be

$$\phi = \frac{(E + R_a I) \times 10^8}{4fS}.$$

The flux in the pole or magnet core and the field yoke is greater than this, due to the leakage between poles. The ratio of the flux in the field to that in the armature is known as the leakage coefficient. It may be easily determined for each particular machine by the method given in the article on *Generators, Alternating-Current;* or if ample margin in the strength of field is always allowed it may be assumed without great error.

Rating in kw.	Leakage coefficient
2.5	1.2 -1.5
5	1.18-1.4
10	1.16-1.35
25	1.15-1.30
50	1.14-1.28
100	1.12-1.20
500	1.08-1.15

18. Cross-Section of Magnetic Path. — Usual Flux Densities. — Usual magnetic densities for determining the proportions of the magnetic circuit are given in the accompanying table.

Part	Material	Lines per sq. in.
Field yoke	Steel	70,000–100,000
Field yoke	Cast iron	35,000– 70,000
Pole core	Steel	70,000–100,000
Air gap	Air	40,000– 70,000
Armature teeth	Steel laminations	90,000–125,000
Armature core	Steel laminations	60,000– 90,000

The proper cross-section of each part is found by dividing the flux in that part by an assumed density. (*See also following paragraph.*) In most cases one dimension, as the length along the shaft, is known and the other is thus fixed.

19. Excitation. — Calculation of Magnetic Circuit. — It is advisable to calculate the densities and excitation for a value of flux corresponding to $E + R_a I$ at the speed at full load, as this is the most important condition. It is common practice to assume a loss of voltage of 5 per cent at full load instead of accurately predetermining the value.

Values of Flux in Different Parts of Magnetic Circuit. — In the armature teeth and air gap the total flux is equal to the calculated value of the useful flux. In the armature core the flux divides and we have one-half the useful flux in each branch. The field flux or flux in pole cores is greater than the useful flux and is equal to the useful flux multiplied by the leakage coefficient (ν). (*See section* 17.) In the yoke the flux divides and each branch contains one-half as much as the pole core.

Ampere-Turns. — The number of ampere-turns required for excitation is best calculated by the assistance of a tabulation like the following, in which the symbols refer to the dimensions shown in Fig. 15.

1	2	3	4	5	6	7
Part	Flux	Cross-Section	Flux density	Amp.-turns per in.	Length of path	No load amp.-turns per pole
Field yoke....	$\frac{\nu\phi}{2}$	de	......	....	i	......
Pole core......	$\nu\phi$	$0.785\,f^2$ or fb	......	....	c	......
Air gap........	ϕ	aL_p	......	$0.313\,B$	g	......
Arm. teeth....	ϕ	$Tt_0\,l_i$	......	....	n	......
Arm. core.....	$\frac{\phi}{2}$	hl_i	......	....	k	......

Net ampere-turns for flux: .....

The dimensions in column 3 can be taken directly from a sketch similar to Fig. 15. If the pole is circular its area is $0.785\,f^2$ and if it is square its cross-

section is fb. The length of pole shoe L_p is usually about 0.5 inch less than L, the length of armature core.

The effective cross-section of the armature teeth is found as follows: At any given time only a portion of the teeth are carrying flux. These are known as

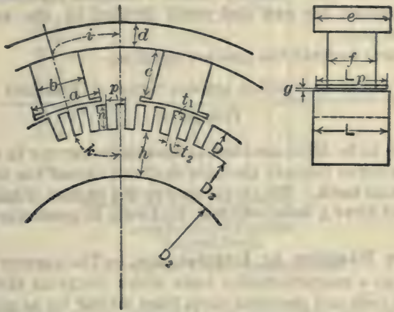

Fig. 15. Dimensions of Magnetic Circuit

the teeth under one pole, T, and are slightly greater than those actually under the pole, on account of the tendency of the flux to spread,

$$T = \frac{a + 2g}{p_1},$$

where a = pole arc,
g = length of gap,
p_1 = pitch of slots at face.

The effective or equivalent cross-section of the teeth is not that which gives the average density but that which gives the mean excitation required and is (see Fig. 15)

$$\frac{(t_1 + 2t_2)}{3} l_i = t_0 l_i.$$

The factor l_i is introduced to give the effective length of iron, excluding that space occupied by the insulation between laminations.

$$l_i = 0.9 \times (L - \text{space of air ducts}).$$

Thus the equivalent cross-section of armature teeth = $T t_0 l_i$.

Column 4 is the flux (column 2) divided by the cross-section (column 3).

Column 5 contains the ampere-turns per inch which are to be taken from saturation curves for the respective materials as given in the article on *Magnetic Properties of Iron*. The magnetizing force for a one-inch gap is always 0.313 B.

Column 6 gives the lengths of the respective paths as shown on Fig. 15.

Column 7 contains the ampere-turns required for each part of the path and is obtained by multiplying the values in column 5 by the respective lengths in column 6. The sum of all values in column 7 is the excitation in ampere-turns required on *each pole* of the field to establish the necessary flux in the machine. With no current flowing in the armature this would give a voltage about 5 per cent greater than the rated voltage of the machine. See section 21 below for compensation for armature reaction.

20. Stability Factor. — It is necessary at this point in the design to check the relations to see if the machine is liable to be unstable or require an excessive shift of the brushes. If interpoles are to be used this is not important, but if interpoles are not to be used certain conditions must be fulfilled. The criterion of these conditions is the "stability factor" which is the quotient of the field ampere-turns required for gap and teeth divided by the armature ampere-turns beneath the pole.

The ampere-turns beneath the pole are equal to

$$\frac{(\text{Armature reaction ampere-turns}) \times (\text{Pole arc})}{\text{Pole pitch}}.$$

The conditions to be filled are that at maximum current in the armature the armature ampere-turns beneath the pole shall not exceed the field ampere-turns required for gap and teeth. This is covered by the custom of making the stability factor at full load have a value of from 1.3 to 1.5 in generators and 1.5 to 3 in motors.

21. Armature Reaction or Interference. — The current in the armature conductors sets up a magnetomotive force which reacts on the magnetomotive force of the field coils and prevents them from setting up as much flux as when there is no current in the armature. It is, therefore, necessary to have the field strength at any load greater than at no load by an amount sufficient to overcome or neutralize the armature reaction, or to provide special auxiliary poles for the purpose. There are four methods of estimating the effect of armature interference and overcoming it.

(*a*) It is common practice in the manufacture of d-c. machines to provide a field coil of such ample capacity that it is sure to be more than sufficient to overcome armature interference and set up the flux desired. This field is then adjusted in practice by putting an adjustable resistance in series with the shunt field and a suitable resistance in multiple with the series field. For this practice it is usual to provide a total number of field ampere-turns per pole (F) equal to the net ampere-turns per pole at full load required for flux, as calculated under item No. 19, plus a number of field ampere-turns equal to 40 per cent of the armature reaction ampere-turns as calculated under item No. 9.

(*b*) The effect of armature reaction is divided into back ampere-turns and cross ampere-turns. The excitation ampere-turns per pole to overcome the back ampere-turns (caused by shifting the brushes) is

$$A' = \frac{(\text{Armature reaction}) \times (\text{Brush shift in segments}) \times 2p}{\text{Total commutator segments}}$$

The excitation to overcome the effect of the cross ampere-turns depends upon many variables but in machines designed in accordance with standard practice it bears a fairly uniform relation to the cross ampere-turns, depending upon the value of the ampere-turns in gap and teeth, as follows:

Cross ampere-turns = (armature reaction ampere-turns) − back ampere-turns).

Field ampere-turns to compensate for cross ampere-turns is then $B' = K \times$ (cross ampere-turns).

Where K has values as shown in Fig. 16 (*given by Cramp, see Bibliography*). Thus the total field ampere-turns at full load is then equal to

$$F = (\text{Net ampere-turns for flux, section 19}) + A' + B'.$$

(*c*) It is possible by working with the saturation curve of the machine to actually determine the effect of the cross ampere-turns on the saturation in

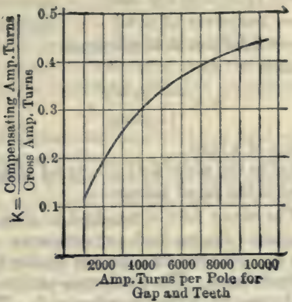

Fig. 16. Constant for Cross Ampere-turns

the teeth and pole face. This method is too elaborate and complicated to be discussed here and is of more value for educational than for practical purposes. It is discussed very fully in the books by S. P. Thompson and E. Arnold (*see Bibliography, below*).

(*d*) Interpoles may be used as described in section 23.

Excitation for Various Loads. — At no load the excitation required would be only that necessary to produce a flux corresponding to the rated terminal voltage. At any load corresponding to a current I the excitation in ampere-turns must be equal to the sum of: (1) that required to produce a flux corresponding to $E + RI$, where R is the total resistance of the armature between brushes, the brush resistance and the resistance of the series field; (2) that required to overcome the back ampere-turns caused by I, and (3) that required to overcome the cross ampere-turns due to I. The process of calculation indicated in sections 19, 20 and 21 for full-load conditions is, therefore, repeated for other load conditions.

22. Field Winding. — If the machine has a series field this should be laid out first, since the shunt-field coils are sometimes placed outside of, and concentric with, the series-field coils.

Series Field. — For a compound-wound machine the ampere-turns required in the series field are equal to the difference between the full-load ampere-turns and the no-load ampere-turns. As it is usual to shunt about 25 per cent of the current through a shunt resistance the number of series turns per pole for each coil is

$$t_1 = \frac{\text{Ampere-turns required}}{0.75 \times \text{Load current}}.$$

The cross-section of each turn in square inches is taken equal to the full-load current divided by 1000.

This winding is usually wound in copper strap like a ribbon, the width being equal to the total length of field spool if it is wound under the shunt winding, or is arbitrarily chosen if the two coils are placed side by side. The thickness of each turn is the quotient of the cross-section divided by the width.

The coils are wound with an extra odd half turn to facilitate connection. Thus the depth of winding is

$$(\text{Thickness} + 0.016 \text{ for insulation})\ (t_1 + 1).$$

Allowance for a layer of insulation inside and outside adds about 0.375 inch to the depth of winding.

The mean length of turn l_f' is (see Fig. 15):

For square pole l_f' = (periphery of pole) + 4 (depth of winding).

For round pole $l_f' = \pi$ (f + depth of winding).

The resistance per spool is

$$r_1 = \frac{0.0093\, t_1 l_f'}{12{,}000\, q_1},$$

where q_1 is the cross-section of a turn.

The total drop in voltage in the series field is $0.75\, r_1 I$ which should be between 0.5 per cent and 1 per cent of the rated voltage.

Shunt Field. — The shunt field is required to give a certain number of ampere-turns, usually equal to the no-load ampere-turns. Let this number of ampere-turns per pole be F. At 1000 amperes per square inch the cross-section of copper in a coil would be $F/1000$. The total cross-section of the coil, including insulation, is determined by means of the space factor f'. Then the cross-section of the coil $= \frac{F}{1000\, f'}$, where f' has a value of 0.5 if No. 18 B. & S. wire is used, 0.55 for No. 12, 0.60 for No. 6, and from 0.6 to 0.8 where copper ribbon is used. Where very small wires are used f' becomes as small as 0.25 (*see article on Electromagnet Windings*).

The depth of winding will be

$$b_2 = \frac{F}{1000\, f' c_2},$$

where c_2 = length of coil (*see Fig. 17*).

The superficial area will be (see Fig. 17) $A_2 = 2\,(f + b + 4\, b_2)c_2$.

The rise in temperature in ° C. of the coil will be from 60° to 80° per watt per square inch of A_2; thus for 40° rise on the spool the watts (w_2) per spool should be from $0.5\, A_2$ to $0.6\, A_2$ depending upon the ventilating effect of the armature.

Fig. 17. Dimensions of Field Coils

Voltage per spool under normal conditions, allowing 25 per cent to spare for rheostat, will be

$$e_2 = \frac{0.75\, E}{p}.$$

Current in shunt field $= i = \frac{w_2}{e_2}$.

Turns per spool $= t_2 = \frac{F}{i}$.

Mean length of turn, in in. $= l_f = 2\,(f + b + 2\, b_2)$ or $= \pi\,(f + b_2)$.

Resistance per spool $= r_2 = \frac{e_2}{i}$.

Cross-section of conductor in sq. in. =

$$q_2 = \frac{0.0093\, l_f t_2}{12{,}000\, r_2}.$$

The size of wire having a cross-section as nearly equal to q_2 as possible is chosen and arranged in layers. Single cotton-covered wire is used. The number

of turns per layer is found by dividing c_2 by the outside diameter of the single cotton-covered wire.

Certain allowances must be made for the thickness of the spool and of the collars or flanges.

After these details of arrangement are settled it is advisable to recalculate the mean length of turn, the resistance, and the heating to be sure that the practical details have not interfered with the preliminary assumptions of watts per square inch.

23. **Interpoles or Commutating Poles** are used where either the armature reaction or the reactance voltage is very great. If the armature reaction at full load is greater than 80 per cent of the field ampere-turns at no load, then interpoles should be considered. If the reactance voltage is greater than 2.5 volts, interpoles should be considered.

The design of the interpoles may be roughly made in accordance with the practice in some of the large companies by making its width equal to the width of two slots and two teeth, and proportioning the winding to have 1.4 times as many ampere-turns as the armature reaction ampere-turns (AR). The axial length of the pole is frequently less than that of the armature, the gap density should be about 30,000 and the iron or steel of the pole should not be saturated when 1.5 times the full-load current is flowing through its windings.

In practice a shunt having inductance is connected across the terminals of the interpole winding and adjusted for sparkless commutation. When the interpoles are used on a compound-wound generator it is possible to decrease the strength of the series coils to such a value as will produce the proper flux for the desired $E + rI$, letting the interpoles take care of the armature reaction. Series coils having 20 per cent as many ampere-turns as the armature will be satisfactory.

Exact Design of Interpoles. — A more exact method starts with the value of the reactance voltage, e, and an assumed density in the air gap of $B_1 = 30{,}000$. The axial length of the interpole is then

$$L_1 = \frac{10^8 \times e}{kB_1 v_a},$$

where

v_a = the peripheral velocity of armature in inches per second,
k = number of inductors in series in short-circuited coil = $2\,t$ (*see formula for reactance voltage, section 16*).

The width (a_1) of pole at face is made equal to or a little greater than twice the pitch of armature slots. The flux per pole in gap is $\phi_1 = B_1L_1a_1$.

The leakage coefficient is higher for interpoles than for the main poles and is in the neighborhood of 1.6 to 1.8.

The magnetic density is kept low in the pole core so that there will be margin for overload.

The necessary exciting ampere-turns to force this flux through the gap and to take care of the leakage and increased densities in the field and armature core are calculated. Let this value be A''.

Let the armature reaction ampere-turns per pole be B''.

Then the total ampere-turns on the interpoles should be $A'' + B''$.

The number of turns per pole is $(A'' + B'')$ divided by the full-load current of the machine, and the winding itself is laid out in the manner employed in laying out the series field.

PREDETERMINATION OF PERFORMANCE FROM DESIGN. — The steps involved in predetermining what will be the performance of a given design are the following:

24. Core-loss. — The core-loss consists of hysteresis and eddy current losses in the armature teeth and core. It is best calculated by taking from the curves in the article on *Magnetic Properties of Iron* the hysteresis loss per cubic inch at one cycle per second for the proper density and the eddy loss per cubic inch at one cycle per second for the proper density and thickness of sheet. These values are substituted in the following formulæ.

Total loss in core $= V_c C_h f + V_c C_e f^2$,
Total loss in teeth $= V_t C_h f + V_t C_e f^2$,

where

V_c and V_t = volume in cubic inches of core and teeth respectively,
C_h = constant from hysteresis loss curve for proper density,
C_e = constant from eddy loss curve for proper density, and thickness of sheet.
f = frequency.

A rough value for V_c and V_t can be conveniently obtained from the values in the table for the calculation of the excitation, section 19. Then

$V_t = p \times$ (area path) $\times$ (length of path),
$V_c = 2\,p \times$ (area path) $\times$ (length of path).

The calculation of the core-loss is one of the most unreliable and unsatisfactory steps in the design, as the value calculated by theoretical principles has to be multiplied by a factor having a value between 1.5 and 3 to obtain the value of the core-loss that will be found in the finished machine, on account of the pulsations of flux caused by the teeth. This factor is obtained in commercial practice from tests on machines similar to the one being designed.

25. Friction and Windage can be calculated by formula for any particular line of similar machines but no method applicable in general can be given. Machines are designed so that the value of the friction loss very closely approximates those given in the table in section 28. The true friction varies directly as the speed, and the windage as the square of the speed. In shunt machines the speed does not change much with the load so this loss may be assumed constant in this class of machines (*see Motors, Direct-current*).

Stray Power Loss. — This term is sometimes used to include both the core-loss and friction and windage.

26. Excitation Loss or RI^2 in Shunt Field. — Since in a self-excited machine the loss in the field rheostat is charged against the machine the total loss is equal to the terminal voltage multiplied by the current in the shunt field. The loss is usually constant and only subject to the arbitrary variations due to hand regulation.

27. RI^2 Loss in Armature Circuit, including the brushes, series field and connections, varies as the square of the armature current, and, therefore, approximately as the square of the load, and also irregularly to a small extent due to the variation of brush contact resistance with the value of the current. The temperature of the copper must also be considered. (*See Resistance and Conductance.*)

28. Efficiency. — The efficiency is the ratio of the output to the sum of the output and all losses. It is predetermined by estimating the value of each loss for the particular condition of load under consideration, as described in the preceding paragraphs.

If P = the output in watts and the symbols A, B, C and D represent respectively the values in *watts* of the various losses described in sections 24, 25, 26 and 27 for the load P, then

$$\text{Per cent efficiency} = \frac{10^2 \times P}{P + A + B + C + D}.$$

The efficiency is a maximum for that load at which the variable loss D is equal to the sum of the constant losses $A + B + C$. Hence the desirability of making the constant losses small in a machine which is to be operated most of the time at a small load.

Usual Efficiencies and Losses. — An idea of a reasonable value for the efficiency of d-c. machines of various sizes and the distribution of the losses is given in the following table. It must be remembered that the efficiency of a machine of any given size may vary throughout a wide range depending upon the speed, weight and cost.

Rating, kw.	Efficiency, per cent	Friction (total), per cent	Excitation, per cent	Core-loss, per cent	Arm. RI^2, per cent
1	80	6	6	4	4
5	84	5	4.2	3.2	3.6
10	86	4	3.6	3.0	3.4
20	88	3	3.0	2.8	3.2
50	90	2.6	2.2	2.2	3.0
100	91.4	2.3	2.0	1.7	2.6
200	92	2.2	1.8	1.6	2.4
500	93	2	1.6	1.4	2.0

29. Regulation is not very important in a d-c. generator as compounding provides a means of obtaining any desired voltage at full load. Most generators are designed with a series field which will give 15 per cent overcompounding at full load. If less compounding effect is desired a certain portion of the load current is shunted by the series field, which adjustment is made by trial.

The inherent regulation of a shunt generator may be predetermined as follows: Let C (*see section 19*) be the total excitation in ampere-turns per pole to give a voltage E at the terminals at full load. C is made up of the m.m.f. to produce flux for $E + RI$ and the m.m.f. to overcome armature reaction. Let E_0 be the voltage at no load for an excitation C as shown by the no-load saturation curve. Then

$$\text{Per cent regulation} = 100\,\frac{E_0 - E}{E}.$$

30. Heating.— The rise in temperature of the field coils was found in section 22. The rise in temperature of the armature conductors was roughly checked in section 14, but it is necessary to determine more accurately the probable rise in temperature of the armature conductors as the heat due to the core-loss affects the rise in temperature of the armature windings. The following method given by Arnold takes into account all the variables in this complex problem and with a correct choice of constants gives very reliable results. The dimensions are as indicated in Fig. 18.

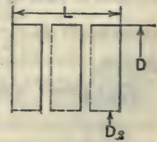

Fig. 18. Radiating Surface of Core

The surface of the armature core itself is taken as the sum of the outer cylindrical surface, the two annular surfaces at the ends, and one-half of the annular surfaces in the air ducts. Only one-half the surface in the ducts is used since this surface is not as effective as the external surfaces. It is assumed that the energy dissipated by this surface consists of the core-loss and that portion of the armature R_aI^2 which occurs in the portion of the conductors embedded in the slots.

A part of the heat due to R_aI^2 in the conductors flows from the copper through the slot insulation to the iron, if the temperature of the copper is higher than that of the iron. Sometimes, however, at light loads for instance, the temperature of the iron is greater than that of the copper and the flow of heat is reversed. This condition is taken care of in the following method.

The remainder of the heat due to R_aI^2 is dissipated through the insulation on the end connections to the surrounding air which is in motion due to the peripheral speed of the armature.

Power in watts dissipated by surface of armature core is

$$P_1 = \text{core-loss} + R_aI^2\left(\frac{2L}{l}\right),$$

I = full-load current; R_a = resistance of armature winding; L = total length of core; l = mean length of armature turns.

Effective radiating surface of armature core in square inches is

$$A_i = \pi DL + \frac{\pi}{4}(D^2 - D_2^2)(2 + d).$$

D = outside diameter of armature (Fig. 15); D_2 = inside diameter of armature; d = number of air ducts.

Rise in temperature in ° C. of armature core above air is

$$T_i = \frac{P_1}{A_i}t.$$

t = 30 for narrow, or high-speed armature; = 40 for long or slow-speed armature.

Permeance of heat path from copper to iron through slot is

$$M = \frac{U_sLs}{k_1d_1},$$

where

U_s = perimeter of slot insulation; s = total number of slots; k_1 = 200 to 250; d_1 = thickness of insulation in slot; L = length of core.

Heat power in watts transferred from copper to iron or vice versa is

$$Q_1 = M(T_c - T_i),$$

where

T_c = rise in temperature of copper above air

Permeance of heat path from copper to air at end connections is

$$N = \frac{U_cl_cZ}{k_2d_2 + k_3},$$

where

U_c = perimeter of end connection; l_c = length of end connection; Z = total number of bars or coil sides; k_2 = 400 to 500; d_2 = thickness of insulation on end connection; $k_3 = \frac{170}{1 + 0.00025\,V}$; V = periphery speed of armature, feet per minute.

Heat power in watts transferred from copper to air by end connections is

$$Q_2 = NT_c.$$

Total power of R_aI^2 in armature winding is

$$P_2 = Q_1 + Q_2 = M(T_c - T_i) + NT_c.$$

Whence, by transposal, the temperature rise, in °C., of copper of conductor above surrounding air is

$$T_c = \frac{P_2 + MT_i}{M+N}.$$

Explanation of Constants and Choice of Values:

t is the rise in temperature per watt per square inch at the surface of the armature or the specific drop in thermal potential from iron to air. It is affected by the peripheral speed and by the length of the armature.

k_1 is the drop in thermal potential per watt per square inch for slot insulation having a thickness of one inch. For any other thickness d_1, the drop is k_1d_1.

k_2 is the drop in thermal potential per watt per square inch for insulation on end connections when the thickness is one inch, k_2d_2 for any thickness d_2.

k_3 is the drop in thermal potential per watt per square inch between the surface of the insulation and the surrounding air. For stationary conditions $k_3 = 170$. As the peripheral velocity increases k_3 decreases.

Usually four-coil sides are bound together in the slot but separate on leaving the slot. Hence the reason for using s and Z in the different equations.

TESTING OF DIRECT-CURRENT MACHINES. — Direct-current machines are judged by four characteristics: efficiency, regulation, commutation and heating. It is necessary to subject a machine to actual full-load conditions to determine its heating and commutation and it is desirable to do so to determine the regulation, but the efficiency is determined more accurately by the "separate loss method" which does not involve loading the machine.

The customary tests on d-c. machines are: (*see also Standardization Rules.*)

(1). Resistance measurements, cold and hot. (2). Saturation curve. (3). Core-loss and friction test. (4.) Load run for commutation and regulation. (5). Heat runs. (6). Compounding test. (7). Insulation test.

1. Resistance Measurements — The first measurements are made when the machine is at the same temperature as the room, that is, after it has been idle from 12 to 24 hours depending upon its size. This is in order that the relation between the resistance and temperature may be accurately known. The resistance of the shunt-field, series-field and armature winding proper are measured.

Resistance of Lap Winding. — In measuring the resistance of the armature winding it is preferable to measure between two diametrically opposite points of the commutator of a multiple-wound armature. The effective resistance of the armature is calculated from this by the formula

$$R_a = \frac{4\,R'}{p^2},$$

where R' = resistance measured as above, and p = number of poles.

Resistance of Wave Winding. — In measuring two-circuit series-drum windings it is necessary to measure between two commutator segments separated by a distance equal to the periphery of the commutator divided by the number of poles. This will give the effective resistance of the armature.

Brush Contact Resistance. — The resistance of the brushes and brush contact is calculated from data such as given in section 15 in the above section on *Design;* a measurement made with the actual circuit is not very satisfactory. However, it is sometimes the practice to measure the resistance of the entire armature circuit from brush to brush by the drop in potential method for several values of current, the armature being at rest. This is not accurate, however, as the resistance of brush contacts depends upon the speed.

2. **Saturation Curve.** — The saturation curve is not of immediate interest in determining the quality of a machine but is more particularly of interest to the designing engineer. It is made by driving the armature at the proper speed and supplying current to the shunt fields from a separate source of potential. As the current in the shunt field increases the potential generated by the armature varies and this is measured by a voltmeter.

On account of the existence of the hysteresis loop (*see Magnetic Properties of Iron*) it is necessary to increase the field current gradually and never to reduce the value at any step until the maximum value has been reached. The curve is plotted with volts as ordinates, and field current or ampere turns as abscissæ and shows the typical knee curve of all saturation curves. Different curves are obtained with increasing and decreasing excitation as shown in Fig. 19.

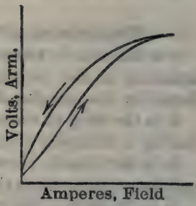

Fig. 19. Saturation Curve

3. **Core-Loss and Friction, or Stray Power.** — These tests are made with the same arrangement as used in determining the saturation curve, and may be made at the same time. The machine to be tested is driven by a small motor having a capacity about 10 per cent of the rating of the machine under test. All the losses and constants of this motor must be known.

First the machine under test is driven at the desired speed with no current in the fields and the power taken by the driving motor noted. The mechanical output of the driving motor is calculated and this gives the friction loss of the large machine.

Then the large machine is excited and the power to drive it is determined. This power represents the core-loss plus the friction loss. Deducting the friction loss already determined, the values of the core-loss for various values of excitation are found. See also *Magnetic Properties of Iron.*

The combined core-loss and friction loss constitute the stray power loss.

4. **Load Runs.** — The machine is run and the excitation adjusted to give the proper voltage at no load. The load is then added and the terminal voltage noted. If E_0 = voltage at no load and E = voltage at full load, the regulation is $\frac{E_0 - E}{E}$.

If the machine has not commutating poles it is necessary to make a preliminary test in order to set the brushes at that position which gives the best compromise in sparking at no load and full load. Commutation is judged by observing the action of the brushes at full load and 150 per cent load. The conclusions are a matter of judgment and experience, although degrees of sparking have been arbitrarily agreed upon and are represented in a chart or series of pictures.

5. **Heat Runs** are made by operating the machine at full load for a period of time until the temperatures of the various parts that can be noted during operation have become constant. The greater the capacity of the machine the longer will be the time necessary.

Dead-load and Pumping-back Methods. — Heat runs may be made either by the "Dead Load" method in which a resistance load, such as a water-rheostat, is connected to the terminals and full rated load power is required to drive the machine; or by the Hopkinson or "Pumping-back" method in which two machines having similar characteristics are run together, one acting as a generator to supply electrical power to the motor which in turn drives the first by means of a belt or similar mechanical connection. For this test a "loss-supply" is required which may consist of a source of either electrical power or

mechanical power. The amount of power required is from 10 to 20 per cent of the rating of one of the machines. The connections are shown in Fig. 20, for electrical loss supply and Fig. 21 for mechanical loss supply.

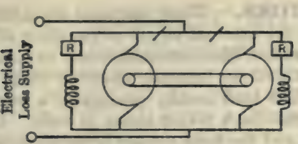

Fig. 20. Hopkinson Load Test, Electrical Supply of Losses

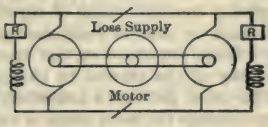

Fig. 21. Hopkinson Load Test, Mechanical Supply of Losses

Thermometer Readings. — During the heat run there are taken at stated periods, readings of thermometers which show the temperature of the frame, field coils, bearings and surrounding air. After the heat run the thermometers are placed on various parts of the machine, the bulbs being protected from radiation by small pads of cotton, and the following temperatures are noted:

Armature-core surface;	Field coils;
Armature-core ventilating ducts;	Bearings;
Armature conductors or winding;	Frame;
Commutator surface;	Room.
Pole tips;	

The resistance of all electrical circuits should be measured and the average temperature of the copper calculated from the formula,

$$t_1 = \frac{R_1}{R_0}(239 + t_0) - 239,$$

where R_1 and R_0 are the hot and cold resistances respectively, and t_1 and t_0 the hot and cold temperatures respectively, 98 per cent conductivity copper being assumed. See article on *Copper*.

6. Compounding Test. — In order to adjust the current in the series field so that a compound-wound generator will give specified voltages at no load and full load, the machine is first operated at no load and the current in the shunt field is adjusted to give the desired no-load voltage. The load is then put on and usually it will be found that at full load the terminal voltage is too great. Strips of German silver or other resistance metal are then connected in multiple with the series field until by shunting current from the series field the voltage is reduced to such value as is desired. This shunt resistance is then made up in permanent form and connected in circuit. Before making the no-load adjustment it is desirable to overexcite the shunt field for a moment in order to over come the hysteresis. See also section on *Parallel Operation* below.

7. Insulation Tests. — After the heat runs it is customary to apply a high-potential test to the insulation of the machine. This consists of applying an alternating potential between each electrical circuit and the frame of the machine for one minute. The value of the potential depends upon the capacity and rated voltage of the machine under test and is definitely specified in the Standardization Rules of the A. I. E. E. (*q. v.*).

EXAMPLES OF DESIGN AND PERFORMANCE. — In the tables on pp. 701 and 702 will be found the essential data including mechanical, electrical and magnetic characteristics of four examples of direct-current machines. In

these data are included all the factors necessary to determine the efficiency regulation, commutation and heating of the machines. Dimensions are in inches and weights in pounds. Performance data are deduced from tests.

SERIES ARC-LIGHT GENERATORS. — These machines are intended to give a constant value of current in the external circuit irrespective of the resistance or counter e.m.f. in that circuit. This is accomplished by causing the voltage impressed on the external circuit to decrease whenever there is a tendency of the current to increase. The load usually consists of a number (20 to 50) of arc lamps connected in series, each taking about 40 volts. Each lamp requires the same value of current and the total voltage is proportional to the number of lamps in use. If a lamp is no longer needed it is short-circuited by a switch and the current in the remainder of the lamps remains the same because the generator automatically reduces the voltage supplied. This regulation of the current is obtained by two or more of the following methods: (1) high armature reaction which tends to weaken the field whenever the current increases; (2) movable brushes to vary the number of turns in series between brushes; (3) a series field arranged so that turns may be shunted, short-circuited or cut out; and (4) a high degree of saturation in the magnetic circuit of the field so that the flux changes very slightly for considerable change of m.m.f.

The pole face density is usually quite high and the path of the flux in the field is of high reluctance, whereas the path of the armature flux is of low reluctance; thus there is much field distortion. The machines are usually wound for 2000 to 4000 volts. The distorting effect of armature reaction is usually relied upon to take care of sudden and small variations of the load, and large and lasting variations of the load are taken care of either by moving the brushes to another position giving either a lower or higher voltage, or by commutating or shunting some of the turns of the series field.

Brush Arc Machine. — This was the first arc-light machine to be brought out. It consists of a ring armature having a winding of the open-circuit type. There are a number of spool-wound coils on the armature, diametrically opposite coils being connected in series to an independent pair of commutator segments. The brushes make a series connection of the various groups of coils, the coils of highest e.m.f. being connected in series, those of medium e.m.f. in multiple and those of low e.m.f. being left out of circuit. The numerous field poles are on both sides of the ring armature. The two poles facing each other are of the same polarity so that the flux flows along the ring of the armature from one pole to the next pole on the same side of the ring. Regulation is obtained by shunting field turns and shifting the brushes for good commutation. A rotary oil pump is used as a regulator to move the handle of the field regulator and to move the brushes.

Thomson-Houston Arc Dynamo. — This machine contains a spherical-shaped armature with a three-part open-circuit winding. The three windings being spaced at 120° on a ring armature and connected in "Y," the terminals going to the segments of a three-part commutator. The commutator has air spaces between segments and a jet of air to blow out any arcs between brushes and segments. There are four brushes on the commutator, connected in pairs. At some part of each revolution two coils are in multiple, at other positions only one. A relay moves the brushes so that as the load increases the positive and negative brushes move farther apart thus giving more voltage. As one brush of a pair moves forward the other moves backward thus keeping them symmetrical with respect to the neutral axis. The field structure is of a hollow, cage-like construction.

Wood Arc-light Machine. — This generator has a closed-circuit Gramme ring armature with a commutator having a large number of segments. Regu-

MECHANICAL DATA ON D-C. MACHINES

Dimensions in Inches, Weights in Pounds

Type	1 Motor	2 Generator	3 Generator	4 Generator
Poles	2	6	6	8
Rating, kw	2.25	35	500	250
Revolutions per minute	1200	1050	900	150
Volts	115	125	275	250
Current	23.4	280	1830	1000
Armature diameter at face	7.13	20.25	31.5	45
Armature diameter at back	1.63	13.5	19.5	32
Armature, total length	4.69	3.38	14.	21.5
Air ducts	....	1×0.38	5–0.375	7×0.5
Slots, number	34	109	108	128
Slots, dimensions	0.95×0.22	1×0.28	1.12×0.36	1.32×0.52
Conductors per slot	28	2	2	6
Conductors, size	d=0.054	0.16×0.32	0.26×0.4	0.5×0.125
Conductors in multiple	4	2	6	8
Type winding	Drum	S.D.	M. D.	M.D.
Pitch of coils	1	1	100%	1
Air-gap length	0.06	0.156	0.188	0.312
Pole arc	6.83	7.38	11.5	13.25
Pole length	4.25	3.13	13.5	21
Magnet-core length	4.25	3.13	13.5	21
Magnet, width	3.94	4.83	8.5	8.13
Magnet, radial length	3.44	5.25	10	10
Yoke length	6	8	15	21
Yoke, radial length	2.75	2.5	8	5
Shunt spool, turns	2210	481	900	372
Shunt spool, size conductors	d=0.018	d=0.083	B.&S.No.10	0.144×0.156
Series spool, turns	28	9.5	1.5	4.5
Series spool, size conductors	0.14×0.13	5.06×0.05	2.5×0.75	9.25×0.075
Commutator diameter	5	13.5	20.75	30
Commutator length	3.13	6.25	13.5	14.5
Commutator, number segments	34	109	108	384
Studs × brushes	2×2	6×3	6×10	8×8
Dimensions each brush	0.75×1	0.75×1.38	1.25×1.25	0.75×1.25
Interpole arc	0.87	....	2	....
Interpole dimensions	0.87×3	....	2×13	....
Interpole, size conductors	0.09×0.09	....	2.5×0.75	....

ELECTRICAL AND MAGNETIC DATA ON TYPICAL D-C. MACHINES

Percentages are all in Terms of Rated or Full-load Values

Type		1 Motor	2 Generator	3 Generator	4 Generator
Rating	kw.	2.25	35	500	250
Rated voltage	volts	115	125	275	250
Rated current	amperes	23.4	280	1830	1000
Flux per pole	maxwells	1.12×10^6	1.19×10^6	8.8	13.5×10^6
Leakage coefficient	....	1.2	1.13	1.18	1.13
Excitation amp. turns, no load	amp. turns	1040	3470	7300	6525
Excitation amp. turns, full load	amp. turns	....	5270	.....	9140
Armature reaction	amp. turns	1390	2540	5500	6000
Stability factor	....	0.9	1.55	.....	1.36
Excitation to balance arm. reaction	amp. turns	....	1090	.....	1150
Excitation at 110% volts	amp. turns	1145	4200	.....	7800
Volts per bar	volts	6.8	6.8	15.2	5.2
Reactance volts	volts	5.7	1.6	7	1.01
Friction loss, brush	watts	60	460	3800	485
Friction loss, other	watts	100	600	7000	....
Core-loss	watts	60	1680	8300	2815
Armature res. at 25° C.	ohms	0.27	0.013	0.0016	0.006
Brush res. at full load, 25° C.	ohms	0.044	0.0072	0.00104	0.0026
Series-field res. at 25° C.	ohms	0.05	0.0033	0.00018	0.00137
Interpole res. at 25° C.	ohms	0.123	....	0.00034	....
Shunt-field res. at 25° C.	ohms	150	9	25.2	9.2
Friction loss	per cent	6	2.6	2.	0.2
Core-loss	per cent	2	4.1	1.56	1
Shunt-field loss	per cent	3	2.2	0.42	1.6
Arm. copper loss	per cent	6.5	4.9	2.20	4.1
Efficiency at full load	per cent	82.5	86.2	93.8	93.1
Rise in temp. by thermometer:					
Armature	° C.	....	....	44	18
Field	° C.	....	....	24	15
Rise in temp. by resistance:					
Armature	° C.	....	....	44	....
Field	° C.	....	....	33	....

lation is obtained by moving the brushes so as to vary the voltage available and by the use of a high armature reaction balancing the field m.m.f.

HOMO-POLAR GENERATORS. — This type of machine is also sometimes called "acyclic" and was formerly incorrectly called uni-polar. Its method of operation is based on the principle of the Faraday disk, which consisted of a copper disk revolving about an axis and projecting between the poles of a magnet. By this rotation in a magnetic field an e.m.f. is set up between the axis and the periphery of the disk, and if brushes bearing on these two parts are connected to an external circuit a current will flow. The peculiar characteristic of a homo polar machine is that each conductor always cuts the flux in the same direction, consequently the e.m.f. induced in it is always in the same direction and is not alternating as in the usual direct-current machine. Thus no commutator is required.

This absence of a commutator is the feature which makes the homo-polar machine attractive. The commutator presents many difficulties in high-speed machines to be driven by steam turbines. It is for this application that recent attempts to develop a successful homo-polar machine have been directed. Instead of a commutator, collector rings with brushes are used to collect the current from the moving conductors. These collector rings, however, present difficulties in construction and operation on account of the high peripheral speed at which they must run. The rings are subject to a considerable centrifugal force and there is a tendency of the current to arc between the brush and the collector on account of the high rubbing speed.

Voltage. — The voltage of such a machine is not only unidirectional as in all direct-current machines but is really constant. But since there can be only a few conductors in series, the voltage generated is very low. The voltage generated per disk or inductor is

$$E = Blv\,10^{-8},$$

where B = magnetic lines per sq. cm., l = length of conductor in cm., v = velocity of conductor in cm. per sec.,
This is more conveniently expressed,

$$E = \frac{NZ\phi 10^{-8}}{60} \text{ volts},$$

where N = revolutions per minute, Z = conductors in series, ϕ = total flux traversing gap.

Radial and Axial Types. — There are two types of homo-polar machines, the radial and the axial. The radial type (Fig. 22) is like the Faraday disc and

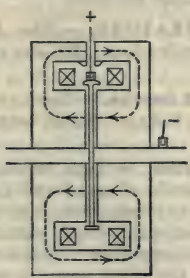

Fig. 22. Radial Type of Homo-polar Generator

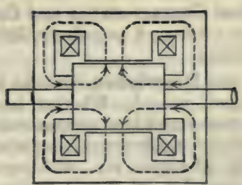

Fig. 23. Axial Type of Homo-polar Generator

consists of a disc revolving between the two poles of a cylindrical magnet. Brushes bear on the outer rim and the shaft to collect the current. The disc may be made of steel to reduce the reluctance of the magnetic path. The voltage of such a machine is limited to 10 or 15 volts, but the current may be very large. A variation of this type has two discs on the same shaft and the magnetic path so arranged that the voltage of the two discs may be added in series by brushes bearing on the peripheries of the two discs. The axial type (Fig. 23) consists of a cylindrical steel armature with copper bars in the surface, the whole revolving in a cylindrical field so arranged that the magnetic flux flows outward from the armature in a radial direction at all points. The several conductors on the armature are connected to slip-rings at both ends, and by means of brushes and stationary conductors, these conductors are connected in series. The voltage of such a machine may be from 40 to 50 volts per conductor.

Data on Large Axial Type Machine. — In the *Trans. A.I.E.E., Vol. 24*, Noeggerath describes a machine of the axial type rated at 300 kw. for 500 volts at 3000 r.p.m. The armature has 12 conductors connected in series for 500 volts. The diameter of the armature is 19 inches and the length 12 inches. The peripheral velocity is 15,000 ft. per min. The armature is of cast steel and has 24 cast steel collector-rings on it. The stationary conductors connecting the collector-rings together are placed in the face of the pole and thus their m.m.f. may be used to balance the armature reaction.

Excitation. — The armature reaction of such machines is very high and has only a distorting effect. However, it weakens the field as the cross magnetisation weakens one part more than it strengthens another due to saturation. By a proper arrangement of the movable connections between the collector-rings and stationary conductors a m.m.f. may be set up which will strengthen the field and thus the machine may be compounded. These machines may be made self-exciting but the resistance of the shunt fields must be very low in order that the machine may pick up on starting as the resistance in the brush contacts is high. The drop in voltage in each brush contact is about 0.8 volt at full load but is higher before the current flows. It sometimes requires from 10 to 20 times the normal voltage of brush contact to start the current.

Losses. — Such a machine as described above has an efficiency of approximately 90 per cent at full load. The losses are made up principally of friction and I^2R in the brushes. The field I^2R is low as the air gap is small for mechanical considerations. The armature I^2R is almost negligible due to the few turns. The eddy losses are low if the flux density is constant in one zone around the armature but the density may vary along an element of the cylinder. The total weight of the machine is about the same as for the usual d-c. machine but the proportion of copper is much lower than in the usual machines.

SPECIFICATIONS FOR D-C. GENERATORS. — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Principal Characteristics and Conditions of Service. — Service for which it is to be used, such as railway, lighting, etc. Voltage. Rated output, kilowatts. Speed.

Style and Description, Details of Construction. — Type, whether shunt or compound wound. Whether interpole. Details of speed and governing of prime mover and how generator is connected to prime mover, e.g., direct or belt driven by steam turbine, reciprocating engine, water wheel, water turbine, gas engine, oil engine, etc., with vertical or horizontal shaft. If field rheostat is to be supplied, its characteristics, including the effect upon the generator

voltage of each step and of all steps: accessibility of armature. If belt driven, specify pulley details.

Work to be Done by Other Contractors. — *See Generators, Alternating-Current,* under same heading.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*). Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at ¼, ½, ¾, 1, and 1¼ loads. (Whether rheostat losses to be included in calculating efficiencies.) High potential tests of insulation. Requirements regarding effect of moisture upon insulation. When run at constant rated speed, the load may be varied from zero to a stated per cent of rated load without causing more than a stated variation of voltage, the field rheostat being kept constant.

INSTALLATION. — In installing a d-c. machine the following precautions must be observed:

1. For large machines with foundations the foundation bolts must be provided in accordance with drawings.
2. Bearings must be lined up and well cleaned before being filled with oil.
3. The armature must be properly centered so that the air gap is correct at all points. Taper wedges are used to measure the gap. The magnet frame should be bolted to the base.
4. The field coils must be properly connected. Test for polarity with a compass in order to make sure that no field coils are reversed. For a self-exciting generator there is one particular connection of the field to the armature for each direction of rotation.
5. The commutator must be smooth and polished; use sandpaper to polish the commutator, never use emery cloth.
6. The brushes must be properly and accurately spaced around the commutator. They must be sandpapered and fitted to the curvature of the commutator. The pressure on the brushes must be adjusted to the correct value which is usually 1.5 to 2 pounds per square inch of contact surface.
7. The machine must be thoroughly dried out by heating and the insulation measured as a check.

OPERATION. — In starting up a single generator it is sometimes necessary to "charge" the field by separately exciting the shunt fields for a moment to set up residual magnetism.

To cause the machine to "pick up" or generate voltage by self-excitation it is necessary to cut out or short-circuit most of the resistance of the regulating rheostat connected in series with the shunt field. If the total resistance of the shunt-field circuit exceeds a certain critical value the machine will not "pick up," however much time is allowed.

Parallel Operation. — In order to operate a power station under economical conditions it is necessary to have a number of machines whose aggregate capacity is equal to the maximum demand on the station. As the demand varies the number of machines in operation is adjusted so that the machines running are operating at a load near their rating, and, therefore, at a good efficiency.

Shunt Generators. — In order to operate shunt generators in parallel, that is, feeding the same bus-bars, it is only necessary to adjust them all to the same polarity and voltage, connect them to the bus-bars, and adjust the division of load by strengthening the field of the underloaded machine if the voltage of the bus-bars is low. If the voltage of the bus-bars is high, weaken the field of the overloaded machine.

Compound Generators, Equalizer Connection. — In order to operate compound-wound machines in parallel it is necessary to provide an equalizer connection which makes a common connection on all the machines at the point between the armature and the series field as shown in Fig. 24.

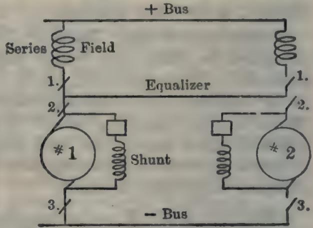

Fig. 24. Parallel Operation of Compound-wound Generators

The function of the equalizer is to divide the load current at all times in the proper proportion between the series fields of the different generators. This prevents the machines from acting as series generators or differential motors which would cause short-circuits.

For compound-wound machines to operate successfully in multiple all machines connected to one set of bus-bars must have the same amount of compounding as well as the same voltage at no load. Due to saturation in the magnetic circuit it is not always possible to make the compounding curve a straight line, i.e., the increase in voltage directly proportional to the load. It is, therefore, necessary to investigate the compounding curves of machines before they are operated in parallel. With unlike compounding curves one machine may become overloaded while another is under-loaded, unless the field current of one machine is adjusted.

In connecting a machine in parallel with those in operation it is necessary to see that it has the proper polarity and voltage, that the equalizer circuit is made,

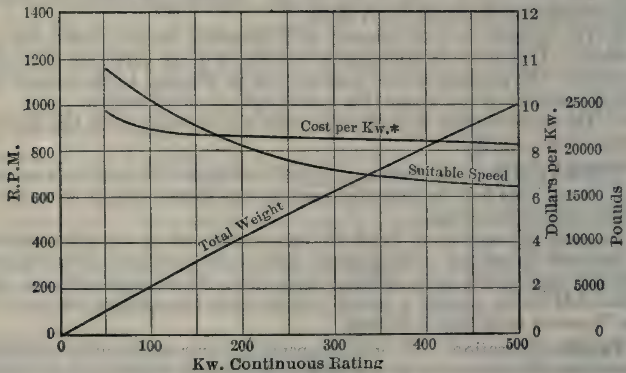

Fig. 25. Data on High-speed D-C. Machines

and that the switches are closed in the order 1, 2, 3, as shown in Fig. 24. If any other order is used the effect is the same as having no equalizer.

In shutting down one machine switch No. 3 must be opened first, and then No. 2 and No. 1.

COST, WEIGHT AND SPEED (Pre-war prices).— In Figs, 25, 26 and 27 will be found the weight and cost per kilowatt of direct-current generators for usual or standard speeds. Fig. 25 gives the weight and cost of a line of high-speed machines for the suitable speeds shown by the curve. Fig. 26 gives the same information for a line of moderate-speed machines, generators or

* 1922 prices about 50 per cent higher than given by this curve.

motors, and Fig. 27 for a line of slow-speed generators designed for direct connection to steam engines.

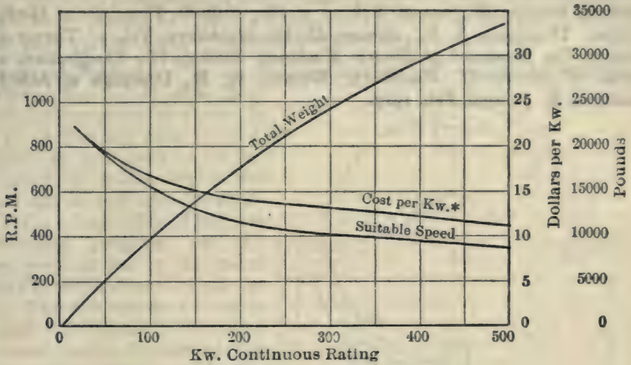

Fig. 26. Data on Moderate-speed D-C. Machines

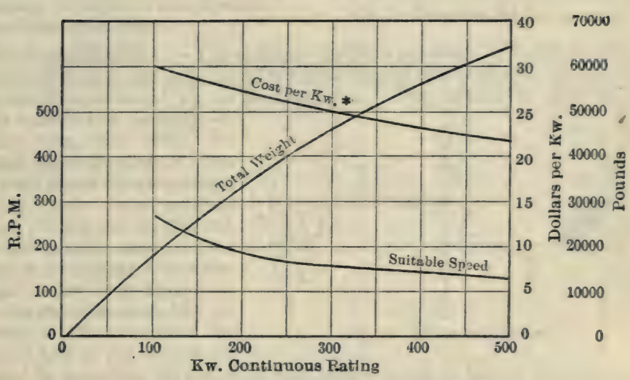

Fig. 27. Data on Slow-speed D-C. Machines

BIBLIOGRAPHY. — The following references will be found valuable in studying the design of direct-current machinery and as reference books to be used in seeking solutions to special intricate problems:

A.I.E.E. Convention Papers, Feb. 1913, Vol. 32; *Am. Technical Society, Cyclopedia of Applied Electricity*, Vol. 3; Arnold, E. *Gleichstromtechnik* Vols. 1 and 2; Cramp W. C. C., *Machine Design;* Crocker and Wheeler, *Management of Electric Machinery;* Fleming and Johnson, *Insulation and Design of El. Windings;* Fisher-Hinen, *Gleichstrom Maschinen;* Gray, A., *Electric Machine Design;* Hawkins and Wallis, *The Dynamo;* Hobart. H. M., *Dynamo Design;* Hobart and Ellis, *Armature Construction;* Hobart and Ellis, *High Speed Dynamo-Electric Machinery;* Kapp, G., *Dynamo Maschinenen;* Lamme, B. G., *A.I.E.E.* 1913, Aug. 1915, Sept. 1915; Langsdorf, A. S., *Principles of D.C. Machines;* Livings-

* 1922 prices about 50 per cent higher than given by this curve

ton, R., *Mechanical Design and Construction of Generators;* Murdock, W. H. F., *Ventilation of Electric Machines;* Parshall and Hobart, *Electrical Machine Design;* Smith, S. P., Design, *London Electrician*, June 1916; Smith, C. H., *Choice of Brushes*, El. Journal, Vol. 13, p. 263; Still, A., *Principles of Electrical Design;* Thompson, S. P., *Dynamo-Electric Machinery*, Vol. 1; Turner and Hobart, *The Insulation of Electric Machines;* Walker, M., *Specifications and Design of Dynamo-El. Machinery;* Weaver, W. B., *Operation of Interpole Machines*, E Journal, Feb. 1917.

GENERATORS, STATIC. — *(See also Generators, Alternating-current; X-Rays.)* In electro-therapeutic and X-ray work a unidirectional high-frequency current is desired and is often obtained by a kind of electric generator called a static generator. There are two types of static generators available for the purpose, the frictional machine and the influence machine.

Frictional Machine. — In the frictional machine two dissimilar substances are rubbed together in some form of rotating apparatus as shown in Fig. 1. A rotating glass plate *A* is rubbed on both sides at *BB* by two pieces of leather which are greased and covered with the amalgam 1 Zn, 1 Sn, 2 Hg. The glass is electrified positively and the charge is drawn off by the metal points *CC* which almost touch the glass. Two silk aprons *DD* cover the glass between the rubbers and the metal points, to prevent the leakage of the charge into the air. The rubbers are connected to ground to remove the negative charge which accumulates upon them.

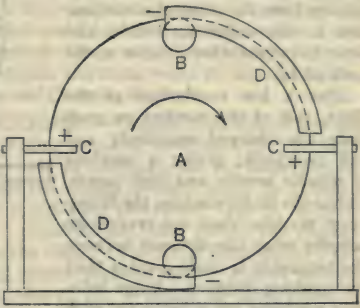

Fig. 1. Frictional Machine

Influence Machine. — The influence machine generates an e.m.f. by electrostatic induction, the principle of the action being clearly shown (see Fig. 2) in a machine invented by G. Belli in 1831. Two spheres or disks *AA*, called carriers and normally insulated from each other, are rotated about the shaft *B*. The two fixed plates *CC* are charged initially from some external source. When the carriers *AA* take the position shown in the figure, a momentary connection is made to the neutralizing wire *E* at the spring contacts *DD*. As a result of this connection between *A* and *A*, the carriers are charged by induction, the charge on each being opposite to that of the adjacent plate. When the carriers rotate still further, the connections at *DD* are broken and the charges induced on the carriers become isolated. When the carriers are rotated through half a revolution from the position shown, they strike the springs *FF* and add their charges to the field plates *CC*.

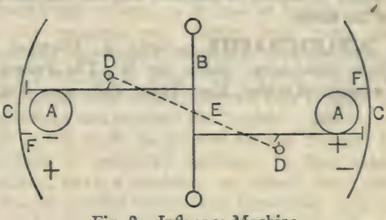

Fig. 2. Influence Machine

Wimshurst Machine. — Many forms of influence machines based upon the above principles have been devised. The most successful designs were made by Toepler, Holtz and Wimshurst. The Wimshurst machine, shown in Fig. 3, has superseded most of the other forms because of its self-exciting powers and suitability for work in all conditions of atmosphere. Two glass disks, *A* and *B*, are rotated close together and in opposite directions. A certain number of tin-foil carriers *CCC*, are mounted upon the outside surface of each plate. Neutralizing conductors *DD* for each disk are placed at right angles to each other. The collecting combs *EE* are connected to a condenser *F* and a spark gap *G* across which the discharge takes place. If two diametrically opposite

carriers on the back plate are charged positively and negatively respectively, opposite charges will be induced upon the adjacent carriers on the front plate if they are connected by the neutralizing wire. The induced charges are isolated when this connection is broken and are drawn off at the collecting combs *EE*. In the Wimshurst machine induced charges in moving from the point of electrification to the collecting combs also serve to induce other charges on the back plate.

After a few revolutions of the disks, half of the carriers on each plate are charged negatively and half positively, giving a machine of reliable service and high power. Unlike the Holtz machine the Wimshurst machine does not reverse its polarity when the distance between the terminals is made greater than the sparking distance. Large influence machines are constructed with several plates revolving in opposite directions and connected in parallel, the whole being contained in a glass case to protect the plates from dust. An eight-plate Wimshurst machine gives a spark of eight inches and a twelve-plate machine will give a spark of 13⅝ inches. The plates in these machines are approximately 30 inches in diameter and the machines give 6 sparks for each revolution of the disks.

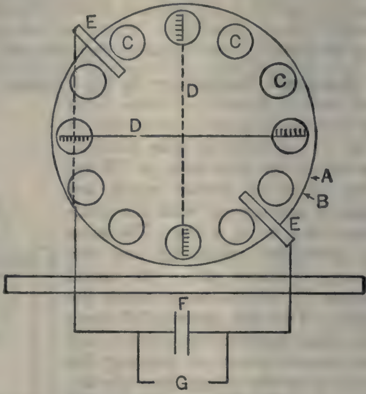

Fig. 3. Wimshurst Machine

BIBLIOGRAPHY. — Gray. J., *Electrical Influence Machines;* Mason, H. *Static Electricity;* Holtz, Dr. W., *The Priority of Invention of the Double-plate Influence Machine*, Elec. Tech., Zeit. 1904, Vol. 25, p.

GREEK ALPHABET.

Α	α	Alpha.	Ν	ν	Nu.
Β	β	Beta.	Ξ	ξ	Xi.
Γ	γ	Gamma.	Ο	ο	Omicron.
Δ	δ	Delta.	Π	π	Pi.
Ε	ε	Epsilon.	Ρ	ρ	Rho.
Ζ	ζ	Zeta.	Σ	σ	Sigma.
Η	η	Eta.	Τ	τ	Tau.
Θ	θ	Theta.	Υ	υ	Upsilon.
Ι	ι	Iota.	Φ	φ	Phi.
Κ	κ	Kappa.	Χ	χ	Chi.
Λ	λ	Lambda.	Ψ	ψ	Psi.
Μ	μ	Mu.	Ω	ω	Omega.

GROUND CONNECTIONS. — *(See also Grounding of Electric Circuits.)* A ground connection is a conductor buried in the earth and connected by conductors to other conductors or apparatus which are to be maintained at earth potential, irrespective of the current which may flow through it. Usually a circuit is not grounded except at such points that under normal conditions of operation no appreciable current flows to ground. A current may, however, be established through a ground connection due (1) to a lightning discharge, (2) to an accidental grounding of a live wire of a grounded circuit, and (3) to a cross between the grounded circuit and a high voltage ungrounded circuit; in the last instance the current through the ground connection is only the relatively small charging current between the other wires of the high voltage system and the ground *(see Capacity and Charging Current)*.

USE OF GROUND CONNECTIONS. — Ground connections are used for the following purposes: (1) for lightning arresters, to afford a path to ground for the lightning; (2) for lightning rods on buildings, for the same purpose; (3) for steel transmission poles to give the current a short circuit to ground in case of insulator failure. Also as a lightning protection; (4) to bring the neutral points of circuits to ground potential and thus reduce the potential stresses of the system and minimize the danger of accident from shock; (5) to bring transformer cases, instrument cases, conduits, etc. to ground potential and thereby reduce the danger from shock; (6) to obtain an earth return for telegraph circuits; (7) for wireless telegraph systems to obtain the use of the earth's electrostatic capacity.

GROUNDING TO CONDUCTORS USED FOR OTHER PURPOSES. — Where an extensive system of water pipes buried in the earth is available, a satisfactory ground connection can be made directly to the pipes. Gas pipes are not suitable for ground connections, as in case of broken joints the resulting arc might ignite the escaping gas. Steam heat pipes are heat insulated (and therefore partially electrically insulated) from the earth. Where there are two ground connections, one to an electric railway track and the other to a water pipe, there is danger that railway current may thereby be introduced into the water piping system and produce electrolysis. Sheaths of cables are not good ground as the cables might be injured by the currents in their sheaths, either from electrolysis or unforseen heating. In fact, cable sheaths, like other buried conductors, have a tendency to collect stray currents from electric railways and special ground connections sometimes have to be made to permit these currents to escape without producing electrolysis. Steel frames of buildings present only a limited surface to ground at the foundations and are therefore only good when the current to be discharged will be proportionally small.

Connections to Piping System. — The National Electrical Code contains the following requirements:

Size of Ground Wires. — Not to be smaller than No. 6 A.W.G. copper for lightning arresters and must be large enough to carry current caused by or following discharge of arrester (Rule 15*Al*); not to be smaller than No. 8 A.W.G. for direct-current systems and must have at least one-fifth the carrying capacity of the grounded conductor, and not to be smaller than No. 8 A.W.G. for alternating-current systems but must have at least one-fifth the carrying capacity of the wire to which it is attached, except that it need not be larger than No. 0 A.W.G. (Rule *Ak*).

Method of Making Ground Connections.—In connecting a ground wire to a piping system, the wire should be sweated into a lug attached to an approved clamp, and the clamp firmly bolted to the water pipe after all rust and scale

have been removed; or be soldered into a brass plug and the plug screwed into a pipe or pipe fitting.

GROUNDING TO SPECIAL PLATES, PIPES, ETC. — When suitable buried conductors are not already available, direct connection to the ground may be made (1) by burying a plate in the earth, (2) by burying a wire or ribbon in the earth, or (3) by driving a pipe vertically into the earth. The last form of ground connection is now generally accepted as the best. These three methods of direct grounding are described in detail in the section below on *Design*.

Factors Affecting the Impedance of Ground Connections. — The impedance opposing the flow of current into the ground through a ground connection depends (1) upon the extent of metal surface in contact with the earth, (2) the thoroughness of this contact, (3) the wetness of the earth, (4) the distance between this ground connection and the ground connections (if any *) at which the current leaves the earth.

The effect of the distance upon the impedance between the electrodes is an important one, but is frequently overlooked. If the electrodes are close together, the impedance will be low on account of the shortness of the current path. If the distance between electrodes be increased, the impedance will be increased until a certain point is reached, after which it will steadily decrease on account of the increase in the cross-sectional area of the earth path. In experiments by the Cunliffes (*Jour. I.E.E., 1909, Vol. 43, p. 449*) the "resistance" (probably meaning impedance) reached a maximum when the ground plates were about 25 feet apart, the decrease in "resistance" being quite sudden for smaller or greater distances, as shown in Fig. 1. A curve by E. E. F. Creighton (*G. E. Rev., 1912, Vol. 15, p. 69*) shows the resistance between pipe grounds at various distances apart, the maximum resistance being again attained at 25 feet.

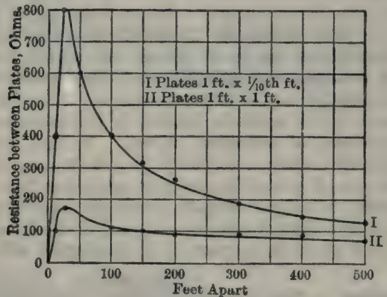

Fig. 1

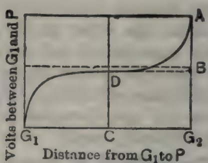

Fig. 2.

Definition of the Resistance and Impedance of a Single Ground Connection. — If the distance between ground connections be fixed and the drop of potential from one of them G_1 and a point P between them be plotted against the distance of P from G_1 a curve such as G_1DA, Fig. 2, will be obtained, if the two ground connections G_1 and G_2 are similar. If, on the other hand, the ground connection G_1 has very much more perfect and extensive contact with the earth, the curve of potential drop will have the shape G_1DB. Both curves have the peculiarity of being steep near the electrodes and of becoming flat between them, indicating that the greater part of the resistance occurs near the electrodes,

* The circuit may be completed by a displacement current, through the air, from the surface of the earth.

where the current is restricted to a limited area, practically no drop occurring where the current spreads out.

The impedance of the ground connection G_1 is then defined as the ratio of the potential drop CD, to the current entering the ground at G_1, and similarly for the ground connection G_2. The resistance may be approximated by measuring the impedance at different frequencies, and by extrapolation, finding the impedance at zero frequency. This will be the resistance.

Resistance of Some Typical Ground Connections. — The resistance of ground connections varies so greatly with the condition and nature of the soil, that data on resistance have only local application. The following tests are presented merely to give a general idea of the results which are obtained in practice.

Authority	Time of year	Description of soil	Type of ground connection	Dimensions	Depth	Ohms
					ft.	
Hoxie.....	Feb.–June	Surface loam	Plate	1 ft.×1 ft.	..	1940
	Feb.–June	Moist black loam	Plate	6 ft.×2 ft. 3 in.	6	113
	Feb.–June	River bottom	Plate	1 ft.×1 ft.	..	132
	Feb.–June	Gravelly soil	Pipe	1¼ in.	5	630
	Feb.–June	Gravelly soil	30 Pipes	1¼ in each	5	13
	Feb.–June	Swampy soil	7 Pipes	1¼ in each	5	15
Del Mar...	Summer	Rock & loam fill	Plate	15 in.×15 in.	6	155
Hayden...	Aug.–Sept.	Clay loam	Pipe	2½ in.	3¾	26.1
Hayden...	Feb.–Mch.	Clay loam	Pipe	2½ in.	3¾	120
Hayden...	June–July	Clay loam	Pipe	2½ in.	3.1	35.4
	Mch.–Apr.	Clay loam	Pipe	2½ in.	3.1	240

E. E. F. Creighton (*G. E. Review, 1912, Vol. 15, p. 67*) says: "If an iron pipe one inch in diameter is driven into normally moist earth to a depth of 6 or 8 feet, it will usually have a resistance of about 15 ohms. Eight ohms may be considered unusually low, while dry soils may give a resistance of 56 ohms and upwards." Creighton also states that after a depth of several feet in the conducting stratum has been reached, each additional foot decreases the total resistance by the factor $d/2$ where d is the depth in feet.

DESIGN OF GROUND CONNECTIONS. — It is as a rule impracticable to install a ground connection of sufficiently low conductivity to allow the passage of the maximum current it may have to carry without producing a considerable rise of voltage. The conductivity which a ground connection should have is therefore largely a matter of experience and judgment rather than a matter of exact calculation. Cheapness and durability usually determine the material, usually iron or copper, and the thickness.

Copper Plates. — Formerly the most common form of ground connection consisted of a copper plate embedded in charcoal or coke in moist earth. The use of coke is questionable, for the reason that the sulphur in the coke is likely to corrode the copper plate and thus increase its resistance. Some engineers are of the opinion that neither charcoal nor coke is of any particular use in connection with a ground plate. The corrosion of copper ground plates is greatly reduced by coating with tin. As the conductance of a ground connection is

not proportional to the area of the metal plate, the use of a single large plate is an uneconomical method of obtaining a low resistance.

Copper Wire or Ribbon. — The wire or ribbon has the advantage of a lower resistance than a plate for the same area. The wire has the further advantage that no special material or connection is required. The wire may be laid out in a straight line or coiled with turns wide apart into a plane spiral. The latter form is sometimes made by wrapping a number of turns of wire around the butt of a pole before setting it.

Pipes Driven into the Ground. — The driven pipe or rod is probably the best of all forms of direct ground connections. The pipe is usually an iron one, often galvanized or sherardized, from ¾ inch to 2 inches in nominal size, driven 6 feet or more into the moistest ground available. The diameter of a pipe has little effect upon its earth resistance; thus a pipe 2 inches in diameter has a resistance only 6 to 12 per cent less than a pipe 1 inch in diameter (E. E. F. Creighton, *G. E. Rev., 1912, Vol. 15, p. 14*). To facilitate driving, the pipe is sometimes provided with a pointed casting at the lower end and cap at the upper end. The pipe has the advantage that it is cheap to install, as no hole is dug, and being driven vertically, it can tap any conducting stratum of the earth which is practically accessible. It is highly efficient as regards conductivity per square foot of surface exposed to the earth, and while the resistance of a single pipe will exceed that of a large plate, a higher and more permanent conductivity can be obtained from several pipes driven at some distance apart and connected in multiple, than from an equal expenditure in money on a single large ground plate. Ground pipes or plates in multiple should be not less than 6 feet, and preferably 10 feet, apart, in order to avoid superimposing two zones of high current density.

If the ground available is naturally dry the conductivity may be improved by cupping out the soil around the pipe at the surface and adding a few pounds of salt and water. A neater form of earth unit is shown in Fig. 3. A cylinder of metal or earthenware of any available diameter is set around the pipe at the surface of the ground and covered by a lid. This receptacle will hold the salt. Its advantages lie in the easy construction of the connection and protection of surrounding vegetation from the saline water.

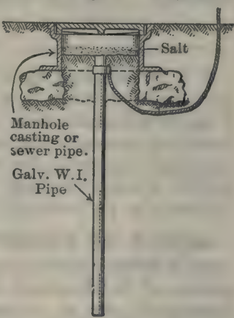

Fig. 3.

Pipe Grounds for Lightning Arresters. — The General Electric Company recommends a large number of grounds at each installation. These numerous ground connections are joined together by a copper connection. It sometimes happens that a good ground cannot be conveniently made near the arrester, or that a better one can be made at a more distant point. In this case it is recommended that the principal ground be made at the more distant point, but that a ground of some sort, the best possible under the conditions, be made directly underneath the arrester.

Connections to Ground Plates and Pipes. — The wire connecting the apparatus to be grounded to the buried conductor should lead directly to the latter with as few bends as possible; this is particularly important in grounding lightning arresters, in which case a flat strip is also better than a round wire. All exposed joints and those buried in the ground should be made in such a way that they will not rust off. The buried connection from a ground plate to the surface should be so protected that it will not rust off or be cut off when exca-

vations are made for other purposes. It is also necessary that the connections be protected from mechanical injury and theft, from the surface of the ground to above the level at which it can be reached. To insure reliability, connections should as far as possible be made so that their continuity can be determined by inspection.

TESTS OF GROUND CONNECTIONS. — Ground connections should be frequently tested to determine their continuity and occasionally to determine their impedance. Alternating current should be used, in order to avoid the effects of polarization. In order to pass a current through a ground connection it is necessary to have a second connection in order to complete the circuit. Where a large water-piping system is available for the return connection, an approximate measure of the resistance can be obtained by passing a current from the ground connection to an accessible part of the piping, say a hydrant, and computing the resistance by dividing the volts by the amperes. The resistance thus obtained includes both the resistance to be measured and that of the connection through the piping system. As the latter is probably much the smaller and may be insignificant, the result gives an approximate idea of the resistance and locates a discontinuity as an infinite resistance, that is, no current flowing. Where there are two similar grounds, current may be passed from one to another and the sum of the two resistances, or the average resistance, obtained.

When a more accurate determination of the individual resistances of two or more ground connections is desired, the connections shown in Fig. 4 may be employed and a curve obtained by test, similar to that shown in Fig. 2. The apparatus required comprises a source of alternating current, a rod P which may be driven into the earth, a high resistance wire BD stretched between the two ground connections G_1 and G_2, an ammeter A and a telephone receiver T. The contact C is slid along the resistance wire until the telephone receiver is silent. The drop of potential between G_1 and the rod is then the same as between G_1 and the contact C. The drop between G_1 and P is therefore equal to

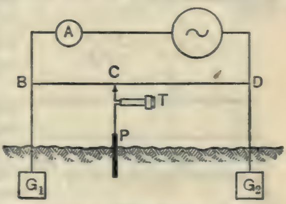

Fig. 4.

$$\frac{(\text{length of wire } BC)}{(\text{length of wire } BD)} \times (\text{total drop from } G_1 \text{ to } G_2).$$

By taking a series of observations with P driven into the ground at various points the curve of potential drop is readily plotted and the resistance of each ground connection, as defined above, readily obtained.

CARE OF GROUND PLATES AND PIPES. — The greatest trouble with ground plates and pipes arises from electrolysis, and periodic resistance measurements should be made as described above. When a ground connection shows an abnormally high resistance, it is usual to supplement it with a new ground connection, as it seldom pays to remove the old ones. Ground connections deteriorate rapidly if equipped with salt boxes; but even under the worst conditions they are likely to last many years. Practically no accurate measurements of the resistance of ground connections were made until quite recently, so that little is known about their life performance. J. L. R. Hayden cites the case of some pipe grounds which maintained the same average resistance for three years.

SPECIFICATIONS FOR GROUND PIPES AND PLATES. — *(See also article on Specifications.)* In the case of ground plates, the following data

should be specified: Material of plate (usually copper); size of plate, area and thickness; depth at which the plate shall be buried; amount of charcoal below and above it; method of connecting the plate to its cable or wire (usually soldered and bolted); whether the plate shall also be connected to the track rails and if so, how.

In the case of pipe ground connections, the following data should be specified: Diameter of pipe; whether galvanized, sherardized, etc.; depth to which the pipe shall be driven; method of finishing the top of the pipe; height the pipe shall project above the ground; whether the pipe shall be connected to the track rails and if so, how.

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GROUND DETECTORS AND ARCING-GROUND SUPPRESSORS. — A ground detector is a device for indicating an accidental ground on a transmission line; an arcing-ground suppressor is an arrangement of switches and relays by means of which an arc tending to maintain an accidental ground is automatically extinguished

GROUND DETECTORS FOR NORMALLY UNGROUNDED SYSTEMS. — On ungrounded two-wire direct-current systems a voltmeter of the central-zero type connected between the ground and the middle point of a resistance joining the two line wires serves as a satisfactory ground detector. The direction of the deflection will indicate on which wire the ground has taken place, and the magnitude will indicate the nature of the ground. For alternating-current circuits, this method is unsatisfactory on account of capacity effects and also because of the higher voltages commonly involved.

Electrostatic Ground Detectors. — For single-phase ungrounded systems the simplest form of ground detector consists of two electrostatic voltmeters, each connected between one line wire and the earth. A ground on either wire causes an inequality in the readings of the two meters.

The more common form of ground detector operates as an electrostatic differential voltmeter. Fig. 1 shows diagrammatically the arrangement for a single-phase instrument. The movable vane V is connected to the earth, the fixed plates S_1 and S_2 are connected one to each of the two mains. Vane V will be equally attracted by the two plates unless the insulation resistances of the two line wires to ground are not equal, but will deflect from its normal position to one side or the other when a ground occurs.

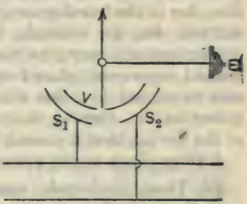

Fig. 1. Single-phase Electrostatic Ground Detector

The three-phase electrostatic ground detector for ungrounded circuits is made in two principal forms. One form consists of three separate elements, each similar to that used in the single-phase instrument, all three elements being in one case but having individual scales. Another form consists of a movable spherical vane, connected to the earth, and three fixed vanes, each of which is connected to one line wire. The movable vane in its normal position is equally distant from each one of the fixed vanes, but, in case of a ground on one line wire, it moves away from the fixed vane which is connected to the line wire on which the ground occurs. The principal parts of this instrument are clearly visible through the glass front of the instrument cover; hence no pointer and scale are needed. Two single-phase detectors will work satisfactorily on three-phase circuits, if the junction point of the two instruments is connected through a condenser to the third line wire.

Voltage Range. — Commercial sizes of electrostatic ground detectors range from 650 volts to 22,000 volts for single-phase circuits, and from 1150 volts to 22,000 volts for three-phase circuits.

Connections. — Electrostatic ground detectors are usually not connected directly to the line wires, especially if the voltage is above 3300. Some types are furnished with special terminal studs containing high-resistance rods of graphite through which the instrument is connected to the line wires. Other types are connected to the circuit through condensers, commonly of tubular form. Although, in this case, the condensers practically insulate the instrument from the line, the leads connecting the detector and the condensers must be treated as high-tension conductors, i.e., they should be properly separated and

far enough from neighboring metal in order that disturbing capacity effects may not be introduced. Therefore, the leads should be fairly short and must not be enclosed in metal conduit.

GROUND DETECTORS FOR NORMALLY GROUNDED SYSTEMS. — The methods outlined above, depending on the variable potential difference between mains and earth, do not apply to earthed systems, e.g., three-wire circuits with grounded neutral, or four-wire, three-phase circuits with grounded neutral. The most usual method for earthed circuits is to connect an ammeter in the earth circuit between the earth plate and the system; hence any leakage current will have to pass through the ammeter. A low-range instrument is desired if slight grounds are to be detected. In order to protect the instrument from excessive currents, an arrangement may be used by which the instrument is short-circuited as soon as the current reaches a certain value, or a resistance may be inserted in the circuit to reduce the current to a safe value. By the second method a short-circuit due to a ground would be suppressed without interruption of service on the line in question. At the same time, however, the pressure on the other line wires with respect to earth would be raised to a higher value.

ARCING-GROUND SUPPRESSORS. — Interruptions to service and damage to apparatus are frequently caused by grounds which tend to persist as arcs, due to the burning away of the conductor. Arcs of this kind may be intermittent, due, for example, to the swinging of the conductor back and forth. Arcing grounds may give rise to disastrous surges in the system. The arcing-ground suppressor, placed on the main bus at the generating station, removes arcs to ground on a line wire by automatically grounding that line wire at the power station, thereby short-circuiting the arc. It is obvious, from the nature of the device, that it cannot be used on earthed systems.

A three-phase arcing-ground suppressor consists of the following parts:

(*a*) Three single-pole, motor-operated, oil circuit-breakers each connecting one phase of the power house main bus to the ground. These circuit-breakers are provided with interlocking relays, each of which has three contacts, one of which closes the tripping circuit of its own oil circuit-breaker while the other two open the tripping circuits of the other two circuit-breakers. Hence it is impossible to have more than one oil circuit-breaker closed at the same time. For use with overhead systems each circuit-breaker is equipped with the second-stroke lock device described later. The oil circuit-breakers are provided with the usual remote-control switches with red and green lights to be placed on the station switchboard.

(*b*) Three single-pole, single-throw disconnecting switches for disconnecting the oil circuit-breakers.

(*c*) An electromagnetic selective relay for detecting the ground phase and operating the proper oil circuit-breaker. This selective relay is connected to the main bus through three Y-connected transformers of which the neutral on the high-tension side is grounded.

Operation when used with Overhead Lines. — The arcing-ground suppressor is suitable for overhead lines only when metal towers or poles of relatively low resistance are used, as on a wooden pole line the resistance of the pole is liable to prevent sufficient current flowing to ground to reduce the potential sufficiently to operate the phase selective relay.

The operation is as follows: If a ground occurs on one wire in the system, the unbalancing in the potentials of the grounded Y-connected transformers causes the phase selective relay to be operated in such a way that the proper interlocking relay is thereby energized. This closes the corresponding oil circuit-breaker,

thereby grounding the line wire, on which the arcing ground has occurred, through a metallic circuit. Thus the arcing ground is almost instantly cleared, and the oil circuit-breaker opens automatically after a fractional part of a second, and remains open, provided the normal state of insulation of the line is reëstablished after the suppression of the arc to ground. Whenever an arc is established or broken, high-frequency oscillations are set up, the duration of which depends on the amount of damping resistance in the circuit (*see Transmission Lines*). In order to eliminate the dangers from such oscillations, a damping resistance placed in the switch pot is thrown in series before the switch rod closes the main contact to ground and after it has opened the main contact.

If the line insulation is permanently broken down by the arc, the line potential will establish a second arc to ground as soon as the metallic ground through the oil switch is opened. The second-stroke lock device then operates; it locks the oil switch as it closes, provided the second closing of the switch occurs immediately after it has been opened. In this case the system remains grounded on one side, and can continue to operate with the metallic ground. After the trouble has been cleared, the oil switch is opened by the attendant.

Operation when used with Underground Cables. — When the arcing-ground suppressor is used for the protection of cables, the second-stroke lock device is omitted, as it is desirable for the oil circuit-breaker not to be opened after it has once been closed until the feeder has been cleared, the reason being that in a cable the distance of a conductor from the sheath is usually so small that puncture of the insulation produces a permanent fault, and the normal difference of potential is sufficient to reëstablish an arc, even if it were automatically extinguished. The arcing-ground suppressor on cable systems, therefore, minimizes the injury due to a puncture in the cable and prevents the trouble from spreading, and suppresses dangerous high-frequency surges due to arcing grounds. It is not intended to clear short-circuits between line wires.

Location of Arcing-ground Suppressor. — The best location for the arcing-ground suppressor is at the power house, and directly connected to the high-tension bus, where it may be under the immediate observation of the station operator, where a direct-current supply is available, and where the more important switching operations in case of grounds are usually performed.

Costs.— A single-phase electrostatic ground detector costs from $30 to $60, depending upon the voltage, and a three-phase electrostatic ground detector costs from $50 to $150. A three-phase arcing-ground suppressor for voltages from 11,000 to 110,000 costs approximately from $2200 to $6000.

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GROUNDING OF ELECTRIC CIRCUITS. — (*See also Generators, Alternating-current; Ground Connections; Transformers.*) Some electric circuits are operated insulated, that is with no intentional electrical connection from any point of the circuit to the ground, while others are grounded by one or more conductors provided for the purpose.

Among the considerations influencing the decision of this point are: (1) danger of shock to persons touching the circuit; (2) danger of fire from escaping current, spark, or arc; (3) elimination of electrical oscillations set up in a circuit due to the sudden changes in current resulting from the making and breaking of an arc, usually to ground, usually referred to as an "arcing ground;" such grounds may give abnormal voltages particularly on a-c. circuits where the arc forms and is extinguished twice every cycle; (4) decreased strain on circuit insulation in case of accidental ground; (5) the utilization of the ground and conductors on or in the ground for carrying return currents; (6) the danger of electrolysis and disturbance of telegraph and telephone systems by currents escaping to the ground.

CURRENT PRACTICE REGARDING GROUNDING. — While the practice of different companies is not consistent with respect to insulating or grounding of various classes of circuits, the following rules probably cover the best practice at the present time.

(1) Low-voltage circuits should be insulated where they are in no way exposed to direct or indirect crossing with other circuits of higher voltage.

(2) Low-voltage circuits should be grounded if there is any danger of crossing with circuits of higher voltage.

(3) Intermediate voltage circuits should be insulated when practicable.

(4) Intermediate voltage circuits if grounded (600-volt railway circuits, for example) should be handled only by experienced people and should be inaccessible to the public.

(5) High-voltage circuits are always dangerous and may be insulated or grounded according to other conditions than safety from shock.

The reasons for the above practice are discussed below.

ELECTRIC SHOCK FROM GROUNDED AND UNGROUNDED CIRCUITS. — Of the various considerations affecting the desirability of grounding a circuit, that of shock is perhaps the most important. (*See also article on Shock, Electric.*) A person touching a circuit at any two points between which there is a difference of potential will receive a shock. The danger or severity of shock from touching simultaneously two line wires of a circuit is not affected by grounding the circuit, but the danger or severity of shock from touching simultaneously either wire and the ground does depend upon whether the circuit is grounded or not.

It should be noted that the sensitiveness of different people to shock varies a great deal and that the severity of a shock depends as much on the surface resistance of the skin (whether dry or wet), and on the parts and organs of the body through which the current passes, as on the voltage. It is therefore impossible to say that any voltage used in practice is so low that under no condition can it give a dangerous shock. However, considering ordinary conditions and the result of a majority of the shocks, circuits of voltages up to and including 220-volts may be considered as not liable to cause a serious shock, whether grounded or ungrounded, and may be referred to as "low-voltage" circuits.

Voltage to Ground of Grounded Circuit. — When a circuit is intentionally grounded, the voltage of the grounded point is made permanently that of the

earth, and the potential of every other point of the circuit becomes fixed with respect to the ground and may be readily calculated (*see Kirchhoff's Laws in index*). Each conductor then carries a definite, predetermined risk, instead of an indefinite risk, which may be nothing or may be very great.

Voltage to Ground of Ungrounded Circuit. — Under normal conditions the average potential of an insulated circuit is the same as that of the ground, the positive parts of a circuit having a potential above and the negative parts below that of the ground. These differences of potential cause the positive and negative wires to be charged like the plates of condensers, the positive wire and earth forming one condenser and the earth and negative wire the other. The positive and negative charges on the wires will be equal and the resultant charge on the earth will be zero in all cases. These condensers are in series and if the circuit is symmetrical the two condensers will have equal capacity. Then the voltage of the positive wire will be as much higher than that of the earth as that of the negative wire is below earth potential. In an unsymmetrical circuit the potentials of the positive and negative wires with respect to earth will be inversely as the capacities.

Current Through Ground Connection. — In case of a ground, either intentional or accidental, the grounded point of the circuit is brought to ground potential and a change is made in the voltage to ground of the positive and negative parts of the circuit, and the resultant charge on the earth is no longer zero. This charge must come by conduction from the circuit and must therefore enter the earth through the ground connection. As long as there is a change in voltage of the wires of the circuit due to the ground connection a current will flow through the ground connection. Such a current is only momentary in the case of a d-c. circuit, but flows as long as the ground continues with alternating currents. See also article on *Capacity and Charging Current.*

The number of amperes which flows through such a ground connection is roughly proportional to the voltage of the system and its electrostatic capacity to ground. The electrostatic capacity is proportional to the extent of the circuit (miles of line). As the miles of line of commercial circuits ordinarily increase about in proportion to the voltage, it follows that the amperes to ground will be about as the square of the voltage of the circuit. The energy which can be liberated at the point of accidental ground will be proportional to this current multiplied by the voltage, that is, will vary as the cube of the voltage. In addition to current due to the capacity to ground there may be a leakage current due to imperfect insulation.

Shock from Single Contact with High-voltage Circuits. — Circuits having a normal line voltage of 11,000 volts or over are generally of sufficient extent (and therefore have sufficient electrostatic capacity) so that a fatal shock may be received from wire to ground, due to the current passing to ground through the body, even though the circuit be otherwise perfectly insulated from ground.

Shock from Intermediate Voltage Circuits. — In general the power available for shock in the current escaping from an insulated circuit to ground is small compared to that in case of contact with the two wires of the circuit. Consequently, there is a class of circuits whose voltages are high enough to give serious or fatal shocks where contact is made with two points of the circuit, but not high enough to give serious (or sometimes even appreciable) shocks from wire to ground when the circuit is ungrounded. These circuits are intermediate between the low-voltage (220-volt) and high-voltage (11,000-volt) circuits above mentioned. This class includes 440- and 550-volt power circuits and sometimes 1100- and 2200-volt primary circuits, if leakage is small.

Shock from Low-voltage Circuit Crossed (in Contact) With High-voltage Circuit. — When an insulated circuit becomes crossed with a circuit of higher voltage it becomes charged to the voltage of the latter circuit at the point of contact. It becomes practically a part of the higher voltage circuit and is equally dangerous to touch. As its insulation is not designed to stand such abnormally high voltage, there is danger of shock even through the insulation of the conductor.

When a grounded circuit is crossed with one wire of a higher voltage circuit, this wire tends to come to ground potential, and the voltage of the several parts of the low-voltage circuit with respect to the ground remains as before. In cases where a very large amount of current escapes from the high-voltage circuit, this will be appreciably modified by the addition of a voltage due to the resistance or impedance drop in the low-voltage circuit due to the current from the high-voltage circuit from point of contact to ground. If, for example, an outside wire of a three-wire, 110/220-volt, alternating-current secondary circuit, with grounded neutral, should become crossed with a 11,000-volt grounded circuit, a large amount of current may flow into it, which can only escape to the ground after passing through the lamps on one side of the circuit, or through the transformer secondary. The impedance of the path through the lamps and transformer may be high enough so that the voltage of the crossed outside wire of the secondary may be raised to a dangerous voltage above that of the grounded neutral.

Conclusions regarding Effect of Grounding with respect to Shock. — The considerations on which above classification is based lead to the conclusions:

(1) The insulating of low-voltage (under 220 volts) circuits decreases the danger of shock under normal conditions, but as such shocks would rarely be serious, there is little gain in safety through the insulation of the circuit.

(2) The grounding of low-voltage (under 220 volts) circuits greatly decreases the danger of fatal shocks in cases where the low-voltage circuits become crossed with one of higher voltage.

(3) The insulating of intermediate voltage circuits (440 to 2200 volts) greatly decreases the danger of shock under ordinary conditions because most shocks are obtained from touching one conductor, and in such circuits the electrostatic capacity is ordinarily so small that no appreciable current flows to ground.

(4) The grounding of intermediate voltage circuits (440 to 2200 volts) does not greatly decrease the danger of fatal shock in case the circuit becomes crossed with one of higher voltage because the normal voltage of the circuit when grounded is itself dangerous.

(5) Neither the insulating nor grounding of a high-voltage circuit (above 11,000 volts) materially changes the danger of shock which is very great in either case.

DANGER OF FIRE FROM GROUNDED AND UNGROUNDED CIRCUITS. — In general, conditions which diminish the danger of electric shocks also diminish the danger of fire.

The Rules and Requirements of the National Board of Underwriters is the principal authority on the subject of grounding from the standpoint of fire hazard. These rules, given in the National Electrical Code (1920 edition), are abstracted in the following table.

GROUNDING OF HIGH-VOLTAGE CIRCUITS. — The grounding of low-voltage circuits is governed by considerations of danger of shock and fire, while that of high-voltage circuits is for the purpose of increasing the reliability (continuity of service over the line), or of decreasing the cost of trans-

Type of Current	Type of circuit	Grounding	Point of attachment of ground
D–C	Two wire (*a*)	Prohibited	At one station only
	Two wire (*b*)	Optional	At one station only
	Three wire (*c*)	Optional	Neutral, at one or more supply stations but not at individual services or within buildings served
	Three wire (*d*)	Compulsory	
A–C	Any system (*e*) P.D. between grounded point and any other point in the circuit, does not exceed 150 V.	Compulsory	Each service is near the transformers but not otherwise within buildings. Single-phase three-wire systems should have their neutrals grounded. Single-phase two-wire systems may have the ground on either conductor. In the case of two- or three-phase systems the ground should be on the point of the system which causes the lowest voltage from ground of unguarded current carrying parts of connected devices
	P.D. between grounded point and any other point in the circuit exceeds 150 V.	Optional	
	(*f*)	Optional	
	Furnace circuits	Optional	

(*a*) Exposed to leakage or induction from aerial circuits of higher voltage.
(*b*) Unexposed to leakage or induction from aerial circuits of higher voltage.
(*c*) Unexposed to leakage or induction from aerial circuits above 600 V.
(*d*) Exposed to leakage or induction from aerial circuits above 600 V.
(*e*) Exposed to leakage or induction from aerial circuits, etc., above 600 V.
(*f*) Unexposed to leakage or induction from aerial circuits, etc., above 600 V.

mission by decreasing or limiting the voltage strain on the line insulator and transformer insulation; see also article on *Generators, Alternating-current* and section on *Transformer Connections* in the article on *Transformers*.

Eliminating Arcing Grounds by Metallic Grounding. — In normally ungrounded high-voltage circuits the current to ground is sometimes sufficient to maintain an arc between one of the wires and a grounded conductor near it but not quite in contact with it, producing a so-called "arcing" ground. Such grounds have been found to produce destructive voltages on the circuit. By metallically grounding the neutral or one wire of the circuit the rise of voltage is reduced. This is perhaps the main reason why many high-voltage circuits are grounded.

Limiting Insulation Strain by Grounding the Neutral. — In high-voltage circuits the cost of insulators is an important element. In an ungrounded circuit each insulator must be large enough to stand full line voltage (between wires), for any phase may become grounded accidentally. Where the neutral of the circuit is grounded the maximum voltage on any insulator can never exceed the normal voltage to ground. The voltages to neutral of a single- or two-phase system is 50 per cent of the voltage between lines, and the voltage to neutral of a three-phase system is 58 per cent of the voltage between lines. A circuit with grounded neutral therefore requires insulators of from 50 to 58

per cent of those for same circuit with insulated neutral, or for the same size insulator the line voltage can be from 73 to 100 per cent higher with grounded neutral than without. These figures neglect the fact that the maximum strain is constant for grounded neutral, and occurs only for a short time at irregular intervals with insulated neutral. In practice the same insulators are usually used whether the neutral is grounded or not, the factor of safety being higher when the neutral is grounded.

Limiting Short-circuit Current by Resistance.— In a grounded circuit every accidental ground becomes a short-circuit. As such short-circuits may be destructive to generating machinery, the current is sometimes limited by a resistance in the ground connection (usually made at the neutral). The amount of resistance required depends on the per cent of short-circuit current which is permissable. The resistance has, however, the disadvantage that it increases the strain on the insulation, so that as the short-circuit current is reduced, the benefit to the insulation is also reduced. Where the circuit is grounded as a remedy for arcing grounds and no increased factor of safety on the insulator for normal condition of operation is desired, the use of resistance in the ground connection is allowable.

Use of Ground for Return Circuit. — Connections to ground are little used as return paths for the normal current of the circuit, except for railway work. Formerly many 500-volt direct-current power circuits, operated from railway circuits which were necessarily grounded, used the ground as a return circuit, though usually only for small or outlying motors. Such connections may cause electrolysis in underground pipes and arcs, in cases where the pipes or other foreign conductors over which the currents are returning, are broken. Present practice on light and power distribution is to provide complete metallic circuits for all currents which will flow in a circuit under normal operating conditions. Where one conductor is to be intentionally grounded the escape of current into the ground may be prevented by grounding it at only one point, or where, on account of great extent of circuit, grounds on same conductor are necessary at several points, the escape of current will be reduced to a minimum, by making the ground connection on the neutral instead of an outside wire.

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GUTTA-PERCHA. — (*See also Insulating Materials; Telegraph Instruments and Circuits; Wires and Cables, Insulated.*) Gutta-percha is derived from the milky secretion or latex of the bark of certain trees of the order of Sapotaceæ, especially the Dichopsia Gutta, found chiefly in the Straits Settlements and Malaccan Archipelago. The trees are felled immediately after the rainy season, and the gutta or gum collected as it exudes from incisions in the bark. Latex is also extracted from the leaves by digesting them in toluol. However it may be extracted, the latex is boiled in water and it is then ready for export. Gutta-percha is becoming quite scarce and practically the whole available supply is used by British cable makers.

The chemical composition of gutta-percha is represented by the formula $C_{10}H_{16}$. It resembles dark brown leather at temperatures between 0° C. and 27° C. At higher temperatures it softens, and at 65° C. it is plastic and capable of being molded or rolled. On cooling it returns to the non-plastic condition.

Gutta-percha oxidizes when exposed to the air, changing from dark brown or black to yellowish grey and becoming brittle.

PREPARATION OF GUTTA-PERCHA INSULATION. — For insulating purposes gutta-percha is shredded and squeezed in warm water. It is then kneaded and strained through fine-wire gauze and rolled into sheets. Its further refinement is carried on differently by various manufacturers, the processes being more or less trade secrets. Like rubber it is applied to the wire either by a tubing machine or by strips. Unlike rubber it is used in the pure state without mixture with minerals. Gutta-percha is less porous than rubber and therefore more waterproof, a quality which makes it the best material for submarine cables. Its specific gravity is just above unity.

SPECIFIC RESISTANCE. — The constant K in the formula

$$M = K \log \frac{D}{d}$$

has the value 900 approximately, at 75° F. after one minute electrification. See also article on *Rubber*.

Temperature Coefficient of Resistance. — The temperature coefficient of resistance of gutta-percha is of the same nature as that of rubber (*see article on Rubber*), i.e.,

$$R_T = R_{75}\epsilon^{(75-T)C},$$

where R_{75} is the resistance at 75° F., R_T the resistance at T° F. and C a constant which varies from 0.065 to 0.085. For values of ϵ^x see *Exponential Functions*.

Effect of Pressure upon Resistance. — Gutta-percha being used principally for submarine cables, the effect of pressure upon its resistance is important. Let R = its resistance at atmospheric pressure, R_p = resistance under pressure of p pound per square inch.
Then

$$R_p = R\,(1 - 0.00023\,p).$$

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HEAT AND THERMAL PROPERTIES. — (*See also Temperature and Thermometers; Thermodynamics, Principles of.*) Heat is said to be added to a body, or the body is said to absorb heat, (1) whenever its temperature rises, (2) whenever it passes from a solid to a liquid state, or (3) whenever it passes from a liquid to a gaseous state; when the reverse of these changes takes place the body is said to lose or to give out heat. A body can also absorb (or give out) heat without any of these changes taking place, provided it gives out (or absorbs) at each instant an amount of energy of some other form equivalent to that absorbed (or given out) as heat. The heat absorbed or given out by a body in virtue of a change in its temperature is called "sensible" heat; heat absorbed or given out in passing from one state to another is called "latent" heat. Experiment shows that heat may be considered as a form of energy.

Symbol for Heat (H). — Both H and Q are commonly used to designate quantity of heat. The symbol H is used throughout this article.

UNITS OF HEAT. — There are several arbitrarily chosen units of heat, viz.,

15° Gram-Calorie or Small Calorie. — The heat necessary to raise the temperature of 1 gram of water from 14.5° C. to 15.5° C. This is the unit commonly employed in scientific work.

Mean Gram-Calorie or Small Calorie. — The $\frac{1}{100}$th part of the heat required to raise the temperature of 1 gram of *water* from 0° C. to 100° C., the latent heat of fusion and boiling not being included. According to Marks and Davis (*Steam Tables and Diagrams, N. Y., 1912*), the 15° calorie and the mean small calorie differ by less than one-tenth of one per cent.

Kilogram-Calorie or Large Calorie. — The heat required to raise the temperature of 1 kilogram of water from 14.5° C. to 15.5° or the $\frac{1}{100}$th part of the heat required to raise the temperature of 1 kilogram of water from 0° C. to 100° C. The relation between the gram-calorie and the kilogram-calorie is then

1 kilogram-calorie = 1000 gram-calories.

Ostwald Calorie. — The heat required to raise the temperature of 1 gram of water from 0° C. to 100° C. This unit is frequently used by electrochemists.

1 Ostwald calorie = 100 mean gram-calories.

British Thermal Unit (B.t.u.). — The heat required to raise the temperature of 1 pound of water 1° F. There is no general agreement as to *which* degree of temperature shall be used; Peabody uses the degree from 62° F. to 63° F. Marks and Davis define the British thermal unit as the $\frac{1}{180}$th part of the heat required to raise the temperature of 1 pound of water from 32° F. to 212° F. The B.t.u. as defined by Marks and Davis is about 0.13 per cent greater than the B.t.u. as defined by Peabody. Marks and Davis's definition is adopted throughout this book; the difference between the two is negligible for ordinary practical work. The relation between the B.t.u. and the mean kilogram-calorie is

1 B.t.u. = 0.25200 kilogram-calories.

Mechanical Equivalent of Heat. — This is the name given to the experimentally determined conversion factor between any heat unit and any unit of mechanical work; see *Units and Conversion Factors*. The fundamental relation is

$$1 \text{ mean gram-calorie} = 4.1834 \times 10^7 \text{ ergs.}$$

This is the value given by Marks and Davis.

THERMAL CAPACITY AND SPECIFIC HEAT.—The "thermal capacity" of a body is defined as the heat absorbed by the body per unit increase in its temperature, there being during this change in temperature no change of state (e.g., no change from solid to liquid or from liquid to gaseous form or no chemical change) and no transfer of heat energy from the body in question to other bodies. The thermal capacity *per unit mass* of a substance is approximately constant, but increases slightly with increase in temperature; in the case of iron the increase with temperature is quite marked. Calling C the thermal capacity per unit mass of a substance the heat absorbed by a homogeneous mass M when its temperature increases from t_1 to t_2 is

$$H = CM\,(t_2 - t_1) \qquad (1)$$

provided C is constant.

The mean thermal capacity per unit mass of water (between 0° C. and 100° C.), when expressed in mean gram-calories per gram per degree centigrade, is numerically equal to unity. The ratio of the thermal capacity per unit mass of any substance to the mean thermal capacity of water is called the "specific heat" of the substance. The specific heat of a substance does not depend upon the units in which the various quantities are measured; its thermal capacity per unit mass does. When heat is expressed in mean gram-calories, mass in grams and temperature in degrees centigrade, the thermal capacity per unit mass is equal to its specific heat; compare with density and specific gravity.

Calculation of Heat Absorbed or Given Out.—

C = specific heat (gram-calories per gram per °C.),
M = mass heated,
$t_2 - t_1$ = rise of temperature.

Then for any set of units the heat absorbed is (2)

$$H = kCM\,(t_2 - t_1),$$

where k has the following values:

VALUES OF k

Unit of heat or energy	Unit of mass	Temperature scale	Value of k
Gram-calorie	Gram	Centigrade	1.000
Kilogram-calorie	Kilogram	Centigrade	1.000
B.t.u	Pound	Centigrade	1.800
B.t.u	Pound	Fahrenheit	1.000
Watt-second (joule)	Gram	Centigrade	4.183
Watt-second (joule)	Pound	Fahrenheit	1054
Kilowatt-hour	Kilogram	Centigrade	1.162×10^{-3}
Kilowatt-hour	Pound	Fahrenheit	2.928×10^{-4}

Values of Specific Heat. — In the table below are given the values of the specific heat for the more common substances used in engineering work. These numbers are also equal to the thermal capacity per unit mass, when mass is expressed in grams, temperature in degrees centigrade and heat in gram-calories.

Specific Heats at Constant Volume and Constant Pressure. — In general, when a body absorbs heat an expansion (contraction in a few cases) results and the body does work on whatever opposes this expansion, part of the heat absorbed being thus converted into mechanical work; see *Ther-*

TABLE I.—SPECIFIC HEAT OF SOME COMMON SUBSTANCES

(*From Landolt-Börnstein Tables; see also article on Pyrometers.*)

Substance	Temperature, ° C.	Specific heat C.	Substance	Temperature, ° C.	Specific heat C.
Air (*a*)	−102 to 440	0.237	Ice	− 78 to −18	0.463
Aluminum	15 to 435	0.236	Iridium	0 to 100	0.032
Ammonia	23 to 216	0.520	Iron, cast	18 to 100	0.113
Antimony	22 to 600	0.052	Lead	17 to 100	0.031
Asbestos	20 to 98	0.195	Manganin (*e*)	18	0.097
Bismuth	−79 to 100	0.029	Manganin (*e*)	100	0.100
Brass (*e*)	20 to 100	0.092	Marble	0 to 100	0.206
Bronze (*c*)	20 to 100	0.104	Mercury	0	0.0335
Carbon (gas carbon)	20 to 1040	0.315	Mercury	100	0.0326
Carbon (graphite)	0 to 3000	0.535	Mica	20 to 98	0.208
Carbon dioxide (*a*)	− 78 to 7	0.184	Molybdenum	20 to 550	0.072
Carbon dioxide (*a*)	0 to 200	0.215	Nickel	0 to 105	0.108
Carbon monoxide (*a*)	23 to 198	0.243	Nitrogen (*a*)	0 to 200	0.244
Cement (Portland)	28 to 30	0.271	Nitrous oxide (*a*)	13 to 172	0.231
Chlorine	13 to 202	0.124	Oxygen (*a*)	20 to 440	0.224
Cobalt	15 to 350	0.109	Osmium	19 to 98	0.031
Concrete, tamped	20 to 100	0.156	Palladium	0 to 100	0.059
Concrete, tamped	800	0.219	Palladium	0 to 1265	0.071
Constantan (*a*)	18	0.098	Paraffin	25 to 30	0.589
Constantan (*a*)	100	0.102	Petroleum	21 to 58	0.511
Copper	−188 to 20	0.080	Petroleum	18 to 99	0.498
Copper	0 to 100	0.094	Platinum	0 to 100	0.032
Copper	300	0.098	Rhodium	10 to 97	0.058
Copper	900	0.126	Silver	0 to 260	0.057
Cork	....	0.485	Steam (*g*)	....	.....
Cotton	0 to 100	0.362	Steel	20 to 100	0.118
Ebonite	....	0.339	Tantalum	−185 to 20	0.033
German silver	0 to 100	0.095	Tin	17 to 100	0.056
Glass	0 to 19	0.171	Tungsten	20 to 100	0.034
Glass	56 to 78	0.192	Wax (yellow)	26 to 42	0.820
Gold	0 to 100	0.032	Wood's metal (*f*)	5 to 50	0.035
Hydrogen (*a*)	− 28 to 198	3.41	Wool	....	0.393
			Zinc	20 to 100	0.093

(*a*) At constant pressure of 1 atmosphere.
(*b*) 60 Cu + 40 Zn.
(*c*) 88.7 Cu + 11.3 Al.
(*d*) 60 Cu + 40 Ni.
(*e*) 84 Cu + 4 Ni + 12 Mn.
(*f*) 25.85 Pb + 6.99 Cd + 52.43 Bi + 14.73 Sn.
(*g*) See article on *Steam.*

modynamics, Principles of. In the case of solids and liquids the external work done is practically negligible. In the case of gases and vapors, however, the external work done is appreciable. The specific heat of a gas or vapor when kept at constant volume, so that it can do no external work, is called the specific heat at constant volume, and is usually designated by the symbol C_v. The specific heat of a gas or vapor when it is allowed to expand at constant pressure is called the specific heat at constant pressure, and is usually designated by the symbol C_p. The ratio

$$\gamma = \frac{C_p}{C_v} \tag{3}$$

has very nearly the same value, 1.40 approximately, for all ordinary (diatomic) gases; see Table II.

TABLE II. — VALUE OF $\gamma = \frac{C_p}{C_v}$ FOR SOME GASES AT ATMOSPHERIC PRESSURE

(From Landolt-Börnstein Tables.)

Gas	Temperature, ° C.	$\gamma = \frac{C_p}{C_v}$	Gas	Temperature, ° C.	$\gamma = \frac{C_p}{C_v}$
Air	−181	1.34	Chlorine	20–340	1.32
Air	0	1.40	Hydrogen	......	1.40
Air	900	1.39	Nitrogen	......	1.40
Ammonia	0–100	1.30	Nitrous oxide	0–100	1.29
Carbon dioxide	0–100	1.30	Oxygen	......	1.40
Carbon monoxide	0–100	1.40			

Recalescence. — When heat is supplied at a uniform rate to a piece of iron or steel it is found that the rate of increase of temperature gradually increases (i.e., uniform increase of specific heat) until a certain temperature is reached at which the rise of temperature is suddenly and in most cases greatly retarded or even completely arrested. The reverse of this effect occurs when the sample is cooled down from a temperature above this point, and under certain conditions there occurs a spontaneous reheating during the cooling. Any point at which there is an abrupt change in the slope of a heating or cooling curve is called a "recalescence" point; there is a very marked recalescence point for most irons and steels and also one or more points at which the same effect occurs but to a lesser degree. The major recalescence point of ordinary iron or steel is usually between 750° C. and 850° C.

MELTING OR FREEZING POINT AND HEAT OF FUSION. — Certain chemically simple substances when heated to a definite temperature pass from the solid to the liquid state with no increase in temperature during this change in state, provided the solid and liquid are kept thoroughly mixed, but the change is accompanied by a considerable absorption of heat. The temperature at which the change takes place is called the melting point or freezing point (the reverse change takes place at the same temperature), and the heat absorbed per unit mass is called the heat of fusion or heat of liquefaction; this same amount of heat is given out when the body solidifies. In the case of many substances, however, there is no definite melting point, the change from one state to the other being gradual; such substances begin to melt at a lower

tempera ure than that at which solidification begins during cooling. The melting points and heats of fusion for some common substances are given in Table III. The values printed in bold face type are recommended by the Bureau of Standards as suitable for pyrometer calibration.

TABLE III.—FUSION AND VAPORIZATION

(At Atmospheric Pressure, i.e., 760 mm. Mercury.)

Substance and References	Melting point, ° C.*	Heat of fusion, gr-cal. per gr.†	Boiling pt., ° C.*	Heat of vap't'n gr-cal. per gr.†
Aluminum (2)	**658.7**	76.8	1800	....
Ammonia (2, 3)	−75	108	−33.5	321.3
Antimony (2)	**630**	....	1440	....
Bismuth (2)	271	12.64	1430	....
Brass	900 ±	....	....	....
Bronze	900 ±	....	....	....
Cadmium (2)	321	13.66	778	....
Carbon (1, 3)	over 3600	....	over 3600	....
Carbon dioxide (3)	−79	....	−79	....
Carbon monoxide (2, 3)	−203	....	−190	51.2
Chlorine (2, 3)	−102	22.96	−33.6	61.9
Chromium (2)	1610	....	2200	....
Cobalt (2)	1480	....	....	....
Copper (2)	**1083**	42.0	2310	....
German silver	1100 ±	....	....	....
Glass, flint	1300	....	....	....
Gold (2)	**1063**	....	....	....
Gutta percha	100	....	....	....
Hydrogen (1, 3)	−259	....	−252.6	....
Iridium (2)	2350(?)	....	....	....
Iron (2)	1530	23.0 to 33.0	2450	....
Lead (2)	**327**	5.36	1525	....
Manganese (2)	1230	....	1900	....
Marble	2500 ±	....	....	....
Mercury (2)	−38.87	2.82	357	65.0

* Let t_c be the value in ° C.; then the value in ° F. is $t_f = 32 + 1.8\,t_c$.

† Let H be the value in gram-calories per gram; then the corresponding heat of fusion or of vaporization

In kg-cal. per kg. is	$1.000\ H$,
In watt-seconds per gram is	$4.183\ H$,
In kw-hr. per kg. is	$1.162 \times 10^{-3}\ H$,
In kw-hr. per lb. is	$5.271 \times 10^{-4}\ H$,
In kw-hr. per ton (2000 lbs.) is	$1.054\ H$.

References: (1) Bureau of Standards, Cir. No. 35; (2) Smithsonian Physical. Tables, 1920; (3) Landolt-Börnstein-Roth, Physikalisch–Chemische Tabellen. 1912.

TABLE III.—FUSION AND VAPORIZATION — Continued

(*At Atmospheric Pressure, i.e., 760 mm. Mercury.*)

Substance and References	Melting point, ° C.*	Heat of fusion, gr-cal. per. gr.†	Boiling pt., ° C.*	Heat of vap't'n gr-cal. per gr †
Molybdenum (2)	2535	....	....	....
Nickel (2)	1452	4.64	2325	....
Nitrogen (2)	−211	....	−195	47.65
Nitric oxide (3)	−160.6	....	−153	....
Oxygen (1, 3)	−218	....	−182.7	50.97
Osmium (2)	2700	....	2600	....
Palladium (2, 3)	1545	36.3	2535	....
Paraffin (2)	52.4	35.1	....	....
Platinum (2)	1755	27.2	3910	....
Rhodium (2)	1950	....	2500	....
Rubber	100	....	....	....
Selenium (2)	217	....	690	....
Silicon (2)	1420	....	....	....
Silver (2, 3)	961	21.07	1955	....
Steel	1300 to 1475	....	....	....
Sulphur (2, 3)	106.8 to 112.8	9.37	444.7	362.0
Tantalum	2900	....	....	....
Tin (2)	231.9	14	2270	....
Tungsten (2)	3400	....	5830	....
Vanadium (2)	1720	....	....	....
Wax, bees (2)	62	42.3	....	....
Wood's metal (2)	75.5	8.40	....	....
Zinc (2)	419.4	28.13	930	....

* Let t_c be the value in ° C.; then the value in ° F. is $t_f = 32 + 1.8\, t_c$.

† Let H be the value in gram-calories per gram; then the corresponding heat of fusion or of vaporization

In kg-cal. per kg is	$1.000\, H$,
In watt-seconds per gram is	$4.183\, H$,
In kw-hr. per kg. is	$1.162 \times 10^{-3}\, H$,
In kw-hr. per lb. is	$5.271 \times 10^{-4}\, H$,
In kw-hr. per ton (2000 lbs.) is	$1.054\, H$.

References: (1) Bureau of Standards, Cir. No. 35; (2) Smithsonian Physical Tables, 1920; (3) Landolt-Börnstein-Roth, Physikalisch-Chemische Tabellen, 1912.

Freezing Mixtures. — The addition of an impurity to a liquid lowers the freezing point, a common example of which is the lowering of the freezing point of water by the addition of salt. Also, when certain substances go into solution the temperature of the solution is lowered. Some common freezing mixtures are the following, taken from *Hütte*.

FREEZING MIXTURES

Mixture	Parts by weight	Decrease of temp., ° C. From	Decrease of temp., ° C. To	Parts by weight	Decrease of temp., ° C. From	Decrease of temp., ° C. To
Snow	3	0	−17.7	1	0	−18
Common salt (NaCl)	1			1		
Snow	2	0	−33	1	0	−42
Calcium chloride ($CaCl_2$)	3			2		
Snow	3	0	−37	..	..	....
Potassium hydrate (KOH)	4					
Water	1	+10	−16	..	..	....
Ammonium nitrate (NH_4NO_3)	1					
Water	16			1		
Sal ammoniac (NH_4Cl)	5	+10	−12	1	+8	−24
Saltpeter (KNO_3)	5			1		

VAPORIZATION. — Above the surface of any liquid there always exists a certain amount of the substance in a gaseous form, i.e., as a vapor, the amount of which depends upon the nature of the substance and upon the temperature and the pressure in the space occupied by the vapor and such other gases (e.g., air) as may be present. In the case of a simple liquid evaporating into a space from which all other gases and vapors have been removed, the evaporation ceases when a definite pressure is established in this space, this equilibrium pressure depending only upon the temperature at which this space is maintained. This statement is true only when there always remains some unevaporated liquid; if all the liquid evaporates, then the equilibrium pressure also depends upon the mass of the vapor and the space which it occupies; as long as some liquid remains, the equilibrium pressure depends only upon the temperature and is independent of the volume of the space and the mass of the vapor which occupies it. This equilibrium pressure for any given temperature is called the (normal) "vapor pressure" or "vapor tension" at that temperature, and the vapor is said to be "saturated," i.e., each unit volume of the space contains the greatest possible mass of vapor which can occupy it at this particular temperature. Diminishing the volume of the space (at constant temperature) occupied by a saturated vapor causes some of the vapor to condense, and what is left remains saturated at the same pressure and temperature. Increasing the volume of the space (at constant temperature) causes more of the liquid to evaporate and the vapor still remains saturated at the same pressure and temperature, provided always that some liquid is left.

Boiling Point and Heat of Vaporization. — The temperature corresponding to any given pressure at which a vapor is completely saturated is called the "boiling point" of the liquid at this pressure. The temperature of a liquid which is "boiling" in the ordinary sense of the term is in general greater than the temperature of the saturated vapor above it, since the vapor in the bubbles formed is under a greater pressure than the vapor above the surface. The quantity of heat required to convert unit mass of a liquid into vapor at a given pressure is called the heat of vaporization or heat of evaporation of the liquid at this pressure; the same quantity of heat is given out by unit mass of the vapor when it condenses at this same pressure. The boiling points and heats of vaporization generally given are for normal atmospheric pressure, viz., 76 cm.

mercury. Values of these two quantities for some common substances are given in Table III.

Unsaturated or Superheated Vapors; Gases.—When the pressure exerted by a vapor against the walls of the containing vessel is less than the saturation or "normal" vapor pressure, the vapor is said to be unsaturated or superheated. This state of affairs may be brought about either (1) by increasing the volume of a saturated vapor when there is no longer any liquid left to evaporate, keeping the temperature constant, or (2) by raising the temperature of such a saturated vapor, keeping the pressure constant, or (3) by a proper combination of (1) and (2). The ordinary so-called "permanent" gases are superheated vapors, the degree of superheat being very great. The distinction ordinarily made between a gas and a vapor is that a gas is far removed, with respect to temperature and pressure, from the saturated state, whereas a comparatively small increase in the pressure or decrease in the temperature of a vapor will saturate it.

Laws of Perfect Gases. — Ordinary gases, such as air and superheated vapors (the greater the superheat the more nearly do the relations hold), are found to obey *approximately* the following "law,"

$$pV = \frac{MRT}{\mu}, \tag{4}$$

where p = absolute pressure of the gas,
V = volume occupied by it,
M = mass of the gas,
T = absolute temperature; see Table V below,
μ = molecular weight of the gas;* see Table IV below,
R = a constant for all gases, called the "gas constant," whose value depends only upon the units in which the various quantities are expressed; see Table V.

TABLE IV. — MOLECULAR WEIGHTS OF GASES

Gas	μ	Gas	μ
Air (75.5 N+23.2 O+1.3 A)...	28.98	Chlorine (Cl_2)...............	70.92
Acetylene (C_2H_2)	26.02	Hydrogen (H_2)..............	2.016
Ammonia (NH_3)............	17.03	Nitrogen (N_2)...............	28.02
Carbon dioxide (CO_2).......	44.00	Nitrous oxide (N_2O).........	44.02
Carbon monoxide (CO).....	28.00	Oxygen (O_2).................	32.00

The above relation can be deduced from purely thermodynamic relations (*see Thermodynamics, Principles of*) on the assumptions: (1) that the product pV at constant temperature is a constant for any particular gas (*Boyle's or Mariotte's Law*), (2) that the intrinsic energy per unit mass of a gas depends only on its temperature, being independent of the volume and nature of the gas (*Joule's Gas Law*), and (3) that at constant volume the specific heat of the gas is inde-

* For a mixture (without chemical reaction) of perfect gases of different molecular weights, the equivalent molecular weight of the mixture is

$$\mu = \frac{M_1 + M_2 + M_3 + \ldots}{\frac{M_1}{\mu_1} + \frac{M_2}{\mu_2} + \frac{M_3}{\mu_3} + \ldots},$$

where M_1, M_2, M_3, etc., are the masses of the individual constituents and μ_1, μ_2, μ_3, etc., their molecular weights.

pendent of its temperature. A gas which satisfies the above conditions is called a "perfect" gas; all ordinary gases and highly superheated vapors satisfy these conditions approximately.

As a consequence of the above law the following relations must hold for a perfect gas; they also apply approximately to ordinary gases. In addition to the symbols above, let

C_v = specific heat at constant volume,
C_p = specific heat at constant pressure,
$\gamma = \dfrac{C_p}{C_v}$,
W_{12} = work done on the gas when its pressure changes from p_1 to p_2
H_{12} = heat absorbed by the gas when its pressure changes from p_1 to p_2
J = mechanical equivalent of heat.

Then for a perfect gas, for an *isothermal change*, i.e., a change at constant temperature $(T_1 = T_2)$

$$W_{12} = p_1 V_1 \log_\epsilon \left(\frac{p_2}{p_1}\right), \quad H_{12} = \frac{W_{12}}{J}, \tag{5}$$

and for an *adiabatic change*, i.e., no heat passes in or out of the gas $(H_{12} = 0)$

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}, \quad \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}.$$

$$W_{12} = JC_vM\,(t_1 - t_2) = \frac{p_1 V_1\,(t_1 - t_2)}{T_1\,(\gamma - 1)}. \tag{6}$$

Any one of the four sets of units given in the following table (or any other *consistent* set) may be used in these formulas.

TABLE V. — VALUES OF THE GAS CONSTANT R AND MECHANICAL EQUIVALENT J

Notation	Units and constants			
M=mass	kg.	grams	lb.	gram
V=volume	cu. meter	cu. cm.	cu. ft.	cu. cm.
p=absolute press.	kg. per sq. m.	dyne per sq. cm.	lb. per sq. ft.	cm. of Hg.
t=temperature	° C.	° C.	° F.	° C.
H=quantity of heat	kg-cal.	gm-cal.	B.t.u.	gram-cal.
W=work	m-kg.	erg	ft-lb.	meter-gram
T=absolute temp.=	t+273	t+273	t+460	t+273
R=gas constant=	849	0.832×10^8	1547	0.624×10^4
J=mechan. equiv.=	427	4.19×10^7	778	427

Gas Saturated with Vapor. — Experience shows that when a substance evaporates into a space already occupied by a gas (e.g., water evaporating into the atmosphere), evaporation ceases for any given temperature when a definite pressure (depending upon the temperature and the nature of the vapor and the gas) is established in the mixture; or, if the pressure and temperature are maintained constant, then evaporation ceases when a definite amount of vapor has been produced. When a mixture of a gas and a vapor contains this maximum mass of vapor per unit volume of the mixture, the gas is said to be saturated with the vapor. Such a mixture may contain less of the vapor per unit

volume than this maximum amount; the ratio of the mass of vapor per unit volume actually present to the maximum possible mass per unit volume at any given pressure and temperature is called the "relative humidity" of the mixture. The relative humidity of a mixture (e.g., air and water vapor) can be determined by the use of a dry- and wet- bulb thermometer. (*See the Hygrometric Tables of the U.S. Weather Bureau.*)

Calculation of the Amount of Moisture in Saturated Air.— The relations given below are approximate but sufficiently accurate for practical work; they apply to a mixture of any gas and vapor which do not act chemically upon each other. Let

t = temperature of the mixture,
p = absolute pressure of the mixture,
T = absolute temperature corresponding to t, see Table V above,
R = gas constant, from Table V above,
29 = molecular weight of air; see Table IV above,
p_w = normal vapor pressure of steam at the temperature t, to be taken directly from steam tables, see article on *Steam*,
δ_w = mass of unit volume of steam at the pressure p_w; also to be taken from steam tables,
$\delta_a = \dfrac{29\,(p - p_w)}{RT}$ = mass of unit volume of air at the pressure $(p - p_w)$ and temperature t,
$\delta_m = \delta_w + \delta_a$ = mass of unit volume of the mixture.

Example. — (1) What is the weight (mass) in lb. per cu. ft. of air saturated at normal atmospheric pressure (14.70 lb. per sq. in.) with water vapor at 100° F.? (2) What is the weight in lb. of the water contained in 1 cu. ft. of the mixture? From the above formulas $p = 14.70 \times 144$ lb. per sq. ft.; $p_w = 0.946 \times 144$ lb. per sq. ft.; $\delta_w = 0.002851$ lb. per cu. ft.; $T = 460 + 100 = 560°$ F.; $R = 1547$; $\delta_a = \dfrac{29\,(14.7 - 0.946) \times 144}{1547 \times 560} = 0.06630$ lb. per cu. ft.; $\delta_m = 0.002851 + 0.06630 = 0.06915$ lb. per cu. ft. Answer: The mixture weighs 0.06915 lb. per cu. ft. and contains 0.002851 lb. of moisture per cu. ft.

Dalton's Law; Partial Pressures. — The above calculation is based upon the experimentally determined relation, known as Dalton's Law, that, in a mixture of several gases or vapors which do not react chemically upon each other, the total pressure for a given volume of the mixture is approximately equal to the sum of the individual pressures which each gas or vapor would produce if it alone filled this volume. For example, if a mixture of three gases or vapors A, B and C fill, when mixed, a volume V and produce a pressure p, then if the gas or vapor A by itself would exert a pressure p_a when it alone occupied this volume V, and gas B by itself would exert a pressure p_b when it alone occupied this volume V, and similarly for gas or vapor C, then

$$p = p_a + p_b + p_c. \tag{7}$$

The pressures p_a, p_b and p_c are called "partial" pressures. Dalton's Law holds only approximately, but is sufficiently accurate for most practical calculations.

Sublimation. — When a substance passes directly from a solid to a gaseous state the phenomenon is called sublimation. A common instance of this is the sublimation of solid carbonic acid, which passes from solid to gaseous form at atmospheric pressure at a temperature of − 79° C. and absorbs 140 gram-calories per gram or kilogram-calories per kilogram.

TRANSFER OF HEAT. — When a body is at a higher temperature than the surrounding bodies, energy is transferred from the hotter to the colder

bodies, as is manifested by the changes in temperature or state which tend to, or actually do, take place, even though the intervening space is entirely void of matter. The energy thus transferred from one body to another through empty space is called "radiant energy," or "radiant heat," and is similar in nature to the energy radiated in the form of light waves and electromagnetic waves. The waves of radiant heat have a length greater than that of light waves and less than that of the ordinary electromagnetic waves used in wireless telegraphy. Radiant heat is absorbed by, transmitted through and reflected by, ordinary matter in much the same way that light waves are absorbed, transmitted and reflected. Matter which is transparent to light waves, however, may be practically opaque to heat waves; e.g., water absorbs practically all the heat waves which fall upon it.

When a hot and a cold body are separated by a fluid which is free to circulate, heat is transferred from the hot to the cold body by currents of the fluid itself flowing from one to the other; similarly, all parts of a fluid which is being heated quickly come to approximately the same temperature. This transfer of heat by currents of the fluid itself is called "convection."

In the case of a hot and a cold body separated by a solid the transfer of heat, which may be very rapid, particularly when the separating medium is a metal, is probably due to an extremely rapid to-and-fro motion of the molecules which constitute the medium. In any event, the process is essentially different from the transfer of heat either by radiation or by convection; it is described by the term "conduction" of heat. In a fluid heat is transferred both by conduction and convection.

An excellent brief treatment of Heat Transmission is given by I. Langmuir, *A.I.E.E.,Trans. 32, p. 301, Feb. 1913,* including numerical data on radiation, conduction and convection.

Radiation, Absorption and Reflection of Heat. — The rate at which a hotter body radiates heat, and a colder body absorbs heat, depends upon the state of the surfaces of the bodies as well as on their temperatures. The rate of radiation and of absorption are increased by darkness and roughness of the surfaces of the bodies, and diminished by smoothness and polish. For this reason the covering of steam pipes and boilers should be smooth and of a light color; uncovered pipes and steam-cylinder covers should be polished.

The heat radiated by a body at a given temperature T to surrounding bodies at a lower temperature is equal to the heat which this body would absorb at this same temperature T from surrounding bodies at a higher temperature. When a given quantity of radiant heat strikes a body, only part of the heat is, as a rule, absorbed, the rest being reflected.

Let H_i be the incident heat, H_r the reflected heat, and H_a the absorbed heat, at temperature t, and let H_e be the heat which the body would emit at this same temperature to bodies at a lower temperature; then

$$H_i = H_r + H_a,$$

$$H_a = H_e.$$

Definition of "Black Body"; Stefan-Boltzman Law. — A "black body" is defined as one that absorbs all radiations falling upon it, neither reflecting nor transmitting any. The radiation of such a body is a function of the temperature alone, and is identical with the radiation inside an inclosure all parts of which have the same temperature. By heating the walls of an inclosure as uniformly as possible and observing the radiation through a very small opening, a practical realization of a black body is obtained.

The radiation from such a body is found to obey the following law

$$E = K\left[\left(\frac{T}{1000}\right)^4 - \left(\frac{T_0}{1000}\right)^4\right]At, \tag{8}$$

where E is the total energy radiated in time t from such a body at an *absolute* temperature T to surrounding bodies maintained at an *absolute* temperature T_0. A is the area of the surface of the body and K is a constant. This relation is known as the Stefan-Boltzman Law. When the *absolute* temperature T is over three times the absolute temperature T_0, this relation may be written

$$K\left(\frac{T}{1000}\right)^4 At.$$

The value of K given in the Smithsonian Physical Tables (1914) is 1.374 when E is expressed in gram-calories, T in centigrade degrees and t in seconds. For other units K has the following values

VALUES OF K

Unit of energy (E)	Unit of area (A)	Unit of time (t)	Temperature scale	Absolute zero	K =
Gram-calorie......	sq. cm.	second	Centigrade	−273	1.374
Kg-calories........	sq. meter	hour	Centigrade	−273	0.000495
B.t.u.............	sq. ft.	hour	Fahrenheit	−460	0.00174
Watt-seconds.....	sq. cm.	second	Centigrade	−273	5.75
Kw-hr...........	sq. in.	hour	Centigrade	−273	370.

Radiating and Reflecting Powers. — The ratio of the heat radiated per unit area by any surface at a given temperature T to the heat radiated per unit area by an absolutely black surface at this same temperature T is called the radiating power, or relative emissivity, of the surface at that temperature. The difference between this ratio and unity is a measure of the heat which would be reflected by this surface at the same temperature, and is defined as the reflecting power of the surface. The radiating power of a surface depends upon the temperature of the surface; at very high temperatures the radiating power of every surface approaches the value of unity, i.e., at high temperatures the total energy radiated by any surface approaches in value the total energy radiated by an absolutely black body. The table on page 738, taken in part from Kent's *Mechanical Engineer's Pocketbook* and in part from Langmuir's paper (*Trans. A.I.E.E., 1913*) gives the approximate value of the radiating power of some common surfaces at ordinary temperatures.

Oiling a polished surface may increase its radiating power from 2 to 3 times, but oiling does not seriously affect the radiating power of a rough surface.

Conduction of Heat. — Whenever a difference of temperature is maintained between any two parts of the same body there is a transfer of heat from the hotter to the colder part by the process described by the term "conduction," as distinguished from radiation and convection.

Consider a flat layer *within* a substance, the two sides of the layer being parallel and its thickness small compared with its area. Let one side of this be maintained at a constant temperature T, the same at all points of this surface, and the other side of the layer be maintained at a constant temperature T_1;

TABLE VI.—APPROXIMATE RADIATING AND ABSORBING POWERS

Surface	Radiating or absorbing power	Surface	Radiating or absorbing power
Lampblack	1.00	Zinc, polished	0.19
Water	1.00	Steel, polished	0.17
Carbonate of lead	1.00	Platinum, polished	0.24
Writing-paper	0.98	Platinum in sheet	0.17
Ivory, jet, marble	0.93–0.98	Tin	0.15
Ordinary glass	0.90	Brass, cast, dead polished	0.11
Ice	0.85	Brass, bright polished	0.07
Gum lac	0.72	Copper, varnished	0.14
Silver-leaf on glass	0.27	Copper, hammered	0.07
Cast iron, bright polished	0.25	Copper, oxidized	0.74
Cast iron, oxidized	0.65	Copper, calorized	0.21
Aluminum paint on cast iron	0.47	Monel metal, polished	0.40
Gold enamel on cast iron	0.39	Monel metal, oxidized	0.45
Mercury, about	0.23	Gold, plated	0.05
Wrought iron, polished	0.23	Gold on polished steel	0.03
		Silver, polished bright	0.03

let A be the area of the layer (i.e., of one of its flat surfaces) and x the thickness of the layer. Then the amount of heat transferred through the layer in time t is,

$$H = \frac{K\,A\,(T - T_1)t}{x}, \tag{9}$$

where K, called the "thermal conductivity," is approximately a constant for a given material, and is independent of the temperature difference $T - T_1$, when this difference is small; K is not constant, however, for wide temperature variations, particularly in the cases of gases (see paragraph following Table VIII). Values of K are given in Tables VII and VIII. The reciprocal of the thermal conductivity, viz., $\rho = 1/K$, is called the thermal resistivity.

The values of K given in the tables below are the values of this factor when H is expressed in gram-calories, A in sq. cm., x in cm., $(T - T_1)$ in ° C., and t in seconds; i.e., K is the number of gram-calories transmitted per second through

Unit of heat	Unit of area	Unit of thickness	Unit of time	Unit of temp. diff.	Multiply K by
Gram-calorie	sq. cm.	cm.	second	° C.	1.000
Kg-calorie	sq. m.	cm.	hour	° C.	3.600×10^4
B.t.u	sq. ft.	inch	hour	° F.	2902
Watt-seconds	sq. cm.	cm.	second	° C.	4.183
Watt-seconds	sq. ft.	inch	second	° F.	850.8
Kw-hr	sq. m.	cm.	hour	° C.	41.83
Kw-hr	sq. ft.	inch	hour	° F.	0.8508

a cube 1 cm. on each edge when a difference of temperature of 1° C. is maintained between opposite faces of this cube. For other units the value of K as given should be multiplied by the factor noted in the table at bottom of preceding page.

Conduction through Other than Thin Layers. — The formula for H given above is applicable only *when the lines of flow are straight and perpendicular to A and the temperatures T and T_1 are temperatures of the surfaces of the substance itself,* not the temperatures of a fluid, for example, in contact with these surfaces. Failure to take these facts into consideration accounts for some of the inconsistencies in the reported values of thermal conductivity from tests. To calculate the flow of heat in other cases, formulas analogous to those for electric resistance and conductance must be employed; see *Resistance and Conductance;* also Langmuir, Adams and Meikle, *Trans. Am. Electrochem Soc.*, Vol. 24 (1913).

Temperature Coefficient of Thermal Conductivity. — In the case of metals the variation of the internal thermal conductivity with temperature may be expressed with a fair degree of approximation by the relation

$$K = K_0 (1 + at), \tag{10}$$

where K_0 is the conductivity at 0° C., say, and K is the conductivity at any other temperature of t° C., and a is a constant. The coefficient a may be either positive or negative; its value for some of the common metals is given in Table VII.

Values of Thermal Conductivity (K) of Materials. — In the following tables, VII and VIII, are given the thermal conductivity of certain common non-metallic and metallic substances respectively. The data are from the following sources: Landolt-Börnstein, *Physikalish-Chemische Tabellen*, 1912; Nusselt, W., *Zeit. Ver. Deutch. Eng.*, June, 1908; Hering, Carl, *Trans. A.I.E.E.*, 1910; Randolph, C. P., *Trans. Am. Electrochem. Society*, 1912; Ordway, *Trans. Am. Soc. Mech. Eng.*, 1884–85; Coleman, J. J., *Engineering*, Sept. 5, 1884; Scott, H. G., *Power*, 1902; Wolff, *Jour. Frank. Inst.* 1893; Peclet, *Practical Treatise on Heat; Met. & Chem. Eng.*, Feb. 1909, p. 72; Brill, G. M., *Trans. Am. Soc. Mech. Eng.*, Vol. 16, p. 827; *Smithsonian Physical Tables*, 1920; Langmuir, *A.I.E.E., Trans.*, 32, p. 301, Feb. 1913.

Thermal Conductivity of Laminated Steel. — The thermal conductivity of a laminated iron core perpendicular to the laminations depends upon the conductivity of the iron and the oxide scale or varnish on the laminations and also upon the pressure on the laminations. G. E. Luke (*El. World, 70, p. 562, Sept. 22, 1917*) gives the following values for K in gram-calories per cm.^{3} per °C.

Material	Pressure, pounds per square inch			
	25	50	75	100
Silicon steel, 15.5 mils thick, 3 per cent oxide layer........	0.00096	0.00115	0.00125	0.00132
Ordinary varnished steel, 18.35 mils thick, 5 per cent varnish layer......................	0.00094	0.00098	0.00109	0.00120

TABLE VII.—THERMAL CONDUCTIVITY OF NON-METALLIC SUBSTANCES

Substance	Temp. range, ° C. From*	Temp. range, ° C. To	Therm. conduct. K ** From †	Therm. conduct. K ** To†	Temperature coefficient per ° C. a
Air (see paragraph following Table VIII.)					
Asbestos	100	500	0.00016	0.00043	....
Asbestos paper	......	......	0.00043	0.00060	....
Brick building	15	1100	0.00149	....	....
Brick, dust	15	30	0.000461	....	....
Brick, fire	0	1300	0.00140	0.0054	....
Carborundum, brick	150	1200	0.0032	0.027	....
Cardboard	Below 0	....	0.000394	....	....
Cement, Portland	35	90	0.000712	0.00217	....
Chalk	.....	....	0.00219	....	....
Cloth, empire	.....	....	0.00060	....	....
Concrete, slag	50	....	0.00081	....	....
Cork	20	20000	0.000153	0.000717	....
Cotton batting, loose	.....	....	0.000096	....	....
Cotton batting, tightly packed	.....	....	0.000072	....	....
Ebonite	6	90	0.00038	....	−0.0019
Eiderdown, loose	.....	....	0.000107	....	....
Eiderdown, tightly packed	150	....	0.000045	....	....
Feathers	20	155	0.000163	....	....
Felt	21	175	0.000087	0.000225	....
Flannel	50	....	0.00012	....	....
Fullerboard	.....	....	0.00034	....	....
Glass	.....	....	0.0026	0.0036	....
Glycerine	.....	....	0.00057	....	....
Granite	100	200	0.0045	0.0097	....
Hair	20	155	0.000148	....	....
Ice	−160	0	0.0066	0.0050	....
Infusorial earth	20	450	0.000216	0.00354	....
Lampblack	100	500	0.0000756	0.000109	....
Leather	.....	....	0.00015	0.00042	....
Limestone	40	350	0.0046	0.0035	....
Linen	.....	....	0.00021	....	....
Liquids, hydrocarbons, oils, etc	.....	....	about 0.0003	....	....
Magnesia, carb	20	188	0.000175	....	....
Magnesia, calcined	20	155	0.000165	0.000173	....
Magnesia, asbestos	100	400	0.000162	0.000178	....
Marble	15	30	0.00770	0.00910	−0.0005
Mica, pure	.....	....	0.00086	....	....
Mica, paper	.....	....	0.00038	....	....
Micanite	.....	....	0.00024	0.00029	....
Oil, paraffin	.....	....	0.00033	....	....

THERMAL VII.—THERMAL CONDUCTIVITY OF NON-METALLIC SUBSTANCES—*Continued*

Substance	Temp. range, ° C.		Therm. conduct. K**		Temperature coefficient per° C. a
	From*	To	From†	To†	
Oil, castor	.....	....	0.000425	....	....
Oil, petroleum	0	34	0.000355	0.000382	+0.0110
Oil, turpentine	13	....	0.000325	....	+0.0067
Paper	.....	....	0.00029	0.00031	....
Paper, treated	.....	....	0.00034	0.00041	....
Paraffin	.....	....	0.000473	0.00062	....
Pasteboard	.....	....	0.000450	....	....
Plaster	.....	....	0.00130	....	....
Plaster of Paris	20	155	0.000425	....	....
Plumbago	20	155	0.00100	....	....
Poplox, made from Na_2SiO_3	200	500	0.0000920	0.000162	....
Porcelain	95	....	0.00249	....	....
Pumice stone	20	155	0.000428	....	....
Quartz glass	.....	....	0.0036	....	....
Rubber, Para	.....	....	0.00038	....	....
Rubber, vulcanized	.....	....	0.00034	0.00054	....
Sand	20	155	0.000855	0.000867	....
Sawdust	.....	....	0.000123	0.000152	....
Shellac	.....	....	0.00060	....	....
Silk	50	100	0.000095	0.000141	....
Slag	50	....	0.00264	....	....
Slate	94	....	0.0048	....	....
Snow	.....	....	0.000060	0.00115	....
Strawboard	.....	....	0.000330	....	....
Tape, treated cloth	.....	....	0.00036	0.00065	....
Tape, rubber	.....	....	0.00103	....	....
Water	0	25	0.00150	0.00136	....
Wood	.....	....	0.0000878	0.00038	....
Wool, sheep's, loose	.....	....	0.000117	....	....
Wool, sheep's, tightly packed	.....	....	0.000055	....	....
Wool, mineral	0	175	0.0000930	0.000128	....
Wool, steel	100	....	0.000192	0.000216	....
Woolen	100	....	0.0000553	0.000119	....

* When only one temperature is given the measurement was made at that temperature.

† Range of determination by different experimenters except where only one value of K is given, in which case the range is that to which the given value of K applies.

** In gram-calories per centimeter-cube per degree centigrade per second; see table on page 738 for multiplying factors when other units are employed.

TABLE VIII.—THERMAL CONDUCTIVITY OF METALS AND VARIOUS FORMS OF CARBON

Substance	Temp. range, ° C.		Therm. conduct. K*		Temp. coeff. per ° C. a
	From†	To	From‡	To‡	
Aluminum	18	100	0.480	0.492	+0.0030
Aluminum	200	400	0.545	0.760	+0.0020
Aluminum	500	600	0.885	1.01	+0.0014
Brass, yellow	0	100	0.204	0.254	+0.0024
Brass, red	0	100	0.246	0.283	+0.0015
Calorite	20	250	0.038	......	....
Carbon, amorphous	100	360	0.089	......	....
Carbon, amorphous	100	842	0.129	......	....
Carbon, Ach. Graph.	100	390	0.338	......	....
Carbon, Ach. Graph.	100	914	0.291	......	....
Carbon, graphite brick	300	700	0.024	......	....
Charcoal	20	155	0.00019	......	....
Coal	.....	.....	0.00030	......	....
Constantan	18	100	0.054	0.064	+0.00227
Copper	−54	−14	0.921	1.059	+0.0053 to
Copper	74	167	0.914	1.024	+0.00047
Copper	100	197	1.043	......	....
Copper	100	837	0.858	......	....
Copper, commercial	18	.....	0.835	......	....
German silver	0	100	0.070	0.089	+0.0027
Gold	100	.....	0.703	......	−0.00007
Iron	0	.....	0.167	0.207	−0.00023 to
Iron	100	.....	0.142	0.163	0.0001
Iron	200	.....	0.136	......	....
Iron	100	727	0.202	......	....
Iron	100	1245	0.191	......	....
Iron, cast	100	.....	0.096	......	....
Lead	0	100	0.0836	0.0764	−0.00086 to −0.00016
Manganin	18	100	0.52	0.63	+0.0027
Mercury	0	50	0.0148	0.0189	+0.0055
Nickel	18	.....	0.142	......	−0.00031
Nickel	300	.....	0.126	......	....
Nickel	1200	.....	0.058	......	....
Platinum	18	100	0.166	0.173	+0.00051
Platinoid	18	.....	0.060	......	....
Silver	18	100	1.006	0.992	−0.00017
Steel	.....	.....	0.062	0.111	−0.0006
Steel, transformer (a)	20	250	0.077	......	....
Tin	0	100	0.155	0.145	−0.0007
Zinc	18	100	0.265	0.262	−0.00016

* In gram-calories per centimeter-cube per degree centigrade per second; see table on page 738 for multiplying factors when other units are employed.

† When only one temp. is given the measurement was made at that temp.

‡ Range of determinations by different experimenters except where only one value of K is given, in which case the range is that to which the given value of K applies.

(a) See also paragraph *Thermal Conductivity of Laminated Steel* on page 739.

Thermal Conductivity of Gases. — The thermal conductivity of a gas depends not only upon the nature of the gas but also upon its specific heat at constant volume, C_v, and its absolute temperature T. Langmuir (*Phys. Rev. 34, p. 408, 1912*) gives the following values for the *watts* conducted from a plane surface through an adhering film A sq. cm. in area and B centimeters thick

$$W = \frac{A}{B}(\phi_2 - \phi_1), \tag{11}$$

where ϕ_1 and ϕ_2 are functions of the absolute temperatures at the two surfaces of the film, as given in the following table.

VALUES OF ϕ

Abs. Temp. ° C.	Hydrogen	Air	Mercury Vapor
0	0.0000	0.0000	
100	0.0329	0.0041	
200	0.1294	0.0168	
300	0.278	0.0387	
400	0.470	0.0669	
500	0.700	0.1017	0.0165
700	1.261	0.189	0.0356
900	1.961	0.297	0.0621
1100	2.787	0.426	0.0941
1300	3.726	0.576	0.1333
1500	4.787	0.744	0.1783
1700	5.945	0.931	0.228
1900	7.255	1.138	0.284
2100	8.655	1.363	0.345
2300	10.18	1.608	0.411
2500	11.82	1.871	0.481
2700	13.56		0.556
2900	15.54		0.636
3100	17.42		0.719
3300	19.50		0.807
3500	21.79		0.898

Convection. — According to Langmuir (*Trans. A.I.E.E., 31, p. 1229, 1912; 32, p. 301, 1913*) convection in a gas or liquid consists essentially in conduction of heat through a thin film of the fluid of definite thickness, in which the heat carried by motion of the fluid is negligible compared to that carried by conduction, and outside of which the temperature is maintained uniform because of convection currents. In the case of a vertical plane surface in air, when the surface is over 30° C. above the surrounding air, the heat loss by "free" convection (no forced circulation of the surrounding air) is equal to that conducted through a film of still air 4.3 millimeters thick adhering to the surface, taking the temperatures of the two faces of the film the temperature of the surface and the temperature of the room air respectively. Equation (11) should be used for this calculation, taking $x = 0.43$.

For a horizontal plane surface radiating upward the convection loss is about 10 per cent greater, and for a horizontal plan surface radiating downward approximately 50 per cent less, than from a vertical surface.

According to H. T. Gillette (*Eng. and Cont.*, *47*, *p. 573*, *June 27*, *1917*) the equivalent film thickness for water is 1.27 millimeters when the water is not stirred mechanically. The equivalent film thickness for steam is also apparently much less than that for air.

The film thickness of 4.3 millimeters for air is for air at ordinary room temperature and barometric pressure, with no forced ventilation. The film thickness is directly proportional to the absolute temperature of the surrounding air, and varies approximately inversely as the square root of the absolute pressure.

The free convection from a round wire or cylinder of any diameter may also be calculated in a similar manner, except that the thickness of the air film depends upon the diameter of the wire, and account must be taken of the fact that the lines of heat flow are radial. Langmuir gives the following formula for the watts carried off by convection from a length of 1 cm. of wire

$$W = s(\phi_2 - \phi_1), \tag{12}$$

where ϕ_2 and ϕ_1 correspond to the absolute temperature T of the wire and the absolute tenperature T_1 of the surrounding air (see table on p. 743), and s is the shape factor, defined by the relation

$$\frac{a}{B} = \frac{s}{\pi}\epsilon^{-\frac{2\pi}{s}}, \tag{13}$$

where a is the diameter of the wire, in centimeters, B the film thickness for a plane surface, in centimeters, and ϵ the Napierian base. The values of this space factor for wires in air at ordinary room temperatures and atmospheric pressure are given in the following table.

VALUES OF s

$\frac{a}{B}$	s	$\frac{a}{B}$	s	$\frac{a}{B}$	s
0.02	1.85	0.5	5.21	5.0	21.0
0.04	2.22	1.0	7.40	6.0	24.2
0.06	2.47	1.5	9.30	7.0	27.5
0.08	2.67	2.0	11.1	8.0	30.7
0.10	2.85	2.5	12.8	9.0	34.0
0.20	3.60	3.0	14.5	10.0	37.2
0.30	4.20	3.5	16.1	12.0	43.5
0.40	4.75	4.0	17.8	14.0	50.0

Forced Convection. — Langmuir finds (l.c.) that the convection from plane surfaces in a stream of air is equal to that in still air multiplied by the factor $\sqrt{\frac{v + 33}{33}}$, where v is the velocity of the air in cm. per sec.

Total Heat Transfer. — Numerous formulas have been proposed in recent years for the total transfer of heat (radiation, conduction and convection) from pipes to the surrounding gas or vapor; see C. H. Herter, *Am. Soc. Refrig. Eng. J.*, 4, p. 308, Nov. 1917; L. H. Fry, *A.S.M.E.J.*, 39, p. 843, Oct. 1917; Kreisinger and Barkley, *Bur. of Mines*, *Tech. Paper*, No. 114, 1915; E. E. Wilson, *A.S.M.E.J.*, 37, p. 546, Sept. 1915.

EXPANSION DUE TO HEATING. — Most substances expand when heated, but water between 0° degrees and 4° C., quartz glass below − 84° C. and a few other substances contract with increase of temperature. When the temperature of a solid is changed by rapid cooling, slow changes in its dimensions continue long after it has attained the same uniform temperature throughout. This effect, which is particularly marked in glass, is known as "thermal hysteresis." It can be largely eliminated by prolonged heating at a high temperature followed by a very gradual cooling, i.e., by annealing.

The volume of a physically homogeneous substance which is not subjected to any treatment causing more or less permanent changes in its structure, is a definite function of its temperature and the pressure or tension in it. In general, under constant pressure or tension the amount of expansion caused by a given change in temperature depends upon both the material of the body and its initial temperature.

Coefficient of Expansion of Gases. — In the case of gases, however, there is a remarkable uniformity, all the so-called permanent gases expanding about $\frac{1}{273}$ part of their *initial volume at 0° C.* per degree centigrade increase of temperature, irrespective of the pressure, provided this remains constant throughout. That is, for any of the ordinary gases,

$$V = V_0\left(1 + \frac{t_c}{273}\right),$$

where V_0 is the volume at 0° C., and V the volume at any other temperature t_c° C., the pressure being the same at both temperatures. If the temperature is expressed in Fahrenheit degrees and V_0 is the volume at 0° F., then

$$V = V_0\left(1 + \frac{t_f}{460}\right).$$

Note that − 273 and − 460 are the absolute zeros on the centigrade and Fahrenheit scales respectively.

Coefficients of Linear Expansion of Liquids and Solids. — Let l_0 be length of a rod, or of column of liquid, at any standard temperature, say 0° C., and l be the length at any other temperature t. Then the exact relation between l and l_0 may be expressed as a series of the form

$$l = l_0\,(1 + at + bt^2 + \ldots),$$

where a, b, etc., are constant coefficients for any given material and *fixed reference temperature*. As a rule the coefficients of the powers of t above the first are much smaller than the first coefficient, and for small ranges of temperature the approximate formula

$$l = l_0\,(1 + at)$$

is usually employed. The coefficient a in this formula is practically a constant for any range of temperature not exceeding 100° C.; it is sometimes called the "mean coefficient of linear expansion" between the limits of temperature chosen. For example, between 0° C. and 100° C., the mean coefficient of linear expansion is defined by the relation

$$a_0 = \frac{l_{100} - l_0}{100\, l_0},$$

where $l_{100} - l_0$ is equal to the change in length produced by increasing the temperature from 0° C., to 100° C.

It should be noted that for each value of the standard or reference temperature a different value of a must be used. For example, if a rod has a length l_1

TABLE IX. — COEFFICIENTS OF LINEAR EXPANSION

$l = l_0 (1 + at + bt^2)$, temperature in °C.*

a_{20} = "true" coefficient at 20°C.

(*From Landolt-Börnstein's Tables, 1912 Edition.*)

Substance	Temp., ° C.		a	b	a_{20}
	From	To			
Aluminum	0	610	0.235×10^{-4}	0.707×10^{-8}	0.238×10^{-4}
Brass (73.7 Cu+24.2 Zn +1.5 Sn+0.6 Pb)	0	80	0.179×10^{-4}	0.456×10^{-8}	0.181×10^{-4}
Bronze (81.2 Cu+8.6 Zn +9.9 Sn+0.2 Pb)	0	80	0.176×10^{-4}	0.469×10^{-8}	0.177×10^{-4}
Carbon, gas-carbon	40	....	0.054×10^{-4}	....	....
Carbon, graphite	40	....	0.079×10^{-4}	....	....
Constantan (60 Cu+40 Ni)	0	500	0.148×10^{-4}	0.402×10^{-8}	0.150×10^{-4}
Copper	0	625	0.167×10^{-4}	0.403×10^{-8}	0.169×10^{-4}
Glass, Jena	0	100	0.077×10^{-4}	0.350×10^{-8}	0.079×10^{-4}
Glass, French	2	100	0.072×10^{-4}	0.544×10^{-8}	0.075×10^{-4}
Gold	9	95	0.136×10^{-4}	1.12×10^{-8}	0.140×10^{-4}
German silver	0	100	0.184×10^{-4}	....	....
Ice	−27	−2	0.514×10^{-4}	....	....
Iron, cast	0	625	0.098×10^{-4}	0.566×10^{-8}	0.102×10^{-4}
Iron, wrought	0	500	0.117×10^{-4}	0.525×10^{-8}	0.119×10^{-4}
Lead	14	94	0.273×10^{-4}	0.74×10^{-8}	0.276×10^{-4}
Marble, white	15	100	0.117×10^{-4}	....	....
Mica, parallel to cleavage	5	80	0.077×10^{-4}	1.200×10^{-8}	0.082×10^{-4}
Mica, perpendicular to cleavage	4	82	0.076×10^{-4}	0.490×10^{-8}	0.079×10^{-4}
Nickel	0	1000	0.135×10^{-4}	0.332×10^{-8}	0.136×10^{-4}
Nickel steel (24% Ni)	0	38	0.175×10^{-4}	0.711×10^{-8}	0.178×10^{-4}
Phosphor bronze (97.6 Cu +2.2 Sn+0.2 P)	0	80	0.167×10^{-4}	0.462×10^{-8}	0.168×10^{-4}
Platinum	0	1000	0.0887×10^{-4}	0.1324×10^{-8}	0.0892×10^{-4}
Porcelain, Berlin	20	100	0.027×10^{-4}	0.306×10^{-8}	0.028×10^{-4}
Porcelain, Bayeux	0	600	0.034×10^{-4}	0.107×10^{-8}	0.035×10^{-4}
Rubber, hard	17	25	0.77×10^{-4}	....	....
Rubber, hard	25	35	0.84×10^{-4}	....	....
Silver	0	750	0.1827×10^{-4}	0.4793×10^{-8}	0.1846×10^{-4}
Steel	0	300	0.092×10^{-4}	0.336×10^{-8}	0.093×10^{-4}
Tin	8	95	0.203×10^{-4}	2.63×10^{-8}	0.214×10^{-4}
Vulcanite	0	18	0.636×10^{-4}	....	....
Zinc	9	96	0.274×10^{-4}	2.34×10^{-8}	0.284×10^{-4}

* When the temperature is expressed in Fahrenheit degrees, the formulas become

$$l = l_{32}\left[1 + \frac{a(t_f - 32)}{1.8} + \frac{b\,(t_f - 32)^2}{3.24}\right],$$

$$a_{68} = \frac{a_{20}}{1.8},$$

where a, b and a_{20} have the values given in the above table.

at t_1 degrees and a length l at t degrees, and a_0 is the mean coefficient of linear expansion referred to 0° C., then $l_1 = l_0(1 + a_1 t_1)$ and $l = l_0(1 + a_0 t)$, whence

$$l = l_1 [1 + a_1 (t - t_1)],$$

where $$a_1 = a_0/(1 + a_0 t_1).$$

If t_1 is so large that $a_0 t_1$ is appreciable compared with unity then a_1 is not equal to a_0.

True Coefficient of Linear Expansion. — The "true" coefficient of linear expansion at any temperature t is defined as the *rate* of increase of length with increase in temperature divided by the length at this temperature t, or

$$a = \frac{1}{l} \cdot \frac{dl}{dt}.$$

The "true" coefficient a_t any temperature depends on the temperature.

Coefficients of Cubical Expansion of Liquids and Solids. — The volume or cubical expansion of liquids and solids may be expressed in exactly the same manner as the linear expansion, viz.,

$$V = V_0 (1 + \alpha t + \beta t^2 + \ . \ . \ .)$$

or approximately

$$V = V_0 (1 + \alpha t),$$

and the true coefficient at any temperature t is defined as

$$\alpha_t = \frac{1}{V} \frac{dV}{dt},$$

where V and V_0 are the volumes at t degrees and at the reference temperature respectively.

As a first approximation, when the coefficient of linear expansion is small and the temperature rise not excessive, the volume coefficient may be taken equal to 3 times the linear coefficient.

A few volume coefficients, determined from direct experiment, are given in Table X. For Water see article on *Weights of Materials*.

TABLE X. — COEFFICIENTS OF CUBICAL EXPANSION

$V = V_0 (1 + \alpha t + \beta t^2)$, temperatures in ° C.
α_{20} = "true" coefficient at 20° C.
(*From Landolt-Börnstein's Tables, 1912 edition.*)

Substance	Temp., ° C. From	Temp., ° C. To	α	β	α_{20}
Caoutchouc, crude gray..	0	75	6.62×10^{-4}	24.2×10^{-8}	6.80×10^{-4}
Gutta-percha, pure rolled.	0	40	4.96×10^{-4}	496×10^{-8}	6.94×10^{-4}
Paraffin..................	0	33	5.84×10^{-4}	99.2×10^{-8}	5.88×10^{-4}
Petroleum, sp. gr. 0.8467..	24	120	8.99×10^{-4}	140×10^{-8}	9.55×10^{-4}
Wax, white solid..........	10	57	10.7×10^{-4}	-5580×10^{-8}	3.06×10^{-4}

Mercury (— 10 to 300° C.):

$$V = V_0(1 + 1.805553 \times 10^{-4} t + 1.2444 \times 10^{-8} t^2 + 2.539 \times 10^{-11} t^3)$$

FLASH POINT OF OILS.—The flash point of an oil is the temperature to which the oil must be raised before the vapor immediately above it will take fire upon the application of a flame. This temperature depends to an appreciable extent upon the size of the flame, the method of applying it, and the shape and dimensions of the containing vessel; see *Oil, Transformer*. The following values of the flash point for various oils are taken from J. Lewkowitisch, *Chemical Technology and Analysis of Oils, Fats and Waxes*.

TABLE XI.—FLASH POINT OF OILS

Oils	Spec. grav. at 60° F.	Flash point ° F.
Mineral oils:		
Refined American	0.875 to 0.920	325 to 425
Refined Russian	0.895 to 0.915	300 to 425
Scotch	0.875 to 0.895	300 to 350
Natural (dark) American	0.880 to 0.895	325 to 425
Natural (dark) Russian	0.910 to 0.915	250 to 300
Natural filtered American	0.885 to 0.905	450 to 575
Animal oils:		
Sperm	0.8804 to 0.8807	446 to 457
Lard	0.9172	494
Tallow	0.951	265
Neat's foot	0.9178	470
White whale	0.9207	476
Vegetable oils:		
Castor	0.963	275
Linseed	0.930	285
Olive	0.914	305
Rape, crude	0.920	265
Rape, refined	0.911	305

HEATS OF FORMATION, COMBUSTION AND SOLUTION.—When two or more substances react chemically heat is generally either given out or absorbed. When heat is given out the reaction is called "exothermic" and when heat is absorbed the reaction is called "endothermic." The following tables give the heats of reaction for some of the more important industrial processes; see *Landolt-Börnstein's Tables* for more complete tables. A minus sign indicates an endothermic reaction; i.e., an absorption of heat. When two values are given these refer to independent measurements.

The values given are the total amounts of heat in kilogram-calories given out or absorbed per mol (*see Electrochemistry, Principles of*) of the product, starting with the proportions of the reacting substances represented by the left-hand member of the reaction equation, the temperature at the beginning and at the end of the reaction being the same, i.e., room temperature. For example, starting with 1 gram-atom of oxygen (16 kilograms, say) and 1 gram-atom of calcium (40.07 kilograms) both at room temperature, then the net amount of heat given out in the formation of one mole ($16 + 40.07 - 56.07$ kilograms) of calcium oxide when its temperature is brought back to room temperature is 130.9 kilogram-calories or 2.33 kilogram-calories per kilogram of calcium oxide.

TABLE XII. — HEATS OF FORMATION OF IMPORTANT COMPOUNDS

(*At room temperature unless otherwise stated.*)

Product	Equation	Kilogram-Calories* evolved per mol of product
Acetylene	$2C+2H=C_2H_2$	−47.8 to −58.1
Aluminium hydroxide	$2Al+3O+3H_2O=2Al(OH)_3$	194.5 to 196.5
" "	$Al+3O+3H=Al(OH)_3$	297.0
" oxide	$2Al+3O=Al_2O_3$	380.2
Ammonia gas	$N+3H=NH_3$	11.9 to 12.2
Barium dioxide	$BaO+O=BaO_2$	18.4
" "	$Ba+2O=BaO_2$	145.5
" hydroxide	$Ba+2O+2H=Ba(OH)_2$	217.0
" monoxide	$Ba+O=BaO$	133.4 to 126.4
Calcium carbide	$Ca+2C=CaC_2$	−7.25
" oxide	$Ca+O=CaO$	145.0 to 151.9
Carbon dioxide	C amorphous$+2O=CO_2$	96.96–97.65
" "	C graphite$+2O=CO_2$	94.8
" monoxide	C amorphous$+O=CO$	29.0
Copper chloride	$Cu+2Cl=CuCl_2$	51.63–51.4
" sulphate	$Cu+S+4O=CuSO_4$	182.6
Cupric oxide	$Cu+O=CuO$	37.2
Cuprous oxide	$2Cu+O=Cu_2O$	40.8–43.8
Ferric hydroxide	$2Fe+3O+3H_2O=2Fe(OH)_3$	95.6
" oxide	$2Fe+3O=Fe_2O_3$ (dried at 400°)	65.2
	$2Fe+3O=Fe_2O_3$ (heated to 1000°)	64.8
Ferrous chloride	$Fe+2Cl=FeCl_2$	82.2
" hydroxide	$Fe+O+H_2O=Fe(OH)_2$	68.3
" oxide	$Fe+O=FeO$	65.7
" sulphate	$Fe+S+4O+7H_2O=FeSO_4\cdot 7H_2O$	240.1
Lead dioxide	$PbO+O=PbO_2$	12.6
" monoxide	$Pb+O=PbO$	50.3
" sulphate	$Pb+S+4O=PbSO_4$	216.2
Magnesium chloride	$Mg+2Cl=MgCl_2$	151.0
" hydroxide	$Mg+2O+2H=Mg(OH)_2$	217.5
" oxide	$Mg+O=MgO$	143.3
Nickel chloride	$Ni+2Cl=NiCl_2$	74.5
" hydroxide	$Ni+O+H_2O=Ni(OH)_2$	60.8
" oxide	$Ni+O=NiO$	57.9
" sulphate	$Ni+S+4O+7H_2O=NiSO_4\cdot 7H_2O$	233.6
Nitric oxide	$N+O=NO$	−21.6
Nitsic acid	$N+H+3O=HNO_3$ liquid	41.5
	$N+H+3O=HNO_3$ dissolved	49.1

* 1 Kg-cal. = 1000 gram-cal. = 3.968 B.t.u. = 4183 watt-seconds (joules) = 1.162×10^{-3} kw-hr.

TABLE XII. — HEATS OF FORMATION OF IMPORTANT COMPOUNDS — *Continued*

(*At room temperature unless otherwise stated.*)

Product	Equation	Kilogram-Calories* evolved per mol of product
Nitrogen dioxide	$N+2O=NO_2$ at 22° C.	−1.7
" "	$N + 2O = NO_2$ at 200°	− 7.9
Ozone	$1\frac{1}{2} O_2=O_3$	−34.1
Potassium chlorate	$K+Cl+3O=KClO_3$	95.8– 93.8
" chloride	$K+Cl=KCl$	105.6
" hydroxide	$K_2O+H_2O+Aq.=2\,KOH$ (Aq.)	67.4
" "	$K+O+H=KOH$	103.2 to 104.6
" hypochlorite	$K+Cl+O+Aq.=KClO$ (Aq.)	89.4 to 88.0
" oxide	$2\,K+O=K_2O$	97.1
" perchlorate	$K+Cl+4O=KClO_4$	113.5
Silver nitrate	$Ag+N+3O=AgNO_3$	28.7
" oxide	$2\,Ag+O=Ag_2O$	5.9– 7.0
Sodium chlorate	$Na+Cl+3O=NaClO_3$	86.7 to 84.8
" chloride	$Na+Cl=NaCl$	97.8
" hydroxide	$Na_2O+H_2O+Aq.=2\,NaOH$ (Aq.)	63.9 to 56.5
" "	$Na+O+H=NaOH$	101.9 to 102.7
" hypochlorite	$Na+Cl+O+Aq.=NaClO$ (Aq.)	83.4 to 84.7
" oxide	$2\,Na+O=Na_2O$	100.3 to 91.0
" perchlorate	$Na+Cl+4\,O=NaClO_4$	100.3
Sulphuric acid	$S+4\,O+2\,H=H_2SO_4$ liquid	192.9
Sulphurous oxide	S solid$+2O=SO_2$ (gas)	71.1
" "	S solid$+2\,O=SO_2$ (dissolved)	78.8
Tin (stannous) chloride	$Sn+2\,Cl=SnCl_2$	80.8
" " oxide	$Sn+O=SnO$	67.6 to 66.2
Water	2 H gas+O gas$=H_2O$ at 18° C.	68.36
"	2 H gas+O gas$=H_2O$ at 0° C.	68.25 to 69.0
Zinc chloride and H	$Zn+2\,HCl(aq.)=ZnCl_2(aq.)+H_2$	34.2
" oxide	$Zn+O=ZnO$	85.0 to 85.4
" sulphate	$Zn+S+4\,O=ZnSO_4$	231.1 to 229.6

* 1 Kg-cal. = 1000 gram-cal. = 3.968 B.t.u. = 4183 watt-seconds (joules) = 1.162×10^{-3} kw.-hr.

TABLE XIII. — HEATS OF COMBUSTION (COMPLETE) OF SOME IMPORTANT INORGANIC SUBSTANCES*

(*At temperature 18° C. and constant pressure.*)

Name	Formula	Heat of combustion in kilogram-calories per mol
Methane	CH_4	213.5
Acetylene	C_2H_2	315.7
Methyl alcohol	CH_3OH	170.7
Ethyl alcohol	C_2H_5OH	326.1
Nitroglycerine	$C_3H_5N_3O_9$	361.2
Acetone	$CH_3-CO-CH_3$	427.3

* See also article on *Fuels.*

TABLE XIV. — HEATS OF SOLUTION

(*Room temperature; anhydrous salts unless otherwise noted.*)

Compound	Formula	Mols of water per mol of compound dissolved	Kilogram-calories evolved per mol of compound dissolved
Copper chloride	$CuCl_2$	600	11.1
" sulphate	$CuSO_4$	400	15.8
" " (crystals)	$CuSO_4 \cdot 5\,H_2O$	400	−2.75
Ferrous chloride	$FeCl_2$	350	17.9
" sulphate (crystals)	$FeSO_4 \cdot 7\,H_2O$	400	−4.5
Magnesium chloride	$MgCl_2$	800	35.9
Nickel chloride	$NiCl_2$	400	19.2
" " (crystals)	$NiCl_2 \cdot 6\,H_2O$	400	−1.16
" sulphate (crystals)	$NiSO_4 \cdot 7\,H_2O$	800	−4.25
Potassium chloride	KCl	200	−4.4
" chlorate	$KClO_3$	400	−10.02
" perchlorate	$KClO_4$	200–400	−12.1
Sodium chloride	$NaCl$	100	−1.03
" chlorate	$NaClO_3$	180–360	−5.6
" perchlorate	$NaClO_4$	200–400	−3.5
Silver nitrate	$AgNO_3$	200	−5.44
Zinc chloride	$ZnCl_2$	300	15.6
" sulphate	$ZnSO_4$	400	18.4
" " (crystals)	$ZnSO_4 \cdot 7\,H_2O$	400	−4.26

BIBLIOGRAPHY. — See references in text.

HEATING OF BUILDINGS. — (*See also Railways, Energy Requirements for.*) The problem of determining the amount of heat necessary to maintain a room at a given temperature above the out-door temperature resolves itself into the determination of the amount of heat transmitted through the walls, ceiling and floor; and the amount of heat necessary to raise the incoming ventilating air to the temperature of the air in the room.

HEAT REQUIRED TO WARM INCOMING AIR.— When the room is occupied by a number of people, the quantity of ventilating air required is often calculated on the basis of 1800 cu. ft. per person per hour. If this value is greater than that which leaks in through the cracks at doors, windows, etc., forced ventilation should be employed. $\frac{1}{56}$ B.t.u. is required to heat 1 cu.ft. of air 1° F.

Let

t = temperature of incoming air, ° F.,
T = temperature of room, ° F.,
Q = cubic feet of air per hour leaving room at room temperature,

Then the B.t.u. per hour required to heat this air to room temperature, which is also the amount of heat carried off by this air, is

$$H_v = \frac{Q(T - t)}{56} \tag{1}$$

Infiltration of Air. — The heat required to warm the air which leaks in through cracks at windows and doors may be determined either by finding the volume of air Q which enters in this manner and applying equation (1), or the heat required may be expressed directly in terms of the size and length of cracks. Following the first method Carpenter proceeds as follows: Let

C = cubical contents of room in cubic feet,
N = number of air changes per hour,

then

$$Q = NC \tag{2}$$

where, for well constructed buildings, N has the values given in the following table.

Kind of room	N
Halls	3
Rooms on first floor	2
Rooms on second floor	1
Offices and stores, first floor	2 to 3
Offices and stores, second floor	1.5 to 2
Churches and public assembly rooms	0.75 to 2
Large rooms with small exposure	0.5 to 1

By far the greater portion of the infiltration is due to loose-fitting sash. The actual sash clearance determines the infiltration loss in any given case. In the general run of well constructed buildings this clearance should not exceed $\frac{1}{32}''$ while for poor construction the clearance may be as much as $\frac{1}{16}''$. The heat loss by infiltration may be approximately determined by the use of the following values:

Good construction, 1/32″ sash clearance . 1.2 B.t.u. per hr. per ft. of crack per ° F.
Poor construction, 1/16″ sash clearance . 2.4 B.t.u. per hr. per ft. of crack per ° F.
Weather-stripped sash. . . . . 0.15 B.t.u. per hr. per ft. of crack per ° F.

Infiltration as calculated by this method is determined by the total lineal feet of sash and door clearance existing in the one outside wall having the greatest amount of glass and door surface, and not upon the total number of feet of crack or clearance in all of the outside walls.

HEAT TRANSMITTED DIRECTLY THROUGH WALLS, CEILING AND FLOOR. — This heat may be conveniently referred to as the heat lost by "radiation," although it is of course conducted through the walls or other surfaces and then carried off both by radiation and convection. Let

S = radiating surface, in square feet,
t = temperature, ° F., of outside air,
T = temperature, ° F., of inside air,
K = B.t.u. lost per hour per square foot of radiating surface (wall, window, ceiling or floor) per ° F. difference in temperature.

Then the heat lost due to radiation is

$$H_r = \Sigma\, KS\,(T - t),$$

where the summation includes all the radiating surfaces. Certain additions are usually made to the value of H_r as thus calculated to allow for the nature of the exposure.

A rough approximation to the total heat lost by radiation may be made by taking $K = \frac{1}{4}$ for all wall surface, $K = 1$ for all window surfaces and neglecting floor and ceiling (Carpenter's rule).

A more accurate method is to use the proper value of K for each kind of wall and type of window. Probably the most accurate values of K are those given by Harding and Willard, who take into account not only the nature of the material of which the wall is made, but also the nature of both the inside and outside surface of the wall and the wind velocity of the outside air at the wall. For a wall of uniform structure and having a thickness of x inches, Harding and Willard give for K the value

$$K = \frac{1}{\dfrac{1}{K_1} + \dfrac{1}{K_2} + \dfrac{x}{C}}, \tag{3}$$

where the constants K_1, K_2, and C $\left(\text{or } \dfrac{x}{C}\right)$ are given in the following tables.

The value of $\dfrac{x}{C}$ for thin metal plates, building paper or glass is so small that it may be safely neglected in the calculations. If the wall is composed of several layers of different materials in contact with one another (no air spaces) then

$$K = \frac{1}{\dfrac{1}{K_1} + \dfrac{1}{K_2} + \left(\dfrac{x_1}{C_1} + \dfrac{x_2}{C_2} + \dfrac{x_3}{C_3} + \text{etc.}\right)}, \tag{4}$$

in which x_1, x_2, x_3, etc., are the thickness of the various materials in inches, C_1, C_2, C_3, etc., are the corresponding values of C.

VALUES OF K_1 FOR STILL AIR

Brickwork	1.35 to 1.40	Glass Window	1.50
Concrete	1.30	Sheet Asbestos	1.40
Corkboard	1.25	Magnesia Board	1.45
Cement Plaster Finish	0.93	Wood (finished surface)	1.40

The average value of K_1 from above data is 1.34.

The value of K_2 increases with the velocity of air over the surface. The value of K_2, for brickwork and wood, for various velocities of air or wind movement may be obtained by multiplying the values of K_1 from the above table by the following factors:

Velocity, miles per hour	Multipliers of K_1	
	Brickwork	Wood
5	2.38	2.19
10	3.20	2.71
15	3.76	2.95
20	4.22	3.02

In practice the exposed walls of a building are not subjected to an average wind movement of more than 15 miles per hour in extremely cold weather.

VALUES OF C AND $\frac{x}{C}$

Material	Condition	Weight pounds per sq. in.	C	$\frac{x}{C}$	Authority *
Asbestos board, corrugated	Dry	20.4	0.48	....	1
Asbestos sheet	Dry	48.3	0.29	....	1
Brickwork	Dry	132	4.00	....	1
Brickwork	Moist	......	5.00	....	1
Brickwork	......	......	3.42	....	3
Concrete, cinder, 1 : 2 : 4	......	......	2.35	....	2
Concrete, stone, 1 : 2 : 4	Dry	140	8.30	....	1
Concrete, stone, 1 : 2 : 4	......	......	6.25	....	2
Cork, granulated	Packed	6.25	0.35	....	..
Cork, board	Dry	9.7	0.32	....	1
Glass 0.085 in. thick	......	......	....	24.3	1
Hollow tile, 2-inch	Note (*a*)	......	....	0.99	1
Hollow tile, 4-inch	Note (*b*)	......	....	0.61	1
Hollow tile, 6-inch	Note (*a*)	......	....	0.47	1
Magnesia board	Dry	13.5	0.51	....	1
Mineral wool	Packed	16.3	0.35	....	..
Mortar	......	......	8.00	....	..
Sandstone	......	......	9.00	....	3
Window, 76.3 per cent glass	......	......	....	8.64	1
Window, double	½-in. air	......	....	1.04	1
Wood (fir)	Dry	33.4	1.00	....	1

* Authorities: (1) Harding and Willard; (2) Norton; (3) Poensgen.

NOTES —(*a*) Plastered both sides; (*b*) Plastered both sides with ready prepared gravel roofing applied to one side only.

Temperature of Inside Air at Ceiling. — It is recommended that an increase of approximately 15 per cent be made to the specified inside temperature for the temperature at the ceiling for ceiling or wall heights not exceeding 15 feet, and 30 per cent for ceiling heights of 20 feet or more in estimating the heat loss of roofs. Thus, if 65° is the specified inside temperature to be maintained in a room the height of which is 20 feet, the temperature of the air in contact with the under side of the roof may be assumed as 65° + 30 per cent or 85°

Exposure Factor. — To compensate for other than southern exposures, the following more or less arbitrary factors, by which the calaulated heat transmission loss of the walls is to be multiplied, are quire commonly used:

North, northeast and northwest exposure where winds are to be counted upon as an important factor.......	= 1.15 to 1.30
East or west walls moderately exposed................	= 1.10 to 1.20

TOTAL HEAT REQUIRED. — If there are no sources of heat already in the room the total number of B.t.u. which must be supplied per hour by the heating device installed is

$$H = H_v + H_r,$$

where H_v and H_r have the values given above, H_r including the allowances for exposure, etc. If there are a number of people in the room, or a number of lamps, or electric machinery (as in a power house or substation) the additional B.t.u. required in excess of the heat given off by the people or apparatus is

$$H = H_m,$$

where H_m is the B.t.u. given out by the people and apparatus, and is calculated as follows:

HEAT SUPPLIED BY PERSONS, LIGHTS, MOTORS, MACHINERY, ETC. — A man at rest gives off about 400 B.t.u. per hour., and at work about 500 B.t.u. per hour. This quantity of heat is not of sufficient importance to be taken into account except in assembly halls and theaters.

Lamps. — The heat supplied by the electric lamps in a room is equal to

$$3.415 \times (\text{watts per lamp}) \times (\text{number of lamps})$$

The heat supplied by gas lamps may be calculated from the following data:

1 cu. ft. producer gas............................	150 B.t.u.
1 cu. ft. illuminating gas.........................	700 B.t.u.
1 cu. ft. natural gas.............................	1000 B.t.u.

A Welsbach burner averages 3 cu. ft. of gas per hour and a fish tail burner 5 cu. ft per hour.

Motors and Machinery. — Motors and the machinery which they drive, if both are located in the room, convert all of the electrical energy supplied into heat, which is retained in the room if the product being manufactured is not removed until temperature is the same as the room temperature. In this case

$$\text{B.t.u. supplied per hour} = 2550 \left(\frac{\text{H.P. Output to Motor}}{\text{Efficiency of Motor}}\right).$$

If power is transmitted to the machinery from the outside, then only the heat equivalent of the brake horsepower supplied is used, in which case

$$\text{B.t.u. supplied per hour} = 2550 \times (\text{Brake Horse-power}).$$

In high-powered mills the heat given out by the machinery is the chief source of heating and is frequently sufficient to overheat the building even in zero weather, thus requiring cooling by ventilation the year round.

In a transformer substation, or other room from which electric energy is transmitted, the heat supplied by the electric apparatus is of course only the losses in the apparatus. Note that a loss of 1 kw. is equivalent to 3415 B.t.u. per hour.

HEATING APPARATUS. — For dwellings, shops, etc., hot-air, hot-water or steam-heating systems are usually employed; for large buildings the last two only. In substations or power houses (e.g., a hydro-electric station) where the heat losses are not sufficient to provide enough heat, either steam or electric heaters are used.

Harding and Willard give the following values of K (= B.t.u. radiated per square foot per hour per ° F. difference in temperature between steam and room air) for direct steam radiators. These values of K were determined from tests on such radiators standing exposed in air at 70° with steam at 220° with a standard temperature difference of 150°.

VALUES OF K FOR DIRECT RADIATORS

Type Radiator	Height of Radiators			
	20 and 22 ins.	26 ins.	32 ins.	38 ins.
1 column	1.95	1.90	1.85	1.80
2 column	1.80	1.75	1.70	1.65
3 column	1.70	1.65	1.60	1.55
4 column	1.60	1.55	1.50	1.45
Flue, 42 square feet	....	....	....	....
Window	1.85	....	....	....
Pipe coils	2.00	....	....	1.57*
Wall (horizontal)	1.95	....	....	....
Wall (vertical)	1.90	....	....	....

* Air entering flues at 70 and leaving same at 152°. (Allen.)

K increases (1) as height of radiator is reduced and (2) as number of columns or width of radiator decreases. The variation in the value of K is approximately 0.2 per cent per degree F. above or below the standard range of 150° F. Thus if a 3-column, 38-in. high, direct radiator is to be used in a room kept at 60° with steam at 230° we would have a temperature difference of 170°, or 20° above standard, and the value of K would become $(1.55 + 0.002 \times 20 \times 1.55) = 1.61$, and each square foot of radiation would give off $1.61 \times 170 = 274$ B.t.u. per hour.

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HEATING AND COOKING BY ELECTRICITY. — (*See also Heat and Thermal Properties; Insulating Materials; Resistance and Conductance; Wires, Resistance.*) Electricity may be used to generate heat for any heating or cooking operation that can be performed with any other source of heat. The readiness with which the heat can be applied to exactly the place needed, and in exactly the quantities needed, usually gives results so far superior to those obtained with other heating agents, that the increase in the cost of the heat is often overbalanced by the increased benefits obtained in other directions.

DESIGN OF HEATING UNIT. — Let R be the resistance of the unit in ohms, and I the current, in amperes. Then the rate at which heat is developed in the unit is

$$P = 3.413\,R\,I^2 \qquad \text{B.t.u. per hour.} \qquad (1)$$

Let ρ = resistivity of the wire of which the device is made, in ohms per mil-foot, l = length of wire in feet, and s = cross-section of wire in circular mils. Then

$$R = \rho\frac{l}{s}. \qquad (2)$$

Values of ρ for some of the metals commonly used in heating units are given in Table I (*see also article on Wires, Resistance*).

Temperature Elevation for Given Power Input. — When a given heating device reaches a constant temperature, the rate at which heat is carried off

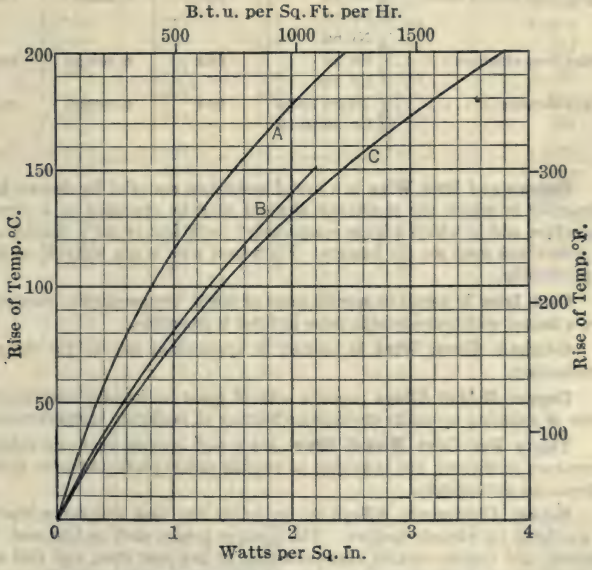

Fig. 1. Total Radiation and Convection in Still Air at 21° C.
A = Dull Nickel: B = Rough Black Body; C = 2.3 In. W.I. Pipe.

from its surface by radiation and convection (assuming the heat lost by thermal conduction is negligible) is equal to the rate at which heat is developed in the

device. The rate at which heat is lost by radiation and convection both depend upon the extent and the nature of the surface and upon the difference in temperature between this surface and the surrounding air; see Fig. 1 and also the article on *Heat and Thermal Properties.*

Resistance Materials. — Metallic resistance materials are usually used either in the form of rod, wire or ribbon, although cast shapes may be employed. The material chosen depends upon the temperature, the operating conditions, space available and first cost. The following table contains some of the principal materials used

TABLE I.—DATA ON RESISTANCE MATERIALS

Material	Chemical composition (approx.) per cent	ρ Spec. Res. ohms per cir-mil-ft.	Temp. coefficient per deg. F.	Max. Temp. deg. F.
Pure nickel	....	69	0.0039	....
Iron wire	....	69	0.00625	....
30 per cent German silver	....	280	...	....
Copper-nickel	Ni 60 to 70 Cu 40 to 30	280	0.000004	1000
30 per cent nickel steel	Ni 70 Fe 30	500	0.0007	1000
Nickel-iron-chrome	Ni 60 Fe 28 Cr 12	650	0.00020	1600
Nickel-chrome	Ni 80 to 84 Cr 20 to 16	600	0.00006	2000

Galvanized Iron Wire is a useful resistance material for electric heater applications in which the resistance element may be operated at a very low temperature and in which a large temperature coefficient is not a disadvantage. It is sometimes used for air heaters. Plain iron wire is not suitable as it will rust too readily.

Cast Iron is useful in special cases of lower temperatures, particularly where a heater with considerable mass in itself is desirable.

German Silver Wire is limited in application due to the danger of crystallization.

Copper-Nickel Alloys produce wire of great uniformity and flexibility, capable of standing up under continuous heating at moderate temperatures.

Thirty Per Cent Nickel Steel has a high specific heat and fairly low temperature coefficient, but is subject to rusting unless protected from moisture and from the atmosphere.

Nickel Chromium Alloys are probably the best resistance materials now available for electric heaters. The cheaper grades such as Chromel "C," Nichrome, and Calido contain approximately 25 per cent iron, and this lowers their resistance to oxidation as compared to the better grades, such as Kromore, Chromel "A," and Rayo, containing practically no other materials than nickel and chromium.

Nickel-Chromium alloys are covered with a coat of oxide that protects the material from further oxidation. This oxide does not scale off on repeated

TABLE II.—NICKEL-CHROMIUM ALLOYS

B. & S. gauge	Diam. inches	Area, circular mils	Feet per pound	Resistance (75° F.) ohms per ft.	
				Nickel-iron chromium alloy	Nickel-chromium alloy
4	0.204	41,616	8.6	0.0157	0.0151
5	0.182	33,124	10.8	0.0198	0.0190
6	0.162	26,244	13.6	0.0250	0.0240
7	0.144	20,736	17.2	0.0316	0.0304
8	0.128	16,384	21.8	0.0400	0.0384
9	0.114	12,996	27.4	0.0504	0.0485
10	0.102	10,404	34.2	0.0630	0.0605
11	0.091	8,281	43.0	0.0791	0.0760
12	0.081	6,561	54.3	0.1000	0.0961
13	0.072	5,184	68.7	0.1263	0.1215
14	0.064	4,096	87.0	0.160	0.154
15	0.057	3,249	110	0.202	0.194
16	0.051	2,601	137	0.252	0.242
17	0.045	2,025	176	0.324	0.312
18	0.040	1,600	222	0.409	0.394
19	0.036	1,296	275	0.505	0.486
20	0.032	1,024	348	0.640	0.605
21	0.0285	812	439	0.800	0.765
22	0.0253	640	557	1.01	0.970
23	0.0226	510	699	1.27	1.215
24	0.0201	404	884	1.61	1.53
25	0.0179	320	1,110	2.03	1.90
26	0.0159	252	1,410	2.57	2.41
27	0.0142	201	1,770	3.78	3.03
28	0.0126	158	2,250	4.03	3.85
29	0.0113	127	2,790	5.02	4.77
30	0.0100	100	3,560	6.40	6.10
31	0.0089	79	4,500	8.09	7.70
32	0.0080	64	5,570	9.98	9.52
33	0.0071	50	7,070	12.7	12.06
34	0.0063	39	8,990	16.1	15.4
35	0.0056	31	11,380	20.4	19.5
36	0.0050	25	14,250	25.6	24.4
37	0.0045	20	17,600	31.6	30.1
38	0.0040	16	22,300	40.0	38.2
39	0.0035	12	29,100	52.2	49.8
40	0.0030	9	39,600	71.2	67.8

heating and cooling. The resistance of various grades and sizes is shown in Table II.

Insulating Materials. — Insulating material must be chosen with a view to the use to which it is to be put.

Mica is one of the best electrical insulators for heating purposes. Care must be exercised, however, in elements in which the resistor is to exceed 500° C., because of the effect of the heat in the mica. Mica, being a crystalline material, will dehydrate if heated to the proper temperature. Some grades of mica begin to dehydrate near 500° C., while others will stand up to 1000° C. During dehydration, the electrical resistance of mica is reduced to such a low value that it becomes a good conductor. Furthermore, the dehydrated mica loses its mechanical strength. During the process of dehydration water vapor is given off which will attack the resistance material.

Mica is used in the form of solid plates of natural mica, and also in the form of built-up plates, using small pieces of natural mica and binding them together with a suitable bond. Since natural mica can be obtained in quantities only in small sizes, any support or insulator larger than a few inches in either direction must be made of built-up mica.

The bonds for built-up mica are of two classes, volatile and non-volatile. Volatile bonds require heating to drive them out before the element is ready for use. Non-volatile bonds do not bake out and are to be preferred because they do not require this operation, and also because the small pieces of mica are held together more securely. The bond should be of such material as not to attack the resistance material at the working temperature.

Porcelain possesses high electrical resistance, but only certain grades are suitable for high temperature uses, because of the danger of cracking while undergoing the wide variations in temperature encountered. The insulator must be designed with care to avoid such cracking, even with the best of materials.

"Lava" is an excellent material but is much more expensive than porcelain.

Asbestos is quite hygroscopic, which interferes with its use in heating appliances, except in special cases. It is also incapable of withstanding high temperatures.

Asbestos Lumber and Other Moulded Compositions containing asbestos and other fibers have various special uses, but none are suitable for high temperatures.

HOUSEHOLD APPLIANCES. — The following is a partial list of the usual household appliances and the energy required for each.

3-lb. flat-iron	250 to 300	watts
6-lb. flat-iron	550	"
9-lb. flat-iron	635	"
Toaster	450 to 550	"
Table stoves	500 to 600	"
Table grills	600	"
Percolators	350 to 400	"
Waffle irons	600	"
Water cup	300 to 500	"
Reflector radiator	600	"
Warming pad	25 to 50	"
Hair curler	15 to 20	"
Immersion water heater	300 to 500	"
Ranges	5000 to 8000	"

Electric Flat-iron. — The heating element is usually of flat nickel-iron-chromium ribbon approximately 1/16 inch wide by 0.005 inch thick. The sup-

port for this ribbon is a flat plate of mica which is similar in shape to the bottem of the iron. The insulating plates are of mica also and are slightly larger than the ribbon form or support. The resistance ribbon is wound so as to distribute the heat over the ironing surface as desired. The usual ground test for this class of devices is 900 volts A.C. cold and 500 volts A.C. at working temperature. In elements of this type of construction the heat density of the ribbon is approximately 30 watts per square inch of total surface of the ribbon. The density of the heat distributed over the bottom area of the iron averages approximately 27.5 watts per square inch.

The usual working temperature of the bottom surface of a flat-iron is about 225° C. (437° F.) while the temperature of the element is about 350° C. (662° F.). If the iron is not used and the current left turned on, the temperature will rise until the heat radiated equals the input. This will occur when the bottom surface is near 600° C. (1112° F.), at which time the element will be near 700° C. (1292° F.).

Toasters. — Toasters of the vertical type usually have vertical heaters, similar in principle of construction to flat-iron heaters, except that the wire is exposed to radiate freely upon the bread.

Household Ranges. — Electric ranges are made in a variety of shapes and sizes to suit the various needs of the users. A range consists of one or more ovens and one or more open hot plates or burners. The type most generally used is the "Cabinet" type, having the oven on the same level as the top burners, and located at either end of the range. Other types have the oven beneath the open burners, and others the oven elevated above them corresponding to similar types of gas ranges.

Ovens are insulated with an insulating material such as mineral wool or air cell asbestos to prevent heat loss. They are equipped with one or two heaters according to the design. The maximum input per cubic inch varies from 0.40 to 0.70 watt per cubic inch for the ordinary sizes. Oven heaters are either of the open coil, enclosed coil, or sheathed wire construction.

Top burners for open cooking are from 6 to 10 inches in diameter, averaging about 8 inches. These burners operate at from 20 to 30 watts per square inch of upper surface. They are of various types, such as open coils supported in grooves in a porcelain disc, coils enclosed in a circular casting and insulated with mica or porcelain, or cast-in type of enclosed element.

The total capacity of the usual range is from 5000 to 8000 watts depending on the size of the oven and number of open burners.

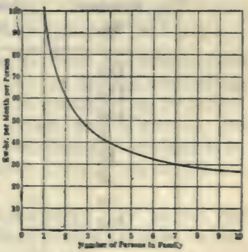

Fig. 2.

The Cost of Cooking depends on the number of persons in the family, the kind of meals served, the skill of the user, and the rate for current. The curve in Fig. 2 gives an idea of the power requirements for electric ranges under average conditions. This does not include the power required for lights, extra appliances, water heating, etc.

INDUSTRIAL APPLIANCES. — Electrically heated appliances have been developed for a very large number of uses. The following list contains only a few of the more common devices.

Tailor's Pressing Irons	Glue Pots
Solder Pots	Sealing Wax Heaters

Soldering Irons
Branding Irons
Hatter's Irons
Velouring Stoves
Hot Tables
Press Blocks
Chocolate Warmers

Usually the cost of the energy to operate such devices is immaterial in view of the great advantage gained in speed, convenience, safety, and uniformity of operation.

INDUSTRIAL APPLICATION UNITS. — These are small heater units, complete in themselves with supports and terminals which have been designed to meet a wide variety of applications of heat to machines and appliances in which heated surfaces or spaces are essential to their successful operation. Usually the application involves only minor changes in the construction of the machine to accommodate the electric units. Terminals are provided for readily connecting up the units.

Such units are usually provided in standard voltages of 110 and 220 volts, although the larger units for ovens are suitable for operation in series up to 550 volts. Some of the types and applications of these units are described in the following paragraphs.

Cartridge Heaters. — A tubular unit having terminals at one or both ends and designed to be inserted in holes drilled in castings which require heating, such as embossing presses, hot moulding presses, shoe machinery irons, hat machine irons, etc., is called a cartridge heater. Nickel chrome resistance material is used for the heater and is wound on a central core of porcelain or lava and insulated from the outer tube of brass by cement or mica. Cartridge heaters may be used in applications up to 300° C.

G. E. CARTRIDGE TYPE UNITS

Symbol No.	Length in inches	Diameter in inches	Watts
142	2.39	0.498	100
151	2.51	0.748	200
102	3.9	0.933	250
9	4.87	0.933	70
11	4.87	0.933	140
48	4.87	0.933	440
63	5.	1.291	300
103	5.	1.291	600
104	5.	1.291	900
162	8.5	1.291	1000

Steel-clad Heaters. — This is a flat type of element consisting of a mica form on which a resistance element is wound, insulated with flat plates of mica, and enclosed in a casing of sheet steel pressed flat on the element. Terminals are usually located at one end .

This unit is made in a wide variety of sizes and is suitable for hot-press heating, hot tables, heating rolls, small ovens, etc., in which the uniform distribution of heat is desired. A few of the standard sizes available are given listed in the following table.

WESTINGHOUSE STEEL-CLAD HEATERS

Length of heater, inches	Width, inches	Thickness, inches	Watts	Volts
10	2 1/4	3/16	350	55
12	2 1/4	3/16	500	110
15	2 1/4	3/16	650	110
21	2 1/4	3/16	1000	110
12	1 1/8	1/8	175	110
24	1 1/8	1/8	350	110

Space Heaters. — These are similar to the steel-clad heaters, except that one terminal is located at each end. The 24-inch length is a standard size and is suitable for a wide variety of applications, of which the following is a partial list. The unit is 1 1/2 inches wide, 3/16 inch thick, and is supplied in two designs, one for 110 volts and the other for 220 volts, the rating being 500 watts in either case.

Small Industrial Ovens	Watchmens' Houses
Blue-print Driers	Elevator Cabs
Hoist Cabs	Laundry Driers
Ticket Booths	

Industrial Oven Heaters. — These heaters are designed for the larger sizes of industrial ovens. They are of nickel-chromium ribbon wound over porcelain bushings and assembled into a metal frame. Exposed heating ribbon is used to give free radiation into the oven space. The heating is done partly by radiation and partly by convection. Terminals are provided at the ends for connections. Units are usually made up for 110-volt and 220-volt service. Two 220-volt units may be connected in series for 440 volts. Complete sets of suitable connection straps and insulators are provided to accompany the heaters.

INDUSTRIAL ELECTRIC OVENS. — Electric heat has been applied to ovens for japanning, lithographing, paint drying, lacquer dyeing, core baking, armature baking and many other purposes requiring temperatures up to 600° F. The absence of fumes and gases, reduction in fire hazard, ease of control, and increased production have in many cases justified even increased costs of heat. Ovens are of the usual kiln type, the semi-continuous type, or the continuous type.

A Kiln-type Oven is usually rectangular in shape with a door at one end into which the material to be baked is loaded. When the oven has been loaded, the door is closed and current turned on. When the baking has been completed, the current is turned off, door opened and work removed.

A Semi-continuous Oven has doors at both ends and a new load is pushed in at one end while the finished load is removed from the other end.

In a Continuous Oven the work is carried through the oven on a continuous conveyor loading at one end and discharging at the other at a uniform speed.

Heaters are installed along the side walls or on the floor. A space of 8 to 10 inches from the wall or floor should be provided. Over the heaters a screen of expanded metal or wire mesh can be installed to prevent work dropping on the heaters and connections

Control is usually by magnetic switches. Door switches are sometimes provided so that when the doors are opened the main switches also open. A thermostat is usually used to regulate the temperature by turning the current on or off as may be needed. This thermostat operates through a relay to control the main magnetic switches.

110, 220, or 440 volts of any frequency and either single phase, two phase, or three phase can be used. Direct current up to 550 volts can be used also.

Heat Required for Industrial Oven. — In calculating the heat requirements, the following losses and uses of the heat must be carefully determined.

Heat to Raise the Temperature of the Work, Including Trucks. — The total weight of the material to be baked, the initial and final temperatures, and the specific heat must be known. Dividing the heat required by the time gives the heat per hour needed. The japan or paint can be ignored. The heat required to heat the trucks or hangers is calculated in the same way for each load.

Heat to Raise the Temperature of Oven Walls. — The insulating material in an oven wall absorbs heat in the period of heating up even though it be fairly good heat insulator. The usual method is to average the temperatures of the inner and outer surfaces, and subtract the initial temperature to obtain the average temperature rise. This is multiplied by the weight and specific heat to obtain the total heat required. Dividing this by the time allowed for the heating operation gives the rate at which heat must be supplied.

The interior framework and oven lining of metal are heated on the average to the same temperature as the work in the oven, and can be calculated in the same way.

In the case of a kiln-type oven the interior cools down somewhat during the period when the doors are open for emptying and reloading. For this reason a portion of the heat must be replaced with each charge. This may be from one-quarter to one-half of that required on the first bake.

Radiation Losses from the Oven. — This depends upon the wall and door construction. It is of prime importance that the amount of metal extending through the oven walls from the inner to the outer surfaces be reduced to a minimum. Average radiation loss from a metal oven with two inches of insulation (air-cell asbestos or magnesia) and door at one end only will be approximately 50 watts per square foot of total outside surface of oven, with an inside temperature of 400° F. Exact figures for radiation losses cannot be given, as it is controlled to such a great extent by the design of doors, corners, vents, etc., as well as by the thickness and character of the insulation.

Ventilation. — The amount of ventilation required depends upon the product to be baked. Ovens for japanning, paint drying, core baking and all other operations that give off gases or vapors require ventilation. Where a paint or japan is used that contains a volatile solvent of combustible nature, good ventilation is necessary to reduce the danger of fire or explosion. This danger is much more remote with an electrical oven than with a gas or other fuel-heated oven because of the uniform temperature regulation obtained with an electric oven. Ventilation also removes the gases given off during baking and improves the surface obtained.

For japanning ovens from ten to twenty changes of air per hour are usually allowed. To calculate the heat required the volume of the oven is multiplied by the weight of a cubic foot of air at the oven temperature, times the temperature rise, times the specific heat of air, times the number of changes per hour.

Totals. — Adding together the heats required for each of the above throughout the time allowed will give the total heat. Dividing by the num-

ber of hours gives the rate the heat will be required. If the heat is calculated in B.t.u., divide this rate by 3415 to get the power input in kw.

It is impracticable to give average figures of inputs for various oven sizes and applications. As an example, however, it may be stated that for japanning ovens at 400° F., two-hour baking period, the input would be around 100 watts per cubic foot of volume of the oven.

Bread-baking Ovens. — Electrically heated bread-baking ovens are made in two types, shelf type and reel type.

Shelf-type ovens are made with from one to five or six shelves. The greater the number of shelves, the more bread capacity per square foot of floor space is obtained. Heaters are usually placed in each shelf, although in the smaller sizes only one heater located under the bottom shelf may be used. In the latter case convection currents in the air in the oven are depended on to carry the heat to the shelves above. Ducts are provided at the sides or back to assist the circulation.

Reel-type ovens have a reel carrying six to eight bread trays which rotates inside the oven. The heaters are placed on the bottom.

Bread ovens vary in capacity from 30 to 600 loaves and from 4 kw. to 75 kw.

The average energy consumption will be from 100 to 200 watt-hours per one pound loaf, depending upon the size of oven, kind of bread and skill of operator.

TEMPERING AND ANNEALING FURNACES. — Furnaces for these purposes, capable of temperatures up to 2000° F., are available in various sizes. They are usually constructed of a fireclay lining, either solid or sectional, to which the heaters are mounted. This is surrounded by a wall of heat insulating material. Heaters are usually of the highest grade of nickel-chromium material in the form of heavy wire or rod. In order to utilize a heavy cross-section of resistor in small furnaces a voltage lower than the usual 110 volts is provided by means of a small transformer.

To determine the proper size of furnace it is necessary to calculate the amount of heat required to raise the temperature of the material and also to estimate the heat losses from the furnace. The frequency of opening and closing doors will affect the rate of heating, as the radiation through an open door is very high at such temperatures.

Manufacturers' catalogs should be consulted for more detailed information.

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HOISTS, ELECTRIC. — (*See also Cranes; Elevators; Motors, Industrial Applications of; Telpherage.*) A hoist is a machine for raising and lowering weights. The most common application is in mines, to which service the following article is devoted.

The hoists most generally used are of the drum type, the drums being either cylindrical or conical or a combination of the two. As a rule two drums operating in balance are used for each hoist. The weight of the skip, or cage and car carrying the ore, is balanced by a similar empty skip which is lowered in a second compartment simultaneously with the hoisting of the loaded skip in the first, the loaded skip being dumped at the top and the empty one loaded at the bottom, and the cycle being repeated. The difference between balanced and unbalanced hoisting is that the static load when working unbalanced is greater than when working balanced, and the change of load due to the rope is only half as great.

Preliminary Data. — In dealing with hoisting problems the following data are required:

Present and ultimate depth of shaft.
If inclined, give angle of inclination with horizontal, or grade, in per cent.
Weight of material per trip.
Weight of skip or cage and car.
Diameter and weight of rope.
Size of tail rope, if used.
Rope speed.
Number of trips per hour.
Time required for loading and unloading.
Double or single drum.
Diameter and weight of drums.
Motor geared or direct connected.
Hoist balanced or unbalanced.
If balanced, will unbalanced operation ever be necessary?

Besides the above, the ordinary data of the power supply system must be known; also the restrictions as to permissible peak loads, so as to make it possible to decide whether a load equalizing set will be required.

For cylindrical hoists the power required for the various periods of the duty cycle can be figured directly in horse-power. For conical drums the problem is somewhat more complicated and a moment diagram is usually first figured and plotted, and then converted into horse-power.

SYSTEMS OF ELECTRIC HOISTING. — Of the large number of systems of electric hoisting which have been proposed, by far the greater majority can be included in the three following systems:

1. Those driven by induction motors.
2. Those driven by direct-current motors, power for which is supplied by a motor-generator set.
3. Those driven by direct-current motors, power for which is supplied by a flywheel motor-generator set.

1. Induction-motor System. — In the first and simplest system the speed of the direct-connected induction motor is controlled by a variable resistance in the rotor circuit. The advantages of this system are: simplicity, low first cost, high motor efficiency when the hoist is running at full speed (approximately 90 per cent), no power consumed when the hoist is at rest. The disadvantages are: low motor efficiency during acceleration (approximately 45 per cent), no power is returned to supply system during retardation,

large power consumption for small movements of cage, fluctuation of power demand.

2. Direct-current Motor System. — In the second system the hoist is driven by a separately excited direct-current motor, receiving power from the alternating-current supply system through a synchronous or induction motor-generator set. The hoist motor is controlled by varying the voltage of the generator, which is separately excited, one generator being used for each motor. The advantages of the system are: high motor efficiency during acceleration, return of large portion of energy during retardation. The disadvantages are: low combined efficiency of the equipment when running at full speed (probably not better than 82 per cent), loss in motor-generator set when hoist is at rest, high initial rotor expense, fluctuation of power demand.

3. Direct-current System with Flywheel. — The thirds ystem takes advantage of the low first cost and efficiency of the flywheel (*see Flywheels for Load Equalization*) as a means for storing and returning large quantities of power for short intervals. This system is similar to the second, except for the addition of a flywheel to the induction-motor-generator set, and an automatic regulator for varying its speed. In its most common form this regulator consists of a water rheostat connected in series with the induction-motor rotor. The resistance is varied by means of movable electrodes suspended from an arm mounted on the shaft of a small induction motor, which is connected in series either directly or through series transformers, with the induction motor of the flywheel set. The regulator motor is so connected that its torque opposes the weight of the electrodes, which are partially counterbalanced to reduce the size of the *regulator* motor to a minimum, and to permit of an adjustment of the regulator for different values of line current. When the line current exceeds the value for which the regulator is adjusted, the torque of the motor overbalances the weight of the electrodes, lifting them and inserting resistance in the armature circuit of the induction motor. This causes it to slow down, and allows the flywheel to assist in driving the generator during the peak loads.

Steam vs. Electric Systems. — A comparison between a steam system and these three electrical systems is given in the following table, in which the fuel and ore ratios for each are given for a small installation hoisting from a 2000-foot level, and a large installation hoisting from a 6000-foot level.

In addition to the saving in fuel which may be realized by the use of electric hoists instead of steam hoists, there is a very material reduction in the labor, the

Installation	Hoisting system	Coal burned, tons per day	Ore hoisted, tons per day	Ratio tons ore to tons coal
Small, hoisting from 2000-foot level.	Steam	47	1780	40
	Electric, first system	13	1780	137
	Electric, second system	15	1780	119
	Electric, third system	16	1780	110
Large, hoisting from 6000-foot level.	Steam	65.5	1580	24
	Electric, first system	23	1580	69
	Electric, second system	24	1580	66
	Electric, third system	25	1580	63

cost of which is chargeable against the hoist. This may amount to the wages of one or two men in the boiler house if power is developed by the mining company, or of the whole boiler house force if power is purchased, and frequently the wages of one man in the hoist house.

Power Consumption. — So many factors enter into the cost of electric hoisting that each individual case must be analyzed separately. An approximate estimate of the power consumption would be from 1 ¼ to 2 ¼ kilowatt-hours per 1000 ton-feet, the tonnage, in the case of unbalanced hoisting, including the weight of the ore and skip, while for balanced hoisting only the ore.

SPECIFICATIONS FOR ELECTRIC HOIST. — The following memoranda are intended to assist in writing specifications. See also article on *Elevators, Electric* and the article on *Specifications*.

Principal Characteristics and Conditions of Service. — General description and use of hoist. Motor voltage and frequency (*see articles on Motors*). Mechanical rating, i.e., load to be lifted a specified height in a specified time. Proportion of time above load will be carried and of time machine is at rest.

Details of Construction. — Length of hoisting rope and whether or not rope is to supplied. If so, details of rope and of hook or other device. Brake requirements. Lubrication details. Materials of principal parts.

Performance and Tests. — Maximum load to be lifted. Maximum speed descending. How tests are to be conducted.

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HYDRAULICS. — (*See also Dams; Hydrology; Pipes and Piping; Power Stations, Hydro-Electric Turbines, Hydraulic; Turbines, Hydraulic, Speed Regulation of.*) Hydraulics comprises the principles and laws governing the motion and mechanical reactions of liquid bodies. There are two general divisions of the subject: Hydrostatics, which refers to liquids at rest; and Hydrodynamics, which refers to liquids in motion. This article will be confined to that portion of the subject which relates more directly to the problems met with in the study of water-powers.

Head and Pressure. — In water power terminology the word *head* means the difference in level of water between two points. It is usually expressed in feet. The pressure of water in a pipe for instance, is usually given as so many feet head and not as so many pounds per square inch. The relation between head in feet, h, and pressure in pounds per square inch, p, is as follows:

$$h = 2.31\,p,$$

$$p = 0.433\,h.$$

These formulas assume 62.4 lb. as the weight of 1 cu. ft. of water. See also *Units and Conversion Factors.*

FLOW THROUGH PIPES. — See articles on *Pipes and Piping* and *Water Wheels, Speed Regulation of.*

FLOW THROUGH OPEN CHANNELS. — The term channel includes all forms of closed conduits when flowing partly full as well as ordinary stream beds or canals. The rate of flow in such a case depends upon the slope given to the water surface. It is difficult to treat analytically the flow in natural channels, because of the wide divergence in shape of cross-section and nature of sides and bottom of channel. Only artificial channels are considered here, although the same formulas apply to any section of natural channel which has the same cross-section throughout.

Velocity and Friction Head. — The total head required to transport water in an open channel is used in creating velocity and in overcoming friction.

The "velocity head," or head required to impart a given velocity of flow to water, may be expressed by the equation,

$$h_v = \frac{v^2}{2g},$$

where h_v is the head in feet, v is the average linear velocity in the channel in feet per second, and g is the acceleration of gravity, or 32.16. The theoretical head required to increase the velocity from v_1 to v_2 is therefore,

$$h_v = \frac{v_2^2 - v_1^2}{2g}.$$

All changes in velocity should be made gradually to prevent the formation of eddies which result in additional friction loss.

The difference in elevation between the water surface at any two points in a channel of constant cross-section is the "friction head" or head required to overcome friction between those points. The friction head per unit of length depends upon the cross-section, shape and roughness of the channel.

Chezy's Formula. — This formula gives the relation between the linear velocity of the water, hydraulic radius and the slope and a coefficient which must be determined experimentally, viz.,

$$v = C\sqrt{rs},$$

where v = linear velocity of water in feet per second, r = hydraulic radius in feet, s = slope of stream (i.e., the difference in elevation between two points in the water surface divided by the slope distance between the two points measured along the surface) and C is an experimentally determined coefficient which may also be expressed by the empirical formula given below.

Hydraulic Radius. — This term is used to designate the quotient of the area of a cross-section, e.g., $ABCD$ in Fig. 1, by the *wetted* perimeter of the channel walls and bottom, $AB + BC + CD$ in Fig. 1.

Fig. 1.

Fig. 2.

Best Section to Use. — Hydraulically, the most advantageous cross-section to use is that having the maximum value of the hydraulic radius r. The semi-circle is therefore the best but is the most difficult to construct and to maintain. Of trapezoidal cross-sections the half hexagon is the best and of rectangular cross-sections the half square. In unlined earth channels the trapezoidal cross-section must be used. Of these sections the best to use is shown in Fig. 2. The angle α is determined from the character of the soil. The sides and bottom are made tangent to a semi-circle having its center at the water surface.

Practically, however, the area to be adopted must be that which will result in a minimum construction *cost* for the same *slope* in water surface, and this applies to canals, flumes and all other types of channels. In Fig. 1, for example, if the surface of the ground were considerably above the water surface, a minimum of excavation would correspond to a relatively deeper and narrower channel having a slightly larger area to retain the same slope for the given discharge.

Kutter's Formula. — This is the formula in common use for expressing the coefficient C in Chezy's formula in terms of the slope, hydraulic radius and the characteristics of the channel.

$$C = \frac{41.65 + \dfrac{0.00281}{s} + \dfrac{1.811}{n}}{1 + \left(41.65 + \dfrac{0.00281}{s}\right)\dfrac{n}{\sqrt{r}}},$$

where r and s are as above, and n is the "coefficient of roughness" depending upon the characteristics of the sides and bottom of the channel. Approximate values of n are given below. For a more extended list see *Bibliography*.

	Values of n
Best type of metal flumes	0.011
Concrete lining of smoothest kind, and well constructed clean wooden flumes with surfaced lumber	0.012
Clean wooden flumes of unplaned lumber, and average troweled concrete linings	0.015
Average concrete linings, not troweled, having prominent form marks, best conditions of canals in earth and best brickwork	0.016
Very rough concrete linings with deposits of gravel and moss, inferior brickwork, good rubble	0.018
Well constructed canals in firm earth or fine packed gravel free from vegetation	0.020
Worst type of ordinary metal flumes	0.022

	Values of n
Average canals in earth	0.023
Canals in earth with occasional weeds and stones, and smoothest canals in rock cut	0.025
Canals in earth with a rather heavy growth of weeds and large rocks in bottom	0.030
Canals in bad condition and average canals in rock cut	0.035
Canals in rather rough rock cut	0.040

FLOW OVER WEIRS. — In hydraulics the term "weir" is used in general to designate any kind of a dam across a stream, and in particular to designate a vertical notch, usually rectangular, by means of which the quantity of water passing a given point may be determined, Fig. 3.

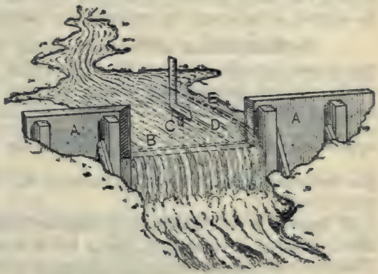

Fig. 3.

Definitions Regarding Weirs. — The "crest" is the horizontal edge of the weir. The "head" on a weir is not the depth of water over the crest, but is the difference in level between the crest and the reservoir surface, measured far enough away from the weir to avoid the surface curve or sharp slope near the crest. A "fully contracted" weir is one in which each end of the weir is at a distance of at least 3 times the head from the sides of the channel. A "suppressed" weir is one in which the sides of the channel (smooth and vertical) form the end of the weir. The "velocity of approach" is the velocity of the water in the channel leading to the weir. For a given head on the weir the quantity of water passing the weir depends somewhat upon the velocity of approach.

Weir Formulas. — The weir formulas of Francis, Bazin, Fletey and Stearns, Smith and others are each based on somewhat different experimental conditions. Therefore, to measure accurately the discharge over sharp-crested weirs, that formula should be used which is the result of experiments on weirs of the same general type and made under similar conditions.

Francis' formulas are given below. They are applicable only to vertical, rectangular weirs having

(1) Sharp, smooth, bevelled edges on the downstream side as shown in Fig. 3 (or a thin metal plate bolted to a wooden board may be used for the edge of the weir).
(2) The crest of the weir at least three times the head above the bottom of the channel.
(3) A length of crest at least three times the head on it.
(4) A head not greater than 2 feet nor less than 0.5 foot.
(5) A velocity of approach not greater than one foot per second.

Let Q = cu. ft. of water per sec. flowing over weir,
H = measured head on weir, in ft.,
h = head in ft., due to velocity of approach,
H_1 = "effective" head on weir, in ft.,
b = breadth of weir (length of crest) in ft.

Then for a *fully contracted* weir

$$Q = 3.33\ (b - 0.2\ H)\ H_1^{3/2}$$

and for a *suppressed* weir

$$Q = 3.33\ bH_1^{3/2},$$

when the cross-section of the stream immediately above the weir is more than 6 bH the effect of the velocity of approach is negligible and H_1 may be taken equal to H. When the cross-section of the stream is less than 6 bH, then

$$H_1^{3/2} = (H + h)^{3/2} = h^{3/2}.$$

For the determination of the discharge over weirs having other than sharp crests, the same general equations may be used; but the coefficient, 3.33 must be altered as indicated by experiments (*see Horton's paper listed in Bibliography*).

Determination of Velocity Head (h). — First take $H_1 = H$, and from the above formula calculate Q. This value of Q divided by the cross-section A of the channel immediately above the weir gives an approximate value for the velocity of approach V. The head due to the velocity of approach is then approximately $h = \dfrac{V^2}{64.32}$. Recompute the discharge Q, calculating $H_1^{3/2}$ from the measured head H and this value of h. This will usually give the discharge very closely.

If desired, a new value of V may be found from this new value of Q and a third value of Q computed, using for h a value based on the new value of V. This method will be recognized as one of successive approximations. Generally one, and never more than two, recomputations are all that are necessary. In using the weir formulas given it is necessary to adhere to this method strictly, as the constants in the formulas were derived by using this method.

OTHER METHODS OF MEASURING FLOW. — The rate of discharge of a channel or natural stream may also be measured by means of floats, a Pitot tube, or a current meter. These devices are briefly described below.

Floats. — When a stream is of reasonably regular cross-section for a little distance, this method is applicable. Sticks about 1¼ inches in diameter, sufficiently weighted at one end so that they will float upright in the water, should be used. Surface floats are unreliable because of the influence of wind, skin friction of the water, etc. The floats are started at the beginning of a measured course and the time which they take to reach the end of the course is accurately taken. The velocity of the stream is thus obtained; this multiplied by the area of the cross-section gives the rate of discharge.

The floats should be started from various points in the width of the stream in order to obtain the average velocity as nearly as possible. They should be long enough to come as close as practicable to the bottom of the stream without touching, in order to average the velocities, which are variable throughout the depth of the stream. This method is less accurate than the use of the current meter.

Current Meters. — The ordinary current meter is essentially a wheel revolved by the water and held with its axis perpendicular to the direction of flow by means of vanes. Some means of recording the revolutions are provided. The meter must be rated by drawing it through still water at various measured speeds. Elaborate apparatus for meter rating is maintained by the Government and by various universities. Measurements of the velocity of the stream are made at frequent intervals across the stream and at varying depths, and the average velocity determined; this multiplied by the cross-sectional area of the stream for the height of water existing at the time gives the rate of discharge.

Pitot Tube. — This is merely an open tube having a right-angle bend, the tube is placed in the stream in the position shown in Fig. 4, the lower end being directed against the current. The height h to which the water rises in the tube above the surface of the stream is equal to the velocity head, or the linear velocity of the stream is

$$v = \sqrt{2\,gh},$$

where h is in ft., $g = 32.16$ ft. per sec. per sec., and v in ft. per sec. This formula does not apply to the flow through a pipe, since the pressure head must also be taken into account. For measuring the flow in pipes a differential gage of this type may be used, but the Venturi meter (see below) is better suited to such measurements.

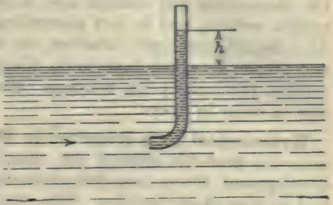

Fig. 4. Pitot Tube

Venturi Meter. — This meter, Fig. 5, serves as a simple and accurate means of measuring the discharge through a pipe. The meter consists essentially of an hour-glass-shaped section of pipe, with smoothly rounded internal walls, into which are fitted two gages as shown. The planes of the internal openings of gage pipes are at right angles to the direction of the current. Referring to Fig. 5, let

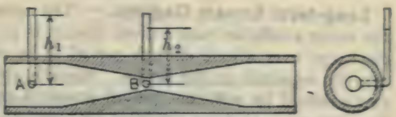

Fig. 5. Venturi Meter

a_1 = area, in sq. ft., of the cross-section at A,
a_2 = area in sq. ft., of the cross-section at B,
h_1 = reading of gage at A, ft. of water column,
h_2 = reading of gage at B, ft. of water column,
g = acceleration due to gravity, ft. per sec. per sec.,
C = a coefficient, ranging from 0.94 to 1.00, which takes into account the friction in the meter.

Then the discharge through the pipe in cu. ft. per sec. is

$$Q = C\frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2\,g\,(h_1 - h_2)}.$$

Ordinary pressure gages may be used at A and B, and their reading converted to feet of head, with a correction added for height of gage above center line of meter.

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HYDROLOGY. — (*See also Hydraulics; Power Stations, Hydro-Electric.*) In the broadest sense, hydrology is the science of water, including its properties, the phenomena and natural laws associated with it, its distribution over the earth's surface, and in fact everything relating to water as a physical entity. As ordinarily employed, however, the term is limited to include only those laws and properties which have to do directly with its distribution over the earth's surface, particularly the relations between rain-fall, natural drainage, and the flow of water in rivers.

Hydrological Data Required in Water-power Engineering. — To determine the probable amount and character of power output of a water-power development, it is necessary to estimate the probable usable run-off of the stream and its flow characteristics. The characteristics of a stream are usually represented by its minimum flow and its variation from day to day and year to year.

ESTIMATING STREAM FLOW. — The probable discharge characteristics of a stream may be estimated in various ways depending upon the data available.

Long-term Stream Gagings. — The most accurate method of estimating the water available from a given stream is to make direct measurements of the discharge over a long enough period to be reasonably representative of what may be expected in the future. Records covering a period of ten to twenty years should be sufficient for all practical purposes, if rainfall records over a much longer period are available for comparison.

Such measurements are made at a suitable place and the elevation of water surface or "gage height" is recorded for a sufficient number of discharges to determine a "rating curve," see Fig. 1. It is then only necessary to make daily observations of the gage height to obtain a daily record of stream flow.

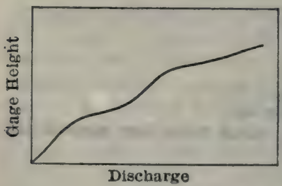

Fig. 1.

The U. S. Government has maintained gaging stations for a number of years on many streams in this country. These records are published in the Annual Reports of the U. S. Geological Survey (1888 to 1900) and in the Water Supply and Irrigation Papers (1896 to date) also issued by the U. S. Geol. Survey.

Short-term Stream Gagings. — Short-term stream gagings, while useful for comparative purposes, are, in themselves, of little value in estimating accurately the probable future discharge. If, however, a near-by stream of similar characteristics or another point on the same stream has been gaged over a considerable period, the relative discharge characteristics can be determined approximately from a comparison of simultaneous average monthly discharges as indicated in Fig. 2. The longer records of the nearby stream can then be used if each average monthly discharge is modified according to the curve.

In the absence of long-term records on a nearby stream, a rough estimate of future discharge can be obtained from a study of long-term rain-fall records, but the result so obtained is not satisfactory and should be used with caution.

No Stream Gagings. — If no gagings on the stream have been made, it becomes necessary to estimate the discharge from discharge records of nearby streams of similar characteristics. Such records, however, must be modified as indicated by a comparison of rain-fall records; or, in the absence of rain-fall records, a comparison of the rain-producing characteristics on the two water-

sheds, and a study of the physical features of those areas which affect the "run-off," or percentage of rain-fall which finds its way to the stream.

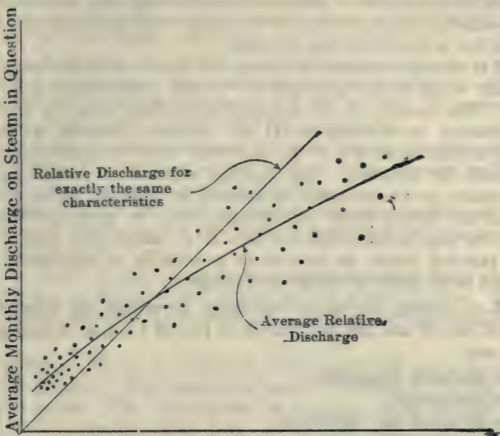

Fig. 2.

In the absence of discharge records on nearby streams, there is little hope of estimating accurately the probable future discharge. Even if long term rain-fall records are available, there is no satisfactory method by which the run-off can be determined even approximately.

RAIN-FALL AND RUN-OFF. — The discharge of a stream bears some relation to rain-fall on the tributary drainage area and depends upon the percentage of rain-fall which ultimately finds its way to the stream. Of the total rain-fall over a given period, a portion is lost by evaporation—directly to the atmosphere or utilized by vegetation. The rest appears as run-off.

As mentioned heretofore, it is necessary, in the absence of stream-flow records, to use the records of nearby streams modified in accordance with the relative rain-fall and run-off characteristics of the two drainage areas. The necessary modification is difficult to determine accurately, and becomes increasingly difficult the farther apart the two streams, particularly if on opposite sides of a large range of hills. A brief discussion of the numerous agencies affecting rain-fall and run-off are given in the following paragraphs.

Rain-fall.* — Rainfall varies materially in different localities, not only in annual precipitation but in its seasonal distribution. The U. S. Weather Bureau has published data and maps indicating the variation in mean annual precipitation throughout this country. The influences most affecting the deviations of local rain-fall from the general mean are the local topographical characteristics, the direction of prevailing winds and the altitude.

Moisture-laden winds, diverted upwards by mountain ranges, expand, cool, and condensation appears in the form of rain or snow. Thus the side of the range subjected to the prevailing winds will have the greater rain-fall.

In general, rain-fall seems to increase with the altitude; but in some cases

* Including snow, ice, etc., i.e., total precipitation.

the effect of the topographical features on the prevailing winds may modify the general law. A knowledge of the conditions affecting local rain-fall are useful in the interpretation of published rain-fall data and particularly in comparing the rain-fall characteristics of two areas where sufficient rain-fall gaging stations are not available for that purpose.

Rain-fall is usually expressed in inches, the number of inches being the depth of rain water which would be caught in a vessel with vertical sides set out in the open. The records of rain-fall may be obtained from the United States Weather Bureau in Washington, D. C. Records covering as extended a period and as wide a portion of the watershed under consideration as is practicable should be obtained. Too absolute dependence should not be placed on these results, as the intensity of rain-fall at different places in the drainage area of the stream under consideration may be quite variable, not only in general, but also in particular storms. It is conceivable that a station for recording rain-fall might be located at a point in an area where the annual precipitation is far above or below the average.

Run-off. — Among the many influences governing the relation between rain-fall and run-off and the distribution of run-off for a given period, the most important are:

The geological formation,
The topography,
The nature of the vegetation,
The extent of lakes and swamps,
The amount of previous ground saturation,
The rate and amount of rainfall,
The mean temperature and wind velocity.

Run-off passes to the stream as surface flow or ground water flow. In rocky or clayey, hilly country the run-off passes quickly to the stream or its tributaries, and a relatively small percentage of the rain-fall disappears in evaporation. Such drainage areas therefore are characterized by a larger percentage of run-off; but a considerable divergence between maximum and minimum flow. The latter characteristic makes them less attractive for power purposes unless large storage reservoirs are used.

Flat or rolling, impervious areas with many lakes or swamps are subject to considerable evaporation but the run-off is more uniform.

The best condition is a flat or rolling country with pervious soil. With such conditions a large part of the rain-fall seeps quickly underground with comparatively little evaporation. Ground water takes months in some cases to reach the stream and affords a supply during seasons of drought.

Evaporation is considerably more constant than run-off. Consequently the percentage run-off is usually much larger for high rain-falls than for low. Evaporation is known to increase with the temperature, although many other meteorological conditions affect it greatly.

The influences controlling run-off are so numerous and inter-connected that it is practically impossible to estimate closely the monthly or seasonal stream flow from rain-fall data; although a rough approximation of total annual run-off can be obtained. A knowledge of the factors affecting rain-fall and run-off are particularly useful if modified discharge records on one stream are to be used in estimating the probable discharge of another stream.

STORAGE RESERVOIRS. — "Primary" or uninterrupted power corresponds to the minimum flow of the stream. "Secondary" power corresponds to the flow above the minimum, or that power which is available only part of the time. Primary power is sometimes essential to the fulfillment of contracts.

The natural minimum flow of small streams is usually not large enough to permit of the successful installation of primary power developments. In such cases it becomes necessary to install turbines having a capacity considerably in excess of the minimum natural flow, and to provide steam or other auxiliary power to supplement the water power during periods of drought, or to provide storage reservoirs to hold back the excess or unusable flow until such times as the natural flow is insufficient. The reservoirs may be required to supply a deficiency extending over a few months or a series of years, depending upon the extent of the development in relation to the natural flow.

For use in fixing the required size of the reservoir, an investigation should be made to determine the minimum season, year or series of years, as the case may be, which may be expected to occur in the future.

Storage reservoirs are also used to partially regulate the flow of the stream. When thus used, they augment the primary power and increase somewhat the total available secondary power.

Allowance should always be made for evaporation from large storage reservoirs, which may modify the discharge records of previous years. Evaporation from free water surfaces varies from 20 to 100 inches, depending upon local conditions.

Load Factor. — All water is wasted which passes the water power plant in excess of the demand. The demand is fixed by the capacity of the installation and the load factor. The "load factor" is the ratio of the average load to the peak load, and ordinarily varies from 30 per cent to 80 per cent, depending upon the nature of the market. The load factor varies from day to day and from season to season. The daily load factor on Sundays is usually considerably lower than on week days, particularly in winter, when a large part of the load is used for lighting and all mills are shut down. Lighting, railway loads and many other loads vary considerably with the seasons. In most developments, the pond at the power dam is large enough to regulate the daily and week-end flow with in the capacity of the plant, so that it is only necessary to consider the average weekly or monthly load in determining the power available and the required capacity of storage reservoirs.

Hydrographs. — Fig. 3 represents a hydrograph of a stream showing a continuous graph of the discharge. The ordinates indicate the "second feet" or

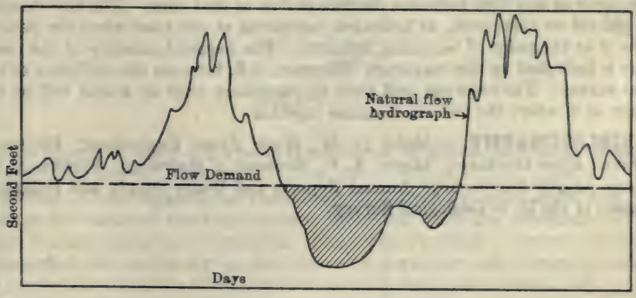

Fig. 3. Hydrograph.

cubic feet per second flowing at any time during the period covered. For large reservoirs, average monthly flow may be plotted instead of average daily flow with a considerable saving of labor. The period should cover a length of time

sufficient to indicate the operation of the reservoir during the time of minimum flow. The flow demand, or flow corresponding to the average power demand is also indicated. This is not necessarily a straight, horizontal line as shown; but may vary with the seasons as hereinbefore explained. The shaded area represents the deficiency of natural flow and hence the amount of storage required to provide continuous or primary power. It is measured in "second-feet-days."

One second-foot-day requires 86,400 cubic feet of reservoir capacity. The maximum draft on the reservoir occurs at the end of the shaded area, after which there must be sufficient excess natural flow above the demand line to fill the reservoir again.

Mass Curves. — Another method of indicating storage requirements is by means of a mass curve, Fig. 4. Each ordinate indicates the total natural flow

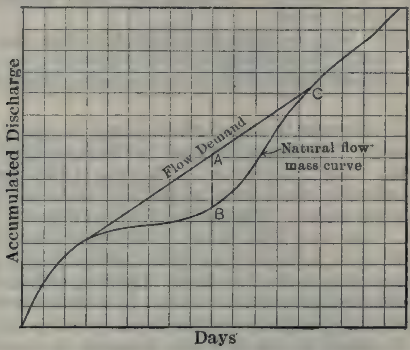

Fig. 4. Mass Curve.

since the date of the beginning of the curve, expressed in cubic feet, second-feet-days, second-feet months or other terms as may be convenient. The slope of the curve at any time indicates the rate of flow at that time. The flow demand is laid off on this curve, as indicated, beginning at the time when the natural flow is at the point of becoming deficient. The required capacity of the reservoir is indicated by the maximum difference, AB, between the ordinates to the two curves. The reservoir will have its maximum draft at B and will be full again at C where the two curves come together.

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HYPERBOLIC FUNCTIONS. — (*See also Derivatives; Integrals; Series, Mathematical.*) Hyperbolic functions are an extension of the trigonometric functions to those cases where the use of the latter gives rise to imaginary or complex angles. From the relations,

$$\cos x = \frac{\epsilon^{jx} + \epsilon^{-jx}}{2}$$

$$\sin x = \frac{\epsilon^{jx} - \epsilon^{-jx}}{2j}$$

where $j = \sqrt{-1}$, it follows that, putting $x = jz$:

$$\cos jz = \frac{\epsilon^{z} + \epsilon^{-z}}{2} \qquad (1)$$

$$-j \sin jz = \frac{\epsilon^{z} - \epsilon^{-z}}{2} \qquad (2)$$

Expressions (1) and (2) are both real quantities when z is real, that is, when the angle jz is imaginary. The first expression is called the hyperbolic cosine of z, abbreviated and pronounced "cosh"; the second expression is called the hyperbolic sine of z, abbreviated sinh and pronounced "shin." Hence, using x for the variable,

$$\sinh x = \frac{\epsilon^{x} - \epsilon^{-x}}{2}$$

$$\cosh x = \frac{\epsilon^{x} + \epsilon^{-x}}{2}$$

The hyperbolic tangent, cotangent, secant and cosecant are defined as follows:

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

The hyperbolic angle x is a number analogous to radians in circular measure; it is never expressed in degrees.

Period of the Hyperbolic Functions. — Adding 2π to an angle does not change the value of the trigonometric functions; they are therefore said to have a period equal to 2π radians. Hyperbolic functions, however, have no true period, but adding $2\pi j$ to the hyperbolic angle does not change the values of the functions; hence these functions have an imaginary period, $2\pi j$.

Table of Hyperbolic Functions.* — Below is given a table of hyperbolic functions.

* More complete tables of hyperbolic functions may be found in the *Smithsonian Mathematical Tables*. by G. F. Becker and C. E. Van Orstrand, Washington, 1909 and in *Tables of Complex Hyperbolic and Circular Functions*, by A. E. Kennelly, Harvard Univ. Press, 1914.

HYPERBOLIC FUNCTIONS 0.00-1.49

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0701	0.0801	0.0901
0.0	cosh	1.0000	1.0001	1.0002	1.0005	1.0008	1.0013	1.0018	1.0025	1.0032	1.0041
	tanh	0.0000	0.0100	0.0200	0.0300	0.0400	0.0500	0.0599	0.0699	0.0798	0.0898
	sinh	0.1002	0.1102	0.1203	0.1304	0.1405	0.1506	0.1607	0.1708	0.1810	0.1911
0.1	cosh	1.0050	1.0061	1.0072	1.0085	1.0098	1.0113	1.0128	1.0145	1.0162	1.0181
	tanh	0.0997	0.1096	0.1194	0.1293	0.1391	0.1489	0.1587	0.1684	0.1781	0.1878
	sinh	0.2013	0.2115	0.2218	0.2320	0.2423	0.2526	0.2629	0.2733	0.2837	0.2941
0.2	cosh	1.0201	1.0221	1.0243	1.0266	1.0289	1.0314	1.0340	1.0367	1.0395	1.0423
	tanh	0.1974	0.2070	0.2165	0.2260	0.2355	0.2449	0.2543	0.2636	0.2729	0.2821
	sinh	0.3045	0.3150	0.3255	0.3360	0.3466	0.3572	0.3678	0.3785	0.3892	0.4000
0.3	cosh	1.0453	1.0484	1.0516	1.0549	1.0584	1.0619	1.0655	1.0692	1.0731	1.0770
	tanh	0.2913	0.3004	0.3095	0.3185	0.3275	0.3364	0.3452	0.3540	0.3627	0.3714
	sinh	0.4108	0.4216	0.4325	0.4434	0.4543	0.4653	0.4764	0.4875	0.4986	0.5098
0.4	cosh	1.0811	1.0852	1.0895	1.0939	1.0984	1.1030	1.1077	1.1125	1.1174	1.1225
	tanh	0.3800	0.3885	0.3969	0.4053	0.4136	0.4219	0.4301	0.4382	0.4462	0.4542
	sinh	0.5211	0.5324	0.5438	0.5552	0.5666	0.5782	0.5897	0.6014	0.6131	0.6248
0.5	cosh	1.1276	1.1329	1.1383	1.1438	1.1494	1.1551	1.1609	1.1669	1.1730	1.1792
	tanh	0.4621	0.4700	0.4777	0.4854	0.4930	0.5005	0.5080	0.5154	0.5227	0.5299
	sinh	0.6367	0.6485	0.6605	0.6725	0.6846	0.6967	0.7090	0.7213	0.7336	0.7461
0.6	cosh	1.1855	1.1919	1.1984	1.2051	1.2119	1.2188	1.2258	1.2330	1.2402	1.2476
	tanh	0.5370	0.5441	0.5511	0.5581	0.5649	0.5717	0.5784	0.5850	0.5915	0.5980
	sinh	0.7586	0.7712	0.7838	0.7966	0.8094	0.8223	0.8353	0.8484	0.8615	0.8748
0.7	cosh	1.2552	1.2628	1.2706	1.2785	1.2865	1.2947	1.3030	1.3114	1.3199	1.3286
	tanh	0.6044	0.6107	0.6169	0.6231	0.6292	0.6352	0.6411	0.6469	0.6527	0.6584
	sinh	0.8881	0.9015	0.9150	0.9286	0.9423	0.9561	0.9700	0.9840	0.9981	1.0122
0.8	cosh	1.3374	1.3464	1.3555	1.3647	1.3740	1.3835	1.3932	1.4029	1.4128	1.4229
	tanh	0.6640	0.6696	0.6751	0.6805	0.6858	0.6911	0.6963	0.7014	0.7064	0.7114
	sinh	1.0265	1.0409	1.0554	1.0700	1.0847	1.0995	1.1144	1.1294	1.1446	1.1598
0.9	cosh	1.4331	1.4434	1.4539	1.4645	1.4753	1.4862	1.4973	1.5085	1.5199	1.5314
	tanh	0.7163	0.7211	0.7259	0.7306	0.7352	0.7398	0.7443	0.7487	0.7531	0.7574
	sinh	1.1752	1.1907	1.2063	1.2220	1.2379	1.2539	1.2700	1.2862	1.3025	1.3190
1.0	cosh	1.5431	1.5549	1.5669	1.5790	1.5913	1.6038	1.6164	1.6292	1.6421	1.6552
	tanh	0.7616	0.7658	0.7699	0.7739	0.7779	0.7818	0.7857	0.7895	0.7932	0.7969
	sinh	1.3356	1.3524	1.3693	1.3863	1.4035	1.4208	1.4382	1.4558	1.4735	1.4914
1.1	cosh	1.6685	1.6820	1.6956	1.7093	1.7233	1.7374	1.7517	1.7662	1.7808	1.7956
	tanh	0.8005	0.8041	0.8076	0.8110	0.8144	0.8178	0.8210	0.8243	0.8275	0.8306
	sinh	1.5095	1.5276	1.5460	1.5645	1.5831	1.6019	1.6209	1.6400	1.6593	1.6788
1.2	cosh	1.8107	1.8258	1.8412	1.8568	1.8725	1.8884	1.9045	1.9208	1.9373	1.9540
	tanh	0.8337	0.8367	0.8397	0.8426	0.8455	0.8483	0.8511	0.8538	0.8565	0.8591
	sinh	1.6984	1.7182	1.7381	1.7583	1.7786	1.7991	1.8198	1.8406	1.8617	1.8829
1.3	cosh	1.9709	1.9880	2.0053	2.0028	2.0404	2.0583	2.0764	2.0947	2.1132	2.1320
	tanh	0.8617	0.8643	0.8668	0.8693	0.8717	0.8741	0.8764	0.8787	0.8810	0.8832
	sinh	1.9043	1.9259	1.9477	1.9697	1.9919	2.0143	2.0369	2.0597	2.0827	2.1059
1.4	cosh	2.1509	2.1700	2.1894	2.2090	2.2288	2.2488	2.2691	2.2896	2.3103	2.3312
	tanh	0.8854	0.8875	0.8896	0.8917	0.8937	0.8957	0.8977	0.8996	0.9015	0.9033

HYPERBOLIC FUNCTIONS 1.50 - 2.99

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	2.1293	2.1529	2.1768	2.2008	2.2251	2.2496	2.2743	2.2993	2.3245	2.3499
1.5	cosh	2.3524	2.3738	2.3955	2.4174	2.4395	2.4619	2.4845	2.5074	2.5305	2.5538
	tanh	0.9052	0.9069	0.9087	0.9104	0.9121	0.9138	0.9154	0.9170	0.9186	0.9202
	sinh	2.3756	2.4015	2.4276	2.4540	2.4806	2.5075	2.5346	2.5620	2.5896	2.6175
1.6	cosh	2.5775	2.6013	2.6255	2.6499	2.6746	2.6995	2.7247	2.7502	2.7760	2.8020
	tanh	0.9217	0.9232	0.9246	0.9261	0.9275	0.9289	0.9302	0.9316	0.9329	0.9342
	sinh	2.6456	2.6740	2.7027	2.7317	2.7609	2.7904	2.8202	2.8503	2.8806	2.9112
1.7	cosh	2.8283	2.8549	2.8818	2.9090	2.9364	2.9642	2.9922	3.0206	3.0493	3.0782
	tanh	0.9354	0.9367	0.9379	0.9391	0.9402	0.9414	0.9425	0.9436	0.9447	0.9458
	sinh	2.9422	2.9734	3.0049	3.0367	3.0689	3.1013	3.1340	3.1671	3.2005	3.2341
1.8	cosh	3.1075	3.1371	3.1669	3.1972	3.2277	3.2585	3.2897	3.3212	3.3530	3.3852
	tanh	0.9468	0.9478	0.9488	0.9498	0.9508	0.9518	0.9527	0.9536	0.9545	0.9554
	sinh	3.2682	3.3025	3.3372	3.3722	3.4075	3.4432	3.4792	3.5156	3.5523	3.5894
1.9	cosh	3.4177	3.4506	3.4838	3.5173	3.5512	3.5855	3.6201	3.6551	3.6904	3.7261
	tanh	0.9562	0.9571	0.9579	0.9587	0.9595	0.9603	0.9611	0.9619	0.9626	0.9633
	sinh	3.6269	3.6647	3.7028	3.7414	3.7803	3.8196	3.8593	3.8993	3.9398	3.9806
2.0	cosh	3.7622	3.7987	3.8355	3.8727	3.9103	3.9483	3.9867	4.0255	4.0647	4.1043
	tanh	0.9640	0.9647	0.9654	0.9661	0.9668	0.6974	0.9680	0.9686	0.9693	0.9699
	sinh	4.0219	4.0635	4.1056	4.1480	4.1909	4.2342	4.2779	4.3221	4.3666	4.4117
2.1	cosh	4.1443	4.1847	4.2256	4.2668	4.3085	4.3507	4.3932	4.4362	4.4797	4.5236
	tanh	0.9705	0.9710	0.9716	0.9722	0.9727	0.9732	0.9738	0.9743	0.9748	0.9752
	sinh	4.4571	4.5030	4.5494	4.5962	4.6434	4.6912	4.7394	4.7880	4.8372	4.8868
2.2	cosh	4.5679	4.6127	4.6580	4.7037	4.7499	4.7966	4.8437	4.8914	4.9395	4.9881
	tanh	0.9757	0.9762	0.9767	0.9771	0.9776	0.9780	0.9785	0.9789	0.9793	0.9797
	sinh	4.9370	4.9876	5.0387	5.0903	5.1425	5.1951	5.2483	5.3020	5.3562	5.4109
2.3	cosh	5.0372	5.0868	5.1370	5.1876	5.2388	5.2905	5.3427	5.3954	5.4487	5.5026
	tanh	0.9801	0.9805	0.9809	0.9812	0.9816	0.9820	0.9823	0.9827	0.9830	0.9834
	sinh	5.4662	5.5221	5.5785	5.6354	5.6929	5.7510	5.8097	5.8689	5.9288	5.9892
2.4	cosh	5.5569	5.6119	5.6674	5.7235	5.7801	5.8373	5.8951	5.9535	6.0125	6.0721
	tanh	0.9837	0.9840	0.9843	0.9846	0.9849	0.9852	0.9855	0.9858	0.9861	0.9864
	sinh	6.0502	6.1118	6.1741	6.2369	6.3004	6.3645	6.4293	6.4946	6.5607	6.6274
2.5	cosh	6.1323	6.1931	6.2545	6.3166	6.3793	6.4426	6.5066	6.5712	6.6365	6.7024
	tanh	0.9866	0.9869	0.9871	0.9874	0.9876	0.9879	0.9881	0.9884	0.9886	0.9888
	sinh	6.6947	6.7628	6.8315	6.9009	6.9709	7.0417	7.1132	7.1854	7.2583	7.3319
2.6	cosh	6.7690	6.8363	6.9043	6.9729	7.0423	7.1123	7.1831	7.2546	7.3268	7.3998
	tanh	0.9890	0.9892	0,9895	0.9897	0.9899	0.9801	0.9903	0.9905	0.9906	0.9908
	sinh	7.4063	7.4814	7.5572	7.6338	7.7112	7.7894	7.8683	7.9480	8.0285	8.1098
2.7	cosh	7.4735	7.5479	7.6231	7.6991	7.7758	7.8533	7.9316	8.0106	8.0905	8.1712
	tanh	0.9910	0.9912	0.9914	0.9915	0.9917	0.9919	0.9920	0.9922	0.9923	0.9925
	sinh	8.1919	8.2749	8.3586	8.4432	8.5287	8.6150	8.7021	8.7902	8.8791	8.9689
2.8	cosh	8.2527	8.3351	8.4182	8.5022	8.5871	8.6728	8.7594	8.8469	8.9352	9.0244
	tanh	0.9926	0.9928	0.9929	0.9931	0.9932	0.9933	0.9935	0.9936	0.9937	0.9938
	sinh	9.0596	9.1512	9.2437	9.3371	9.4315	9.5268	9.6231	9.7203	9.8185	9.9177
2.9	cosh	9.1146	9.2056	9.2976	9.3905	9.4844	9.5792	9.6749	9.7716	9.8693	9.9680
	tanh	0.9940	0.9941	0.9942	0.9953	0.9944	0.9945	0.9946	0.9948	0.9949	0.9950

HYPERBOLIC FUNCTIONS 3.00-4.49

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	10.018	10.119	10.221	10.324	10.429	10.534	10.640	10.748	10.856	10.966
3.0	cosh	10.068	10.168	10.270	10.373	10.476	10.581	10.687	10.794	10.902	11.011
	tanh	0.9951	0.9952	0.9953	0.9953	0.9954	0.9955	0.9956	0.9957	0.9958	0.9959
	sinh	11.076	11.188	11.301	11.415	11.530	11.647	11.764	11.883	12.003	12.124
3.1	cosh	11.121	11.233	11.345	11.459	11.574	11.689	11.806	11.925	12.044	12.165
	tanh	0.9960	0.9960	0.9961	0.9962	0.9963	0.9963	0.9964	0.9965	0.9966	0.9966
	sinh	12.246	12.369	12.494	12.620	12.747	12.876	13.006	13.137	13.269	13.403
3.2	cosh	12.287	12.410	12.534	12.660	12.786	12.915	13.044	13.175	13.307	13.440
	tanh	0.9967	0.9968	0.9968	0.9969	0.9969	0.9970	0.9971	0.9971	0.9972	0.9972
	sinh	13.538	13.674	13.812	13.951	14.092	14.234	14.377	14.522	14.668	14.816
3.3	cosh	13.575	13.711	13.848	13.987	14.127	14.269	14.412	14.556	14.702	14.850
	tanh	0.9973	0.9973	0.9974	0.9974	0.9975	0.9975	0.9976	0.9976	0.9977	0.9977
	sinh	14.965	15.116	15.268	15.422	15.577	15.734	15.893	16.053	16.215	16.378
3.4	cosh	14.999	15.149	15.301	15.455	15.610	15.766	15.924	16.084	16.245	16.408
	tanh	0.9978	0.9978	0.9979	0.9979	0.9979	0.9980	0.9980	0.9981	0.9981	0.9981
	sinh	16.543	16.709	16.877	17.047	17.219	17.392	17.567	17.744	17.923	18.103
3.5	cosh	16.573	16.739	16.907	17.077	17.248	17.421	17.596	17.772	17.951	18.131
	tanh	0.9982	0.9982	0.9983	0.9983	0.9983	0.9984	0.9984	0.9984	0.9985	0.9985
	sinh	18.285	18.470	18.655	18.843	19.033	19.224	19.418	19.613	19.811	20.010
3.6	cosh	18.313	18.497	18.682	18.870	19.059	19.250	19.444	19.639	19.836	20.035
	tanh	0.9985	0.9985	0.9986	0.9986	0.9986	0.9987	0.9987	0.9987	0.9987	0.9988
	sinh	20.211	20.415	20.620	20.828	21.037	21.249	21.463	21.679	21.897	22.117
3.7	cosh	20.236	20.439	20.644	20.852	21.061	21.272	21.486	21.702	21.919	22.139
	tanh	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990
	sinh	22.339	22.564	22.791	23.020	23.252	23.486	23.722	23.961	24.202	24.445
3.8	cosh	22.362	22.586	22.813	23.042	23.273	23.507	23.743	23.982	24.222	24.466
	tanh	0.9990	0.9990	0.9990	0.9991	0.9991	0.9991	0.9991	0.9991	0.9992	0.9992
	sinh	24.691	24.939	25.190	25.444	25.700	25.958	26.219	26.483	26.749	27.018
3.9	cosh	24.711	24.959	25.210	25.463	25.719	25.977	26.238	26.502	26.768	27.037
	tanh	0.9992	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993	0.9993
	sinh	27.290	27.564	27.842	28.122	28.404	28.690	28.979	29.270	29.564	29.862
4.0	cosh	27.308	27.583	27.860	28.139	28.422	28.707	28.996	29.287	29.581	29.878
	tanh	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994	0.9994
	sinh	30.162	30.465	30.772	31.081	31.393	31.709	32.028	32.350	32.675	33.004
4.1	cosh	30.178	30.482	30.788	31.097	31.409	31.725	32.044	32.365	32.691	33.019
	tanh	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995	0.9995
	sinh	33.336	33.671	34.009	34.351	34.697	35.046	35.398	35.754	36.113	36.476
4.2	cosh	33.351	33.686	34.024	34.366	34.711	35.060	35.412	35.768	36.127	36.490
	tanh	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996
	sinh	36.843	37.214	37.588	37.965	38.347	38.733	39.122	39.515	39.913	40.314
4.3	cosh	36.857	37.227	37.601	37.979	38.360	38.746	39.135	39.528	39.925	40.326
	tanh	0.9996	0.9996	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997
	sinh	40.719	41.129	41.542	41.960	42.382	42.808	43.238	43.673	44.112	44.555
4.4	cosh	40.732	41.141	41.554	41.972	42.393	42.819	43.250	43.684	44.123	44.566
	tanh	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

HYPERBOLIC FUNCTIONS 4.50-5.99

Angle		0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	sinh	45.003	45.455	45.912	46.374	46.840	47.311	47.787	48.267	48.752	49.242
4.5	cosh	45.014	45.466	45.923	46.385	46.851	47.321	47.797	48.277	48.762	49.252
	tanh	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
	sinh	49.737	50.237	50.742	51.252	51.767	52.288	52.813	53.344	53.880	54.422
4.6	cosh	49.747	50.247	50.752	51.262	51.777	52.297	52.823	53.354	53.890	54.431
	tanh	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
	sinh	54.969	55.522	56.080	56.643	57.213	57.788	58.369	58.955	59.548	60.147
4.7	cosh	54.978	55.531	56.089	56.652	57.221	57.796	58.377	58.964	59.556	60.155
	tanh	0.9998	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	60.751	61.362	61.979	62.601	63.231	63.866	64.508	65.157	65.812	66.473
4.8	cosh	60.759	61.370	61.987	62.609	63.239	63.874	64.516	65.164	65.819	66.481
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	67.141	67.816	68.498	69.186	69.882	70.584	71.293	72.010	72.734	73.465
4.9	cosh	67.149	67.823	68.505	69.193	69.889	70.591	71.300	72.017	72.741	73.472
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	74.203	74.949	75.702	76.463	77.232	78.008	78.792	79.584	80.384	81.192
5.0	cosh	74.210	74.956	75.709	76.470	77.238	78.014	78.798	79.590	80.390	81.198
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	82.008	82.832	83.665	84.506	85.355	86.213	87.079	87.955	88.839	89.732
5.1	cosh	82.014	82.838	83.671	84.512	85.361	86.219	87.085	87.960	88.844	89.737
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	sinh	90.633	91.544	92.464	93.394	94.332	95.281	96.238	97.205	98.182	99.169
5.2	cosh	90.639	91.550	92.470	93.399	94.338	95.286	96.243	97.211	98.188	99.174
	tanh	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
	sinh	100.17	101.17	102.19	103.22	104.25	105.30	106.36	107.43	108.51	109.60
5.3	cosh	100.17	101.18	102.19	103.22	104.26	105.31	106.67	107.43	108.51	109.60
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	110.70	111.81	112.94	114.07	115.22	116.38	117.55	118.73	119.92	121.13
5.4	cosh	110.71	111.82	112.94	114.08	115.22	116.38	117.55	118.73	119.93	121.13
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	122.34	123.57	124.82	126.07	127.34	128.62	129.91	131.22	132.53	133.87
5.5	cosh	122.35	123.58	124.82	126.07	127.34	128.62	129.91	131.22	132.54	133.87
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	135.21	136.57	137.94	139.33	140.73	142.14	143.57	145.02	146.47	147.95
5.6	cosh	135.22	136.57	137.95	139.33	140.73	142.15	143.58	145.02	146.48	147.95
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	149.43	150.93	152.45	153.98	155.53	157.09	158.67	160.27	161.88	163.51
5.7	cosh	149.44	150.94	152.45	153.99	155.53	157.10	158.68	160.27	161.88	163.51
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	165.15	166.81	168.48	170.18	171.89	173.62	175.36	177.12	178.90	180.70
5.8	cosh	165.15	166.81	168.49	170.18	171.89	173.62	175.36	177.13	178.91	180.70
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	sinh	182.52	184.35	186.20	188.08	189.97	191.88	193.80	195.75	197.72	199.71
5.9	cosh	182.52	184.35	186.21	188.08	189.97	191.88	193.81	195.75	197.72	199.71
	tanh	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Example. — $\sinh 0.83 = 0.9286$, $\cosh 0.83 = 1.3647$, $\tanh 0.83 = 0.6805$.

Approximate Formulas. — Note that for x less than 0.1,

$$\sinh x = x \text{ with an error of less than 0.2 per cent.}$$

$$\cosh x = 1 + \frac{x^2}{2} \text{ with an error of less than 0.09 per cent.}$$

For x greater than 6,

$$\sinh x = \cosh x = \frac{\epsilon^x}{2} = \frac{1}{2} \log_{10}{}^{-1} (0.43429\, x)$$

with an error of less than 0.01 per cent.

Anti-Functions. — If $a = \sinh x$, then x is the angle whose hyperbolic sine is a; this may be expressed symbolically

$$x = \sinh^{-1} a$$

which is read "x equals the angle whose hyperbolic sine is a." The angle x is also called the "anti-hyperbolic sine" or the "inverse hyperbolic sine" of a. Similarly for the other hyperbolic functions. (See *Trigonometric Functions, sub-heading Anti-functions.*) The following relations exist between the anti-hyperbolic functions and the natural logarithms:

$$\sinh^{-1} x = \log \left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1} x = \log \left(x + \sqrt{x^2 - 1}\right)$$

$$\tanh^{-1} x = \frac{1}{2} \log \left(\frac{1 + x}{1 - x}\right)$$

Relations among Functions of the Same Angle. —

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \frac{1}{\cosh^2 x}$$

$$\coth^2 x - 1 = \frac{1}{\sinh^2 x}$$

$$\sinh (-x) = -\sinh x$$

$$\cosh (-x) = \cosh x$$

$$\tanh (-x) = -\tanh x.$$

See also the definitions given above.

Sum and Difference of Two Angles. —

$$\sinh (x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh (x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\tanh (x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$

$$\sinh (x - y) = \sinh x \cosh y - \cosh x \sinh y$$

$$\cosh (x - y) = \cosh x \cosh y - \sinh x \sinh y$$

$$\tanh (x - y) = \frac{\tanh x - \tanh y}{1 - \tanh x \tanh y}$$

Product of the Functions of Two Angles. —

$$\sinh x \sinh y = \tfrac{1}{2} [\cosh (x + y) - \cosh (x - y)]$$

$$\sinh x \cosh y = \tfrac{1}{2} [\sinh (x + y) + \sinh (x - y)]$$

$$\cosh x \sinh y = \tfrac{1}{2} [\sinh (x + y) - \sinh (x - y)]$$

$$\cosh x \cosh y = \tfrac{1}{2} [\cosh (x + y) + \cosh (x - y)]$$

Functions of Twice an Angle. —

$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \sinh^2 x + \cosh^2 x$$
$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

Functions of Half an Angle. —

$$\sinh \frac{x}{2} = \sqrt{\frac{\cosh x - 1}{2}}$$
$$\cosh \frac{x}{2} = \sqrt{\frac{\cosh x + 1}{2}}$$
$$\tanh \frac{x}{2} = \sqrt{\frac{\cosh x - 1}{\cosh x + 1}}$$

Functions of Three Times an Angle. —

$$\sinh 3x = 3 \sinh x + 4 \sinh^3 x$$
$$\cosh 3x = 4 \cosh^3 x - 3 \cosh x$$
$$\tanh 3x = \frac{3 \tanh x + \tanh^3 x}{1 + 3 \tanh^2 x}$$

Relations between Hyperbolic and Trigonometric Functions. —

$$\sinh (jx) = j \sin x \qquad \sin(jx) = j \sinh x$$
$$\cosh (jx) = \cos x \qquad \cos (jx) = \cosh x$$
$$\tanh (jx) = j \tan x \qquad \tan (jx) = j \tanh x$$
$$\sinh^{-1} jx = j \sin^{-1} x \qquad \sin^{-1} jx = j \sinh^{-1} x$$
$$\tanh^{-1} jx = j \tan^{-1} x \qquad \tan^{-1} jx = j \tanh^{-1} x.$$
$$\cosh^{-1} jx = j \cos^{-1} jx = \log (x + \sqrt{1 + x^2}) - j \frac{\pi}{2}$$

Hyperbolic Functions of a Complex Angle. —

$$\sinh (x + jy) = \sinh x \cos y + j \cosh x \sin y = M\epsilon^{j\theta}$$

where $$M = \sqrt{\frac{\cosh 2x - \cos 2y}{2}} \quad \text{and} \quad \tan \theta = \frac{\tan y}{\tanh x}.$$

$$\cosh (x + jy) = \cosh x \cos y + j \sinh x \sin y = N\epsilon^{j\phi}$$

where $$N = \sqrt{\frac{\cosh 2x + \cos 2y}{2}} \quad \text{and} \quad \tan \phi = \tanh x \cdot \tan y.$$

$$\tanh (x + jy) = \frac{\sinh x \cos y + j \cosh x \sin y}{\cosh x \cos y + j \sinh x \sin v} = P\epsilon^{j\psi}$$

where $$P = \sqrt{\frac{\cosh 2x - \cos 2y}{\cosh 2x + \cos 2y}} \quad \text{and} \quad \psi = \tan^{-1} \left[\frac{\sin 2y}{\sinh 2x}\right].$$

$$\tanh^{-1} (A\epsilon^{j\alpha}) = B_1 + jB_2,$$

where $$B_1 = \tfrac{1}{2} \tanh^{-1} \left[\frac{2A \cos \alpha}{1 + A^2}\right] \quad \text{and} \quad B_2 = \tfrac{1}{2} \tan^{-1} \left[\frac{2A \sin \alpha}{1 - A^2}\right].$$

[W. A. Del Mar.]

IGNITION, ELECTRIC, FOR INTERNAL COMBUSTION ENGINES. — (See also *Batteries, Primary; Batteries, Storage; Internal Combustion Engines; Starting and Lighting Systems.*) It is the function of the ignition system of an internal combustion engine to initiate the burning of the charge of fuel within the engine cylinder at the proper instant in the cycle; i.e., when the explosive mixture of gasoline vapor and air is at or near the point of maximum compression. The typical system, see Fig. 1, comprises

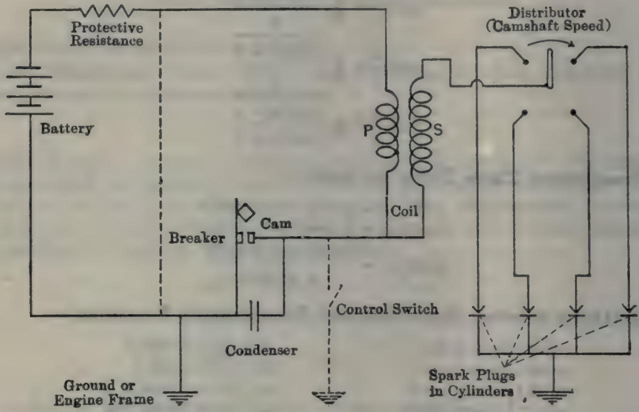

Fig. 1. Circuits of Typical Single-spark, Jump-spark Ignition System. Solid lines show battery system. If the battery and resistance are omitted, the dotted connections added, and the "coil" taken to represent the windings of a magneto, the circuits become those of a high-tension magneto ignition system.

(1) a source of electrical energy (battery or magneto generator); (2) a timer for correlating the production of the spark with the rotation of the engine (this may be either a "timer" or "commutator" which closes the primary circuit of a vibrator system at the proper instant, or a "breaker" which interrupts the primary circuit of an induction coil); (3) an induction coil (which may be incorporated as part of the magneto) for transforming the energy to a high voltage; (4) a distributor for leading the high voltage discharge to the successive cylinders in the proper sequence; and (5) a spark plug in each cylinder to provide a fixed insulated gap at which the igniting spark is produced.

IGNITION OF GASES. — Combustible gases and vapors when mixed with air in the proper proportion (15 of air to 1 of gasoline by weight is approximately the theoretical combining ratio) become explosive. That is, a flame started in the mixture is self-propagating with a velocity which varies greatly with the material and the proportions of the mixture, and is greater as the pressure and temperature of the mixture are increased, but so far as is now known is independent of the character of the igniting source. For gasoline engines the velocity is of the order of 10 meters per second, and consequently the spark should be timed to occur several thousandths of a second before the piston reaches upper dead center, so that the charge may be practically all ignited at the beginning of the power stroke. Since this desired *time* interval corresponds to a different *angular* interval at different engine speeds, means are usually pro-

vided for either manually or automatically advancing the angular position of the timer mechanism with increase of speed. At starting the spark should always be retarded nearly to dead-center to avoid "back firing."

To cause ignition of a gaseous mixture a certain intensity of spark is required (*see Paterson and Campbell, Proc. Phys. Soc., London, 31, p. 168, 1919*), which however does not correspond to a fixed amount of energy. At high voltages the quantity of electricity passing the gap may be made much less than in inverse proportion to the voltage, and still produce ignition. The intensity needed with a reasonable mixture of gasoline and air is 0.002 joule at 5000 volts. This is roughly the same as the electrostatic energy stored in a capacity of 200×10^{-6} microfarads, which is roughly the capacity of the usual spark plug and its connected circuit, when charged to a sparking voltage of 5000 volts. Consequently, if the ignition system produces a spark at all, ignition is practically certain to result, and the supposed advantages of a "hot" or "fat" spark are mainly illusory. In cases where carburetion is imperfect, as with heavy fuels or when starting a cold engine, much of the charge is in the form of spray rather than vapor. Under such conditions it is quite possible that a spark of long duration or great energy content may be advantageous by vaporizing some of the fuel. It should be noted that the hottest magneto spark (0.2 joule) contains sufficient heat to vaporize only 0.0007 cu. cm. of gasoline, and that an equal amount of heat is supplied by the combustion of only 0.000006 cu. cm. of gasoline.

Kinds of Spark. — An electric spark may be defined as the passage of an electric current through a gaseous medium and the distinction between it and an arc is not of importance here. There are two main types of spark:

(a) **The Condenser or Disruptive Spark,** such as is produced by the sudden discharge of a condenser through a circuit of small resistance and inductance. The current in such cases is of great amplitude, oscillates in direction with high frequency and is rapidly damped out, the entire discharge lasting only a few millionths of a second. The sparks in air are bluish white, noisy and exert violent pressure effects, but will not readily ignite solid materials such as cotton or paper. It is, however, as effective as the other type of spark in igniting gas mixtures.

(b) **The Induction Spark or Arc** is produced by the discharge of magnetic energy from a coil. The current is of small value (0.1 ampere or less, in ignition circuits), is unidirectional, and may last for several thousandths of a second. The spark is reddish or orange, quiet, of greater apparent "thickness" and will, if of sufficient intensity, readily ignite paper or cotton.

Most ignition systems give a combination of both kinds of spark. The coil winding, distributor leads, and spark plug form a condenser which discharges a type (a) spark which is followed by a type (b) spark from the coil.

TYPES OF IGNITION SYSTEMS. — Electric ignition systems may be classified into:

Make and Break or Low-tension Systems, in which the current from a battery flows through a coil of high inductance and through a pair of contacts inside the engine cylinder. At the proper point in the cycle the contacts are separated by a suitable mechanism and the resulting arc ignites the mixture. A low-voltage magneto with a single winding is often used in place of the battery and inductance coil. The mechanical complexity of this system limits it to slow-speed, low-compression engines. It is now little used.

Jump Spark or High-tension Systems, in which a sufficiently high voltage is induced in the secondary winding of an induction coil or magneto, by

the breaking of current in a primary winding, to produce a spark across a gap of fixed length inside the cylinder. Jump spark systems may be further classified as:

Vibrator Systems, in which the primary current is interrupted repeatedly by a vibrating contact during the part of the stroke when a spark is required. The contact is vibrated as in an electric bell or buzzer either by the magnetic flux from the core of the main induction coil, or by a small auxiliary buzzer or relay. The frequency of the vibrations is of the order of several hundred per second, but even then in high-speed engines there would be time for only a few vibrations during the entire expansion stroke, and the net result is the same as in a single-spark system.

Single-spark Systems are those in which a single spark is produced at each engine cycle by the mechanical opening of the primary circuit contacts. This is by far the most common type of system.

Ignition systems may also be classified according to the primary source of energy as battery, magneto, and dual:

Battery Systems draw current (0.5 to 5 amperes) at low voltage (4 to 24 volts; usually 6) from a battery of storage or dry cells through the primary of an induction coil and the "breaker" contacts (see Fig. 1). In the so-called **closed circuit** type the breaker is closed for a fixed fraction of the engine cycle which, at moderate engine speeds, gives a time of closure about equal to the electrical time constant $\left(\frac{L}{R}\right)$ of the primary circuit. Consequently the primary current has time to build up to nearly its final value $\frac{E}{R}$ before the circuit is broken. At higher speeds this final value is not reached and the current at "break" and the spark energy are correspondingly decreased. At standstill the full value of current would flow continuously and produce excessive heating in the circuit. This is obviated in some systems by a series resistance of high temperature coefficient which is so heated by the excess current that the resulting increase of resistance limits the standstill current to a moderate value.

In the **open-circuit** type the breaker is closed for a period which is short compared to the time constant of the primary circuit and which is independent of engine speed. The maximum current would be very large but is never attained and the spark energy is independent of engine speed.

MAGNETOS are electric generators which have a permanent magnet field structure and are supplied with mechanical energy from the driving shaft of the engine. Magnetos are classified as "low tension" or "high tension." In the former a single winding generates current at relatively low voltage which is transformed to high voltage by an external coil as with a battery system. In the latter a secondary winding is placed over the primary and the armature serves the double function of generator and transformer.

Magnetos are also classified as **one-spark** or **two-spark.** In the former one end of the secondary is grounded through the primary and the other end is connected through a single distributor to one spark plug at a time. In a "two-spark" magneto both ends of the secondary winding are highly insulated and connected through the two parts of a double distributor to two spark plugs in the same cylinder. The ignition of the charge from two widely separated points in the combustion space, secured either in this way or by two separate magnetos, hastens the rapidity of inflammation of the charge and may increase the engine power several per cent.

Magnetos may be classified according to their construction as follows:

"**Shuttle or H-Armature Type.** — This most widely used construction is shown in Fig. 2. The permanent magnet *M* is provided with soft iron or laminated pole pieces *N S* each of which subtends an arc of about 90°. The armature core is of the cross-section indicated and is laminated as much as possible. The cheeks at the ends of the winding extend axially to give a total length of rotor about three times the width of the central yoke. Ball-bearings are universally used and the air gaps reduced to a minimum. Around the core is wound a primary coil of few turns (approx. 150) of coarse wire (No. 22) and outside of this a secondary coil of many turns (approx. 10,000) of fine wire (No. 40). The layers are insulated with oiled paper and at intervals with oiled silk. The outside layers and turns require additional insulation because of the high voltage between turns which is produced at the instant of passage of the spark. The circuits of the magneto system are closely analogous to those in a battery system and are shown dotted in Fig. 1. The outer end of the secondary is connected by a well-insulated slip-ring and carbon brush to the central point of the distributor.

Fig. 2. Cross-section of Shuttle Armature, High-tension Magneto.

The distributor may be either of the **brush type**, in which a carbon brush carried on the rotating central arm rubs in succession against metal inserts set flush in the fixed insulating block which are in turn connected to the spark plugs in the successive cylinders; or of the **jump type** in which the arm passes near but does not touch (about 0.010 inch clearance) the fixed projecting inserts. The latter construction avoids trouble from the accumulation of carbon dust on the track and the resulting flashing of the spark between inserts. The main distributor block is preferably made of a molded phenolic material to avoid softening under heat, but the surface over which the brush rubs should be of a rubber compound to reduce arcing troubles.

One terminal of the primary winding is connected to the armature core and grounded to the frame of the machine by a carbon brush. The other end is connected to an insulated block which supports one of the breaker contact points. The other contact point is carried on a grounded lever which is actuated by a cam acting on a fiber cam-follower so as to open the circuit abruptly at two opposite points in each revolution. The contact surfaces are usually of platinum-iridium alloy though tungsten is used to a considerable extent in battery systems. A condenser of 0.1 to 0.2 microfarad capacity mounted in the armature is connected in parallel with the contacts and serves to reduce the sparking which would otherwise occur. This condenser is usually of mica, though paper is used to some extent. It should be capable of withstanding a test voltage of 350 volts a-c. A brush bearing on the insulated block may be connected through the control switch to ground, thus permanently short-circuiting the primary and rendering the device inoperative when it is desired to stop the engine. This type of magneto furnishes two sparks per revolution and for a 4-cylinder, 4-cycle engine must be driven at crankshaft speed.

Polar Inductor Type. — In this type (Fig. 3) the coils are stationary and are wound on a laminated core with extended pole pieces. Between these there rotates a pair of soft iron blocks which are carried on opposite sides of a non-magnetic shaft. The blocks are enlarged at opposite ends to form disks which rotate in close proximity to the respective poles of the U-shaped permanent magnet. The axis of rotation is along the line joining the poles of the mag-

net and perpendicular to the axis of the coil and to the line joining the pole pieces of the inductor. This type has the advantage of having the coil stationary, and it is consequently easier to insulate the high voltage leads. The type as described gives two sparks per revolution, but constructions using four or more

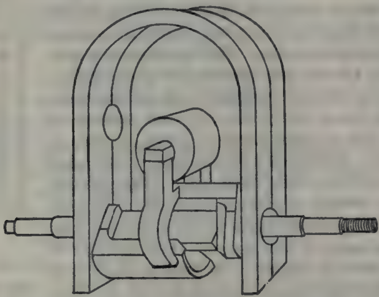

Fig. 3. Schematic Drawing of Polar Inductor-type Magneto.

rotor blocks of alternate polarity give a correspondingly greater number of sparks per revolution.

Sleeve-inductor Type. — This machine contains a field system and shuttle armature like the shuttle type but the armature is fixed in position with its magnetic axis at right angles to that of the field. Between the two structures, in what corresponds to the greatly widened air gap, there rotates a cylindrical non-magnetic sleeve containing two soft iron segments subtending approximately 90°. A complete rotation of the sleeve reverses the flux through the armature core four times, so that by a proper cam four sparks per revolution can be produced.

Oscillating Magnetos. — Among the other numerous types there may be mentioned the oscillating magnetos in which the armature is rotated through a fraction of a revolution against a set of springs which on release snap it back quickly across the position of flux reversal, and thus produce a spark.

Still another type of igniter properly classed as a magneto has only a single winding of many turns of fine wire. The translatory motion of a pair of soft iron cores into and out of magnetic contact with the poles of a permanent magnet reverses the flux through the coil so rapidly that sufficient voltage is generated to produce a spark. Both of these devices are, however, limited in speed and are used mainly on stationary engines.

Impulse Starter Coupling. — This attachment is often inserted in the shaft connecting the engine with any of the foregoing types of magneto to assist in starting, by producing a sudden rotation of the magneto armature at the proper time when the engine is being cranked slowly. At low speeds a latch prevents the motion of the magneto rotor until the driving shaft has rotated through a considerable angle and has wound up a spring located between the driving member and the rotor. When the engine has turned to upper dead center, the latch is displaced by a cam. The released rotor is spun rapidly by the spring past the firing position and a spark is produced in the cylinder. A centrifugal device renders the latch entirely inoperative at speeds above 150–200 r.p.m. and the magneto is then directly driven from the engine. The angular velocity during the impulse is equivalent to a speed of about 800 r.p.m.

DUAL IGNITION SYSTEMS involve the combination of two more or less independent ignition systems on a single engine. The terms "dual," "duplex," and "double" are used in different senses by different makers to designate such combinations the purpose of which is to provide: (1) The addition to magneto ignition of some auxiliary to assist in starting; (2) A factor of safety in case of failure of one system, and (3) a slight increase of engine power resulting from two sparks in a single cylinder. Among the many possible combinations are the following:

(*a*) A high-tension magneto system with a battery which can be connected in series with the magneto primary and breaker for starting.

(*b*) A high-tension magneto with a small auxiliary hand-driven magneto delivering their secondary current through the same distributor segments. With this arrangement, which is widely used on heavy truck and airplane engines, the hand magneto is usually connected to an auxiliary distributor arm which is retarded one segment behind the regular arm, thus avoiding back-fire.

(*c*) A low-tension magneto and a battery feeding a common induction coil, distributor and plugs.

(*d*) A high-tension magneto fitted with an additional breaker mechanism. This is in series with a battery and induction coil, the secondary of which supplies the same distributor as the magneto, only one system being operative at a time.

(*e*) Two completely independent systems supplying separate sets of spark plugs. These may be two similar magnetos or battery systems as on most airplane engines or a magneto and a battery system.

(*f*) Battery systems having both vibratory and single-spark breakers but with other elements in common are sometimes called "dual," and if the vibrator is of high pitch are called "high frequency."

HIGH-FREQUENCY SYSTEMS, properly so called, such as the Lodge and the Dean systems, utilize the oscillating discharge of a condenser to produce frequencies of several thousand cycles per second. The former system is closely analogous to the series gap system and is very effective in firing spark plugs which are heavily shunted by carbon deposits.

CYCLE OF OPERATION. — The detailed operation of any type of jump spark ignition system can be most clearly understood by splitting up the complete cycle of operation into a number of separate periods during each of which the phenomena proceed under fairly definite electrical conditions, each period in turn being separated from the preceding and following one by an abrupt change of conditions.

Period 1 includes the building up of primary current and the consequent storage of magnetic energy, as a result of either the impressed voltage from the battery, or the voltage generated by the rotation of the magneto armature. In the former case the current builds up exponentially and gradually approaches its closed circuit value. In the case of the magneto, although the e.m.f. wave generated by the rotation is very peaked, the short circuit current wave at moderate and high speeds is rather flat-topped and sufficient current exists to produce a spark if "break" occurs over a range of about 90 electrical degrees in each half wave. At slower speeds the current wave is much more peaked and a spark can be produced over only a narrow timing range. To obviate this limitation magnetos are frequently so constructed that the pole pieces (or in polar-inductor types the coil and core) can be rotated with the breaker cam, and the "break" occurs always at the same point in the electrical cycle, though it is shifted with respect to the engine cycle.

The normal current at break is usually about 5 amperes in either battery or

magneto systems. The inductance of the latter is, however, usually considerably greater, so that the available magnetic energy $\frac{1}{2}Lb^2$, and hence the heat energy in the spark is correspondingly larger.

The discharge of the battery on closed circuit, or the operation of the magneto as a short-circuited a-c. generator, is suddenly interrupted by the opening of the breaker contacts by the cam and Period 2 begins.

Period 2. — This is of extremely short duration (of the order of 0.00005 second), and extends from the opening of the primary contacts to the initiation of the spark at the spark plug, but is of great importance, since the production or failure of the spark is determined by whether or not the induced secondary voltage during the period rises to a value equal to the breakdown voltage of the spark plug gap.

At the beginning of the period the primary current continues to flow unabated but into the primary condenser instead of through the breaker contacts. This current rapidly charges the condenser, raising the voltage at its terminals and introducing a back e.m.f. into the primary circuit. This causes a decrease in the primary current and in the magneto-motive force acting on the core and hence in the resultant flux. This decrease in flux induces a voltage in the secondary which is greater than that in the primary in approximately the ratio of turns $\left(u = \frac{S_2}{S_1} = 50 \text{ to } 60\right)$; and this voltage in turn sets up a secondary current which charges the secondary condenser (capacity 2×10^{-4} microfarad approximately) of which the secondary winding, distributor and spark plug leads form one "plate," and the grounded frame and engine the other "plate." This transformation of energy from magnetic to electrostatic form, and its storage. in primary and secondary condenser, continues until either the secondary voltage is high enough to jump the spark gap, or until all the energy is in the electrostatic form. In normal operation the former result occurs when only a small part (about 10 per cent) of the magnetic energy has been transformed When, however, the system by reason of too small current at break or too long spark gap is just at the limit of sparking, the following relation holds:

$$E_s = I_b u \sqrt{\frac{L_1}{C_1 + u^2 C_2}}, \qquad \text{approx.} \tag{1}$$

Where

E_s = limiting secondary voltage (volts)
I_b = primary current at break (amperes)
u = ratio of turns secondary to primary
L_1 = self-inductance of primary (henries)
C_1 = capacity of primary condenser (farads)
C_2 = capacity of secondary condenser (farads)

The above equation is useful as indicating the variation of E_s with I_b, u, C_1 and C_2 but, in computing absolute voltages, the result must be corrected by a factor of from 0.5 to 0.7 to allow for losses

In most magnetos

$$C_2 = \frac{C_1}{u^2} \qquad \text{roughly} \tag{2}$$

In case the secondary terminals are shunted by a resistance, as is the case when the spark plugs are fouled with carbon deposits, the maximum voltage attained is much less than that given by equation (1), but can be roughly estimated from the following considerations: If the secondary were short-circuited and a current equal to $\frac{I_b}{u}$ were induced in it after "break," the magnetomotive force

acting on the core would be the same as before "break," and there would be no decrease of flux and no induced e.m.f. Consequently this current $\frac{I_b}{u}$ is the limiting value which the *secondary winding* can ever supply. A fouling resistance (S) sufficiently low to produce misfiring practically constitutes such a short-circuit, and the voltage across such a resistance is therefore always somewhat less than the corresponding IR drop, viz.,

$$E_s = \frac{I_b}{u} S. \tag{3}$$

For most systems the current $\frac{I_b}{u}$ is about 0.1 ampere and hence a 3000 volt spark gap would not fire if shunted by 30,000 ohms.

Period 3. — After the attainment of breakdown voltage of the spark gap during Period 2, the gap becomes conducting and the secondary condenser discharges through it, giving a condenser type of discharge, which, as explained above, is probably always intense enough to produce ignition.

Period 4. — After the completion of the condenser spark the magnetic energy still left in the coil continues to deliver an inductive spark for a duration of several thousandths of a second, until the entire energy is dissipated. The secondary current during this period decreases linearly with time, giving a wave roughly triangular in shape. The spark delivered by a magneto is usually of decidedly longer duration than that for a battery system, both because of the greater amount of stored magnetic energy, and also because the voltage generated in the secondary circuit by rotation may (in case the spark is fully advanced) force an additional flow of current across the gap.

SPARK PLUGS. — The function of the spark plug is to maintain within the combustion chamber a gap of definite breakdown voltage between two electrodes, one of which is almost universally grounded on the cylinder wall. The three principal requirements are that the sparking voltage should not exceed about 6000 volts; the insulation resistance should be at least 100,000 ohms, and the plug should be practically gas-tight. In the most severe cases (aircraft engines, etc.) the plug must satisfy these requirements while under pressure of 500 to 1000 pounds per square inch, while immersed in a medium which alternates rapidly in temperature between 0 and 2500° C., and in an atmosphere which tends to deposit soot on the surface of the insulator.

Spark-plug Troubles. — The various manners in which a spark plug may become inoperative are:

(1) **Fouling with Carbon Deposit Causing a Short Circuit.** — This is probably the most frequent cause of trouble and results from excess oil, too rich a mixture, or a plug whose insulator operates at too low a temperature. It may be avoided by either (a) so shaping the insulator that part of it is hot enough to burn clean, or (b) by so shielding the insulator with baffles that it is not exposed to oil. (*See section on Auxiliary Gap below.*)

(2) **Fouling with Oil Causing Open Circuit.** — The breakdown voltage of oil is very much greater than that of an equal thickness of compressed gas, so that the accumulation of a drop or film of oil on the electrodes may increase the breakdown voltage of the gap to a value in excess of that generated by the coil. This usually develops at light loads and on starting, where an excess of oil is supplied to the cylinder. Small electrodes so shaped as to drain the oil away from the gap tend to reduce this trouble.

(3) **Cracking of Insulator.** — This may result from mechanical shock or differential thermal expansion. The trouble is absent in mica plugs, and can be greatly reduced in others by the use of the special porcelains recently developed for spark plugs, which have relatively high mechanical strength and thermal conductivity and small thermal expansion.

(4) **Pre-ignition.** — This occurs in hot high-compression engines and results in loss of power and overheating. The remedy is short thick electrodes and thorough cooling of the plug.

(5) **Conduction through the Insulator.** — At high temperatures all ceramic bodies become to some extent conductors of electricity and different varieties of porcelain differ greatly in this property. The better grades, however, show so little of this effect that it is seldom of serious importance in engine operation.

(6) **Minor Troubles,** such as warping and corrosion of the electrodes, may usually be avoided by proper design and material.

Construction of Spark Plugs. — Spark plugs are built in a very great variety of forms, most of which show very little improvement over the simple and conventional type (Fig. 4). Numerous tests have shown that gas-tightness is a matter of workmanship rather than of design. The insulator and the shell are usually held together either by a threaded bushing, as indicated, or by crimping the edge of the metal shell over the shoulder of the insulator. The latter construction is rather cheaper, but prevents the removal of the insulator for cleaning, and also, unless carefully done, is liable to set up undue strains in the porcelain. Either construction can be made satisfactorily tight. The inner end of the shell is frequently nearly closed, leaving a small annular spark gap around the electrode. Such "closed end" plugs are suitable for "oily" engines. Steel is usually preferable to brass for the shell, as the latter conducts more heat to the outer part of the insulator.

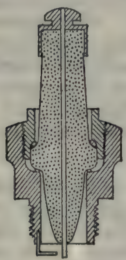

Fig. 4. Cross-section Typical Two-piece Spark Plug.

Mica and various ceramic bodies (steatite, lava, glass, quartz, etc.) are used as insulators. The former has the advantage of freedom from cracking, but gives a surface which is somewhat more liable to collect carbon.

The electrodes are usually of nickel or nickel-manganese alloy, but occasionally copper or silver is used in very hot engines. They are made in a great variety of shapes, although this has no observable effect on the character of the spark except as it affects the breakdown voltage of the gap. This voltage depends on the length of the gap, the shape of the electrodes, the density of the gas, the temperature, and possibly the materials of the electrodes, and upon the rate at which the voltage is applied. The voltage increases nearly linearly with the density of the gas, whether the density results from increased pressure or lowered temperature. For the usual gap length (0.020 inch) the sparking voltage is between 2000 and 6000 volts, depending on the compression ratio of the engine. When the electrodes are at a higher temperature than the gas, the sparking voltage is very considerably decreased below the value to be expected from the density.

Auxiliary Gaps. — If a well-insulated spark gap of proper length is connected in series with a spark plug which is so heavily shunted by carbon that it normally does not fire, the plug will often be found to spark satisfactorily. The beneficial effect of this simple device is due to the fact that it prevents a flow of secondary current during Period 2 (see *Cycle of Operation* above), until the volt-

age at the magneto (or coil) terminals is high enough to break down the external gap. When this occurs this full voltage is suddenly applied to the spark plug gap, and this will also spark if its break down voltage is less than that of the external gap. On the other hand, in cases where failure to fire is caused by insufficient coil voltage, or oil fouling on the plug electrodes, the introduction of a series gap is a positive detriment. Devices of this type are widely advertised as "Spark Intensifiers" or "Energizers" and extravagant claims are made as to increase in engine power and economy, which, however, are justified only in cases where there has been serious misfiring previous to their installation.

TESTING. — The principal overall test of an ignition system is its ability to fire a set of standard spark gaps satisfactorily, as many gaps being provided as there are engine cylinders. The most generally used (but far from satisfactory) gap is the "three-point" gap shown in Fig. 5. "The third or teaser electrode in this type of gap is left insulated from the rest of the circuit but when properly adjusted draws a tiny pilot spark which provides ionization for the main spark gap. Such a spark gap with 5-mm. spacings between the main electrodes has a sparking voltage of about 8000 volts, and is much more regular in its operation when subjected to the sudden impulse from a spark coil than are most forms of two-point gap.

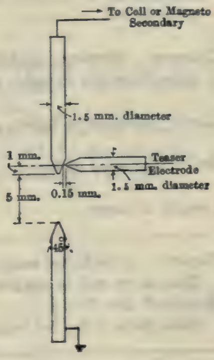

Fig. 5.

The system should fire a set of these gaps regularly and without excessive arcing at the breaker at speeds above that at which it will normally operate. Magnetos are also tested for the low-speed limit at which they will just fire such gaps regularly. This limit should for most purposes be lower than 150 r.p.m. Similar tests may be made with the gaps shunted with 100,000 ohms to imitate the conditions of carbon fouling. Spark plugs should if possible be tested by long runs both at load and idling in the type of engine in which they are to be used, since conditions of oil, temperature, etc., vary greatly. Plugs when tested in the laboratory should show less than 1 cc. per second gas leakage under a pressure of 15 kg. per square centimeter (225 pounds per square inch); should show no cracks when the insulators are quenched in water after being heated to 150° C.; and should withstand 25,000 volts a-c. between the center electrode and the outer surface of the insulator when tested cold under oil. (See Report No. 51–III, National Advisory Committee for Aeronautics.)

DIMENSIONS AND COSTS. — The standard dimensions adopted by the Society of Automotive Engineers for magnetos for 4 and 6 cylinder engines are as follows:

DIMENSIONS OF MAGNETOS, S.A.E. STANDARD

	Dimension, millimeters	Dimension, inches
Height of shaft for 4 or 6 cylinder magnetos	45	1.771
Distance from center of front base plate holes to large end of shaft taper	53	2.086
Distance from center of front base plate holes to center of rear base plate holes	50	1.968
Distance between centers of base plate holes left to right	50	1.968
Large diameter of taper	15	0.590
Small diameter of taper	12	0.472
Length of taper	15	0.590
Taper 1 : 5 (included angle) 11° 30′. Thread on end of magneto shaft	⅜ in., 16 threads per inch, U. S. Std.	
Woodruff key No. 3. Base plate holes for bolts	⅜ in., 16 threads per inch, U. S. Std.	

Three forms of spark-plug threads are in general use in the United States, the dimensions standardized by the Society of Automotive Engineers being as follows:

SPARK-PLUG THREADS

	⅞ in.–18 Size		Metric Size		½ in. Size
	Max. inches	Min. inches	Max. mm.	Min. mm.	
Outside diameter	0.875	0.872	17.97	17.85	....
Pitch diameter	0.839	0.836	17.00	16.88	.7584 at small end
Root diameter	0.803	0.800	15.86	15.74	....
Pitch	18 per inch		1.5 millimeters		14 per inch
Depth of thread	....		....		0.571 inch
Taper	None		None		1 in 16 on diam.
Angle of thread	60°		60°		60°

COSTS. — The following are approximate costs as of 1922:

High-tension Magnetos		
Small 1-cylinder type	$35	– $45
4-cylinder medium grade	55	– 80
4-cylinder high grade	100	– 185
6 and 8-cylinder high grade	120	– 200
Dual or two-spark type	95	– 260
Oscillating igniters	20	– 40
Battery Systems:		
Coil alone	6	– 10
Breaker and distributor	8	– 16
Generator	50 approx.	
Storage battery	40 approx.	
Spark Plugs	0.35–	2.00

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ILLUMINATION, INTERIOR. — (*See also Illumination, Laws of; Illumination, Street; Vision, Laws of.*) The chief problems in interior illumination are: (1) to render visible certain objects or surfaces so as to avoid fatigue and injury of the eyes, and to enhance in effectiveness all operations dependent on good vision; (2) to reveal in their true values the architecture and decoration of interiors; and (3) to produce intrinsic artistic effects through the control of the color, intensity and direction of light. Engineering practice has been developed mainly with reference to the first of these problems, though the other two are often of chief importance and must at all times be considered. For a discussion of the hygienic aspects of illumination, see *Vision, Laws of.*

Advantages of Good Light. — The advantages of good light are so apparent that a list of its principal effects may seem commonplace. These effects are of such great importance, however, particularly in their relation to factory work, that they should receive careful attention. The main advantages of good light, including the effects of bright and cheerful surroundings in factory buildings, include the following:

1. A tendency to reduce accidents.
2. Promotes greater accuracy in workmanship.
3. Contributes to increased production for the same labor cost.
4. Tends to reduce eye strain.
5. Helps to improve working and living conditions.
6. Affords greater contentment among the employees.
7. Order and neatness around the shop are encouraged.
8. Supervision of the men is made easier.

It is rather interesting to note that items 4, 5, 6, 7 and 8 all have a certain bearing on accident prevention in factory work.

Reference Surfaces. — Illumination problems are usually worked out by reference to one or more planes or surfaces whose illumination can be taken as an index to the meeting of the general requirements. Unless special considerations dictate otherwise the plane selected is usually horizontal and at a height of from 2½ to 3 feet above the floor, corresponding with that of table tops, desks, counters, work benches, etc. In many cases other surfaces, as the floor, walls, faces of shelving, stacks of books, etc., are more significant than an arbitrarily chosen horizontal plane. The proper reference surface for the illumination of show windows is usually a surface inclined or curving upward from the base of the window. In many art galleries and on the stage of a theatre the proper reference planes are vertical. The most significant reference planes for any problem should be selected by a study of the special requirements.

Coefficient of Utilization. — The ratio of the luminous flux received by a reference plane to that produced by the light sources used for its illumination is known as the coefficient of utilization of the system. The light received by a reference plane is often termed useful flux to distinguish it from the total flux output of illuminants. The above terms may be very misleading if used indiscriminately, for the flux received by other surfaces than a reference plane is usually essential to the effectiveness of a system of illumination.

DIRECT, INDIRECT AND SEMI-INDIRECT ILLUMINATION. — Methods of illumination may be classified according to the manner in which light is delivered to the working surfaces. Direct light is that received directly from lamps and their accessory shades and reflectors. Indirect light is that received by reflection from some extended diffusing surface, as a ceiling or wall. Systems of illumination are either direct, indirect or semi-indirect. In the direct system the illuminants, including their reflectors and shades, are exposed and

deliver a considerable part of their light in the direction of the reference areas. Indirect light from the walls and ceilings is received as an auxiliary component. In the direct system the lower surfaces of a room have the highest illumination. The illuminants produce shadows, the sharpness of which vary inversely with the diffusion of light. Polished or glazed surfaces may produce an annoying glare in certain lines of vision, due to the specular reflection of the exposed light sources. Unshaded lamps are a source of great annoyance and fatigue. The shadows of direct lighting intensify and in some cases exaggerate relief effects, and may be made to serve as aids to vision where color differences are slight.

The indirect system of lighting conceals all primary sources and distributes light from large diffusing surfaces, usually white ceilings and upper walls. The upper surfaces of the room have the highest illumination. Shadows are eliminated or greatly reduced in intensity. Glare from visible light sources is avoided and the specular reflection of glazed surfaces is generally inappreciable. The uniformity of illumination on the lower surfaces of a room is very great. The great diffusion of light in the indirect system is frequently stated to closely reproduce daylight. This is not strictly true, however, for daylight though diffused is directed and gives a diminishing gradation of illumination from the lower to the upper part of the room. The faintness of shadows with indirect lighting tends to flatten relief effects.

Semi-indirect lighting mediates between the two extremes. The light sources are shielded from direct vision by inverted bowl reflectors of translucent glass, which transmit downward a moderate amount of direct light and reflect the remainder to the ceiling. There is an absence of agreement as to the most favorable ratio of direct to indirect light. It is desirable that the brightness of the translucent bowls shall equal or slightly exceed that of the ceiling. The direct component should be adequate to create a normal relief effect and to bring up the illumination of working surfaces to a value above that of the background. The experiments of T. W. Rolph (*Trans. Ill. Eng. Soc., Vol. 7, pp. 234 and 549*) indicate that very satisfactory results are secured with a direct component of about 15 per cent.

Still another system which has proved very satisfactory in certain cases is that known as the luminous bowl fixture system, in which the component of direct light through the under bowl may be controlled by the choice of density for the small inner diffuser. By this means it is possible to keep the brightness of the under bowl of the fixture down to practically any degree that may be thought desirable. This fixture is of particular value where Mazda C lamps, with their high brightness, are employed within the bowl of the fixture.

In comparing the various systems outlined a large allowance must be made for differences of psychological effect. Critics of indirect lighting assert that the complete elimination of shadows and the reversal of the customary gradation of brightness are very annoying, distract attention, distort the sense of distance and promote fatigue. Others find the same aspects advantageous and hold that the elimination of glare by specular reflection outweighs other considerations. The unfavorable qualities of direct lighting avoided by the indirect method are largely abuses due to unshaded and poorly-located lamps rather than inherent faults. Several investigators have studied the relative illumination intensities required for equally effective vision by the different systems. The observed differences are inconsistent and are probably due to incidental conditions, such as unequal contrast between working surfaces and backgrounds and to direct glare from visible light sources. It is agreed that with equal skill in design direct lighting ranks first, semi-indirect lighting second and indirect lighting third in coefficient of utilization.

Desirable Color and Direction of Illumination.—There is no well-defined color standard in illumination. Daylight white is desirable for exact

color matching and for color printing, but is not essential for other purposes. The Moore carbon-dioxide tube reproduces daylight white with great fidelity and also closely approaches daylight in diffusion. True white may also be obtained from arc and incandescent lamps by the aid of color screens to filter out the hues present to excess. Magnetite arcs and nitrogen-filled tungsten lamps are best adapted to this process. Of the illuminants in common use the white flame arc and the intensified carbon arc give the nearest approach to white. The nitrogen-filled tungsten lamp surpasses other incandescent lamps in whiteness. The ordinary tungsten lamp and the best grade of Welsbach mantles are somewhat unlike in tone, but differ about equally from white. Light having a predominant hue, as that of the mercury arc, is superior for the revelation of fine detail. Light of amber tint is regarded as softer and warmer than light of bluish or greenish tint. The tint of indirect light is modified by the color of the reflecting walls and ceilings.

In determining the direction of light the avoidance of glare and shadows in the visual field is most important. In the display of art objects and architectural details the dominant direction should be carefully chosen to produce a correct sense of relief. An inversion of light and shade by artificial light as compared with daylight is especially to be avoided. Head and hand shadows are especially to be avoided in reading and in mechanical operations. The old rule of light from above the left shoulder is an excellent one. It has been found advantageous in schoolrooms, offices and drafting rooms to locate desks and tables with the windows to the left of those at work and to displace the artificial light sources from symmetrical positions toward the windows in order to give a directive effect similar to daylight.

QUANTITY OF LIGHT AND NUMBER OF ILLUMINANTS FOR DIRECT LIGHTING. — The intensities of illumination required for various purposes are given in the article on *Vision, Laws of*. The simplest illumination problems are those in which a uniform illumination of E foot candles is desired on a horizontal plane of A square feet. The useful flux required is then A times E. If a factor of utilization K is known or assumed for the conditions of the room the total flux F, in lumens, to be produced by the illuminants is

$$F = \frac{AE}{K}.$$

The total watts of electric power or the cubic feet per hour of gas consumption to produce the required flux is found by dividing the required flux by the lumens per watt or per cubic foot per hour given by the type of light source under consideration.

LUMENS ON REFERENCE PLANE PER WATT, ELECTRIC LAMPS

(Lamps and glassware are assumed to be clean and at a height above the reference plane not exceeding 15 feet.)

Lamp	Reflector or globe	Ceiling	Walls	Effective lumens per watt
Tungsten	Silvered reflector	Light	Light	6.1
Tungsten	Prismatic reflector	Light	Light	5.0
Tungsten	Prismatic reflector	Light	Dark	4.0
Tungsten	Enamelled reflector	Light	Light	3.5
Tungsten	Enamelled reflector	Light	Dark	3.0
Gem	Prismatic reflector	Light	Light	2.2
Gem	Prismatic reflector	Light	Dark	1.8
5-amp. d-c. arc	Opal inner globe	Light	Medium	2.0
Mercury arc	Enamelled reflector	Medium	Medium	5.5

LUMENS ON REFERENCE PLANE PER CUBIC FOOT OF GAS PER HOUR

(Lamps and glassware assumed to be clean; new mantles; height above reference plane not exceeding 15 feet; gas, 700 B.t.u. per cu. ft.)

Lamp	Reflector or globe	Ceiling	Walls	Effective lumens per cu. ft. per hour
Upright mantle	Opal globe	Light	Light	49
Upright mantle	Opal globe	Light	Dark	27
Upright mantle	Opal reflector	Light	Light	85
Upright mantle	Opal reflector	Light	Dark	50
Inverted mantle	Prismatic reflector	Light	Light	140
Inverted mantle	Prismatic reflector	Light	Dark	128
Inverted mantle	Roughed ball	Light	Light	101
Inverted mantle	Roughed ball	Light	Dark	75
4-mantle upright arc	Alabaster globe	Light	Light	66
4-mantle upright arc	Alabaster globe	Light	Dark	48
5-mantle inverted arc	Alabaster globe	Light	Light	87
5-mantle inverted arc	Alabaster globe	Light	Dark	65

For detailed data on the lumens per watt or per unit of gas consumption given by various illuminants, see *Lamps, Incandescent; Lamps, Arc* and *Gas Lighting*.

Estimates of Effective Flux. — The preliminary estimates for most problems in interior illumination can be made with fair accuracy from the preceding tables, which give empirical values based on laboratory and service tests. (*See Trans. Ill. Eng. Soc., Vol. 3, p. 518, and Vol. 4, pp. 321, 849, 885.*)

Location of Illuminants. — Having found the total wattage or rate of gas consumption to be provided the next step is to select a proper number and arrangement of lamps. It is seldom possible to produce a uniform illumination from a single source along a radial distance greater than the height of the lamp above the plane. It is practically impossible to avoid shadows from a single central unit. A symmetrical and uniform spacing of several lamps is necessary for uniform illumination by direct lighting. The use of many small units with close spacing promotes uniformity and reduces shadows. The use of few large units widely spaced promotes economy. In many cases a compromise must be made between the two plans. The various spacing schemes in vogue are largely reducible to three types, viz.: (*a*) a long and narrow room with a single row of lamps on the center line, the spacing about equal to the width of the room; (*b*) a rectangular room divided into squares with lamps located at the center of each, the width of each square preferably not greater than twice the height of suspension of the lamps above the reference plane; (*c*) lamps at corners of equilateral triangles and spaced at not more than twice suspension height spacing to walls being half the spacing from lamp to lamp. Plans of these schemes are shown in Fig. 1.

In determining the spacing of lamps very careful attention should be given to the structural divisions of ceiling space. Each bay or division may properly have a symmetrical spacing as this will greatly reduce the shadows cast by pillars and beams. It is possible to secure a very uniform direct illumination from any unit giving a symmetrical distribution if a certain critical ratio of height to spacing is exceeded. The value of this critical ratio depends on the

form of light distribution, as shown by the prototype curves of Fig. 10 in the article on *Illumination, Laws of*. These critical ratios, while of interest, are scarcely ever so exact as to warrant the placing of too much dependence upon definite numerical figures, and it should also be remembered that where uniform illumination is necessary, direction of light and shadows are usually of importance as well as uniformity. According to Harrison, it is difficult to obtain illumination results which are satisfactory in all of these respects with a spacing ratio greater than 1.67.

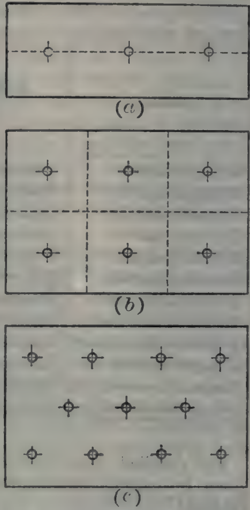

Fig. 1. Spacing Schemes for Direct Lighting

Keeping in mind that there are numerous types of reflectors such as those of metal, opal glass, and the like, the following notes on Holophane and on X-ray units are included merely to illustrate the meaning of the critical ratio mentioned in the preceding paragraph.

Holophane reflectors for tungsten lamps are made in three types, viz., extensive, for wide spacing, the critical ratio of height to spacing being 0.5; intensive, for medium spacing, the critical ratio being 0.67; focussing, for high suspension and close spacing, the critical ratio being 1.33. A spacing chart for Holophane and for X-ray reflectors is shown in Fig. 2. With a ratio of height to spacing below the critical value the light is spotted. With a higher ratio the illumination remains uniform, but with some loss of utilization efficiency, due to the increased absorption of light by the walls. The efficiency does not follow an inverse square relation to the height in any case and is but slightly affected

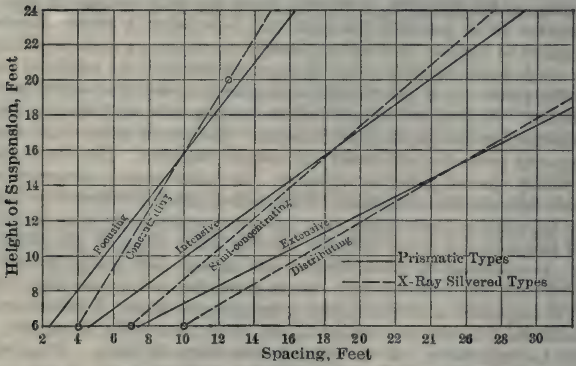

Fig. 2. Maximum Spacing of Reflectors for Uniform Illumination

when the walls are of light color and the reflectors give a strong downward distribution. In very large rooms the effect of suspension height on utilization efficiency is trifling. The suspension height should if possible be sufficient to remove the light sources from direct lines of vision.

Single-lamp Units vs. Clusters. — A single-lamp electric unit is less expensive to install and maintain and is usualy more efficient than a cluster

of several lamps having the same total candle-power. The cross-absorption of light in clusters may be as high as 10 per cent. Gas clusters of the so-called "arc" type have an advantage over single-mantle lamps in heat conservation and facility of remote control. In fact, single-mantle lamps offer a small range of candle-power.

Point-by-point Calculations of Illumination. — Such calculations are very useful in checking up the distribution of illumination. The method of calculating is described in the article on *Illumination, Laws of*.

METHODS OF INDIRECT LIGHTING. — The process of determining the power required to produce a given illumination of any area by indirect means is the same as that outlined above for direct lighting, a suitable value of the efficiency of illumination being used. There are two types of indirect lighting, the bowl and the cove. Indirect lighting from suspended bowls affords greater flexibility in light distribution than cove lighting and is capable of producing more uniform and efficient effects. The efficiencies of utilization obtainable with indirect bowl lighting are given by the National X-ray Reflector Co. as ranging from 0.20 to 0.32 with dark walls and from 0.24 to 0.34 with light walls for ratios of minimum floor dimension to ceiling height of from 1.0 to 3.5; the ceilings are assumed to be painted white and an allowance of 20 per cent for loss of light by dust and lamp aging is included in these values. The proper suspension height for indirect bowl units is approximately three-quarters the height of the room. In spacing the units a symmetrical arrangement at the centers of ceiling squares is highly desirable. The maximum dimension of such a spacing square or rectangle should not exceed a certain ratio to the ceiling height for the best results, viz., for ceiling heights below 12 feet, the maximum ratio is 1.5; for ceiling heights from 12 to 17 feet, the maximum ratio is 1.75; above 17 feet, 2.0.

In the cove type of indirect lighting the lamps are placed in the trough of the cove with axes horizontal, and are backed with a trough reflector of high efficiency. Light is thus thrown to the upper surface of the cove and from it reflected into the room. Cove lighting about the base of a flat-domed ceiling is somewhat more effective than with a flat ceiling. Dust tends to collect on lamps and reflecting surfaces and seriously reduces the average efficiency of the system.

MAINTENANCE OF EFFICIENCY. — Loss of efficiency in lighting systems may be due to the aging of lamps, failure to maintain proper lamp voltage, and the collection of dirt on lamps, reflectors, shades and the reflecting surfaces of the room. Systematic inspection and cleaning are essential if the most economical results are to be secured from large systems. Lamps should be replaced when the bulbs become badly blackened. Globes and reflectors should be cleaned at least once a month for most effective service. C. E. Clewell, *Factory Lighting*, p. 48, reports that a depreciation test of an office installation of tungsten lamps in glass reflectors showed a gradual loss of efficiency which reached a steady value of 19 per cent in 30 days. A similar test in a factory showed a gradual reduction of efficiency which reached a steady value of 48 per cent in 24 days.

In the table on page 804 are given the approximate losses expressed as percentages of the initial illumination on the working plane are taken from *Lighting Data* of the Edison Lamp Works by A. L. Powell for April, 1920.

Arc lamps depreciate in efficiency between trimmings and cleaning from 15 to 30 per cent due to the collection of ash on the inner globes. Arcs are relatively little affected by outside dirt and are quite generally preferred to incandescent lamps in very smoky and dusty locations. Careful attention should be given

	4 weeks	8 weeks	12 weeks	16 weeks	20 weeks
RLM standard dome..	5	8	10	12	14
Dense opal bowl direct lighting..........	7	10	13	16	19
Prismatic bowl direct lighting............	9	13	16	19	22
Light density opal bowl direct lighting......	12	18	24	28	30
Semi-indirect.........	14	22	29	35	40
Totally indirect.......	20	29	37	44	50

to the ceilings of rooms with indirect lighting to prevent the accumulation of dust and the loss of whiteness.

SPECIAL LIGHTING PROBLEMS. — In art galleries for the exhibition of paintings a moderate general illumination should be provided from sources giving good diffusion. Paintings should be illuminated by direct light received from concealed sources several feet in front of, and slightly above, the level of the paintings. The direction of this light should be carefully studied to avoid specular reflection from glass or glossy portions of paint. For good color effects the light should approach as near as possible the color of daylight. Sculpture is effectively lighted by indirect or semi-indirect methods, but the illuminants may properly be suspended along the side of the room containing the windows to obtain directed illumination sufficient to reproduce the relief effects of daylight.

In ritualistic churches it is important to produce a brilliant illumination of the sanctuary with a large vertical component by means of concealed lamps. The distribution of lamps in the nave should be chosen to reveal the architectural effects of the structure. Large and low-hanging pendant fixtures in the axis of the room are generally undesirable. Bracket clusters, groups of small incandescent lamps worked into the capitals of pillars, and small pendant fixtures in the arcades can be used with excellent effect. In non-ritualistic churches adequate reading light is desired at all pews. Brilliant light sources in the field of vision are especially to be avoided. If a balcony is used the light sources should be hung high and thoroughly diffused. The platform should be brightly lighted, but by light sources not visible to the auditors.

Stages and platforms in theatres and public halls should have high illumination, especially in vertical planes, but the light sources should be entirely concealed.

Office desks should be lighted with a view to preventing head and hand shadows and glare from glossy surfaces. Well-diffused general lighting with a dominant component from above the left shoulder is perhaps the best solution of the problem. When local lighting is required a pendant lamp in a deep conical reflector of green flashed opal glass suspended at a height above the desk of 2 to 2.5 feet and near the left edge of the desk will fulfill all requirements.

Show windows should be lighted by entirely concealed light sources placed at or above the upper edge of the window. Illumination should be designed for a surface which is inclined or concave upward from the lower edge of the window. On brilliantly lighted business streets a very high illumination of 20 foot-candles or more is necessary to attract attention. On less brilliantly-lighted streets the illumination may be reduced in proportion. The lamps should be backed by highly efficient reflectors. The form of light distribution

desired depends on the dimension of the window, the degree of concentration increasing with the ratio of the height to the depth of the window space. Tests reported by H. B Wheeler (*Ill. Eng. Soc., Vol. 8, p. 555*) show that the coefficients of utilization on the trim surface depend on the ratio of height to depth of the window space. The results cited range from 58 per cent for a height to depth ratio of 1, to 42 per cent for a ratio of 2. Show cases in the aisles of stores afford a problem akin to show-window lighting. In brightly-lighted rooms the illumination of the interior of the case must be very high to gain attention. Entirely concealed lamps are required and these may properly be placed in shallow trough reflectors in the upper dihedral angles of the case. Tublar and linolite lamps are appropriate, due to their space economy.

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ILLUMINATION, LAWS OF. — (*See also Angles; Photometric Quantities; Vision, Laws of.*) Light may come from either a point source or a finite source. Serious errors may result from the application of the laws of point sources to the illumination from finite areas, due to the non-parallel rays. The errors are negligible, however, when the distance separating the light source and the illuminated surface exceeds ten times the greatest projected dimension of either in a plane normal to the mean direction of light. For definitions of quantities see *Photometric Quantities.*

LIGHT FROM A POINT SOURCE. — When the light comes from a point source the "inverse square" and the "cosine" laws apply:

Inverse Square Law. — The illumination from a point source varies inversely as the square of the distance from the source. This law rests on the geometrical fact that the spherical area intercepting light flux varies as the square of its radius.

Cosine Law. — The illumination received by an element of surface varies as the cosine of the angle of incidence. The basis of this law is also geometrical. These two laws may be stated thus:

$$E = \frac{I \cos \alpha}{l^2},$$

where E = intensity of illumination in foot-candles; I = intensity of the light source in candle-power; α = angle of incidence of light on the illuminated area; and l = distance in feet from the light source to the point illuminated.

Calculation of Light Flux and Mean Intensities from Point Sources. — Let I_θ be the mean intensity of a light source at an angle θ from the axis about which the light distribution is symmetrical, usually the vertical axis. The flux within an elementary zone at this angle of inclination is then

$$dF = I_\theta \, d\omega = 2\pi I_\theta \sin \theta \, d\theta.$$

The flux within any finite zone about the axis of distribution may be found by integration when the law of distribution is known. In general, however, it is necessary to employ graphical means of integration for this purpose. The mean intensity within any finite zone cannot be found by direct average, since each beam bears a weight proportional to $\sin \theta$. The graphical methods described below are the most serviceable.

Protractor Methods. — This method is adapted to quick approximations of mean zonal, mean hemispherical and mean spherical intensities. The method assumes a sphere of light distribution divided into zones of equal solid angular content, i.e., of equal altitude. A direct average of the mean intensities of the several zones gives the general mean for all the zones included, since each component zone bears equal weight. For convenience it may be assumed generally that the mean intensity in any zone agrees closely with that at the angle which bisects the area of the zone. Fig. 1 shows a convenient subdivision of the sphere based on this assumption. To obtain the mean spherical candle-power the intensities at all the angles indicated by radial lines are averaged. This result multiplied by 4π gives the total flux from the source. Similarly the average of the intensities at the angles indicated in either hemisphere gives the mean hemispherical candle-power, which may be multiplied by 2π to obtain the hemispherical light flux. The average taken at the three lowest positions gives the mean intensity

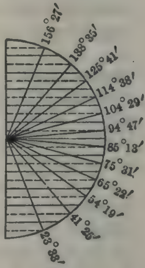
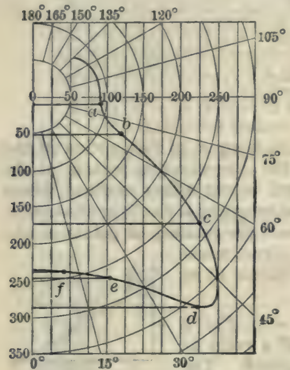


Fig. 1.

below 60° and this value, multiplied by π, gives the flux within the $0° - 60°$ zone. For convenience in practice a transparent protractor may be prepared with the reference angles shown by black lines. This protractor may be placed over the polar distribution curve and the values for averaging read off directly. The accuracy of the method depends on the regularity of the distribution curve.

Flux-o-lite Diagram. — (Fig. 2.) This diagram is constructed from the polar distribution curve by drawing vertical reference lines tangent to the candle-power circles, with uniform intermediate divisions as desired. To find the flux within any zone the horizontal projection of the mid-zone intensity is measured on the scale created by the vertical reference lines. Multiply this value by the constant which corresponds to the arc of the zone in the accompanying table.



$a = 90$ $b = 118$ $c = 225$
$d = 223$ $e = 103$ $f = 41$
Sum $= 800$ $F_{0h} = 800 \times 1.64 = 1312$
$I_{mlh} = 1312 \div 6.2832 = 208.8$

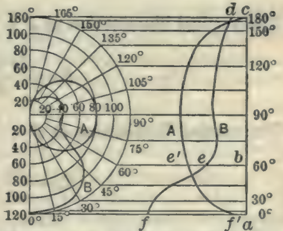
Fig. 2. Flux-o-lite Diagram

The flux within any angular limits is found by summing the components from the several zones included. For accuracy a large number of small component zones should be taken. For proof, it is readily shown that the solid angular content of any zone is equal to

Arc of zone, degrees	Constant
5	0.548
10	1.098
15	1.64
20	2.18
25	2.72
30	3.25

$$4\pi \sin\theta \sin\frac{n}{2},$$

where n is the arc of the zone in degrees and θ the bisecting angle measured from the vertical axis. The above constants in each case equal $4\pi \sin n/2$ and the horizontal projections read from the diagram are in each case equal to the assumed mean intensity of the zone times the sine of the bisecting angle.



A = Bare 80 c.p. Lamp B = Same with Reflector
Area $abce'f' = 15000$ For A, $F_s = 785.4$ $I_s = 62.5$
Area $abcdef = 12080$ For B, $F_s = 632.9$ $I_s = 50.3$
Absorption of Reflector $= 152.5 = 19.52\%$

Fig. 3. The Rousseau Diagram

Rousseau Diagram. — (Fig. 3.) The Rousseau diagram admits of considerable accuracy, but involves the measurement of area. The arcs of the several zones of distribution are projected horizontally on the line *abc*. The intercepts equal the altitudes of the several zones and are therefore proportional to their several solid angular contents, with a total *abc* equal to 4π. On each projection line is laid off from *abc* a length equal to the radius of the polar

curve at the corresponding angle. The terminal points are then connected by the smooth curve *def*. The area *abcdef* so inclosed represents the total flux, since

$$\text{Area } abcdef = \int_0^{4\pi} I \, d\omega = 4\pi I_s.$$

The total flux is

$$F_s = \frac{4\pi \times \text{area } abcdef}{\text{length } ac}.$$

The flux in any zone, e.g., between 0° and 60°, is

$$F_z = \frac{4\pi \times \text{area } abef}{\text{length } ac}.$$

The mean intensity between any angular limits is equal to the portion of the area *abcdef* between those limits divided by the corresponding portion of the base line *abc*. This method is to be preferred to the others outlined where accuracy of a high order is desired, as in the measurement of the light absorption of reflectors, globes, shades, etc. In such cases the number of points of the photometric distribution curve determined by direct measure and referred by projection to the line *def* should be as large as practicable.

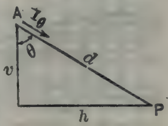


Fig. 4.

Calculation of Illumination at Points. — In Fig. 4 let A be the location of an illuminant and P a point at which the illumination from A is to be determined. The three most important cases of this problem refer respectively to the illumination of elements of surface at P located in horizontal and vertical planes and in a plane normal to the light path AP. From the laws of inverse squares and of cosines (*see above*) the horizontal illumination at P is

$$E_h = \frac{I\theta \cos\theta}{d^2} = \frac{I\theta \cos^3\theta}{v^2}.$$

The vertical illumination at P is

$$E_v = \frac{I\theta \sin\theta}{d^2} = \frac{I\theta \sin^3\theta}{h^2}.$$

The normal illumination at P is

$$E_n = \frac{I\theta}{d^2} = \frac{I\theta \cos^2\theta}{v^2} = \frac{I\theta \sin^2\theta}{h^2}.$$

Calculations of this type are facilitated by a chart (Fig. 5) showing the value of θ for various values of v and h, and by tables giving the values of the illumination constants $\cos^3\theta/v^2$, $\sin^3\theta/h^2$, and $\cos^2\theta/v^2$ for various values of v and h. Such tables are given below.

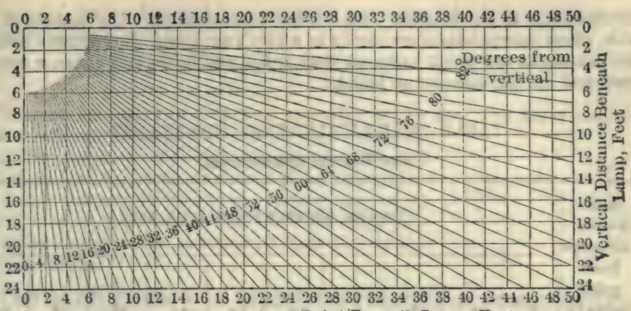


Fig. 5. Angle of Effective Beams for Point-by-point Calculations

TABLE OF ILLUMINATION CONSTANTS

For Horizontal and Vertical Illumination

Horizontal distance from point beneath lamp, feet	Vertical distance below lamp, feet									
	6	7	8	9	10	12	14	16	18	20
0	0.0278	0.0204	0.0156	0.0127	0.0100	0.00695	0.00510	0.00391	0.00309	0.00250
1	0.0266	0.0198	0.0152	0.0122	0.0099	0.00687	0.00505	0.00388	0.00307	0.00249
2	0.0236	0.0182	0.0143	0.0115	0.0094	0.00665	0.00491	0.00382	0.00303	0.00246
3	0.0200	0.0158	0.0129	0.0106	0.0088	0.00635	0.00476	0.00371	0.00296	0.00241
4	0.0160	0.0134	0.0112	0.0094	0.0080	0.00592	0.00453	0.00357	0.00288	0.00235
5	0.0126	0.0111	0.0094	0.0082	0.0072	0.00547	0.00426	0.00340	0.00278	0.00228
6	0.0098	0.0089	0.0080	0.0071	0.0063	0.00496	0.00397	0.00320	0.00264	0.00220
8	0.0060	0.0058	0.0055	0.00515	0.00476	0.00400	0.00333	0.00279	0.00235	0.00200
10	0.0038	0.00385	0.00382	0.00369	0.00353	0.00312	0.00274	0.00238	0.00211	0.00179
12	0.0025	0.00261	0.00266	0.00268	0.00263	0.00245	0.00222	0.00200	0.00178	0.00158
14	0.0017	0.00182	0.00191	0.00195	0.00196	0.00190	0.00180	0.00167	0.00152	0.00138
16	0.0012	0.00131	0.00140	0.00146	0.00149	0.00151	0.00145	0.00138	0.00129	0.00119
18	0.00088	0.00097	0.00105	0.00110	0.00114	0.00118	0.00118	0.00114	0.00109	0.00103
20	0.00066	0.00074	0.00080	0.00085	0.00089	0.00095	0.00097	0.00095	0.00093	0.00088
24	0.00038	0.00044	0.00050	0.00053	0.00057	0.00062	0.00065	0.00067	0.00067	0.00066
28	....	0.00028	0.00032	0.00036	0.00039	0.00043	0.00046	0.00048	0.00049	0.00049
32	....	....	0.00022	0.00024	0.00027	0.00030	0.00033	0.00035	0.00036	0.00037
36	....	....	....	0.00017	0.00019	0.00022	0.00024	0.00026	0.00028	0.00029
40	....	....	....	....	0.00014	0.00017	0.00019	0.00020	0.00021	0.00022
45	....	....	....	....	....	0.00012	0.00014	0.00015	0.00016	0.00017
50	....	....	....	....	....	....	0.00010	0.00011	0.00012	0.00013

The horizontal illumination at any point equals the intensity of the light source in its direction (see Fig. 5) multiplied by the constant in the above table

corresponding to its location. With a few exceptions near the bottom of the table, direct interpolations are correct to within 2 per cent. To compute vertical illumination at any point, exchange the vertical and horizontal distance components and use the constant so found.

TABLE OF ILLUMINATION CONSTANTS

For Normal Illumination

Horizontal distance from point beneath lamp, feet	Vertical distance below lamp, feet									
	6	7	8	9	10	12	14	16	18	20
0	0.0278	0.0204	0.0156	0.0127	0.0100	0.00695	0.00510	0.00391	0.00309	0.00250
1	0.0270	0.0200	0.0154	0.0122	0.00990	0.00690	0.00507	0.00389	0.00308	0.00249
2	0.0250	0.0189	0.0147	0.0118	0.00962	0.00675	0.00500	0.00384	0.00304	0.00247
3	0.0222	0.0172	0.0137	0.0111	0.00917	0.00653	0.00487	0.00377	0.00300	0.00244
4	0.0200	0.0154	0.0125	0.0103	0.00842	0.00625	0.00472	0.00367	0.00294	0.00240
5	0.0164	0.0135	0.0112	0.00943	0.00800	0.00592	0.00452	0.00356	0.00286	0.00235
6	0.0139	0.01175	0.01000	0.00855	0.00735	0.00555	0.00431	0.00342	0.00278	0.00229
8	0.0100	0.00885	0.00780	0.00689	0.00610	0.00480	0.00385	0.00312	0.00258	0.00215
10	0.00735	0.00672	0.00610	0.00552	0.00500	0.00410	0.00338	0.00281	0.00235	0.00200
12	0.00556	0.00518	0.00480	0.00444	0.00410	0.00347	0.00294	0.00250	0.00214	0.00184
14	0.00431	0.00408	0.00385	0.00361	0.00338	0.00294	0.00255	0.00221	0.00192	0.00168
16	0.00342	0.00327	0.00312	0.00297	0.00281	0.00250	0.00221	0.00195	0.00172	0.00152
18	0.00278	0.00268	0.00258	0.00247	0.00235	0.00214	0.00192	0.00172	0.00154	0.00138
20	0.00229	0.00222	0.00215	0.00208	0.00200	0.00184	0.00168	0.00152	0.00138	0.00125
24	0.00163	0.00160	0.00156	0.00152	0.00148	0.00139	0.00129	0.00120	0.00111	0.00102
28	0.00122	0.00120	0.00118	0.00115	0.00113	0.00108	0.00102	0.00096	0.00090	0.00084
32	0.00094	0.00093	0.00092	0.00090	0.00089	0.00086	0.00082	0.00078	0.00074	0.00070
36	0.00075	0.00074	0.00074	0.00073	0.00072	0.00069	0.00067	0.00064	0.00062	0.00059
40	0.00061	0.00061	0.00060	0.00059	0.00058	0.00057	0.00056	0.00054	0.00052	0.00050
45	0.00049	0.00048	0.00048	0.00047	0.00047	0.00046	0.00045	0.00044	0.00043	0.00041
50	0.00039	0.00039	0.00039	0.00039	0.00038	0.00038	0.00037	0.00036	0.00035	0.00034

The normal illumination at any point equals the intensity of the light source in its direction (see Fig. 5) multiplied by the constant in the above table corresponding to its location.

Calculation of Solid Angle Subtended by Illuminated Area. — In many cases it is desired to compute the total flux of light which falls upon a plane area from an approximate point source. This problem resolves itself into two elements, (1) to determine the mean intensity acting toward the illuminated plane, to which the methods previously described apply, and (2) to determine the solid angle subtended by the area illuminated at the source of light. The following theorems apply to the latter problem.

The solid angle subtended by a circle with the light source in its axis (Fig. 6) is

$$\omega = 2\pi\,(1 - \cos\alpha).$$

The solid angle subtended by a rectangle, one corner of which is directly beneath the source of light (Fig. 7), is

$$\omega = \tan^{-1}\frac{ac}{hd}.$$

When the projection of S falls on one edge of the area, the latter may be divided into two rectangles to meet the above theorem. When the projection falls within the area, the area may be divided into four such sections. When the projection falls outside ac in Fig. 7, the surface may be extended to meet the theorem and the solid angle subtended by the extension substracted from that of the combined areas.

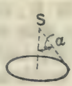


Fig. 6.

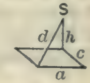


Fig. 7.

STANDARD FORMS OF LIGHT DISTRIBUTION FOR UNIFORM ILLUMINATION. — The most important phase of this problem, viz., the uniform illumination of a horizontal plane by one and by many illuminants, will be outlined. The form of light distribution required to obtain uniform horizontal illumination from a single lamp is found by reversing the expression for horizontal illumination,

$$E_h = \frac{I_\theta \cos^3\theta}{v^2} \qquad \text{to} \qquad I_\theta = \frac{E_h v^2}{\cos^3\theta},$$

taking E_h and v as constants. The resulting form of polar curve for various ratios of the limiting horizontal distance h' to v are shown in Fig. 8. It is apparent that practical difficulties limit the range of application of this method. The more general case must be solved by the use of a considerable number of lamps uniformly spaced above the area illuminated. The type of horizontal illumination curve shown in Fig. 9 adequately meets this condition, as shown by the resultant illumination curves along the side and diagonal of the square included by the points directly under four equally spaced lamps. To obtain the form of illumination curve shown for various ratios of horizontal spacing s to vertical distance to the lamps v, the forms of light distribution shown in Fig. 10 are required.

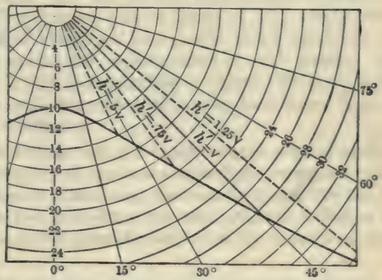


Fig. 8. Light Distribution Required to Obtain Uniform Horizontal Illumination from One Source

The ratios of spacing to height indicated in the figure are to be considered as the maximum ratios with which uniform illumination is obtained. With smaller ratios the uniform condition still exists. With a fixed spacing varying the height within the limit imposed by the ratio affects neither the uniformity nor the inten-

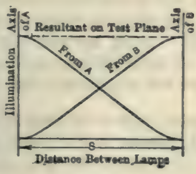


Fig. 9.

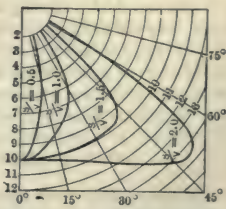


Fig. 10. Curves of Light Distribution for Uniform Illumination from many Sources

sity of illumination, except in that portion of the room lying outside the outer row of lamps. By aid of well-designed reflectors it is possible to closely approximate the typical distribution curves for certain ratios of spacing to height and this ratio is important as a criterion in the selection of reflecting devices.

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ILLUMINATION, STREET. — (*See also Illumination, Interior; Illumination, Laws of; Lamps, Arc; Lamps, Incandescent; Lighting Plants.*) Street lighting is intended to promote public safety, facilitate travel and business, and reveal architectural effects during hours of natural darkness. Experience shows that good street lighting attracts traffic and stimulates retail trade. The most important factors affecting the degree of street illumination required are the density of traffic, prevalence of retail business, degree of police supervision needed and the architectural importance of the thoroughfare. In moderate and dim light vision depends primarily on differences of brightness and is but slightly assisted by color distinctions. Objects may be seen directly and in some detail against a darker background, but are seen in mass or silhouette against a brighter background.

Requirements of Street Lighting. — The most important streets require sufficient illumination for direct and detail vision to clearly reveal vehicles, persons, obstructions, irregularities of the pavement, and to permit the easy reading of timepieces and addresses, which calls for an average of about 0.5 foot-candle and a minimum of about 0.2 foot-candle on the most important, working surface. Uniform illumination without deep shadows, which may be obtained by well-diffused light sources with fairly close spacing, is highly desirable on important streets.

Secondary streets with moderate evening traffic and orderly conditions require greater illumination at intersections than at intermediate points. The silhouette aspect of vision is very important in such streets and emphasizes the need of fairly bright and even roadway illumination without spots of deep shade from foliage. An average illumination of about 0.1 foot-candle and a minimum of about 0.06 foot-candle meet the reasonable requirements of such streets. In this range of intensities the eye is highly susceptible to glare. Brilliant light sources without diffusing globes and suspended at a low level often largely defeat their purpose under these conditions.

Minor streets with scattered buildings and infrequent travel require mainly beacon lighting, equivalent in intensity to moderate moonlight, or normal illumination varying from a minimum of about 0.02 foot-candle to an average of about 0.03 foot-candle. At such low intensities the need of good diffusion at light sources is most acute, but is quite generally neglected for reasons of supposed economy.

Reference Planes. — Street illumination in America is usually referred to normal reference planes (i.e., perpendicular to the light rays), and to horizontal planes in Europe. There is wide diversity in the elevation of reference planes, the most common levels being that of the pavement and that of a plane four feet above the pavement. The use of normal planes of reference takes satisfactory account of the light from but one source, while it is possible to sum up the light received from all directions on horizontal planes. It is evident from the cosine law of incidence that the horizontal illumination from low and distant lamps is less in magnitude and more difficult of exact measurement than the normal component. Normal illumination is a more useful index to the visibility of upright objects. Small objects as cards and timepieces are instinctively held normal to the rays of the nearest lamp. The vertical component of street illumination should have due consideration for its importance in revealing the architecture of buildings and assisting in the recognition of persons.

TYPES OF ILLUMINANTS. — The illuminants most extensively used in street lighting are as follows: (*For detailed descriptions and data see Lamps, Arc, Lamps, Incandescent; and Gas Lighting.*)

Electric Arc Lamps on Series Circuits, including (*a*) carbon arcs of open d-c., enclosed d-c. and enclosed a-c. types, with standard currents of 6.6, 7.5 and 9.6 amperes; (*b*) flame carbon arcs of open d-c., enclosed d-c. and enclosed a-c. types, with standard currents of 6.6, 7.5 and 9.6 amperes; and (*c*) metallic or luminous arcs of open d-c. type with standard currents 4.0, 5.5 and 6.6 amperes, arranged for suspension and pedestal mounting.

Electric Arc Lamps on Multiple Circuits at or near 110 volts, including (*a*) carbon arcs of open d-c., enclosed d-c. and enclosed a-c. types; (*b*) flame carbon of open d-c., enclosed d-c. and enclosed a-c. types; and (*c*) quartz tube mercury arcs.

Electric Incandescent Lamps, Series Type, are made only in the gas filled design with standard currents of 5.5, 6.6, 7.5, 15 and 20 amperes, and a range of candle-power of from 60 to 1000.

Electric Incandescent Lamps, Multiple Type at or near 110 volts of from 10 to 1000 watts.

High-pressure Gas Lamps of upright and inverted types, including (*a*) compressed gas, (*b*) compressed air and (*c*) compressed gas and air types.

Low-pressure Gas Lamps of upright and inverted types, including (*a*) single mantle lamps and (*b*) clusters of 3, 4 and 5 mantles.

Naphtha Vapor Mantle Lamps of self-contained, upright type with nominal candle-power of from 45 to 60.

Relative Advantages of Various Street Illuminants. — The advantages of electric lighting over gas lighting are (1) superior flexibility in sizes and possible locations, (2) greater ease of maintenance, (3) availability of white color, and (4) ease of control from central points with series circuits. The advantages of gas over electricity are (1) lower probability of interruption by accident, (2) steadiness of light and (3) the low intrinsic brilliancy of mantles.

Both electric and gas lamps suffer fluctuations due to unsteady pressure, but these conditions are more easily regulated in electric systems. Skillful maintenance is necessary to insure the proper efficiency of all street lamps, but affords a simpler problem in the case of electric lamps. High-pressure gas lamps require maintenance of an exceptionally high order.

Gas lamps are usually lighted by hand or by local automatic clock devices, though a few methods of central control have been devised. Naphtha vapor lamps are difficult to maintain in good efficiency and require costly attention, as each lamp must be filled by hand and heated by a blast torch before lighting. Such lamps should be used only where electric or gas lighting from central systems is not available.

Series electric lamps tend to economy in power distribution and to simplicity of central control. Separate mains must be used for all circuits requiring central control. Lamps operated in multiple from regular mains are lighted and extinguished by hand or by special clock switches. The high voltage of series circuits involves elements of danger and liabilities to interruption by accidents not found in parallel systems. Parallel systems can employ metal poles and grounded supports with greater safety. Incandescent electric lamps provide a very wide range of candle-power and involve smaller expense for maintenance and renewals than arc or gas lamps. On well-regulated circuits incandescent lamps surpass all other street illuminants in steadiness. The effectiveness of incandescent lamps as commonly installed without diffusing shades is often badly impaired by glare.

The color of light is relatively less important in street lighting than in interiors. White is generally preferred, as satisfying a sense of naturalness. Yellow and green have superior penetrating power in fog and smoke.

Small vs. Large Units. — Large units are generally more efficient than

small units and involve relatively less expense for installation, maintenance and operation per unit of light production. With proper spacing the light of small units can be more completely utilized on roadways and produces a more uniform illumination. Small units lend themselves more readily to decorative schemes and to the lighting of shady streets and curving roadways. The effect of numerous small units at low levels, especially when unshaded, is distinctly more obtrusive and glaring than a small number of high power sources hung fairly high and well diffused.

Ratings of Street Lamps. — Street lamps are variously rated in candle-power, watts and rate of gas consumption. Much confusion is caused by loose usage of the term candle-power, which in different cases refers to mean horizontal, maximum, mean spherical, mean lower hemispherical and merely nominal values. Horizontal and maximum candle-power or that at any specified angle are significant only when comparing lamps having a definite form of light distribution in common. Mean spherical candle-power gives the gross light output of a lamp without any index to the effectiveness with which it may be utilized. It is an appropriate rating for a bare luminous element without accessories. The mean lower hemispherical candle-power is perhaps the best single index to the available light output of a lamp, but it cannot be conveniently measured in service. Candle-power ratings are sometimes of a purely nominal nature, e.g., the 1200 and 2000 candle-power ratings applied to carbon arc lamps denote lamps consuming 330 and 480 watts respectively.

The rated candle-power of gas and gasoline mantle lamps usually represents the maximum intensity which a given type of lamp can produce commercially. This rating almost invariably exceeds the average results in service by a wide margin due to imperfect maintenance, and can serve only as an index to the type of lamp referred to.

Incandescent electric lamps are rated in nominal horizontal candle-power, on the arbitrary bases of 1 candle-power for each 10 lumens of the actual initial performance. Such a nominal rating corresponds approximately to the old actual horizontal candle-power rating.

The candle-power rating of incandescent lamps installed on streets with reflectors is often taken to imply the actual average intensity at some angle between 15° and 25° below horizontal. A test of 130 incandescent lamps with flat enameled metal reflectors in service, which was made by the writer in 1913, showed the average candle-power of such units at 25° below horizontal to be 111.6 per cent of the rated horizontal candle-power of the lamps.

It should be noted that with the development of the Mazda C lamp a dome reflector has superseded to some extent the older flat radial reflector.

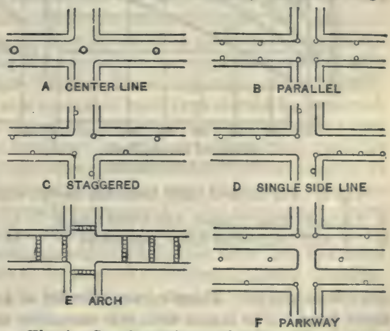


Fig. 1. Spacing Schemes for Street Lamps

Indefinite candle-power specifications in lighting contracts have caused much litigation. Arc lamps of all types are preferably rated in watts, as affording the only accurately measurable index to their performance. The specification that a reasonable average of candle-power as indicated by a prescribed photometric test shall be maintained is recognized as a valuable check on the quality of maintenence and service.

SPACING OF LAMPS. — The plans of lamp spacing most widely used include: (1) center suspension at street intersections only; (2) center suspension at street intersections with one intermediate center suspension; (3) center suspension at street intersections and one intermediate curb unit; (4) curb units, located opposite each other; (5) curb units, staggered; and (6) center park-way with a line of posts down its center. Several examples are shown diagrammatically in Fig. 1. Plan A, or single center line spacing is the most effective method of using high-power units. Lamps may be hung from span wires between poles or buildings or from mast arms projecting over roadways. In some cases lamps are mounted directly on poles or standards in a central parkway or series of safety isles. Center suspension is especially desirable when only one lamp is placed at an intersection. Plan B, or parallel spacing, and Plan C, or staggered spacing, are best adapted to the brilliant lighting of main thoroughfares with high-power units and the lighting of ordinary streets with small units. The staggered arrangement favors the meeting of a definite minimum requirement of roadway illumination with the smallest number of lamps per mile. Plan D, or single side line spacing, sacrifices symmetry and uniformity to the convenience of running electric circuits or gas piping on but one side of the street. Plan E, or arch lighting, is adapted only to incandescent electric lamps; it tends to give a street a festive appearance, but is not an effective method of illumination. Plan F, or parkway lighting, is of obvious value where the roadway is divided as shown.

Side vs. Center Mounting. — Fig. 2 shows the effect of center and side mounting on the per cent of the total light which falls within the limits of streets

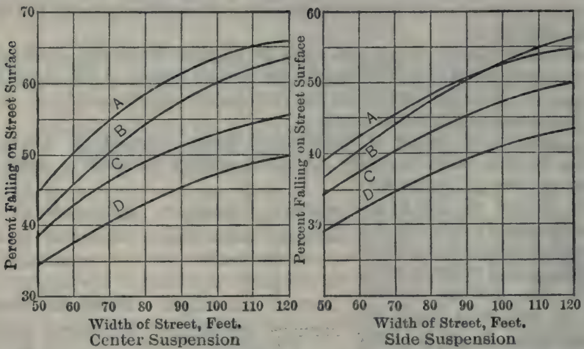


Fig. 2. Per Cent of Light from Arc Lamps which falls within Limits of the Street

A. 9.6 amp. Open Carbon Arc, Clear Globe
B. 4 amp. Magnetite Arc, Clear Globe
C. 6.6 amp. Magnetite Arc, Opal Globe
D. 6.6 amp. Enclosed Carbon Arc, Opal Globe

of various widths. These curves all refer to a mounting height of 20 feet above street level. The lamps with side mounting are assumed to be over a curb line which is distant from the nearer street line by an amount equal to one-fifth the total width of the street. The higher efficiency of center mounting is apparent. (*Data by P. S. Millar, Trans. Ill. Eng. Soc., Vol. 5, p. 658.*)

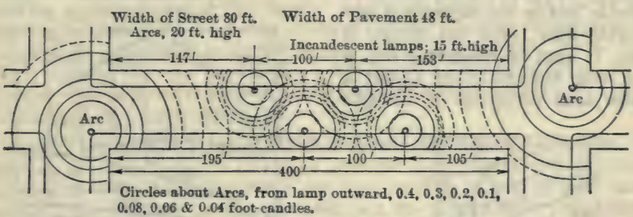
It is obvious that mounting the lamp at the street intersection permits the light to shine up and down both streets and hence utilizes a much larger portion of the light than when the lamp is not mounted at the street intersection. For

this reason, street intersection units, usually on suspension mounting, are likely to continue in use for a considerable time in the future.

Various attempts have been made to increase the percentage of light thrown on the street by reflectors of special design which deflect lengthwise of the street light which would otherwise be thrown to the sides. These have had a very limited acceptance.

Excellent use of large units at intersections and small units at intermediate points may be made on secondary streets, especially where foliage interferes with the distribution of light at distances. On curved roadways it is preferable to locate the lamps on the outer sides of the curves, as they are then visible at greater distances.

Height of Suspension. — The spacing of lamps should be laid out with due regard to the height of suspension of the lamps and the degree of minimum illumination to be provided. In many cities a standard height of mounting is employed with each type of lamp. When such is the case, spacing problems are readily solved by the aid of diagrams such as shown in Fig. 3. Taking account of the suspension height and the light distribution curve of each lamp in its normal service condition, the horizontal distances from the axis of the lamp at



Circles from incandescent lamps outward, 0.1, 0.08, 0.06 & 0.04 foot-candles.

Fig. 3. Spacing and Illumination Plan for a Minimum Normal Illumination of 0.04 Foot-candle

which various normal intensities of illumination are produced at an appropriate level, as that of the street or at a height four feet above the street, are computed by the point-by-point method. Using these distances as radii, circles are drawn to scale about a center representing the axis of the lamp and are marked with the corresponding intensity of illumination. These diagrams are preferably made on translucent tracing cloth and several of each type should be prepared. A plan of the space to be lighted is drawn to the same scale on a separate paper and the circle diagrams are used as templates for various trial spacings until the desired result is obtained. The circles then furnish points for the plotting of illumination profiles or contours at the center line and curb lines, from which the approximate average illumination is readily computed. The illustration shows an appropriate location for 4-ampere magnetite arcs, with globes at street intersections, and 60-candle-power series incandescent lamps with flat enameled metal reflectors at intermediate points, to produce a minimum normal illumination of 0.04 f.c. on a section of street 80 feet wide and 480 feet long lying between centers of intersecting streets.

Selection of Height of Suspension. — The most appropriate suspension height for a street lamp depends on its light output, form of distribution and intrinsic brilliancy, and upon the degree of minimum illumination required. It is practically impossible to realize uniform illumination from large arcs and high-pressure gas lamps without higher suspension than that commonly employed. The usual result is a bright spot immediately about the lamp and low

illumination at mid-points. The contrast so produced reduces the effectiveness of the light.

A high ratio of suspension height to horizontal spacing tends to improve the uniformity of light distribution and to increase the distance over which a single lamp can provide illumination above a specified minimum value. A high ratio of height to distance is especially desirable when the maximum light intensity of the lamp is more than 30° below the horizontal.

The following example may be taken. In Fig. 4 polar curve (*a*) refers to a 510-watt magnetite arc with clear globe and (*b*) to a 450-watt d-c. flame arc with vertical electrodes and a light opal globe. The illumination curves show the normal illumination at various distances for various suspension heights. In case (*a*) the maximum intensity is but 10° below the horizontal and little gain at distant points is obtained by employing a suspension height exceeding 25 feet. In case (*b*), however, the maximum intensity is much lower on the polar curve and a much higher suspension could advantageously be employed. Fig. 5 shows the effect of height of suspension of these two lamps on the normal illumination at various distances.

The actual suspension height is often determined by the exigencies of trimming, by the length of wooden poles available, and by the necessity of distributing light below the foliage of trees. It must be recognized that a lamp with high mounting throws a smaller total percentage of its light on the street, and that the gain is from the better distribution to distant points.

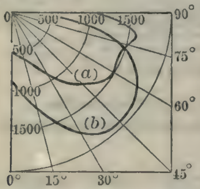
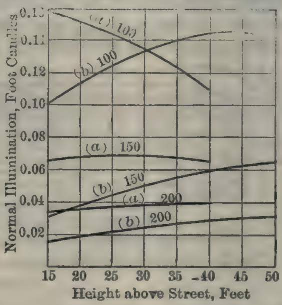


Fig. 4. Polar Light Distribution Curves of (a) 6.6-ampere Magnetite Arc with Clear Globe and (b) 450-watt Enclosed Flame Arc



Numbers on curves are horizontal distances (feet) from source

Fig. 5. Effect of Height of Suspension of Arc Lamps of Fig. 4 on Normal Illumination at Various Distances

In drawing conclusions concerning mounting heights from the foregoing, it is well to remember that the influence of greater mounting heights is to permit wider spacing and that wider spacing tends to decrease installation and operating costs. The decreased blinding influence which is likely to result from greater mounting heights is worthy of careful consideration.

Sweet suggests the use of the letter "M" for the ratio of spacing distance to mounting height and the approximate relation between various values of "M" and uniformity of illumination on the street surface according to Sweet is given in the table on the top of page 819.

LIGHTING SCHEDULES. — All-night schedules provide for a yearly total of from 3830 to 4000 hours of operation. The former total results from lighting 30 minutes after sunset and extinguishing an hour before sunrise. The latter involves one half-hour longer burning each night. The "Philadelphia moonlight schedule" allows lamps to remain unlighted on nights when the moon

Value of M	Approximate ratio of maximum to minimum illumination on the street surface	
	Equipment other than holophane refractor	Holophane refractor
16	300 to 1	33 to 1
12	125 to 1	14 to 1
8	40 to 1	5 to 1
6	18 to 1	2 to 1

is full or nearly so and on other nights the lamps are lighted one hour before moonset and extinguished one hour after moonrise. This schedule calls for an annual total of 2000 hours of operation. The "Frund system" ignores the moon until midnight, after which the provision of the moonlight schedule apply, making an annual total of 3000 hours. As a large portion of the cost of street lighting is due to fixed charges the saving of reduced schedules as compared with an all-night schedule is much less in proportion than the reduction of hours. Each year the *Electrical World* issues a detailed set of lighting schedules for the ensuing year, separate tables being furnished for northern, middle and southern latitudes of the United States.

COST OF STREET LIGHTING (Pre-war figures). — The cost of street lighting is usually based on an annual price per lamp or fixture for a specified number of hours of burning. This cost may properly include all fixed charges on the lamps and their accessories as well as a due proportion of the charges on poles, lines, transformers and other appliances assignable to the service, all costs of maintenence and renewals involved in the service, and a reasonable charge for the energy supplied. These costs naturally vary with the character of the distributing system, value of poles, rate of depreciation of lamps and the cost of producing electrical energy. Depreciation rates on street lamps and special appliances used in supplying them are apt to reach or exceed 10 per cent, due to rapid obsolescence. The average prices charged per year for various types of street lamps operated on all-night schedules in a representative group of cities are given below:

PRICES PER YEAR FOR ELECTRIC STREET LAMPS; ALL-NIGHT SCHEDULE

Type of lamp	Cities averaged	Average rate	Maximum rate	Minimum rate
4-amp. magnetite arc.	27	$63.00	$80.00	$45.00
6.6-amp. enc. carbon arc.........	40	87.00	100.00	74.00
7.5-amp. enc. a-c. carbon arc....	16	70.00	85.00	60.00
80-c.p. series tungsten lamp......	6	29.50	39.00	18.50
60-c.p. series tungsten lamp......	16	23.25	28.00	15.00
40-c.p. series tungsten lamp......	4	20.00	28.00	18.50
32-c.p. series tungsten lamp	16	18.00	22.50	12.50

The increasing use of large Mazda C lamps makes the cost of such units of interest. The following rates are taken from one of the large Public Utilities and may be taken as representative.

Number of lamps	400 C. P.	600 C. P.	1000 C. P.
1 to 250 lamps	$66.00	$80.00	$104.00
251 to 750 lamps	61.88	75.00	97.50
751 to 1250 lamps	57.75	70.00	91.00
1251 to 1750 lamps	53.63	65.00	84.50
Over 1750 lamps	49.50	60.00	78.00

Single-mantle gas lamps are usually furnished on an all-night schedule for prices ranging from $20 to $25 per annum. In many of the smaller cities special display systems of incandescent electric lamps are operated on main business streets, the expense being assumed jointly by the city and the occupants of abutting property. The lamps are usually arranged in arches, festoons or in clusters on pedestals, and are often operated from constant-potential mains to permit the turning off of part of the lamps after midnight. These systems are usually maintained by the operating company and charged for on a flat rate basis.

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INDETERMINATE FORMS. — (*See also Derivatives.*) Let $f(x)$ and $F(x)$ be any two functions of x and let $y = \frac{f(x)}{F(x)}$. For certain values of x, both $f(x)$ and $F(x)$ may be zero, making y equal to $\frac{0}{0}$, an expression which, considered alone, may be anything between 0 and ∞. Its true value may, however, be determined from the nature of the functions $f(x)$ and $F(x)$ by the following process, which involves finding the derivatives of the two functions. If $y_1 = \frac{0}{0}$ when $x = x_1$, then

$$y_1 = \left.\frac{\frac{d}{dx} f(x)}{\frac{d}{dx} F(x)}\right|_{x=x_1}$$

which expression may have a perfectly determinate value. For example if

$$y = \frac{x^2 - 4}{x^3 - 8}$$

and $x = 2$, then

$$y_1 = \frac{0}{0} = \left.\frac{2x}{3x^2}\right|_{x_1=2} = \frac{4}{12} = \frac{1}{3}.$$

If the ratio of the derivatives is still indeterminate, differentiate numerator and denominator again and, if necessary, repeat the process until a determinate form is obtained.

When $y = \frac{f(x)}{F(x)}$ reduces to the indeterminate form $\frac{\infty}{\infty}$ for any particular value x_1 of x, the corresponding value of y is

$$y_1 = \left.\frac{\frac{d}{dx}\left(\frac{1}{F(x)}\right)}{\frac{d}{dx}\left(\frac{1}{f(x)}\right)}\right|_{x=x_1}$$

When $y = f(x) \times F(x)$ reduces to the indeterminate form $0 \times \infty$ for any particular value x_1 of x, the corresponding value of y is

$$y_1 = \left.\frac{\frac{d}{dx} f(x)}{\frac{d}{dx}\left(\frac{1}{F(x)}\right)}\right|_{x=x_1}$$

INDUCTANCE AND INDUCTIVE REACTANCE. — (*See also Alternating Currents; Electricity and Magnetism, Principles of; Skin Effect; Transmission Lines.*) The phenomena of self and mutual inductance are described in the article on *Electricity and Magnetism, Principles of.* In general, when the current i_1 in a circuit, No. 1 say, is varying at the rate $\frac{di_1}{dt}$, a potential drop, in the direction of the current (or *back* e.m.f.) is induced in the circuit, which may be written

$$v_{11} = L_1 \frac{di_1}{dt},$$

where L_1 is called the coefficient of self induction, or self inductance, or simply the inductance, of the circuit. Similarly, when a second circuit, No. 2 say, is in the vicinity of No. 1 and the current i_2 in No. 2,* is varying at the rate $\frac{di_2}{dt}$, an additional potential drop is induced in circuit No. 1 equal to

$$v_{12} = M_{12} \frac{di_2}{dt},$$

where M_{12} is called the coefficient of mutual induction, or simply the mutual inductance, of one circuit with respect to the other.

When the currents in both instances are sine-wave currents of effective values I_1 and I_2 respectively and of frequency f, the effective values of these potential drops are

$$V_{11} = x_1 I_1 \qquad \text{and} \qquad V_{12} = x_{12} I_2 \tag{1}$$

and V_{11} leads I_1 by 90 degrees, and V_{12} leads i_2 by 90 degrees, and

$$x_1 = 2\pi f L_1 \qquad \text{and} \qquad x_{12} = 2\pi f M_{12}. \tag{1a}$$

x_1 is called the inductive self-reactance, or simply the inductive reactance of circuit No. 1. (*see Alternating Currents*), and x_{12} is called the inductive mutual reactance of one circuit with respect to the other. The mutual inductance, and therefore the mutual reactance, between any two circuits is the same for No. 1 with respect to No. 2 as for No. 2 with respect to No. 1, i.e., $M_{12} = M_{21}$ and $x_{12} = x_{21}$.

The total inductive† drop in any circuit due to the *variation of the current* in this and in any number of neighboring circuits is

$$v_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + \mathrm{M}_{13} \frac{di_3}{dt} + \text{etc.} \tag{2}$$

For sine-wave currents of frequency f the total inductive* drop is, in *vector notation* (*see Alternating Currents*),

$$V_1 = j\,(x_1 I_1 + x_{12} I_2 + x_{13} I_3 + \ldots). \tag{2a}$$

Units of Inductance and Reactance. — The practical unit of inductance is the henry, the c.g.s. electromagnetic unit the abhenry (also called a "centimeter"), and the c.g.s. electrostatic unit the stathenry. Inductances are fre-

* i_2 is to be considered positive with respect to i_1 when the flux lines threading No 1 due to the current in No. 2 thread through No. 1 in the same direction as the flux lines due to the current in No. 1.

† In addition to these drops there are also the resistance drop and such other back e.m.f.'s as may be present. See *Alternating Currents.*

quently expressed in thousandths of a henry, i.e., in millihenrys. The units of reactance are the same as the units of resistance (q.v.) See *Units and Conversion Factors* for the interrelations of the various units.

Total Inductance of Two or More Circuits in Series. — When several circuits, having the coefficients L_1, L_2, L_3, etc., and M_{12}, M_{13}, M_{23}, etc., are connected in series the currents $i_1 = i_2 = i_3 =$ etc. $= i$, say, and the total induced e.m.f. is $v = v_1 + v_2 + v_3$, etc., or

$$v = \left[(L_1 + L_2 + L_3 + \text{etc.}) + 2\,(M_{12} + M_{13} + M_{23} + \text{etc.}) \right] \frac{di}{dt},$$

whence the resultant or total inductance is

$$L = (L_1 + L_2 + L_3 + \text{etc}) + 2\,(M_{12} + M_{13} + M_{23} + \text{etc.}). \tag{3}$$

This relation makes possible the accurate calculation of the self inductance of a coil of a number of turns when the self inductance of each turn and the mutual inductance of each pair of turns are known.

RELATION BETWEEN FLUX, CURRENT AND INDUCTANCE. — When the permeability of the conductors and of the medium between and surrounding them is constant (e.g., when the conductors and medium are non-magnetic substances), the self and mutual inductances are constants, independent of the values of the currents, for any given arrangement, shape and size of the circuits. Under these conditions the total number of linkages (*see Electricity and Magnetism, Principles of*) between the flux lines linking any particular circuit, No. 1, and the turns forming that circuit may be expressed by the relation

$$\lambda_1 = L_1 i_1 + M_{12} i_2 + M_{13} i_3 + \text{etc.}, \tag{4}$$

where

$L_1 i_1 = \lambda_{11} =$ the linkages between the flux lines due to the current i_1 in circuit No. 1 and the turns of circuit No. 1,

$M_{12} i_2 = \lambda_{12} =$ the linkages between the flux lines due to the current i_2 in circuit No. 2 and the turns of circuit No. 1, etc.

Hence the common definition of inductance as "linkages per unit current." In certain simple cases the linkages per unit current may be calculated from the configuration of the circuit or circuits, starting from the fundamental relations given by equations (35), (36) and (29) in the article on *Electricity and Magnetism, Principles of*.

Internal Flux, Internal Inductance and Internal Reactance. — In the case of a wire of finite cross-section the various "filaments" of which the wire may be considered as made up are not all linked by the same number of flux lines, and consequently the back e.m.f.'s induced in the various filaments are different, tending to produce a non-uniform distribution of current. This variation in the induced e.m.f. from filament to filament is due only to that portion of the total flux which actually cuts the wire in question; it may therefore be called the "internal flux," and that portion of the inductance or reactance corresponding to this internal flux may be designated as the "internal inductance" and "internal reactance" respectively. Although the internal flux tends to produce a non-uniformity in the distribution of current, this effect is counteracted by the resistance of the wire and for ordinary non-magnetic wires at frequencies under 60 cycles per second the current remains practically uniformly distributed over the cross-section of the wire. See article on *Skin Effect*.

FORMULAS FOR SELF INDUCTANCE.—The following formulas, unless otherwise noted, are taken from a very comprehensive paper on the calculation

of inductance by Rosa and Grover in the *Bull. Bur. Stand.*, Vol. 8, No. 1, p. 1, 1912, in which the accuracy of the various formulas is thoroughly discussed and many others given, as well as tables to minimize the labor of calculation. See also Dwight, H. B., *New Formulas for Reactance Coils*, Proc. A.I.E.E., Sept., 1919.

Unless otherwise stated *all formulas are in c.g.s. electromagnetic units;* the conversion factors are given in the article on *Units and Conversion Factors*.

Self Inductance of a Single Circular Turn formed by a Wire of Circular Cross-Section. — Let a = mean radius of the turn, in centimeters, r = radius of the wire, in centimeters; then the self inductance is

$$L = 4\pi a\left[\left(1+\frac{r^2}{8a^2}\right)\log_e\frac{8a}{r}+\frac{r^2}{24a^2}-1.75\right]. \qquad (5)$$

This formula is derived from an infinite series in $\frac{r}{a}$, hence for $\frac{r}{a}$ large it is approximate only; however, for $\frac{r}{a}$ less than 0.1 which covers all ordinary cases, the error is less than 1 part in 100,000.

Mutual Inductance of Two Co-axial Circles. — Dimensions as in Fig. 1, all in centimeters. Put

$$k = \sqrt{1-\left(\frac{m_2}{m_1}\right)^2}, \qquad k_1 = \frac{m_1 - m_2}{m_1 + m_2}.$$

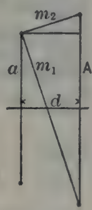

Fig. 1.

For $k < 0.2$ use the formula

$$M = \frac{\pi^2 k^3}{4}\sqrt{Aa}\left[1+\frac{3}{4}k^2+\frac{75}{128}k^4+\frac{245}{512}k^6+\ldots\right], \qquad (6)$$

the general term in the brackets being

$$\left(\frac{3\cdot5\cdot7\ldots(2n+1)}{4\cdot6\cdot8\ldots(2n+2)}\right)^2\frac{(2n+2)}{(2n-1)}k^{2n}.$$

For $k > 0.2$ use the formula

$$M = 2\pi^2 k_1^{3/2}\sqrt{Aa}\left[1+\frac{3}{8}k_1^2+\frac{15}{64}k_1^4+\frac{175}{1024}k_1^6+\ldots\right], \qquad (6a)$$

the general term in the brackets being

$$\left(\frac{n+1}{2n+1}\right)\left[\frac{3\cdot5\cdot7\ldots(2n+1)}{4\cdot6\cdot8\ldots(2n+2)}\right]^2 k^{2n}.$$

Self Inductance of a Long Solenoid* of Circular Cross-Section. — Let

l = axial length of solenoid in centimeters,
n_1 = number of turns per centimeter length, i.e., total number of turns = $n_1 l$,
a = mean radius of the solenoid in centimeters

Then for $\frac{a}{l}$ small, the self inductance is, to a first approximation,

$$L = 4\pi^2 n_1^2 a^2 l. \qquad (7)$$

There is a considerable error in this formula, due to the end effect, but the variations in L due to changes in l are almost exactly proportional to the changes in

* By a solenoid is meant a coil in which the wire forms a uniform, straight cylindrical helix.

l, and hence this formula may be used for calculating the corresponding variations in L as long as $\frac{a}{l}$ remains small.

Self Inductance of a Single-layer Short Solenoid of Circular Cross-Section. — For such a solenoid (see Fig. 2) the summation formula, equation (3), becomes when there are n turns,

$$L = nL_1 + 2\,(n-1)\,M_{12} + 2\,(n-2)\,M_{13} + 2\,(n-3)\,M_{14} + \ldots + 2\,M_{1n}, \qquad (8)$$

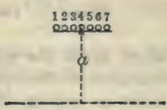

Fig. 2.

where L_1 is the self inductance of a single turn, M_{12} is the mutual inductance of the first and second turns or any two adjacent turns, M_{13} is the mutual inductance of the first and third or of any two turns separated by one, etc., and M_{1n} is the mutual inductance of the first and last turns. L_1 may be calculated from equation (5) and the M's from equation (6) or (6a). The general equation (3) may also be used to calculate the self inductance of a coil of any number of layers, but the calculation becomes tedious.

Self Inductance of Circular Coil of Rectangular Section (Fig. 3). — Dimensions as in Fig. 3, all in centimeters, also

N = total number of turns,
$R = 0.2235\,(b + c)$.

Fig. 3.

Then to a degree of approximation sufficient for most practical purposes

$$L = 4\pi a N^2 \left\{ \log_e \frac{8\,a}{R}\left(1 + \frac{3\,R^2}{16\,a^2}\right) - \left(2 + \frac{R^2}{16\,a^2}\right)\right\}. \qquad (9)$$

See also Doggett, L. A., *Elec. W.*, *63, p. 259, Jan. 31, 1914.*

Mutual Inductance of Two Concentric Co-axial Solenoids.* — All dimensions being in centimeters, let

l = one-half the length of the *shorter* coil,
x = one-half the length of the *longer* coil,
A = radius of *outer* coil (which may be either the shorter or longer coil),
a = radius of the *inner* coil,
$d = x\sqrt{1 + \left(\frac{A}{x}\right)^2}$,

N_1 and N_2 = the *total* number of turns in the first and second coils respectively.

Then as a first approximation, for $\frac{A}{x}$ small, the mutual inductance between the two coils is

$$M = \frac{2\pi^2\,a^2\,N_1N_2}{d}. \qquad (10)$$

By calculating a sufficient number of terms the mutual inductance may be obtained exactly from the formula

$$M = \frac{2\pi^2 a^2 N_1 N_2}{d}\left[1 - \frac{A^2}{2\,d^4}\,\frac{4\,l^2 - 3\,a^2}{4} - \frac{A^2\,(4\,x^2 - 3\,A^2)}{8\,d^8}\cdot\frac{8\,l^4 - 20\,l^2a^2 + 5a^4}{8} - \frac{A^2(8\,x^4 - 20\,x^2A^2 + 5\,A^4)}{16\,d^{12}}\cdot\frac{(64\,l^6 - 336\,l^4a^2 + 280\,l^2a^4 - 35\,a^6)}{64} - \ldots\right]. \quad (10a)$$

* One solenoid inside the other with their centers coinciding.

Mutual Induction of Two Co-axial Circular Coils Each of Rectangular Section (Fig. 4). — Dimensions as in Fig. 4, all in centimeters. N_1 and N_2 represent the total number of turns on the two coils respectively. The following formula, known as the "formula of quadratures," is sufficiently exact for most practical cases:

$$M = \frac{N_1 N_2}{6}(M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8 - 2M_0), \qquad (11)$$

where M_0 is the mutual inductance of the two central turns o_1 and o_2 in Fig. 4. M_1 is the mutual inductance of the circle through o_2 and the circle through 1, M_5 is the mutual inductance of the circle through o_1 and the circle through 5, etc. These mutual inductances M_0, M_1 . . . M_8 may be calculated from equation (6) or (6a). When the sections of the two coils are small compared with their distance apart, $M_0 = M_1 = \ldots M_8$ and equation (11) becomes

$$M = N_1 N_2 M_0. \qquad (11a)$$

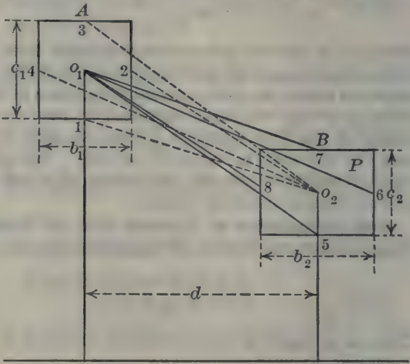

Fig. 4.

INDUCTANCE OF STRAIGHT CONDUCTORS. — The inductance of a circuit formed by straight conductors, such as the wires of a transmission line, may be considered from two points of view, viz., (1) each conductor may be looked upon as having a certain *self* inductance independent of the position of the other conductors forming the circuit or circuits and a *mutual* inductance with each of these conductors or (2) when the conductors are symmetrically arranged and the currents in them bear a fixed relation to one-another, as in a two-wire single-phase or symmetrical three-wire three-phase transmission line, each conductor may be looked upon as having a *total* inductance which takes into account both its self inductance and its mutual inductance with respect to the other wires. In the case of a 2-wire line the total inductance of the loop formed by the two conductors is twice the total inductance of *each* conductor; in the case of more than two wires the "loop inductance" has no simple physical meaning.

In the following paragraphs are given the formulas for *self*, *mutual* and *total* inductance *per conductor* for straight conductors of various sections and for various arrangements of such conductors. *The formulas are all in c.g.s. electromagnetic units* unless otherwise specified; see *Units and Conversion Factors*.

Self Inductance of a Single Straight Round Wire, Return Neglected. — Let

l = length of wire, in centimeters,
d = diameter of round wire, in centimeters,
$r = \frac{d}{2}$ = radius of round wire, in centimeters.

Then for a round wire the self inductance is

$$L' = 2\left[l \log_e \frac{l + \sqrt{l^2 + r^2}}{r} - \sqrt{l^2 + r^2} + \frac{l}{4} + r\right] \qquad (12)$$

$$= 2l\left[\log_\epsilon \frac{2l}{r} - \frac{3}{4}\right] \text{ approximately.} \tag{12a}$$

Where the permeability of the wire is μ, and that of the medium outside is unity and the frequency low (12a) appears in the form

$$L' = 2l\left[\log_\epsilon \frac{2l}{r} - 1 + \frac{\mu}{4}\right]. \tag{12b}$$

This last formula is of theoretical interest only, as the value to assign to μ is doubtful and when such wires are used even for currents of moderate frequencies, the skin effect is appreciable; see article on *Skin Effect.*

External and Internal Self Inductance of a Round Wire. — The self inductance of a round wire may be considered as made up of two parts, viz., the inductance due to the flux *external* to the wire and that due to the flux *within* the wire. The first or "external" inductance is

$$L_e = 2l\left[\log_\epsilon \frac{2l}{r} - 1\right], \tag{13}$$

and the "internal" self inductance is

$$L_i = l\frac{\mu}{2}. \tag{13a}$$

Self Inductance of a Hollow Tube of Circular Section, Return Neglected. — The *external* inductance is the same as for a solid wire, i.e., equation (13), taking for r the external radius of the tube. The *internal* inductance of the tube, putting r_2 = external radius and r_1 = internal radius, is

$$L_i = 2\,\mu\,l\left[\frac{r_1^4}{(r_2^2 - r_1^2)^2}\log_\epsilon \frac{r_2}{r_1} - \frac{1}{4}\,\frac{3\,r_1^2 - r_2^2}{r_2^2 - r_1^2}\right]. \tag{14}$$

The term in the square brackets is always less than ¼, i.e., the internal inductance of a hollow tube is always less than the internal inductance of a solid wire; see equation (13a). In the limit, where $r_1 = r_2$ (tube with infinitely thin walls), the *internal* inductance is zero.

Self Inductance of a Straight Bar or Strip, Return Neglected. — For a straight bar of a non-magnetic substance and of rectangular cross-section the self inductance, neglecting the return circuit, is

$$L' = 2l\left[\log_\epsilon \frac{2l}{a+b} + \frac{1}{2} + \frac{0.2235\,(a+b)}{l}\right], \tag{15}$$

where a and b are the lengths of the two edges, in centimeters.

This formula is not applicable to thin strips at close spacing; see H. B. Dwight, *El. Rev. and West. Elec. 70, p. 1087, June 30, 1917.*

Mutual Inductance of Two Parallel Straight Wires or Bars. — See Fig. 5. Same notation as above and in addition let

D = distance between centers of the two wires, in centimeters.

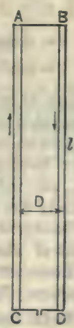

Fig. 5.

Then the mutual inductance between the two is

$$M = 2\left[l\log_\epsilon \frac{l + \sqrt{l^2 + D^2}}{D} - \sqrt{l^2 + D^2} + D\right] \tag{16}$$

$$= 2l\left[\log_\epsilon \frac{2l}{D} - 1 + \frac{D}{l}\right] \text{approximately,} \tag{16a}$$

when the length l is great in comparison with D.

Equation (16), which is an exact expression when the wires have no appreciable cross-section, is not an exact expression for the mutual inductance of two parallel cylindrical wires, but is not appreciably in error even when the section is large and D is small if l is great compared with D.

Equation (16) is also applicable, with a practically negligible error to bars of rectangular section and in fact to the mutual inductance between any two parallel conductors of any section and external to each other, e.g., between an overhead wire and a rail, the distance D being the distance* between the center of gravity of the two sections.

Mutual Inductance Between a Tube and an Interior Wire. — Using the same notation as for equation (14) above the mutual inductance in this case is

$$M = 2l \left[\log_\epsilon 2l - \frac{r_2^2 \log_\epsilon r_2 - r_1^2 \log_\epsilon r_1}{r_2^2 - r_1^2} - \frac{1}{2} \right]. \tag{16b}$$

Total Inductance of a Two-wire Transmission Line. — Let the two wires (Fig. 5) be designated as No. 1 and No. 2 and the currents as i_1 and i_2. Since $i_2 = -i_1$, from equation (2) the total inductive drop in each wire is

$$v_1 = (L_1 - M_{12}) \frac{di_1}{dt}.$$

The total inductance of each wire is then $L = L_1 - M_{12}$, where L_1 is given by equation (12) or (14) and M_{12} by equation (16). For a length so great that d and D are negligible compared with l, this total inductance per wire for equal round wires becomes

$$L = 2l \left[\log_\epsilon \frac{2D}{d} + \frac{\mu}{4} \right],$$

where d is the diameter of each wire. The total inductance of the line per unit length of *wire*† is

$$\left.\begin{aligned} L &= \frac{\mu}{2} + 2 \log_\epsilon \frac{2D}{d} && \text{abhenries per centimeter} \\ &= 0.01524\mu + 0.14037 \log_{10} 2D/d && \text{millihenries per 1000 feet} \\ &= 0.08047\mu + 0.74113 \log_{10} 2D/d && \text{millihenries per mile,} \end{aligned}\right\} \tag{17}$$

where D and d may be expressed in any units of length provided they are both expressed in the *same* units. The formulas given in equation (17) also apply approximately to stranded wires, provided d is taken as the diameter of the solid wire having a cross-section equal to that of the copper in the stranded wire. i.e., the inductance of a No. 0000 stranded wire on a given spacing is approximately the same as that of a No. 9000 solid wire on the same spacing.

Tables of L and the corresponding reactances for 25 and 60 cycles for various sizes of wires and various spacings are given below.

The above formula is not applicable to conductors at close spacing when the frequency is high; see F. B. Silsbie, *Elec. W., 68, p. 125, July 15, 1916.*

Total Inductance of a Rectangle. — Let L_a = self-inductance of long side, L_b = self-inductance of short side (calculated from the proper formulas, 12 to 15) and let M_a = mutual inductance between the two long sides and M_b = the mutual inductance between the two short sides (calculated from formula 16); then the total inductance of the rectangle is

$$L = 2(L_a - M_a) + 2(L_b - M_b)$$

* Accurately, the geometrical mean distance between the two areas; see *Bull. Bur. Stand.*, 1912, Vol. 8, pp. 125 and 126. For round wires, solid or tubular, the geometrical mean distance between them is exactly the distance between their centers.

† To obtain the total inductance of both wires multiply by *twice* the length of the line.

SELF INDUCTANCE OF SOLID NON-MAGNETIC WIRES*

Millihenries per 1000 FEET of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.05750	0.1245	0.1667	0.1915	0.2090	0.2337	0.2512	0.2648
750,000	0.8660	0.06627	0.1332	0.1755	0.2002	0.2178	0.2425	0.2600	0.2736
500,000	0.7071	0.07863	0.1456	0.1879	0.2126	0.2301	0.2548	0.2724	0.2860
350,000	0.5916	0.08950	0.1565	0.1987	0.2235	0.2410	0.2657	0.2832	0.2968
250,000	0.5000	0.09976	0.1667	0.2090	0.2337	0.2512	0.2760	0.2935	0.3071
0000	0.4600	0.1048	0.1718	0.2141	0.2388	0.2563	0.2810	0.2986	0.3122
000	0.4096	0.1119	0.1789	0.2211	0.2459	0.2634	0.2881	0.3057	0.3193
00	0.3648	0.1190	0.1860	0.2282	0.2529	0.2705	0.2952	0.3127	0.3263
0	0.3249	0.1260	0.1930	0.2353	0.2600	0.2775	0.3022	0.3198	0.3334
1	0.2893	0.1331	0.2001	0.2423	0.2671	0.2846	0.3093	0.3269	0.3405
2	0.2576	0.1402	0.2072	0.2494	0.2741	0.2917	0.3164	0.3339	0.3475
4	0.2043	0.1543	0.2213	0.2635	0.2883	0.3058	0.3305	0.3481	0.3617
6	0.1620	0.1685	0.2354	0.2777	0.3024	0.3199	0.3447	0.3622	0.3758
8	0.1285	0.1826	0.2496	0.2918	0.3165	0.3341	0.3588	0.3763	0.3899
10	0.1019	0.1967	0.2637	0.3060	0.3307	0.3482	0.3729	0.3905	0.4041
12	0.08081	0.2109	0.2778	0.3201	0.3448	0.3623	0.3871	0.4046	0.4182
14	0.06408	0.2250	0.2920	0.3342	0.3590	0.3765	0.4012	0.4187	0.4323
16	0.05082	0.2391	0.3061	0.3484	0.3731	0.3906	0.4153	0.4329	0.4465

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.2760	0.2935	0.3071	0.3182	0.3358	0.3494	0.3741	0.3916	0.4052
750,000	0.2847	0.3023	0.3159	0.3270	0.3445	0.3581	0.3828	0.4004	0.4140
500,000	0.2971	0.3146	0.3282	0.3393	0.3569	0.3705	0.3952	0.4127	0.4263
350,000	0.3080	0.3255	0.3391	0.3502	0.3678	0.3814	0.4061	0.4236	0.4372
250,000	0.3182	0.3358	0.3494	0.3605	0.3780	0.3916	0.4163	0.4339	0.4475
0000	0.3233	0.3408	0.3544	0.3656	0.3831	0.3967	0.4214	0.4390	0.4526
000	0.3304	0.3479	0.3615	0.3726	0.3902	0.4038	0.4285	0.4460	0.4596
00	0.3374	0.3550	0.3686	0.3797	0.3972	0.4108	0.4356	0.4531	0.4667
0	0.3445	0.3620	0.3756	0.3867	0.4043	0.4179	0.4426	0.4601	0.4737
1	0.3516	0.3691	0.3827	0.3938	0.4114	0.4250	0.4497	0.4672	0.4808
2	0.3586	0.3762	0.3898	0.4009	0.4184	0.4320	0.4568	0.4743	0.4879
4	0.3728	0.3903	0.4039	0.4150	0.4326	0.4462	0.4709	0.4884	0.5020
6	0.3869	0.4045	0.4181	0.4292	0.4467	0.4603	0.4850	0.5026	0.5162
8	0.4011	0.4186	0.4322	0.4433	0.4608	0.4744	0.4992	0.5167	0.5303
10	0.4152	0.4327	0.4463	0.4574	0.4750	0.4886	0.5133	0.5308	0.5444
12	0.4293	0.4469	0.4605	0.4716	0.4891	0.5027	0.5274	0.5450	0.5586
14	0.4435	0.4610	0.4746	0.4857	0.5033	0.5169	0.5416	0.5591	0.5727
16	0.4576	0.4751	0.4887	0.4998	0.5174	0.5310	0.5557	0.5732	0.5868

* The inductances given in this table also apply, with a practically negligible error (about 1 per cent) to ordinary stranded wires of the *same cross-section*.

SELF INDUCTANCE OF SOLID NON-MAGNETIC WIRES*

Millihenries per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.3036	0.6572	0.8803	1.011	1.103	1.234	1.327	1.398
750,000	0.8660	0.3499	0.7035	0.9266	1.057	1.150	1.280	1.373	1.445
500,000	0.7071	0.4152	0.7688	0.9919	1.122	1.215	1.346	1.438	1.510
350,000	0.5916	0.4726	0.8262	1.049	1.180	1.272	1.403	1.496	1.567
250,000	0.5000	0.5267	0.8803	1.103	1.234	1.327	1.457	1.550	1.622
0000	0.4600	0.5536	0.9072	1.130	1.261	1.353	1.484	1.577	1.648
000	0.4096	0.5909	0.9445	1.168	1.298	1.391	1.521	1.614	1.686
00	0.3648	0.6282	0.9818	1.205	1.335	1.428	1.559	1.651	1.723
0	0.3249	0.6654	1.019	1.242	1.373	1.465	1.596	1.688	1.760
1	0.2893	0.7029	1.057	1.280	1.410	1.503	1.633	1.726	1.798
2	0.2576	0.7402	1.094	1.317	1.447	1.540	1.671	1.763	1.835
4	0.2043	0.8148	1.168	1.392	1.522	1.615	1.745	1.838	1.910
6	0.1620	0.8894	1.243	1.466	1.597	1.689	1.820	1.912	1.984
8	0.1285	0.9641	1.318	1.541	1.671	1.764	1.894	1.987	2.059
10	0.1019	1.039	1.392	1.615	1.746	1.839	1.969	2.062	2.134
12	0.08081	1.113	1.467	1.690	1.821	1.913	2.044	2.136	2.208
14	0.06408	1.188	1.542	1.765	1.895	1.988	2.118	2.211	2.283
16	0.05082	1.263	1.616	1.839	1.970	2.062	2.193	2.286	2.357

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	1.457	1.550	1.622	1.680	1.773	1.845	1.975	2.068	2.140
750,000	1.503	1.596	1.668	1.726	1.819	1.891	2.021	2.114	2.186
500,000	1.569	1.661	1.733	1.792	1.884	1.956	2.087	2.179	2.251
350,000	1.626	1.719	1.791	1.849	1.942	2.014	2.144	2.237	2.309
250,000	1.680	1.773	1.845	1.903	1.996	2.068	2.198	2.291	2.363
0000	1.707	1.800	1.872	1.930	2.023	2.095	2.225	2.318	2.390
000	1.744	1.837	1.909	1.967	2.060	2.132	2.262	2.355	2.427
00	1.782	1.874	1.946	2.005	2.097	2.169	2.300	2.392	2.464
0	1.819	1.911	1.983	2.042	2.135	2.206	2.337	2.430	2.501
1	1.856	1.949	2.021	2.079	2.172	2.244	2.374	2.467	2.539
2	1.894	1.986	2.058	2.117	2.209	2.281	2.412	2.504	2.576
4	1.968	2.061	2.133	2.191	2.284	2.356	2.486	2.579	2.651
6	2.043	2.135	2.207	2.266	2.359	2.430	2.561	2.654	2.725
8	2.118	2.210	2.282	2.341	2.433	2.505	2.636	2.728	2.800
10	2.192	2.285	2.357	2.415	2.508	2.580	2.710	2.803	2.875
12	2.267	2.359	2.431	2.490	2.582	2.654	2.785	2.877	2.949
14	2.341	2.434	2.506	2.565	2.657	2.729	2.860	2.952	3.024
16	2.416	2.509	2.581	2.639	2.732	2.804	2.934	3.027	3.099

* The inductances given in this table also apply, with a practically negligible error (about 1 per cent) to ordinary stranded wires of the *same cross-section.*

25-CYCLE REACTANCE OF SOLID NON-MAGNETIC WIRES*

Ohms per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.04770	0.1032	0.1383	0.1588	0.1733	0.1939	0.2085	0.2196
750,000	0.8660	0.05497	0.1105	0.1456	0.1661	0.1807	0.2011	0.2157	0.2270
500,000	0.7071	0.06523	0.1208	0.1558	0.1763	0.1909	0.2115	0.2259	0.2372
350,000	0.5916	0.07425	0.1298	0.1648	0.1854	0.1998	0.2204	0.2350	0.2462
250,000	0.5000	0.08274	0.1383	0.1733	0.1939	0.2085	0.2289	0.2435	0.2548
0000	0.4600	0.08697	0.1425	0.1775	0.1981	0.2126	0.2331	0.2477	0.2589
000	0.4096	0.09283	0.1484	0.1835	0.2039	0.2185	0.2389	0.2536	0.2649
00	0.3648	0.09869	0.1542	0.1893	0.2097	0.2243	0.2449	0.2594	0.2707
0	0.3249	0.1045	0.1601	0.1951	0.2157	0.2302	0.2507	0.2652	0.2765
1	0.2893	0.1101	0.1661	0.2011	0.2215	0.2361	0.2565	0.2712	0.2825
2	0.2576	0.1163	0.1719	0.2069	0.2273	0.2419	0.2625	0.2770	0.2883
4	0.2043	0.1280	0.1835	0.2187	0.2391	0.2537	0.2741	0.2887	0.3001
6	0.1620	0.1397	0.1953	0.2303	0.2509	0.2653	0.2859	0.3004	0.3117
8	0.1285	0.1515	0.2071	0.2421	0.2625	0.2771	0.2975	0.3122	0.3235
10	0.1019	0.1632	0.2187	0.2537	0.2743	0.2889	0.3093	0.3239	0.3353
12	0.08081	0.1749	0.2305	0.2655	0.2861	0.3005	0.3211	0.3356	0.3469
14	0.06408	0.1866	0.2422	0.2773	0.2977	0.3123	0.3327	0.3473	0.3587
16	0.05082	0.1984	0.2539	0.2889	0.3095	0.3239	0.3445	0.3591	0.3703

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.2289	0.2435	0.2548	0.2639	0.2785	0.2898	0.3103	0.3249	0.3362
750,000	0.2361	0.2507	0.2620	0.2712	0.2858	0.2971	0.3175	0.3321	0.3434
500 000	0.2465	0.2609	0.2723	0.2815	0.2960	0.3073	0.3279	0.3423	0.3536
350,000	0.2554	0.2701	0.2814	0.2905	0.3051	0.3164	0.3368	0.3514	0.3627
250,000	0.2639	0.2785	0.2898	0.2990	0.3136	0.3249	0.3453	0.3599	0.3712
0000	0.2682	0.2828	0.2941	0.3032	0.3178	0.3291	0.3495	0.3642	0.3755
000	0.2740	0.2886	0.2999	0.3090	0.3236	0.3349	0.3554	0.3700	0.3813
00	0.2800	0.2944	0.3057	0.3150	0.3294	0.3407	0.3613	0.3758	0.3871
0	0.2858	0.3002	0.3115	0.3208	0.3354	0.3466	0.3671	0.3818	0.3929
1	0.2916	0.3062	0.3175	0.3266	0.3412	0.3525	0.3730	0.3876	0.3989
2	0.2975	0.3120	0.3233	0.3326	0.3470	0.3583	0.3789	0.3934	0.4047
4	0.3092	0.3238	0.3351	0.3442	0.3588	0.3701	0.3906	0.4052	0.4165
6	0.3210	0.3354	0.3467	0.3560	0.3706	0.3818	0.4023	0.4169	0.4281
8	0.3327	0.3472	0.3585	0.3678	0.3822	0.3935	0.4141	0.4286	0.4399
10	0.3444	0.3590	0.3703	0.3794	0.3940	0.4053	0.4257	0.4404	0.4517
12	0.3561	0.3706	0.3819	0.3912	0.4056	0.4169	0.4375	0.4520	0.4633
14	0.3678	0.3824	0.3937	0.4030	0.4174	0.4287	0.4493	0.4638	0.4751
16	0.3796	0.3942	0.4055	0.4146	0.4292	0.4405	0.4609	0.4755	0.4869

* The reactances given in this table also apply, with a practically negligible error (about 1 per cent) to ordinary stranded wires of the *same cross-section.*

60-CYCLE REACTANCE OF SOLID NON-MAGNETIC WIRES*

Ohms per MILE of each wire of a single-phase or of a symmetrical three-phase line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches	Inches between wires, center to center							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.1145	0.2478	0.3319	0.3811	0.4158	0.4652	0.5003	0.5270
750,000	0.8660	0.1319	0.2652	0.3493	0.3985	0.4336	0.4826	0.5176	0.5448
500,000	0.7071	0.1565	0.2898	0.3739	0.4230	0.4581	0.5074	0.5421	0.5693
350,000	0.5916	0.1782	0.3115	0.3955	0.4449	0.4795	0.5289	0.5640	0.5908
250,000	0.5000	0.1986	0.3319	0.4158	0.4652	0.5003	0.5493	0.5844	0.6115
0000	0.4600	0.2087	0.3420	0.4260	0.4754	0.5101	0.5595	0.5945	0.6213
000	0.4096	0.2228	0.3561	0.4403	0.4893	0.5244	0.5734	0.6085	0.6356
00	0.3648	0.2368	0.3701	0.4543	0.5033	0.5384	0.5877	0.6224	0.6496
0	0.3249	0.2509	0.3842	0.4682	0.5176	0.5523	0.6017	0.6364	0.6635
1	0.2893	0.2650	0.3985	0.4826	0.5316	0.5666	0.6156	0.6507	0.6778
2	0.2576	0.2791	0.4124	0.4965	0.5455	0.5806	0.6300	0.6647	0.6918
4	0.2043	0.3072	0.4403	0.5248	0.5738	0.6089	0.6579	0.6929	0.7201
6	0.1620	0.3353	0.4686	0.5527	0.6021	0.6368	0.6861	0.7208	0.7480
8	0.1285	0.3635	0.4969	0.5810	0.6300	0.6650	0.7140	0.7491	0.7762
10	0.1019	0.3917	0.5248	0.6089	0.6582	0.6933	0.7423	0.7774	0.8045
12	0.08081	0.4196	0.5531	0.6371	0.6865	0.7212	0.7706	0.8053	0.8324
14	0.06408	0.4479	0.5813	0.6654	0.7144	0.7495	0.7985	0.8335	0.8607
16	0.05082	0.4762	0.6092	0.6933	0.7427	0.7774	0.8268	0.8618	0.8886

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center								
	3	4	5	6	8	10	15	20	25
1,000,000	0.5493	0.5844	0.6115	0.6334	0.6684	0.6956	0.7446	0.7796	0.8068
750,000	0.5666	0.6017	0.6288	0.6507	0.6858	0.7129	0.7619	0.7970	0.8241
500,000	0.5915	0.6262	0.6533	0.6756	0.7103	0.7374	0.7868	0.8215	0.8486
350,000	0.6130	0.6481	0.6752	0.6971	0.7321	0.7593	0.8083	0.8433	0.8705
250,000	0.6334	0.6684	0.6956	0.7174	0.7525	0.7796	0.8286	0.8637	0.8909
0000	0.6435	0.6786	0.7057	0.7276	0.7627	0.7898	0.8388	0.8739	0.9010
000	0.6575	0.6925	0.7196	0.7416	0.7766	0.8038	0.8528	0.8878	0.9150
00	0.6718	0.7065	0.7336	0.7559	0.7906	0.8177	0.8671	0.9018	0.9289
0	0.6858	0.7204	0.7476	0.7698	0.8049	0.8317	0.8810	0.9161	0.9429
1	0.6997	0.7348	0.7619	0.7838	0.8188	0.8460	0.8950	0.9301	0.9572
2	0.7140	0.7487	0.7759	0.7981	0.8328	0.8599	0.9093	0.9440	0.9712
4	0.7419	0.7770	0.8041	0.8260	0.8611	0.8882	0.9372	0.9723	0.9994
6	0.7702	0.8049	0.8320	0.8543	0.8893	0.9161	0.9655	1.001	1.027
8	0.7985	0.8332	0.8603	0.8826	0.9172	0.9444	0.9938	1.028	1.056
10	0.8264	0.8614	0.8886	0.9105	0.9455	0.9727	1.022	1.057	1.084
12	0.8547	0.8893	0.9165	0.9387	0.9734	1.001	1.050	1.085	1.112
14	0.8826	0.9176	0.9448	0.9670	1.002	1.029	1.078	1.113	1.140
16	0.9108	0.9459	0.9730	0.9949	1.030	1.057	1.106	1.141	1.168

* The reactances given in this table also apply, with a practically negligible error (about 1 per cent) to ordinary stranded wires of the *same cross-section*.

Total Inductance of a Concentric Cable.—In a similar manner the total inductance of a concentric main may be found from equations (3), (13), (14) and (16b), which give for the total inductance per unit length of *cable*

$$L = 2\left[\log_\epsilon \frac{d_2}{d_1} + \frac{d_3^4}{(d_3^2 - d_2^2)^2}\log_\epsilon \frac{d_3}{d_2} - \frac{d_3^2}{2\,(d_3^2 - d_2^2)}\right] \text{ abhenries per centimter,}$$

where d_1 = diameter of internal conductor, assumed solid, d_2 = internal diameter of outer conductor and d_3 = external diameter of outer conductor.

Inductive Drop in a Three-wire Transmission Line. — Let the wires be designated as Nos. 1, 2, and 3; and the three currents in the direction away from the generator as i_1, i_2 and i_3. From equation (1) the inductive drop in each wire* is

$$\left.\begin{aligned} v_1 &= L_1\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} + M_{13}\frac{di_3}{dt}, \\ v_2 &= M_{12}\frac{di_1}{dt} + L_2\frac{di_2}{dt} + M_{23}\frac{di_3}{dt}, \\ v_3 &= M_{13}\frac{di_1}{dt} + M_{23}\frac{di_2}{dt} + L_3\frac{di_3}{dt}. \end{aligned}\right\} \qquad (18)$$

The values of the L's and M's in terms of the radii and distances apart of the three wires are given in equations (12) or (14) and (16).

Equilateral Triangle Arrangement. — For ordinary three-phase work when there is no neutral current, the sum of the three line currents at any instant is equal to zero, whence

$$(i_2 + i_3) = -\,i_1,\ (i_1 + i_3) = -\,i_2, \text{ and } (i_1 + i_2) = -\,i_3.$$

When the three wires are all of the same size and are arranged so that they form the three edges of an equilateral prism, a common arrangement, $L_1 = L_2 = L_3$ and $M_{12} = M_{13} = M_{23}$, and the inductive drops per unit length in the three wires are respectively

$$v_1 = L\frac{di_1}{dt},\quad v_2 = L\frac{di_2}{dt},\quad v_3 = L\frac{di_3}{dt}, \qquad (19)$$

where L has here the same value as in equation (17), numerical values of which are given in the accompanying tables.

For a sine-wave current of effective value I and frequency f, the inductive drop in each wire per unit length of line has the effective value

$$V = 2\pi fLI \qquad (19a)$$

and leads the current in this particular wire by 90 degrees, irrespective of whether the load be balanced or not, the only condition being that no current returns to the generator through any other conductor than the three line wires.

Three Parallel Wires in the Same Plane. — Let the three wires of the three-phase system have equal diameters d, and let No. 2 be the middle wire, and let Nos. 1 and 3 be at equal distances (D between centers) from No. 2 and on opposite sides of No. 2. Under these conditions, considering unit length of line, put

L = same values as in (17), taking D as the distance between either outer and the middle wire,

and $M = 2\log_\epsilon 2 = 1.3863$ abhenries per centimeter
$= 0.04225$ millihenries per 1000 feet
$= 0.2231$ millihenries per mile.

* Note that in vector notation $\frac{di}{dt} = j\,2\pi fI$ where f is the frequency, I the current and $j = \sqrt{-1}$.

Then equations (18) for the inductive drops in the three wires become, provided only that $i_1 + i_2 + i_3 = 0$,

$$\left.\begin{aligned} v_1 &= L\frac{di_1}{dt} - M\frac{di_3}{dt}, \\ v_2 &= L\frac{di_2}{dt}, \\ v_3 &= L\frac{di_3}{dt} - M\frac{di_1}{dt}. \end{aligned}\right\} \qquad (20)$$

Such an arrangement, therefore, causes an unbalancing of the system, but when the wires are far apart, so that L is large compared with M, the effect is slight. It can be avoided by transposing the wires at intervals.

Inductance of Overhead Wires With Earth Return. — This case is not susceptible of a definite solution, since the inductance depends upon the distribution of the return current in the earth. When the current returns through one or more rails immediately below the wire, the leakage current to the earth may be neglected and the wire and rails* treated as linear conductors, applying the formulas given above. When there is no metallic return circuit, an approximate solution may be obtained by considering the earth as equivalent to the "images" of the overhead wires in the plane of the earth's surface, i.e., considering the return circuit as consisting of the same number of wires as there are overhead, these fictitious return wires being the same distance below the earth as the actual wires are above it. The value of the inductances as thus calculated can never be greater than the actual inductances but will usually be slightly less than the actual values.

EFFECT OF FREQUENCY ON INDUCTANCE. — See article on *Skin Effect*.

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* The *external* self-inductance of a rail is practically the same as that of a round wire, taking for r the perimeter of the rail divided by 2π; see equation (13). The *internal* self-inductance depends upon the permeability μ and the frequency of the current. The internal inductance, however, is small compared with the external inductance and for, approximate calculations may be neglected. See *Rails, Track and Third; Trolley Systems, Overhead.*

INDUCTION COILS. — (*See also Electricity and Magnetism, Principles of; Electromagnet Windings; Ignition, Electric.*) An induction coil is a device which transforms a low direct e.m.f. to a high alternating e.m.f. of unsymmetrical form. The single-winding coil, called a primary coil, is used extensively in gas-engine ignition and in automatic gas lighting, and the double-winding coil, called a secondary coil, is used for the excitation of X-ray tubes, gas-engine ignition, automatic gas lighting, electrotherapeutics and wireless telegraphy.

PRIMARY INDUCTION COIL. — The primary induction coil consists of a single coil wound upon an iron core made up of a compact bundle of soft-iron wires. To obtain a spark from such a coil it is connected in series with a battery and some kind of "make-and-break" contact. When the circuit is closed, the current increases gradually according to the expression,

$$i_t = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right),$$

where i_t equals the current at any time t seconds after the circuit is closed, E equals the e.m.f. of the battery, R equals the resistance in ohms and L equals the inductance in henries of the entire circuit, and ϵ the base of the natural system of logarithms. The shape of the current curve at "make" is shown in the curve AB (Fig. 1),* which is an oscillograph record taken when an e.m.f. of 4 volts is impressed upon a primary coil of 1 ohm resistance and 0.01 henry inductance.

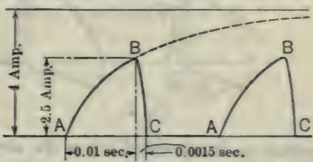

Fig. 1.

If the circuit is opened when the current reaches some value, such as B (Fig. 1), the current decreases rapidly to zero as shown in the curve BC. Since the e.m.f. induced in such a coil at any instant equals $L\frac{di}{dt}$ when the current falls from B to C, the e.m.f. induced in the coil will be many times that of the battery e.m.f. and a spark will be established between the open contacts at "break." For use in connection with gas engines the "make and break" contacts are located within the cylinder. Reliable ignition is obtained when about 0.02 joule is dissipated in the spark at "break." Since the energy dissipated as heat in the spark is converted from the electromagnetic energy stored up in the coil, it follows that the coil must have an inductance and carry a current at "break" such that the energy stored up, i.e., $\frac{1}{2}LI^2$, will exceed the required value of 0.02 joule. In practice it is customary to design the coil and time of contact so that $\frac{1}{2}LI^2$ equals about 0.04 joule, such coils having an efficiency of about 50 per cent. One-half of the energy put into the coil is then used up in heating the conductors and metallic parts by I^2R and hysteresis losses.

SECONDARY INDUCTION COIL. — The secondary coil has two separate windings wound about an iron core, one of few turns called the primary winding and the other of many turns called the secondary winding. The primary winding is connected in series with a battery and an interrupter (described below). The spark is produced between the terminals of the secondary winding. In most cases a condenser is shunted across the interrupter to prevent sparking at

* This and the following oscillograph records are due to Bailey, B. F., *Elec. W. 1910, Vol. LV., p. 943.*

that point. Since the e.m.f. induced in each turn linked by the magnetic flux is the same, the e.m.f. induced in the secondary winding, neglecting leakage, will equal $\frac{N_s}{N_p}$ times that induced in the primary winding, where N_s and N_p equal the respective number of turns on the secondary and primary windings. Secondary coils which give sparks as long as 5 feet have been constructed in this manner.

Effect of Shunting Interrupter with Capacity.— (*See also Ignition, Electric.*) When the interrupter is shunted by a capacity the e.m.f. induced in the secondary winding becomes oscillatory. This effect of the capacity is well shown in the oscillograph records given in Figs. 2 and 3. In each case a high non-inductive

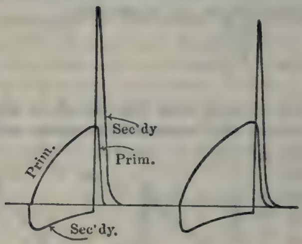

Fig. 2.

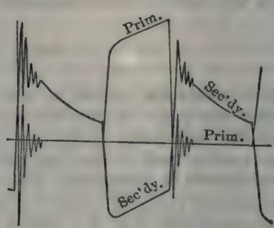

Fig. 3.

resistance is connected across the secondary terminals, and the curves indicate the variation of the primary current and the secondary e.m.f. In Fig. 2 the interrupter is unshunted by a condenser, and as a result the secondary e.m.f. increases at "break" and decreases to zero without oscillation. In Fig. 3 the interrupter is shunted by a condenser and the secondary e.m.f. becomes oscillatory. The form of the e.m.f. curve in any actual case is not shown accurately by Fig. 3 except at the time of "break," since the resistance between the terminals of a spark gap decreases rapidly after the spark is established. In Fig. 4 a low resistance is connected across the secondary terminals and shows the general form of the e.m.f. curve after the spark is established.

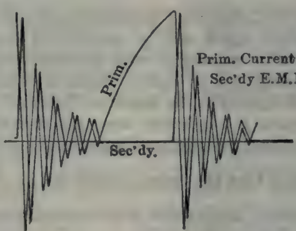

Fig. 4.

Although not shown by the curves owing to difference in scale, the capacity shunted around the interrupter also increases the secondary e.m.f. above the value obtained without the capacity. Assuming that no spark is formed at the interrupter, it may be shown that the e.m.f. of the secondary equals $\frac{N_s}{N_p}\sqrt{\frac{L}{C}}I_b$, where L equals the inductance of the primary in henries, C equals the capacity of the condenser in farads and I_b equals the primary current at "break" in amperes. If C is made so low that a spark appears at the interrupter, the above formula no longer holds and the secondary e.m.f. will be greatly reduced. C should then be made as small as possible but of sufficient size to suppress the spark at the interrupter.

Insulation. — The conductors in the secondary winding must be heavily insulated to withstand the very high e.m.f. induced in that winding. In most

cases a double-covered silk insulation impregnated with some insulating compound is used. In large induction coils the secondary winding is built up of several flat coils insulated from each other by ebonite or fiber disks. The coils are so wound that the electrical connections are made alternately at the top and at the bottom of the respective coils.

Dimensions of Three-inch Coil. — The general dimensions of parts of a 3-inch induction coil, as reported by S. R. Bottone, *Radiography* (*London, 1898*), are given following: Iron, bundle of No. 20 (B. & S. G.) annealed iron wire, 1¼ in. diameter, 13 in. long. Primary winding, four layers of No. 12 double silk-covered copper wire, about 4¼ lb. Ebonite tube over primary, 12 in. long, 2 in. inside diameter, 2½ in. outside. Two ebonite heads, 5 in. square, ½ in. thick. Seven vulcanized fiber circlets (for sections), 4½ in. diameter, ⅛ in. thick, 2½ in. central aperture. Secondary winding, 4 lb. No. 36 double-silk-covered wire. Platinum-tip contact breaker, height from base to center of iron hammer, 2½ in., size of iron head of hammer ¾ in. diameter, ¾ in. long. Base (fitted with false bottom to contain condenser), 18 in. long by 9 in. wide by 2¾ in. deep. Condenser, 144 sheets of tinfoil, size 6 by 12 in., interleaved with 144 sheets of paraffined paper 8 by 13 in.

Dimensions of Six-inch Coil.— Similar dimensions for a 6-inch induction coil, as reported by Bottone, are given following: Iron, bundle of No. 16 (B. & S. G.) annealed iron wire, 1½ in. diameter, 15 in. long. Primary winding, four layer of No. 12 double silk-covered copper wire, about 5 lb. Ebonite tube over primary 14 in. long, 2¼ in. inside diameter, 2¾ in. outside. Ebonite heads 6 in. square, ¾ in. thick. Seven vulcanized fiber circlets 5¼ in. diameter, ⅛ in. thick, with 2¾ in. central hole. Secondary winding, 7 lb. No. 38 double-silk-covered copper wire. Platinum-tip contact breaker, height from base to center of hammer, 3 in. size of hammer head, 1 in. diameter, 1 in. long. Base (fitted with false bottom to contain condenser), 20 in. long by 10 in. wide by 3½ in. deep. Condenser, 144 sheets of tinfoil, size 6 by 12 in., interleaved with 144 sheets of paraffined paper, 8 by 13 in.

Interrupters. — An interrupter should be so designed that it will close the circuit for a definite time by easy adjustment and will open the circuit at the end of that time as quickly as possible. Two distinct types, the mechanical and the electrolytic interrupter, are in common use. Mechanical interrupters may be divided into the following forms: hammer, atonic, commutator and mercury interrupters. Of the electrolytic interrupters the Wehnelt type is the most popular.

Hammer Interrupter. — In the hammer interrupter, shown in Fig. 5, the circuit is opened at *A* when the core *B* attracts the iron mass *C* mounted at the free end of the spring blade *D*. Since the core attracts the iron mass only when the circuit is closed and loses its attraction when the circuit is opened, the spring blade is set in vibration and opens and closes the circuit in rapid succession. This type is used extensively in connection with small coils. The contacts at *A* are usually tipped with platinum to withstand the intense heat developed by sparking. The rapidity of the "break" is not great enough for large coils, and when a large current must be broken, trouble is experienced in keeping the contact points in good condition.

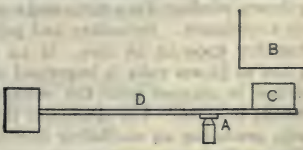

Fig. 5. Hammer Interrupter

Atonic Interrupter. — In the atonic interrupter, shown in Fig. 6, the free end of the iron strip *P* is attracted as in the hammer interrupter and is

returned to its original position by the spring *R*. The circuit is opened at the contacts *ab* when the free end of *P* strikes the blade *L*. Interrupters constructed in this manner open the circuit quicker than do the hammer type since the attracted member is moving faster at the instant of "break." The period of the atonic interrupter may be varied within wide limits by regulating the tension on the spring *R* by means of the thumb-screw *M*.

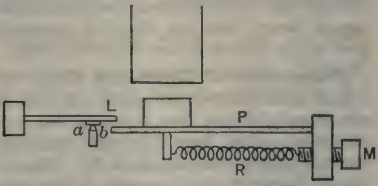

Fig. 6. Atonic Interrupter

Commutator Interrupters are often used when the primary current is too large to be broken by the hammer or atonic interrupters. A brush bearing upon a revolving disk, built up of conducting and insulating segments, makes and breaks the circuit at any desired rate, depending upon the speed of the motor which drives the disk.

Mercury Interrupters are specially adapted to circuits of high e.m.f. In the *plunger* type a pointed electrode is alternately immersed and withdrawn from a cup of mercury, the moving electrode being set in vibration magnetically as in the hammer interrupter, or by the positive action of a cam driven by a motor. The mercury cup is covered with a layer of oil so that the spark is quickly extinguished when the moving electrode is withdrawn into the oil. In the *turbine* type a small stream of mercury is directed upon the periphery of a revolving toothed wheel. The circuit is made when the mercury stream strikes a tooth and is broken when the mercury stream passes through a slot between the teeth. The toothed wheel is rotated at high speed by a motor which also drives a small mercury pump.

Wehnelt Interrupter. — The Wehnelt interrupter has two fixed electrodes which are immersed in dilute sulphuric acid. The anode consists of a small platinum wire insulated by glass except at its tip end. The cathode usually consists of a sheet of lead. If the electrolyte is well circulated, an interrupter of this kind will give about 450 interruptions per second with 24 volts impressed upon the primary circuit. When used with large coils the electrolyte heats up quickly, and a cooling coil is sometimes immersed in the electrolyte to control the temperature.

Tesla Coil. — In some cases, as in wireless telegraphy, electro-therapeutics, etc., where a unidirectional e.m.f. is not required, but an e.m.f. of high frequency is desired, a Tesla coil may be used in place of an induction coil. The construction of a simple Tesla coil is shown in Fig. 7. In the primary circuit a primary winding of few turns and a small spark gap are shunted by a condenser. Secondary and primary windings are wound together upon an air core. If an alternating e.m.f. of from 5,000 to 10,000 volts is impressed upon the primary the condenser is charged until the voltage across the primary gap breaks it down. The condenser then discharges through the gap, producing an oscillating current in the primary winding. The oscillating current in the primary induces a high-frequency e.m.f. in the secondary, which is sufficient to break down the long secondary spark gap.

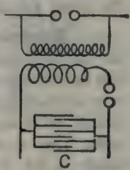

Fig. 7. Tesla Coil

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INSULATING MATERIALS, MISCELLANEOUS. — (*See also Cambric, Varnished; Electricity and Magnetism, Principles of; Gutta Percha; Insulating Materials, Testing of; Paper, Impregnated; Rubber; Wires and Cables, Insulated.*) Materials used in the insulation of electrical apparatus may be classified as follows (*see also Standardization Rules of A.I.E.E.*):

1. Vitreous, including glass, enamel, etc., 2. Stony, such as slate, marble, mica, asbestos, porcelain, etc., 3. Osseus, such as bone and ivory, 4. Resinous, including shellac, resins, copal and other gum, 5. Bituminous, as bitumen, asphaltum, pitch, etc., 6. Waxy, including bees-wax, paraffin, etc., 7. Elastic, such as india-rubber, ebonite, gutta-percha, etc., 8. Oily, including various oils and fats of animal and vegetable origin as mineral petroleum, 9. Cellulose, including dry wood and paper, cotton, celluloid, etc., 10. Silk and allied animal tissue such as catgut, 11. Sulphur, 12. Amber, 13. Molded insulations.

SELECTION OF INSULATING MATERIAL. — In selecting an insulating material for any special purpose and in predetermining accurately the behavior of such a material under working conditions, it is desirable that the following electrical, physical and chemical properties of the material should be known as far as they apply to the use in question.

Electric.—

Dielectric strength (kilovolts per mm. at puncture).
Dielectric loss.
Specific inductive capacity; Direct-current; Alternating-current.
Conductivity or resistivity. Surface leakage.
Temperature coefficient of resistance.

Physical.—

Specific gravity and specific weight.
Hardness.
Toughness.
Brittleness.
Ductility.
Workability.
Flexibility (with reference to varnished cloths, etc.).
Strength (tensile, compressive and shearing).
Fracture (fibrous, crystalline or amorphous).
Ability to take polish.
Melting point.
Shrinkage.
Adhesiveness.
Effect of high temperature (electric arc).
Effect of low temperature.
Artificial aging test.
Porosity.
Viscosity.
Flash and fire test.
Film-making power.
Rate of drying.
Penetration.
Absorption of moisture.
Amount of volatile matter given off at prescribed temperatures.

Chemical.—

Proximate composition.
Solubility in water, oil, etc.
Effect of moisture and moist air.
Weathering qualities.
Chemical effect upon metals in contact.
Presence of acid.

DIELECTRIC STRENGTH. — According to best authorities, an insulating material should be homogeneous and free from moisture in order to have high dielectric strength. In addition, some authorities claim it should also have a low specific inductive capacity (permittivity). These claims are based on experiments made on some of the more important insulations. Whether they apply to all insulations in general is a question which requires more extensive experiments before it can be answered definitely. However, enough experi-

ments have been made to warrant that these claims be given consideration in choosing insulating material for uses in which very high dielectric strength is required.

The breakdown voltage is influenced by the size and shape of electrodes, the frequency and wave shape of the applied voltage, time of voltage application, and the temperature of the insulation. The literature on the subject is not always clear on these points nor is there a standard practice adopted.

The Continental Fiber Company has adopted the practice of using blunt needle points on test specimens over 0.125 inch thick and disc electrodes 1 inch in diameter for test specimens 0.125 inch thick or less, and raising the voltage at the rate of 5000 volts per second from zero to puncture voltage.

Table I illustrates the effect of the thickness of the test specimen upon the dielectric strength of porcelain. The effect may be greater or less for other insulating materials. It is obvious, however, that a standard thickness of the test specimen should be adopted in order to obtain comparable values of dielectric strength of the various insulations.

TABLE I.—DIELECTRIC STRENGTH OF PORCELAIN FOR DIFFERENT THICKNESSES OF TEST SPECIMEN

(Peek: Dielectric Phenomena in High Voltage Engineering)

Total thickness, mm.	Kv./mm.
0.5	16.0
1.0	14.5
2.0	12.2
5.0	11.0
10.0	9.6
15.0	9.2

Table II illustrates the effect of the time of application of voltage. It also shows the effect of temperature on the dielectric strength. The values are

TABLE II.—VARIATION OF DIELECTRIC STRENGTH WITH THE TIME OF APPLIED VOLTAGE

(Peek: Dielectric Phenomena in High Voltage Engineering)

Material	Thickness, mm.	Time to puncture, min.	25° C. kv/mm.	100° C. kv/mm.
Oil-impregnated paper, 30 layers, 60-cycle, 10 cm. diameter discs. round edges.	1.90	"Inst."	39.4	32.0
		1	33.1	27.3
		2	31.0	25.7
		4	29.2	24.5
		6	28.2	23.6
		10	26.8	22.7
		20	25.5	21.6
		40	23.6	20.5
		60	22.7	19.7
		80	22.1	19.3
		100	21.6	19.0

given for paper which was thoroughly dried prior to impregnation with oil. If an insulation contains moisture the effect of temperature may be opposite from that shown here for paper (see article on *Fibre*). It follows, therefore, that insulations should preferably be tested at a definite relative humidity at a definite temperature and the time of application of voltage should be standardized in order to obtain comparable results.

Table III gives the dielectric strength of various insulating materials. The values given were not all obtained under the same test conditions. The literature from which the values were obtained seldom stated the test conditions. The table, therefore, cannot be used to compare absolutely the dielectric strength of insulating materials but will serve only as a guide in choosing materials of a given order of dielectric strength.

SURFACE LEAKAGE is due to a film of moisture on the surface of the insulation. This moisture may contain soluble salts and condensed gases, such as ammonia, and carbon dioxide from the air. This film may indeed be very thin; for example, on cleaned quartz glass, it has been found to be from 3 to 6 $\times$ 10^{-6} mm. Waxy materials have no surface film because the moisture condensed on the surface tends to draw into drops instead of spreading over the surface. Surface resistivity according to Curtis is independent of voltage and also temperature, the relative humidity being constant. The temperature range which he investigated was 25° to 30° C. with relative humidity at 25 per cent.

Surface resistivity of most insulating materials decreases upon exposure to sunlight or ultra-violet light. The results obtained by Curtis indicate that a few hours of exposure to ultra-violet light will produce the same or sometimes a greater effect than would be produced in years by sunlight.

Surface resistivity is defined as the resistance between two opposite edges of a surface film, one centimeter square. The values in Table III are those obtained by Curtis at temperatures from 25° to 29° C. and are given for 30 and 90 per cent relative humidity. The test voltage was 200 volts.

On high voltages, apparent surface leakage should be distinguished from actual surface leakage. The surface moisture film under high voltage is soon dispelled by the heat generated with the leakage current. Then a new leakage film is produced by the oxidation of the insulation with ozone generated at the surface due to corona formed by local ionization of the surrounding air. If the surface of the insulation is covered with an oil such as blown linseed oil prior to the application of the voltage the oxidized film can be greatly reduced, and the arc-over or flash-over voltage will be increased. Even though all the actual surface leakage were removed, the flash-over voltage would still be limited because of the apparent surface leakage, which, for a given leakage distance, depends upon the position and shape of the electrodes, relative specific inductive capacities of air and the insulation and the shape of the insulation (contour of the surface). If no surface leakage at all existed the flash-over voltage would then be the breakdown voltage of the air between the electrodes. (Adapted from Peek, *Dielectric Phenomena in High Voltage Engineering*).

As very little data are available on the flash-over voltage of insulations, this property is not included in Table III.

VOLUME RESISTIVITY of insulating materials in general follows Ohm's law. The only exception of any consequence is a few vegetable oils, as for example, castor oil in which the resistance decreases with increase in the applied voltage. The volume resistivity varies with the amount of moisture present and with the temperature. In Shrader's work on the effect of moisture, he found, for example, an insulating material, which is generally considered non-hygroscopic, had a volume resistivity of 26 $\times$ 10^{10} ohm-cm. ordinarily, but after being heated in vacuum, had a resistivity of 8 $\times$ 10^{14} ohm-cm., or 3000

TABLE III.—ELECTRICAL PROPERTIES OF INSULATING MATERIALS

Material	Dielectric strength			Resistivity				S.I.C.	
	Specimen thickness, mm.	Kv. per mm. (a)	Authority	Volume ohm-cm.	Authority	Surface (b) Ohms, 30 %	Surface (b) Ohms, 90 %	Air unity	Authority (p. 856)
Amberite	.....	....	...	5×10^{16}	26	2×10^{15}	4×10^{12}	.....	...
Ambroin	0.84	6.0	2	1.7×10^{13}	2	......	......	.....	...
Asbestos paper	1.2	4.2	3	1.6×10^{11}	24	......	......	.....	...
Asphalt (Byerlyte)	3.6	14.0	4	....	...	......	......	2.7	25
Bakelite, C-1	.....	up to 27.5	5	over 2×10^{14}	5	......	......	5.2 to 9.9	5
Bakelite, wood molding mixture	.....	17.7 to 21.6	5	1×10^{12}	5	......	......	4.5 to 5.5	5
Bakelite, asbestos molding mixture	.....	up to 9.8	5	4×10^{11}	5	......	......	.....	...
Bakelite, Continental	3.2	15.7	7	....	...	......	......	.....	...
Bakelite, Micarta-213	.....	up to 31.4	6	5×10^{11}	5	......	......	5	6
Bakelite, Micarta-21D	.....	5.9	6	....	...	......	......	5	6
Bakelite, Micarta-21H	.....	15.7	6	....	...	......	......	5	6
Bakelite-Dilecto-X	3.2	25.6	7	1×10^{12}	7	......	......	5	7
Bakelite-Dilecto-XX	3.2	25.6	7	1×10^{12}	7	......	......	5	7
Celluloid (clear)	0.25	12 to 28	1	2×10^{10}	26	8×10^{10}	2×10^{9}	.....	...
Celluloid (colored)	0.25	10.2 to 18.9	1	....	...	......	......	.....	...
Ceresin	.....	....	...	over 5×10^{18}	26	8×10^{16}	8×10^{16}	.....	...
Condensite (molded)	5.7	19.7	8	4×10^{10}	26	7×10^{11}	6×10^{9}	.....	...
Condensite (celoron)	5.7	29.5	8	....	....	......	......	.....	...
Conite	0.13	15.7	7	....	...	......	......	.....	...
Copal	3.0	3.2	3	....	...	......	......	.....	...

Electrose (black)	.....	....	...	1×10^{14}	26	3×10^{12}	8×10^{9}	.....	...
Electrose (yellow)	.....	....	...	5×10^{15}	26	1×10^{15}	5×10^{8}	.....	...
Empire cloth, canvas	.41	28.9	9	....	...	......	......	.....	...
Empire cloth, linen	.15	54.0	9	....	...	......	......	.....	...
Empire cloth, muslin	.38	48.0	9	....	...	......	......	.....	...
Empire cloth, silk	.15	48.0	9	....	...	......	......	.....	...
Faturan	3	10	10	....	...	......	......	.....	...
Fiber, vulcanized, including hard fiber, all colors	0.79	8.9 to 16.7	12	5 to 20×10^{9}	26	3×10^{10}	1×10^{7}	5	...
	3.2	4.9 to 10.8	12	5 to 20×10^{9}	26	3×10^{10}	1×10^{7}	5	...
	6.4	3.9 to 8.9	12	5 to 20×10^{9}	26	3×10^{10}	1×10^{7}	5	...
	12.7	3 to 5.9	12	5 to 20×10^{9}	26	3×10^{10}	1×10^{7}	5	...
Galalith (white)	.....	6 to 8.5	13	1×10^{10}	26	6×10^{10}	6×10^{8}	.....	...
Glass (ordinary)	.....	8 to 9	13	9×10^{13}	27	......	......	5.5 to 9.1	25
Glass (plate)	.....	....	...	2×10^{13}	26	3×10^{13}	2×10^{7}	5.5 to 9.1	25
Gummon	.....	....	...	3×10^{12}	26	5×10^{12}	3×10^{8}	.....	...
Hermit	.....	2 to 3	23	1×10^{10}	26	3×10^{10}	5×10^{8}	.....	...
Jute (impregnated)	6	1.2	14	....	...	......	......	3 to 4	28
Lava	.....	3 to 10	15	2×10^{10}	26	6×10^{11}	1×10^{8}	.....	...
Litholite	4.5	4.5	16	....	...	......	......	.....	...
Marble	.....	6.5	13	1 to 100×10^{9}	26	8×10^{10}	2×10^{7}	8.3	29
Mica	.6	21 to 28	13	.04 to 200×10^{15}	26	2×10^{13}	3×10^{9}	5 to 7	40
Micabond, plate	1.6	37.5	17	....	...	......	......	.....	...
Micabond, flexible	1.6	23.1	17	....	...	......	......	.....	...
Micanite, plate	1.6	37.5	18	....	...	......	......	.....	...
Micanite, flexible	1.6	23.1	18	....	...	......	......	.....	...
Minerallac	.....	39.4	19	5×10^{9}	8	......	......	2.1	8
Paper	0.13	8.7	40	5×10^{4}	...	......	......	2.6	40
Paraffin (parowax)	.....	11.5	13	1×10^{16}	26	1.5×10^{16}	5×10^{15}	2.1	31

(*a*) To obtain volts per mil multiply kilovolts per millimeter by 25.4. (*b*) At 30 per cent and 90 per cent relative humidity.

TABLE III.—ELECTRICAL PROPERTIES OF INSULATING MATERIALS—Continued

Material	Dielectric strength			Resistivity				S.I.C.	
	Specimen thickness, mm.	K.V. per mm. (a)	Authority	Volume ohm-cm.	Authority	Surface (b) Ohms, 30%	Surface (b) Ohms, 90%	Air unity	Authority (p. 856)
Paraffin (special)	.....	....	...	over 5×10^{18}	26	over 1×10^{18}	over 1×10^{18}	.....	...
Porcelain	20	8.0	21	3×10^{14}	26	4×10^{13}	5×10^{8}	4.4	21
Pressboard (oiled)	0.25	39.3	40	....	...	......	......	5.0	32
Pressboard (oiled)	1.58	29.2	40	....	...	......	......	5.0	32
Pressboard (oiled)	3.17	21.1	40	....	...	......	......	5.0	32
Pressboard (varnished)	0.25	26.3	40	....	...	......	......	3	32
Pressboard (varnished)	1.58	15.5	40	....	...	......	......	3	32
Pressboard (varnished)	3.17	9.5	40	....	...	......	......	3	32
Presspahn	.....	5.2 to 9.3	33	1×10^{10}	34	......	......	.....	...
Redmanol (molded)	5.1	11.8 to 18.5	8	2.5 to 25×10^{10}	8	......	......	.....	...
Redmanol (laminated)	0.8	41 to 51	8	2.2×10^{11}	8	......	......	.....	...
Rosin	.....	....	...	5×10^{16}	26	8×10^{14}	2×10^{14}	2.5	38
Rubber (hard)	0.5	70	37	1×10^{18}	26	6×10^{15}	1×10^{9}	2.0 to 3.5	30
Shellac	.....	....	...	1×10^{16}	26	2×10^{14}	6×10^{9}	3.0 to 3.7	39
Slate	10.3	1.3	22	1×10^{18}	26	2×10^{8}	1×10^{7}	6.6 to 7.4	35
Stabalite	.....	....	...	3×10^{13}	26	6×10^{13}	4×10^{7}	.....	...
Sulphur	.....	....	...	1×10^{17}	26	1×10^{16}	1×10^{14}	2.9 to 3.2	39
Tegit	.....	....	...	2×10^{12}	26	2×10^{12}	5×10^{7}	.....	...
Vulcabeston	1.9	3.15 to 7.1	20–23	2×10^{10}	26	1×10^{10}	3×10^{9}	.....	...
Wood (maple), paraffined	15.2	4.6	32	3×10^{10}	26	1×10^{12}	2×10^{9}	4.1	32

(a) To obtain volts per mil multiply kilovolts per millimeter by 25.4.
(b) At 30 per cent and 90 per cent relative humidity.

times as great. As to the effect of temperature, Curtis found that in 53 of the insulations which he investigated, the ratio of the resistivity at 20° C. to that at 30° C. varied from 1 to 5.3.

The values given in Table III are for room temperature at ordinary atmospheric humidity. The unit used is the resistance to the current flowing through the material between two opposite faces of a centimeter cube, no surface leakage being present.

DIELECTRIC LOSS is due to the following causes:

With either d-c. or a-c.

1. Ionization of occluded gases.
2. I^2R losses due to leakage current.

With a-c. only.

3. Oscillatory movement of particles of higher or lower specific capacity than the surrounding medium, due to the charges induced on them being alternately attracted and repelled by the electrodes.
4. I^2R losses due to displacement current.

Generally, dielectric loss varies as the square of the voltage up to a certain dielectric stress at which the loss begins to increase faster than the square of the voltage. The increase occurs when the occluded gases or other particles of low specific inductive capacity begin to ionize. This has the effect of greatly increasing the dielectric stress on the particles of high specific capacity which soon rupture. Dielectric loss therefore has a bearing upon the dielectric strength of insulations especially when the voltage is applied indefinitely.

Table IV gives dielectric loss of varnished cloth and oil treated paper. No data are available on insulations in general.

The power-factor of insulation is the ratio of the power dissipated in it (i.e., the dielectric loss) to the apparent power delivered to it.

As the charging current and therefore the apparent power increases, with the specific capacity, it is obvious that low power factor may result either from low dielectric loss or high specific capacity. The power factor is therefore not a good indication of the quality of insulation.

THE PHYSICAL PROPERTIES of insulating materials are given in Table V. The degree of hardness is not listed because it has been determined for only a very limited number of materials. Furthermore, there seems to be no agreement in the unit chosen to express the degree of hardness. Transverse strength is not given because no standard method for making this test has been adopted. Very few values can be found in the literature for shearing strength and so this property is also omitted.

Souder and Hidnert found that the expansion of insulating materials is a function of the temperature, medium in which the material is heated (air or oil), time, rapidity of heating or cooling. Of these factors the medium is the most important for materials which absorb moisture in considerable quantities. The values given for the linear temperature coefficient of expansion are those obtained in an air medium with the exception of Continental-Bakelite which, when heated in air, has an irregular expansion.

MOLDED INSULATIONS may be classified as Hot Molded Insulations and Cold Molded Insulations.

Hot Molded Insulations are subjected to heat and pressure simultaneously in the mold. Three classes of binders are used.

1. Gums, such as shellac and copal which become soft and plastic when heated and hard when cold. These gums are mixed with short asbestos fiber, cotton fiber, ground mica, infusorial earth, barytes or any of a great variety of materials.

Authorities for Table III: (1) Hobart and Turner. (2) Physicalische Technische Reichsanstalt. (3) Steinmetz. (4) Byerly & Sons, Manufacturers. (5) General Bakelite Co. (6) Westinghouse. (7) Continental Fiber Co. (8) Electrical Testing Laboratory. (9) Mica Insulator Co. (10) Bütlemann. (11) Formica Insulation Co. (12) William Eves, 3d. (13) Walter. (14 Baur. (15) American Lava Corporation. (16) Symons. (17) Chicago Mica Co. (18) Mica Insulation Co. (19) Minerallac Electric Co. (20) Canfield & Robinson. (21) The Locke Insulator Mfg. Co. (22) Massachusetts Institute of Technology. (23) Electrician. (24) Whittaker's Pocket Book. (25) Pirani. (26) Curtis. (27) Table of French Physical Society. (28) Coyne & Howe. (29) Schmidt. (30) Various. (31) Zietkowsky. (32) Hendrick. (33) Kinzbrunner. (34) Stadt Lab., Munich. (35) Schulze. (36) E. Müller. (37) C. C. Paterson, E. H. Raynor, A. Kinnes. (38) Boltzmann. (39) Wüllner. (40) Peek.

TABLE IV.—DIELECTRIC LOSSES

(Peek: Dielectric Phenomena in High Voltage Engineering)

(Effective Sine Wave 60 Cycles)

Total thickness mm.	Insulation	No. of layers	Temp. °C.	Volts per mm.	Watts per cu. cm.
4.0	Varnished cloth..............	15	25	4,000	0.005
				6,000	0.015
				8,000	0.035
				10,000	0.060
				12,000	0.090
4.0	Varnished cloth..............	15	90	4,000	0.025
				6,000	0.075
				8,000	0.150
				10,000	0.240
				12,000	0.350
2.5	Oil-treated paper.............	30	25	10,000	0.040
				14,000	0.070
2.5	Oil-treated paper.............	30	60	10,000	0.043
				14,000	0.080
2.5	Oil-treated paper.............	30	90	10,000	0.050
				14,000	0.100
2.5	Oil-treated paper.............	30	120	10,000	0.067
				14,000	0.138

These losses may be lower or very much higher, depending upon the condition of the insulation.

TABLE V.—PHYSICAL PROPERTIES OF INSULATING MATERIALS

Material	Specific gravity	Tensile strength pounds per square inch	Compressive strength, pounds per square inch	Linear temperature, coefficient per °C.
Ambroin, Grade A. F	....	2,140	2,680	
		up to	up to	
Bakelite, C–1	1.27	11,000	32,000	.00011
Wood molding mixture	1.34	4,000	36,000	.00003
Asbestos molding mixture	1.89	4,200	36,000	.00002
Micarta, 213	1.36	15,000	*22,000	.00002
Micarta, 21D	1.36	10,000	*21,000	.00002
Micarta, 21H	1.36	10,000	*20,000	.00020
Dielecto-X	1.32	12,000	*20,000	.000052
Dielecto–XX	to 1.38	....	†43,000	....
Continental Bakelite	1.36	8,500	*23,000	‡.00003
Grade CBL			†41,000	
Condensite (molded)	....	5,000	30,000	.00002
				to .00004
Condensite, celoron	....	13,000	40,000	....
Fiber	1.2	9,000	25,000	.000027
	to 1.5	to 14,000	to 43,000	....
Formica, Grade P	1.25	20,000	50,000	.00001
				to .00003
Lava	2.50	....	20,000	....
	to 2.70	....	to 30,000	....
Marble	2.50	....	9,000	.00001
	to 2.80	....	to 22,000	to .00002
Porcelain	2.30	3,000	20,000	.000002
	to 2.50	....	....	to .00002
Redmanol (moulded)	1.32	3,500	43,000	....
		to 7,200	to 55,000	....
Redmanol (laminated)	1.30	13,000	*26,000	....
	to 1.40	to 31,000	to 49,000†	....
Slate	2.65	....	11,000	....
			to 30,000	....
Wood, Maple	....	8,000	6,000	....
		to 12,000	to 8,000	....
Wood, Walnut	....	8,000	4,000	....
		to 14,000	to 8,000	....

* parallel to lamination.
† perpendicular to laminations.
‡ measured in an oil bath.

A proper combination of gums and fillers is selected to obtain the desired results, These materials are mixed by the use of a grinder, hot rollers or a solvent which is afterwards driven off. The mixed material is heated to fuse the binders and then placed in a heated mold. The charged mold while hot is pressed in a cold

press to produce the desired form and then allowed to cool and harden under pressure. Insulations molded thus are generally mechanically weak, have high dielectric strength, soften and distort at about 40 to 60° C. and low in cost.

2. Synthetic Resin such as that produced by the mixture of phenol and formaldehyde in the presence of ammonia. These chemicals combine to form a liquid varnish which has the remarkable property of hardening upon heating without softening upon subsequent heating even when heated to carbonization. A variety of other chemicals can be used. In this pure form the insulation is translucent with a beautiful variety of colors and takes a high polish and is known as synthetic amber. Examples of this product are on the market: bakelite, condensite, and redmanol. A molding mixture consists of this resin, wood flour, asbestos fiber or other materials, molded in a hot press and allowed to set. The resin in solution may be spread on paper or fabric which is then pressed together and heated to drive off the solvent. An example of such a product is micarta. Insulations of this group have great mechanical strength, high dielectric strength and in most cases, resist temperatures up to 150° C.

3. Vulcanizing Agent. Hard rubber or vulcanite is an example of this group. Crude rubber is mixed with fillers and a vulcanizing agent, such as sulphur, and then vulcanized in molds under heat and pressure. These insulations have high resistivity, fair mechanical strength, and distort under slight loads even at room temperature.

Cold Molded Insulations are made by the use of two classes of binders.

1. Binder dissolved to mix with a filler, the solvent of which is expelled after the mold is completed. The binder may be (*a*) a pitch dissolved in benzine or naphtha, (*b*) phenolic resin dissolved in alcohol or benzol, heat being continued after the solvent is driven off the mixture to polymerize or harden the resin and (*c*) silicate of soda dissolved in water, the water being expelled from the mixture in a drying oven. The filler used with these binders is generally asbestos. The characteristics of each are: (*a*) soften at 60° C., fair in mechanical and in dielectric strength, and low cost; (*b*) does not soften up to 150° C., and not as strong mechanically, nor as good electrically as the hot molded insulations; (*c*) weak mechanically and electrically, resists high temperatures, but hygroscopic.

2. Set by chemical action: Examples of this group are: (*a*) Portland cement with asbestos fiber, the excess water being driven off by heating; and (*b*) lime and silica mixed with water properly treated and used with a filler of asbestos or magnesia. The properties of each are (*a*) not weakened by moisture, fair electrically up to 400° C., mechanically fair, porous in structure, but can be made weatherproof with a coating which will limit the temperature in use: (*b*) mechanically weak and resists very high temperature.

MISCELLANEOUS INSULATIONS. — Insulating materials are often described in general terms without specific data. A material is said to be "heat proof" which may mean that it will not deteriorate when used continuously at some temperature within the range of 30° and 250° C. Dielectric strength is sometimes quoted without reference to the thickness of the test specimen. If the test has been made on a very thin specimen, the dielectric strength obtained will be misleading to those who are not aware of the law of the increase of dielectric strength with the decrease of the thickness of the test specimen. Claims may be made that an insulating material is insoluble in certain solvents and is weatherproof. Such claims are not absolute. They are only general and refer to ordinary conditions found in the use of insulations. If the working conditions are unusual and severe, specific information should be obtained on the appropriateness of the insulation under consideration.

The more important types of insulation are described in alphabetical order in the following pages.

Air has a low conductivity and specific inductive capacity, and a dielectric strength sufficient for most purposes except very high voltage apparatus and transmission lines; see articles on *Corona* and *Spark Gap*.

Amberite or ambroid is made by compressing scrap amber. It is now in extensive use and as far as surface leakage is concerned is the equal of native amber. In using it the surface should be well cleaned to avoid surface leakage due to the deposition of deliquescent salts in handling it with the fingers.

Ambroin is made by baking in a vacuum silicate of sodium, asbestos, fossil copals, etc., mixed with alcohol. It is moulded in heated moulds, under high pressure. Certain grades of Ambroin will resist very high temperatures and are suitable for use in arc shields. Ambroin is only slightly hygroscopic as compared with other insulators of the same nature. (Adapted from Hobart and Turner.)

Asbestos is a mineral consisting chiefly of silica, magnesia, lime, alumina water and oxide of iron. The structure is of innumerable fibers the ultimate fiber of which is thought to be a single row of the molecular structure of the crystal. The fibers are exceedingly smooth and glossy and have very little friction to hold them together when they are spun into a yarn, this resulting in a low tensile strength. This difficulty, however, has been partly overcome so that now threads are made which have fair tensile strength but not equal to cotton. The more important varieties of asbestos are amianthus and amphibole and are used in the form of asbestos paper, cardboard, yarn, cloth and as a filler in moulding mixtures.

Asbestos contains small particles of iron oxide or grit which cannot be entirely removed and affect to a slight degree its insulating qualities. It is hygroscopic and should therefore not be used on high voltage, in general not over 3300 volts. It is unaffected by oils, acids and alkalies and withstands very high temperatures. According to Stifler its insulating qualities break down at 1000° C., but recover after it is cooled. Above 1000° C. it loses its mechanical strength and melts at about 1300° C. It has an extensive use as a heat insulator.

In Europe some of the trade names of asbestos impregnated with pitches, rosins, sulphur, paraffin, cellon and phenolic rosin are eshallite, ambroin, tenazite, agalite, australite and festonite.

Asphalt is a mineral pitch found in geological formation in various parts of the world. Asphalt is used (1) in the manufacture of insulating varnishes and japans, (2) for the impregnation of hygroscopic non-waterproof insulating materials and (3) as an insulating covering for cables. The various grades of asphalt used for electrical work are called Trinidad, Bitumen, Elaterite, Gilsonite, Byerlyte and Manjak. Asphalt insulators possess a high insulation resistance, dielectric strength, flexibility and mechanical toughness, are very cheap, and are unaffected by moisture.

Pure asphalt (solid bitumen) softens at from 90° to 100° C. and is not recommended for use for immersion in hot oils, or where it would be subjected to centrifugal stress. Certain grades of Byerlyte melt at temperatures ranging from 200° to 350° F., while the melting point of Gilsonite varies from 230° to 400° F. Symons states that the dielectric strength of pure asphalt is 30,000 volts per inch (test made on sheet 1/16th inch thick).

Bakelite is a synthetic organic substance resulting from the chemical condensation of phenols and formaldehyde (General Bakelite Co., N. Y.). It may be applied as a liquid or used as a solid. The liquid Bakelite is useful for the impregnation of porous materials, for enameling under heat and pressure and as a binding agent for moulded compounds. The solid Bakelite is unaffected by water, steam, oils and almost all chemicals. It does not melt or soften at ordinary

machine temperatures; it is destroyed only at temperatures in the vicinity of 300° C. It is easily and accurately moulded and may be made to take any color. It compares favorably with rubber and gutta-percha in every way except that it is not as flexible as those substances. The moulded insulation will withstand a temperature of 90° to 450° C., depending on the filler used.

Bakelite-Dilecto is made by hot pressing paper which has been saturated with raw bakelite. It is furnished in five different grades in the form of sheets, tubes and rods. If immersed in water for eight days, it will absorb from 1 to 6 per cent of its weight of water, depending upon the grade. In general, it is considered non-hygroscopic and resists ozone indefinitely. It withstands a temperature of 105° C. continuously and 150° C. for short periods of time without deterioration. Its chemical inertness is similar to that of Bakelite. It can be worked with any machine shop tool and takes a splendid polish.

Bakelite Micarta is primarily an insulating material but can be used for many other purposes as for example, pinions and gears, the wearing properties of which are better than rawhide. It is furnished in the form of sheets, rods, and tubes and can be machined accurately and takes a good polish. For short periods of time, it will withstand a temperature of 140° C. Its chemical inertness is similar to that of Bakelite.

Bitumen. — See section on *Asphalt.*

Byerlyte. — See section on *Asphalt.*

Cambric, Varnished. — See separate article on *Cambric, Varnished.*

Cellon is obtained by heating cellulose in a closed glass vessel with acetic anhydride to 180° C. or 110° C. if hydrocellulose is used. When sulphuric acid is used as a catalyser, the reaction occurs at about 55° C. The resulting mass is thinned by adding acetic acid. The cellon is precipitated by adding water, benzine or ether and afterwards washed thoroughly with water. It is then dried at a temperature not exceeding 100° C. The finished product is furnished in either a hard or flexible form in any desired color. Its specific gravity is 1.35 and its dielectric strength is 30 kv. per mm. It can be shaped with any tool, polished and becomes elastic in boiling water and can then be pressed into moulds.

Celluloid (xylonite) is a dried solution of gun-cotton (pyrolin) and oil. It may be machined or moulded into any form by softening in boiling water. Its dielectric strength is very much reduced at high temperatures, and it is very combustible. It is only slightly hygroscopic.

Coal-tar Pitch is the residue remaining after the fractional distillation of coal tar. It flows at low temperature and when cold is quite brittle. Symons gives its dielectric strength as 2000 volts per millimeter.

Condensite is a hard infusible substance, the chief constituent of which is a resinous gum, made by the reaction between phenol and formaldehyde. Condensite is produced by combining this gum under heat and pressure with a hardening agent. It is supplied in three grades; for plastic moulding, for impregnating and as a cement. It can, in general, be moulded to an accuracy of .002 in. plus or minus, with all kinds of metal inserts.

It is non-inflammable, non-corrosive, and infusible, insoluble in oils, most acids and other solvents and shrinks but one-fifth of one per cent in moulding. The impregnating material is used in connection with metal, wood, paper, cardboard, rubber, leather, etc. The plastic cement is used for fastening together the parts of porcelain insulators, for sealing terminals in porcelain bases, etc.

The value given for dielectric strength in Table I was obtained between blunt needle points, normal temperature, oil immersion, 60 cycles, and the voltage applied for about 10 seconds. Flash-over voltage as tested by the Bureau of

Standards at radio frequency was 25 kv. between skirted stubs ¾ in. apart on a specimen 4 inches by 4 inches by ⅛ inch thick.

The carbonization point is 287° C. Condensite will withstand temperatures of 148° to 215° C. continuously, depending upon the filler used. The shearing strength is 5000 pounds per square inch.

Condensite-Cellulac is claimed to be non-hygroscopic and impervious to oils or acids. It has no corrosive effects at ordinary temperatures and can be used in the weather. It is furnished in sheets, rods, tubes and in plastic uncured state for filling irregular spaces in which it will afterwards harden.

Condensite Celoron is a new insulating material on the market. The composition of this material has not as yet been published. It has a high dielectric strength and has excellent physical properties.

Conduline is made especially for insulating cable joints and end bells. Aside from having good insulating qualities, it has very desirable physical properties. It does not become brittle at low temperatures but remains plastic. The upper temperature limit of plasticity is 95° C. when it suddenly changes over to a very mobile liquid state completed at 110° C. When this filling compound is used in connection with a cable cell, known on the market as "Conducell," cable joints are obtained which are claimed to be uniform in quality and construction.

Continental Bakelite is made by hot pressing canvas or cloth which has been saturated with raw Bakelite. It is furnished in two grades, CB and CBL, in the form of sheets, rods and tubes. It is non-hygroscopic. It absorbs only 2 per cent of its weight of water when immersed for eight days. It withstands 105° C., continuously and 150° C. periodically without deterioration. It can be machined and takes a good polish. Its chemical inertness is equal to that of Bakelite.

Copal is a resinous substance, which makes a colorless varnish when dissolved in alcohol, oil of turpentine or linseed oil. Copal is easily fused, is very inflammable and is quite brittle when cold.

Cornimit is made from fish offals by a Danish process. It is not as hygroscopic as galalith and has higher electrical resistance. The cost of it is very low compared with other insulations.

Dorrite can be moulded or stamped into any commercial size or shape. It is said to be fire-proof, acid-proof and water-proof. A specimen 3.2 mm. thick, and 7.6 cm. square, after being immersed in water for 70 hours is reported to withstand 40 kv. without signs of leakage or change.

Ebonestos is an English product made of asbestos fiber impregnated and coated with various gums and is moulded accurately under heat and pressure. It is furnished in four grades: (*a*) Bright and natural finish, hard and short grained, and low softening point. (*b*) Dull black finish, hard and short grained, softening point 100° C. (*c*) Tough fibrous material with varnish finish and softening point of 80° C. (*d*) Coarse fibrous fire-proof material. A specimen of grade "A" after being immersed for 48 hours in brine had an electrical resistance of 5×10^{13} ohm-cm.

Ebonite. — See section on *Rubber, Hard*.

Elaterite. — See section on *Asphalt*.

Electrobestos is an English product and is an asbestos compound which will withstand temperatures up to 900° C. at 500 volts. It is used in heating and cooking devices and arc lamps.

Electrose has been in use for thirty-two years. It is a composite insulation which does not tarnish metals because no sulphur is used in its vulcanization. Its compressive strength is 10,000 pounds per square inch. Its temperature

coefficient of expansion is very low and it can therefore be used for mounting electrical parts accurately such as the diaphragm and magnet in telephone cases. It possesses hardness and toughness without brittleness, takes a smooth polished surface and is moisture, water and oil proof. It does not shrink or warp under ordinary conditions. A piece 12 inches by 12 inches and 1/8 inch thick arcs over at 75,000 volts but does not puncture between brass spheres 3/8 inch in diameter.

It can be moulded into a great variety of shapes. It is used extensively in all types of insulators, bushings and terminals, for railway work, power transmission, radio and telephone transmission over a wide range of frequencies and voltages.

Empire Cloth consists of a closely-woven cambric coated with two or more films of an oxidized oil, prepared by a special process. Canvas, duck, linen, silk, and paper are also coated by the same process. (See *Varnished Cambric.*)

Erinoid is an English product made from solids of milk and is similar to Galalith and resembles horn in appearance. It is furnished in sheets, tubes, and rods in various colors including a striated appearance. Its insulating properties are good if not used in damp places. It can be shaped with tools and polished by rotating in a barrel with Erinoid shavings and pumice stone.

Eswelite is somewhat similar to Bakelite except that it is brittle and becomes flexible in boiling water. According to Bütlemann its specific gravity is 1.2 and its dielectric strength is 15 kv. per mm.

Fiber is pure cotton cellulose chemically treated to form a hard bone-like material. It is known by various trade names such as Fiber, Vulcanized Fiber, Hard Fiber, Horn Fiber. These fibers are all essentially the same and vary only in quality and uniformity with the skill and experience of the manufacturer. Fiber of best quality has its origin in old rags because the natural oil of the virgin cotton fiber is deleterious and cannot be removed by easy chemical process. This oil is removed in the wearing and washing of the garment. All dirt and grit must be removed from the rags. The quality also depends on the assortment of the rags.

"Vulcanized Fiber sheets are made by passing cotton rag paper through a strong acid or chloride bath and rolling it up on a large drum where each layer of paper sticks to the layer beneath it. When the proper thickness is obtained the acid soaked material is cut from the drum and cut in half forming two sheets of raw fibre. These sheets are put through a soaking process in large wooden tubs, each subsequent tub containing a weaker solution, the last tub containing pure water." (Catalogue of The Continental Fiber Co.)

The soaking process requires from one week to one year depending on the thickness of the sheet. A very small amount of acid will remain in spite of all the soaking. The sheets are air dried and seasoned at a constant temperature. During the drying, they warp and shrink to one-half the original thickness. They are flattened in a steam heated hydraulic press and then calendered to exact thickness.

According to Almy the mechanical and physical properties of fiber varies between wide limits according to the manipulation of the chemical treatment and the varying quality of the original paper or rags. A fiber for a particular use should therefore be selected with great care.

Fiber is not waterproof. It will absorb water but is not injured beyond warping by either hot or cold water because it returns to its original dimensions and properties when dried. Dilute acids and alkalis cause no other effects than water, but concentrated acids cause disintegration. Organic solvents and oils have no effect whatever. At a sustained temperature of 80° to 100° C., it loses its water of condition and becomes brittle and chars in a short period of time at 200° C.

According to Eves, the dielectric strength is about the same for all colors of a given fiber. The tendency of the iron salts in the coloring matter to lower the dielectric strength is offset by the loading effect of the coloring matter which gives the fiber greater homogeneity. The dielectric strength of thin specimens of fiber increase with increase in temperature whereas it decreases with increase in temperature with thick specimens. The moisture is dispelled from the thin specimens at the higher temperatures but is retained in the thicker specimens. This difference in the retained moisture explains the variations in dielectric strength as the temperature is increased. Fig. 1 gives the average breakdown voltage for different thicknesses of fiber of all colors.

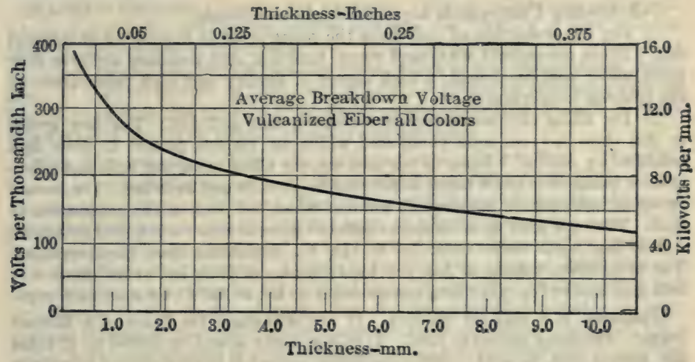

Fig. 1.

Fiber is used very extensively. One of the greatest uses of fiber, either for mechanical or electrical purposes, is for railway signal insulation. The Railway Signal Association's specification requires a specific gravity of 1.3 to 1.5; an absorption when immersed in water at 70° F. for twenty-four hours, not to exceed 45 per cent by weight for 3.18 mm. fiber, 30 per cent for 4.76 mm. and 26 per cent for 6.35 mm. fiber; a tensile strength of 5000 pounds per square inch; a dielectric strength of 1280 volts per millimeter for all thicknesses under an application of voltage for one minute between disc electrodes 2.54 cm. in diameter; withstand bending into a circle of a radius ten times the thickness without cracking or splitting.

Formica is furnished in three grades in the form of sheets, tubes and rods, and can be moulded. Its physical and chemical properties are claimed to be similar to those of redmanol. Its electrical and mechanical properties as given by the manufacturer are: dielectric strength, 39 kv. per mm. Resistivity, 2.2×10^{13} ohm-cm., tensile and compressive strength, averaging 20,000 pounds per square inch, specific gravity of 1.25 to 1.35 and a linear coefficient of expansion of .00002 per degree centigrade. It will withstand a temperature of 150° C. continuously.

Fuller-Board. — See section on *Pressboard*.

Galalith is a German product made by heating skimmed milk with caustic soda and precipitating the casein with acid. The casein plates after being dried are saturated with formaldehyde and dried again resulting in a translucent sheet of yellow-white color. It is rather hygroscopic but serves as a good insulator in dry places. It can be shaped cold after being softened with hot water. Its specific gravity is 1.32.

Gilsonite. — See section on *Asphalt*.

Glass is a silicate of soda or potash combined with certain metallic oxides, usually lead oxide. Owing to the condensation of moisture upon its surface, glass has a very large surface leakage. Rain water roughens its surface and the resulting accumulation of dirt reduces its insulating qualities. According to Gray and Dobbie the specific resistance of potash glass is higher than that of soda glass. Annealed glass has in general a higher specific resistance than unannealed glass. As compared with porcelain, glass is inferior in mechanical strength; whereas porcelain is only chipped by a blow, glass is easily cracked or shattered.

G-E Sealing Compounds are divided into two classes:

(1) For the protection of coils from the penetration of moisture and mineral oils. These compounds withstand rough handling, and possesses uniform flow point under continued heat, a high degree of fluidity and high penetration at the treating temperature.

(2) For filling cavities to exclude moisture, dust and dirt. The first group in this class includes solid compound which are melted, poured in place and solidified by cooling. Some of the uses are, for filling cable end bells and joints and for filling over screw heads in receptacles, sockets and switches. The second group includes plastic compounds some of which set without the application of heat. They are used to fill in spaces between wires of armatures and of field coils.

The flow point varies from 65° to 250° C., depending upon the compound. The breakdown voltage of No. 227 joint compound is 75 kv. at 65° for a 0.25 inch gap, and of No. 239 filling compound is 40 kv. at 80° C. for a 0.2 inch gap.

Gummon is a coal tar product. It will withstand the flame of a Bunsen burner, remaining perfectly hard. It is not easily sawed or drilled as it takes the temper out of steel. Other insulating materials having similar properties are agalite and australite.

Gutta-Percha. — See article on *Gutta-Percha*.

Hemit is furnished in four grades. Grade A can be moulded accurately with metal inserts and is claimed to be less brittle than porcelain. It withstands a temperature of 318° C. continuously and sudden changes of temperature without cracking. It is non-hygroscopic and not affected by dilute acids. Grade B can be shaped with tools or moulded. It is hygroscopic and does not resist acid. It will withstand a temperature of 300° C. continuously and sudden changes of temperature. Grade C 1 is moulded in sheets and is non-hygroscopic, but does not resist acids. It withstands a continuous temperature of 425° C. and exposures to the electric arc and sudden changes of temperature. Grade C 2 is moulded in sheets and should not be exposed to moisture which reduces its dielectric strength. It is unaffected by the electric arc and withstands a continuous temperature of 1100° C. and sudden changes of temperature.

Jute is a fiber obtained from the inner bark of certain trees growing in India. Commercial jute is usually softened and rendered less brittle by impregnating it with paraffin or some similar mineral oil. It is used extensively as a filler in lead-covered cables and as a constituent of certain kinds of paper and press-board.

Kalanite is an English product furnished in three grades. (A) in sheets and rods which can be shaped with tools, (B) a moulding mixture and (E) in sheets which are fire resisting and used for arc shields and switch boxes.

Lava is a mineral talc, machined in its natural condition and then baked at a temperature of 1100° C. to a condition of extreme hardness. It is then unaffected by any subsequent temperature short of its baking temperature. It is slowly attacked by hydrochloric acid but is not affected by other acids or alkalies.

Its dimensions are unchanged by absorption of moisture and it has a negligible coefficient of expansion with temperature.

Lavite. — See section on *Lava*.

Leatheroid. — See section on *Fiber*.

Litholite is an insulating material resembling psychiloid in structure. It is made in sheets and is pressed into any form. It is softened by sulphuric acid and distilled water and is reduced to a pulp if immersed in caustic soda. It is tough but inflammable. It is used for commutator rings, bushings, washers, formers, bobbins and for square and round tubing.

Marble is the name given to any limestone which is sufficiently compact to admit of a polish. While pure marble is white, the presence of iron oxide or other impurities give it different colors. It is used principally for switchboard work and should not contain metallic veins, which reduce its insulating qualities. If used on circuits of 1000 volts or more, it should be saturated with an insulating varnish and baked. It shows oil spots and for that reason it is sometimes stained black and given a so-called marine finish.

Mica is an anhydrous silicate of aluminium and potash or sodium. It crystallizes in a laminated mass, some grades of which may be subdivided down to a thickness of 0.0008 millimeter. The ultimate thickness of cleavage layers is unknown and may be finally but one layer of the molecular structure. It is useful as an insulator because of its high insulating qualities and its ability to withstand high temperatures. Owing to its impurity, lack of flexibility and excessive surface leakage in the natural state, the laminæ are separated and sorted into various grades of purity and are then cemented together to form plate or flexible reconstructed mica of any thickness or purity. Moulded mica is used in the manufacture of overhead line material and in the making of moulded pieces as a substitute for hard rubber. It softens at 136° F. Moulded mica is insoluble in water, but is affected by certain oils and all acids and alkalies to some extent. Mica loses its insulating qualities at a temperature of 1000° C., but recovers them after cooling. Andrews showed that the dielectric strength of mica is greatly reduced when immersed in or coated with oil.

Micabond Cloth, Paper and Plate are forms of reconstructed mica. Micabond cloth is India sheet mica faced on one side with muslin and on the other with Japanese insulating paper, bonded together with a special binder. It is furnished in three different grades varying in flexibility and in thicknesses from .005 inch to 0.125 inch. Among its uses is the insulation of field coils and transformer coils. Micabond paper is made in the same way as the cloth except paper of different grades are used on both sides of the mica. It is furnished in six different grades in the form of sheets and tapes and in thickness from 10 to 20 mils. Micabond plate is made of mica sheets bonded with orange shellac. Grades 101 and 111 are furnished in sheets from .01 to 0.125 inch thick and are used for armature slots, spools for magnets and other similar purposes. These grades can be moulded. Grades 102 and 104 are also furnished in sheets which are very carefully milled to exact thickness varying from .01 to 0.125 inch and are intended especially for the insulation of commutator segments. These grades cannot be moulded. Grade 21 is made by a special process and is recommended for use in electric flat iron and ranges. Micabond is also furnished in the form of tubes, washers and commutator rings.

Micanite is also a reconstructed mica. White mica of India and amber mica of Canada are used. The mica films are very carefully inspected for purity and the bonding material is carefully selected to give the best results. It is furnished in the same forms and in the same grades and dimensions as micabond. Its uses are the same as those of micabond.

Minerallac is furnished in four grades all of which are impervious to water and show no alkali or acid reactions. Grade No. 1 is an impregnating compound of the consistency of molasses with a flash point of 230° C. Grade No. 2 is a semi-solid compound used in cable end bells and potheads or for impregnating coils, fiber and wood for use on high voltage. Grades No. 2-A is used in place of No. 2 in warm climates because of its higher flow point. Grade No. 20 is a semi-solid compound for use on voltages below 2000 volts. Grade No. 64 is a mineral insulating compound for use in high voltage cable joints. Its temperature coefficient of expansion is low and its dielectric energy loss is low.

	No. 2	No. 2–A	No. 20	No. 64
Flow point, °C........	48	59	75	68
Drip point, °C........	63	85	127	91
Flash point, °C........	200	243	348	182

Oil. — See article on *Oil, Transformer.*

Paper (*See article on Paper, Impregnated.*) The various kinds of paper used for insulation may be divided into two classes according to the method of manufacture:

(*a*) Fiber or pulp compressed into paper, examples of which are Kraft paper and Fuller board. This class of paper may receive a sizing treatment of a glue solution to harden the surface.

(*b*) Fiber specially digested or parchmentized to give a close grained cellulose texture, an example of which is fish paper. This class of paper must be carefully washed out in the manufacturing process to remove all digesting chemicals, otherwise the product becomes very brittle and disintegrates with age.

The following list gives the name, origin and method of manufacture of the well-known papers in use:

Name	Origin	Method of manufacture
Linen paper	Linen rags	Compressed and sized
Kraft paper	Wood pulp	Sulphate treatment unsized
Rope paper	Old rope	Compressed and unsized
Press board	Cotton rags	Compressed and sized
Fuller board	Cotton rags	Compressed and unsized
Fish paper	Cotton rags	Digested with sulphuric acid
Fiber	Cotton rags	Digested with zinc chloride
Asbestos paper	Asbestos and cotton	Compressed and unsized

Paper is very hygroscopic and should not be used for insulation unless it is impregnated or sealed. It is used extensively in telephone cables sealed with a lead sheath. Paraffined paper is used as a dielectric in electric condensers. Impregnated with oils, it is used in power cables in which it is also protected with a lead sheath. Many other uses might be listed but these are the most important.

Paraffin is a product obtained by the destructive distillation of petroleum shale. The dielectric strength of cloth, wood and paper is increased by impregnation with paraffin and the materials are rendered less hygroscopic. Paraffin

is acidproof but is very inflammable and has a melting point depending upon the grade. The commercial grades of paraffin have melting points varying from 47° to 52° C. Its insulating properties are excellent. The surface leakage is very low and is practically constant up to 90 per cent relative humidity. A special grade of paraffin has such high electric resistance that it cannot be measured by any methods yet conceived. Its melting point is 58° C.

Paxolin is an English trade name for an insulation which is similar to bakelite micarta.

Pitch is a residue from vegetable oils. Examples are cottonseed pitch and wood pitch. Coal tar residues are often referred to as pitches. Pitch is used mainly as a binder in moulded insulations.

Porcelain (*See article on Insulation for Overhead Lines.*) Porcelain is distinguished from other forms of earthenware by a vitrified and non-porous structure. It is composed of china clay, ball clay, flint and felspar. China clay, sometimes called kaolin, is slightly plastic whereas ball clay is very tough and plastic. By a proper combination of the two the desired plasticity is obtained. Flint is in the form of pure sand or quartz ground so finely as to be entirely free from any gritty feeling. Feldspar is a natural rock and is ground as fine as the flint. These four ingredients are mixed in proper proportions and passed through the manufacturing processes which require very skillful care and experience upon which depends the quality of the product. Such articles as knobs, cleats and lamp sockets are made by a dry process and are not suitable for high voltage because of their porosity. Insulators are made by a wet process and are non-porous which is necessary for high voltage use. The glaze is provided to protect the porcelain from dust and deterioration in the weather and should have the same temperature coefficient of expansion as the porcelain.

In the dry process the great variety of forms of the product is obtained by pressing the material into a steel mold. Articles made with this process shrink 1/32 inch per inch in firing. The shape of the insulator made by the wet process is obtained by turning it down with machine tools before firing. It shrinks about 1/16 inch per inch in the firing process. The shrinkage is caused by the material fusing at the high temperature attained and filling the voids between the finely divided particles.

The mechanical strength depends upon the flint content, the heat resistance upon the clay and the dielectric strength upon the feldspar which unfortunately also adds brittleness. In a special type of porcelain for spark plugs, magnesia is used which adds mechanical strength at high temperatures. Porcelain, because of its low tensile strength, should be used under a compressive strain. It is comparatively inexpensive, chemically inert and not sensitive to changes of temperature. The fracture of good porcelain is conchoidal, fine grained, white and bright.

Pressboard is similar to paper in its construction, except that it is thicker and less flexible. It is usually hygroscopic unless impregnated with some moisture repellant. It is pressed into many useful forms, and is used as an insulator, principally in connection with low voltages. See sections on ***Fiber***, ***Paper***, ***etc.***, for other properties.

The dielectric strength of oiled pressboard is not very definite. It depends a great deal upon the time of application of the stress. The dielectric strength of varnished pressboard is low but uniform and depends largely on the varnish film.

Presspahn is a type of pressboard made in Germany. It is strong and can be easily bent. It is usually impregnated and boiled in pure linseed oil, thinned with benzine. Other insulations which are similar to presspahn are pertinax, casta and repelite. These sometimes contain inserts of linen or cotton.

Redmanol is a synthetic resin or amber, made of phenol and anhydrous hexamethylenetetramine. It is chemically inert, non-corrosive and non-inflammable. Its physical and chemical properties remain unchanged with age, but its dielectric strength increases with age. It is manufactured in four forms: molded, laminated, transparent and paint.

Moulded Redmanol is furnished in a number of different grades and can be moulded very accurately with metal inserts. These grades do not warp and some of them with proper fillers can be used continuously in air at temperatures up to 260° C. and in super-heated steam up to 343° C. The properties vary with the different grades.

Laminated Redmanol is tough, strong and flexible and can be shaped with tools very accurately and takes a smooth polish. It does not warp or deteriorate with age or upon exposure to heat and weather. It is recommended as an insulator to withstand the effect of salt sea air. It is available in sheets, tubes and rods in different grades with different properties.

Transparent Redmanol has a refractive index of 1.66 and therefore possesses brilliancy and lustre when polished.

Redmanol paint is used for coating electrolytic baths and vats containing solutions of not over 50 per cent of nitric acid, and as a protective coating against ammonia and sulphur dioxide fumes. These coatings withstand indefinitely a temperature of 230° C. and are not affected by boiling water or strong salt solutions.

Resin is the oxidized exudation of certain trees or plants and may be of recent origin or exist as a fossil. Some of the common resins which are used for insulating purposes are mastic, shellac, jalap, colophony, storax, amber, copal, kauri-gum, gutta-percha, gum arabic, chicle and dammar. Most resins are insoluble in water but dissolve in alcohol, ether, etc., forming varnishes of high dielectric strength. The resin varnishes are limited in their use by the fact that they become brittle at low temperatures, and will disintegrate if under vibration. Resins are hygroscopic and in general possess undesirable acid properties.

Rubber. — (See article on *Rubber*.) Hard rubber is frequently referred to as ebonite or vulcanite. According to Paterson, Rayner and Kinnes, the best grade of hard rubber is composed of 65 per cent of para rubber and 35 per cent of sulphur. They made up a number of specimens accordingly and also a number with 15 per cent of various adulterants and in addition obtained one specimen in the open market. Their results show that a good grade can generally be identified by the specific gravity and yield point. The specific gravity of the specimens made up without adulterants was 1.2 and that of the open market was 1.5. The yield point of the former was 75° C. and the latter 56° C. Adulterants will usually increase the specific gravity.

The yield point was obtained by supporting a specimen 25 mm. wide and 10 mm. thick as a cantilever beam with a one pound weight fixed at 15 cm. from the support in an oven and observing the yield after two hours heating at each temperature. The temperature at which the specimen yields and takes a permanent set is the yield point. A high grade will yield not more than 20 mm. in two hours at 70°C.

The dielectric strength test should be applied under oil and by imbedding snugly two spheres 51 mm. in diameter in a specimen 10 cm. thick until the spheres are 0.5 mm. apart. The best grade of hard rubber should show a dielectric strength of 110 kv. per mm.

French chalk or steatite as an adulterant will not materially affect the dielectric strength but the product will be spotty and will not take a good polish.

The surface resistivity of rubber decreases upon exposure to light. Curtis

found that a high surface resistivity could be restored by washing the surface with a dilute solution of ammonia and rinsing it thoroughly with water. The surface should be thoroughly wiped with a cloth until it is dry. The appearance of the surface can be improved by rubbing it, after it is dry, with waste oiled with a light lubricating oil.

Sindanyo is an English product made of asbestos with a special binder. It is manufactured in the form of sheets under pressure and then afterwards impregnated with an insulating compound. The sheets are furnished in two grades and of dimensions 3 feet by 4 feet or 3.5 feet by 8 feet and 0.125 inch to 2 inches thick. The Ebony grade is a substitute for slate and marble. Its dielectric properties are better than marble or slate as it is non-hygroscopic and free of metallic veins. As its mechanical strength is greater than that of either marble or slate it can be used in thinner slabs for the same purpose. It will not swell or warp under severe conditions and withstands temperature up to 200° C.

The plain grade is fire resisting and has good mechanical strength but unstable dielectric properties. It will withstand 900° C. without disintegration, warping or diminution of mechanical strength. It is used for arc deflectors and insulation in heating units.

Slate is any rock having a fissile structure, the common variety being composed principally of silica, alumina, and oxides of iron. It is hygroscopic and should be boiled in paraffin. It is often permeated by metallic veins, making it unfit for use unless the electrical connections are insulated by bushings. It is useful for switchboard and switch-base work owing to its desirable mechanical and fireproof qualities. Its dielectric strength decreases rapidly as the temperature increases, and at a high temperature slate becomes a conductor. When a high voltage is impressed upon a piece of slate for some time, the slate usually is not punctured but, due to the consequent rise in temperature, the slate acts as a short-circuit to the impressed voltage. The breakdown is thus only apparent as the specimens regain their dielectric properties after cooling.

Stanol is an English product made of asbestos and a rubber compound. It is furnished in sheets 1/32 inch thick and 50 inches by 60 inches. It is flexible and is used for lining controller cases and switch boxes. Its dielectric strength is 3.9 kv. per mm.

Varnished Cambric. — See article on *Cambric, Varnished.*

Varnishes, Insulating, are made in two classes: (1) baking varnishes, which harden by oxidation when subjected to artificial heat and (2) air-drying varnishes and similar compounds, which harden or set by evaporation of the solvent. Varnishes and similar compounds are manufactured in a number of grades adapted for use in armature, field and transformer coils, magnet and arc lamp spools, transformer bushings, insulating cloth, paper and fiber, armature and transformer stampings, asbestos and magnesia board, cut-out boxes, switch and panel boards, mica plate, flexible mica, mica cloth and paper, storage batteries, weatherproof wire, iron and metal fittings, moulding, pot-heads, taped connections, etc. Some of the trade designations are P. and B., S. P. C., S. V. W., G. P., H. R., Voltalac, W. P., Glinolac, Benolite, Dolph's, Ohmlac, Minerallac, Nico, M.C., Insullac, Enamelac, Armalac, Chatterton, Linolac, M. I. C., etc.

Vulcabeston is an English product often called "J. P. Bakelite" and is furnished in the form of sheets or special moulded forms. One grade is made of asbestos fiber and rubber as a binder. It is slightly elastic and has a tensile strength of 3600 pounds per square inch. It is claimed to withstand a temperature of 145° C. without physical changes. When immersed in cold water for 48 hours it absorbs 1.75 per cent of its weight of water. Grade No. 201 is

made of asbestos fiber with a special phenolic gum as a binder. It has a tensile strength of 9000 pounds per square inch and will withstand a temperature of 260° C. without undergoing physical changes. It absorbs 0.6 per cent of its weight of water when immersed in cold water for 48 hours.

Vulcanite. — See article on *Rubber*.

Wood. — Thoroughly dried hard wood, if impregnated with an insulating material, makes a good insulator. The woods commonly used are maple, cherry, ash and yellow pine. For use in transformers, wood is usually impregnated with transformer oil, and when used in air, the wood is impregnated with paraffin or rosin.

Other Insulations which are in very general use but not described in published literature are: Carvanite, Faturan, Kalanite, Pilit, Solidite, Stabelite and Tegit.

Information on the various insulating materials described were obtained from the authorities cited in the bibliography, from the following manufacturers: American Lava Corporation, American Vulcanized Fiber Company, Chicago Mica Company, Condensite Company of America, Continental Fiber Company, Delaware Hard Fiber Company, Diamond State Fiber Company, Formica Insulation Company, General Bakelite Company, General Electric Company, M. Kirchberger & Co., Mica Insulator Company, Minerallac Electric Co., National Fiber and Insulation Company, Redmanol Chemical Products Company, J. Spaulding & Sons Company, D. M. Steward Manufacturing Company, R. Thomas & Sons Company, and Westinghouse Electric and Manufacturing Company.

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INSULATING MATERIALS, TESTING OF. — (*See also Insulating Materials, Properties of.*) The most important electrical properties of an insulating material are its dielectric strength, specific inductive capacity, resistivity, dielectric loss, and power factor. Some of the more common methods of measuring these quantities are described below.

DIELECTRIC STRENGTH. — The dielectric strength of an insulating material is defined as the puncturing voltage per unit thickness, the thickness usually being measured in centimeters. The voltage required is in general higher than can be conveniently obtained with direct-current sources of e.m.f. and alternating e.m.f.'s are therefore used in such tests. The materials to be tested are placed between electrodes connected to the high-tension terminals of a transformer, which receives power from a low-voltage alternator.

Testing Transformers and Alternators. — Transformers for such use are made to deliver voltages up to 500,000 volts. For the purpose of comparison and computation of the maximum e.m.f. from the effective value, it is essential that the alternator should give a sine-wave voltage at all loads. Both transformer and alternator should be large enough to operate with good voltage regulation at all testing loads, so that no distortion of the wave-form will be produced by the charging current. Skinner suggests the following ratings of testing transformers:

Max. test voltage	Cap. of trans. in kw.*	Max. test voltage	Cap. of trans. in kw.
2,000	1	50,000	50
6,000	3	100,000	100
10,000	5	150,000	150
30,000	30	250,000	250

* Testing transformers are usually rated by their four-hour capacity.

The alternator should have a distributed field winding and in order that the combination of alternator and transformer may respond to any instantaneous load, the impedance of each should be low; that is, the speed of the alternator should be high and the e.m.f. induced per turn in the transformer should be high.

Form of Electrodes. — Many investigators have demonstrated that the form and size of the electrodes used are important factors of the test. If the edges of the electrodes are rounded, the resulting brush discharge heats the specimen so that it breaks down at a low voltage. If the edges of the electrodes are not rounded, the increased flux density at the edges produces an excessive strain on the dielectric under the edges of the electrodes. Most authorities favor the use of flat electrodes with slightly rounded edges. Hendricks suggests flat electrodes with corners rounded to a radius equal to one-tenth of the diameter of the flat face. Kinzbrunner has discussed in detail the effects of various shapes of electrodes.

For testing oils or other liquids two standards composed of definitely shaped terminals supported in a liquid container are commonly used. In one the testing terminals consists of brass balls ½ inch in diameter fastened to rods 3/16 inch in diameter and placed vertically in a glass tube. The distance between the balls

is adjustable, a distance of 0.15 inch, however, being considered standard. In the other standard the terminals consist of brass disks 1 inch in diameter mounted on rods ⅜ inch in diameter and placed horizontally in a small box made of Bakelite or some other insulating material. The spacing of the disks in this case is also adjustable, although 0.1 inch is adopted as a standard Average dry oil should not break down in the ball electrode standard at less than 60,000 volts and in the disk electrode standard at less than 30,000 volts.

Time of Electrification. — Materials may be tested by impressing the puncturing voltage, 1. instantaneously, 2. in steps of one-minute duration or 3. in several instantaneous applications of decreasing voltage and increasing time of electrification. The question of the proper time of electrification is at present undecided, although many investigators favor the one-minute electrification. For arguments against this standard, see paper by Kinzbrunner (reference in Bibliography at end of this article).

Control of Voltage. — The voltage impressed between the test electrodes may be controlled 1. by varying the excitation of the alternator, 2. by inserting resistance in series with the low-voltage terminals of the transformer, 3. by varying the number of active turns on the low-tension side of the transformer, or by an induction regulator in series with the transformer primary. A combination of the first and third methods is very commonly used. In the second method the voltage wave is made more peaked as resistance is introduced; in the third method, if used alone, the circuit must be broken as connections are made to the tap leads. By varying the field excitation of the alternator and changing the tap connections between tests, the shape of the voltage wave is kept constant and the variation of field saturation in the alternator is reduced to a minimum. The last method is probably the most commonly used and gives entirely satisfactory results.

Measurement of Voltage. — The voltage impressed upon the material under test may be determined by 1. a variable spark gap shunted across the electrodes, 2. by a voltmeter and an extension coil or multiplier connected across the electrodes, 3. by an electrostatic voltmeter connected across the electrodes, 4. by a voltmeter connected across the low-tension terminals of the transformer, the reading of which is to be multiplied by the ratio of transformation of the transformer, 5. by a potential transformer and a voltmeter connected across the terminals and 6. by a special voltmeter winding placed in the middle of the high-tension winding of the power transformer. The needle spark-gap method, while convenient because of its indication of the maximum rather than the effective value of the voltage, is tedious in use, and its readings are dependent upon the time of electrification, circulation of air, the condition of the needles, etc. The sphere spark gap is preferable; see *Spark Gap*. The use of the voltmeter and extension coil, while flexible and convenient, is not recommended because of the load which is placed upon the transformer and the possibility of leakage in the extension coil. The use of an electrostatic voltmeter is desirable, except that at high voltages the moving element must be immersed in oil, making the instrument sluggish in action, and may frequently break down. In any method involving the ratio of transformation of the transformer, the results are questionable owing to the assumption of a constant ratio of transformation at all loads. The use of the potential transformer and voltmeter is prohibited in most cases by the cost of the instrument transformer and the excessive distorting load introduced by such an arrangement.

A voltmeter winding in the power transformer furnishes as a rule the most accurate and efficient method of measurement, the chief source of error being due to leakage flux and impedance drop in the transformer.

Causes of Variations in Results. — The puncturing voltage of an insulating material is affected by its previous history, precise condition when tested, size, thickness, form, uniformity of electrostatic field, temperature, time of electrification, frequency and the surrounding medium. The puncturing voltage is affected more by temperature than any other factor, and since the temperature of a specimen is increased greatly under the action of the corona discharge (*see Corona*) it is desirable that corona discharge should be eliminated. To prevent the heating action of the corona, 1. guard rings may be supplied, as described by Ryan, Norris and Hoxie, 2. the electrodes may be imbedded in the dielectric when possible, 3. the "picein drop" method described by Walter may be used on the specimen, or 4. the electrodes and test specimen may be immersed in oil as recommended by Hendricks. In the last case materials easily permeated by oils should be glazed over with a varnish before testing. While the temperature of insulation in practice may range from $-25°$ C. to $+125°$ C., tests between $+25°$ C. and $+100°$ C. should suffice to indicate average working conditions.

SPECIFIC INDUCTIVE CAPACITY OR DIELECTRIC CONSTANTS. — (*See also Condensers.*) The specific inductive capacity of any substance is the ratio of the capacity of a condenser, in which the substance in question fills the space between the plates, to the capacity of the same condenser when the space between the plates is filled with afr. This ratio, designated by K, varies with the temperature of the substance, the time of electrification used in the test and the frequency, when tested with alternating currents. The specific inductive capacity of most substances decreases as the temperature increases, except in the case of a few substances. The specific inductive capacity in general increases with the time of electrification and decreases as the frequency increases. The determination of the specific inductive capacity from test involves the determination of the capacity of a condenser of known dimensions. A plate or cylindrical condenser is commonly used for solid material and a spherical condenser for liquids. Having determined the capacity of any condenser of known dimensions from test, the specific inductive capacity of the dielectric used can be obtained by substituting the known constants in the condenser formulas given in the article on *Capacity and Charging Current*.

Various methods of measuring capacity are given in the article on *Condensers, Electric*.

The Standards of the American Institute of Electrical Engineers recommend that cable capacity measurements be made with alternating current. The specific inductive capacity with alternating current, therefore, is of such practical importance that a detailed description of the method is given below.

If the dielectric under consideration is a liquid a standard condenser C_s is not required. A reading is first obtained on the container with air as a dielectric and then again on the same container with the unknown liquid as a dielectric. The specific capacity is

$$K = \frac{R_a}{R_x}.$$

R_a is the reading on R with air in the container and R_x is the corresponding reading with the unknown dielectric.

Alternating-Current Bridge Method.—A Wheatstone bridge as shown in Fig. 1 is used. C_1 is a mica or an air condenser, R and R_2 are resistances wound non-inductively, L is a variable inductance and L_2 may be a fixed inductance. The detector may be either a telephone or a vibration galvanometer, the latter being essential for 60-cycle work. As the capacity to be measured is small, a substitution method should be used by inserting, first the unknown condenser C_x in the arm C and varying R and L until a balance is obtained, and

then a standard condenser C_s in the same arm and obtaining a balance again by varying R and L, but leaving C_2, L_2 and R_2 undisturbed. Then

$$C_x = \frac{R_s}{R_x} C_s,$$

where R_x and R_s are corresponding readings obtained on R with C_x and Cs.

The power factor is equal to sin θ_x or tan θ_x when θ_x is small, and may be computed from the following equation:

$$\tan (O_x - O_s) = 2\left(\frac{L_x}{R_x} - \frac{L_s}{R_s}\right),$$

where O_s is the angle by which the current falls short of being 90° ahead of the e.m.f. impressed on the standard condenser, and is zero for an air condenser and seldom more than 5′ for a good mica condenser. L_x and L_s are readings obtained on L with C_x and C_s respectively.

If only an approximate value of the specific inductive capacity is desired, the inductances L and L_2 may be omitted in Fig. 1. In this case the current in the detector will not be zero when the bridge is balanced but will be a minimum.

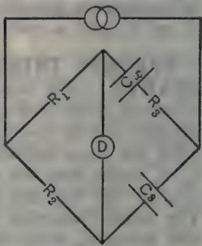

Fig. 1.

Where a telephone detector is used and high degree of accuracy is required, the bridge should be shielded so that the observer will be at the same potential as the telephone. Also a guard ring should be provided in the condenser under test, the ring being on the side nearer the detector and connected to the junction of L and L_2.

INSULATION RESISTANCE. — The method commonly employed is one of substitution. A standard resistance R_s (usually 1/10 megohm) connected in series with a D'Arsonval galvanometer is connected across a direct-current source of e.m.f. of about 500 volts and the deflection D_s of the galvanometer noted. The galvanometer must have a sensitiveness of about 1×10^9 and the deflection should be directly proportional to the current. If the galvanometer is shunted, an Ayrton universal shunt should be used (*see Shunts*) so that the damping of the galvanometer will be independent of the multiplying power. The deflection D_x is then noted when the unknown resistance R_x is connected in place of the standard resistance. Then

$$R_x = \frac{D_s}{D_x} R_s.$$

The usual errors of this method are due to leakage and absorption. Leakage may be reduced considerably by the use of a guard ring, so connected as to shunt the leakage current around the galvanometer. The deflection of the galvanometer will vary with time due to absorption, and the resistance of the material will apparently vary with the time of electrification. To facilitate comparison of the insulation resistance of dielectrics, it has become common practice to take the resistance obtained after one minute's electrification. Owing to the large negative resistivity temperature coefficient of most dielectrics, it is essential that the temperature of the material be noted when tested.

ENERGY LOSS IN THE DIELECTRIC—EFFECTIVE CONDUCTANCE. — Energy loss in dielectrics is difficult to measure on account of both the small amount to be measured and the low power factor. The compensated

dynamometer wattmeter method and the electrostatic wattmeter method have found the most favor. Sometimes the dielectric loss is expressed as the effective a-c. conductance, which is equal to the watts lost divided by the square of the voltage.

COMPENSATED DYNAMOMETER, WATTMETER METHOD. — (Fig. 2.) The test sample is subjected to a high voltage from a test transformer and the power absorbed by it is measured by means of a delicate, mirror type

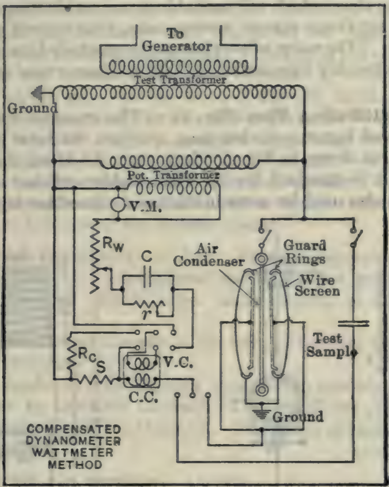

Fig. 2.

wattmeter. The inductance of the potential transformer and wattmeter potential coil *V. C.* is neutralized by means of a condenser *C*. The effective capacity of this condenser may be varied by changing the value of the shunting resistance *r*. Before making a test a zero-loss air condenser is substituted for the test sample and the resistance *r* is varied until the wattmeter shows zero deflection. The test sample is then substituted for the air condenser and the test repeated, the deflection indicating the power lost in the sample.

To measure the current, the potential coil of the wattmeter is connected across the resistance *S*, through the variable resistance R_c. This puts the current in the two coils in phase and the instrument will read as an ammeter.

The wattmeter is calibrated by direct-current, using a potentiometer and standard cell. It must be separately calibrated if used as an ammeter.

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INSULATOR PINS. — (*See also Cross Arms; Insulators for Overhead Lines.*) Insulator pins are made of wood, iron and various combinations of wood, iron and porcelain.

WOODEN PINS. — Wooden pins are usually made of locust but sometimes of oak, birch, maple or eucalyptus.

The forest service reports: "Black locust is admitted to be the best of all woods used for insulator pins, but of late years other woods are being brought into use. Among those which will probably prove satisfactory as substitutes for the black locust are Osage orange, various oaks, yellow birch, gum, hard maple, elm, etc." Also: "The value of eucalyptus, particularly blue gum, for insulator pins has been thoroughly demonstrated. After fifteen years' service, sound pins are still in use."

Standard Distribution Pins (Fig. 1). — The standard wooden pin for distribution lines is of locust, 9 inches long, 1½-inch diameter shank, and 1 inch diameter thread, as shown in Fig. 1. It is often described simply as a "1½-in. by 9-in. pin." It is standard for use with double-petticoat deep-groove glass insulators and is also used for some insulators designed for higher voltages.

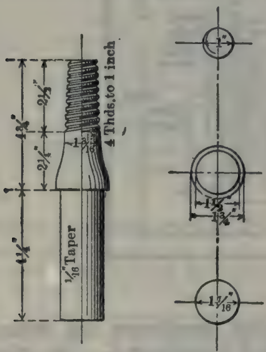


Fig. 1. Standard Locust Pin

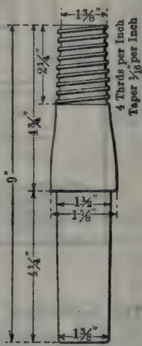


Fig. 2. Insulator Pin for 13,000 Volts

High-voltage Distribution Pins (Fig. 2). — For voltages above 2200 and not exceeding 13,200 it is desirable to have pins with 1½ inches diameter shanks to fit standard cross arms, but with larger threads, as shown in Fig. 2.

High-voltage Transmission Pins. — For voltages above 13,200 and not exceeding 25,000 wooden pins are used to a considerable extent. These pins are special and their length must be sufficient to give the clearance from insulator to cross arm required by the design of insulator used. The shanks should be larger than those of standard pins.

On high voltages wooden pins are too weak for the high stress imposed by the great pin leverage of large insulators and present practice is toward metal pins to secure the required strength. The use of paraffined wood pins has been practically discontinued.

IRON DISTRIBUTION PINS. — Iron pins (malleable iron and drop-forged iron) are made with 1½ inch shanks and 1-inch threads so as to be interchangeable with standard wooden pins. They are used at points of heavy strain.

Pins with Iron Shanks consisting of ½-inch, ⅝-inch or ¾-inch iron bolts are also used. These have a nut and washer on the lower end used for pulling

the base of the pin firmly into contact with the top of the arm. This type of pin has the advantage that the smaller hole removes less wood from the arm. For heavy stresses the shank of the pin should be stiff, or the base of the pin broad to prevent local crushing of wooden cross arms due to the pin leverage.

ATTACHMENT OF INSULATORS TO PINS. — Insulators are usually attached to wooden pins by a screw thread having a pitch of 4 threads per inch. The threaded portion of the pin is tapered, the diameter increasing downward from the top about 1/16-inch per inch of length. Standard thread diameters are 1 inch for distribution insulators and 1 3/8 inches for high-tension insulators measured at the small end over the thread. Good fit in the thread between pins and insulators is not commonly attained because: (1) the threads are often imperfectly cut on the pins, (2) the wood of the pin shrinks so that it ceases to be circular or warps so that the axis of the pin is not truly straight, (3) the porcelain of the insulator warps in manufacture so that the hole ceases to be circular or the surface of the thread may not be smooth. Where a pin does not screw into the insulator to full depth intended, or where it is loose or bears unequally, the strength of the attachment is much reduced. The weakest point is usually at the root of the bottom thread.

Attachment of Insulators to Iron Pins. — Insulators are sometimes attached to iron pins by a screw thread similar to that used for wooden pins. The thread on the iron pin is often made of lead to get a more uniform bearing.

In later types the thread is made in the form of a coiled spring to better distribute the load stresses.

High-tension insulators are frequently cemented to iron pins. The pin hole of the insulator may be threaded as usual. The end of the pin is usually corrugated or fluted instead of being threaded. The pin is grouted to insulator with neat Portland cement.

ATTACHMENT OF PINS TO ARMS. — The standard wooden distribution pin has a shank of 1 1/2-inch nominal diameter. The actual diameter at the top is the same as the nominal diameter, the shank tapering so that the bottom is about 1/16 inch smaller. The pin hole in the cross arm is bored to the nominal diameter but often becomes smaller (across the grain) due to shrinkage of the wood, so that a standard pin, although smaller than nominal size of the pin hole except at extreme top, will often make a good driving fit. Pins which are too small (or pin holes which are too large) will make a loose fit and should not be used. Pins are often made considerably oversize at the top and undersize at the bottom to allow for inaccuracy of workmanship. Such pins will not drive completely in, and they weaken the construction by increasing the pin leverage and decreasing the bearing surface between the pin and cross arm. At the top of the shank a shoulder should be provided to limit the distance the pin can be driven in. For maximum strength the shank should be long enough to go completely through the arm and the taper should be small enough so that it will bear against the arm for the full depth without leaning far to one side. The standard 1 1/2-inch by 9-inch pin has a 4 1/4-inch shank length corresponding to the old "standard" depth of cross arm, and, therefore, does not develop the full strength of the later standard arms which are 4 1/2 inches or 4 3/4 inches deep.

Pins are prevented from pulling out of the arm (especially where the wire exerts an uplift on the insulator and pin) by driving a nail (usually six-penny) through arm and pin.

Wooden pins for high-tension insulators are made with larger diameter shanks, 2 inch, 2 1/4 inch and 2 1/2 inch being used.

Comparative Strength of Some Special High-tension Pins. — The following are the results of tests made on pins fitted into wooden cross arms. The pins broke at the top of the shank.

Material	Dimensions of shank, inches		Pin leverage, inches from center of stress to top of arm	Breaking strength, pounds
	Diameter	Length		
Oak	2¼	6	12	140
Oak	2¼	6	16	120
Oak	2½	6	12	205
Oak	2½	6	16	120
Eucalyptus	2¼	6	about 10	260
Red oak	2½	7¼	12½	495
White oak	2½	7¼	12½	738
Locust	2½	7¼	12½	1121

COSTS. — The following figures, giving the cost, are rough approximations.

Material	Dimensions in inches			Approximate cost per 1000
	Diam., Shank	Overall length	Diam., Top	
Standard Oak (Fig. 1)	1½	9	1	$ 7.50
Standard Locust (Fig. 1)	1½	9	1	14.00
Special Locust	1½	10¾	1⅜	22.00
Solid Drop-forged Galvanized Iron	1½	9	1⅜	500.00

BIBLIOGRAPHY. — *Report of Committee on Overhead Line Construction,* Trans. N.E.L.A., 1911; Lindquist, R. A., *Transmission Line Construction,* N. Y., 1912; *Forest Service Circular, No. 179;* Ohio Brass Co., *Catalog No. 12;* Western Electric Co., *Catalog Bulletin No. 74;* Westinghouse General Supply Catalog.

INSULATORS FOR OVERHEAD LINES. — (*See also Distribution Lines; Insulator Pins; Transmission Lines.*) Insulators for overhead lines may be classified as insulators for distribution lines, pin insulators for transmission lines and suspension insulators for transmission lines. The general features of design common to all classes of line insulators will first be considered.

DESIGN OF LINE INSULATORS. — Glass is commonly used for making small low-voltage insulators and porcelain for large insulators. Porcelain has greater mechanical strength, condenses less moisture on its surface and parts may be cemented together, but faults cannot be as readily detected as in glass. Glass insulators are often completely shattered by a blow while porcelain insulators usually are only chipped under similar circumstances. Various compositions are also used to a limited extent for line insulators.

Properties of Porcelain and Glass. — Following are some of the more important characteristics of porcelain and glass:

Property	Glass	Porcelain
Tensile strength, lb. per sq. in	2,500–9,000	1,500–2,200
Crushing strength, lb. per sq. in	6,000–10,000	14,000–16,000
Elastic limit, lb. per sq. in	3,200 approx.	....
Modulus of elasticity, lbs. per sq. in	8,000,000	2,500,000 (*b*)
Coeff. of expansion per °F	0.0000046	0.00000585 (*b*)
Coeff. of expansion per °C	....	0.00000413 (*a*)
Weight per cubic foot in lbs	160	155
Puncturing strength in volts per inch		
for very thin samples	300,000	400,000
for commercial sizes and quantities	150,000	225,000
Specific inductive capacity (air = 1)	5 to 10 (*a*)	4.38 (*a*)

(*a*) Smithsonian Physical Tables. (*b*) Austin, A. O., Proc. N.E.L.A., 1913.

In American practice porcelain parts are made from ¼ inch to 2½ inches in thickness. In European practice porcelain is used in thicknesses up to 1 inch. The working voltage usually averages from 10,000 to 20,000 volts for each thickness of porcelain used. 15,000 volts is about an average figure, so that a 30,000-volt insulator usually has two parts, a 40,000-volt insulator 3 parts and a 60,000-volt insulator 4 parts.

An insulator must be designed to stand extreme and sudden temperature changes, sleet and rain, as well as smoke, dust, and often special conditions such as salt fogs, salt water sprays, and chemical fumes, without deterioration from chemical action, breakage from mechanical stresses or electrical failure. The design of high-tension insulators is a process of compromise between requirements often antagonistic in nature.

Some of the principal points in the electrical design of high-voltage insulators are:

Shape. — The insulating surfaces should conform to the flow lines of the electrostatic field, the surfaces of the rain sheds or petticoats should conform to equipotential surfaces, the leakage resistance per shell should be about equal, the plane of mechanical rupture should not coincide with the plane of electrical stress, and the unit should have approximately equal capacitances per shell.

Thickness to Resist Puncture. — The porcelain must be thick enough to resist puncture by the combined working voltage of the line and any probable

transients whose time lag to spark over is great. If this thickness is greater than desirable from a manufacturing standpoint, two or more pieces are used to give the proper aggregate thickness.

Arcing Distance; Free Arcing. — The porcelain must extend beyond the charged conducting connections (i.e., tie wire or cap at the top and pin at the bottom) sufficiently so that the distance between the connections through the air around the porcelain is greater than the arcing distance of the maximum voltage to be carried. The arcing distance required for a given voltage may be determined roughly (but only very roughly) from the tables of arcing distances between needle points; see article on *Spark Gap*. The greater radius of curvature (compared with needle points) of such metal parts as the insulator pin decreases the potential gradient at the terminals. Also the porcelain has a much greater specific inductive capacity than the air and its proximity to the arcing path disturbs the electrostatic field through the air. Surface charges on the porcelain because of surface leakage or corona also modify the field.

Free arcing is the property of arcing over along a line which does not touch the porcelain body from the point where the arc leaves the metal cap to where it strikes the metal pin. Where the arc touches any part of the porcelain the great heat fractures the porcelain in a few seconds; hence the desirability of designing the insulator so that it is free arcing. A properly designed insulator will arc over as a whole before any individual part (i.e., shell or unit) arcs over. In many defective designs the insulator will fail by some parts arcing over, thereby increasing the voltage on others which then fail by puncture or arcing over.

Factor of Safety Against Puncture. — The thickness of a porcelain part must be so related to the distance around it that it will arc over before it will puncture. The ratio of puncture strength to arc over-voltage is the factor of safety of the part, or of the insulator, against puncture.

This ratio should be high to give sufficient margin to protect the insulator from puncture by the transients before mentioned.

Guard Rings and Rods. — Where an insulator is not naturally "free arcing" from cap to pin, properly located rings or rods will divert the arc so as to accomplish an equivalent result. Rings used for this purpose are known as Nicholson guard rings. Where a pin insulator is improperly designed so that it is liable to puncture before flashing over, a Nicholson ring around the base may be used to redistribute the potential or to reduce the arcing distance so as to reduce trouble from puncture. Such rings and rods are also sometimes used in connection with suspension insulators.

Leakage Surface. — Leakage surface is ordinarily measured as the number of (linear) inches from cap (or equivalent) to pin taken radially along the surface; see Fig. 4. This, however, is only a rough measure as it neglects the varying width of leakage path. The figures in the accompanying table on pin insulators from a manufacturers' catalog give approximately the total amount of leakage surface used in practice. The amount of surface allowed varies considerably between different designs and makes.

Rated voltage	Length of leakage surface, inches
5,000	4
6,600	5½
10,000	6¾
23,000	12
44,000	29
66,000	53

Spread of Petticoats of Pin Insulators. — In two concentric shells the two surfaces which lie opposite to each other are at different potentials

except where they are cemented together. The difference in potential between two points on opposite surfaces is greater the further they are removed from the joint. Unless the shells diverge correspondingly so as to increase the distance between the shells as the potential increases the air will break down and part of the leakage surface will be short-circuited by a corona discharge. This divergence is shown in Fig. 4, where the top is a disc made slightly convex to shed water and the inner shells are a series of cones which radiate from the top of the pin.

Color and Glazing. — Brown, slate and white are the common colors used in glazing porcelain. Brown is the most common color since it is more of an aid in determining faults. Slate-colored glazing of the same color as galvanizing on towers makes insulators a less conspicuous target for malicious destruction.

Cementing of Insulators. — Insulator parts are cemented together with neat freshly burned and finely ground Portland cement which is usually allowed to set under water from ten to fourteen days before testing mechanically. The more freshly burned and finely ground the cement and the higher the temperature, the less time required for setting. The cemented surfaces of porcelain are unglazed and corrugated to obtain good bond.

Faults in Insulators. — The more common faults in porcelain are porosity, folds and flaws in moulding and the development of checks and hair cracks in process of drying, incomplete and non-uniform glazing, warping, air bubbles, conducting impurities, under- and over-firing and chipping of edges. Only 50 to 75 per cent of moulded shapes ordinarily pass final test and even a less number of the more difficult shapes. Inspection and testing are essential to eliminate faults in both design and manufacture.

DISTRIBUTION-LINE INSULATORS, STANDARD (Fig. 1). — For ordinary distribution circuits, including constant-potential circuits up to 2200 volts nominal and series-arc circuits of all voltages, the double-petticoat deep-groove (D.P.D.G.) insulator, of the type shown in Fig. 1, is standard.* Such insulators are usually of glass, though occasionally of porcelain, and in cases where there are heavy stresses, as at dead ends where iron pins are required, the insulators are made of moulded mica. The dimensions vary with different manufacturers.

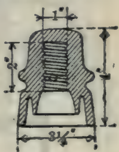


Fig. 1.

These insulators are used with the standard pin shown in Fig. 1 in the article on *Insulator Pins*.

Pony Insulators. — These are small insulators used for telegraph and telephone wires. They are unsuitable for electric light and power wires. The groove is too small (sometimes only ¼ in.) to properly support the larger wires used for lighting and the leakage surface is insufficient for proper insulation.

Insulators with Top Groove. — The standard distribution insulator has a groove in the side to which the wire is tied. For very heavy cables special insulators with groove in top are used. Insulators for higher voltage usually have both side and top groove, the former used where there is horizontal stress due to angle in line, and the latter on straight parts of line where the weight is the principal force.

DISTRIBUTION-LINE INSULATORS, HIGH-VOLTAGE (Fig. 2). — For circuits exceeding 2200 volts but not exceeding 13,200 volts nominal, it is convenient to have insulators which can be used on the standard distribution

* The same size insulator is used for 110-volt and 2200-volt circuit. This has the advantage that only one kind of insulator is kept in stock, and a wire can be transferred from use on a low-voltage circuit to one of higher voltage without reinsulating it.

pins. A number of insulators are made with 1-in. threads to fit the standard 1½ in. by 9-in. pin. It is better, however, to use stronger special pins with 1⅜-in. top; see article on *Insulator Pins*. Fig. 2 shows an insulator which has been used on these voltages. It is used with the pin shown in Fig. 2 in the article on *Insulator Pins*. The manufacturer rates this insulator at 23,000 volts with a factor of safety (wet) of 2.

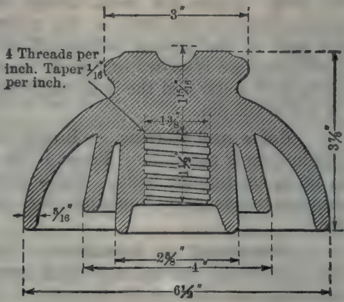


Fig. 2.

TRANSMISSION-LINE INSULATORS, PIN-TYPE (Figs. 3 and 4).—The pin-type insulators are similar in mechanical construction to the low-voltage insulators used for distribution lines, but are larger and designed for heavier mechanical stresses as well as higher voltages. The conductor is supported on a top groove except where an angle in the line causes a lateral pressure, in which case the side groove is used. For voltages up to about 10,000 volts the insulator usually consists of but a single porcelain part. For higher voltages from two to five parts are cemented together. The size and expense of pin-type insulators increase rapidly with the voltage so that while they may be used for operating potentials of 70,000 and even more, their use is generally restricted to potentials of not over 50,000. Typical three-part insulators are shown in Figs. 3 and 4. The separate parts are called parts, shells or cones and are numbered in order beginning with the top.

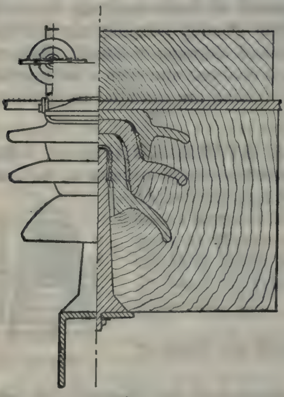


Fig. 3.

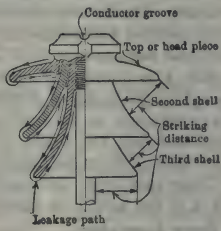


Fig. 4.

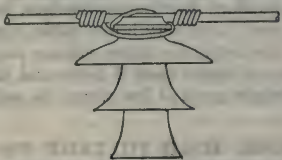


Fig. 5.

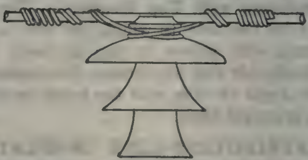


Fig. 6.

On a cost per mile basis, the suspension insulator construction is often less

than the pin type for pressures above 44,000 volts, due to the longer spans thus made possible.

Tie Wires and Clamps. — Tie wires consisting of the same material as conductors are commonly used for fastening conductor to insulator. Two common types of ties are shown in Figs. 5 (single tie wire) and 6 (double tie wire). For copper conductors the tie wires are usually soft-copper wires which are three sizes on the A.W.G. (B. & S. gage) smaller than the conductor. Metal caps carrying clamps for the conductor are cemented to tops of insulators when more strength is required than can be obtained by tie wires.

TRANSMISSION-LINE INSULATORS, SUSPENSION TYPE (Figs. 7 to 10). — The suspension type insulator is always used in tension, the connections at the two ends being made so that the insulator is free to swing in any direction; the insulator takes such a position that its axis coincides with the direction of the mechanical stress.

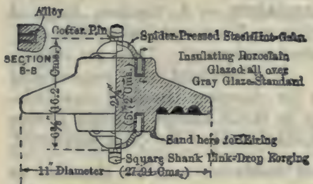


Fig. 7.

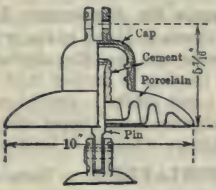


Fig. 8.

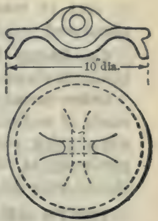
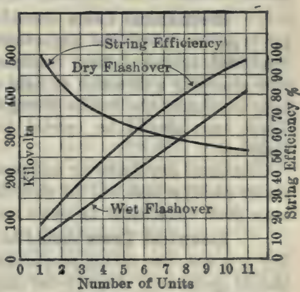


Fig. 9.

This type is used hanging below the cross arm with axis vertical as a suspension insulator for sustaining the weight of cable at points where there is little horizontal force, and also with axis approximately horizontal as a "strain" or "dead-end" insulator at points where the horizontal force predominates.



[Fig. 10.

The suspension type consists of one or more complete insulators, called "units," connected in a string. Each unit consists of one or more insulating parts usually with a metal cap above and a metal pin below, all cemented together. Three typical designs now in general use are shown, Figs. 7 to 9.

Relation Between Electrical Strength of Single Unit and a String of Units. — The potential per unit required to flash-over a suspension insulator composed of units of the same design decreases with an increasing number of units as shown in Fig. 10. This is due to an unequal potential gradient. See *Bibliography*.

Connections Between Suspension Units. — The caps and pins of suspension insulators are commonly made of galvanized malleable iron castings, but where strength is required the pins are steel drop forgings, and pressed steel for these fittings is coming into general use.

The common types of connection between the pin of one insulator and the cap of the one next below are ball and socket, clevis and pin (Figs. 7 and 8) and hook and eye. The hook and eye is the simplest type, but there is danger of its becoming unhooked and, in order to obtain strength, the design requires more space between units than other types of connection. The clevis and pin has the disadvantage of loose parts. The ball-and-socket connection requires special terminal fittings and a rigid connection to prevent the clamp from turning when used on deadend insulators for conductors which tend to untwist. It can, however, be designed with less space between units than other types.

A type of suspension insulator which does not require caps and pins is shown in Fig. 9. These units are connected together by cable loops. This was one of the first types of suspension insulators. It is not commonly used on account of cost of manufacturing. A very close inspection is often required to find punctured units that are not shattered by the failure.

Conductor Clamps for Suspension Insulators. — A common form of insulator clamp is shown in Fig. 8 in the article on *Wires and Cables, Bare*.

Other forms are in use (see Manufacturers'catalogues). They are usually made of malleable castings or pressed steel. A requirement to be remembered when using steel cored cables is provision for clamping the steel core independently of the aluminum or copper wires in strain clamps. Strain clamps should have an ultimate strength at least equal to that of the cable supported.

TESTS OF INSULATORS. — Tests of insulators may be classified as (1) design tests, made on a very few insulators to determine the characteristics of new designs, and (2) routine tests made on each insulator manufactured to detect defects of material or workmanship.

Design Tests. — Design tests are quite expensive for very high-voltage insulators because of the difficulty of securing suitable testing apparatus.

Mechanical Tests should be made to determine the strength. With pin-type insulators the important point is the strength of pin and insulator combined against a force (representing a horizontal wire pull) applied at the wire groove in a direction at right angles to the axis of the pin. For suspension-type insulators the tensile strength is the important mechanical consideration. These tests should be carried to the point of destruction of a number of samples.

Combined dry flashover and tension tests are important to determine the load under which a unit will fail electrically. Proper design should produce an insulator good electrically up to its maximum mechanical strength.

Electrical Tests should be made for puncture strength, arc-over voltage dry and wet, free arcing properties and corona formation. The electrodes used should conform in shape to the cap and pin used in practice so that all surface exposed to puncture when the insulator is in use will be tested. Design tests of the electrical properties of suspension insulators should be made on complete strings of insulators as well as on single units, assembled under field conditions as to proximity of towers, etc.

Puncture Test. — As the air surrounding a properly designed insulator acts as a safety valve, it is necessary to immerse the insulator in oil in order to test the material of which the insulator is composed up to its puncture strength at normal frequency. Such a test may be used to ascertain the margin by which the puncture strength exceeds the flash-over voltage and to verify the dielectric strength of the porcelain. Tests in air made with high frequencies, however, are found to puncture insulators which arc over at normal operating frequencies. The frequencies used for normal tests include all commercial frequencies or say

from 25 to 125 cycles per second with 60 cycles the most common. For high-frequency testing a 200,000 cycle oscillator is found suitable.

Arcing Tests. — Arcing tests, both dry and wet, are made to determine the voltage at which an insulator will arc over when the voltage is raised with the insulator in its normally dry condition and when wet as in rainy weather. The dry arcing test is the most common one made because of its simplicity, the voltage merely being raised on cap and pin until arc over occurs. Wet test in new designs is very important as there is danger of puncture or arc over of individual parts which appear safe on dry test. The standard precipitation for a wet test is 1/5 inch of distilled water per minute inclined at an angle of 45° with the axis of the insulator. The wet arcing test usually gives a lower arc-over voltage and hence gives a measure of the electrical factor of safety of the line (ratio of voltage at which failure occurs to working voltage) under the weather conditions when the factor is usually the lowest.

Free Arcing Tests. — In order to test the free-arcing properties the testing transformer must be large enough so that the flash at time of arc over will continue as a true power arc; even then the phenomena of fracture will rarely occur, even if the insulator is not free-arcing, because of the comparative feebleness of the power used in testing.

Corona Formation is indicated by the emission of a sound (i.e., the insulator is not "quiet") and may be still more accurately detected by the light from the corona when insulator is tested in the dark. Corona is considered an evidence of defective design and it is considered desirable that the voltage of corona formation should not be much below the arc-over voltage and certainly not below working voltage. However, many and perhaps most, high-voltage insulators show some corona at working voltage, though the amount is so small that it is difficult to detect.

Routine Tests. — No high-voltage insulator should be used without having been tested at the factory. Mechanical tests, except on a few selected samples, are not ordinarily made on pin-type insulators. On suspension insulators a tension test on each unit is desirable, but should not exceed about one-half the expected ultimate strength, for this is sufficient to eliminate defective ones, and most insulators are permanently injured electrically at a point below this ultimate strength.

Flash-over or arcing tests should be made on each porcelain part before assembly (so that no poor part will get into any insulator) and on the whole insulator when assembled. Each part should be tested to flash-over potential for five minutes, on normal frequency, or two minutes at 200,000 cycles.

The cup-shaped parts (also complete pin-type insulators) may be tested by setting them inverted in a pan of water used as one electrode, and partly filling the cup with water for the other. A large number of similar parts are ordinarily tested simultaneously in this way. Assembled suspension units are tested by using pin and cap as electrodes. The string of units forming a complete suspension insulator is not tested, except for a few selected samples (design tests). Factory tests of parts can be made at the factory with a transformer which will give a moderately high voltage, and no special measuring instruments are necessary, as the fact that parts are tested to arc-over voltage is evidenced by the fact that the arc-over is visible. A certain number of all units fired should be submitted to a test for porosity by immersing broken units in fuchsine dye under pressure of about 200 pounds per square inch for about two hours.

Testing on the Line. — Periodic testing of insulators on the line should be carried out to eliminate deteriorating units before they fail and interrupt service.

SPECIFICATIONS FOR INSULATORS. — (*See also Article on Specifications.*) Several specifications for high-voltage insulators have been suggested as suitable for "Standard Specifications"; see papers by Peek, Sanford and Thomas in *Transactions of A.I.E.E.*, 1913, Vol. 32–2, pp. 1457–1508. Specifications covering the inspection and tests of porcelain high-tension line insulators for over 25,000 volts, which had been prepared by the High Tension Transmission Committee of the A.I.E.E., were presented at the 1914 annual convention of the Institute. The reader is referred to the above-mentioned papers for details.

INSTALLATION OF INSULATORS. — See the articles on *Distribution Lines* and *Transmission Lines.*

PIN TYPE

Material	Operating voltage, volts	Test voltage		Diameter, inches	Height, inches	Number of parts	Weight, pounds	Cost, dollars
		Wet, volts	Dry, volts					
Glass......	110–2200	......	......	3¼	4	1	1¼	.08
Porcelain .	13,200	40,000	80,000	6½	3¾	2	3⅜	.36
Porcelain .	22,000	45,000	72,000	7	5	2	5	.81
Porcelain .	33,000	60,000	90,000	9	8	2 or 3	8	1.62
Porcelain .	44,000	80,000	110,000	10½	10	3	13	2.03
Porcelain .	50,000	95,000	120,000	12	11	3	18	4.28
Porcelain .	60,000	115,000	150,000	14	13	4	27	5.22

SUSPENSION TYPE UNITS

Diameter, inches	Number of parts	Spacing, inches	Test voltage		Ultimate strength, pounds	Working stress, pounds	Weight, pounds	Cost, dollars
			Wet, volts	Dry, volts				
10	1	5½	50,000	75,000	8,000	4,000	11	2.20
11	1	6¾	50,000	100,000	9,000	4,500	15½	2.45
12	1	[illegible]½	50,000	75,000	9,000	4,500	13	3.15
14	2	9	65,000	90,000	12,000	6,000	20	5.50

WEIGHTS AND COSTS. — The approximate dimensions, weights and costs of glass and porcelain pin-type insulators and for porcelain suspension units for different operating and test voltages are given in the preceding table.

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Vol. 15, 1912; Musten, *Insulator Depreciation and Effects of Operation*, A.I.E.E., Vol. 33, p. 1731; *Present Practice in the Design and Manufacture of High Tension Insulators*, A.I.E.E., Vol. 36, p. 545; *Factors Producing Reliability in the Suspension Insulator*, Proc. N.E.L.A., Vol. 32-2, pp. 1457-1508, 1913; Brundage, *Expansion Effects as a Cause of Deterioration in Suspension Insulators*, A.I.E.E., Vol. 36, p. 535; Flaherty, *Testing for Defective Insulators in High Tension Transmission Lines*, A.I.E.E., Vol. 35, p. 1095; Clark, *Experiments on Porcelain Suspension Insulator Units*, A.I.E.E., Vol. 35, p. 1453; Woodbridge, *Investigation of Suspension Insulator Depreciation*, A.I.E.E., Vol. 35, p. 1467; Creighton, *Electrical Porcelain*, A.I.E.E., Vol. 34, p. 465; Imlay and Thomas, *High Frequency Tests of Line Insulators*, Vol. 31, p. 2121; Ackerman, *High Tension Insulators*, Electrical World, Vol. 73, p. 116, Jan. 18, 1919; Marvin, *A New Method of Grading Porcelain Insulators*, A.I.E.E., Trans., Vol. 35, p. 745; Hunt, *Megger and Other Tests on Suspension Insulators*, A.I.E.E., Trans. Vol. 35, p. 735; Ryan, *Unit Voltage Duties in Long Suspension Insulators*, Journal A.I.E.E., July, 1920; *Ceramics in Relation to Durability of Porcelain Suspension Insulators*, A.I.E.E., Vol. 35, p. 1437; Feder, *Surface Leakage as a Factor in Insulator Design*, Journal A.I.E.E., Sept., 1920; Brand, *High Tension Insulator Tests: A Study of Design Factors*, General Electric Review, Vol. 14, 1913. Fortesque and Fransworth, *Air as an Insulator When in the Presence of Insulating Bodies of Higher Specific Inductive Capacity*, A.I.E.E., Vol. 33, p. 893; Sothman, *Comparative Tests on High Tension Insulators*, A.I.E.E., Vol. 31, p. 2169; Peaslee, *Factors Controlling the Design and Selection of Suspension Insulators*, Journal A.I.E.E., Vol. 76, June, 1920; Gilchrist and Klinefelter, *Applications of Theory and Practice to the Design of Transmission Line Insulators*, Electric Journal, Vol. 16, p. 8; *Experimental Investigation of Porcelain Mixes*, Electric Journal, Vol. 15, p. 77; *Design of Porcelain Insulators from a Theoretical Standpoint*, Electric Journal, Vol. 15, p. 443; *Design of Porcelain Insulators from a Ceramic Standpoint*, Electric Journal, Vol. 15, p. 489; *A Comparison of Different Methods of Testing Electrical Porcelain*, Electric Journal, Vol. 12, p. 282.

INTEGRALS. — (*See also Derivatives; Equations, Differential.*) If the function $y = f(x)$ be plotted as a curve (see Fig. 1) and the axis of x be divided into a number of very small sections, each of width dx, the area between the curve and the axis of x may be conceived as the sum of the products obtained by multiplying each length dx by the corresponding altitude or ordinate y. Then

$$\text{Area} = \int y\,dx,$$

where $\int$ stands for "sum."

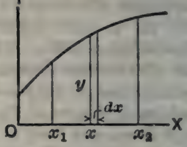

Fig. 1.

The Definite Integral. — The area between the values x_1 and x_2 is written

$$\int_{x_1}^{x_2} y\,dx.$$

Such an expression is called a "definite integral."

It is evident from the figure that the area of the curve between x_1 and x_2 is equal to the difference between the area from o to x_2 and the area from o to x_1. Hence calling the former area $F(x_2)$ and the latter area $F(x_1)$, we have

$$\int_{x_1}^{x_2} y \cdot dx = F(x_2) - F(x_1).$$

This equation is the general expression for any definite integral.

Indefinite Integrals. — The above equation may also be written

$$F(x_2) = \int_{x_1}^{x_2} y\,dx + F(x_1)$$

and if x_2 is considered a variable, the subscripts are omitted, as follows:

$$F(x) = \int y\,dx + A,$$

where A is a constant for any given reference point x_1. This constant A is an arbitrary constant, since any point x_1 may be chosen as the reference point, but when a point is once chosen, A is fixed. The expression $\int y\,dx$ is called an "indefinite integral" and A is called the "integration constant."

From the definition of a derivative (*see Derivatives*), the derivative of $F(x)$ is

$$\frac{dF(x)}{dx} = \frac{\int_{x_1}^{x+dx} y dx - \int_{x_1}^{x} y\,dx}{dx}$$

and from the figure, it is evident that the difference between the area from x_1 to $x + dx$ and the area from x_1 to x is simply $y\,dx$. Hence

$$\frac{dF(x)}{dx} = y,$$

that is, the integral $F(x)$ of any function y, with respect to x, must be such a function that when differentiated with respect to x, the result is the function y.

Formulas for Integration. — u, v, x and z are variables; a, m and n are constants.

$$\int (u+v)\,dx = \int u\,dx + \int v\,dx$$

$$\int y\,dx = \int y\,\frac{dx}{dz}\,dz$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{F'(x)}{F(x)} = \log F(x)$$

where $F'(x)$ stands for $\frac{dF(x)}{dx}$.

TABLE OF INTEGRALS

Function $f(x)$	**Integral** $\int f(x)dx$	**Function** $f(x)$	**Integral** $\int f(x)dx$
x^m	$\frac{1}{m+1}x^{m+1}$	$\frac{1}{\sqrt{a^2-b^2x^2}}$	$\frac{1}{b}\sin^{-1}\frac{b}{a}x$
$\frac{1}{ax}$	$\frac{1}{a}\log_\epsilon x$	$\frac{1}{\sqrt{a^2+b^2x^2}}$	$\frac{1}{b}\sinh^{-1}\frac{b}{a}x$
ϵ^{ax}	$\frac{1}{a}\epsilon^{ax}$	$\frac{1}{a^2+b^2x^2}$	$\frac{1}{ab}\tan^{-1}\frac{bx}{a}$
a^{bx}	$\frac{1}{b\log a}a^{bx}$	$\frac{1}{(a^2-b^2x^2)_{bx<a}}$	$\frac{1}{ab}\tanh^{-1}\frac{bx}{a}$
$\cos ax$	$\frac{1}{a}\sin ax$	$\frac{1}{(a^2-b^2x^2)_{bx>a}}$	$\frac{1}{ab}\tanh^{-1}\frac{a}{bx}$
$\sin ax$	$-\frac{1}{a}\cos ax$	$\frac{1}{(x-a)(x-b)}$	$\frac{1}{a-b}\log\frac{x-a}{x-b}$
$\tan ax$	$-\frac{1}{a}\log(\cos ax)$	$\frac{1}{\sqrt{2ax-x^2}}$	$2\sin^{-1}\sqrt{\frac{x}{2a}}$
$\cosh ax$	$\frac{1}{a}\sinh ax$	$\frac{1}{x\sqrt{x^2-a^2}}$	$-\frac{1}{a}\sin^{-1}\frac{a}{x}$
$\sinh ax$	$\frac{1}{a}\cosh ax$	$\frac{1}{x\sqrt{x^2-a^2}}$	$\frac{1}{a}\cos^{-1}\frac{a}{x}$
$\tanh ax$	$\frac{1}{a}\log(\cosh ax)$	$\frac{1}{x\sqrt{a^2+x^2}}$	$-\frac{1}{a}\sinh^{-1}\frac{a}{x}$
$\tan x\sec x$	$\sec x$	$\frac{1}{x\sqrt{a^2-x^2}}$	$-\frac{1}{a}\cosh^{-1}\frac{a}{x}$
$\sec^2 ax$	$\frac{1}{a}\tan ax$	$\frac{x}{\sqrt{a^2\pm x^2}}$	$\pm\sqrt{a^2\pm x^2}$
$\frac{1}{\cos^2 ax}$	$\frac{1}{a}\tan ax$	$\frac{x}{\sqrt{x^2-a^2}}$	$\sqrt{x^2-a^2}$
$\frac{1}{\sin^2 ax}$	$-\frac{1}{a}\cot ax$		

Function $f(x)$	Integral $\int f(x)dx$
$\sqrt{a^2-x^2}$	$\frac{1}{2}\left(x\sqrt{a^2-x^2}+a^2\sin^{-1}\frac{x}{a}\right)$
$\sqrt{x^2+a^2}$	$\frac{1}{2}\left[x\sqrt{x^2+a^2}+a^2\sinh^{-1}\frac{x}{a}\right]$
$\sqrt{x^2-a^2}$	$\frac{1}{2}\left[x\sqrt{x^2-a^2}-u^2\cosh^{-1}\frac{x}{a}\right]$
$\sin ax \sin bx$	$\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)}$
$\cos ax \cos bx$	$\frac{\sin(a-b)x}{2(a-b)}+\frac{\sin(a+b)x}{2(a+b)}$
$\sin^2 ax$	$\frac{1}{2}x-\frac{1}{4a}\sin 2ax$
$\cos^2 ax$	$\frac{1}{2}x+\frac{1}{4a}\sin 2ax$
$\sinh ax \sinh bx$	$\frac{\sinh(a-b)x}{2(a-b)}-\frac{\sinh(a+b)x}{2(a+b)}$
$\cosh ax \cosh bx$	$\frac{\sinh(a-b)x}{2(a-b)}+\frac{\sinh(a+b)x}{2(a+b)}$
$\sinh^2 ax$	$\frac{1}{2}x-\frac{1}{4a}\sinh 2ax$
$\cosh^2 ax$	$\frac{1}{2}x+\frac{1}{4a}\sinh 2ax$
$\epsilon^{ax}\sin bx$	$\frac{\epsilon^{ax}(a\sin bx-b\cos bx)}{a^2+b^2}$
$\epsilon^{ax}\cos bx$	$\frac{\epsilon^{ax}(a\cos bx+b\sin bx)}{a^2+b^2}$
$\frac{\epsilon^{mx}}{x}$	$\log_\epsilon x+\frac{mx}{1}+\frac{m^2x^2}{2!}+\frac{m^3x^3}{3!}+\text{etc.}$
$\sin^m x\cos^n x$	$-\int \sin^{m-1}\cos^n x\, d\cos x$

Double Integrals. — Just as the area of a surface may be represented by the expression

$$\int y \cdot dx$$

so the volume of a solid may be represented by

$$\int\left[\int y \cdot dx\right] dz$$

or adopting the usual notation, by

$$\int\int y\, dx \cdot dz$$

taken between limits determined by the data of the problem.

[W. A. Del Mar.]

INTEREST, ANNUITIES AND SINKING FUND. — Let

P = principal invested in dollars;
n = number of years principal is invested;
r = rate of interest, per cent per annum;
A = total amount of principal and interest at end of n years.

Note. — The following relations hold irrespective of the unit of time selected, provided r is taken as the interest earned per $100 during that time.

Simple Interest. — If the principal is invested at simple interest, then

$$A = \left(1 + \frac{nr}{100}\right)P.$$

Compound Interest. — If the principal is invested at compound interest, i.e., if the interest earned each year is invested at the end of that year at the same rate as the original principal, then the total amount due at the end of n years is

$$A = PR^n,$$

where

$$R = 1 + \frac{r}{100}.$$

R^n is the amount of the principal of 1 dollar and interest at the end of n years. The following table gives the value of R^n for n ranging from 1 to 50 years and r from 3 to 6 per cent.

VALUES OF $R^n = \left(1 + \frac{r}{100}\right)^n$

Years n	Per cent interest = r				Years n	Per cent interest = r			
	3	4	5	6		3	4	5	6
1	1.03	1.04	1.05	1.06	16	1.6047	1.8730	2.1829	2.5403
2	1.0609	1.0816	1.1025	1.1236	17	1.6528	1.9479	2.2920	2.6928
3	1.0927	1.1249	1.1576	1.1910	18	1.7024	2.0258	2.4066	2.8543
4	1.1255	1.1699	1.2155	1.2625	19	1.7535	2.1068	2.5269	3.0256
5	1.1593	1.2166	1.2763	1.3382	20	1.8061	2.1911	2.6533	3.2071
6	1.1941	1.2653	1.3401	1.4185	21	1.8603	2.2787	2.7859	3.3995
7	1.2299	1.3159	1.4071	1.5036	22	1.9161	2.3699	2.9252	3.6035
8	1.2668	1.3686	1.4774	1.5938	23	1.9736	2.4647	3.0715	3.8197
9	1.3048	1.4233	1.5513	1.6895	24	2.0328	2.5633	3.2251	4.0487
10	1.3439	1.4802	1.6289	1.7908	25	2.0937	2.6658	3.3863	4.2919
11	1.3842	1.5394	1.7103	1.8983	30	2.4272	3.2433	4.3219	5.7435
12	1.4258	1.6010	1.7958	2.0122	35	2.8138	3.9460	5.5159	7.6862
13	1.4685	1.6651	1.8856	2.1329	40	3.2620	4.8009	7.0398	10.2858
14	1.5126	1.7317	1.9799	2.2609	45	3.7815	5.8410	8.9847	13.7648
15	1.5580	1.8009	2.0789	2.3965	50	4.3838	7.1064	11.4670	18.4204

The following table gives the number of years required for a given principal to double itself at compound interest.

Interest Rate	3	4	5	6
Years to Double	23.5	17.7	14.2	11.9

Annuities. — An annuity is a fixed sum of money paid yearly, or at other equal times agreed upon.

One dollar invested at interest at r per cent at the beginning of every year will at the end of n years amount to

$$R \cdot \frac{R^n - 1}{R - 1} \text{ dollars,}$$

where $R = 1 + r/100$, the interest being compounded at the end of each year.

One dollar invested at the beginning of a period of n years will yield at the end of each year an annuity of

$$\frac{R^n (R-1)}{R^n - 1} \text{ dollars,}$$

where $R = 1 + r/100$, the interest being compounded at the end of each year.

Sinking Fund. — A sinking fund is a fund built up from fixed yearly payments or annuities. Sinking funds are usually provided to retire bonds, which are issued for a given number of years. The annuity required to retire a bond of $1000 at the end of n years is

$$1000\left(\frac{R-1}{R^n - 1}\right).$$

Values of this annuity for various rates of interest and for various values of n are given in the following table.

ANNUITY REQUIRED TO REDEEM $1000

At end of years	Rate of interest, per cent								
	2	2½	3	3½	4	4½	5	5½	6
2	495.05	493.78	492.69	491.42	490.20	489.00	487.80	486.62	485.43
3	326.72	325.14	323.56	321.94	320.36	318.77	317.21	315.63	314.10
4	242.63	240.84	239.02	237.26	235.50	233.74	232.01	230.29	228.60
5	192.16	190.24	188.35	186.49	184.63	182.79	180.98	179.13	177.39
6	158.53	156.56	154.61	152.67	150.79	148.88	147.02	145.18	143.36
7	134.52	132.49	130.51	128.57	126.61	124.67	122.82	120.96	119.13
8	116.51	114.47	112.46	110.48	108.53	106.60	104.72	102.86	101.03
9	102.52	100.46	98.44	96.44	94.49	92.57	90.69	88.83	87.02
10	91.33	89.25	87.24	85.24	83.29	81.38	79.50	77.67	75.87
11	82.18	80.11	78.07	76.09	74.15	72.25	70.39	68.57	66.79
12	74.56	72.49	70.46	68.48	66.55	64.67	62.83	61.03	59.28
13	68.12	66.05	64.03	62.06	60.14	58.27	56.45	54.68	52.96
14	62.60	60.54	58.53	56.57	54.67	52.82	51.02	49.28	47.58
15	57.83	55.77	53.77	51.82	49.94	48.11	46.34	44.62	42.96
16	53.65	51.60	49.61	47.68	45.82	44.01	42.27	40.58	38.95
17	49.97	47.93	45.95	44.04	42.20	40.42	38.70	37.04	35.44
18	46.70	44.67	42.71	40.82	38.99	37.24	35.54	33.92	32.36
19	43.78	41.76	39.81	37.94	36.14	34.40	32.75	31.15	29.62
20	41.15	39.14	37.22	35.36	33.58	31.87	30.24	28.68	27.18
25	31.22	29.27	27.43	25.67	24.01	22.44	20.95	19.55	18.23
30	24.65	22.78	21.02	19.37	17.83	16.39	15.05	13.80	12.65
35	20.00	18.20	16.54	15.00	13.58	12.27	11.07	9.97	8.97
40	16.55	14.84	13.26	11.83	10.52	9.34	8.28	7.32	6.46
45	13.91	12.27	10.78	9.45	8.26	7.20	6.26	5.43	4.70
50	11.82	10.26	8.87	7.63	6.55	5.00	4.78	4.06	3.44

INTERNAL-COMBUSTION ENGINES. — (*See also Gas*; *Gas Producers*, *Power Stations*, *Gas-Electric*.) An internal-combustion engine is an engine in which combustible gas, vapor, or oil is burned in a cylinder, generating a high temperature and high pressure in the gases of combustion, which expand behind a piston driving it forward. (Rotary gas engines or gas turbines are still, 1922, in the experimental stage.)

Gas Engine. — A gas engine in its simplest form is similar to a reciprocating single-acting steam engine, comprising a cylinder, a piston, crank, flywheel, etc. The pressure to move the piston is obtained by exploding or burning with great rapidity a compressed mixture of air and combustible gas.

Oil and Gasoline Engines. — The lighter distillates of petroleum, such as gasoline, are easily vaporized at moderate temperatures, and a gasoline engine differs from a gas engine only in having an atomizer attached, for spraying a fine jet of the liquid into the air-admission pipe. With kerosene and other heavier distillates, or crude oils, it is necessary to provide some method of atomizing and vaporizing the oil at a high temperature, such as injecting it into a hot vaporizing chamber at the end of the cylinder, or into a chamber heated by the exhaust gases.

Diesel Oil Engine. — The distinguishing features of the Diesel engine are: It compresses air only, to a predetermined temperature above the firing point of the fuel. This fuel is blown as a cloud of vapor (by air from a separate small compressor) into the cylinder when compression has been completed, ignites spontaneously without explosion, solely by reason of the heat of the air generated by the compression, and burns steadily with no essential rise in pressure. The temperature of gases, developed and rejected, is much lower than with engines of the explosive type. The engine uses crude oil and residual petroleum products.

Diesel engines sizes run from 6¾ by 8⅝ inches, two-cycle, four cylinder, 110 b.h.p. for four cylinders to 32.2 × 39.4 inches, two-cycle, one cylinder, 1250 b.h.p. for one cylinder

The largest for land service are four-cylinder engines of approximately 2400 b.h.p. These engines are running for electric light service with admirable results on a guaranteed oil consumption not to exceed 0.4 pound of oil per brake horse-power-hour. They are of the two-cycle type. Four-cycle engines have an oil consumption of practically 90 per cent of this figure, or about 0.36 pound of oil per brake horse-power-hour.

The thermal efficiency of these engines is between 30 and 40 per cent. Twenty-five to 30 per cent of the heat is carried away in the cooling water and the rest in the exhaust gases.

The variation of economy with variation of load as shown by tests is reported to be:

Per cent rated load	30	50	75	100	120
Lb. oil per b.h.p. hour	0.71	0.55	0.46	0.43	0.44

Alcohol Engines. — Due to the relatively high cost of alcohol, even when "denatured," alcohol engines are not used extensively in this country.

CLASSIFICATION OF INTERNAL-COMBUSTION ENGINES. — Internal-combustion engines are classified as four-cycle and two-cycle engines, single- and double-acting engines, and single-and multi-cylinder engines.

Four-cycle Engines. — In what is known as a four-cycle engine, one ignition of gas takes place in one end of the cylinder every two revolutions of the flywheel, or every two double strokes. The following sequence of operations takes place during four consecutive strokes: (*a*) inspiration of a mixture of gas and air during an entire stroke; (*b*) compression during the second (return) stroke; (*c*) ignition at or near the dead-point and expansion during the third stroke; (*d*) expulsion of the burned gas during the fourth (return) stroke.

Fig. 1 is an indicator diagram of a four-cycle engine. *AB*, the lower line, shows the admission of the mixture, at a pressure slightly below the atmosphere on account of the resistance of the inlet valve. *BC* is the compression into the clearance space, ignition taking place at *C* and combustion with increase of pressure continuing from *C* to *D*. The gradual termination of the combustion is shown by the rounded corner at *D*. *DE* is the expansion line, *EF* the line of pressure drop as the exhaust valve opens, and *FA* the line of expulsion of the burned gases, the pressure being slightly above the atmosphere on account of the resistance of the exhaust valve.

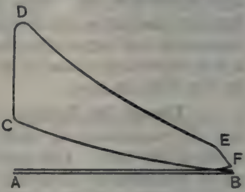

Fig. 1. Indicator Diagram of Four-cycle Engine

Two-cycle Engine. — In a two-cycle single-acting engine an explosion takes place with every revolution, or with each forward stroke of the piston. Referring to the diagram, Fig. 1, and beginning at *E*, when the exhaust port begins to open to allow the burned gases to escape, the pressure drops rapidly to *F*. Before the end of the stroke is reached an inlet port opens, admitting a mixture of gas and air from a reservoir in which it has been compressed. This mixture being under pressure assists in driving the burned gases out through the exhaust port. The inlet port and the exhaust port close early in the return stroke and during the remainder of the stroke *BC* the mixture, which may include some of the burned gas, is compressed and the ignition takes place at *C*, as in the four-cycle engine.

In one form of the two-cycle engine only compressed air is admitted while the exhaust port is open, the fuel gas being admitted under pressure after the exhaust port is closed. By this means a greater proportion of the burned gases is swept out of the cylinder. This operation is known as "scavenging."

Single- and Double-acting Engines. — These terms have the same significance as in the case of steam engines (q.v.). Comparatively few large engines are single-acting.

Multi-cylinder Engines. — In small multi-cylinder engines the cylinders are in "parallel," i.e., each has a separate piston connected to the crank shaft. In large engines two cylinders are frequently arranged in tandem. The term "twin cylinder" is used to designate two cylinders operating in parallel as contrasted with the "two-cylinder tandem" arrangement, in which the two cylinders are in "series." A "twin-tandem" engine has four cylinders in all, or two pairs of tandem cylinders.

RATING. — In contrast to steam prime movers the gas engine has a definite limit of power with the usual methods of governing and its economy improves until this limit is almost reached. This characteristic makes a maximum rating more significant than a normal rating with overload capacity. As ordinarily rated, however, engines can develop 20 per cent overload for brief periods and will care for momentary swings much greater. Due to their low mechanical efficiency (75 to 85 per cent) gas engines are preferably rated in terms of their brake horse-power instead of indicated horse-power.

Rating of Automobile Engines. — Automobile engines are single acting and as a rule multi-cylinder, the cylinders all being in parallel. The American Licensed Automobile Manufacturers Association have adopted the following arbitrary formula for the rating of automobile engines

$$\text{Brake horse-power} = \frac{D^2N}{2.5},$$

where D = diameter of each cylinder in inches, N = number of cylinders, This rating is usually referred to as the A.L.A.M. rating. It corresponds approximately to the actual brake horse-power when the piston speed is 1000 feet per minute and the mean effective pressure is 55 pounds per square inch.

The following ratings are derived from the formula:

Bore, in.	2½	3	3½	4	4½	5	5½	6
Bore, mm.	64	76	89	102	114	127	140	152
Horse-power, 1 cylinder	2½	3.6	4.9	6.4	8.1	10	12.1	14.4
Horse-power, 2 cylinders	5	7.2	9.8	12.8	16.2	20	24.2	28.8
Horse-power, 4 cylinders	10	14.4	19.6	25.6	32.4	40	48.4	57.6
Horse-power, 6 cylinders	15	21.6	29.4	38.4	48.6	60	72.6	86.4

DESIGN AND CONSTRUCTION. — Only a few of the more important features in the design and construction of internal-combustion engines can be noted here. See *Bibliography* at end of this article for references to various treatises on the subject.

Arrangement of Cylinders in Large Engines. — The number and arrangement of cylinders depends upon the space, the power required and the uniformity of rotation to be produced. Vertical-cylinder engines are built in sizes up to 750 horse-power. They require less floor space than the horizontal types but the superior mechanical properties of the latter warrant their selection when space economy is not important. The tandem double-acting arrangement is standard in horizontal engines. As it is not practicable to develop more than 1500 horse-power in a single cylinder, engines of the largest sizes are twin, tandem, double-acting.

Cooling is one of the most important problems of gas-engine design. In the small sizes provision for air cooling or a simple water jacket about the cylinder walls suffices, but the large sizes require the cooling of the piston, piston rod, and valve stems. The superficial area of the cylinders increases less rapidly than their cubical contents and this fact tends to aggravate the cooling problem as the size of the cylinders is increased. Cooling is more difficult with rich gases than with lean. One practical result of these difficulties is found in the fact that engines of large capacity are little if any cheaper per horse-power than small ones and that maintenance is apt to be proportionately larger. The amount of cooling water required per horse-power-hour is stated by different investigators to be from 0.75 to 1.00 cubic foot. Figures for a large number of commercial plants average 2.4 cubic feet. This high average is undoubtedly due to the fact that water cost little in most of these plants.

Ignition. — The "hot-tube" method of igniting the compressed mixture of gas and air in the cylinder is practically obsolete, and electric systems are used instead. Of these the "make-and-break" and the "jump-spark" systems are in common use. In the former two insulated contact pieces are located in the end of the cylinder, and through them an electric current passes while they are in contact. A spark coil is included in the circuit, and when the circuit is suddenly broken at the proper time for ignition, by mechanism operated from

the valve-gear shaft, a spark is made at the contacts, which ignites the gas. In the "jump-spark" system two insulated terminals separated about 0.03 inch apart are located in the cylinder, and the secondary or high-tension current of an induction coil causes a spark to jump across the space between them when the circuit of the primary current is closed by mechanism operated by the engine. See article on *Ignition, Electric*.

In some oil engines the mixture of air and oil vapor is ignited automatically, by the temperature generated by compression of the vapor, in a chamber at the end of the cylinder, called the vaporizer, which is not water-jacketed and therefore is kept hot by the repeated ignitions. Before starting the engine the vaporizer is heated by a Bunsen burner or other means.

Timing. — By adjusting the cam or other mechanism operated by the valve-gear shaft for causing ignition, the time at which the ignition takes place, with reference to the end of the compression stroke, can be regulated. The mixture is usually ignited before the end of the stroke, the advance depending upon the inflammability of the mixture and on the speed of the engine.

Governing. — Two methods of governing the speed of an engine are in common use, the "hit-and-miss" and the throttling methods. In the former the engine receives its usual charge of air and gas only when the engine is running at or below its normal speed; at higher speeds the admission of the charge is suspended until the engine regains its normal speed. One method of accomplishing this is to interpose between the valve rod and its cam or other operating mechanism, a push rod, or other piece, the position of which with reference to the end of the valve rod is controlled by a centrifugal governor so that it hits the valve rod if the speed is at or below normal and misses it if the speed is above normal.

The throttling method of regulating is similar to that used in throttling steam engines; the quantity of mixture admitted at each charge being varied by varying the position of a butterfly valve in the inlet pipe. Cut-off methods of governing are also used, such as varying the time of closing the admission valve during the suction stroke, or varying the time of admission of the gas alone, or "quality regulation."

Lubrication of cylinders is best provided by a time force-feed system injecting oil between the piston rings at dead-center positions. The supply should be closely regulated, as an excess carbonizes on the cylinder walls and becomes a source of trouble. Bearings are commonly ring-oiled or flooded.

Owing to the high temperatures that prevail in the cylinder of the internal combustion engine, the question of proper lubrication is a serious one. Cylinder oil should be exceedingly pure, free from acids, and composed of hydrocarbons that leave no residue after combustion. Only mineral oils, therefore, are suitable for the purpose. The ignition point of good cylinder oil should not be lower than 535° F. The losses in power due to poor lubrication of gas engines may amount to 10 or 15 per cent.

The amount of oil required per horsepower-hour varies with the character of the installation and the method of operation. For full load 24-hour service, the proportion per horsepower-hour is, of course, greater than for a plant running under a light load for a 9 or 10-hour day. The average of several figures given by the engine manufacturers for the amount of engine oil required is 0.5 gallon per 1000 horsepower-hour. The operators of commercial plants report figures, however, that average about double this amount, and an equal amount for cylinder oil.

Starting Devices. — In large engines starting is now universally accomplished by compressed air. In some small stationary engines an explosive cartridge is inserted into the cylinder head and set off by percussion.

Electric self-starters are now almost universally used for automobile engines in pleasure cars (see *Starting and Lighting Systems for Automobiles*).

Speeds. — Small gasoline engines are usually designed for speeds of from 300 to 1000 revolutions per minute. Large gas engines, for central-station service, usually have speeds about as follows:

100 h.p.	275 rev. per min.
300 h.p.	200 rev. per min.
500 h.p.	150 rev. per min.
1000 h.p.	100 rev. per min.

POWER DEVELOPED BY AN INTERNAL COMBUSTION ENGINE. — In order to calculate the power developed by an internal-combustion engine it is necessary to know the mean effective pressure in the engine cylinder, the dimensions of the cylinder and either (1) the length of stroke and number of explosions per minute, or (2) the piston speed and number of explosions per revolution.

Mean Effective Pressure. — The mean effective pressure can be obtained from the indicator diagram (*see Steam Engines*), or if a test diagram is not available the mean effective pressure either has to be assumed from a knowledge of that found in other engines of the same type and working under the same conditions as those of the design, or it may be calculated from the ideal air diagram and modified by the use of a coefficient or diagram factor depending on the kind of fuel used and the compression pressure.

The following figures are given by C. P. Poole as a rough approximate guide to the mean effective pressures obtained with different fuels in a four-cycle engine. In a two-cycle engine the mean effective pressure of the pump diagram should be subtracted. The delivery pressure is usually from 4 to 8 pounds per square inch above the atmosphere, and the corresponding mean effective pressure of the pump about 3.8 to 7.

PROBABLE MEAN EFFECTIVE PRESSURE, POUNDS PER SQUARE INCH, FOUR-CYCLE ENGINES

Kind of fuel	Compression pressure	Engine horse-power						
		5	10	25	50	100	250	500
Anthracite Producer gas	100	..	55	60	65	70	75	80
	130	..	65	70	75	80	85	90
	160	..	..	..	80	85	90	90
Mond Producer gas	100	..	..	60	65	65	70	75
	130	..	65	65	70	75	80	85
	160	..	..	75	80	85	90	90
Natural and Illuminating gas	65	..	60	65	70	80	85	..
	100	..	75	80	90	90	95	100
	130	..	..	..	..	100	105	110
Gasoline vapor	65	70	75	80	85	..	..	..
	100	85	90	90	95	..	..	..
Kerosene spray	65	50	55	60	65	..	..	..
	115	70	75	80	85	..	..	..

Number of Explosions per Revolution. — The number of explosions per revolution depends upon the type of engine as given in the accompanying table.

Formulas for Horse-power Output. — (*See also Lemp, H., B.H.P. Formula for Internal Combustion Engines, G. E. Rev. 22, p. 808, Oct., 1919.*) Let

p = mean effective pressure, lb. per sq. in.
d = diameter of cylinder bore, inches,
N = number of revolutions per minute,
e = total number of explosions per revolution,
S = piston speed, feet per minute,
L = length of stroke, feet,
ϵ = mechanical efficiency, as a fraction,
P_i = indicated horse-power.

Type of engine	Explosions per rev. per cylinder
Single-acting, 4-cycle..	0.5
Single-acting, 2-cycle..	1.0
Double-acting, 4-cycle.	1.0
Double-acting, 2-cycle.	2.0

Then

$$P_i = \frac{pLd^2eN}{42,000} = \frac{pd^2eS}{84,000}.$$

The brake horse-power is

$$\text{Brake h.p.} = \epsilon P_i.$$

The brake horse-power is frequently expressed by the formula

$$\text{Brake h.p.} = Cd^2,$$

where $C = peS\epsilon/84,000$ is called the "engine constant." For $\epsilon = 0.84$ and 2 explosions per revolution the values of C for various mean effective pressures are as follows:

VALUES OF C FOR 2 EXPLOSIONS PER REVOLUTION

M.E.P., lb. per sq. in.	Piston speed, feet per minute					
	500	600	700	800	900	1000
50	0.50	0.60	0.70	0.80	0.90	1.00
60	0.60	0.72	0.84	0.96	1.08	1.20
70	0.70	0.84	0.98	1.12	1.26	1.40
80	0.80	0.96	1.12	1.28	1.44	1.60
90	0.90	1.08	1.26	1.44	1.62	1.80
100	1.00	1.20	1.40	1.60	1.80	2.00
110	1.10	1.32	1.54	1.76	1.98	2.20

These values of C apply to 4-cylinders, 4-cycle, single-acting, to 2-cylinders, 2-cycle, single-acting, and to 1-cylinder, 2-cycle, double-acting. For single cylinders, 4-cycle, single-acting, divide by 4; for single cylinders, 4-cycle, double-acting, or 2-cycle, single-acting, divide by 2.

TESTING INTERNAL-COMBUSTION ENGINES. — The test of a gas or oil engine includes the measurement of its power by a friction brake, of the number of cubic feet of gas or pounds of oil used per brake horse-power

hour, and of the calorific value of the gas or oil. In connection with the tests indicator diagrams may be taken, the air supply and the jacket cooling water may be measured, the exhaust gas analyzed, the temperatures of air, water and gas taken, and a heat balance calculated showing the thermal efficiency and the several sources of loss of energy. For details see *Code of 1922 of the A.S.M.E.*

EFFICIENCY AND FUEL CONSUMPTION. — The thermal efficiency of an internal-combustion engine is considerably higher than that of the best reciprocating engine or steam turbine. It is probable that the maximum thermal efficiency of such engines under the most favorable operating conditions is about 38 per cent, based on the indicated horse-power or 30 per cent, based on the brake horse-power. The mechanical efficiency of an internal combustion engine is lower than that of a reciprocating engine, ranging from 70 to 85 per cent as against 90 to 95 per cent for a high-class reciprocating engine and 92 to 97 per cent for steam turbines.

Thermal Efficiency. — The conditions which appear to give the highest thermal efficiency in gas and oil engines are: (1) high temperature of cooling water in the jackets; (2) high pressure at the end of compression; (3) lean mixture; (4) proper timing of the ignition; (5) maximum load. The higher economy of a lean mixture may be due to the fact that high compressions may be used with such a mixture, while with rich mixtures high-compression pressures cannot be used without danger of preignition.

Other things being equal, the hotter the walls of the cylinder the less heat is transferred into them from the hot gases, and therefore the higher the efficiency. Cool walls, however, allow of higher compression without preignition, and high compression is a cause of high efficiency. Cool walls also tend to give the engine greater capacity, since with hot walls the fuel mixture expands more on entering the cylinder, reducing the weight of charge admitted in the suction stroke.

Distribution of Heat Losses. — The heat losses are: (1) the heat carried away in the jacket water, (2) that carried away in the waste gases, and (3) that lost by radiation. The relative amounts of these three losses vary greatly, depending on the size of the engine and on the amount of water used for cooling. Carpenter and Diederichs quote the following:

Ratio of compression	R.p.m.	M.E.P., lb. per sq. in.	Ratio air to gas	Heating value of charge, B.t.u.	Work done by 1 B.t.u., ft. lb.	Exhaust temp., deg. F.	Distribution of heat losses, per cent		
							Work	Jacket water	Exhaust
2.67	187	54.3	7.11	18.5	140	1022	18.0	51.2	30.8
2.67	247	51.5	7.35	17.4	141	1137	18.1	45.6	36.3
4.32	187	69.3	7.43	17.0	190	867	24.4	53.8	21.8
4.32	247	65.2	7.40	16.8	184	992	23.7	49.5	26.8

showing that the distribution of the heat losses varies with the rate of compression and with the speed.

Over-all Efficiency and Fuel per Brake Horse-Power. — The over-all

efficiency of internal-combustion engines is the product of the thermal efficiency by the mechanical efficiency. A more convenient expression is

$$\text{Over-all efficiency} = \frac{2546}{HQ} \qquad \text{as decimal fraction}$$

where H = heat of combustion, B.t.u. in one cubic foot, and Q = number of cubic feet of gas supplied per brake horse-power hour. In the case of a liquid fuel it is more convenient to take H = heat of combustion, B.t.u. per pound, and Q = pounds of liquid supplied per brake horse-power hour. See articles on *Gas and Fuels*. The product HQ in either case is the number of B.t.u. per brake horse-power.

In Fig. 2 are given the results of tests on several internal-combustion engines using different kinds of fuels, and also, for comparison, the performance of three steam turbines. These curves were supplied by Prof. W. E. Wickenden.

These curves bring out very strikingly the relatively poor performance of

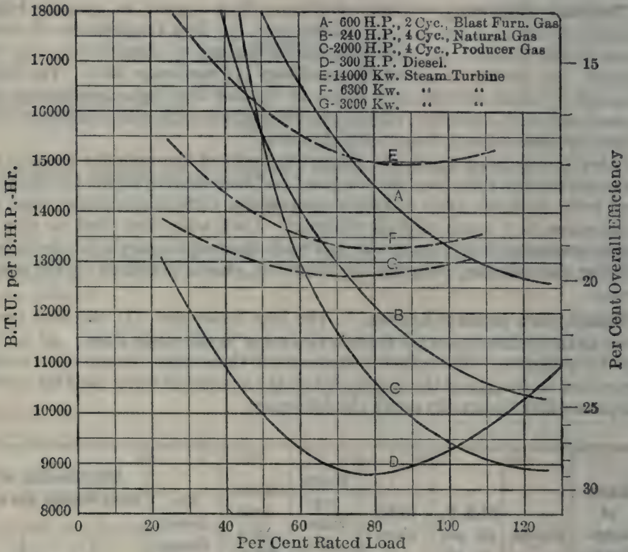

Fig. 2. Fuel Consumption of Gas and Oil Engines

internal-combustion engines at light loads. Under working conditions gas engines are expected to produce a brake horse-power hour for every 9000 to 12,000 B.t.u. in the fuel. The average of several quotations from different manufacturers is as follows:

Per cent rated load....................	100	75	50	25
B.t.u. per b.h.p.-hr....................	10,500	12,000	14,700	20,000

Coal per Brake Horse-power Hour. — Figures giving the coal consumption of a producer per brake horse-power hour of the gas engine supplied

from it are of little value unless the type of producer and type of engine as well as the quality of the coal are stated. Let H = B.t.u. per pound coal, ϵ_1 = efficiency of producer (*see Gas Producers*), ϵ_2 = over-all efficiency of gas engine. Then

$$\text{pounds coal per brake horse-power hour} = \frac{2546}{H\epsilon_1\epsilon_2}.$$

For example, if the heating value of the coal used is 14,000, the over-all efficiency of the gas engine 30 per cent and the efficiency of the producer 80 per cent, then $2546/14{,}000 \times 0.3 \times 0.8 = 0.76$ pound coal will be required per brake horse-power hour. Figures as low as 0.7 pound coal per brake horse-power hour have been obtained, but they are exceptional.

Average consumptions are:

Fuel	Lb. per b.h.p.-hr.	
	As fired	Dry
Anthracite	1.3	1.3
Bituminous coal	1.3	1.2
Lignite	2.0	1.6
Peat	2.6	2.0
Wood	3.3	...

GAS AND OIL ENGINE TROUBLES. — Among the causes of troubles are: the variable composition of the fuel; too much or too little air supplied; compression ratio not right for the kind of fuel; ignition timer set too late or too early; preignition; backfiring; electrical and mechanical troubles with the igniting system; carbon deposits in the cylinder and on the igniting contacts.

COST OF GAS ENGINES (Pre-war figures). — Differences in the ratings of gas engines, the partial development of the art and the close competition of builders make costs quite variable. The following methods of estimating are derived from a number of installations and probably represent average conditions for electrical generation. Let P = brake horse-power rating; then

City Gas or Natural Gas Engines, cost in dollars..... $36 \times P$

Producer Gas Engines:
- 20 to 100 horse-power, cost in dollars........... $400 + 40 \times P$
- 100 to 500 horse-power, cost in dollars........... $2500 + 24 \times P$
- 500 to 3000 horse-power, cost in dollars........... $34 \times P - 2500$

Four-cycle Diesel Engines:
- 100 to 500 horse-power, cost in dollars........... $3000 + 47 \times P$
- 600 to 1000 horse-power, cost in dollars........... $3000 + 45 \times P$

Two-cycle Diesel Engines:
- About 20 per cent less than four-cycle for the larger sizes.

BIBLIOGRAPHY. — For theory of the internal-combustion engine, see paper by Dugald Clerk, *Proc. Inst. C. E.*, 1882, Vol. 69; and Van Nostrand's *Science Series No. 62*. See also Wood's *Thermodynamics*. Standard works on gas engines are: *A Text-book on Gas, Air, and Oil Engines*, by Bryan Donkin; *The Gas and Oil Engine*, by Dugald Clerk; *Internal Combustion Engines*, by Carpenter and Diederichs; *Gas Engine Design*, by C. E. Lucke; *Gas and Petro-*

leum Engines, by W. Robinson; *The Modern Gas Engine and the Gas Producer*, by A. M. Levin; *The Gas Engine*, by C. P. Poole; and *Internal Combustion Engines*, by Hugo Guldner, translated by H. Diederichs. For practical operation of gas and oil engines, see *The Gas Engine*, by F. R. Jones, and *The Gas Engine Handbook*, by E. W. Roberts. For descriptions of large gas engines using blast-furnace gas, see papers in *Proc. Iron and Steel Inst.*, and *Trans. A.S.M.E.* See also Fernald and Orrok, *Engineering of Power Plants;* Adams, W. H., *The Diesel Engine and its Applications in So. California*, A.S.M.E. Trans., Vol. 37, p. 447; Goldingham, A. H., *The Heavy Oil Engine*, A.S.M.E. Trans., Vol. 37, p. 477.

IRON, PIG AND CAST. — (*See also Iron, Wrought; Castings, Iron and Steel; Magnetic Properties of Iron; Steel.*) Chemically pure iron is not a commercial product. Iron of a very high degree of purity may be obtained, however, by electrolytic deposition. The microscopical constituent of pure iron is called ferrite, and pure iron is said by metallographists to be composed of polyhedral crystalline grains of ferrite. Commercial products are pig iron, cast iron, malleable cast iron, wrought iron and steel.

PIG IRON. — Pig iron is the crude product obtained from iron ore by smelting in a blast furnace. The name is applied either to the molten material or to rough castings varying in length from 30 inches to 36 inches and in cross-section from about 10 sq. in. to 30 sq. in. A mass of piled pig iron 8 by 10 by 12 ft. weighs approximately 100 tons. Pig iron is made by burning a mixture of iron ore, coke or other fuel and limestone in a high furnace through which a blast of heated air is forced under pressure. The liquid iron when drawn from the furnace is either run into sand molds formed on the ground near the furnace, or into cast-iron molds forming a part of a conveyor chain which receives the molten iron, chills it by contact, and delivers it at the car. A pig is the bar of iron formed in either of these types of molds. The molten iron is also occasionally conveyed to the steel furnace without being allowed to cool.

Classification. — Pig iron may be classified as follows:

a. Gray, white or mottled as determined by the color of its fracture and its chemical composition.

b. Coke, anthracite or charcoal iron as determined by the fuel used in reducing the ore.

c. Foundry iron, Bessemer iron, etc., according to the use to which it is to be put.

Use of Pig Iron. — The product of the blast furnace is irregular in composition and physical properties and is not used directly. By forming it into pigs it is possible to grade it, and also transport it to foundries and iron and steel furnaces. Pig iron is sometimes used by engineers because of its great weight to form counterweights in unbalanced structures such as movable bridges, but concrete is more economical for this purpose.

Strength. — See below in the section on *Cast Iron.*

Specifications for Pig Iron. — See Standard Specifications for Foundry Pig Iron in American Society for Testing Materials Standards.

Cost. — The cost of pig iron is an important factor in commerce. Quotations are given weekly in *The Iron Age* and in the first issue of each month of the *Engineering News-Record.* The fluctuations in price are very considerable, even in normal times, Bessemer pig at Pittsburg varying during 1912 from \$14.90 to \$18.15 per long ton of 2240 pounds. Since the World War the increase has been great. Bessemer pig was quoted at \$50.46; Pittsburgh, Aug. 31, 1920, and at \$26.96, May 16, 1922.

CAST IRON. — Cast iron may be defined as iron containing so much carbon that it is not malleable at any temperature, specifically, iron cast into articles of specific form and purpose as distinguished from pig iron. Except for special cases castings are made of gray iron. Cast iron is made from pig iron by melting the latter and casting in sand or iron (chill) molds. The furnace commonly used is a cupola furnace although reverberatory furnaces are sometimes employed.

Composition of Cast Iron. — The composition of an iron casting is identical with that of the pig iron from which it is made and usually consists of metallic

iron (ferrite) accompanied by from 2.5 per cent to 4 per cent (by weight) of carbon, together with silicon, phosphorus, manganese and other impurities. The carbon may be chemically combined with the iron, giving homogeneous white iron, or some of it may be precipitated in cooling in the form of graphite, making gray iron or mottled iron. The influence of carbon upon the properties of the cast iron is of great importance and may be summed up as follows:

Uncombined Carbon or Graphite. — Porosity and workability increase and shrinkage decreases, with an increase in the percentage of graphite. Strength also generally decreases with an increase in graphite, although it should be noted that an iron may be weak by having too much combined carbon and that treatment which changes some of this combined carbon to graphite increases its strength.

Combined Carbon. — In slowly-cooled white iron all the carbon occurs in the form of an alloy, called cementite, which forms in cooling, and the iron partakes of the characteristics of cementite, being hard and brittle, the latter characteristic increasing with an increase in the percentage of the cementite. The carbon in this condition is called combined carbon.

Silicon. — This acts as a precipitant of carbon, drawing it out of combination into graphitic form. The maximum precipitation occurs with from 2.5 per cent to 3.5 per cent of silicon. Beyond this percentage the opposite effect is noted. Increase of silicon up to 3.5 per cent therefore softens the iron, decreases shrinkage, imparts fluidity and reduces the strength. The maximum density of gray iron occurs with about 1 per cent of silicon; more than 2 per cent causes porous iron.

Sulphur. — Sulphur increases the amount of combined carbon, its effect in this direction being far greater than that of silicon in reducing the combined carbon, it being generally considered that 0.01 per cent *S* will neutralize 0.15 per cent Si. The amount of sulphur should be restricted to a very low percentage. In the Standard Specifications of the A.S.T.M. the sulphur content is limited to not over 0.08 per cent for light castings, not over 0.10 per cent for medium castings and not over 0.12 per cent for heavy castings.

Manganese. — This increases the total carbon and the proportion of carbon in combined form, although by combining with sulphur and reducing the effect of the latter, the net result may be a decrease in the combined carbon. It strengthens the iron if below 1 per cent, strengthens but increases brittleness if between 1 per cent and 1.5 per cent, and if over 1.5 per cent decreases strength and toughness and increases hardness and shrinkage.

Phosphorus. — This causes expansion after solidification, therefore making the iron useful for very thin castings. It, however, tends to make the iron weak and brittle and should be kept below the following values:

	Per cent
Chilled castings	0.3
Malleable castings	0.2
Gray castings	0.7

Use of Cast Iron. — Cast iron should in general not be used where tensile or bending strength is required. Its cheapness and high crushing strength make it useful for heavy parts of machinery, and for pieces where intricate patterns not easily made by tools are needed. It is frequently used for water pipes, despite its low tensile strength, and often for columns in buildings, although cast-iron columns have been superseded to a considerable extent by steel columns, except for the simplest forms of construction. Car wheels of cast iron with the outer surface chilled by iron molds are much used.

Compressive Strength. — The ultimate compressive strength of cast iron is from 60,000 to 200,000 lb. per sq. in. A safe working value for cases where column action cannot occur is 16,000 lb. per sq. in. For columns the following formula may be used if the applied load is not eccentric.*

$$\frac{P}{A} = 6100 - 32\frac{l}{d},$$

in which

P = total allowable load in pounds,
A = cross-section area in square inches,
l = length unrestrained against lateral deflection in inches,
d = diameter, or shortest side of rectangular column, in inches.

Tensile Strength. — Ultimate strength may vary from 15,000 to 35,000 lb. per sq. in., but is ordinarily from 18,000 to 22,000 lb. per sq. in. For cast-iron water pipes, it is usual to specify 3,300 lb. per sq. in. or 1/5 of the tensile strength, assuming the latter to be 16,500 lb. per sq. in.

Modulus of Elasticity varies from 10,000,000 to 30,000,000 lb. per sq. in. For ordinary foundry iron the modulus of elasticity may be taken as from 12,000,000 to 15,000,000 lb. per sq. in.

Elastic Limit.— Cast iron has no well-defined elastic limit.

Specifications for Cast Iron. — See *American Society for Testing Materials Standards.*

Cost of Cast Iron. — See *Castings, Iron and Steel.*

MALLEABLE CAST IRON. — This is the name given to cast iron which has had a portion of the combined carbon changed to graphitic carbon in the form of a fine powder by reheating white iron to a temperature somewhat below the melting point. The process used in the United States generally eliminates the carbon entirely from the outer layer. Malleable cast iron is used for parts of agricultural machinery, pipe fittings which have to be threaded, plow shares, etc. It has a much higher tensile strength (from 40,000 to 50,000 lb. per sq. in.) than ordinary cast iron and is more ductile. It is frequently sold as some special form of steel, and should be carefully guarded against by purchasers of small articles such as bevel gears, hammers and automobile drop forgings.

Specifications for Malleable Cast Iron. — See Standard Specifications for Malleable Castings in *American Society for Testing Materials Standards.*

Cost of Malleable Iron Castings. — See *Castings, Iron and Steel.*

BIBLIOGRAPHY. — See *Bibliography* in article on *Steel.*

* See article on *Structures, Simple,* for effect of eccentricity on building columns. It should be noted that the allowable loads on cast-iron columns as given in the building laws of various cities, and summarized in the article on *Buildings, Allowable Unit Stresses in*, are in many instances too high for conservative practice.

IRON, WROUGHT. — (*See also Iron, Pig and Cast; Magnetic Properties of Iron; Steel.*) If commercial iron is mechanically mixed with a suitable amount of slag (*see article on Steel*) there results a malleable material called wrought iron which does not harden when suddenly cooled. It melts at a full white heat, but becomes pasty at a lower temperature, in which condition it can be readily welded. It is ductile when cold.

PROCESS OF MANUFACTURE. — Practically all wrought iron is produced from pig iron by indirect processes although direct processes for production from the ore exist. These indirect processes may be divided into two general classes based upon the type of furnace used: (*a*) reverberatory or puddling furnaces; and (*b*) charcoal hearths. The best iron is made upon hearths, but puddling furnaces produce the larger quantity. The essential difference between the two processes is that in hearths the chief source of oxidation is atmospheric air, and the fuel is burned in contact with the iron, while in puddling furnaces the chief source of oxygen is magnetic oxide of iron, and the fuel is burned in a chamber separate from that containing the iron. A description of these processes follows.

Puddling. — This method consists of melting pig iron in a reverberatory furnace heated either by coal or natural gas. The furnace hearth is lined with oxide of iron. The pig is exposed for about two hours to the continuous action of a flame hot enough to melt it and to remove most of the impurities, but not hot enough to keep pure iron in a molten state. By the action of the flame the molten iron becomes less fusible and finally pasty. After reaching this condition it is puddled by being worked into balls by hand labor. It is then taken from the furnace and squeezed or hammered into blooms, and then rolled into small bars about ¾ inch thick and from 2 to 6 inches wide, called "muck bars." After cooling, these muck bars are cut into short pieces about 2 feet in length, piled into bundles, fastened by iron wire, reheated to welding heat and re-rolled into merchant bars. If the iron is subjected to a second piling, heating and re-rolling, it is called "double refined iron."

Charcoal Hearths. — The following are the more important hearth processes.

1. **Finery Process.** — Charcoal fineries produce "knobbled" iron of a high degree of softness which is much used for boiler tubes.

In the finery process the pig iron is first melted down in a coke or charcoal refinery to remove the silicon, phosphorus, and sulphur, and is then transferred in a molten condition to a charcoal hearth which is still hot from its previous charge. Damp charcoal is thrown in, a low-pressure, unheated blast turned on and the metal agitated to keep it in contact with the blast. After an hour or more the metal is collected into a ball and hammered to remove some of the slag, cut up and reheated in piles. Gray iron may be used in this process.

2. **Walloon Process.** — The Walloon process is used in Sweden for producing wrought iron from Dannemora pig iron, the resulting product being shipped to England, particularly to Sheffield, for conversion into blister steel for use in fine toolmaking.

In the Walloon process long pigs are melted gradually by being pushed forward into a charcoal fire. The molten iron drops through the blast, becoming decarburized, and collects in a pasty mass at the bottom of the furnace. The partially refined iron is then raised to the top of the charcoal fire, and melted down with the addition of rich slag and hammer scale. The metal is then balled, reheated and hammered.

3. Lancashire Process. — The Lancashire process is used principally in Sweden, but is also used in the United States.

The Lancashire process somewhat resembles the Walloon process. Pig iron is melted between two layers of charcoal, the liquid dropping down through the blast and becoming oxidized. The molten metal collects in a pasty mass at the furnace bottom where it is allowed to remain for twenty or twenty-five minutes; it is then mixed with decarburizing slag and remelted in a similar manner. Finally the pasty mass is removed from the hearth and hammered.

Busheled Iron. — "Busheled iron" is made of scrap instead of pig iron. The scrap is heated in a furnace, squeezed and rolled into bars. The resulting product is of inferior quality.

COMPOSITION. — Wrought iron consists of a mass of ferrite interspersed with elongated particles of slag. Commercial shapes of wrought iron, such as plates and rods, are made from piles of muck bars, and have a fibrous character since they consist of a series of welds.

USE. — Wrought iron has been gradually replaced by steel for most structural purposes. Good quality of iron, however, is still in demand where toughness and ductility are necessary, and where welding or other blacksmith work is to be done. Iron of a high degree of purity is sometimes specified for use where a non-corrosive material is needed.

GRADES. — Wrought iron is on sale in the eastern states under the following classification:

Norway (or Swedish Iron). — Best grade; used for fine wrought work and machine work. It is particularly fibrous.

Double Refined or Best Refined. — This is the best domestic iron. It is used for forging, welding or machine work.

Common Iron.— This is the cheapest grade. Used for nails, horseshoes, etc. It does not weld as readily as other grades.

STRENGTH AND ELASTICITY; WEIGHT. — The values stated in the specifications which follow indicate the tensile strength of wrought iron of the grades specified. The following values are from tests made upon merchant iron at the Massachusetts Institute of Technology, and show the variations that may occur between the various grades bought in the market without specifications. The specifications which follow show the strength which may be obtained in material of the grades specified therein.

TENSILE STRENGTH, MODULUS OF ELASTICITY, ELONGATION, ETC., OF WROUGHT IRON

(M.I.T. Tests)

Item	Single Refined	Double Refined	Swedish
Ultimate strength, lb. per sq. in.....	47,000	51,700	40,600
Elastic limit........................	32,000	25,800	20,900
Yield Point........................	35,000		
Modulus of Elasticity................	27,700,000	29,700,000	29,100,000
Per cent of elongation in 10 inches..	....	22.7	26.4
Reduction per cent..................	20.6	37.5	75.0

Compressive Strength. — For all practical purposes this may be taken as the yield point, or from 3000 lb. to 4000 lb. above the elastic limit.

Weight. — Wrought iron weighs 480 lb. per cu. ft. A bar 1 yard in length and 1 sq. in. in cross-section weighs 10 lb.

STANDARD SPECIFICATIONS FOR WROUGHT IRON. — See *American Society for Testing Materials Standards.*

COST. — In the *Iron Age* for May 18, 1922, refined iron bars were quoted at 2.10 cents per pound, f.o.b. Pittsburgh.

BIBLIOGRAPHY. — See *Bibliography* in article on *Steel.*

LAMPS, ARC. — (*See also Arc, Electric; Distribution and Transmission Systems; Lamps, Incandescent; Illumination, Street; Photometric Quantities; Photometry; Rectifiers.*) The theory of the electric arc is discussed in detail in the article on *Arc, Electric*. Arc rectifiers are discussed in the article on *Rectifiers*. This article deals with the use of the electric arc as a source of illumination.

The positive and negative electrodes are termed respectively anode and cathode. The active conducting medium of the arc is supplied by the electro-vaporization of the cathode. The wasting of the anode is due only to its oxidation and can be prevented without interrupting the arc by cooling or by isolation from air. With the exception of the carbon arc, it is more difficult to maintain an arc with alternating than with direct current.

Voltage Drop Across an Arc. — The voltage drop across an arc comprises three elements, (1) a sensibly constant drop at the anode, (2) a similar but much smaller drop at the cathode and (3) a variable drop in the arc stream. The electrode drops depend only on the nature of the electrodes. The drop in the arc stream varies with the current according to Ohm's law, but the relation is greatly complicated by the changes in the resistance of the arc stream. Steinmetz (*Radiation, Light and Illumination, p. 139*) proposes the following general expression for arc conduction in air:

$$E = E_0 + \frac{k\,(l + l_1)}{\sqrt{I}},$$

where E is the total voltage, E_0 the electrode drop, l the arc length in inches, I the current, and k and l_1 constants depending on the cathode material.

For carbon electrodes, $E_0 = 36$, $k = 130$, $l_1 = 0.33$.
For magnetite-copper electrodes, $E_0 = 30$, $k = 123$, $l_1 = 0.05$.

Steadying Resistance. — When the arc is employed in a series circuit its instability is overcome by supplying it with a constant current. Arcs in multiple on constant-potential circuits must each be compensated by a considerable ballast of stable series resistance or reactance; see article on *Arc, Electric*.

Power Factor. — Due to the distortion of the current wave by the varying resistance of the arc, the alternating-current carbon arc has a power factor of about 85 per cent, although the current and voltage pass through their zero values simultaneously; see *Alternating Currents*. Reactive ballast coils used in constant-potential lamps reduce the over-all power factor to values between 60 and 75 per cent.

Sources of Luminosity. — There are three distinct modes of light production by electric arcs, viz., (1) by incandescence of the electrodes due to their high temperature, (2) by the luminescence in the arc of salts derived from mineralized carbon electrodes, and (3) by the luminescence in the arc of the conducting vapors from the cathode. The first is exemplified by the ordinary carbon arc. The second mode is exemplified by the flame arc. The color of the light and its efficiency depend on the nature of the luminescent salts and the temperature attained in the arc. Carbon electrodes heavily mineralized with calcium salts yield a yellow light of high efficiency. Barium salts yield white light and strontium red, but the efficiencies are lower than those attained with yellow light. The third mode is exemplified in the metallic arcs, the chief practical representatives being the magnetite and the mercury arcs. Here the color of the light depends solely on the natural spectrum of the cathode vapor and the efficiency depends on the richness of that spectrum in highly luminous components.

Steadiness of Light. — Light from arc lamps is inferior to that from incandescent lamps in the matter of steadiness. The conditions requisite for

steady light are constant arc length, constant current, fixity of arc position, uniform feed, homogeneous electrodes and complete protection from drafts. When well ballasted the mercury arc in the vacuum tube best meets these conditions. The alternating-current arc displays instantaneous variations in intensity which are very annoying at low frequencies. Arcs should not be operated at less than 40 cycles per second where considerations of hygiene are important. The three-phase arc is free from this defect.

CARBON ARC LAMPS.— Unless used for indirect light the upper electrode of a direct-current carbon arc should be the anode in order to gain the fullest advantage of downward light distribution from the crater. In the alternating-current arc semi-craters of lower temperature are formed on both electrodes, resulting in lower efficiency of light production than in the direct-current arc. The performance of typical carbon arc lamps is given in Table I and Fig. 1.

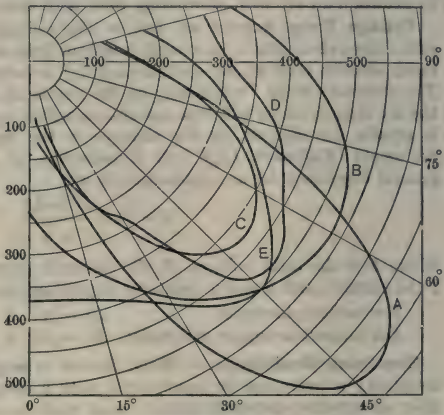

Fig. 1. Light Distribution of Typical Carbon Arcs.

A. 6.6-amp. series D-C. open arc, clear globe.

B. 6.6-amp. series D-C. enclosed arc, opal inner, and clear outer globe.

C. 7.5-amp. series A-C. enclosed arc, opal inner, and clear outer globe, small reflector.

D. 5.5-amp. multiple D-C. enclosed arc, 110 volts, opal inner, clear outer globe.

E. 5-amp. multiple D-C. intensified arc, 110 volts, opal inner, and clear outer globe.

Open Carbon Arcs. — The open arc is characterized by the rapid oxidation of the electrodes due to the free access of air. The anode of a direct-current open arc wastes at a rate between 1 and 2 inches per hour and the cathode at about half the rate of the anode. The electrodes of open alternating-current arcs burn from 1 to 1.5 inches per hour. A life per trim sufficient for all-night operation may be obtained by the use of a single pair of very long, heavy carbons or by providing two sets for successive consumption. The open arc is very sensitive to drafts. It is operated with short arc length, low voltage, usually 45 to 55, and heavy current, usually 6.6 to 10 amperes, to improve the steadiness of the light. The short arc length impedes downward light distribution and the intense crater casts harsh shadows. The erratic travel of the arc about the electrodes makes the light very unsteady. The use of the open arc is now confined to series circuits.

Inclosed Carbon Arcs. — The inclosed carbon arc with closely restricted air supply affords a greatly increased electrode life over the open type. It is also much less liable to extinction by wind, and hence can effectively employ greater arc length, higher voltage and lower current than the open arc. The crater of the inclosed arc is less pronounced, entailing some loss of efficiency compared with the open arc, but the greater length of the arc reduces the interference of the lower carbon with downward distribution. The light is much steadier than

TABLE I. PERFORMANCE OF TYPICAL CARBON ARC LAMPS

Item	Multiple d-c. inclosed	Multiple d-c. inclosed	Intensified d-c. inclosed	Multiple 220-volt d-c. inclosed	Multiple a-c. inclosed
Terminal volts	110	110	110	220	110
Volts at arc	80	80	80	150	72
Amperes	5	6.5	5	3.25	6
Watts	550	715	550	715	430
Power factor	....	....	....	....	0.65
Diameter of electrodes, inches	½	½	Two ¼ One ⅜	½	½
Reflectors and glassware.	Porcelain reflector. Opal inner, no outer globe.	Porcelain reflector. Opal inner, no outer globe.	Porcelain reflector. Opal inner, no outer globe.	Porcelain reflector. Opal inner, no outer globe.	Porcelain reflector. Opal inner, no outer globe.
Life per trim	150 hours	100 hours	75 hours	150 hours	125 hours
Maximum C.P.	400 at 60°	580 at 60°	600 at 35°	240 at 60°	310 at 60°
M.S.C.P.	215	318	225	160	160
Watts per M.S.C.P.	2.56	2.25	2.44	4.44	2.68
M.L.H.C.P.	379	559	414	215	276
Watts per M.L.H.C.P.	1.45	1.28	1.33	3.33	1.56

Item	Multiple a-c. inclosed	Series d-c. open	Series d-c. open	Series d-c. inclosed	Series a-c. inclosed	Series a-c. inclosed
Terminal volts	110	50	50	75	77	77
Volts at arc	72	48	48	73	72	72
Amperes	7.5	6.6	9.6	6.6	6.6	7.5
Watts	540	330	480	495	425	480
Power factor	0.65	.....	.....	......	0.84	0.84
Diameter of electrodes, inches	½	½ 2 pairs	⅝ 2 pairs	½	½	½
Reflectors and glassware.	Porcelain reflector. Opal inner, no outer globe	Clear globe.	Clear globe.	Porcelain enamel reflector, Clear globes.	Porcelain enamel reflector, Clear globes.	Porcelain enamel reflector. Clear globes.
Life per trim	100 hours	18 hours	18 hours	125 hours	125 hours	100 hours
Maximum C.P.	410 at 60°	720 at 45°	1250 at 45°	530 at 60°	250 at 70°	305 at 70°
M.S.C.P.	215	265	460	290	144	173
Watts per M.S.C.P.	2.51	1.25	1.02	1.71	2.95	2.77
M.L.H.C.P.	371	395	690	479	232	291
Watts per M.L.H.C.P.	1.45	0.82	0.71	1.03	1.83	1.65

that from the open arc. It is bluer in color, due to the greater proportion derived from the arc proper. Electrodes of moderate length afford a life per trim of from 80 to 100 hours in alternating-current lamps and of 100 to 150 hours in direct-current lamps. The range of currents in common practice is from 3 to 7.5 amperes.

Intensified Carbon Arc. — The intensified carbon arc, used only on direct-current circuits, employs a short, thick vertical lower carbon as cathode and two long, thin, inclined carbons as anode. The upper carbons converge just above the cathode and feed downward at the rate of consumption without the aid of any regulating mechanism. The lower carbon is controlled by a feeding solenoid and tends to maintain an arc of constant length at a fixed level. The high current density in the positives produces a very intense dual crater which emits light of somewhat whiter quality and of higher efficiency than that of the ordinary inclosed arc. The fixed level of the arc and the absence of crater travel aid in the effective use of reflectors and diffusing globes and in the maintenance of a definite form of light distribution. Under standard conditions of use the 5-ampere arc has a life per trim of from 80 to 100 hours.

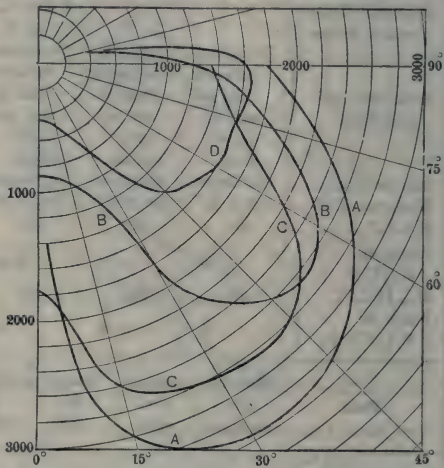

Fig. 2. Light Distribution of Typical Flame Arcs

A. 10-amp., 55-volt, D-C. yellow flame arc, clear globe, inclined electrodes.

B. 10-amp., 63-volt, A-C. yellow flame arc, clear inner, opalescent outer globe, vertical electrodes, enclosed type.

C. 10-amp., 55-volt, D-C. yellow flame arc, opal globe, inclined electrodes, open type.

D. 10-amp., 55-volt. A-C. white flame arc, clear inner and outer globes, vertical electrodes, enclosed type.

FLAME ARC LAMPS. — The flame arc depends for light production on the luminescence of salts impregnated in carbon electrodes. In its electrical characteristics it is essentially a carbon arc. The evolution of this type of lamp has been along two structural lines, one having vertical electrodes, introduced by Blondel, and the other having converging electrodes, introduced by Bremer. In both types both electrodes are fed by the regulating mechanism and the arc is maintained at a fixed level beneath a vitreous canopy or economizer which serves to conserve the heat of the arc, deflect the fumes to the ports at the periphery of the housing, and to assist in the downward reflection of light. Flame arcs may be either open or inclosed. The performance of typical flame arcs is given in Table II and Fig. 2.

Open-flame Arcs. — The open type provides positive ventilation to sweep the fumes of the arc from the arc inclosure and to prevent their deposit on the inclosing globe. The unrestricted air supply causes a rapid wasting of the electrodes. To obtain a practicable life per trim the electrodes are made either

TABLE II. — PERFORMANCE OF TYPICAL FLAME ARCS

Item	Open d-c. inclined carbons	Open a-c. inclined carbons	Open d-c. vertical carbons	Open d-c. vertical carbons
Terminal volts	55	55	110	78
Volts at arc	45	45	75	75
Amperes	12	12	6.5	6.5
Watts	660	500	715	510
Power factor	......	0.75	......	......
Diameter of electrodes inches.	1/4	1/4	1/2	1/2
Kind of electrodes	Impregnated yellow.	Impregnated yellow.	Lower impregnated.	Lower impregnated.
Glassware	Light opal globe.	Light opal globe.	Alba globe.	Alba globe.
Life per trim, hours	17	17	20	30
Maximum C.P.	2150 at 0°	1350 at 0°	2425 at 60°	2350 at 60°
M.L.H.C.P.	1890	1270	2058	2050
Watts per M.L.H.C.P.	0.35	0.39	0.35	0.25

Item	Inclosed d-c. multiple vert. carbons	Inclosed a-c. series vert. carbons	Inclosed a-c. multiple vert. carbons
Terminal volts	110	60	110
Volts at arc	70	55	70
Amperes	6.5	10	7.5
Watts	715	450	510
Power factor	....	0.75	0.62
Diameter of electrodes, inches.	7/8	7/8	7/8
Kind of electrodes	Impregnated yellow.	Impregnated yellow.	Impregnated yellow.
Glassware	Clear inner, opalescent outer globe.	Clear inner, opalescent outer globe.	Clear inner, opalescent outer globe.
Life per trim, hours	100–125	100–125	100–125
Maximum C.P.	....	....	....
M.L.H.C.P.	1740	1595	1600
Watts per M.L.H.C.P.	0.41	0.28	0.32

very long or of very great cross-section. The co-axial arrangement of very long electrodes is obviously impracticable; hence the converging arrangement. The slender electrodes afford additional advantages through the high current density at the arc, absence of arc travel and low heat loss by conduction. A magnetic blow coil is employed to hold the arc in its proper position below the electrode tips. Lamps of this type have a maximum luminous intensity in the downward direction, and hence must be hung very high to produce uniform illumination. The open-flame arc with converging electrodes has a life per trim of from 10 to 15 hours. The open type with vertical electrodes has a life per trim of from 12 to 18 hours, but produces a less steady light than the converging type, due to arc travel.

The ventilating system of open-flame arcs requires very careful design. Unless the fumes are positively removed from the inclosing globe, the accumulation of powder on the globe during the life of a single pair of electrodes may reduce the light transmitted by as much as 40 per cent.

Lamps of the open type are greatly handicapped by the high cost of maintenance and electrodes.

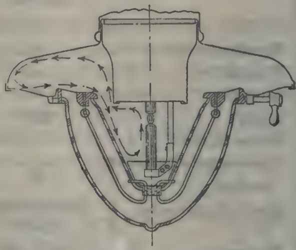

Fig. 3. Method of Condensation of Gas in Inclosed Flame Arc

Inclosed Flame Arcs. — The inclosed type of flame arc is provided with a condensing chamber into which the fumes of the arc are carried by convection, as shown in Fig. 3. The gases are thus cooled and freed from their solids without rapid ingress of air. The life of the electrodes is thus prolonged about ten times, giving a life per trim of from 100 to 150 hours. The electrodes used in inclosed flame arcs are less highly mineralized than those used in open arcs and the gain in life is accompanied by some loss in efficiency. The converging carbon type of flame arc has not been developed in the inclosed form.

MAGNETITE ARC LAMPS. — The magnetite arc depends for light production solely on the luminescence of the conducting vapors produced at the cathode. The cathode consists of a thin iron tube closely packed with a uniform powdered mixture of magnetite, oxide of titanium and oxide of chromium. The positive electrode is a block of copper of large heat-radiating capacity. The magnetite produces an arc of excellent conductivity, but of relatively lean spectrum. The titanium oxide enriches the spectrum of the arc, improving its color and efficiency. The oxide of chromium serves to restrain the rate of vaporization of the active elements and so prolong their life. The resulting light is of excellently balanced white color. The copper anode is not consumed and requires infrequent replacement. The cathode has a life of from 100 to 250 hours depending on the current and arc length. It is not practicable to operate the magnetite arc in an inclosure with limited air access, due to the need of ventilation to remove the copious brown fumes. The light of the magnetite arc is relatively unsteady, a feature which is aggravated by the intermittent method of feeding employed.

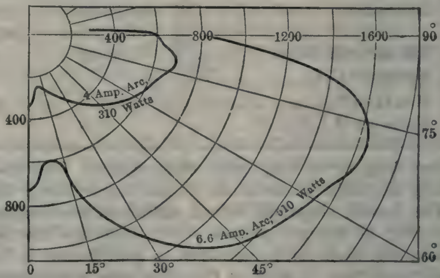

Fig. 4. Light Distribution of Magnetic Arc in Clear Globes

The two commercial types of magnetite arc differ principally in the electrode arrangement. In the General Electric type the positive is the upper; in the Westinghouse type the negative is the upper. The advantages claimed for each type conflict sharply.

Unlike other types of arc lamps the magnetite arc is poorly adapted to interior use on constant potential circuits. Its field is therefore largely confined to street lighting on constant-current circuits. While the magnetite arc is solely a direct-current device, it is readily adapted to use in connection with an

alternating-current supply by the use of the mercury arc rectifier combined with the constant-current transformer; see *Rectifiers*.

The voltage range per lamp on series circuits is from 75 to 80. The standard currents are 4, 5 and 6.6 amperes. The performance of typical magnetite arcs is given in Table III and Fig. 4.

TABLE III. — PERFORMANCE OF TYPICAL MAGNETITE ARCS

Item	Series type	Series type	Multiple type
Terminal volts	78	78	110
Volts at arc	75	75	75
Amperes	4	6.6	6.5
Watts	312	515	715
Glassware	Clear globe.	Clear globe.	Clear globe.
Life per trim, hours	175	125	80
Maximum C.P.	700 at 80°	1650 at 80°	1650 at 80°
M.L.H.C.P.	545	1340	1340
Watts per M.L.H.C.P.	0.59	0.38	0.53

LOSS OF CANDLE POWER OF CARBON AND FLAME ARCS. — The light output of all types of arc lamps falls off gradually during the life of each set of electrodes, due to the accumulation of ash and fumes on the glassware. The amount of this loss varies greatly with the conditions. Matthews (*Nat. Elec. Light Assn., 1901, p. 296*) reported tests of inclosed carbon arcs of which the following results are typical, viz., reduction of light output in 100 hours, best grades of carbons 5 per cent, low-grade carbons 30 per cent. In carbon arcs this loss depends chiefly on the purity of the carbons employed. The loss in flame arcs depends on the scheme of ventilation and the extent to which the carbons are mineralized. Composite results of many tests on inclosed flame arcs reported in *Good Lighting*, Vol. 7, p. 515, show a loss in 100 hours burning of 23.5 per cent of the initial mean lower hemispherical candlepower. Tests on General Electric inclosed white flame arcs made at the Massachusetts Institute of Technology (*Wright and Sprowls, 1912*) showed the average reduction of light in 150 hours burning to be 18 per cent. The loss of light in magnetite arcs is relatively small, due to the positive character of the ventilation. The importance of very careful cleaning of glassware of arc lamps at each trimming is apparent from the above data.

EFFECTS OF DIFFUSING GLOBES. — The intrinsic brilliancy of all arcs is sufficiently high to seriously interfere with good vision when in the direct field of view. Diffusing globes reduce the light by from 25 to 40 per cent through absorption, but generally serve to increase the effectiveness of vision to a degree which fully compensates for the loss of light. Diffusing globes tend to modify light distribution toward greater uniformity at various vertical angles; hence pronounced side-wise distribution must be attained by a properly designed reflector.

REGULATING MECHANISMS FOR CARBON AND FLAME ARCS. — The regulating mechanism of an arc lamp serves the following functions: (1) to strike the arc upon lighting, (2) to feed one or both electrodes at the rate of consumption and (3) to maintain approximately constant the arc length, voltage and current. Feeding systems are of two general classes. In the first the electrodes are fed by gravity upon the releasing of a clutch controlled by solenoids. As the feeding steps are small and frequent the process approaches a uniform continuous feed. In the second type the arc is restruck at each feeding, as in the magnetite arc.

Gravity-feed Mechanism for Multiple Carbon Arcs.—The mechanisms of constant-potential lamps are practically all of the gravity-feed type. The commonest type has a ballast of resistance or reactance in series with a solenoid and with the arc itself; see Fig. 5. When the arc is not lighted the carbons are in contact. When the switch is closed the solenoid is energized and draws its plunger, tightens the clutch and strikes the arc by drawing the carbons apart. As the arc lengthens the current falls until the forces on the plunger are balanced. As the arc lengthens by burning away, the current tends to fall, releasing the clutch and permitting the carbon to feed until checked by the rising of the plunger. In a well-adjusted lamp this process is practically continuous. In alternating-current mechanisms of this type the plungers are laminated and the ballast consists of a reactive coil.

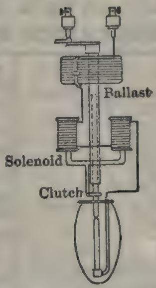

Fig. 5. Multiple Carbon Arc Lamp

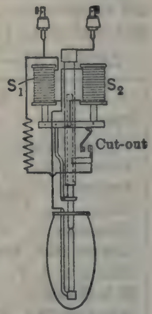

Fig. 6. Series Carbon Arc Lamp

Gravity-feed Mechanism for Series Carbon Arcs.—The commonest type of mechanism for gravity-feed lamps of the series class employs the differential action of two solenoids to control the feeding clutch. Fig. 6 shows this type of mechanism as applied to the inclosed carbon arc. It provides a solenoid S_1 in series with the arc, a solenoid S_2 and resistance coil in parallel with the arc, and a short-circuiting cut-out. The solenoids act differentially on the clutch. The shunt solenoid when strengthened tends to release the clutch, the series solenoid to lift it.

When the lamp is idle the carbons are normally in contact. When current is switched on, the series solenoid is energized and draws out the arc. The cut-out opens, S_2 is energized and tends to oppose the drawing action of S_1, equilibrium being attained when the arc has the proper length. As the carbons burn apart S_2 is strengthened and releases the clutch momentarily, allowing the upper carbon to feed by gravity. If the circuit through the arc is accidentally broken, S_2 alone is energized and the cut-out is immediately closed, preventing the interruption of the circuit. Alternating-current mechanisms have laminated plungers, but differ in no essentials from those in direct-current lamps.

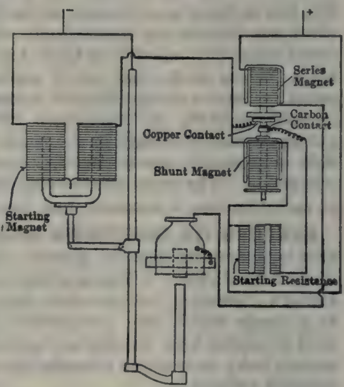

Fig. 7. Mechanism and Circuits of G. E. 4-amp. Arc Lamp

Intermittent Feed Mechanism.—The intermittent type of feeding mechanism for series lamps is shown in Fig. 7. When the current is switched on, the starting magnets are energized, bring the electrodes together and cause the arc to strike. As the electrode burns away the arc gradually increases in length

and potential drop until the shunt coil is sufficiently energized to momentarily short-circuit the arc and cause the striking process to be repeated. As the electrodes come together with considerable force the accumulated slag is broken from the end of the cathode.

Feed Mechanisms for Flame Arcs. — The mechanisms of flame arcs with vertical carbons differ only in details from those of carbon arcs of analogous type, with the exception that the feeding device is dual and acts on both carbons, tending to maintain the arc at a fixed level. Mechanisms for converging carbon flame arcs exist in great variety. The most generally-used type for constant potential operation has a series coil whose plunger acts laterally on one electrode to draw out the arc and gravity feed. In many cases an escapement clutch controlled by a shunt solenoid is used to regulate the rate of feed. In another type the rate of feed is controlled by a metal fin attached to each electrode. The base of this fin rests against a refractory lug. The fin is vaporized away by the heat of the arc at a rate exactly corresponding to the required rate of electrode feed.

MERCURY-VAPOR LAMPS. — (*See also Rectifiers.*) The luminous element of this type of lamp is a luminescent arc, in a highly evacuated tube of glass or quartz, formed between a mercury cathode and an anode of mercury or other metal not attacked by it. The voltage required to sustain such an arc, when the cathode (negative electrode) is mercury, consists of a constant drop at the electrode of about 13 volts and an arc stream voltage which is directly proportional to the length. The arc voltage per inch increases with the vapor pressure. At low vapor pressures the arc is an unstable conductor and the arc voltage per inch falls with a rising current. At high pressures, as in the quartz-tube lamp, the arc voltage rises very rapidly with the current.

The large output of actinic radiation from mercury-arc lamps gives them special advantages in the fields of photography and blue-printing. The large ultra-violet radiation from quartz-tube lamps not inclosed in glass may be utilized in a wide variety of processes for bleaching, sterilization, etc.

Power Factor of Mercury-arc Lamps. — The mercury arc can be sustained at ordinary voltages only when the mercury electrode is the cathode; this principle is utilized in the mercury-arc rectifier (*see Rectifiers*).

To operate a mercury-arc lamp with alternating current the tube is provided with two anodes which are attached to the respective terminals of a reactance coil bridging the line. The cathode is attached to the middle point of this reactance bridge. The power factor of this type of arrangement, including the wave distortion and the lag produced by the reactance coil, is from 50 to 60 per cent. More recent types have power factors from 50 to 85 per cent, depending upon the regulation of the line. In other words, when the line voltage is varying to any great extent, say from 10 to 15 volts from normal, the lower power factor lamp is used. When the line voltage is kept within say 5 volts, the higher power factor lamp can be used as less regulation inductance is then required.

Glass-tube Mercury-arc Lamps. — The glass-tube type of lamp operates at a low vapor pressure and temperature. Its spectrum is that characteristic of the mercury arc which is entirely deficient in red lines and has a great preponderance of yellow, green and violet.

The very low intrinsic brilliancy of this type of lamp renders diffusing glassware unnecessary. Its simple color composition tends to enhance the acuity of vision. For general use the spectrum is perceptibly improved by combining with the arc a parabolic reflector coated with rhodamine enamel, which by fluorescence converts part of the blue and violet radiation incident upon it to red and orange rays.

Methods of Starting. — The mercury arc is started either by tilting the tube until mercury from the cathodes makes metallic contact with the anode and strikes the arc or by breaking down the gap by a high-potential spark discharge. As installed for general lighting the mechanism usually includes an automatic starter operating on one of the two methods referred to. The tilting mechanism is operated by a solenoid which is cut out as the arc is struck. The spark-discharge device is essentially an automatic quick-break switch which opens the circuit of an induction coil so as to cause it to discharge through the tube. The tube is mounted in an inclined or vertical position. The mercury vapor condenses at the top in a condensing chamber and is restored to the cathode by gravity.

Performance of Mercury-arc Lamps. — Performance data on several typical lamps of this class are given in Table IV.

TABLE IV.—CANDLE-POWER CHARACTERISTICS OF COOPER HEWITT MERCURY VAPOR LAMPS

Lamps for Alternating-current Circuits

Rating of lamp in average watts	Voltage	Type	Length of tube in inches	Mean lower hemi-spherical c.p.	Watts per mean lower hemi-spherical c.p.	Total lumens	Lumens per watt
210	100–125	E	35	400	0.53	3,179	15.14
380	100–125	F	50	850	0.45	6,283	16.53
	For Direct-current Circuits						
	Series on						
192	100–125	H	21	300	0.64	2,388	12.43
385	100–125	HH	21	600	0.64	4,712	12.23
385	100–125	K	45	700	0.55	5,529	14.36
220	100–125	L	35	400	0.55	3,142	14.28
385	100–125	P	50	850	0.45	6,283	16.31
770	200–250	PP	50 (2)	1600	0.48	12,400	16.10
	Quartz Lamps for Direct-current Circuits						
726	200–240	Z	4	2400	0.3	18,839	25.96

Life of Glass Tubes. — Tubes fail ultimately through breakage or through deterioration of vacuum. The life per tube varies considerably, but is usually several thousand hours of operation.

Loss of Candle Power with Age. — The light output of glass-tube lamps deteriorates quite rapidly during the early portion of tube life. Tests reported by Harrison (*Trans. Ill. Eng. Soc., Vol. 6, p. 545*) give the following average value on two type *H* tubes operated at 3.3 amperes:

Per cent initial candle power...	100	83	80	77	73	70.5	67
Hours burning..............	0	250	500	1000	2000	3000	4000

Quartz-tube Mercury-arc Lamps. — The quartz-tube lamp operates at a high temperature and pressure. Its luminescent spectrum is richer than that of the low-pressure lamp and has superposed upon it a continuous spectrum of incandescence including all luminous elements. The light of the high-pressure lamp is therefore much nearer white than that of the low-pressure lamp. On account of the high pressure of the arc in the quartz tube the length required for circuits of commercial voltage is very much less than the length of glass-tube arcs. The quartz-tube lamp emits a considerable amount of invisible ultra-violet radiation which is capable of doing serious injury to the eye. Ample protection is afforded by surrounding the tube by an envelope of clear glass. Quartz tubes are nominally indestructible, but may require occasional repumping.

COST OF ARC LAMPS (Pre-war prices). — Open carbon-arc lamps are no longer sold. Magnetite and metallic oxide arc lamps cost from $23 to $27. The first cost of flame-arc lamps is from $33 to $45. Glass-tube mercury-arc lamps cost complete, including reactance coil, from $27 to $35. The renewal cost of short glass tubes is approximately $9, that of long glass tubes $12; the average cost per lamp per year will run around $3 per lamp.

Annual Costs. — The following comparative costs of arc-lamp operation under street lighting conditions for a total of 4000 hours (one year, all night service) are given by Friedman (*Elec. World, Vol. 55, p. 1071*).

ANNUAL COST OF OPERATING ARC LAMPS

Type......... Amperes..........	Open d-c. 9.6	Open d-c. 6.6	Enc. a-c. 7.5	Enc. d-c. 6.6	Enc. a-c. 6.6	Magnetite 4.0	Magnetite 6.6
Electrodes.........	$5.50	$5.50	$1.50	$1.20	$1.20	$1.55	$2.85
Trimming.........	6.00	6.00	2.00	2.00	2.00	1.00	2.00
Repairs...........	2.50	2.50	1.00	1.00	1.00	0.75	0.75
Inner globes.......	......	......	0.45	0.45	0.45	......	......
Outer globes......	0.30	0.30	0.30	0.30	0.30	0.50	0.50
Renewals of station equipment	1.50	1.50	......	1.50	......	2.00	3.00
Energy, 1.5 cts. per kw-hr	50.00	32.70	34.50	42.90	30.30	22.80	38.82
Totals.........	$65.80	$48.50	$39.75	$49.35	$35.25	$28.60	$47.92

The above costs do not include fixed charges. Interest may properly be taken at 6 per cent and depreciation at from 7.5 to 10 per cent.

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LAMPS, INCANDESCENT ELECTRIC. — (*See also Illumination, Laws of; Illumination, Interior; Illumination, Street.*) Incandescence denotes the emission of light by a solid or liquid body due to its temperature elevation. The performance of incandescent bodies is referred to a standard *black body*, or one which, at any temperature, emits the maximum possible intensity of radiation at each wave-length of the spectrum. The percentage of luminous radiation from incandescence is low, but increases markedly with the temperature. Hyde computes the luminous radiation from a black body as 1.9 per cent at 2000° C. Abs., 8.8 per cent at 2500° C., 16.9 per cent at 3000° C., 32.5 per cent at 4000° C., 44 per cent at 5000° C. and a maximum of 50 per cent at 6500° C. Certain materials excel the black body in relative light radiation due to *selectivity* or the relative depression of heat radiation as compared with light.

The efficiency of the incandescent lamp depends on: (a) the temperature which its filament can sustain without an undue rate of decay; (b) the degree of selectivity manifested; and (c) the prevention of thermal leakage by conduction and convection. These conditions require a filament of the highest obtainable melting point and lowest vapor tension, which must be supported by a system of low thermal conductivity. So fully are the two latter conditions met that the thermal leakage of modern lamps is less than 5 per cent.

Electrical and Mechanical Properties of Lamps. — The specific resistance of the filament material should be relatively high to insure favorable dimensions in units of low power and high voltage. A relatively large positive temperature coefficient of resistance is desirable as this tends to lessen the variations in performance caused by voltage fluctuations. The filament should be of great mechanical strength, especially in its resistance to fracture. The material required should be obtainable in ample quantities, readily workable and uniform in its finished state. Experience indicates that alloys are distinctly inferior to pure metals and metalloids in all essential features.

Vacuum. — Incandescent lamps require the highest practicable vacuum for the chemical and thermal protection of the filament. In the manufacture of lamps the first stage of evacuation is mechanically produced. Final exhaustion is produced chemically by burning within the inclosure a small amount of phosphorus compound. During the latter stage the filament is raised to incandescence to drive off residual gases. To prevent the loss of vacuum through the unequal expansion of the glass and the leading-in wires the latter are made of a copper-steel alloy.

TYPES OF LAMPS. — There are described in the following paragraphs the types of incandescent lamps which have become of commercial importance. Of the several types the tungsten filament is the most widely used.

Carbon Lamps. — Carbon vaporizes at about 3900° C. and exists in a variety of forms which vary greatly in other physical properties. *Base carbon*, prepared as lamp filaments by carbonizing squirted threads of dissolved cellulose, is of low conductivity, high vapor tension and of negative temperature coefficient of resistance. *Graphitic carbon*, which is deposited from hydrocarbon vapors on incandescent filaments of base carbon, has a high conductivity, low vapor tension and a small positive temperature coefficient. *Metallized carbon*, prepared from the two preceding types by heat treatment in an electric furnace, has a greatly increased conductivity, a small positive temperature coefficient, low vapor tension and a fair degree of selectivity. The common carbon lamp has a core of base carbon on which a shell of graphitic carbon has been deposited by the process of flashing. The hot resistance of this type is

approximately half the value cold. The chief advantage of this type is its low cost and great ruggedness.

Gem Lamps. — These have filaments of metallized carbon prepared from the two preceding forms by heat treatment in an electric furnace. Such a filament has a greatly increased conductivity, a pronounced positive temperature coefficient, low vapor tension and a fair degree of selectivity. These features give the *gem* lamp a higher efficiency and a better color of light than the carbon lamps. The hot resistance of the gem lamp is 2.6 times its value cold.

Tungsten Lamps. — Tungsten melts at about 3200 deg. C.; has a low vapor tension, its atomic weight being 184; has a conductivity of 6.2 microhms per cm. cube at 25° C. for the hard-drawn variety and 5 microhms for the annealed; has a mean temperature coefficient of + 0.0051 between 0° and 170° C. and is selective in radiation to a valuable degree. The hot resistance of the filament is from 10.8 to 13 times the cold resistance, depending on the type of lamp. Tungsten is exceedingly hard and is worked with difficulty. Early filaments were produced by sintering finely divided tungsten reduced from tungstic oxide; they were extremely fragile and were uncertain in performance.

Drawn wire filaments superseded the former pressed filaments and revolutionized lamp manufacture, simplifying the processes very considerably and reducing the cost. It is possible to draw filaments exactly to size, which increases the practical efficiency by eliminating weak points and also makes it possible for the first time in lamp manufacture to produce all lamps of a lot for a predetermined voltage. Some early difficulties with erratic blackening of bulbs have been eliminated by the introduction of special chemicals known as "getters" in the bulb. The tungsten lamps made by the principal American manufacturers are commonly known as "Mazda B" lamps.

Non-Vacuum Tungsten Filament Lamps. — This type of lamp has a closely coiled helical filament of drawn tungsten wire mounted in a glass chamber filled with nitrogen or other inert gas. The pressure of the gas retards the decay of the filament so that it may be operated with a satisfactory life at a higher temperature than is practicable in a vacuum. The gain in radiant efficiency so obtained is offset in part by the convection of heat from the filament by the gas. When the diameter of the filament is minute there is little or no net gain in efficiency. When the filament is relatively heavy the net efficiency may be doubled. The helical coiling of the filament increases its effective diameter as a radiant and simplifies the problem of its support, for the filament is distinctly soft when incandescent. The gas-filled lamp has an elongated bulb, the upper portion of which serves as a cooling chamber. The walls of this chamber receive the black deposit from the filament, but are so placed that they absorb but little of the useful light. The larger sizes of gas-filled lamps are standard for operation in a pendent position only. If it is desired to operate these at some other angle, they should be ordered for up-right or universal burning, the lamps in such cases being supplied with slightly modified filament supports. Such lamps are much more brilliant than vacuum lamps and should be fully shaded. The light of the gas-filled lamp is decidedly whiter than that of the vacuum tungsten lamp. The gas-filled tungsten filament lamps manufactured by the principal American firms are commercially known as "Mazda C" lamps.

Use of Various Types of Lamps. — The following table shows the relative use of the various types of lamps produced for domestic sale in recent years, as secured from various sources. It will be noted that the inefficient types have been rapidly superseded by the more efficient "Mazda" lamps. For 1919, the total sales of incandescent lamps, excluding miniature lamps, amounted, accord-

ing to Schroeder, to 183,000,000 for domestic use, 170,000,000 of these being of the tungsten filament type.

RELATIVE USE OF VARIOUS TYPES OF LAMPS

Type of lamp	1907	1910	1913	1916	1919
	per cent	per cent	per cent	per cent	per cent
Carbon	93.27	63.08	11.85	16.32	7.0
Gem	5.88	14.88	31.41		
Tantalum	0.75	3.57	.09	.....	.....
Mazda	0.10	18.47	55.65	83.68	93.0

TYPES OF BULBS AND FILAMENTS. — The more common forms of bulbs and filaments are illustrated in Fig. 1. In the bulb designation, such

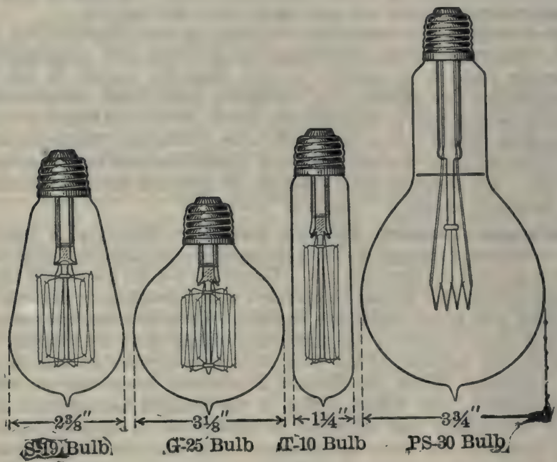

Fig. 1. Bulb Classification (Lamps Illustrated ¼ Scale)

as S-19, the letter indicates the type of bulb, and the number the diameter of the bulb in eighths of an inch. "S" indicates a straight side bulb, "G" a round, or globular, bulb, "T" a tubular bulb, and "PS" a pear-shaped bulb.

Coiled Filaments. — One of the results of the use of ductile tungsten in the manufacture of Mazda lamps, has been the possibility of winding the tungsten wire around a mandrel, thereby producing a helically coiled filament. The first application of this was the so-called "focus" type of lamp, in which the filament was concentrated into a small space, more or less approximating a point source. In this way the automobile and the locomotive headlight lamps became available, also a more effective stereopticon lamp.

The advantages of the concentrated light source in connection with lenses and reflectors is illustrated in the following table, according to Stickney, which shows the maximum beam candle-power obtained with a 16-in. parabolic

reflector with lamps of approximately 100 watts, but with widely varying filament dimensions. In these tests the lamps were focussed so as to give maximum beam candle-power and operated at 100 mean spherical candle-power.

BEAM CANDLE-POWERS (G. H. Stickney)

Mazda lamp used				Light source dimensions mm.		Beam candle power
Volt	Watt	Bulb	Type	Dia.	Length	
6	108	G–30	C headlight	2.0	6.5	462,000
32	100	G–30	C headlight	5.0	5.0	223,000
110	100	G–25	C stereopticon	6.5	6.5	142,000
110	100	G–30	B stereopticon	8.0	8.0	32,600
110	100	PS–25	C regular	25	0.5	12,700
110	100	G–35	B regular	30	68	3,800

The effect of coiling the filament, however, is most pronounced in gas-filled bulbs.

Focus Type Lamps. — Lamps of this type are designed for use with lenses and parabolic reflectors, and are employed with stereopticons, small moving, picture machines, signals, and for spotlighting, floodlighting, headlighting, and the like.

Miniature Lamps. — This term covers a wide variety of small lamps used mostly for special purposes. Such lamps are usually operated at low voltage and are provided with small size bases. Important uses of miniature lamps include decorative schemes like the Christmas tree and in "flashlights."

Colored Lamps. — Daylight Lamps. — For color matching, photography and general decorative purposes, various colored lamps are available. Colors can be produced either by dipping the bulbs or by the use of colored glass in the bulbs. The former is less expensive but the latter is more permanent. A line of lamps known as "Mazda C-2" or "Daylight" is standard. These are of the regular gas-filled construction, but the filament is operated at a higher temperature so as to produce whiter light, and a specially determined blue glass bulb is used. The light from these lamps is approximately daylight in character.

LAMP RATINGS AND PERFORMANCE. — Constant potential lamps are rated primarily at the watts and voltage at which they are designed to operate most economically. Constant current lamps are rated by candle-power and amperes. Secondary ratings are given in lumens per watt and watts per spherical candle-power. From these values the light output in total lumens and the mean spherical candle-power can readily be determined. The candle-power rating of street-series or constant-current lamps is nominal on the basis of 1 candle-power per 10 lumens output; thus, an 80 candle-power lamp is an 800 lumen lamp, while a 600 candle-power lamp is a 6000 lumen lamp.

Daylight Mazda Lamps are made in the following sizes: 75, 100, 150, 200, 300 and 500 watts, these lamps haviug the following lumen ratings respectively: 600, 875, 1400, 2000, 3360 and 5600 lumens.

DATA ON CONSTANT POTENTIAL LAMPS 110 TO 125 VOLTS

Watts	Lumens	Lumens per watt	Watts per spherical candle-power	Watts per horizontal candle-power	Reduction factor	Position of burning
"MAZDA B" LAMPS—REGULAR TYPE						
10	75	7.52	1.67	1.30	0.78	Any
15	125	8.32	1.51	1.18	0.78	Any
25	226	9.04	1.39	1.08	0.78	Any
40	372	9.31	1.35	1.05	0.78	Any
50	480	9.59	1.31	1.02	0.78	Any
60	575	9.59	1.31	1.02	0.78	Any
"MAZDA C" LAMPS—REGULAR TYPE						
50	450	White mazda lamp				Any
75	865	11.53	1.09	....	....	Any
100	1,260	12.57	1.00	....	....	Any
150	2,040	13.66	0.92	....	....	Tip down
200	3,100	15.51	0.81	....	....	Tip down
300	4,840	16.11	0.78	....	....	Tip down
500	8,750	17.45	0.72	....	....	Tip down
750	13,900	18.48	0.68	....	....	Tip down
1000	19,300	19.33	0.65	....	....	Tip down
"MAZDA B" LAMPS—ROUND BULBS						
15	126	8.38	1.50	1.20	0.80	Any
25	220	8.79	1.43	1.14	0.80	Any
25	232	9.31	1.35	1.08	0.80	Any
40	392	9.82	1.28	1.02	0.80	Any
"MAZDA B" LAMPS—TUBULAR BULBS						
25	222	8.91	1.41	1.10	0.78	Any
40	362	....	....	....	....	Any

In addition to the lamps listed in the following tables, a complete line of Mazda B and Mazda C lamps are available for operation on constant potential circuits of 220 to 250 volts. Other lamps are standard for stereopticon service, locomotive and railway headlights, floodlights, train lighting, country home lighting, sign lighting, and similar special service.

Candle-power Distribution. — In earlier practice nearly all incandescent

DATA ON MAZDA STREET SERIES LAMPS

Nominal candle power	Lumens	Lumens per watt	Watts per spherical candle power	Average volts	Average watts	Position of burning
"MAZDA C" LAMPS—4.0 AMPERES—NOT FOR RECTIFIER CIRCUITS						
32	320	10.93	1.15	7.3	29.3	Any
40	400	11.42	1.10	8.7	35.0	Any
60	600	12.57	1.00	11.9	47.8	Any
80	800	13.23	0.95	15.1	60.4	Any
100	1,000	13.96	0.90	17.9	71.6	Any
250	2,500	13.00	0.90	44.7	179.0	Tip down
400	4,000	13.00	0.90	71.5	287.0	Tip down
"MAZDA C" LAMPS—5.5 AMPERES—NOT FOR RECTIFIER CIRCUITS						
32	320	10.47	1.20	5.6	30.5	Any
40	400	11.42	1.10	6.4	35.0	Any
60	600	12.32	1.02	8.9	48.7	Any
80	800	13.37	0.94	10.9	59.9	Any
100	1,000	13.96	0.90	13.0	71.6	Any
250	2,500	15.32	0.82	29.7	163.0	Tip down
400	4,000	15.32	0.82	47.4	261.0	Tip down
"MAZDA C" LAMPS—6.6 AMPERES—NOT FOR RECTIFIER CIRCUITS						
32	320	10.05	1.25	4.8	31.8	Any
40	400	10.93	1.15	5.5	36.6	Any
60	600	12.20	1.03	7.4	49.2	Any
80	800	13.23	0.95	9.2	60.5	Any
100	1,000	13.96	0.90	10.9	71.6	Any
250	2,500	16.11	0.78	23.5	155.0	Tip down
400	4,000	16.11	0.78	37.6	248.0	Tip down
600	6,000	16.11	0.78	56.4	372.0	Tip down
"MAZDA C" LAMPS—7.5 AMPERES—NOT FOR RECTIFIER CIRCUITS						
32	320	9.82	1.28	4.4	32.6	Any
40	400	10.74	1.17	5.0	37.2	Any
60	600	11.86	1.06	6.8	50.6	Any
80	800	13.09	0.96	8.2	61.1	Any
100	1,000	13.96	0.90	9.6	71.6	Any
250	2,500	15.71	0.80	21.2	159.0	Tip down
400	4,000	16.11	0.78	33.1	248.0	Tip down
600	6,000	16.53	0.76	48.4	363.0	Tip down
"MAZDA C" LAMPS—15 AMPERES						
400	4,000	17.95	0.70	14.9	223.0	Tip down
"MADZA C" LAMPS—20 AMPERES						
600	6,000	18.76	0.67	16.0	320.0	Tip down
1000	10,000	19.33	0.65	25.9	517.0	Tip down
1500	15,000	19.95	0.63	37.6	752.0	Tip down

lamps had the same distribution of candle-power, thus causing the mean horizontal candle-power to bear practically a fixed relation to the mean spherical candle-power and to the total light output. In recent practice

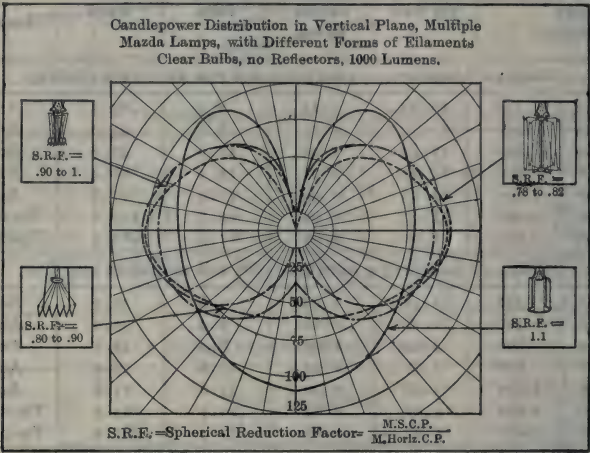

Fig. 2.

various forms of filaments are used so that the mean horizontal candle-power is now no longer a representative measure of the light output of the lamp. This feature is illustrated in the following diagram (Fig. 2), due to G. H. Stickney.

Performance with Voltage Variations. — The very high rate of change of light produced and of filament life with temperature causes incandescent lamps to be extremely sensitive to variations in voltage. A high positive temperature coefficient of resistance, as in the tungsten filament, affords a partial compensation to voltage variations and increases the stability of the lamp's performance. The relations of candle-power, watts, watts per candle and life to voltage are shown for modern types of lamps in Fig. 3. High voltage increases the candle-power and efficiency, but greatly reduces the life of the lamp. Operating conditions are always a compromise between these three factors. Under ordinary conditions circuit voltage and lamp voltage should agree closely. The effect of low voltage on candle-power is marked. Satisfactory service requires that the voltage be maintained within 3 per cent of the rated value for the lamp. The sensitiveness of life to voltage emphasizes the utter futility of life tests with imperfect voltage regulation.

The relations between lamp performance and voltage may be very completely expressed by a series of simple proportions and exponents:

$$\frac{I_1}{I_2} = \left[\frac{V_1}{V_2}\right]^a; \frac{W_1}{W_2} = \left[\frac{V_1}{V_2}\right]^b; \frac{E_1}{E_2} = \left[\frac{V_2}{V_1}\right]^c; \frac{L_1}{L_2} = \left[\frac{V_2}{V_1}\right]^d,$$

where V = voltage, I = candle-power, W = watts, E = watts per candle, L = life.

The values of the exponents in the above expressions, as determined by the Nat. El. Lamp Assoc., are as follows:

	a	b	c	d
Carbon	5.55	2.05	3.51	20.5
Gem	4.80	1.75	3.06	17.6
Tungsten	3.68	1.59	2.10	13.8

These exponents apply to variations within a moderately limited range and it should be kept in mind that an extreme variation in voltage would not follow the same law.

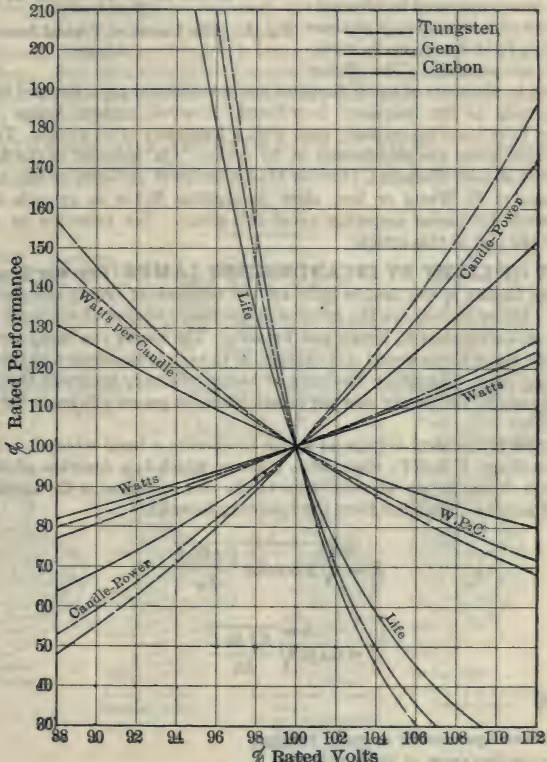

Fig. 3. Characteristic Curves of Incandescent Electric Lamps

Life Performance. — The rated life of incandescent lamps applies to the average of a very large number and not to individual performance. All lamps deteriorate with continued burning in candle-power, efficiency and strength. This decay is partly due to the slow vaporization of the filament and partly to

growing light absorption in a black deposit on the inner side of the bulb. The frosting of bulbs, while not accelerating filament decay, hastens the decline of candle-power and efficiency due to the added absorption by the bulb deposit of internally reflected light and the greater tendency of the lamp to accumulate exterior dirt. The effect of bowl frosting is practically negligible. The life curves of Fig. 4 are typical of present-day lamps.

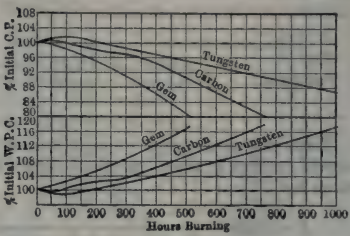

Fig. 4. Life Curves of Typical Incandescent Lamps

Effect of Frequency. — Incandescent lamps operated by alternating current of low frequency produce a flickering light due to cyclic variations of temperature. The flicker is greatest in filaments of small diameter and low thermal capacity, and increases in magnitude as the frequency is reduced. 110-volt carbon lamps of low efficiency have been successfully used with a frequency of 25 cycles. Tungsten street series lamps are satisfactory at 25 cycles. On account of the low heat capacity of slender filaments, 110-volt Mazda lamps of 25 watts or less and 220-volt lamps of 60 Watts or less, show perceptible flicker on 25 cycle circuits. Lamps made for higher amperage avoid this effect. See reference in *Bibliography* at the end of this article.

COST OF LIGHT BY INCANDESCENT LAMPS (Pre-war figures).— The main factors in the cost of light are the renewals of lamps and the cost of energy. Minor factors are the labor of inspection, the cost of renewals and the interest on investment in lamps and fixtures. The longer the lamp remains in service the less is the rewewal cost chargeable to each lumen-hour and the greater the energy cost. Operating the lamp at better efficiency increases the renewal cost and reduces the energy cost per lumen-hour; at poorer efficiency the reverse is true.

The most economical voltage at which to operate a lamp which is rated at a definite voltage V is RV, where R is a factor which is a function of the ratio of the cost of energy per kilowatt-hour to the cost of renewing the lamp. This factor R may be calculated from the following formula.

$$R = \sqrt[12.22]{0.000208 \frac{EID_eL}{D_r}}$$

$$= 0.139 \sqrt[12.22]{\frac{EID_eL}{D_r}}$$

in which

E = watts per candle at rated voltage,
I = candle-power at rated voltage,
D_e = cost of energy per kilowatt-hour in dollars,
D_r = cost of lamp renewal in dollars,
L = life of lamp in hours at rated voltage.

DIFFUSERS FOR INCANDESCENT LAMPS. — Diffusers soften light and reduce its brilliancy by the increase in the area of its apparent source. This is at the expense of considerable absorption (*see table on Absorption of*

Light by Glassware in article on Illumination, Laws of). The chief diffusing media are: ground glass, produced by sand-blasting or acid etching; translucent glass of various types, such as alabaster, opaline, opal, milk, etc., which has in its structure minutely-divided mineral oxides; and prismatic glass. These media are used in frosted bulbs, globes and bowl reflectors partially covering the lamp. The diffusion from ground glass is inferior to that from other media, as the light source is partially visible as a bright spot in high contrast to the surrounding area. Opaline and alabaster transmit a subdued image of the filament but diffuse a considerable portion of the light very effectively. Opal and milk glass are truly diffusing, but may be highly absorbing if the glass is very thick. Recent developments have enabled glass manufacturers to blow this glass very thin, which is a combination that gives excellent diffusion with low absorption. Prismatic globes transmit by refraction innumerable images of small elements of the filament distributed over the entire surface of the globe. When viewed directly the diffusion is imperfect, but to averted vision the effect is excellent. The refracting prisms enable the light distribution to be modified throughout a considerable range. Other diffusing globes tend to equalize light intensity in all directions.

REFLECTORS FOR INCANDESCENT LAMPS.— Reflectors serve primarily to modify light distribution and incidentally to shade the illuminant,

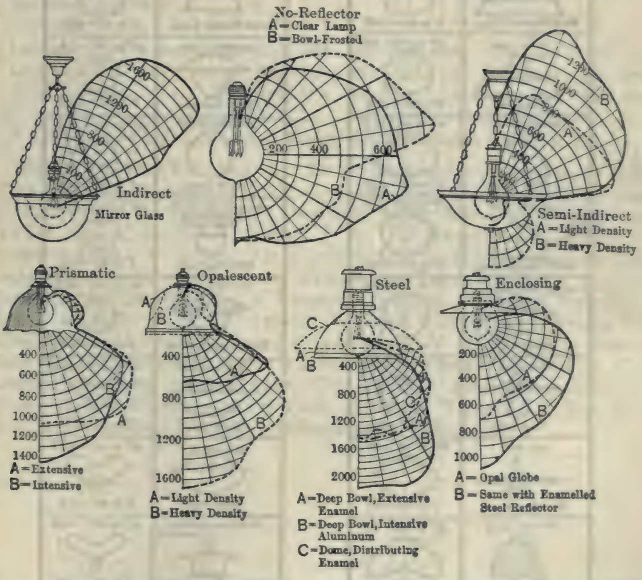

Fig. 5. Candle-power Distribution of Typical Reflector and Globe Equipments—500-watt (Mazda C) Lamps.

(Data furnished by Illum. Eng. Laboratory, Schenectady.)

to diffuse its light partially and to produce artistic effects. The chief reflecting agencies are polished metal, silvered glass, prismatic glass, opal glass, enam-

elled metal and aluminum paint. Polished metals, silvered glass and prismatic glass afford a wide latitude in the distribution of light, depending on the contour and depth of the bowl. Diffusely-reflecting surfaces afford less range of control and are not capable of producing strong downward concentration. Prismatic and opal glass reflectors transmit a moderate amount of diffused light and afford some direct illumination for surfaces not receiving the directed beams from the unit. Practically all reflectors are designed for mounting at a definite position with reference to the luminous center of the filament and produce their normal distribution only in this position. The table below gives data on the mean performance of representative groups of reflectors for incandescent lamps.

The curves shown in Fig. 5 indicate the distribution of light from various forms of equipment when used with Mazda C lamps, according to G. H. Stickney.

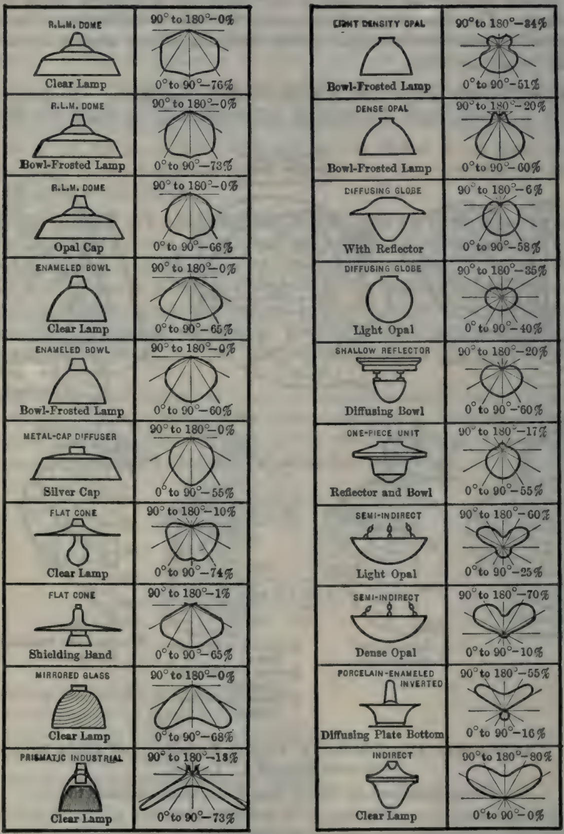

Fig. 6. Examples of Typical Incandescent Lighting Equipment.

In Fig. 6 are shown additional examples of typical incandescent lighting equipment.

MEAN PERFORMANCE OF TYPICAL GROUPS OF REFLECTORS FOR MAZDA B INCANDESCENT ELECTRIC LAMPS *

Item	Clear lamp	Industrial types, metal				Opalescent glass types				
		Flat dome enamel, 5 makes, 7 types	Radially fluted enamel, 2 makes, 2 types	Bowl enameled, 2 makes, 2 types	Bowl aluminized, 4 makes, 5 types	Blown bowl, 6 makes, 9 types	Pressed bowl, 8 makes, 16 types	Blown flared, 5 makes, 7 types	Pressed flared, 7 makes, 9 types	Prismatic bowl, 4 makes, 11 types
Mean lower hemispherical c-p.	70.7	104.0	92.6	81.1	75.5	72.3	73.0	88.3	85.7	80.6
Mean spherical c-p.	67.3	54.0	61.7	40.7	37.8	56.0	57.6	58.5	60.0	58.1
Per cent total flux absorbed	1	20	9	40	44	17	14	13	11	13
Lumens, 0°–60° zone	177	398	287	442	419	303	304	349	295	331
Lumens, 0°–90° zone	444	654	583	510	475	454	458	553	537	506
Total lumens	846	677	774	510	475	703	723	735	755	732

* Table adapted from data by A. L. Powell, *Gen. Elec. Review*, Vol. 15, p. 717.

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LIGHTING PLANTS. — (*See also Distribution and Transmission Systems; Illumination, Interior; Illumination, Street; Lamps, Arc; Lamps, Incandescent; Power Stations; Substations.*) Electric lighting systems divide broadly into two classes, viz., series or constant-current, and parallel or constant-potential. The former system is appropriate where a large number of similar lamps in scattered locations are to be controlled simultaneously and where a high voltage can be safely applied to the circuit, as in street lighting. The parallel system is extremely flexible, is adapted only to low voltages and permits the independent control of single lamps and groups.

More recently there has been a tendency in street lighting to adopt a scheme possessing characteristics of both the series and the constant-potential systems. These combination systems have been termed "constant-potential series" systems by Dr. Steinmetz in their relation to street lighting distribution problems.

LOAD CONDITIONS. — The load of a lighting plant varies greatly with the season and with the hours of the day. There is usually a sharply-defined peak load in the early evening which reaches its annual maximum in midwinter. Storms attended by darkness often give rise to sudden and heavy increases of load. Such conditions demand generating equipment of great flexibility in operation. Sufficient generating capacity must be available to carry the maximum peak load and to provide an ample reserve against accidents. The brief duration of the peak load in many plants makes desirable the selection of types of equipment having large temporary overload capacity in order to decrease the reserve needed. Typical load curves are shown in Figs. 1 and 2.

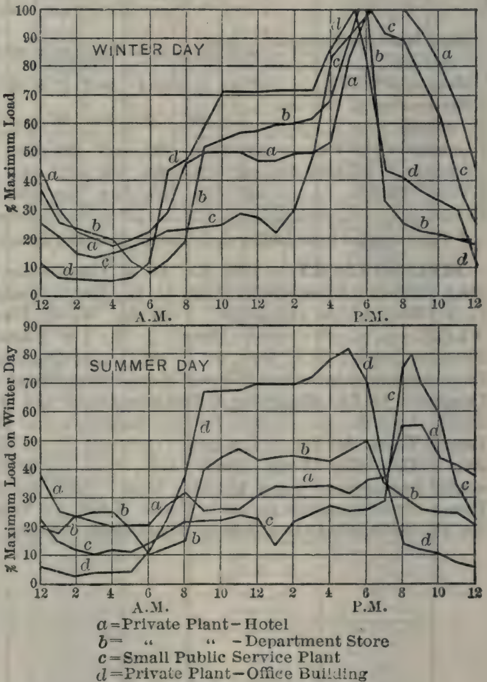

Fig. 1. Load Curves of Lighting Plants

Load Factor and Diversity Factor.— (*See also Standardization Rules of the A.I.E.E.*) The *load factor* of a machine, plant or system is the ratio of the average power to the maximum power during a certain period of time. The *diversity factor* is the ratio of the sum of the maximum power demands of the subdivisions of any system or part of a system

to the maximum demand of the whole system or the part of the system under consideration measured at the point of supply. The investment in a lighting system is largely determined by the peak load to be provided for. The annual charges on this investment are practically independent of the output; hence the investment expense per kilowatt-hour varies in nearly inverse ratio to the annual load factor. The average annual load factors of lighting installations, of various types, as given by E. W. Lloyd (*Nat. El. Lt. Assoc., 1909, Vol. 2, p. 586*) range from 5 to 29 per cent, with the general average of complete systems less than 20 per cent. The economic advantages of high load factor can be secured in lighting service only by combining it with loads having non-coincident peaks, as commercial power and railway systems. Fig. 2 shows typical load curves and load factors for a typical winter and a typical summer day of (*a*) a city system of lighting and power, (*b*) a city street and elevated railway system and (*c*) the combined load of (*a*) and (*b*). (*Trans. A.I.E.E., 1912, Vol. 31, p. 240.*) The advantage of the diversity factor of these loads is apparent.

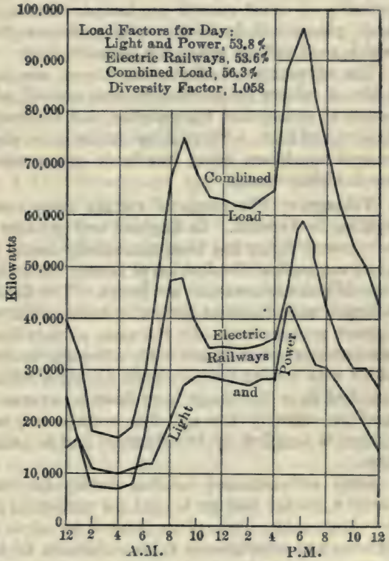

Fig. 2. Load Curves of Composite System on Winter Day. Daily Load Factor: Light and Power 53.8%, Electric Railways, 53.6%, Combined Load 56.3%, Diversity Factor, 1.058

In a general sense an appropriate scale of charges for electric energy should be graded according to the degree to which the load contributes to the system's peak as well as according to the energy consumed. Off-peak loads may appropriately receive low rates. The gain in load factor from diversity often makes it possible for a lighting station to sell energy at wholesale to a railroad system at a cost less than that at which the latter system could generate it independently.

Diversity factors exist between the various subdivisions of a lighting system and have an important influence on the rated capacity and investment required in the several elements of a system. H. B. Gear (*Trans. A.I.E.E., 1910, Vol. 29, p. 375*) reports the diversity factors of various classes of service and branches of the system of the Commonwealth Edison Company of Chicago to be:

DIVERSITY FACTORS OF A LIGHTING SYSTEM

Subdivision of the System	Residence lighting	Commercial power	Scattered power	Large users
Substation to feeders	1.15	1.15	1.15	1.15
Feeders to transformers	1.8	1.25	2.0	1.25
Transformers to meters	3.0	1.6	1.1	1.0
Total diversity factor	6.20	2.30	2.53	1.44

Although these data are ten years old, they represent present conditions quite accurately. For a rediscussion of the question, see an article by H. B. Gear on "Increasing Demand Factor of Residence Consumers" on page 9 of the *Electrical World* for July 6, 1918.

DIRECT- VS. ALTERNATING-CURRENT SYSTEMS. — (*See also Distribution and Transmission Systems.*) Direct current has the following advantages for lighting: (1) safety, since the lines are in no way associated with high-voltage conductors; (2) freedom from power factor, reactance and skin effect, which results in superior voltage regulation in heavily loaded low-voltage circuits; (3) the direct availability of the storage battery as a reserve and a load regulator; (4) the self-exciting and self-regulating features of direct-current generators; (5) the superiority of direct-current motors for adjustable speed service and for the operation of elevators and cranes; and (6) the marked superiority of direct-current arc lamps. Direct current is generally used in isolated plants and in congested city districts because of the greater ease with which good voltage regulation is maintained.

The advantages of alternating current are chiefly those incidental to its flexibility of voltage transformation and control, which makes possible an extended range of economical transmission and the independent regulation of separate feeders and lines. Alternating-current generators are less expensive than direct-current machines, being free from commutator limitations and adaptable to much higher speeds.

Voltage. — The range of voltage from 100 to 125 is standard for electric lighting in America. In England and to some extent on the continent the 200- to 250-volt range has been extensively used. The higher range lends itself to more economical distribution of power, the lower range to the superior construction of incandescent and arc lamps. The Edison 3-wire system of distribution is largely used in direct-current systems. The voltage generated for alternating-current systems is usually the same as that used for primary distribution when the latter is below 15,000 volts, although steam turbo-alternators are often provided with raising compensators having a ratio of 1 to 2 when the line voltage supplied is in the range from 6000 to 15,000. In most systems of small and medium capacity the a-c. line voltage is 2300. The secondary distribution system is supplied by transformers and is usually at 110 to 115 volts between lines.

Small self-contained lighting outfits for train-lighting and similar service where a storage battery is used for regulating purposes are usually designed for a voltage range from 25 to 35 or from 50 to 65, and use lamps of the train-lighting and compensator types. See also *Lighting of Trains by Electricity.*

Phases. — The polyphase system affords distinct advantages in the first cost and voltage regulation of both generating equipment and lines. Primary distribution in all modern a-c. systems is either 2-phase or 3-phase, the latter predominating largely. Secondary distribution from transformers is either single-phase, 2-wire or 3-wire; 2-phase, 3-wire or 4-wire; or 3-phase, 3-wire or 4-wire. 6-phase connections are used only as a link between high voltage systemes and synchronous converters. See also *Converters, Synchronous; Transformers.*

Frequency. — The standard lighting frequency in America is 60 cycles, and that in Europe 50 cycles. A frequency of 25 cycles has many advantages for overhead transmission and for conversion to direct current by synchronous converters. Power transmitted at 25 cycles is frequently converted to 60 cycles for lighting purposes by the use of frequency changing motor generators. In a few cases where a large load of induction motors is associated with a lighting system the compromise frequency of 40 cycles is employed.

Although the performance of electric lamps for interior lighting is not entirely satisfactory on 25 cycles, due to flicker in the light, certain industrial lighting systems have been operated with a fair degree of satisfaction on this frequency. It is obvious that a higher frequency for interior lighting is desirable.

PRIME MOVERS. — *(See also Gas Engines; Steam Engines; Steam Turbines; Water Wheels.)* Prime movers for lighting plants should provide approximately constant speed without variation in angular velocity during each revolution. The steam turbine has a great advantage over the reciprocating engine for large alternators, due to its lower first cost and superior operating economy. It has marked incidental advantages due to lighter foundations, smaller space requirements, elimination of fly-wheel, ease of attendance, and lower cost of maintenance. The piston engine is preferred for the driving of direct-current generators, because of its more appropriate speed.

Internal combustion engines surpass steam power in thermal efficiency, but are handicapped by higher first cost and small range of overload capacity. Their net advantage over steam power may be large when the cost of coal is very high, when a supply of natural or by-product gas is available at low cost and when the load factor of the system is unusually good. Internal combustion engines do not compete successfully with steam units in plants furnishing both heating and lighting service where a supply of exhaust steam has large value.

The crude-oil engine involves a high investment and heavy maintenance expense, but is in other respects an excellent type of motive power where oil is a more economical fuel than coal. Gas from a public source of supply and gasoline are used to a large extent in very small lighting plants to operate internal combustion engines. They are relatively expensive fuels, but may afford a saving by displacing a gas-producing plant on the premises. Gas-engines in lighting plants should be controlled by close-regulating governors and should be supplied with fly-wheels of considerable inertia.

Waterwheels of all types are well adapted to the driving of lighting generators if controlled by close-regulating governors.

Overload Capacities. — Steam prime movers are capable of carrying continuous overload capacities of 50 per cent or more whereas gas engines and waterwheels are very limited in this respect. A large overload capacity enables a plant to carry a smaller reserve equipment for emergencies. Current practice favors the use of a sufficient boiler or gas producer capacity to carry the average load with high efficiency and resort is had to forcing during peaks of short duration. In the average of a large number of modern stations of medium capacity 0.4 boiler horse-power is provided for each kilowatt of generating capacity.

GENERATORS. — *(See also separate articles on Generators.)* Direct-current generators are usually compound-wound and are operated at the voltage of the distributing system. A heavy direct current can usually be more economically secured by the conversion of alternating current than by direct generation. This method is an economic necessity when the direct-current load is more than one mile distant from the station. Edison 3-wire systems may be supplied by sets of two generators in series, by special 3-wire generators or by standard 2-wire generators operated in conjunction with voltage balancing sets.

Polyphase alternating-current generators are chosen for low cost and good regulation. Small low-speed alternators are usually provided with individual exciters, often on the same shaft, but large alternators are supplied with exciting current from a central system comprising several exciters in parallel.

Constant-current d-c. generators are now practically obsolete. Power for series circuits is now most economically obtained from constant-potential a-c. generating sources through constant-current transformers. Direct current for

series circuits is obtained by operating the mercury arc rectifier in series with the secondary circuit of the constant-current transformer.

DISTRIBUTION. — (*See also Distribution and Transmission Systems.*) Distribution systems are designed with a view to the greatest possible reliability of service and the closest regulation of voltage consistent with reasonable investment. The 3-wire system is standard for local d-c. service. An interconnected network of mains is used in extensive 3-wire systems and is fed at many points by feeders from stations or substations. Alternating current is distributed at 2300 volts or more, 3-phase, to transformers, from which secondary distribution is made to consumers by 2-wire and 3-wire systems. To obtain good voltage regulation on polyphase systems it is necessary to maintain a close balance of loads.

SUBSTATIONS. — (*See also Substations, Lighting.*) Substations are used for the following purposes: (*a*) to receive alternating current from high voltage transmission lines or feeders and reduce it in voltage for local distribution; (*b*) to convert alternating current from feeders into direct current for use in the adjacent territory; (*c*) to convert alternating current received from feeders at 25 cycles to 60 cycles for distribution by local lighting circuits; (*d*) transform alternating current from constant potential sources to constant current, either direct or alternating, for the supply of arc lighting circuits; (*e*) to house storage batteries used as load regulators or reserves on d-c. systems; and (*f*) to house feeder regulators for the control of voltage on lighting circuits. Conversion to direct current is accomplished either by motor-generator sets or by synchronous converters. Motor-generator sets may be either of the synchronous or the induction type and are usually operated without step-down transformers. Synchronous converters are usually 3-phase or 6-phase and are always associated with voltage-lowering transformers. Converters may be of the split-pole type, the synchronous booster type or the compound type to provide automatic control of d-c. voltage, or may be operated in conjunction with voltage regulators similar to those used on a-c. feeders.

Motor Generators versus Synchronous Converters. — (*See also Motor Generators; Converters, Synchronous.*) Motor-generator equipment is more expensive and less efficient than converter equipment, but occupies less space,* affords more synchronous condenser capacity for power factor correction and is more reliable in its operation, especially at an a-c. frequency of 60 cycles. Synchronous converters are used almost exclusively for 25-cycle conversion and are gaining in use at 60 cycles. Each converter is usually supplied with its own bank of lowering transformers. Converters of larger capacity than 500 kw. are usually operated 6-phase. Converters supplying 3-wire d-c. systems are adjusted to give from 220 to 250 volts, d-c., and the neutral wire is derived from the neutral connection of the transformers.

Storage Batteries. — (*See also articles on Batteries, Storage.*) Storage batteries are extensively used in stations and substations as a reserve against the failure of the primary source of direct current. It is difficult to operate the battery as a load regulator on constant-voltage systems, especially by a floating connection. A booster or end-cell arrangement is required to compensate for the sloping volt-ampere characteristic. The substation battery commonly floats on the bus bars fully charged and assumes the load if the voltage of the bus bars falls a certain amount. The battery falls in voltage as its discharge proceeds and additional cells must be added in series to sustain the line voltage. The end-cell switch for this purpose may be either hand-operated or automatic. If automatic, it is controlled by a relay switch operated by a solenoid connected across the main bus bars.

* This comparison is on the basis of converter and transformers versus motor-generator set without transformers.

PROTECTIVE EQUIPMENT.—(*See also Switchgear Equipment for Power Stations; Lightning Protectors.*) As lighting systems are required to render practically uninterrupted service the protective appliances are intended to disconnect apparatus and circuits only in extreme emergencies and then only those in serious danger. Generators and exciters are not as a rule provided with automatic circuit breakers. Radial feeders and transmission lines have circuit breakers which open with extreme overloads, often with inverse time elements. Feeders and transformers operated in parallel at both ends are provided with selective relays which open the circuit breakers at both ends in case of trouble, and so prevent the disturbance of other circuits.

VOLTAGE REGULATION. — Aside from the condition in which the illuminants are installed and maintained, the quality of lighting service is almost solely a matter of continuity of service and close regulation of voltage. The best service standards require that the variation of voltage at the lamps shall at no time exceed 2 per cent on either side of the mean. Service with less than 3.5 per cent variation may be considered fair and with more than 5 per cent decidedly poor.

In direct-current systems fairly constant voltage at the loads may be obtained by the use of generators properly over-compounded. A much more sensitive method of control is afforded by the Tirrill regulator (*see Regulators*), which automatically corrects all but the most transient voltage fluctuations of the generators. Synchronous converters have fixed ratios of voltage conversion, but may be regulated to provide approximately constant voltage at load centers by methods cited above in the paragraph on *Substations*. A plan of voltage regulation commonly used in 3-wire networks is to provide in the station or substation two sets of bus bars differing somewhat in voltage. All feeders terminate in selector switches, by which they may be connected with either set of bus bars as desired. The more heavily loaded feeders are connected to the higher bus, the others to the lower bus, with the result of nearly uniform voltage throughout the network of mains.

The following voltage-control methods are employed in a-c. systems: (1) to regulate all generators by hand or by Tirrill regulators to constant voltage or to a voltage rising in proportion to the load; (2) to provide not only approximate regulation of generators but to equip each outgoing line with feeder regulators of the switch or induction type, to be automatically controlled if desired, so that each feeder is caused to supply approximately constant voltage at its center of load. The latter system is the more flexible and economical for extensive distribution systems, since each feeder is independently regulated and may be proportioned in cross-section with regard to economy rather than inherent voltage regulation. Single-phase regulators are preferred for close control if the loads are subject to unbalancing.

As checks on voltage regulation at load centers voltmeters should be provided at stations or substations to indicate conditions at these centers. The voltmeters may be connected to the feeders at the distributing centers by small pressure leads or to the feeders at the station through compensators which allow for the voltage drop in the feeders. When a power load of low power factor is associated with an a-c. lighting system, voltage regulation and economy in transmission can often be assisted by the use of the synchronous condenser to compensate the lagging current; see *Converters, Synchronous*.

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LIGHTING OF TRAINS BY ELECTRICITY. — Electricity is now used almost exclusively by the railroads of the United States for the lighting of trains.

At the present time there are over 20,000 cars lighted by electricity and gas. The gas cars are being converted to electricity as quickly as conditions permit.

Systems Used. — The following systems are used for the purpose of electric lighting:

a. Axle Generator System.
b. Head End System.
c. Straight Storage System.

Axle Generator System. — This method of lighting is the most popular. The principal parts of the axle generator system are:

(1) *An axle generator* mounted either on the truck or under the car body and driven by means of a belt from a pulley on the truck axle.

The generator is usually wound for 40 volts and delivers current to 16 cells of lead storage battery through suitable regulating devices. The capacity of the generators vary from 1 kw. to 4 kw., depending upon the service requirements. A 1 kw. machine is found to be of sufficient capacity for a baggage car. An ordinary coach requires a 2 kw. machine while the additional load on a dining car often requires a generator of 4 kw. capacity.

(2) *A storage battery*, usually of 16 cells, of from 100 to 300 amperes hours capacity.

The battery is installed in a suitable ventilated box hung under the car body. The amount of charge is regulated by an automatic device mounted on the panel board inside the car. When the car reaches a predetermined speed an automatic switch cuts in and connects the generator to the battery and automatically disconnects the generator from the battery when the car speed falls below a predetermined limit.

(3) *A regulator panel* on which are mounted the apparatus controlling the output of the generator and a lamp regulator which holds a constant voltage on the lamps.

(4) *A drive* which consists of a belt connecting properly proportioned pulleys on the generator and on the car axle.

The development of a suitable shaft drive is at present occupying the minds of axle generator manufacturers. This would obviate the use of belts and eliminate one of the sources of trouble especially in winter weather.

(5) *A suspension.* — The former general practice of suspending the axle generator from the truck is being rapidly superseded by the body suspension.

By mounting the generator under the car body a much lighter form of suspension can be used. The machine is also better protected from shock due to rough track, crossing switches, etc., than when mounted on the truck.

Head End System. — The so-called Head End Systems may be divided into three classes, viz.:

a. A Straight Generator System.
b. A Steam Generator Set with Batteries.
c. An Axle Driven Generator Set with Batteries.

(*a*) *The straight generator system* consists of a direct connected engine, set usually installed in a baggage car next to the locomotive, steam being furnished through a reducing valve at 90 pounds pressure. Current is furnished at 64 volts. A loop system of wiring is used in order to give uniform lamp voltage. This system is applicable only when trains run solid between terminals. The

principal disadvantage of this system is that the train is without light as soon as the locomotive is disconnected, or the set becomes inoperative.

(b) *The steam generator set with storage batteries* is used to good advantage on roads with long runs and solid trains.

The set is generally of the turbo generator type—80 volt and a capacity of 35 kw.

The usual practice is to have three or four cars in the train each equipped with a 64 volt, 300 ampere hour battery. The batteries ensure the lighting of all cars in the train at all times.

(5) *The axle driven generator set with batteries* has a generator mounted in the baggage car and driven by means of a chain drive through the car floor from the truck axle.

The generator is 80 volts, 20 kw. capacity. The battery equipment and other apparatus is the same as generally used in the axle generator system.

Straight Storage System. — This is the most simple method of lighting by electricity.

Each car is equipped with one or two sets of 16 cell storage batteries suspended under the car body and arranged for convenient charging at terminal yards.

This system requires elaborate charging facilities at terminals where it is necessary to "place" cars and either hold them long enough to give the battery proper time to charge or to have spare batteries fully charged to replace the discharged ones.

At first sight this system appears inexpensive to operate. However, when the maintenance and operation, together with fixed charges of the charging plant are considered, it is perhaps the most expensive method of lighting cars.

Car Lighting. — Considerable study has been given to car illumination during the past few years. When cars were first lighted electrically no shades were used to conceal the glare of the lamp filament from the eye. Now great care is taken to protect against objectionable glare and give as far as possible uniform illumination on the reading plane.

Lamps. — Depending upon the style of fixtures employed, and their location in the car, lamps range from 15 to 100 watts. The Mazda B type of lamp is used almost exclusively.

The standard voltages in use are:

Straight storage and head end systems	60–64 volt
Axle generator system	30–32 volt

There have been great improvements in the manufacture of lamps since the introduction of electricity for car lighting. The old type carbon filament operating at from 3.5 to 4 watts per candle power has been replaced by Mazda lamps operating at approximately 1 watt per candle power.

Fixtures. — As in ordinary lighting three standard types of fixtures are used: viz.: direct, semi-indirect, and indirect. Fixtures are in some case suspended from the center deck while in other cases side deck mounting is preferred.

The Post Office Department of the U. S. Government after many exhaustive tests has issued a rigid specification for the lighting of railway mail cars. This specification covers the location of light units, limits for illuminating values, emergency lighting requirements, etc.

General. — From an operating standpoint the axle generator system is more flexible than the others. A car so equipped may be operated on any division independent of charging facilities at the division terminal points or train line connections between cars; it may also be cut out of one train at a junction

point for connection to another train without special arrangements being made to supply light from an outside source.

COSTS. — The approximate first costs of the various sizes of axle generator equipments are shown below together with the approximate average maintenance costs. In this connection it should be noted that the costs given are present day (1921) figures and may be subject to considerable modification due to variation in local and market conditions.

First Costs:

Type of car	Kw. rating of generator	First cost including battery
Baggage	1	$900
Coach	2	1200
Diner	4	1900

Maintenance Costs:

Type of car	Per 1000 miles	Per car per month
Average	$4.00–$5.00	$17.00–$22.00

The above figures do not include interest or depreciation on investment for equipment or maintenance facilities.

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LIGHTNING PROTECTORS: GROUND WIRES, ARRESTERS, CHOKE COILS. — (*See also Distribution Lines; Ground Connections; Power Stations; Substations; Switch Gear Equipment for Power Stations; Transmission Lines.*) Abnormal rises of voltage in transmission and distribution circuits may be set up either by lightning discharges or by electrical oscillations caused by switching loads on and off the lines. The latter type of disturbances have been called "internal lightning." High-frequency oscillations are usually of small power and are generally called "statics." The term "surge" is used for any kind of oscillation.

A "ground wire" is a grounded wire which is run above and parallel to the main wires of an aërial line. A "lightning arrester" is any device shunted between wires or from wire to ground, which device under normal voltage permits practically no current to flow, but which becomes a fairly good conductor, usually by the formation of an arc, when the voltage rises a given amount above normal. A "choke coil" is a coil of wire of comparatively low resistance and small inductance, which is placed in series with the line. The coil has a very small impedance to normal line frequencies, but a high impedance to high-frequency oscillations.

GROUND WIRES. — A line completely inclosed in a grounded metallic sheath, as, for example, a lead-covered cable, is completely protected from induced electrostatic charges (*see Electricity and Magnetism, Principles of*). The ground wire used over aërial transmission lines acts as a partial screen and is one of the best means of protecting aërial lines against lightning. The protection is not perfect, however, since the wire forms only a partial screen.

Where systems operate with a thoroughly grounded neutral this neutral wire can form the overhead grounded conductor and the system becomes practically a three-phase four-wire system allowing the use of single-phase transformers of 58 per cent normal line voltage to be connected between any phase wire and the neutral. See articles on *Distribution Circuits; Transmission Lines.*

LIGHTNING ARRESTERS. — A lightning arrester consists essentially of a spark gap so set that excess voltage will cause the gap to arc over, allowing the charge due to this voltage to pass to ground. There is combined with the gap some means of suppressing the power arc which follows and which tends to continue after the abnormal voltage has ceased.

An ideal lightning arrester should take no current at the ordinary operating potential, but at any potential much higher than ordinary there should pass enough current to limit the abnormal potential to some fixed safe value. When the abnormal potential ceases the arrester should stop taking current from the line. The closer an arrester approaches these ideal conditions the better the arrester.

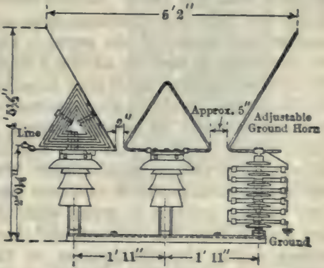

Fig. 1. Horn-gap Arrester

This arrester shown consists essentially of two horn gaps in series with a resistance to ground. One side of one of the horn gaps is made up into the form of a traingular choke coil and the sharp bend in this coil increases the tendency of the lightning to jump across the first air gap rather than to pass around the choke coil and thence to the apparatus to be protected. For 70,000 volt ser-

vice there is an air gap of approximately two inches between the horn formed by the choke coil and the intermediate horn, and a further air gap of approximately five inches between the intermediate horn and the horn connecting through the resistors to the ground. For lower voltages the intermediate horn is omitted and the distance between the knees of the horn depends largely upon the voltage between line and ground, and has been based principally on actual test. It varies between about 1/16 inch to 3/32 inch for 2200 volts to about 6½ to 10 inches for 110,000 volts.

The resistor in the ground circuit can be made either in the form of a resistance rod or in the form of a water resistor, the latter being the type usually employed in Europe. These water resistors usually are made of earthenware tubes containing a mixture of about 70 per cent glycerin and 30 per cent water, and the resistance is usually selected to allow approximately 1 ampere to flow to ground at the time of a lightning discharge.

The operation of the horn gap is based on the fact that a short-circuit once started at the base travels upward due to the heated gases and to the force exerted on the arc by the magnetic field of the current until the arc is ruptured by attenuation. On circuits of high voltage this rupture sometimes takes a second or two, but seems to act with but little disturbance of the line. The angle between the horizontal and the straight portion of the knee ranges from 55 to 60 degrees. The curvature of the knee should have a radius of from 3 to 5 inches.

Multipath Arresters have been developed for a-c. and d-c. service for voltages not exceeding 1000 by the use of a carborundum block fastened between two terminal plates, thus allowing the static discharge to spread itself over a number of minute discharge paths. The normal voltage between the line and the ground is divided into so many minute gaps that the voltage across each gap is too small to maintain an arc after the discharge has passed.

Nonarcing Arresters (Fig. 2) based on the discovery of "nonarcing metal" by Mr. A. J. Wurts, formed the first successful high-voltage arresters. The peculiar property of this metal is that an alternating current will not maintain an arc between adjacent cylinders of the metal provided the voltage is not too high and that the power current which follows the lightning discharge does not vaporize too much of the metal. The first condition is met by having a fairly large number of very small gaps in series, and the second condition gives no trouble where the amount of power current is comparatively small as was the case on the early high-voltage installations. For large amonuts of power it becomes necessary to use resistors in series with the spark gaps to limit the current, and these resistors reduce the effectiveness of the arresters. For very hight voltages different schemes are used to reduce the number of gaps required. It has been found that by shunting a certain number of these gaps by a non-inductive resistor the effectiveness of the arrester is increased.

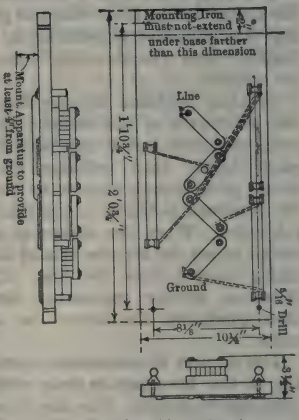

Fig. 2. Nonarching Lightning Arrester

Fig. 2 shows an arrester of this design intended for service on 6600-volt lines where the capacity does not exceed 2000 kv-a. The nonarcing cylinders are

held between porcelain insulators in such a way that there is an air gap of about 1/32 inch between adjacent cylinders in each one of the four sets of seven cylinders. The marble slab forming the base of the arrester also has mounted on it three graphite resistance rods shunting some of the gaps. Modifications of this scheme were used for the "low equivalent," "multigap," "multiphase" and similar "shunted gap" arresters that were installed before the electrolytic arresters were brought out and are still giving good satisfaction in many plants operating at voltages as high as 88,000 volts.

Electrolytic Arresters (Fig. 3) have been found after experimental research and operating experience of many years to be the best suited for high voltages up to the highest in actual service. See also *Power Stations, Hydroelectric.*

The arrester itself consists essentially of a system of nested cup shaped trays filled with liquid electrolyte arranged and supported as shown in Fig. 3 and immersed in oil in a steel tank. The liquid electrolyte forms on the surface of the trays an insulating film (aluminum hydroxide) of high electrostatic capacity. The resistance of the film permits only a very small current to flow at normal voltage, but breaks down at abnormal voltages; on cessation of the abnormal voltage the film regains its original resistance. The electrostatic capacity permits only a small current to flow at normal frequency, but permits very large current to flow at high frequency. Thus the arrester relieves both high voltage and high frequency. The oil provides a covering and insulating medium and prevents evaporation of the electrolyte. The volume of the oil is sufficient to absorb the heat due to a long continuous discharge

To prevent undue heating of the electrolytic elements, these arresters are provided with suitable gaps which insulate the electrolytic element from the line.

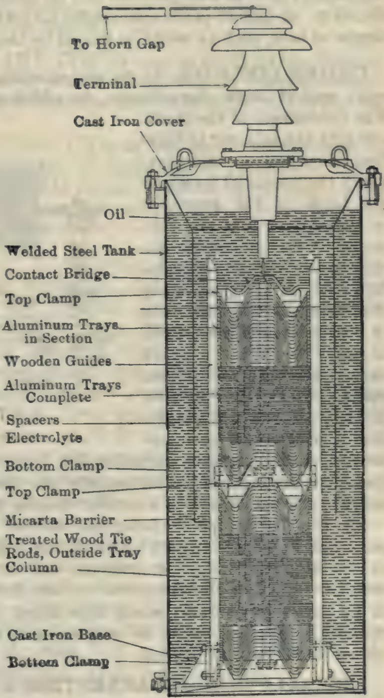

Fig. 3. Electrolytic Arrester

To maintain the film on the trays it is necessary periodically to charge the arrester; this is done by closing the gaps and permitting current to flow into the element. This gap is so arranged that it not only acts as a convenient device for charging the arrester, but also serves as a disconnecting switch by opening wide the gaps.

There are two general types of gaps in use, the horn or sphere gap, and the impulse gap. The horn or sphere gap when used at settings less than their

diameter, discharge high frequency surges at values nearly the same as the 60-cycle discharge value of a plain horn.

The most improved gap known to the art at this time is the impulse gap. This gap in addition to the advantages of the sphere gaps, selects the high frequency surges and discharges them at voltages of about 0.6 of the normal 60 cycle discharge value of the gap. This property is obtained by an ingenious arrangement of resistors and condensers. In the commercial form these condensers take the form of line insulators.

Owing to the use of the liquid electrolyte, the necessity for periodical charging and the comparatively high price of the aluminum cell electrolytic arrester, other types of arrester such as the oxide film have been developed. This oxide film arrester depends for its functioning on the fact that certain dry chemical compounds such as lead peroxide can be readily changed from a very good conductor to an almost perfect nonconductor (litharge) by the application of a slight degree of heat, such as would be caused by the passage of a lightning discharge.

CHOKE COILS (Fig. 4) are an important element in the protection of circuits against static disturbances, in addition to the lightning arrester itself. The inductance of the choke coil acts as a reflector to high-frequency waves and prevents the potential to which the leads of the generator or transformer coil are subjected from undergoing excessive or abrupt changes. As at the operating frequency the value of volts per turn in a choke coil is very small, a surge may cause a spark to pass momentarily between turns but no arc will be formed. This is not usually true of a generator or transformer. Although extra insulation for the end turns of generators and transformers is desirable, it cannot entirely take the place of choke coils but frequently permits the use of coils of a smaller choking power.

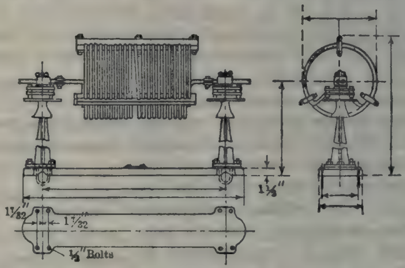

Fig. 4. Choke Coil

Fig. 4 shows a type of choke coil built in normal capacities up to 600 amperes and in voltage ratings up to 132,000 volts. Modifications of this design can be supplied for heavier currents and for higher voltages. These coils are made of copper or aluminum rod in the form of a cylinder 6 inches, 9 inches, or 15 inches in diameter wound into about 20 turns. These are arranged for supporing on insulators, or for suspension in the line circuits and are suitable for indoor or for outdoor service.

INSTALLATION OF ARRESTERS is dependent on their design. The electrolytic type is usually arranged for outdoor service, as is the oxide film type. The other types are usually mounted indoors and are connected in circuit at the point where the lines pass from the building. Nearly all kinds of arresters can be furnished for outdoor service, if desired.

Ground Connection. — (*See also article on Ground Connections.*) One of the most important features of a lightning-arrester installation is the securing of a satisfactory ground connection to enable the static electricity to pass readily into the earth. With a poor ground connection the value of lightning arresters and choke coils is greatly reduced.

A common method of securing a ground connection is to solder or rivet the ground wire to a large tinned copper plate which is buried in several layers of crushed coke or charcoal in permanently damp earth. Wrought-iron pipes driven deep into the moist earth will also make a good ground. In hydraulic plants the ground should include a connection to the penstock or some other portion of the piping system.

SPECIFICATION FOR LIGHTNING ARRESTERS. — *(See also article on Specifications.)* As the services to which lightning arresters are subjected are not capable of reproduction for testing purposes, lightning arresters cannot be specified in terms of performance. Hence it is necessary for the engineer to specify the type and essential features of construction after an examination of the various types. When calling for proposals, the following details should be stated: station or out-of-door service, voltage and frequency of circuit; details as to where arresters are to be located.

DIMENSIONS, WEIGHTS AND COSTS vary so much for the different voltages, class of service, whether grounded or ungrounded neutral, indoors or outdoors, that it is impossible to tabulate this information in the limited space available.

Lightning protection in a power house is almost independent of its capacity but depends on the voltage and number of feeders to be protected. A 200 ampere, 2300 volt, 3 phase feeder would require lightning protection costing about $12, while a 200 ampere, 110,000 volt feeder would require lightning protection costing about $5700.

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LOCOMOTIVES, ELECTRIC. — (*See also Cars, Electric; Collectors, Current; Locomotives, Steam; Railways, Energy Requirements and Motor Capacity for.*) For many years a new type of electric locomotive was designed for nearly every new proposition. However, with the very numerous applications of electric locomotives of from 30 to 50 tons weight for slow freight and switching service on interurban roads and in terminals, the double-truck bogie type finally demonstrated its superior fitness and came to be adopted almost universally in America for all work involving speeds less than 45 m.p.h. For higher speeds special provision must be made for guiding the locomotive around curves by the addition of guiding trucks or axles, for placing as much of the weight of the motors as possible on springs and for raising the center of gravity to a reasonable height (five feet or over). With a low center of gravity every side-wise movement of the mass of the locomotive strikes a blow side-wise on the track, but with a high center of gravity a side swaying is transformed into a downward thrust on the track. As the track is not usually designed to withstand great side thrusts it is better to avoid a low center of gravity in high-speed locomotives.

The coefficient of adhesion for electric locomotives has usual and safe values of from 18 to 22 per cent. It is higher for electric than for steam locomotives on account of the uniform torque of the electric motor. With clean dry rails the coefficient for electric locomotives may be as high as from 30 to 40 per cent.

CLASSIFICATION. — Locomotives are usually classified by the arrangement of their wheels and the subdivision of the wheels into driving wheels and guiding wheels. A series of numerals is used, each numeral representing a group of wheels of one form usually on one truck. Thus 4–4–0 designates a locomotive having four wheels on a guiding truck, four driving wheels connected and no trailing wheels or truck. This is the common "American" type of steam passenger locomotive. An ordinary double-truck four-motor freight locomotive or motor car would be designated as 4–0–4. (*See also Locomotives, Steam.*)

TYPES OF MOTORS.— Locomotives are built with various types of motors and operate from various systems of electrical distribution, e. g.,

a. Direct-current Motors at 600, 1200, 1500 or 2400 volts; for the higher voltages two motors are operated in series.

b. Single-phase Motors for 250 volts and 15 or 25 cycles, connected to the secondary of a transformer which receives its power from a 6000- or 11,000-volt trolley line.

c. Three-phase Induction Motors operating at about 500 volts supplied by the secondary of a transformer whose primary is connected to a 6000- or 11,000-volt three-phase trolley.

CONTROL SYSTEMS. — The control of all modern electric locomotives is by the multiple-unit system (q.v.), as the currents required are too large or the voltage too high for a drum control (q.v.). The possibility of one motorman operating and controlling two or three locomotive units at the same time is also advantageous.

TYPES OF ELECTRIC LOCOMOTIVES.— The simplest form of electric locomotive is a box car with a motor geared to each of the four axles on two bogie trucks. Such a locomotive would be geared for a very low speed and hence high tractive effort. To prevent slipping of the wheels the car would be weighted down with ballast or a load of freight.

The most common type of electric locomotive consists of a cab and framing carried on two heavy four-wheel trucks, each axle carrying a geared motor. Extra strength is provided in the cab framing to transmit the tractive effort of

the motors to the couplers. The complete locomotive may weigh from 30 to 50 tons and is equipped with motors having an aggregate capacity of approximately 500 horse-power. It will haul trains of about 20 cars, weighing about 500 tons, at speeds from 20 to 30 miles per hour. The usual tractive effort at the one-hour rating of the motors is from 10,000 to 15,000 lb. Such locomotives are well adapted for switching purposes in terminal freight yards and for hauling freight trains on interurban electric railways.

Detroit River Tunnel Locomotives. — A further development of this type of locomotive is exemplified in the large locomotive used for pusher and grade service in the Detroit River Tunnel of the Michigan Central R.R. and other similar installations. Each of these locomotives weighs from 100 to 120 tons, all on drivers, and consists of two four-wheel trucks carrying geared motors of from 300 to 500 horse-power each. The two trucks are coupled together by a pin or hinge which causes them to guide each other. This is called the "Articulated" type. The cab, containing all the control and auxiliary apparatus, is mounted on the trucks. The Detroit River Tunnel locomotives operate on 600 volts d-c. Locomotives of this type are now in operation also on 2400 volts d-c. and on 6000 volts three-phase a-c. They are limited in speed to about 40 miles per hr. on account of their lack of guiding trucks.

New York Central R.R. Locomotives. — The locomotives constructed in 1910 have a leading and a trailing two-axle guiding truck and in the middle four driving axles with gearless motors. The armature of each motor is mounted directly on the driving axle and the bi-polar field of the motor forms a part of the mechanical frame work of the locomotive. The magnetic flux passes through all four motors in series and returns by the side frames. The motors are wound for 600 volts d-c. and have very large air gaps to allow for the play between the armature and the field as the wheels pass over irregularities in the tracks. The field structure is spring borne but the armatures are not.

The locomotives constructed in 1913 consist of two sections articulated, with two two-axle trucks on each section. One truck on each section is rigid and the other is a bogie or guiding truck. Every axle carries a motor and all wheels are of the same size. The single cab is carried on a king pin on each section.

New Haven R.R. Locomotives. — The type of electric locomotive adopted by the N. Y., N. H., & H. R.R. in 1911 has four gearless single-phase series motors, the armatures being mounted on quills concentric with the driving axles and driving the wheels by means of springs. The locomotive has two trucks, each having four large driving wheels and two smaller guiding wheels. The motors are designed to operate at from 250 to 300 volts either a-c. or d-c., two in series on the line for d-c. and on the secondary of a transformer for a-c. The line voltage is 11,000 at 25 cycles.

Pennsylvania R.R. Locomotives. — The locomotives of the Pennsylvania R.R. consist of two similar units coupled back to back. Each unit has a two-axle guiding truck and has two driving axles rigid with the body frame. There is one large motor per unit, mounted in the cab and driving the wheels by means of an inclined connecting rod from the motor to a jack shaft and horizontal side rods from the jack shaft to the drivers. The connecting rods on opposite sides are placed at right angles to avoid dead centers.

Chicago, Milwaukee & St. Paul Locomotives. — The freight locomotives delivered in 1916 (see table) consist of two similar units coupled together, each unit having a leading bogie truck of idle axles for guiding, and two four-wheel trucks with all axles driving. A motor is geared to each of the four driving axles of each unit. The motors are designed to operate with 1500 volts d-c. and two motors are permanently connected in series for 3000 volts, the line

LOCOMOTIVES, ELECTRIC

	Bush Terminal Steel Switcher	Pied-mont	Detroit River Tunnel	Great North-ern	Penn-sylvania Termi-nal	St. Paul	Norfolk and West-ern	N. Y. Central	New Haven	New Haven	St. Paul	St. Paul
Date		1911	1909	1908	1910	1916	1914	1918	1919	1919	1920	1920
System	D-C.	D-C.	D-C.	3-phase	D-C.	D-C.	A-C.	D-C.	A-C.	A-C.	D-C.	D-C.
Trolley voltage	500	1,500	600	6,000	600	3,000	11,000	600	11,000	11,000	3,000	3,000
Service	Frt.	Frt.	Frt. and Pass.	Frt. and Pass.	Pass.	Frt. and Pass.	Frt.	Pass.	Pass.	Frt.	Pass.	Pass.
Total weight, tons	40	55	100	115	166	284	264	134	173	141	265	275
Number of motors	4	4	4	4	2	8	8	8	12	4	12	6
Horse-power per motor *	90	185	275	375	1,250	452	410	325	170	345	270	700
Weight of electrical equipment, tons	14	19	27	55	64	124	112	44	81	....	117	100
Weight on drivers, tons	40	55	100	115	104	225	224	134	115	97	229	168
Diameter of drivers, inches	36	37	48	60	72	52	62	36	63	54	44	68
Rigid wheel-base, ft., ins.	6′–6″	7′–4″	9′–6″	11′	8′	10′–6″	11′–6″	5′	16′–9″	10′	13′–11″	16′–9″
Total wheel-base, ft., ins.	22′	25′	27′–6″	31′–9″	55′–11″	102′–8″	43′	46′–5″	59′–0″	54′	67′	79′–10″
Tractive effort, hour, lbs.	17,000	13,700	35,000	38,000	25,000	132,000	87,400	20,000	21,000	27,000	46,000	66,000
Speed per same, mile per hr.	8	20	12	15	32	10.2	14	49	36	23	26.4	22.7
Classification	404	0440	0440	0440	4444	444+ 444	2442	4444	262+ 262	4444	6886	462+ 264

* One hour rating.

potential. Special arrangements are provided for regenerative braking on downgrades by using the main motors as separately excited generators. The passenger locomotives of that date are similar except as to gear ratio. In 1920 two new types of passenger locomotives were installed, one using gearless motors like the New York Central and the other using twin motors geared to each driving axle (see *Bibliography*).

SPECIFICATION FOR ELECTRIC LOCOMOTIVE. — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

On important high-speed systems it is usual for the design of the locomotive to be worked out by the purchaser and manufacturer working in collaboration, and in such cases, the design is usually specified in detail. In other cases, it is more usual to specify the operating characteristics and leave the design to the manufacturer. The following memoranda are to assist in the preparation of a specification of the latter type.

General Description of Service. — Whether for direct- or alternating-current, single-phase or three-phase, freight or passenger hauling, overhead trolley or third rail. Line voltage, etc.

Specific Details of Work to be Performed by Locomotive. — Weight of cars loaded and empty. Maximum train weight. Average train weight. Time to make typical run of stated length. Number and duration of stops in typical run. Ton miles per day per locomotive. Maximum speed on level with average load. Maximum speed on maximum grade with maximum load. Acceleration (miles per hour per second), with maximum load. Hours per day in regular service. Amount of time in shifting and yard service.

Profile and Plan of Line. — Grades and curves.

Clearances and Limiting Dimensions.—Gauge of track, clearance diagram of right-of-way. Maximum and minimum height of trolley wire or third-rail location. Height of coupler. Wheels, tread and flange (M.C.B. or special). Weight of rail. Minimum radius of curve. Wheel diameter. Maximum permissible weight per running foot of right-of-way.

Operating Characteristics. — Absence of nosing or lateral swing. Absence of rail pounding. Temperature-rise limitations. Efficiency.

Control. — See specifications under *Control Systems for Railway Motors*.

Motors. — See specifications in articles on *Motors*.

Air Brake. — Straight, automatic or combined. General characteristics.

COST, WEIGHT AND DIMENSIONS. — Electric locomotives cost about $800 per ton weight, which is considerably more than steam locomotives, but they are cheaper to operate and maintain as they can make more mileage per day. It costs, roughly, between 5 and 10 cents per locomotive mile to operate electric locomotives.

The characteristics of the most prominent types of electric locomotives are given in the table, page 938.

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LOCOMOTIVES, STEAM. — (*See also Locomotives, Electric; Railways, Energy Requirements and Motor Capacity for.*) Locomotives are classified broadly by the number of truck and driving wheels. Each arrangement of wheels has a definite name, but the various wheel arrangements are so numerous that the "Whyte classification" has come into extensive use.

Whyte Classification. — According to this scheme of classification three numbers are used; the first indicates the total number of leading or truck wheels in front of the drivers, the second indicates the total number of drivers, and the third indicates the total number of wheels behind the drivers and under the locomotive proper. Tender wheels are not included. This classification may be extended, by the use of more than three numbers, to cover engines of the articulated, or Mallet, type. The classification and names of some of the more important types are given in the table on the next page, the weights being in short tons. The tabular values are for comparatively recent engines of the largest size. Each type of engine has been made in a wide variety of sizes, ranging mainly downwards from the given weights to approximately half these values for standard gauge track, and much lower for narrow gauge or industrial tracks. The weight of the tender loaded ranges approximately from 60 to 100 per cent of the weight of the engine.

DESIGN AND PERFORMANCE. — A steam locomotive is essentially a moving power plant and as such it must transport its fuel and water with it. Its limitations as to size and weight are determined by roadway clearances, the curves upon which it must run, and the strength of the track supporting it.

Locomotive engines may be either simple or compound, but compound locomotives are not as extensively used now as formerly, except in the case of the articulated types.

The load upon a driving axle is generally limited to 30 tons, but this weight has been exceeded in a number of recent engines of large size.

Maximum Power. — The limit of power of a steam locomotive is measured by the size and steam-producing capacity of its boiler (*see Boilers*). Owing to the necessarily small grate area, coal must be burned very rapidly under an induced draft. The rate of combustion is quite commonly as high as 100 pounds and sometimes exceeds 200 pounds per square foot of grate area per hour. The use of brick arches and superheaters largely increases the power, adding from 30 to 40 per cent in some cases. On account of difficulties of lubrication, wear of valves, etc., piston valves must be used with superheaters. The use of superheaters is practically universal in modern high-powered engines.

The maximum horse-power which can be exerted by a steam locomotive is in some cases 4000, although this is unusual; from 2000 to 2500 is a fairer maximum figure for ordinary cases. The largest tenders have a capacity of 20,000 gallons of water and 20 tons of coal.

Tractive Effort. — The tractive effort of a single-expansion steam locomotive at slow speed is computed from the following formula:

$$\text{Tractive effort} = \frac{P \times D^2 \times S}{d},$$

where P = mean effective pressure in cylinders, in lbs. per sq. in. (usually taken as 0.85 of boiler pressure),
D = diameter of the piston, in inches,
S = stroke, in inches,
d = diameter of driving wheels, in inches.

CLASSIFICATION OF STEAM LOCOMOTIVES

Type	Wheel arrangement	Name	Total wheels	Kind of service	Approximate weights, tons, engine only	Load on drivers, tons per axle
0-4-0	∠ O O	4-wheel switcher	4	Switching	50	25.0
0-6-0	∠ O O O	6-wheel switcher	6	Switching	83	27.7
0-8-0	∠ O O O O	8-wheel switcher	8	Switching	107	26.8
2-6-0	∠ o O O O	Mogul	8	Freight	100	28.0
2-8-0	∠ o O O O O	Consolidation	10	Freight	148	33.5
2-6-2	∠ o O O O o	Prairie	10	Freight	124	29.2
2-8-2	∠ o O O O O o	Mikado	12	Freight	163	30.0
2-10-2	∠ o O O O O O o	Santa Fe	14	Freight	209	33.7
4-4-0	∠ o o O O	American	8	Passenger	87	30.1
4-6-0	∠ o o O O O	10 wheel	10	Passenger and freight	109	28.3
4-8-0	∠ o o O O O O	12 wheel	12	Freight	131	26.4
4-4-2	∠ o o O O o	Atlantic	10	Passenger	120	34.3
4-6-2	∠ o o O O O o	Pacific	12	Passenger and freight	156	34.1
4-8-2	∠ o o O O O O o	Mountain	14	Passenger and freight	184	30.4
0-6-6-0	∠ O O O O O O	Articulated	12	Freight	176	29.3
0-8-8-0	∠ O O O O O O O O	Articulated	16	Freight	240	30.0
2-6-6-2	∠ o O O O O O O o	Articulated	16	Freight	224	29.8
2-8-8-2	∠ o O O O O O O O O o	Articulated	20	Freight	280	31.2

See remarks under Whyte's classification on preceding page.

Tonnage Rating. — By the tonnage rating of a locomotive is commonly meant the weight, in tons, of the train which it can pull, exclusive of the weight of the locomotive, but including the weight of the tender. Short tons (2000 lb.) are used. Let:

F = maximum tractive effort, usually taken as the tractive effort corresponding to a mean effective pressure equal to 0.85 of boiler pressure,
G = maximum grade, per cent.
L = weight of locomotive, in tons,
R = locomotive resistance, lbs. per ton,
r = train resistance, lbs. per ton,
T = "tonnage rating," i.e., weight behind locomotive.

Then

$$T = \frac{F - (R + 20\,G)\,L}{r + 20\,G}.$$

See article on *Railways, Energy Requirements and Motor Capacity for*, for values of r and R. On heavy grades this value of the tonnage rating will be limited by the ability of the fireman to keep up full steam pressure. This difficulty has been very satisfactorily overcome in recent years by equipping the engines with mechanical stokers.

Empty or partially loaded cars have a larger resistance per ton than fully loaded cars. With all empty cars on a straight track the capacity of an engine may be reduced as much as 47 per cent on a level stretch, 15 per cent on a 1 per cent grade, and 8.6 per cent on a 2 per cent grade. The corresponding figures with half loading would be 18, 4, and 2.3 per cent.

According to an extended series of experiments reported in a paper read before the American Society of Civil Engineers in December, 1902, by Mr. A. C. Dennis, freight train resistance is practically constant between 7 and 35 miles per hour, and may be regarded as made up of two parts—the dead load tonnage (cars) multiplied by the normal grade and curve resistance in pounds per ton plus 8.9, and the live load tonnage (freight) multiplied by the normal grade and curve resistance in pounds per ton plus 2.6, for any degree of loading. This is equivalent to regarding one ton of dead load as having a greater resistance than one ton of live load, with a variable ratio depending on grade and curvature. It is often convenient to consider the combined value of the live load and the product of the dead load by this ratio. This result is known as the tonnage rating of the train, with the understanding that a rating ton is a live load ton or its equivalent in the above sense.

The tonnage rating of a locomotive when used in connection with the above method of computation means its capacity in rating tons under the proposed conditions, a quantity which changes with grade and curvature, but remains constant under any condition of loading. The locomotive and tender are considered as dead load tonnage in computing their tractive resistance.

Reduction of Tonnage Rating in Stormy Weather. — Weather conditions also affect the steam locomotive's capacity. The worst condition is cold weather with a heavy wind at right angles to the track. This side wind always makes engines steam badly and increases flange friction against the rails. It is a common cause of delays.

Tonnage ratings are reduced a variable amount under severe weather conditions. Some roads have fixed rules and some have not. The maximum reduction, taken from a table published in Bulletin No. 59 of the University of Illinois Engineering Experiment Station, is 30 per cent for temperatures around 15 degrees below zero. On lines having heavy grades the grade resistance is

such a large proportion of the total work to be done that the train resistance becomes a very small factor. Consequently the tonnage ratings for roads having very heavy grades are practically the same for all seasons and weather conditions.

Unbalanced Forces. — The necessity of partially balancing the heavy reciprocating parts by weights located on the wheels produces unbalanced vertical forces which are hard on roadbed and bridges.

Center of Gravity and Stability.— The boiler is necessarily placed almost wholly above the wheels and the center of gravity of the whole engine is therefore a considerable distance above the rails. Careful tests have shown this to be a positive advantage so far as ease of maintenance of track is concerned. By proper elevation of outer rails on curves, the overturning tendency of the centrifugal force may be wholly or sufficiently neutralized. This overturning tendency is much less than most engineers suppose. It is safe to state that it is impossible to *overturn* a locomotive at any practicable speed upon any ordinary curve by the action of centrifugal force unless the conditions are such (as in a cross-over, for instance) as to set up a rolling or oscillatory motion of the engine about its longitudinal axis. Generally speaking, the tender is the part of the train most likely to leave the rails. This is due to its varying and shifting load of coal and water.

Typical Large Locomotives. — The locomotives listed on the following page are typical of modern locomotive development. The information for the first three engines was kindly furnished by the Baldwin Locomotive Works, and for the last two was obtained from Bulletin No. 7 of the Locomotive Superheater Company. The first two engines were built by the Baldwin Company, the Pacific type for the Lehigh Valley Railroad, and the Mountain type for the A. T. and S. F. Railway. The Consolidation engine is now under construction by the Baldwin Company for the Western Maryland Railway, and is a very large engine for this type. The fourth engine is the heavy Mikado and the fifth the heavy Santa Fe of the United States Railroad Administration Standard Locomotives. All five engines are equipped with superheaters, piston valves and Walschaert valve gear. The first engine burns a mixture of hard and soft coal while the remaining four are soft coal burners. In computing the tractive effort of these engines the mean effective pressure is taken as 0.85 of boiler pressure. The total effective heating surface is taken as the total boiler heating surface plus 1.5 times the superheater heating surface.

OPERATION OF STEAM LOCOMOTIVES. — Below are given some of the more important facts in regard to steam locomotive operation.

Location of Coal and Water Stations. — Coal and water stations for replenishing the tender are required at intervals, depending upon the topography of the line and volume of traffic. On fairly level roads coaling stations are required at intervals of 50 or 60 miles for freight engines and 120 miles for passenger engines. Water stations are needed at about 50-mile intervals for passenger and 25-mile intervals for freight service. These distances may be much reduced on heavy grades. The location of water stations is also a matter of available water supply. Where possible water should be taken at regular stops.

Fuel and Water Consumption. — This is very variable depending upon topography and alignment of road, kind of service, frequency of stops, whether single or double track, climatic conditions, weight of trains, etc. The number of pounds of coal burned per locomotive mile averages about 104 for passenger, 208 for freight, 130 for mixed, 108 for switching and 150 for all types of service. The actual water evaporated varies from 4.5 to 6.5 lbs. per pound of coal burned The first figure is for a coal consumption of 200 lbs. and the last for 65 lbs. per sq. ft. of grate per hour.

TYPICAL STEAM LOCOMOTIVES

Item	Pacific (4-6-2)	Mountain (4-8-2)	Consolidation (2-8-0)	Mikado (2-8-2)	Santa Fe (2-10-2)
Service	Passenger	Passenger	Freight	Freight	Freight
Weight, engine and tender, working order, lbs.	469,100	610,100	526,000	497,000	586,100
Weight, engine only, working order, lbs.	311,900	367,700	296,000	325,000	380,000
Total weight on drivers, lbs.	204,560	243,100	268,000	240,000	293,000
Total tractive effort, lbs.	48,722	54,085	68,262	59,801	73,829
Weight on drivers ÷ tractive effort	4.20	4.49	3.93	4.01	3.97
Diameter of driving wheels	73″	69″	61″	63″	63″
Length of driving wheel base	13′ 8″	18′ 0″	17′ 6″	16′ 9″	22′ 4″
Total wheel base of engine	36′ 0″	39′ 5″	27′ 3″	36′ 1″	42′ 2″
Total wheel base of engine and tender	68′ 0⅜″	76′ 8⅝″	74′ 1¼″	71′ 9½″	82′ 10½″
Size of cylinders	27″ × 28″	28″ × 28″	27″ × 32″	27″ × 32″	30″ × 32″
Type of boiler	Conical wagon top	Conical wagon top	Straight top	Conical wagon top	Conical wagon top
Working pressure of boiler, lbs. per sq. in.	205	200	210	190	190
Diameter of boiler	83¾″	82″	88″	86″	88″
Size of firebox	120 1/16″ × 114¼″	122¼″ × 84¼″	112″ × 96¼″	120⅛″ × 84¼″	132⅛″ × 96¼″
Grate area, sq. ft.	95.2	71.5	74.9	70.8	88.2
Number of tubes	45-254	43-254	50-240	45-247	50-271
Diameters of tubes	5½″-2¼″	5½″-2¼″	5½″-2¼″	5½″-2¼″	5½″-2¼″
Heating surface of tubes, sq. ft.	3,734	4,428	3,242	3,978	4,727
Heating surface of firebox, sq. ft.	244	246	237	319	429
Minor heating surfaces, sq. ft.	138	128	30		
Total heating surface of boiler, sq. ft.	4,116	4,802	3,509	4,297	5,156
Heating surface of superheater, sq. ft.	980	1,086	870	993	1,230
Total equivalent heating surface, sq. ft.	5,586	6,431	4,814	5,786	7,001
Water capacity of tender, U. S. gals.	8,000	12,000	15,000	10,000	12,000
Coal capacity of tender, tons	15	16	16	16	16

See remarks under **Typical Large Locomotives** on preceding page.

In some parts of the United States oil is used for fuel on account of the high cost of coal. It results in a greater expense for upkeep of boilers on account of the intense heat, but in the far west and southwest is economical on the whole.

Use of Different Kinds of Coal. — Exhaust nozzles and draft appliances can be arranged to suit any one of various kinds of coal, and arrangements should be made to use only the kind of coal for which the engine is "drafted," as other kinds are burned at a less efficiency and consequently at what is probably a larger expense even though the cost per ton of the improper coal is less.

Idle Steaming. — Whenever a locomotive is standing idle under steam, coal is being consumed. This idle time should be reduced to a minimum by avoiding firing up a long while before an engine is needed, and by keeping trains moving while they are on the road. Roads having traffic largely of one kind can show better results in this particular than those having a mixed traffic composed of equal parts of all kinds.

Lubrication. — The friction in a locomotive is large, ranging from about 20 per cent of the total power of the engine to a maximum of 50 per cent or more under certain conditions of passenger service. Probably 35 per cent is a fair average. In many cases an appreciable economy can be instituted by increasing the quantity of oil allowed for an engine. Most motive power officials are strangely blind in this respect. Generally oil consumption is kept at a minimum, and enough coal is burned to overcome the resulting friction. The proper method is to so arrange the ratio of oil to coal that the total expense for the two is a minimum.

Blowing-off. — Boilers must be blown off and washed out at intervals depending upon the quality of water used.

COSTS (Pre-war figures). — The first cost of a steam locomotive is from 6 to 7 cents per pound of weight, including tender.

Annual Costs. — The cost of maintenence and repairs ranges from $2000 to $3500 per locomotive per year, an average figure being $2600.

The average life of a locomotive is about 20 years; hence in addition to the cost of maintenance and repairs an annual depreciation of 5 per cent of the first cost should be charged against the locomotive. In addition, an interest charge of, say, 5 per cent of the first cost should be included.

The operating costs are as follows, the figures being for ordinary mixed service on a trunk line with moderate grades:

Wages of crew,	10.6 cents per loco.-mile
Coal, at $3.00 per ton,	18.9 cents per loco.-mile
Oil, waste, etc.,	0.3 cent per loco.-mile
Wipers,	0.5 cent per loco.-mile
Repairs,	11.3 cents per loco.-mile
Total operating cost,	41.6 cents per loco.-mile

The average cost of operation of a locomotive, including wages of conductor and flagman, is about $50 per day, on the basis of 100 miles being a day's work.

The total annual cost of a locomotive costing initially $20,000 and covering an average of 100 miles per day would then be roughly:

Operation	$10,000
Maintenance and repairs	2,600
Depreciation	1,000
Interest	1,000
Total	$14,600

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LOGARITHMS. — The logarithm of a number A, to a given base b, is the power n to which that base b must be raised in order to equal the number A. Thus, if $b^n = A$, then n is the logarithm of A to the base b, which may be written $n = \log_b A$. From this definition the following properties are readily deduced, where A and B are any two numbers.

$$\log AB = \log A + \log B.$$

$$\log \frac{A}{B} = \log A - \log B.$$

$$\log A^n = n \log A.$$

Characteristic and Mantissa. — These three equations hold irrespective of what is chosen as the base of the logarithms, provided the same base is used for each logarithm. In the common or Brigg's system of logarithms the number 10 is chosen as the base. In such a system the logarithm of any number may be expressed directly in terms of the logarithm of a number (including decimal fractions as numbers) between 1 and 10. For example:

$$\begin{aligned} \log_{10} 376.42 &= \log (100 \times 3.7642) \\ &= \log 100 + \log 3.7642 \\ &= 2 + \log 3.7642, \end{aligned}$$

since the power to which 10 must be raised to give 100 is 2. Similarly, $\log_{10} 3764.2 = 3 + \log 3.7642$. The logarithms of all numbers between 1 and 10 are less than unity. The whole number or integer part of a logarithm is called its "characteristic" and the fractional part its "mantissa." The characteristic of the logarithm of a number less than unity is negative. For example:

$$\begin{aligned} \log 0.037642 &= \log (\tfrac{1}{100} \times 3.7642) \\ &= \log 1 - \log 100 + \log 3.7642 \\ &= -2 + \log 3.7642. \end{aligned}$$

In general, *the characteristic of a number greater than unity is positive and is one less than the number of figures to the left of the decimal, while the characteristic of a number less than unity is negative and is one greater than the number of ciphers between the decimal and the first significant figure.* A table of logarithms gives the mantissas only, the characteristics being determined by the above rule. Such a table is given below.

Antilogarithms. — If $n = \log A$, then A is the number whose logarithm is n. This may be written symbolically

$$A = \log^{-1} n.$$

A is then called the antilogarithm or inverse logarithm of n. The antilogarithm of a number (i.e., the number which has the given number for its logarithm) is found from a table of logarithms by finding the number in the margin corresponding to the decimal point of the given number in the table, and fixing the decimal point by the rule given above. Example:

$$\log^{-1} 1.6464 = 44.3.$$

Use of Logarithms. — Logarithms are used in the processes of multiplication, division, raising to powers and taking roots. For example, to find the product of two numbers take from the table the logarithms of the two numbers,

COMMON LOGARITHMS OF NUMBERS

No.	0	1	2	3	4	5	6	7	8	9
10	00000	00432	00860	01283	01703	02119	02531	02938	03342	03743
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301
14	14613	14921	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17897	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21218	21484	21748	22010	22271	22530	22788
17	23045	23299	23552	23804	24054	24303	24551	24797	25042	25285
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014
21	32222	32428	32633	32838	33041	33243	33445	33646	33845	34044
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983
23	36173	36361	36548	36735	36921	37106	37291	37474	37657	37839
24	38021	38201	38381	38560	38739	38916	39093	39269	39445	39619
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560
28	44716	44870	45024	45178	45331	45484	45636	45788	45939	46089
29	46240	46389	46538	46686	46834	46982	47129	47275	47421	47567
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020
34	53148	53275	53402	53529	53655	53781	53907	54033	54157	54282
35	54407	54530	54654	54777	54900	55022	55145	55266	55388	55509
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172
41	61278	61384	61489	61595	61700	61804	61909	62013	62118	62221
42	62325	62428	62531	62634	62736	62838	62941	63042	63144	63245
43	63347	63447	63548	63648	63749	63848	63948	64048	64147	64246
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181
46	66276	66370	66464	66558	66651	66745	66838	66931	67024	67117
47	67210	67302	67394	67486	67577	67669	67760	67851	67942	68033
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930
49	69020	69108	69196	69284	69372	69460	69548	69635	69722	69810
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345
53	72428	72509	72591	72672	72754	72835	72916	72997	73078	73158
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957

COMMON LOGARITHMS OF NUMBERS

No.	0	1	2	3	4	5	6	7	8	9
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74741
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447
70	84509	84571	84633	84695	84757	84818	84880	84941	85003	85064
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272
73	86332	86391	86451	86510	86569	86628	86687	86746	86805	86864
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448
75	87506	87564	87621	87679	87737	87794	87852	87909	87966	88024
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890
85	92941	92993	93044	93095	93146	93196	93247	93298	93348	93399
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856
91	95904	95951	95999	96047	96094	96142	96189	96236	96284	96331
92	96378	96426	96473	96520	96567	96614	96661	96708	96754	96801
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726
95	97772	97818	97863	97909	97954	98000	98045	98091	98136	98181
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956

add these logarithms, and then from the table find the number of which this is the logarithm. The position of the decimal point is fixed by the value of the mantissa in the sum of the two logarithms. By adding a whole number to a mantissa and subtracting the same number from the characteristic the mantissa of the final result can always be kept positive.

Examples. — Multiply 376.2 by 0.587:

$$\begin{aligned} \log 376.2 &= 2 + .57541 \\ \log 0.587 &= -1 + .76863 \end{aligned}$$

$$\text{Adding gives } 1 + 1.34404 = 2.34404$$

therefore $376.2 \times 0.587 = 220.8.$

Divide 37.62 by 587:

$$\begin{aligned} \log 37.62 &= 1 + .57541 = \phantom{2+{}} 1.57541 \\ \log 587 &= 2 + .76863 = 2 + .76863 \end{aligned}$$

$$\text{Subtracting gives} -2 + .80678$$

therefore $$\frac{37.62}{587} = 0.06409.$$

NATURAL LOGARITHMS. — (*See also Roots and Powers.*) The base of the so-called natural system of logarithms is the value of the expression $\left(1 + \frac{1}{n}\right)^n$ when n is taken equal to infinity. The numerical value of this expression is found by expanding $\left(1 + \frac{1}{n}\right)^n$ by the binomial theorem (*see Series*), and is equal to 2.718282 +. This number is usually represented by the symbol ϵ, that is,

$$\epsilon = 2.718282 +, \text{ or } 2.718 \text{ approximately.}$$

Logarithms to this base are readily calculated by means of Taylor's series (*see Series*); also this number ϵ enters in a very simple manner into various mathematical and physical relations (*see Equations, Differential; Transient Electric Phenomena; Hyperbolic Functions; Trigonometric Functions*).

The relation between a logarithm of any number A to any base b and the logarithm of A to any other base a is

$$\log_b A = \frac{\log_a A}{\log_a b}.$$

Hence $$\log_\epsilon A = 2.30259 \log_{10} A.$$

From this last relation the natural logarithm of any number may be found directly from the table of common logarithms.

Example. —

$$\log_\epsilon 376.2 = 2.3026 \times 2.57541 = 5.930.$$

The natural logarithm may also be taken directly from the table of exponential functions (q.v.), remembering that $\log_\epsilon A$ is the number in the margin of the table corresponding to the number equal to A in the columns of the table.

The symbol "ln" is frequently used for the natural logarithm and the symbol "log" without a subscript is usually employed as an abbreviation for the logarithm to the base 10.

LUBRICANTS AND LUBRICATION. — (*See also Bearings; Friction.*) Ordinary lubricants may be classified as follows:

Vegetable Oils. — Commonly employed vegetable oils are linseed, cottonseed, rape and castor. Vegetable oils decompose at comparatively low temperatures. They are used chiefly for compounding with mineral oils.

Animal Fats. — Animal fats ordinarily employed for lubrication are tallow, neat's-foot, lard, sperm, wool grease and fish oil. Like vegetable oils they decompose at comparatively low temperatures and are used chiefly for compounding with mineral oils.

Mineral Oils. — These are all petroleum products, and form the whole or the greater part of most of the lubricants employed.

Solid Lubricants. — Dry graphite, soapstone and mica are sometimes used as lubricants for slow-speed work when the bearing surface is restricted in area and the load to be carried is very large.

"Deflocculated" Graphite. — In 1906 E. G. Acheson discovered a process of producing a fine, pure, unctuous graphite, which when heated with a solution of tannin would remain suspended in water for months. The graphite thus suspended in water, known as "aquedag" has been successfully used as a lubricant (*Trans. A.I.E.E., 1907*). Acheson's "deflocculated" graphite, as the graphite in the finely divided form is called, has also been suspended in oil, the oil emulsion being known as "oildag," making an excellent lubricant.

Greases. — Compounds of oils and fats containing sufficient soap to form a more or less solid mass at ordinary temperatures are called greases. Lime soda or lead soaps are used in these compounds. For very high pressures graphite, soapstone and mica are sometimes added to the grease.

QUALIFICATIONS OF A GOOD LUBRICANT. — The generally accepted conditions of a good lubricant are as follows:

1. "Body" (i.e., viscosity) enough to prevent the surfaces to which it is applied from coming in contact with each other.
2. Freedom from corrosive acid, of either mineral or animal origin.
3. As fluid as possible consistent with sufficient "body."
4. Low coefficient of friction (as determined in a standard bearing).
5. High "flash" and burning points.
6. Freedom from all materials liable to produce oxidation or "gumming."

The examinations to be made to verify the above are both chemical and mechanical, and are usually arranged in the following order:

1. Identification of the oil, whether a simple mineral oil, or animal oil, or a mixture. 2. Density. 3. Viscosity. 4. Flash point. 5. Burning point. 6. Acidity. 7. Coefficient of friction. 8. Cold test.

Test for Fats. — Heat a small quantity of the oil in a small test tube 15 minutes with small pieces of metallic sodium or caustic potash. If fatty oil is present, a soapy mass will form at the top. (*Gebhardt.*)

Test for Tarry Matter. — Dissolve a small quantity of the oil in from 10 to 20 times its bulk of gasoline; tar and other insoluble matter will be precipitated. (*Gebhardt.*)

Specific Gravity. — This is usually made with a hydrometer, graduated according to the Baumé scale. At 60° F.

$$\text{Specific gravity} = \frac{140}{130 + \text{degrees Baumé}}.$$

Viscosity. — Viscosity, or internal friction, is usually determined by observing the time required for a given amount of oil to flow through a standard orifice. By "specific viscosity" is meant the ratio of the time for the oil to run out to that required for an equal volume of water at 60° F. The temperature of the oil should always be observed and stated. Engine oils are usually tested for viscosity at 70° F. and cylinder oils at 212° F. (*Gebhardt.*)

Flash Point. — The flash point is determined by heating a sample of oil in a cup at the rate of 15° F. per minute until a spark will ignite the vapor; the corresponding temperature is the flash point. The flash point as thus determined depends to some extent upon the surface exposed, the size of the spark, the distance between spark and surface of oil and the dimensions of the cup. (*Gebhardt.*)

Burning Point or Fire Test. — By continuing the application of heat and noting the temperature at which the oil itself takes fire and continues to burn, the burning point is determined.

Acidity. — The presence of free acid is determined by shaking up equal quantities of oil and water and testing with litmus paper.

Cold Test. — The cold test is the temperature at which the oil will just flow.

Friction Test. — The coefficient of friction as determined from friction-testing machines gives but little information concerning the action of the oil under the widely different conditions found in practice. (*Gebhardt.*)

Properties of Vegetable and Animal Oils. — The following data are taken from Gebhardt's *Steam Power Plant Engineering.*

Kind of oil	Specific gravity		Flash test, ° F.
	Water as 1.00	Baumé	
Lard	0.9175	23	505
Sperm	0.8815	29	478
Tallow	0.9080	24.5	540
Cottonseed	0.9210	22	518
Linseed	0.9299	19	505
Castor	0.9639	15	...
Palm	0.9046	25	405
Rape-seed	0.9155	23	...

Properties of mineral oils as compounded for ordinary use are given below.

Grease Lubricants. — Tests made on an Olsen lubricant testing machine at Cornell University are reported in *Power*, Nov. 9, 1909. It was found that some of the commercial greases stood much higher pressures than the oils tested, and that the coefficients of friction at moderate loads were often as low as those of the oils. The journal of the testing machine was 3¾ inches diameter, 3½ inches long, and the babbitt bearing shoe had a projected area of 5.8 square inches. The speed was 240 revolutions per minute and each test lasted one hour, except when the bearing showed overheating. The following are the coefficients of friction obtained in the tests:

RELATIVE VALUES OF FRICTION COEFFICIENT WITH GREASES AND OILS

Lb. per sq. in.	Mineral grease	Animal grease	Graphite grease	Mineral grease	Engine oil	Engine oil	Grease	Grease
86.2	0.024	0.023	0.04	0.023	0.019	0.015	0.020	0.025
172.4	0.021	0.023	0.05	0.018	0.04	0.022	0.015	0.022
258.6	0.021	0.023	....	0.018	0.06	0.037	0.014	0.020
344.8	0.025	0.025	....	0.019	....	....	0.017	0.020
431.0	0.050	0.035	....	0.028	....	....	0.026	0.019

APPLICATIONS OF VARIOUS TYPES OF LUBRICANTS. — The type of lubricant to use in any case depends upon:

1. The cost due to consumption of lubricant.
2. The saving in annual cost due to lessening of wear of bearings, guides and other rubbing surfaces.
3. The cost of the energy saved (as the result of decreased friction losses) due to the use of the lubricant.

For minimum annual cost the sum of the last two items should equal the first. Estimates of this kind are difficult to make, and the result is that the kind of lubricant used in any specific case is usually determined by experience.

The following table, from a paper in *Power*, December, 1905, p. 750, gives the kind of lubricant ordinarily employed for various purposes, together with their approximate characteristics. The cold test of all these oils, except oil for refrigerating machinery, is given as 30° F. Refrigerating-machinery oil should not solidify above 0° F.

METHODS OF LUBRICATION. — The commonest type of "lubricator" on engines or dynamos is the simple oil cup with sides of glass, so that the level of the oil in the cup can be seen. Any type of lubricator in which the flow of oil can be seen is known as a "sight feed." The flow of oil is regulated by a needle valve in the base of the cup.

Lubrication of Crossheads, Crank Pins, etc. — In applying oil to rubbing surfaces, both of which are in motion, various devices are used by means of which the oil cup can be kept at rest. A stationary oil cup may be used with a "wiper" on the moving member, or a "telescopic," "pendulum" or "centrifugal" oiler may be employed. Sometimes the crank, connecting rod and crossheads are inclosed in a casing the bottom of which is filled with oil so that at each revolution the end of the connecting rod splashes oil over all the parts.

Oil Rings and Chains. — The bearings of small high-speed engines and dynamos are frequently provided with rings or chains running loosely over the journal and dipping into an oil bath in the pedestal below the bearing. The rotation of the journal gives enough motion to the rings to enable them to carry up sufficient oil from the bath to keep the bearing surfaces bathed in oil.

Cylinder Lubrication. — The oil must be forced into steam cylinders against the steam pressure. This is usually accomplished by means of specially constructed cylinder cups, hydrostatic lubricators or force pumps. A brief description of the more common forms of these devices will be found in Gebhardt's ***Steam Power Plant Engineering.***

APPLICATIONS AND CHARACTERISTICS OF VARIOUS OILS

Kind of oil and application	Specific gravity, Baumé	Flash test, ° F.	Fire test, ° F.	Viscosity at 70° F. (Water = 1)
High-pressure cylinder oils: For cylinders using dry steam at from 110 to 210 lb.	25–24.5	600–610	645–660	175–205
General cylinder oil: For cylinders using dry steam at from 75 to 100 lb. Also for air compressor cylinders when the oil is made from steam-refined mineral stock and has a viscosity of 200.	26–25.5	550–585	600–630	180–190
*Wet cylinder oil** For cylinders using moist steam, especially in compound- and triple-expansion engines.	25.8–25.3	560–585	600–630	150–185
Gas-engine cylinder oil†	26.5	320	350	300
Automobile gas-engine oil‡	29.5	430	485	195
Heavy engine and machinery oils: For heavy slides and bearings, shafting and horizontal surfaces.	30.5–29.5	400	440–450	170–195
General engine and machinery oils: For high-speed dynamos and other comparatively heavy machines.	30.8–30	400–420	450–470	175–190
Fine and light machine oils: For fine work, such as printing presses, sewing machines, typewriters, spindles, etc.	32.5–30.2	400	440	110–160
Cutting oils: For cutting tools, screws, etc.	27–23	410–420	475–480	210–175
Refrigerating machine oils	30.2	200	225	165
Wet service and marine oils§	28	430	475	230
Greases: Various kinds, used in special work requiring high pressures and low velocities.	......	....	....	......

* May contain not over 2 to 6 per cent of refined acidless tallow oil in the high-pressure oils and not over 6 to 12 per cent in the low-pressure oils.

† Neutral mineral oil compounded with soap. The soap will not decompose at high heat, and although not a lubricant serves as a vehicle for carrying some oil.

‡ Owing to lack of body this oil will not deposit carbon on the sparking points.

§ May contain 30 to 40 per cent of pure strained lard oil.

Oil-feed Systems. — In power plants oil is supplied continuously to the bearing surfaces of the various engines and generators by means of an oil-feed system, comprising essentially a supply tank, pump and the necessary piping. Oil filters and purifiers are also used in connection with the oiling system, to eliminate the impurities which collect in it due to dust, wear of bearings, exposure to the heat and to the atmosphere. See article on *Power Stations*.

AMOUNT OF OIL REQUIRED FOR ENGINES. — J. H. Spoor, in *Power*, Jan. 4, 1910, has made a study of a great number of records of the amount of oil used for lubricating cylinders of different engines, and has reduced them to a systematic basis, i.e., the number of pints of oil used in a 10-hour day for different areas of surface lubricated. The surface is determined in square inches by multiplying the circumference of the cylinder by the length of stroke. The results are plotted in a series of curves for different types of engines, and approximate average figures taken from these curves are given below:

PINTS OF CYLINDER OIL IN 10 HOURS

Type of Engine	Square inches lubricated								
	1000	2000	3000	4000	6000	8000	10,000	12,000	18,000
Automatic high speed	2	....	....	....	....	....	....	....	....
Simple slide valve..	0.5	....	....	....	....	....	....	....	....
Compound..........	....	2	....	3.5	4.3	5	5.5	6	6.5
Corliss:									
Average...........	0.9	1.65	2.25	3.75	....	....	....	....	....
Maximum.........	1.2	2.25	....	....	....	....	....	....	....
Minimum.........	....	1.00	....	....	....	....	....	....	....

As shown in the figures under 2000, Corliss, a certain engine may take 2¼ times as much oil as another engine of the same size. The difference may be due to smoothness of cylinder surface, kind and pressure of piston rings, quality of oil, method of introducing the lubricant, etc. Variations in speed of a given type of engine and in steam pressure do not appear to make much difference, but the small automatic high-speed engine takes more oil than any other type. Vertical marine engines are commonly run without any cylinder oil, except that used occasionally to swab the piston rods.

The amount of engine oil required will of course depend upon the number of cups on the engine and the size of the various bearings. For a 1000-h.p. Corliss engine the Vacuum Oil Company state that the amount of engine oil would not exceed twice the amount of cylinder oil required.

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MACHINE TOOLS, ELECTRICAL OPERATION OF. — (*See also Motors, Industrial Applications of.*) When the work of equipping a machine tool with motor drive is undertaken, there are certain features which should be taken into account and properly analyzed, as the conditions of operation generally vary greatly with the product manufactured. If a tool is intended for a certain specialized kind of work, information on the following points should be given:

1. The exact class of work which the tool is to accomplish.
2. If the power required to remove the metal is not known, then a statement should be made as to the approximate feed and cutting speeds to be taken.
3. Careful analysis should be made of the time required to load and unload the machine, to determine the feasibility of employing auxiliary means other than manual labor for loading the tools.

From this information an approximate determination can be made as to the intermittency of operation of the tool, in order to decide whether an intermittently rated motor or a continuously rated motor will be required. From a knowledge of the physical shape of the work, a determination can be made as to whether an adjustable-speed motor will result in economy of time, if used on this particular class of tool. The tool builder can then decide upon the proper type of controller, and its most desirable location from an operating point of view for the workman.

If a special type of tool is not desired and it is preferable to purchase one with such characteristics that it can be used for general manufacturing, one should determine as nearly as possible the range of material or work for which it will be used in straight manufacturing operations.

TYPE OF MOTOR. — The following table will aid in the choice of the proper motor for machine tool application.

It must be kept in mind that various circumstances, such as size or roughness of work, flywheel capacity, etc., may call for radical departures in choice of motors, this list being compiled to meet average conditions.

Shunt Motors are used in the following cases: when the work is of a fairly steady nature; when considerable range of adjustment of speed is required, as on lathes and boring mills, and on group and line-shaft drives, etc.

Compound-wound Motors are used where there are sudden calls for excessive power of short duration, as on belt driven planers, punch presses, etc. When a 50 per cent shunt, 50 per cent series, field is used, a motor is obtained which will develop nearly as much torque per ampere as a series motor with the advantage that light load speed will not be greater than 150 per cent full load speed.

Series Motors should be used where speed regulation is not essential and where excessive starting torque and slow starting speeds are required, as, for instance, in moving carriages of large lathes, in raising and lowering the cross rails of planers and boring mills, and for operating cranes.

When in doubt as to the choice of compound or series motors of small horsepower, the choice might be determined by the simplicity of control in favor of the series motor. Series motors, however, should never be used when the motor can run without load, as the speed would accelerate beyond the point of safety.

Induction Motor. — The alternating-current motor of the squirrel-cage rotor type corresponds to the constant-speed, shunt, direct-current motor, but with a high-resistance rotor it approaches more closely the characteristics of a compound direct-current motor. It is understood that the variable-speed machines, checked in this list under the alternating-current squirrel-cage rotor column, have the necessary mechanical speed changes.

MOTORS FOR MACHINE TOOLS

Tool	D-C.			A-C. (See Footnotes)		
	Shunt	Comp., %	Series	×	✱	⊕
Bolt cutter	✓	....	....	×	..	..
Bolt and rivet header	....	20	....	×	✱	..
Bulldozers	....	20	....	×	✱	..
Boring machines	✓	....	....	×	..	..
Boring mills	✓	....	....	×	..	..
Raising cross rails on boring mills and planers	....	20–50	✓	..	✱	..
Boring bars	✓	....	....	×	..	..
Bending machines	....	20–50	....	×	✱	..
Bending rolls	....	50	....	..	..	⊕
Corrugating rolls	....	20–50	....	×	✱	..
Centering machines	✓	....	....	×	..	..
Chucking machines	✓	....	....	×	..	..
Boring, milling and drilling machines	✓	....	....	×	..	..
Drill, radial	✓	....	....	×	..	..
Drill press	✓	....	....	×	..	..
Grinder — tool, etc	✓	....	....	×	..	..
Grinder — castings	✓	....	....	×	..	..
Gear cutters	✓	....	....	×	..	..
Hammers — drop	....	20	....	..	✱	..
Keyseater — milling — broach	✓	....	....	×	..	..
Keyseater — reciprocating	✓	20	....	×	..	..
Lathes	✓	....	....	×	..	✢
Lathe carriages	....	....	✓	..	✱	..
Milling machines	✓	....	....	×	..	..
Heavy slab milling	✓	....	....	×	..	..
Pipe cutters	✓	....	....	×	..	..
Punch presses *	....	20	....	×	✱	..
Planers — belt driven	....	20	....	×	..	..
Planers — reversing motor	✓	....	....	×	..	..
Planers — rotary	✓	....	....	×	..	..
Saw — small circular	✓	....	....	×	..	..
Saw — cold bar and I beam	....	20	....	×	..	..
Saw — hot	....	20	....	×	..	..
Screw machine	✓	....	....	×	..	..
Shapers	✓	20	....	×	..	..
Shears	....	20	....	×	✱	..
Shears — rotary type	✓	....	....	×	..	..
Swaging	✓	20	..	×	✱	..
Tappers	✓	....	....	×	..	..
Tumbling barrels or mills—indiv.	....	20	....	×	..	..
Tumbling barrels or mills—group	✓	....	....	×	..	..

× Squirrel-cage rotor. ✱ Squirrel-cage rotor — high starting torque.

⊕ Slip-ring induction motor with external rotor resistance.

✢ Might be used for tire lathes as it allows slowing down when cutting hard spots.

* Small punch presses running at high speed can be driven by shunt motors.

The slip-ring induction motor with external rotor resistance would be used for variable speed, but this must not be construed to mean that it corresponds to a direct-current, adjustable-speed motor, as it has the characteristics of a direct-current shunt motor with armature control.

The self-contained, rotor-resistance type would be used for line-shaft drives, and for groups when of sufficient size.

Multi-speed Alternating-current Motors are those giving a number of definite speeds, usually 600 and 1200 or 600, 900, 1200 and 1800 r.p.m., and are made for both constant horse-power and constant torque. These motors would be used where alternating current only was available, or direct current limited; and the speed range of the motor, together with one or two change gears, would give the required speeds.

Shaft Couplings. — In connection with the selection of motors, standard shafts and shaft extensions should be chosen so that spare parts and interchanges may be made with the least cost and time. A number of standard shafts and shaft extensions are shown in the sketches in Fig. 1.

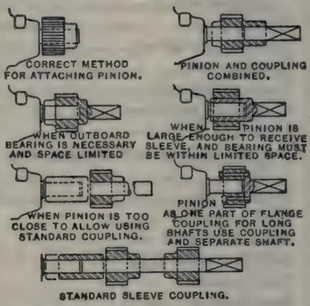

Fig. 1. Standard Shaft Couplings

CONTROL EQUIPMENT. — The choice of control, whether it be for old or new tools, in the majority of cases is fully as important as that of the motor. In selecting the control it is necessary to consider the nature of the work, the accessibility of the controller to the operator, the method of attaching it to the tool and in some cases its relative position to other tools; for instance, an open-type starting rheostat should not be exposed to danger of short-circuit from flying chips.

When installing controllers, accessibility in case of accident should be kept in mind, even though of little importance as far as starting up is concerned. The starting apparatus should be placed where the motor or some of the moving parts can be seen by the operator. On individual motor-driven tools, where the motor is started and stopped many times a day or where the starting conditions are of a severe nature, or where tools are edged along, drum-type controllers with extra heavy starting resistance should be used. For adjustable speed motors, using the drum-type control, the field control should be through fingers making contact on segments of the controller drum and not by sliding contacts on a dial. Motors above 40 or 50 horse-power under these severe conditions are best operated by a master controller which operates contactors for cutting out steps of starting resistance, and if adjustable speed, the field control should be taken care of by fingers making contact on segments of the drum. This class of starting apparatus will stand any quantity of abuse and, by the addition of a simple current limit relay device, becomes practically a fool-proof protection for the motor. There are cases where it might be advantageous to use master controllers and contactors even with smaller motors. The controlling apparatus as well as the motor in the case of individual drives should be attached directly to the tool when possible. This arrangement allows moving the tool by simply disconnecting the leads and connecting them in the new position. In case of portable tools this, of course, is an absolute necessity.

Upon the convenient arrangement of the control depends, to a considerable degree, the output of the tool. The importance of the arrangement from the standpoint of the operator cannot be ignored, since the output of a tool will be

materially increased when an operator can start and stop the tool and obtain at all times maximum cutting speeds by simply turning a handle. The controller must be placed in a safe position and should be accessible for repairs, which very often means that some arrangement is necessary to bring the operating handle within easy access of the operator.

The convenience of control, which bears directly on production, is ignored in the majority of tools where the control is of the greatest importance. A familiar illustration of the convenience of control is the arrangement so commonly seen on lathes, whereby the operating handle travels with the tool carriage and allows the operator at all times a complete control of his tool.

APPLICATION OF REVERSING MOTORS. — One of the most interesting motor applications is the use of reversing motors for machine tools. The large increase in production due to this form of drive on planers is generally appreciated, but the application of the reversing motor drive in its various forms (which is almost unlimited) is not so well understood. This reversing drive is applicable not only to planers, new and old, but to screw, worm and rack-driven slotters, keyseaters, turret lathes, wire and tube drawing machinery, grinder tables, lapping machines and to boring mills, when machining projections which are short in comparison with the total travel of the mill or when machining surfaces where projections prevent a complete revolution.

The motors recommended for this service are of the commutating pole type with a speed range usually of from 300 to 1200 r.p.m. for the small sizes and 250 to 1000 r.p.m. for the larger sizes. Other speeds can be obtained when required. The speeds given allow the motor, in the majority of cases, to be coupled direct to the driving shaft of the machine.

Starting, stopping and reversing are accomplished in remarkably short periods of time. It is considered the best practice to bring the motor to rest by means of dynamic braking.

On all reversing motor equipments, an emergency brake must be provided so the motor will be brought to rest quickly, when the voltage fails. Undervoltage protection should always be used.

Reversing Planers. — These equipments should provide quick reversals, the motor being brought to rest before each reversal, preferably by dynamic braking. This method will insure the motor, and consequently, the planer, coming to rest at practically the same point, for the various conditions encountered. This is especially important when planing up to a shoulder or in a pocket, where any overtravel would damage the work or the machine.

It is advisable to use an equipment so interlocked that it is impossible to connect the panel to the power supply with the master switch in a running position. If this feature is not used, an unexpected start is liable to result. "Jogging" by means of a portable push button is desirable for setting up work.

The "cutting" and "returning" speeds should be independently adjustable, and thirty-five or more cutting speeds provided.

Example. — In Fig. 2 curves *A*, *B* and *C* are for motor No. 2 and show the ampere input of a 10 horse-power, 1250 r.p.m. motor driving a 36-inch modern type planer through shifting belts. *A* is the return stroke of the planer table at a speed of 68 feet per minute; *C*, the cutting stroke, without cut, at 33.3 feet per minute; *B*, the same cycle but with a cut slightly less than 10 horse-power. The lost time on the cutting stroke, due to the belt slipping, is plainly seen, the cutting speed falling off from 33.3 feet to 29.4 feet or 13 per cent. Curves *E*, *F* and *G* are made on the same machine when driven by motor No. 1, a 10-horse-power, 250 to 1000 r.p.m. reversing motor, direct connected. These curves are superimposed on the above curves for comparative purposes. No attempt was made in this set of curves to duplicate the slow cutting or return speeds of the

belt-driven machines as the comparison would have shown power differences only. *E* is the return stroke of the planer table at a speed of 88.7 feet per minute;

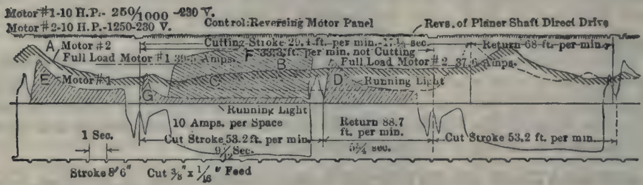

Fig. 2. Tests on 36 in. by 10 ft. Reversible Planer

G, the cutting stroke at 53.2 feet per minute; *F*, the same cycle but with a cut of approximately 13 horse-power. The speed drop in this case is motor slow-down only. For comparison the speed of motor No. 1, curve *F*, was chosen as the most economical speed under the conditions which the test was made. The loss in time of the belt drive as compared with the direct-connected reversing-motor drive (the depth of cut and feed being the same in both cases) is 62 per cent.

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MAGNETIC PROPERTIES OF IRON AND OTHER METALS.

— (*See also Electricity and Magnetism, Principles of; Magnetic Testing.*) For the definitions of magnetizing force and flux density see *Electricity and Magnetism, Principles of;* for the relations of the various units in which these quantities are expressed see *Units and Conversion Factors.*

Hysteresis Loop — Residual Magnetism and Coercive Force. — When a magnetic substance which is not magnetized initially is placed in a magnetic field the intensity of which is increased from O to H_m, the flux density produced

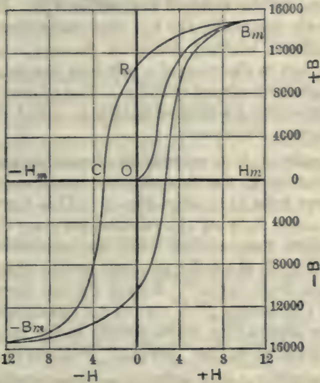

Fig. 1.

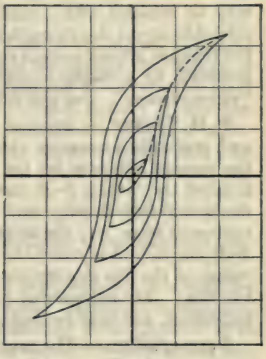

Fig. 2.

in the magnetic substance increases in the manner shown by the curve OB_m in Fig. 1. If the magnetizing force is then decreased to a value H the flux density does not return to the value corresponding to this value of H on the ascending curve, but decreases less rapidly than it increased. This phenomenon is known as "magnetic hysteresis."

When the magnetizing force is reduced to zero the flux density in general has a considerable value (OR in Fig. 1); this value of B is called the "remanent" or "residual" magnetism. To reduce the flux density to zero the magnetizing force must be reversed and increased in the reversed direction to a value OC, called the "coercive force." As the magnetizing force is still further increased in the reversed direction to a value numerically equal to the positive maximum, and then decreased to zero, reversed, and increased again to H_m, the flux density passes through the cycle of values represented by the closed loop. This closed loop is called the "hysteresis loop."

If the iron (or other magnetic substance) is not originally unmagnetized, this hysteresis loop will be shifted above or below the axis of H, but after a number of reversals of the magnetizing force between given positive and negative values, the loop will become practically symmetrical with this axis, particularly if the iron is continually jarred. In the armatures of electrical machines and the cores of transformers, in which the field intensity reverses a large number of times every second, and the iron is continually jarred, the relation between flux density and field intensity after a short interval of time is represented by a symmetrical loop of the form shown in Fig. 1.

The area inclosed by the hysteresis loop depends upon the maximum value of the flux density reached during the cycle, but the general shape remains

about the same. Fig. 2 shows a series of loops corresponding to various values of the maximum flux density. The area of the loop is also different for various kinds of iron or steel. $\frac{1}{4\pi}$ times the area of this loop when B and H are plotted to scale is equal to the ergs of heat developed in the iron per cubic centimeter per cycle. (*See section on Hysteresis Loss, below.*)

Permanent Magnets — Retentiveness. — An examination of this hysteresis loop also makes clear how a bar of steel may be permanently magnetized by placing it in a magnetic field. For, when the bar is removed from the field it retains a flux density approximately equal to the residual magnetism OR. In the case of a cast-iron or steel bar, properly hardened, the bar thus magnetized may be handled with comparative roughness without reducing to any considerable extent the strength of its poles, but in the case of a soft-iron bar even the slightest jar will cause it to lose its magnetism almost entirely. The property possessed by a magnetic substance of retaining its magnetization is called its "retentiveness." See also article on *Magnets, Permanent.*

B-H Curves. — Starting with a sample of iron completely demagnetized, and gradually increasing the magnetizing force, the flux density increases in the manner indicated by the middle curve from O to B in Fig. 1. This curve is called the "rising B–H curve." The curve showing the relation between the maximum flux density and the maximum magnetizing force for successive hysteresis loops, i.e., the dotted curve in Fig. 2, is similar in shape to this rising B–H curve, and is called the "alternating B–H curve." Except for the lower values of the flux density the rising and alternating B–H curves are practically identical. For engineering purposes, the alternating characteristic is the more important, and may be conveniently referred to as the "normal" B–H curve.

Permeability. — The magnetic permeability μ of a substance corresponding to any degree of magnetization is usually defined as the quotient of the flux density B by the magnetizing force H, that is

$$\mu = \frac{B}{H}.$$

On account of the hysteresis effect, however, this quotient may have any value within wide limits depending upon how the magnetization is produced. Consequently a more restricted definition of permeability is required in the case of highly magnetic substances like iron and steel. For such substances the normal permeability for any value of the flux density is taken as the ratio of $\frac{B}{H}$ corresponding to this flux density on the normal B–H curve.

The relation between permeability and magnetizing force or flux density is a complex one. As the flux density increases, μ reaches a maximum at a relatively low flux density and then decreases ultimately to unity when the magnetizing force reaches a value so large that the intensity of magnetization J is negligible in comparison with H. This limiting condition, however, is never reached in practice.

Reluctivity. — Magnetic reluctivity is defined as the reciprocal of magnetic permeability, viz.,

$$\text{Reluctivity} = \rho = \frac{1}{\mu} = \frac{H}{B}.$$

Typical reluctivity curves are given in Fig. 6, corresponding to the B–H curves in Figs. 3 to 5.

Intensity of Magnetization. — The difference between the flux density B

in a magnetic substance corresponding to a given magnetizing force H and the flux density (numerically equal to H) which this same magnetizing force H would produce in free space, divided by 4π, is called the intensity of magnetization, viz.,

$$J = \frac{B - H}{4\pi}.$$

Susceptibility. — The quotient of the intensity of magnetization J divided by the magnetizing force H is called the magnetic susceptibility κ corresponding to this magnetizing force; viz.,

$$\kappa = \frac{J}{H} \quad \text{whence} \qquad \mu = 1 + 4\pi\kappa.$$

Magnetic Saturation. — Experiment shows that up to the highest values of the magnetizing force which have yet been produced, the normal flux density B always increases with an increase in the magnetizing force H. However, such experiments indicate that the difference between the flux density B and the magnetizing force H, that is, the difference $(B-H)$, approaches a definite maximum limit, dependent solely upon the nature of the material under test. In other words, for any given magnetic substance there is a definite limit to the intensity of magnetization $\left(J = \frac{B - H}{4\pi}\right)$ which can be established in it. When this limiting value of the intensity of magnetization is reached, the substance is said to be "magnetically saturated."

The difference $(B-H)$, when B and H are both expressed in c.g.s. electromagnetic units, may be conveniently designated as the "metallic" component of the flux density corresponding to a given magnetizing force H, since it is equal to the excess of the resultant flux density over the flux density which this same magnetizing force would produce in free space. The maximum, or saturation, value of this metallic flux density is about 20,000 gausses for soft iron and steel, 15,000 gausses for cast iron, 12,000 gausses for cobalt, and 6000 gausses for nickel. To increase the flux density beyond these values requires an increase in the magnetizing force equal to the increase in flux density desired.

An inspection of the reluctivity curves in Fig. 6 shows that for values of the magnetizing force greater than about 40 ampere-turns per inch (or 20 gilberts per centimeter), the relation between reluctivity and magnetizing force is approximately a straight line one. Due to the fact that there is a definite magnetic saturation point, the reluctivity curves actually have a slight downward droop. If instead of the reluctivity $\rho = \frac{H}{B}$, the ratio $\frac{H}{B - H}$ is plotted against the magnetizing force, it is found that for very high values of H, this ratio bears an exact linear relation to the magnetizing force H, viz., that

$$\frac{H}{B - H} = \alpha + \sigma H \qquad (5)$$

where α and σ are constants for any given material. Steinmetz calls the constant α the "coefficient of magnetic hardness" and the constant σ the "coefficient of magnetic saturation."

FACTORS AFFECTING THE PERMEABILITY AND HYSTERESIS LOSS. — The normal permeability and the hysteresis loss (the latter is proportional to the area of the hysteresis loop) depend to a very great extent upon the physical structure and chemical constitution of the sample and the heat treatment to which it has been subjected. It has also been recently discovered (*Pender and Jones, Phys. Rev., 1913*) that when sheet steel is annealed in an alternating magnetic field, the permeability is increased in certain cases as much

as 50 per cent, but there is no appreciable change in the hysteresis loss. The B–H curves of two samples taken from the same lot of material may even differ considerably. The permeability and hysteresis loss also depend to a slight extent upon whether the sheets are magnetized in the direction of rolling, or transverse thereto, being higher in the latter case.

Temperature and Aging. — Permeability and hysteresis also depend upon the temperature of the sample at the time the observations are taken, though the variation due to ordinary changes of temperature is slight. For very high temperatures, however, all magnetic substances become practically non-magnetic. This temperature corresponds to the major recalescence point, which is about 750° C. for steel of the quality used in armature and transformer punchings. When steel is kept continuously at a moderately high temperature (100° C.), the hysteresis loop also gradually increases in size, and therefore the energy loss in the magnetic circuits of electric machines due to hysteresis increases with time. This effect is called "aging." There is practically no aging of silicon steel.

Chemical Composition of "Electrical" Sheet Steel. — Sheet-steel manufacturers make a special grade of sheet for electrical purposes, which they sell under various trade names. Such steel is always low in carbon content and, except silicon in the so-called silicon steels, all impurities are reduced to small amounts. Silicon steel, i.e., steel containing about 3 per cent silicon is also extensively used, particularly for the magnetic circuit of transformers. The permeability of this steel is somewhat lower, in the useful range of flux densities, than that of ordinary electrical steel, but the area of the hysteresis loop, and therefore the hysteresis loss, is from 40 per cent to 60 per cent less; the specific resistance of silicon steel is also about 3 times greater than that of ordinary electrical steel, resulting in a reduction of about 70 per cent in the eddy current loss (*see curves below*). Aluminium has much the same effect as silicon, but the aluminium alloy is not so easily rolled.

The following are typical chemical analyses of the two kinds of electrical steel, but it should be understood that considerable variations in the proportions of the various constituents occur in practice.

	Ordinary electrical steel *	Silicon steel
	Per cent	Per cent
Silicon	0.01	3.46
Phosphorus	0.08	0.04
Manganese	0.50	0.13
Sulphur	0.03	0.02
Carbon	0.06	0.06

* Parshall & Hobart, Electric Machine Design.

Of a large number of electrical steels analyzed by the Bureau of Standards (*Lloyd and Fisher, Trans. A.I.E.E., 1909, Vol. 28, p. 463*) none showed more than the slightest trace of vanadium.

Annealing of Sheet Steel. — Sheet steel as it comes from the rolling mill may be greatly improved in magnetic properties by proper annealing. The chief requirement seems to be that the steel be brought to a temperature about 100° C. above the major recalescence point, i.e., to a temperature of about

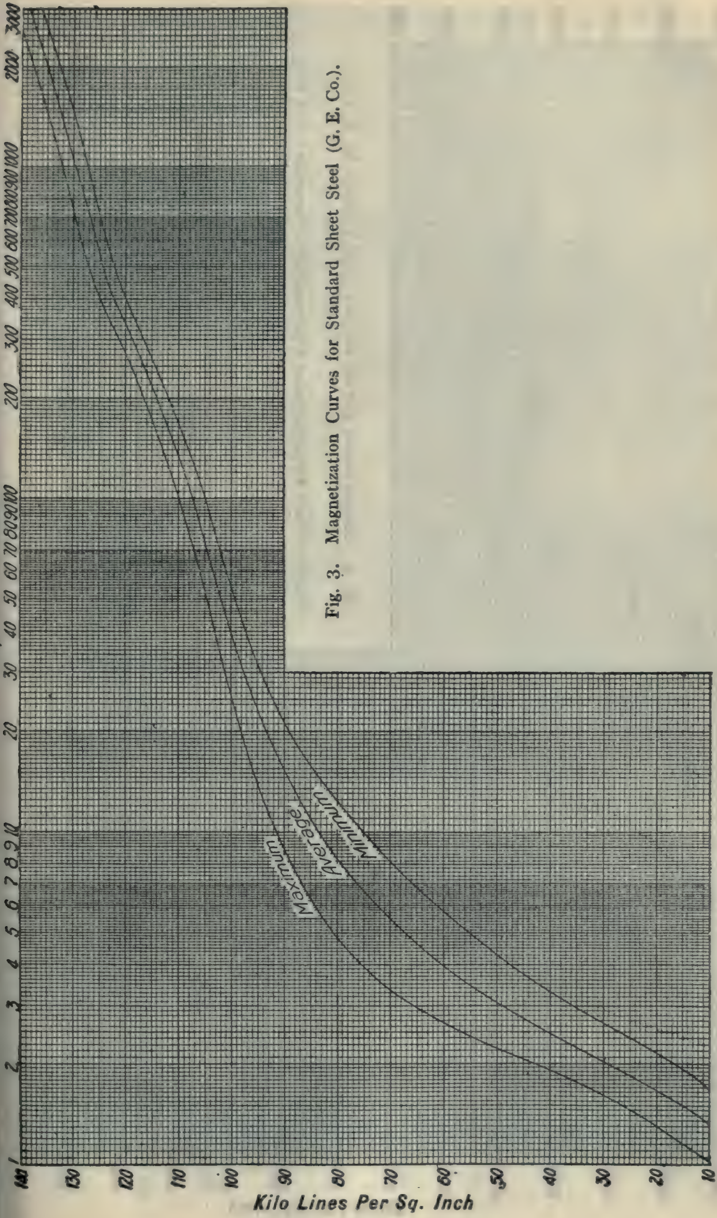

Fig. 3. Magnetization Curves for Standard Sheet Steel (G. E. Co.).

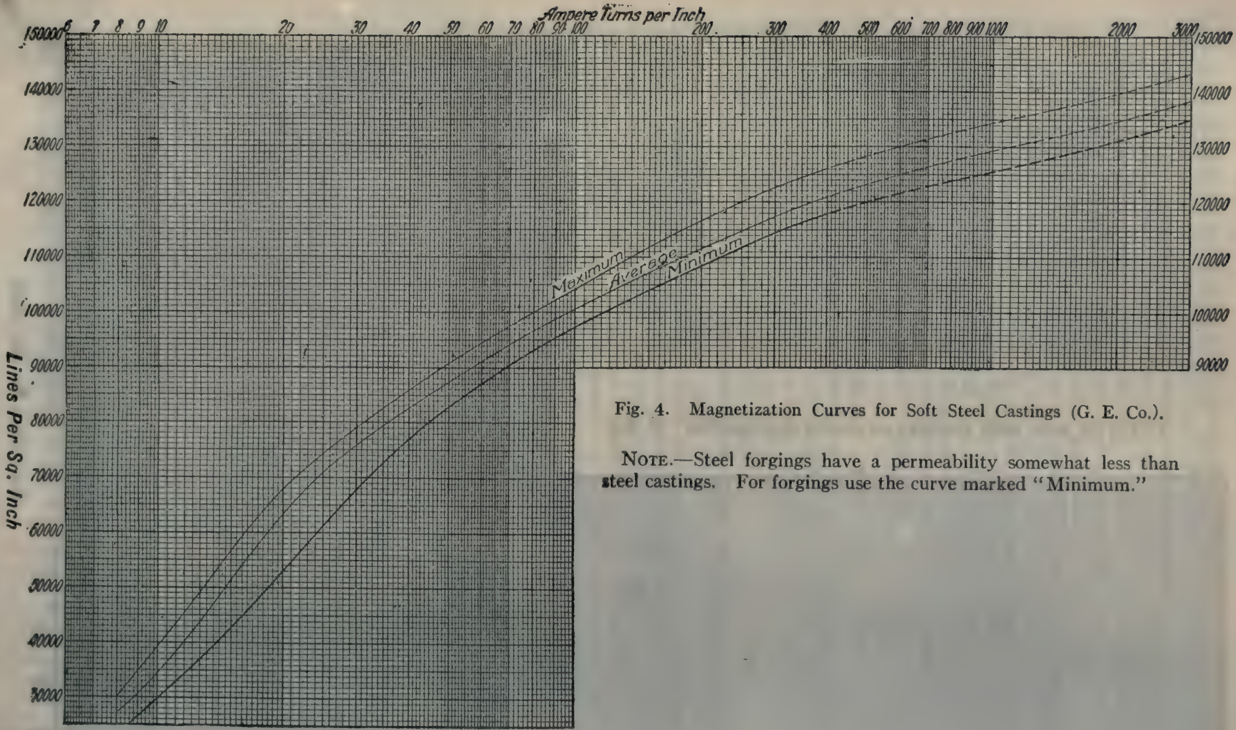

Fig. 4. Magnetization Curves for Soft Steel Castings (G. E. Co.).

NOTE.—Steel forgings have a permeability somewhat less than steel castings. For forgings use the curve marked "Minimum."

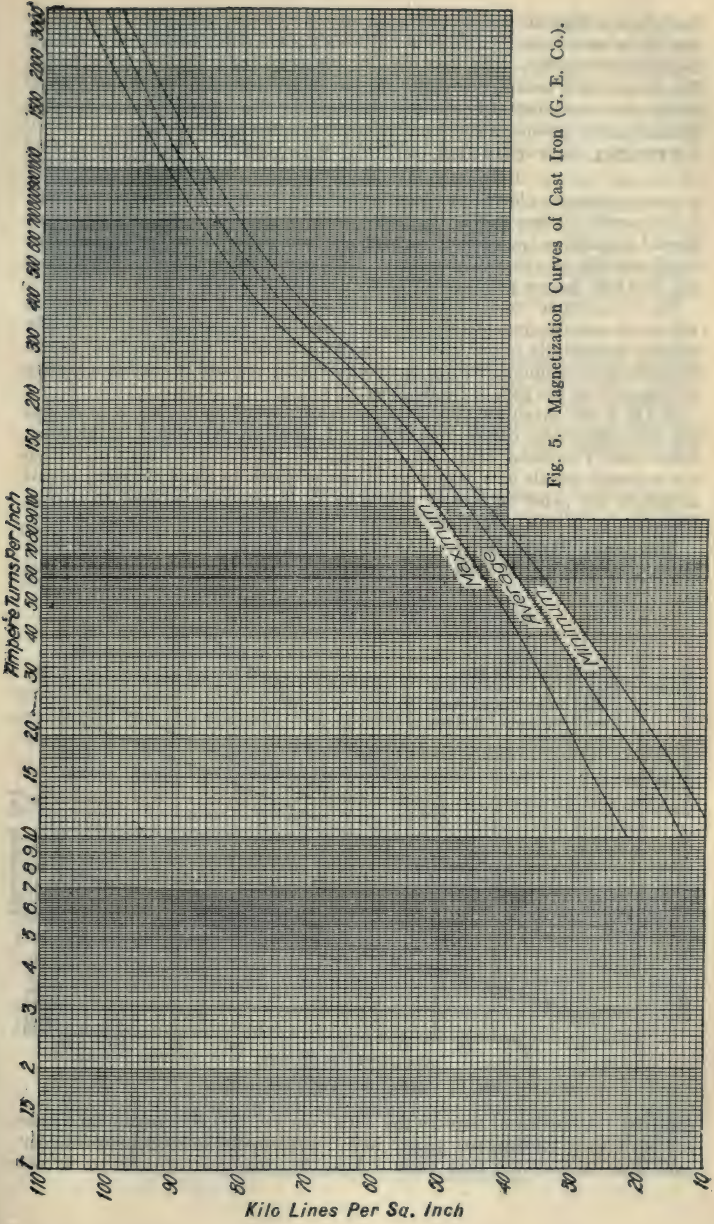

Fig. 5. Magnetization Curves of Cast Iron (G. E. Co.).

850° C., and then allowed to cool slowly down to a temperature of from 100 to 150° C., when it may then be removed from the annealing furnace and allowed to cool more rapidly. The time required for annealing is from 12 to 36 hours. The annealing is usually done after the punchings have been made, thus eliminating the hardening at the edges produced by the cutting; otherwise this hardening may produce a considerable increase in the hysteresis loss.

TYPICAL *B-H* CURVES. — In Figs. 3 to 5 are given the standard *B-H* curves used by one of the large manufacturing companies. The abscissas (H) of these curves are plotted to a logarithmic scale, in order to increase the range of the curves. These curves represent results obtainable under ordinary commercial conditions (joints not included) on iron and steel of the composition found suitable for electrical purposes. Ordinary commercial iron or steel will not, as a rule, have a permeability as high as given by these curves.

The "maximum" and "minimum" curves in Figs. 3 to 5 do not refer to individual samples, but to the average of, say, the ten highest and ten lowest samples respectively in one hundred samples included in the "average" curve. Whether the maximum, average or minimum curve should be used is a matter of judgment on the part of the designer.

In Fig. 6 are given the reluctivity curves corresponding to Figs. 3 to 5, and also the reluctivity curves for nickel and cobalt as determined by Fleming, Ashton and Tomlinson (*Phil. Mag., 1899, Vol. 48, p. 271*). Alloys of certain non-magnetic metals have been found to be magnetic to about the same extent as nickel; see paper on *Heusler Alloys* by E. B. Stephenson, *Bull. 47 (1911), University of Illinois, Eng. Exp. Stat.*

Maximum Permeability. — As indicated by the curves in Fig. 3, the maximum permeability of sheet steel occurs below the range of commercial flux

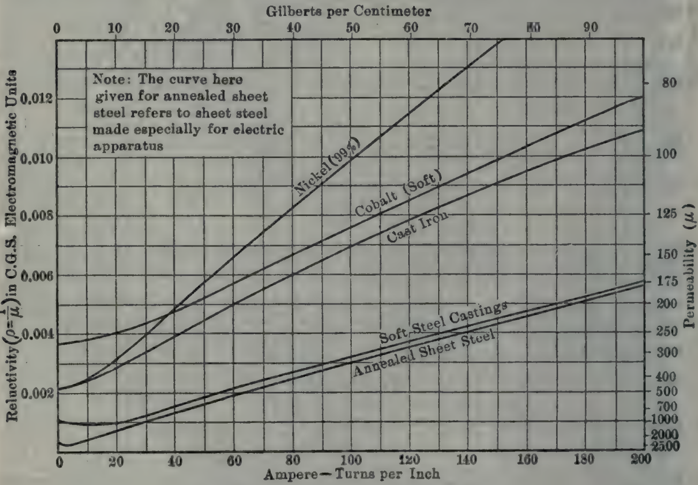

Fig. 6. Reluctivity and Permeability

densities. A number of tests by the author showed that this maximum occurs at from 6 to 10 kilolines per square inch, the permeability decreasing rapidly with flux densities less than these values, the complete curve being similar

in shape to the curves shown in Fig. 6. The author has obtained a maximum permeability of about 9000 c.g.s. units with ordinary electrical sheet steel, and a maximum of 13,000 c.g.s. units with silicon steel carefully annealed in small lots in an alternating magnetic field (*Pender and Jones, Phys. Rev., 1913*). A permeability as high as 72,000 has been obtained by Yensen for pure iron heat-treated in a vacuum (*Elec. Rev., London, 80, p. 539, 1917*). These exceptionally high permeabilities, however, are not obtained in practice under commercial conditions of annealing.

CORE-LOSSES. — When a varying magnetic field is established in a magnetic substance a certain amount of heat is developed due (1) to hysteresis and (2) to the electric currents induced in the conducting mass. The induced currents are called eddy currents, and the total amount of energy dissipated as a result of hysteresis and eddy currents is known as the core-loss, i.e.,

$$\text{Core-loss} = \text{hysteresis loss} + \text{eddy-current loss}.$$

Hysteresis Exponent. — As already noted, the hysteresis loss is proportional to the area of the hysteresis loop. Steinmetz (*Elec. Eng., 1890; Trans. A. I. E. E., 1892, Vol. 9, p. 3*) found from a series of tests on a large number of samples that the area of the hysteresis loop for a given sample is approximately proportional to the 1.6 power of the maximum flux density corresponding to the tip of the loop.

Hysteresis Coefficient. — If the magnetic field throughout the iron is *uniform*, and the hysteresis exponent is assumed constant and equal to 1.6, then the power loss can be expressed by the formula

$$P_h = KVfB^{1\cdot 6} \quad \text{or} \quad P_h = KWfB^{1\cdot 6},$$

where V = volume of iron,
f = frequency of alternating field, in cycles per second,
B = maximum flux density,
W = weight or mass of the iron.

The value of the factor K depends upon the units in which the other quantities are expressed. If V is in cubic centimeters, B in gausses, f in cycles per second and P_h in ergs per second the formula is usually written

$$P_h = \eta VfB^{1\cdot 6}.$$

That is, the loss in ergs per cycle per cubic centimeter is $\eta B^{1\cdot 6}$. The factor η is known as Steinmetz's hysteresis coefficient and the quality of iron or steel with respect to hysteresis is frequently expressed in terms of this constant.

The hysteresis coefficient η is not absolutely constant, even for a given sample, at least if there is any heterogeneity in the material of the sample, as, for example, an appreciable amount of scale on the sheets of which the sample may be composed. Due to such heterogeneity the value of η increases with the flux density, the amount of increase depending upon the degree of heterogeneity. See Ball, J. D., *Investigation of Magnetic Laws for Steel and Other Materials, Journal of the Franklin Institute, April, 1916.*

The hysteresis loss is also sometimes stated as the watts per pound at 10,000 gausses and 60 cycles. Let w_1 = hysteresis loss in watts per pound at 10,000 gausses and 60 cycles. Then the corresponding value of η is:

For Specific Gravity = d	Specific Gravity = 7.7	Specific Gravity = 7.5
$\eta = 0.000146\, w_1 d$	$\eta = 0.0013\, w_1$	$\eta = 0.00110\, w_1$.

Note that w_1 is the hysteresis loss only, excluding the eddy-current loss, which is discussed below.

The following table gives the value of K in terms of η when the various quantities in the formula for hysteresis loss are in the units stated.

FORMULA FOR CALCULATING HYSTERESIS LOSS

$$P_h = KVfB^{1.6}$$

Power, P_h	Volume, V	Frequency, f	Flux density, B	$K =$
Ergs per sec.	Cu. cm.	Cycles per sec.	Gausses	η
Watts	Cu. cm.	Cycles per sec.	Kilogausses	$0.00631\,\eta$
Watts	Cu. in.	Cycles per sec.	Kilogausses	$0.1035\,\eta$
Watts	Cu. in.	Cycles per sec.	Kilolines per sq. in.	$0.00525\,\eta$

$$P_h = KWfB^{1.6}$$

Power, P_h	Weight, W	Frequency, f	Flux density, B	$K =$
Ergs per sec.	Grams	Cycles per sec.	Gausses	$\frac{\eta}{d}$
Watts	Kilograms	Cycles per sec.	Kilogausses	$6.31\,\frac{\eta}{d}$
Watts	Pounds	Cycles per sec.	Kilogausses	$2.86\,\frac{\eta}{d}$
Watts	Pounds	Cycles per sec.	Kilolines per sq. in.	$0.145\,\frac{\eta}{d}$

d = specific gravity = 7.7 for ordinary electrical sheets = 7.5 for silicon steel.

Hysteresis Loss for Various Substances. — The hysteresis loss in iron and steel and other magnetic metals depends to so great extent upon their chemical composition, physical structure, heat treatment, etc., that average

VALUES OF HYSTERESIS COEFFICIENT

η = ergs per cycle per cubic centimeter for B = 1 gauss

w_1 = watts per pound at 60 cycles for B = 10,000 gausses

Metal	Values of η		Values of w_1	
	From	To	From	To
Silicon steel, annealed sheets	0.0006	0.0015	0.55	1.36
Ordinary electrical sheets, annealed	0.00095	0.004	0.84	3.5
Soft cast steel	0.003	0.012	2.7	11
Cast iron	0.011	0.016	10	14
Forged steel	0.015	0.025	13	22
Hard cast steel	......	0.028	....	25
Cobalt	0.012	......	11	....
Nickel	0.013	0.040	12	35

values have no significance. The preceding table is intended to give the range in the value of the hysteresis coefficient for iron and steel used in electrical machinery; figures for nickel and cobalt are also included-

The extreme low values of η can seldom be uniformly realized in practice. One of the large manufacturing companies uses for design purposes the values $\eta = 0.00145$ for silicon steel and $\eta = 0.0033$ for ordinary electrical sheets, these values allowing a considerable margin for variations in the quality of the steel. For close design the particular quality of steel to be used should be carefully tested (*see Magnetic Testing*) and the test results used.

In Fig. 7 are plotted hysteresis loss curves for various values of η assuming the $B^{1.6}$ law.

Eddy-Current Loss. — When an alternating magnetic field is established in a conducting material alternating currents are set up in it due to the alter-

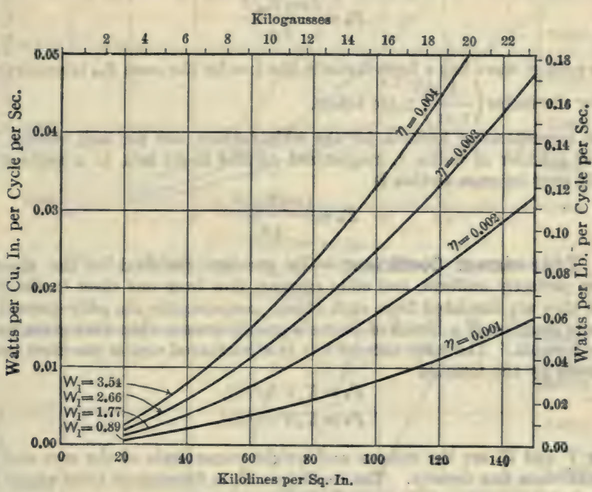

Fig. 7. Hysteresis Loss

nating e.m.f. produced by the alternating field. Consider a sheet of iron, Fig. 8, subjected to a *uniformly* distributed alternating field parallel to its faces. The dots indicate the flux lines perpendicular to the page and the loops the induced currents. Let

B = flux density, in gausses,
x = thickness of plate, in centimeters,
ρ = specific resistance of plate, in abohms per cm-cube,
$\frac{dB}{dt}$ = rate of change of flux density with time, gausses per second.

Then the *instantaneous* power loss per cubic centimeter of the sheet, when the thickness x is very small (1 per cent or less) compared with the width of the plate, is

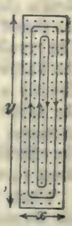

Fig. 8.

$$p = \frac{x^2}{12\rho}\left(\frac{dB}{dt}\right)^2.$$

The *average* power loss per cubic centimeter for a complete cycle of variation of the flux, in ergs per second, is

$$P_\epsilon = \frac{x^2}{12\,\rho} \times \left(\text{root-mean-square value of } \frac{dB}{dt}\right)^2.$$

It should be noted that this average loss depends upon the form factor (*see Alternating Currents*) of the rate of change of flux density, the latter being proportional to the counter e.m.f. induced by the varying field of the magnetizing coil.

If the induced voltage and therefore the flux density varies according to a sine function of the time, i.e., if $B = B_m \sin(2\pi ft)$, where B_m is the maximum value of the flux density, f the frequency, and t the time, then the average power loss due to eddy currents per cubic centimeter is

$$P_\epsilon = \frac{(\pi x f B_m)^2}{6\rho}.$$

If the voltage wave has a form-factor h, the loss for the *same* B_m is greater than this by the factor $\left(\frac{h}{1.11}\right)^2$; see below.

The corresponding formula for the eddy-current loss per unit volume in a wire or cylinder of radius r, magnetized *parallel* to its axis, to a *uniform* flux density over its cross section is

$$P_\epsilon = \frac{(\pi r f B_m)^2}{4\rho}.$$

Eddy-current Coefficient. — In practice the field in the sheets is seldom perfectly uniform, and eddy currents flow from one sheet to the other, unless they are insulated from each other; consequently the eddy-current loss in a core made up of a bunch of sheets is usually greater than that given by the above formula. The eddy-current loss in a laminated core is therefore usually expressed by the formula

$$P_\epsilon = K_1 V\,(xfB)^2,$$

or

$$P_\epsilon = K_1 W\,(xfB)^2,$$

where V and W are the volume and weight respectively of the core and B is the maximum flux density. The coefficient K_1 is determined from actual tests on a suitable sample (*see Magnetic Testing*).

The coefficient K_1 (aside from its dependence upon the units in which the various quantities are measured) depends upon the specific resistance of the iron, the wave shape of the induced voltage, the distribution of flux in the sheets, the degree to which the sheets are insulated from each other, and upon the shape of the magnetic circuit in so far as this affects the distribution of flux. The results of the tests by Lloyd and Fisher also indicate that K_1 depends upon the value of B, or, in other words, that the eddy-current loss does not vary directly as the square of the maximum flux density, all other conditions being the same. At high frequencies, above 100 cycles per second, the loss also increases less rapidly than the square of the frequency, due to the increase in the effective resistance of the sheets as the result of an action similar to the skin effect (q.v.) in a wire carrying a rapidly alternating current.

When the loss P_ϵ is expressed in watts, V in cubic centimeters, f in cycles per second, x in centimeters and B in gausses, the coefficient K_1 may be represented by the symbol ϵ, and the formula for eddy-current loss becomes

$$P_\epsilon = \epsilon V\,(xfB)^2.$$

The eddy-current loss is frequently stated as the watts per pound at 10,000 gausses and 60 cycles for a thickness of sheet corresponding to No. 29 on the sheet-steel gage (= 0.0141 inch = 0.0358 centimeter). Let w_2 = eddy-current

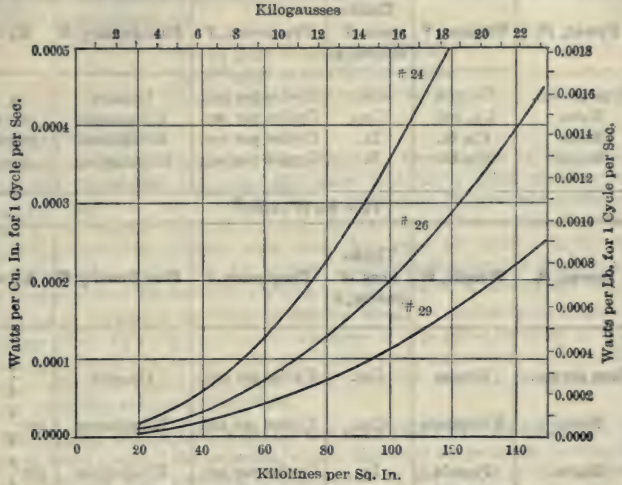

Fig. 9. Eddy-current Loss in Ordinary Electrical Sheet Steel

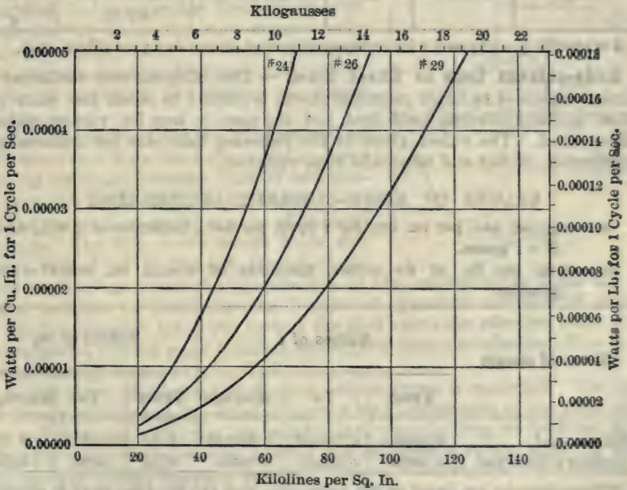

Fig. 10. Eddy-current Loss in Silicon Sheet Steel

loss in watts per pound at 10,000 gauzses and 60 cycles for sheets 0.0141 inch thick. Then the corresponding value of ϵ is:

Specific Gravity = d	Specific Gravity = 7.6	Specific Gravity = 7.5
$\epsilon = 0.0000478\, w_2 d$	$\epsilon = 0.000368\, w_2$	$\epsilon = 0.000359\, w_2$

FORMULA FOR CALCULATING EDDY-CURRENT LOSS *

$$P_\epsilon = K_1 V (xfB)^2$$

Power, P_ϵ	Volume, V	Thickness of sheets, x	Frequency, f	Flux density, B	$K_1 =$
Ergs per sec.	Cu. cm.	Cm.	Cycles per sec.	Gausses	ϵ
Watts	Cu. cm.	Cm.	Cycles per sec.	Kilogausses	0.1ϵ
Watts	Cu. in.	In.	Cycles per sec.	Kilogausses	10.58ϵ
Watts	Cu. in.	In.	Cycles per sec.	Kilolines per sq. in.	0.254ϵ

$$P_\epsilon = K_1 W (xfB)^2$$

Power, P_ϵ	Weight, W	Thickness of sheets, x	Frequency, f	Flux Density, B	$K_1 =$
Ergs per sec.	Grams	Cm.	Cycles per sec.	Gausses	$\frac{\epsilon}{d}$
Watts	Kilograms	Cm.	Cycles per sec.	Kilogausses	$100\frac{\epsilon}{d}$
Watts	Pounds	In.	Cycles per sec.	Kilogausses	$292\frac{\epsilon}{d}$
Watts	Pounds	In.	Cycles per sec.	Kilolines per sq. in.	$7.01\frac{\epsilon}{d}$

d = specific gravity = 7.7 for ordinary electrical sheets = 7.5 for silicon-steel.

Eddy-current Loss in Sheet Steel. — The eddy-current coefficient for various makes of ordinary electrical sheets is subject to much the same variation as the hysteresis coefficient, and the same is true for various makes of silicon steel. The values given in the following table are for uniform space distribution of flux and sinusoidal time variation.

VALUES OF EDDY-CURRENT COEFFICIENT †

ϵ = ergs per sec. per cu. cm. for 1 cycle per sec., thickness of 1 cm., and B = 1 gauss.

w_2 = watts per lb. at 60 cycles, thickness of 0.0141 in., and B = 10,000 gausses.

Kinds of sheets	Values of ϵ			Values of w_2		
	From	To	Average	From	To	Average †
Silicon steel.........	0.000043	0.000098	0.000065	0.12	0.27	0.180
Ordinary electrical..	0.00012	0.00025	0.00022	0.34	0.70	0.608

* These formulas are based on the assumption that the flux-time wave is sinusoidal. When this is not the case the total eddy-current loss is the sum of the losses which the several harmonics (including the fundamental) in the flux-time wave would produce separately.

† From tests by L. T. Robinson, *Trans. A. I. E. E., 1911, Vol. 30, p. 741* and Lloyd and Fisher, *Bull. Bur. Stds., 1909, Vol. 5, p. 453.* The "average" values are those given by Mr. Robinson for "standard" and "alloyed iron" respectively.

Eddy-current-loss curves for ordinary electrical sheets are plotted in Fig. 9, and for silicon steel in Fig. 10, these being based on Robinson's values of w_2 and the formula given above, *which assumes uniform distribution of flux in the sheets and a sine-wave variation with time.* The numbers on the curves are the gage numbers of the sheets, viz.,

No. 29 gage = 0.0141 inch thick,
No. 26 gage = 0.0188 inch thick,
No. 24 gage = 0.0250 inch thick,

See *Gages, Sheet Metal.*

Effect of Wave Form on Core-Loss. — Experiments by M. G. Lloyd (*Bull. Bur. Stds., 1908, Vol. 5, p. 381*) show that for a given maximum flux density in the core the hysteresis loss is practically independent of the wave form of the counter induced e.m.f. in the exciting coil, provided the voltage wave does not pass through zero more than twice per cycle On the other hand, the eddy-current loss for a given maximum flux density depends upon the magnitudes of the harmonics which may be present in the voltage wave.

Effect of Direction of Grain in Silicon Steel. — According to Chubb and Spooner (*Elec. J., 13, p. 393, 1916*), whenever it is possible to avoid it, silicon sheet-steel should not be used with the lines of force perpendicular to the direction in which the sheets have been rolled, since the magnetic properties under these conditions are not at their best.

EXCITING CURRENT ANGLE OF HYSTERETIC ADVANCE.—For a given wave shape of the impressed e.m.f., the wave shape of the exciting current (neglecting eddy currents) is determined by the shape of the hysteresis loop, since the exciting current is proportional to the magnetizing force H. Curve I in Fig. 11 shows the wave shape of the exciting current for a sine wave impressed e.m.f. calculated from hysteresis loop in Fig. 2 for a maximum flux density of 10,000 gausses.

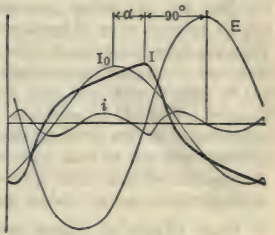

Fig. 11.

The sine curve I_0 is the fundamental of this wave (*see Wave Analysis*), and the curve i is the difference between I and I_0 and consists chiefly of the third harmonic.

The curve E is the sine wave of induced voltage. The flux curve would be a sine curve shifted 90° to the left (ahead of E). The angle α by which the fundamental of the current wave leads the flux wave is called by Steinmetz the "angle of hysteretic advance of phase."

For flux densities below the knee of the B–H curve the effective value of the fundamental current wave I_0 differs but slightly from the effective value of the actual current wave.

In the above discussion the effect of eddy currents is neglected. The effect of these is to increase the exciting current by a component of the same shape as, and in phase with, the impressed e.m.f., thus causing an increase in the effective value of the exciting current and an increase in the angle by which the exciting current leads the flux.

Magnetizing and In-phase Components of Exciting Current. — Let

P_0 = total core-loss,
E = back e.m.f. induced in the exciting coil,
I = exciting current,
$\cos\phi = P_0/EI$.

Then $I \sin \phi$ is called the magnetizing component of the exciting current and $I \cos \phi$ the in-phase (or energy) component. For magnetizations below the knee of the B–H curve

$$\phi = 90 - \alpha',$$

where α' is the total angle of advance of the exciting current ahead of the flux due to both hysteresis and eddy-current losses.

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MAGNETIC TESTING. — (*See also Electricity and Magnetism, Principles of; Magnetic Properties of Iron and Other Metals.*) The ordinary magnetic tests of iron and other substances are: (1) The determination of the normal *B–H* curves and hysteresis loop, and (2) The determination of the core-losses, i.e., hysteresis and eddy-current losses. The following is a brief table of contents of this article:

DETERMINATION OF THE NORMAL *B–H* CURVES AND HYSTERESIS LOOP. — Under certain conditions, noted below, the magnetizing force H may be calculated in terms of the number of turns per unit length and the current. The flux density B may be measured in any one of three ways: (1) By winding on the sample a secondary or "test" coil and connecting in series with it a ballistic galvanometer or equivalent device (e.g., a fluxmeter, q.v.) and noting the deflection of this instrument when the current in the primary or "magnetizing" coil is changed in value or reversed; (2) by measuring the mechanical force required to separate one end of the sample from a yoke which, with the sample, forms a closed magnetic circuit; and (3) by measuring the change in resistance of a spiral of bismuth inserted in an air gap between two parts of the sample or between the sample and a yoke. (4) In addition the permeability of two samples may be compared by a device known as a permeability bridge, in which the detector is a magnetometer or compass. These four methods may be designated respectively as the ballistic, traction, bismuth spiral and bridge methods respectively. The first is best suited for precision measurements, the last three for rapid shop tests where great accuracy is not demanded.

Magnetic Circuit of Testing Apparatus. — The simplest form of circuit is a straight bar, which is placed inside a solenoid or helix, the flux returning through the air. With this type of circuit, however, unless the bar and coil are very long, the demagnetizing action of the magnetic poles formed at the ends and along the sides of the bar render it impossible to calculate with accuracy the resultant magnetizing force in the bar. If the sample is made in the form of a closed iron ring, the mean magnetizing force over the cross-section of the metal may be calculated, but unless the radial thickness of the metal is small compared with the radius of the ring, the magnetizing force will be appreciably greater near the inner edge than near the outer edge, and as the permeability is a function of the magnetizing force, an appreciable error may be introduced. Also, the difficulty of winding the coils on a ring makes its use objectionable.

Bar and Yoke. — As a compromise, the sample is usually made in the form of a rod (or bunch of straight strips, in the case of sheet steel) which is fitted into a massive yoke of low reluctance which completes the magnetic circuit. The magnetizing coil is wound on the sample only. As a first approximation, in case the joints between sample and yoke are well made, the reluctance of the yoke and joints may be neglected, and the magnetizing force per unit length of the sample taken as the total magneto-motive force divided by the length of the sample. Methods have been devised for correcting the effect of the yoke and joints as described below.

Demagnetizing the Sample. — After the sample has been mounted ready for test (inserted in the yoke, if a bar and yoke method is employed) the circuit is thoroughly demagnetized by an alternating magnetizing force, preferably of about 1 period per second, which is gradually reduced from an initial intensity which establishes an induction well beyond the point of maximum induction to be measured to a final value somewhat lower than the lowest induction to be measured.

Ordinary Ballistic Method. — The connections are shown in Fig. 1. A ring sample is here shown, but this may be replaced by a straight sample

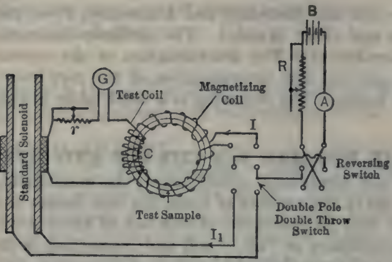

Fig. 1. Connections for Ballistic Method of Measuring Permeability

or by a straight bar in a yoke, Fig. 2. The test coil on the sample, the ballistic galvanometer G, the secondary coil of the standard solenoid and a resistance r are all connected permanently in series. This

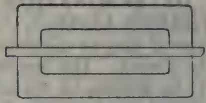

Fig. 2. Bar and Yoke

resistance is adjusted so that the galvanometer shows a suitable deflection when the current in the magnetizing coil of the test sample is reversed. The galvanometer is then calibrated (*see Galvanometers*), keeping this resistance unchanged. A double-pole double-throw switch is provided for connecting the battery circuit at will to the magnetizing coil on the test sample or to the primary coil of the standard solenoid. The sample should be in place when the galvanometer is checked, as the hysteresis and eddy-current losses due to the variable current in the primary during the test have the same effect as an increase in the resistance of the galvanometer circuit. (*See Burrows, C. W., Bull. Bur. Standards, 1909, Vol. 6, p. 31.*)

Precautions. — For a high degree of precision the sample should be placed with its axis perpendicular to the earth's field, the apparatus should be protected from mechanical vibration, and should be sufficiently remote from any strong magnets (such as the permanent magnets in ammeters and voltmeters) in order not to be affected by their magnetic fields.

Calibration of Galvanometer. — Let

N_1 = number of turns in primary of standard solenoid *per centimeter length* (axial),
n_1 = total number of turns in secondary coil of standard solenoid,
n = total number of turns in secondary coil on test sample,
A_1 = mean cross-section, in square centimeters of primary coil of standard solenoid if secondary is on the outside; if the secondary is inside the primary A_1 is the mean cross-section of the secondary,
A = cross-section in square centimeters of the test sample,
a = mean cross-section in square centimeters of the magnetizing coil on test sample if the test coil is on the outside; if the test coil is on the inside a is the mean cross-section of the test coil,
I_1 = current established through primary of solenoid,
D = deflection of galvanometer when this current is reversed,
B = flux density in test sample corresponding to a deflection D when the current through the magnetizing coil on the sample is reversed,
H = magnetizing force in sample corresponding to the flux density B.

The flux in maxwells established by the current I_1 through the primary of the solenoid is then $0.4\pi N_1 I_1 A_1$, and the number of linkages of this flux with the secondary of the standard solenoid, i.e., with the galvanometer circuit, is $0.4\pi n_1 N_1 A_1 I_1$. When the double-throw switch is thrown to connect the battery to the magnetizing coil on the test sample, establishing in this coil a current I and a flux density B, the number of linkages between the flux in the test sample

and the galvanometer circuit will be $BnA + Hn(a - A)$. If these two linkages are equal, as they will be if the galvanometer shows the same deflection D when I is reversed through the magnetizing coil on the test sample as when I_1 is reversed in the primary of the solenoid, then

$$B = \frac{0.4\pi n_1 N_1 A_1}{nA} I_1 - \frac{a - A}{A} H. \qquad (1)$$

Or, putting

$$K = \frac{0.4\pi n_1 N_1 A_1}{nA},$$

which is a constant for a given solenoid and sample, then

$$B = KI_1 - \frac{a - A}{A} H. \qquad (2)$$

The correction term $\left(\frac{a - A}{A}\right) H$ is usually negligible except for very low values of the flux density.

Hence, by sending various currents I_1 through the primary of the solenoid, reversing these currents and noting the deflection D, a curve may be plotted showing the relation between B ($= KI_1$) and D. Then, when a given current is reversed through the magnetizing coil on the test sample and the deflection D noted, the flux density corresponding to this current may be read directly from the curve. This curve will be practically a straight line unless the damping of the galvanometer is excessive.

The calibration curve holds only for a constant total resistance in the galvanometer circuit (*see Galvanometers*). If it is necessary to change this resistance in order to alter the sensibility of the galvanometer a new calibration curve must be obtained.

Calculation of Magnetizing Force. — Let

N = number of magnetizing turns per centimeter of mean circumference (*see Fig. 1*). If a straight sample with or without a yoke is used, N is taken as the total number of magnetizing turns divided by the free length of the sample in inches.

I = current in magnetizing coil in amperes.

Then the magnetizing force in gilberts per centimeter is

$$H = 0.4\pi NI. \qquad (3)$$

This formula represents the *average* magnetizing force over the section of the ring, if a ring sample is used. If a straight sample is used, the formula is approximate only, due to the reluctance of the return circuit through the air or yoke.

Determination of Normal *B–H* Curve by Ballistic Method. — The sample is first demagnetized as described above, then the lowest magnetizing force to be used is applied and reversed many times until the iron is brought to a cyclic magnetic state, that is, until the reversal of the magnetizing force reverses the direction of magnetization without changing its magnitude. The number of reversals required to establish a cyclic condition in soft steel ranges from about 10 at high flux densities to several hundred at low flux densities. In general the harder the steel the fewer the number of reversals required. The galvanometer deflection is then noted, and the corresponding value of B taken from the calibration curve and H calculated from the formula given above. As a check, it is well to carry the iron through the demagnetizing process again, reduce to a cyclic state, and redetermine the point.

Higher points on the curve may be obtained in a similar manner, but it is not necessary to demagnetize the sample between successive points unless it accidentally becomes magnetized above the point being determined.

Determination of Hysteresis Loop by the Ballistic Method. — The sample is first demagnetized and then the maximum magnetizing force (corresponding to the tip of the loop) is applied and a cyclic condition established as described in the preceding paragraph. By suddenly inserting an additional resistance in the magnetizing circuit, e.g., by moving the slider on the resistance R so that less of this resistance is short-circuited, the magnetizing force is reduced to any desired value, and the change in flux density corresponding to this change in magnetizing force is determined by noting the galvanometer deflection. If the calibration curve of the galvanometer has been obtained by the method of reversals, as described above, the change in flux density is twice the value of B as read from the calibration curve, since the actual change in flux corresponding to any ordinate of the calibration curve is twice this ordinate.

The next point on the hysteresis loop is determined in the same manner, first bringing the iron to a cyclic state under the maximum magnetizing force corresponding to the tip of the loop. Data with negative values of the magnetizing force are obtained by reversing the currents in addition to making the adjustments already described. The points on the hysteresis curve may be taken in any order.

In this method of obtaining hysteresis data, the measured quantity is the change in induction when the magnetization is changed at one step from a maximum to any other given point on the hysteresis loop. This method is comparatively free from the irregularities due to the slow creep that occurs when a magnetizing force is applied slowly or changed by small steps. It is also free from irregularities due to variations in the size of step in the "step-by-step" method, in which the magnetizing force is changed from one value to the next lower without restoring it each time to the maximum value. (*See Burrows, Bull. Bur. Standards, 1909, Vol. 6, p. 1.*)

Zero Ballistic Methods for Determining Flux Density. — By using an independent battery circuit to energize the primary of the standard solenoid and establishing the currents I_1 and I at the same time and reversing them simultaneously, it is possible by having the relative directions of these currents right, so to adjust I_1 that for a given value of I the net discharge through the galvanometer is zero, and no deflection occurs. Under this condition B may be calculated directly from equation (1) or (2) above.

Instead of using a standard solenoid with fixed coils, an adjustable mutual inductance, previously calibrated, may be used, and a constant current maintained through its primary, the adjustment being effected by moving the secondary. Let

M = mutual inductance of standard, in henries,
I = current in amperes, in primary of mutual inductance,
n = total number of turns in test coil on sample,
A = cross-section of sample in square centimeters,
B = flux density in sample for balance.

Then

$$B = 10^8 \frac{MI}{nA} = K_1 M,$$

where $K_1 = \dfrac{10^8 I}{nA}$ = a constant for a given test coil and sample and given current in the primary of the mutual inductance.

The design of a standard mutual inductance suitable for this purpose is described in detail by Burrows, *Bull. Bur. Stds., 1909, Vol. 6, p. 31.*

Burrows' Compensated Double-Yoke Method. — This method, described in detail in Vol. 6 of the Bulletins of the Bureau of Standards (*Reprint No. 117*), is the standard method used by the Bureau of Standards and has also been adopted as the standard method of testing permeability by the American Society for Testing Materials, see *Proc. Am. Soc. Test. Mat., 1912.* The following description of the method is taken from Burrows' paper.

Fig. 3 shows the relative positions of the magnetizing and test coils when double yokes and double rods are used. The lower rod is the one under test.

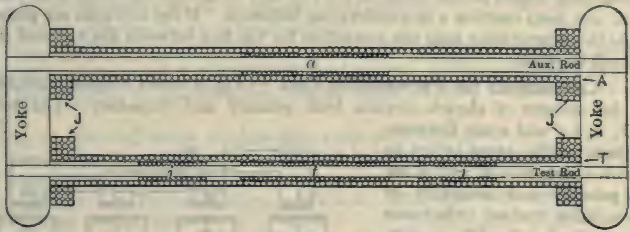

Fig. 3. Double Yoke and Bars

The upper rod is an auxiliary rod of approximately the same magnetic properties as the test specimen. T and A are the two main magnetizing coils, one wound over each rod. Over the four joints are wound four short coils J, each about 1.5 centimeters long. These are connected in series and used as a single coil. t and a are the two test coils surrounding test and auxiliary specimen respectively. j is the end test coil distributed with one-half over each end of the test rod.

Test Samples. — When rods are to be tested these should preferably be of square cross-section, ⅜ by ⅜ inch (0.9525 cm.) and 30 cm. long. With rods of square cross-section it is possible to make a more perfect joint between the rods and yokes. Sheet steel is tested by making up two equal bundles of strips 3 centimeters wide by 50 centimeters in length, having a total weight of 5 kilograms (*Stand. Spec. of Am. Soc. Test. Mat., June, 1912*). Each bundle will be about 2.25 centimeters thick. One bundle takes the place of the test rod, the other takes the place of the auxiliary rod.

Yokes. — The yokes are of soft iron, provided with suitable clamps for holding the sample firmly and making a good magnetic joint. When sheet steel is being tested, the yokes may be made of short strips of the material 3 centimeters wide, the magnetic circuit in this case being a rectangle made up of strips, the joints at the four corners being alternately butt and lap in successive layers.

Magnetizing Coils. — No. 18 A. W. G. double-cotton-covered copper wire is a convenient size. A coil made of ten layers of this wire will stand continuously a current of 1.7 amperes, corresponding to a magnetizing force of $H =$ 171 gilberts per centimeter. For short periods twice this current may be safely employed, giving a magnetizing force of about 350 gilberts per centimeter. The length and diameter of the coils will depend upon the dimensions of the test sample (see above). If the coils are made of round cross-section the hollow core on which they are wound should have a shallow screw thread, 8 threads to the centimeter, cut on the outside. The first layer is wound in this thread, and succeeding layers wound in the same direction between adjacent wires in the previous layer. The various layers are then connected in series. The mag-

netizing force at the center of such a coil of ten layers, assuming perfect compensation for the rest of the magnetic circuit, is

$$H = 0.4\pi \times 8 \times 10\, I = 100.53\, I,$$

where I is the current in the coil.

Test Coils. — The test coils are made of fine wire (enameled or silk covered) wound on thin cores of paper, cloth or slotted metal. Coils t and a are placed over the middle portions of the test rod and auxiliary rod, respectively. Over the two ends of the test rod are placed the two halves of a third test coil j. These three test coils have the same number of turns and are spread over a considerable length of rod, so as to prevent any irregularities which may exist in the iron from exerting a preponderating influence. If the test coils are placed inside the magnetizing coils the correction for the flux between the rod and the coil will be small.

Connections for Permeability Test. — Fig. 4 shows diagrammatically the full scheme of electric circuits both primary and secondary. The coils T, J, A, t, j and a are the same as those of the same letters in Fig. 3. The coils M and m are the primary and secondary of the variable mutual inductance (or standard solenoid) used to balance the e.m.f. induced in the test coil.

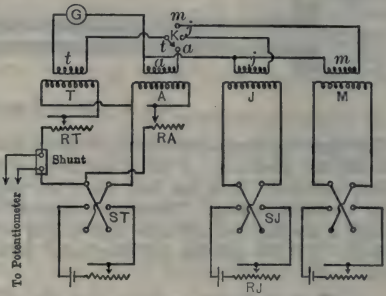

Fig. 4. Connections for Burrows' Method of Testing Permeability

Compensation. — Compensation is secured when the flux across every section of the iron circuit is the same, i.e., when there is no leakage. This condition may be closely realized by adjusting the currents in the three magnetizing coils separately.

The switches ST and SJ are reversed repeatedly, and the resistance RA and RJ adjusted until the three test coils, t, j and a, indicate the same change in flux when the magnetizing currents are simultaneously reversed, i.e., switches ST and SJ reversed simultaneously. With the key K on the point t–a, the equality of flux in the test and auxiliary rods is secured first by adjusting RA until the galvanometer shows no deflection on reversing ST. Then with the key K on the point t–j the flux near the magnetic joints is adjusted to uniformity.

Measurement of Induction. — To measure the induction K is moved to the point t–m. A reversal of the magnetizing forces produces an impulsive electromotive force acting on the galvanometer, which may be measured as a deflection or may be compensated for by reversing simultaneously a suitable current through a variable mutual inductance, or standard solenoid M.

Measurement of Magnetizing Force. — For accurate work the current should be measured by means of a potentiometer (*see Potentiometers*). If the shunt used for measuring the current has a resistance of 1.0053 ohms and the magnetizing coil T 80 turns per centimeter, then

$$H = 100\, V,$$

where V is the fall of potential across this shunt.

STANDARD INDUCTION TESTS OF THE A.S.T.M. (1921 Edition). — The normal magnetic induction is the induction produced by a magnetizing force in a given piece of magnetic material which has been previously demagnetized and then subjected to many reversals of the given magnetizing force.

Both the induction B and the magnetizing force H shall be expressed in terms of the C. G. S. electromagnetic unit (gauss).

Sheet Metal. — The standard normal induction data for sheet material shall consist of the magnetizing forces corresponding to inductions of 2000 4000, 6000, 8000, 10,000, 12,000, 14,000, 16,000, 18,000, 20,000 gausses, or such as may be obtained without exceeding a magnetizing force of 200 gausses.

The following details are to be observed:

The permeability sample for sheet material shall consist of an even number of strips cut parallel to the direction of rolling, and even number of strips cut perpendicular to that direction selected from material sampled as for core loss.

The sample shall weigh not less than 1 nor more than 2 kg.

The magnetic circuit shall be a rectangle having the test material for one pair of opposite sides, and the same or different material for the other pair, which may be shorter. The joints at each corner are alternately butt and lap, or may be clamped on the edges.

The magneto-motive force is applied in two sections. The main magnetizing coils shall consist of two equal uniformly-wound solenoids surrounding the test material. The compensating coils shall consist of four short coils, each having the same number of turns wound closely over the ends of the magnetizing coils.

The test coil surrounds the middle portion of each bundle of test material. Four other test coils each of half the number of turns are placed over the four positions of the test material, approximately midway between the yokes and the center. The two center test coils are joined in series and the four end test coils are joined in series. The corresponding ballistic deflections, due to these two sets of test coils, are measures of the magnetic fluxes through the underlying portions of the magnetic circuit. By connecting the two test coils so that the induced electromotive force opposes that of the four coils, and adjusting the current through the compensating magnetizing coils so that there is no resulting ballistic deflection, an approximate uniformity of flux is secured through the greater portion of the test material, and the induction may be measured ballistically in the regular manner. The magnetizing force when the flux is adjusted to uniformity is that calculated from the uniform winding of the main magnetizing solenoids.

The cross-section of the magnetic circuit is determined as in the standard core-loss test.

Rods. — The standard test for rods for use in electromagnets shall consist of the magnetizing forces corresponding to induction of 2000, 4000, 6000, 8000, 10,000, 12,000, 14,000, 16,000, 18,000, 20,000 gausses, or such as may be obtained without exceeding a magnetizing force of 200 gausses.

The standard data for material intended for permanent magnets shall consist of the normal induction (B), the normal residual induction (B_r), and the normal coercive force (H_ϵ) corresponding to a magnetizing force of 200 gausses.

Standard magnetic data shall be determined by the Burrows compensated double-yoke method. (For further details see *A.S.T.M. Standards.*)

Permeability Bridge. — A double-bar and yoke arrangement, such as shown in Fig. 3, may be used without compensating or test coils to obtain a fairly

accurate measure of the permeability of a sample, provided the B–H curve of one of the rods is known. A small magnetic needle or compass placed in the air space between the two rods is used as a detector of magnetic leakage from one rod to the other. The currents in the two magnetizing coils are adjusted until the needle shows no deflection when the currents in the two coils are reversed. The total flux in each rod will then be the same. The magnetizing forces in the two rods are then calculated from the currents in the magnetizing coils (*equation 3 above*) and the flux from the B–H curve of the standard sample.

PERMEAMETERS FOR SHOP TESTS. — A great number of approximate methods have been devised for the rapid testing of permeability in shop work, where a high degree of precision is not required. Some of these are briefly described below.

Thompson Permeameter (Fig. 5). — In this instrument the flux density is measured in terms of the force required to separate a rod S from a yoke A, when the magnetic circuit formed by the rod and yoke is magnetized by a current in the coil B. The handle E is turned until the sample is pulled away from the yoke and reading on the balance taken at the instant of break.

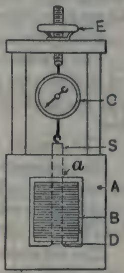

Fig. 5. Thompson Permeameter

Let

I = amperes in magnetizing coil,
N = number of turns in magnetizing coil,
l = "equivalent" length of magnetic circuit in inches = distance from a to D plus from 10 to 20 per cent to allow for reluctance of yoke and joints,
P = pull in pounds as read by balance,
A = cross-section of rod S in square inches.

Then the magnetizing force is

$$H = \frac{0.4\pi NI}{2.54\,l},$$

and the flux density is, within the accuracy of measurement by the instrument,

$$B = 1320\sqrt{\frac{P}{A}}.$$

There are several errors in the instrument, however, notably the unavoidable air gap between the sample and the top of the yoke and the contact at D, which makes the instrument unsuitable for the accurate determination of the absolute values of the qualities of iron and steel. It is, however, extremely useful for the comparison of samples where exact absolute values are not required. More nearly absolute values can be obtained by properly calibrating the instrument by using a standard sample the permeability of which at various flux densities has been measured by a more accurate method.

Fahy Permeameter. — This apparatus (Fig. 6) is H-shaped with test coils ST spanning the free ends of the H and a magnetizing coil M wound on the cross-piece. D and D' are also test coils. $SDD'T$ have the same number of turns and are connected in series and in such a manner that the integrated e.m.f.'s induced in S and D' when a magnetizing current is reversed in M are in the same direction but in opposition to those simultaneously induced in D and T. The permeameter is initially adjusted so that when the coils S and T encircle air the differential effect of the coils D and D' is zero. In use, S encircles a standard bar bridging the gap on the left and T the test-bar bridging the gap on the right. In the cross-bar is an air-gap J, designed to create a demagnetizing force within

the core itself and which is adjustable to suit requirements. Magnetomotive force can also be applied in the compensating coils C and the currents in M and C are independently adjustable. When current is flowing in both M and C one magnetic circuit has its m.m.f. increased and that of the other circuit is diminished by an equal amount.

Thus by varying the current in the compensating coils the difference in fluxes linking the test coils D' and T can be made equal to the difference in fluxes linking S and D. When these flux differences are equal, the leakage flux through the air path which is in parallel with the path of the coil T is equal to the leakage through the air path in parallel with S. Since these two paths are symmetrical and carry the same flux, the same difference of magnetic potential exists between their ends. Coil H has a relatively large number of turns. It is wound on the same form as S and may be used to measure the difference of magnetic potential in this region, and hence the mean magnetizing force, which will be that obtaining along the path of T in which the test specimen lies.

With respect to consistency of repetition, effect of length of test specimen, reluctance of joints, effect of iron in the vicinity, position of specimen on the pole-faces, and strain effect the instrument represents a distinct advance on direct-reading permeameters in common use. Normal induction measurements of solid bars show errors no greater than 5 per cent of the magnetizing force required for a given induction. Commercial materials are seldom uniform enough to warrant better precision than 5 per cent. The instrument is also adaptable to the measurement of sheet materials. For a complete description of the instrument and its use see *Bull. Bur. Stands., 14, p 267, 1917.*

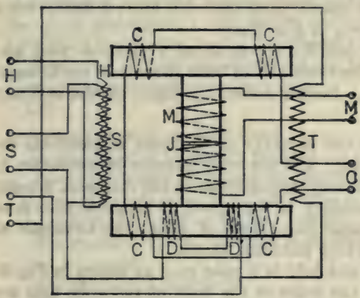

Fig. 6. Fahy's Permeameter

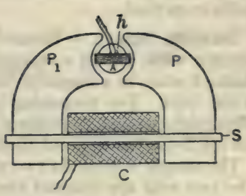

Fig. 7. Koepsel Permeameter

Koepsel Permeameter (Fig. 7). — The magnetic circuit consists of the sample S and two heavy soft-iron pole pieces P and P_1, with a gap and soft-iron core A (similar to the gap and core in the voltmeter). A small coil h is suspended in the gap and connected to an auxiliary current supply. The sample is magnetized by current passing through the magnetizing coil C. The flux produced in the magnetic circuit causes the coil h to deflect an amount proportional to that flux, for a constant strength of auxiliary current. The deflection is indicated by a pointer, attached to the coil h, swinging over a scale that is calibrated to read directly in flux densities. The calibration of the instrument does not hold for wide variations in the permeability of the samples since the leakage depends upon this permeability. Also the magnetizing force can be calculated only approximately. These difficulties can be overcome in a measure by placing compensating coils on the yokes P and P_1 and sending

sufficient current through these to compensate for the leakage. For detailed description see *Bull. Bur. Stands., 11, p. 101, 1914.*

Esterline Permeameter. — This instrument is similar to the Koepsel apparatus except that the moving coil and core (*h* and *A* in Fig. 7) are replaced by a small direct-current armature. The apparatus then becomes a small separately-excited dynamo. The armature is driven by an auxiliary motor at a constant speed. The voltage across the armature is then directly proportional to the total flux cutting the armature conductors. Leakage is avoided by the use of compensating coils on the pole pieces. The current in these compensating coils is adjusted until a small compass placed near the magnetic circuit of the apparatus shows no deflection when the magnetizing and compensating currents are simultaneously reversed. The apparatus is described in detail in *Proc. Am. Soc. Test. Mat., 1906, Vol. 6, p. 320.*

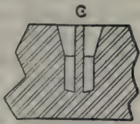

Fig. 8. Drilled Hole for Drysdale Permeameter

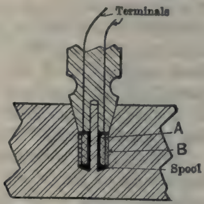

Fig. 9. Drysdale Permeameter

Drysdale Permeameter (Fig. 9). — This device is particularly useful in determining the permeability of large generator frame castings or other large masses of metal without the preparation of samples. A hole is drilled in the casting at any point desired with a special drill that will leave a hole with a core in the center similar to that shown at *C* in Fig. 8.

A magnetizing coil *A* and a test coil *B* are wound on a soft-iron plug and inserted into this hole as shown. The terminals of both these coils are brought out through small holes in the plug. Measurements are made by the ordinary ballistic method.

Bismuth Spiral. — Bismuth has the property of changing in electrical resistance when put in a magnetic field. The per cent increase in resistance in bismuth when in a magnetic field is nearly proportional to the magnetic density of the field. For measuring permeability, a flat spiral of bismuth wire non-inductively wound is fastened between two plates of mica, the terminals of the coil being brought out through a long insulated handle.

The sample is made in two pieces and held in a yoke such as shown in Fig. 2. The two rods are inserted through the holes in the ends of the yoke and are pushed in until only a small air gap is left between their opposing ends. The bismuth spiral is inserted in this gap. The resistance of the spiral is measured with a Wheatstone's bridge and the flux density determined from a curve furnished with the spiral giving the variation in resistance with varying density. The calculated value of the magnetizing force must be corrected for the reluctance of the yoke and air gap.

CORE-LOSS MEASUREMENTS. — A great number of methods have been suggested for determining the core-loss in a sample of iron or steel when subjected to an alternating field. These methods may be classified as: (1) hysteresis loop method; (2) wattmeter method; (3) mechanical torque methods. Of these the wattmeter method, when a properly constructed magnetic circuit is employed, gives the most reliable results. The hysteresis loop method is tedious and gives no knowledge of the eddy-current loss. The mechanical torque methods necessitate an air gap in the magnetic circuit with a resulting induction in the specimen which is far from uniform; these latter methods, however, are frequently used where high degree of accuracy may be sacrificed

to speed in testing, e.g., in comparative shop tests.

Hysteresis Loss from Hysteresis Loop.—The hysteresis loop may be determined by any of the methods described above. The loop is plotted on cross-section paper and integrated. Plot the magnetizing force, in gilberts per centimeter, to a scale of h gilberts per centimeter equal to 1 inch and the flux density to a scale of b gausses equal to 1 inch. Then if A is the area of the loop in square inches, the hysteresis loss, in ergs per cycle per cubic centimeter of iron, is

$$W = \frac{hbA}{4\pi}.$$

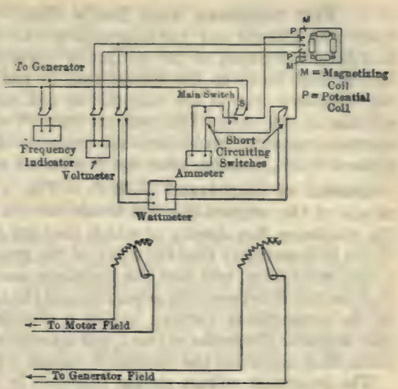

Fig. 10. Connections for Core-loss Tests

Principle of Wattmeter Method of Determining Core-Loss. — Fig. 10 is a complete diagram of connections. The principle of this method is as follows: The sample to be tested is inserted in a magnetizing coil of a known number of turns, under* which is wound a secondary coil having either the same number of turns or a multiple thereof. The primary coil is connected in series with a source of alternating e.m.f. (which can be adjusted without changing its wave form) and the current coil of a wattmeter. The potential coil of the wattmeter is connected to the secondary coil on the test sample. A voltmeter is also connected across this secondary coil.

Let

N_1 = number of primary turns,
N_2 = number of secondary turns,
A = cross-section of sample in square centimeters,
f = frequency in cycles per second,
h = form factor of secondary voltage,
B = maximum flux density in sample, assumed constant throughout its length,
V = reading of voltmeter,
P = reading of wattmeter,
$R = r + \frac{R_1 R_2}{R_1 + R_2}$, where r is the resistance of the secondary coil, R_1 the resistance of the voltmeter and R_2 the resistance of the potential coil of wattmeter.

The resistance R should be sufficiently large so that the heat developed (V^2/R) in the secondary circuit is small compared to the core-loss, and the resistance r of the secondary coil should also be small compared with R. Then if the instruments are calibrated to read correctly and P and V are read simultaneously, the total core-loss is, assuming r negligible compared with R,

$$P_c = \frac{N_1}{N_2} P - \frac{V^2}{R},$$

and the maximum flux density is

$$B = \frac{10^8\, V}{4\, hfAN_2}.$$

* The two windings may also be "sandwiched."

Test Specimen with Single Coil. — The core-loss test can also be made with a single coil (the magnetizing coil) on the sample with the voltmeter and potential circuit of the wattmeter connected to the terminals of this coil. In this case, however, the voltage read by the voltmeter must be corrected for the resistance drop in the magnetizing coil, and the wattmeter reading for the rI^2 loss in this coil. The two-coil method also has the advantage that the wattmeter reading can be made any multiple of the actual loss by using a proper ratio of N_2/N_1, thus enabling one to measure small losses with greater accuracy.

Control of Impressed E.M.F. — The e.m.f. impressed across the terminals of the magnetizing coil should have a sine wave form for all values of the maximum flux density at which tests are to be made. To secure this condition it is necessary that the e.m.f. of the generator supplying the current have a sine wave form, and that the resistance and reactance of both the generator and the circuit between the generator and the test sample be as small as possible, as the magnetizing current, having a distorted wave form due to hysteresis (*see Magnetic Properties of Iron*), will introduce a voltage drop having a non-sine form, thus distorting the impressed voltage wave.

The impressed voltage should therefore be controlled through the field of a generator having a large capacity compared to the power taken by the test sample, or instead of varying the field a transformer with suitable taps may be used, provided the resistance and leakage reactance of the transformer are small. In either case, the generator should give a sine wave at all field excitations within the range used. If this condition cannot be realized the hysteresis and eddy-current losses should be separated by measuring the total loss at two frequencies and the total loss then corrected for form factor (see below).

Form of Specimen for Wattmeter Test. — The specimen may have one of three forms: (1) It may be in the form of straight strips, the flux lines returning through the air; (2) straight strips may be used with a yoke; and (3) the specimen may be arranged to form a closed magnetic circuit in itself.

The first form gives a distribution of flux which is far from uniform. It is possible, however, by determining the flux distribution in the sample by means of an exploring coil to correct the observed losses for this flux variation (*see Robinson, L. T., Trans. A.I.E.E., 1911, Vol. 30, p. 741*).

The second form gives a more uniform flux, but it is necessary to distinguish between the energy supplied to the specimen and that supplied to the yoke. This can only be done satisfactorily by knowing the constants of the yoke, and only then by having the distribution of flux uniform, a condition difficult to secure.

Consequently, for accurate measurements the third form is the most reliable, although for factory use the first or second may prove more convenient where accuracy can be sacrificed for other considerations. The material used should be cut in a form such that only a small part of it is contiguous to a cut edge since all methods of cutting have a hardening effect upon the material bordering upon the cut. This means that the strip, whether straight or in ring form, should not be too narrow. This condition may be dispensed with if all specimens are annealed under definite conditions after cutting to size, and prior to testing.

Two general forms of magnetic circuit are available. The material may be stamped into rings (as in the Esterline apparatus), or the circuit may be built up from straight strips as in the Epstein apparatus. Leakage is most effectually avoided by using rings. With this form of specimen, however, the flux density will not be uniform unless rings of very great diameter are employed, and in the latter case there is a very great waste of material. The non-uniformity of flux existing in rings of small diameter, even when uniformly wound, and the errors resulting therefrom, are discussed by Loyd, M. G., *Bull. Bur. Stds., 1909, Vol. 5, p. 435*. The use of rings is thus restricted to cases where the

material is annealed after stamping, and the radial width of the ring should be very small in comparison to its diameter. When rings are employed, the labor of winding each specimen separately with a magnetizing coil may be obviated by the use of the apparatus of Esterline (*see p. 925*) or Möllinger (*E. T. Z., 1901, Vol. 22, p. 379*).

Epstein Method.—This method has been adopted as the standard in Germany and by the American Society for Testing Materials in this country. It is used for commercial testing by both the General Electric and Westinghouse Companies. The method is clearly described in the standard specifications for making core-loss tests given in the following paragraph.

A.S.T.M. Standard Core-loss Tests. (**1921 Edition.**) — The power consumption in electrical sheet steel when subjected to an alternating magnetization is known as the core-loss. The standard core-loss is the total power in watts consumed in each kilogram of material at a temperature of 25° C., when subjected to a harmonically-varying induction having a maximum of 10,000 gausses and a frequency of 60 cycles per second, when measured as specified below. It is represented by the symbol $W_{10/60}$.

The ageing coefficient is the percentage change in the standard core-loss after continued heating at 100° C. for 600 hours.

The standard core-loss shall be measured under the following conditions:

The magnetic circuit consists of 10 kilograms (22 pounds) of the test material, cut with a sharp shear into strips 50 centimeters (19 11/16 inches) long and 3 centimeters (1 3/16 inches) wide, half parallel and half at right angles to the direction of rolling, made up into four equal bundles, two containing material parallel and two containing material at right angles to the direction of rolling, and finally built into the four sides of a square with butt joints and opposite sides consisting of material cut in the same manner. No insulation other than the natural scale of the material (except in the case of scale-free material) shall be used between laminations, but the corner joints shall be separated by tough paper 0.01 centimeter (0.004 inch) thick.*

The magnetizing winding shall consist of four solenoids surrounding the four sides of the magnetic circuit and joined in series. A secondary coil shall be used for energizing the voltmeter and the potential coil of the wattmeter.

These solenoids shall be wound on a form of any non-magnetic non-conducting material of the following dimensions:

Inside cross-section	4 by 4 cm.
Thickness of wall	not over 0.3 cm.
Winding length	42 cm.

The primary winding on each solenoid shall consist of 150 turns of copper wire uniformly wound over the 42-centimeter length. The total resistance of the magnetizing winding shall be between 0.3 and 0.5 ohm. The secondary wind-

* The purpose of this paper is to prevent the exposed end of the laminations being forced into the spaces between those in the adjacent side of the other part of the sample and to prevent the formation of eddy currents at the corners which may not be confined to the thickness of the laminations if the paper is not used. The certainty of the measurements is thereby improved by a small but definite amount over the results which are obtained without the paper in the joints. This improvement is of course accomplished at the expense of some increase of leakage flux at the corners and consequently greater departure from absolute uniformity of flux distribution along the length of the sample, also the conditions imposed upon wattmeter are more severe as the general power factor is lower. The flux distribution over a section of the sample is more uniform with the paper. The net result is a definite gain in accuracy.

(*L. T. Robinson.*)

ing of 150 turns of copper wire on each solenoid shall be similarly wound beneath the primary winding. Its resistance shall not exceed 1 ohm.

A voltmeter and the voltage coil of a wattmeter shall be connected in parallel to the terminals of the secondary winding of the apparatus. The current coil of the wattmeter shall be connected in series with the primary winding.

A sine-wave electromotive force shall be applied to the primary winding and adjusted until the voltage of the secondary circuit is given by the equation ,

$$E = \frac{4fNnBM}{4lD\,10^8},$$

in which

f = form factor of primary e.m.f. = 1.11 for sine wave,
N = number of secondary turns = 600,
n = number of cycles per second = 60,
B = maximum induction = 10,000
M = total mass in grams = 10,000,
l = length of strips in centimeters = 50,
D = specific gravity = 7.5 for high-resistance steel
= 7.7 for low-resistance steel,
E = 106.6 volts for high-resistance steel for sine voltage
= 103.8 volts for low-resistance steel for sine voltage.

A specific gravity of 7.5 is assumed for all steels having a resistance of over 2 ohms per metergram, and 7.7 for all steels having a resistance of less than 2 ohms per metergram. These steels are designated as high- and low-resistance steels, respectively.

The wattmeter gives the power consumed in the iron and the secondary circuit. The loss in the secondary circuit is given in terms of the total resistance and voltage. Subtracting this correction term from the total power gives the net power consumed in the steel as hysteresis and eddy-current loss. Dividing this value by ten gives the core-loss in watts per kilogram.

Sampling. — The core loss material shall be cut from two or more sheets taken at random from the shipment. The strips should be distributed symmetrically over the sheet, as nearly as may be practicable. It is recommended that a test sample shall represent not more than 5000 kg. (11,000 pounds).

The Procedure. — 1. Cut the test material into strips 3 by 50 centimeters as indicated under *Sampling.*

2. Place on the balance a pile of strips weighing 2.5 kilograms. Add a second pile of the same kind, bringing the weight up to 5 kilograms. In each case the weight is taken to the nearest strip. Add in succession two piles of 2.5 kilograms each, of the other kind of strips, bringing the weight up to 7.5 kilograms and 10 kilograms respectively.

3. Secure each bundle by string or tape (not wire) and insert in the apparatus as indicated.

4. Apply the alternating voltage to the primary coil and tap the joints together until the current has a minimum value, as shown by an ammeter in series. Then clamp the corners firmly by some suitable device.

5. Shunt the ammeter and adjust the primary current until the voltmeter indicates the proper value. This adjustment may be made by an auto-transformer by varying the field of the alternator, or by both, but not by the insertion of resistance or inductance in the primary circuit. Simultaneously the frequency must be adjusted to 60 cycles.

6. Read the wattmeter.

7. Calculations. Subtract from the wattmeter reading the instrument losses, which will be constant for any set of instruments and voltage, and divide by 10. The result is the standard core-loss.

Bureau of Standards' Method. — (*Bull. Bur. Stds., 1909, Vol. 5, p. 453; Trans. A.I.E.E., 1909, Vol. 28, p. 439.*) This is a modification of the Epstein method. It differs from the latter in the use of a smaller test specimen, from 1.5 to 2 kilograms (about 4 pounds) of strips 25.4 by 5 centimeters (10 by 2 inches), and in the use of a different form of joint between the four bundles of strips. In other respects it is essentially the same as the Epstein method.

Fig. 11 shows the arrangement of the joint. At the corners of the square, short pieces of test material are bent at right angles and interleaved between the strips of adjacent bundles, as shown in the figure. There are as many of

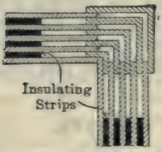

Fig. 11. Detail of Joint Used by N. B. S.

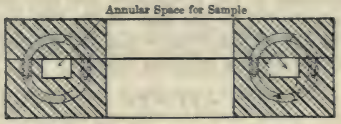

Fig. 12. Esterline Apparatus

these corner pieces as there are test pieces, and they are graduated in length so as to give a uniform lap of about 2 millimeters. A special clamp is tightened over these laps, so as to give a good magnetic joint. The object of these corner pieces is to reduce the leakage and thereby obtain a more nearly uniform flux throughout the sample. The loss in these corner pieces is small and can be calculated to a sufficient degree of accuracy and allowed for.

Esterline Apparatus For Testing Iron Rings (Fig. 12). — This apparatus is designed for testing rings made up of punchings from sheet steel. It is essentially a solenoid made in the form of a doughnut, but divided into two halves. The sample is inserted by lifting off the top half; the top half is then replaced and the teeth formed by the projecting ends of each upper half-turn fit into little sockets formed at the ends of the corresponding lower half-turn. (*Esterline, J. W., Proc. Am. Soc. Test. Mat., 1906, Vol. 6, p. 320.*)

Ewing Hysteresis Tester. — Strips 5/8 by 3 inches are cut from the sheets to be tested and a bundle of about 7 is used in each test. This sample is rotated at a relatively low speed (to avoid eddy currents) between the poles of a permanent magnet, the magnet being pivoted at the center of rotation and carrying a pointer that deflects over a scale, the deflection depending on the hysteresis loss in the sample. Two standard samples of known hysteresis properties are furnished with the instrument, so that the scale may be calibrated from time to time.

The Blondel Hysteresis Tester. — This apparatus is similar in principle to the Ewing hysteresis tester, but the magnet is revolved instead of the sample. The samples are made in the form of rings which are mounted on a pivoted spindle between the poles of a U-shaped permanent magnet. When the magnet is revolved the sample tends to follow it but is restrained by a spiral spring, and therefore it turns only through an angle such that the torque due to the hysteresis loss is balanced by the opposing torque of the spring.

The Holden-Esterline Core-loss Tester. — This instrument is similar to the Blondel hysteresis tester previously described but an electromagnet is substituted for the permanent magnet and this magnet is driven by a motor at the required frequency. The spring is arranged with a torsion head and pointer that is set to zero when the instrument is not in use.

The angle of torsion necessary to bring the sample back to its zero position

when the instrument is operating is read on a scale calibrated directly in watts loss. The speed of rotation being high the eddy-current losses are appreciable, and therefore the instrument reads the combined loss due to hysteresis and eddy currents at the frequency corresponding to the speed of rotation.

SEPARATION OF HYSTERESIS AND EDDY-CURRENT LOSSES. —When the counter e.m.f. induced in the winding on any magnetic core is sinusoidal, the eddy-current and hysteresis losses may be separated by making measurements of the total loss at two frequencies f_1 and f_2, at voltages E_1 and E_2, respectively proportional to these frequencies. Let the values of the total core-loss at these two frequencies be P_1 and P_2. Since the voltages are taken proportional to the frequencies and the voltage is sinusoidal, the maximum flux density will be the same in the two measurements. Let P_e be the eddy-current loss at the frequency f_1 and P_h the hysteresis loss at this frequency. Then the following relations hold:

$$P_1 = P_e + P_h, \qquad P_2 = \left(\frac{f_2}{f_1}\right)^2 P_e + \left(\frac{f_2}{f_1}\right) P_h.$$

Putting $a = \frac{f_2}{f_1}$ and solving for P_e and P_h, gives

$$P_e = \frac{a P_1 - P_2}{a(1 - a)}, \qquad P_h = \frac{P_2 - a^2 P_1}{a(1 - a)}.$$

IRON-LOSS VOLTMETER. —As pointed out in the article on *Magnetic Properties of Iron,* the core-loss depends upon the form of the impressed voltage wave. The dotted curves *A*, *B* and *C* in Fig. 13 show the variation in core-loss with form factor when the eddy-current loss is 14 per cent, 20 per cent and 30 per cent of the total loss. The loss corresponding to a sine-wave voltage (form factor 1.11) is taken as 100 per cent.

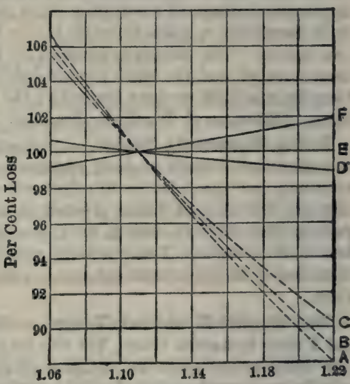

Fig. 13.

In making commercial tests on transformers it is frequently inconvenient to obtain a sine-wave voltage, and it is also equally inconvenient to obtain the form factor of the actual voltage available and the relative value of the eddy-current loss. Yet, for comparative purposes, the core-loss should be referred to a standard sine-wave form. The iron-loss voltmeter, devised by L. W. Chubb (*Trans. A.I.E.E., 1909, Vol. 28, p. 417*) is an instrument designed to read, when connected to a circuit in which the voltage has *any wave form whatever*, the value of the sine-wave voltage which would produce the same total loss as produced by the actual voltage. The meter is calibrated for one frequency only, usually 60 cycles per second. When such an instrument is connected across the transformer and the voltage adjusted until the instrument reads the rated voltage of the transformer, the core-loss read by a wattmeter connected in the usual manner will be the core-loss corresponding to a sine-wave voltage having an effective value equal to the voltage

read by the iron-loss voltmeter, irrespective of what the reading of an ordinary voltmeter connected across the line may be.

Chubb's iron-loss voltmeter is essentially an ordinary wattmeter, the current coil of which is connected in series with a winding on a small laminated iron core, these two elements in series being connected directly across the line. The potential circuit of the wattmeter is also connected across the line to the same two terminals as the first or series circuit. The meter therefore has but two terminals. The deflection of the moving element of such a meter, when connected across the supply mains, is proportional to the total power absorbed by the circuits and the iron core. Neglecting the small resistance loss in the series circuit, it is possible, by adjusting the resistance in the potential circuit and the number of turns on the iron ring, to make the equivalent eddy-current loss in the instrument (including the resistance loss in the potential circuit as an eddy-current loss, since it depends on the effective value of the voltage in the same way as the eddy-current loss in the sample) any desired proportion of the total loss. These adjustments are so made that at about 0.6 full-scale deflection the equivalent eddy-current loss on a pure sine-wave voltage at 60 cycles is 20 per cent of the total, this being the average proportion of the hysteresis loss to total loss in commercial 60-cycle transformers.

The instrument is then calibrated to read directly in volts by connecting it in parallel with an ordinary alternating-current voltmeter on a pure sine-wave voltage of the required frequency.

If the percentage hysteresis loss in the transformer under test is different from that of the iron-loss voltmeter, a slight error will be introduced. Curves *D* and *F* in Fig. 13 show the error when the eddy-current loss in the transformer is 14 per cent and 30 per cent respectively, instead of 20 per cent.

The frequency need only approximate the normal values; the final core loss determined at the voltage indicated by the iron-loss voltmeter will be the same as would be obtained on a sine wave of normal frequency and voltage.

Adjustment of Form Factor. — Chubb also gives a method of obtaining a form factor of 1.11 from a wave of any shape. This is shown in Fig. 14. *T* is the transformer under test, *W* an ordinary wattmeter, V_1 the iron-loss volt-meter, V_2 an ordinary a-c. voltmeter, *R* a variable resistance, *L* a variable inductance, *C* an aluminum electrolytic condenser having a critical voltage less than that impressed across it. By varying *L* and *R* the readings of V_1 and *V* may be made to agree; under these conditions the form factor of the voltage across the transformer is 1.11. The wave however will not in general be a sine wave.

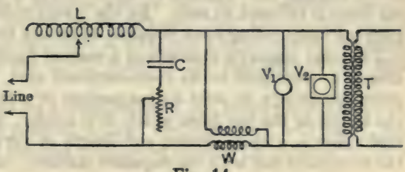

Fig. 14.

BIBLIOGRAPHY. — Gray, A., *Absolute Measurements of Electricity and Magnetism;* Edgecumbe, K., *Industrial Measuring Instruments;* Karapetoff, V., *Experimental Electrical Engineering; Magnetic Testing at the Bureau of Standards*, Circ. No. 17; Burrows, C. W., *Magnetic Analysis in the Study of Steel Products*, Met. and Chem. Eng., 17, p. 130, 1917; *Magnetic Testing of Straight Rods in Intense Magnetic Fields*, Bull. Bur. Stands., 15, p. 625, 1920; Ytterberg, A., *Method of Determining Hysteresis Loops*, El. World, 65, p. 212, Jan. 23, 1915. Also the references in the text.

MAGNETS, PERMANENT. — (*See also Magnetic Properties of Iron and Other Metals; Magnetic Testing.*) A permanent magnet is a piece of iron or steel which has been magnetized and which retains a relatively large part of its magnetism when the external magnetizing force is removed. The simplest form of permanent magnet is the bar magnet, used principally for experimental and instructional purposes and in the compass and certain types of galvanometers, etc. U-type permanent magnets are largely used in electrical measuring instruments and in the construction of magnetos.

The materials used for permanent magnets are (1) cast iron, (2) carbon-steel (0.15 to 0.3 per cent carbon), and (3) alloy steel containing carbon together with tungsten, molybdenum, chromium, or vanadium.

Cast iron magnets are used principally where weight is not objectionable.

Prior to 1914 high grade permanent magnets were made almost universally of alloy steel containing principally carbon, tungsten and chromium, and in which the percentage of manganese, sulphur and phosphorus was kept within the lowest practical limits. Since that date, chromium alloys have been produced from which it is possible to make magnets equal to those made of tungsten steel.

While the chromium alloys do not permit of as wide a range in manufacturing and heat-treating conditions as tungsten steels, with proper and uniform conditions the product shows greater uniformity than that made of tungsten steel.

Carbon-steel magnets have about 75 per cent of the strength of tungsten steel magnets of the same size.

Permanent magnets are usually made by cutting and forming the raw material, and hardening it by heating to a temperature, dependent upon the nature of the steel, usually in the neighborhood of 900° C., and cooling quickly by plunging into cold water or brine, and magnetizing.

Residual Induction and Coercive Force. — The two magnetic qualities of fundamental importance in a permanent magnet are (1) the residual induction of magnet and (2) its coercive force or retentivity. By residual induction is meant the flux density which exists in the material when the external magnetizing force is reduced to zero. By the coercive force or retentivity is meant the external magnetizing force required to reduce the induction to zero.

The total magnetic strength of a magnet having a given residual flux per unit area is proportional to its cross-sectional area, and the ability of a magnet of given retentivity to hold its magnetism is proportional to its length; that is, the length of the bar from which it is made. For any magnetic circuit considered there is a cross-section and length of permanent magnet which will give the maximum flux with the requisite stability, using a minimum of magnet steel.

Values of Residual Induction and Coercive Force. — In the *Physical Review* for Dec., 1920 (p. 496), Honda and Saito give data on "K.S." magnet steel, a cobalt-tungsten-chromium steel having the composition:

C...........	0.4 to 0.8 per cent
Co...........	30 to 40 per cent
W...........	5 to 9 per cent
Cr...........	1.5 to 3 per cent

For samples 20 cm. long and 0.5 cm. diameter (dimensional ratio of 40), heated to 950° C. and quenched in heavy oil, and then magnetized in a field of 1500 gilberts per centimeter, the residual induction ranged from 7800 to 11,500, depending upon the composition of the sample. The coercive force ranged from 257 to 226 gilberts per centimeter. This coercive force is stated by Honda and Saito to be about three times greater than that of the best tungsten steel, and the residual induction to be also greater than that of tungsten steel.

The data given by Honda and Saito also show that for dimensional ratios greater than about 18, the residual induction is independent of the value of the dimensional ratio, whereas for smaller dimensional ratios the residual induction is approximately proportional to this ratio, the relation being

$$B = 650\frac{l}{d},$$

where B is the residual induction, l the length, and d the diameter of the specimen. For a dimensional ratio of 15, K. S. magnet steel is about 1.8 times stronger than that of tungsten steel.

Effect of Air Gap. — The useful flux of a permanent magnet operating across an air-gap is always less than the flux which the magnet would give with no air-gap between its poles, or when magnetically "short circuited." On account of the very high reluctance, or magnetic resistance, of air the residual magnetism of a magnet of a given length is lessened as the air path between the poles is increased. This must be given careful consideration in the design of apparatus embodying the use of permanent magnets.

Since the introduction of an air gap between the poles of a permanent magnet has the effect of partially demagnetizing it, and the extent of the demagnetizing depends upon the relative length of the magnet and the air gap, it is important that the ratio of length of the magnet to the length of the air gap be such that the magnet will not be demagnetized by the air gap to the point where magnetization becomes unstable.

In some types of apparatus, such as magnetos, the magnet is opposed by a coil of wire carrying current. In designing permanent magnets for machines of this type it is important to take into account the effects of both air gap and coil.

Magnetization. — The process of magnetizing should be such as to fully saturate the magnet uniformally throughout its entire length. This can best be done by having the proper shaped and size coils to slip over each leg of the magnet, in the case of U-shaped magnets, completing the magnetic circuit by placing a keeper over the magnet gap and passing current through the coils.

The magnetizing apparatus should be so arranged that the magnet can be slipped out of the magnetizing coils and a soft iron keeper put on it before removing the keeper that was put on to complete the magnetic circuit while magnetizing.

U-shaped magnets magnetized by a very common but incorrect method, that of placing the magnets on the poles of a large electromagnet, will have large losses due to a large portion of the flux circulating around the magnet in paths that are not useful.

Aging and Magnetic Treatment. — One of the main conditions to be met by permanent magnets used in most electrical measuring instruments is that the magnet be stable and not subject to change with time, otherwise the instrument will go out of calibration. To secure this condition requires (1) a physical treatment of the steel after quenching, which makes further changes in its structure, molecular condition or magnetic qualities improbable, and (2) a magnetic treatment which will make changes in its magnetization under the condition of ordinary operation improbable.

When a magnet is hardened it undergoes rapid structural changes and for a time after quenching there is a change in condition taking place, rapid at first, and gradually decreasing until the physical properties become stable. This stability of the structural state of the steel can be hastened by physical treatment which usually consists in boiling the magnet in water for about 40 hours. This is called aging or maturing.

The magnetic treatment of the permanent magnet is usually carried out as follows: After the magnetic circuit on which the permanent magnet is to operate is completely assembled, a demagnetizing force is applied which removes a portion of the flux. It is next subjected to a magnetizing force and then to a demagnetizing force. This has the effect of reducing the residual induction somewhat, but makes the magnet more stable.

Magnet Meter.— The magnet meter provides a simple and effective method for the commercial testing of permanent magnets. This instrument consists of an ordinary millivoltmeter with the permanent magnet removed and having soft iron extensions which are attached to the pole pieces of the meter and extend through the meter case. The moving coil of this meter is connected in series with a battery, rheostat and milliammeter.

A magnet of standard strength is placed against the soft iron extensions of the meter pole pieces and the current through the magnet meter coil adjusted, by means of the rheostat, until the pointer of the instrument deflects to some arbitrary point on the scale. The standard magnet is now replaced with the magnet to be tested. With the current kept the same as in the first test the deflection of the instrument is noted.

BIBLIOGRAPHY. — A considerable portion of the material in this article is taken from a pamphlet on *Permanent Magnets* published by the Esterline Co., Indianapolis. See also Kelly, F. C., *Permanent Magnets*, G. E. Rev., 20, p. 569, 1917; Thompson, S. P., *Magnetism of Permanent Magnets*, Inst. Elec. Eng. J. 50, p. 80, 1913 (contains complete bibliography up to 1913); Honda and Saito, *Phys. Rev.*, Dec., 1920; Morgan, J. D., *Testing Magneto Magnets*, Engin., 103, p. 559, 1917.

MAXIMA AND MINIMA. — (*See also Derivatives; Series, Mathematical.*) Let y be any function of a variable x, then y will be a maximum or minimum for any value of x which satisfies

$$\frac{dy}{dx} = 0 \tag{1}$$

provided $\frac{d^2y}{dx^2}$ is not zero. If the second derivative $\frac{d^2y}{dx^2}$ is positive for this value of x, then the corresponding value of y is a minimum; if this second derivative is negative, the corresponding value of y is a maximum.

In case $\frac{d^2y}{dx^2}$ is also zero for the value of x which satisfies (1), the corresponding value of y is not a maximum or minimum unless $\frac{d^3y}{dx^3}$ is also zero and $\frac{d^4y}{dx^4}$ is not zero. When $\frac{d^3y}{dx^3} = 0$, y is a minimum if $\frac{d^4y}{dx^4}$ is positive and a maximum if $\frac{d^4y}{dx^4}$ is negative. In case $\frac{d^4y}{dx^4}$ is also zero, similar relations must hold for the fifth and sixth derivatives, etc.

Example. — Find the maximum and minimum values of

$$y = 2x^3 - 9x^2 + 12x - 3,$$

then

$$\frac{dy}{dx} = 6x^2 - 18x + 12 = 0,$$

$$\frac{d^2y}{dx^2} = 12x - 18,$$

whence y is maximum or minimum for $x^2 - 3x + 2 = 0$, that is, for $x = 2$, or $x = 1$. For $x = 2$, $\frac{d^2y}{dx^2}$ is positive; for $x = 1$, $\frac{d^2y}{dx^2}$ is negative; hence the maximum value of y is 2 and occurs for $x = 1$, while the minimum value of y is 1 and occurs for $x = 2$.

[W. A. Del Mar.]

MECHANICS, PRINCIPLES OF. — (*See also Structures, Simple; Units and Conversion Factors.*) In this article are given the definitions of the more commonly-used mechanical quantities together with a statement of the quantitative laws in accord with which displacements and motion of matter (including deformations) take place. The interrelations of the various units employed for measuring any particular quantity are given in the article on *Units and Conversion Factors.*

DEFINITIONS. — The various terms relating to the displacement and motion of matter and to forces producing these displacements and motions are the following:

Scalar and Vector Quantities. — A mechanical quantity which is not directed in space, e.g., mass, energy, etc., is called a scalar quantity. A mechanical quantity which has a space direction as well as magnitude, e.g., velocity, force, etc., is called a vector quantity. A scalar quantity may be positive or negative, that is, has two "senses," but to specify a vector quantity like force it is necessary to specify not only its magnitude but its direction with respect to one or more fixed lines of reference or axes. Scalar quantities may be treated as ordinary algebraic quantities, and added and subtracted in the usual way. Vector quantities must be added and subtracted *vectorially,* as described in the article on *Vectors.*

Linear Displacement (l). — When a particle P moves from a point A to any other point B, the distance, measured along a straight line, from A to B is called the linear displacement of the point P.* Any unit of length may be used as a unit of displacement; see *Units and Conversion Factors.*

Angular Displacement (θ). — Let a particle P move from a point A to some other point B, and let OX be any arbitrarily-chosen line, or axis. Draw planes through A and OX and through B and OX. Then the angle between the planes AOX and BOX is called the angular displacement of P about OX. * Angular displacement may be measured in degrees, radians or turns; see *Angles* and *Units and Conversion Factors.*

Linear Velocity (v) **and Speed** (s). — The linear velocity or speed of a particle P is the rate of increase with time of the linear displacement of P, i.e.

$$v = \frac{dl}{dt} \qquad \text{or} \qquad s = \frac{dl}{dt}, \tag{1}$$

where dl is the linear displacement in time dt.
Any unit of length per any unit of time may be used as a unit of velocity or speed; see *Units and Conversion Factors.*

Angular Velocity or Speed (ω). — The angular velocity or speed of a particle about a given axis X is the rate of increase with time of its angular displacement about this axis, i.e,

$$\omega = \frac{d\theta}{dt}, \tag{2}$$

where $d\theta$ is the angular displacement in time dt. Any unit of angle per any unit of time may be used as a unit of angular speed; see *Units and Conversion Factors.*

* The positions of A and B (and of OX also in the case of angular displacement) must be referred to some system of coördinates; when the system of coördinates moves, the displacement as above defined is the displacement *relative to* this system of coördinates.

Linear Acceleration (a). — The linear acceleration of a particle P is the rate of increase with time of the linear velocity of P, i.e.,

$$a = \frac{dv}{dt} = \frac{d^2l}{dt^2}, \tag{3}$$

where dv is the increase of linear velocity in time dt. Any unit of velocity per any unit of time may be used as a unit of acceleration, e.g., centimeters per second per second, miles per hour per hour, etc.; see *Units and Conversion Factors*.

Acceleration Due to Gravity (g). **Gravitational Acceleration Constant** (g_0). — At any given place on the earth a body falling freely in a vacuum has a constant linear acceleration, independent of its size, shape or material. This acceleration is called the acceleration due to gravity (g), and the particular value of this acceleration at 45 degrees latitude and sea level is called the gravitational acceleration constant (g_0). The value of g_0 as adopted by international agreement * is

$$g_0 = 980.665 \text{ cm. per sec. per sec.}$$
$$g_0 = 32.1739 \text{ ft. per sec. per sec.}$$

The value of g for any other location varies but slightly from this value, being at sea level approximately 0.3 per cent less at the equator, 0.3 per cent greater at the poles, and decreasing at the rate of about 0.01 per cent per 1000 feet increase in elevation. See *Landolt-Bornstein's Tables*.

Angular Acceleration (α). — The angular acceleration of a particle P about an axis X is the rate of increase with time of the angular velocity of P about X, i.e.,

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}, \tag{4}$$

where $d\omega$ is the increase in angular velocity in time dt. Any unit of angular velocity per any unit of time may be used as a unit of acceleration, e.g., degrees per second per second, turns per second per second, etc.; see *Units and Conversion Factors*.

Mass or Weight.* — Two bodies are said to have equal masses or weights,† irrespective of their volume, shape or chemical composition, if, when they are suspended simultaneously in a vacuum, one from each end of an equal-armed balance, there is no tipping of the beam of the balance from its original position. This criterion for the equality of two masses holds only in case the bodies and the balance are neither electrically charged nor magnetized, both bodies are supported at the same distance from the earth, and the equilibrium of the balance is not affected by the presence of any other bodies (except the earth) in the vicinity. This is an entirely arbitrary definition, but mass as thus defined is found to be a fundamental property of a body irrespective of its shape, physical state or relation to other bodies. The units of mass or weight and their interrelations are given in the article on *Units and Conversion Factors*.

Density (δ) **and Specific Gravity.** — The density of a uniform substance is defined as the mass of the substance per unit volume. When the substance

* *Troisième Conf. Gen. des Poids et Mes., 1901, p. 66.* See Note 5, p. 3 of this book.

† The term weight is also used for the force exerted by the earth on a portion of matter; in this sense weight is not independent of the relation of the given portion of matter to other bodies, and the term mass is therefore preferable when referring to quantity of matter, since it has not acquired a double meaning. However, engineers almost invariably use the word weight instead of mass for quantity of matter. See the article on *Units and Conversion Factors* for a further discussion of this point.

is not uniform, its density at any point is defined as the mass of an infinitesimally small volume taken about the point divided by this volume; i.e., calling dv the volume and dm the mass of this volume, the density is

$$\delta = \frac{dm}{dv}. \tag{5}$$

In the c.g.s. system the standard unit of density is the gram per cubic centimeter, but any other unit of mass per any unit of volume may be used in either the metric or English system; see *Units and Conversion Factors*. For values of the density of various materials see article on *Weights of Materials*.

The specific gravity of a substance is defined as the ratio of the weight of a given volume of that substance to the weight of an equal volume of water or air. Water is used as the standard of reference for solids and liquids, and air at 0° C. and 760 mm. mercury pressure as the standard of reference for gases. Strictly, the temperature of the water also should be specified, but this is not always done. The variation in the weight of a given volume of water with temperature is slight and for many purposes negligible. When 4° C. is taken as the standard water temperature the specific gravity of a substance is numerically equal to its density in grams per cubic centimeter to within 25 parts in 1,000,000, a difference which is very much less than the degree of precision of ordinary measurements of density.

Surface and Lineal Densities. — Density as defined above is the *volume* density of a body. It is sometimes convenient to use factors which represent the mass or weight of a substance (or other physical quantity) per unit of surface or per unit of length, e.g., the weight per square foot of a plate or the weight per foot of a wire.

Center of Mass, Center of Gravity, or Center of Inertia. — A body of mass M which has any size or shape may be considered as made up of a number of small particles of masses m_1, m_2, m_3, etc., such that $m_1 + m_2 + m_3 + \cdots = M$. These particles may be considered as small as desired, that is, each particle may be considered so small that it occupies but a point in space. Choose any three mutually perpendicular planes, fixed with respect to one another, and represent by x_1, y_1 and z_1 the perpendicular distances of the particle m_1 from these planes respectively, and by x_2, y_2 and z_2 the perpendicular distances of the particle m_2 from these three planes respectively, and so on for the other particles. Then the point whose distances from these three planes are respectively

$$\begin{aligned} X &= \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{M} \\ Y &= \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{M} \\ Z &= \frac{m_1z_1 + m_2z_2 + m_3z_3 + \cdots}{M} \end{aligned} \tag{6}$$

is defined as the center of mass, or center of gravity, or center of inertia, of the body. The name center of gravity arises from the fact that the vertical forces acting on all the various particles of a body, due to the pull of the earth, may be considered equivalent to a single force, equal to the sum of these individual forces, applied to the body at this point. The center of mass of a perfectly rigid body is a fixed point with respect to every point in the body; it may, however, lie either within or without the body.

Equations (6) may also be written in terms of the calculus, thus:

$$X = \int \frac{\delta x\, dv}{M}, \quad Y = \int \frac{\delta y\, dv}{M}, \quad Z = \int \frac{\delta z\, dv}{M}, \tag{6a}$$

where dv represents an elementary volume of the body at any point, x, y and z the distances of this point from the three planes of reference, δ the density of the body at this point, and M the total mass of the body. When the density is uniform throughout,

$$X = \int \frac{x\,dv}{V}, \quad Y = \int \frac{y\,dv}{V}, \quad Z = \int \frac{z\,dv}{V},$$

where V is the total volume of the body.

Centroid or Center of Gravity of a Plane Section. — This is defined as the point whose coördinates in the plane of the section are

$$X = \int \frac{x\,dA}{A} \quad \text{and} \quad Y = \int \frac{y\,dA}{A} \tag{6b}$$

referred to any two fixed mutually perpendicular lines in this plane, A being the total area of the section and dA any elementary area of the section. Physically, the center of gravity of a plane section may be defined as the center of gravity of a thin plate having the same shape as that of the section and of uniform thickness and specific gravity. See article on *Structures, Simple,* for the position of center of gravity for various plane sections.

Center of Gravity of Cylinders and Prisms. — The center of gravity of a cylinder or prism with parallel end surfaces and of *any* shaped cross-section, solid or hollow, is the centroid of that cross-section of the cylinder or prism which is halfway between the two ends, provided the cylinder has a uniform density throughout.

Moment of Inertia (I). — Consider any axis of reference X and any particle of mass m at a distance r from this axis; then the product mr^2 is called the moment of inertia of the particle m about the axis X. The moment of inertia of an extended body or system of bodies about any axis is

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots, \tag{7}$$

the summation including products mr^2 for all the particles of the body or system of bodies, r being the distance of the particle from the axis. This may also be written in the notation of the calculus as

$$I = \int r^2\,dm, \tag{7a}$$

where dm represents the mass of any particle and r is the distance from the axis. In case the body has a uniform density δ throughout this may also be written

$$I = \delta \int r^2\,dv, \tag{7b}$$

where dv represents an elementary volume of the body.

Units of Moment of Inertia of Bodies. — The moment of inertia of a body is of the nature of, or has the "dimensions" of, $(\text{length})^2 \times (\text{mass})$. When the mass is expressed in pounds and the distance r in feet then the moment of inertia may be said to be expressed in $(\text{foot})^2$-pounds, and similarly for any other units of mass and length. In applying the formulas below the same set of units must be used throughout. See *Units and Conversion Factors.*

Moment of Inertia of a Plane Section. — The moment of inertia of a plane section about any axis is defined by the relation

$$I = \int r^2\,dA, \tag{7c}$$

where dA represents any elementary area of this section and r the distance of this elementary area from the axis. When the axis is chosen *perpendicular* to the plane of the section *through its center of gravity* the moment of inertia about this axis is called the "polar" moment of inertia. Physically, the moment of inertia of a plane section about any axis may be defined as the moment of inertia about this same axis of a thin plate having the same shape as that of the section and having unit mass per unit of area.

The unit of moment of inertia of a plane surface has the dimensions of length to the fourth power.

Let X and Y be two mutually perpendicular axes in the plane of the section *passing through its center of gravity* and let Z be the axis through the center of gravity perpendicular to this plane and therefore also perpendicular to X and Y; let I_x and I_y be the moments of inertia of the plane section about X and Y respectively, and I_z be the polar moment of inertia of the section. Then

$$I_z = I_x + I_y. \tag{7d}$$

Values of I_x and I_y for various plane figures are given in the article on *Structures, Simple.*

Let X be any axis in the plane of a section, X_0 an axis through the center of gravity of the section parallel to X, x the distance between the two parallel axes X and X_0, I_0 the moment of inertia of the section about X_0, and A the area of the section. Then the moment of inertia of the section about X is

$$I_x = I_0 + x^2 A. \tag{7e}$$

Principal Axes and Principal Moments of a Plane Section. — For every plane surface there are two rectangular axes passing through its center of gravity and lying in the surface, such that about one of these axes the moment of inertia is less, and about the other greater, than that about any other axis in the given surface. These two axes are called the *principal axes* and the moments the *principal moments of inertia.* If the plane figure has an axis of symmetry, this axis is one of the principal axes; e.g., a rectangle has two axes of symmetry, one parallel to each side, hence each of these axes is a principal axis. In the case of a circle any axis may be considered a principal axis. The principal axes for various sections are given in the article on *Structures, Simple.*

Moment of Inertia of Cylinders and Prisms. — Let the cylinder or prism be of any cross-section whatever, solid or hollow; let its density be δ, constant throughout; let the length of its own axis between the parallel end surfaces be l; and let I_s be the moment of inertia of the cross-section of the cylinder or prism about any axis X parallel to the length l; then the moment of inertia of the entire cylinder or prism about this axis X is

$$I_c = \delta l I_s. \tag{7f}$$

The density δ must be referred to a volume unit based upon the same unit of length as that in which the dimensions of the cylinder or prism are expressed. For example, if I_s is expressed in centimeters-to-the-fourth-power, δ must be expressed in mass per cubic centimeter, and l must be expressed in centimeters; if δ is in grams per cubic centimeter, then the moment of inertia is in (centimeter)^2-grams.

Radius of Gyration (ρ). — The radius of gyration of a body about any axis is defined by the relation

$$I = M\rho^2 \quad \text{or} \quad \rho = \sqrt{\frac{I}{M}}, \tag{8}$$

where M is the mass of the body and I its moment of inertia about the given axis. When the body has a uniform density throughout the radius of gyration may also be written

$$\rho = \sqrt{\frac{\int r^2 dv}{V}}, \tag{8a}$$

where V is the volume of the body. Radius of gyration is of the same nature as length and is therefore expressed in the same units as length.

Radius of Gyration of Plane Section. — The radius of gyration of a plane section about any axis is defined by the relation

$$\rho = \sqrt{\frac{\int r^2 dA}{A}}, \tag{8b}$$

where A is the total area of the section, dA an elementary area of this section and r the distance of dA from the axis. Values of ρ_x and ρ_y for various sections are given in the article on *Structures, Simple*.

From formula (7d) it follows that if ρ_x and ρ_y represent the radii of gyration of a plane section about two mutually perpendicular axes *passing through the center of gravity* of the section, then the radius of gyration ρ_z of this section about an axis through its center of gravity *perpendicular* to the plane of the section is

$$\rho_z = \sqrt{\rho_x^2 + \rho_y^2}. \tag{8c}$$

For example, for a circular section $\rho_x = \rho_y = \frac{d}{4}$, where d is the diameter of the section. Hence the polar radius of gyration about a perpendicular axis through the center of the circle is $\rho_z = \frac{\sqrt{2}}{4} d$.

From formula (7c) it also follows that if ρ_0 is the radius of gyration about any axis X_0, then the radius of gyration about a *parallel* axis X at a distance x from X_0 is

$$\rho_x = \sqrt{\rho_0^2 + x^2}. \tag{8d}$$

Radius of Gyration of Cylinders and Prisms. — The radius of gyration of a cylinder or prism about any axis X parallel to its own axis is the same as the radius of gyration about the axis X of any plane section of the cylinder or prism, provided the end surfaces are parallel and the density is uniform throughout. This relation holds for a prism or cylinder of any shape of cross-section, whether solid or hollow.

Linear Momentum (mv). — When the *center of mass* of a body of mass m is moving with a linear velocity v with respect to any point P, the product mv is called the linear momentum of the body with respect to this point. Units of linear momentum are of the same nature or dimensions as the units of power; see *Units and Conversion Factors*.

Experience shows that in any system of bodies composed of two bodies or groups of bodies A and B, any change in the linear momentum of the first body or group of bodies A with respect to the center of mass of the system is accompanied by an equal and opposite change in the linear momentum of the second body or group of bodies B with respect to the center of mass of the system, and vice versa. This fact of experience is known as the ***Principle of the Conservation of Linear Momentum.***

Angular Momentum ($I\omega$). — When a body has a moment of inertia I about any axis X and is rotating about this axis with an angular velocity ω with respect to any point P, the product $I\omega$ is called the angular momentum of the body about this axis with respect to the given point. Units of angular momentum are of the same nature or dimensions as those of linear momentum and of power; see *Units and Conversion Factors*.

Experience shows that in any system of bodies composed of two bodies or groups of bodies A and B, any change in the angular momentum of the first body or group of bodies A about any axis X is accompanied by an equal and opposite change in the angular momentum of the second body or group of bodies B about this same axis, and vice versa. This fact of experience is known as the *Principle of the Conservation of Angular Momentum.*

Force (f). — In general terms a force is that which produces or tends to produce a change in the state of rest or motion of a body, or "a force is pull or a push." The nature of force is not thoroughly understood, but the effects of a force, e.g., change in motion, the extension or compression of a spring, etc., are readily measured. From the principle of the conservation of linear momentum stated above it follows that any change in the motion of a body or portion of a body A is always accompanied by a change in the motion of some other body or portion of the same or of some other body B. Hence the body A is said to exert a force on the body B, and vice versa.

In a system composed of but two bodies A and B, uninfluenced by the presence of any other bodies, the measure of the force produced on the body A by the body B may be taken as the rate of change with time of the linear momentum of A with respect to the center of mass of the system formed by A and B. The measure of the force produced on B by A is similarly defined. From the principle of the conservation of linear momentum it then follows that the force produced on A by B is equal and opposite to the force produced on B by A, or "action and reaction are equal and opposite."

Stated in a formula the above definition is that the mutual force between any two bodies A and B whose motion is uninfluenced by any other forces is

$$f = m_1 a_1 = -m_2 a_2, \tag{9}$$

where m_1 and m_2 are the masses of A and B respectively, and a_1 and a_2 are the accelerations of the centers of mass of A and B respectively with respect to the center of mass of the system formed by A and B together.

Force of Gravitation. — For example, consider the case of a mass m at any point above the earth. The earth exerts a force on m and m exerts an equal and opposite force on the earth, and if there are no other forces acting m will move toward the joint center of mass of the earth and the body m, and similarly the earth will move toward this same point. However, as the earth has a mass many times that of any body at or near its surface, the center of mass of the system formed by m and the earth is practically that of the earth itself, and the motion of the earth with respect to this point is inappreciable. Hence the force exerted by the earth on the mass m is, within the limits of error of observation,

$$f = mg, \tag{9a}$$

where g is the acceleration of the center of mass of m with respect to the center of the earth, or with respect to any point fixed with respect to the surface of the earth in the vertical line through the center of mass of m.

Units of Force. — The rational unit of force is that force which will give unit linear acceleration to unit mass; this unit of force is called the "absolute" unit of force. When the mass is expressed in grams and the acceleration in centimeters per second per second, the corresponding absolute unit

of force is called the "dyne;" when the mass is expressed in pounds and the acceleration in feet per second per second, the corresponding absolute unit of force is called the "poundal."

The relation expressed by equation (9a), however, suggests another unit of force which for many purposes is very convenient, and in fact is the unit ordinarily used by engineers. This unit, called the "gravitational" unit of force, is the force exerted by the earth on unit mass; since the value of this force in terms of the absolute unit varies with latitude and elevation, it is also necessary to specify a definite place of measurement of this unit force, or better a *definite* value of the acceleration g, which shall be used in evaluating this force in terms of the absolute unit. The value of g adopted by international agreement is $g = 980.665$ cm. per sec. per sec. = 32.1739 ft. per sec. per sec., which (see Note 5 at top of p. 3 of this book) is equal to the acceleration due to gravity at 45 degrees latitude and sea-level; see also above under *Acceleration.* Unfortunately the gravitational units of force are given the same names as the units of mass to which they refer; for example a force of 1 pound is the pull exerted by the earth on a mass of 1 pound at 45 degrees latitude and sea-level. This double use of names often leads to confusion unless one keeps clearly in mind the definitions of unit mass and unit force. See *Units and Conversion Factors.*

When the force acting on a body is expressed in absolute units, the relation between force, mass and linear acceleration is

$$f = ma \quad \text{absolute units.} \qquad \text{(9b)}$$

When the force is expressed in gravitational units, the relation between force, mass and linear acceleration is

$$f = \frac{m}{g_0} \cdot a \quad \text{gravitational units,} \qquad \text{(9c)}$$

where g_0 stands for the numerical factor 32.1739 when f is in pounds, m in pounds and a in feet per second per second; similarly when f is expressed in grams, m in grams and a in centimeters per second per second g_0 stands for 980.665.

Measurements of Force. — One seldom has to deal with a simple system of but two bodies, and consequently the above definitions of the measure of a force can seldom be applied to an actual measurement. The value of these definitions is that they fix the unit of force. Forces are measured by balancing the pull of an unknown force against a known force (e.g., gravity or the reacting force of a spring), the principles involved being deduced from the fundamental principle of the conservation of linear and angular momentum when applied to bodies in equilibrium; see the section on *Conditions for Equilibrium* below.

Weight and Force. — The word weight, as already noted, is used to designate both mass and force. A weight of 10 pounds, say, as ordinarily used in reference to "how much" of a substance, means that the piece of matter in question has a mass of 10 pounds. A weight of 10 pounds used in reference to the pull produced by the earth on a piece of matter means that this pull is a force of 10 pounds. From the above definitions it follows that, neglecting the slight variation with latitude and elevation, the *numerical* values of weight used in the two senses are equal, provided mass is expressed in absolute units and force in gravitational units, i.e., a mass of 10 pounds also weighs 10 pounds, but weighs 321.739 *poundals;* see above under *Units of Force.*

Point of Application and Line of Action of a Force. — Any particle or point of a body which is acted upon directly by some external force (e.g.,

a string attached to the body at this point) is said to be the point of application of this external force. A line drawn through this point in the direction of the force is called the line of action of the force. In general, whenever a solid body is acted upon by any number of external forces applied at various points the motion produced is the same as that which could be produced by *not more than two* single external forces. When the points of application of these two resultant forces coincide, then the actual motion produced is the same as that which would be produced by a *single* force acting at that point.

Impulse of a Force. — The impulse of a force is defined as the time integral of the force, i.e., if a force f is applied for a time t, then the impulse is $\int_0^t f\,dt$, account being taken of both the variation of the magnitude and direction of the force during the given time interval. The total change in the linear momentum of a body in any time t is equal to the impulse of the resultant force acting on it during this time. The conception of the impulse of a force is useful in dealing with suddenly applied forces which continue but a short time.

Concurrent and Coplanar Forces. — When the lines of action of all the external forces acting on a body meet in a point these forces are said to form a concurrent system of forces. When all the external forces acting on a body lie in the same plane they are said to form a coplanar system of forces.

Pressure (p). — The perpendicular component of the force per unit area exerted on any surface is called the pressure on that surface. Let df be the perpendicular or normal component of the force acting on an area dA, then the pressure at dA is

$$p = \frac{df}{dA}. \tag{10}$$

The term pressure is sometimes used as a synonym for force and what is here defined as pressure is called the "intensity of pressure." The rational unit of pressure is force per unit area, e.g., dynes per sq. cm. or pounds per sq. in., but a number of other arbitrary units are employed, such as, one inch of water column, one millimeter of mercury column, one atmosphere, etc. A dyne per square inch is also a barie. A standard atmosphere is the pressure that will support a column of mercury 76 centimeters = 29.9212 inches high at 0° C. at a place where $g = 980.665$ centimeters per second per second; using Thiesen and Scheel's determination of the density of mercury at 0° C. as 13.59545 grams per cubic centimeter (*Zeitsch. f. Instrkde., 1898, Vol. 18, p. 138*) a standard atmosphere is equivalent to 14.6964 pounds per square inch. See *Units and Conversion Factors.*

Absolute Pressure. — The atmosphere exerts a pressure (ranging from about 29 to 30 inches of mercury, depending upon the weather conditions and elevation) upon every surface with which it is in contact. The total pressure on a surface including that applied by any artificial means and that of the atmosphere is called the absolute pressure on the surface.

Torque or Moment of a Force (T). — Consider any axis X and any force acting at a point P at a perpendicular distance r from this axis; let f be the *component* of this force perpendicular to the plane through X and P. Then the product

$$T = rf \tag{11}$$

is called the moment of the force, or the torque due to this force, about the axis X. The distance r is called the "lever arm" or simply the "arm" of the force about this axis. Both absolute and gravitational units of torque are

used, the units being of the same nature or dimensions as the units of energy (see below), but the two words in the compound names of the energy units are usually reversed. For example the foot-pound is an energy unit but the corresponding unit of torque is called the pound-foot. The conversion factors, however, are identical; see *Units and Conversion Factors*. Torque is also frequently expressed as "torque at unit radius," e.g., a torque of so-many pounds at 1 foot radius is a common expression; a more exact expression for the measure of torque would be "force at unit radius."

Torques about the same axis can be added algebraically, and torques about several axes meeting in a point can be added vectorially.

Couples. — Two equal and opposite parallel forces which are not concurrent (i.e., do not act along the same line) are said to form a couple, and the strength of the couple is defined as the product of either force by the perpendicular distance between their lines of action. The torque produced by a couple about any axis perpendicular to the plane of the two forces is equal to the strength of the couple.

Torque and Angular Acceleration. — The relation between torque, moment of inertia and angular acceleration is the same as that between force, mass and linear acceleration, i.e.,

$$T = I\alpha, \qquad \text{absolute units,} \tag{12}$$

where T is the torque in absolute units, I the moment of inertia and α the angular acceleration, all about the same axis. When the torque is expressed in gravitational units the relation is

$$T = \frac{I}{g_0}\alpha, \qquad \text{gravitational units,} \tag{12a}$$

where g_0 is a number numerically equal to 32.1739 when T is expressed in pound-feet, I in (foot)^2-pounds and α in radians per second per second; similarly for the centimeter-gram-second units $g_0 = 980.665$.

Work and Energy. — When the point at which a force is applied to a body A moves* with respect to the agent producing the force, in such a manner that the force has a component in the direction of the displacement of its application point, the force is said to do work; the body B exerting the force on A is also said to do mechanical work on the body A. As a measure of the mechanical work dW done by the force f, when its point of application moves a distance dl with respect to the body producing the force, is taken the product of the displacement dl by the component of the force f in the direction of this displacement, i.e.,

$$dW = (f \cos\theta)\, dl, \tag{13}$$

where θ is the angle between the direction of the force and the direction of the displacement. For a finite displacement the mechanical work done is

$$W = \int_0^l f\,(\cos\theta)\, dl. \tag{13a}$$

When mechanical work is done on a body a change in the state of motion or in some other condition of the body is always produced. Like changes can also be produced by other means. For example, when two bodies are rubbed together rapidly, mechanical work is done on them by the agent which produces the force which moves one over the other against the opposing force due to friction. As a result, the temperature of the bodies is raised. The

* Either as a result of the motion of the body A as a whole or in consequence of the motion of that part of A to which the force is applied.

temperature of the two bodies may also be raised by placing them near or in contact with a hotter body, without there being any appreciable amount of *mechanical* work done. In general, any change produced in a body or system of bodies by any means whatever, which change can also be produced directly by doing mechanical work on that body or system, or indirectly by doing mechanical work on some other body or system, is said to be due to a transfer of "energy" *to* that body or system; or work * is said to be done on that body or system, irrespective of the means whereby the change is produced.

As a measure of the gain in energy corresponding to any change in a body or system of bodies is taken the amount of mechanical work which would have to be done by a mechanical force to produce this change and no other. For example, water can be heated by stirring it rapidly with a paddle driven by some external force; the total mechanical work done by this force can be expressed in terms of the value of the force, the number of revolutions of the paddle and the diameter of the pulley attached to the paddle, and the mass of the water and the resultant temperature rise can be measured. By making proper corrections for the work done against the various frictional forces other than the opposing force due to the stirring of the water itself and for the loss of energy by radiation, the amount of work required to raise the temperature of a pound of water, say, one degree Fahrenheit can be determined. By this means a very convenient secondary unit of energy can be expressed in terms of the unit of mechanical work, and the secondary unit can be used for expressing the amount of energy involved in various heat effects. See *Heat and Thermal Properties*.

Whenever a change takes place in a body or system of bodies which is the *reverse* of the change which can be produced in it by doing work *on* it, the body or system is said to *lose* energy, or energy is said to be transferred *from* it, or it is said *to do* work. As a measure of the amount of energy lost by the body or system when a given change takes place in it is taken the amount of work which would have to be done on it to restore it to its original condition.

Units of Work and Energy. — The fundamental unit of mechanical work in the c.g.s. system is the work done by a force of 1 dyne when its point of application is displaced (with respect to the agent producing the force) a distance of 1 centimeter in the direction of the force; this unit is called the "erg." The "joule" is equal to 10^7 ergs, by definition. The fundamental unit of mechanical work in the English gravitational system is the foot-pound, which is the work done by a force of 1 pound when its point of application is displaced a distance of 1 foot in the direction of the force; or 1 foot-pound is the work required to raise a mass of 1 pound a distance of 1 foot at a place where the acceleration due to gravity is 980.665 cm. per sec. per sec. or 32.1739 ft. per sec. per sec. Energy is expressed in the same units as mechanical work and in addition various heat units, such as the British thermal unit, the large and small calories, etc., are used; see *Heat and Thermal Properties*. For the relations among the various units see *Units and Conversion Factors*.

Principle of the Conservation of Energy. — Experience indicates that the amount of energy, as above defined, which can be transferred from a body or system of bodies to which no energy is added, is limited; i.e., the energy "possessed by" or "associated with" any body or system of bodies is *finite* in amount. As a rule, only a relatively small portion of the energy

* The word "work" is used by some writers to signify mechanical work only, but the term is a very convenient one to use in referring to the transfer of energy by other means as well, i.e., any change resulting in a transfer of energy to a body may be said to result from the doing of "work" on the body, whether the change is produced by mechanical, electrical or other means.

associated with a body or system can be transferred to another body or system. Experience also justifies the assumption that whenever one body or system of bodies *gains* energy, some other body or system *loses* an *exactly equal amount* of energy. In every instance where this assumption can be tested directly it is found to hold, and every deduction from it has been found to be in accord with experimental fact. Hence this assumption is accepted as a fundamental principle of nature.

Kinetic Energy. — Work is required to set a body in motion, for while its motion is changing it is accelerating and therefore a force must be exerted upon it. From the definitions of acceleration, force and work given above it follows immediately that the work required to change the *linear* velocity of a body from v_0 to v_1 is

$$W_t = \frac{1}{2} m (v_1^2 - v_0^2) \qquad \text{absolute units,}$$

or

$$W_t = \frac{1}{2}\frac{m}{g_0} (v_1^2 - v_0^2) \qquad \text{gravitational units,}$$

where m is the mass of the body and g_0 is the gravitational constant. Similarly, the work required to change the angular velocity of a rigid body about a given axis from ω_0 to ω_1 is

$$W_r = \frac{1}{2} I (\omega_1^2 - \omega_0^2) \qquad \text{absolute units,}$$

or

$$W_r = \frac{1}{2}\frac{I}{g_0} (\omega_1^2 - \omega_0^2) \qquad \text{gravitational units,}$$

where I is the moment of inertia of the body about this axis and g_0 the gravitational constant. The expression

$$\frac{1}{2} mv^2 \qquad \text{or} \qquad \frac{1}{2}\frac{m}{g_0} v^2 \tag{14}$$

is called the "kinetic energy of translation" of the body, and the expression

$$\frac{1}{2} I\omega^2 \qquad \text{or} \qquad \frac{1}{2}\frac{I}{g_0} \omega^2 \tag{14a}$$

is called the "kinetic energy of rotation" of the body, both referred to the point with respect to which the velocities are measured.

Total Kinetic Energy of a System of Bodies. — The total kinetic energy of a moving system of bodies is equal to (1) the kinetic energy of the entire system moving with a velocity equal to the velocity of the center of mass of the system plus (2) the sum of the kinetic energies, rotational and translational, of each constituent body of the system due to the relative motion of this constituent body with respect to the center of mass of the whole system. For example, the total kinetic energy of a railway car is

$$W = \frac{1}{2} Mv^2 + \frac{1}{2} \Sigma I\omega^2 \qquad \text{absolute units,} \tag{14b}$$

$$W = \frac{1}{2}\frac{M}{g_0} v^2 + \frac{1}{2 g_0} \Sigma I\omega^2 \qquad \text{gravitational units,} \tag{14c}$$

where M is the entire mass of the car with full equipment, I the moment of inertia, ω the angular velocity of any rotating part and the summation sign Σ indicates the sum of the products $I\omega^2$ for all the rotating parts. See also *Railways, Energy Requirements for.*

Potential Energy. — The energy possessed by a body in virtue of its position with respect to the earth is usually called potential energy. More generally the term potential energy is used to designate any form of energy other than kinetic energy. In absolute units the increase in the potential energy of a body when it is raised a vertical distance h is mgh; in gravitational units the increase in potential energy is mh.

Power. — By power is meant the time rate of doing work or the time rate of change of energy. Let dW be the work done in time dt, then the power is

$$P = \frac{dW}{dt}. \tag{15}$$

The power produced by a force f or a torque T can also be expressed as

$$P = fv \qquad \text{or} \qquad P = T\omega, \tag{15a}$$

where v is the linear velocity of the point of application of the force and ω is the angular velocity of the point of application of the torque. The rational unit of power is the unit of work done or of energy transferred per unit time, such as 1 erg per second, 1 joule per second, 1 foot-pound per second, 1 British thermal unit per second, and the like. 1 joule per second is called the watt, which is also equal to the power corresponding to 1 ampere and a potential difference of 1 volt (*see Units, Practical Electrical*). The horse-power (English and American) is defined* as the power corresponding to 550 foot-pounds per second or 33,000 foot-pounds per minute. The metric horse-power, also called cheval-vapeur, force de cheval, Pferde-kraft, is defined as 75 kilogram-meters per second. The boiler horse-power is defined in the article on *Boilers, Steam.* The interrelations of the various units of power are given in the article on *Units and Conversion Factors.*

CONDITIONS FOR EQUILIBRIUM. — A body or system of bodies is said to be in equilibrium with respect to any external body (e.g., the earth) when (1) there is no change in the motion of the center of mass of the system with respect to this external body and (2) when there is no change in the total angular momentum of the body or system about any axis fixed with respect to this external body. These two conditions require (1) that the resultant of all the external forces† acting on the body or system be zero and (2) that the resultant of all the moments of these external forces, or torques, about any axis acting on the body or system be zero. These two conditions are most conveniently expressed by choosing three mutually perpendicular axes fixed with reference to the body of reference (e.g., the earth) and resolving all the forces into components F_x, F_y and F_z parallel to these three axes and calculating the moments or torques T_x, T_y and T_z of each of these forces about these three axes; then, using the symbol Σ to indicate the algebraic summation of the individual x, y or z components,

$$\left.\begin{array}{ll} \Sigma F_x = 0 \qquad & \Sigma T_x = 0 \\ \Sigma F_y = 0 \qquad & \Sigma T_y = 0 \\ \Sigma F_z = 0 \qquad & \Sigma T_z = 0 \end{array}\right\} \tag{16}$$

* The Bureau of Standards recommends the adoption of 746 watts as the definition of a horse-power. The older definition given above makes a horse-power 745.701 watts using the standard value of g_0 given above, namely, 980.665 cm. per sec. per sec. and the legal values of the foot and the pound (*see Units and Conversion Factors*). As this older definition is the one generally employed by all classes of engineers, it seems preferable to the author to retain it. See *Circular No. 34 of the Bureau of Standards* and discussions in the various technical journals during 1912 and 1913.

† i.e., forces any one of which would, if acting alone, produce a motion of the body or system or of some part of it with respect to the reference body.

These two sets of conditions constitute the basis of the entire subject of statics Certain elementary applications are given in the article on *Structures, Simple.*

Stability. — When a body or system of bodies is in equilibrium and the state is such that when the body or system is displaced slightly it returns of itself to its original condition, the equilibrium is said to be stable; if when displaced slightly the body or system moves farther from its original condition, the equilibrium is said to be unstable; if when displaced slightly it remains in that condition, the equilibrium is said to be stable. The condition for stability may be expressed in a number of ways, viz.: (1) when the potential energy of the system with respect to the body of reference (e.g., the earth) is a minimum the equilibrium is stable; (2) when the forces acting are all gravitational the equilibrium is stable when the center of mass is in the lowest possible position; or (3) when the body or system rests on a number of points, the equilibrium is stable only when the resultant of all the forces acting on the body including its own weight, but excluding the supporting forces, cuts the smallest polygon which can be drawn including all the points of support.

MOTION OF A PARTICLE. — By a particle of matter is meant a portion of matter so small that it may be considered as occupying but a point in space. Let m be the mass of the particle, v its linear velocity at any instant with respect to any arbitrarily chosen set of axes of reference, dv the increase in its velocity in time dt (including both change in direction as well as in magnitude), and let F be the resultant of all the external forces, in absolute units,* acting on the particle. Then the motion of the particle is completely specified by the equation

$$F = m\frac{dv}{dt},$$

where dv is taken to include the change in *direction* as well as the magnitude of the velocity. To take into account the variation of the direction of v as well as the variation in its magnitude, the resultant force F and the velocities may be resolved into components along the three axes, in which case this equation breaks up into the three equations

$$F_x = m\frac{dv_x}{dt}, \quad F_y = m\frac{dv_y}{dt}, \quad F_z = m\frac{dv_z}{dt}.$$

Since the velocity along any axis is equal to the time rate of displacement along that axis, $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, and $v_z = \frac{dz}{dt}$, whence these equations may also be written

$$\frac{d^2x}{dt^2} = \frac{F_x}{m}, \qquad \frac{d^2y}{dt^2} = \frac{F_y}{m}, \qquad \frac{d^2z}{dt^2} = \frac{F_z}{m}. \tag{17}$$

Hence when the forces can be expressed in terms of the coördinates of the point which the particle occupies at each instant, the displacements and velocities can be determined by solving these equations. Note that $\frac{F_x}{m}$, $\frac{F_y}{m}$ and $\frac{F_z}{m}$ are the accelerations a_x, a_y and a_z along the three axes.

Rectilinear Motion. — The simplest case is that of a particle acted upon by a force F which remains constant in direction. One of the axes, say the

* When F is expressed in gravitational units replace m by $\frac{m}{g_0}$, where g_0 is the gravitational constant.

X axis, may be taken parallel to the line of action of the force, in which case equations (17) reduce to the single equation

$$\frac{d^2x}{dt^2} = \frac{F}{m}, \tag{17a}$$

and there is no acceleration along either of the other axes; i.e., the particle moves along the line of action of the force. If the force is also constant in magnitude, $\frac{F}{m} = a$, a constant. In this case the solution of (17a) is (*see Equations*):

$$\left.\begin{aligned} x - x_0 &= \frac{1}{2} a (t - t_0)^2 + v_0 (t - t_0), \\ v - v_0 &= a (t - t_0), \end{aligned}\right\} \tag{17b}$$

where $t - t_0$ represents any interval of time, $x - x_0$ the displacement of the particle during this interval and $v - v_0$ the change in velocity of the particle during this interval. The displacement $x - x_0$ may also be written

$$x - x_0 = \frac{v^2 - v_0^2}{2a}. \tag{17c}$$

Curvilinear Motion; Motion in a Circle. — Consider the special case of a particle moving in a circle of radius r with a constant tangential or peripheral velocity v. Choose the Z axis perpendicular to the plane of the circle. Let θ be the angle (measured counter-clockwise) which a line drawn from the center of the circle to the particle makes at any instant with the X axis, then $(90 - \theta)$ is the angle which this line makes with the Y axis at this instant. At any instant, then,

$$v_x = -v \sin \theta \qquad \text{and} \qquad v_y = v \cos \theta$$

and these components are not constant since θ changes with time. The corresponding accelerations along the two axes are

$$a_x = -v \cos \theta \frac{d\theta}{dt} \qquad \text{and} \qquad a_y = -v \sin \theta \frac{d\theta}{dt}.$$

both of which components are *toward* the center of the circle. But $\frac{d\theta}{dt}$ is the angular velocity of the particle about the Z axis, and this is equal to the peripheral velocity divided by the radius, or $\frac{d\theta}{dt} = \frac{v}{r}$. Whence

$$a_x = -\frac{v^2}{r} \cos \theta \qquad \text{and} \qquad a_y = -\frac{v^2}{r} \sin \theta,$$

and therefore the resultant acceleration has the numerical value

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r} \tag{17d}$$

and is *toward* the center of the circle. Hence to cause a particle to rotate in a circle a force equal to $\frac{mv^2}{r}$ must be applied to it in the direction *toward* the center, and the particle in turn pulls away from the center with an equal and opposite force which produces a tension outward along the radius in whatever (e.g., a string) holds the particle to the center. The tendency of the particle to pull outward from the center is called the centrifugal force, and its value

is $\frac{mv^2}{r}$; the equal inward pull required to make the particle move in the circular orbit is called the centripetal force.

Simple Harmonic Motion. — (*See also Wave Analysis.*) By simple harmonic motion is meant a motion such that the acceleration of the moving particle or point at each instant is proportional to, but in the opposite direction from, the displacement. Such motion may be either rectilinear or a rotation about a fixed axis. Rectilinear simple harmonic motion may also be defined, see Fig. 1, as the motion of the projection P on a given diameter YY', of a point Q which moves with a constant angular velocity ω around a circle of radius A. The motion of the bob of a simple pendulum, the position of the end of a piston rod, etc., are examples of simple rectilinear harmonic motion (very nearly).

Referring to the circle diagram in Fig. 1, P_0 represents the projection of the moving point Q at time $t = 0$. The angle made by OQ at any instant with

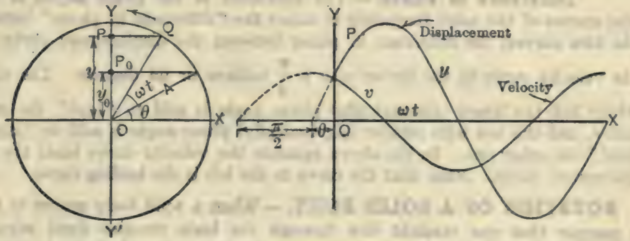

Fig. 1. Harmonic Motion

OX is then $(\omega t + \theta)$. The linear velocity of Q is $A\omega$, and the *component* of this velocity at any instant *along* OY, i.e., the velocity of P, is

$$v = \frac{dy}{dt} = A\omega \cos(\omega t + \theta). \tag{18}$$

Whence the displacement y of P from O is, by integration,

$$y = A \sin(\omega t + \theta), \tag{18a}$$

and the acceleration along OY is, by differentiation,

$$a = \frac{d^2y}{dt^2} = -\omega^2 A \sin(\omega t + \theta), \tag{18b}$$

whence $a = -\omega^2 y$, which agrees with the first definition of harmonic motion.

Period and Frequency. — The period of any kind of an oscillation is defined as the time (usually in seconds) taken for the oscillating point to pass through a complete cycle of values back again to the starting point. The frequency is the number of complete cycles per unit time (usually per second). In the case of a harmonic oscillation the period is

$$T = \frac{2\pi}{\omega} \tag{18c}$$

and the frequency is

$$f = \frac{\omega}{2\pi}. \tag{18d}$$

Amplitude. — The "sine curves" to the right of the circle diagram show the variation with time of the displacement and the velocity (along OY) of the projected point P. Displacement and velocity are plotted along the Y axis and ωt, which is proportional to time, is plotted along the X axis. These curves bear a definite relation to each other and to the "origin" of time, i.e., to the point O. The maximum ordinate of each curve is called the "amplitude" of the curve. The amplitude of the displacement curve is A, the radius of the circle; the amplitude of the velocity curve is ωA.

Phase. — The distance expressed in angular measure, from the origin to the point at which the curve first crosses the X axis in the *rising* direction is called the "phase angle" of the curve. The phase angle is taken positive when it is measured to the *left* from the origin, and negative when measured to the right.* The phase angle of the displacement curve is θ, and the phase angle of the velocity curve in the above case is $\frac{\pi}{2}+\theta$.

Difference in Phase. — The difference in the phase angles of two sine curves of the same frequency is called the "difference in phase" between the two curves; the difference in phase between the displacement curve and the velocity curve in the above case is $\frac{\pi}{2}$ radians or 90 degrees. The curve which has the larger (algebraically) phase angle is said to "lead" the other curve, and the one with smaller (algebraically) phase angle is said to "lag behind" the other one. In the above example the velocity curve leads the displacement curve. Note that the curve to the left is the leading curve.

ROTATION OF A SOLID BODY. — When a solid body moves in such a manner that one straight line through the body remains fixed with respect to any given set of axes of reference, its motion is called simple rotation with respect to these axes. For such motion the external forces must be equivalent to two equal and opposite forces which have a moment only about the axis of rotation. Let T be the value of this moment or torque, I the moment of inertia of the body about this axis and $d\omega$ the increase in the angular velocity about this axis in time dt (the same for each point of the body), then the motion is completely specified by the equation

$$\left.\begin{aligned} & T = I\frac{d\omega}{dt} && \text{(for } T \text{ in absolute units)} \\ \text{or}\quad & T = \frac{I}{g_0}\frac{d\omega}{dt} && \text{(for } T \text{ in gravitational units).} \end{aligned}\right\} \qquad (19)$$

For constant torque and therefore constant angular acceleration, i.e., for $\frac{d\omega}{dt} = \alpha =$ constant, the change in angular velocity in the interval of time (t_2-t_1) is

$$\omega_2-\omega_1 = \alpha\,(t_2-t_1), \qquad (19a)$$

and the angle turned through in the interval (t_2-t_1) is

$$\theta_2-\theta_1 = \frac{1}{2}\,(\omega_2+\omega_1)\,(t_2-t_1) = \frac{\omega_2^2-\omega_1^2}{2\,\alpha}. \qquad (19b)$$

THREE-DIMENSIONAL MOTION. — The motion of a solid body may in the most general case be expressed in terms of a translation of its center and a rotation of the body around an axis through its center of mass. The motion

* The opposite convention is used by some writers, in which case the equation for the sine curve is $y = A\sin(\omega t-\theta)$.

of the center of mass produced by any number of external forces is the same as would be produced by a single force, equal to the vector sum of the actual forces, acting on a single particle occupying the position of the center of mass in the actual body and having a mass equal to the entire mass of this body. The rotation can be determined by considering the center of mass as fixed and the actual forces then applied. That is, the translation of the center of mass and the rotation about the axis through it can be considered independently of each other; the actual motion is then the resultant of these two types of motion. See *American Civil Engineers' Pocket Book* for a brief treatment of this subject.

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MENSURATION. — (*See also Angles; Equations; Trigonometry.*) The term mensuration is used in this article to include the relations between the areas and volumes of geometric figures and their linear dimensions.

Triangle. —

$$\text{Area} = \tfrac{1}{2}\,(\text{base}) \times (\text{perpendicular height})$$
$$= \sqrt{s\,(s-a)\,(s-b)\,(s-c)},$$

where a, b and c are the lengths of the three sides respectively, and $s = \tfrac{1}{2}(a+b+c)$.

Trapezoid. —

$$\text{Area} = \left(\frac{a+b}{2}\right) d,$$

where a and b are the lengths of the parallel sides respectively, and d their distance apart.

Parallelogram. —

$$\text{Area} = (\text{base}) \times (\text{perpendicular height}).$$

Parabola. —

$$\text{Area} = \tfrac{2}{3}\,(\text{area of circumscribing triangle}).$$

Cycloid. —

$$\text{Area} = \tfrac{3}{4}\pi \times (\text{altitude})^2,$$

the altitude being the diameter of the rolling circle.

Circle. —

$$\text{Circumference} = 2\pi r = \pi d,$$

$$\text{Area} = \pi r^2 = \frac{\pi}{4} d^2,$$

where r is the radius and d the diameter.

$$\text{Area of segment} = \frac{r^2}{2}\,(\theta - \sin\theta),$$

where θ is the angle in radians (*see Angles*) subtended by the arc of the segment. If n is the height of the segment, measured along the radius perpendicular to the chord,

$$\text{Area of segment} = \pi r^2 M - A\,(r-n),$$

where

$$A = \sqrt{n\,(2r-n)} \quad \text{and} \quad M = \frac{1}{180} \sin^{-1}\left(\frac{A}{r}\right).$$

Ellipse. —

$$\text{Area} = \pi ab,$$

where a and b are the principal semi-axes.

Prism with Parallel Sides and Parallel Ends. —

$$\text{Volume} = (\text{area of end}) \times (\text{Perpendicular Distance between Ends}).$$

Right Circular Cylinder. —

$$\text{Volume} = \frac{\pi}{4} d^2 l,$$

where d is the diameter, and l the length.

$$\text{Total surface of right cylinder} = \pi d\,(l + \tfrac{1}{2} d).$$

Right Circular Cone. —

$$\text{Volume} = \tfrac{1}{3}\,(\text{area of base}) \times (\text{height}),$$
$$= \tfrac{1}{3}\,(\text{volume of circumscribing cylinder}),$$

where r is the radius of base and h the height of the cone.

$$\text{Area of curved surface of a right circular cone} = \pi r\sqrt{h^2 + r^2}.$$

Right Pyramid. —

$$\text{Volume} = \tfrac{1}{3}\,(\text{area of base}) \times (\text{height}),$$
$$\text{Volume of frustum of pyramid} = \tfrac{1}{3}\,(\text{height})\,(A + a + \sqrt{aA}),$$

where A and a are the areas of the ends respectively.

Sphere. —

$$r = \text{radius},$$
$$\text{Area of surface} = 4\pi r^2$$
$$= \tfrac{2}{3}\,(\text{total area of circumscribing cylinder}).$$

Area of the surface of a zone of a sphere = area of zone of the same height as this zone projected on to a cylinder.

$$\text{Volume} = \tfrac{4}{3}\pi r^3$$
$$= \tfrac{2}{3}\,(\text{volume of circumscribing cylinder}).$$

Volume of a frustum of a sphere $= \pi r^2\,(k \mp h) - \frac{\pi}{3}\,(k^3 \mp h^3)$, where k is the distance of its outer face from center and h the distance of its inner face from the center, the negative signs in the brackets to be used if both faces are on the same side of the center and the positive signs if on opposite sides of the center.

Ellipsoid. —

$$\text{Volume} = \tfrac{4}{3}\,\pi\,abc,$$

where a, b and c are the three principal semi-axes respectively.

Paraboloid. —

Volume of a paraboloid of revolution equals one half that of the circumscribing cylinder.

MOTOR-CONVERTERS. — (*See also Converters, Synchronous; Motors, Induction; Standardization Rules.*) A motor-converter is a combination of an induction motor and synchronous converter, with the secondary of the motor and the armature of the converter mounted upon the same shaft and connected together electrically without slip rings. The induction motor receives all the a-c. power, transforms a part of it into mechanical power delivered to the shaft, and also acts as a transformer delivering the rest of the power in electrical form at a lower frequency from its secondary to the armature of the converter. It operates like two induction motors in "concatenation" or "cascade." The object is to obtain the steadiness of a 30-cycle converter on a 60-cycle circuit.

The speed of a motor converter depends upon the supply frequency and varies inversely as the sum of the number of poles in both machines. Thus if a combination of a 6-pole motor and 6-pole converter be operated from a 60-cycle circuit the speed of the armature will be 600 r.p.m. The primary of the induction motor will operate at 60 cycles but the secondary will supply the armature of the converter with 30 cycles. Thus the converter may be built with the good design constants of a 30-cycle machine.

This combination is larger than a converter but smaller than a motor-generator set and its efficiency is lower than that of a converter. It is used somewhat in Europe but not much in the United States.

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MOTOR-GENERATORS. — (*See also Converters, Synchronous; Frequency Changers; Generators; Motor; Standardization Rules.*) A motor-generator set is a combination of a motor and a generator having separate fields and armatures but mounted on the same shaft with common base and bearings. Combinations of various types of motors and generators are used; some of the more important combinations and their applications are described below.

Frequency Changers. — See separate article on *Frequency Changers.*

Direct-current Motor Driving Direct-current Generator. — These sets are used when it is desired to convert low-voltage direct current into high voltage direct current, or vice versa; they are used in preference to a dynamotor (q.v.) when good regulation is desired in the secondary circuit.

Direct-current Motor Driving Alternating-current Generator. — These sets are used for converting direct into alternating current. See also the section on *Inverted Synchronous Converters* in the article on *Converters, Synchronous.*

Induction Motor Driving Direct-current Generator. — The induction motor may be wound for potentials as high as 13,000 volts and the transformation from alternating current at this voltage to direct current may be made without the use of transformers. Since the induction motor has a decreasing speed with increasing load the direct-current generator must be compounded to give good regulation; with proper compounding excellent regulation may be obtained. The induction-motor-generator set is sometimes used in preference to a synchronous converter when the service requires specially good regulation. However, the efficiency of such a set (about 85 per cent at rated load) is less than that of a synchronous converter, even if no transformers are required by the motor-generator set. The motor-generator set also occupies from 50 per cent to 80 per cent more floor space, weighs from 30 per cent to 50 per cent more, and costs from 25 per cent to 50 per cent more than a synchronous converter, in spite of the fact that they are designed to operate at the highest practicable speeds.

Costs and Speeds of Induction-motor-generator Sets. — Very roughly the prices and suitable speeds are (in 1922) as follows:

Kw. capacity	R.p.m.	Cost per kw.
200	750	$33
500	500	24
1000	300	19

These prices are about 50 per cent higher than pre-war prices.

Synchronous Motor Driving Direct-current Generator. — This combination is preferable in many instances to the preceding, because it operates at constant speed and because the field of the synchronous motor may be adjusted to make use of line compounding or to compensate for low power factor in other apparatus on the circuit (*see Motors, Synchronous*). Provision must be made for the direct-current excitation for the synchronous motor. If the direct-current generator is wound for too high a potential a special exciter must be provided. Since there is no load on the set at starting the synchronous motor may be started in the usual manner.

SPECIFICATION FOR MOTOR-GENERATORS. — The following memoranda are intended to assist in writing specifications. See also articles on *Specifications; Motors;* and *Generators.*

General description and use of machine. Motor: see specifications in articles on *Motors*. Generator: see specifications in articles on *Generators*. Whether or not motor and generator are to be on common bed-plate. Exciter for generator field. Limiting over-all dimensions.

BIBLIOGRAPHY. — Taylor, J. B., *Parallel Operation of Synchronous Motor-Generator Sets*, Trans. A.I.E.E., Vol. 25, p. 113; Lincoln, P. M., *Motor-Generators vs. Synchronous Converters*, Trans. A.I.E.E., Vol. 26, p. 303; Allen, E. W., *Rotary Converters vs. Motor-Generators*, Elec. World, Nov. 14, 1908; Weingreen, J., *Electric Power Plant Engineering*. See also Bibliography in article on *Frequency Changers*.

MOTORS, ALTERNATING-CURRENT COMMUTATOR.—(*See also Motors, Direct-current; Motors, Polyphase-induction; Motors, Single-phase Induction; Standardization Rules.*) There are several different types of alternating-current commutator motors designed to operate on single-phase circuits, but they differ chiefly in the electrical connections employed. They may be divided into two general classes, series motors and repulsion motors. While these motors differ in their connections and in slight details in their characteristics, they all have the general characteristics of the d-c. series motor, that is, increasing torque with decreasing speed and a high efficiency over a considerable range of speed. Alternating-current commutator motors with shunt motor characteristics are also used to a limited extent abroad.

GENERAL CHARACTERISTICS.—The torque of any a-c. commutator motor is constant in direction, but pulsating in value, and its average value

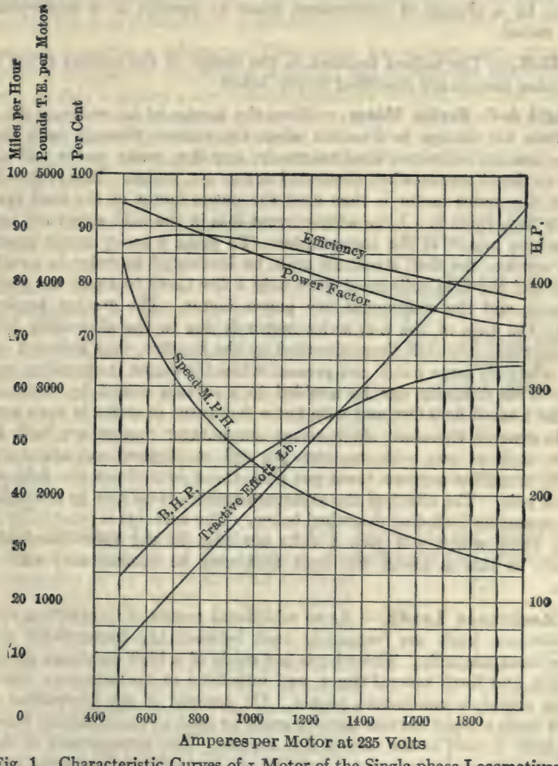

Fig. 1. Characteristic Curves of 1 Motor of the Single-phase Locomotives of the N. Y., N. H. & H. R. R. (St. Ry. Jour., Apr. 1906.)

is proportional to the product of the effective value of the flux and the effective value of the armature current. The direction of the torque may be changed

by changing the direction of the current in the field with respect to the armature, or vice versa. The power factor increases with increase of the speed and therefore decreases with increase of load. The efficiency, while not as good as that of a d-c. motor of the same rating is, however, fairly high. The motors have in addition to the losses common to d-c. motors a core-loss in the field, increased core-loss in the armature, increased commutation loss and increased RI^2 loss in special windings. In Fig. 1 are given the characteristic curves of the a-c. compensated series motors used on the single-phase locomotives of the New York, New Haven & Hartford Railroad.

APPLICATIONS. — The most general application of a-c. commutator motors in large sizes is in railway and hoisting work; see *Railways, Electric*, and *Hoists, Electric*. The same principles of operation are made use of in the devices for starting single-phase induction motors (*see Motors, Single-phase Induction*), the motor being brought up to speed as an a-c. commutator motor and then by a change of connections made to operate as a single-phase induction motor.

DESIGN. — The salient features in the design of the various types of a-c. commutator motors are described briefly below.

Straight A-C. Series Motor. — Since the torque of an ordinary d-c. series motor does not change in direction when the current through both the field and the armature reverses simultaneously, any d-c. series motor will develop a uni-directional torque when connected across a-c. mains. However, when an ordinary d-c. series motor is thus used the power factor of the load taken by it is very low, there is a large eddy-current loss in the field structure, and violent sparking occurs at the commutator. To make a series motor practicable for a-c. service, the field structure must be laminated in order to avoid eddy currents and the field coils must have only a few turns to avoid too great self-inductance, and the consequent low power factor. The greater tendency to spark in the case of the a-c. series motor is due to the *alternating* field flux which interlinks the coils short-circuited by the brushes, thus inducing in these coils a relatively large e.m.f. not present when the motor is operating on a d-c. circuit. This difficulty can be avoided to a certain extent by designing the motor for a small field flux and with but a few turns in series in each armature coil. In general, therefore, single-phase commutator motors are built for low voltages, such as 200, with one turn per coil, multiple-wound armatures and with the armature ampere turns per pole about four times the field ampere turns per pole. The effect of the armature ampere turns can be neutralized by a "compensating" winding described below. The field flux is practically limited to that value which will give 4 volts per turn in the short-circuited armature coil, as this is about the limit that may be commutated with carbon brushes.

Resistance Leads. — As an additional means of preventing sparking, high-resistance leads are frequently used between the commutator segments and the armature coils. These leads are made of a high-resistance metal strip bent back and forth several times, and imbedded in the armature slots along with the armature conductors proper. On account of the dissipation of heat in these leads a lower current density must be used in the armature conductors proper than is used in the case of d-c. motors. The arrangement of the leads is such that the main or useful current passes through two high-resistance leads in multiple as it enters the armature, while the undesirable short-circuited current passes through two high-resistance leads in series. It should be noted, however, that at any instant there is current only in those leads connected to coils which are being short-circuited at this instant.

Compensating Winding. — To obviate the high armature reaction in an a-c. series motor and at the same time to improve the power factor, a "compensating" winding is usually employed. This consists of a distributed winding imbedded in slots in the pole faces and connected usually in series (Fig. 2) with the main field winding and armature in such a manner that the current through it sets up a magnetomotive force which practically neutralizes the effect of the armature ampere turns. When the compensating winding is connected in series with the field and armature, as shown in Fig. 2, the motor is said to be "conductively compensated." An "inductively compensated" motor has this winding short-circuited upon itself and the current in it is induced from the armature by transformer action. Inductive compensation is not operative on d-c. circuits, but is as satisfactory as conductive compensation for a-c. operation.

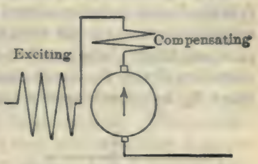

Fig. 2. Compensated Series Motor

Thomson Repulsion Motor. — This motor has a stationary structure or field with a completely distributed winding, which may be wound for any voltage. In this is placed a low-voltage armature designed with all the refinements necessary for single-phase commutator work. The brushes bearing on this commutator are short-circuited upon themselves and are so placed that the line connecting the positive and negative brushes makes an angle α with the neutral axis of the field. The field turns lying within the angle $(90 - \alpha)$ induce a current in the armature winding by transformer action and the field turns lying within the angle α constitute the "exciting" turns and set up the necessary flux to produce the driving force. The arrangement is equivalent to the circuits shown in Fig. 3, although actually there is but a single field winding. This motor then acts exactly like the combination of a transformer and a series motor in one structure. It may be reversed by changing the position of the brushes or by shifting the points of connection of the external circuit to the field or stator winding. This motor operates particularly well near synchronous speed as then it has practically a rotating magnetic field and no excessive commutation difficulties, but at starting and at low speeds the commutation is not as good as that of the compensated series type.

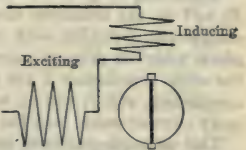

Fig. 3. Thomson Repulsion Motor

Winter-Eichberg-Latour (Fig. 4). — This motor has the special attribute that, due to the rotation of the armature conductors of the circuit B_2 in the transformer flux set up by the inducing windings, an e.m.f. is generated which is almost exactly 180° in time phase from the e.m.f. of self induction caused by the motor flux. It therefore neutralizes this inductive drop and improves the power factor of the machine. Were it not for the incidental leakage reactance the power factor would be unity at synchronous speed when the ampere turns of the inducing and exciting windings are equal. By a proper adjustment of the series transformer T this lagging voltage may be made to exceed the inductive drop at speeds near and above synchronism and cause the power factor to be anti-inductive.

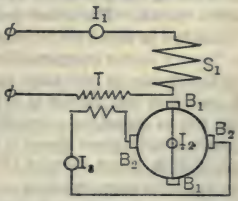

Fig. 4. Winter-Eichberg-Latour Compensated Repulsion Motor.

The principle of this action is shown in Figs. 5 and 6, where ϕ_1 shows the transformer flux set up by the inducing winding and ϕ_2 is the motor flux produced by the series exciting current I_3 in the armature circuit B_2, in phase with I_3 and I_1. I_1 is the line or primary curent in the inducing winding S_1. I_2 is the transformer secondary current in the armature circuit B_1. The resultant of these two mf.m.'s produces the transformer flux ϕ_1. E_1 is the voltage consumed by rotation of the armature circuit B_1, due to cutting ϕ_2 and is in phase with ϕ_2, transferred to primary S_1, by transformer action. E_2 is the e.m.f. consumed by self-induction in

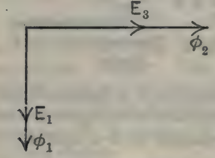

Fig. 5. Position in Space, Compensated Repulsion Motor.

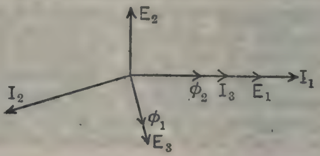

Fig. 6. Time-phase Diagram, Compensated Repulsion Motor.

B_2 by the alternation of ϕ_2. E_3 is the e.m.f. consumed by rotation in B_2 due to cutting the transformer flux ϕ_1.

Neglecting drops due to resistance and leakage reactance it can be seen that E_2 and E_3 are inductive drops in the armature, opposite in effects, and may completely neutralize each other, leaving the resultant primary voltage E_1 in phase with the primary current and giving unity power factor. By means of the series transformer T the current I_3 and thus the flux ϕ_2 may be decreased in comparison with I_1 and ϕ_1, which would make E_3 greater than E_2 and the primary current would lead the resultant line voltage

This principle has been employed in the construction of a number of types of small (1 to 20 h.p.) single-phase motors for industrial use, and these show a great superiority over the straight single-phase induction motor in the matter of starting torque and power-factor.

Wagner Repulsion-Starting, Induction Running Motor (Fig. 7).—The stator has two windings, the main or primary S_1 is connected in series with the exciting brushes B_2 on the armature as in the series motor. This gives the starting characteristics of the series a-c. motor. The armatrue has two sets of brushes. The main pair B_1 in line with the primary flux is short circuited, thus giving compensation as in the compensated series motor. The other pair of brushes B_2 is connected in series with the primary and across a "power factor compensating coil" S_2, wound on the stator in the same axis as the primary. A switch in this circuit is left open when starting, but is closed by a centrifugal governor just before synchronous speed is reached. Closing this switch short-circuits this pair of brushes through the compensating coil and makes the armature a two-phase rotor, and thus eliminates series-motor action and makes the action that of a true single-phase induction motor. The compensating coil being in circuit causes the anti-inductive effect in this coil to react on the primary as in the compensated repulsion motor, and give a good power-factor or

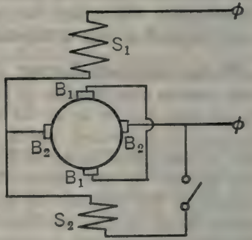

Fig. 7. Repulsion-starting, Induction-running.

leading current. By adjusting the number of turns or current in this coil S_2 the power factor may be regulated.

To prevent the motor from running away in case the centrifugal switch should fail to act, and to relieve the brushes under load, a regular squirrel-cage winding is placed in the bottom of the armature slots underneath the commutated winding and separated from it by magnetic separators to magnify the magnetic leakage and render the squirrel-cage winding ineffective at starting. This motor is very satisfactory in practice, has a starting torque 1.5 times full load torque for a starting current of three times full load current. The power-factor is anti-inductive at light loads, unity at full load and inductive at overloads. The efficiency at full load is about 80 per cent in the 5 and 10 h.p. sizes. Reversal of rotation is effected by changing the connections to the exciting brushes. Small motors may be thrown directly on the line for starting. Large motors have a series resistance or reactance to limit the starting current to a reasonable amount.

Repulsion-Induction (R. I.) Motor of the General Electric Co. — This motor is shown diagrammatically in Fig. 8. It has two windings on the stator, the main or primary S_1 being connected directly and solely across the line as in any induction motor. The armature has two equally spaced sets of brushes displaced a small angle (20°) from the axis of the primary flux, thus obtaining the repulsion motor starting effect. The main brushes B_1 are short circuited and the auxiliary brushes B_2 connected permanently in series with the "power factor compensating coil" S_2 on the stator. The method of operation is best understood by considering the motor as being a repulsion motor and a single phase induction motor on the same shaft. At starting the repulsion-motor torque is great and the induction-motor torque is zero. Just below synchronism the induction-motor torque is great and repulsion-motor torque small. Above synchronism the repulsion-motor torque still exists but the induction-motor torque has become negative (generator). Thus at no load the repulsion motor pulls the induction motor above synchronism until the positive torque equals the negative torque and friction. At full load the speed is nearly synchronous. The compensating coil introduces an anti-inductive effect described above, and by varying the number of turns included, the power-factor may be made unity at any load. The efficiency is about 80 per cent in a 5 h.p. motor. Reversal of rotation is effected by moving the brushes or, if to be done often, by introducing a special coil in the primary whose connection is changed for reversal. An ordinary starting box is used to limit the current to about three times full load current for 1.5 times full load torque.

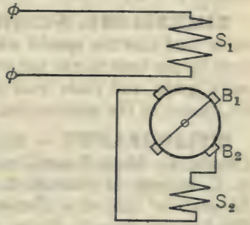

Fig. 8. Repulsion-Induction (R.I.) Motor.

Series Repulsion Motor. — Since the repulsion motor has better operating characteristics at a speed near synchronism than the series motor it is better to run with repulsion motor connections. On the other hand, the series motor has better operating characteristics during starting conditions. These facts have lead to the development of the "series repulsion" motor by one of the manufacturing companies. In the control of this motor a compensator with numerous taps is used and the connections so arranged that during starting the armature receives a very large current while the field is excited below the normal value. This gives a considerable torque with little trouble from commutation. As the motor speeds up, the connections are gradually changed until at full speed

the motor operates practically as a repulsion motor with the good commutating characteristics of that motor.

Polyphase A-C. Commutator Motors. — The series a-c. motor may be constructed three-phase by providing three exciting and three compensating windings on the stator for each pair of poles and providing three equally spaced sets of brushes per pair of poles on the armature and connecting these in series to the three phase supply as in any "Y" connected machine. This machine has the typical variable speed characteristics of the series motor. The speed may also be varied by changing the position of the brushes.

In order to secure good commutation the armature must be wound for a low voltage and it therefore becomes convenient to connect the stator and rotor by means of a transformer instead of direct conduction. With this arrangement it is possible to use any desired voltage on the line. There has been a considerable development in the application of these motors in the last few years.

The armature and stator may also be connected in parallel with each other (preferable by means of a transformer) thus giving a polyphase shunt motor with constant speed characteristics. This type, however, has not found much commercial application so far.

DIMENSIONS, WEIGHT AND COSTS. — Due to the low flux densities used and the special windings required an a-c.commutator motor weighs from 50 per cent to 100 per cent more, occupies from 25 per cent to 50 per cent more space, and costs from 50 per cent to 100 per cent more than a 600-volt d-c. motor of the same rating; see *Motors, Direct-current.*

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MOTORS, DIRECT-CURRENT. — (*See also Alternating Currents; Electricity and Magnetism, Principles of; Generators, Direct-current; Motors, Industrial Applications of; Standardization Rules.*) A motor is a dynamo-electric machine for converting electrical power into mechanical power; that is, it performs the converse function of a generator. Direct-current generators and motors are always interchangeable in function, although a machine which is designed specifically for a motor would probably not make a first-class generator and vice versa. Motors of less than 5 horse-power are usually bipolar; larger machines are multipolar.

The following is a brief outline of the contents of this article:

CLASSIFICATION. — There are four types of direct-current motors, differentiated by their characteristics and the connection of the exciting windings or circuits.

Shunt Motor (Fig. 1). — This motor has only one exciting winding, which is connected across the armature terminals and is thus in parallel or in shunt with the armature. The field winding consists of a large number of turns of fine wire on each pole, and usually the windings on all the poles are connected in series in one circuit. The current in the field depends upon the line voltage and upon the resistance of the field winding. The resistance of the field winding is purposely made high so that the field current will be between 1 per cent and 5 per cent of the full-load current of the motor. The characteristic of the shunt motor is a fairly constant speed for all reasonable values of load.

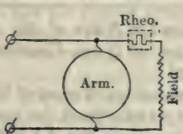

Fig. 1. Shunt Motor

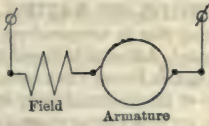

Fig. 2. Series Motor

Series Motor (Fig. 2). — This motor has only one exciting winding, which is connected in series with the armature so that all the current flows through the field as well as the armature. The field winding consists of a few turns of thick wire on each pole and the windings on all poles are connected in series. The current in the field depends upon the load and is thus large with heavy load and small with light load. The resistance of the field winding is purposely made low so that the loss of voltage and power in that circuit will be small. The characteristics of a series motor are a speed varying with every change in load,

high speed at light load and low speed at heavy load. The efficiency is high throughout a wide range of speed. The speed will be dangerously high at no load; thus a series motor must always be connected rigidly to its load. Since the torque is high at low speeds this motor is particularly adapted to work requiring frequent starting.

Compound (or Cumulative) Motor. — This motor has both a series winding and a shunt winding on each pole, wound and connected so that the two windings assist each other in the production of magnetism. It is a combination of a shunt and a series motor designed to give the good starting qualities of the series motor and to avoid the danger of excessive speed at light loads. See also *Motors, Industrial Applications of.*

Differential Motor. — This motor has a shunt and a series winding connected so that they oppose each other in the production of magnetism. The motor therefore has poor starting qualities, increases in speed with increase in load but has no tendency to run at a dangerously high speed. The applications of this motor are very limited.

Inclosed vs. Open Type. — These terms refer to the mechanical housing of the motor. The open type has all its parts freely exposed to the air and is therefore well ventilated. It is intended to be used indoors and in protected places. The inclosed type is intended to be used in exposed locations where there is a liability of dampness or dirt. Special means must be provided to circulate the air inside the machine, but even then an inclosed motor is larger and more expensive than an open motor of the same capacity.

The relative capacities in output of open, semi-inclosed, and totally inclosed motors are shown by the accompanying data on one of a line of typical commercial motors. In general an inclosed motor weighs about 15 per cent more than an open motor of the same capacity in spite of the fact that it is allowed to operate at 15° C. higher temperature by commercial convention.

Type	Output, h.p.	Temp. rise, °C.	Weight in lb. for 700 r.p.m. and given temp. rise
Open	10	40	970
Semi-inclosed	8	40	....
Totally inclosed	5.75	55	1100

METHODS OF RATING. — See *Standardization Rules of the A.I.E.E.* for limiting rise in temperature and for classification of types, Constant Speed, Multi-Speed, Adjustable Speed and Variable Speed.

The Electric Power Club, which includes representatives of most of the manufacturers of electric machinery in the U. S. A., has standardized, for commercial practice, certain temperature ratings which are based on the A.I.E.E. Rules. These regulations, which are for limiting temperatures as measured by thermometer with continuous duty, and the corresponding A.I.E.E. rules are:

	Max. rise on commutator	Max. rise on other parts
Open type	45° C.	40° C.
Protected type	55	50
Enclosed type	60	55
A.I.E.E. Rules	55	50

VOLTAGE AND CURRENT. — Usual values of voltage for direct-current motors are:

110–125 for small motors on lighting circuits.
220–250 for motors in factories, shops, etc.; on power mains or on the outside mains of a three-wire system.
500–600 for general railway work.
1200 for special railway installations.

The current required for any motor is found by the relation

$$\text{Current} = \frac{\text{Output in h.p.} \times 746}{\text{Efficiency} \times \text{Voltage}}.$$

Usual values for the efficiencies of motors of various sizes are given below.

APPLICATIONS OF MOTORS. — This subject is treated in detail in a separate article on *Motors, Industrial Applications of* (q.v.). The chief applications of continuous-current motors are the following:

Shunt Motor. — Driving shafting, machine tools, blowers, reciprocating pumps; motor generators.

Series Motors. — Railway and all other transportation work; hoists; cranes.

Compound Motor. — Elevators, hoists and machinery that must be started often.

Differential Motor. — Very special applications of small units for peculiar speed conditions.

PRINCIPLES. — The principles upon which a direct-current motor operates are the same as those upon which a direct-current generator operates (*see Generators, Direct-current*). These principles are briefly as follows:

Force Acting on Conductor. — A conductor of length l carrying a current I and placed in a magnetic field having a flux density B is acted upon by a force which is proportional to BlI, which force is in a direction at right angles to the direction of the magnetic flux and at right angles to the length of the conductor.

This in practical form gives the relation

$$T = \frac{p\phi ZI}{852\, m \times 10^6}.$$

T = torque of an armature in pounds at one-foot radius.
p = number of poles.
m = number of armature paths between brushes.
ϕ = flux per pole in armature (lines).
Z = number of active conductors or inductors on armature.
I = current taken by the armature from line.

This torque is exerted whenever a current flows and is independent of the speed. The core-loss and friction absorb some of the torque so that the torque at the pulley is slightly less than the value given by the formula.

Counter E.M.F. in Conductor. — A conductor of length l moving with a velocity V in a magnetic field of density B has induced in it an electromotive force E proportional to BlV. In practice as soon as the armature starts to move a counter e.m.f. is induced in its conductors which has the value

$$E = \frac{p\phi Zn}{m \times 10^8},$$

where n = revolutions per second.

Thus as soon as the armature moves, this counter e.m.f. tends to stop the flow of current and the impressed e.m.f. must be increased to maintain the flow of current.

The relation between current and counter e.m.f. is given by the equation

$$E_i = E + IR,$$

where E_i = impressed e.m.f.,
E = counter or generated e.m.f.,
R = resistance of armature circuit.

In practice R is made as small as possible so that E and E_i are as nearly equal as possible.

Reversing Rotation.— From a consideration of the equation for the torque it is evident that torque is proportional to the product ϕI, i.e., to the product of the flux in the armature by the current. If the direction of the current through the armature is reversed, that is, if I becomes negative, the product becomes negative and the torque is in the opposite direction. If the direction of the flux is changed (the armature current being unchanged) the direction of torque is reversed. But if both ϕ and I are reversed the torque is not reversed. From this follows the rule for reversing the direction of rotation of any direct-current motor; viz., change the direction of flow of current in either the field or armature winding but not in both.

Speed Control. — From the equation for counter e.m.f. it follows that the speed is proportional to E/ϕ. That is, the speed varies directly as the counter e.m.f. and inversely as the flux. Thus to reduce the speed, decrease the counter e.m.f. by decreasing the e.m.f. impressed on the armature or increase the flux by increasing the field current. To increase the speed perform the converse. To decrease the e.m.f. impressed on the armature a resistance may be inserted between the source of potential and the armature terminals. This is the customary manner of controlling the speed during the starting of motors. (*See also section below on Starting of D-C. Motors.*)

SHUNT MOTOR. — Since the flux in the armature of a shunt motor is practically independent of load, the characteristics of the motor are: approximately constant speed for all reasonable variations of load, torque directly proportional to the armature current irrespective of speed, efficiency high throughout a wide range of load but for only a small range of speed see Fig. 3.

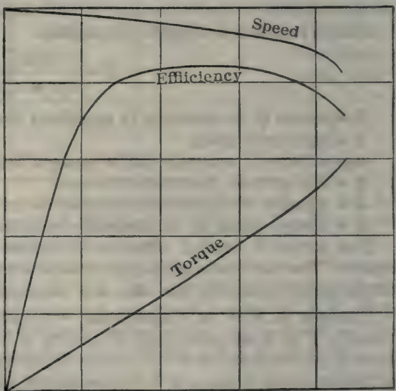

Fig. 3. Speed, Torque and Efficiency Characteristics of a Shunt Motor

Design of Shunt Motor. — The method of design and calculation of shunt motors is the same as for direct-current generators (*see Generators, Direct-current*) except for the minor details noted below.

The armature reaction of a motor is in the opposite direction to that of a generator running in the same direction, and thus the field is distorted in the opposite direction. Hence, if the brushes are to be moved to assist commutation they must be moved in a direction opposite to the direction of rotation of the armature.

The effect of armature reaction is to weaken the field. This causes a tendency to increase the speed and also causes bad commutation.

The stability factor of a motor must be greater than that of a generator because when the motor drops in speed as the load comes on, the field must be weakened to increase the speed. Hence the field is liable to be operated at an excitation less than normal.

Testing of Shunt Motors. — (*See also Standardization Rules of the A.I.E.E.*) The tests on shunt motors may be divided into two classes: (*a*) Commercial, to determine the qualities and serviceability of particular motors; and (*b*) Special, to determine the general characteristics and actions of a type of motor. Commercial tests are the following:

Resistance measurements.
Stray power test, including core-loss and friction.
Input-output test for heating, efficiency and commutation.
Insulation test.

Resistance Measurements are made with the machine cold and later after the heat run. The resistances of the armature winding and field winding are measured, and the brush-contact resistance may be measured but is usually calculated (*see Generators, Direct-current*). For any value of current the resistance losses (RI^2) are calculated from the measured hot resistances.

"Stray Power" Test. — The term "stray power" is applied to the lumped sum of the core-loss and the loss due to friction, bearings and windage. The stray power of a d-c. machine can be determined approximately by impressing normal voltage on the field and letting the machine run as a motor without load, varying the voltage impressed on the armature from about 10 per cent above normal to about 10 per cent below normal. The speed, armature voltage and armature current for each adjustment are observed. Then the stray power for any induced voltage is equal to the armature input less the corresponding RI^2, where R is the armature resistance and I the armature current. The value of stray power for any given load on the motor is then equal to the measured value corresponding to the same counter e.m.f. E, where E is calculated from the impressed voltage E_i by the relation $E = E_i - RI$, I being the armature current for this load and R the armature resistance as before.

If it is desired to determine the stray power more accurately by taking into account the effect of armature reaction, the field current may be adjusted so that the speed on the above run is the same as the load speed. This gives a flux of the same average value as when the machine is under load.

Calculation of Efficiency from Losses. — Let P = total output in kilowatts; R_a = resistance of armature, including brushes; I_a = armature current; I_f = field current; E_i = impressed voltage; S = stray power. Then the per cent efficiency is

$$\epsilon = \frac{100\,P}{E_i(I_a + I_f)} = \frac{100\,P}{P + E_iI_f + I_a^2R_a + S}.$$

Input-output Test with Prony Brake. — The input-output test may be made either by means of a prony or band brake on a pulley, or by using a d-c. generator as a load. If the brake test is made the output is

$$\text{Watts} = \frac{PLN}{7.04},$$

where P = net pull in pounds, L = lever arm or radius in feet, N = revolutions per minute.

Input-output Test with Generator as Load. — With large motors it is desirable to use a generator as a load in making an input-output test. In

this case it is necessary to know the resistance of the generator armature circuit. It is also desirable to have the generator separately excited and to maintain a constant excitation throughout the entire test.

The input of the motor and the output of the generator, together with the speeds of both machines, are observed. A "counter-torque" test must also be made to determine the belt friction loss and the core-loss and friction of the generator. This is performed by making two tests as follows:

(*a*) The motor input is observed when driving the unloaded generator at normal speed first through the regular leather belt and second through a light cotton belt. The difference in input to the motor in the first and second cases gives the belt-friction loss. As this loss is comparatively small, it may frequently be neglected.

(*b*) A regular stray power test (*see above*) is made on the generator when entirely disconnected from the motor. This gives the core-loss and friction of the load machine (generator).

Then for any load during the load run the output of the motor under test is

$$P_1 = P_2 + R_2I^2 + S + F,$$

where P_1 = output in watts of motor, machine 1.
P_2 = output in watts of generator, machine 2.
R_2I^2 = loss in armature winding of generator.
S = stray power of generator for speed and induced voltage at observed load.
F = belt-friction loss.

The ratio of this motor output to the electrical input as observed gives the efficiency of the motor.

Heat Run. — From the input-output test it is also possible to determine the speed regulation, commutation features and heating. The heat run may also be made by "bucking" two machines as described in the article on *Generators, Direct-current.* Small motors will reach a constant temperature in a short time and the heat run need only last 5 or 6 hours for a 100-h.p. motor. A thermometer is usually placed on the machine in a safe and accessible place and read every half hour until it indicates no further rise in temperature.

Insulation Test. — The margin of safety on a 110- or 220-volt motor is usually so great that it is not necessary to make an insulation test. If the motor has been exposed to dampness it may be desirable to make the test after the motor has been thoroughly dried out. The method is indicated in the *Standardization Rules of the A.I.E.E.* (q.v.)

Special Tests. — As a special test there may be obtained a saturation curve of the machine and possibly the distribution of potential around the commutator. These are of particular interest in an adjustable speed motor. In some of these motors with commutating poles there may exist some very high voltages between bars which are not evident except in the bar-to-bar potential test.

SERIES MOTOR. — Since the flux in the series motor is produced by the load current, the flux increases with the current. The torque is proportional to the product ϕI and therefore increases more rapidly than the current. Thus four times full-load torque can be obtained with from two to three times full-load current.

The characteristics of the series motor are: increase of torque faster than increase of current, variation in speed inversely as the load, and high efficiency throughout a wide range of speed as well as load, see Fig. 4.

Design of Series Motors. — In general the method of calculation is the same as for a direct-current generator (*see Generators, Direct-current*). The special considerations are:

A series motor is usually designed to have a large output and low speed at the one-hour rating. At any lesser output the speed will be higher, so the peripheral velocity must be quite moderate at the rated load and speed.

Since the speed is very nearly inversely proportional to the flux the speed curve depends on the shape of the saturation curve, to which very careful attention is paid in designing. By exactly fixing the flux for two extreme values of current the speed for these two values of current is fixed.

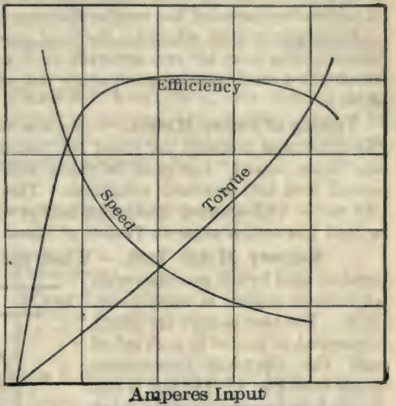

Fig. 4. Speed, Torque and Efficiency Characteristics of a Series Motor

The relations between the speed and current of a series motor are shown by the formulas:

$$E_i = E + IR,$$

$$E = \frac{p\phi Zn}{m\,10^8},$$

$$n = \frac{m(E_i - IR) \times 10^8}{p\phi Z},$$

where

E_i = impressed voltage,
E = counter e.m.f. induced in armature,
R = total resistance of armature and field,
I = current taken by motor,
p = number of poles,
m = number of parallel paths between positive and negative brush sets,
Z = total number of conductors on armature,
n = speed of armature in revolutions per second,
ϕ = total flux per pole in maxwells.

Since the current in the field of a series motor is the same as that in the armature, the ratio of the turns in each is the same as the ratio of ampere-turns or m.m.f's. Thus if the magnetomotive force per pole of the field is to be 1.5 times that of the armature the number of turns will be 1.5 times the number of turns in series in the armature.

Since a series motor is usually an inclosed motor with a one-hour rating its rise in temperature and rating are a direct function of the watts lost and the ability of the mass of the motor to store up this heat energy. In a one-hour run the amount of energy radiated is only about 10 per cent of the amount stored in the mass. For a rise in temperature of 75° C. in one hour there should be about 0.4 pound of material for each watt of loss. This assumes reasonable provision in the construction of the motor for the transfer of the heat from the armature to the field and frame.

Much attention has been directed recently to the ventilation of these motors by drawing air from outside the motor by means of fan blades on the armature and by circulation of the air inside the motor through definite paths. This has considerably increased the weight efficiency of these motors.

In railway motors, which are the most general application of the series motor, commutating poles are very generally used, as this construction makes it possible to obtain a much greater momentary output from a motor of a given size (*see below*).

Testing of Series Motors. — (*See also Standardization Rules of the A.I.E.E.*) To determine properly the speed and torque characteristics of a series motor an "input-output" test must be made, which involves subjecting the motor to actual load and overload conditions. This may be accomplished by running the motor with a prony brake as a load or with a direct-connected generator as a load (*see section above on Testing of Shunt Motors*).

Railway Motor Test. — When two similar motors are available the method used by the manufacturers of railway motors is most desirable. The two motors are direct connected, or geared to each other, and the electrical connections made as in Fig. 5. The test is run through by keeping constant rated voltage on the motor and regulating the load on the motor by changing the load on the generator.

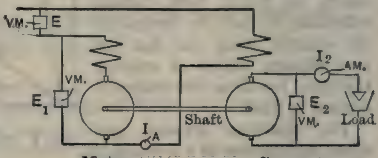

Fig. 5. Railway Motor Load Test

As the two machines are operating under almost exactly the same conditions, their efficiencies are very nearly the same. Thus

$$\text{Efficiency of set} = \frac{E_2 I_2}{EI},$$

$$\text{Efficiency of each motor} = \sqrt{\frac{E_2 I_2}{EI}}.$$

The speed and torque curves should be made for both directions of rotation of the armature as an incorrect brush setting will give results differing with the direction of rotation. The direction of rotation is changed by reversing the connections of either the field or the armature of the motor.

Commutation is observed during the speed and torque test.

The heat run is made with the same arrangement as the speed-torque test. In making the heat run the motor must start cold or at room temperature. The covers of the inspection openings of railway motors are customarily left open during the heat run.

Losses and Efficiency of Series Motors. — For a more accurate determination of the efficiency and losses the following special tests are made:

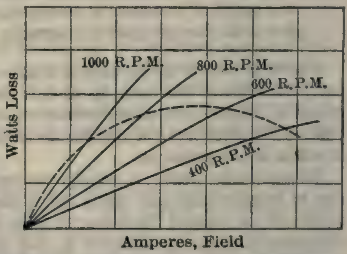

Fig. 6. Core-loss Curves of Series Motor

1. Resistance of armature, brushes and field. These tests are similar to those for a shunt motor, see above.

2. Core-loss test. On account of the variable speed and variable field of a series motor this test consists in repeating the usual core-loss test as described

above for a generator at several different speeds. The field strength is varied step by step throughout the maximum range for each speed. Fig. 6 shows the curves for these different runs and the dotted line connects the points on the different curves that apply to the normal speed curve of the motor.

Insulation Tests. — See *Generators, Direct-current,* and *Standardization Rules of the A.I.E.E.*

STARTING OF DIRECT-CURRENT MOTORS. — A starting box, Fig. 8, or rheostat is always employed in starting direct-current motors in order to reduce the voltage impressed on the motor when it is not running at a high enough speed to generate the proper counter e.m.f.

Let E_i = line voltage,
E = counter e.m.f. (approximately proportional to speed),
I = current,
r = resistance of armature circuit,
R = resistance of starting box or rheostat.

Then

$$I = \frac{E_i - E}{r + R}.$$

At the first instant the motor armature is stationary and $E = 0$; thus $I = \frac{E_i}{r + R}$, and the value of R is determined by the desired value of I. As the motor accelerates, E increases. If R remained constant I would decrease to such a small value that there would not be sufficient torque to accelerate the load. The current, and therefore the torque, can be brought back to their original values by changing R to such a value R_1, that

$$I = \frac{E_i - E}{r + R_1}.$$

Fig. 7 shows the sudden rise in current when the resistance is changed and the gradual decrease in current as the speed increases. The number of steps necessary depends upon the ratio of the maximum allowable instantaneous value of the current to the final constant value, upon the value of the armature resistance and upon the inertia of the load.

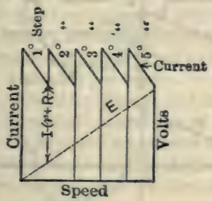

Fig. 7. Motor Current During Starting

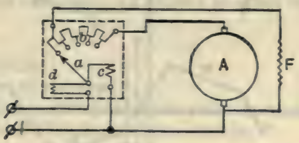

Fig. 8. Starting Box Connections for Shunt Motor

Starting Box.—(*See also article on Rheostats*). A starting box usually contains the following features, as indicated in Fig. 8: (a) a means of opening and closing the circuit supplying all the current to the motor including the field current; (b) a set of resistance steps in series with the motor armature and a means of short-circuiting this resistance step by step; (c) a magnet coil connected across the motor terminals to open the circuit if the impressed voltage fails or falls below a specified value (low-voltage release); (d) a magnet coil carrying the main current to actuate a spring and open the circuit if the current exceeds a

specified value (overload release). The usual connections of a starting box to the line and motor are shown in Fig. 8.

SPEED CONTROL. — There are three methods of varying and controlling the speed of d-c. motors, namely, potential, rheostatic and field systems.

Potential Control or Multi-voltage System. — By means of several generators and several wires various definite voltages are made available, such as 240, 180, 120, etc. By connecting the motor to the 240-volt circuit full speed is obtained; by connecting to the 180-volt circuit ¾ speed is obtained, etc. The shunt field circuit is left connected at all times to a circuit of the proper voltage. A shunt motor with normal field excitation will be stable, that is, it will operate constantly, at the fractional speed. The efficiency will be good at the fractional speeds. A series motor controlled in this manner will be unstable, but for a given torque the speed will be roughly proportional to the voltage.

Rheostatic Control. — A rheostat in series with the armature will reduce the voltage impressed on the armature by an amount proportional to the current, and thereby reduce the speed. The speed is unstable with this arrangement, changing with every change of load, and the efficiency is poor.

Field Control. — By increasing the resistance in series with the field of a shunt motor the speed is increased due to the weakening of the field. If the motor has commutating poles to assure good commutation the speed may be varied in a ratio of 1 to 2, and even 1 to 3 in small sizes. The shunt motor is stable with this method of control and the efficiency is good. In a series motor the field may be shunted by a resistance to increase the speed but the motor is not stable and this practice is not to be recommended.

USE OF COMMUTATING POLES (INTERPOLES) IN VARIABLE-SPEED MOTORS. — In motors intended to be operated over a large variation in speed, obtained by changing the field strength, and in motors which are to be subjected to heavy overloads, it is necessary to use commutating poles in order to obtain good commutation. In a motor without commutating poles the field strength must always be a certain percentage greater than the armature strength to prevent a shifting of the field flux and of the neutral point. Thus,

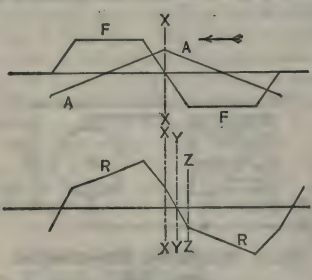

Fig. 9. Flux Distribution without Commutating Poles

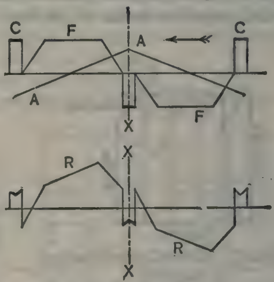

Fig. 10. Flux Distribution with Commutating Poles

if, in Fig. 9, F represents the distribution of field flux when existing alone and A represents a strong armature flux existing alone, then R shows the distribution of the resultant flux when both field and armature are excited.

It will be noticed that the neutral point has been shifted from XX at no load in a direction against rotation to YY at the load considered. The brushes would have to be shifted from XX at no load to a point ZZ beyond YY at load

in order that they shall commutate a coil in a flux which is producing a voltage helpful to commutation.

Resultant Flux with Commutating Poles. — If, however, commutating poles are placed between the main poles and excited with the armature current they will maintain at the geometrical neutral a flux of the direction and value necessary to give good commutation. In Fig. 10, *F* and *A* represent the field and armature flux separately as before and *C* the commutating pole flux that would exist at full load. When at full load these fluxes are combined there exists the resultant flux shown at *R*.

It will be noticed that there remains at the neutral point a small flux of the proper polarity and magnitude to provide an e.m.f. to reverse the current and give good commutation and it is not necessary to move the brushes.

The commutating pole must be of the same polarity as the pole towards which the brush would have to be moved if there were no commutating poles. In fact the principle of commutating poles is nothing more than bringing to the brush a part of the pole instead of moving the brush to the pole. Thus the polarity of the commutating pole is different during motor action from that during generator action. If the windings on the commutating poles are connected in series with the armature the conditions will be correct for either motor or generator action.

INSTALLATION AND ERECTION. — In the installation and erection of a direct-current motor there are certain features which must receive careful attention in order that the machine shall operate properly and not deteriorate with undue rapidity. Although this procedure varies with different motors according to their mechanical construction the following brief memorandum of points to be looked after will be found useful:

1. Base bolted down.
2. Bearings clean and filled with oil.
3. Bearings lined up.
4. Magnet frame bolted to base.
5. Field coils secured in place.
6. Field coils tested for open circuit, wrong connection and polarity.
7. Armature in place.
8. Air gap adjusted by shimming.
9. Measure resistance of armature and field.
10. Measure insulation resistance.
11. Brushes properly fitted and spaced and pressure adjusted to about 1.5 to 2 pounds per brush.
12. Commutator smooth and true.
13. Substantial connections of field circuit.
14. Field adjusted for correct direction of rotation.

The motor must be protected from moisture during shipment and if by accident it becomes damp it must be dried out before it is subjected to a voltage.

OPERATION. — In the operation of a direct-current motor several factors should be considered.

Care. — All motors should be frequently inspected and the following points noted:

1. Bearings filled with proper amount of oil.
2. Brushes securely held in proper position.
3. Brushes fit properly.
4. Commutator smooth: Danger of "high mica" or the insulation between commutator bars projecting above the bars.
5. Air gap true.
6. Commutator not worn in grooves.

Troubles. — In the following paragraphs is given a concise list of the troubles that may be experienced in operating continuous-current motors and their causes as given by Crocker and Wheeler in *Management of Electrical Machinery.*

1. *Sparking at the Commutator.* — Causes: Armature carrying overload. Brushes improperly spaced. Brushes not at proper position. Rough commutator. Poor brush contact. Internal short or open circuit. Field too weak. Unequal strength of poles. Vibration.

2. *Heating of Commutator and Brushes.* — Sparking. Bearing trouble. Bad connections. Brush friction too great.

3. *Heating of Armature.* — Overload. Internal short circuit, moisture or ground. Reversed coil. Excessive eddy currents.

4. *Heating of Field.* — Internal short circuit.

5. *Heating of Bearings.* — Bearings dry or dirty. Shaft out of true. Bearings out of line. Thrust due to belt. Unbalanced magnetic pull.

6. *Noise.* — Armature not balanced. Brushes dry or not set at proper angle. Armature strikes.

7. *Speed Too Low.* — Wrong voltage. Overload. Armature strikes. Bearing too tight.

8. *Speed Too High.* — Wrong voltage. Field too weak.

9. *Motor Stops or Fails to Start.* — Overload, open circuit, wrong connection.

SPECIFICATIONS FOR D-C. MOTORS FOR INDUSTRIAL USE. — (*See next section for Specifications for Railway Motors.*) The following memoranda are intended to assist in writing specifications. See also article on *Specifications.*

Principal Characteristics and Conditions of Service. — Use to which motor is to be put, kind of load and method of drive. Voltage. Rating, horse-power. Speed.

Style and Description; Details of Construction. — Whether to be open, semi-inclosed or inclosed. Whether to be series, shunt or compound wound; if shunt wound, whether shunt field rheostat is to be supplied; if compound wound, state whether cumulative or differential. Requirements regarding pulley or length of shaft. Whether rails are required. Whether starting rheostat is to be supplied, and if so, its general characteristics.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*). Temperature rises upon which ratings are to be based. Details of overload. Efficiency at 25, 50, 75, 100, and 125 per cent load; whether rheostat losses are to be included in calculating efficiencies. Starting torque with full-load current, pound-feet. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation. Regulation; the supply voltage being constant, and the field rheostat fixed, a variation of load from zero to . . . per cent of full-rated load shall cause a variation of speed not greater than . . . per cent. The shunt field rheostat to give speed variation of . . . per cent in steps not greater than . . . per cent and not less than . . . steps, and to carry the current for any speed continuously without undue heating.

SPECIFICATIONS FOR SERIES RAILWAY MOTORS. — The following memoranda are intended to assist in writing specifications. See also article on *Specifications.*

Principal Characteristics and Conditions of Service. — General statement of the service, giving type of cars, whether current is direct or alternating,

or both, etc. The motor shall be designed for normal operation at . . . volts and shall operate safety at . . . volts.

Style and Description; Details of Construction. — The frame shall be designed so as to allow the easy removal of armature and field coils. It shall be provided with openings at both ends, and both above and below the shaft, which will enable the inside of the motor to be readily inspected and cleaned. Bearings shall be designed so that lubricant cannot enter the frame, and shall be so located that they may be easily emptied and cleaned. The diameter of the driving axle on which the motor is to be mounted shall be . . . inches. Whether motor is to be of interpole type. Whether natural or forced ventilation.

Brushes and Brush Holder. — The brush holders shall be readily removable through the hand holes. The springs holding the brushes against commutator shall not be relied on to carry current. The brushes shall be staggered or provided with adjustment parallel to the armature shaft so as to prevent the formation of ridges on the commutator.

Clearances.—The minimum distance between motor frame and back of wheel flanges shall be . . . inches, the minimum distance between bottom of motor and top of rail when tires are new shall be . . . inches.

Gears and Gear Case (if any).— Single or double reduction; what gear wheels shall be mounted on; material of wheel and pinion; description of teeth, whether cut or cast, and width of face of wheel or pinion; gear case, material, how suspended, oil-tightness.

Suspension of Motors.—General description and requirements, location of lugs on motor frames.

Data to be Furnished by Bidder. — The armature will be bound with . . . bands. Material and dimensions of the bands. Dimensions of openings in the frame. The brush holders will be adjustable so as to allow . . . inch wear with uniform pressure on the brushes, after the diameter of the commutator has been reduced by . . . inches. The current density in the carbon brushes will not exceed . . . amperes per square inch at normal rated load. The gear ratio will be . . .

Performance and Tests. — (*See also Specifications in article on Locomotives, Electric.*) Either the nominal rating and the continuous ratings at ½, ¾ and full voltage should be specified, or the following data supplied.

Line voltage. . . . .
Number of motors per car. . . . .
Weight of loaded car, exclusive of motors and control equipment.
Diameter of driving wheels. . . . .
Schedule speed. . . . .
Distance between stops. . . . .
Duration of stops. . . . .
Acceleration miles per hr. per sec. . . . .
Retardation, miles per hr. per sec. . . . .

The engineer should also give a diagram of grades and curves.

Motor Characteristics. — The bidder shall submit diagrams showing speed, tractive effort, efficiency, RI^2 losses, core losses, and any other information bearing on the performance of the motor. Requirements regarding the effect of moisture upon insulation.

Tests. — The motor shall be tested at the Manufacturer's works in the presence of the Engineer's inspector. (In the case of new motor developments it is good practice to make the tests under service conditions; but for standard motors a stand test at the factory is sufficient.) A complete series of tests shall be made upon the first motor manufactured under this specification. These tests shall confirm all the statements made by the bidder in relation to operating characteristics. Should the motor fail to comply with any of these statements, the defects shall be corrected and any changes in construction or design which may be necessary to accomplish this shall be made at the contractor's expense. The first motor shall be submitted to a flashing test to determine the susceptibility of the motor to flash-over on opening the maximum specified line voltage across the motor when running at maximum speed. After the acceptance of the first equipment, any other motors to be supplied under this specification shall be submitted to an approved stand test. The insulation of the armature windings, commutator and field windings, shall be subjected to stated alternating voltages (*see Standardization Rules of the A.I.E.E.*) for a period of one minute.

PERFORMANCE, WEIGHT AND COSTS (Pre-war prices) *. — Usual values of the efficiency and losses, and also values of the weight, speed and cost of shunt and series motors are given in the following tables:

PERFORMANCE OF SHUNT MOTORS

H.P.	Full load efficiency, per cent	Field I^2R, per cent	Friction, per cent	Core-loss, per cent	Armature I^2R, per cent
0.5	70	5	10	5	10
1	79	4	8	4	5
2	82	3.5	7	3.5	4
5	85	3	5	3	4
10	87	2.5	4.5	2	4
20	88	2	4	2	4
25	89	2	3	2	4

WEIGHT, SPEED AND COST OF SHUNT MOTORS

H.P.	Speed, r.p.m.	Weight, pounds	Cost *	Speed, r.p.m.	Weight, pounds	Cost *
0.5	2200	60	$30	....	....	...
1	2000	100	50	1675	125	$60
2	1650	210	75	1175	250	90
5	1100	550	150	920	600	165
10	850	950	240	700	1100	280
20	680	1650	410	550	1900	470
25	650	1950	500	500	2300	600

* For 1922 prices add 50 per cent.

PERFORMANCE OF SERIES MOTORS

Commutating Pole Railway Type

H.P.*	Full-load efficiency, per cent	Field I^2R, per cent	Friction, per cent†	Core-loss, per cent	Armature I^2R, per cent
40	79	6.5	5.3	2.2	7.0
50	82	5.0	5.0	2.1	5.9
75	84	4.5	5.0	2.0	4.5
100	85	4.3	5.0	2.0	3.7
150	86.5	3.5	5.0	2.0	3.0

* H.P. for 75° C. rise in one hour.
† Friction includes loss in gearing.

WEIGHT, SPEED AND COST OF SERIES MOTORS

Commutating Pole Railway Type

H.P.*	Speed, r.p.m.	Lb., weight per h.p. †	Cost, dollars per hp.‡
40	750	57	11
50	700	53	10.5
75	650	44	8.5
100	625	37	7
150	600	33	6.4

* H.P. for 75° C. rise in one hour.
† Weight includes cast-steel frame and gear pinion and gear case.
‡ For 1922 prices add 50 per cent.

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MOTORS, INDUSTRIAL APPLICATIONS OF.—(*See also Bearings; Belts and Belting; Blowers and Compressors; Chains and Chain Drive; Conveyors; Couplings, Direct; Cranes; Dredges, Electrically-Operated; Elevators, Electric; Fans; Flywheels for Load Equalization; Gears and Gearing; Hoists, Electric; Machine Tools, Electrical Operation of; Motors, A-C. Commutator, Direct-Current, Polyphase Induction, Single-Phase Induction, Synchronous; Pumps and Pumping Engines; Printing Presses; Ropes and Rope Drive; Shafting; Shovels, Electrical Operation of; Steel Mills, Electric Drive of; Telpherage; Unloaders, Coal and Ore; Valves.*) The application of the electric motor for driving industrial machinery, either individually or in groups, was at first thought to be of value merely in the saving of power through the elimination of the losses due to friction in line shaftings or other forms of mechanical transmissions. A still higher economy has, however, been found to lie in the remarkable effect that it has in increasing the output of the production.

ADVANTAGES OF ELECTRIC DRIVE. — The more important advantages of electric drive are the following:

Location of Machines. — The various machines can be placed in almost any desired position and the use of portable tools is readily made possible, as, for example, when a portable drill and slotter are brought to a heavy casting, the slotter being applied to the outside of the piece at the same time the inside is being drilled.

Head Room. — A clear head room is obtained by the elimination of belting. This gives better illumination and ventilation and permits overhead cranes to be used freely, which is of greatest importance in any factory as it greatly facilitates the handling of the material, resulting in a considerable saving in time and labor, and thus increasing the output.

The constant source of dripping oil from overhead bearings and shafting is eliminated, and the danger which always accompanies the use of belts is overcome.

Centralized Power. — Power can most readily be distributed from a central supply station to the different buildings, and changes or additions to the system can always be made without difficulty.

Reliability. — The electric system offers greater reliability than belt drive. A breakdown is usually confined to a single machine, but with belting and a shafting a breakdown will generally cause a shutdown of a considerable portion of the equipment.

Study of Machine Performance. — Meters of either the recording or indicating type can be installed easily where desired and the performance of every individual machine ascertained. This is a very important point in all industrial undertakings, as it is then possible to maintain all the machinery in the best operating condition. Any excess power taken is at once readily detected and the defect can be promptly corrected. An accurate record can also be kept of the cost of power for the different operations.

GROUP VERSUS INDIVIDUAL DRIVE. — There are two general systems of drive, namely, group and individual, and there is still a diversity of opinion as to the relative advantages of the two. The group drive is to a certain extent an outgrowth of the older system of line shafting. When such a system is to be changed to an electric drive, the most obvious and simple way of making the change is to split the shafting up into such sections as would be most convenient, and drive each of these by means of a comparatively large motor. On the other hand, it may frequently happen that it is necessary to operate only one machine of the entire group for a considerable time, as in overtime work,

and to do this it would then be necessary to keep the motor and the line shafting of the whole group running. Since the efficiency of the motor at this light load would be small, and the friction losses of the entire drive would have to be supplied, it is evident that such a method of operation would result in a waste of power and be most inefficient. Modern installations, therefore, indicate a tendency toward the use of both the group and individual drive.

Influence of Character of Load. — It is generally agreed that all large tools or other machinery should be equipped with individual motors, especially if their service is of an intermittent nature. With the group drive there are two distinct loads, the variable of the machines and the friction of the line shafting and belting. The lower the machine load factor, the greater becomes the percentage of friction load and the more inefficient the group transmission.

Influence of Speed. — Wide ranges of control and the possible variations of speed are reasons which in many cases are sufficient in themselves for the selection of an individual drive. With group drive the methods of speed control for the individual machines are obviously more limited. It is then generally accomplished by shifting of belts on cone pulleys or by change of gears. Both of these methods, however, take a considerably longer time than the simple manipulation of the controller with the individual electric drive.

With individual motor drive it is possible to obtain very fine speed graduations, this benefit, of course, only being derived with a variable speed motor. Another advantage is the fact that it is possible to speed up a machine with a proportional increase in power. This may be necessary whenever a change is made from carbon to high-speed steel.

Influence of Relative Cost. — The increased cost of installation is one of the principal factors that prevent the general installation of individual drives. With this drive the total horse-power rating of the motors installed in the plant will be considerably greater than with group drive, but the maximum power demand of the plant is approximately the same in either case. If power is purchased the price should be based on the actual maximum power demand and not, as sometimes is required, on the total connected horse-power capacity of the motors. This latter method would obviously give a lower basic rate for the group drive, although the higher efficiency of the individual drive would considerably reduce the actual power consumed.

The question of whether or not group or individual drive is to be installed is thus a financial one and each case must be properly analyzed. Individual drive necessarily means a larger investment, but in nearly all cases a much greater percentage income will be realized than if line-shaft drive were employed.

GENERAL CONSIDERATIONS IN THE SELECTION OF MOTORS. — The conditions of capacity and efficiency are both of importance in any motor installation and should therefore be given careful consideration. The installation of a motor having too large a capacity should in general be avoided, unless an increase in the load is to be expected in the near future, because the efficiency of a motor is usually a maximum at its normal rated output, decreasing above and below this point. With alternating-current motors the effect of the power factor must furthermore be considered. This decreases rapidly below normal load, and on account of its bad effect on the regulation of the system it should be kept as high as possible, which can only be done by operating the motors as nearly fully loaded as possible. Ordinarily, however, it is possible to so group the machinery that the motors may be operated near their rated output at all times. Too small a motor is naturally also very undesirable, as it would then in all probability be subject to overloads, which may result in overheating and a burn-out of the motor, causing a shutdown, not only of the motor itself, but also of the machinery which it drives. The op-

erating conditions of the plant may furthermore be such, as for example in steel mills, that the failure of a single motor may necessitate the shutting down of the entire mill.

The selection of suitable motors requires not only complete information on the power required to drive each group of machines or each machine individually but also a thorough knowledge of the motor design and its inherent characteristics to meet the requirements of the load. Some machines will require motors with very heavy starting torque, although running under light load when up to speed, while for others the requirements may be just the opposite. With a varying speed motor the torque-speed characteristics should agree as nearly as possible with the load which the motor is to drive, and the characteristics of adjustable-speed motors as influenced by different systems of control should also be carefully investigated. See also section below on *Data Required to Determine Type and Size of Motor.*

Influence of Motor Efficiency. — A motor of high efficiency is obviously desirable, and it is generally an easy matter to estimate the saving incurred by the installation of a motor of high efficiency as compared to a less efficient one. When a motor is operated for a considerable part of the time on light load, this fact must be given due consideration in the comparison on account of the variation in the motor efficiency for different loads.

Influence of Torque and Speed. — In order to obtain the most satisfactory results from motor drive it is essential that the type as well as the size of the motor be properly adapted to the work contemplated. This is especially important in the case of individual drive, where a wrong selection of the proper motor would be more serious than in group drive. The size of the motor may be ample to operate the machine under normal load but it may not be able to develop a sufficient starting torque, or it may draw a too excessive starting current from the line. For example, to start and accelerate the bridge of a crane requires a motor capable of developing a high starting torque, but after the bridge is accelerated comparatively little power is required to keep it in motion.

The condition of maximum torque must also be given due consideration. A motor driving a heavy punch may, in spite of the flywheel, develop insufficient torque to keep up the speed. As a rule, however, where the motor is large enough for starting and normal operation, but not large enough for the maximum overload required for perhaps only a second or two, the addition of a suitable flywheel will sometimes cut down the maximum torque required. In other cases it may be necessary to install a motor larger than necessary for the average work.

The proper speed regulation is also of importance and a motor must be selected which is best meeting these requirements. The size of the motor is also influenced by the cycle of operation, i.e., whether the load is continuous or intermittent. Careful consideration should be given to this point, and, as previously mentioned, motors will undoubtedly soon be rated to conform to the particular service for which they may be required. See also section below on *Classification of Motors According to Speed.*

Alternating Versus Direct Current. — The choice of alternating or direct current depends largely on local conditions and on the service requirements. With all other conditions the same, the alternating-current system offers many advantages when the distances over which the power must be transmitted are large. A higher transmission voltage may be selected which will diminish the amount of copper needed in the line conductors. The conditions may be such that high-voltage motors can be used, but in other instances step-down transformers may have to be provided and the expense of these as well as other auxiliary apparatus connected therewith must then be considered. It is gener-

ally conceded that the alternating-current system is more reliable than the direct current. This is mostly due to the absence of commutator trouble and to the rugged design of the induction motor.

There is no reason for installing direct-current motors except where an adjustable speed service is required, and in small plants where such a service is predominating the direct-current system would naturally be the one to install. If a varying-speed feature is only required intermittently, the phase-wound induction motor may be used to advantage, but if the motor must run at reduced speed a considerable portion of the time, the varying-speed, brush-shifting motor should be given careful consideration. As a rule, a considerable part of industrial machinery will require a constant-speed service, for which the alternating-current motor is admirably adapted, and should direct current be required it can be obtained by installing a motor-generator set.

As a rule it may be said that the alternating-current system should be selected if possible. This would furthermore permit of throwing over to a central-station service, in case it should be found that power could be more economically purchased from the central station than be generated on the premises.

CHARACTERISTICS OF MOTORS AFFECTING THEIR APPLICATIONS. — (*See also the separate articles on Motors.*) Motors are divided into two classes — direct-current and alternating-current, according to the system from which they are operated. The direct-current motors are further subdivided into three types, namely, series, shunt and compound motors.

There are also three general types of alternating-current motors, namely, induction, synchronous and commutator motors.

Series Motor. — This motor is used when a powerful starting torque and rapid acceleration are required, without an excessive instantaneous demand of energy. The torque is practically independent of the voltage and at low flux densities varies directly as the square of the current, but as the magnetization approaches saturation it becomes more nearly proportional to the first power of the current. The maximum torque exists at low speed, this being the most valuable feature of the series motor. Dangerously high speeds may be attained by the armature with very light loads, and series motors should for this reason be either geared or direct connected to the load.

Speed Control of Series Motor. — The speed of a series motor on constant potential varies automatically with the load, increasing as the load decreases. The speed may, however, be adjusted if some means of varying the impressed voltage is provided. As the work required of a series motor is very often intermittent in character, the insertion of resistance in the armature circuit to reduce the speed is permissible from an economic standpoint in such cases. In others, such as railway work, where two or four motors are used, reduced voltage is most readily and economically obtained by connecting the motors in series or in series-parallel.

Shunt Motor. — This motor has good starting characteristics and a practically constant speed, varying only slightly with load changes. The speed can, however, be adjusted, either by changing the e.m.f. impressed on the armature, or by changing the field flux.

Speed Adjustment by Armature-voltage Control, i.e., by changing the e.m.f. impressed on the armature, does not change the full-load torque which the motor is capable of exerting, since the rated torque depends only upon field flux and rated armature current. These methods are therefore constant-torque methods and are properly adapted to loads in which the torque remains constant regardless of speed. The method most generally used for varying the impressed e.m.f. with a single-voltage system is by means of inserting resistance in series with the armature. The efficiency with this method is, of course,

very low at slow speeds. The speed regulation with varying loads may also be very poor.

There are several systems of controlling the motor speeds by applying different voltages, such as by the use of three-wire generators or two-wire generators with balancer sets or by the Ward-Leonard system. This latter system, which is the most practical, consists of a constant-speed motor driving a generator which supplies current to the motor whose speed is to be adjusted. This arrangement is very satisfactory but on account of the expense of providing three full-sized machines instead of one to perform the work, the cost may be prohibitive except with very large motors, such as for hoists, etc.

Speed Adjustment by Shunt-field Control, i.e., by inserting resistance in the shunt-field circuit, is the simplest of all methods of speed variation, but with ordinary shunt motors, the range of speed variation by this means is small. Where a variation of more than from 20 to 30 per cent is desired, a motor of modified design and of a certain increased size is generally required because, the field must be more powerful with respect to the armature than in the case of standard single-speed motors. Varying or adjustable speed motors of the field-weakening type are not constant torque, but constant-output motors, i.e., the torque falls proportionally as the speed increases.

A speed variation up to 3 to 1 meets, as a rule, all requirements and such motors can readily be obtained in commercial sizes. Should a greater speed variation be desired, say 4 to 1 or 5 to 1, it is possible to accomplish this by the commutating-pole shunt motor with field control only. A combined field and armature control would, however, be a better method.

Compound Motor. — This motor is provided with both a series and a shunt field. The two fields are usually connected so that they act in the same direction, in which case the motor is called a "cumulative" compound motor. "Differential" compound motors, with the two fields opposing, are sometimes employed for special services. The cumulative, or ordinary, compound motor combines the characteristics of the shunt and series motors, having a speed not extremely variable under load changes, but developing a powerful starting torque and an increasing torque with decreasing load. Motors having a comparatively weak series field are employed extensively in shop practice where the motor may be required to start under heavy load but must maintain an approximately constant speed after starting, or when the load is removed. The heavily compounded motor is used where powerful starting torque and rapid acceleration are necessary, with a speed not varying too widely under load changes, such as for rolling mills, etc.

The speed control employed with compound motors may be any of the various methods explained in connection with the shunt motor. For certain service the control may be entirely rheostatic, the series winding being cut out after the motor has come up to speed.

Induction Motor. — The induction motor is essentially a constant-speed machine, although the speed may be varied either by varying the applied stator frequency or by introducing resistance in the rotor circuit. It is built in two distinct types, namely, the squirrel-cage and the phase-wound.

Squirrel-cage Motor. — The squirrel-cage type is used for constant-speed service with infrequent starting. It has a relatively small starting torque per ampere and draws a large starting current from the line. By increasing the resistance of the rotor, it may however also be built in the smaller sizes for a high starting torque, rapid acceleration and frequent starting, for such applications as sugar and laundry centrifugals, etc., where simplicity of control is desirable. They are also used for operating punches, shears, etc., where a fly-wheel is provided for storing the energy.

Induction Motor with Wound Rotor. — For service requiring high starting torque combined with moderate starting current a motor with the wound type of rotor is best adapted. A motor with the resistance mounted inside the rotor should not be used to operate machinery having large inertia or excessive static friction, since full starting current may be required for a long period before the apparatus attains full speed, and, as the capacity of the internal resistance is small, excessive temperatures may result. This type of motor is, as a rule, not built above 200 horse-power due to mechanical difficulties involved in connection with the internal resistance.

A motor with external resistance should be used for moderate and large sizes. The rotor must then be provided with collector rings and brushes. The contact resistance of these as well as the leads and the controller fingers, which are in the circuit all the time, may impair the efficiency and regulation of the motor especially if the controller and the resistance are located some distance from the motor. The phase-wound induction motor with an external variable rotor resistance is best adapted for a varying-speed service, as the losses necessary to obtain reduced speeds are external to the motor itself.

Multi-speed Induction Motors. — It often happens that the service is such that two or three speeds will be satisfactory for the operation of the machinery and that these speeds must be independent of the load. Under such conditions multi-speed motors can frequently be used. In these motors the different synchronous speeds are produced by changing the number of poles in the magnetic circuit. Each of these speeds is fixed, if no resistance is used in the secondary circuit. With multi-speed motors, as with single-speed motors however, resistance may be used in the secondary circuit for varying the speed.

A change of the number of poles may be produced in any of the following ways:

1. By the use of single magnetic and electric circuits, changing the number of poles by regrouping the coils.
2. By the use of single magnetic circuits and independent electric circuits.
3. By means of separate magnetic and electric circuits, the so-called Cascade connection.

Synchronous Motor. — The speed of a synchronous motor is constant, being fixed by the number of poles and the frequency of the applied voltage. The single-phase type is not self-starting and the polyphase type has in itself a very poor starting torque. They may, however, be made self-starting in the same manner as squirrel-cage induction motors, by the use of an ammortisseur or cage winding, similar in construction to that used for induction motors.

The speed-torque curve of a synchronous motor is similar to that of an induction motor except that the torque values are lower for a given resistance of rotor winding on account of the construction of the machine. The starting winding must be designed with both the load at start and the load at synchronous speed in mind, because too great a slip may cause the motor to shut down when the field is put on. It is, however, seldom that the same motor will be called upon to start a heavy load and at the same time synchronize a heavy load, as the load usually consists principally of either static friction, as in the use of motor-generator sets, line shafting, etc., or it comes up with the speed as in the case of a fan blower or centrifugal pump. The former case would be met by a high-resistance squirrel-cage winding and the latter would require a low resistance.

Single-phase Series Motor. — This type of commutator motor has a very powerful starting torque, high power factor and relatively high efficiency. It is most generally used for traction work, the speed being controlled by varying

the applied voltage, which can most readily be done by means of an auto-transformer with a number of taps.

Repulsion Induction Motor. — This type of commutator motor has a limited speed and an increase of torque with decrease in speed. The action of the compensating field insures a power factor approximately unity at full load and closely approaching unity over a wide range in load. In addition it serves to restrict the maximum no-load speed and also permits, where varying speed service is involved, an increase over the synchronous speed.

Starting of Repulsion Motors. — A repulsion motor, if started by directly closing the line switch, will develop about 2½ times full-load torque. The starting current corresponding to full-load starting torque is from 2 to 2¼ times full-load running current. As a general rule, starting boxes are not required up to and including 2 horsepower rating. From 2 to 5 horsepower the use of a rheostat is optional, dependent upon the degree and care to be exercised in maintaining voltage regulation. Starting boxes should, however, preferably be used on sizes above 5 horse-power, especially where light and power circuits are combined.

Reversible Repulsion Motors. — The repulsion motor may be designed for reversible service. This is accomplished by adding an auxiliary reversing winding spaced 90 degrees from the main field winding and connected in series with it. By reversing the relative polarity of the two windings, the direction of rotation is changed in a simpler manner than by mechanical shifting of the brush holder yoke. Instant reversal may be effected from full speed in one direction to full speed in the other, about 200 per cent of normal running torque being developed at moment of speed reversal in either direction.

Varying-speed Repulsion Motors. — In addition to the constant-speed repulsion motor, two other types are also available, one for constant-torque and varying-speed service, the other for adjustable speed independent of torque In general, varying-speed repulsion motors are not applicable to lathes, boring mills or similar machines where the service requires adjustable speed and constant horse-power at all speeds below and above normal. When a certain amount of varying speed is required at approximately constant torque, such as driving fans, blowers, printing presses, etc., the repulsion motor successfully meets a wide field of application.

POWER RATING OF MOTORS. — (*See also Standardization Rules of the A.I.E.E.*) These rules recommend that with the exception of railway motors, all motor ratings shall be expressed in kilowatts (Kw.) available at the shaft, On account of the hitherto prevailing practice of expressing mechanical output in horsepower, it is, however, also recommended that for machinery of this class the rating may, for the present, be expressed both in kilowatts and in horsepower, as follows: Kw. ——— h.p. ———. The horsepower rating of a motor may for practical purposes be taken as $\frac{4}{3}$ of the kilowatt rating.

It is also highly desirable that the motor ratings should closely conform to the actual service requirements, and for this reason the Standardization Rules also recommend the following two kinds of ratings:

1. **Continuous Rating,** when the motor shall be able to operate continuously at its rated output, without exceeding any of the guaranteed limitations.

2. **Short-Time Rating,** when the motor shall be able to operate at its rated output during a limited period, to be specified in each case, without exceeding the guaranteed limitations. Such service includes runs alternating with stoppages of sufficient duration to ensure substantial cooling.

SPEED CLASSIFICATION OF MOTORS. — Motors may, for industrial application, be classified with reference to their speed characteristics. The wording adopted from the standardization rules of the American Inst. of Electrical Engineers is as follows:

Constant Speed Motors, whose speed is either constant or does not materially vary; such as synchronous motors, induction motors with small slip, and ordinary direct-current shunt motors

Multispeed Motor (Change Speed Motor). — A multispeed motor (change speed motor) is a motor which can be operated at any one of several distinct speeds (these speeds being practically independent of the load), but which cannot be operated at intermediate speeds.

Adjustable Speed Motors, in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load; such as shunt motors designed for a considerable range of speed variation.

Varying-Speed Motors, or motors in which the speed varies with the load, ordinarily decreasing when the load increases; such as series motors, compound wound motors, and series-shunt motors. As a sub-class of varying-speed motors, may be cited adjustable varying-speed motors, or motors in which the speed can be varied over a considerable range at any given load, but when once adjusted, varies with the load; such as compound wound motors arranged for adjustment of speed by varying the strength of the shunt field.

DATA REQUIRED TO DETERMINE TYPE AND SIZE OF MOTOR. — In selecting a motor for a certain application complete information must be had with regard to the machines to be driven. This is the first step and in some respects one of the most important parts of the problem, as without complete information on the subject it becomes very difficult, and in certain instances utterly impossible, to intelligently select or design a motor and control equipment which will satisfactorily fulfill the conditions to be met in actual operation. In order to facilitate the work of obtaining such information, an outline of the points to be investigated is given below.

1. Description of Machine to be Driven. —

- *a.* Individual or group drive.
- *b.* Photographs, drawings and sketches of machines and connections as complete as advisable, especially such drawings as indicate the size of heavy flywheels or rotating masses whose speed must be varied, and also drawings indicating gearings, transmission devices, etc.
- *c.* Limiting features of product. What are the points in quality and characteristics of product which fix the condition of operation? e.g. limiting speed of tool or mechanism, etc. *Example:* A 26-inch lathe used for finishing requires approximately 3 horse-power to drive. The same lathe for roughing shafting, when equipped with high-speed tool steel and two tools, needs approximately 30 horse-power.
- *d.* Can electric drive approach nearer the conditions wanted?
- *e.* If group drive can be considered for several machines, some of which are only to be operated intermittently, give notes with regard to the latter machines.
- *f.* What arrangements can be made so that intermittently-operated machines need not operate simultaneously, thus giving a smoother load curve and allowing the use of one small motor to drive several intermittent machines?

2. Cycle of Operation. —

a. Starting condition. Torque at start of day, and start of each cycle. Is frequent starting and stopping necessary? *Example:* The adjustment of large boring mills affects the controller, etc. Starting torque may be measured approximately if necessary by adjusting a rough beam to shafting. Find distance from axis at which the weight just starts machine. Knowing the weight in pounds the torque may be calculated.

b. Curves of loads, speeds, torque, maximum and average conditions through one complete cycle. Where necessary note power required with machines both loaded and unloaded. From this may be determined losses in shafting and transmission.

c. Time in operation, days per year, per month, per week, hours per day.

d. Reversing conditions, their frequency, their time, full or partial. Some apparatus, e.g., lathes, printing presses (flat-bed type), etc., only require slight backward movement to release tool or to adjust cylinders.

e. Starting. Flywheel or line shafting, friction clutches, etc.

f. Are speeds, torques, accelerations, etc., fixed by conditions of work or of driving machine?

g. Can any parts of cycle be varied with benefit? This should be investigated carefully.

3. Present Method of Drive. —

a. Prime mover or source of power.

b. Method of speed variation and speed change or control. Must machine be shut down to change speed?

c. Adaptability to gearing, etc.

d. Speed of machine, size of pulley, size of belt, size of gear or chain.

4. Mechanical Transmission. —

a. Rope, belt, chain, gear, direct connection. Direction of pull on belt or gear. Is belt or chain pull on the top or bottom? Is belt tightner advisable?

b. Clutches. Crab clutch, friction or couplings. Rigid, flexible or insulated.

c. Method of speed variation.

d. Brakes, electric. Solenoid or magnetic. Regenerative control, dead load or pumping back.

e. Brakes, mechanical. Band-post brake, disc brake, automatic safety brake, steam, hydraulic, air- or hand-operated.

5. Conditions of Location. —

a. Near external source of heat, furnace, etc.

b. Character of dust, conductive, magnetic or wearing, marble or stone, etc.

c. Possibility of fire or explosion to be caused by sparks from motor. Combustible flyings, e.g., cotton mills.

d. Explosive gases. Coal gas, benzine fumes, etc.

e. Presence of injurious gases. Acid fumes or salt air, e.g., SO_2, Cl, etc.

f. Dampness and moisture.

g. Insurance regulations. Obtain and forward copy of State Insurance Rules, or other state or local regulations, e.g., mining laws, municipal laws. Consider also personal danger or liability due to use of high-voltage apparatus.

h. Accessibility for inspection and repairs, oiling, etc.

i. Ventilation.

j. Allowable space for installation, also space for transportation. Down mine shaft, through doorways, etc.

k. Foundations and how attached to same.

l. Sudden temperature variations. This may cause condensation of moisture on windings and insulation.

m. If controlled from distance where will controller or switch be placed? Give approximate length of leads necessary. Must they pass through or under water, in building, on poles, underground?

6. Control. —

a. Intelligence (probable) of operators.

b. Is entire range of operation visible to operator or are special automatic features desirable?

c. Regenerative Control — See item No. 4-*d;* also *Cycle of Operation,* item No. 2.

7. Overload and Safety Devices. —

a. Can electric automatic devices be made to supersede mechanical overload safety devices, such as slipping clutches, braking shafts or crabs?

b. Should these devices be time limit or instantaneous? If former, how long?

c. Are safety devices necessary, other than those to protect overload — e.g., overrun of hoist, stoppage or cessation of load. Must emergency stop be employed, and where placed?

d. Are undervoltage releases required or advisable?

8. Probable Cost of Present Method of Operation. —

a. *Steam Operation.*

1. Coal. Tons per day or month; quality, cost, distance shipped.
2. Feed and other water. City mains or pumped; source, quality.
3. Is steam required for other purposes beside power? e.g., digesters, heating, etc.
4. Can steam be generated from waste gases or other waste products?
5. Distance of transmission, outdoors and indoors.

b. *Air.* Distance of transmission, air pressure, how obtained?

c. *Water Power.* Has electric installation to compete with water power? If so, report separately on this aspect.

9. Strength of Present Equipment. —

a. Strength of line shafting, foundation, transmission or gearing, machine parts.

b. Give estimate of limiting horse-power of items in (*a*), using sketch and dimensions where necessary. The above items should be considered with regard to any change in speed or torque made necessary or desirable, due to the electric drive.

10. Generating Station or Source of Electric Power. — Capacity of station or feeder, frequency, voltage, location, voltage or frequency variations.

BIBLIOGRAPHY. — It is impossible in the limited space available to give references to all the numerous papers in the technical journals dealing with the industrial applications of motors. The reader should consult the files of the *Transactions of the American Institute of Electrical Engineers, American Society of Mechanical Engineers, National Electric Light Association; also the Electrical World, General Electric Review, Electric Journal, Electrical Review, Power, Power and Engineer, Electrical Engineer, Engineering Magazine, Cassiers' Magazine.*

MOTORS, POLYPHASE INDUCTION. — (*See also Electricity and Magnetism, Principles of; Generators; Motors, A-C. Commutator, Direct-current, Single-phase Induction and Synchronous; Motors, Industrial Applications of; Standardization Rules; Transformers.*) An induction motor may be either single, two, or three phase. Single-phase induction motors are treated in another article (*q.v.*). The induction motor is essentially a polyphase transformer with the secondary free to move; the electric energy transferred to the secondary is transformed by this motion directly into mechanical energy.

The following is a brief table of the contents of this article:

DEFINITIONS AND PRINCIPLE OF OPERATION. — Certain terms used in connection with the induction motor can best be defined by a brief statement of the principles underlying its operation.

Primary and Secondary. — By the primary of an induction motor is meant that part which receives energy by direct connection to the source of electric energy; the other member is called the secondary.

Stator and Rotor. — That member of an induction motor which remains stationary, whether it be the primary or secondary, is called the "stator," and the revolving member is called the "rotor." In most machines the primary is the stator. A "squirrel cage" rotor is one in which the conductors are straight bars of copper all connected together at each end of the rotor by copper rings.

Poles of an Induction Motor. — In Fig. 1 is given a diagram of the primary winding of a two-phase induction motor. The small numbered circles represent the conductors forming the winding of one phase, the small black circles the conductors forming the winding of the second phase. The numbers opposite the circles give the order in which the current in phase I passes through the conductors, a cross indicating that the current goes down into the page and the open circles that the current is coming up.

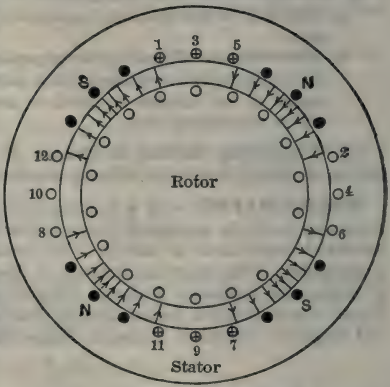

Fig. 1. Elementary Induction Motor

The diagram is drawn to represent that instant at which the current in the second phase is zero. At this instant the distribution of flux in the air gap will be roughly as indicated by the lines with arrows on them; that is, the flux will leave the stator iron in the two regions marked N and enter it in the two regions marked S. Consequently, the current in the winding of phase 1, which consists of 4 bands or groups of conductors, will produce 4 polar regions, or 4 poles, on the stator. As will be shown below, the combined effect of the two phase currents in the two windings is merely to cause a rotation of these polar regions. The "number of poles" is always equal to the number of bands of conductors into which the total winding of each phase is divided. The bands of conductors forming one phase usually overlap the bands of conductors forming the other phase, there being then two or more conductors per slot.

Rotation of Magnetic Flux. — In Fig. 1 is shown the distribution of flux in the gap when the current in phase 2 is zero; the curve marked A in Fig. 2 represents this same state of affairs, the cylindrical surface of the stator here being bent out into a plane, and the ordinates of the curve giving the value of

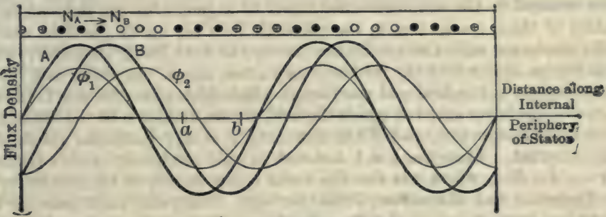

Fig. 2. Rotating Field

the flux density in the gap at each point of this surface. The flux distribution is not a smooth curve as shown, but approximates such a curve.

Next consider the case when the current in phase 1 has decreased to, say, 0.7 of its maximum value and the current in phase 2 has increased to 0.7 of its maximum value (this corresponds to ⅛ of a cycle). Then the flux distribution due to phase 1 remains in the same position as before but is reduced at each point of the gap to 0.7 of its maximum value, i.e., reduces to the curve marked ϕ_1. Similarly, the flux due to the current in phase 2 is similar in shape to the flux due to the current in phase 1, but its position is to the right of the latter by an amount equal to the width of one of the bands of conductors. The distribution of the flux due to the current in phase 2 is then as shown by the curve ϕ_2, the ordinates of which at the instant under consideration are 0.7 of their maximum values.

The resultant flux in the air gap at this instant is then the sum of the curves ϕ_1, and ϕ_2, namely the curve B. That is, the effect of the two fluxes due to the two phases is a resultant flux shifted forward, or moved around the gap, a distance equal to ⅛ the distance between successive north poles, but this resultant flux curve has the same shape and maximum value as before. This is strictly true only if the windings are distributed with absolute uniformity over the internal periphery of the stator. An extension of this analysis will show that the resultant flux remains constant in value at all times but travels around the air gap with a speed of

$$N = \frac{120f}{p} \text{ rev. per min.,}$$

where f is the frequency of the supply and p the number of poles.

This same result holds for a three-phase machine.

Synchronous Speed. — The speed of rotation of the air-gap flux, namely the speed N given by the above formula, is called "synchronous" speed. At light loads the speed of the rotor is very nearly equal to this speed.

Slip. — The slip of an induction motor is the ratio of the difference between the actual speed (N_1) of the rotor and synchronous speed (N) to the synchronous speed (N), i.e.,

$$s = \frac{N - N_1}{N}.$$

The slip may be expressed as a fraction or as a per cent. The slip at standstill is unity; at no load it is very nearly zero. An induction motor driven at a speed higher than its synchronous speed has a negative slip; such is the case in an induction generator.

Electromotive Forces in Secondary. — The electromotive forces in the secondary of a polyphase induction motor are induced by the rotation of the flux produced by the currents in the primary windings, just as the electromotive forces induced in the armature conductors of a generator are induced by the rotation of these conductors in the magnetic field set up by the field winding. In the generator only the conductors move, the field being stationary, but in the induction motor both the field and conductors move. In either case it is the *relative* motion of the field and conductors which determines the e.m.f. induced.

Let v be the linear speed at which the field moves, and let v_1 be the linear speed of the rotor conductors, and B the flux density at any particular conductor C at any instant. Then the e.m.f. induced in this conductor at this instant is $B(v - v_1)l = Blsv$, where s is the slip and l is the length of the conductor (*see also Electricity and Magnetism, Principles of*). The rotor electromotive force is therefore proportional to the slip. As the rotor turns, the conductor C moves slower than the rotating flux, and the state of affairs is just the same as if the flux remained at rest and the conductor moved through the field in the gap at a speed of $v - v_1 = sv$. Hence the electromotive force induced in each rotor conductor is alternating, since the flux which it cuts varies from a positive maximum to a negative maximum, and a consideration of the relative speed of the conductor and the flux will show that the frequency of this induced electromotive force is the frequency in the primary multiplied by the slip.

That is, the secondary electromotive force is proportional to the slip and has a frequency equal to the product of the slip by the frequency in the primary.

Secondary Current and Torque. — The current set up in each rotor conductor by this electromotive force will be practically in phase with this electromotive force, since the rotor conductors have but a small reactance, particularly when the rotor is revolving at a speed near synchronism. Hence the current in any chosen rotor conductor at any instant is proportional to the electromotive force in this conductor at that instant, which in turn is proportional to the flux density at this conductor at this instant. Since the force produced by a magnetic field on a conductor is equal to the product of the flux density by the current by the length of the conductor (*see Electricity and Magnetism, Principles of*), the force acting at any instant on any rotor conductor will then be equal to

$$f = Bli = Bl\left(\frac{Blsv}{r}\right) = \frac{B^2l^2sv}{r},$$

where r is the resistance of the conductor. Since the current and the flux density both change signs at the same time (being in phase) the direction of this force will always be in the same direction and will consequently drive the rotor against whatever opposing force may exist.

Not only is the force on each conductor always in the same direction, but the *total* force acting on *all* the conductors is practically constant for a given value of the slip. Consider any two rotor conductors which are a distance apart equal to ¼th the distance measured along the periphery of the rotor between successive north poles, for example, at a and b in Fig. 2. Then, assuming a sine-wave distribution of flux in the air gap, and calling B_m the maximum flux density, the flux density at a is $B_a = B_m \sin x$ and the flux density at b is $B_b = B_m \cos x$, where x is a function of the distance measured from some fixed point in the air gap. Then the total force on the two conductors at a and b is

$$\frac{l^2 sv B_m^2}{r} (\sin^2 x + \cos^2 x) = \frac{l^2 sv B_m^2}{r}$$

and is therefore constant, since B_m is a constant. Similarly for any other two conductors this same distance apart. Hence the total force on all the conductors is constant. On any practical machine the flux distribution is not an exact sine wave, and there is a slight pulsation in the total force, and therefore in the torque, but this pulsation is extremely small.

Magnetizing Current. — The currents set up in the secondary of an induction motor produce a rotating flux, which travels with the same speed with respect to the primary and in the same direction as the flux set up by the current in the primary, but the direction of this secondary flux at any point in the air gap is opposite to the direction of the primary flux. Hence the resultant flux when there is current in the secondary is equal to the difference of these two fluxes, and this difference remains practically constant irrespective of the secondary currents, just as the resultant flux in a transformer is practically independent of the secondary current. The primary current which would be necessary to produce this *resultant* flux is called the "magnetizing" current, and is very nearly equal to the current in the primary when the motor is running without load, in which case the current in the secondary is extremely small.

METHODS OF RATING. — The Standardization Rules of the A.I.E.E. up to 1914 recommended that the rating of an induction motor should be the load in horse-power which it will deliver continuously at the shaft with a maximum rise in temperature of any part not exceeding 50° C. by thermometer. In commercial practice three variations of this have been developed to suit different conditions, as noted below. See, however, the new ratings recommended in the proposed rules of 1914, given in the article on *Standardization Rules of the A.I.E.E.*

A-Rating. — For cases where there are no excessive overloads and where the load is fairly steady, it is customary to guarantee that the motor will operate continuously at its rated load with a maximum rise in temperature of 40° C., and that subsequently it will deliver a load 25 per cent greater than the rated for two hours with a maximum rise in temperature not exceeding 55° C.

B-Rating. — For cases where there are frequent overloads and for intermittent service, that is, a low load factor, it is customary to guarantee that a motor will deliver its rated load continuously with a rise in temperature not to exceed 35° C., and that it will deliver an overload of 50 per cent for two hours with a maximum rise in temperature not exceeding 55° C.

One-hour Rating. — Certain motors for special intermittent work, as for hoists, elevators, etc., are rated in accordance with the output they will give for one hour.

Starting and Break-down Torque. — In addition to the ability to carry its rated load without excessive heating and with reasonable constants, such as

efficiency, power factor and slip, it is advisable to make sure that the motor is able to start such loads as must be brought up to speed with the motor, as good starting ability in an induction motor involves certain complications and expenses. This subject is treated at length in the section below on *Methods of Starting*. Another important point is that the motor should be able to carry momentary overloads without "breaking down" as it is called, which means gradually decreasing in speed to a standstill when the load is excessive. To be sure of this qualification we must know the maximum output of the motor, which should be at least 50 per cent greater than the rated output.

VOLTAGE. — Motors may be wound for any voltage up to 13,000 but the great majority and all the small motors are wound for voltages of 110, 220 or 440 volts between lines.

FREQUENCY AND SPEED. — Induction motors may be built for any frequency. The higher frequencies are satisfactory in those cases where the load never exceeds normal conditions. Lower frequencies, such as 25, are more favorable where frequent overloads are met with or large starting torques are required.

The speed of the rotor of an induction motor at normal loads approaches within 5 to 10 per cent of the synchronous speed. The synchronous speed is fixed by the frequency of the system and the number of poles of the winding (*see above*) so that for a given frequency of the supply circuit there are only certain speeds available. Thus for 25 cycles we have

1500 for 2 poles;	500 for 6 poles;
750 for 4 poles;	375 for 8 poles, etc.

PHASE CONNECTIONS. — Two-phase or quarter-phase motors are usually wound with independent phase windings. Three-phase motors are connected in Y or Δ, depending upon the convenience of the designing engineer.

In a single-phase and two-phase motor the voltage and current per phase are the same as the voltage between lines and current in line; in a Y-connected three-phase motor the current per phase is equal to the line current, and the voltage per phase is equal to the line voltage divided by $\sqrt{3}$; in a Δ-connected three-phase motor the current per phase is equal to the line current divided by $\sqrt{3}$, and the voltage per phase is equal to the line voltage.

CURRENTS TAKEN BY MOTORS. — Let

P_0 = horse-power output;
I = current in each line;
ϵ = efficiency as a decimal fraction;
$\cos\phi$ = power factor as a decimal fraction;
E = voltage between lines (between one outside wire and the middle wire for three-wire two-phase line).

Then for

$$\text{Two phase: } I = \frac{373\,P_0}{\epsilon E\cos\phi};$$

$$\text{Three phase: } I = \frac{431\,P_0}{\epsilon E\cos\phi}.$$

Usual efficiencies and power factors at full rated load for polyphase motors are as follows:

POWER FACTORS AND EFFICIENCIES

Horse-power	25 cycles		60 cycles	
	Efficiency	Power factor	Efficiency	Power factor
1	0.79	0.78	0.78	0.78
5	0.85	0.88	0.82	0.88
20	0.88	0.91	0.84	0.91
50	0.90	0.92	0.87	0.92
100	0.905	0.925	0.89	0.92
200	0.91	0.925	0.905	0.92

DESIGN. — The methods of calculating two-phase and three-phase motors are practically the same. Most induction motors on single-phase circuits are made with polyphase windings, as the extra winding is necessary in starting.

The factors which must be considered in the design of an induction motor are the same as those considered for a synchronous generator with the addition of the power factor. It is desirable to have a high power factor but a high power factor requires a generous use of material, a small air gap, and a careful arrangement of windings. A high power factor at light load requires a small magnetizing current, and a high power factor at overloads requires a low value of leakage flux.

A small value of magnetizing current is obtained by using a small air gap and a large value of diameter per pole.

A low value of leakage flux is obtained by using a large value of diameter per pole and by subdividing the windings in a large number of slots.

Preliminary Choice of Main Dimensions. — In the discussion below the following symbols are employed:

E = volts per phase;
I = full-load current per phase;
D = diameter of armature, in inches;
L = length of armature, in inches;
p = number of poles;
f = frequency in cycles per second;
g = length of gap, in inches;
B = average flux density in gap, in lines per square inch;
ϕ = flux per pole, in maxwells;
S = turns in series per phase;
N = revolutions per minute;
α = quotient of no-load current divided by full-load current (ranges from 0.45 for a 1-horse-power motor to 0.25 for a 200-horse-power motor);
σ = ampere-conductors per inch of periphery. The values given in the article on *Generators, Alternating-Current,* also apply to the induction motor.
T = ratio of width of tooth at face to slot pitch (T varies from 0.6 to 0.7 for open slots and from 0.9 to 1.0 for overhung slots);
q = ampere-conductors per slot.

Diameter and Length of Rotor. — The first problem in the design of an induction motor is the estimation of the proper diameter of armature. There are three methods of determining the proper value of this very important dimension:

(a.) Reference to machines already built, which shows that the diameter varies directly with the number of poles and inversely as the frequency. Thus for customary values of diameter in inches divided by the number of poles:

for 25 cycles, $D/p = 5$ to 6 inches;
60 cycles, $D/p = 2.5$ to 3 inches.

(b.) The peripheral speed, being a direct function of the diameter and speed, determines the diameter. As the value of the peripheral speed may vary from 3000 to 10,000 feet per minute, depending on the mechanical construction, this function is not very definite. For lack of more definite figures, 5000 feet per minute may be considered a good average figure for peripheral speed.

(c.) On account of the effect of the magnetizing current on the power factor it is desirable to limit the value of this current to a certain percentage of full-load current (*for values see p. 1066*). To accomplish this there is a certain minimum limit to D and L which is expressed approximately in the equation

$$D^3L = \frac{28.7 \times 10^{12} fgk}{\alpha \sigma^2 N^2} \times (\text{Kv-a. rating}).$$

k = ratio of the actual excitation current to the excitation current for the air gap; $k = 1.2$ for 60 cycles and $k = 1.4$ for 40 cycles.

This gives a relation between D and L. Assuming that for best economy L is equal to the pole pitch $\left(L = \frac{\pi D}{p}\right)$ a value for D is obtained. This formula also indicates the effect on the general design which would result from radical changes in any of the quantities.

The formula given for the preliminary calculation of dimensions of an alternating-current generator (*see article on Generators, Alternating-Current*) may also be used if proper values for T and L be selected.

Air Gap (g) varies in length according to the diameter, usual values being given in the accompanying table.

Diameter, inches	Air gap, inch
0 to 12	0.02 to 0.03
12 to 26	0.03 to 0.04
26 to 50	0.04 to 0.06
50 to 100	0.06 to 0.10
100 up	Diameter/1000

Average Flux Density in Gap (B). — The usual value of the average flux density in the air gap for 25 cycle machines is 30,000 lines per sq. in. and for 60 cycle machines is 25,000 lines per sq. in.

Conductors per Slot (c). — The "effective" number of conductors per slot is the number of conductors *in series* per slot, i.e., if each phase is made up of two windings in parallel, the two conductors of the two parallel windings are counted as one conductor. Hence the effective conductors per slot are

$$c = \frac{2S \times (\text{Number of phases})}{(\text{Total number of slots})}.$$

The permissible number of ampere conductors per slot (q) depends upon the permissible current density per square inch of copper, usual values of which are

Size of motor	Voltage	Amperes per sq. in.
Small	Low	3000
Small	High	2000
Large	Low	2000
Large	High	1600
Large	Above 6000	1000

Usual values of q are

Size of motor h.p.	Value of q
Up to 5	Up to 250
5 to 50	250 to 350
50 to 100	350 to 450
100 to 200	450 to 600
Greater	Up to 800

Number and Dimensions of Slots. — The number of slots is

$$\frac{\pi D\sigma}{q}.$$

The conductors should be arranged to give slots having a depth about four times the width, and the width of slot should be about ½ to ⅔ of the pitch of slots at the gap. Machines of small diameter will have slots smaller with respect to the pitch and machines of large diameter will have slots occupying more than ⅔ of the pitch.

The slots of one member at least (usually the rotor) should be overhung, i.e., partly closed at the opening. It is better if both members have partly closed slots, as this reduces the magnetizing current and improves the power factor, but it entails a more expensive method of winding.

The dimensions of the slots are determined by the size of conductors and amount of insulation. The allowance to be made is about as follows:

Type of slot	Voltage	Coil sides per slot	Allowance for insulation	
			Vertical	Horizontal
			in.	in.
Straight	500	2	0.30	0.04
Straight	2200	2	0.45	0.15
Straight	6000	2	0.65	0.25
Overhung	500	4	0.40	0.15
Overhung	2200	4	0.50	0.20

The depth of the slot is found by adding to the total depth of cotton-insulated copper the vertical dimension given in the table.

Turns in Series per Phase (S) may be calculated from the formula

$$S = \frac{\pi D \sigma}{2 I \times (\text{number of phases})}$$

or from the formula

$$S = \frac{10^8 E}{4.44 \phi f k},$$

where ϕ is the flux per pole and is given by the formula

$$\phi = \frac{0.7 \pi B D L}{(\text{number of poles})}$$

and k is a constant depending on the wave form and winding distribution (*see next paragraph*). The value of S as obtained from these two formulas must check in the final design.

Flux per pole (ϕ). — The flux per pole is given by the formula

$$\phi = \frac{10^8 E}{4.44 f S k},$$

where k, called the distribution constant, has the values given in the following table, provided the flux distribution in the air gap is sinusoidal.

DISTRIBUTION CONSTANT k

Two phase					Three phase				
Slots per pole	Per cent winding pitch				Slots per pole	Per cent winding pitch			
	100	75	67	50		100	75	67	50
	k	k	k	k		k	k	k	k
2	1.00	....	....	0.71	3	1.00	....	0.87	....
4	0.93	0.85	....	0.66	6	0.97	....	0.84	0.69
6	0.91	....	0.79	0.64	9	0.96	....	0.83	0.68
8	0.905	0.84	....	0.64	12	0.96	0.89	0.83	0.68
12	0.90	0.836	0.78	0.63	18	0.958	0.885	0.83	0.68
Many	0.90	0.833	0.78	0.63	Many	0.958	0.885	0.83	0.68

Choice of Phase Connection. — The decision whether a motor shall be Δ or Y connected depends on such minor details of design as the convenience of arranging the conductors in the slots. For instance, if for 110 volts and Δ connection a desirable flux value and number of conductors would require 7 conductors per slot, a Y connection having 64 volts per phase and 4 conductors per slot could be substituted and would give a practical winding. 64 volts per phase, Y connected, gives 110 volts between lines.

Magnetic Circuit. — A tentative layout of the magnetic circuit is next made in the same manner as described in the article on *Generators, Alternating-*

Current. The values there given for usual values of the flux density also apply to the *maximum* instantaneous flux density in the magnetic circuit of an induction motor. The magnetic circuit must have such dimensions that the exciting current will not be too large, and the slots must be of sufficient size to accommodate conductors of necessary size (*see above*).

Maximum Efficiency and Power Factor. — Since induction motors frequently operate on an intermittent or variable load, it is desirable that the efficiency and power factor be high at fractional loads. It is therefore quite usual to design the motors so that the maximum efficiency comes at ¾ load and maximum power factor at less than full load. This is accomplished with regard to efficiency by making the core-loss and friction small (the core-loss is made small by using low flux densities) and with regard to power factor by making the magnetizing current small by using a small air gap.

PREDETERMINATION OF PERFORMANCE OF AN INDUCTION MOTOR FROM ITS DIMENSIONS. — From the above calculations a preliminary drawing of the motor to scale may be laid out. The next step is to calculate its performance, i.e., predetermine what will be the efficiency, the power factor, and the temperature rise in the various parts. Examples of specific design and tested performance are given below.

Calculation of Exciting Current. — The first step is the calculation of the exciting current. This current is practically constant at all loads, and is equal to the no load current, i.e., the current taken by the motor when it is running light. The exciting current has two components, the magnetizing current, which leads the induced voltage by 90° and a component in phase with the induced voltage, which supplies the core-loss and friction. The magnetizing current is much the larger component, and for preliminary calculations may be taken equal to the exciting current.

Magnetizing Current. — The magnetizing current is calculated by determining the flux density in each part of the magnetic circuit and the ampere turns required for each part. This is most easily done by means of the following tabulation:

MAGNETIC DENSITIES AND M.M.F.'S

(For dimensions refer to Fig. 3.)

Part	Flux per pole	Area	B_{max}	A.T. per inch	Length path	Total A.T.
Stator core.....	$\phi/2$	$h_1 \times l$	$1 \times B_{avg}$	....	$\frac{\pi D_1}{2 \times \text{poles}}$	....
Stator teeth....	ϕ	$r_1 \times l \times \frac{\text{slots}}{\text{poles}}$	$1.57 \times B_{avg}$	....	n_1	....
Air gap.........	ϕ	See below	$1.57 \times B_{avg}$	....	g	....
Rotor teeth....	ϕ	$r_2 \times l \times \frac{\text{slots}}{\text{poles}}$	$1.57 \times B_{avg}$	....	n_2	....
Rotor core.....	$\phi/2$	$h_2 \times l$	$1 \times B_{avg}$	....	$\frac{\pi D_3}{2 \times \text{poles}}$	....
					Total.....	......

Stator Core or Yoke. — The flux divides in the core, one-half going each way. The maximum and average* densities are practically the same and equal to ($\phi/2$ divided by the radial depth h_1 times the effective length l). The material is laminated steel of high permeability. The ampere turns per inch length of path are obtained from a magnetization curve of the steel, see *Magnetic Properties of Iron.* The length of path is indeterminate but closely approximates one-half of the pole pitch measured on the circle of diameter D_1. By multiplying the ampere turns per inch obtained from the magnetization curve by the length of path, the ampere turns required to send the flux through this path are obtained.

Stator teeth. — The average density in the teeth is first obtained. The effective area of one tooth is the area one-third the distance from the face to the root of the tooth as shown at r_1. This gives the average magnetizing force rather than the average density. The total cross section of the path in the teeth is therefore $r_1 \times l \times$ (the number of teeth per pole). Due to the peaked or sinusoidal space distribution of the flux the maximum density in the teeth is 1.57 times the average density. The ampere turns required depend upon the maximum density. From the proper curve determine the ampere turns per inch necessary to establish this density, and by multiplying this quantity by the distance n_1 in inches, the ampere turns for the stator teeth are found.

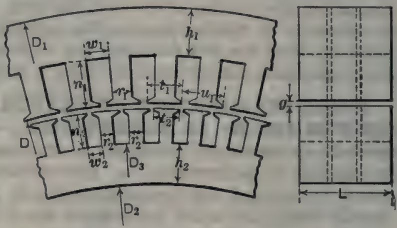

Fig. 3. Dimensions of Magnetic Circuit

Air Gap. — The flux is not uniformly distributed in the air gap, especially if the openings of the slots are fairly large. The flux passes through each tooth and spreads out from the iron when it leaves the teeth to cross the gap. The peripheries of the stator and rotor present unequal and dissimilar surfaces passing each other. The lesser of the two is the one that must be considered. The effective area of the air gap is

$$\frac{(t+g)\, l \times (\text{total number of slots})}{0.9\, p}$$

and the maximum density = $\dfrac{1.57 \times \phi}{\text{gap area}}$

(the $t+g$ allows for the spreading of the flux in one direction and $l/0.9$ allows for the spreading in the other direction).

The ampere turns per inch are $0.313\, B_{max}$. The length of gap being known, the total ampere turns for the gap are found.

Rotor Teeth. — The calculation of the ampere turns for the rotor teeth follows the same method as the calculation for the stator teeth.

Rotor Core. — The same as for the stator core.

The effective value of the magnetizing current is then

$$I_m = \frac{(\text{Total ampere turns}) \times p}{2\sqrt{2}\, S} \quad \text{for three phases}$$

$$\text{and } I_m = \frac{(\text{Total ampere turns}) \times p}{\sqrt{2}\, S} \quad \text{for two phases}$$

* By average is here meant the average over the cross section of the maximum instantaneous value of the flux density, which, of course, alternates between fixed positive and negative values.

Resistance per Phase of Primary Winding. — The resistance to direct current, or ohmic resistance, is given by the formula

$$r_1 = \frac{0.0093\, S \times (\text{mean length of turn})}{12{,}000\, an},$$

where

S = turns in series per phase,

a = cross section of one conductor in sq. in.,

n = no. of conductors or circuits in parallel,

and 0.0093 is the resistance at 60° C. of a conductor 1000 feet long and 1 square inch cross section. The mean length of turn is approximately (see Fig. 3)

$$2\,L + 10\frac{D}{p} \times (\text{pitch as a fraction}).$$

Due to the eddy currents set up in the primary conductors by the total flux and to eddy currents set up in the core by the leakage flux, the "effective" resistance of the primary winding is about 15 per cent greater than the value calculated by the above formula.

Resistance per Phase of Secondary Winding. — In a wound rotor the resistance per phase is found in the same manner as the resistance per phase of the stator winding. The secondary resistance reduced to primary is then equal to $\left(\frac{\text{number prim. turns per phase}}{\text{number sec. turns per phase}}\right)^2 \times$ (actual effective sec. resistance), when the secondary has the same number of phases as the primary.

The squirrel-cage rotor, in which each slot contains one bar and all bars are short-circuited at each end by a ring presents a more complicated problem. It may be solved as follows:

$$\text{Amp.-cond. per sec. slot} = \frac{\pi D \sigma}{(\text{number of sec. slots})}.$$

σ = amp.-cond. per inch for full-load current.

$$\text{Current density in sec. bars} = \frac{(\text{amp.-cond. per slot})}{(\text{area of one bar})}.$$

Watts lost in bars = 0.775×10^{-6} (vol. of bars) (amp. per sq. in.)^2.

Area of bars per pole = (area of one bar) (number of bars per pole).

$$\text{Current density in rings} = \frac{(\text{current density in bars})\,(\text{area of bars per pole})}{4\,(\text{area of ring})}.$$

Watts lost in rings = 0.775×10^{-6} (vol. of both rings) (density in rings)^2.

$$\left.\begin{array}{c}\text{Resistance of sec. in terms of primary for a}\\ \text{three-phase motor}\end{array}\right\} = \frac{\text{total loss}}{3\,(\text{full load prim. current})^2}.$$

Leakage Reactance.— When the motor is loaded the currents in the secondary set up a counter m.m.f. which causes part of the flux to pass along the air gap instead of into the secondary core. This flux does not interlink both members and is therefore a leakage or useless flux. It is proportional to the load currents and to the permeance of this path. As the greater portion of the path is in the air and is of high reluctance, therefore that part of the path in the iron may be neglected. The flux in both the primary and secondary must be calculated.

Referring to the diagram Fig. 4, the permeance of these two paths are

$$P_1 = \left[\frac{o_1}{3\,w_1} + \frac{2\,r_1}{w_1 + q_1} + \frac{p_1}{q_1} + \frac{t_1 - q_2}{6\,g} + 0.37\,\frac{l_{s1}}{L}\log_{10}\frac{1.5\,l_{s1}}{V_{s1}}\right] L,$$

$$P_2 = \left[\frac{o_2}{3\,w_2} + \frac{2\,r_2}{w_2 + q_2} + \frac{p_2}{q_2} + \frac{t_2 - q_1}{6\,g} + 0.37\,\frac{l_{s2}}{L}\log_{10}\frac{1.5\,l_{s2}}{V_{s2}}\right] L$$

where

g = length of gap in inches,
l_3 = length of end connection at one end in inches,
V_3 = perimeter of a *band* of end connection per pole per phase, in inches,
L = total length of iron in inches.

In these formulas the subscript 1 refers to the primary and subscript 2 to the secondary. The first three terms give the slot reactance, the fourth term gives the "zigzag" or "tooth-tip" reactance, and the last term the reactance of the end connections.

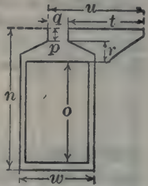

Fig. 4. Slot Dimensions

The primary reactance in ohms per phase is

$$x_1 = 3.2\, P_1 k c_1^2 s_1 \times 2\,\pi f 10^{-8}.$$

The secondary reactance in ohms per phase in terms of the primary turns is

$$x_2 = 3.2\, P_2 k c_2^2 s_2 \times 2\pi f 10^{-8}\left(\frac{c_1 s_1}{c_2 s_2}\right)^2,$$

where

c_1 and c_2 are the conductors in series per slot (see p. 1058),
s_1 and s_2 are the slots per phase,
f = primary frequency,
k = winding distribution constant (*see above*).

Losses in Induction Motor. — The losses in an induction motor are

Core-loss;
Friction, bearing and windage;
Primary copper loss;
Secondary copper loss.

The first two of these are approximately constant for all loads and the last two vary as the square of the current per phase.

Core-loss. — The distribution of the magnetic flux in an induction motor is quite irregular both in the core and in the teeth. The losses are therefore greater and their calculation more involved than in machines having uniform density in each part. In the primary the frequency of the passage of the secondary teeth introduces pulsations which increase the losses. In the secondary the frequency, being proportional to the slip, is so low that the core-loss is negligible.

In order to avoid too lengthy and complex calculations use is made of empirical constants, by which the easily calculated losses are multiplied to derive the practical loss. The total loss consists of hysteresis and eddy loss in the primary core and primary teeth. The loss in watts per cubic inch for one cycle per second at any magnetic density is found by the curves given in the article on *Magnetic Properties of Iron.*

The losses are then calculated as follows:

Hysteresis Loss:

In primary core $= k_h C_1 f V_c$ watts,
In primary teeth $= k_h C_2 f V_t$ watts.

Eddy Loss:

In primary core $= k_e C_3 f^2 V_c$ watts,
In primary teeth $= k_e C_4 f^2 V_t$ watts,

where V = volume in cu. in., C = respective loss per cu. in. from curves, f = frequency, k_h = empirical constant, 1 to 1.5, k_e = empirical constant, 3 to 4.

The higher values of k_h and k_e are to be used where open slots are used and where the frequency of the passage of the secondary teeth past one primary tooth is high. The core-loss is the sum of all the above losses.

Friction and Windage Loss varies greatly with the style of motor and form of structure, bearings, etc. The loss is made up of bearing friction and wind friction. The latter may be large purposely, as the motor may be designed with fan blades in order to circulate the air for the purpose of keeping the motor cool by ventilation. While each manufacturer has a formula which will calculate the friction loss correctly for machines built according to a particular plan, no general formula can be assumed. The nearest approach to an estimate is obtained from the percentage given in the table below.

Primary and Secondary Copper Losses. — The calculation of the effective resistances of the primary and secondary windings is given above. The primary or secondary copper loss is equal to the product of the number of phases, the effective primary or secondary resistance per phase, and the square of the current per phase. The total copper loss is the sum of the primary and secondary copper losses.

Power Factor. — The power factor ($\cos\phi$) of an induction motor for a given current input (I) may be calculated roughly from the formula

$$\cos\phi = \cos(\alpha + \beta),$$

where $$\alpha = \sin^{-1}\left(\frac{I_m}{I}\right) \text{ and } \beta = \sin^{-1}\left(\frac{XI}{E}\right),$$

and I_m = magnetizing current per phase,
I = total current per phase,
$X = (x_1 + x_2)$ = total reactance per phase, the secondary reactance x_2 being reduced to the primary turns,
E = voltage per phase.

For a more exact formula for power factor and for the usual values of the power factor at rated load, see *below.*

Efficiency. — The efficiency (as a fraction) for a given current input is

$$\epsilon = 1 - \frac{F + C + m(r_1 + r_2)I^2}{mEI\cos\phi},$$

where F = friction loss, in watts; C = total core-loss, in watts; m = number of phases; r_1 = effective primary resistance per phase; r_2 = effective secondary resistance per phase reduced to primary; E = voltage per phase; I = current per phase; $\cos\phi$ = power factor.

The corresponding horse-power output is then

$$P_0 = \frac{\epsilon mEI\cos\phi}{746} \text{ horse-power.}$$

Usual values of the efficiency are given above in section on *Currents taken by Motors.* Usual values of the component losses for 60-cycle motors are given in the following table.

In 25-cycle motors the exciting current is usually greater than in 60-cycle motors and the IX drop less. Thus the power factor will be lower at light load and higher at overloads than in 60-cycle motors.

Rating, h.p.	$\frac{Im}{I}$	$\frac{XI}{E}$	$\frac{\text{Friction loss}}{\text{Input}}$	$\frac{\text{Core-loss}}{\text{Input}}$	$\frac{r_1 I}{E}$	$\frac{r_2 I}{E}$
1	0.45	0.20	0.06	0.05	0.05	0.05
5	0.35	0.14	0.035	0.04	0.04	0.04
20	0.30	0.13	0.025	0.035	0.035	0.035
50	0.27	0.11	0.015	0.03	0.025	0.03
100	0.26	0.11	0.015	0.03	0.025	0.03
200	0.25	0.11	0.015	0.03	0.025	0.03

The efficiency of 60-cycle motors naturally tends to be higher than of 25-cycle motors. This quality is usually sacrificed by economizing in material and making the 60-cycle motors lighter and cheaper, but of about the same efficiency as the 25-cycle motors.

Slip and Speed. — The slip for any given current is approximately

$$s = \frac{r_2 I}{E},$$

where the symbols are defined in the preceding paragraph.

The synchronous speed is

$$N = \frac{120 f}{p} \text{ rev. per min.},$$

where f is the frequency of the supply and p the number of poles. The actual speed of the motor is then

$$N_1 = (1 - s)\, N.$$

Values of Other Characteristics. — The energy component of the exciting current is

$$I_e = \frac{(\text{Total core-loss}) + (\text{Total friction loss})}{mE},$$

where m is the number of phases and E the volts per phase.

The total exciting current per phase is then

$$I_{00} = \sqrt{I_e^2 + I_m^2},$$

where I_m is the magnetizing current per phase.

The impedance per phase at standstill ("short-circuit" impedance) is

$$Z = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2},$$

where r_1 and r_2 are the effective values of the primary and secondary resistances per phase, the latter being reduced to primary turns and x_1 and x_2 the primary and secondary reactances per phase, the latter being reduced to primary turns.

The current per phase at standstill ("short-circuit" current) is

$$I_s = \frac{E}{Z}.$$

The starting torque in pounds at 1-foot radius is

$$T_s = \frac{7.06\, m r_2 E^2}{N Z^2}.$$

where m, r_2, E, and Z are as defined above, and N is the synchronous speed in revolutions per minute.

The slip at maximum output is

$$s_m = \frac{r_2}{r_1 + Z}.$$

The maximum output in watts is

$$P_m = \frac{0.5\, mE^2}{1.3\,(r_1 + r_2) + Z} \text{ watts.}$$

This should be from 1.5 to 3 times the rating of the motor. A 25-cycle motor usually has a greater maximum output or overload capacity than a 60-cycle motor.

Heating. — Since in most motors the primary member is subject to both a core-loss and a copper loss and is stationary, the heating of this member is very important. As the secondary contains only a copper loss and is usually revolving, its rise in temperature is usually much less than that of the primary.

The problem therefore consists of analyzing the flow of heat and drop in thermal potential in the primary as the energy lost in the windings flows to the iron of the core through the insulation in the slots and to the air around the projecting end connections. The core itself is maintained at a temperature above the air by the core-loss. It is therefore necessary to calculate first the rise in temperature of the iron of the core caused by the core-loss and that portion of the copper loss which is conducted to the core through the slot insulation. It is then necessary to calculate the rise in temperature of the copper above the iron and above the air around the end connections.

Since at times in every motor, when the core-loss is great compared with the copper loss, the iron may be hotter than the copper, it is necessary to distinguish between that portion of the power (P_1) which passes between copper and iron (or vice versa), and that portion (P_2) which passes from copper direct to the air. The analysis must take account of the fact that sometimes the heat due to core-loss passes to the copper and is dissipated by the end connections.

The method given here is based on the more exact and elaborate treatment given by Arnold in Vol. V of his treatise *Wechselstromtechnik*, but is simplified by assuming that the copper conductors have such a high heat conductivity that they have the same temperature throughout their length. The result is a value for the average temperature of the copper such as would be found by a resistance measurement. For a method of determining the maximum temperature in any spot the reader is referred to Arnold's treatise.

Temperature Rise of Iron of Core (T_i). —The heat which is dissipated by the core surface consists normally of the core-loss and that part of the primary copper loss occurring in the portion of the winding embedded in the slots. It is

$$H = \text{core-loss} + mI^2r_1\left(\frac{2L}{t}\right),$$

where m = number of phases,
I = primary current per phase,
r_1 = effective resistance of primary per phase,
L = length of armature core between heads,
t = mean length of a primary turn.

The surface consists of the outer cylindrical surface of the core, the two annular surfaces at the ends, and the surfaces in the air ducts, see Fig. 3. Since the

surface in the air ducts is not as effective as the others, only half the air-duct surface is used.

The area is $A_i = \pi D_1 L + \frac{\pi}{4}(D_1^2 - D^2)(2 + d)$,

where D_1 = outside diameter of stator in inches,
D = inside diameter of stator in inches,
d = number of air ducts.

The rise in temperature of the iron of the core will then be $T_i = \frac{kH}{A_i}$, in degrees Cent., where k ranges from 30 for narrow machines to 50 for long machines.

Temperature Rise of Copper (T_c). — A part of the heat due to the copper loss flows from the copper to the iron through the slot insulation, due to the small difference in temperature between the copper and the iron, $T_c - T_i$, where T_c is the rise in temperature of the copper above the air. The power in watts so dissipated is

$$P_1 = A\,(T_c - T_i),$$

where

$$A = \frac{U_s L S_1}{d_1 k_1}$$

and U_s = perimeter of slot insulation,
S_1 = total number of slots,
d_1 = thickness of slot insulation,
k_1 = a constant, 200 to 250 (say, 210).

Sometimes T_i is greater than T_c and P_1 becomes negative, which means that some of the energy of the core-loss flows to the windings and is dissipated by the end windings.

Another portion, P_2, of the copper loss flows through the insulation of the end connections to the air. Here there are paths, insulation and air, in series, across which there is a drop in temperature of T_c°C. The flow of energy to the air is affected by the movement of the air, which, in turn, is a function of the peripheral speed V of the rotor. The value of P_2 is

$$P_2 = BT_c,$$

where

$$B = \frac{U_c l_c Z_1}{d_2 k_2 + k_3}$$

and U_c = perimeter of end connections, in inches,
l_c = length of end connections, in inches,
Z_1 = total number of coils,
d_2 = thickness of insulation on same, in inches,
k_2 = a constant, 400 to 500 (say, 430),
k_3 = resistance to flow of heat from insulation to air and is given by the formula,

$$k_3 = \frac{170}{1 + 0.015\,V};$$

where V = peripheral speed of rotor in feet per second.

The total copper loss is $P = P_1 + P_2$, whence

$$T_c = \frac{P + AT_i}{A + B}.$$

This is approximately the average difference in temperature between copper and air as it would be determined by a measurement of resistance.

TESTING OF INDUCTION MOTORS. — (*See also Standardization Rules of A.I.E.E.*) Induction motors may be given either an "input-output" test at load under working conditions, from which the efficiency and power factor may be determined, or the motor may be given a no-load excitation and no-load short-circuit test, from which all the characteristics may be calculated. The latter method requires very little power and is preferable in the case of large motors, where it would be expensive to supply power and inconvenient to dissipate the energy.

Excitation Test. — In this test the machine is operated without load at constant frequency with the voltage varied through a range from 30 per cent above rated value down to as low a voltage as will cause the machine to rotate. The current in each phase is noted and by means of two wattmeters the power required is noted. This test is sometimes also made with the machine operating single phase for the purpose of checking the core-loss. The curves are plotted with voltage as abscissas and amperes and watts as ordinates. At very low voltages the core-loss is negligible and therefore the watts input may be taken as friction loss. Around normal voltage the real core-loss may be determined by subtracting from the input the small copper and the friction loss.

Short-circuit Impedance Test. — In this test the starting resistance, if there is any, is short-circuited and the armature is blocked to prevent rotation. A low voltage is applied to the primary until the ammeter in the primary circuit shows a current of from 1 to 1.5 times the full-load value. Two wattmeters are used to read the power input. As the leakage flux, and hence the reactance, varies with the relative position of the rotor and stator teeth, it is customary to take readings with the rotor in several positions. Sometimes the rotor is allowed to revolve very slowly at, say, two to three revolutions per minute. From this test the impedance (Z) of the machine is obtained by dividing the volts per phase by the current per phase; the combined effective resistance (R) of primary and secondary is found by dividing the watts input per phase by the square of the current per phase. This latter is the effective value of the resistance, since the watts input includes all losses due to eddy currents. From these values the total reactance (primary and secondary) of the motor is $X = \sqrt{Z^2 - R^2}$. With the results of the two preceding tests and the measured resistances of the two members, the characteristics of the motor may be calculated either by the Steinmetz Method or the Circle Diagram. (*See below under Performance, Calculation of.*)

Stationary Torque Test (Fig. 5). — This test may be made at the same time the impedance test is made and consists in the measurement of the torque of the motor by means of a brake arm and spring balance. When the current of the machine has been adjusted to a suitable value the brake arm (to which a known weight has been attached to overcome bearing friction) and the spring balance are allowed to move downward through a small arc, during which the spring balance will register a pull (P_1) equal to torque (T) plus the weight (W) minus the friction (F), that is, $P_1 = T + W - F$. The spring balance and brake arm are then raised against the torque through the same small arc and the spring balance will then register a pull (P_2) equal to the torque plus the weight plus the friction, that is, $P_2 = T + W + F$. From these two readings the friction can be eliminated

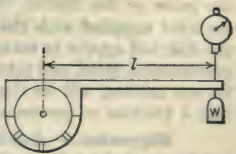

Fig. 5. Stationary Torque Test

and the actual torque of the armature on the shaft is determined. This should be done at two or three equally spaced positions of the rotor, as the torque varies considerably with the position. The corrected torque of the machine is found by solving the two equations for T and multiplying T by the lever arm of the brake in feet; this gives the torque in pounds at 1-foot radius.

Load Test. — To make this test on a small machine a prony brake is required, while for a large machine a direct-current generator may be conveniently employed as a load. The induction motor and generator are direct-connected if possible, otherwise belted together. All the constants of the generator are determined, so that its losses under any condition may be calculated. The motor is then allowed to drive the generator, the rated voltage being impressed upon the motor, and the generator is loaded by means of a rheostat or a water-box so that any load may be obtained. The voltage across each phase of the motor, the current in each phase, the total watts input (by two meters), speed, and, if possible, slip (by a slipmeter, *see below*) are all read. Care must be taken that correct voltage and frequency are supplied to the motor. The load on the motor is increased step by step to the maximum output point, which is easily known, since when it is reached a decrease in speed is not accompanied by any increase in output of the motor. The motor is still stable at the maximum output condition, since maximum torque occurs at a lower speed than maximum output.

After the load run the direct-current generator is run as a motor with the same field strength as before, to determine the mechanical losses, first driving the induction motor at the proper speed, and then running alone at the same speed or speeds at which it ran in the load test. Knowing the no-load friction of the induction motor as previously determined in the *Excitation Test*, the increase in friction due to the belt and load is determined and half of it charged against the motor. Let

EI = the output of the direct-current generator in watts,
I^2R = the hot resistance loss in the direct-current armature,
CT = the counter torque or stray power losses in the direct-current generator plus one-half belt loss,
P_0 = watts input to induction motor.

$$\text{Then efficiency of the motor} = \frac{EI + I^2R + CT}{P_0}.$$

From these tests, curves may be plotted for the speed, efficiency, power factor current and torque of the induction motor for any horse-power output.

Use of Stroboscope to Measure Slip. — One type of slipmeter consists of a disk which is attached to the motor shaft and on which alternate sectors of black and white are shown, preferably as many black sectors as there are poles on the motor. If this rotating disk is illuminated by means of an arc light supplied with the frequency impressed on the primary of the motor, the disk will appear to rotate at a speed proportional to the difference between synchronous speed and actual running speed. The number of these revolutions for one minute, will be the slip in turns per minute, which may be translated into a fraction or percentage of synchronous speed.

Slipmeter. — Another type of slipmeter has two disks of insulating material mounted on the same frame, one carrying a wiping finger which makes contact with a button on the other disk. One of the disks is connected to a small synchronous motor which is driven from the same power source as the induction motor under test; the other disk is arranged to be readily attached to the shaft of the induction motor. The circuit of a miniature tungsten lamp is closed when the wiping finger and button are in contact. The number of flashes

of the lamp per minute gives the slip in revolutions of the induction motor behind the synchronous speed.

Heat Runs. — On machines of moderate size heat runs are advisable. but on large size machines heat runs are expensive. It is not always necessary to make a heat run in order to know whether the motor is properly designed, as the losses can be accurately calculated from the excitation and short-circuit tests, and the machine may be run without load but with losses equivalent in value to the losses at full load. The usual heat run consists in operating the machine at a certain output for a period from three to six hours, depending on the size, measuring the resistance before and after the run and taking temperatures by thermometer on the following parts after the run: primary winding, the iron surfaces in the air gap, secondary winding, bearings, frame.

A heat run which will indicate whether there is anything radically wrong with a motor consists in operating it for an hour or two with a voltage 15 per cent above normal, but without load.

Insulation Tests. — High-potential tests are made on the primary in the manner described under *Testing* in the article on *Generators, Alternating-Current.* The value of the high potential for the primary is chosen in accordance with its rated voltage from the *Standardization Rules of the A.I.E.E.* (q.v.). The potential applied to the secondary, however, has no relation to the rated voltage of the machine; as the working potential in the secondary is low, 1000 to 1500 volts is the usual range of testing potential for the secondary.

PERFORMANCE, CALCULATION OF. — The performance of an induction motor at any load may be determined directly from the load tests described above, or the performance at any load may be calculated from the excitation and short-circuit tests by either of the two methods given below. These methods are also applicable to the calculation of performance from the values of the constants calculated from the dimensions of the machine. The first method is given in detail by Steinmetz in his *Elements of Electrical Engineering,* and the Heyland Circle Diagram is given by McAllister in his *Alternating Current Motors.* The former is recommended where accuracy is desired and the latter (graphical) for the student desiring a general understanding of the relations.

Steinmetz Method. — This method is based upon the equivalent circuit of a transformer, as given in the article on *Transformers.* Let

E = impressed voltage per phase,
m = number of phases,
r_1, r_2, x_1, x_2 = resistance and reactance of primary and secondary respectively, per phase and reduced to terms of primary,
s = slip as a decimal fraction (assumed),
g = primary no-load conductance $= \dfrac{\text{core-loss}}{mE^2}$,
b = primary no-load susceptance $= \dfrac{I_m}{E}$.

Assume a slip s and calculate

$$a_1 = \frac{sr_2}{s^2x_2^2 + r_2^2},$$
$$a_2 = \frac{s^2x_2}{s^2x_2^2 + r_2^2},$$
$$g_1 = g + a_1,$$
$$b_1 = b + a_2,$$
$$c_1 = 1 + r_1g_1 + x_1b_1,$$
$$c_2 = g_1x_1 - b_1r_1.$$

Then

Counter e.m.f. per phase	$e = \dfrac{E}{\sqrt{c_1^2 + c_2^2}}$,
Current per phase	$I = e\sqrt{g_1^2 + b_1^2}$,
Total volt-ampere input to motor	$P' = mEI$,
Watts input to motor	$P = me^2\,(g_1c_1 - b_1c_2)$,
Watts output of armature	$P_0' = me^2a_1\,(1 - s)$,
Watts output of pulley	$P_0 = P_0' -$ (friction in watts),
Efficiency	$\epsilon = \dfrac{P_0}{P}$,
Power factor	$\cos\phi = \dfrac{P}{P'}$.

Circle Diagram or Graphical Method (Figs. 6 and 7). — The method of the circle diagram is based on the fact that the electrical reactions in an induction motor (or transformer) may be represented without any great error by the reactions in two parallel inductive circuits as in Fig. 6. I_e and I_m represent the two components of the exciting current, which is assumed constant. R_m and X_m represent the total resistance and reactance (both assumed constant) of the motor and are in series with a variable resistance R_L representing the load.

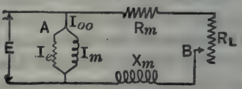

Fig. 6. Equivalent Circuits

The vector relations of the currents are as shown in Fig. 7, where

$OE =$ the impressed voltage per phase,
$OM =$ the current I_{oo} in A,
$MP =$ the variable current in B (not drawn).

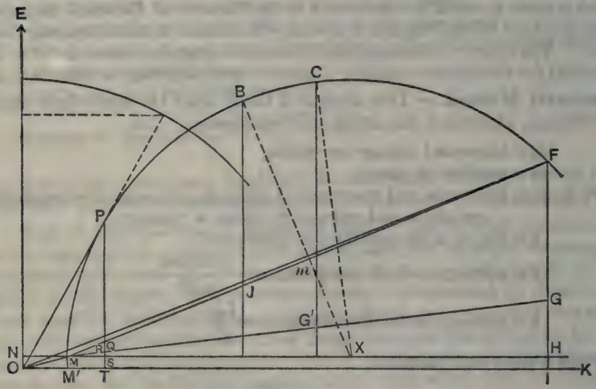

Fig. 7. Circle Diagram

Since the current represented by MP is equal to $\dfrac{E}{\sqrt{(R_m + R_L)^2 + X_m^2}}$, as R_L varies the point P will describe a circle through M, P and F, where $MF = \dfrac{E}{\sqrt{R_m^2 + X_m^2}}$. The total or resultant current will then be OP.

Heyland Circle Diagram (Fig. 7). — The circle diagram upon which the following discussion is based is a modification of Heyland's transformer, or induction motor diagram.

Let
OE = impressed voltage per phase.
OK be at 90° to OE.
OM = exciting current per phase, drawn in phase and magnitude.
OF = short circuit current per phase, drawn in proper phase and magnitude.
IF = energy component of OF.
Join M and F; then MF is the secondary short-circuit current in terms of primary circuit.
Bisect MF at m.
Draw mX perpendicular to MF at middle point, intersecting NH at X.
With X as center and either XM or XF as radius, draw the semicircle MCF. This is the locus of the primary current.
Since $OE \times IF$ = watts input at standstill, draw HG such that $OE \times HG$ = primary I^2R at standstill.
Then $OE \times GF$ = secondary I^2R at standstill.
Draw GM. The vertical distance between GM and NH at any point gives a current which if multiplied by OE gives the power loss in primary.
Choose any point P on circle; then OP = current per phase.
Then cos $\measuredangle POE = \cos\phi$ = power factor.
MP (not drawn) = secondary current reduced to primary turns.
$PT \times OE$ = power input to primary, in watts.
$TS \times OE$ = no-load loss, in watts.
$QT \times OE$ = total motor loss.
$RT \times OE$ = total primary loss in watts. These
$RS \times OE$ = primary copper loss, in watts. are
$PR \times OE$ = secondary input, in watts.
$QR \times OE$ = secondary copper loss, in watts. all
$QP \times OE$ = motor output, in watts. per
$OM \div OP$ = per cent magnetizing current.
$M'T \div OP$ = per cent leakage reactance voltage. phase
Draw XC perpendicular to MG and CG' perpendicular to NH.
Then $CG' \times OE$ = maximum torque in synchronous watts.
Draw XB perpendicular to MF and BJ perpendicular to NH.
Then $BJ \times OE$ = maximum output in watts.
$QR \div RP$ = per cent slip (in case of induction motor).
$OP \times OE$ = volt amperes input.

Torque in pounds at 1-foot radius T_s = synchronous watts $\times\, m \times \dfrac{7.06}{N}$, where m = number of phases and N = synchronous speed in rev. per min.

Characteristic Curves. — The observed or calculated values (by either of the above methods) of the slip, power factor, efficiency, apparent efficiency, current per phase and torque may be plotted as ordinates with horse-power output at the shaft as abscissas. An example of the characteristic curves thus plotted is shown in Fig. 8. Usual values of the various quantities for different sizes of motors at rated load are given above. The "power factor" shows the relation between the true power input of the machine and the apparent power, called the "volt-amperes." A poor power factor does not involve any greater registration of the watt-hour meter or cost of energy to operate the motor, but it does involve poor regulation of voltage in the system as a whole and larger capacity of wiring, transformers, etc. The "apparent efficiency" is equal to the product of the power factor and efficiency, or is equal

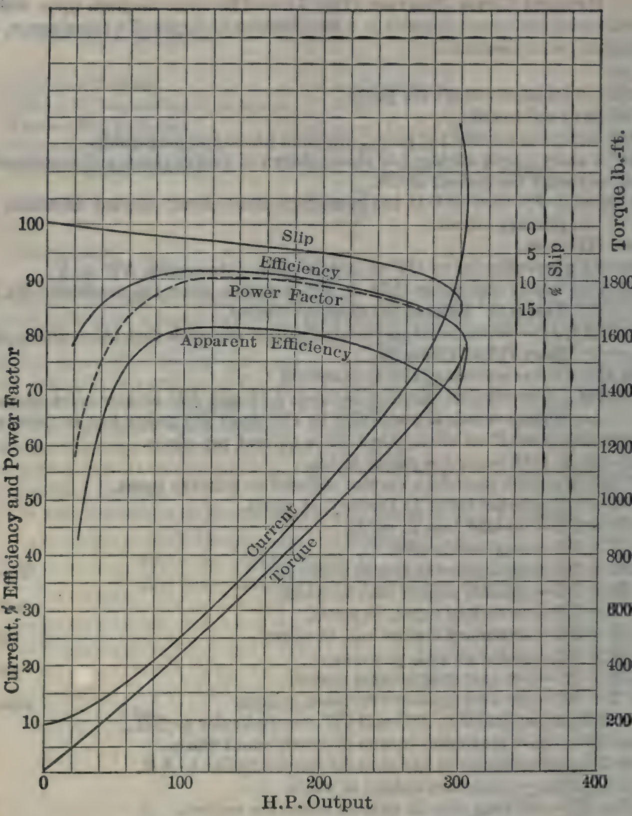

Fig. 8. Characteristic Curves of an Induction Motor

to the ratio of output in watts divided by input in volt-amperes. Its value determines the actual capacity of the lines and transformers supplying the motor.

EXAMPLES OF DESIGN AND PERFORMANCE. — In the accompanying tables are given the essential data for both mechanical and electrical features of six three-phase induction motors. The list of items will be found useful as a guide in collecting data on various machines. Performance data are deduced from tests.

INDUCTION MOTORS

Mechanical Data. (Dimensions in Inches)

Type	1	2	3	4	5	6
	3-ph.	3-ph.	3-ph.	3-ph.	3-ph.	3-ph.
Number of poles......	4	4	4	6	6	8
Rating in h. p........	5	15	30	5	10	20
Revs. per min........	750	750	750	1200	1200	900
Prim. volts bet. lines..	440	220	220	220	440	220
Prim. connections.....	Y	Y	Δ	Δ	Δ	Δ
Frequency............	25	25	25	60	60	60
Stator						
Outer diam. punchings	17	21	28	17	18.25	26
Inner diam. punchings	12.06	15.07	19.07	12.094	11	19.07
Total length of iron...	6	7.5	8	5	3.75	6
No. of ducts..........	0	0	0	0	0	0
Width of each duct...	....	....	....	....	....	....
Total no. slots........	60	72	72	54	72	96
Depth of slot..........	1.25	1.25	1.44	1.56	1.5	1.25
Width of slot..........	0.32	0.34	0.39	0.50	0.3	0.33
Width of slot at face...	0.32	0.34	0.39	0.25	0.3	0.33
Wires per slot.........	36	16	16	64	30	16
Size of wire.........	No. 14 B. & S.	No. 10 B. & S.	No. 8 B. & S.	No. 14 B. & S.	No. 15 B. & S.	No. 13 B. W. G
Wires in multiple.....	1	2	2	2	1	2
Turns in series per ph..	360	96	96	288	360	128
Per cent coil pitch.....	100	72	67	100	100	75
Rotor						
Outer diam. punchings	12	15	19	12	10.95	19
Inner diam. punchings	8	11	12	8.5	6	14
Total no. slots........	37	47	67	72	127	71
Depth of slots.........	0.56	0.47	0.47	0.94	0.5	0.47
Width of slots.........	0.56	0.56	0.56	0.34	0.12	0.56
Width of slots at face.	0.063	0.063	0.063	0.19	0.02	0.063
Wires per slot.........	1	1	1	4	1	1
Size of wire or bar....	0.5×0.45	0.35×0.5	0.35×0.5	0.34×0.11	0.35×0.09	0.35×0.5
No. in multiple.......	1	1	1	4	1	1
Cross section each ring, sq. in.	1.3	0.94	2	....	0.198	1.2
Resistance rel. to copper.................	2	2	2	....	1	2
Air gap on one side...	0.03	0.035	0.035	0.047	0.025	0.035

INDUCTION MOTORS

Electrical Data. (All quantities per phase)

Type	1	2	3	4	5	6
Volts per phase, E	254	127	220	220	440	220
Current per ph. at rating	6.6	38	42	8.1	7.3	31.2
Flux per pole, megalines	0.636	1.2	2.07	0.29	0.48	0.645
Magnetizing current I_m	2.1	9	13	2.64	2.4	12.3
Friction, watts	80	175	530	150	295	390
Core-loss, watts	150	390	1240	180	210	760
Primary res. at 60° C., ohms	2.86	0.193	0.13	1.15	2.78	0.213
Second. res. at 60° C., ohms	1.76	0.18	0.17	1.11	3.5	0.21
Short-circuit current	36	212	362	29.4	26	205
Y = (Sh. cir. cur.) $\div I_m$	17	23.6	28	11.2	10.8	16.6
Reactance per ph.	5.52	0.55	0.54	8	5.05	1.10
Eff. at rating	0.83	0.853	0.883	0.83	0.86	0.875
Power factor at rating	0.905	0.91	0.917	0.845	0.90	0.83
$I_m \div I$	0.32	0.24	0.31	0.33	0.33	0.39
Fr. $\div P_0$	0.018	0.013	0.02	0.033	0.034	0.023
Core-loss $\div P_0$	0.033	0.029	0.049	0.04	0.024	0.044
$IR_1 \div E$	0.074	0.058	0.025	0.042	0.046	0.03
$IR_2 \div E$	0.046	0.054	0.032	0.041	0.058	0.03
$IX \div E$	0.143	0.165	0.103	0.29	0.084	0.156
Slip, per cent	0.047	0.042	0.032	0.03	0.038	0.03

METHODS OF STARTING. — In order to start an induction motor of any size without injurious heating either a resistance must be connected in the secondary circuit or the voltage impressed on the primary must be reduced. The two general methods of starting induction motors are known as "Potential Control" and "Rheostatic Control." The same methods are used for speed control (*see below*).

Starting by "Potential Control" Method (Fig. 9). — This method consists of reducing and regulating the voltage impressed on the primary, usually by means of a starting compensator or auto-transformer which provides one or two fractional voltages. In order to make use of this method the secondary must be of higher resistance than with other methods of starting. For this reason and for the reason that there is no need of making any change in the windings, a squirrel-cage rotor winding is customarily used with this method of starting. This winding is made up of one bar per slot, and all bars are connected at both ends to rings. To start the motor the primary is connected to taps on the compensator which give a voltage of from ½ to ⅔ the rated voltage if it is a small motor, and ⅓ to ½ if it is a large motor. A small motor may be brought up to full speed on this voltage but a large motor may require an intermediate step. The connections are shown in Fig. 9. It is customary to adjust the motor resistance and starting voltages to give the relations in the accompanying table.

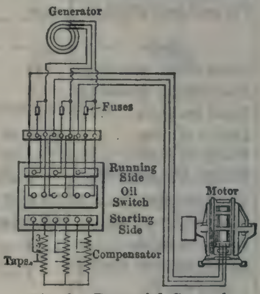

Fig. 9. Potential Control

Voltage on Motor, per cent	Current in line, per cent	Starting torque, per cent
33	75	22
50	175	50
66	300	85
100	700	200

Boucherot Winding. — This type of winding for the rotors of induction motors is a device designed to give good torque at starting by means of high secondary resistance and inherently to reduce this resistance as the motor accelerates, so that at full speed the secondary resistance will be low, the slip small and the efficiency high.

The device consists of two independent windings on the secondary (usually squirrel cage), one, a high resistance winding of brass or German silver bars is placed near the periphery of the rotor in the usual form of slots. The other winding is of low resistance (large copper bars) and is placed in slots like tunnels below the bottom of the surface slots. A narrow slit or shaft extends from the bottom of the outer slot to the top of the inner slot.

The outer winding acts like any high resistance winding, giving good starting conditions. The inner winding is so highly inductive that at stand-still, when the secondary has full frequency, it has a very high reactance, carries very little current and contributes but little to the action of the motor. At speeds near synchronism the secondary frequency is very low (of the order of one cycle per second), the reactance is low, and the inner low-resistance winding carries most of the current; consequently under running conditions the motor has practically the same characteristics as one having a single low-resistance secondary.

Starting by "Rheostatic Control" Method (Fig. 10). — Better apparent torque efficiency, that is to say, more torque for a given current, is obtained by inserting in the secondary circuit a much greater resistance than can be left permanently in circuit. This is accomplished by having a special starting resistance connected in series with the armature winding and a switch for short-circuiting the resistance either step by step or as a whole, as the motor speeds up. There are two practical methods of doing this:

The first is intended to be used only when the torque required at starting is not very great, in which case the starting resistance may be small and located inside the armature spider. The switch lever is so arranged that the resistance can be short-circuited in steps while the armature is revolving. This obviates the need of collector rings and external connections.

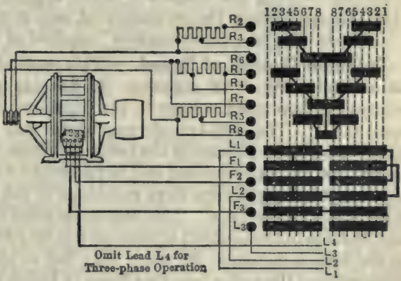

Fig. 10. Rheostatic Control

The second method consists in bringing the three terminals of the secondary winding to collector rings. From brushes bearing on these rings conductors lead to external resistances with steps or taps so that the resistance may be

short-circuited gradually. This scheme is used where a large starting torque (greater than full load torque) is required. It may be used also for speed control as shown in Fig. 12

The proper value of resistance per phase in the secondary is determined by the relation

Torque in pounds at 1 foot =

$$\frac{E^2 r_x}{Z^2} \times \frac{7.06\, m}{\text{r.p.m.}} = \frac{7.06\, m r_x E^2}{N Z^2},$$

where

E = primary voltage per phase,

r_x = total secondary resistance per phase in terms of primary,

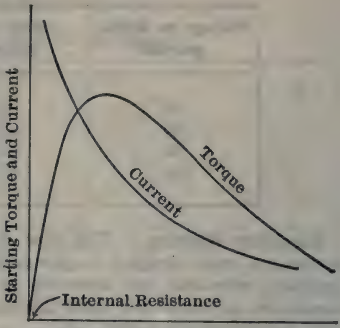

Fig. 11. Starting Resistance, Torque and Current

$Z = \sqrt{(r_1 + r_x)^2 + X^2}$, where r_1 is the resistance per phase of primary and X the total reactance per phase of both primary and secondary,

m = number of phases,

N = synchronous speed in revolutions per minute.

The relation between starting torque and total resistance of the secondary is shown by the curve in Fig. 11.

SPEED CONTROL OF INDUCTION MOTORS. — (*See also Hoists, Electric.*) The speed of an induction motor may be controlled in five ways:

(a) By varying the potential applied to the primary of motor having a suitable permanent resistance in the secondary.
(b) By varying the resistance in the secondary circuit.
(c) By changing the connections of the primary winding in a manner to change the number of poles.
(d) By varying the frequency of the applied voltage.
(e) By connecting the secondary of one motor to the primary of another, called the "concatenation" method of control.

Potential Control of Speed. — This method is an elaboration of the potential-control method of starting. A suitable resistance or auto-transformer reduces the impressed voltage to the fractional value desired (two-thirds is the usual ratio). The auto-transformer wastes less power than a resistance but is more expensive. Because of the reduced voltage the flux and torque-per-ampere are reduced and more current will be required for a given torque. The induction motor should have a very large resistance in the secondary, which is preferably of the squirrel-cage type. This resistance gives the motor a speed characteristic such that its full-load speed is some 10 per cent less than that of a normal motor. As the load is increased the speed may fall to about 30 per cent of the no-load value without the motor breaking down or falling out of step, which in the normal motor usually takes place at about 80 per cent of the full-load speed. This motor is not stable in speed as each slight change in the load will cause a change in the speed more or less inversely proportional to the load. The advantages of this method of control are the simplicity of the connections and devices. The disadvantages are the greatly increased heating in the motor itself with the decreased speed. Thus the motor must be larger than if other speed-control methods are used. The table given below shows the efficiencies obtained.

Rheostatic Control of Speed. — With this method (an elaboration of the method of the same name for starting) the secondary or rotor must have a definite winding (which costs more than the squirrel-cage winding used in the preceding method) with slip rings and brushes to lead out the current. The friction and resistance losses due to these brushes decrease the efficiency of the motor a slight amount. The action of this method is based on the principle that in an induction motor the drop in speed for any given torque is proportional to the resistance of the secondary circuit.

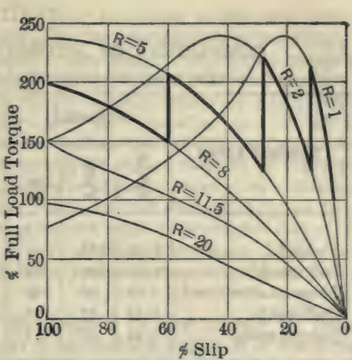

Fig. 12. Torque-speed Curves

Assuming a motor which has at full speed a net resistance of the secondary proper of 1 ohm, the speed-torque curve would be as shown for $R = 1$ in Fig. 12. This motor would have a slip of 5 per cent for full-load torque and of 25 per cent for maximum torque, a torque at starting of 80 per cent of full-load torque and a very large starting current. If by some means this resistance is doubled the speed-torque curve would be as shown for $R = 2$, which shows a slip of 10 per cent for full-load torque and of 50 per cent for maximum torque. For $R = 5$, $R = 8$, etc., there would be other speed curves. By starting with a resistance of 8 ohms a torque of about 200 per cent of full-load torque would be obtained at starting with about twice full-load current. By allowing the motor to follow this curve until the torque has dropped to the full-load value, the motor would reach 60 per cent of synchronous speed, or 40 per cent slip. Then by reducing the resistance in steps the torque and speed would follow the heavy zigzag line until the motor reached full speed.

With this method of control the torque per ampere remains practically constant as in a shunt motor regulated by resistance in the armature circuit. The efficiency varies directly with the speed as shown in the table below.

The advantages of this method are the higher efficiency and particularly the smaller losses in the motor itself. The losses are in the rheostat. The disadvantages are the necessity of collector rings, brushes, controllers, etc. The motor is not stable at any fractional speed but the speed will change with every change of load.

Change of Poles to Control Speed. — By a proper design of the windings an induction motor may be made to operate with either 4 or 8 poles, 6 or 12 poles, or even 4 or 6 poles or 6 or 8 poles. This is accomplished by a throw-over switch to which taps from the windings are brought. In this arrangement the pitch of the primary winding must be made a compromise between the proper value for the different numbers of poles, and therefore the constants of the motor are not as good as those of a standard motor. It is also necessary to use a squirrel-cage armature, since this is suitable for any number of poles without change of connections. This type of motor operates advantageously only at the two speeds corresponding to the two arrangements of poles and is stable at each of these speeds. If a wider range is desired the potential-control scheme may be combined with it. The speeds and efficiencies with this scheme of control are given in the following table. At half-speed the efficiency is almost double that obtained with the other methods, but the losses in the motor are greater than with the rheostatic control.

COMPARISON OF METHODS OF SPEED CONTROL

(For constant torque equal to torque at full load)

Nominal speed	Potential			Rheostatic			Change of poles		
	Speed	Volts	Eff.	Speed	Volts	Eff.	Speed	Volts	Eff.
No load	1.00	100	..	1.00	100	..	1.00	100	..
Full speed	0.89	100	81	0.96	100	86	0.96	100	86
Three-quarter speed	0.67	66	59	0.72	100	65	....	...	..
Half speed	0.45	57	37	0.48	100	43	0.48	100	74
Quarter speed	0.22	56	17	0.24	100	22	....	...	..

Change of Frequency Method of Control. — The speed of an induction motor at any load varies directly with the frequency of the supply circuit. If two circuits from two alternators of different frequencies are provided the motors may be connected to one circuit for one speed and to the other circuit for another speed. This method of speed control requires as many separate generators and circuits as the number of speeds desired. It is, therefore, costly and not widely used.

Concatenation Control or Cascade Control. — If two motors have definite windings in both the primary and secondary and are rigidly connected to a common shaft, they may be operated at a fractional speed corresponding to a number of poles equal to the sum of the number of poles on the two motors. With the concatenation control the primary (stator) of motor No. 1 is connected to the supply circuit, the secondary (rotor) of No. 1 is connected to the primary (rotor) of No. 2, and the secondary (stator) of No. 2 is connected to a resistance which is eventually short-circuited. When the two motors have the same number of poles, which is the usual commercial condition, motor No. 1 transforms half the power into mechanical power at the shaft at *half speed* and the remainder into electrical power at half frequency. Motor No. 2 receives this electrical power and transforms it into mechanical power at this same speed. If the number of poles is different, the speed in r.p.m. of the combination is

$$N = \frac{120f}{p_1 + p_2}.$$

These simple relations are exact only on the assumption of no losses and the secondary of No. 2 short-circuited.

The objections to this method of control are that the first motor has to carry the magnetizing current of both motors, thus having a low power factor. The first motor must receive the power for both motors; therefore, it must be the larger of the two and specially designed or there is an inefficient use of material in the second motor. If the two motors are alike (the usual commercial condition) the torque of the two motors in concatenation is not as great as that of the two motors in parallel.

The efficiency at fractional speed is, however, better than with the rheostatic method of control. Only two "free-running" speeds are available with two motors having the same number of poles. This scheme has been adopted frequently for the speed regulation of induction motors used on electric locomotives in foreign countries. In such applications it is frequently the custom to allow one motor to be idle at the higher speed.

INSTALLATION AND ERECTION. — Induction motors are usually built, even in large sizes, as a unit including the bearings, which are usually a part of the end frame of the motor proper. They may be either direct-connected or belted to their load, but the latter method is more general, since each induction motor can only be built for a certain definite speed corresponding to a certain number of poles. The smaller motors need no foundation, and in fact are frequently attached to the wall or to the ceiling, the bearings and end shield being made in such a manner that they may be turned through 90 degrees or 180 degrees so that the oil rings will operate properly under these conditions. In most motors reasonable ventilation, free from dust and dirt, must be available. For certain applications such as cement mills or mines, the motors are built totally inclosed and may then be even submerged in water. In this case, of course, a motor of a given rating is larger and more expensive than one of the open type.

OPERATION. — Small motors are designed to start merely by closing the main switch. With larger motors if the starting switch is at the proper position, potential may be applied to the motor and the starting resistance gradually cut out by moving the switch.

Induction motors are very sensitive to variation in the impressed voltage. A decrease in impressed voltage from 100 to 80 would cause the maximum output and maximum starting torque to decrease from 100 to 64 and roughly would cause a proportional increase in the heating for a given load.

Unequal voltages in the different phases also cause a decrease in the maximum output and an increase in the heating for any given output. An unbalancing of 25 per cent in voltage would double the heating effect at full load, i.e., would give the same heating as an overload of 50 per cent.

Care in Starting. — Before starting the motor for the first time it is desirable to make sure that the starting device is in operating condition and in the proper position, in order that the motor should not become injuriously heated. Attention should be paid to having the wiring so proportioned as to carry the starting current without an excessive drop in voltage (*see above*).

Faults. — Some of the more common faults occurring in induction motors, together with their signs and remedies, are the following.

Secondary Open-circuited. — The motor will not start and will not take a current greater than the exciting current. The cause is probably due to the starting resistance not being connected in.

One Phase of Secondary Open-circuited. — The motor has a tendency to remain at half synchronous speed although the current is apparently normal. If the armature is blocked it will be found that the current in the three phases will be unbalanced.

One Phase of Primary Open. — The motor will not start and the current will be unbalanced.

One Phase of Primary Reversed. — The currents in the primary will be very much unbalanced when the motor is running and the starting torque will be very slight.

Short-circuited Coil in Primary. — There will be humming when potential is applied to the motor and excessive local heating around the short-circuited coil.

Vibration. — Vibration due to mechanical unbalancing is chiefly noticeable at high speeds and particularly in high-speed machines. If the vibration is due to magnetic unbalancing it is probably caused by inequality in the air gap at different portions of the circumference and with different positions of the armature. This may be detected by measuring the air gap with taper wedges

at various points around the circumference, first with the armature in one position and then in several other positions.

SPECIFICATIONS FOR INDUCTION MOTORS. — The following memoranda are intended to assist in writing specifications. See also article on *Specifications*.

Principal Characteristics and Conditions of Service. — Use to which motor is to be put; kind of load and method of drive. Voltage and number of phases. Rating, horse-power. Frequency and speed.

Style and Description; Details of Construction. — Whether to be open, semi-inclosed or inclosed. Requirements regarding pulley or length of shaft. Whether rails are required. Method of starting; compensator, external resistance or internal resistance; whether motor is to be run at speeds other than full speed. Whether the starting devices are to be supplied.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*). Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load. Starting torque with full-load current, pound-feet. High-potential tests of insulation. Requirements regarding effect of moisture upon insulation.

WEIGHT AND COST (Pre-war prices)*. — The weight and cost of an induction motor vary with the type of its armature winding and the character of its mechanical frame and housing, as well as with the speed, frequency and voltage.

The curves in Figs. 13 and 14 give an idea of the weights and costs of a line of 25-cycle and 60-cycle motors respectively, having squirrel-cage rotors (the

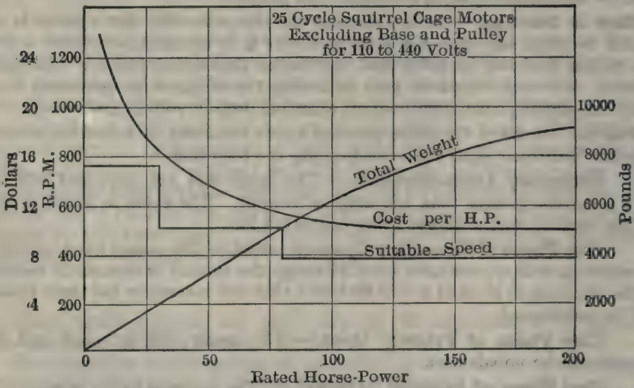

Fig. 13. Weight, Cost* and Suitable Speed of 25-cycle Induction Motor

simplest and least expensive type) and with the simplest form of mechanical frame. They therefore represent minimum values.

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* For 1922 prices add 50 per cent.

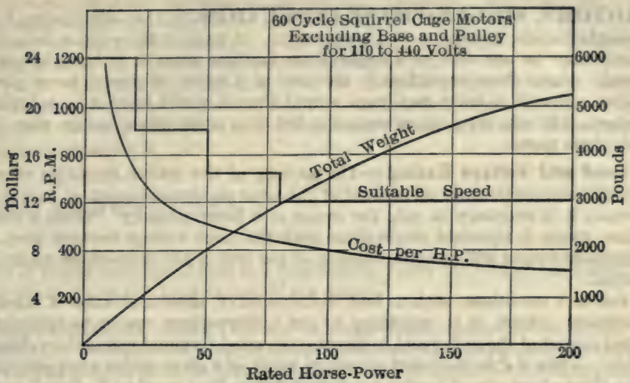

Fig. 14. Weight, Cost * and Suitable Speed of 60-cycle Induction Motor.

A-C. Phenomena; Thompson, S. P., *Dynamo-Electric Machinery,* Vol. II; Smith, L. W., *Effect of Voltage and Frequency Variations,* Elec. J., Mar., 1917; Smith, H. L., *Electrical Design of Induction Motors,* Electrician, Aug. 6, 1915; Crosby, F. B., *Speed Control of Induction Motors,* G. E. Rev., June, 1914.

* For 1922 prices add 50 per cent.

MOTORS, SINGLE-PHASE INDUCTION. — (*See also Motors, Polyphase Induction; Motors, A-C. Commutator*). A two- or three-phase induction motor may be operated as a single-phase machine after it is brought up to speed. Under these conditions it operates at a lower efficiency, lower power factor and with a lower maximum output than it would have as a polyphase motor. The slip for a given output is less in a single-phase motor than in a polyphase motor.

Load and Voltage Rating. — On account of the poorer operating characteristics and particularly on account of the lower maximum output or maximum torque, it is necessary to rate the motor at a lower capacity. When a three-phase motor is operated single-phase with the same voltage between lines, its maximum output will be approximately 40 per cent of the three-phase maximum output.

For best conditions, such as best distribution of losses and ratio of rated to maximum output, it is customary to use a three-phase motor, to reduce the rated output of the motor and to increase the rated terminal voltage in a definite ratio. Thus, if P be the rated output, in watts, of a given motor when operating on a three-phase circuit having a voltage between lines equal to E, then it would be advisable to operate the motor single-phase with a voltage between lines equal to $1.3\ E$, and to assign the motor a rating of from $0.67\ P$ to $0.75\ P$.

This will result in a distribution of losses in the motor quite similar to that which obtains during three-phase operation. The maximum output as a single-phase motor (at the higher voltage) will be about 67 per cent of the maximum output of the three-phase motor. The efficiency and power factor will be reasonable.

In case the voltage of the single-phase supply circuit must be the same, the winding of the motor is changed to give about 75 per cent as many turns in series per phase as for normal three-phase operation.

Exciting Current and Power Factor. — At a given voltage between terminals the volt-amperes input at no load for excitation are practically the same for single-phase and polyphase operation. Thus the no-load current of a single-phase motor is considerably greater than when operating polyphase. The increase in the applied voltage or the decrease in the number of turns makes a still greater increase in the magnetizing current. Thus the power factor of a single-phase motor is very poor at light loads and not very good at rated load.

CALCULATION OF PERFORMANCES. — To predetermine the characteristics of a single-phase motor it is calculated as a three-phase motor for the same voltage between lines as the single-phase circuit. The magnetizing current, core-loss, resistance per phase, and reactance per phase are all calculated as usual. The motor primary may be connected either delta or Y, but for purposes of calculation it is desirable to pro-rate the constants of a Y-connected motor on the assumption that it is delta connected.

To pro-rate the resistance and reactance per phase of primary and secondary, multiply the values for a Y-connection by 3 to obtain the values for the equivalent delta constants. The voltage per phase is presumed to increase in the ratio 1 to 1.73, while the voltage between lines remains the same.

The magnetizing current and the core-loss current are found by dividing the 3-phase values by 1.73.

The single-phase magnetizing current is found by dividing the volt-amperes excitation for three-phase conditions by the single-phase voltage. The core-loss in watts remains practically the same single-phase and the energy component of the single-phase exciting current is equal to core-loss watts divided by rated voltage.

The resistance and reactance of the primary of a single-phase motor are taken the same as the equivalent delta values per phase.

The resistance and reactance of the secondary of a single-phase motor are taken as ⅛ of the three-phase equivalent delta values.

These values are then substituted in the formulæ of the Steinmetz method (*see Motors, Polyphase Induction*) and the characteristics calculated for several assumed values of slip. The only difference in calculation is that the torque in synchronous watts is

$$T = e^2 a_1 (1 - s)$$

and the output of the armature is

$$P = e^2 a_1 (1 - s)^2.$$

It is of course understood that in a single-phase motor the output calculated for one phase is also the total output of the motor.

METHODS OF STARTING. — A single-phase motor has no torque at standstill. It must be started by some device such as a phase-splitting device. It may be started in either direction and as soon as it starts to rotate a slight torque develops which increases with the speed. When this torque has reached a great enough value to overcome the friction and inertia the special starting apparatus may be disconnected and the motor will continue to accelerate to its proper speed.

Starting of Small Motors.— Small motors may be started without auxiliary electric circuits by giving the armature a spin by hand, after which (if there is no load) they will accelerate. Certain small motors are designed with a loose pulley which is clutched at a predetermined speed of the armature by means of a centrifugal governor. These motors are provided with "shading coils" or a small external phase-splitting device, to give them just enough starting torque to overcome their own friction.

Starting of Large Motors. — A phase-splitting device is generally used; either a reactor and resistor or a condenser and resistor may be employed. Commutator devices are also used.

Use of Reactance Coil and Resistor. — The connections employed are shown in Fig. 1. This device consists of a resistance and a reactance connected so as to advance the phase of the e.m.f. impressed on one circuit of the motor and retard the phase of the e.m.f. on another circuit, while the line voltage is impressed on the third winding. This may be accomplished either by connecting the resistance and reactance across the line terminals and in multiple with the motor windings as shown in Fig. 1 A, or by connecting the resistance and reactance in series with the respective windings as shown in Fig. 1 B.

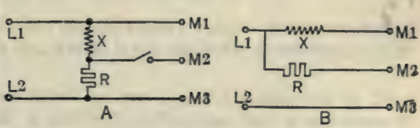

Fig. 1.

In these figures L_1 and L_2 represent line terminals, R is the resistance coil, X the reactance coil and M_1, M_2 and M_3 the three motor terminals. The former method gives a greater starting torque but takes a greater current in proportion to the torque. The latter method is more efficient. The motor must have sufficient resistance in its secondary circuit to give good starting characteristics as a three-phase motor.

The principle of this device is that the voltages between the outside terminals and the middle point are out of phase with each other and form a somewhat

flattened vector triangle similar to that of an unbalanced three-phase system. The ratio of the starting torque obtained with such a device to the normal starting torque with balanced three-phase voltages is the same as the ratio of the altitude of the triangle of vector voltages to the altitude of the equilateral triangle. Thus in Fig. 2 the ratio of altitudes is $30/95 = 0.316$; thus the single-phase starting torque would be 31.6 per cent of the three-phase starting torque.

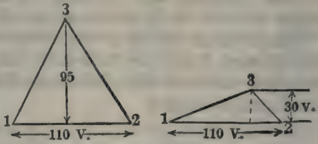

Fig. 2.

Use of Condenser and Resistor. — If a condenser of the proper capacity is substituted for the resistance the starting torque will be increased and the efficiency improved, but this is much more expensive and frequently involves the addition of a transformer across the condenser.

Use of Commutator. — Another method of starting single-phase motors involves the use of a commutator which permits the motor to start as a repulsion motor (which has good starting qualities; see *Motors, A-C. Commutator*), and after the motor has reached a considerable speed the brushes are removed from the commutator and a short-circuited squirrel-cage winding comes into play.

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MOTORS, SYNCHRONOUS.—(*See also Alternating Currents; Generators, Alternating Current; Motors, Industrial Applications of; Motors, Polyphase Induction.*) Any alternator will operate as a motor. If two synchronous alternators are connected in parallel to bus-bars supplying a load, and the driving power be removed from one prime mover, the alternator connected to this prime mover will continue to run at the same speed as before, taking power from the other alternator and driving its own prime mover or other apparatus coupled to it; i.e., this alternator acts as motor. The speed of such a motor depends solely upon the speed of the generator or generators supplying electric energy to it; it is therefore said to run in "synchronism" with the source of supply and is called a synchronous motor. The synchronous speed of a synchronous motor having p poles, when supplied with a current of a frequency of f cycles per second, is

$$N = \frac{120f}{p} \qquad \text{rev. per min.}$$

If the load on such a motor increases the speed will not decrease, unless the load reaches such an excessive value that the maximum output or "pull-out torque" is reached, when the motor will drop out of step and come to rest, while the current taken will increase to short-circuit value and the torque decrease to a negligible value.

Differences between an Alternator and Synchronous Motor. — The difference in construction between an alternator and a synchronous motor is that the latter has placed in the face of the field poles, a squirrel-cage winding, which is intended to give good starting torque and to prevent hunting while running. A synchronous motor usually operates better with a higher value of armature reaction than that of a well-designed generator of the same kilowatt rating. This increase in armature reaction is usually obtained in practice by operating the machine as a motor at lower voltage than that for which it would be operated as a generator. Thus a standard 2300-volt generator will operate very satisfactorily as a motor at 2080 volts, and as these are the natural values of the generated and delivered voltages, this characteristic of the synchronous motor fits in very well with customary distribution practice. Thus a standard generator may have a squirrel-cage winding added to its poles and become a good synchronous motor.

Field Excitation. — Synchronous motors always require direct current for field excitation and if a suitable d-c. source is not available an exciter must be provided.

Number of Phases. — Synchronous motors may be single, two or three phase. The single-phase motor is not self-starting and has a considerably lower efficiency than the polyphase motor. It is also more liable to hunt (*see below*) and be unstable, and is therefore far less desirable than a polyphase motor. The two-phase and three-phase motors are very similar in all their characteristics. There is a slight economy in the three-phase over the two-phase motor as there is in the three-phase over the two-phase generator (q.v.).

Terminal Voltage. — Since synchronous motors are usually built with a revolving field and a stationary armature, it is possible to insulate the armature winding for voltages as high as 13,000 and thus obviate the need of transformers in many cases.

Advantages and Disadvantages of Synchronous Motors. — The advantages of synchronous motors as compared to induction motors are: higher efficiency, higher power factor, power factor may be controlled, constant speed,

high voltage, lower cost. The disadvantages are: need of an exciter, will not start under load, possibility of hunting.

Relations of Voltage and Current. — The relations between line voltage and phase voltage are the same as in a-c. generators (q.v.). The current in each line of a three-phase motor is

$$I = \frac{746\,P}{\sqrt{3}\,\epsilon\,E\cos\theta},$$

where

P = horse-power output,
E = voltage between lines,
ϵ = efficiency at load assumed,
$\cos\theta$ = power factor (may be unity).

Usual values for efficiency are about the same as for a-c. generators (q.v.).

APPLICATIONS. — In order to transform from alternating to direct current, or from one kind of alternating current to another differing in frequency, potential or phase relation, motor-generator sets, consisting of a synchronous motor direct connected to one or more generators, are often employed. By this means the potential of the secondary or distribution circuit is independent of the variation in potential of the primary circuit supplying power to the motor. In certain cases it is desired to take power from a 25-cycle circuit and supply power at 60 cycles for lighting purposes. Here a synchronous motor-generator set would be used. Such a set is frequently called a "frequency changer" (see *Frequency Changers*). In some applications of electric drive by induction motors one synchronous motor is installed for the purpose of making it take leading current in order to neutralize the lagging current taken by the induction motors. This effect is produced by over-exciting the fields of the synchronous motor. Such a synchronous motor is called a "rotary phase modifier" or "rotary condenser." It may incidentally be used to drive any machinery that does not require a large starting torque.

GENERAL PRINCIPLES. — In any synchronous generator or motor when a current flows in the armature there is a loss of voltage proportional to IZ_0, where I is the current and Z_0 is a hypothetical quantity called the synchronous impedance, which includes the effect of the resistance, the leakage reactance and the demagnetizing effect of the armature current. This quantity is obtained by the synchronous impedance test (*see below*) and is expressed in complex quantities as $Z_0 = r + jx_0$ and in algebra $Z_0^2 = r^2 + x_0^2$ where r is the effective resistance per phase and x_0 is the synchronous reactance per phase. IZ_0 is therefore the drop in voltage per phase in the armature, and this voltage and the current differ in phase by an angle θ, where $\tan\theta = x_0/r$ and $\cos\theta = r/Z_0$. If a synchronous motor is running and generating a counter e.m.f. e and is connected to bus-bars of voltage E, the current flowing in the armature will be proportional to the vector difference between E and e and inversely proportional to Z_0, all taken per phase.

Vector Relations for Motor Action. — In Fig. 1 let E represent the bus-bar or line voltage (per phase) impressed on the terminals of a synchronous motor. Let e represent the counter e.m.f. of the motor and in this case assume $e < E$ and directly opposed to E. Then the difference between E and e will set up a current in the armature of the motor. IZ_0 will be the voltage and the current will lag behind IZ_0 by an angle θ where $\cos\theta = \frac{r}{Z_0}$. Thus the motor takes a current I lagging behind the impressed voltage E.

Fig. 2 shows the relations when $e > E$, then IZ_0 will be reversed in phase as indicated. I will always lag behind IZ_0 by the angle θ and will be found drawn upward. I lags behind e and IZ_0 but I leads the impressed e.m.f. E by an angle $(180° - \theta)$. Thus when the field excitation is increased so that e tends to

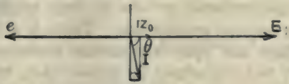

Fig. 1.

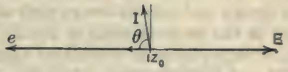

Fig. 2.

become greater than E, the machine takes a current leading with respect to the impressed e.m.f. As in a synchronous generator the field excitation required for a given terminal voltage depends upon the phase relation of the external circuit or the load, so conversely in a synchronous motor the phase relation of the current into the armature at a given terminal voltage depends upon the field excitation and the load.

Fig. 3 shows the relations when e is more than 180 degrees behind E, that is, e is behind the position it had in the preceding examples by an angle α. The vector resultant of E and e will be IZ_0 as shown, leading E. The current I will lag behind IZ_0 by the same angle θ, but is now almost in phase with E and lagging only slightly. Thus power ($EI \cos \phi$) is being sent into the ma-

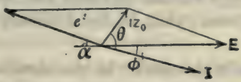

Fig. 3.

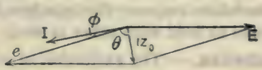

Fig. 4.

chine and it acts as a motor, transforming electrical power into mechanical power. When the machine is running as in Figs. 1 and 2 the power is very small. If, however, a mechanical load is applied, the armature will drop back in position by a slight amount. This causes e to drop back in phase and the machine immediately draws power from the line.

If, as in Fig. 4, power is applied to make the armature move forward and cause e to advance in the opposite direction, the resultant IZ_0 is thrown downward and I, lagging behind IZ_0 by θ, is thrown around almost in phase with e. The machine then becomes a generator, transforming the mechanical power applied into electrical power $eI \cos \phi$.

Synchronous Position. — In the discussion of the action of a synchronous motor it is convenient to employ the term "synchronous position," by which is meant the position which any definite point on the revolving member occupies at the same period of each cycle of time. It is only necessary for the machine to change in synchronous position by a very slight angle α to cause a large energy current to flow. If α should become 90 degrees, theoretically the power would become a maximum, and any increase in α means that θ becomes greater than 45 degrees, the power decreasing and the machine falling out of step. When α becomes 180 degrees a total e.m.f. equal to the sum of e and E is short-circuited by Z_0 and the current is enormous and the power factor low.

Maximum Torque. — If the speed changes for a short instant of time sufficiently to allow a point on the armature to drop back in synchronous position one-half the pitch of one pole (90 electrical degrees), the motor torque will increase from zero to the maximum available in the motor. Thus, for any torque less than the maximum the armature (or revolving field) need only change in speed sufficiently to drop back some distance less than one-half the pitch of a

pole. If the load demands a torque greater than this maximum the armature will drop back more than 90 degrees and will fall out of step and come to rest. In most synchronous motors the maximum torque is about 6 times normal torque.

DESIGN AND CALCULATIONS. — (See also *Generators, Alternating Current.*) The design and calculation of synchronous motors is very much like the design and calculation of alternating-current generators. There is a difference in the proportioning of certain details and there are certain features that are of importance in generators that are not important in motors (i.e., regulation) and the converse is also true.

Armature Reaction. — The armature reaction of a synchronous motor is expressed as $\frac{1.5\sqrt{2}SIk_2k_3}{p}$ for a three-phase machine and $\frac{\sqrt{2}SIk_2k_3}{p}$ for a two phase machine, where S = turns in series per phase, I = full-load current per phase, k_2 and k_3 have the values given in the article on *Generators, Alternating-current*, and p = number of poles. The armature reaction of a motor is usually 30 per cent to 50 per cent greater than that of a generator of the same rating and frequency. The higher value gives greater synchronizing torque per ampere, a better starting torque, and reduces the cross-currents between machines in case of hunting. The armature-reaction ampere turns at full load may be equal to the field ampere turns at no load. Too great an armature reaction is objectionable, because it reduces the energy transfer between two machines and therefore reduces the synchronizing power, that is, the tendency of the machines to hold each other in step.

Excitation. — The excitation of a motor is calculated in the same manner as the excitation of a generator. The magnetic densities are usually a little less. The capacity of the field winding depends upon whether the synchronous motor is to be used as an ordinary motor or to regulate the power factor of a system by over-excitation.

Leakage Reactance. — The leakage reactance may be higher in a motor than in a generator as regulation is not of so great importance. However, too great a reactance will reduce the starting torque of the motor.

Short-circuit Current and Synchronous Impedance. — The short-circuit current of the motor depends upon the leakage reactance and the armature reaction. The short-circuit current and the synchronous impedance may be predetermined from the no-load saturation curve and the calculated leakage reactance per phase.

Let F = excitation in ampere turns per pole for which it is desired to find the short-circuited current,
I = rated current per phase,
x = leakage reactance in ohms per phase,
E = voltage per phase due to F at no load,
S = turns in series per phase,
p = number of poles.

Then the ampere-turns synchronous impedance for full-load current is

$$\frac{2.12\,SI}{p} + \frac{IxF}{E},$$

and the short-circuit current with excitation F is

$$I_0 = \frac{EpF}{2.12\,SE + xpF}.$$

The synchronous impedance at this excitation is $Z_0 = E/I_0$, and the syn-

chronous reactance is $x_0 = \sqrt{Z_0^2 - r^2}$, where r is the resistance of the armature per phase.

Efficiency and Losses. — The losses are predetermined as in a generator. They are: A = friction, B = excitation or field RI^2, C = core-loss, D = armature RI^2; then

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + A + B + C + D}.$$

Phase Characteristics or V-Curves. — The phase characteristic is a curve showing the variation in armature current for any given load with varying field excitation. Fig. 5 shows the shape of the curves, which may be determined both by calculation and test. The phase characteristic for any particular load has the general shape of the letter V, and the group of such curves for various loads are frequently called the "V-curves" of the machine. There are two methods of calculating the phase characteristics, the electromotive force method and the magneto-motive force method.

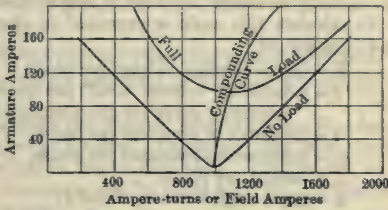

Fig. 5. Phase Characteristics of Synchronous Motor

Electromotive Force Method. —

Let E = line voltage per phase,

e = motor counter e.m.f. per phase corresponding to the excitation to be used,

i = component of current in phase with e,

ei = mechanical output of armature,

i_1 = reactive component of current, positive for leading, negative for lagging,

r = armature resistance per phase,

x_0 = synchronous reactance per phase,

then $E^2 = (e + ri - x_0 i_1)^2 + (x_0 i + r i_1)^2 = e_1{}^2 + e_2{}^2$.

E, r and x_0 are constant, e is set by the value of the field current assumed, then i = (watts output)/e and i_1 remains the only unknown quantity. Solving for i_1 the total current $I = \sqrt{i^2 + i_1{}^2}$ and the power is $e_1 i - e_2 i_1$ (for i_1 lagging). If e is greater than E, then i_1 is leading, and the power factor is $(e_1 i + e_2 i_1)/EI$.

Magneto-motive Force Method. —

Let E = line voltage per phase,

F = field ampere turns per pole for this voltage,

D = any value of field ampere turns per pole,

I = armature current per phase,

i = energy component of current = (power input) ÷ E,

p = number of poles,

ν = field leakage coefficient of machine,

S = turns in series per phase,

i_1 = wattless component of current for excitation D.

Then

$$pF = pD - 2.12\, S i_1 \nu \text{ and}$$

$$i_1 = \frac{pD - pF}{2.12\, S\nu}.$$

and

$I = \sqrt{i^2 + i_1^2}$ is the current in armature for power input Ei and for the excitation D assumed. If i_1 is negative ($F > D$) then i_1 is lagging. If $D > F$ then i_1 is positive and leading.

Angular Lag Due to Load (α). — For any load on a synchronous generator there is an angular advance of the generated e.m.f. ahead of the terminal e.m.f., and in a synchronous motor there is an angular lag of the generated e.m.f. behind the terminal e.m.f. This phase displacement is accompanied by a shift in the synchronous position of the armature, which may be calculated and actually measured (*see Tests of Synchronous Motors below*).

To calculate this angle α expressed in electrical degrees (360 degrees per pair of poles), let

E = line or terminal voltage per phase,
e = induced or counter e.m.f. per phase; may be taken from no-load saturation curve for given excitation,
r = resistance of armature per phase,
x_0 = synchronous reactance per phase,
I = current per phase,
ϕ = phase angle between E and I.

Then

$$e^2 = (E - Ir\cos\phi - Ix_0\sin\phi)^2 + (Ir\sin\phi - Ix_0\cos\phi)^2,$$

and

$$\alpha = \sin^{-1}\left[\frac{(Ir\sin\phi - Ix_0\cos\phi)}{e}\right],$$

or roughly

$$\sin\alpha = \frac{I\cos\phi}{\text{short-circuit current}}.$$

The mechanical displacement of the armature for the load $EI\cos\phi$ per phase is $2\,\alpha/p$.

Synchronizing Torque.—The synchronizing power of a machine is a measure of the ability of a machine to keep in step with its supply circuit. It may be expressed in terms of torque per degree of displacement. If P is the kw. output of a motor and α is the angular displacement of the armature in electrical degrees for this load, then

$$\text{Motor torque} = \frac{7050 \times (\text{kw.})}{\text{rev. per min.}},$$

$$\sin\alpha = \frac{(Ir\sin\phi - Ix_0\cos\phi)}{e},$$

and

$$\text{Synchronizing torque} = \frac{7050 \times (\text{kw.})}{\alpha \times (\text{rev. per min.})}.$$

A high resistance between machines reduces the synchronizing torque as it reduces E. A reactance between machines is not as bad as resistance. Increasing the excitation increases e and improves the synchronizing torque.

Hunting; Natural Period. — The rotating part of every synchronous machine acts like a pendulum, tending to swing ahead and behind its normal synchronous position. The mass of the armature (and its flywheel) acts like the mass of the pendulum, and the torque of the machine being proportional to the displacement (α) corresponds to a spring or gravity acting on a pendulum. Such a combination has an "electro-mechanical period" of its own, and if the frequency of this period is in tune with any other pulsating force in the system, such as engine impulses, "hunting" or "surging" may occur.

Boucherot and Kapp have shown that the natural period of any synchronous machine expressed in seconds or fraction of a second is given by the formula:

$$T = 0.308\, N \sqrt{\frac{Wk^2}{fgmEI_0}},$$

where N = r.p.m. of revolving part,

W = total weight of revolving part including any flywheel in pounds,

k = radius of gyration of W in feet,

f = frequency of current, cycles per second,

g = acceleration of gravity, in ft. per sec. per sec. = 32.2,

m = number of phases,

E = terminal voltage per phase,

I_0 = short-circuit current of machine per phase and at excitation used.

The frequency of the natural period expressed in impulses per minute is $f_1 = 60/T$.

The formula shows that the greater the flywheel effect kW^2 (*see Flywheels*) the longer will be the periodic time of a swing. The greater the short-circuit current or excitation the shorter will be the periodic time. The periodic time may be increased by connecting reactance coils in series with the machine between its terminals and the bus-bars.

If T_0 is the periodic time of any other pulsating force in the system, as the strokes of a steam engine, the danger of hunting is greatest when

$$T/T_0 = 1/4,\ 1/3,\ 1/2,\ 1.$$

A tendency to hunt is damped by solid pole pieces, bridges between poles or, best of all, a squirrel-cage winding in the pole face.

Maximum Output. — As the current which would flow during maximum output of a synchronous motor is so great that it would burn up the windings in case this output should last more than a fraction of a second, the value of the maximum output is only of theoretical interest. In practice the maximum output (for a given voltage) is only reached under two conditions: (1) when, due to extraneous causes, the line voltage decreases to a fractional value, the maximum output decreasing as the square of the voltage; (2) when, due to hunting or pulsation, the flow of energy into and out of the machine reaches excessive values momentarily. In one of these swings the power may reach the value of the maximum output or exceed it and the machine shut down. Although the power of the machine drops off gradually after the point of maximum output has been reached, the motor is unstable in this region and is more than likely to shut down when the condition is reached.

Starting Torque. — Synchronous motors may be started either as hysteresis * or induction motors. In the former case the motor requires a high voltage and takes a small current, but as the torque is very slight this method is seldom used in practice. When starting as an induction motor a high armature reaction is desirable and a low leakage reactance. A squirrel-cage winding in the pole face must be provided, and as in an induction motor this squirrel-cage winding must have a cross-section approximating a certain value. If the cross-section is too large, the currents will be excessive and the starting torque not the best. If the cross-section is too small, the currents will be small and the starting torque not the best. While the cross-section of the squirrel-cage winding may be roughly predetermined by treating the machine as an induction motor, this

* A piece of iron in a rotating field has a torque produced on it due to the hysteresis and eddy currents set up in it. Such an arrangement may be called a "hysteresis" motor; an open-circuited field of a polyphase synchronous motor therefore forms a hysteresis motor.

method is not accurate because the construction of thc field renders the calculation of the leakage reactance inaccurate. It is thus mueh better determined empirically.

Rotary Phase Modifiers or Rotary Condensers. — Synchronous motors are sometimes used to improve the power factor and reduce the line current of an installation of a number of induction motors. If a factory has an installation of 100 kv-a. of induction motors having an average power factor of 71 per cent and taking I amperes, then by installing a synchronous motor of 100 kv-a. rating, designed to be overexcited, the power available will be doubled and the line current only increased by 41 per cent or to $1.41\ I$. Such a machine is called a "rotary condenser" and if it is rated at 100 kv-a. it may give 71 kw. of power and 71 kv-a. to balance the reactive effect of the inductive apparatus. Other relations may be obtained in accordance with the principle of vector combinations.

Let P_1 = true power taken by the induction motors, kw.,
Q_1 = reactive power taken by the induction motors, kv-a.,
$L = \sqrt{P_1^2 + Q_1^2}$ = total kv-a. of induction motors,
P_2 = true power taken by rotary condenser, kw.,
Q_2 = reactive power taken by rotary condenser, kv-a.,
$K = \sqrt{P_2^2 + Q_2^2}$ = total kv-a. of condenser,

then

$$\text{Line kv-a.} = \sqrt{(P_1 + P_2)^2 + (Q_2 - O_1)^2}.$$

TESTS OF SYNCHRONOUS MOTORS. — Certain tests on synchronous motors are the same as those made on an a-c. generator; the methods of carrying out such tests are described in the article on *Generators, Alternating Current.* The first five of the following tests are of this character:

1. Resistance of armature and field circuits both cold and hot.
2. Saturation curve at no load and under special circumstances at full load.
3. Core-loss.
4. Short-circuit or synchronous impedance.
5. Insulation tests.

6. Phase Characteristics or V-Curves at no load, full load and any other specified load. The machine is operated as a motor with the specified load kept constaut throughout the run. The voltage and frequency impressed upon the motor are also kept constant. The current in the field is varied from the minimum at which the motor will operate to the maximum (from ¼ normal to 1½ normal) and the variation in current input to armature noted. Readings are taken of load, volts armature, amperes armature and amperes field. A curve is plotted with amperes armature as ordinates, and amperes or ampere turns per pole in field as abscissae. This gives the characteristic V-curves of the synchronous motor (see Fig. 5). The point of minimum current input for each load is very clearly shown. At this point the power factor is unity. At lesser values of field current the armature current is lagging and the power factor poorer. At greater values of field current the armature current is leading. The point of minimum current occurs at a higher value of field current for the greater loads because the field excitation must be increased with the load to overcome the armature reaction due to the load current.

Compounding Curve. — The curve connecting the points of minimum armature current in the group of V-curves is called the compounding curve for unity power factor.

7. **Starting Tests.** — A low voltage is impressed on the armature and gradually increased until the motor starts. The field circuit is open in two or more places and a high potential voltmeter connected across one section to determine the voltage induced in the field spools by the rotating magnetic flux. The test is repeated for several different initial positions of the revolving part and a record is made of the time required for the machine to reach synchronism. The time at which synchronous speed is reached may be determined by the fact that the induced voltage in the field becomes zero. Readings are taken of volts armature, amperes armature, initial position, maximum volts field, time to reach synchronism. See also section on *Starting*, below.

8. **Armature Phase Position.** — The phase position of the armature discussed previously may be measured, although the item is only of theoretical interest and not of commercial importance. A synchronous motor is supplied with power from a special alternating-current generator, and on the end of the shaft of each machine is placed a contact-making disk, as shown in Fig. 6.

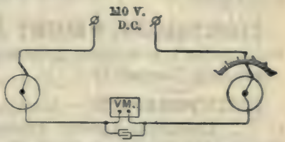

Fig. 6. Contact Method of Measuring Armature Phase Position

A voltmeter is connected in series with a source of direct current, the two disks, and the brushes pressing on the disks. The voltmeter reading is a maximum when the two brushes are in contact with the metal strips at the same time. The brush on the motor may be moved over a graduated scale or arc, so that its position may be varied and the actual angular movement measured. The brush on the generator remains stationary. As the load on the motor is increased it will be found necessary to move its brush in order to keep the voltmeter reading at its maximum value, and the angle β through which the brush has been moved for the change in load gives the phase position of the motor armature with respect to the generator armature. It will be found that the value of β is directly proportional to the load. The difference in phase of the electromotive forces in electrical degrees will be β multiplied by the number of pairs of poles of the motor. If hunting exists, the maximum angle of swing may be determined by moving the motor brush first one way as far as the effect may be noticed, and then in the other direction. (*See Morecroft, Laboratory Manual.*)

SPECIFICATIONS. — Synchronous motors are rated in the same manner as synchronous generators, and the same heating limits and specifications apply. (*See Generators, Alternating Current.*) It is customary to specify the value of the current taken by the motor in starting with no load other than the friction of its own bearings, or its own friction plus that of the machine to which it is connected, in case it is part of a motor-generator set. It is also sometimes mentioned in the specifications that the motor will not hunt providing the total resistance drop between the generator and motor is less than some specified value (10 per cent or 15 per cent).

INSTALLATION AND OPERATION. — The precautions to be taken in installation are the same as for a-c. generators (q.v.). Direct current must be provided for excitation. If a synchronous motor is operated on a polyphase system having unbalanced voltages it will take unequal currents in the different lines and tend to balance the voltages. These unequal currents, however, increase the heating somewhat for a given load.

Starting. — Provision must be made for a reduced voltage for starting the motor, either by means of taps on the transformers or by means of a starting compensator (q.v.). Large motors require two steps in starting, ⅓ and ⅔

of normal voltage. Small motors will start with one step at ½ voltage. The field circuit is opened by the field "break up" switch and a voltage applied to the armature. When the armature current has decreased from its large value at starting to a reasonable value the voltage is increased, step by step. When the motor has reached maximum speed the field is excited and the motor pulls into synchronism. The field current is then adjusted until the condition of minimum armature current is found or until the power-factor indicator records unity power factor. The load may then be applied.

If a synchronous motor is to be started often (several times a day) it is desirable to provide a special starting motor which brings the synchronous motor up to a speed a little above synchronism. Synchronizing must then be effected as in a-c. generators. Such a starting motor would require only about 30 per cent of the full-load current of the motor, and therefore have very little effect on the regulation of the system and not cause disturbance to the lights and other motors on the system. It usually requires less than a minute to bring a motor up to speed.

DIMENSIONS, WEIGHT AND COSTS. — These are approximately the same as for synchronous generators of the same kv-a. rating. (See *Generators, Alternating-Currents.*)

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OHMMETERS. — (*See also Bridges for Electrical Measurements; Galvanometers.*) The name ohmmeter is applied to any portable device designed for the direct measurement of electrical resistance. The simplest form of ohmmeter consists of a slide-wire bridge, battery and galvanometer all mounted in a portable case. Another form of ohmmeter utilizes a special form of galvanometer whose deflection, measured on a properly calibrated scale, gives the value of the unknown resistance directly in ohms.

BRIDGE TYPE OF OHMMETER (Fig. 1). — This ohmmeter is based on the Wheatstone-bridge principle, but differs from most Wheatstone bridges in that the rheostat resistance R is kept constant and bridge arms, formed by the wire W, are varied by moving a contact C. Ohmmeters of this type are made in many varied forms. In some a straight slide wire is stretched over a cardboard scale and contact on the wire made with a metallic stylus. In others a slide wire is wound on an insulating cylinder. The scale may be equally divided, or may be divided to read directly the ratio of the resistance measured to the resistance of the fixed rheostat resistance R.

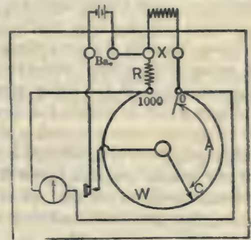

Fig. 1. Slide Wire Ohmmeter

EVERSHED MEGGER (Fig. 2). — This instrument is of the galvanometer type. It is principally useful for the measurement of insulation resistances and other high resistances where extremely high accuracy is not required. The scale is graduated to read directly in megohms, whence the name.

Referring to Fig. 2, D is a small hand-driven d-c. magneto and G a special form of moving-coil instrument. One pair of bar magnets supplied the flux for both. The galvanometer contains three coils rigidly attached at a fixed angular distance apart to the shaft which carries the pointer P. The unknown resistance is connected at X.

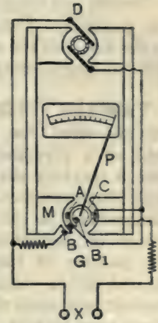

Fig. 2. Evershed Megger

The coils BB_1, called the pressure coils, are permanently connected through a resistance across the terminals of D. The current coil A is made to move through an annular gap in such a manner that the field in which it moves is uniform, whereas the pressure coils BB_1 move from a position midway between the poles, where the field is at a minimum, into a stronger and stronger field, the connections being such that the torque due to the current in the current coil is opposed by the torque due to the current in the pressure coil. With no current in the current coil, that is to say, when the resistance to be measured is infinite, current through the pressure coils will cause them to come to rest with their plane at right angles to the magnetic field. When the current through the current coil is increased by putting in lower resistances the current coil drags the moving system round in a clockwise direction; since the pressure coils come into a stronger and stronger field, the resistance to this motion becomes greater and greater. Hence a definite position is assumed by the system for the particular resistance at X. An increase in voltage would increase the current in both current and pressure coils in the same proportion; consequently the instrument is independent of the voltage of the generator.

In the cheaper instruments the generator gives a variable voltage depending upon the speed of rotation of the handle. In the higher priced instruments

the generator is designed to give a constant voltage for a considerable range in the speed of driving. The voltage at which an insulation test is made is often of great importance (*see Resistance and Conductance*). Constant-voltage meggers are especially useful for testing insulation resistance associated with a large electrostatic capacity, where a varying test voltage would cause charging and discharging currents to flow with resulting fluctuations in the readings.

WESTON DIRECT-READING OHMMETER. — This is a permanent-magnet moving-coil instrument having two windings on the moving coil and an adjustable magnetic shunt. Terminals are provided for the unknown resistance and an auxiliary battery. Before use, the instrument is checked by placing a plug in the checking position. On closing the key, a current from the battery divides through the two windings. The ampere-turns of one winding, which is in series with a fixed resistance, tend to deflect the pointer up scale, while the ampere-turns of the other winding tend to return it to zero. With a battery voltage which is within the designed limits, the magnetic shunt may be adjusted to bring the pointer to full-scale position. If the plug be now shifted to the low-range position, a resistance equal to that of the low range is removed from the circuit whose ampere-turns tend to deflect the moving coil down scale. If a resistance equal to that of the low range be now connected across the "X" posts, the previous condition will be restored, and the pointer will stand at the full scale position. For any value of resistance less than this the opposing ampere-turns will be greater, and will bring the coil back to a lower position on the scale. For zero resistance across the "X" posts, the ampere-turns of the two windings will balance each other, and the coil will stand at the zero of the scale, corresponding to the unstressed condition of the springs.

Double and triple ranges are available by shifting the plug to the corresponding positions. This form of ohmmeter is very rapid in operation. Variations in the e.m.f. of the auxiliary battery are taken care of by periodically repeating the checking operation.

VAWTER INDICATING OHMMETER. — This instrument is of the galvanometer type, containing two coils carried by a shaft and moving in the field of a permanent magnet. The core is specially shaped to give such scale characteristics as may be desired. Connections to the coils are made by spirals of negligible torque, so that the position of the pointer does not depend on the value of the auxiliary current nor upon the strength of the permanent magnet. The instrument can therefore be operated on a generator source of voltage. This ohmmeter is made in portable form with from one to four ranges; in switchboard form, applicable for determining the resistance of the windings of electrical machines under load, and as a rail-bond tester.

KELVIN BRIDGE OHMMETER (Fig. 3). — This is a portable type direct reading ohmmeter used for measuring low resistances.

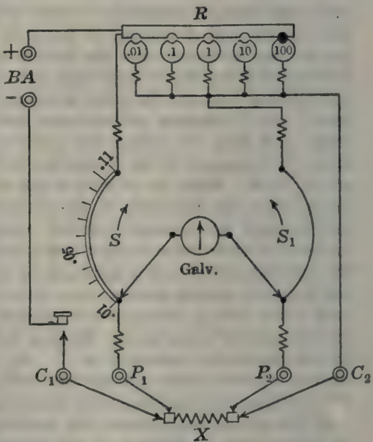

Fig. 3.

Current and potential leads are provided which are connected to the unknown resistance.

The two slide wires S and S_1 are mounted on the same disc and move together. The reading on a scale mounted on this disc gives the ratio of x/R. The reading multiplied by the value of R plugged in gives the value of x between potential points.

COST, RANGE AND ACCURACY. — Bridge type ohmmeters range in price from $60 for ohmmeters having a range of from 0 to 100 ohms and an accuracy of about 2 per cent to $80 for ohmmeters having a range from 0 to 10,000 ohms and an accuracy of ½ per cent. Meggers range in price from $225 for variable voltage meggers having a range of from 0 to 10 megohms and an accuracy of 5 per cent to $530 for constant pressure meggers having a range of from 10 to 2000 megohms and an accuracy of about 4 per cent. Kelvin bridge ohmmeters having a range of .0001 ohm to 11 ohms cost about $100. The above prices are approximate for the year 1921 and should be used only as a rough guide.

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OIL, TRANSFORMER. — (*See also Transformers; Circuit Breakers; Insulating Materials, Testing of.*) Oil is used in transformers, electrolytic arresters and feeder regulators as an insulating and cooling medium, and in oil-circuit breakers as an insulating and extinguishing medium. For these various uses, it must have certain general characteristics as follows:

(1) It must have high dielectric strength.

(2) It must have sufficiently high flash point to prevent its ignition during the opening of circuit breakers, and the failure of transformers, feeder regulators or electrolytic lighting arresters.

(3) Where oil is used for cooling purposes, it must have low viscosity to allow free circulation and must be free from deposit to prevent the obstruction of oil ducts.

(4) For use in oil circuit breakers it must have high viscosity to prevent undue splashing.

(5) For use in electrolytic arresters the oil should not emulsify with the electrolyte.

Mr. D. B. Rushmore gives the following as the characteristics of oil usually furnished with large, high-voltage, water-cooled transformers:

Flash point	130° C.
Burning point	145° C.
Freezing point	−15° C.
Color	white
Spec. gravity at 15.5° C.	0.830
Viscosity (Saybolt 40° C.)*	40

Dielectric Strength and Specific Inductive Capacity. — The dielectric strength of clean dry oil should not be less than twenty-two thousand (22,000) volts, when tested between one-inch disks set one-tenth (0.10) inch apart.

Oil in service, except in electrolytic lightning arresters, should be filtered when it fails to stand a pressure of sixteen thousand five hundred (16,500) volts when tested between one-inch disks set one-tenth (0.10) inch apart.

According to Tobey, the dielectric strength of transformer oil increases with increase of temperature, the increase from 30° to 90° C. being about 25 per cent.

On the other hand, the insulation resistance of oil decreases very rapidly as the temperature increases. In a typical test Tobey found the insulation resistance of a given sample to fall from 1200 megohms at 50° C. to 250 megohms at 90° C.

The specific inductive capacity of transformer oil is about 2.5 between 25 and 100° C.

Dryness and Purity. — Great care should be taken to keep the oil absolutely dry under all conditions. The presence of water in as small amount as one one-hundredth of one per cent is sufficient to impair seriously the dielectric strength of the oil.

Great care should be taken to remove from oil before it is used, foreign material, such as scale from iron drums, which may become dislodged during shipment; also to keep foreign material from entering the oil when the apparatus is in service.

The presence of mineral acid or of alkali in insulating oil is not permissible as they are corrosive or destructive in their action upon the materials of the apparatus. An oil practically neutral is preferable. It should contain no free sulphur, because sulphur in its free form attacks copper. All petroleum oils contain sulphur compounds, but in this form it is not objectionable.

* The time in seconds required for a certain definite quantity of oil to flow through a definite opening as measured with a Saybolt viscosimeter.

Deposit or Sludge. — An oil for transformers and feeder regulators should be as free from deposit as possible. Deposit is objectionable primarily because it clings to the coil windings and fills the coil ducts, thus seriously affecting the cooling action.

The forming of deposit is a function of the temperature and length of time the oil has been used. Different grades of oil differ as to the temperature at which deposit will appear. A satisfactory oil should not show a deposit or any change other than a possible darkening of color when operated continuously under temperature conditions experienced during actual operation.

Pour Point or Freezing Point. — The pour point or freezing point is usually of no great importance, except for oil for apparatus where there is a movement of parts. The oil used in transformers, lightning arresters and regulators usually has a pour point of about 0° C. For outdoor installation of regulators and oil circuit breakers in extremely cold climates and all oil fuse cutouts and moving coil constant current transformers, the oil used should have a pour point of about −30° C.

PURIFYING AND DRYING. — All new oil as it is taken from the drums or tank cars, and all old oil that has been in use for some time, that shows the presence of sediment or dirt, should be filtered. In placing the oil in the transformer case it should be run in through about two thicknesses of finely woven cambric, which has been washed to remove all sizing and thoroughly dried.

The presence of large quantities of moisture in transformer oil is easily detected, since water is heavier than oil and will settle to the bottom, if allowed to stand. If not present in sufficient quantity to settle in this manner, the oil may still contain considerable moisture in a suspended state. If a sample of clean oil shows too low a dielectric strength, there is moisture present and the oil must be dried.

The best method of purifying oil is by means of one of the commercial filter presses produced by the various manufacturing companies. The oil is treated in a filter press by being forced through several layers of blotting paper which remove all moisture and solid matter held in suspension by the oil.

In filtering oil it is preferable to filter from one tank and discharge into another, as in this way the oil is given a more complete treatment than if drawn from the bottom and discharged into the top of the same tank. The latter method is, however, very useful in cases where apparatus cannot be removed from service long enough to empty it completely.

For satisfactory operation of the press, it is important to see that the filter papers are changed with sufficient frequency and are properly assembled. Only new clean filter paper should be used, and it should be thoroughly dried in an oven before being placed in the press. Filter paper will take up moisture from the air rapidly and must therefore be stored under dry oil in case it cannot be used directly from the oven.

If a drying outfit is not available, the oil may be dried in a fairly satisfactory, although slow and inconvenient manner, by passing it through clean, dry, unslaked lime and filtering afterward to remove all particles of foreign matter. Various other methods have been suggested or tried, but they are not to be recommended. For example, passing hot, dry air through the oil is not desirable because of the difficulty of removing all the moisture from the air, and the heating of the oil for a considerable length of time involves the liability of injuring it by such treatment.

TESTING. — The most important and frequently made test of transformer oil is the test for dielectric strength. Additional tests are: (*a*) viscosity, (*b*) flash point, (*c*) fire point, (*d*) pour point, (*e*) acidity, and (*f*) sulphur content.

Complete directions for making all these tests are given in *A.S.T.M. Tentative Standards*, D 117–21 T.

The following brief description of the test for dielectric strength is adapted from Insturction Book 5094–B, published by the Westinghouse Electric and Manufacturing Company.

For testing oil for dielectric strength, use some standard form of testing device such as that shown in Fig. 1. The gap terminals are flat disks, 1 inch in diameter, with square edges. The standard gap setting is 0.10 inch.

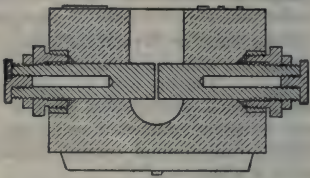

Fig. 1. Oil Test Cup

The following precautions should be observed: Wipe out the testing cup with a clean, dry cloth, being careful to remove any particles of cotton fiber that may adhere to the surface. Adjust and fix the gap in its correct setting and then rinse the cup thoroughly with benzine or gasoline, being particularly careful to see that no particles of dirt remain. Then give a final rinsing with a portion of the oil to be tested immediately before placing the testing sample of oil in the cup.

The temperature of both the oil and the testing cup should be approximately the same and between the limits of 20° C. and 30° C.

Pour the test sample into the testing cup and allow it to stand until all minute air bubbles have had time to escape. This will require two or three minutes. The oil level should not be less than 1 inch above the electrodes.

Apply the testing voltage, starting at a low value and increasing steadily at the rate of approximately 3000 volts per second without opening the circuit until breakdown takes place. The testing transformer should have a capacity of at least 1 k-va.

After each break, jar the testing cup but do not cause the oil to be agitated sufficiently to take up air bubbles. This will tend to loosen the bubbles of carbonized oil from between the electrodes. Do not introduce any device into the oil for stirring, as it is likely to cause low breakdown voltages on account of introducing minute particles of foreign matter or moisture.

Repeat the application of voltage until five breakdowns have been made and use the average value as the breakdown value of the test sample of oil.

Sometimes a small, bright spark passes across the gap, but does not form an arc or cause circuit breaker or fuse to open the circuit. Such discharges should be disregarded in recording results. They are not likely to occur if sufficient time is allowed for all air to escape before applying voltage.

The testing cup should preferably be used in connection with a regular high voltage testing outfit, provided with suitable means for regulating the testing voltage. If a testing outfit is not available, a generator and a commercial high-voltage transformer may be used, control of the testing voltage being obtained by variation of the generator field. If such an arrangement is used a high resistance (about 1 ohm for every volt of testing voltage) should be placed in series with the testing cup.

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OSCILLOGRAPHS. — (*See also Braun Tube; Wave Analysis.*) The oscillograph is essentially a galvanometer of very short period. It is applied in the observation of potentials or currents, as voltmeter or ammeter, mainly when variations are too rapid to be indicated by the more usual instruments; for example in observation of potential waves of generators, potential and current waves in inductive apparatus, short-circuits or switching of transmission lines. As in many cases it is necessary to have two or more curves taken together in their relative phase relation, more than one element is necessary in a practical instrument; it is regularly built as a three-element oscillograph, giving one, two or three curves, as may be required, on the record. One element may be used as a chronograph by connecting it to record a timing wave from an a-c. source of known frequency.

Types of Oscillographs. — The most common form of oscillograph is the moving-coil type, the "coil" being two small thin strips or ribbons arranged very close together, thus forming the two sides of a coil of one turn. A moving iron vane can also be used, but this type has not received extended practical application. The electrostatic oscillograph, which is particularly well adapted for direct observation of high potentials, consists of two insulated strips maintained at a constant potential difference, these strips being caused to vibrate by the varying force of attraction and repulsion due to the charges on two fixed plates connected to the varying high-potential source. The vibrator is similar in construction to the vibrator of the ordinary oscillograph, except that the two strips are connected by a light insulating thread. The strips are connected to the terminals of a storage battery, the middle of which is kept at a potential midway between the potential of the two plates, by being connected between two equal condensers in series across the attracting plates. Still another form of oscillograph is the cathode ray, or Braun, tube (*see Bibliography for references*). This last type of oscillograph finds its most valuable application in the laboratory study of high-frequency and high-voltage phenomena which are beyond the range of the ordinary oscillograph. The practical application of this type has been found rather difficult.

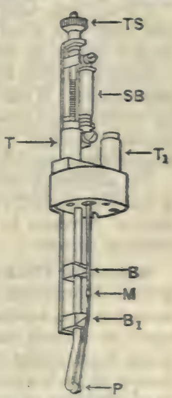

Fig. 1. Oscillograph Vibrator. B,B_1 bridges for supporting strip; P, pulley; SB, spring balance indicating tension; TS, tension screw; M, mirror

DESIGN OF MOVING-COIL OSCILLOGRAPH. — The ordinary or moving-coil oscillograph is described in detail below.

Construction of Vibrator. — The vibrator, or moving element (Fig. 1), consists of two strips of flattened wire stretched over bridges, with a very small mirror cemented directly to the strips; the arrangement constitutes a one-turn galvanometer coil of elementary form. The vibrator is placed between the wedge-shaped poles of an electromagnet or of a permanent magnet. It is immersed in a liquid which provides critical damping. The vibrator conductor passes from one terminal post T over the bridges BB_1 in narrow grooves, over a pulley P, back over the bridges in grooves very close to the former, to the other terminal post T_1. The width of the strip is usually 0.005 inch to 0.007 inch. The mirror M is cemented to the strips midway between the bridges, its usual size being 0.060 inch by 0.017 inch. Tension is applied to the strips, for the ordinary vibrator 6 oz. for the two strips, indicated by a small spring balance SB. The vibrator is readily rewired in case of break or burn-out.

For efficient operation vibrators are made interchangeable. This is necessary to make expeditious operation possible, and to minimize delay due to accidental destruction of vibrator parts.

Construction of Field Magnets. — The galvanometer field is an electromagnet with wedge-shaped poles. The vibrator is so mounted that the strips lie in the narrow air gap between these poles. A direct current through the winding of the electromagnet produces a very strong field in this air gap. The ampere turns of the electromagnet are sufficient to saturate the pole tips and render the strength of the field practically independent of the voltage applied to the electromagnet windings, at least for ordinary voltage variations. One terminal of the vibrator may be connected electrically to the core of its field magnet, each core being insulated from its field winding for a working pressure of 2300 volts.

A permanent magnet can be substituted for the electromagnet as the oscillograph field. This construction is practicable for two elements. The permanent magnet oscillograph is somewhat inferior in sensibility, and greatly inferior in insulation between elements, compared with the electromagnetic type of oscillograph.

Optical System. — The light for the oscillograph from a projection arc lamp enters the case at the shutter aperture, and, after being reflected toward the vibrator cell by a total reflecting prism (one for each element), passes through a slit which adjusts the width of the image. It is then reflected by the small vibrator mirror, and passes through a cylindrical lens to the image on screen or film. Very careful adjustment of the optical system is essential to secure records of good photographic intensity.

The arc lamp used as a source of light for the oscillograph is usually a hand-feed lamp with small carbons at right angles, taking 5 to 8 amperes, with *solid positive* carbon in the horizontal position. The light is rendered parallel by a simple projection lens. Any convenient arc lamp, however, may be used with good results. If only an a-c. source is available for the arc, records of fluctuating intensity are obtained, which, however, are usually legible throughout their length. Sunlight source with a heliostat, when circumstances permit, gives photographic records of superior intensity.

The astigmatic optical system of the oscillograph, due to the cylindrical lens, is quite a distinctive one; the geometrical light source for the image in the vertical direction is the adjustable slit, in the horizontal direction it is the vibrator mirror.

Means for Obtaining Photographic and Visual Records. — The photographic record is taken on a moving film or plate. The most practical arrangement is a film, having a length of about 12 inches, on a drum driven at suitable speed by a small motor. A contactor opens the shutter for one revolution of the drum, giving exposure once over the film. The exposure can be adjusted to start at the beginning of the film, or if a record is taken in response to a signal, as of switching or short-circuit, the exposure can be started instantaneously at any part of the film. A long film is desirable in some cases, as in transmission line switching, where the whole disturbance lasting perhaps several seconds is to be taken. By the use of a suitable attachment for long films, records can be taken on films 3 to 5 feet in length.

The motion of the spot of light reflected from the vibrator mirror may be projected as a standing wave on a tracing table, for examination or demonstration. An oscillating mirror, actuated by a cam driven by a small synchronous motor, is given a uniform angular velocity during alternate cycles of the current or voltage observed, and draws out the wave longitudinally. Only alternate waves or cycles are utilized, the intervening waves being cut off by a revolving screen on the motor shaft during the return motion of the mirror. The recur-

rences of the waves are so rapid as to produce an image sensibly continuous to the eye. The synchronous mirror is removed by shift of a simple mechanism to permit photographic record to be taken in the usual manner by revolving film.

Free Period of Vibrator. — The free period of the vibrator is about 1/6,000 second, but some have been constructed to have a free period as high as 1/10,000 second. The higher the free period the less the distortion of the higher harmonics in the current and voltage waves. A free period of 1/6,000 second, is ample to secure practically accurate values of all harmonics having a frequency of less than 1200 cycles per second; with a free period of 1/10,000 second harmonics having a frequency as high as 2000 cycles per second are recorded with practical accuracy.

The free period of a vibrator depends upon the moment of inertia of the vibrating system and upon the tension on the strips; the less the moment of inertia and the greater the tension, the shorter will be the free period. The free period of a given vibrator may therefore be slightly shortened by increasing the tension on the strips, but this in turn decreases the sensibility, i.e., the deflection for unit current through the strips.

To measure the free period of a vibrator it is placed in the cell between the poles of magnet, in the usual way, but with no damping liquid in the cell. It is connected to a d-c. source through an interrupter which makes or breaks the circuit one or more times during the exposure of a film running at as high a speed as possible. The free vibration in decreasing amplitudes is shown on the film, and the period is readily counted, the film speed being known.

Sensibility of Oscillograph. — An oscillograph having a free period of 1/6,000 second requires from 0.1 to 0.2 ampere through the vibrator to give a curve of good amplitude; or about 0.006 ampere is required to give a millimeter deflection from the zero line on the film or tracing table. As ordinarily constructed a deflection of 45 mm. on each side of the zero may be obtained, but a deflection of more than 30 mm. is seldom necessary. The resistance of the vibrator is from 1 to 1.5 ohms; hence to obtain a deflection of 30 mm. about 0.2 volt is required across the vibrator terminals. The sensibility of a vibrator may be much increased by using thinner strips and smaller mirrors, but such arrangements are more delicate and should be used only to meet special requirements.

CONNECTIONS AND ADJUSTMENTS. — For potential, or voltage, curves the oscillograph is connected similarly to a voltmeter; **E** in Fig. 2. A suitable amplitude of curve is obtained by an external adjustable resistance.

For current curves the oscillograph is connected similarly to a millivoltmeter across a non-inductive shunt; **I** in Fig. 2. A shunt potential drop of at least 0.1 volt is required, but for convenient adjustment it should be larger, 0.5 volt being a suitable value where practicable. If the current measured is less than 0.2 ampere, no shunt is used, the whole current being taken by the vibrator.

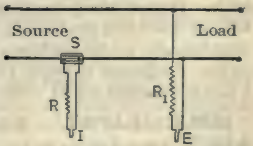

Fig. 2

For waves of flux distribution in generators or motors, the oscillograph is connected to give the voltage of an exploring coil; sometimes in d-c. machines the voltage between commutator bars can be used.

Potential and current transformers are used where the line voltages and currents are too high for practical or convenient direct observation. The oscillograph potential circuit can be connected to the potential transformer along with voltmeter or other instruments, as it is only a fraction of the rated

potential transformer load. For current curves a shunt is connected in the current transformer secondary circuit.

Adjustment of Vibrator and Optical System. — Individual vibrators are provided with movements for vertical and horizontal adjustment of the image. The beam of light is brought vertically to the middle, or axis, of the cylindrical lens; this adjustment is left unchanged, but should be examined occasionally to insure its correctness; this may conveniently be done by viewing the position of the light spot on a thin sheet of paper held against the case where the light comes through to form the image on film or ground-glass screen; the light spot should appear central in the aperture. The horizontal adjustment brings the image to the proper position on the record, and is changed freely according to circumstances. When two curves are to be taken on the record, the images are usually placed for clearness so as not to overlap. If, however, the phase relation is important they are placed with their zero positions near together; it is better not to make them coincident, however, as good superposition is not always secured. After the principal exposure, an auxiliary exposure for zero line is taken with vibrator circuit opened, except in occasional records where the zero line is unnecessary.

Quantitative Measurements with Oscillograph. — The values of the curves may be obtained quantitatively when necessary by reference to a d-c. measurement. For potential curves an observation is taken of a d-c. source of known voltage; for current curves similarly an observation is taken of a measured d-c. current. During these d-c. measurements the resistances in the vibrator circuit should be the same as used during the observations. If a number of observations are taken at different resistances, two or three d-c. calibrations can be made with known resistances, and the calibration for the other resistances computed, assuming the deflection proportional to the total resistance of the vibrator circuit. When potential and current transformers are used, the voltages and currents as directly measured are reduced to terms of line voltages and currents by the transformer ratios. In case of short-circuit currents, where it is not practicable to obtain d-c. currents comparable in amount with those of the observation, a measured d-c. voltage, as from dry cells, can be applied to the oscillograph leads detached from the shunts, the corresponding currents being computed from the shunt resistances.

In many cases, the curve is self-calibrating, a portion of the curve being at a constant d-c. value which can be measured by d-c. instruments, as for instance on generator-field voltage and current curves of short-circuits.

COST OF OSCILLOGRAPH (Pre-war cost).— An oscillograph complete with a three-element electromagnet galvanometer, optical system, shutter and shutter-operating mechanism, motor and countershaft, photographic and tracing attachments, six film-holders, and the following repair parts: 6 extra suspension strips; 6 vibrator mirrors, 1 box special gold-leaf fuses, 1 bottle mirror cement, 1 bottle damping liquid, costs about $650.

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PAPER, IMPREGNATED. — (*See also Insulating Materials, Wires and Cables, Insulated.*) Wires and cables may be insulated by winding paper ribbon helically around the conductor in successive layers until the desired thickness of insulation is obtained. The paper-covered cable, after being thoroughly dried in a hot vacuum dryer is immersed in oil until saturated, and then passed through a lead press, which covers it with a continuous sheathing of lead. The paper serves the triple function of affording a conveyance for the oil, adding to the insulating value of the oil, and being a mechanical separator between the conductor and sheath.

THE PAPER. — The most commonly used paper for such cables is that made of Manila hemp or musa textilis, a fiber grown in the Philippine Islands for rope making. The original fibers are about 6 millimeters long and have a diameter of 0.024 millimeter but the length is materially decreased in the process of beating described below. The paper made from this fiber is not necessarily the best for cable insulation, but the fact that it has stood the test of twenty years use, makes American manufacturers slow to try others. Hemp papers are often called Manilas, and at the present time mixtures of hemp and musa have come into general use, a common proportion being one of hemp and two of musa. Jute is sometimes added, but as it is not permanent and adds nothing to the strength, it should be avoided. German manufacturers are using paper containing long fiber wood-pulp in large quantities or even made exclusively of such wood-pulp, and claim that they are better than Manila papers (C. Beaver). American manila paper used for cables is made of old Manila rope.

MANUFACTURE OF THE PAPER. — The process of manufacturing Manila paper is as follows: The fibers are cut up, placed in a boiler with lime and caustic soda and boiled under a pressure of from 30 to 50 pounds for between 5 and 10 hours. They are then emptied out, washed free of alkali and put in a beaker where the fibers are partially disintegrated and washed. Sometimes bleaching is performed at this stage, by adding a solution of chloride of lime and again washing. The material is then put into the beater, where it is reduced to the condition necessary for the paper machine. After suitably diluting the material it is passed over sand tables, where gritty matter is deposited and then through strainers, where any coarse particles are retained. It then passes in a continuous flow to the endless wire of the Fourdrinier machine, where the fibers are deposited in the form of a sheet; then to the couch rolls and to the press rolls, where the water is squeezed out. Finally it passes over a series of drying cylinders where it emerges dry and is taken upon reels.

NECESSARY PROPERTIES OF THE PAPER. — The paper should have the following qualities.

1. Porosity, in order that it may hold a large amount of oil.

2. Strength and elasticity, in order that it may not break either while being wound or while suffering shrinkage in the desiccator or while being installed. Impregnation with oil should not impair the strength.

3. Freedom from excess of mineral matter, not only to give greater porosity but to avoid electrical weakness.

4. Freedom from alkalies, in order that the impregnating oils shall not be saponified.

5. Freedom from metallic salts, such as chlorides and sulphates, in order to avoid short-circuiting the oil by an electrolyte and also to avoid injuring the conductor.

STRENGTH OF THE PAPER. — The tensile strength of Manila rope paper is usually about 6000 pounds per square inch, and the elongation at the

breaking-point is about 3 to 4 per cent. The resistance to folding and to tearing are, however, much more important than the resistance to straight pulling, but as no standard methods of test have been agreed upon, it is not practicable to give average values for these quantities. The tearing strength of paper 8 mils thick is between 6 and 18 ounces, depending upon its humidity. The maximum strength is obtained with about 12 per cent of moisture.

OIL FOR IMPREGNATING. — Two classes of oils are used for impregnating the paper, rosin oil containing rosin in solution, and a special grade of carefully dried mineral oil or petrolatum, with or without rosin and other substances in solution. The rosin oil has higher dielectric strength at low temperatures, but does not maintain it at high temperatures; furthermore it occasions greater dielectric losses, an important consideration in high voltage cables.

SPECIFIC RESISTANCE. — The specific resistance of impregnated paper, depends upon its dryness and upon the nature and quantity of the substances dissolved in the oil. The value of K in the formula $M = K \log \frac{D}{d}$ (see article on rubber) varies from 500 to 1500, the usual value being about 800.

POWER FACTOR. — The power factor depends greatly upon the temperature, the dryness and the resistivity of the impregnating compound. In the case of paper impregnated with petrolatum compound, the accompanying values are typical.

Temperature, deg. C.	Power factor, per cent	
	Minimum	Maximum
40	0.4	2.5
60	0.9	7.5
75	1.5	13.0
80	1.7	15.0
85	2.0	18.0
100	3.4	30.0

TEMPERATURE COEFFICIENT OF RESISTANCE. — The effect of temperature changes upon the resistance of oiled paper is greater than upon rubber and less than upon varnished cloth. The table below is representative of a typical paper insulation, but the properties of different makes vary considerably.

Temperature °F.	Per cent of resistance at 60°
60	100
65	62.5
70	42.6
75	30.0
80	23.3
85	18.5
90	15.2

DIELECTRIC STRENGTH. — The dielectric strength of impregnated paper insulation is a very indefinite quantity. It is greatly reduced by crinkling and by the presence of moisture and it decreases rapidly with increasing temperature. The dielectric strength across the cellulose fiber is much greater than along them. It is therefore not surprising to find different experimenters reporting widely different values of this quantity. Thus, E. Jona says that the average commercial impregnated paper subjected to electric stress for an hour with progressingly increasing potential, will stand from 8 to 10 kilovolts (effective a–c.) per millimeter and that it is not uncommon to find samples with 20 or 30 per cent greater dielectric strength. E. J. Berg gives 250 to 300 kilovolts per inch, or 10 to 12 kilovolts per millimeter. P. Human says that high-grade

impregnated paper will stand 20 kilovolts per millimeter. The usual testing strength H, in the formula

$$H = \frac{E}{r \log_\epsilon \frac{R}{r}}$$

is about 4 kilovolts per millimeter or 100 kilovolts per inch, but the validity of this formula when applied to paper saturated with ionizable oils is very doubtful. F. Fernie (Beauia, Sept., 1921), says that the failure of single-conductor impregnated paper cables occurs when the stress *at the sheath* is about 100 kv. per cm. as calculated by the formula

$$H = \frac{E}{R \log_\epsilon \frac{R}{r}}.$$

In the case of triplex sector cables, failure occurs when the average stress between conductors exceeds about 100 kv. per cm. When impregnated paper is punctured electrically, the paper itself chars, so that even though the oil tends to flow into and repair the gap, the short-circuit is maintained (*see Wires and Cables, Insulated*).

SPECIFIC INDUCTIVE CAPACITY. — The specific inductive capacity of dry Manila paper (without oil) is from 1.7 to 1.8 while that of Manila paper impregnated with petrolatum compound is between 2.75 and 3.5 at ordinary temperatures, when measured with alternating current.

SPECIFICATIONS. — Specifications for impregnated paper insulation for wires and cables will be found in the article on *Wires and Cables, Insulated.*

BIBLIOGRAPHY. — Beadle, C. and Stevens, H. P., *The Composition and Durability of Cable Papers*, Electrician (London), Vol. 63, 1909; Cross, C. F. and Bevan, E. J., *Paper Making*, London, 1916; Dawe, E. A., *Paper and its Uses*, London; Sindall, R. W., *Paper Technology*, London; Sutermeister, E., *Chemistry of Pulp and Paper Making*, New York; Witham, J. S., *Modern Pulp and Paper Making*, New York, 1920.

PERMUTATIONS AND COMBINATIONS. — (*See also Factorials.*) Each of the *arrangements* which can be made by taking some or all of a number of things is called a permutation.

Each of the *groups* or *selections* which can be made by taking some or all of a number of things is called a combination.

The number of permutations of n things taken r at a time is

$$n(n-1)(n-2)\ \dots\ (n-r+1).$$

The number of combinations of n things taken r at a time is

$$\frac{n(n-1)(n-2)\ \dots\ (n-r+1)}{r(r-1)(r-2)\ \dots\ 3.2.1}.$$

The number of combinations of n things taken r at a time is equal to the number of combinations of n things taken $(n-r)$ at a time.

PHASE CONVERTERS AND PHASE BALANCERS. — (*See also Generators, Alternating-current; Motors, Polyphase Induction; Motors, Synchronous; Transformers.*) — A phase converter is a machine for converting single-phase power into polyphase power, or vice versa. A phase balancer is a machine for balancing the load on one or more polyphase generators when these generators are supplying an unbalanced load to the circuit to which they are connected.

PHASE CONVERTER. — The phase converter is a single unit two-phase induction motor, or a two-phase synchronous motor with an extra heavy squirrel cage.

Polyphase to Single-phase Conversion. — It is well known that a two-phase induction motor or synchronous motor after once started will continue to run on one phase. The disconnected phase of the motor may be used as a source of single-phase power. The voltage and current of this phase are nearly 90 degrees displaced from the voltage and current of the phase connected to the supply.

In order to change from two-phase to single-phase the connections shown in Fig. 1 are used. Phase A of the converter is connected across one phase of the supply and phase B in series with the load across the other phase. The voltage of B adds to the voltage of the supply, making the total voltage at the load $2V$. When the current in the load is I the volt-amperes in B are VI, and the volt-amperes in phase A are approximately VI. The volt-ampere capacity of the machine must equal the volt-amperes of the load. If the current or the power-factor of the load changes, the currents and power-factors of phases A and B of the machine change an almost equal amount. The action of this machine is entirely automatic, no complicated regulators being required to control its action.

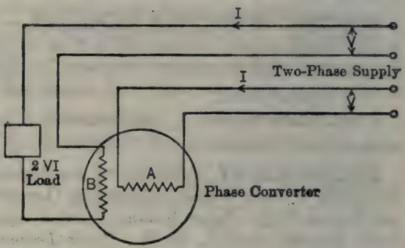

Fig. 1.

The machine is started as a quarter-phase induction motor, and after it is up to speed one of the phases is reversed and connected to the supply in series with the load. The converter is located near the single-phase load. The phase converter is well adapted to supply electric furnaces or electric welding outfits with single-phase power from polyphase systems.

Single-phase to Polyphase Conversion. — The Norfolk and Western Electric Locomotives receive single-phase power through a single trolley. Three-phase induction motors are used to drive these locomotives. The connecting link between the single-phase and polyphase system is a two-phase induction motor operated as a phase converter. The connections to change from single-phase to an approximately balanced three-phase system are shown in Fig. 2.

The machine is driven by power supplied to phase A. Phase B corresponds to the teaser coil of a transformer T-connection. This phase has an e.m.f. induced in it of the proper magnitude and phase position to make the voltages across the three phases approximately balanced.

The phase converters on the Norfolk and Western are started by single-phase commutator motors. The operating speed of the converter is high, permitting a design of minimum weight for a given output. The converter in

addition to furnishing power to polyphase motors may be used to drive auxiliary apparatus. Polyphase motors receiving their power from phase converters may be used for regenerative braking.

PHASE BALANCER (Fig. 3).—The so-called shunt phase balancer consists of a large and a small unit mounted on the same shaft. The large unit has the same capacity as the unbalanced load, and resembles a synchronous motor, with an extra heavy squirrel cage attached to the rotor. The small unit has an ordinary polyphase winding on the stator, and two separate windings displaced 90 electrical degrees on a cylindrical rotor. The rotor for the small machine is wound for the same number of poles as the large unit. The strength and position of these poles is determined by relative amounts of the excitation furnished by two exciters connected to the separate rotor windings.

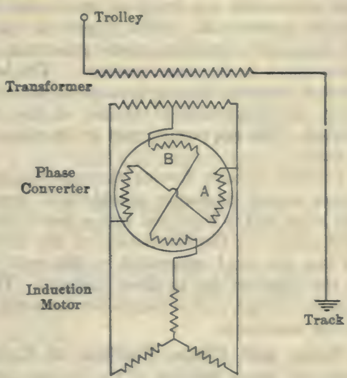

Fig. 2.

The stator windings of the two units have the same current rating and are connected in series in opposite phase rotation as shown in Fig. 3.

The phase balancer connected in this way to an unbalanced polyphase system will, if properly adjusted, take balanced current from the system and enable the main generators to deliver balanced currents. This machine will automatically and almost instantly balance any single-phase load, at any power-factor, on any phase, provided the net unbalancing is within the capacity of the machine. It is well adapted to cases where a large single-phase load is taken from polyphase generators for traction purposes. For example, the Philadelphia Electric Co. furnishes single-phase power to the Paoli Division of the Pennsylvania R.R. The outputs of the polyphase generators are balanced by two 5000 kv-a. phase balancers.

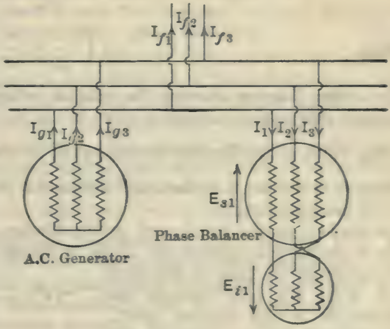

Fig. 3.

The currents taken by a phase balancer can be resolved into "synchronous" and "inverse" components. The synchronous components have the same phase rotation as the bus-bar voltages and are required to keep the machine turning. The inverse components have a phase rotation opposite that of the bus-bar voltages, and are responsible for the balancing action.

It is well known that the addition of an unbalanced polyphase load to a

balanced polyphase load of inverse phase rotation results in a balanced polyphase load. With respect to the synchronous component of the current the large unit acts like a synchronous motor. By proper excitation the synchronous component can be made small and of negligible value compared to the inverse component. With respect to the inverse component of the current the large unit acts like a motor generator. That is motor and generator action takes place in the same machine. The large unit of the phase balancer with inverse currents flowing through it serves as a medium for transferring power from the under-loaded phases of the system to those which are overloaded. The net power input to the phase balancer due to the inverse currents is zero.

Inverse currents flowing through the armature of an ordinary synchronous motor would encounter considerable impedance. To reduce this impedance the large unit is equipped with a heavy squirrel cage on the rotor. The small unit assists in the circulation of the inverse currents through the balancer. In fact the e.m.f. generated by this unit overcomes the total impedance drop in the balancer due to these currents. This drop is small compared with the supply voltage, and accounts for the lower voltage rating and smaller capacity of the small unit.

Value and Phase Position of Currents in Balancer. — The magnitude and phase position of the inverse currents required to balance an unbalanced polyphase system depends on the magnitude of the unbalancing. If I_{f1} and I_{f2} represent the feeder currents in two of the feeders in Fig. 3, the value of inverse current required to balance the system, in vector notation, is:

$$I = \frac{-j\rho^2 I_{f_1} + j I_{f_2}}{\sqrt{3}},$$

where

$$\rho = \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right).$$

The vector diagram showing the relation between the supply voltages, feeder currents and inverse currents is given in Fig. 4.

The value of the inverse current taken by the balancer is also equal to $\frac{E_i}{Z}$, where E_i is the e.m.f. of the small unit and Z is the impedance of the balancer to the inverse currents.

If the load on the unbalanced system changes, the value and the phase position of the inverse currents are likely to change. It is evident that automatic regulators in the field of the small unit must not only regulate the magnitude of the induced e.m.f., but also its phase position with respect to the bus-bar voltages. The magnitude of the e.m.f. of the small unit is determined by the current in the rotor (field) windings, and the phase position of the e.m.f. is determined by relative values of the currents in these windings. The entire action of the small unit is controlled by two special voltage regulators. The regulators in the Philadelphia Elec-

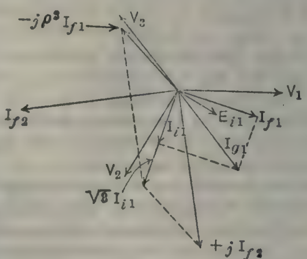

Fig. 4.

tric installation each control a 1.6 kw. exciter and each small exciter controls a 50 kw. exciter.

The phase balancer is comparatively easy to operate. It may be brought up to speed by induction motor action. When started in this way a special switch must be provided to change the connection between the machines, so they will have the same phase rotation, and a proper starting resistance must be inserted in the rotor of the small unit.

The phase balancer may be installed in the power house or placed anywhere along the line. A phase balancer when installed should be protected from short circuits by suitable relays and circuit breakers. The impedance of a phase balancer to inverse currents is very small and a short circuit on one of its phases might ruin an unprotected machine.

Rating, Dimensions and Weight of the Phase Balancers of Philadelphia Electric Co. — Large unit 5000 kv-a. 6-pole, 13,200-volt, 3-phase 25-cycle, 219 amperes.

Small unit 550 kv-a., 6-pole, 840-volt, 3-phase, 25-cycle, 219 amperes.

One of these balancers will balance 5000 kv-a. continuously, 6000 kv-a. for one hour, and 12,000 kv-a. for 5 minutes.

Weight of complete set 172,660 pounds.

Overall dimensions, 20 feet 9 inches by 12 feet 2 inches by 10 feet 1 inch high.

COMPARISON OF METHODS FOR OBTAINING SINGLE-PHASE POWER. — A power company furnishing single-phase power to its customers can do it in the following ways:

1. By a single-phase alternator driven by a steam turbine.
2. By a single-phase alternator driven by an induction motor.
3. By a phase converter connected to a polyphase supply.
4. By taking single-phase directly from the bus-bars of a polyphase system and balancing the generator outputs with a phase balancer.

Method 1 is reliable, but not very economical, because it is an established fact that it is cheaper to generate polyphase power than single-phase power. Method 2 has the disadvantage that the single-phase system is not independent of the polyphase system. Method 3 is good where the load is sufficient to permit the purchase of the converter. This method does not enable power-factor correction. Method 4 is most flexible, because the single-phase load can be taken from any phase. With this method power-factor correction is possible, but has the disadvantage of requiring expensive regulating apparatus.

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PHOTOMETRIC QUANTITIES. — *(See also Illumination, Laws of; Photometry; Vision, Laws of.)* The names and sense of the common photometric units are conventional, though a uniform value of the basic unit of the system, viz., the international candle, was established in 1909 by the national standards laboratories of Great Britain, France and the United States. The standard unit in Germany is the Hefner unit, but the relation that 1 Hefner unit equals 0.9 international candle was officially recognized. The terminology and definitions that follow are in agreement with those recognized by the Geneva Congress in 1896 and the proposals made by the Committee on Nomenclature and Standards of the Illuminating Engineering Society in 1912 *(Trans. Ill. Eng. Soc., Vol. 7, p. 723. See also Standardization Rules of the A.I.E.E.)*

LIGHT FLUX is a measure of the rate of flow of light from a luminous body. It is not identical with the flow of radiant energy but is an evaluation of radiation in terms of the corresponding light sensation. Light flux is proportional to two factors, power radiated and a stimulus coefficient $K\lambda$. The stimulus coefficient varies with the wave-length of radiation as shown in Fig. 1 in the article on *Vision, Laws of*. *(See also Phil. Mag., Vol. 24, p. 853.)*

Lumen. — The lumen, or unit of light flux, denotes the light radiating within one steradian from a source having a uniform luminous intensity of one candle. The steradian, or unit solid angle, is the angular space subtended at the center of a sphere by a portion of its surface equal to its radius squared. An entire sphere includes 12.5664 steradians and a hemisphere 6.2832 steradians. The symbol for luminous flux is F, that for luminous intensity I, and that for steradians ω. The following relations hold:

Flux in any solid angle,	$F = \int I\, d\omega;$
Total flux from source,	$F_s = 12.5664\, I_{ms};$
Flux in lower hemisphere,	$F_{lh} = 6.2832\, I_{mlh};$
Flux in upper hemisphere,	$F_{uh} = 6.2832\, I_{muh};$
Flux in any zone	$F_z = \omega_z\, I_{mz};$

where the I's have the designations given in the following paragraph.

LUMINOUS INTENSITY, commonly termed candle-power, denotes the solid angular density of light flux emitted in the direction considered, or

$$I = \frac{dF}{d\omega}.$$

Although luminous intensity refers in the strict sense to a single direction, mean intensities within certain limits are widely used. The following designations are employed in this article and also the articles on *Illumination, Laws of*, and *Photometry*.

Mean horizontal intensity,	I_h.
Mean spherical intensity,	I_s.
Mean zonal intensity,	I_{mz}.
Mean upper hemispherical intensity,	I_{muh}.
Mean lower hemispherical intensity,	I_{mlh}.

International Candle. — The international candle is the official unit of luminous intensity in France, Great Britain and the United States. The Hefner unit, which is standard in Germany equals 0.9 international candle. For further discussion of photometric standards see *Photometry*.

Distribution Curves. — The luminous intensities at various angles about a light source are commonly represented by polar curves of horizontal and vertical

distribution. Caution should be observed in interpreting such curves. The mean horizontal intensity is equal to the mean polar radius of the curve of horizontal distribution. The mean polar radius and the inclosed area of the curve of mean vertical distribution are entirely lacking in significance. For methods of computing mean intensities from vertical distribution curves, see *Illumination, Laws of.*

Spherical Reduction Factor. — This factor is the ratio of the mean spherical intensity of an illuminant to its mean horizontal intensity.

ILLUMINATION denotes the density of light flux intercepted by a surface or traversing an area in space. The unit of illumination corresponds to unit flux per unit area and is commonly one lumen per square foot, or foot-candle. The metric unit of illumination is the meter-candle, or lumen per square meter, sometimes called the "lux." Based on a suggestion by Blondel, the unit of one lumen per square centimeter is designated by the title "phot." One foot-candle is 10.764 meter-candles and one meter-candle is 0.0929 foot-candle.

For methods of measuring illumination see *Photometry;* for the calculation of illumination, see *Illumination, Laws of;* for values of illumination required for various purposes, see *Vision, Laws of.*

INTRINSIC BRILLIANCY OF COMMON ILLUMINANTS

Illuminant	Candles per sq. in.	Candles per sq. cm.
Crater, carbon arc	84,000	13,000
Magnetite arc	4,000	620
Nernst glower	3,010	470
Incandescent electric lamps:		
Tungsten, 1.25 watts per c-p.	1,060	164
Graphitized carbon, 2.5 watts per c-p.	750	120
Tantalum, 2 watts per c-p.	580	90
Carbon, 3.1 watts per c-p.	485	75
Carbon, 3.5 watts per c-p.	400	63
Acetylene flame, 1-foot burner	53	8.2
Acetylene flame, 0.25-foot burner	33	5.1
Welsbach mantle	31	4.8
Welsbach mantle, mesh	56	8.7
Mercury arc	14.9	2.3
Kerosene flame	9.0	1.4
Gas flame	2.7	0.4
Frosted tungsten lamp, tip	1.67	0.26
Frosted tungsten lamp, side	6.0	0.93

BRIGHTNESS OR INTRINSIC BRILLIANCY is the luminous intensity per unit area of a surface projected on a plane normal to the line of sight. It is measured in candles per square inch or per square centimeter of projected area. Let b denote surface brightness, S the area and θ the angle between the normal to the surface and the line of sight, then

$$b = \frac{dI}{dS \cos \theta}.$$

The specific luminous radiation is closely akin to surface brightness as it denotes the flux emitted per unit area. If the emission agrees with the cosine

law the specific luminous radiation E' and the surface brightness normal to the surface b_0 bear the relation

$$E' = \pi b_0.$$

The brightness of illuminated surfaces is conveniently expressed in lumens emitted per square foot, but this should be clearly distinguished from illumination, or lumens received per square foot.

Mean values of brightness or brilliancy of various illuminants are given in the preceding table (*Ives and Luckeish, Elec. W., Vol. 57*).

The presence in the field of view of illuminants of high brilliancy depresses to a marked degree the sensibility and acuity of vision. The upper limit of brightness consistent with best vision has been variously estimated between 4 and 7.5 candles per square inch. (*See Glare in article on Vision, Laws of.*) For method of measurement of surface brightness see Ives and Luckeish, *ref. cit.*

REFLECTION OF LIGHT. — Reflection is regular or diffuse according as the reflector is a polished surface or a matte. Regular reflection is characterized by equal angles of incidence and reflection, the formation of images and the invisibility of the reflecting surface. Diffuse reflection scatters light in all directions, produces no images and renders the reflecting surface luminous. The intensity of light reflected by a perfect matt surface varies in proportion to the cosine of the angle of departure from the normal, and the total flux reflected equals π times the normal intensity. However, some regular reflection always accompanies diffuse reflection and these laws can be applied only approximately. Fair diffusing plates can be prepared from barium sulphate, magnesium oxide or other white materials of extremely fine and even grain. Opal glass with both surfaces carefully depolished is fairly satisfactory for diffuse reflection and transmission of light. Wall coverings of paper, plaster, fabric, paint and kalsomine deviate considerably from the cosine law of diffusion and its corollary and caution must be observed invariably in applying these laws to calculations of illumination. All materials not white, gray or black are selective and vary in their reflecting power with the spectral composition of the light received.

The following approximate values of reflection factors have been collected from various sources and are useful in calculations dealing with reflected light:

Reflector	Factors	Reflectors	Factors
Polished silver	0.92	Orange yellow paper	0.34
Silvered mirror	0.80	Light green paper	0.25
White blotting paper	0.80	Light pink paper	0.25
White bond paper	0.75	Light blue paper	0.18
White kalsomine	0.75	Medium blue paper	0.12
Flat white paint	0.66	Dull green paper	0.08
Chrome-yellow paper	0.62	Light red paper	0.10
Cream paper	0.56	Medium red paper	0.08
Flat cream paint	0.53	Medium brown paper	0.08
Flat ivory paint	0.50	Deep red paper	0.06
Light buff paper	0.45		

The above values refer to light having the spectrum of commercial incandescent illuminants. In general flat paints are slightly below papers of the same tone in reflecting power, whereas glossy paints exceed papers by 0.10 to 0.15 on account of regular reflection.

For further information on brightness values and on the percents of reflected

light from various surfaces see "Principles of Interior Illumination" by Cravath, Harrison and Pierce, and "Modern Lighting Accessories," by W. F. Little, *Illuminating Engineering Practice*, 1916 Lectures, I. E. S.—U. P.

A device for the measurement of diffuse reflection factor applying to matte surfaces which follow Lambert's law fairly closely, is described by Nutting, *Trans. Ill. Eng. Soc., Vol. 7, p. 412.* The more satisfactory method however is that in which the integrating sphere is used.

TRANSMISSION OF LIGHT.— Transmitting medias possess qualities quite analogous to reflecting surfaces with respect to regular and diffuse transmission and selective absorption depending on color. The absorption of light in the varieties of glassware most used in the lighting art is approximately as follows (*see Trans. Ill. Eng. Soc., Vol. 6, p. 98*):

Kind of glass	Absorption in per cent	Kind of glass	Absorption in per cent
Clear	5–12	Ground	20–30
Light sand blast	10–20	Medium opalescent	25–40
Alabaster	10–20	Heavy opalescent	30–60
Canary	15–20	Flame	30–60
Light blue alabaster	15–25	Signal green	80–90
Heavy blue alabaster	15–30	Ruby	85–90
Ribbed	15–30	Cobalt blue	90–95
Opaline	15–40		

It should be remembered in interpreting the values in this Table, that the absorption percentages are, in reality, 100 per cent less the transmission, which in itself, does not represent true absorption, but loss by reflection and absorption taken together. For example, the true absorption of clear glass is probably in the neighborhood of 1 per cent or less, depending upon its thickness, but losses take place also by reflections of the incident light away from the surface, which make the apparent absorption percentages higher, as in the above table.

BIBLIOGRAPHY. — Johns Hopkins University *Lectures on Illuminating Engineering*, Vol. 1; Barrows, W. E., *Light, Illumination and Photometry;* Liebenthal, E., *Praktische Photometry;* Wickenden, W. E., *Illumination and Photometry.* Numerous papers in *Trans. Ill. Eng. Soc., Bull. Bur. Stand.*, and *Elec. World;* also *Bibliography* in the article on *Photometry.*

PHOTOMETRY. — (*See also Illumination, Laws of; Vision, Laws of; for definitions of quantities and symbols see Photometric Quantities.*) Photometry is the science of light measurement. Its chief aspects are the determination of (*a*) the intensity of light sources in definite directions, (*b*) mean vertical light distribution, (*c*) the mean intensity and light flux emitted within specified limits of distribution, (*d*) the illumination of surfaces, (*e*) the reflecting properties of surfaces and (*f*) the analysis of the color of light. Its methods are essentially visual and consist in equating the brightness of two diffusing surfaces. As an instrument of physical measurement the eye lacks precision as an estimating device, but serves well in the judgment of equality. Photometric devices are of four general types, according to the method of comparison employed, viz., (1) by equality of brightness of two surfaces visible simultaneously, (2) by equality of contrast between two pairs of surfaces differing slightly in brightness, (3) by the disappearance of flicker when two surfaces are viewed in rapid alternation, and (4) by acuity of vision. The latter process does not admit of precision and is used only for rough comparisons.

Methods of Obtaining Photometric Balance. — The methods of obtaining photometric balance are: (1) varying the distance of one or both light sources from the surfaces compared, according to the law of inverse squares (*see Illumination, Laws of*); (2) varying the angle of incidence of light according to the cosine law (*ibid*); (3) varying the proportion of light received by means of a sector disk or slit and collimator; and (4) by means of the Nicol prism. In cases (1) and (2), given two diffusing surfaces illuminated to equal brightness b by respective light sources of intensities I_1 and I_2; the reflection or transmission coefficients of the surfaces as K_1 and K_2; the angles of incidence as α_1 and α_2; and the distances from the light sources d_1 and d_2 respectively; then if $\alpha_1 = \alpha_2$ and $K_1 = K_2$ and I_2 is the known intensity of a standard lamp,

$$I_1 = \frac{I_2 d_1^2}{d_2^2}.$$

If α_1 and α_2 or K_1 and K_2 differ slightly, a sensibly correct result is obtained by interchanging the light sources for half the observations and using the mean distances in the inverse square relation.

STANDARD LAMPS. — A primary standard possesses two essential qualities, definiteness and reproducibility from specifications. None of the existing standards of luminous intensity meet both requirements. Of the flame standards, the Hefner and pentane lamps are approximately reproducible and fairly definite with standard atmospheric conditions and fuel. The incandescent electric lamp is capable of definite calibration, but is not strictly reproducible. The present unit of luminous intensity, the international candle, is derived from the mean intensity of a group of incandescent electric lamps maintained by the U. S. Bureau of Standards, in coöperation with similar custodians in France and Great Britain.

Hefner Standard. — The Hefner standard which is the official standard of Germany and which has been extensively used in the United States, is a wick lamp burning amyl acetate of definitely specified chemical and physical properties. Its standard intensity is 0.9 international candle. Expressing the actual intensity as I, corrections for various atmospheric conditions are made as follows:

For variations from the standard flame height of 40 mm.

$$I = 1 + 0.025\,(h - 40) \text{ or } I = 1 - 0.034\,(40 - h)$$

for heights greater or less than 40 mm. respectively, where h is the flame height in millimeters. For barometric variations

$$I = 1 + 0.00011\ (b - 760),$$

where b is the barometric pressure in millimeters. For atmospheric humidity

$$I = 1.049 - 0.0055\ x,$$

where x signifies the liters of water vapor per cubic meter of air at 760 mm. and free from CO_2.

For atmospheric vitiation by CO_2

$$I = 1.012 - 0.0072\ y,$$

where y signifies the liters of CO_2 per cubic meter of dry air.

The objections to the Hefner standard are its low intensity, its reddish color, its flabby flame and its sensitiveness to variation in flame height. The element of uncertainty associated with it is at best not less than 2 per cent.

Pentane Standard. — The pentane standard, as represented by the Vernon-Harcourt type, is essentially an argand burner supplied with pentane-air gas and preheated air. The fuel is formed by passing air over pentane in a saturator box subdivided by baffles. The burner is surmounted at a height of 47 mm. by a cylindrical chimney. An annular chamber surrounding the chimney supplies the interior of the flame with preheated air. The flame is shielded from drafts by a conical, blackened hood having a slit at one side through which the flame is exposed. The chimney is fitted with a mica window showing a gauge line to which the flame height is closely adjusted. Under standard atmospheric conditions, viz., a barometric pressure of 760 mm. and a humidity of 8 liters of water vapor per cubic meter of air the flame should have a horizontal intensity of 10 candles. Experience indicates, however, that it is usually less by 2 to 4 per cent and that it is desirable to calibrate the lamp against a more definite standard. Variations from the above standard atmospheric conditions may be corrected by the equation:

$$I = I_n\,[1 - 0.00567\ (e - 8) + 0.0008\ (b - 760)],$$

where I is the actual intensity, I_n the standard intensity, e the liters of water vapor per cubic meter of air and b the barometric height.

The color and intensity of the flame are convenient for practical purposes, especially for the measurement of the illuminating power of gas. An added advantage in the testing of luminous flames of all sorts resides in the fact that their intensities are affected by atmospheric variations in a manner corresponding quite closely to the changes in the pentane lamp, whereby somewhat troublesome corrections are avoided.

Carcel Lamp. — The Carcel lamp is the recognized standard of luminous intensity in France. It has a central-draft ring burner fitted with a wick of the light-house type burning colza oil. Its standard intensity is 9.61 international candles.

Standard candles are now discredited for all accurate work, though still used extensively in the routine testing of gas.

Incandescent Electric-lamp Standard. — The incandescent electric lamp is superior to all other working standards where corrections for flame luminosity due to atmospheric variations are not required. Carefully selected carbon-filament lamps aged by burning until the hot resistance is constant maintain their candle-power sensibly unchanged for a period of 10 hours or more. With the most precise photometric apparatus such lamps may be standardized

with the mean error of any single determination not exceeding say from 0.5 to 1.0 per cent.

Lamps are standardized for use in a fixed position or in rotation about the principal axis. No difference exists between the precision of the two methods. Lamps are standardized in terms of a definite voltage or current and are only incidentally calibrated for power consumption. It is desirable to have lamps standardized in terms of both voltage and current for the constancy of the lamp can be relied upon as long as both standard conditions exist simultaneously. Carbon filament lamps were formerly widely used as standards, although the Mazda B lamp is even more satisfactory from the standpoint of permanency than the carbon, and the Mazda C lamps are now used with very good results. It is best to keep on hand a group of well-seasoned lamps of all commercial types and to select from these the most appropriate working standard for calibration against the primary carbon standard, so that the latter is used only for checking.

Other Standards have had a very limited use. Among these are the acetylene flame; the Methven screen, which is essentially a sharply-defined portion of a luminous gas flame, calibrated against a primary standard; and the Elliot lamp, which is a kerosene lamp of the student type having a limited portion of its flame exposed by a screen. The utility of these standards is largely in the routine testing of gas.

Relations of Luminous Standards. — The relative intensities of the several standards under standard conditions are given in the following table:

	International candle	Hefner	10-c-p. pentane	Carcel	Bougie decimale	English candle	German candle
International candle	1.00	1.11	0.10	0.104	1.00	0.96	0.95
Hefner	0.90	1.00	0.09	0.0936	0.90	0.864	0.855
10-c-p. pentane	10.00	11.11	1.00	1.04	10.0	9.6	9.5
Carcel	9.61	10.66	0.96	1.00	9.6	9.24	9.19
Bougie decimale	1.00	1.11	0.10	0.104	1.00	0.96	0.95
English candle	1.04	1.154	0.104	0.1	1.04	1.00	0.98
German candle	1.055	1.17	0.105	0.109	1.055	1.02	1.00

SIGHT BOXES. — A photometric sight box consists of two diffusing surfaces and accessories to facilitate the comparison of brightness. The types described following are of greatest utility.

Bunsen Sight Box. — The bunsen screen is a disk of white diffusing paper, a well-defined region of which is made translucent by impregnation with paraffine or other material. The disk is set transversely in a sight box of blackened interior, as shown on the plan in Fig. 1. Light from the sources to be compared enters the apertures A–A, and falls normally on the disk surfaces. Dihedral mirrors M_1 and M_2 enable both sides of the disk to be viewed at the sight tube T. The opaque portion of the disk reflects diffusely, while the translucent region partially reflects and partially transmits the light received. A photometric balance exists when the two sides of the disk appear alike. If both lights are alike in color and the absorption of both regions of the disk is equal, the boundary disappears and both sides appear uniformly bright. With unequal absorption, balance exists when equal contrast exist between the opaque and translucent regions on both sides of the disk. The contrast principle is of distinct advantage with slight color differences. The sensitiveness of the screen depends largely on the definition of the boundary of the impregnated portion.

Leeson Disk. — This is a useful modification of the Bunsen type built up by pasting opaque paper disks with accurately matched star-shaped apertures on the two sides of disk of translucent paper. By a careful selection of the materials the disk may be made to embody either the equality-of-brightness or the equality-of-contrast principle. The paper used should agree closely with the cosine law of diffusion. The later types of Leeson disks are not pasted, but the two outer thin sheets are pressed tightly against the centre opaque sheet with glass. This makes a very sensitive and satisfactory type of disk.

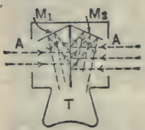

Fig. 1.

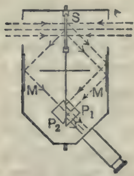

Fig. 2.

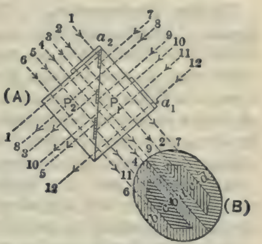

Fig. 3.

Lummer-Brodhun Sight Box. — The plan of this box is shown in Fig. 2. An opaque diffusely-reflecting screen S receives light from the sources to be compared and reflects it along the paths indicated by aid of the mirrors M–M and the prisms P_1–P_2. The prisms present to the eye a composite field in which the brightness of the two sides of S can be conveniently compared. Fig. 2 also shows the arrangement for equality-of-brightness working. The prisms are in optical contact over an elliptical portion of their hypotenusal faces, and the remainder of one is cut away. The central portion of the field is illuminated by direct transmission through the contact area, the outer portion by total reflection from the face of the uncut prism.

For equality-of-contrast working the arrangement of the prisms is as shown in Fig. 3. The hypotenusal face of P_2 is recessed over the area shaded in (B). That of P_1 is plane. Two thin glass absorbing strips a_1 and a_2 are set before the faces of P_1 and P_2 as shown in the plan (A). By tracing the several paths indicated it is seen that the field has the appearance of (B) and that the regions shaded are darkened by the absorption of light in a_1 and a_2. In a state of balance b and d appear equally bright and c and e equally dark in contrast. The degree of contrast created by a_1 and a is ordinarily about 8 per cent, this being in accordance with Fresnel's formula. A method of securing less than 8 per cent would be to place a sheet of glass over the entire side of the cube and to increase the thickness of the glass (either by cementing two pieces together, or by grinding) over the contrast field. A degree of contrast as low as 2 per cent has been secured by this method, which has been found satisfactory.

Accuracy of adjustment and cleanliness of all parts are essential in photometers of the Lummer-Brodhun type.

Comparison of Bunsen, Leeson, and Lummer-Brodhun Boxes. — Bunsen, Leeson and Lummer-Brodhun sight boxes should be so mounted as to permit the complete reversal of the optical system about its axis of symmetry in order that optical asymmetry may be corrected as explained above. The Bunsen and Leeson types are binocular and therefore less fatiguing in a long

series of observations than the monocular Lummer-Brodhun type. Furthermore, they are the more readily balanced with slightly flickering light. The Lummer-Brodhun contrast type excels in sensitiveness and general utility with steady lights of equal and slightly dissimilar color.

It may be stated, however, that the complete reversal system is seldom used in practical photometry, with the exception of measurements of gas flames, where the substitution method cannot be readily followed. In the opinion of practical operators of photometric equipment, there is relatively no difference between steady and flickering light for the different types of photometers.

Flicker Photometer. — The flicker photometer affords the most reliable means of comparing light sources of distinctly unlike color. A field of view alternately illuminated by two such sources displays a flickering appearance which may be due to color dissimilarity or to difference in brightness. Above a moderate rate of alternation the color sensations blend and the disappearance of flicker is a true indication of equal brightness, as conclusively established by Ives, who recommends the following conditions as suited to the best precision: (*a*) a field illumination of 25 meter-candles; (*b*) a photometric field of 2 in. diameter; and (*c*) a background field about 25 in. in diameter surrounding the photometric field and about equal to it in brightness. The latter provision, though not essential to precision, is an aid to comfort.

Reference should also be made to the work done with the Ives-Kingsbury flicker photometer, which has a very ingenious eyepiece, made so that it may be attached to the contrast type of Lummer-Brodhun photometer with the contrast glasses removed. The approximate specifications are as follows:

The flicker field shall subtend an angle of approximately 2 degrees as seen from the eyepiece. The field shall have an effective brightness after allowance for all loss of lens absorption and reflection of 2 ½ millilamberts, and the flicker field is to be surrounded by an illuminated field of approximately the same brightness, the pupil aperture to be 5 millilamberts in diameter.

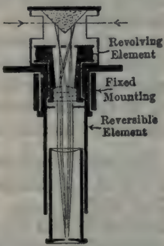

Fig. 4.

Bechstein Flicker Photometer (Fig. 4). — This photometer employs a train of lens and prism oscillating before a fixed diffusing wedge. The field of view consists of a circle and ring, alternately illuminated by the respective sides of the wedge as the lens system revolves. The highest sensitiveness exists at the lowest speed at which the flicker can be made to disappear.

Wild Flicker Photometer (Fig. 5). — The Wild flicker photometer is the simplest of this class. It consists of a disk D of white diffusing paper, one half of which is made translucent by impregnation. This disk is revolved about an axis slightly at one side of the path of light and a mirror M reflects one side of the disk to the sight tube T. A high degree of precision is claimed by Wild for this device.

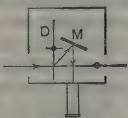

Fig 5.

Color-equalizing Filters. — The simplest method of comparison for lights of dissimilar color is to employ a Lummer-Brodhun photometer in connection with a set of glass filters of graded tints, selecting one which renders the light of the standard lamp equal in color to that of the lamp under measurement. The absorption of each screen for the light of the standard lamp must be determined by means of the flicker photometer, but the inconvenience of using the latter instrument is limited to the process of calibration.

Sector Disk. — The sector disk affords a most convenient means of reducing

the intensity of the light received from an illuminant on the photometric screen. It consists of one or more disks with sector apertures revolving on a common axis. By advancing one disk with respect to the other the net aperture may be altered at will. By Talbot's law the intensity of transmitted to incident light equals the ratio of the total angular opening to 360 degrees. The sector disk has the advantage over other absorbing media in that it is adjustable, is not affected by time, and is independent of color.

Photometric Bar. — The devices above described are best suited for use in connection with a photometric bar or bench, which should be level and straight, and preferably greater than 100 inches in length. A plan of the layout is shown in Fig. 6. For convenience the sight box and at least one of the lamps should be mounted on movable carriages. The bench should be provided with a series of screens D of dead black material having graded apertures along the photometric axis and with solid screens at the ends. These screens should completely occlude from the sight box all extraneous light, and should protect the eye of the operator from the direct light of the lamps. If these conditions are met the photometric room need not be blackened. The sight box should have a dark background, however, and all light in the room should be well diffused. The bar should be provided with a scale of equal divisions and a scale reading directly the ratio of the inverse squares of the distances from the ends of the bar. For use with a standard lamp of definite value in a fixed position a direct reading scale of candle-power is readily obtained from the ratio scale.

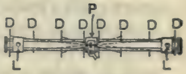

Fig. 6.

Connections for Testing Incandescent Electric Lamps. — In testing incandescent electric lamps the effects of voltage fluctuation must be reduced to a minimum. A storage battery or special generator of very close regulation should be employed if possible. In practically all cases double circuit photometers are used, and the simpler the wiring the better.

Manipulation of Photometric Bar. — Direct comparison between the test lamp and the standard may be made with the two in fixed positions by moving the sight box to a point of balance. In this case half the observations should be made with the sight box reversed. For the substitution method a fixed socket is provided at one end of the scale. The second socket and the sight box are on movable carriages coupled at a fixed distance. A well-seasoned lamp is placed in the movable socket and adjusted to a suitable voltage, which is subsequently held constant. The standard lamp is placed in the fixed socket and a balance secured by setting the movable carriages. The standard lamp is then removed and test lamps substituted in turn, a balance point being observed for each. The intensity of each test lamp equals that of the standard multiplied by the direct ratio of the squares of their distances from the screen at the times of balance. Reversals are unnecessary in the substitution method. The comparison lamp should be checked against the standard at intervals.

For further information on the *substitution method* see Wickenden, W. E., *Illumination and Photometry*, N. Y., pages 56 and 57.

MEASUREMENT OF PHOTOMETRIC DISTRIBUTION. — Mean horizontal intensity is measured with the lamp rotating about its vertical axis. Special mountings, driven by motor or hand wheel, are provided for this purpose. Rotators are made universal by provision for the turning of the lamp by definite angular steps about its luminous center in a vertical plane including the photometric axis. The speed of horizontal rotation should be only sufficient to equalize differences in intensity.

Mirror Rotators. — In testing the light distribution of arcs, heavy reflector

units, gas lamps, etc., which must remain in an upright position, mirror rotators are generally employed to direct the light from any desired vertical angle toward the photometer. A three-mirror device is shown schematically in Fig. 7. A two-mirror arrangement could also be used. In either case the lamp remains stationary, or is revolved about its vertical axis only, while the mirror system is turned by steps about the photometric axis. Numerous other devices for this purpose are described in standard works on photometry.

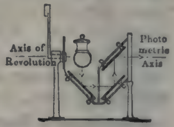

Fig. 7.

INTEGRATING PHOTOMETERS. — An integrating photometer enables the measurement of mean spherical or mean hemispherical intensity, or of the total flux of light from an illuminant, to be made by a single observation.

Sphere Photometry. — Since the sphere is practically the standard method of photometry, the following notes are important in their relation to the practical testing by this method. One type of sphere known as the Ulbricht Integrating Sphere is shown in Fig. 8, in which it will be seen that the apparatus consists of a hollow sphere whose inner surface is coated with a white material giving approximately true diffusion. In the following notes on sphere photometry, the type developed by the Electrical Testing Laboratories will be referred to.

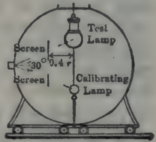

Fig. 8.

Sphere Theory.—The theory of the sphere photometer is that the illumination on any part of its interior surface when a lighted lamp is introduced into it at any location is made up of two parts: (*a*) the direct light from the lamp; and (*b*) the light diffusely reflected from all parts of the sphere wall which is proportional to the total flux. It is only necessary therefore to measure (*b*) and this may be done by putting a translucent milk glass in a window contiguous with the inner wall of the sphere. (*a*) is screened off from the window by an opaque screen. Another method is to measure (*b*) by viewing directly as the test surface a portion of the inner surface of the sphere.

Equipment. — The **Photometer.** — When translucent windows are used, the photometer head may be arranged so that the sphere window forms one side of the photometer field and the comparison lamp window the other side.

In the selection of a **comparison lamp** it is desirable that its construction should be as nearly perfect as possible with the filament lying in a single plane, preferably without an anchor support or with supports such that variable cooling may be obviated.

In connection with the **screen** practice has shown that the best position for the screen inside the sphere is two-thirds the distance from the lamp to the window. This screen should be the smallest that will adequately screen the largest lamp.

The lumen **scale** is computed with relation to its location from the comparison lamp window. This position is fixed and must not be changed.

The **slide aperture** used in conjunction with the 40-inch sphere as developed by the Electrical Testing Laboratories is proportioned to the scale and is made adjustable.

Photometer Calibration. — **Slide Apertures.** — When the photometer is calibrated the apertures are adjusted so that a continuous range is given for the measurement of lamps from 10 to 1000 watts. The size of the lamp bulb and base are likely to introduce discrepancies, however, which prevent the use of this range in its entirety with one setting.

As far as **color** is concerned, for the photometry of Mazda *B* lamps the comparison lamp should be operated at such a temperature or efficiency as to produce a color match with the average size of lamp. In most cases the sphere coating is slightly selective, absorbing more light toward the blue end of the spectrum, therefore the comparison lamp may be operated at a somewhat lower temperature than the test lamp and still give a good color match and prolong the life of the comparison lamp.

When the color match has been secured, the **intensity** may be adjusted by varying the glass either in the comparison lamp window, sphere window or both. All the glasses should be non-selective in so far as practicable. The brightness of the field for best sensibility should be equal to or slightly brighter than the immediate surroundings. In a dark room the average should not be less than 2.5 millilamberts or the equivalent of three foot-candles on a white blotter.

As to **filters** for the photometry of Mazda C lamps, or other lamps where a color difference is involved, glass light filters may be used to correct the color of the test lamp to that of the comparison lamp or vice versa. Theoretically the color of the comparison lamp should remain unchanged and the filter should be placed between it and the window. Under these conditions the transmission of the filter will remain fixed, regardless of the color of the test lamp. However, as a practical matter, it is more convenient to change the color of the light from the test lamp, because Mazda C lamps are usually high in candle-power to begin with, and the filter reduces the intensity of the test surface and also changes the color. Therefore it is preferable to allow the color of the comparison lamp to remain unchanged as the transmission factor of the filter will remain constant regardless of the color of the test lamp. The filter should be placed between the prism and the window and not between the lamp and the window for the reason that light travels more rapidly through air than through glass. Should it be placed between the lamp and window it would be equivalent to increasing the distance between the lamp window thus introducing error. Also the distance between the filter and window should be sufficient to reduce the inter-reflection to a negligible quantity, two inches is usually sufficient. However, this applies only when the transmission of the filter has been determined independent of the photometer, namely, not calibrated in place in the photometer.

Regarding **alignment,** all optical parts, track and comparison lamp carriage should be kept in alignment. A non-uniform field is an indication of poor alignment or soiled test plate. The prism is screened with a small diaphragm but slightly larger than the field so that a small shift in the prism may cause a dark shadow on the field.

In selecting a **coating** for the sphere, a number of points must be borne in mind. The surface must have good diffusing qualities; it must be reasonably non-selective in its absorption; it must have a relatively high reflection factor and it must be permanent in color. These qualifications necessitate a pure white pigment with a colorless binder. A number of experiments have been conducted using zinc oxide, lead oxide, magnesium carbonate and the like, with binders of cellulose, sodium silicate, white shellac, and the like. The Bureau of Standards has developed a formula for a coating which has been found very satisfactory though difficult to make and apply. For all practical purposes a good grade of calcimine carefully applied with a wide camel's hair brush makes a reasonably good coating easily applied and always available. A little Prussian blue mixed with the calcimine will somewhat lower its reflection factor but will make it less selective.

Possible Reasons for Erroneous Results. — Bulb absorption. — The substitution method should be adhered to even more rigidly in the sphere

than on the bar photometer. A discolored or tinted bulb may absorb some of the reflected light which it intercepts thus lowering the photometer setting. This is not an error though it may render an erroneous result. To photometer a lamp of this character, if the substitution method cannot be applied, a checking lamp may be introduced into the sphere and measured, first alone and second with the discolored or tinted lamp cold. The difference in reading represents the per cent of the reflected light absorbed by the latter. In making these measurements, the direct light should be screened off from the discolored or tinted lamp. A simple method is to place the two base to base with the test lamp in the regular socket.

A change in the size or shape of a bulb or base of the test lamp may give a wrong reading. The proper standard lamp or the use of the method described for measuring tinted lamps may be used with good results.

As to the **coating,** it should be kept in mind that the inside surface of the sphere under the best of conditions will collect dust and this accumulation tends to settle in the lower half of the sphere with denser accumulation in the bottom of the sphere. This may change the character of the light distribution between standard lamps and the lamps tested and give fallacious results. The solution of the difficulty is properly to maintain the surface by frequent cleanings or recoatings.

Undue voltage drop in the **lamp socket** contact may give rise to incorrect results. A double center contact should be employed and the shell of the socket kept clean.

Screening. — Photometer. — Stray light reaching the test plate either from an extraneous source or reflected from the photometer lamp may give an incorrect answer. To test for stray light, the test plate should be removed and the eye or a mirror substituted in order to see if light from any source other than the comparison lamp may reach the window. A perfectly protected test plate should not be visible from any position from which light may come other than the comparison lamp.

In screening the **sphere** it must be remembered that it is only the reflected and not the direct light that is to be measured, therefore, it is well to be certain that the screen in the sphere is performing its function. A hole should be provided in the sphere so that the shadow cast by this screen may be seen when the test lamp is in place.

An improperly located **scale** will give fallacious results but it is easily discovered and remedied. A gauge with a forked end to span the comparison lamp and reach the comparison lamp window should be a part of the equipment of every sphere.

Extension of Range. — If lamps are to be tested that are beyond the limits of the scale, the range may be extended in two ways. If the range is to be changed temporarily and but slightly, the comparison lamp voltage may be raised or lowered until the lamps can be read on the scale. Standard lamps are then read and the factor which is obtained, applied. The range may be considerably extended by changing the glass in the comparison lamp window. If the brightness of this window is to be decreased, one thickness of denser opal glass (or several thicknesses) may be employed. On the other hand, if it is to be increased, several thicknesses of ground glass may be substituted. In any case, the distance from the scale to the window should be measured from the first diffusing surface.

Verification. — A slight dust accumulation on the sphere window may cause the light emitted through the slide apertures to vary. If the substitution method is not adhered to, it is necessary to verify these apertures from time to

time. This can be done by using the largest opening as standard and reading on the other scales in turn. If they are not in agreement, the substitution method should be used throughout. Simple and accurate methods of determining the position of the scale, with a moving comparison lamp, is available with this type of photometer.

Standard Lamps. — A very important part of the sphere equipment is a complete range of standard lamps. There should be a sufficient number of lamps to permit the use of the substitution method and the extent to which the substitution method must be followed will depend largely upon the grade of electrical instruments used.

The qualifications for a standard lamp demand that it must pass a most rigid inspection for appearance in every particular. The construction may differ considerably from the same type of commercial lamp, for instance, lamps where copper supports are employed must be so made that each clamp is tight in order to obviate a variable cooling effect. Where spring supports are used, they must be such that too much tension is not placed upon the filament, at the same time they must be tight enough to prevent locked filaments. It is hardly necessary to mention the vacuum for type B lamps and the proper treatment of the type C lamps as any slight defect will impair the constancy of the standard.

Standardization values may be relied upon only for a limited period of time, therefore, lamps should be verified every three months if they are in current use. If they are used only occasionally they may be returned less frequently. To prolong the life as much as possible without materially changing the color, lamps are standardized at slightly lower than rated efficiency. If lamps are subjected to a higher voltage than labelled it is readily seen that the life will be shortened and the standardization values changed. It is good practice, therefore, always to put resistance in the circuit when changing lamps.

PORTABLE PHOTOMETERS. — Weber Photometer. — Portable photometers exist in great variety and have for their purpose the measurement of illumination and of the intensity of light sources in place. The great majority are modifications of the Weber photometer, shown schematically in Fig. 9. This comprises two cylindrical tubes of blackened interior, one fixed and the other attached to it at right angles by means of a sleeve to allow rotation. A Lummer-Brodhun prism device is placed at the junction of their axes, and permits the brightness of two translucent glass plates P_1 and P_2 to be compared at the eyepiece E. P_1 is illuminated by an external source L_1 at a distance d_1. P_2 is movable along the tube by a knurled head and is illuminated by a small standard lamp L mounted in a convenient housing. Assuming both plates to be illuminated by light incident normally at a state of balance, then

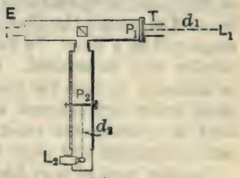

Fig. 9.

$$I_1 = \frac{I_2 T_2}{T_1} \times \frac{d_1^2}{d_2^2},$$

where T_1 and T_2 are the respective brightness coefficients of the two plates. A calibration for intensity measurements is readily obtained by the use of a standard lamp as I_1 and by observing the scale readings d_2 giving a balance with various values of d_1, keeping I_2 constant.

Two methods are available for the measurement of illumination. In the first the terminal tube T is removed and a flush test plate of depolished milk glass fitted in its place. This test plate is placed in the position where the illumination is to be tested and the instrument balanced as usual.

For the second method of illumination measurement the terminal tube is not removed, but the plate P_1 is omitted. A large, white diffusely reflecting card is placed in the position where the illumination is to be tested and the tube T pointed in its direction so that the entire cone of light entering T arises from the test tube. So long as this condition is met the inclination and distance of T is not important.

The first method is preferable where it is possible to make the attached test plate coincide with the position of the test. In the second method it is difficult to avoid interference with light which should reach the test plate.

Sharp-Millar Photometer. (Fig. 10). — This photometer is a modified Weber instrument which is extensively used in America. It comprises an elongated wooden box divided into two compartments, one of which contains a fixed Lummer-Brodhun photometric cube and the other a lamp carriage movable along the box by turning a knurled head H. Observations are made at E at one side of the box. The two compartments are separated by a ground-glass window whose brightness is balanced against illumination from an outside source admitted through an elbow tube T. Stray light in the lamp compartment is screened from this window by a series of diaphragms with central apertures. The elbow tube T is fitted on a collar and may be turned to any desired inclination. At the elbow is a circular, reversible plate one side of which is a mirror for measurements of illumination and the other a white diffusing surface for measurements of candle-power. In the former case the end of the tube is fitted with a flush plate of depolished milk glass. The range of the photometer is controlled by a pair of glass absorbing screens mounted in the compartment with the Lummer-Brodhun cube. These plates are respectively of high- and low-absorbing power. They may be turned so that either one may be used to reduce the illumination of either part of the field. But one may be employed at a time. In this way a range from 0.004 foot-candle to 2000 foot-candles may be secured. In the most recent design the need of a voltmeter or ammeter to keep the standard lamp constant is obviated by a small Wheatstone bridge arrangement with a telephone receiver, whereby the lamp may be kept at constant hot resistance. The Sharp-Millar photometer may be used with a detached test plate similar to that described for the Weber photometer.

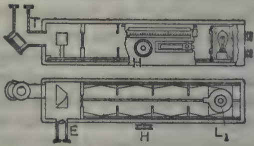

Fig. 10.

Another type of portable photometer which has come into fairly wide use is that known as the Macbeth Illuminometer. This instrument has certain features which differ from the Sharp-Millar type, although in principle the two instruments are fairly similar.

COLORIMETRY. — The Ives colorimeter is an instrument for tri-chromatic color analysis. (*See Vision, Laws of.*) It consists essentially of an oblong box, at one end of which are placed four slits, one clear, and the three others equipped respectively with red, green and blue screens. By means of levers the openings of the three colored slits can be altered to read by scales from 0 to 100. By rotating a wheel of lenses the three colors are mixed. The observer views a divided field, one part consisting of the mixture of the three primary colors and the other of the color to be matched as viewed through the clear slit. To make a measurement, the three colored slits are opened until white is matched, and the scales are set to read 100 for each color. Then any color matched by moving the levers can be read off in terms of the per cent of red, green and blue necessary to match white. The precision of the instrument is from 2 to 5 per cent under favorable conditions.

REFLECTING POWER OF SURFACES. — See section on *Reflection of Light* in the article on *Photometric Quantities.*

PRECAUTIONS IN PHOTOMETRIC OBSERVATIONS. — The eyes of the observer should be constantly shaded from bright light to maintain their sensitiveness in a state of dark adaptation. For best photometric sensibility a screen illumination of about 2 foot-candles is desirable. At low intensities the Purkinje effect (see *Vision, Laws of*) may prove disturbing. Should a daylight photometer be used, a field or screen brightness far in excess of 1.7 millilamberts would not only be advisable but necessary.

The precision of photometric settings may often be improved by a process of narrowing down between points equally out of balance. Many good photometricians reject their first observation in a set as untrustworthy. Not more than three figures in the result are significant.

A photometer bar need not be more than 18 to 20 inches in length. The distance from a large unit to the screen should not be less than about ten times the over-all maximum dimension of the unit under test.

The voltage of electric lamps, or current, in the case of series lamps, should be measured by the most accurate device obtainable. In life tests of incandescent lamps exact regulation of voltage is of the utmost importance and a sensitive, automatic regulator is most desirable. In tests of illuminants in place, the voltage, current or power, or the gas consumption and pressure should be ascertained and recorded if possible. In tests of gas illuminants the volume consumed should be reduced to the corresponding volume at a temperature of 60° F., and a barometric pressure of 30 inches.

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PI (π), VALUE OF. — The letter π is used to represent the ratio of the circumference of a circle to its diameter; it is an incommensurable quantity. Its value is 3.14159265. . . . The value 3.1416 is sufficiently accurate for all ordinary purposes, and for rough calculations the value 22/7 is convenient. The following factors frequently occur:

$\pi = 3.14159$ $2\pi = 6.28319$	$\frac{\pi}{4} = 0.785398$	$\frac{1}{2\pi} = 0.159155$
$3\pi = 9.42478$ $4\pi = 12.56637$	$\frac{\pi}{6} = 0.523599$	$\frac{1}{4\pi} = 0.079578$
$\frac{\pi}{2} = 1.57080$	$\frac{4\pi}{3} = 4.18879$	$\pi^2 = 9.86960$ $\pi^3 = 31.00628$
$\frac{\pi}{3} = 1.04720$	$\frac{1}{\pi} = 0.318310$	$\sqrt{\pi} = 1.77245$ $\sqrt[3]{\pi} = 1.46459$

PIPES AND PIPING. — (*See also Boilers; Electrolysis of Grounded Structures; Hydraulics, Principles of; Power Stations; Valves.*) In the following table is given a list of the metals ordinarily used for steam, gas and water pipes, together with their average weight per cubic inch, their tensile strength and the expansion per 100 feet for various temperature differences. The expansions are based on data given in Gebhardt's *Steam Power Plant Engineering*.

TENSILE STRENGTH, WEIGHT AND EXPANSION OF PIPE MATERIALS

Material	Lb. per cu. in.	Tensile strength, lb. per sq. in.	Expansion (in. per 100 ft.) Temp. rise above 60° F.				
			100°	200°	300°	400°	500°
Brass, wrought.	0.30	50,000	1.15	2.41	3.80	5.38	7.11
Copper, wrought	0.32	30,000	1.08	2.26	3.56	5.05	6.66
Iron, cast.......	0.26	18,000	0.72	1.50	2.38	3.36	4.44
Iron, wrought...	0.28	50,000	0.79	1.65	2.61	3.70	4.89
Lead...........	0.41	1600 to 2400	....	....	....	....	....
Steel, mild......	0.28	65,000	0.79	1.65	2.61	3.70	4.89

The material for steam pipe, whether high or low pressure, is now almost uniformly open-hearth steel. This may be made by the acid or basic process, but Bessemer pipe or wrought-iron pipe should not be used if the best results are to be obtained. The use of Bessemer-steel pipe brings in difficulty in flanging and bending is usually uncertain at the welds. It has in its composition rather more phosphorus and sulphur than is considered good when severe strains are to be placed on the material. Wrought-iron pipe, when it can be obtained, may be very good for certain uses but it is almost impossible to flange a piece of wrought-iron pipe satisfactorily, and its use is now confined mainly to unimportant work at localities close to the place where the pipe is made. Steel pipe when used for oil or salt water is often galvanized and its thickness should be proportioned to meet the pressures in use.

Where warm water is to be distributed, cast-iron pipe has been and is the standard. Cast-iron pipe, when properly made, has proved to be the best for large and small water mains for either low or high pressures. Where the water pipe is small or where many bends are required, or where the heat and wear are excessive, bronze pipe has been substituted for cast-iron with very good results. The smaller sizes of pipe used in oiling systems are almost invariably made of brass. The use of copper pipe for steam work has been almost entirely superseded, the introduction of superheated steam with the resulting action of the high temperature on the copper rendering it unfit for such employment. There are many stations in which nothing but open-hearth steel and cast-iron piping are used and it may be noted that this practice is increasing and these materials will be the standard for the future.

DIMENSIONS AND WEIGHT OF COMMERCIAL PIPE. — The size of iron and steel pipes is usually specified in terms of the "nominal" inside

diameter. The actual inside diameter is usually greater than the "nominal," the percentage difference being the greatest for small sizes. The thickness of wall and weight per lineal foot of a given size of pipe varies over a considerable range, due to processes of manufacture. Manufactures specify that "full weight" pipe may have a variation of from 5 per cent above to 5 per cent below nominal or table weight, but "merchant pipe," which is the ordinary pipe carried by jobbers and manufacturers, is almost invariably from 5 to 10 per cent under the nominal weight.

In drawing specifications for pipe, engineers should be careful to state what grade of pipe is desired, whether "merchant," full weight, or extra strong, and in the case of cast-iron pipe the class according to the table below.

DIMENSIONS AND WEIGHT OF CAST-IRON PIPE

Nominal inside diam., in.	Class A		Class B		Class C		Class D	
	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.	Thickness, in.	Lb. per ft.
3	0.39	14.5	0.42	16.2	0.45	17.1	0.48	18.0
4	0.42	20.0	0.45	21.7	0.48	23.3	0.52	25.0
6	0.44	30.8	0.48	33.3	0.51	35.8	0.55	38.3
8	0.46	42.9	0.51	47.5	0.56	52.1	0.60	55.8
10	0.50	57.1	0.57	63.8	0.62	70.8	0.68	76.7
12	0.54	72.5	0.62	82.1	0.68	91.7	0.75	100.0
14	0.57	89.6	0.66	102.5	0.74	116.7	0.82	129.2
16	0.60	108.3	0.70	125.0	0.80	143.8	0.89	158.3
18	0.64	129.2	0.75	150.0	0.87	175.0	0.96	191.7
20	0.67	150.0	0.80	175.0	0.92	208.3	1.03	229.2
24	0.76	204.2	0.89	233.3	1.04	279.2	1.16	306.7
30	0.88	291.7	1.03	333.3	1.20	400.0	1.37	450.0
36	0.99	391.7	1.15	454.2	1.36	545.8	1.58	625.0
42	1.10	512.5	1.28	591.7	1.54	716.7	1.78	825.0
48	1.26	666.7	1.42	750.0	1.71	908.3	1.96	1050.0
54	1.35	800.0	1.55	933.3	1.90	1141.7	2.23	1341.7
60	1.39	916.7	1.67	1104.2	2.00	1341.7	2.38	1583.3
72	1.62	1283.4	1.95	1545.8	2.39	1904.2	....	......
84	1.72	1633.4	2.22	2104.2	....	......	....	......

The safe working pressures recommended for the four classes are:

SAFE WORKING PRESSURES, CAST-IRON PIPE

Unit of Pressure	A	B	C	D
Pounds per square inch........	43	86	130	173
Head of water, feet...........	100	200	300	400

Cast-iron Pipe. — The dimensions and weights given above are taken from the catalogue of the U. S. Cast-iron Pipe and Foundry Co. (1908). The weights are figured on the basis of a pipe length of 12 feet, and include proportional part of weights of standard sockets.

Welded Pipe. — The first table following is based on Briggs' Standard for sizes up to 10 inches, and upon the National Tube Co.'s Standard above 10 inches (1910). Weights per foot, up to and including 15 inches internal diameter, are based upon a length of 20 feet including the coupling; weights given for larger sizes are for plain end pipe.

The second table gives the dimensions of "Extra Strong" and "Double Extra Strong" welded tubes (*National Tube Co., 1902*).

DIMENSIONS AND WEIGHT OF WELDED PIPE

Size, nominal internal diam., in.	Diameter in inches		Thickness of metal, in.	Number of threads per in.	Weight of pipe per lin. ft., lb.
	Actual external	Approx. internal			
1/8	0.405	0.269	0.068	27	0.246
1/4	0.540	0.364	0.088	18	0.426
3/8	0.675	0.493	0.091	18	0.570
1/2	0.840	0.622	0.109	14	0.855
3/4	1.050	0.824	0.113	14	1.14
1	1.315	1.049	0.133	11 1/2	1.69
1 1/4	1.660	1.380	0.140	11 1/2	2.29
1 1/2	1.900	1.610	0.145	11 1/2	2.74
2	2.375	2.067	0.154	11 1/2	3.69
2 1/2	2.875	2.469	0.203	8	5.85
3	3.500	3.068	0.216	8	7.66
3 1/2	4.000	3.548	0.226	8	9.24
4	4.500	4.026	0.237	8	10.9
4 1/2	5.000	4.506	0.247	8	12.7
5	5.563	5.047	0.258	8	14.9
6	6.625	6.065	0.280	8	19.2
7	7.625	7.023	0.301	8	23.8
8	8.625	7.981	0.322	8	28.9
9	9.625	8.941	0.342	8	34.3
10	10.750	10.020	0.365	8	41.2
11	11.750	11.000	0.375	8	46.4
12	12.750	12.000	0.375	8	50.9
....	14.000	13.250	0.375	8	56.1
....	15.000	14.250	0.375	8	60.7
....	16.000	15.250	0.375	8	64.9
....	18.000	17.250	0.375	....	70.6
....	20.000	19.250	0.375	....	78.6
....	22.000	21.250	0.375	....	86.6
....	24.000	23.250	0.375	....	94.6

DIMENSIONS OF "EXTRA STRONG" AND "DOUBLE EXTRA STRONG" WELDED TUBES

Nominal diam., in.	Actual outside diam., in.	Thickness, extra strong, in.	Thickness, double extra strong, in.	Actual inside diam., extra strong, in.	Actual inside diam., double extra strong, in.
1/8	0.405	0.100	.....	0.205	.....
1/4	0.54	0.123	.....	0.294	.....
3/8	0.675	0.127	.....	0.421	.....
1/2	0.84	0.149	0.298	0.542	0.244
3/4	1.05	0.157	0.314	0.736	0.422
1	1.315	0.182	0.364	0.951	0.587
1 1/4	1.66	0.194	0.388	1.272	0.884
1 1/2	1.9	0.203	0.406	1.494	1.088
2	2.375	0.221	0.442	1.933	1.491
2 1/2	2.875	0.280	0.560	2.315	1.755
3	3.5	0.304	0.608	2.892	2.284
3 1/2	4.0	0.321	0.642	3.358	2.716
4	4.5	0.341	0.682	3.818	3.136

Riveted Pipes. — Large pipes are frequently made of sheets of boiler steel with riveted joints, with longitudinal, circumferential or spiral seams. The following tables give the necessary data regarding dimensions, rivets, etc. The first table is taken from a catalogue of the Abendroth & Root Mfg. Co.

SHEET IRON AND RIVETS REQUIRED FOR RIVETED PIPES

No. sq. ft. of iron required to make 100 lin. ft. punched and formed sheets when put together			Approx. No. of rivets 1 in. apart required for 100 lin. ft. punched and formed sheets	No. sq. ft. of iron required to make 100 lin. ft. punched and formed sheets when put together			Approx. No. of rivets 1 in. apart required for 100 lin. ft. punched and formed sheets
Diam. in in.	Width of lap in in.	Square feet		Diam. in in.	Width of lap in in.	Square feet	
3	1	90	1600	14	1 1/2	397	2800
4	1	116	1700	15	1 1/2	423	2900
5	1 1/2	150	1800	16	1 1/2	452	3000
6	1 1/2	178	1900	18	1 1/2	506	3200
7	1 1/2	206	2000	20	1 1/2	562	3500
8	1 1/2	234	2200	22	1 1/2	617	3700
9	1 1/2	258	2300	24	1 1/2	670	3900
10	1 1/2	289	2400	26	1 1/2	725	4100
11	1 1/2	314	2500	28	1 1/2	779	4400
12	1 1/2	343	2600	30	1 1/2	836	4600
13	1 1/2	369	2700	36	1 1/2	998	5200

THICKNESS AND WEIGHT PER FOOT OF SHEET IRON

No. of gauge, B.W.G.	Thickness, in.	Weight in lb., black	Weight in lb., galvanized	No. of gauge, B.W.G.	Thickness, in.	Weight in lb., black	Weight in lb., galvanized
26	0.018	0.80	0.91	18	0.049	1.82	2.16
24	0.022	1.00	1.16	16	0.065	2.50	2.67
22	0.028	1.25	1.40	14	0.083	3.12	3.34
20	0.035	1.56	1.67	12	0.109	4.37	4.73

Wooden Stave Pipes are usually built up in place. Staves of redwood, fir, yellow pine, and spruce are used. The staves range from 1¼ to 2½ inches in thickness and from 6 to 8 inches in width; these are held in place by steel bands ranging from ⅜ to ¾ inch in diameter. The interior surfaces of the staves wear smoother by the action of the flowing water and do not become fouled. Stave pipes have been installed in sizes ranging from 18 to 144 inches in diameter, and for heads up to 300 feet.

FORMULAS FOR WEIGHT, CIRCUMFERENCE, SURFACE, CONTAINED VOLUME AND SAFE PRESSURE. — Let

A = external surface per lineal foot in square feet,
C = external circumference in inches,
D = internal diameter in inches,
D_0 = external diameter in inches,
f = factor of safety,
H = safe head in feet of water,
P = safe pressure in pounds per square inch,
S = tensile strength in pounds per square inch,
T = thickness of wall in inches,
V = volume of contents (water, steam or gas) per lineal foot in cubic feet,
w = specific weight of metal in pounds per cubic inch,
W = weight of metal per lineal foot in pounds.

Then

$$A = 0.262\,D_0, \qquad C = 3.14\,D_0, \qquad D_0 = D + 2\,T,$$

$$H = \frac{4.62\,ST}{Df}, \qquad P = \frac{2\,ST}{Df},$$

$$T = \frac{D_0 - D}{2} = \frac{DfP}{2\,S} = \frac{DfH}{4.62\,S},$$

$$V = 0.00545\,D^2, \quad W = 9.42\,w\,(D_0^2 - D^2) = 37.7\,wT\,(D + T).$$

The values of the specific weight w and tensile strength S are given in the table at the beginning of this article. The tensile strength of riveted pipe is about 70 per cent of the tensile strength of the metal.

JOINTS. — Pipe joints have been a great source of trouble in the past and the various kinds and "standards" have been as many almost as there were individual engineers. For low-pressure pipe work the screwed joint with the standard pipe thread and cast-iron flanges has been and is the standard for the best work. For pressures above 100 lb., however, another type of joint should

be adopted if the best work is desired. For this purpose there has been no joint found better than the so-called Van Stone joint. This is made by flanging the end of the pipe against the outside of a steel or cast-iron flange. There are many varieties of welded flanges in which the flange is welded directly to the pipe, but these do not seem to have been as popular or as good in construction as the so-called Van Stone, although many people use them. All welded flanges have the disadvantage that they cannot be turned on the pipe, making great care necessary to avoid mistakes in drilling them.

The best joints are made by grinding the seats to a perfect surface and then bolting them together without a gasket. This, however, takes a high-grade mechanic and has been satisfactory only when made in the proper manner. Instead of grinding the faces, it is now considered at least as good to fine-tool finish them and insert a gasket which in the best work has been made of very soft steel approximately 1/100 inch thick. Duralite and other indurated fibers make good gaskets. Copper gaskets appear to deteriorate very rapidly in this position and are not used on high-pressure work as much as formerly. The tongue-and-groove joint cannot be recommended for steam work as it is almost impossible to bring two joints to the same degree of tightness. For the lower steam pressures copper gaskets work very nicely and are now standard. These are usually stamped with corrugations which flatten out when the bolts are tightened up, assuring a surface practically the whole width of the face. For exhaust work rubber with wire insertion such as the " Rainbow " is mostly used. For water, whether hot or cold, the " Common Sense " or other babbitt composition gaskets are quite satisfactory. The gasket made up of a soft lead ring with a copper wire ring outside of it has also been largely used with very good results and " Rainbow " gaskets are satisfactory when the pressures are not too high.

PIPE FITTINGS. — For low-pressure work, either steam, exhaust or water, the Master Steam Fitters' Association has adopted a standard of pipe fittings which is used practically throughout the United States. It is only for pressures higher than 300 pounds that special fittings are required. Up to 200 pounds steam pressure with no superheat, cast iron or gun iron forms the ideal material for pipe fittings and is practically the only material in use. With the advent of superheated steam, however, the cast-iron fittings soon proved themselves to be useless with the high heats and semi-steel and steel fittings were tried with the best of results. To-day no plant using superheated steam installs cast-iron fittings for high-temperature work. All fittings should be provided with proper means of draining and drainage pockets or outlets should be placed at the lowest points for the attachment of the drainage system.

For dimensions and weights of various pipe fittings see *Kent's Mechanical Engineers' Pocket Book*. See also article in this book on *Valves*.

PIPE COVERINGS. — To prevent loss of heat by radiation, steam and feed-water pipes are usually protected with a cover, 1 inch or more in thickness, of loose non-conducting material. About 3 B.t.u. per square foot per hour per degree difference in temperature is radiated from a bare pipe under ordinary conditions. By the use of any good commercial covering from 75 to 85 per cent of this loss may be prevented. Pipe covering is usually applied in sections, moulded to fit the pipe, and held in place by bands. The covering for fittings and valves is usually applied in plastic form.

Cost of Pipe Covering (Pre-war figures). — Eighty-five per cent magnesia 1 inch thick costs in the neighborhood of 30 cents per square foot in place. In general such covering costs about one-half list price including labor. One writer states that one man will cover 100 feet of straight pipe

or 40 fittings per day up to 4-inch pipe size. Above 4 inch the cost per 100 feet of pipe length will be greater owing to the increased labor of handling.

FLOW OF WATER THROUGH PIPES. — The pressure required to force a stream of water through a pipe is usually expressed in terms of the height of a column of water which would produce a static pressure equal to this pressure. A pressure of

1 pound per square inch = 2.31 feet of water column,
1 foot of water column = 0.433 pound per square inch.

Friction and Velocity Head. — The head required to overcome the resistance of a pipe is called the "friction head," and for a given pipe and velocity of flow is proportional to the length of the pipe. The head required to overcome the resistance at the entrance to a pipe is called the "entry head;" in the case of long pipes the entry head is negligible in comparison with the friction head. In addition to these two heads, a certain pressure, and therefore a corresponding head, is required to produce any change in the velocity of flow, as, for example, when water enters a pipe from a reservoir. This velocity head is equal to $\frac{V_1^2 - V_2^2}{64.4}$, when the velocity changes from V_1 to V_2 feet per second. A decrease in velocity gives rise to a *negative* velocity head. In long pipes the velocity head is negligible.

Hydraulic Grade Line. — Imagine a horizontal line drawn over a pipe line from a reservoir, and let this horizontal line be at the same elevation as the surface of the reservoir. From each point of this horizontal line drop perpendiculars equal in length to the loss of head between this point and the reservoir. The locus of the foot of these perpendiculars is called the "hydraulic grade line." In a pipe leading from a reservoir no part of the length of the pipe should be above the hydraulic grade line. If the pipe has vertical curves, valves should be provided at the high points to permit the escape of the air which tends to collect at the top of such curves, otherwise the pipe may become "air-bound," i.e., water will not flow although the supply is higher than the outlet.

Formulas Connecting Velocity, Discharge, and Head. —Various formulas have been proposed to express the relation between velocity, discharge and head. Let

d = diameter of pipe in inches,
D = diameter in feet,
H = loss of head in pipe in feet of water,
L = length of pipe in feet,
p = loss of pressure in pipe in pounds per square inch,
Q = discharge in cubic feet per second,
V = velocity of flow in feet per second.

Unwin gives the following formula:

$$V = 4.012\sqrt{\frac{DH}{fL}},$$

where f is the coefficient of friction. This coefficient depends upon the smoothness and cleanness of the pipe and also upon whether the pipe is straight or crooked, and upon the velocity. (*See Kent's Mechanical Engineers' Pocket Book.*) Rankine gives the following formula for f for smooth, clean, straight pipe:

$$f = 0.005\left(1 + \frac{1}{12\,D}\right).$$

This formula is approximate only, since it does not take into account the velocity of flow. It is sufficiently accurate, however, for velocities up to 6 feet per second.

Combining these two formulas the following are obtained:

$$V = 16.4\,d\sqrt{\frac{H}{L\,(d+1)}} = 25\,d\sqrt{\frac{p}{L\,(d+1)}},$$

$$V = 57\,D\sqrt{\frac{H}{L\,(D+0.083)}} = 87\,D\sqrt{\frac{p}{L\,(D+0.083)}},$$

$$Q = 0.00545\,d^2V = 0.089\,d^3\sqrt{\frac{H}{L\,(d+1)}} = 0.136\,d^3\sqrt{\frac{p}{L\,(d+1)}},$$

$$Q = 0.785\,D^2V = 44.7\,D^3\sqrt{\frac{H}{L\,(D+0.083)}} = 68\,D^3\sqrt{\frac{p}{L\,(D+0.083)}}.$$

When the pipe is not completely filled with water, the same formulas hold provided d and D are taken as 4 times the hydraulic radius. The "hydraulic radius" is defined as the ratio of the area of the cross section of the water to that portion of the perimeter of the pipe in contact with the water.

These formulas apply approximately to any kind of clean, straight pipe, provided the interior surface is smooth. The formulas involving the head H also apply approximately to any kind of liquid or gas, provided H is taken as the height of a column of the given liquid or gas which will produce a static pressure equal to the fall in pressure in the pipe. The coefficients in the formulas involving the drop of pressure p should then be multiplied by $\sqrt{\frac{62.4}{w}}$, where w = weight in pounds of 1 cubic foot of the given liquid or gas, and 62.4 = weight in pounds of 1 cubic foot of water.

These formulas are for *new, clean, straight pipes*. For cast-iron pipes that have been in service a number of years the loss of head will be larger on account of corrosion and incrustation, and the value of H in the formulas should be multiplied under average conditions by the factors opposite; but they must be used with much discretion, for some waters corrode pipes much more rapidly than others.

10 years	1.3
20 "	1.6
30 "	2.0
50 "	2.6
75 "	3.4

The same figures may be used for wrought-iron pipes which are not subject to a frequent change of water.

From the above formulas for velocity the loss in head due to pipe resistance is

$$H = \frac{LV^2\,(d+1)}{270\,d^2} = \frac{LV^2\,(D+0.083)}{3250\,D^2}.$$

William Cox (*Amer. Mach.*, 1893) gives the following formula for pipes over 6 inches in diameter:

$$H = \frac{L\,(V^2 + 1.25\,V - 0.5)}{300\,d}.$$

The Pelton Water Wheel Co. advocates the following formula for riveted pipe for pipes over 6 inches in diameter:

$$H = \frac{L(V^2 + 1.25V - 0.5)}{250d}.$$

Effect of Curves and Valves.—The resistance of curves and valves may be allowed for approximately by taking for L in the above formulas the actual length of the pipe plus a length equal to

$$\frac{kd^2}{d+1} \text{ feet, or } \frac{KD^2}{D+0.083} \text{ feet,}$$

for each curve or valve, where k and K have the following values:

	45° angle	90° angle	Gate valve	Globe valve	Angle valve
$k=$	0.8	4.2	0.8	8.1	12.4
$K=$	9	49	9	96	148

These coefficients are based on data given by Gebhardt (*Steam Power Plant Engineering*). Variations of 100 per cent or more from the values given may be expected, depending on the radius of the bends and design of the valves.

According to Briggs the effect of each right-angle bend is equivalent to increasing the length 40 diameters, and the effect of each globe valve is equivalent to increasing the length 60 diameters.

Water-Hammer. — From the formula given by Prof. I. P. Church the pressure developed by the instantaneous closing of a valve in a water pipe is

$$P_i = \frac{63.5\,V}{\sqrt{1 + \frac{300{,}000\,d}{MT}}} \text{ pounds per square inch,}$$

where M = modulus of elasticity of the pipe material and the other symbols are as above.

FLOW OF AIR AND GAS THROUGH PIPES. — (*See also preceding section.*) Let

d = internal diameter of pipe in inches,
L = length of pipe in feet,
p = fall of pressure in the pipe in pounds per square inch,
Q = discharge in cubic feet per second,
V = linear velocity in feet per second,
w = weight of 1 cubic foot of air or gas in pounds.

Then from Unwin's formula, using the same coefficients as for the flow of water, *when the fall of pressure is small*,

$$V = 198\,d\sqrt{\frac{p}{wL(d+1)}}. \qquad (1)$$

$$Q = 1.08\,d^3\sqrt{\frac{p}{wL(d+1)}}. \qquad (2)$$

For air: $w = \frac{2.70P}{460+t}$; for any other gas: $w = \frac{2.70GP}{460+t}$,

where P = absolute pressure in pounds per square inch and t = temperature in degrees F and G = the specific gravity of the gas referred to air as unity. For ordinary illuminating gas $G = 0.65$.

Various authorities give different values of the numerical coefficients in the formulas for V and Q (*see Kent's Mechanical Engineers' Pocket Book*). The values given above, which are the same as for the flow of water ($w = 62.4$), are sufficiently accurate for rough calculations. The formula for Q may also be written

$$Q = Cd^2 \sqrt{\frac{dp}{wL}}, \tag{3}$$

where $C = 1.08 \sqrt{\frac{d}{d+1}}$, corresponding to the coefficient 1.08 in equation (2).

C is frequently taken as unity for all sizes of pipes.

When the *fall of pressure is large*, the above formulas for Q give approximately the quantity per second at the *average* pressure in the pipe, provided the weight per cubic foot is taken corresponding to this *average* pressure. Or, putting P = the pressure in pounds per square inch at the *outlet* of the pipe, and w = weight per cubic foot at this pressure P, p being the fall in pressure, then the discharge in cubic feet per second at the pressure P is approximately (from equation 2)

$$Q = 1.08\, d^3 \sqrt{\frac{p\,(P + 0.5\,p)}{PwL\,(d+1)}}.$$

For d large compared with unity, $\frac{d}{d+1}$ may be taken sensibly equal to unity, and this expression may be written

$$Q = 1.08\, d^2 \sqrt{\frac{p\,(P + 0.5\,p)\,d}{PwL}}.$$

For $P = 14.7$ pounds per square inch, and $t = 60°$ F., then for air

$$Q_a = 1.02\, d^2 \sqrt{\frac{p\,(P + 0.5\,p)\,d}{L}}, \text{ cubic feet per second,}$$

and for illuminating gas of 0.65 specific gravity,

$$Q_g = 1.26\, d^2 \sqrt{\frac{p\,(P + 0.5\,p)\,d}{L}} \text{ cubic feet per second.}$$

Instead of the coefficients 1.02 and 1.26 the following values correspond to the coefficients in the formulas used by the authorities named:

	Instead of 1.02	Instead of 1.26
Wm. Cox (*Am. Mach.*, 1902)	0.95	1.18
J. E. Johnson (*Am. Mach.*, 1899)	0.96	1.19
E. A. Rix (*Pac. Coast Gas Assoc.*, 1905)	1.05	1.30

Effect of Bends, Valves, etc. — The Norwalk Iron Works Co. give the following table; radius of elbow and length of pipe are both expressed in terms of the pipe diameter:

Radius of elbow........	5	3	2	1½	1¼	1	¾	½
Equivalent lengths of straight pipe.........	7.85	8.24	9.03	10.36	12.72	17.51	35.09	121.2

W. L. Saunders (*Compressed Air*, 1902) gives the following figures for the length of pipe in feet equivalent to each of the items listed:

Diam. of pipe, in..	1	1½	2	2½	3	3½	4	5	6	7	8	10
Globe valves.....	2	4	7	10	13	16	20	28	36	44	53	70
Elbows and tees..	2	3	5	7	9	11	13	19	24	30	35	47

FLOW OF STEAM. — The formulas given in the preceding section apply only approximately to the flow of steam. Putting v = volume of 1 pound of steam in cubic feet (*see tables under Steam*), the above formula for flow in cubic feet per second becomes

$$Q = 1.08\, d^3 \sqrt{\frac{vp}{L\,(d+1)}},$$

and the corresponding flow in pounds per second is

$$W = 1.08\, d^3 \sqrt{\frac{p}{vL\,(d+1)}}.$$

G. H. Babcock, in *Steam*, gives the flow in pounds per minute as*

$$W = 87 \sqrt{\frac{wpd^5}{L\left(1+\frac{3.6}{d}\right)}}.$$

The corresponding flow in cubic feet per second may be written

$$Q = 1.45\, d^3 \sqrt{\frac{p}{wL\,(d+3.6)}}.$$

For a given drop in pressure this formula gives a less flow for pipes under 2 inches in diameter and a greater flow for pipes over 2 inches in diameter than is given by the corresponding formula for air or gas.

Size of Steam Mains. — Formerly steam mains of 18 to 24 in. in diameter were considered necessary in small stations and the radiation owing to the poor quality of pipe covering then used was enormous. At the present time very few mains are put in of larger size than 14 in. I. D. and in some cases a 10 or 12-in. main is considered sufficient. The drop in pressure between boiler and prime mover is considerably larger than formerly but the actual heat loss due to friction and radiation is very much less than it was. This reduction in pipe sizes also brought in great economies in the upkeep cost of the steam lines as with the high pressures carried at the present day it would be almost impossible to keep a 20 in. or 24-in. main tight under the conditions of actual service.

* w is pounds per cubic foot at entrance pressure.

For years a standard practice for the speed of steam in steam lines was 4000 ft. per minute as the minimum, 6000 as the average and 8000 as the maximum. This was considered the standard in the days when 125 pounds steam pressure was carried without superheat. At the present time with 200 pounds pressure and superheat which may extend as high as 200° F. above the saturation temperature, the minimum steam speeds are much higher and very few engineers are using as the minimum speed less than 8000 ft. per minute, the maximum in some cases running as high as 18,000 ft. with no bad results.

Many formulas have been used for determining pipe sizes for steam engines, but most of them are now obsolete. In "Power" for January 19, 1915, F. W. Salmon presents results secured by plotting the necessary data from a large number of successful plants.

Let A = area of pipe bore in square inches,
d = diameter of pipe bore in inches,
W = average pounds of steam per hour,
C = a constant for a given pressure in the pipe,
K = a constant $= \frac{\pi \times C}{4}$,

then

$$W = A \times C = d^2 \times K,$$

or

$$d^2 = \frac{W}{K}.$$

The values of C and K are given in the following table.

Vacuum, in. of Hg	C	K	Gage pressure lb. per sq. in.	C	K
28	50	39.2	80	267	210.0
26	84	66.0	100	275	216.0
24	105	82.5	125	284	223.0
22	122	95.7	150	291	229.0
20	134	102.0	175	298	234.0
18	144	113.0	200	304	239.0
16	151	118.6			
13	162	127.0			
6	176	138.0			
0	187	147.0			

EXPANSION OF PIPE. — The coefficient of expansion for ordinary steam pipe is about 0.000006 of its length for each degree F. A rough and ready rule is to allow ½ in. for every 100 ft. for every 100° F. difference of temperature. All pipe lines should be laid out to take care of this expansion, and to this end large radius bends should be employed wherever possible. It is usual to cut the pipe so it will be the right length when it is hot, making up the joints and pulling them together, so that an initial stress is put in the cold pipe. When the pipe becomes hot, this stress disappears and the pipe will then be in equilibrium. Suitable hangers should be provided every 10 or 12 ft. to support the pipe in its proper place. These may consist of a band around the pipe with a

rod hanger from some of the floor beams overhead, or the pipe may be supported from below on a roller. Anchors should also be provided at certain places so that the direction of expansion may be controlled. On very long lines sliding expansion joints become necessary, or the corrugated-steel expansion joint may be used.

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POLES FOR OVERHEAD LINES. — (*See also Cross Arms; Distribution Lines; Insulator Pins; Insulators; Transmission Lines.*) The following is a brief table of contents of this article:

Methods of Specifying Pole Dimensions. — A pole for supporting an overhead line is usually specified by its *total* or "nominal" length and by the diameter of its top; e.g., a 40-ft 7-in. top pole. When set, the distance a pole stands above the surface of the ground is less than the nominal length by the amount it sets in the ground. Poles are standard in lengths which are multiples of 5 feet. The ordinary range of length is from 30 to 60 feet.

The top of a pole is sometimes specified by inches circumference instead of by diameter. Poles are standard in diameters which are even multiples of one inch. The ordinary range of top diameter is from 7 to 8 inches except on the Pacific Coast, where from 8 to 10 inches is common.

Taper of Poles. — The taper of various kinds of poles, specified as the difference, measured in inches, between two circumferences 10 feet apart, is given as follows in *Forest Service Bull. No. 84:* Chestnut (Maryland), 3.8 to 4.0; Northern white cedar (Michigan), 5.2; Western yellow pine (California), 4.0; Lodgepole pine (Montana), 3.0; Loblolly pine (Texas), 2.4; Western red cedar (Washington), 3.5. Trees grown upon a high elevation have a greater taper in the trunk than trees grown lower down.

WOODS USED FOR POLES AND CROSS ARMS. — (Based on publication of the Forest Service, see *Bibliography.*) Of the timbers used for poles, chestnut and northern, southern and Idaho cedar easily rank first. Longleaf and shortleaf pine, red cedar, cypress, redwood, locust, catalpa and several of the oaks are used, but in much smaller numbers, and their employment is generally confined to the region of their growth. Still other timbers are used, but in numbers insignificant in comparison with those mentioned above.

For cross arms, longleaf, shortleaf and loblolly pines of the South and Norway pine of the North are most largely used, while the demand for cedar, cypress, spruce and red fir is but little less. Again, as in the case of pole timbers, a third group may be formed of those timbers used in small numbers and very locally.

In 1915, of the 4,077,964 poles purchased in the United States, 61.8 per cent were cedar, 16 per cent were chestnut, the remainder being oak, pine, cypress, etc.

Desirable Timber for Poles. — The several qualities which timber must possess to adapt it to use for poles are stated to be: Durability in contact with the soil, minimum weight, straightness coupled with relatively small size and little taper. The wood must be soft, so that the spikes of a climber may enter readily and at the same time it must have strength to support considerable

weight. These qualities are admirably combined in cedar and in juniper, which commercially is a cedar; no other woods possess so many.

Uncertainties in Names of Timber Trees. — The terms cedar, pine, etc., used in describing poles and cross arms and even the apparently more exact terms, such as white cedar, yellow pine, etc., each cover several kinds of trees and have different meanings in different localities.

There are at least eight pines (of the thirty-five native ones) in the market. some of which so closely resemble each other in their minute structure that they can hardly be told apart; and yet they differ in quality and should be used separately, although they are often mixed or confounded in the trade.

Referring to the use of yellow pine as the material for cross arms the Committee on Overhead Line Construction of the N. E. L. A., says: "Yellow pine is understood to cover what is commonly known as longleaf pine. It is understood that the term is descriptive of quality rather than of botanical species." Forestry Bulletin No. 10 states: " 'Yellow pine,' is applied in the trade to all the Southern lumber pines; in the Northeast it is also applied to the pitch pine; in the West it refers mostly to bull pine. 'Yellow longleaf pine,' 'Georgia pine,' chiefly used in advertisement, refers to longleaf pine."

Timbers Ordinarily Used for Poles and Cross Arms. — The principal timber trees from which poles and cross arms are obtained are briefly described below in accordance with the names used in the publications of the Forest Service.

Chestnut. — Chestnut ranks next to the cedars in the quantity of poles used. The reported number purchased in 1909 was 608,000. The sapwood is very narrow, usually from about 1/8 to 3/8 of an inch wide. Chestnut is widely distributed throughout the entire Appalachian mountain region. A small territory embracing parts of Pennsylvania, Maryland, Virginia and West Virginia furnishes nearly all the chestnut poles. Chestnut is not so straight as cedar and is liable to be knotty. It has greater strength, but this advantage is more than counterbalanced by its greater weight, which prohibits long shipments.

Northern White Cedar or Arborvitæ. — This species is very commonly used for poles throughout the central and eastern portion of the United States. The principal source of supply is in the states bordering the Great Lakes. It makes a very desirable pole on account of its durability but is high-priced. The sapwood varies from 1/2 to 1 inch in thickness. A very large portion of northern white cedar poles have unsound butts. Northern white cedar, common in the northern woods of New England, New York and the Lake States, occurs as far south as North Carolina and Tennessee, but only in the mountains where the elevation is sufficiently great to permit northern species to thrive.

On account of its strength, lightness, durability and form it is the most desirable pole timber. Arborvitæ is extremely slow in growth. The sapwood zone is narrow at the butt and gradually widens as the top is approached. The average time it takes to produce a 30-foot arborvitæ pole is about 190 years.

Southern White Cedar. — The number of southern white cedar poles purchased in 1909 was 44,000. The woods known under the general name of "cedar" comprise a number of distinct species which differ in their durability, the white cedar of the southern swamps being somewhat less durable than the cedar of the Lake States. The sapwood, which is usually from 1/2 to 1 inch wide, decays very quickly. Southern white cedar, though sometimes found as far north as southern Maine, is of commercial importance chiefly south of Delaware and New Jersey.

Red Cedar. — A small to medium-sized tree scattered through the forests, or, in the West, sparsely covering extensive areas (cedar brakes). The

red cedar is the most widely distributed conifer of the United States, occurring from the Atlantic to the Pacific and from Florida to Minnesota, but attains a suitable size for lumber only in the Southern and more especially in the Gulf States.

Juniper. — The term Juniper is commonly used by telephone men for southern white cedar; the term also is applied to red cedar. Juniper poles come from Virginia, the Carolinas and other South Atlantic States.

Western Red Cedar. — The light and durable western red cedar is much used for poles on the Pacific coast and throughout the Northwest. Also it competes to a certain extent with northern white cedar in the East, its form and size making it especially desirable for the larger classes of poles. The principal points of production are northern Idaho and western Washington. The relative durability of western red cedar and northern white cedar under similar conditions is not known, and the testimony by pole users on this point is somewhat contradictory.

Cypress. — The cypress is a large deciduous tree, occupying much of the swamp and overflow land along the coast and rivers of the Southern States. Cypress is usually considered a durable wood, and the heartwood is, in fact, one of the most durable of our native species. The sapwood, however, decays quickly and this seriously weakens the pole. The width of the sapwood on pole-size trees is from ¾ of an inch to 1¼ inches. Cypress frequently is too large for use as a pole and has greater value for lumber. Even when its general diameter is small enough the butt will often be so big that it adds too much weight.

Longleaf Pine. — Large tree; forms extensive forests and furnishes the hardest and strongest pine lumber in the market. Coast region from North Carolina to Texas. The longleaf pine is strikingly heavy, hard and resinous, and usually very regular and narrow ringed, showing little sapwood, and differing in this respect from the shortleaf pine and loblolly pine, which usually have wider rings and more sapwood, the latter excelling in that respect.

Shortleaf Pine. — Resembles loblolly pine; often approaches in its wood the Norway pine. The common lumber pine of Missouri and Arkansas, North Carolina to Texas and Missouri.

Loblolly Pine. — Large-sized tree; forms extensive forests; wider-ringed, coarser, lighter, softer, with more sapwood than the longleaf pine, but the two often confounded. This is the common lumber pine from Virginia to South Carolina and is found extensively in Arkansas and Texas, Southern States, Virginia to Texas.

This pine is not durable when used as a pole unless treated with preservatives, but because of its cheapness and ease of impregnation is very desirable if preservative treatment is contemplated. Its distribution, ease of reproduction and rapidity of growth insure a steady and cheap supply. When this timber is used it is necessary to treat the entire pole instead of only the butt, especially in the warmer and more humid localities of the South.

Norway Pine.—Large-sized tree; never forming forests, usually scattered or in small groves, together with white pine; largely sapwood and hence not durable. Minnesota to Michigan; also in New England to Pennsylvania. The Norway pine, which may be confounded with the shortleaf pine can be distinguished by being much lighter and softer. It may also, but more rarely, be confounded with heavier white pine, but for the sharper definition of the annual ring, weight and hardness.

Western Yellow Pine. — Western yellow pine is used for poles to a limited extent in certain parts of the Southwest, where the high cost of more

durable pole timbers makes it necessary to find a cheaper substitute. The life of this timber, untreated, is very short. In the upper part of the San Joaquin Valley of California, where a study of this species was made, untreated pine poles last only two or three years; but since the wood when not exposed to the soil is fairly durable, it is believed that a butt treatment with a good wood preservative will result in a pole that will give good service. A butt-treated pine pole costs considerably less than an untreated cedar pole in this locality.

Lodgepole Pine. — Lodgepole pine is cut to a limited extent for poles. It grows at high altitudes in the Rocky Mountains. It decays quickly in contact with the soil, but is durable when not so exposed. The tree grows tall and straight, with very little taper and makes a well-shaped pole. In certain parts of the West, where there are large bodies of fire-killed lodgepole that remain standing for many years, sound and thoroughly seasoned, conditions for effective treatment are excellent. If given a butt treatment, this dead timber makes a durable pole, and in many localities the cost of the pine pole plus the cost of the treatment is less than that of the Idaho cedar untreated. The sapwood of pole-sized timber may be an inch or an inch and a quarter thick.

DEFECTS IN WOOD USED FOR POLES AND CROSS ARMS. — (*See also Timber.*) The following are the defects in timber which are frequently referred to in specifications for poles and cross arms.

Pith. — The pith of a tree is the central core about which the annual rings are formed. It goes through the tree from top to bottom and branches into the limbs. The pith is quite thick, usually 1/8 to 1/5 inch in Norway pine and in the southern species, though much less so in white pine and is very thin 1/15 to 1/25 inch in cypress, cedar and larch. The pith of the tree is the weakest part on account of the many knots which it invariably and necessarily contains.

Sapwood. — The sapwood of a tree is a zone of wood next to the bark, 1 to 3 or more inches wide and containing 30 to 50 or more annular rings (in coniferous trees). It is of lighter color than the inner, darker part of the log which is the heartwood. Sapwood changes to heartwood as the tree grows.

The width of the sapwood is small for longleaf and white pine and great for loblolly and Norway pines. In old trees of longleaf pine the sapwood forms about 40 per cent of the merchantable log, while in the loblolly and in all young (coniferous) trees the bulk of the wood is sapwood.

Sapwood, being the normal condition of the outer rings of a tree, is not a "defect" in poles, where the whole cross section of the tree (except bark) is used. Being weaker and more liable to decay it is considered a "defect" in pins and cross arms, which are better if made from the heartwood only.

Cup-Shakes. — These are cracks extending circumferentially at one or more places, caused by the separation of the annual rings.

Doatiness. — This is a speckled stain found in beech, American oak and other timber, due to incipient decay. It is produced by imperfect seasoning or by exposure for a long period to a stagnant atmosphere.

Heart-Shakes.— These are splits or clefts occurring in the center of the tree. They are common in nearly every variety of timber and are very serious when they twist in the length, as they interfere with the conversion of the tree into boards or scantlings. They sometimes divide the log in two for a few feet from the end.

Star-Shakes. — When several heart-shakes occur in one tree they are called star-shakes from the appearance produced by their radiation from the center.

Wind-Cracks. — Shakes or splits on the sides of a balk (a log which has been squared off) of timber, caused by shrinkage of the exterior surface, are called wind-cracks.

Dry Rot. — Dry rot is a special form of decay in timber caused by the growth of a fungus which spreads over the surface like a close network of threads, white, yellow or brown, and causes the inside to perish and crumble. Causes which render timber favorable to the growth of this fungus are; large proportion of sapwood; felled at wrong season when full of sap; if cut down in the spring or fall of the year instead of in midwinter or midsummer, when the sap is at rest; stacked for seasoning without sufficient air spaces being left; fixed before thoroughly seasoned; painted or varnished while containing moisture. (Six preceding definitions from *Carpentry and Joinery* by *Paul N. Hasluck.*)

Sound Knot. — A sound knot is one which is solid across its face and which is as hard as the wood surrounding it; it may be either red or black, and is so fixed by growth or position that it will retain its place in the piece.

Loose Knot. — A loose knot is one not firmly held in place by growth or position.

Pith Knot. — A pith knot is a sound knot with a pith hole not more than one-fourth of an inch in diameter at the center.

Encased Knot. — An encased knot is one which is surrounded wholly or in part by bark or pitch. Where the encasement is less than one-eighth of an inch in width on both sides, not exceeding one-half the circumference of the knot, it shall be considered a sound knot.

Rotten Knot. — A rotten knot is one not as hard as the wood it is in.

Pin Knot.— A pin knot is a sound knot not over one-half inch in diameter.

Spike Knot.— A spike knot is one sawn in a lengthwise direction. The mean or average width shall be considered in measuring these knots.

Pitch Pocket. — A pitch pocket is an opening between the grain of the wood containing more or less pitch or bark.

Pitch Streak. — A pitch streak is a well-defined accumulation of pitch at one point in the piece. When not sufficient to develop a well-defined streak, or where the fiber between grains — that is, the coarse-grained fiber, usually termed "spring wood" — is not saturated with pitch, it shall not be considered a defect.

Wane.—Wane is bark, or lack of wood from any cause, on edges of timber.

Shakes. — Shakes are splits in timber which usually cause a separation of the wood between annual rings.

Checks.— Checks are splits in timber, which usually cause a separation of the wood across annual rings. (Last twelve definitions are those used in the timber-test work of the Forest Service in describing defects. *Forest Service Circular 38, Revised.*)

Wind Shake. — A crack or incoherence in timber produced by violent winds while the timber was growing.

Wind. — A turn or bend. A piece of timber is out of wind when it is perfectly straight or flat.

Warped.— Twisted out of shape by seasoning.

Cat-Faces. — Old wounds, partially overgrown, leaving a long, narrow, dead surface exposed.

VOLUME AND WEIGHT OF POLES.—A quick way to find the approximate volume of a pole is to multiply the area of the circle at the center of gravity

by the length of the pole. The formula for the volume, considering a pole as a frustrum of a cone is

$$v = \frac{\pi}{1728}(d_1^2 + d_1d_2 + d_2^2)h,$$

where v = volume in cubic feet; d_1 = diameter at butt in inches; d_2 = diameter at top in inches; h = length of pole in feet.

The following table gives the volume of some standard poles:

Kind of pole	Nominal size		Volume, cubic feet
	Diameter, inches	Length, feet	
Chestnut	7	30	20.0
Southern white cedar	7	30	20.8
Northern white cedar	7	30	17.6
Western red cedar	8	40	27.3
Western yellow pine	8	40	26.0

The weight of a pole may be found by multiplying its volume in cubic feet by its weight per cubic foot. The following table gives the weight per cubic foot.

WEIGHT PER CUBIC FOOT OF POLES

Kind of pole	When cut		When seasoned	
	Weight,* pounds per cubic foot	Moisture, per cent of dry weight	Weight,* pounds per cubic foot	Moisture, per cent of dry weight
Southern white cedar	38.9	88	25.0	21
Chestnut (N. C.)	56.5	101	43.2	54
Chestnut (N. J.)	51.8	85	42.2	50
Chestnut (Pa.)	54.0	92	40.7	45
Chestnut (Md.)	56.4	86	44.9	48
Northern white cedar	34.2	90	22.9	27
Western red cedar	42.4	133	23.5	29
Western yellow pine	66.6	154	30.3	16

* Including contained moisture.

SEASONING.— Poles should be seasoned because it increases their resistance to decay, increases their strength and decreases their weight. The strength of partially seasoned timber, other things being equal, increases as the amount of moisture it contains decreases. Thoroughly seasoned timber of small sizes is sometimes three or even four times as strong as the same timber, when green.

Seasoning of poles reduces their weight, commonly from 16 to 30 per cent, and even more for some species, with a corresponding decrease in the cost of transportation. Thorough seasoning is essential if the poles are to be treated with preservatives. The percentage of moisture in a pole when cut varies with the season when cut as shown in the table at top of p. 1149.

In general, poles cut during the spring and summer lose weight most rapidly. Poles cut during autumn and winter lose weight less rapidly, but more regularly.

Too rapid seasoning may be detrimental to the timber by causing excessive checking. Shrinkage of poles during seasoning is very slight and does not exceed 1 per cent on the circumference.

MOISTURE CONTENT WHEN CUT, PER CENT OF DRY WEIGHT

Kind of pole	Spring	Summer	Autumn	Winter
Southern white cedar (N. C.) .	68	77	87	88
Chestnut (N. C.).............	97	91	95	101
Chestnut (N. J.).............	81	83	81	85
Chestnut (Pa.)...............	89	92	88	88
Chestnut (Md.)...............	83	84	85	86
Northern white cedar (Mich.).	77	82	79	90
Western red cedar (Cal.)......	...	133	...	...
Western yellow pine (Cal.).....	149	147	145	154

The time in months required for poles cut at different periods of the year to season to approximately air-dry weight is as follows:

TIME REQUIRED FOR SEASONING

Kind of pole	Spring	Summer	Autumn	Winter	Moisture content* seasoned
	Months	Months	Months	Months	Per cent
Chestnut (Md.).............	5	4	8	7	55
Southern white cedar (N. C.)..................	3	3	8	5	26
Northern white cedar (Mich.).................	12	9	7	6	37
Western red cedar (Cal.)....	....	....	....	....	43
Western yellow pine (Cal.)..	5	3	9	6	25

* The average amount of moisture remaining in the poles after seasoning as above in per cent of the weight of the dry wood.

ROOFING. — If the top of a pole is left flat rain water will not run off rapidly and will penetrate the pole by following the grain, causing early decay of the pole top. Poles are accordingly "roofed" by cutting the top to give an inclined surface which is sometimes conical but usually is merely two inclined planes meeting in a horizontal ridge. The angle of the planes with the horizontal is usually 45 degrees. Where a bracket is to be bolted to top of pole a flat strip from ½ to 1 inch in width is sometimes left, instead of a sharp ridge, in order to leave more material in the pole top where the strain from the upper bracket bolt comes. Roofs should be painted to close the grain which is porous, in order to prevent the entrance of water.

PRESERVATIVE TREATMENT. — The forest service estimate that it requires 190 years to grow a 30-foot cedar pole whose average life, when set in the ground in its natural state, does not exceed 15 years. They also find that 8 per cent of the poles in use have to be replaced annually. Taking into account the number of poles in use, they conclude that the enormous demand must soon

deplete the supply and have made extensive experiments on preservative treatment as a means of conserving the pole supply.

Preservative treatments are used to increase the resistance of poles to decay. The advantages of such treatment are:

1. It increases the life of the pole.
2. It makes possible the use of smaller poles as less allowance need be made for decay.
3. It makes possible the use of species of timber not naturally durable.

The butt of the pole near the ground line is most subject to decay and treatment of the butt alone is usually deemed sufficient. In some of the Southern states the whole pole is subject to decay, in which case the whole pole is treated.

Cause of the Decay of Timber. — Decay of wood is due to low forms of plant life called fungi. The germs of decay are not inherent in the wood. The wood-destroying fungi start from the outside, either from adjacent rotten wood or by spores, which correspond to seeds, being carried by the wind and deposited on the surface. While the fungi from these spores begin at the "outside" of the wood, this surface must be understood to include all holes or cracks which the spores may enter.

Fungi require for their growth and development air, heat, moisture and food. Warmth, preferably between 60° and 100° F., favors decay. Cold retards it and temperatures above 150° F. prevent it. Under water or deep under the surface of the ground where the air is excluded, decay does not take place. Ordinarily wood which is seasoned until it is air-dry does not contain sufficient moisture to support the growth of fungi.

Preservatives. — The best method of checking the growth of fungi is to deprive them of food. This can be done by injecting poisonous substances into the timber. These substances are called preservatives. Of the many antiseptics which have been proposed for the preservation of timber only four have been largely used with success in the United States. These are creosote, zinc chlorid, corrosive sublimate and copper sulphate. Copper sulphate has fallen into almost total disuse. At present creosote and zinc chloride, pure or in mixture, are the only preservatives which are in general use.

Corrosive Sublimate (Bichlorid of mercury). — This is used in the so-called "kyanizing" process. This process consists in steeping the timber in a dilute solution of corrosive sublimate long enough to insure thorough penetration.

Zinc Chloride. — Zinc chloride is an excellent antiseptic; it is obtained by dissolving metallic zinc in hydrochloric acid. This is further diluted by water before it is used for wood preservation. Zinc chloride is much cheaper than creosote, and since it is shipped in the form of a solid the freight charges are considerably less. Zinc chloride is soluble in water, being in fact, injected into the timber in water solution and so when timber treated with it is exposed to moisture the leaching out of the salt is only a question of time. Hence zinc chloride is most commonly used in comparatively dry situations.

Creosote. — Creosote is a by-product of coal tar, which is produced at most plants for the manufacture of illuminating gas and at by-product coke-oven plants. Wood tar, when distilled in a similar manner, gives "wood creosote," which like that derived from coal tar, possesses strong antiseptic properties. There is also on the market a so-called creosote, a by-product of water-gas tar or tar manufactured from kerosene oils, which, for wood preservation, is probably inferior to the true creosote. In general, however, by "creosote" is meant the dead oil of coal tar.

Creosote is not a single chemical compound but a mixture of a number of compounds. Not only does the relative proportion of the several constituents vary, but some may be absent or other compounds, not normally constituents of creosote, may be present. Creosote proper is the fraction of oil passing over between 240° C. and 270° C. during the first distillation of the crude coal tar. In practice, however, many of the creosote oils of commerce contain considerable amounts of materials having boiling points higher than 270° C. and lower than 240° C. Some commercial creosotes are rather thin oils, some are almost entirely solid with naphthalene and some are heavy oils with a large proportion of high-boiling constituents.

An analysis of creosote in well-preserved timbers (*Forest Service Circular 98*), led to the conclusions that light oils, boiling below 205° C., will not remain in timber, but that heavy oils, containing a high percentage of anthracene oil, will remain almost indefinitely and protect the wood from decay and boring animals.

The cost of creosote in carload lots (including transportation) is (1921) about 40 cents per gallon for points east of the Mississippi River and in the vicinity of the Gulf ports west of the Mississippi. West of the Rocky Mountains the cost is about 50 cents per gallon. A gallon of creosote is estimated to be 8½ to 9 pounds.

Patented Preservation. — There are many other patented substances known by various names, but most of them have for their base creosote or zinc chloride.

Carbolineum. — Carbolineum, like creosote, is derived from the distillation of coal tar. The compounds included in carbolineum are derived from the coal tar at a higher temperature of distillation and are therefore somewhat different.

Crude Petroleum. — Crude petroleum has been experimented with but there is little definite knowledge of its value as a wood preservative.

Methods of Treatment. — The methods of applying the preservatives to the pole are the brush treatment, open tank treatment and pressure tank treatment.

The brush treatment is applied to a part of the butt at the ground line, the open-tank treatment to the whole butt and the pressure-tank treatment to the whole pole.

The brush treatment is least expensive and gives the least protection and the pressure tank is most expensive and gives the most protection.

For a full description of these methods of treatment and estimates of cost see *U. S. Forest Service Circulars, Nos. 84, 147, etc.*

An estimate in 1921 for a two-coat brush treatment of the butts with creosote is approximately 50 cents per pole. Another way in which this may be figured is on the basis of the amount of creosote and the amount of labor required to treat one pole, as follows: It takes 2 to 3 pounds of creosote (¼ to ⅓ pound per square foot of surface treated) for one pole, and an efficient man about ½ hour to apply it. The cost of open-tank treatment of the butt with creosote varies from $2 for a 25-foot 7-inch pole to $13 on a 60-foot 8-inch pole. This cost mounts rapidly for large poles because the butt is not only of greater diameter but a greater depth has to be treated.

LIFE OF POLES. — Statistics compiled by the National Electric Light Association give the following figures for the average life of *untreated* poles; the figures for *butt-treated* poles are according to estimates by the U. S. Forest Service.

Untreated	Years	Butt-treated	Years
Cedar	13.5	Chestnut	20
Chestnut	12.0	Western cedar	20
Cypress	9.0	Northern white cedar	22
Pine	6.5	Pine, in dry climates	20
Juniper	8.5		

Records of the German Postal and Telegraph Department covering 52 years show an average life of 20.6 years for creosoted pine poles.

SPECIFICATIONS FOR POLES. — In the report of the Committee on Overhead Line Construction of the National Electric Light Association (abbreviated N.E.L.A.) are given very complete specifications for chestnut, eastern white cedar and yellow pine poles. Space does not permit of the incorporation of these specifications here; copies may be had from the National Electric Light Association, 33 W. 39th St., New York.

FORCES ACTING ON A POLE.—A pole is subject to the following forces:

(1) Vertical forces due to weight of pole, wires, sleet, etc., and to downward pull of guys.
(2) Lateral horizontal forces due to wind across line on pole, wire, sleet, etc.
(3) Longitudinal horizontal forces due to unbalanced pull of wires.
(4) Torsional forces due to unbalanced pull of wires.

A pole is strong as regards the vertical forces but weak for horizontal forces and the cross arms are weak for the torsional forces. The theory of good line work is, therefore, first to reduce the horizontal and torsional forces as much as possible by balancing the stresses and second to convert remaining unbalanced horizontal stresses into vertical stresses on the pole by the use of guys.

In practice the lateral horizontal force of the wind is one which cannot ordinarily be provided for by guys. Calculations for strength of poles, when made, are ordinarily limited to the effect of side wind.

Breaking of Pole by Cross Wind. — The principal forces tending to break a pole are wind pressures on pole and conductors when the wind blows transversely. These tend to break it by cross bending.

Let M_1 = moment of the wind on the pole,
M_2 = moment of the wind on the wires,
M = moment of resistance of the pole.

Then the condition that the pole shall not break is that

$$M_1 + M_2 < M.$$

The calculation of M_1, M_2 and M is given below.

Moment of Wind on Pole (M_1). — Moment at ground level due to wind pressure on pole is

$$M_1 = \frac{P_1 H_1^2 (D_1 + 2\,D_2)}{72},$$

M_1 = moment at the ground in pound-feet,
P_1 = wind pressure in pounds per sq. ft. of projected area of pole,
H_1 = height of pole in feet,
D_1 = diameter of pole at ground in inches,
D_2 = diameter of pole at top in inches.

The maximum bending moment due to horizontal forces at the top of the pole is ordinarily assumed to be at the ground level: it is really a little below ground level and opposite the center of pressure of the resistance furnished by the ground.

Moment of Wind on Wires (M_2). — Moment at ground level due to wind pressure on the wires is

$$M_2 = \frac{P_2 H_2 n d\,(S_1 + S_2)}{24},$$

M_2 = moment at the ground in lb.-ft.,
P_2 = wind pressure in pounds per sq. ft. of projected area of wires,
H_2 = height of wires above ground in feet,
n = number of wires,
d = diameter of wires (including ice) in inches,
S_1 and S_2 = lengths of adjacent spans in feet.

Where wires are of different diameters or at different levels the formula is to be applied to each size and each level separately and moments summed.

FIBER STRESS AND BREAKING LOAD

Kind of timber	Test specimen	Fiber stress at elastic limit, pounds per square inch	Fiber stress at rupture,* pounds per square inch	Actual force at rupture, pounds
Arborvitae (1)	2 by 2 by 30 in.	2600	4250	....
Cedar:				
Red, western	25-ft. pole	....	5000 (5)	1310 (3)
Red, western (4)	25- to 35-ft. poles	....	....	2215
Red, western (4)	25- to 35-ft. poles	....	....	1930
Oregon (4)	25- to 35-ft. poles	....	....	3040
White, Maine (2)	29- to 31.5-ft. poles.	....	3200 to 5600	1235 to 2650
White, Maine (2)	....	....	3600 (5)	....
Chestnut, Conn. (2)	29- to 31.5-ft. poles.	....	4500 to 9780	1540 to 3240
Cypress	....	4430 (1)	7110 (1)	....
Pine:	....	....	5000 (5)	....
Lodgepole (1)	2 by 2 by 30 in.	3080	5130	....
Lodgepole (3)	25-ft. pole	....	....	1430
Longleaf (1)	2 by 2 by 30 in.	5090	8630	....
Shortleaf (1)	2 by 2 by 30 in.	4360	7710	....
Southern, dense (5)	....	....	6500	....
Yellow, Cal. (1)	2 by 2 by 30 in.	3180	5180	....
Redwood (5)	....	....	3600	....
Spruce, Engelmann (3)	25-ft. pole	....	....	1405

(1) *Forest Service Cir., No. 213;* green, clear pieces.
(2) L. W. Winchester, *Elec. W., March 16, 1911;* top circumference 17 to 24.5 inches, poles set in ground from 4 to 6 ft., force applied 22 ft. to 26 ft. above ground.
(3) *Forest Service Cir., No. 204;* 7 in. top diameter, force applied at top.
(4) *Pac. Tel. & Tel. Co.;* from 6 to 9 in. top diameter, force applied at top.
(5) *National Electrical Safety Code.*

* Modulus of Rupture.

Moment of Resistance (M). — The moment of resistance or strength of a circular pole for cross bending is

$$M = \frac{f\pi D^3}{384} \quad \text{or} \quad = \frac{fD^3}{122}.$$

M = moment of resistance of the section considered in lb-.ft.,
f = fiber stress in pounds per sq. in.,
D = diameter of pole in inches.

The maximum allowable moment M is found by using the maximum allowable value for the fiber stress f.

Fiber Stress (f) **and Actual Tests of Strength.** — The table on page 1153 gives the value of the fiber stress for various kinds of timber and the actual breaking load from tests of a number of poles.

Weakest Point of a Pole. — A pole is approximately a truncated cone in shape. For a bending force applied at one end such a cone is weakest at the point where the diameter is 3/2 the diameter at the point (near the small end) where the force is applied. A pole with 8-inch diameter at the cross arm is, therefore, weakest where it is 12 inches in diameter and may be expected to break at this point provided this point is above the place where maximum bending occurs. If it is less than 12 inches in diameter at the point of maximum bending then the break may be expected here. This rule must be considered approximate as it neglects the fact that the pole is not homogeneous, i.e., outer annual rings are sapwood and inner are heartwood, and also neglects effect of knots, etc.

ATTACHMENT OF CROSS ARMS TO POLES. — Wooden cross arms are attached to wooden poles:

(1) By gaining the pole, see below.
(2) By one or two lag screws or bolts.
(3) By one or two cross-arm braces.

The forces at the point of attachment which these fastenings must resist are:

(1) A force vertically downward, equal to weight of cross arm, pins, insulators and wire (including sleet).

(2) A horizontal force parallel to axis of arm, equal to pressure of wind blowing across line on wires.

(3) A horizontal force at right angles to axis of arm: (a) toward pole or (b) away from pole and equal to difference in pull of wires on two sides of arm.

(4) A couple in a vertical plane parallel to arm, equal to difference in moments of weight on the two ends of arm.

(5) A couple in a horizontal plane parallel to arm, equal to difference in moments of wire pull on the two ends of arm.

(6) A couple in a vertical plane at right angles to arm, equal to difference in moments of wire pull (caused by pin leverage) in the two directions.

Framing and Hardware. — Typical framing and hardware for the three principal combinations of arms (single arms, double arms and buck arms on corner poles) are shown in Figs. 1, 2 and 3, respectively. These are standard framings of the Stone & Webster Engineering Corp. The arms shown are all 6 pin, but the framing would be the same, except for length of arm for 4-, 8- or 10-pin arms.

Gaining. — A gain is a notch cut in the side of a pole to receive a cross arm. The width (vertical dimension) of the gain should be just large enough for the cross arm. The depth of gain varies from ½ to 1 inch. With gains shallower than ½ inch the cross arm has insufficient support below and the flat bearing

surface at the back is inadequate unless the pole is of larger diameter than usual. Deep gains greatly weaken the top of the pole especially when double arms are used. Gains should be painted before arms are attached to prevent moisture entering the wood through the cut surface.

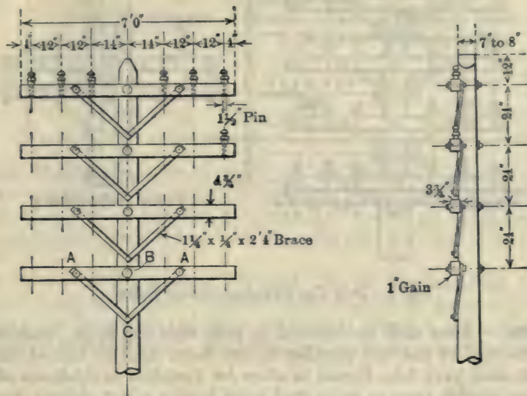

Fig. 1. Pole Top Framing, Single Arm

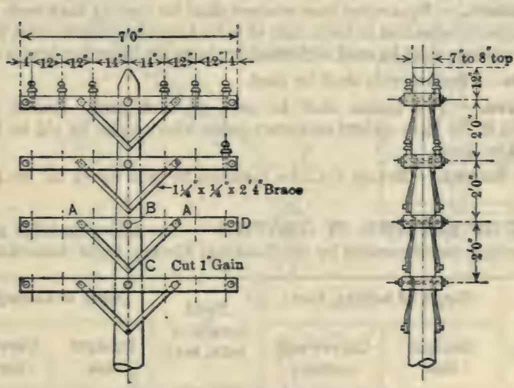

Fig. 2. Pole Top Framing, Double Arms

Specification for Framing. — Gaining shall be as follows:

Top of pole to center of top gain, 12 inches.
Center to center of gains, 24 inches.

Gains for buck arms shall be located centrally between gains for main arms. Gains shall be cut not over 1 inch deep.

Arms shall be fastened to pole by one through bolt. Back of pole shall be flattened to give true bearing surface for washer under head of bolt. Each arm shall be braced by two braces fastened to back side of arm. Ends of double arms shall be separated by spacing blocks and shall be bolted together.

Specification for Hardware. — All hardware shall be galvanized iron.

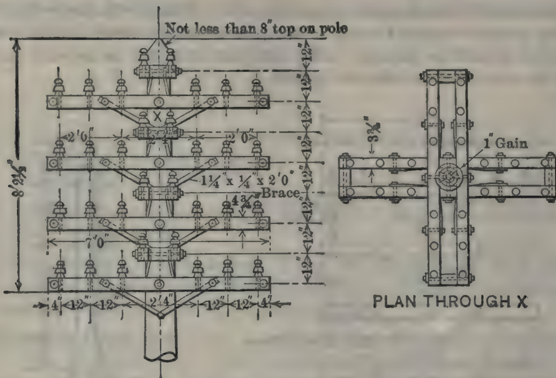

Fig. 3. Pole Top Framing, Corner Poles

Bolts. — Arms shall be fastened to pole with one ⅝-in. machine bolt. Double arms shall be fastened together by one ⅝-in. machine bolt at each end. Above bolts shall have 6-in. thread to allow for variation in thickness of poles. For fastening double arms to poles stud bolts having a nut at each end shall be used. Braces shall be fastened to arms by ⅜-in. machine bolts.

Washers. — Square-cut iron washers shall be used at both ends of bolts fastening arms to poles and at both ends of bolts fastening double arms together. Round-cut washers shall be used with machine bolts for fastening braces to arms.

Nuts. — Square nuts shall be used.

Braces. — Two braces shall be used on each arm. Braces shall be 28 in. by 1¼ in. by ¼ in. except on corner poles, where 24 in. by 1¼ in. by ¼ in. braces shall be used.

Lag Screws. — Braces shall be fastened to pole by ½ in. by 4 in. lag screws.

DEPTH OF SETTING IN GROUND. — The following table gives the depth of setting recommended by the National Electric Light Association.

Total length of pole, feet	Depth of setting, feet		Total length of pole, feet	Depth of setting, feet	
	Straight line	Curves and corners		Straight line	Curves and corners
30	5.0	6.0	60	7.0	7.5
35	5.5	6.0	65	7.5	8.0
40	6.0	6.5	70	7.5	8.0
45	6.5	7.0	75	8.0	8.5
50	6.5	7.0	80	8.0	8.5
55	7.0	7.5			

GUYING OF POLES. — A guy is ordinarily composed of guy wire, clamps, strain insulators, turnbuckles (sometimes) and guy stub or guy anchor.

Guy Wire. — Iron wires are used for guy wires but should always be galvanized (*see Galvanizing*). Solid wire was formerly common but stranded cable

(called "strand') is now generally used. Various sizes have been used but the ⅜-inch is probably the best. The N.E.L.A. specification for the two sizes recognized by them is:

Size	Strands	Size of individual wires, B. W. G.	Ultimate breaking strength
Inch			Pounds
¼	7	14	2300
⅜	7	12	5000

Strain Insulators. — Strain insulators are placed in guys to prevent the lower part of the guy wire, which is accessible to the public, becoming charged through leakage or contact with a conductor. Two strian insulators should be used, one located about 5 feet from the pole and the other 8 feet above the ground.

Two types of strain insulator are used. In one the insulation (usually impregnated wood) is in tension, while in the other (usually porcelain) it is in compression. The former has the disadvantage that failure of the insulation causes mechanical failure of the guy. The latter has the disadvantage that the insulator

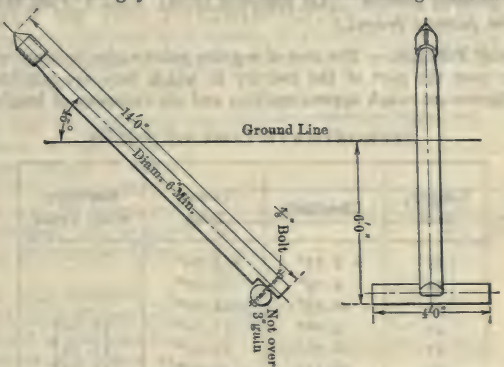

Fig. 4.

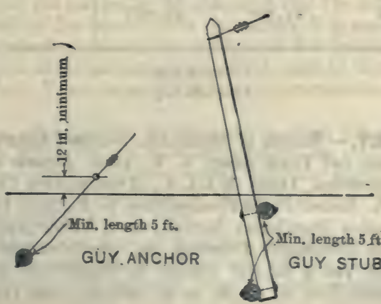

Fig. 5.

may fail and the guy become charged without the failure being readily apparent. The N.E.L.A. specification favors the latter type and requires a mechanical strength of twice the strength of the guy wire and a wet flash over electrical test of four times the line voltage.

Guy Stubs and Guy Anchors. — A common guy stub is shown in Fig. 4. This is made from parts of defective poles and is fastened together with a cross-arm bolt. Fig. 5 shows another common form of guy stub and a common guy anchor.

There are also various forms of patented anchors which screw into the earth or are placed or driven into small holes and then expanded. These are designed to economize labor of installation of standard anchors.

REPAIRING DECAYED POLES. — Where poles have been weakened at the ground line by decay the strength may be restored by cutting off the decayed butt and resetting pole, thus reducing its height by six to eight feet. When reduction of height is not permissible, the pole may be stubbed by setting along side of it a short pole or stub extending a few feet above ground to which the old pole or the undecayed part above ground is bolted or otherwise fastened. This method does not look well and is unsuitable for city distribution lines but has been used for transmission lines. A more recent method is to reinforce the decayed pole by a sleeve of concrete (usually reinforced) extending above and below the decayed portion. (*See Electrical World, April 1, 1909 and Aug. 25, 1913 for Orr patented process.*)

COST OF POLES. — The cost of wooden poles varies between wide limits, depending upon the part of the country in which they are purchased. The following figures are rough approximations and are exclusive of freight.

APPROXIMATE COST OF POLES

Length, feet	Chestnut	Cedar, Iowa	Northern white cedar
25	$ 3.75*	$ 5.50*	......
30	4.00*	10.00*	......
35	6.25*	13.50*	......
40	9.00†	17.00*	$8.00†
45	10.50†	22.00*	11.75†
50	16.75†	26.00*	14.50†
55	18.50†	40.00†	......
60	14.50†	43.00†	......

* 7-inch top.
† 8-inch top.

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POTENTIOMETERS. — (*See also Cells, Standard; Galvanometers; Resistors, Standard.*) A potentiometer is primarily an arrangement of resistances for the accurate comparison of two potential differences by balancing one against the other. In connection with suitable resistance standards (*see Resistors, Standard*) it may also be used for the accurate measurement of electric currents. The accessories for making ordinary d-c. measurements are a standard cell (*see Cells, Standard*) and a galvanometer (q.v.), and suitable keys or switches. For making a-c. measurements certain additional apparatus is required; see below under *Alternating Current Potentiometer.*

USES OF THE POTENTIOMETER. — For the calibration of current, voltage and power-measuring instruments, both d-c. and a-c., the potentiometer is the most accurate and satisfactory instrument available. A technical laboratory relies almost entirely upon the standard Weston (or Clark) cell and a set of standard resistances as its ultimate or primary standards, leaving the testing of the accuracy of these latter to a central standardizing bureau, such as the Bureau of Standards at Washington. It is, of course, convenient to have suitable "precision" ammeters, voltmeters and wattmeters as secondary laboratory standards but, as such secondary standards tend to "lose their calibration," they should be frequently checked against the standard cell and standard resistances by means of a potentiometer.

PRINCIPLE OF THE POTENTIOMETER (Figs. 1 to 3). — The potentiometer in its simplest form consists of a uniform wire stretched over a scale divided into a number of even parts, say 1500. A battery B (Fig. 1) having an e.m.f. of about 2 volts and rheostat R are connected in series with this wire and two contact points A and S are provided, one contact S being movable. G is a galvanometer. At P there may be connected at will a standard cell or any other source of potential difference.

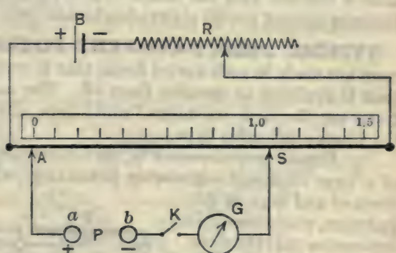

Fig. 1. Simple Potentiometer

At P is first connected a standard cell, say a Weston cell, having an e.m.f. of 1.01830 volts and the contact A is placed at 0 and the contact S at 1.0183 on the scale. By varying the rheostat R it is possible to so adjust the current in the slide wire that the fall of potential between A and S due to the current from the battery B is exactly 1.0183 volts. This can be determined by closing the switch K; if R is properly adjusted there will be no deflection of the galvanometer. Care must be taken that the positive terminals of both B and the standard battery are connected to the same end of the wire.

To measure any other p.d. the standard cell is taken out of circuit and the terminals between which the p.d. is to be measured are connected to a and b respectively. The contact S is then moved until the galvanometer shows a balance. The corresponding position of the contact S, as read on the wire, then gives the value of this p.d. in volts.

Increase of Range by Use of Volt Box or Multiplier. — To measure voltages above 1.5 one must resort to the "volt box" or "multiplier," which is simply a standard high resistance, R_1 (Fig. 2) being provided with taps, so arranged that a definite fraction of the total drop can be measured on the poten-

tiometer. The unknown p.d. is connected at *E*, Fig. 2, the terminals *a* and *b* being the same as the like-lettered terminals in Fig. 1. The potentiometer reading would then be multiplied by 10 or 100 depending on the position of the switch *D*. The resistance R_1 must be sufficiently large so that the current taken by R_1 does not appreciably affect the value of the p.d. in the circuit being tested.

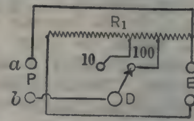

Fig. 2. Volt Box

Increase of Accuracy by Use of Additional Series Resistance. — The accuracy of measurement may be increased by inserting in series with the slide wire a known multiple of the resistance of, say, 1000 divisions of the slide wire. This is equivalent to increasing the length of the slide wire, and therefore, each division on the scale represents a correspondingly smaller fraction of a volt.

Current Measurement. — If it is desired to measure current, the drop across a known low resistance *r* is measured as shown in Fig. 3. The resistance *r* is usually so adjusted that its resistance between the potential terminals PP' is an even fraction of an ohm, in which case the potentiometer reads the current directly, with the exception of the proper pointing of the decimal. For instance, using a 0.01 ohm standard the potentiometer reading is to be multiplied by 100.

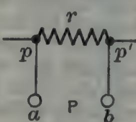

Fig. 3.

Caution. — In using any form of potentiometer a balance with the standard cell in circuit should always be obtained just before and after a balance with the unknown p.d. in circuit. In other words, the potentiometer current should always be checked both before and after a measurement.

TYPICAL DIRECT-CURRENT POTENTIOMETERS. — Potentiometers are made in such varied forms that it would take considerably more space than is available to describe them all. The most generally used direct-current types are the high-resistance "null" type, the low-resistance "null" type and the Brooks deflection type.

The Brooks potentiometer is particularly well adapted to the requirements of a large electrical engineering laboratory, where many instruments must be checked and kept in adjustment, and where it is imperative that the work be done with great speed, combined with a degree of precision ample for engineering work.

Low-resistance Null Type (Fig. 4). — A diagram of the connections of a Leeds and Northrup low-resistance null-type potentiometer is shown in Fig. 4.

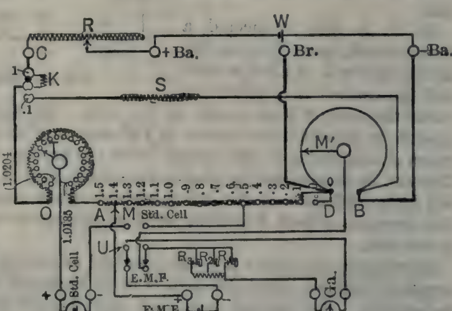

Fig. 4. Leeds & Northrup Potentiometer

Fifteen 5-ohm coils are connected to the studs of a dial switch, the contact *M* arranged to make contact with any stud. The slide wire *DB* consists of 11 turns of manganin wire wound on a marble cylinder 6 inches in diameter. The contact M' is mounted on the inside of a light aluminum hood, which serves to protect the slide wire from dust. It moves over the entire length of the slide

wire and carries a scale from which the number of turns and the fraction of a turn can be easily read.

This potentiometer is designed for use with commercial "standard" Weston cells, which differ slightly in e.m.f. Each one, however, is accompanied by a certificate giving its e.m.f.

The switch T should be set at the stud corresponding to this stated e.m.f. In other words, the switch T is used for adjusting the potentiometer to the particular cell which is used. The rheostat R is then adjusted until no current flows through the galvanometer with the double-throw switch U thrown across the contacts marked Std. Cell.

In closing the galvanometer circuit close the keys R_3, R_2, and R_0 in the order stated. R_3 puts the galvanometer in series with a high resistance, R_2 puts the galvanometer in series with a medium resistance and R_0 puts the galvanometer in circuit without series resistance. If the potentiometer is considerably off balance this may thus be detected without causing a violent deflection of the galvanometer, and a closer balance obtained before the other keys are closed.

Range of Low-resistance Null Type. — With the rheostat R adjusted to give a perfect balance the current through the potentiometer is $1/50$ ampere, and the drop across each coil and across 1000 divisions of the slide wire is consequently 0.1 volt and across 1 division of the slide wire 0.0001 volt.

Range-lowering Device. — The shunt S serves as a range-shifting device, by which the range may be reduced to 0.1 the normal, one division of the slide wire in this case being equal to 0.00001 volt. This shunt has such a resistance that when the plug opposite K is shifted to the lower hole the drop over AD will be $1/10$ its previous value.

High-resistance Null Type (Fig. 5). — A diagram of the connections of a Wolff high-resistance null-type potentiometer is shown in Fig. 5. In this instru-

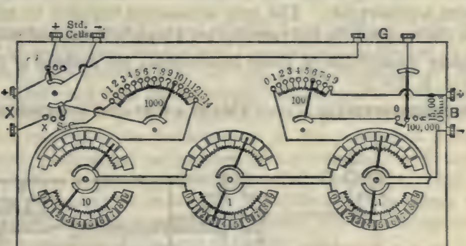

Fig. 5. Wolff's Potentiometer

ment the slide wire is dispensed with altogether. The double-dial switches are so arranged that the total resistance between the terminals B is constant and equal to 15,000 ohms irrespective of the position of the switches. The means whereby this is accomplished is evident upon tracing out the circuits in the diagram. By using this high resistance and keeping it constant, the effect of the contact resistances in the various switches is rendered negligible. The adjustments are made in the same manner as for the simple slide-wire potentiometer, except that instead of moving a sliding contact along a wire, one manipulates the various dial switches. The potentiometer circuit proper and all necessary keys and switches, are provided in this instrument. The accessories necessary for making measurements up to 1.5 volts are a regulating rheostat of about 5000 ohms total resistance, a standard cell, a suitable galvanometer and two cells of storage batteries or other source of e.m.f. of about 4 volts.

Range of High-resistance Null Type. — This potentiometer has a range up to 15 volts in steps of 0.0001 volt and up to 1.5 volts in steps of 0.00001 volt.

Brooks Potentiometer (Fig. 6). — In this instrument no attempt is made to obtain an exact balance; the dial switches are set so near to the null point that

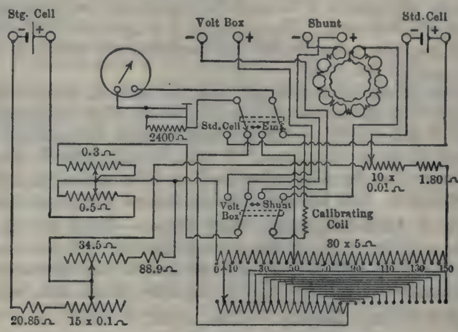

Fig. 6. Connections of Brooks Potentiometer

the galvanometer deflection is small. The galvanometer is so graduated that it gives the amount that must be added to the reading of the dial in order to give the unknown p.d. The potentiometer proper, regulating rheostat, galvanometer and necessary keys and switches are mounted in one box, making the instrument semiportable. The only accessories required to make measurements within the range of the instruments are a standard cell and storage battery.

For complete description of these instruments see *Bulletin of the Bureau of Standards, Vol. 8, No. 2.*

White Potentiometer(Fig. 7). — This is a four-dial low-resistance potenti-

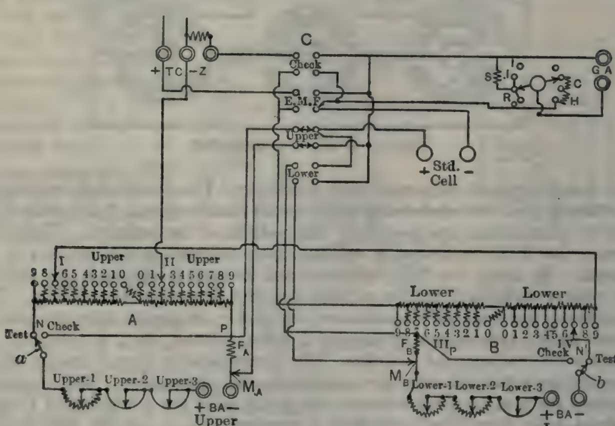

Fig. 7. White Potentiometer.

ometer used for measuring small potential differences to high precision. It is well adapted for making high precision temperature measurements using thermocouples. The potentiometer is so constructed that parasitic thermoelectric forces are entirely eliminated. The resistance in the galvanometer circuit is kept constant by means of compensating coils which enable the instrument to be used as a deflection potentiometer.

A description of this potentiometer is given by Walter P. White in Zeitschrift für Instrumentenkunde 27, 210, 1907.

Queen-Gray Potentiometer (Fig. 8).—This potentiometer is of the "split circuit" type and is a compromise between high- and low-resistance instruments,

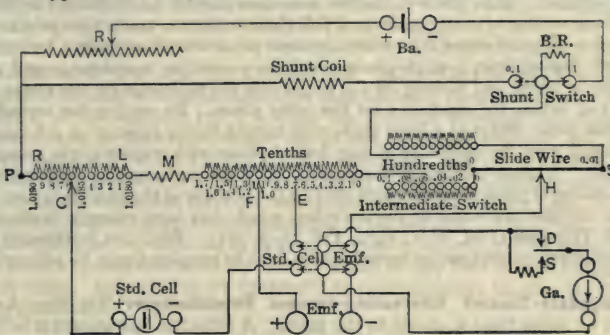

Fig. 8. Queen-Gray Potentiometer.

thereby possessing the advantages of each form without their disadvantages. The current to operate is 0.002 ampere, thus permitting the use of a dry battery. The drop over each resistance in the first group of coils is 0.1 volt, over the second is 0.01 volt and over the slide wire which consists of but one turn is also 0.01 volt. The slide wire is divided into 200 parts or 0.00005 volt. Each scale division can easily be estimated to its 0.2 part, thus making the final reading 0.00001 volt, which can be reduced 0.1 by a shunt coil, thereby making the lowest reading 0.000001 volt. The maximum direct range is 1.8 volts, which can be extended by means of multipliers. It is direct reading with any cadmium cell and has a contained rheostat for accurately controlling the 0.002 ampere regardless of the type of battery used.

ALTERNATING-CURRENT POTENTIOMETER. — The principles involved in the use of the potentiometer for alternating-current measurements are the following: Referring to Fig. 1, a balance is obtained with a battery at B, a standard cell at P and the contact S at the point on the scale corresponding to the e.m.f. of the standard cell, and the current flowing in the potentiometer circuit noted by means of an electrodynamometer or a.c.-d.c. ammeter connected between B and A. The battery B is then replaced by a source of alternating current and the rheostat R adjusted until an alternating current of the same effective value, as read on the electrodynamometer or ammeter, flows through the potentiometer. The standard cell at P is replaced by the unknown alternating p.d. to be measured and the galvanometer by an alternating-current galvanometer (*see Galvanometers*). To obtain a balance, however, it is now not only necessary that the contact S be moved along until the drop between A and S has the same effective value as the unknown p.d., *but also that this drop and the unknown p.d. have the same frequency and are in the same phase.* Con-

sequently the p.d. at B must be supplied from the same source as the p.d. to be measured, and means must be provided for shifting the phase of one with respect to the other.

The coils of an a-c. potentiometer are wound non-inductively, and consequently the p.d. due to the potentiometer current is in phase with this current. Hence the phase adjustment consists in bringing the potentiometer current into phase with the unknown p.d.

Phase-shifting Device. — One means of shifting the phase of the potentiometer current with respect to the unknown p.d. is to use two small alternators mounted on the same shaft, with the field frame of one adjustable with respect to the other. This, however, can be more conveniently accomplished by using a device similar in construction to a polyphase-induction motor, the primary of which is supplied either from a two- or three-phase circuit, or from a two-phase circuit derived from a single-phase circuit by connecting a resistance and condenser in series, and tapping off the p.d. across the resistance for one phase and the p.d. across the condenser for the other phase. By setting the secondary of this phase shifter at a fixed angle with respect to the primary, a p.d. can be obtained from the secondary at any desired phase with respect to the source of supply. A phase-shifting device based on this principle, and so designed that the change of phase resulting from shifting the position of the secondary does not alter the effective value of the secondary e.m.f., has been recently devised by C. V. Drysdale (*Phil. Mag., 1909, Vol. 17, p. 402*). With this instrument the phase of the potentiometer current and p.d. to be measured may be adjusted to 0.1°.

Drysdale-Tinsley Alternating-current Potentiometer (Fig. 9).— A diagram of connections is shown in the figure. A vibration galvanometer is used

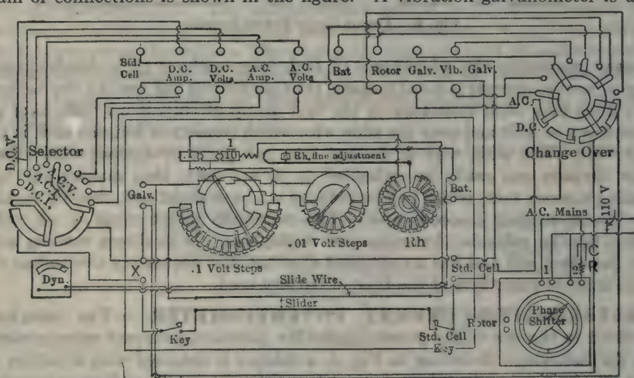

Fig. 9. Drysdale-Tinsley Alternating-current Potentiometer

as an a-c. detector and is tuned to the impressed frequency. The potentiometer is first balanced with an ordinary battery and standard cell and the reading of the electrodynamometer noted. By means of the change-over switch, alternating is substituted for direct current and the reading brought to the same point and held there. The phase of the potentiometer current is then roughly adjusted and the unknown p.d. is balanced as nearly as possible by shifting the potentiometer slides. The balance is then improved by shifting the phase of the potentiometer current and by resetting the slides; thus, by a process of double adjustment, the vibration galvanometer is brought to rest.

As the vibration galvanometer is a tuned instrument, *the frequency of the a-c. supply must be kept constant.* Also, the wave shape of the potentiometer current

and that of the unknown p.d. must be the same, as the vibration galvanometer shows a *balance for the fundamental frequency only.*

Current and Power-Factor Measurements. — By the use of suitable non-inductive shunts alternating currents may also be measured with this potentiometer. Also, by noting the reading of the phase-shifting device corresponding to a balance first of the given p.d. and then for the given current, the phase angle between the p.d. and current can be obtained by taking the difference of the two readings.

Range of Drysdale-Tinsley Potentiometer. — This potentiometer has a range of from 0 to 1.5 volts in steps of 0.001 volt. With suitable non-inductive volt boxes its range may be extended to 750 volts. It may also be used, in conjunction with suitable shunts, for measuring alternating currents of any value.

PRECISION OF POTENTIOMETER MEASUREMENTS. — The precision obtainable with a potentiometer depends upon the accuracy to which the e.m.f. of the standard cell is known, the accuracy to which the various resistance coils in the potentiometer circuit and shunts are adjusted, and the relative magnitude of the various contact resistances, and the proportion of the total resistance in the potentiometer circuit. For a-c. measurements the accuracy is also dependent upon the inductance and capacity of the coils. The Bureau of Standards calibrate the Weston cell to 1/50 per cent. The various coils of a potentiometer are adjusted to different degrees of accuracy, little attention being paid to the absolute accuracy of coils as long as they bear definite relations to each other. Instruments are calibrated as potentiometers to give the following accuracies.

PRECISION OF POTENTIOMETER MEASUREMENTS

Kind of potentiometer	Degree of precision, per cent	
	Voltage measurements	Current measurements
Low-resistance null type	1/50	1/20
High-resistance null type	1/50	1/20
Brooks	1/25	1/10

COST OF POTENTIOMETERS. — The following are approximate costs of the potentiometers proper, exclusive of the standard cell, galvanometer, volt box and shunts for current measurements.

Low-resistance null type (pre-war price)	$240
High-resistance null type, rheostat not included (pre-war price)	250
Brooks deflection type, including galvanometers (pre-war price)	350
White, including rheostats (1920 price)	500

BIBLIOGRAPHY. — A complete bibliography on potentiometers is given in *Circular No. 21, Bur. Stand.*, 1910; H. B. Brooks, *Bull. Bur. Stand.*, Vol. 8, 1911, p. 395. See also Laws, F. A., *Electrical Measuring Instruments;* Pedersen, P. O., *A. C. Potentiometer for Telephone Frequencies*, Electrician, 83, p. 523, 1919; Kennelly and Velander, *A. C. Potentiometer giving Rectangular Components*, J. Frank. Inst., 188, p. 1, 1919; Ryan, H. J., *High Voltage Potentiometer*, Proc. A.I.E.E., 35, p. 1187, 1916.

POWER-FACTOR INDICATORS AND REACTIVE VOLT-AMPERE INDICATORS. — (*See also Alternating Currents; Generators, Alternating-Current; Wattmeters.*) A power-factor indicator is an instrument designed to give a direct reading, at any instant, of the power factor in a circuit or system of circuits as well as to indicate whether the current is leading or lagging; see *Alternating Currents*. The power factor of a single-phase circuit may be calculated from the readings of an ammeter, voltmeter and wattmeter. The power factor of a balanced three-phase circuit can also be calculated directly from the readings of the two wattmeters used to measure the power (*see Wattmeters*). A direct-reading power-factor indicator, however, is usually to be preferred for station purposes, since it gives the power factor directly and also indicates directly whether the current is leading or lagging.

Under certain conditions it is more convenient to obtain a direct measure of the reactive (wattless) power supplied to a circuit; ordinary wattmeters may be thus used as explained below.

Closely allied to power factor meters are the "reactive factor meters" which, however, should not be confused with "reactive volt-ampere meters." Reactive volt-ampere meters are also often called "reactive component indicators," or "reactive component watthour meters." In order to clearly indicate the various instruments involved in phase angle measurements of a-c. power circuits the following table is appended:

Instrument	Measurement	Symbol
Wattmeter	True power	V. A. cos ϕ
Volt-ammeter, K V. A. meter	Apparent power	V A
Reactive volt-ammeter	Reactive component of apparent power or of volt amperes	V. A. sin ϕ
Power factor meter	Angle of lag between *C* and *V*, indicating according to cosine	cos ϕ
Reactive factor meter	Angle of lag between *C* and *V*, indicating according to sine	sin ϕ

SINGLE-PHASE POWER-FACTOR METERS. — Instruments for measuring the power factor of single-phase circuits are provided with one current coil circuit and one potential coil, the latter, however, being generally a double-circuit coil, the two portions being located with axes at right-angles to each other. The one is supplied with current from the potential binding posts through a non-inductive resistance and the other through a highly inductive resistance or "choke coil" so as to receive currents in the divided circuits corresponding in electrical phase angle to the mechanical angle between the coils.

POLYPHASE POWER-FACTOR METERS. — These can be classified as follows:

For Balanced Polyphase Circuits. — Such instruments are provided with a single-current coil connected to one phase of a polyphase circuit, and two or more voltage coils star-connected to each polyphase line. They are calibrated to indicate the cosine of the angle of lag between the line supplying the current coil and the voltage from this line to the artificial neutral point formed by the voltage coils within the instrument. It is assumed that the angle of lag is equal

in all phases within an accuracy sufficient for the purpose for which the instrument is used.

From the above it will be understood that "balanced polyphase" instruments differ from single-phase instruments only as regards the voltage coil connections and the calibration.

For Unbalanced Polyphase Circuits. — The development of such instruments in the United States can be said to date from the adoption of a standard definition by the A.I.E.E. for power factor of unbalanced polyphase circuits (see A.I.E.E. Standards.)

Let the active (power) and reactive (stored) components of the volt-amperes of each phase be represented in Fig. 1 by the small triangles *a*, *b*, *c*, in which the watts are plotted horizontally and the reactive components vertically. Then the total power in the polyphase circuit is OX, the total volt-amperes OA, and the power factor is the cosine of the angle AOX.

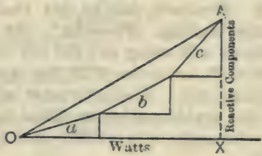

Fig. 1

Power-factor meters for correctly measuring unbalanced polyphase circuits are inherently quite complicated, comprising three voltage circuits and three current circuits or their equivalents. However, by measuring the power factor of each phase separately with instruments having single-phase current coils, the unbalanced-circuit polyphase power factor can be determined.

An instrument intermediate between the balanced and the unbalanced circuit types consists of three star-connected current coils one connected to each line and one voltage coil connected to one phase. Although more accurate on moderate unbalances than the single current-coil instrument, and simpler than the instruments for unbalanced circuits, it is more difficult to connect and calibrate than the single-current coil type. An unbalance of 20 per cent produces an angular error of indication of 7° maximum in such instruments.

MOVING-COIL TYPE. — Power-factor indicators of this type operate on the same principle as wattmeters of the electrodynamometer type, except that the coils are so arranged and so connected to the circuit that a differential action is produced which depends only on the power factor of the load.

In order that the differential action may be correct under all balanced-load conditions, mechanical control forces such as springs must be avoided or made negligibly weak.

Three-phase, Moving-coil Type. — Fig. 2 is a diagram of a three-phase power-factor indicator having three voltage coils and one current coil. C and C' are current-coil sections in series with one leg of circuit and P_1 and P_3 are potential coils connected as shown through the non-inductive resistances r_1 and r_2. Connections into and out of the moving coils are made by light spirals having very small torque, or through the bearings, so that the final position of the moving system depends solely upon the relative forces between the fixed and moving coils.

If the instrument is connected properly, the current coil acts with the two potential coils like the two elements of polyphase wattmeter (see *Wattmeters*). The torques of the two moving elements are developed in opposite directions. The torque of each element depends on the position of its potential coil. The potential coils are fixed with relation to one another at such an angle that the torque of one element always increases when that of the other decreases, due to the movement. Hence the needle will come to rest where the two torques

are equal, and will give a scale reading dependent upon the ratio of the watts supplied to the two elements, i.e., in the case of a balanced load dependent on the ratio $\cos(30 - \theta) \div \cos(30 + \theta)$, where θ is the power-factor angle. Hence, for unity power factor, ($\theta = 0$), the moving element will take up a position which is symmetrical with respect to the two potential coils, i.e., the pointer will take up a vertical position. For 50 per cent power factor, lagging, ($\theta = 60°$), the moving element will swing to the right until the axis of the potential coil P_1 coincides with the axis of the current coil. Similarly, for 50 per cent power factor, leading, ($\theta = -60°$), the moving element will swing to the left until the axis of P_2 coincides with the axis of the current coil. The character and range of the scale can be modified by varying the angle between the two potential coils.

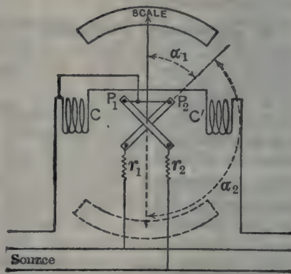

Fig. 2. G. E. Moving-coil Type

Should the direction of flow of power reverse, the pointer would tend to take up the dotted position in Fig. 1. Some instruments are provided with scales to take advantage of this effect.

MOVING-VANE TYPE. — In place of the movable coils this type of power-factor meter contains a movable soft iron vane, which, in the case of polyphase instruments, is magnetized through a stationary coil carrying a current in phase with the current of one phase of the circuit. There is one stationary voltage coil for each phase, the arrangement being such as to produce a rotating field. The iron vane then takes up a position in which the direction of the flux produced in it by the current coil when at a maximum, is coincident with the direction of resultant flux due to the voltage coils. In the three-phase instrument three voltage coils, placed 120° apart, are used; in the two-phase meter two voltage coils at 90° are used. In the single-phase type the iron vane is magnetized by a stationary coil placed in series with the line, while the rotating field is produced by two potential coils 90° apart, one of which is connected to the line through an inductance, the other being connected to the same line through a non-inductive resistance. For the purpose of damping the deflections, an aluminum disk moving in the field of two permanent magnets is attached to the movable system.

REACTIVE WATTLESS VOLT-AMPERE INDICATORS. — Operating conditions on polyphase circuits are sometimes such that the reactive component of the volt-amperes in the circuit does not vary through wide limits of load between light load and full load, whereas the power factor may vary greatly. Under such circumstances it is frequently more convenient to have a direct measure of the reactive volt-amperes rather than the power factor.

Ordinary wattmeters may be used for indicating the reactive component of the volt-amperes, by changing the voltage connections so as in effect to rotate them through 90 electrical degrees, thereby causing the instrument to read $VA \sin\theta$ instead of the usual $VA \cos\theta$ or "active" component usually known as watts or kw. If this rotation is affected without varying the ratio of transformation the calibration of the wattmeter is the same as for ordinary watt measurements. If such connection alters the ratio of transformation, the calibration constant is altered accordingly.

When applied to circuits which may either have lagging or loading current the

zero may be placed in the center of the scale, like zero-center wattmeters which are applied to circuits which are liable to reverse in direction of power flow. Fig. 3 (*a*) shows connections when the phases are rotated without change in calibration being necessary, but requiring voltage transformers having taps. This is the preferable method. Fig. 3 (*b*) shows connections when the phases are rotated requiring special calibration, but not requiring special transformer taps. Fig. 3 (*c* and *d*) shows connections combined with a double-throw switch

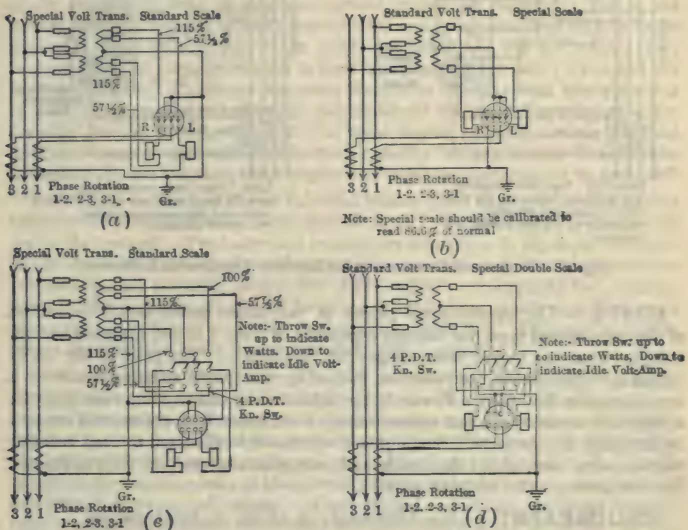

Fig. 3. Diagram of Connections for Reactive Component Indicator.

to read either reactive or active component (*KW*). Fig. 4 shows corresponding connections for 3-phase, 4-wire circuits.

RANGES OF THE VARIOUS TYPES. — The largest capacity of instrument of the moving-coil type for direct connection to the circuit is 100 amperes and 300 volts for the single-phase and 200 amperes and 600 volts for the two-phase and three-phase types. The range of power factor on standard instruments is from 0.60 leading to 0.60 lagging, although instruments with ranges as low as 0.00 leading to 0.00 lagging have been made.

Instruments of the moving-vane type have a current capacity of 5 amperes. They are made for direct connection to circuits of 110, 220 and 440 volts. For all other capacities and voltages the 5-ampere, 110-volt instrument with suitable current and potential transformers is used. The instruments are made for frequencies of either 25 or 60 cycles per second. The range of power factor on the moving-vane type of meter is unlimited.

Power-factor indicators of either type may be connected to instrument transformers which are used in connection with other meters.

Five amperes and 115 volts are standard for wattmeters used as reactive volt-ampere indicators, with transformers for higher ranges. However, single-

phase instruments for direct connection are furnished up to 200 amperes and 600 volts, and polyphase instruments for direct connection, up to 60 amperes and 600 volts.

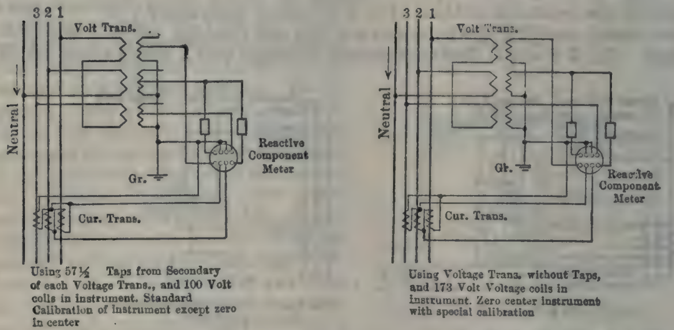

Fig. 4. Diagram of Connections for Reactive Component Indicator.

COSTS. — The approximate prices of polyphase power-factor indicators of the moving-coil type, for current capacities of from 5 to 100 amperes and for voltages from 100 to 600 volts, range from $45 to $75. The price of the corresponding single-phase instruments, including the phase-splitting reactor, is approximately $10 more. Power-factor indicators of the moving-vane type for voltages from 110 to 440 cost approximately from $30 to $40, exclusive of current or potential transformers. Single-phase wattmeters calibrated to read reactive volt-amperes cost about $45 each and polyphase wattmeters thus calibrated cost about $65. (Prices as of 1921.)

BIBLIOGRAPHY. — See *Bibliography* in article on *Synchronizers*.

POWER STATIONS, GAS-ELECTRIC. — (*See also Gas; Gas Engines; Gas Producers; Generators; Power Stations, Steam-Electric; Power Stations, Hydroelectric.*)

Number and Capacity of Units. — As gas engines have little overload capacity a total rating of engines should be installed sufficient to carry the extreme peak load and afford ample emergency reserve. In many cases the kilowatt rating of generators is made but 60 to 65 per cent of the horse-power rating of the engines, in order to provide overload generating capacity. This is not always wise, as the most efficient load of a gas engine is quite near its maximum capacity. The units in very large plants are usually of the horizontal, twin, tandem type and range from 4000 to 6000 h.p. in rating, the latter size being the largest available. Producers are quite flexible in capacity, but heavy forcing is apt to produce clinker and impair the quality of the gas. The total producer capacity usually corresponds with the total engine capacity, unless the load fluctuates greatly and considerable tank capacity for gas storage is available. In most cases not less than three engine and producer units should be provided.

Buildings. — The practice in this field follows quite closely that described in the article on *Power Plants, Steam-Electric* (q.v.). In a few cases producers are set in the open air. Producers and engines are sometimes in separate buildings. In the usual design the engine and producer rooms are parallel and electrical control galleries are provided on the side of the engine room opposite the producer room. The engine room should be spanned by an electrically operated crane with capacity sufficient to handle the heaviest engine part. In some important plants the producer room is also provided with a crane.

Producer Room Lay-out. — The gas producers are usually set in a single row extending the length of the producer room. Each unit comprises one or two gas generators, an evaporator, a scrubbing tower, a gas pump and tar filters grouped so that the path of the gas is toward the engine room. A coal bunker is often provided above the producers, preferably along the outside wall. Down-spouts to the charging bells of the several producers should be somewhat inclined and should have cut-off valves. When desired, automatic weighing hoppers may be attached between the bunker and each down-spout to record the coal consumption of each producer. In some producer rooms a shallow pit is provided along the front of the gas generators so that ashes may be raked into this pit. In other cases ashes are raked out onto the main floor to be carted away. A skeleton gallery of steel connecting the producers at their charging levels is often a convenience.

Engine Room Lay-out. — Horizontal engines are usually set in a single row with piston rods across the engine room. The power end of each unit is on the side of the gas supply and the electrical end on the side nearer the control gallery. In a-c. plants a group of independently driven exciters is placed near the middle of the room. The spaces between units and the clearances at their ends are determined by the room needed for erection, dismantling, making repairs and access during operation.

Piping. — The piping scheme is usually a simple parallel plan. A main gas header is provided and runs the length of the wall between the engine and producer rooms. Branches from this header run to each producer unit, to each engine and to the storage tank. Piping may be of relatively light weight and needs no special provision for expansion, but all joints should be secure, as producer gas is of a poisonous nature. The storage tank may properly be at one end of the building. Each producer is usually set beneath a vent pipe or short steel chimney run a few feet above the roof to discharge the smoke and raw gases produced when starting up.

Space Required. — The following table shows the floor space provided per kw. of plant capacity in modern stations.

Plant capacity, kilowatts	Number of engine units	Number of producer units	Square feet per kilowatt	
			Engine room	Producer room
360	3	2	6.0	2.2
700	2	2	4.5	2.9
1,620	3	2	5.5	1.56
3,000	3	2	3.7	0.93
34,000 (a)	17	0	3.0	.....

(a) Blast-furnace gas plant.

Fuel, Labor and Water. — *Fuel.* — Assuming the coal to have 12,000 B.t.u. per lb., about 1⅙ lb. coal are required per kw-hr. generated at full load; 1⅓ lb. per kw-hr. at ¾ load; and 1⅝ lb. per kw-hr. at ½ load. For coal of any other heating value the weight required is approximately inversely proportional to the B.t.u. per lb.

Labor. — 1 fireman required for each producer in operation and 1 oiler for each engine in operation. Supervision, switchboard attendance, coal handling, etc., as for a steam-electric station. The total cost of labor for operation is usually about 50 per cent of the fuel cost.

Water. — About 9 gallons of water per kw-hr. are required when the supply water is at 50° F., and about 13 gallons per kw-hr. when at 70° F.

COST OF GAS POWER PLANTS. (Pre-war figures.) — The cost per kilowatt of gas power plants varies with the capacity of the plant, the number of units installed and the source of gas supply. The construction cost of engine plants using natural gas ranges from $65.00 to $100.00 per kw., depending on the size of plant. The construction cost of blast-furnace gas plants, including the necessary gas-cleaning equipment, ranges from $85.00 to $120.00 per kw. and of producer plants from $90.00 to $125.00 per kw. according to capacity. The cost per kw. diminishes very slowly above a plant capacity of 12,000 kw.

The reported cost of five blast furnace gas-electric installations is as shown in the following table :

Power plant No.	1	2	3	4	5
No. of units	17	2	3	4	5
Cap. kw., max. con. rating.....	40,000	4,500	9,000	9,000	11,400
Cap. b.h.p. max. con. rating....	56,400	6,400	12,800	12,800	16,300
Cost of installation per kw. max. con. rating:	$	$	$	$	$
(a) Buildings..............	9.87	.....	10.17	10.90	10.52
(b) Eng. equipment	71.78	75.50	72.75	77.78	80.32
(c) Gas cleaning plant......	5.85	16.80	14.40	13.00	12.76
Grand total, power plant, complete, per kw..............	87.50	92.30	97.32	101.86	103.60

Operating Costs of Producer-gas Plants.— The following division of the items making up the cost of operation of producer-gas plants is taken from a typical plant in commercial operation:

	Minimum	Normal	Maximum
Output, kw-hrs. per day	5000	8000	10,000
Fuel, cents per kw-hr	0.25	0.22	0.20
Labor, cents per kw-hr	0.28	0.17	0.14
Supplies and repairs, cents per kw-hr	0.17	0.13	0.11
Operating cost, cents per kw-hr	0.70	0.52	0.45
Fixed charges, cents per kw-hr	0.45	0.28	0.22
Total cost, per kw-hr	1.15	0.80	0.67

Cost of Blast Furnace Gas-Electric Power. — The cost of producing electric power by means of blast-furnace gas engines in one of the large steel plants is reported to be as follows:

Year	1910	1911	1912
Capacity in kw	40,000	40,000	50,000
Kw-hr. produced	116,535,000	157,742,510	286,575,000
Use factor, per cent	33.3	45.0	64.5
Cost of installation, per kw	$88.00	$88.00	$88.00
Annual cost, cents per kw-hr.:			
Labor	0.0678	0.0421	0.0302
Repairs and maintenance	0.0366	0.0305	0.0273
Lubricants	0.0116	0.0100	0.0085
Water	0.0074	0.0057	0.0036
Miscellaneous	0.0064	0.0153	0.0128
Cost of purified gas, cents per kw-hr	0.1951	0.1727	0.1608
Fixed charges, 15 per cent, do.	0.4520	0.3360	0.2310
Grand total at switchboard	0.7769	0.6123	0.4742

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POWER STATIONS, HYDROELECTRIC. — (*See also Power Stations, Gas-Electric; Power Stations, Steam-Electric; Dams; Generators; Hydraulics, Principles of; Hydrology; Pipes and Piping; Switchgear Equipment for Power Stations; Transformers; Water Wheels; etc.*) This article deals primarily with those features in which a hydroelectric plant differs from a steam electric plant; these features arise chiefly from the nature of the prime movers and from the use of the very high voltages (up to 150,000) at which power from hydroelectric stations is transmitted. The individual constituent items, such as generators, water wheels, transformers, switchgear, etc., are treated in the separate articles, dealing with these subjects. The following is a brief table of contents of this article:

LOCATION OF POWER HOUSE. — The location of the power station of a hydroelectric plant is determined primarily by the head utilized and by the location of the dam (*see Dams*). In the case of low head developments the substructure of the power station is usually a portion of the dam itself or a "wing" offset from it. In choosing the location of both the dam and the position of the power station relative thereto, particular attention should be paid to the liability of ice and débris clogging the intakes. For high head developments, using penstocks, the power station is so located as to give the maximum head consistent with economy of construction and availability of a suitable natural channel for carrying off the discharge. Available sites for storage reservoirs should also be considered.

The location of the development generally has to be such that high-tension transmission is necessary, so that provision must also be made for housing the transforming and high-tension switching apparatus. If it is desired to place them indoors, or if it is decided to place the high tension equipment out of doors, an ample area of preferably flat ground should be selected to accommodate, without congestion, this apparatus along with the necessary structures for supporting the buses, etc.

DESIGN OF BUILDING. — (*See also Water Wheels and Their Setting.*) The design of power houses differs greatly, depending on the conditions which are to be met. It is affected to a very great extent by natural conditions, such as the location with respect to the stream, the condition of the soil, etc. Low and high head developments require different types of turbines and these may furthermore be of horizontal or vertical construction, necessitating entirely different lay-outs.

A hydroelectric power-house building is generally divided into two longitudinal bays, a front or main bay, containing the turbines and generators, and a rear bay containing the transformers, switching apparatus, etc.; see Figs. 1 and 1a.

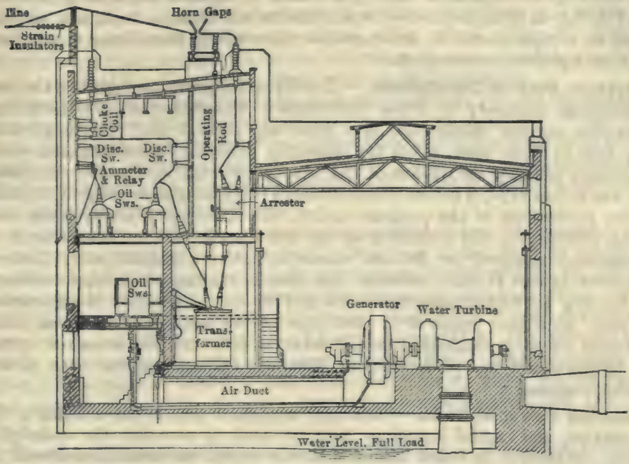

Fig. 1. Cross Section of Power House

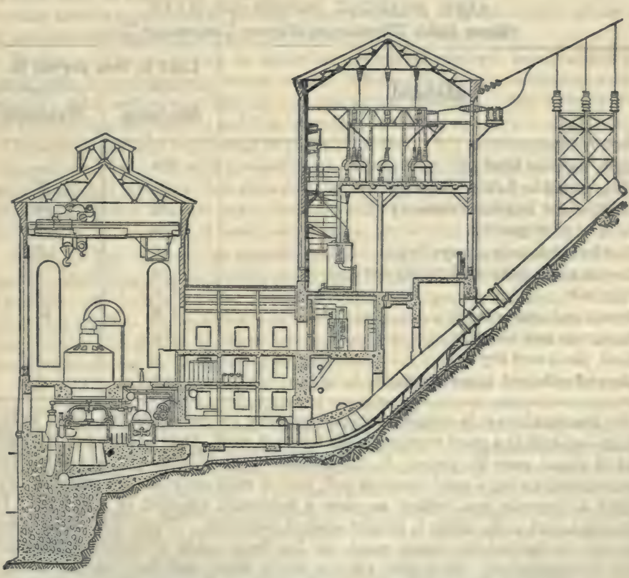

Fig. 1a. Cross Section of Power House; Roof Trusses with Raised Chord

The two bays are separated either by a wall or by a row of supporting columns. The rear bay is divided into two or more floors and these, in turn, into various rooms or compartments to accommodate the step-up transformers, switches, bus-bars, lightning arresters, etc.

Outdoor Step-up Substations. — During recent years, as a result of the cost of buildings necessary to house high-tension transformation and switching apparatus, outdoor substations have come into general use. This is particularly true in those cases where the operating potential exceeds 100,000 volts. Experience confirms the wisdom of this type of construction, for when compared with the indoor type, it is found that although it is a little more difficult to make repairs in the case of the outdoor station, this is largely offset by lower cost of the installation and reduction of the fire hazard. The difference in cost of apparatus designed for the two classes of service is insignificant. The net difference in cost is that between the cost of the large buildings necessary for housing the high-tension equipment and the cost of the light concrete bases and steel structures required in the case of the outdoor installation. The operation of these stations does not seem to be in any way impaired by the low temperatures and snow of our Northern winters nor the heat of our Southern summers.

Bearing Power of Soils. — The most important part of the building is the foundation, and careful soundings must be made to ascertain the underlying strata. If bedrock is found within moderate depth, the foundation should be carried down to the same. The safe bearing loads usually allowed for soils in this country are given in the following table:

SAFE BEARING POWER OF SOILS
(*From Baker "Treatise on Masonry Construction"*)

Material	Load in tons per sq. ft.	
	Minimum	Maximum
Rock, hardest kind	200	..
Rock, equal to Ashler masonry	25	30
Brick, equal to Ashler masonry	15	20
Brick of poor quality	4	7
Clay in thick beds always dry	4	6
Clay in thick beds moderately dry	2	4
Clay, soft	1	[illegible]
Gravel and coarse sand	8	10
Sand, fine and compact	4	6
Sand, clean and dry	2	4
Alluvial soils and uncertain sand	0.5	1

For foundations it is considered good practice to use somewhat lower values. About one-half is a good working basis for such work, thus allowing a maximum load of about 1000 lb. per sq. ft. for ordinary alluvial soils. Clean sharp sand is considered to be a good bearing soil, and it may only be necessary to cover it with a concrete mat, which requires a minimum amount of concrete. For soft or alluvial soils piling is almost always required. These may be of wood, although in the last few years much use has been made of concrete piles, both plain and reinforced. Such piles are less apt to decay and their bearing power is higher, due to their greater friction. They may also be made of larger diameters than can be obtained with wood piles and a less number is therefore required to support a given load.

Design of Foundations. — The weight on a foundation includes the machines, fittings, the weight of the foundation itself, and, in the case of the turbines, the weight due to the water thrust unless this is balanced. Separate foundations should be provided for the different units so as to isolate any failure as far as possible. Concrete is always used for the foundations.

Machinery foundations should be solid, but buildings may be supported on columns or arches so as to economize the concrete (*see Concrete*). Where there is danger of high water in the tail race, the outside foundation walls should necessarily be made water-tight so as to prevent water from entering the basement. For such cases a sump is generally provided into which the seepage may collect and from which it can be pumped out.

Basements. — A basement should be provided below the generator room when vertical turbines are to be used. That part of the floor on which the turbine discharge casings rest should be reinforced by heavy I-beams, and provision should be made for supporting the penstocks and draft tubes. There must be provided an intermediate basement floor which is generally made of concrete and should be carried on I-beams supported by the concrete piers which also support the generators and the main floor. With horizontal turbines no basement is needed. Ventilating and cable ducts for the generators, and tunnels for piping, etc., are, however, often installed below the main floor.

Floors. — No combustible material of any kind should, if possible, be used in the construction of a power house. As the sub-structure of the building is generally built of concrete, it is but natural that the floors should also be of concrete. A dark color is preferable so as to render drops of oil inconspicuous. A tile or mosaic floor is smooth, easy to keep clean and has a very handsome appearance if made to conform with the general interior finish of the station.

Walls. — The walls may be either of reinforced concrete construction or of brick with a steel skeleton frame work. Where future extensions are contemplated a false wall is provided on one end of the building. The interior should be kept as light as possible and it is therefore advisable to apply a smooth surface of cement plaster and whitewash or paint the same. For more important stations the walls may be faced with pressed brick and up from the floor to about ten feet with enameled brick so that they may be readily washed and cleaned. Where the extra expense is warranted, the walls may be entirely lined with enameled brick and a wainscoting of contrasting color, preferably olive-green.

Roof. — The roof of the building should always be supported on the steel trusses, carried on the side walls or on steel columns. The slope should not be excessive, two inches per foot being sufficient with gravel covering. This construction requires less material, and is advantageous when the transmission wires are to enter the station through roof entrance bushings, or where the lightning arrester horns are to be installed on the roof.

The roof covering may simply consist of boards covered with roofing paper, tar and gravel. Reinforced concrete is sometimes used in place of boards so as to make an absolutely fireproof construction. Roofs covered with red tile are often used and present a very pleasing appearance. Corrugated iron roofs are objectionable due to the liability of moisture condensing on the inner surface and dripping into the station. They may also cause the station to be extremely hot in the summer unless an insulating lining is provided below the roof trusses to keep out the heat. With tile or metal roofs it is necessary to provide steeper inclines than with gravel roofs so that the water may run off rapidly. Monitors are sometimes provided so as to give additional ventilating facilities.

Roof trusses with a raised chord, shown in Fig. 1a, are in many instances of great advantage in that they provide an increased headroom without unneces-

sarily raising the walls of the building. This is of special importance in the high-tension part of the station, where ample headroom must be provided for the busses.

Windows. — Good lighting is imperative, and large windows are therefore essential. They should be symmetrically located with regard to the generating units and their design should be such as to harmonize with the building, arched windows being very generally used. Skylights of glass tile placed in the roof will also add considerably to the lighting. The window sashes should preferably be metallic and the glass reinforced with wire netting so as to prevent shattering when broken. Ribbed or non-transparent glass is also desirable, because it keeps out the intense rays of the sun. In order to provide for ventilation, provision should be made so that the windows can be opened, but precaution should be taken so that rain, snow or dust will not blow in on the machinery or apparatus. This is especially important on the switchboard side where the wiring is exposed and it is therefore better practice not to provide any means for opening the windows on that side. For tropical climates the windows which are liable to be opened should be equipped with mosquito screens.

Doors. — The location of the doors is naturally governed by local conditions. One of the openings should be of sufficient size to admit a railroad car, for which tracks should be provided. Very often these doors are of the rolling type, this design being most economical as regards space.

Traveling Crane. — Provision should always be made for supporting the track for a traveling crane, which should span the generator room and run the full length of the station. The track is generally supported on pilasters in the outside wall and on the steel columns separating the generator and switch rooms. There should be ample headroom allowed so that the various machine parts can be readily removed when repairs are to be made. This is especially important with vertical units where the water-wheel rotor is mounted on the same shaft as the generator field, and in which case it should be possible to lift out the whole revolving element by simply removing the top bracket and bearing of the generator.

Miscellaneous Rooms. — Repair rooms, store rooms, offices, toilets, etc., should be provided. Ample stairway provision is essential so as to permit a ready access to important points, such as between the generator room and the switchboard gallery.

LOCATION OF APPARATUS. — The arrangement of the apparatus should be very carefully considered from the standpoint of simplicity and reliability of operation. The purpose of the station being to give reliable service consideration must also be given to the causes of disturbances and means for minimizing their effects. In anticipating these abnormal or so-called emergency conditions, the failure of every piece of apparatus must be considered as a possibility, and a definite plan worked out for limiting the magnitude and area of such disturbances.

Location of Generators. — The turbo-generator units are located on the main floor and are almost always arranged in a line along the long axis of the station. They should be spaced far enough apart so that ample space for passage is provided between them. Horizontal sets may be installed either at right angles or parallel to the long axis, the latter method being necessary for high heads where impulse wheels are used. The arrangement of the rest of the equipment, such as the transformers, may also be a determining factor in regard to which direction the sets should be installed. If one transformer bank, consisting of single-phase units, is to be installed for each generator, the space occupied by them may be of such a length that it would be more economical to install the

turbo-generator sets parallel to the long axis, thus reducing the width of the building.

Location of Turbines. — (*See Water Wheels.*) With horizontal sets the turbines may be located together with the generators in the generator room or in separate wheel chambers built in the dam. The latter practice is only used for very low head developments, where one of the power-house walls forms part of the dam structure. With vertical units the turbines are always located in a basement, the thrust bearing being supported on an intermediate floor below the main floor, unless suspension bearings are used, these being mounted on top of the upper generator-bearing bracket.

Location of Exciters. — The exciters are as a rule installed on the same floor as the main generators and in the center of the station. The advantage of such an arrangement is that the exciters will be located close to the operating switchboard and the amount of copper required for the exciter leads is thus a minimum. The system may readily be sectionalized, one exciter serving the generators located in one-half of the station, and the other the generators on the opposite side. This does not, of course, refer to direct-connected exciters.

Location of Transformers. — Due to their weight, the step-up transformers should preferably be located on the main floor. They are generally installed in isolated compartments in the rear bay, separated from the generating room by fireproof steel curtains. These compartments should be sufficiently large to allow a good ventilation. A car track is provided on the generator-room floor in front of the transformer compartments whose floors are raised so that the transformers can be run out on the car and moved to some convenient place in the station where repairs can readily be made. For large units it may be necessary to provide a hole in the floor above the repair room so as to enable the transformer core to be lifted out of the tank, or a pit may be provided into which the transformer may be lowered so that sufficient headroom is obtained for lifting out the core. Sometimes the repair room is so situated that the main crane cannot be utilized for dismantling the units. In such a case a chainfall supported from a heavy I-beam in the floor above may be provided.

Transformers Installed Out-of-Doors. — Considerable activity has recently taken place in installing transformers and associated high-tension apparatus outdoors. With the exception of the bushings the transformers for such installations differ comparatively little from the indoor type; the only feature out of the ordinary being the necessity of keeping the moisture from entering the transformer cases under the covers and leads. To prevent this the joints have been made with waterproof gaskets and breathing chambers have been provided.

Special precautions must naturally be taken to protect transformers of the outdoor type both from the extreme heat and from the cold in the winter. The former can readily be obtained by providing sunshades, and in certain instances very good results have been obtained by simply painting the cases white. It is more difficult, however, to provide for the cold winter temperatures, especially with water-cooled transformers. With the transformers in service there seems to be no danger of freezing and if such should be the case some sort of heating grids could readily be provided in the bottom of the tanks. The main difficulty lies in the formation of moisture which takes place when the temperature of the transformer is allowed to fall below that of the surrounding air; this applies also to indoor transformers. Precautions must therefore be taken that this does not happen, and may be accomplished by either reducing the water rate at times of cold weather, or by using the cooling water over and over again. Non-freezing oil may be used in such transformers, but its cost is so high that it is almost prohibitive from a commercial standpoint.

Location of Switchboards and Switchgear. — (*See also Switchboards; Switchgear Equipment for Power Stations.*) The different pieces of apparatus comprising the switching equipment are distributed on the various floors in the switch-section of the station, each story being partitioned to suit the various purposes. The operating room with the control switchboard is generally located on the second floor and in such a position that the operator may have an unobstructed view of the station, and be able to readily communicate with the turbine operators. A balcony, somewhat overhanging the generator room in front of the switchboard, is often provided or the operating room is built with a curved front wall extending out over the generator room.

Location of Oil Switches. — The low-tension oil switches are generally of the enclosed type and, together with the low-tension bus-bars (*see Bus-Bars and Bus-bar Structures*), are located generally in compartments on the main floor back of the transformer compartments. The switches themselves should preferably be set opposite the generator and transformer bank which they control, so as to call for as short a connection as possible and in order that these connections may be of equal length. The high-tension oil switches and bus-bars, and also as a rule the lightning arrester tanks, are installed on the floor above, or outdoors as the case may be.

Disconnecting Switches. — It is customary to install disconnecting switches on both sides of an oil switch so that they may be entirely disconnected from the circuit when repairs are to be made on them, when the oil tanks are to be refilled, etc. Disconnecting switches may also be used in a number of cases for changing connections, when this is not to be made under load. Such switches should be provided with locking devices, as experience has shown that the magnetic fields caused by short-circuits may cause disconnecting switches to open, which in turn may cause serious disturbances by the arcs set up.

Spacing of Bus-Bars. — (*See also Bus-Bars and Bus-bar Structures.*) The following spacings are suggested in laying out bus-bar structures:

Voltage	Dimensions in inches			
	Outdoors		Indoors	
	To ground	Between live parts	To ground	Between live parts
2,000– 3,500	3½	4	3	3½
3,501– 7,500	5½	6	4½	5½
7,501– 15,000	9	10	7	9
15,001– 25,000	14	15½	10½	14
25,001– 37,000	19½	22	14½	19½
37,001– 50,000	25½	29	19	25½
50,001– 73,000	36	41	27	36
73,001– 95,000	47	53	34½	47
95,001–115,000	56	64	41	56
115,001–135,000	66	75	48	66
135,001–155,000	75	86	55	75

Location of Lightning Arresters. — (*See also Lightning Protectors; Switchgear Equipment for Power Stations.*) The aluminum arrester is now generally used in all high-voltage stations. Both the arrester tanks and the associated horn gaps may be located within the building, or the horn gaps may be placed outside and the tanks inside, or both may be placed outside, provided there is no danger of the electrolyte freezing. Standard equipments of 27,000 volts and below are usually designed as complete units to be installed inside the station, whereas for those above 27,000 volts the horn gaps should preferably be installed outside the station and the tanks inside. Exception to this rule can be made where there is sufficient space in the station over the gaps.

The arrester tanks should naturally be located close to the line entrances. The horn gaps, when installed out-of-doors, may be placed on the roof of the building if roof-entrance bushings are used, or on a separate structure at the side of the building if wall-entrance bushings are used. The location of the arresters should also be such that the path for the discharge from the line conductors to the arresters and ground will be as straight as possible.

Clearance over Horn Gaps. — Wherever horn gaps are mounted inside the building sufficient clearance should be allowed over them. There is no appreciable arc at the gaps, but in abnormal cases where the film has been allowed to get out of order, the arc may be of considerable size. Where there are no busses or inflammable apparatus, the following are the minimum clearances from the tops of horns to be allowed:

Volts	Clearance, feet
Up to 16,100..........	3
16,101 to 37,900........	4
37,901 to 70,000........	6

These clearances should be materially increased when there are wires, cables, busses or any inflammable material over the horn gaps.

Above 70,000 volts, the horn gaps should never be placed indoors.

Effect of Climatic Conditions on Arresters Installed Out-of-Doors. When aluminum arresters are installed out-of-doors, there is always present the possibility of freezing the electrolyte in cold weather and that of abnormal film dissolution during extremely hot days. The electrolyte is not permanently injured by freezing, but when frozen, the internal resistance of the arrester is considerably increased and hence its discharge rate is materially lowered. This is not of any practical consequence, for during the winter months there is usually a total absence of lightning. The rapid dissolution of the films on the cores of the arrester during warm summer days may be counteracted by more frequent charging. In some locations it may be found advisable to charge two or more times a day. When operating under conditions of high temperature, failure to regularly charge the arrester increases the liability of damage from a heavy electrical discharge through it.

There has recently been introduced a new form of arrester designed to overcome the necessity of periodic charging as in the case of the aluminum arrester. This new arrester is known as the oxide-film type. It requires no charging, has about the same discharge capacity, may be built for either indoor or outdoor operation, and due to the small arc formed across the gaps during discharge, it may be covered in the case of out-door arresters thereby obtaining prac-

tically the same arc-over potential for these outdoor arresters during dry or wet weather.

WATER WHEELS AND GOVERNORS. — The available types of water wheels and governors and the conditions under which they should be used are treated in detail in the articles on *Water Wheels and Their Settings*, and *Water Wheels, Speed Regulation of.*

GENERATORS AND THEIR CONNECTIONS. — (*See also Generators, Alternating-Current.*) Water-wheel-driven alternators are always of the revolving field type. Machines of this type are generally direct-connected to the water wheel. The horizontal construction has been used mostly, but recently the vertical construction has been used to a great extent, especially in low-head developments. The choice is usually determined by hydraulic conditions; see *Water Wheels and Their Settings.*

Use of Power-limiting Reactances. — See article on *Reactance Coils.*

Operation of Generators in Parallel, Synchronizing, etc. — (*See Generators, Alternating-Current; Synchronizers.*)

EXCITATION AND EXCITER SYSTEMS. — It is a good practice to have the combined normal capacity of all the exciters correspond to the excitation required for all the generators, when these are operating at their maximum overload, and at the actual operating power factor. A spare unit to be kept in reserve in case of the break-down of any exciter should generally be provided. This is especially desirable where an uninterrupted service must be secured at any cost and where the exciter units are few in number, as in such a case the shutdown of one exciter would seriously cripple the system.

Amount of Excitation. — The curves in Fig. 2 give approximately the average excitation required for water-wheel-driven alternators of high and slow speeds. It is seen that, as compared to the rating of the generator, the exciter capacity ranges from 0.75 per cent for large high-speed machines to 3 per cent for small slow-speed machines.

"Time Element" of Exciters. — The "time element" of the exciters should be such that the insertion into its field circuit of an external resistance equal to about three times the resistance of its field circuit, will cause the voltage to drop from 125 to 25 volts in from 4 to 6 seconds. This is particularly important when automatic voltage regulators are used (*see below*), for the exciter voltage must respond quickly to the short-circuiting of the field rheostat by the regulator.

Exciter Voltage. — For large installations a 250-volt system of excitation will generally be found more economical than a 125-volt system. This higher voltage will permit the use of smaller exciter and field switches, leads of reduced size may be used between the exciters and the generator field, and the cross-section of the exciter bus-bars will be

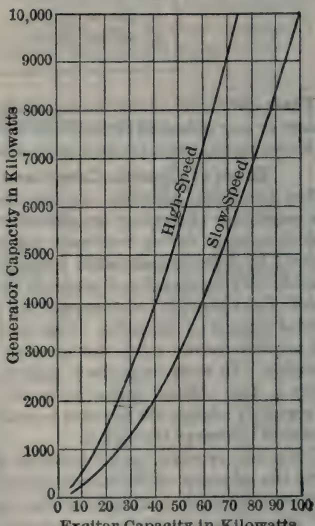

Fig. 2. Excitation Required for Water-wheel-driven Alternators

reduced. A considerable saving can also generally be accomplished in the exciter itself.

Methods of Driving Exciters. — Although the exciters can be either belt-driven or direct-connected to the machines driving them, the latter practice is almost exclusively used except in the very smallest plants. The direct connection may be either to the main generators, to separate water wheels, or to motors. Sometimes (although rarely) an exciter is connected to both a motor and a turbine, the latter running idle when the motor is carrying the load, and the motor running idle when the turbine is doing the work.

Exciters Direct-connected to Main Units. — The practice of installing an exciter direct connected to each generator is very common and desirable. As a rule each exciter is only of sufficient capacity to excite its corresponding generator and is operated as a unit with it. In those stations where it is felt advisable to install spare exciter capacity, this spare capacity usually takes the form of a motor-generator set. The generator of the set is connected to a bus and the switching so arranged that it can be used to excite the field of any one generator in the case of failure of the corresponding direct connected exciter. Fig. 3 shows another arrangement of connection commonly used in which all exciters are paralleled on a common exciter bus and the fields of all machines in operation excited from this bus.

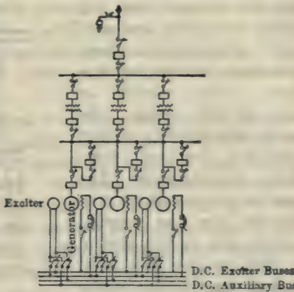

Fig. 3. Exciters Direct Connected to Main Units

This method of employing direct-connected exciters is mostly applicable in the case of medium- and high-speed generators. In the case of low-speed generators the high cost of the slow-speed exciters usually makes the arrangement prohibitive.

Three-exciter System. — A system which is commonly used and which offers a sufficient degree of reliability, is that in which the excitation is obtained from a common source, consisting of as few exciters as possible, Fig. 4. Three units are then generally provided, of which two are all that are needed for supplying the required excitation, the third unit being held in reserve. Sometimes the two exciters normally in service are driven by water wheels, the reserve unit being motor-driven. From the point of view of economy, however, it is evident that two motor-driven units with a water-wheel driven set as the spare will cost less. In the latter case, however, the exciter trouble with the exciter driven by the prime mover would prevent starting up the system, unless a storage battery were provided.

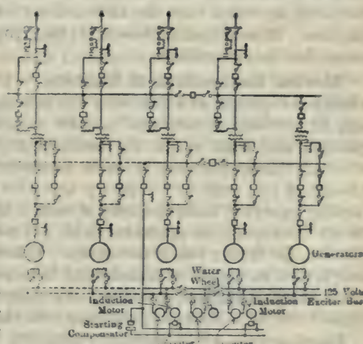

Fig. 4. Three-exciter System

Individual Motor-driven Exciters. — Fig. 5 represents the practice when an individual motor-driven exciter is used for each main unit. Each ex-

citer is connected directly to the field of its respective generator. The exciters are not arranged for parallel operation, but are each provided with its own automatic regulator, so that it is possible to compensate for "wattless" or reactive cross-currents between the generators. In the system shown, the induction motors are fed either from two low-voltage alternating-current generators driven by separate water wheels, or alternatively from the main alternating-current bus-bars through step-down transformers. The auxiliary low-voltage generators are the normal source; the exciter system is thus entirely free from voltage fluctuations or disturbances on the alternating-current system. In another installation of this kind the auxiliary generators are provided for combination drive, one end being connected to a water wheel and the other to an induction motor, which in turn can be connected to the main alternating-current buses through step-down transformers, unless the voltage will permit of a safe operation of the motors without the transformers. In such an arrangement the exciter sets are not provided with independent group connections to the main bus, as shown in the illustration, as it is considered that a breakdown would most commonly be caused by a clogging up of the turbines, in which case the alternating-current units could be driven by the motors. It would seem, however, that the scheme shown in the diagram is more flexible and reliable.

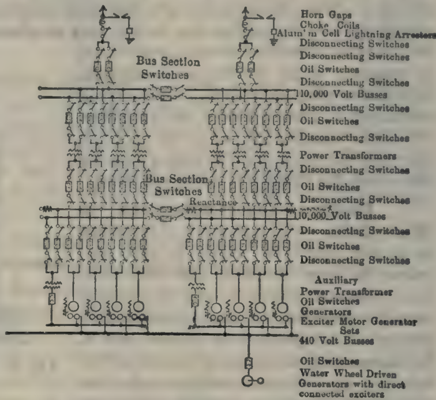

Fig. 5. Individual Motor-driven Exciters

Electric Connections of Exciters. — In those cases where exciters are not chosen to operate as a unit with the generator, it is general practice to provide one or two sets of common bus-bars to which all the exciters are connected in parallel and from which the fields of the different generators are excited, a rheostat being inserted in each generator field circuit. Where exciters are chosen of just sufficient capacity to excite the field of one generator, they are usually operated independently of each other, each with its armature connected to its corresponding generator field. If no spare exciter set is provided in the station, no exciter bus is necessary. Where a spare exciter is installed a single bus is used to permit of transferring it to any one generator field, in which case, the corresponding direct connected exciter is taken out of the field circuit. In these latter two cases, no generator field rheostats are necessary.

Hand-operated field rheostats mounted on the back of the switchboard, or operated by chain drive from a handwheel on the switchboard, are generally used. For large plants the alternator field and the exciter field rheostats are nearly always electrically operated. This method of operation should be used in plants where the main control board is on another floor or distant from the machines.

No automatic overload circuit breakers or fuses should be installed in the exciter circuits. The field current may rise to several times its normal value when the alternator is subject to a short circuit, and an automatic circuit breaker would then naturally open the exciter circuit, which must not be allowed. As

such troubles are rather infrequent, it is considered better practice to risk injury to the exciter rather than to install overload devices which may operate at the wrong time. With exciters operating in parallel on a common bus it may be desirable to have reverse current circuit breakers in the connections between the exciters and the bus.

The alternator field switches should be provided with discharge resistances, and these field switches as well as the exciter switches should be electrically operated in all large plants. The field switches should never open on overload, but may be made to open automatically when the main alternating current circuit breaker opens by the action of reverse power or differential relays in the main alternator leads.

Shunt versus Compound-wound Exciters. — Compound-wound exciters are generally considered preferable for parallel operation with automatic voltage regulators. Non-regulating exciters should be more or less highly saturated to insure a stable parallel operation. However, if such exciters are used with automatic regulators, they will be rather slow to correspond to the changes in field excitation. If a shunt-wound exciter is designed for a low saturation, so as to make it a good regulating exciter, there is a tendency to unstable operation when operating in parallel with other machines and without a regulator. For regulating exciters, which are not to be operated in parallel, the shunt-wound type is entirely satisfactory, provided it has been designed with this point in view, that is, for low saturation.

Storage Battery on Exciter Bus. — The use of storage batteries in connection with exciters is sometimes employed. The storage battery is generally floating on the exciter busses, the pressure of which is kept constant. A separate exciting bus is provided and between this bus and one of the exciter busbars a booster is installed which can be operated to either raise or lower the voltage, its field being controlled by an automatic voltage regulator. In case of failure of the exciters the excitation would be furnished by the storage battery, and the booster in connection with the regulator would take care of the voltage regulation.

AUTOMATIC VOLTAGE REGULATION. — Without some form of automatic voltage regulator it is impossible to take care of the heavy swings in the voltage caused by fluctuating power and railway loads. Even in the case of a purely lighting load it is exceedingly difficult to properly take care of the voltage by hand regulation, especially at peak loads. The present tendency of designing generators for a high internal reactance, in order to reduce destructive short-circuit currents, results furthermore in a rather poor inherent regulation of the generators.

Many different forms of automatic regulators have been devised. Some of them have been designed to operate directly on the alternating-current generator field rheostat by varying the resistance. Such a system has, however, proved to be entirely too sluggish in operation. The most successful type of automatic regulator is the T. A. Regulator in which the regulation is effected entirely in the field circuit of the exciter, by rapidly opening and closing a shunt circuit across the exciter field rheostat. See the article on *Regulators*.

TRANSFORMERS AND THEIR CONNECTIONS. — (*See also article on Transformers.*) The number and size of the transformers depends entirely on the nature of the development and on the conditions to be met. With a moderate voltage development it has in the past been the general practice to install one transformer bank for each generator and having a capacity equal to that of the generator, even if this size was not the most economical.

In some instances transformer capacities are selected so that a bank of transformers corresponds in capacity with the rated carrying capacity of a trans-

mission line. In such cases a bank of transformers is usually operated as a unit with one of the lines. During the early stages of high-tension transmission development this method was almost invariably employed as it permitted the employment of low-tension switching to clear trouble on the high-tension system. In such cases as this the transformers might be considered as a part of the transmission line. With the advancement of the art of building high-potential apparatus and as a result of the experience gained in operating high-tension systems, this manner of operation is gradually giving way to the use of high-tension switching, in which case it is no longer necessary to select transformer banks of the same capacity as that of the transmission lines. The tendency is rather to select them on the basis of the generator capacity.

In order to facilitate moving the transformers in or out of their compartments. wheels should be provided in the base or trucks may be installed on which the transformer will rest. The design should also preferably be such that the complete core and coils, with the cover and leads can be lifted from the tank as a unit. Eyebolts are provided for this purpose and also for lifting the entire transformer filled with oil.

Single- versus Three-phase Transformers. — No specific rule can be given regarding the selection of single-phase or three-phase transformers since both designs are equally reliable; local conditions will generally determine which type is preferable. See the article on *Transformers*.

Use of Auto-Transformers. — (*See also Auto-Transformers.*) Auto-transformers are sometimes used in connection with Y-connected generators for obtaining a moderate rise in the voltage. Where such is the case a path must be provided for the flow of the triple-frequency exciting current, which is required for the normal magnetizing of the transformer. With a grounded generator neutral (*see Generators, Alternating-Current, and Grounding of Electric Circuits*), this can be obtained by also grounding the neutral point of the auto-transformers, although it is also highly desirable to connect the two neutrals together as any ground offers more or less of a resistance. If a sure path for the triple-frequency exciting current is not provided, a third harmonic will appear in the no-load e.m.f. from line to neutral and cause an excessive strain in the windings, which under such conditions should in all cases be insulated for a higher voltage than the normal.

Cooling of Transformers. — Transformers for hydroelectric generating stations should obviously be of the water-cooled type. Ordinarily the water rate to keep a transformer of this type cool is approximately one-half gallon per minute per kw. loss, the temperature of the incoming water being 15° C.

Cooling coils are generally made of extra heavy lap-welded wrought iron pipe with electrically-welded joints. These coils will withstand a hydraulic test of 1000 pounds per square inch. In some cases the quality of water available for cooling purposes may make it necessary to use either brass or copper pipe, in order to avoid corrosion which would prohibit the use of iron pipe.

Drying of Transformers. — It was formerly customary in shipping transformers to pack the cores separately from the tanks. Where the railroad clearances will permit, transformers are now shipped assembled with the oil in the tanks, the cores being securely braced in the tank. In this manner the transformers should arrive at the destination with the insulation and oil practically dry and free from moisture. Where transformers are shipped without the oil in the tanks it is almost invariably necessary to dry them out first. This may be accomplished in several ways, as explained in the article on Transformers (q.v.).

Transformer Oil. — The oil, whether shipped in the transformer case or

separately, should always be tested before it is used in service (*see the article on Oil, Transformer*), and should be dried if it punctures at too low a voltage. Oil for transformers of 40,000 volts and over should be dried before using, if it punctures below 35,000 volts. For transformers having voltages less than 40,000 volts, the oil must be dried if it punctures below 25,000 volts. Where oil is dried it may easily be brought to a puncture of 40,000 volts. If a sample contains sediment, it will puncture at a lower voltage than it would without the sediment.

Transformer Connections. — With the three-phase system the transformers are usually connected in delta or Y, and when the Y-connection is used the neutral may be grounded or not. It is a much-disputed question which connection is to be preferred. In general it may, however, be said that in transmission systems where continuity of service is the most important factor, delta-connected transformers (both primary and secondary) are preferable on account of the increased reliability which such a system affords. See articles on *Transformers* and *Grounding of Electric Circuits*.

For high-voltage systems it is, however, now being generally conceded that the Y-connected system with the neutral grounded is preferable, if not almost essential. The fact that any ground will then constitute a short-circuit followed by a shut-down, is outweighed by the limitations of the rise in voltage caused by such grounds. Modern transmission line apparatus must furthermore be designed to withstand the mechanical strains imposed by short-circuits. With a ground on a delta-connected system it is evident that the neutral is shifted from the center of the delta to one corner, and the charging current, which is a function of the voltage from wire to neutral, is therefore increased in proportion, or about 73 per cent. This increased charging current will in turn cause a corresponding increase in the voltage rise which may take place when the lines are cut in circuit at no load. Actual experience has shown that this voltage may reach prohibitive values which, of course, would not be the case with the Y-connected, grounded system.

SWITCHING EQUIPMENT AND ELECTRICAL CONNECTIONS. — Continuity of service is the most essential part in the protection and operation of a large high-tension power system. A maximum degree of reliability can naturally be obtained by providing reserve and duplicate apparatus, the maximum reliability thus being governed by the permissible investment in the apparatus, by the price paid for the power and by the competitive situation.

In small and medium-size power plants the switching equipment may be of the hand-operated type, mounted directly on the back of the switchboard panels or on a separate framework and operated by hand by means of levers located on the front of the panels. For large modern power houses the switches should, however, always be of the remote-control type. The control board is then located so that the operator may obtain the best view of the station, while the switches and bus-bar structure are installed with regard to convenience of wiring and safety.

The switch and bus-bar structure for large stations may be either of the enclosed or open type, but is mostly a combination of the two.

The entire subject of switchgear equipment is discussed in detail in the separate article on *Switchgear Equipment for Power Stations*, which see.

Sectionalized Low-tension Bus. — (*See also Bus-Bars and Bus-bar Structures.*) The generators should preferably be paralleled on a low-tension bus, and where the total capacity is large the bus should be sectionalized, it being the general practice to limit the normal capacity of each section to from 30,000 to 50,000 kv-a. The sections may be connected by means of automatic switches provided with instantaneous overload relays which in case of trouble in one section will immediately disconnect the same from the other sections and thus

limit the power which the oil switches will have to rupture to the capacity of one section. In some of the recent systems reactance coils have been inserted between the bus sections, and the sectionalizing switches have been made non-automatic. The reactances are selected of such a value as to limit the short-circuit current to an amount that can be safely ruptured by the installed circuit breakers.

In sectionalizing the bus it is desirable to make such provision that sufficient generator capacity to supply the charging current of one transmission line can be entirely separated from the rest and used for testing out the lines. A ring bus will generally insure sufficient flexibility to accomplish this, although for large systems a double bus may be desirable if the extra expense is warranted.

Generator Switches. — The generator switches are usually made non-automatic, as it is of the utmost importance to keep the generators in service, and the possibility of trouble between the generators and buses is rather remote. If, however, automatic protection is desired for the generator switches, the relays (*see Relays*) should be of the definite-time limit type, set very high so as to trip the switches as a last resort, after the automatic switches more remote from the generators have failed to isolate the trouble.

Sometimes the generator switches are provided with reverse energy relays which will cut out from service a damaged generator on the reversal of the power. In the case of large important generator installations the most highly recommended practice, perhaps, is to equip each with differential relay protection. This type of protection is so designed that in case of an internal electrical failure within the generator armature, the generator will be instantly cut out of service. The protection is also usually so arranged that, when the generator main switch opens, the field circuit is simultaneously opened, thereby reducing burning within the machine to a minimum.

Transmission-line Connections. — (*See also Transmission Lines.*) Double transmission lines are nearly always provided to important load centers, and it is then also desirable to so proportion the line conductors that in case of trouble, one line alone or together with a section of the other can take care of the greater part of the load without causing too poor a regulation.

Up until recent years it was customary to recommend that high-tension lines be normally operated electrically apart from one another, each connected at each end to a bank of transformers, the paralleling and switching being done on the low-tension side of the transformers in both generating and substations. This method of operation was early adopted to reduce to a minimum the electrical stresses resulting from high-tension switching. With the growth of high-tension transmission systems with their multiplicity of interconnected transmission lines and as a result of the cost of equipment to adopt a large high-tension net-work for operation in this manner, the method has to a large extent been superseded by the simpler one based upon high-tension switching. Due to the rapid advancement in the art of designing high-tension apparatus and the practical experience gained in its operation, it is now possible to construct high-tension systems with perfect assurance that high-tension switching will give entirely satisfactory results.

All switches should be provided with carefully selected relays, so arranged as to select and cut out of service in the minimum time any circuit in which a fault has occurred so that uninterrupted service may be maintained over the remaining circuits.

LIGHTNING ARRESTERS, EQUIPMENT AND CONNECTIONS.— Aluminum and oxide film lightning arresters are now generally used for lightning protection of high-voltage transmission systems, see article on *Light-*

ning Protectors, for description of the various available types of arresters. The arrester, however, is not a universal protector against all kinds of interruptions. For example, while it meets the usual, and most of the unusual, needs in protection against disruptive potentials from lightning, an arrester located in the station cannot, and is not expected to, protect an insulator out on the line from a lightning flash. Neither is it designed to protect against surges of comparatively low voltage.

Arresters for Grounded and Ungrounded Circuits. — It is important to avoid the mistake of choosing an arrester for a thoroughly grounded neutral when the neutral is only partly grounded; that is to say, grounded through an appreciable resistance. In an arrester for a grounded neutral circuit, each stack of cones normally receives the neutral potential when the arrester discharges; but if a phase becomes accidentally grounded the line voltage is thrown across each of the other stacks of cones until the circuit breaker opens the circuit. Line voltage is 173 per cent of the neutral or normal operating voltage of the cells and therefore about 150 per cent of the permanent critical voltage of each cell. This means that when a grounded phase occurs this 50 per cent excess dynamic potential is short-circuited through the cells until the circuit breaker opens. The amount of energy to be dissipated in the arrester depends upon the kilowatt capacity of the generator, the internal resistance of the cells and the time required to operate the circuit breakers. It is evident that the greater the amount of resistance in the neutral, the longer will be the time required for the circuit breakers to operate. Therefore, in cases when the earthing resistance in the neutral is great enough to prevent the automatic circuit breakers from opening practically instantaneously, an arrester for a non-grounded neutral system should be installed. It is difficult to determine these factors of ground resistance and time elements in the operation of switches and therefore no mistake can be made by adopting the 4-tank arrester even on grounded Y circuits.

Wiring Connections for Lightning Arresters. — The wiring connections of lightning arresters are an important consideration. The discharge circuit should contain minimum impedance and hence must furnish the shortest and most direct path from line to ground. The most severe disturbances which an arrester is called upon to handle are of high frequencies and it is, therefore, imperative to eliminate all unnecessary inductance. The features favorable for low inductance are short length of conductor, large radius bends and large surface of conductor. For wiring high-voltage arresters the use of copper tubing is therefore recommended. Such copper tubing has the advantage over either copper-strip or solid conductors in that it is easily supported, requires fewer insulators and is, therefore, cheaper to install. From arrester to ground it is sometimes more convenient to use copper strip than tubing. Copper strip, say 1.5 in. by 0.03 in., can be fastened to the station wall leading directly down to ground.

Ground Connections for Lightning Arresters. — In all lightning-arrester installations it is of the utmost importance to make proper ground connections since many lightning-arrester troubles can be traced to bad grounds; see article on *Ground Connections*. As noted in the article referred to, a very satisfactory method of making a ground is to drive a number of one-inch iron pipes six or eight feet into the earth about the station, connecting all these pipes together by means of a copper wire, or preferably, by a thin copper strip. A quantity of salt should be placed around each pipe under the surface of the earth and the ground thoroughly moistened with water. It is advisable to connect these earth pipes to the iron framework of the station, and also to any water mains, metal flumes or trolley rails that are available. For the usual size station the following recommendation is made: place three earth pipes equally spaced near each outside wall, making twelve altogether, and place three extra

pipes spaced about six feet apart at a point nearest the arrester. When plates are placed in streams of running water, they should be buried in the mud along the bank in preference to laying them in the stream. Streams with rocky bottoms are to be avoided.

From time to time the resistance of these ground connections should be measured to determine their condition; see article on *Ground Connections*. The resistance of a single pipe ground in good condition has an average value of about 15 ohms. A simple and satisfactory method of keeping account of the condition of the earth connections is to divide the earth pipes into two groups and connect each group to the 110-volt lighting circuit with an ammeter in series. If there is a flow of about 20 amperes the conditions are satisfactory, provided the earth pipes are properly distributed around the station.

Charging of Aluminum Cell Arresters. — This is accomplished as follows:

First. — Operate the charging mechanism so as to bring the charging contact securely against the horn gap and charge for five seconds. The contacts should be so adjusted as to eliminate arcing when gaps are closed. The closing and opening of the charging contact should be performed quickly so as to avoid unnecessary arcing. Note should be made of the size and color of the arc which forms when the contact is broken at the close of the charging period.

Second. — With the horn gaps in normal position reverse the transfer device, thereby interchanging the connections to the ground stack of cones and one of the line stacks.

Third. — The first operation should again be repeated, thus charging the fourth stack of cones, which was originally the ground stack.

When an arrester is first installed and also when one has been off the circuit for several days the initial charging current is sometimes above normal. It is recommended that the cells be charged six or eight times the first day and three times a day during the remainder of the first week. This charging should be performed as just described. After the first week the regular daily charging will usually be found sufficient. It is important that the charging should be done at a time of the day when the line voltage is at a maximum value. The charging period should always be five seconds.

In cases where aluminum lightning arresters are installed in places where the temperature is excessive, it is sometimes advisable to charge the arresters twice a day. This condition will be indicated by an increase in the charging arc and charging current from day to day.

Charging-current Indicator. — The charging-current indicator is a device for measuring the current taken by an alternating-current aluminum arrester during charging; it also indicates the condition of the arrester cells. An arrester in good condition has a charging current of approximately 0.25 ampere on 25-cycle circuits, 0.30 ampere on 40-cycle, and 0.40 ampere on 60-cycle circuits. Should these values be doubled, the arrester must be charged more frequently and the current carefully measured until it comes down to normal. It is only when this additional charging fails to reduce the charging current that an inspection of the cells is necessary.

The essential parts of the charging-current indicator are an ammeter mounted on a specially-constructed switch stick and a set of jacks. These jacks are so connected in the arrester circuit that when the ammeter switch stick is inserted in them and the horn gaps short-circuited, the charging current flows through the meter.

Choke Coils. — Choke coils should always be installed in the power circuit between the lightning arrester and the apparatus to be protected. All choke coils should be very rigidly supported as they are subject to severe mechanical strains when short-circuits occur on the system. See article on *Lightning Protectors*.

Use of Arcing Ground Suppressor. — This apparatus is described in the article on *Ground Detectors and Arcing Ground Suppressors.* One arcing ground suppressor is sufficient for controlling the entire system. When several power stations feed into one transmission system special attention should be given to the best location of the suppressor.

STATION WIRING. — The design and construction of the cabling and wiring system of the station is of equal importance to the rest of the equipment. Experience has shown that in a great number of instances the shut-down of power plants was caused by defective installation of the station wiring. Every cable and wire should have a definite place provided for it in advance, just as much as any other piece of machinery, and wires carrying currents of different voltages should, as far as possible, be kept apart from each other.

Braided vs. Lead-covered Cables. — For generator and low-tension transformer leads, braided or lead-covered cables are used, and may be run in ducts or exposed upon racks in cable-ways or tunnels. Lead-covered cables are necessarily more expensive than braided cables, and their use seems to be justified only in places where protection against water and moisture is required. As the lead sheath is necessarily grounded along its entire length, such cables are more apt to be punctured. As a protection against fire the lead covering is obviously useless and due to its softness it is not very efficient in withstanding mechanical injuries.

Installation of Cables. — (*See also article on Wires and Cables, Insulated.*) In many plants the cables are installed in ducts in the floor, and in such instances the cables should preferably be lead-covered to protect the cables from abrasion when drawn into the ducts. There are, however, several other objections to installing the cables in ducts. With high-voltage single-conductor lead-covered cables, static discharges may take place through the insulation to the lead, which rapidly injures the insulation and a breakdown soon follows. If the cable is not lead-covered a static discharge may take place to the tile duct, this also having a tendency to break down the insulation in time. In multiple-conductor cables this action does not occur, the static activity probably being neutralized. With cables carrying large low-voltage currents the lack of ventilation in the ducts may furthermore cause overheating of the cables.

In a large number of stations the open method of cabling is used. Tunnels or cable subways are then provided in which braided cables are supported in free air upon insulators mounted on racks, these insulators themselves having sufficient insulation to withstand the operating voltage of the cable. Where single-conductor, high-voltage cables are used, they should be separated far enough to prevent static discharges between the cables, and also in order to obtain the best possible ventilation and to minimize trouble in case of short-circuits. The cables should also be rendered fireproof by wrapping them with asbestos tape.

Size of Cables. — Due to the skin effect it is generally considered good practice to limit the size of single-conductor cables to 1,250,000 cir. mils for 25-cycle service and 700,000 cir. mils for 60 cycles. Where the value of current is such that it can be carried safely by one three-conductor cable, this is preferable in every respect to three single-conductor cables. If more than one cable has to be used to carry the current the difficulty of making connections will generally offset the advantages of three-conductor cables, and under such conditions it is usually found to be more advantageous to use single-conductor cables

High-tension Wiring. — Bare wire or tubing supported on post insulators or hung from suspension insulators is generally used for all high-tension wiring. High-tension station wiring is described in detail in the article on *Switch-gear Equipment for Power Stations.*

OPERATION AND OPERATING RECORDS. — The selection and

maintenance of an efficient and reliable operating force is essential. Most modern systems of any size have a method of operation which corresponds to that of a train dispatcher on steam railroads, and where many different plants are attached to the same network, this becomes practically necessary. The directions for operating the different stations and apparatus come from a central source, where the dispatcher has before him a diagram of the whole system and information regarding the capacities of the generators in use and the magnitudes of the loads at the different places of distribution.

Organization of Operating Force. — The organization of the operating force of a hydroelectric generating station is necessarily less complicated than in a steam station. It is determined largely by the location and the arrangement, and there are so many different conditions in such systems that it is impossible to recommend any exact form of organization, as really no two can be quite alike If the station is not too large, it is desired to have the hydraulic superintendent report to the station superintendent, but if the development is of such a magnitude as to require the entire time of a superintendent for each of the departments under consideration, a position is warranted for a man to whom both electric and hydraulic superintendents will report, thus still bringing the responsibility of operation of the two departments under one head.

As a general rule, for the same capacity installed, a plant having horizontal units can get along with a smaller force than one using vertical units. It is a general practice to maintain one man at all times on each of the different levels or floors of the power house, such as the switchboard gallery, the main floor and the basement, where with vertical units the turbines proper as well as the oil pumps and other auxiliaries are located. The man in the basement could in all probability be dispensed with in plants using horizontal units. In addition to these men a chief operator should be provided for each shift, whose duties should carry him to all parts of the building. For a very large station the above force may be entirely inadequate, and for small plants the force may be reduced.

Switching Operations. — The switching operations are determined by the general method of operation. It is desirable to eliminate all high-tension switching under load, due to the fact that such switching may set up surges which may be dangerous to the transformers and other apparatus.

When a line is to be cut into service, the high-tension switches in both the main and substations should be closed first, then the low-tension transformer switch in the generating station should be closed, energizing the transformers and the line, after which the low-tension transformer switch in the substation is closed and the load picked up. In case it becomes necessary to open a high-tension switch in a loaded line, the circuit should if possible first be parallel with another before opening the switch. If, on the other hand, transformers are to be paralleled on both high- and low-tension sides, the low-tension switch should be closed first, assuming that the low-tension bus is energized. Similarly, in cutting out the transformer the low-tension switch should be opened last.

Operating Records. — One of the essential things in connection with the operation of hydroelectric generating stations is the keeping of accurate records. Record sheets should contain only the most important readings, as with complicated forms the attendant generally realizes that a large number of the readings are of no importance and for this reason he becomes very lax in his attention to the readings in general and as a consequence the important ones may suffer

The following description applies to an actual record sheet which has been found to give satisfactory results. The sheet is of the size of ordinary letter paper and is ruled for hourly records of "Water," "Main Units," "Cycles," "Power Factor," "Exciters," "Transformers" and "Floodgates." These items are listed vertically and the sheet is divided into 24 vertical columns, one for

each hour. At the top are given the "Forebay" readings and "Tail Race" readings, the difference between which gives the "Effective Head." Immediately below are listed the indicated kilowatts and per cent gate opening of each generator in service, following which are given the "Total Indicated Kilowatts" and "Total % Gate." The total kilowatt-hours during each hour, as read from the watthour meters, is plotted as a block-curve extending across the face of the sheet.

This serves as a better record for the actual station output than the indicated kilowatts. It has been found necessary, however, to follow the indicated kilowatts to serve as a check on the efficiency and condition of the units in general, from time to time, as well as to determine what capacity would be required for short interval peaks. The station voltage is also plotted as a block curve across the face of the sheet.

The exciters form an individual group, and for each exciter the voltage, current and per cent gate opening are recorded.

Transformer records are limited to the temperatures. These are taken hourly, at which time the oil elevation is noted but not recorded. If the transformer is not in service the column in which the temperature is listed is left blank; if in service the temperature is taken and recorded.

Under the item, "Floodgates" the total opening of the floodgates in feet is recorded, rather than each one separately. This record is maintained daily, the flow of the river at each of the stations being followed very closely.

At the bottom of the sheet appear the daily readings of the various generator and feeder watthour meters taken at midnight of each 24 hours. The following items are also recorded at the bottom of the sheet: "Total Generated," or the total output of the station for 24 hours; the "Maximum Hour Time," or the maximum kw-hr. of any particular hour during the day; the "Maximum Kw. Time," or the maximum indicated kilowatts at any particular instant; the "Average Load," obtained by dividing the total kilowatt hours generated by 24; the "Load Factor," obtained by dividing the "Average Load" by the "Max. Kw. Time"; the "Average Flow of the River in Cubic Feet per Second," calculated each day and converted into "Available Capacity of River," which is shown in kw-hr.; the "Available Capacity of Power House," shown in kw-hr., and determined by calculating the capacity of the machines under the average head for 24 hours; the "Kw-hr. Lost," or the difference between what was actually generated by the machines and what could have been secured from the river during the same number of hours.

Any important notes of operation are entered on the back of each day's log sheet. These notes, together with certain records for log sheets, are also entered each day in a log book kept on the operator's desk at all times, for reference purposes. Weather conditions and temperatures are recorded four times daily, at midnight, 6 A.M., noon and 6 P.M. A rain gauge is provided on the roof of the station, from which records of precipitation covering each 24 hours are obtained

CAPITAL COSTS AND APPROXIMATE DIMENSIONS. — (*See also section below on Capital and Annual Costs of Some Typical Plants.*) The cost of a large water-power development is generally very great. The estimates of the amount of power available are always subject to error and many times are greatly exaggerated. On account of unforeseen obstacles in dam construction, it is always possible that the actual cost will exceed the engineering estimates, and such elements of uncertainty must always be taken into consideration. Moreover, as the large water powers of the country are generally more or less remote from power markets, there is necessitated the construction of expensive high-tension transformer equipments and transmission lines to transmit the power to the point of consumption. This additional cost is often greater than that of the

dam and power-house construction. The provision of large storage reservoirs is often necessary in order to meet the irregularities in the flow of streams, while, on the other hand, the most economical utilization of water power often requires the erection of auxiliary steam plants.

The main items entering into the cost of construction of a hydroelectric power plant are: (1) Dam, (2) Water Conductors, (3) Reservoirs, (4) Power house, (5) Land and Water Rights, (6) Transmission Lines. In addition to the cost of the above physical equipment certain overhead and organization expenses must also be included. These may be classified as follows: (7) Engineering and Contingencies, (8) Administration, (9) Organization, (10) Taxes and Insurance, (11) Interest during Construction, (12) Working Capital.

Range of Total Capital Cost per Horse-Power. — Extensive investigations by the Bureau of Corporations show that the cost of a hydroelectric power development, including the construction of dams, the erection of transmission lines and other equipment, ranged from $50 to $375 per horse-power delivered at the substations. These figures represent extremes, as the usual cost will fall between $100, and $200 per horse-power, depending upon physical conditions and the length of the transmission lines.

Approximate Dimensions and Construction Costs. — It is obvious that there are certain minimum costs that can readily be approximated when the rough dimensions of dams, pipe lines, etc., are available, so that rough minimum figures can readily be made. Such rough figures cannot be expected to take into account expensive contingencies that must be anticipated.

Dams. — Given the length and height of dam, the dam structure itself will have a cost of material that can be estimated roughly, and the ordinary cost of placing such material — including tools and forms — can be added, but the extraordinary labor costs due to the construction of expensive foundations and coffer dams and the cost of placing dam material under difficult conditions can, of course, only be estimated by experienced engineers thoroughly familiar with the local conditions.

Expressing the height of dam in yards as h, the approximate sections of dams and approximate costs of dams not used as weirs or spillways are:

SECTIONS AND COST* OF DAMS NOT USED AS SPILLWAYS

Type of dam	Batter or slope		Approx. section in sq. yards	Approx. cost per cubic yard	Approx. cost per lineal yard
	Common upstream batter horizontal to vert.	Common down stream batter			
1. Earth	2 to 1	3 to 1	$2.5 h^2$	$0.50	$1 25 h^2
2. Crib	1½ to 1½	1½ to 1	$1.5 h^2$	1.50	$2.25 h^2$
3. Rock fill	2 to 1	2 to 1	$2.0 h^2$	2.00	$4.00 h^2$
4. Masonry, straight	Vertical	8/10 to 1	4/10 h^2	12.00	$4.80 h^2$
5. Masonry, arched	0.15 to 1	0.30 to 0	¼ h^2	15.00	$3.50 h^2$

It should be borne in mind that maintenance and depreciation on Types 1, 2 and 3 are far heavier than on Types 4 and 5.

* Pre-war conditions

These figures are intended to cover all costs under ordinary conditions, for the cost per cubic yard is taken sufficiently high; but they cannot be used for extraordinary conditions, producing costs possibly four or five times greater.

Low crib or masonry dams used practically throughout their length as weirs or spillways may easily cost many times these figures.

No figures are given for intakes, owing to the great variety of conditions to be met.

Flumes. — The use of flumes of wood becomes less and less as hydroelectric work becomes more permanent in character. For estimate purposes it is then suggested that the cost of wood-stave or riveted-steel pipe be used instead of using the presumably lower cost of the flume. It is obvious that repairs and depreciation on flumes are heavy, and hence they would seem to have little use in permanent construction.

Low-pressure Pipes. — (*See also Pipes and Piping.*) In hydroelectric work cast-iron pipes are seldom used. Wooden-stave and riveted-steel pipes have been widely used for low pressures. The approximate costs of these, as compiled by A. L. Adams for Chicago, are given in the following tables.

COST * PER FOOT OF WOODEN-STAVE PIPE

Including laying but omitting hauling

Diameter in inches	25-foot head	50-foot head	100-foot head	200-foot head
12	$0.42	$0.49	$0.63	$0.85
18	0.69	0.80	1.02	1.46
24	0.79	0.91	1.14	1.61
30	0.96	1.12	1.44	2.06
36	1.19	1.40	1.82	2.65
42	1.40	1.68	2.23	3.33
48	1.55	1.85	2.46	3.67
54	2.23	2.62	3.43	5.02
60	2.85	3.35	4.37	6.40
66	3.21	3.81	5.00	7.38
72	3.65	4.38	5.83	8.73

COST * PER FOOT OF RIVETED-STEEL PIPE

Including laying but omitting hauling

Diam. in in.	No. 14	No. 12	No. 10	No. 8	No. 6	1/4 in.	5/16 in.	3/8 in.
12	$0.32	$0.38	$0.44	....	....	....	....	....
18	....	0.57	0.65	$0.78	$0.98	....	....	....
24	....	....	0.85	1.04	1.28	$1.55	$1.99	....
30	....	....	....	1.27	1.59	1.93	2.46	$3.04
36	....	....	....	1.55	1.93	2.30	2.92	3.58
42	....	....	....	1.61	2.18	2.66	3.37	4.12
48	....	....	....	....	2.48	3.03	3.83	4.66
54	....	....	....	....	2.80	3.41	4.29	5.21
60	....	....	....	....	....	3.79	4.75	5.74
66	....	....	....	....	....	4.35	5.21	6.29
72	....	....	....	....	....	4.52	5.66	6.83

* Pre-war conditions.

High-pressure Piping. — In high heads the high-pressure piping may be either riveted-steel pipe or lap-welded pipe with bolted flanged joints. It is suggested that a percentage amount be allowed for this, as the cost must vary widely for different conditions.

Tunnels. — Tunnels for hydroelectric work are usually lined. Naturally the cost per cubic yard varies, but for ordinary conditions an average unit cost of $15 per cubic yard will cover all expense including timbering and lining. Allowing a velocity of 10 ft. per second (except in the smallest tunnels), approximate dimensions and costs are as follows:

COST * AND DIMENSIONS OF TUNNELS

Carrying capacity in cubic feet per second	Velocity in feet per second	Net sectional area, square feet	Dimensions in feet, width by height	Approximate slope, feet per 1000 feet	Approximate cost per linear foot
100	3.6	28	4 by 7	0.46	$16.00
500	10.0	50	7 by 7¼	2.0	28.00
1,000	10.0	100	10 by 10	1.5	56.00
1,500	10.0	150	12 by 12½	1.1	85.00
2,000	10.0	200	14 by 14¼	0.9	115.00
5,000	10.0	500	20 by 25	0.6	280.00
10,000	10.0	1000	30 by 33	0.3	500.00

Canals. — In ordinary earth a velocity too small to produce erosion and yet sufficiently great to prevent undue deposit of silt and other matter, and sufficiently great to prevent growth of weeds, should be used. For preliminary calculations a velocity of 2 ft. per second will give approximate results. For this velocity the following approximate figures will serve:

DATA ON AND COST * OF CANALS IN ORDINARY EARTH

Cubic feet per second	Velocity in feet per second, V	Area of wet section, sq. ft.	Water depth, feet	Approximate slope in feet per mile	Approximate cost per running foot	
					Low	High
50	2	25	2.5	4	$0.375	$0.75
100	2	50	3.5	2	0.75	1.50
200	2	100	5	1½	1.50	3.00
300	2	150	6	1	2.25	4.50
400	2	200	7	¾	3.00	6.00
500	2	250	7	¾	3.75	7.50
1000	2	500	10	½	7.50	15.00
1500	2	750	12	½	11.25	22.50
2000	2	1000	12	⅓	15.00	30.00
3000	2	1500	15	¼	22.50	45.00

In rock the velocity in a canal may be much higher, and if the canal be lined 8 ft. per second may be used. For preliminary calculations this velocity will give approximate results. The following approximate figures are on this basis:

* Pre-war conditions.

DATA ON AND COST* OF CANALS IN ROCKS

Cubic feet per second	Velocity in feet per second, V	Area of wet section, sq. ft.	Water depth, feet	Approximate slope in feet per mile	Approximate cost per running foot	
					Low	High
50	8	6.25	2.5	40	$0.32	$1.28
100	8	12.5	3.5	25	0.63	2.50
200	8	25.0	5.0	16	1.25	5.00
300	8	37.5	6.0	12	1.87	7.50
400	8	50.0	7.0	10	2.50	10.00
500	8	62.5	7.0	9	3.25	13.00
1000	8	125.0	10.0	6	6.00	24.00
1500	8	175.0	12.0	4½	8.75	35.00
2000	8	250.0	12.0	3½	12.50	50.00
3000	8	375.0	15.0	3	18.75	75.00

Values are given for lined canals only, since a higher velocity will be allowable without producing too great a lost head and the cost will probably be more favorable.

It must be considered that these figures are wholly approximate since even for a desired useful section the amount of excavation per lineal foot of canal must vary with the character of the route — whether this be flat, rolling or side-hill.

In this connection the following approximate costs based on the annual reports of the United States Reclamation Service are of interest.

APPROXIMATE COST* OF EXCAVATION PER CUBIC YARD

Class	Cost per cubic yard		
	Low	High	Fair value
1. Plowable with 4 horses	$0.098	$1.00	$0.18
1a. Plowable with 6 horses	0.1225	2.00	0.30
2. Indurated material	0.29	2.00	0.60
3. Loose Rock	0.35	3.00	0.75
4. Solid Rock	0.60	5.00	2.00
4a. Excavation below plane of saturation	0.20	3.00	1.80
4b. Solid rock under water	....	....	4.50

Tail Race. — Frequently the cost of the tail race is negligible, but for some developments the cost is an appreciable percentage of the total. No data are given for estimating tail race since those given for canals, pipes and tunnels can be used.

Receivers.— No data are given for approximation of the costs of reservoirs, vent-pipes and surge tanks on account of the varied character of these. Generally speaking the percentage of the total hydro-electric cost to be allowed for these is small and a lump sum or a percentage can be allowed for the same.

Hydroelectric Power Houses. — The following figures upon the approximate space and cost of hydroelectric power houses are wholly approximate, and will give only a rough idea of what may be expected under ordinary conditions, without any allowances for high freight, long haulage, unusually expensive labor charges, etc.; and a wide departure from these figures is to be expected. See also the articles on *Water Wheels*, *Generators*, *Transformers*, *Switchgear*, *etc.*

* Pre-war conditions.

DIMENSIONS AND COST OF POWER HOUSES (Pre-war Conditions)

Item	Low head*			Medium head*			High head*		
	Small†	Medium†	Large†	Small†	Medium†	Large†	Small†	Medium†	Large†
Cu. ft. per kw. for hydraulic apparatus	30	20	10	7	6	5	6	5	4
Cu. ft. per kw. for generators, exciters and switchboards (no transformers)	20	18	15	18	13	10	16	12	10
Cu. ft. per kw. for generators, exciters, transformers and switchboards	30	25	20	28	20	15	26	19	15
Total cu. ft. per kw. for hydroelectric power house (without transformers)	50	38	25	25	19	15	22	17	14
Total cu. ft. per kw. for hydroelectric power house (with transformers)	60	45	30	35	26	20	32	24	19
Approx. cost of power-house building not including foundations (without transformers) per kw:									
Low	$5.00	$3.80	$2.50	$2.50	$1.90	$1.50	$2.20	$1.70	$1.40
High	15.00	11.40	7.50	7.50	5.70	4.50	6.60	5.10	4.20
Approx. cost of power-house building, not including foundations (with transformers) per kw.:									
Low	$6.00	$4.50	$3.00	$3.50	$2.60	$2.00	$3.20	$2.40	$1.90
High	18.00	13.50	9.00	10.50	7.80	6.00	9.60	7.20	5.70
Approx. cost per kw. for hydraulic machinery	15.00	12.00	7.00	12.00	10.00	7.00	10.00	8.00	5.00
Approx. cost per kw. for exciters, generators, switchboards and cables (without transformers)	24.00	15.00	10.00	20.00	12.00	9.50	15.00	9.50	7.50
Approx. cost per kw. for exciters, generators, switchboards, transformers and cables	32.00	22.00	16.00	27.00	17.00	14.00	22.00	14.00	12.00
Approx. cost per kw. for complete power-house and foundations (without transformers)	54.00	38.40	24.50	39.50	27.70	21.00	31.60	22.60	16.70
Approx. cost per kw. for complete power-house and foundations (with transformers)	65.00	47.50	32.00	49.50	34.80	27.00	41.60	29.20	22.70

* Low head = 50 to 200 ft.
Medium head = 200 to 600 ft.
High head = 600 and above.

† Small capacity = 200 to 1000 kw.
Medium capacity = 1000 to 5000 kw.
Large capacity = 5000 kw. and above

It is assumed that the large capacity stations with transformers will be for a line voltage of from 60,000 to 110,000, the medium from 40,000 to 60,000 and the small from 10.000 to 40.000.

Overhead and Organization Expenses. — In addition to the expenditures for the actual construction of the physical plant allowance must be made for the following items to cover overhead and organization expenses.

Engineering and Contingencies. — This item should cover all the cost of engineering, drafting and supervision of construction and of all the items properly chargeable to construction engineering; 5 per cent of the construction cost is a conservative estimate for this item. An equal amount (5%) is also generally allowed to cover contingencies, errors, etc.

Administration. — All items which go to make up the cost of administration for construction, general office expense, etc., come under this head. An additional charge of 5 per cent of the construction cost should also be ample to cover these expenses.

Organization. — This item covers the cost of organization and promotion, such as legal expenses, allowance for brokerage connected with the disposal of the securities, discount on the same, etc., 5 to 10 per cent of the construction cost is usually allowed for this item.

Taxes and Insurance. — Taxes must be provided for until the operation is begun, and usually for some time thereafter, until the income is available for such expenses. Similarly with insurance, including fire, casualty, etc. One per cent of the construction cost is generally allowed to cover these items.

Interest During Construction. — Allowance must also be made for the accrued interest on the idle capital during the construction period.

Working Capital. — A certain amount of money should be provided for working capital, the amount depending on the nature of the business transactions. About 1 per cent of the construction may be allowed to cover this item.

OPERATING COST AND FIXED CHARGES (Pre-war figures). — (*See also section below on Capital and Annual Costs of Some Typical Plants.*) The cost of hydroelectric power can be considered as made up of two parts: the operating expenses and the fixed charges. The former consist of: (1) Labor; (2) Administration; (3) Oil, Waste, etc.; (4) Maintenance and Repairs; and the fixed charges of; (5) Interest; (6) Depreciation; (7) Taxes and Insurance.

It is difficult to give any general figures as to the cost of generating hydroelectric power, on account of the widely varying cost of such developments. The load factor also has a very great bearing on the cost, much more so than with steam plants because the items making up the power cost are affected very little by a change in the load.

The annual operating cost per kilowatt of generator capacity in general (excluding transmission costs) ranges between the following values:

Labor	\$1.00 to \$2.00
Administration	0.25 to 0.50
Oil, Waste, etc.	0.25 to 0.50
Maintenance & Repairs	1.00 to 2.00
Total operation	\$2.50 to \$5.00

Or, considering the annual cost items as percentage of the capital cost, the following figures are representative for a large station for which the capital cost is \$125 per kilowatt, transmission costs excluded:

	Per cent
Labor	1.00
Administration	0.25
Oil, Waste, etc.	0.25
Maintenance & Repairs	1.00
Taxes & Insurance	1.00
Depreciation	3.50
Total	7.00

The cost of power per kilowatt-year is thus 7 per cent of $125 or $8.75. Power at the bus bars of such a station could then be sold for $16, say, per kilowatt-year, leaving $7.25, or approximately 6 per cent for interest on the investment.

Primary and Secondary Power. — Hydroelectric power is generally sold as primary and secondary power. The former must be supplied continuously and is usually marketed by the horse-power-year or kilowatt-year. The secondary power is only available during certain months of the year, and the price obtained for it is necessarily considerably less than for the primary power. To maintain the primary power at the most economical value a steam auxiliary station is usually required; this is frequently obtained by operating the hydroelectric plant in conjunction with an existing steam plant.

CAPITAL AND ANNUAL COSTS OF SOME TYPICAL PLANTS (Pre-war figures). — Below are given data on some typical plants, ranging in size from 750 to 16,000 horse-power.

Minidoka Project. — The following data illustrate the development and power cost of the Minidoka project of the U. S. Reclamation Service:

COST* OF GENERATING STATION

Capacity of station, 7000 kw.
Transmission voltage, 33,000
Hydraulic head, 46 feet

Item	Total cost	Cost per kw.
Building	$82,000	$11.70
Hydraulic machinery	73,000	10.40
Electric machinery	83,000	11.80
Freight and hauling	26,200	3.75
Erection	55,500	7.90
Tailrace	60,000	8.50
Roads and telephone lines	7,300	1.10
Camp and permanent quarters	23,200	3.30
Engineering and incidentals	11,100	1.55
Administration charges, etc	15,000	2.10
Total	$436,300	$62.30

ANNUAL COST* OF OPERATION

Item	Expense per year
Operation:	
Labor	$5,700
Supplies	950
Repairs:	
Labor	900
Supplies and material	300
Superintendence, clerical, camp, etc	1,700
General expense and administration	450
Operating expense	$10,000

* Pre-war conditions.

A depreciation of 5 per cent ($21,800) has also been charged to this development. No taxes or interest is charged, the undertaking being done by the Government. Assuming 7 per cent for interest and taxes the total operating expenses would amount to $62,000. A total of about 15 million kw-hr. were delivered during the year, thus corresponding to a cost of $0.0041 per kw-hr.

Costs* Reported by Ontario Commission. — The following table gives the estimated cost of development and yearly operating expenses of various plants from reports of the Ontario Hydro-Electric Power Commission.†

Location of development	Available head in feet	Developed power in horse-power	Estimated capital cost	Cost per horse-power	Annual operating expenses, including administration	Annual maintenance and repairs	Depreciation	Interest at 4 per cent	Total annual charges
			$	$	$	$	$	$	$
Healey's Falls, Lower Trent River	60	8,000	675,000	84.38	16,875	13,500	13,500	27,000	70,875
Middle Falls, Lower Trent River	30	5,200	475,000	91.37	11,875	9,500	9,500	19,000	49,875
Maitland River (*a*)	80	1,600	325,000	203.12	5,665	2,754	2,755	13,000	24,174
Saugeen River	40	1,333	250,000	187.53	4,840	3,247	3,247	9,984	21,318
Severn River (Big chute) (*b*)	52	4,000	350,000	87.50	17,433	8,571	8,571	14,000	48,575
South River	85	750	115,000	153.33	4,100	2,620	2,620	4,534	13,874
St. Lawrence River, Iroquois, Ont.	12	1,200	179,000	149.16	6,864	5,119	5,118	7,151	24,252
Mississippi River, High Falls "A" (*c*)	78	2,400	195,000	81.25	9,391	3,840	3,841	7,777	24,849
Mississippi River, High Falls "B"	78	1,100	123,000	111.82	6,390	2,491	2,491	4,908	16,280
Dog Lake, Kaministiqula River	310	13,675	832,000	61.00	13,760	16,427	15,927	32,278	79,392
	310	6,840	619,700	91.00	11,296	10,632	10,132	24,787	56,847
Cameron Rapids	39	16,350	815,000	50.00	16,375	17,327	16,727	32,561	82,990
	39	8,250	600,000	73.00	14,390	11,478	10,978	24,008	60,854
Slate Falls	40	3,686	357,600	97.00	6,000	6,634	6,334	14,303	33,271
	40	1,843	260,000	141.00	6,000	3,868	3,669	10,400	23,937

(*a*) Expensive dam.
(*b*) Inexpensive construction of canal and headworks.
(*c*) Includes storage development.

* Pre-war conditions.

† Capital costs cover hydraulic development (such as dams, headworks, pipe lines), power house, hydraulic and electrical equipment, with one spare unit, and step-up transformer station with electrical equipment. It does not include cost of vested rights and land damages or transmission lines.

Company	State	Voltage	Journal	Year
Alabama Pr. Co.	Alabama	110,000	Eng. Record	1914
		44,000	Mfrs. Record	1914
			Elec. Engrg.	1914, '15
			G. E. Review	1916, '19
Aluminum Company of America	Tennessee	150,000	Proc. A.I.E.E.	1920
Appalachian Pr. Co.	Virginia	88,000	Elec. World	1912, '13, '19
			Power	1913, '19
Arizona Pr. Co.	Arizona		Elec. World	1910, '20
			Journ. of El.	1920
California-Oregon Power Co.	California	60,000	Elec. World	1918
			Jour. of Elec.	1918
			Eng. Record	1913
Canadian Light and Power Co.	Canada	44,000	Elec. World	1912
Carolina Power and Light Co.	North Carolina	100,000 60,000	Water Power Chronicle	1913
Colorado Pr. Co.	Col.	100,000	Elec. World	1911, '12
Columbus Pr. Co.	Georgia	66,000	Elec. Engr.	1913
Connecticut Lt. and Power Co.		66,000	Elec. World	1917
Consumers Pr. Co.	Michigan	140,000	Elec. World	1918, '20
		110,000	Power	1918
		72,000	Elec. Review	1919
Ga. Rwy. & Pr. Co.	Georgia	110,000	Elec. World	1913
		60,000	Elec. Engrg.	1914
			G. E. Rev.	1914
Great Western Power Co.	California	100,000	Jour. of El.	1920
		160,000	El. World	1909, '14
Hydro-El. Power Commission of Ontario	Ontario	110,000	Engr. News Rec.	1919
			Engr. World	1919
			El. World	1912
			Elec. News	1915
Idaho Power Co.	Idaho	44,000		
		66,000	Elec. World	1916
Mexican Lt. and Power Co.	Mexico	85,000		1914
Mexican Northern Power Co.	Mexico	110,000	Elec. World	1914
Mississippi River Power Co.	Iowa	110,000	G. E. Review	1914
			El. World	1913, '14
			El. Engr.	1914
			Eng. Record	1912, '13

Company	State	Voltage	Journal	Year
Montana Pr. Co.	Montana	100,000	G. E. Review	1911, '16
			El. World	1915
			El. Rev.	1917
			Elec. World	1915, '20
Montreal Lt. and Power Consolidated.........	Canada	66,000	Elec. Rwy. Jl.	1917
			Can. Engr.	1917
			G. E. Rev.	1916
			Engr. Rec.	1913, '17
New England Pr. Co............	Mass.	66,000	Eng. Record	1913
			G. E. Rev.	1911, 14, '19
			El. World	1913
			Water Chronicle	1914
Northern State Power Co......	Minnesota	50,000	El. World	1907
		60,000	Eng. Record	1914
		110,000	Power	1914
Pacific Gas and El. Co.........	California	110,000	Jour. of El.	1916, '13
		104,000	El. World	1912, '16, '19
		60,000	El. Journ.	1915
Pacific Pr. Lt. Co............	Oregon	66,000	G. E. Rev.	1914
			El. World	1917
Penninsular Pr. Co............	Michigan	66,000	Bul. A.I.E.E.	1915
			El. Rev.	1918
			El. World	1914
			Power	1919
Pa. Water and Power Co......	Pennsylvania	70,000	Power	1913, '19
			Proc. A.I.E.E.	1915
			El. World	1912, '15
			Eng. Record	1915
Portland Rwy. Lt. Pr. Co...	Oregon	57,000	El. Rev.	1920
			Jour. of El.	1913
			El. World	1913, '15
Puget Sound Pr. and Lt. Co.....	Washington	110,000	El. Rev.	1920
		55,000	El. World	1912, '20
			Jour. of El.	1918
Shawinigan Water and Power Co..	Canada	100,000	El. Rev.	1917
			El. World	1912
			El. Times	1920
			El. News	1918

Company	State	Voltage	Journal	Year
Southern Cali-Edison Co.	California	150,000	Jour. of El.	1920
			El. Rev.	1911, '20
		60,000	El. World	'07, '16, '19, '20
			Power	1911
Southern Pow.Co	N. Carolina	100,000	Eng. News	1917
		44,000	G. E. Rev.	1909, '10
			El. Jour.	1911
			El. World	'15, '10, '11, '20
			Eng. News Rec.	1920, '17
			Eng. Record	1917
Southern Sierras Power Co.	Tennessee	120,000	Power	1914
			Elec. Engrg.	1913, '14
Turners Falls Pr. and El. Co.	Mass.	66,000	Elec. Rev.	1917, '19
			El. World	1915, '18
			G. E. Review	1917
Utah Power and Light Co.	Utah	130,000	Jour. of El.	1914
			El. World	1915
			Engr. Record	1913
Washington Water Pr. Co. . .	Washington	60,000	El. Rev. & West. Elec.	1917
		110,000	El. World	1908, '12, '20
			Eng. Record	1912
			Jour. of El.	1914
			El. Rev.	1920
Western Pr. Co. of Canada	Canada	60,000	El. Rev.	1917
			El. World	1912
Wisconsin-Minn. Lt. & Pr. Co. . .	Wisconsin	120,000 66,000	El. World	1916

BIBLIOGRAPHY. — In the preceding table are given references to some of the more important hydroelectric stations. A great amount of valuable data on the control and transmission of energy from hydroelectric stations is also given in the report of the Engineering Data Committee of the A.I E.E., entitled *Engineering Data Relating to High-Tension Transmission Systems* presented at the annual convention of the A.I.E.E., June 25, 1914.

In addition to the above the following articles on the subjects listed contain valuable data: **Exciters and Excitation:** *Elec. World*, 1907, Vol. 49, p. 880; 1912, Vol. 59, p. 1247; *Trans. A.I.E.E.*, 1912, Vol. 31, p. 1841; *G. E. Rev.*, 1912, Vol. 15, p. 626; 1914, Vol. 17, p. 567; *Power Plant Eng.*, 1920, p. 888. **Voltage Regulation:** *G. E. Rev.*, 1912, Vol. 15, p. 468, p. 530; 1912, Vol. 15, p. 44, p. 626; *Trans. A.I.E.E.*, 1912, Vol. 31, p. 1841; *Elec. Jour.*, 1911, Vol. 8, p. 943; 1912, Vol. 9, p. 609; *Elec. World*, 1912, Vol. 60, p. 996. **Station Wiring:** *Elec. Jour.*, 1904, Vol. 1, p. 123; 1906, Vol. 3, p. 412; 1907, Vol. 4, p. 43; *G. E. Rev.*, 1913, Vol. 16, p. 361. **Operation;** *G. E. Rev.*, 1913, Vol. 16, p. 355. **Ventilation of Station;** *G. E. Rev.*, 1914, Vol. 17, p. 572. See also *Bibliography* in articles on the component apparatus.

POWER STATIONS, STEAM-ELECTRIC. — (*See also Power Stations, Gas-electric; Power Stations, Hydroelectric; Boilers; Condensers; Generators; Steam Engines; Steam Turbines; etc.*) The following is a brief table of contents of this article:

LOCATION. — The selection of a site for a power plant depends on the following factors:

(1) The cost and availability of land, adequate in area and suitable in form for present and future needs.

(2) Provision for the economical handling of fuel and ashes. The site should afford navigable water frontage if possible and railroad connection in every case unless the delivery of coal by water is assured at all seasons. In comparing sites the cost of dredging channels and of track construction, including right of way, should be carefully considered.

(3) The nature of the water supply. Ample water supply from natural sources suited to all the needs of the plant is highly desirable. The life, efficiency and cost of maintenance of boilers and condensers depend greatly on the quality of the water supply. Water analyses should be made in connection with preliminary surveys.

(4) The bearing power of the sub-soil for foundations, the probability of costly difficulties in construction, the elevation of ground water, the normal, maximum and minimum stages of adjacent bodies of water.

(5) The general character of the surroundings and the existence of restrictive ordinances relating to smoke, noise, vibration and the movement of coal cars.

(6) Proximity to the load center of gravity, if power is to be distributed at low voltages.

FOUNDATIONS. — (*See also Concrete; Concrete, Reinforced.*) The design of foundations depends on the bearing power of the soil, the concentration of loads and the necessity of suppressing vibration. For table of the bearing power of various soils see the article on *Power Stations, Hydroelectric.*

The taking of borings is usually essential to the proper design of foundations, especially where the site is on alluvial soil near a water frontage. Concentrated loads may reach a maximum of 15 tons per square foot. Foundation footings should rest on rock whenever practicable. Soils of inadequate bearing power are reinforced by driving piles of wood or concrete at points of concentrated load or under the entire foundation, as the soil may require. Rafts of reinforced concrete resting on a soil stratum or on piling are often used to distribute loads, to prevent the flowage of alluvial soil and to reduce the transmission of vibration. The foundation structures proper are usually of reinforced concrete. The following table of safe loads on foundations is given in Snell's *Power House Design.*

Foundations for machinery are usually made separate from those of the walls to reduce the transmission of vibration. Boiler-room equipment is usually carried on the steel framing of the superstructure, but main generating units have separate foundation piers of concrete. Foundations for reciprocating engines have extended bases, and in extreme cases cushions of felt, sand or rubber composition are provided to suppress vibration. Anchor bolts for machinery are

TABLE I. — SAFE LOADS ON FOUNDATIONS

Loads in tons per square foot

Good concrete	4
Steel rails in concrete	8
Concrete piles	12
Ordinary bricks in cement mortar	5
Hard bricks in cement mortar	8
Blue bricks in cement mortar	12

accurately located in concrete work by means of templates. Bolts may be cast in the foundations, or holes may be provided, together with side holes to permit the adjustment of the bolts. When the concrete has set, the machine is lifted into place, aligned and leveled and the interstices run in with cement grout.

The datum line of a power plant should be fixed by consideration of water levels. Tidal limits and flood water stages fix the levels of condenser intake and outlet tunnels, of engine room floors and of furnace grates.

SUPERSTRUCTURE. — A power house should be fireproof in every respect, clean, well-lighted, well-ventilated and well-drained. The skeleton structure is usually of steel with wall panels of brick or of reinforced concrete. All structural members should be computed with a factor of safety of not less than 3 (*see Structures, Simple*). The most heavily-loaded members are the pillars carrying overhead bunkers and those supporting an engine-room crane. The roof should be of fire-proof material, truss-supported and with sufficient pitch to insure good drainage. The roof may properly have glazed monitors to assist in lighting and ventilation. Windows in the generating room and switch houses should be designed to exclude rain when partly open.

Interior walls are usually finished in brick set with close joints and neatly pointed. Glazed brick or tile are appropriately used for walls in generating rooms. The upper portion of such walls should be of light color. The basement floors are usually of smooth cement. Generating room floors should be of tile, brick or other material not tending to form dust. The boiler room floors are usually hard-burned brick or special concrete. The battery rooms should have floors of brick or acid-resisting asphalt. Stairways are preferably of iron and should be provided with non-slip treads.

Recent practice has evolved a standard type of building division, providing a boiler room and a generating room side by side and separated by a solid wall. All electrical control apparatus is placed at the side of the generating room opposite the boiler room on galleries outside the crane span or in a separate section of building. This general ground plan has a number of important advantages. The framing may be proportioned to the loads in the different sections, the boiler plant and generating plant may be extended with equal facility, all wiring is isolated from steam piping, dirt and smoke are excluded from the generating room and accidents may be isolated in the section in which they arise.

BOILER-ROOM LAYOUT. — (*See also Boilers; Chimneys; Draft, Mechanical; Fuel; Smoke Prevention; Steam; Stokers, Mechanical; Feed-water Heaters and Purifiers.*)

Capacity and Number of Boilers. — The rating of boilers is purely nominal and their evaporative capacity is limited largely by the rate at which fuel can be economically burned in their furnaces. A well-designed unit is capable of giving from 75 to 100 pounds of equivalent evaporation per hour per rated horsepower. Considerably higher rates have been maintained in some of the large

central stations. This high capacity demand may result in some sacrifice of efficiency. The boiler plant should have sufficient steaming capacity to operate all steam machinery at its maximum output during the period of peak load, plus reserve capacity to insure against boiler shut-downs. The most economical boiler capacity depends largely on the form of the load curve. It is economical to force boilers to very high outputs during short and severe peaks, due to the reduction of investment and of fuel required for banking fires. With a very even load curve lacking extended banking periods it is most economical to install capacity sufficient to carry the average load at the highest efficiency. In modern public service stations it is customary to draw on boilers up to 200 or 250 per cent of their nominal ratings during peak loads. In a large number of modern stations ranging in capacity from 400 to 10,000 kw. the average boiler installation was found to be 0.4 boiler horse-power per kilowatt of generating capacity. In very large stations of recent design this ratio is from 0.25 to 0.3 boiler horse-power per kilowatt.

The simplest arrangement possible is to group with each generating unit one or two boilers, but this scheme lacks operating flexibility. In large steam plants the boilers are all operated in parallel on a common steam header, though provision is often made to isolate groups of boilers in emergencies. With this arrangement the boiler plant may properly be subdivided into the number of units affording the greatest economy and convenience. Large boilers are usually more efficient than small. The unit costs of boilers, stokers, piping, flues, air ducts and building are apt to be less for large boilers than for small. The crippling of a large boiler withdraws from service a larger portion of the total capacity and correspondingly larger reserve equipment may be needed. There is no evidence to show that small boilers afford greater safety than large. In general a boiler plant should comprise not less than 4 units if continuous operation is anticipated.

Grouping of Boilers. — Two common boiler room plans exist. In one the boilers are ranged in a single or double row facing a firing aisle which runs the length of the plant. In the other there are several lateral firing aisles, each serving a double row of boilers. The former plan is appropriate when the aggregate length of firing aisle does not exceed the length of the generating room. The latter plan lends itself well to the unit or group scheme of connection to generators. When the greatest economy of ground space is necessary, the boiler plant is double-decked, but this plan complicates the handling of fuel and ashes, reduces the natural light and ventilation and requires a building of very heavy framing. A basement space is provided below the boiler room. This space contains the ash hoppers, air ducts for forced draft and ash disposal equipment. Boiler feed pumps, blowers, hotwells, feed water heaters and flues are often placed in this space. The head room of the basement should be not less than 10 feet.

Boiler Spacing and Clearances. — With few exceptions boilers are set in batteries of two, with a space of 5 feet or more between batteries. A space not less than 5 feet wide should be left behind the settings for repair work, access to blow-off valves and minor piping. If the main piping or flues occupy this space it should be at least 8 feet wide. Each firing aisle should afford ample space to withdraw boiler tubes, drums and furnace structures for replacement or repairs. The width is usually between 18 and 25 feet and varies with the type of boiler and furnace. Clear head room above the boilers should be ample to install and repair the main piping and valves and to remove and replace boiler drums. An allowance of from 10 to 12 feet is usually ample. The following table gives minimum allowances of floor space for water-tube boilers of the most widely-used types, set in batteries of two:

TABLE II. — MINIMUM BOILER ROOM SPACE ALLOWANCES FOR WATER-TUBE BOILERS

400-hp. units in batteries of two...........	1.25 sq. ft. per hp.
500-hp. units in batteries of two...........	1.20 sq. ft. per hp.
600-hp. units in batteries of two...........	1.05 sq. ft. per hp.

In power plants of modern design the actual boiler room area per boiler horse power ranges from 0.9 to 2.00 square feet, the average being very nearly 1.4 square feet.

Location of Flues. — Boiler flues may be placed on the floor at the rear of the settings, in the basement space beneath the rear of the boilers, or may be carried overhead. Flues may be of brickwork or of steel plates reinforced with angle-iron stiffeners. In the most compact designs steel flues are used over the rear of the settings. Brick flues should be lined with firebrick. Steel flues are built up from ⅛-inch plates when indoors and of $3/16$-inch plates outdoors. Flues should be as short, air-tight and straight as possible and should preferably have an upward gradient toward the chimney. It is good practice to allow from 4.5 to 5.5 square feet of flue section for each 1000 pounds of coal burned per hour in the boilers served. Branch flues should be equipped with swivel dampers to permit boilers to be shut down independently.

Location of Economizers. — Economizers are most frequently set on a steel gallery above the rear of the boiler setting. Less frequently they are set on the boiler room floor behind the boiler settings or on a floor above the boiler room. Each economizer unit is provided with a by-pass flue below or behind the economizer setting. A soot chamber is provided beneath the tubes and should be from 2 to 2.5 feet in depth. Access must be allowed along the front of the economizer to permit the opening of the cleaning holes in the bottom branch pipes. Clear space of 10 feet or more is required above the setting to permit the withdrawal and replacement of the tubes. Economizers are usually set in brickwork and subdivided into sections, each associated with a battery of boilers. A large central economizer can often be installed at lower cost, but affords less flexibility in operation.

Chimneys and Mechanical Draft Appliances. — (*See also Chimneys; Brick and Brick Masonry; Draft, Mechanical.*) Brick chimneys are usually carried down to foundations independent of the building and outside of its walls. In exceptional cases where greatest space economy is necessary, brick chimneys are carried by steel columns integral with the framing of the boiler house. Steel stacks are usually supported on cast-iron base plates carried by the structural frame work of the boiler house. Fans for forced draft are commonly set at centrally located points on the boiler room floor and distribute air to the various furnaces through ducts of sheet steel beneath the main floor. Induced draft fans are usually installed in duplicate and are located at the bases of the chimneys. Short steel chimneys are generally employed with induced draft systems.

Coal and Ash Handling Equipment. — (*See also Conveyors; Cranes; Fuel; Hoists, Electric; and Unloaders.*)

Coal Storage, External. — Continuity of fuel supply is a vital necessity to power stations. Insurance against interruptions of delivery is commonly made by use of internal bunkers and external coal storage yards. The amount of fuel to be kept in reserve is a local problem and depends on the certainty of delivery, the fluctuation of the market, the rate of deterioration of coal in storage and the danger of spontaneous combustion. Coal exposed to the weather loses heat value at a rate which depends on its content of volatile fuel and which may

exceed 1 per cent per month. Coal stored in deep piles is liable to spontaneous ignition, especially if it contains much sulphur. Both difficulties may be obviated by storing coal in basins under water. Bituminous coal should not be piled deeper than 35 feet unless submerged. As a precautionary measure iron pipes may be sunk into coal piles at intervals and the temperature read periodically by a suspended thermometer. A coal yard is commonly spanned by a gantry crane carrying an automatic grab bucket for distributing and reclaiming.

For the latest information regarding storage of fuel, see *Bituminous Coal Storage*, Univ. of Illinois Bulletin 116, 1920.

Coal Storage, Internal. — Internal storage is provided in overhead bunkers supported by the boiler house framing above the firing aisles. Bunkers are of two general types, suspended steel tanks hung from the framing, and hopper-shaped structures of reinforced concrete incorporated into the building proper. Suspended bunkers are usually concrete-lined and have a limited storage capacity, the practicable limit being about 10 tons per linear foot. Built-in bunkers are best adapted to large storage capacities up to 40 tons per linear foot. Bunkers should be divided by transverse bulkheads to increase their strength and assist in isolating trouble from spontaneous combustion. Hopper bottoms should have a slope of 45° or more to make them self-clearing. Cut-off gates should be provided at point of attachment to down-spouts. Automatic weighing and recording hoppers may be installed between bunkers and down-spouts to good advantage as their records assist in keeping check on boiler performances. Down-spouts should be not less than 12 inches in diameter and should be slightly inclined to lessen the tendency of the coal to pack. The firing aisle is sometimes equipped with an electrically-operated traveling hopper which may draw coal from any desired bunker section and distribute it to the several stokers.

The bunker capacity desirable in a boiler plant depends on the extent of the external storage and the facilities for fuel handling. Bunker capacity sufficient for from 4 days' to 7 days' supply is generally adequate. Very large bunkers are costly and are apt to lead to trouble from spontantous ignition. In computing bunker capacity it is customary to allow 40 cubic feet per ton of coal.

Coal Handling. — The handling of coal which is delivered by rail and is to be delivered directly to overhead bunkers is most readily accomplished by the following plan. The loaded car is run over a track hopper into which it dumps from beneath. The hopper delivers to a crusher which reduces the coal to a uniform size suitable for the use of the stokers. The coal is delivered by the crusher to an elevator which may consist of some type of skip hoist, inclined belt or endless chain of buckets. If a skip hoist or belt is used the coal is dumped into a receiving hopper after its ascent and is finally distributed to the bunker by a horizontal belt, flight or bucket conveyor. A chain of pivoted buckets may be used as both elevator and distributor.

When the boiler room is arranged on the unit plan, i.e., boilers in rows facing transverse firing aisles, each aisle should have its bunker system, track hopper, crusher, elevator and conveyor. Such a conveyor may properly be of the pivoted-bucket type and may serve to elevate and distribute coal or to collect ashes from the ash hoppers beneath the boilers and deliver them to an ash bunker built out over the railroad track.

Reliability is a most important factor in all coal-handling systems and is promoted by making the system mechanically simple and rugged, by installing duplicate sets of equipment and by the sectionalizing of bunkers and conveyor outfits into independent units.

Coal delivered by water is usually handled by a clam-shell or grab-bucket unloader which dumps it into a receiving hopper. After being crushed the coal is conveyed to the storage yard or bunkers by equipment similar to that just described.

Ash Handling. — If ashes are handled by a conveyor when wet or hot the corrosive action on the buckets may make maintenance costly. When the ashes are not handled by a conveyor system it is customary to provide a track running beneath the ash hoppers on which small cars may be run to haul ashes to the dump.

Feed-water Systems. — (*See also Pipes and Piping; Feed-water Heaters and Purifiers; Pumps and Pumping Engines; and Valves.*) Condensing plants usually draw feed water from the hot-wells of the condenser system. When surface condensers are used a small amount of make-up water must be added from outside sources. The water discharged from jet condensers may be used for boiler feeding if of suitable quality. Non-condensing plants operating in connection with steam heating systems usually draw feed water from the return system, with added make-up water as required. Open heaters of all types are placed on the suction side of feed pumps. Closed heaters are placed on the delivery side. Meters are usually placed in the delivery pipe and should be in duplicate if continuous indication is important. Otherwise they should be by-passed, as are all heaters and economizers, to provide for cleaning and repairs during operation.

Water is supplied to boilers through a feed main which is run along the front or rear of the boilers, often in the basement space below. Double and ring mains are occasionally used though the gain in reliability is doubtful. An auxiliary injector main running direct to boilers from the source of cold water is sometimes installed as a reserve. Iron pipe is generally employed with cold water and brass pipe for water above 200° F., or water which has a pitting tendency. Screwed joints are generally used with pipe diameters less than 2 inches. Larger pipes are fitted with screwed flanges.

Feed pumps should be installed in duplicate on each feed main or cross-overs provided between pumps. There is wide divergence in the location of feed pumps. In most plants of the unit type each feed pump is associated with the auxiliaries of a generating unit and is cared for by the turbine operator or oiler. In other cases feed pumps are grouped in a central position on the main floor of the boiler room and are cared for by a water tender. In other cases the feed pumps are placed in a separate basement pump room.

A relief valve should be placed in the pump delivery to prevent strains from excess pressure. The size of feed pipe is usually such as to allow a maximum velocity of from 300 feet to 400 feet per minute. At least two valves, a regulating valve and a check valve, should be placed in each boiler branch.

Condenser Water System. — (*See also Condensers, Steam; Cooling Systems for Power Stations.*) Cooling water for condensers is obtained from an adjacent body of water whenever possible and in other cases from a cooling pond or the basin beneath a cooling tower. In the former case large concrete intake and discharge tunnels are usually provided. These run beneath the generating room in alignment with the intake and discharge pipes of the condensers. The cross-section of these tunnels should be ample to keep the flow of water down to 5 or 6 feet per second. Intakes should have generous openings fitted with trash racks and should if possible be placed at a considerable distance upstream from the discharge outlet. In some cases it is necessary to build a baffle wall in the stream between the two tunnels. A shut-off gate is usually provided in each tunnel to facilitate cleaning and repairs. The tunnels should be of sufficient depth to insure an adequate supply of water at the lowest stage of tide or stream flow. The circulating system of surface condensers is fully enclosed and the work done by the circulating pump is merely that necessary to overcome the fluid friction in the system.

GENERATING ROOM LAY-OUT. — (*See also Condensers, Steam; Cranes; Feed-water Heaters and Purifiers; Generators, Alternating Current; Generators, Direct Current; Lubricants and Lubrication; Pipes and Piping; Pumps and Pumping Engines; Steam Engines; Steam Turbines; and Valves.*)

Capacity and Number of Units. — A generating plant should have sufficient capacity to serve its peak load with any one generating unit shut down. The reserve capacity needed above normal requirements may be provided most economically by selecting types of equipment capable of giving large overloads in emergencies, by maintaining in service condition obsolete machinery which is physically sound but uneconomical, by installing a large reserve storage battery, or by tying in parallel several power plants so that a moderate reserve may be shared in common. More reserve capacity is needed when a plant contains few large units than with many of small size. A smaller number of units than four is disadvantageous, due to the relatively large reduction of capacity by the disabling of one. A larger number than eight units has no inherent advantages. Large units are generally more economical than small units if kept well loaded. If the plant has a normal daily period of very light load it may be economical to employ one small unit well suited to this load.

Types of Equipment.—Steam-turbine generators are used almost exclusively for alternating-current generation in units exceeding 500 kw. The turbine has little or no advantage over the engine in smaller sizes and in non-condensing plants the engine is often superior. On account of its high speed the turbine is poorly adapted to the direct driving of d-c. generators. Large d-c. generators are becoming obsolete in steam plants, for it is usually more economical to generate alternating current in turbine units and convert it to direct current either locally or in distant substations. A self-contained unit comprising boiler, engine and condenser, known as the *locomobile*, has been largely used in very small European plants and to some extent in this country.

Arrangement of Generating Rooms. — The modern standard power house has a long and narrow generating room placed between a boiler house and a switch house or series of electrical control galleries. The generating room is spanned by an electrically-operated crane. The generating units are usually ranged along this room in a single or double row. Each unit comprises a prime mover, electric generator, condenser and the associated pumps and their motive power. In most instances a basement space or series of open pits is provided below the main floor in which the condensers, pumps and most of the piping are located.

Arrangement of Turbines. — In many plants using vertical-shaft turbines the basement has been omitted and the auxiliaries grouped about the base of the turbine on the main floor. In such cases the condenser is either incorporated into the base of the turbine or is placed on the floor immediately beside it. This grouping of equipment on a single floor has several operating advantages. All apparatus has good light. A single operator can give efficient attention to a large group of equipment, and machinery so placed usually receives closer attention than it would in a pit or basement. All pieces can be readily handled by the crane without interference with other apparatus. The ample head room is advantageous in making repairs. The basement plan is especially economical of floor space, but makes the auxiliaries relatively less accessible. Placing condensing equipment in open pits beside the engine or turbine piers makes it accessible for inspection and for handling by the crane.

In determining the arrangement of a generating room it is essential to provide each element with ample space for all needed attention during operation or repairs. Sufficient clear floor space about a unit is usually provided to permit it to be dismantled without removing the parts to a distance. Clear trucking

space is also desirable. The separate pieces of machinery should be so arranged that the crane can be conveniently used in assembling or dismantling any piece without interference with others. Clear overhead space below the crane hook should be sufficient to permit any heavy part to be lifted clear of its setting and carried away. The crane girder and trolley must clear all roof trusses and lighting fixtures. Clearance must be allowed at the generator ends of horizontal turbine units to withdraw the revolving field structures.

Arrangement of Condensers. — In setting surface condensers clear working space must be allowed at both ends to remove the heads and at one end clear space must be allowed to permit the withdrawal and replacement of tubes. Jet condensers should be set in such a manner that the head can be conveniently opened for repairs. Condensers in general should be set below the level of the associated prime mover and as close as possible to its exhaust port. The condenser connection should provide natural drainage for condensed steam. It should have as few joints as practicable to avoid occasion for air leaks. It is usually desirable to provide a copper expansion section in this connection as joints in a rigid pipe are difficult to keep air-tight with varying temperatures. The arrangement of a central condenser serving a group of prime movers is uncommon in electric power plants. Barometric jet condensers are often placed outside of the wall of the building on account of the long tail pipe required.

Floor Space in Generating Rooms. — The floor space provided per kilowatt varies greatly with the size, type and arrangement of equipment. The following data from modern steam-turbine stations are illustrative of the range of best practice:

TABLE III. — SPACE COVERED BY STEAM-TURBINE POWER STATIONS

Station No.	Capacity, kilowatts	Boiler room, square feet per kilowatt	Turbine room square feet per kilowatt	Total, square feet per kilowatt
1	3,000	0.71	0.70	1.41
2	8,000	0.74	0.69	1.43
3	11,000	1.11	0.70	1.81
4	13,500	1.13	0.50	1.63
5	16,000	0.90	0.37	1.27
6	24,000	0.92	0.40	1.32
7	30,000	0.44	0.40	0.84
8	32,000	0.60	0.30	0.90
9	100,000	0.48	0.17	0.65

Oiling Systems. — (*See also Lubricants and Lubrication.*) Steam power plants afford three classes of lubrication problems, viz., wearing surfaces exposed to high temperature steam, as in cylinders, valve chests and stuffing boxes; atmospheric surfaces of open guides and journals; and enclosed surfaces, chiefly journals. Steam surfaces are lubricated with cylinder oils of mineral origin supplied in atomized form by forcing the oil in small quantities into the steam supply pipe. Oil may be supplied by a local force-feed pump or hydrostatic lubricator or may be supplied from a central tank and pump supplying a group of cylinders. Bearings, guides and other exposed surfaces are lubricated by oil or grease supplied by hand or from adjustable feed cups. Enclosed surfaces are usually lubricated by splashing or by flooding with oil from a central source of supply. The flooding system is most efficient. Oil from a central tank is

forced by pump or gravity to the various working parts whence it returns through a drip system to collecting pans and filters. After purification a small amount of make-up oil is added and the reclaimed supply restored to the oiling system.

PIPING SYSTEMS. — (*See also Pipes and Piping; and Valves.*) The various piping systems in a power plant are subdivisible into the following groups: (*a*) High-pressure steam piping between boilers, main units and auxiliary engines; (*b*) exhaust piping to condensers; (*c*) exhaust piping to feed-water heaters; (*d*) atmospheric-exhaust piping; (*e*) feed-water piping; (*f*) cooling-water piping; (*g*) pipe-drainage system and (*h*) oil piping. To facilitate the identification of pipes of various classes it is desirable to paint each a distinctive color.

High-pressure Steam Piping. — The chief considerations in laying out high-pressure piping are (*a*) to produce a reliable system without complexity; (*b*) to make all joints permanently steam tight; (*c*) to take up all expansion strains; (*d*) to maintain proper steam pressure at all points of delivery; (*e*) to reduce to an economic minimum the loss of heat by radiation and (*f*) to drain from the entire system all water of condensation. In many older systems of piping elaborate ring and multiple headers, with numerous by-passes, cross-overs and duplicate-connection branches were employed in the endeavor to promote reliability by making possible the isolation of any fault. Reliability is sought in modern systems by very simple connections with skillful design and the best possible construction.

Unit and Parallel Systems of High-pressure Piping. — The *unit system* and the *parallel system* of connection with their various modifications are now most extensively used. The unit system of piping connects a separate group of boilers to each prime mover and its auxiliaries. The parallel system provides a large steam header running the length of the plant into which all boiler branches deliver and from which all prime movers are supplied. Unit systems are usually provided with cross-overs between unit steam and water headers to permit the parallel operation of different sections in emergencies. Parallel systems are often provided with sectionalizing valves to permit the isolation of any section in case of accident. Diagrammatic sketches of unit and parallel grouping are shown in Figs. 1 and 2.

Size of Piping. — Pipe sizes for high-pressure work are generally determined by the maximum allowable steam velocity. It has been found satisfactory and economical to allow a maximum velocity of 6000 feet per minute with saturated steam and from 9000 to 12,000 feet per minute with superheated steam. The flow to piston engines is intermittent and the pipe size should be proportioned according to the velocity during admission unless a receiver is installed. The steam header of a parallel system serves as a reservoir to equalize pressures and prevent vibrations in the piping. Its cross section is properly proportioned according to the maximum cross flow of steam with any boiler section inoperative.

Joints. — Joints in high-pressure piping are usually of the screwed type for diameters under 3 inches and of the flanged type for larger sizes. Flanged joints may be made between ground faces or by aid of gaskets, the former type being preferable for high-pressure steam. An excellent joint is made by drawing up with loose collar flanges the turned-over ends of pipe sections with faces ground true, often called a Van Stone joint. Such joints are more expensive than rigid flanged joints but the possible swing of the pipe about its axis is a great advantage in the final aligning and connecting up of a system with many branches.

Provisions for Expansion. — Provision for the expansion of pipe without straining joints and fittings is imperative. Between two points which must be rigidly fastened expansion is taken up in expansion loops and bends. The

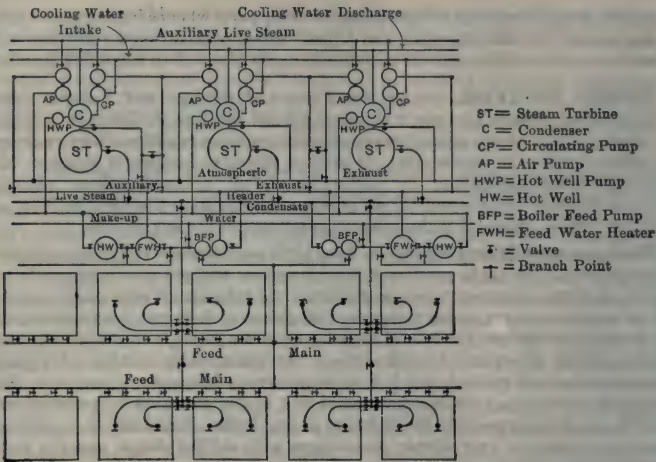

Fig. 1. Parallel Method of Connection

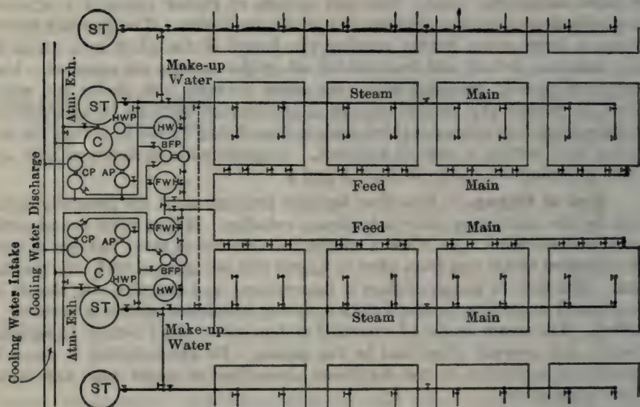

Fig. 2. Unit Method of Connection

radius of such bends should in all cases be not less than five times the pipe's diameter and at each end of the curve there should be a length of straight pipe not less than twice the diameter. Welded flanges are recommended for the attachment of bends. When bends cannot be used recourse must be had to swivel joints or slip joints, which should be avoided if possible. When screwed joints are employed expansion is commonly provided for by the use of sections having single, double or triple swing about a screwed connection. Pipes of considerable length must be anchored at more than one point to prevent vibration. Between each anchor expansion bends are required and the pipe must be support-

ed by hangers, roller brackets or pedestals allowing longitudinal motion. Anchorage is usually made at points of attachment to branches, and fittings with anchor bases are employed for the purpose.

Lagging. — Live-steam pipes, boiler-feed pipes, steam drums, receivers and separators should be covered with heat-insulating material to reduce the radiation losses to a minimum. The loss from bare pipe is approximately 3 B.t.u. per square foot per hour for each degree difference of temperature. Good commercial coverings, such as magnesia, felt, asbestos, mineral wool, etc., will prevent from 75 to 90 per cent of this loss if properly applied.

Drainage. — Saturated steam in passing through pipes undergoes a small amount of condensation due to friction and heat radiation. The presence of water in steam piping is a source of danger, for a water slug, if picked up by the moving column of steam, may be driven with tremendous force against any opposing surface, such as a valve, sharp bend or cylinder, with destructive results. Pipes should be slightly inclined so that water drains away from the prime movers and every point where water may collect must be provided with a drip connection. These drip connections are run through traps to the hot-well so that the hot water may be returned to the boilers. In other systems the drip water is returned directly to the boiler. Separators are often installed at the inlets to reciprocating engines and exhaust steam turbines taking wet or saturated steam, to drain the moisture from the entering steam. Drip connections should always be made to these separators. Bleeder connections to live steam lines are provided in order that the water condensed in warming up the pipe when steam is turned on may be drawn off.

Valves. —Valves for steam pipes are of two general types, gate valves and globe valves. Either type may be outside screw or inside screw, according as the screw of the spindle is outside or inside of the casting. Outside screw types are preferred for high-pressure work as the position of the spindle is then an index showing whether this valve is open or closed.

Check valves are required in boiler connections to prevent steam flowing into the boiler when cold or when its pressure is below that of the header to which it is connected. Emergency valves are often provided to cut off the steam under abnormal conditions, such as the bursting of a pipe or the racing of an engine. These often take the form of a weighted valve which closes itself when a trip is released. Electric motors and hydraulic pistons are sometimes connected to ordinary valves to provide for remote control during emergencies.

Blow-off valves of boilers are subjected to very severe service and are made exceptionally rugged. Such valves must close without leaks, open readily and furnish a free path for the ejection of scale and sediment. The wearing parts of such valves should be readily renewable. Best practice requires the use of two blow-off valves or a valve and a cock. The steam, water and sediment are usually blown through a tank partially filled with water before being exhausted to the air.

A few rules relative to the installation of high-pressure valves and piping may be noted. All valves of a diameter above 6 inches should be by-passed to facilitate opening under pressure. Valve stems are often placed horizontally to lessen the tendency to form water pockets. Angle valves should be selected whenever convenient because of the greater room in them. Branches from mains to boilers should have at least two valves, an ordinary stop valve next to the main and an automatic check valve near the boiler. Valves are best placed at the highest points in the pipe to simplify the drip system. When the flow of steam is intermittent heavy valves should not be placed far to one side of a line joining the points where the pipe is supported as they may cause vibration.

Branches from a main to a prime mover should have a stop valve at the highest point near the main. A receiver-separator from three to four times the volume of the high-pressure cylinder should be placed as close to the throttle of a piston engine as possible to equalize steam flow and drain moisture from the steam. When superheated steam is used cast-steel fittings and valves are preferred to cast iron. Branch pipes are almost invariably connected to the top of the main to prevent water from passing into the branches. When superheated steam is supplied to the main units and saturated steam to the auxiliaries a separate steam main for the latter is employed.

Exhaust Steam Piping. — The size of the exhaust pipe of a prime mover is determined by the permissible back pressure. High vacuum requires ports and exhaust piping of large diameter and the length of pipe to the condenser as short as possible. An atmospheric-relief valve which is normally closed by air pressure against a spring or weighted lever but which opens automatically when the vacuum fails should be installed in the condenser pipe. Atmospheric-exhaust pipe is usually spiral riveted as no precaution against leaks is required. This usually terminates in an enlarged exhaust head which contains baffles to drain from the steam condensed water and oil before it discharges to the air. A common exhaust main for a number of units is often employed. When exhaust steam is used for heating purposes the system is supplied through a back-pressure valve which automatically opens the atmospheric exhaust if the back pressure exceeds that required to operate the heating system and closes when normal back pressure is restored.

GENERATOR AND CONTROL EQUIPMENT. — (*See also Batteries, Storage, Applications of; Bus-Bars and Bus-bar Structures; Circuit Breakers; Generators, Alternating Current; Generators, Direct Current; Reactance Coils; Regulators; Relays; Switches; Switchboards; Switchgear Equipment for Power Stations; Transformers; Transformers, Instrument; Wires and Cables.*)

Direct-current Generators. — Three-wire lighting service may be provided (*a*) by the connection of generators in sets of two in series, (*b*) by the use of three-wire generators with external or internal compensator coils or (*c*) by the use of voltage balancer sets associated with standard two-wire generators. Railway generators are operated at or near 600 or 1200 volts and are grounded at one pole, usually the negative. Series fields and equalizer connections may be on either the positive or the negative side. Equalizer switches are often carried by pedestals near the generator terminals to save wiring to the switchboard. Satisfactory voltage regulation for railway systems is usually provided by the use of compound generators. The close voltage regulation required by lighting systems is best provided by the use of a regulator of the Tirrill type. (*See Regulators.*)

Alternating-current Generators. — Synchronous 3-phase generators are used most extensively. Induction generators are often advantageous in connection with exhaust-steam turbines. Synchronous turbo-alternators are now designed with high internal reactance to prevent excessive transient currents immediately after the creation of a short-circuit. If the internal reactance is inadequate they may be connected to the bus-bars through reactance coils. (*See Reactance Coils.*) In some cases the generators are operated at half the bus-bar voltage and are connected through raising auto-transformers wound with large reactance.

Grounding the Neutral. — Three-phase alternators are usually Y-connected and provision is frequently made for the grounding of one generator at the neutral point to prevent dangerous potential rise should a wire become accidentally grounded. More than one grounded neutral in a group of genera-

ors in parallel is undesirable, due to the possibility of a third-harmonic circulating current passing through the neutral connection.

Air Ducts for Forced Ventilation. — Modern turbo-alternators are designed for forced ventilation. A system of ducts should be provided, either overhead or beneath the floor of the generating room from which each generator may draw a supply of clean outside air. This air is forced through the ventilating spaces of the stator and rotor by the fan action of the rotor. Discharge ducts are also provided in case it is undesirable to discharge the heated air to the generating room. It is desirable to equip such ducts with dampers so that the air may be discharged indoors or outdoors as desired. In many cases where the air supply is dusty it is desirable to install air-conditioning equipment in the intake of the system. See *General Electric Review*, Vol. 16, p. 627 for a full discussion of turbo-generator ventilation.

Excitation. — Fig. 3 shows the exciter capacity required by modern alternators. In small plants each alternator is often provided with its individual exciter driven by the main shaft. Large stations are provided with central

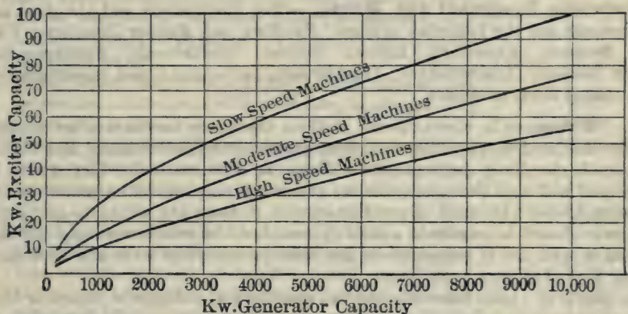

Fig. 3. Exciter Capacity Required by A-C. Generators

systems of excitation comprising not less than two, and generally more, direct-current generators, driven by independent motive power. At least one exciter in every plant should be steam-driven. Three-phase induction motors are quite generally used for the electric driving of exciters. The exciter system is frequently reinforced by a floating storage battery to insure the supply of exciting current in every emergency. (*See Batteries, Storage, Applications of.*) All alternator fields are supplied through adjustable rheostats from a set of excitation bus-bars, except that in a few very large plants the excitation system is sectionalized for the sake of reliability. Exciters are commonly rated at 125 or 250 volts, and are usually compound-wound, with magnetic circuits normally in a state of low saturation. The best position for exciters in large stations is generally near the center of the generating floor.

Voltage Control. — In both alternating- and direct-current stations very sensitive regulation of the bus-bar voltage may be obtained by the use of appropriate types of Tirrill regulators. (*See Regulators.*) When the load variations of different feeders are quite unlike it is often desirable to provide each feeder or group of parallel feeders with independent feeder regulators, which may be hand-operated or may be automatically controlled by voltage relays.

Transformers. — The arrangement of the transformers depends to some extent upon the type used. When it is not convenient or possible to allow above

each transformer clear head-room for crane handling, the transformer cells may be arranged to open at one side along a track and have raised floors at the level of a flat car, so that the assembled units may be slid into or out of place.

Air-blast Transformers. — These are largely used for step-up service in power stations in connection with feeders operating at 20,000 volts or less. Railway transmission lines and feeders operated at more than 20,000 volts are usually supplied through oil-insulated, water-cooled transformers. Air-blast transformers are usually placed above a common pressure pit whose air-supply is drawn from out-doors by electrically operated fans. Each transformer may be equipped with adjustable dampers to regulate the air supply.

Oil-insulated Transformers. — In some cases oil-insulated transformers are isolated in fire-proof cells built of concrete or masonry. The fire-risk is usually not sufficient to warrant isolation for each unit, but it is often desirable to enclose each 3-phase bank. Large oil-insulated transformers should be connected to an oil-drainage system to facilitate the withdrawal and replacement of oil.

Switching Equipment and Wiring. — For descriptions and standard arrangements see the following articles: *Bus-Bars and Bus-bar Structures; Circuit Breakers; Regulators; Relays; Switches; Switchboards; Switchgear for Power Stations; Synchroscopes; Transformers, Instrument;* and the articles on the various kinds of meters.

Wiring. — In direct-current stations the power conductors are usually rubber-insulated copper cables run from generators to switch-board and thence to outside circuits in ducts of tile or fiber. In some cases the duct system is dispensed with and the conductors are carried on open wallracks spaced by insulating knobs. In small alternating-current stations the wiring to and from the switchboard is usually in a duct system, but the switchboard and bus-bar wiring is open and is supported by porcelain insulators on walls and on a light frame work of pipes or angle-irons.

In plants of 12,000 kilowatts and up, operating at voltages of 20,000 or less, it is customary to isolate all conductors of unlike polarity as fully as possible in fireproof cells and barriers. The essential features of such a system of isolation are as follows: — (1) Each horizontal run of conductor is drawn into its individual duct of tile or fibre set in a concrete floor; (2) each bus-bar is mounted in a separate horizontal cell of concrete or masonry; (3) each instrument transformer and disconnecting switch is in a separate cell; (4) each pole of a bottom-connected oil switch or each complete top-connected switch is in its separate cell; and (5) vertical fire-proof barriers are placed between all vertical runs of conductor. Isolation of this nature is seldom practiced in systems above 30,000 volts, but open overhead wiring with generous spacing is employed.

In an enclosed system the power connections are appropriately formed by single-conductor copper cables, insulated with rubber or with cambric tape, and covered with a fire-resisting sheath. Lead sheathing is seldom employed unless the wiring is exposed to dampness. The cross section of such conductors is determined by the safe limits of temperature rise.

COST OF STEAM POWER STATIONS (Pre-war figures). — Power station costs are subject to wide variations with the type and elaborateness of equipment, construction difficulties and rated capacity.

Capital Costs. — Illustrative data showing the range of unit costs of the chief divisions of plant equipment are given by O. S. Lyford, Jr., and R. W. Stoval of the Westinghouse, Church, Kerr Co., in the Electric Journal, 1912, Vol. 9, p. 322, as follows:

TABLE IV. — COSTS OF STEAM-ELECTRIC POWER STATIONS

Capacity, 2000 to 20,000 kw., based on maximum continuous capacity of generators at 50° C. rise

Item	Dollars per kw.	
	High	Low
Preparing Site: Dismantling and removing structures, making construction roads, tracks, etc.	0.25	0.00
Yard Work: Intake and discharge flumes for condensing water, railway siding, grading, fencing, sidewalks	2.50	1.00
Foundations, including foundations for building, stacks and machinery, together with excavation, piling, waterproofing, etc.	6.00	1.00
Boiler Room Equipment, including boilers, stokers, flues, stacks, feed pumps, feed-water heater, economizers, mechanical draft, and all piping and pipe covering except for condenser water	24.00	12.00
Turbine Room Equipment, including steam turbines and generators, condensers with condenser auxiliaries and water piping, oiling system, etc.	22.00	12.00
Electrical Switching Equipment, including exciters, masonry switch structure with all switchboards, switches, instruments, etc., and all wiring except for building lighting	5.00	2.00
Service Equipment, such as cranes, lighting, heating, plumbing, fire protection, compressed air, furniture, permanent tools, coal and ash-handling machinery, etc.	5.00	2.50
Building, including frame, walls, floors, roofs, windows and doors, coal bunkers, but exclusive of foundations, heating, plumbing and lighting	12.00	4.00
Starting Up. — Labor, fuel and supplies for getting plant ready to carry useful load	1.00	0.50
General Charges, such as engineering, purchasing, supervision, clerical work, construction plant and supplies, watchmen, cleaning up	6.00	3.00
Total Cost, except land and interest during construction	83.75	38.00

The same authorities give the costs of foundations at from \$1.25 to \$4.00 per square foot of building area, depending on the nature of the sub-soil. Table III shows that the ground area of buildings ranges from 0.8 to 2.0 square feet per kw. Building costs range from 8 to 12 cents per cubic foot of space, over-all. Power plant buildings range from 50 to 100 cubic feet per kw.

W. E. Wickenden has correlated miscellaneous data on power plant costs in the form of curves, see Fig. 4. The curves are shown in an additive sense, e.g., for a 12,000 kilowatt plant, boiler room equipment and piping cost \$17.50 per kw., generating plant \$37.00 − \$17.50 = \$19.50 per kw., etc., the total cost except land and interest during construction being \$66.00 per kw.

Operating Cost and Fixed Charges. — The cost of producing electrical energy comprises two groups of items, viz., fixed charges on the investment in the power plant to cover interest, depreciation, taxes and insurance, and the

TABLE V. — COSTS OF BOILER ROOM EQUIPMENT.

Item	Dollars per boiler horse-power	
	High	Low
Boilers, except settings	$11.00	$8.00
Superheaters	3.00	0.00
Stokers	5.50	3.00
Masonry settings of boilers	3.50	2.00
Flues	1.50	0.75
Stacks	4.00	2.00
Economizers	4.00	0.00
Mechanical draft	3.00	0.00
Feed pumps	1.50	0.50
Feed heaters	1.00	0.40
All piping and pipe covering	10.00	6.00
Coal chutes and ash hoppers	1.25	0.00
Miscellaneous items	1.00	0.50
Totals	$50.25	$23.15

cost of operating the plant. Fixed charges on the investment range from 10 to 14 per cent of the investment per annum and are practically independent of the load factor of the station. The amount assignable to each kilowatt-hour is therefore inversely proportional to the load factor and equals, in cents,

$$F = \frac{100\,RI}{8760\,K},$$

where I is the investment per kw. of capacity, in dollars, R is the rate of fixed charges expressed as a decimal fraction and K the annual station load factor expressed as a decimal fraction.

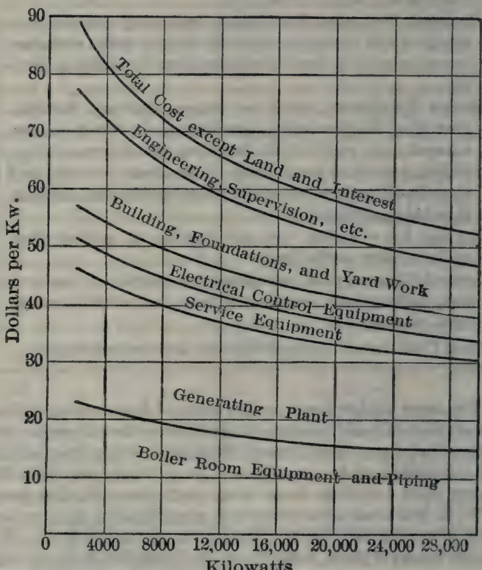

Fig. 4. Approximate Division of Construction Cost of Steam-electric Power Plants. (In additive sense, see text.)

Operating costs include supervision, labor, fuel, other materials consumed and current repairs. The cost of supervision per kw-hr. varies almost inversely

with the load factor. Labor cost per kw-hr. in boiler plants vary in almost direct proportion to the weight of coal burned per kw-hr. Labor cost per kw-hr. in generating rooms varies greatly with the size of the station and the number of units operated, since the number of men required depends more on the number of units of apparatus to be cared for than on the ratings of these units. Large plants are therefore at a large advantage in the item of labor cost. The cost of fuel per kw-hr. varies directly with the price of heat units in the coal burned and inversely with the over-all efficiency of the station. In modern steam turbine stations the cost of supplies other than fuel averages about 20 per cent of the total labor cost. The cost of current repairs in such stations is from $0.75 to $1.50 per annum per kw. of generating capacity. A high load factor is distinctly favorable to good physical efficiency, due to the reduction of stand-by and light-load losses, and to economy in the use of labor. H. G. Stott has pointed out that the production cost per kw-hr. varies inversely with the 4th root of the annual load factor. (*See Proc. A.I.E.E., Vol. 32, p. 1127.*)

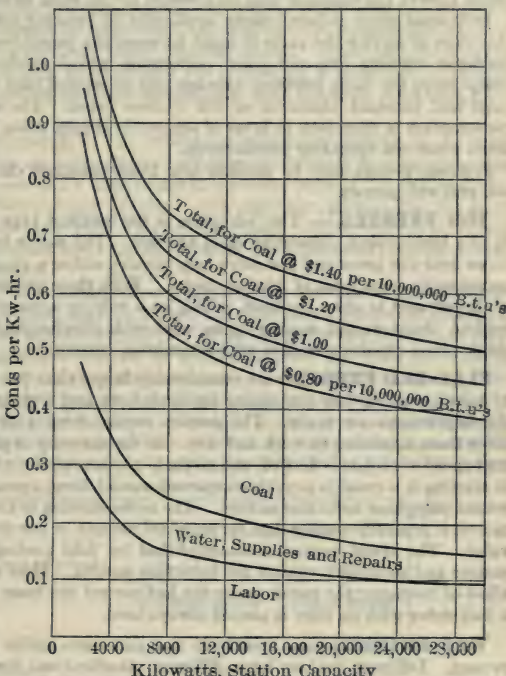

Fig. 5. Approximate Division of Operating Expenses of Steam-electric Power Plants. (In additive sense, see text.)

Wickenden has also correlated the records of the operating costs of a considerable number of railway and public service stations having load factors between 25 and 33 per cent, and from these data has prepared the curves in Fig. 5. (For the B.t.u. equivalent of a ton of coal see article on *Fuel.*) These curves are shown in an additive sense, in the same manner as the curves in Fig. 4.

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PRINTING PRESSES, ELECTRICAL OPERATION OF. — (*See also Motors, Industrial Applications of.*) Owing to the great variety of work performed, printing machinery as a rule requires a certain degree of speed variation in order that with a given equipment and operating force, the maximum of high-grade production may be turned out. All necessary speed variations are most readily, economically and satisfactorily obtained with electric drive and control. Each press can then at all times be under instant control, so that the entire time of the operator may be devoted to the work in hand.

Individual drive is particularly applicable to printing establishments, as in this class of service the ratio is small between the power required to drive the machines running idle and when performing actual productive work. With group drive the ratio between average and connected load is high, due to the large and constant character of the friction losses. The economy in power consumption is, therefore, in favor of separately-driven units, which may be shut down when not operating productively.

Printing presses may be divided into three principal classes, viz., job, flat-bed and web presses.

JOB PRESSES. — The job press is the smallest type and requires from ¼ to 1 horse-power, depending upon the size. The motor is generally mounted on or near the press and the drive may involve either a short belt with idler or occasionally some form of friction device. With these presses a speed variation of 60 per cent is often required, which, as a rule, is accomplished by armature control, shunt-wound motors being used with direct-current installations and single-phase repulsion motors with alternating-current.

FLAT-BED PRESSES are considerably larger than job presses and consist of a reciprocating bed containing the type form and a main cylinder on which the impressions are made. The presses require from 2 to 10 horse-power to drive them according to work and size. In the majority of plants a 50 to 60 per cent speed variation is desired, and as these presses require a considerable torque at starting it is usual to provide compound-wound direct-current motors or phase-wound polyphase induction motors. The series winding of the compound-wound motors is generally weaker than for standard motors, 10 per cent being an average value. The speed control is accomplished by field control for direct-current motors and secondary control for induction motors. Most presses of this type allow of mounting the motor under the bed toward the front of the machine and a belt drive with an idler is almost always used.

WEB PRESSES vary greatly in size and come under the head of rotary presses. Different makers have different classifications for the various sizes, such as the number of decks or webs, as 3-deck, 5-deck, 2-web, etc., or according to the number of groups of which they consist, as quadruple, sextuple, octuple, etc. A modern high-speed sextuple press, for example, has three paper rolls and six plate cylinders, each cylinder generally being four plates wide. The plate cylinders revolve at a maximum speed of 300 r.p.m. A wide range of operating speeds must therefore be provided on the controller as the press will often be called upon to operate at speeds as low as one-half of the above.

All of these presses require a slow threading speed, usually about 10 r.p.m. This speed must be steady as the operators have to thread the paper from roll to roll around the cylinder up over the carriers to the folder, and should the press turn by jerks it might mean the loss of a finger, hand or arm. The control must also provide for "jogging" or "inching along" when making the press ready for service, that is turning the cylinders through a small fraction of a revolution so as to bring them to the desired position for putting on the plates. Brakes must

also be provided so that in case of emergency the press may be brought immediately to rest.

Single-motor Equipment. — Motor equipments for web presses are of either the single or two-motor type. The former is used with small and medium-size direct-current units but is not practicable above 30 or 35 horse-power. The principal objection to single-motor equipments is the difficulty in maintaining a constant slow speed, due to the wide variation in torque during any given revolution of the press cylinders. The waste of power in the series and shunt resistances is also a disadvantage, particularly for the larger sizes. Compound-wound motors are generally used with the single-motor equipments so as to provide for the comparatively-heavy starting torque. Speed regulation down to 10 per cent is readily accomplished by connecting a resistance in parallel with the armature circuit which, in combination with the series resistance, gives a fixed slow speed.

Two-motor Equipment. — The two-motor equipment consists, as the name implies, of two motors, one of small size for driving the press at slow speed through a worm-gear reduction, and a large motor to drive the press at the full-producing speed through ordinary direct gearing. The reduction gearing for the small motor is furthermore provided with an automatic ratchet and pawl clutch, which mechanically disconnects the small motor when the press is being accelerated by the large motor.

Either direct- or alternating-current motors may be used with two-motor equipments. When direct-current is used the small motor should be compound wound to insure sufficient starting torque. It is also a common practice to make the large motor compound wound, so as to accelerate the press more easily. With alternating-current the smaller motor may be of the squirrel-cage type but the larger must be of the phase-wound slip-ring type, as a smooth speed variation from the 10 per cent threading speed to the full running speed is most essential.

The capacity of the motors depends on the size and make of the press, and the accompanying table is only intended to give an approximate idea of the power required:

Type of press	Horse-power	
	Small motor	Large motor
Quadruple	5	35
Sextuple	7	50
Double quadruple	Two 5½	Two 35
Double sextuple	Two 7½	Two 50
Octuple	10	70

Control of Two-motor Drive. — Full automatic control in connection with two-motor drive has been almost universally adopted on larger presses. The essentials of this control are the complete automatic control of the press speed from any number of push-button stations located at different points about the press. From any of these stations it is possible to start the press, increase or decrease the speed or stop it. Five buttons are generally provided with each station marked "Fast," "Slow," "Stop," "Safe" and "Run."

Pressure on the "Fast" or "Slow" button causes the press to speed up or slow down until the button is released, when the press will continue to run at the

speed it has then attained. The manipulation of the "Stop" button will immediately stop the press, through the operation of a dynamic brake on direct-current equipments and a solenoid brake on alternating-current equipments. Pressure on the "Safe" button at any station opens the control circuit and renders the equipment inoperative until the "Run" button at that particular station is closed, releasing the "Fast" button. A "Jog" button is frequently also provided, particularly on a-c. equipments, by means of which the cylinders can be inched along to the desired position when plating is done.

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PROGRESSION. — (*See also Series, Mathematical.*) There are two kinds of progression, arithmetical and geometrical.

ARITHMETICAL PROGRESSION. — Quantities are said to be in arithmetical progression when they increase or decrease by a common difference.

Let a = first term of the progression,
d = the amount by which any term is greater than the next preceding, i.e., the common difference,
n = the number of terms in the progression.

Then the successive terms of the progression are

$$a,\ a + d,\ a + 2d,\ a + 3d, \text{ etc., to } n \text{ terms.}$$

The last term is

$$a + (n - 1)\,d.$$

The sum of the n terms is

$$\frac{n}{2}[2a + (n - 1)\,d].$$

GEOMETRICAL PROGRESSION. — Quantities are said to be in geometrical progression when they increase or decrease by a common ratio. Let

a = the first term of the progression,
r = the ratio of any term to the next preceding term,
n = the number of terms in the progression.

Then the successive terms of the progression are

$$a,\ ar,\ ar^2,\ ar^3, \text{ etc., to } n \text{ terms.}$$

The last term is

$$ar^{n-1}.$$

The sum of the n terms is

$$\frac{a(1 - r^n)}{1 - r}.$$

PUMPS AND PUMPING ENGINES. — (*See also Boilers; Blowers and Compressors; Condensers; Fans; Power Stations; Steam Engines; Steam Turbines.*) Pumps may be classified according to (1) their mode of action as piston, plunger, centrifugal, rotary, jet and direct pressure; (2) their motive power, as engine-driven, turbine-driven, motor-driven, and power-driven (by belt or gearing); (3) the number of cylinders (or their equivalent), as simplex, duplex and triplex; (4) the number of stages or pump elements in series, as single-stage and multi-stage; (5) the mode of connection, as direct-acting, crank-driven and geared. Large steam-driven pumps are usually called pumping engines.

Piston and Plunger Pumps are the most common in use. In the single-acting type, water is taken in on one stroke and discharged on the return stroke. In the double-acting pump water is taken in and discharged on both strokes. In the direct-acting piston or plunger pump the steam piston and the water piston or plunger are both secured to the same piston rod and the steam is used non-expansively. Their steam consumption is consequently high, unless special compensating devices, substitutes for a flywheel, are used, as in the Worthington high-duty and the d'Auria pumping engines. Large piston pumps are usually provided with a crank and flywheel, so that the steam may be used expansively.

Centrifugal Pumps consist essentially of a rotating impeller which draws water in at its center and a stationary casing which guides the water to the discharge outlet. Centrifugal pumps having stationary guide-vanes inside of the casing are called "turbine" pumps. Centrifugal pumps are especially suited for low heads and large volumes, but are also built multi-stage for high heads. They are not as efficient as high-grade pumping engines, but are considerably cheaper.

Rotary Pumps. — Pumps with two parallel geared shafts carrying vanes or impellers which mesh with each other, and other forms of positive-driven apparatus, in which the water is pushed at a moderate velocity, instead of being rotated at a high velocity as in centrifugal pumps, are known as rotary pumps. They have an advantage over reciprocating pumps in being valveless, and over centrifugal pumps in working under widely-varying heads. They are usually not economical, but when carefully designed with the impellers of the correct cycloidal shape, like those used in positive rotary blowers, they give a moderately high efficiency.

Injectors. — The injector is a form of steam pump commonly used for feeding boilers. If a cylindrical tube 1 or 2 inches in diameter is reduced to one-half its diameter, or thereabouts, at one portion of its length, by gradual reduction and enlargement, and a smaller tube or nozzle inserted inside of it, so that the end of the smaller tube approaches the reduced section of the larger tube, then if the larger tube be connected to a supply of water and steam at considerable pressure be introduced through the smaller tube or nozzle, the water will be drawn into the larger tube and ejected from its outer end with such force as to cause it to overcome the pressure in the boiler supplying the steam, and thus to feed water into the boiler. The apparent paradox of the injector is explained by the fact that the violent rush of steam into the water gives the latter a high velocity and the momentum thereby induced cannot be overcome in a limited space without the exertion of a force greater than that of the steam in the boiler. As a boiler feeder the injector has a remarkably high efficiency, for so much of the heat energy of the steam as is not converted into mechanical work is carried into the boiler as heat in the water, but as a pump for ordinary purposes its efficiency is very low, since less than 1 per cent of the thermal

energy of the steam is converted into work. The usefulness of the injector as a boiler feeder is limited by the fact that it will not handle hot water.

Direct-pressure Pumps. — These are of two types, the pulsometer and the air lift. In the pulsometer the water is raised by suction into the pump chamber by the condensation of steam within it, and is then forced into the delivery pipe by the pressure of a new quantity of steam on the surface of the water. Two chambers are used, which work alternately, one raising while the other is discharging. The air-lift pump consists of a vertical water pipe with its lower end submerged in a well and a smaller pipe delivering air into it at the bottom. The rising column in the pipe consists of air mingled with water, the air being in bubbles of various sizes, and therefore lighter than a column of water of the same height; consequently the water in the pipe is raised above the level of the surrounding water. The pulsometer is used for pumping out pumps, drains, etc., and the air lift for pumping from wells.

PERFORMANCE. — The performance of a pump is usually expressed in terms of steam consumption of the steam cylinder or engine driving it, the indicated horse power of the cylinder or engine driving it (or the brake horse power of the motor in case of a motor-driven pump), the mechanical efficiency of the pump, and the slip.

Useful or Water Horse Power. — Let p = difference in pressure in pounds per square inch between inlet and outlet of pump = pressure in pounds per square inch indicated by gauge on force main $\pm$ pressure in pounds per square inch indicated by gauge on suction main (+ if this reads below atmospheric pressure, − if above) + pressure in pounds per square inch corresponding to the distance between the two gauges; H = head in ft. corresponding to p; Q = actual discharge* in cubic feet per minute; W = pounds per minute discharged; w = weight of the fluid per cubic foot. For water at 62° F., w = 62.36 pounds and is about 0.1 per cent greater at 40° F., and 4 per cent less at 212° F. Then the useful or water horse power is

$$P_w = \frac{144\,pQ}{33{,}000} = \frac{pQ}{229} = \frac{pW}{229\,w} = \frac{HW}{33{,}000} = \frac{HwQ}{33{,}000}.$$

For water at 62° F., and approximately at any temperature between 32° and 212° F.,

$$P_w = \frac{pW}{14{,}300} = \frac{HQ}{529}.$$

Slip. — By the slip of a pump is meant the ratio of the difference between the piston displacement and the water delivered to the piston displacement.

Mechanical Efficiency. — By mechanical efficiency is usually meant the ratio of the useful or water horse power to the indicated horse power of the steam cylinder driving it, or to the brake horse power of the motor in the case of a motor-driven pump. Sometimes the useful or water horse power is figured on the basis of the amount of water corresponding to the piston displacement; the actual water delivered is then equal to the piston displacement multiplied by (1 − slip). The power required to deliver water at a given rate in the first case is equal to the useful or water horse power divided by the mechanical efficiency; in the second case the power required is equal to the useful or water

* Q is sometimes taken as equal to the piston displacement, which is greater than the actual water delivered, due to the slip of the water past the piston and valves.

horse power (reckoned on the basis of piston displacement) divided by the product of the mechanical efficiency and (1 − slip). (*See table below.*)

Duty. — The performance of a steam-driven pump is also expressed as the number of foot-pounds of useful work done by the pump (144 pQ × time in minutes) per million B.t.u. in the steam supplied to it above the temperature of the boiler feed water. (If the boiler takes water from several sources at different temperatures, the B.t.u. added by the boiler from each source must be reckoned separately.) This ratio is called the "duty" of the pump. Duty is also sometimes expressed as the useful work in foot-pounds done by the pump per 1000 pounds of dry steam supplied to it.

The efficiency of a pump falls off rapidly with use, due to wear of the moving parts, unless it is kept in first-class condition. The figures in the following table are for average full-load conditions at rated speed in ordinary practice:

EFFICIENCY AND DUTY OF PUMPS.

Type of Pump	Pounds steam per hour per h.p. of Useful Work	Mechanical efficiency	Per cent slip	Duty, million ft. lb. per million B.t.u.
Piston pumps:				
Small duplex, direct-acting	100 to 200	40 to 60	5 to 20	10 to 20
Compound, direct-acting, non-condensing	40 to 80	70 to 90	2 to 20	25 to 50
Ditto with "high duty" attachment	20 to 33	60 to 80	2 to 20	60 to 100
Single cylinder, flywheel, non-condensing	30 to 50	70 to 90	2 to 20	40 to 60
Multi-cylinder, fly wheel, non-condensing	25 to 40	70 to 90	2 to 20	50 to 80
Multi-cylinder, fly wheel, condensing	13 to 20	70 to 95	2 to 20	80 to 160
Direct-connected, motor-driven	....	50 to 80	2 to 20	60 to 80
Geared pumps	....	50 to 90	2 to 20	30 to 90
Centrifugal pumps:				
Single-stage for low heads	22 to 66	40 to 70	....	30 to 90
Multi-stage for high heads	25 to 50	40 to 60	....	40 to 80
Rotary pumps	26 to 50	50 to 80	5 to 20	40 to 75
Pulsometer	100 to 400	....	....	5 to 20
Air lift	....	10 to 40	....	10 to 25

Steam Consumption of Feed Pumps. — With compound steam ends well lagged and covered 100 pounds of steam per indicated horse-power-hour should be safe consumption for feed pumps of large size, while in small pumps, 200 pounds appears to be nearer the mark. The pump efficiency should not be less than 80 per cent.

Performance of Injectors. — Kneass, *Theory of the Injector*, states that the pounds of water delivered (w) per pound of steam supplied is equal, very approximately, to the ratio of the B.t.u. per pound in the steam supplied, reckoned from the temperature (t) of the discharge water to the difference between the temperature (t) of the discharge water, and the temperature (t)

of the suction water. That is, letting r = heat of evaporation, h = heat of liquid, and x = quality of the steam, then

$$w = (rx + h - t + 32) \div (t - t_0).$$

ELECTRIC DRIVE. — (*See also Motors, Industrial Applications of.*) Since the reciprocating pump is essentially a low-speed machine, limited to about fifty revolutions per minute or thereabouts, it requires a speed reduction, as by gearing, for connection to the driving motor. The centrifugal pump, on the other hand, is suitable for direct connection to motors operating at speeds of up to 3500 r.p.m.

Motors for Reciprocating Pumps. — In starting large reciprocating pumps the water may be delivered through a by-pass until the motor is up to speed, when the by-pass is gradually closed and the water delivered into the system. The load at starting, therefore, only consists of the friction losses, and usually does not exceed 25 per cent of the full-load torque. Small- and medium-size pumps may, however, be required to start under full load.

When direct-current motors are used, the compound-wound type is generally selected for single-acting pumps on account of the rather pulsating load, but for double and triplex pumps having steadier load characteristics the shunt-wound type is used to advantage. Both squirrel-cage and phase-wound induction motors are suitable, the latter, as a rule, being selected where it is desirable to reduce the starting current to a minimum or where a somewhat variable speed is required. Synchronous motors may and are frequently used for driving large pumps. By-pass valves must then, however, be provided for reducing the torque at starting as previously mentioned.

Motors for Centrifugal Pumps. — On account of the peculiar characteristics of centrifugal pumps special care is required in the selection of the motor drive. With a reciprocating pump operating at constant speed an increase of the resistance increases the pressure and therefore the load on the motor; but with a centrifugal pump an increase of the resistance reduces the load. The volume of water delivered by a reciprocating pump is not affected by the reduction of the head, but the required power is reduced. A reduction of the head with a centrifugal pump, however, increases the volume of water, and as the efficiency at the same time goes down rapidly, the load increases. It is, therefore, of importance to know what this overload, caused by a reduction of the head, amounts to and the duration of this overload; and the capacity of the motor should as a rule be governed by the low and not the high head conditions.

The condition of starting must also be given careful consideration in selecting the motor. In starting a centrifugal pump the discharge valve may be entirely closed until the motor comes up to speed, so that the motor may start as nearly light as possible. At rest the torque required is small, usually from 15 to 25 per cent of full-load torque, and this drops from 5 to 6 per cent as soon as the machine starts turning over. The pump casing is full of water, however, and as the machine comes up to speed this water is churned around in the casing, causing the motor to load up as it approaches full speed, when with pumps of the usual design it takes from 40 to 50 per cent of full-load torque to drive it even though pumping no water.

Compound-wound direct-current motors, varying-speed brush-shifting motors, and either squirrel-cage or phase-wound induction motors, or synchronous motors are all well adapted for this type of pump and will readily meet the above conditions.

COST OF FEED PUMPS (Pre-war figures). — The cost of feed pumps is a small item in the cost of the station, varying from 20 to 50 cents per kiolwatt of station capacity, or from 15 cents to $1 per boiler horsepower the lower prices

applying to the larger station. Duplex direct-acting pumps, of the Worthington type, and suitable for boiler feeding, vary in capacity from the 6 by 4 by 6 in. 100-gallon pump to the larger pot-valved pumps with compound steam ends, 14 and 20 by 10 by 15 in. delivering 500 gallons per minute. The price varies from about $300 for the small size to around $2000 for the large size, with the intermediate sizes at proportional prices. Centrifugal feed pumps, turbine-driven, run from 200 to 1000 gallons per minute, in about four sizes, and cost about $1500 in the 200 gallons size and about $3200 in the 1000 gallons size. Motor-driven centrifugal pumps are about 10 per cent higher in price, and triplex motor-driven pumps often run from 100 to 200 per cent higher.

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PYROMETERS. — (*See also Temperature and Thermometers.*) A pyrometer is any device for measuring high temperatures. By high temperature as here used is meant a temperature beyond the range of the ordinary mercury thermometer, say 350° C. and up. A great number of pyrometric methods have been proposed, the more important of which will be found treated in detail in Burgess and Le Chatelier's *Measurement of High Temperatures*, New York, 1912. The following brief treatment is adapted from this work.

CLASSIFICATION OF PYROMETERS. — The various types of pyrometers may be classified as follows. Some of these are more fully treated in the following sections.

Gas Pyrometer (Pouillet, Becquerel, Sainte-Claire-Deville, Barus, Chappuis, Holborn, Callendar, Day). — Utilizes the measurement of change in pressure of a gaseous mass kept at constant volume. Its great volume and its fragility render it unsuitable for ordinary measurements; it serves only to give the definition of temperature and should only be used to standardize other pyrometers.

Calorimetric Pyrometer (Regnault, Violle, Le Chatelier, Siemens). — Utilizes the total heat of metals, platinum in the laboratory and nickel in industrial works. It was formerly used for intermittent researches in industrial establishments because its employment demands almost no apprenticeship and because the cost of installation is not great. The cost of a metal ball to withstand over 1000° C. prohibits its use above this temperature in industrial works. Other and more convenient types of pyrometers have almost entirely superseded the calorimetric pyrometer.

Total Radiation Pyrometer (Rosetti, Langley, Boys, Féry, Thwing). — Utilizes the total heat radiated by warm bodies. Its indications are influenced by the variable emissive power of the different substances. Convenient for the evaluation of very high temperatures which no thermometric substance can withstand (electric arc, sun, very hot furnaces), or when it is not convenient to approach the body whose temperature is wanted. Can be made self-registering.

Optical Pyrometer (Becquerel, Le Chatelier, Wanner, Holborn-Kurlbaum, Morse). — Utilizes either the photometric measurement of radiation of a given wave length of a definite portion of the visible spectrum, or the disappearance of a bright filament against an incandescent background. Its indications, as in the preceding case, but to a much less degree, are influenced by variations in emissive power. The intervention of the eye aids greatly the observations, but diminishes notably their precision. This method is mainly employed in industrial works for the determination of the temperatures of bodies difficult of access — for example, of bodies in movement (casting of a metal, the hot metal passing to the rolling mill). Can be used to estimate the highest temperatures and is the best method for use above 1700° C. in laboratory and industrial works.

Electric Resistance Pyrometer (Siemens, Callendar, Waidner and Burgess). — Utilizes the variations of electric resistance of metals (platinum) with the temperature. This method permits of very precise measurements to 1000° C., but requires the employment of fragile apparatus. It merits the preference for very precise investigations in laboratories. As a secondary instrument for the reproduction of a uniform temperature scale throughout the range in which the platinum resistance thermometer can be used, to 1000° C. except in very heavy wire, it is unsurpassed in precision and sensibility. It is also now constructed in convenient form for industrial use.

Thermoelectric Pyrometer (Becquerel, Barus, Le Chatelier). — Utilizes the measurement of electromotive forces developed by the difference in temperature of two similar thermoelectric junctions opposed one to the other. In employing for this measurement a Deprez-d'Arsonval galvanometer with movable coil or a millivoltmeter one has an apparatus easy to handle and of a precision amply sufficient for industrial and many scientific uses. With a potentiometer an instrument is obtained of the highest precision, available for use to 1600° C., or even to 1750° C. with proper precautions. This pyrometer was used for a good many years in scientific laboratories, before it spread into general industrial use, where it also renders most valuable service.

Contraction Pyrometer (Wedgwood). — Utilizes the permanent contraction which clayey materials undergo when submitted to temperatures more or less high. It is employed today only in a few pottery works.

Fusible Cones (Seger). — Utilize the unequal fusibility of earthenware blocks of varied composition. Give only discontinuous indications. Such blocks studied by Seger are spaced so as to have fusing points distant about 20° C. In general use in pottery works and in some similar industries.

Other Pyrometers (Hobson, Uhling-Steinbart, Job, Fournier). — There are a number of other pyrometers which have been found suitable in special cases or which for one reason or another have been found convenient in some particular line of work. Among these are the various industrial instruments based on the relative expansion of metals or of a metal and graphite used in air blasts and metal baths and pyrometers based on the flow or on the pressure of air or vapor.

TOTAL-RADIATION PYROMETERS. — These instruments utilize the radiant heat of *all* wave lengths given off by the body whose temperature is to be measured or by an auxiliary body at the same temperature. By means of a focusing device the radiant heat from a small portion of the hot surface is caused to fall upon a suitable detector. Various devices have been used as detectors, but in modern radiation pyrometers the thermocouple in conjunction with a galvanometer, millivoltmeter or potentiometer is almost universally employed. Instead of the thermocouple a spiral bi-metallic spring, with a pointer attached, is used in Féry's *spiral* pyrometer.

Total-radiation pyrometers can be made self-registering by simply substituting for the indicating galvanometer a suitable recording instrument. .

Conditions of Use. — To obtain accurate results with a radiation pyrometer it should be sighted upon the bottom of a closed-end tube inserted into the furnace or bath. The radiating properties of the bottom of such a tube approach very closely those of a "black body" (*see Heat and Thermal Properties*); i.e., the total energy radiated upon the receiving device is proportional to the difference in the fourth powers of the absolute temperature of the hot body and that of the receiving device. Since the latter temperature is usually low compared with the temperature of the hot body, the radiation is practically proportional to the fourth power of the absolute temperature of the hot body. Hence, if the relation between the absolute temperature T_0 and the deflection D_0 of the galvanometer attached to the thermocouple is known at one temperature, and the deflection is known to be proportional to the amount of heat falling on the thermocouple, then the absolute temperature T corresponding to any other deflection D is

$$T = T_0 \sqrt[4]{\frac{D}{D_0}}.$$

It should be noted that this law does not apply unless the instrument is focused upon a "black body." If the instrument is focused upon objects in the

open air its readings, if calibrated by the above law, will be too low, due to the selective radiating properties of all materials. However, the instrument may be calibrated by comparison with a standard pyrometer, e.g., a thermoelectric pyrometer, to give true surface temperatures when sighted upon any particular kind of surface, but this calibration will not hold for other surfaces.

Féry Radiation Pyrometer. — In this apparatus a concave mirror (gold on glass) is used to focus the rays upon the thermocouple. The gold mirror may be considerably tarnished without seriously influencing the readings; and if the aperture of the furnace sighted upon is of sufficient size and the telescope in focus, the temperature readings are practically independent of the distance. The instrument takes its final reading very promptly with only slight creep.

Foster has also transformed the Féry telescope into a "fixed-focus pyrometer" by putting the thermocouple and the aperture at the *conjugate foci* of the gold mirror. In a similar instrument recently issued by the Brown Pyrometer Company, the sighting of the instrument is facilitated by the use of a finder such as used with photographic cameras. The Féry pyrometer of constant-focus type has been coupled directly to a long closed-end tube by Whipple, so that the closed end may be plunged directly into the hot region or melted metal.

Thwing Radiation Pyrometer. — In Thwing's apparatus the reflecting mirror is replaced by a conical cone which by multiple reflection concentrates the radiation at its apex on one or more thermocouples in series with a portable galvanometer.

OPTICAL PYROMETERS. — These instruments utilize only the visible portion of the spectrum, and their indications depend on the comparison, by the eye, of the equality of brightness of two images, one of the object whose temperature is sought and the other a standard light source. Such instruments are therefore essentially the same as photometers (*see article on Photometers*). As a rule the comparison is made with approximately monochromatic light, the images being viewed through a colored glass, usually red.

Temperature and Intensity of Illumination.— Wien's Law.— Wien has shown that the intensity of the light given out by a "black body" (*see Heat and Thermal Properties*) may be expressed by the formula

$$I = A\epsilon^{-\frac{B}{T}},$$

where A and B are constants for a *given wave length*, ϵ is the base of the natural system of logarithms and T the *absolute temperature*. In general, however, the energy of a given wave length radiated by a body in the open air is less than that of a "black body," i.e., its emissive power is less than unity, and the emissive power varies with the temperature. The above law, however, is usually assumed to apply to such bodies, and although the temperature as thus obtained may differ from the true (gas thermometer) temperature by from 50 to 100° C., a *consistent* scale of temperature is obtained for any *particular* substance observed.

"Black Body" Temperature. — The temperatures indicated by a radiation pyrometer that has been calibrated against a black body, or on the assumption of the laws of "black body" radiation, are known as black-body temperatures. Thus, were a piece of iron and a piece of porcelain both at 1200°, the optical pyrometer, which used the red light emitted by these bodies, would give, as the temperature of these bodies, 1140° and 1100° C. respectively. This means that iron and porcelain at 1200° C. emit red light of the same intensity as is emitted by a black body at 1140° and 1100° C. respectively.

Féry Absorption Pyrometer (Fig. 1). — The optical system of this pyrometer is shown in the figure. pp' are a pair of absorbing-glass wedges, G is a

mirror with only a narrow central strip silvered over, L is the standard light source, focused on the mirror G by the lens l. The resultant field, when observing a small crucible, is then as shown at ab. r is a red glass in the eye-piece. The instrument has a fixed angular aperture, so that no correction has to be made for focusing or for varying distance from furnace. The range of the instrument may be extended by the use of auxiliary absorbing glasses A, A'. The instrument is movable about a horizontal axis, which is a convenience in sighting.

A setting is made by adjusting the thickness of the absorbing glasses pp' by moving the wedges together by means of a micrometer screw. Let x = setting of micrometer screw, T = absolute temperature, then, assuming Wien's law to apply

$$T = \frac{a}{x+b},$$

where a and b are two constants, for a given set of wedges pp', and can be obtained by observing two known temperatures.

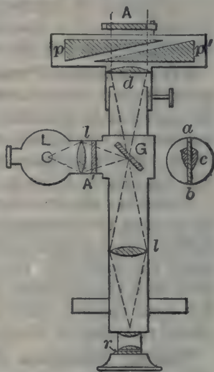

Fig. 1. Féry Absorption Pyrometer

Le Chatelier's Optical Pyrometer. — The Féry pyrometer is a modification of an earlier form devised by Le Chatelier. The latter used an iris diaphragm instead of the wedges, the setting of the instrument being accomplished by changing the aperture. With this arrangement greater sensibility can be obtained, but with the decided disadvantage that the calibration will then hold only for a fixed distance of the instrument from the object viewed.

Wanner Pyrometer (Fig. 2). — In this instrument a Nicol prism is used to vary the relative intensities of the light received from the standard lamp and from the object viewed, the angle through which the prism must be turned to give equal illumination being a measure of the relative intensity of the light emitted from the two sources. The principle is the same as that of a König spectrophotometer (*see Photometers*).

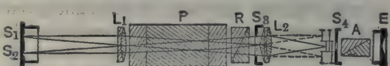

Fig. 2. Wanner Pyrometer

The slit S_1 is illuminated by light from the comparison source, a small 4-volt electric lamp not shown in the figure reaching S_1 after diffuse reflection from a right-angled prism placed before S_1. Light from the object whose temperature is sought enters the slit S_2. If the analyzer is at an angle of 45 degrees with the plane of polarization of each beam, and if the illumination of S_1 and S_2 is of the same brightness, the eye will see a single red circular field of uniform brightness. If one slit receives more light than the other, one-half of the field will brighten, and the two may be brought to equality again by turning the analyzer carrying a graduated scale, which may be calibrated in terms of temperature. Let ϕ = angle through which prism is turned to establish equal illumination, T = absolute temperature of furnace or bath; then, assuming Wien's law to apply,

$$T = \frac{a}{\log(\tan\phi) - b},$$

where a and b are constants determined by calibration. Only two known tem-

peratures need be observed to obtain these constants, but it is safer to plot a calibration curve from several known temperatures.

In the latest form of this instrument the details of its mechanical construction have been improved, and it has been made direct-reading by providing a second scale on the instrument graduated in temperatures, corresponding, of course, to a definite normal point and for a source approximating a black body.

Incandescent Lamp Pyrometers. — In this type of instrument the current through the filament of an incandescent lamp is adjusted until a portion of the filament is of the same color and brightness as the object. When this occurs this part of the filament becomes invisible against the bright background, and the current then becomes a measure of the temperature as given either by a thermocouple or in terms of the intensity of illumination. This principle appears to have been first used by Morse and independently developed by Holborn and Kurlbaum. An absolute match of both color and brightness cannot be made unless monochromatic light is used or unless the lamp filament and viewed object radiate similarly. This instrument is of the Morse type.

Leeds and Northrup Optical Pyrometer. — The principle and manner of using the instrument will be understood by reference to Fig. 3, in which L is a

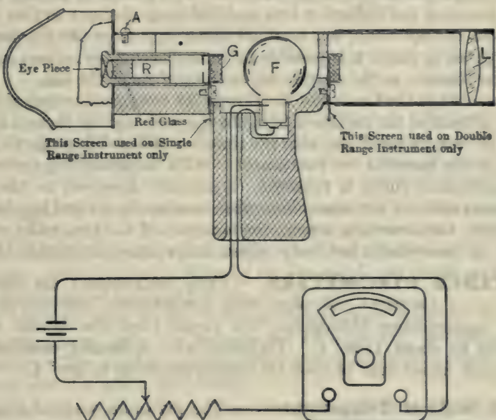

Fig. 3. Leeds and Northrup Optical Pyrometer.

lens, through which radiation from the body whose temperature is to be measured is brought to a focus at the point F. In the plane of the image produced by the lens is placed a tungsten lamp filament. The lamp filament receives current from a small battery contained in a portable case, also containing a rheostat and an accurate milliammeter. The incandescent filament and the image produced by the lens are observed through the eyepiece.

By means of the rheostat the current through the lamp is adjusted until the brightness of the filament is just equal to the brightness of the image produced by the lens L, whereupon the filament blends with or becomes indistinguishable in the background formed by the image of the hot object. This adjustment can be made with great accuracy and certainty, as the effect of radiation upon the eye varies some twenty times faster than does the temperature at 1600° F, and some fourteen times faster at 3400° F. When a balance has been obtained,

the observer notes the reading of the milliammeter. The temperature corresponding to the current is then read from a calibration curve supplied with the instrument.

The red glass R is in for all temperature measurements. An additional red glass G in Fig. 3 is put in when measuring higher temperature where the light emitted becomes dazzling. This also eliminates any question of matching colors or of the observer's ability to distinguish colors. It is further of value in dealing with bodies which do not radiate light in the same composition as that emitted by a black body, since the intensity of radiation of any one color from such bodies increases progressively in a definite manner as the temperature rises. The intensity of this one color can therefore be used as a measure of temperature for the body in question.

Measurement of Very High Temperatures. — In observing bodies at very high temperatures, i.e., above 2500° F., the light received through the lens of the telescope is too blinding for direct observation, even through the red glass of the eyepiece. The intensity of the image also becomes greater than that at which it is practicable to burn the filament.

Some method of reducing the intensity of the light from the hot body must therefore be employed at these temperatures. This is done by placing an absorbing screen between the objective lens and the lamp filament so that it reduces the light from the hot body, but does not affect that from the filament. With the proper screen in place, it is possible to make direct observations on the most brilliant light sources, as the electric arc or the surface of the sun.

The screen used in the Leeds and Northrup high-temperature optical pyrometer for cutting off part of the radiation from the hot body can be thrown into or out of the field of view by revolving with the thumb a milled disk projecting through an opening in the barrel of the instrument. With the absorbing screen in use, a different calibration curve is required. Inasmuch, however, as the range of the instrument without the absorbing screen overlaps by several hundred degrees the range with the absorbing screen, the accuracy of the two scales can always be checked by observing a hot body whose temperature lies within this range.

RESISTANCE PYROMETERS. — These instruments are based upon the variation of electrical resistance with temperature, and are suitable for measuring temperatures throughout the range from the lowest obtainable temperatures to about 1200° C. Platinum wire is usually employed as the resistor, though nickel may be used for temperatures up to 400° C.

Platinum Scale of Temperatures. — The platinum scale of temperatures is defined by the relation

$$p_t = \frac{R_t - R_0}{R_{100} - R_0} \times 100,$$

where R_0 and R_{100} are the resistances of the platinum wire at the temperature of ice and boiling water (760 millimeters mercury pressure) respectively, and R_t is the resistance at any other temperature t. For this scale to agree with the gas thermometer scale would require that the temperature coefficient of resistance of platinum (on the gas scale) be a constant equal to

$$c = \frac{R_{100} - R_0}{100 R_0}.$$

As a matter of fact the temperature coefficient is not constant but the relation between temperature (on the gas scale) and resistance is

$$R_t = R_0 (1 + at - bt^2)$$

where a and b are constants. Consequently the platinum temperature and the gas temperature differ by an amount

$$t - p_t = \delta\left(\frac{t}{100} - 1\right)\frac{t}{100},$$

where

$$\delta = \frac{10{,}000\, b}{a - 100\, b}.$$

The fundamental constants of a platinum thermometer are then R_0, c, and δ. These can be determined by measuring the resistance at three known temperatures, viz., melting point of ice, boiling point of water and boiling point of sulphur (444.7° C. on the gas scale). For the purest platinum $\delta = 1.50$ and $c = 0.0039$. The value of δ increases with the amount of impurity.

CORRECTIONS TO t FOR SMALL CHANGES IN δ

Centigrade scale				Centigrade scale			
	Δt for $\Delta\delta = 0.01$		Δt for $\Delta\delta = 0.01$		Δt for $\Delta\delta = 0.01$		Δt for $\Delta\delta = 0.01$
50	−0.002	300	+0.060	550	+0.247	800	+0.560
100	.000	350	.087	600	.300	850	.637
150	+ .008	400	.120	650	.357	900	.720
200	.020	450	.157	700	.420	950	.807
250	.037	500	.200	750	.487	1000	.900

Computations of t from p_t are made by the table on p. 1238, as if the thermometer had $\delta = 1.50$. The above corrections (Δt) are then applied to the computed values of t for the value of δ proper to the thermometer.

Example. Let $p_t = 470.00$, whence $t = 500.00°$ C, by table on p. 1238. If $\delta = 1.52$, the corrected value of t is 500.40° C.

For a full discussion of the platinum temperature scale and its relation to the gas-thermometer scale, see Burgess and Le Chatelier, *Measurement of High Temperatures*. The brief discussion given above is adapted from this treatise, from which also the tables are taken.

Forms of Resistance Thermometers and Pyrometers. — A complete outfit consists of the thermometer proper or "bulb," the indicating or recording instrument and two or three dry cells for operating the same. Where the indicating or recording instrument is to be in circuit continuously, it is best to operate it from a direct-current lighting circuit or storage-battery system.

With the proper switching device a number of bulbs at widely-separated points may be used with the same indicator or recorder.

Thermometer Proper or Bulb. — Since practically every problem in temperature measurement presents different conditions there can be no general type or form of resistance thermometers. The following descriptions are of some of the thermometers now on the market that have proved satisfactory for certain classes of work.

Resistance thermometers for use below 125° C. are of relatively simple construction, for in this case silk-insulated nickel wire may be used. Such a thermometer is shown in section in Fig. 4. This particular thermometer has an over-all length D of 10¼ inches, a length of winding C of 4½ inches and a

VALUES OF TEMPERATURE CENTIGRADE (t) IN TERMS OF PLATINUM TEMPERATURES (p_t) FOR THERMOMETERS WITH $\delta = 1.500$

p_t	t	Difference for 1° p_t	p_t	t	Difference for 1° p_t	p_t	t	Difference for 1° p_t	p_t	t	Difference for 1° p_t
0	0.000	0.985	250	255.99	1.066	500	534.89	1.170	750	844.26	1.313
10	9.867	0.988	260	266.67	1.070	510	546.62	1.175	760	857.42	1.319
20	19.762	0.991	270	277.38	1.073	520	558.40	1.180	770	870.65	1.326
30	29.687	0.994	280	288.13	1.077	530	570.22	1.185	780	883.95	1.333
40	39.641	0.997	290	298.92	1.081	540	582.10	1.190	790	897.32	1.340
50	49.625	1.000	300	309.75	1.084	550	594.03	1.195	800	910.76	1.347
60	59.639	1.003	310	320.61	1.088	560	606.00	1.200	810	924.28	1.355
70	69.683	1.006	320	331.51	1.092	570	618.03	1.205	820	937.87	1.363
80	79.758	1.009	330	342.46	1.096	580	630.11	1.210	830	951.54	1.370
90	89.863	1.012	340	353.44	1.100	590	642.24	1.216	840	965.28	1.378
100	100.00	1.015	350	364.46	1.104	600	654.43	1.222	850	979.10	1.386
110	110.17	1.018	360	375.52	1.108	610	666.67	1.227	860	993.01	1.394
120	120.37	1.021	370	386.62	1.112	620	678.97	1.232	870	1007.00	1.403
130	130.60	1.024	380	397.76	1.116	630	691.32	1.238	880	1021.07	1.411
140	140.86	1.027	390	408.95	1.120	640	703.73	1.244	890	1035.23	1.420
150	151.16	1.031	400	420.18	1.125	650	716.20	1.250	900	1049.47	1.428
160	161.49	1.034	410	431.45	1.129	660	728.73	1.256	910	1063.80	1.437
170	171.85	1.038	420	442.77	1.134	670	741.32	1.261	920	1078.21	1.445
180	182.25	1.041	430	454.13	1.138	680	753.97	1.267	930	1092.71	1.455
190	192.68	1.044	440	465.53	1.142	690	766.67	1.274	940	1107.31	1.464
200	203.14	1.048	450	476.97	1.146	700	779.44	1.280	950	1122.00	1.474
210	213.64	1.052	460	488.46	1.151	710	792.27	1.286	960	1136.79	1.484
220	224.18	1.055	470	500.00	1.156	720	805.17	1.293	970	1151.69	1.494
230	234.75	1.058	480	511.58	1.160	730	818.13	1.299	980	1166.68	1.503
240	245.35	1.062	490	523.21	1.165	740	831.16	1.306	990	1181.76	1.513
250	255.99	1.066	500	534.89	1.170	750	844.26	1.313	1000	1196.95	1.524

diameter A of 3/8 inch. The same type of thermometer is also made with a number of different lengths from 3¾ inches over-all to 12¼ inches over-all, and one form, designed to be particularly quick acting, is encased in a thin steel tube ¼ inch outside diameter. This type of thermometer is largely used for the measurement of the temperature of air, solids, concrete, water, and grains and fruit in transit. It may also be used as a wet-bulb thermometer.

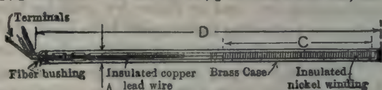

Fig. 4. Resistance Thermometer for Use up to 125° C.

The thermometer shown in Fig. 5 is a platinum resistance bulb for measurement of the highest precision. The winding is of pure platinum wound on a mica cross; the protecting tube is of royal Berlin porcelain. For shop use it is well to protect the porcelain tube with a seamless nickel case. Such a case will maintain its mechanical strength after long exposures to high temperatures. Thermometers of this type may be used up to 1000° C. This type of ther-

mometer is also provided with a special fireproof head and with a steel protecting case in addition to the nickel case. This is necessary where the thermometer is to be used in connection with case hardening.

The Leeds and Northrup Company make a bulb designed especially for measuring the temperature of generators, transformers, etc. The resistance winding of this bulb is about 3 inches long and is mounted on a flexible stem 30 inches or more in length, the width over-all is ½ inch and the thickness 1/25 inch. This resistance coil is wound non-inductively and is therefore unaffected by neighboring magnetic fields.

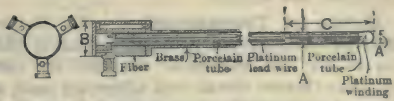

Fig. 5. Thermometer for Use up to 1000° C.

Indicating Devices. — The application of the various bridge, potentiometer, differential galvanometer and ohmmeter methods (q.v.) to the measurement of the resistance of the platinum coil and the methods of compensating for the resistance of the leads are described in detail in Burgess and Le Chatelier's *Measurement of High Temperatures*. Two types of indicators are employed: (1) the balance type which requires the adjustment of a rheostat to produce zero deflection of the indicating instrument, the temperature then being read in terms of the balancing resistance, current or p.d., and (2) the direct-reading type in which the deflection of the instrument, read on a properly calibrated scale, gives the temperature directly.

THERMOELECTRIC PYROMETERS. — These instruments consist of a thermocouple and some device for measuring the e.m.f. generated due to the difference in temperature between the hot and cold junctions of the thermocouple, for which the relation between e.m.f. and temperature difference is known. The thermocouple consists of two parallel wires of different materials joined together at one end but insulated from each other throughout their length. The two ends which are joined form the hot junction and the two open ends are usually referred to as the cold junction, although they are not connected directly to each other. The two cold ends are, however, connected electrically through the device used for measuring the e.m.f. generated, thus forming a closed circuit.

Thermocouple Circuits. — The net e.m.f. tending to establish a current in such a circuit as described above is the algebraic sum of all the contact e.m.f.'s (*see Electricity and Magnetism, Principles of*) in the circuit. Wherever there is a change in the chemical nature or physical properties of the conductors forming such a circuit there exists such a contact e.m.f. These various contact e.m.f.'s are functions of the temperature of the transition points, but if the entire circuit is at the same temperature throughout, the net e.m.f. is zero (provided the circuit is formed solely of *metallic* conductors). If, however, *any* transition point is at a higher temperature than another, the net e.m.f. will in general not be zero. Consequently, when a thermocouple is used to measure the difference in temperature between the hot and cold junctions of the couple itself, care must be taken to eliminate the e.m.f.'s due to the temperature differences between the various other transition points in the circuit.

Transition points occur (1) at the junction of any two dissimilar metals; (2) wherever there is a change in the homogeneity of the structure of any of the metals in the circuit, and (3) wherever there is a change in the temperature of the metal, although it may be perfectly homogeneous. To avoid errors due to the e.m.f.'s at the transition points other than the hot and cold junctions of the couple itself, it is necessary to use for the latter metals which give a relatively *high* e.m.f. for a given temperature difference between the two junctions and

which are also *homogeneous* throughout, and to so dispose the rest of the circuit that the e.m.f.'s produced therein as the result of temperature differences neutralize one another.

Metals for Thermocouples. — For high-precision thermoelectric pyrometers platinum against platinum-iridium or platinum-rhodium alloys (containing 10 per cent iridium or rhodium) are usually employed. For industrial purposes cheaper metals and alloys are used, such as nickel, copper, iron and alloys of these metals; such couples are usually referred to as *base* metal couples, as contrasted with the *noble* metal couples formed of platinum, iridium, etc.

Thermocouples for Industrial Use. — For industrial use cheap and robust couples are required. Such couples may be made either entirely of the base metals or a base-metal couple may be used in series with a short length of a platinum-alloy couple, the latter only being exposed to the highest temperature. In Fig. 7 is shown one form of compound couple.

Fig. 6. Base Metal Couple

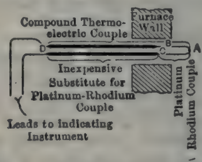

Fig. 7. Compound Couple

In the pipe type couple (Fig. 6) the outside iron pipe is one element of the couple and an alloy wire such as constantan is carried up through the center of this pipe insulated from it throughout the entire length except at the end where it is welded to the pipe forming the hot junction of the couple.

The double wire type couples are usually made of No. 8 wires for temperatures up to 1800° F., and smaller sizes of wires for lower temperatures. The ends are twisted and welded together and porcelain insulators are slipped over the wires. These couples are protected by either a heavy iron pipe or a nichrome pipe. The life of the latter at temperatures above 1800° F. is much greater than that of the iron pipe.

E.M.F.'s of Thermocouples. — By the e.m.f. of a thermocouple is meant the e.m.f. developed in a circuit consisting solely of the metals forming the thermocouple itself; i.e., the e.m.f.'s in the external circuit are not considered. If E_{sa} is the e.m.f. of a couple made of a metal a against any standard metal s, and E_{sb} is the e.m.f. of a couple made of a metal b against this same standard metal s, the e.m.f.'s in each case being for the same temperature of the cold junction (standard temperature is 0° C.) and for the same difference in temperature between the hot and cold junctions, then the e.m.f. of a couple made of a and b is, for these same temperatures

$$E_{ba} = E_{bs} + E_{sa} = E_{bs} - E_{as}.$$

(In this notation *rises* of potential are considered to be in the direction of the subscripts, that is, E_{ba} is the rise of potential from b to a). Consequently the insertion of a metal wire of any material in the circuit of a thermocouple does not change the net e.m.f., *provided this wire is of uniform structure and temperature throughout.* Consequently the hot junction may be soldered with any other metal which will not melt at the temperature to which the junction is exposed.

Electromotive Force Formulas. — No simple relation exists between the e.m.f. (E) of a thermocouple and the temperature difference (t) between

the hot and cold junctions. A number of approximate formulas, however, has been suggested. The following, due to Holman, is one of the simplest.

$$E = mt^n,$$

where m and n are constants, the cold junction being kept at 0° C. For convenience in plotting and calculation this may be written

$$\log E = n \log t + \log m.$$

Holman's formula when applied to couples made of platinum and its alloys of iridium and rhodium gives results accurate to about 2° C. for t between 200° C. and 1200° C. Typical values of n and log m are given below (Cambridge Scientific Instrument Company).

	n	log m
Pt against (Pt 90%, Ir 10%)............	1.102	0.895
Pt against (Pt 90%, Rh 10%)..........	1.189	0.526

The values of these constants, however, differ with different makes of couples, and in any particular case should be obtained from the maker as determined by calibration.

Another formula, due to Holborn and Day, is the following,

$$E = -a + bt + ct^2,$$

where a, b and c are constants. Between 300° C. and 1100° C. this formula gives results accurate to 1° C. when applied to couples made of platinum and the various platinum alloys.

Ideal wire (*see Wires, Resistance*) and copper form a very satisfactory couple of relatively large e.m.f. suitable for a range of temperature between 0° and 400° C. With the cold junction at 0° C. the temperature of the hot junction and the e.m.f. are related as follows:

Temp. Hot Junct., °C.........	50	100	150	200	250	300	350
E.m.f., microvolts...	2.3	4.65	7.05	9.45	11.90	14.43	17.08

Thermoelectric Power (H) of Thermocouples. — The thermoelectric power of a thermocouple is defined as the increase in the e.m.f. developed per degree increase in the difference in temperature between the hot and cold junctions. This quantity, which is equal to $b + 2ct$ in Holborn and Day's formula, depends in general upon the temperature (t) of the hot junction; the values given in the table on page 1242 are close approximations within the range stated.

Measurement of E.M.F. of Thermocouples. — Two methods may be used to measure the electromotive force of a couple: the potentiometer method and the galvanometric method (*see Potentiometers; Galvanometers*). The first is the more accurate and is usually made use of in laboratories. The second method is simpler, but possesses the inconvenience of giving only indirectly the measure of the electromotive force by means of a measurement of current strength.

There are sources of error, however, inherent in the galvanometric method, such as effects of lead resistance and temperature coefficients of leads and galvanometer, which are difficult if not impossible of complete elimination even with the best apparatus available. The potentiometer method, on the other hand, may be made, in so far as the measurements of e.m.f. are concerned, as

THERMOELECTRIC POWERS OF THERMOCOUPLES
Microvolts per °C. increase in temperature, t in °C.

Thermocouple	Thermoelectric power	Temperature range, ° C.	Authority
Pt and 90% Pt, 10% Rh	$4.3+0.0088\,t$	0–1300	Le Chatelier
Pt and 90% Pt, 10% Ir	$11.3+0.0104\,t$	0–1000	Le Chatelier
Pt and Ni	$7.8+0.01325\,t$	300–1300	Burgess
Cu and Ni	$24.4+0.016\,t$	0–235	Pécheux
Cu and Constantan *	$42.3+0.058\,t$	0–320	Pécheux
Pt and Fe (forged)	$2.5+0.0210\,t$	700–1000	Le Chatelier

* 60% Cu, 40% Ni.

exact as may be desired, or so that the only outstanding uncertainties are inherent in the thermocouple itself. These uncertainties, such as inhomogeneity, conduction along the wires, variable zero and actual change of e.m.f., are sometimes overlooked, giving rise to illusory accuracy.

The application of these two methods to the measurement of the e.m.f.'s of thermocouples is described in detail in Burgess and Le Chatelier's *Measurement of High Temperatures*.

Cold-junction Correction.— When the galvanometer method is used, it is often not convenient to keep the junctions of the couple to the lead wires of the galvanometer at a definite temperature, although the galvanometer itself may be so far removed from the furnace that its temperature changes are slight. Except in the roughest kind of work, allowance has to be made for the cold-junction temperatures, which may be measured by an auxiliary thermometer.

Calling t_0 the cold-junction temperature for which the instrument reads correctly, t the observed temperature of the cold junction, the correction to apply to the observed temperature readings of the galvanometer, otherwise supposed to read correctly for a given thermocouple, usually lies between $\frac{1}{3}(t-t_0)$ and $(t-t_0)$, depending on the type of couple and the temperatures of both hot and cold junctions. This question has been treated in detail for several types of couple by C. Offenhaus and E. H. Fischer (*Electrochem. and Met. Ind., 1908, Vol. 6, p. 362*).

Elimination of Cold-junction Correction. — Bristol has also devised an automatic compensator for cold-end temperatures, shown in Fig. 8, consisting of a small glass bulb and capillary tube partially filled with mercury, into which a short loop of fine platinum wire dips. This is inserted in the thermoelectric circuit close to the cold junction. Changes in temperature cause the mercury to expand or contract, cutting in or out resistance in the circuit. This acts in opposition to the change in e.m.f. with temperature at the cold end, so that a balance may be established if the parts are properly designed.

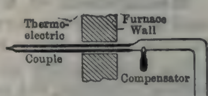

Fig. 8. Bristol's Compensator

In the Thwing instruments the elimination of the temperature variations of the cold ends of the couple, where they can be brought close to the galvanometer, is affected by a device consisting of a compound strip of two metals having unequal coefficients of expansion, so attached to the spring controlling the

pointer that the reading of the galvanometer when no current is flowing is the temperature of the surroundings.

Fig. 9 shows a simplified diagram of connections of the Leeds and Northrup split-circuit potentiometer system with automatic cold-junction compensator. The current from the battery *Ba* divides at *a* and *b*, one-half passing through the upper branch which includes the slide wire *S* and resistances *G* and *B*. The lower branch includes the nickel coil *D* and the resistance *C*. All the resistances, except the nickel coil *D*, are made of manganin wire having a zero temperature coefficient. The current in the two branches is kept constant by adjusting the rheostat *R* until the drop of potential across the coil *C* is equal to the e.m.f. of the standard cell *W*. The galvanometer *Ga* shows when a balance has been obtained. In the recording instrument, this adjustment is made automatically. The resistances *B* and *C* are high and are so chosen that the resistance in the two branches are equal.

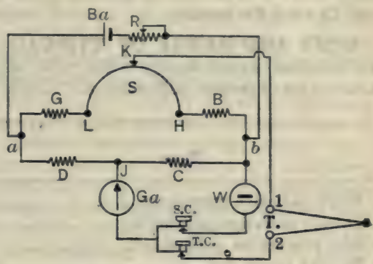

FIG. 9.

The nickel coil, *D*, is located near the cold junction of the thermocouple and has such a value that changes in cold junction temperature are compensated for by changes in the drop of potential across this coil caused by changes in its resistance with temperature.

The value of the resistance of the compensating coil *D* is calculated as follows: Let R = resistance of compensating coil *D* at reference temperature; c = change in e.m.f. of thermocouple per degree change in temperature of cold junction; K = temperature coefficient of nickel composing compensating coil per ohm per degree; i = current in branch including nickel coil; e = change in e.m.f. of thermocouple due to temperature change t of cold junction; e_1 = change in fall of potential across nickel coil due to temperature change t of cold junction. In order to have compensation e must equal e_1, regardless of changes in temperature of the cold junction, which condition is satisfied when

$$R = \frac{c}{Ki}.$$

The value of the resistance *G* is so chosen that the scale starts at 0° or at any other desired temperature. For example, suppose the temperature range is from 0 to 1000° C. The coil *G* will be equal to the nickel coil *D* at 0° *C*. Suppose both the hot junction and the cold junction are at 0° *C*. The e.m.f. of the thermocouple will be zero and the difference of potential between *K* and *J* will also be zero because the resistance of *D* equals that of *G*. If the temperature of the hot junction increases to 1000° C. and the cold junction stays at 0° C., the galvanometer will show a balance when the contact *K* is at the extreme right side of the scale (side marked *H*). Now suppose the temperature of the cold junction increases. The e.m.f. of the couple will decrease but the resistance of the nickel coil increases and causes a decrease in the e.m.f. across the points *J* and *K* that exactly compensates for the decrease in the e.m.f. of the thermocouple. The result is that the contact *K* remains at 1000° on the scale, which is the temperature at the hot junction.

CALIBRATION OF PYROMETERS. — The methods employed are described in detail in Burgess and Le Chatelier's *High Temperature Measurements*. In general, the reading of the pyrometer is observed when the fire end is at the melting or boiling point of some solid or liquid. The melting and boiling points which possess the greatest reliability are given in the article on *Heat and Thermal Properties*.

COST AND RANGE OF TYPICAL PYROMETERS. — These items are indicated *approximately* in the table below. The precision stated is on the assumption of proper calibration and careful use.

Type	Range ° C.	Cost,* dollars
Ferry radiation pyrometer	800 and up	420–564
Thwing radiation pyrometer	800 and up	160–600
Fery absorption pyrometer	900 and up	130–150
Wanner pyrometer	900 and up	225–300
Leeds & Northrup optical pyrometer	650 and up	110–175
Resistance thermometer bulb	Up to 125	10–50
Resistance thermometer bulb	Up to 1000	40–80
Indicator for resistance thermometer	Optional	80–150
Recorder for resistance thermometer	Optional	150–700
Platinum Pt. Rh. Couples	Up to 1600	10 and up
Base metal couples	Up to 1200	5–20
Milli voltmeter indicator	Optional	50–200
Milli voltmeter recorder	Optional	150–700
Potentiometer indicator	Optional	120–200
Potentiometer recorder	Optional	200–700

* As of 1920.

BIBLIOGRAPHY. — Burgess and Le Chatelier's *Measurement of High Temperatures*, N. Y., 1912. In this treatise will also be found a complete bibliography of the entire subject of pyrometry. See also *Symposium on Pyrometry*, Am. Inst. Min. and Met. Eng., 1920, and *Pyrometer Practice*, Bur. Stands. Tech. Paper No. 170.

RADIO COMMUNICATION. — (*See also Alternating Currents; Capacity and Charging Currents; Electricity and Magnetism, Principles of; Electron Theory; Inductance and Inductive Reactance; Skin Effect; Transient Electric Phenomena.*)

RADIATION. — Whenever current flows in an ordinary electric circuit magnetic and electric fields are set up in and around the circuit; the magnetic field is proportional to the current flowing and the electric field is proportional to the difference in electric potential in various parts of the circuit.

These two fields represent a certain amount of energy stored in the space where they exist. The magnetic energy depends upon the self-inductance of the circuit and the current flowing, whereas the electric energy depends upon the capacity (electrostatic) of the parts of the circuit, considered as a condenser, and the potential difference. When the current or potential difference changes the energies change and, if the current and potential difference become zero, the greater part of the magnetic and electric energies are returned to the circuit.

When the current in the circuit is a low-frequency alternating current, practically all the energy which goes out into space is returned when the current and potential become zero. When the frequency is high (and strictly in every case irrespective of the frequency), part of the energy represented by the two fields, is sent out into space and never returns to the circuit. This energy is said to be radiated; as the radiated power varies (for a given circuit) as the square of the frequency, this action may be neglected in circuits where the frequency is less than several thousand; in radio circuits the frequency ranges from 20,000 to 1,000,000 or more, and at these high frequencies the radiated power may constitute the principal loss in the circuit.

ELECTROMAGNETIC WAVES. — The radiated energy travels in the form of electromagnetic waves; such a wave has half of its energy magnetic and half electric, and travels with the velocity of light. If the intensity of the magnetic field is H and that of the electric field is F (these two intensities being so related that magnetic and electric energies per unit volume are equal) the electromagnetic wave becomes self-sustaining by virtue of the fact that an electric field of intensity E travelling with the velocity of light generates at right angles to itself, and at right angles to the direction of wave motion, a magnetic field of intensity H; or conversely field H, travelling with the velocity of light sets up, at right angles to itself and the direction of motion, an electric field F.

The arrangement of electric and magnetic fields around a radiating antenna is indicated in Fig. 1, the magnetic field being shown by dots and the electric

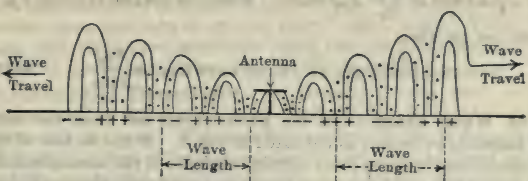

Fig. 1.

field by lines; this arrangement of fields is practically circular around the antenna. The "feet" of the waves slide over the earth's surface (this being considered a conductor) so that in the earth's surface there are around the antenna circular distributions of electric charges, alternately positive and nega-

tive, which bands of charge spread out from the antenna with the velocity of light.

Wave Length. — The waves of radiation travel out from the antenna at fixed velocity, so that the distance between the " feet " remains constant; the distance from a band of positive charge in the earth's surface to the next positive band is termed the wave length. In radio practice the wave lengths emitted vary from perhaps 50 meters to as much as 20,000 meters. The longer waves are used in the large stations engaged in transoceanic communication while the shorter may be used in aeroplane radio sets or in small portable field outfits. The standard wave length used for inter-ship communication is 600 meters; occasionally 300 and 450 meters are used in this work.

The wave length and frequency in any wave propagation are related by the equation $f\lambda = V$, in which f is the frequency, λ is the wave length, and V is the velocity of propagation. For any electromagnetic disturbance, whether light, heat, X rays or radio waves, the velocity of travel is 3×10^8 meters per second. Hence the wave length, in meters, is given by

$$\lambda = \frac{300{,}000{,}000}{f} \quad \text{meters.}$$

The frequency of currents used in radio antennæ lie between 15,000 for the very large stations and 2,000,000 for small portable sets. Ship sets nearly always use a frequency of 500,000 cycles per second.

Energy in Electromagnetic Wave.— If H is the intensity of the magnetic field in gilberts per cm. and F is the intensity of electric field in e.s. units (statvolts per cm.) at any point in the wave front, the energy per cu. cm., is

$$\frac{H^2}{8\pi} + \frac{F^2}{8\pi} = \frac{H^2}{4\pi} = \frac{F^2}{4\pi},$$

the two energies being equal.

As the two energies have a sinusoidal distribution in the wave disturbance, the average value of the energy density will be one-half the maximum density; if therefore F_m is the maximum electric gradiant in the wave front the average energy per cu. cm. is $\frac{F_m{}^2}{8\pi}$ ergs. The energy passing through one sq. cm. per second is therefore equal to $\frac{F_m{}^2}{8\pi} \times 3 \times 10^{10}$ ergs, or the radiated power associated with one sq. cm. of wave front is $\frac{3000}{8\pi} F_m{}^2$ watts. When the electric gradient F_m is measured in *volts per cm.* then

$$\text{Radiated power per sq. cm. of wave front} = \frac{F_m{}^2}{240\pi} \quad \text{watts.}$$

Form of Wave Front. — As the wave travels out from the antenna it gradually increases in height, as indicated in Fig. 1. If the surface of the earth is a good conductor the wave front remains upright; if the atmosphere is highly absorbing (ionization, etc.) the wave front tips backwards, whereas if the atmosphere is non-absorbing and the earth's surface is a poor conductor the wave front tips forwards.

It seems likely that the waves do not extend very high; in the upper rarified atmosphere the ionization is intense, so that the air becomes a fair conductor. Such a conducting layer of air will act as an upper boundary beyond which the waves are able to penetrate with difficulty. Radio communication may there-

fore be regarded as carried on by wave propagation between the earth's surface and this upper layer of conducting air.

Attenuation. — As the electromagnetic wave travels over the earth's surface, the intensity of both electric and magnetic fields continually diminish, due to two factors; the wave front becomes larger in area with increasing distance from the transmitting antenna, thus decreasing the energy density in the wave front, and there is also some energy absorbed in the atomosphere and the earth's surface as the wave progresses. Measurements of the attenuation are difficult to make, and experimental results on this effect are meagre. Duddell and Taylor, in 1905, reported the results of tests over comparatively short distances, and L. W. Austin has reported more extensive tests carried out in 1909. Duddell's results indicated that the field strength in the wave front varied inversely with the distance from the sending station, whereas Austin's experiments showed the field strength falling off more rapidly than this. He deduced from his results a formula which is now regarded as the best obtainable. It is

$$I_r = \frac{A}{d} \epsilon^{-\frac{ad}{\sqrt{\lambda}}},$$

in which I_r = current set up in the receiving antenna; d = distance between transmitting and receiving stations, in km., λ = wave length in km.; a = a constant, ordinarily equal to 0.0015; A = a constant, depending upon heights of the two antennæ, current in the transmitting antenna, etc.

The absorption constant, a, depends upon the amount of ionization present in the air, and condition of the surface over which the wave has travelled, etc. It is least for transmission over sea at night, and maximum in daytime over mountainous country thickly covered with vegetation.

Reflection and Refraction. — Radio waves reflect and refract in the same way as do light waves, allowance being made for the vast difference in wave length; to these two effects many vagaries in radio transmission are undoubtedly due. Thus two ships on opposite sides of a mountainous island are not able to communicate at all, yet if each steams away from the island, thus increasing the distance between them, the transmission becomes normal; such effects are noted by vessels off the shores of Cuba. This effect is probably due to peculiar reflections of the waves by the semi-conducting minerals in the hills over which the waves must travel in crossing the island.

In other cases a very small amount of power may suffice to establish communication over vast distances; it seems quite likely that the upper layers of partly ionized atmosphere concentrate the energy of the electromagnetic waves just as mirrors and lenses concentrate light waves. Of course such effects cannot be relied upon, but they do give at times " silent zones," where the signal disappears, and other zones where the signal may be much more intense than the transmitter power and distance would seem to justify.

Daily Variation in Transmission. — There is a very great difference in radio transmission at different times of day, especially if the line of communication runs east and west. This was first noticed at the Marconi stations at Glace Bay and Clifden. The transmission follows a curve about as shown in Fig. 2; when the line of sunset or sunrise was between the two stations the transmission was very poor. It seemed that the line of sunrise or sunset acted as a barrier across which the waves could not easily pass. This effect was more marked on short waves than on long ones, in fact transmission is always more uncertain on short waves than on long ones. Some exceptions to this have, however, been noted, such as those reported by Taylor and Blatterman, who found 500 meters better than 1500 meters in the night time, but not in the daytime.

The variation in signal strength at a receiving station may occur very suddenly, thus at Chelmsford the received current from Clifden would double in 5 minutes. Other instances are on record of even more rapid changes in signal strength, the transmitting station sending out a constant amount of power. Such effects are often spoken of as the " fading " of the signal.

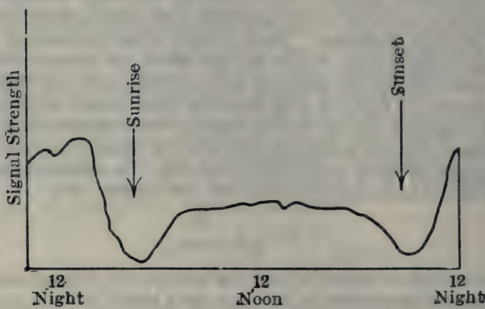

Fig. 2.

Seasonal Variation. — L. W. Austin has reported the results of a long series of tests of transmission over distances of about 100 miles, which show that there is a very large variation in the transmission throughout the year. The transmission was over land, with a wave length of 1000 meters. The average signal in the winter showed about 6 times as much energy as the average signal in summer.

Transmission through Water and Earth. — Calculation of the attenuation of an electromagnetic wave penetrating water would lead to the belief that the penetration can occur only with great loss, and such proves to be the case. The reception of radio signals by a submerged submarine is only possible if long waves are used, and suitable amplifiers are used with the receiving set. It is possible, however (with a good amplifier), for a submarine submerged 30 to 40 feet to detect signals from a long-wave, high-powered station thousands of miles away. Thus submarines on the Atlantic coast of the U. S. are able to pick up signals from Nauen, even though submerged.

Experiments have been reported of picking up radio signals with all the receiving apparatus at the bottom of a mine shaft, indicating that the waves penetrate a considerable distance into the earth's surface; not much reliable information on the subject is at hand.

ANTENNÆ. — The original experiments in radio transmission, by Hertz, used ungrounded sending and receiving systems; this is seldom the case in modern radio sets. Practically the only case of successful ungrounded transmitting antennæ is that of air craft. Attached to the upper plane is a network of wires and through the floor of the cockpit of the plane is suspended a wire generally having a weight on the end. This trailing wire and the network on the wing form the two parts of the antenna. Such antenna systems permit communication between two aeroplanes several miles apart, or between a plane and ground station perhaps 50 miles apart. As mentioned below, an ungrounded coil is often used for a receiving station, but practically never as a transmitting antenna.

Grounded Antennæ. — In Marconi's first successful experiments a single vertical wire was used for antenna, the lower end being connected to ground through the spark gap, as shown in Fig. 3. The high voltage secondary of a spark coil was connected across the gap as indicated by the dotted line. When the coil was operating the antenna was charged to a high voltage by the opening of the interrupter in the primary circuit, thus storing energy in the electric field of the condenser formed by the wire M and the earth. When the voltage on M rose to a sufficient value, the spark gap broke down, and an oscillatory current was set up in the antenna, the frequency of this current being controlled by the height of the wire M. Such a form of antenna radiates very rapidly, so that the oscillatory current for each discharge of the spark gap is of very short duration, perhaps 5 or 10 cycles.

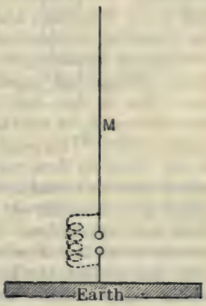

Fig. 3.

Forms of Voltage and Current Distribution. — When a charged antenna is left free to oscillate, the charge on the antenna so flows and rearranges itself that the intensities of potential and current at various parts of the wire follow approximately a sine law distribution. It is evident that in any case the potential of the antenna (with respect to earth) must be zero at the ground end and that the current in the antenna must be zero at the top, or end, of the antenna.

Typical forms of distribution are shown in Fig. 4. At (*a*) are shown the current and potential curves for a simple vertical wire, similar to the original Marconi antenna; such a condition is called ***quarter wave length distribution*** because one-quarter of a complete wave is developed for each of the curves. In (*b*) and (*c*) of Fig. 4 are shown the current and voltage forms when a series inductance has been introduced in the base of the antenna; (*b*) shows the effect for a small inductance and (*c*) that for a much larger one. Sometimes it is necessary to use a condenser in the base of the antenna, for which case the current and potential distributions are somewhat as shown in (*d*) Fig. 4. The node of voltage here shown is not a true node; the phase of the voltage does reverse on the two sides of the point, but the magnitude of the voltage does not go through zero.

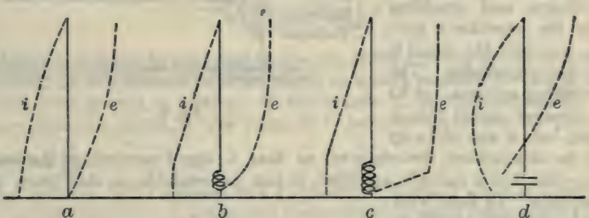

Fig. 4.

Forms of Antennæ. — There are three ordinary forms of transmitting antennæ, as illustrated in Figs. 5, 6, and 7. Fig. 5 represents a T antenna which is the type generally used for ship sets, because of the ease of erection from the two ship's masts. It is practically non-directive, sending about the same amount of radiated power in all directions.

The form of antenna shown in Fig. 6, called the inverted L, is used for stations

designed to communicate with a fixed receiving station; the large Marconi stations for transoceanic communication are of this type. In the largest stations these antenna may be as much as 500 feet high, 1000 feet wide, and more than a mile long.

High powered stations designed to communicate with stations in various directions, use an umbrella shaped antenna, Fig. 7. This type radiates equally well in all directions, and has the possible mechanical advantage of requiring but one mast. These tall masts are from 600 to 800 feet high, generally made of steel lattice construction. This mast may be in three or four sections jointed flexibly to one another, radial guys running out from these junction points. It is practice to insulate these masts from the ground, the base of the mast being supported on a marble ball, or similar construction. The guys of these high power stations are generally broken up into insulated sections, to keep the induced currents in them low.

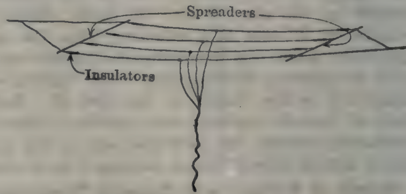

Fig. 5.

Receiving Antennæ. — A few years ago it was customary to use a large antenna for receiving, but to-day such is seldom the case. The present limit on radio reception is the amount of atmospheric disturbance present, this producing a hissing and crackling noise in the operator's telephones, so that the signal is blurred and unreadable. The large receiving antenna "picks up" a stronger signal than a small one, but it also picks up more of the atmospheric disturbances, so that a signal received on the small antenna, after being suitably amplified, is just as readable as that from the large antenna. In fact, with a properly designed amplifier, the signal is amplified more than the disturbing noises, so that the smaller antenna is actually more serviceable in receiving than the large one.

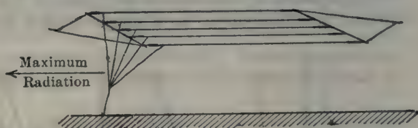

Fig. 6.

Coil Antenna. — The coil antenna is a special form of antenna which is never used for transmitting but which is very useful for receiving. It consists of a few turns (sometimes constructed as shown in Fig. 8) and the receiving apparatus is connected to the terminals a and b, there being no ground connection at all to the receiving circuit. Such a coil must be connected to a powerful amplifier if the

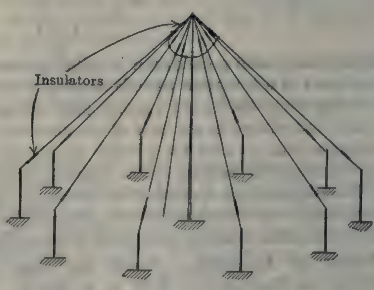

Fig. 7.

signals to be received are from a distant station, as the coil picks up but little energy from the radio wave. The size of coil, as well as the number and spacing of turns, depends upon the wave length to be received; for 600 meters a coil 4 feet square of 12 turns spaced ⅜ inch from each other is suitable, whereas for 10,000 meters a coil 20 feet square of 50 turns spaced 1½ inches is about right. The spacing between the turns must be kept sufficiently great, otherwise the capacity between the turns will be so high that tuning with an attached condenser will be poor; in fact if the turns are too close the natural period of the coil will be greater than that of the signalling current, and the reception will then be very poor.

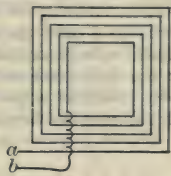

Fig. 8.

Ground Antenna. — A long insulated wire laid along the ground, or even better, at the bottom of a shallow body of water, forms a good receiving antenna, if it is opened at its middle point and the receiving apparatus connected in. The wire should be laid in the direction from which the desired signal comes, as this type of antenna is fairly directive. If laid in salt water the wire should not be covered more than a foot or two, whereas in fresh water the depth may be several feet, without too much attenuation of the signal.

Capacity of an Antenna. — The electrostatic capacity of an antenna depends upon the area covered by the net work of wires, and upon the height from ground, and to a lesser extent upon the size and number of wires used. For an antenna of the forms shown in Figs. 5 and 6, Austin has found the empirical formula,

$$C = \left(4\sqrt{a} + 0.885\frac{a}{h}\right) \times 10^{-5},$$

in which C = capacity in microfarads, a = area of the antenna (width of spreaders multiplied by the distance between them) in square meters; h = height of the antenna from ground, in meters. If the length, l, of the antenna is much greater than the width, b, the value of C must be multiplied by the factor $\left(1 + 0.015\frac{l}{b}\right)$.

The ordinary ship's antenna has a capacity * of about $0.001\mu f$, a small one such as that carried by a submarine about $0.0005\mu f$ while those carried by the battleships may have as much as $0.003\ \mu f$. The large land stations may have a capacity of $0.01\mu f$ or more.

The capacity of an antenna, being distributed, is really a function of the frequency, but if the loading coil used has sufficient inductance to increase the wave length to about four times the natural wave length, the antenna capacity may be treated as lumped and thus independent of the frequency.

Wave-Length Radiated from an Antenna. — If the antenna is connected directly to the ground (no loading coil used), the wave length radiated is very closely 4 times the distance from the ground, up the lead wire (vertical part of the antenna) and so out to the extreme end of the antenna; actually it may be 10 per cent larger than this value. An antenna of the *T*-form, having a height of 50 meters, and a length of top part of 150 meters, would have a wave length about $4 \times \left(50 + \frac{150}{2}\right) \times 1.1 = 550$ meters. This would be called the "natural wave length" of the antenna. If the lead wire is now attached to the end of the

* The abreviation μf is used for microfarad.

top part, thus making an inverted L antenna, the natural wave length would be $4 \times (50+150) \times 1.1 = 880$ meters.

Loading Coil and Condenser. — It is generally not feasible to use an antenna radiating its natural wave length; generally an antenna is worked at from 2 to 4 times its natural wave length. This is accomplished by putting a suitable inductance (called a loading coil) in the base of the antenna, thus increasing the inductance of the oscillating circuit over that inherent in the antenna itself. For such a case the wave length radiated from the antenna is easily calculated from the formula,

$$\lambda = 1885\sqrt{LC} \qquad \text{meters,}$$

in which L is the inductance of the coil in microhenries, and C is the capacity of the antenna in microfarads.

By putting a condenser in series with the base of the antenna, and using only a small loading coil (at least a small coil is nearly always necessary to the operation of the average set, as will appear later), it is possible to make the antenna radiate at less than its natural wave length; such operation is carried out only in emergencies.

Antenna Resistance. — It is impossible to measure the amount of high frequency power supplied to an antenna by a wattmeter or similar test; the power in the antenna is always calculated from the known resistance of the antenna and the reading of the hot wire ammeter in the antenna circuit. A knowledge of the antenna resistance is thus important; in fact it is probably the most important single factor in the whole radio system.

In Fig. 9 are shown two typical antenna resistance curves; that indicated by A being for a ship, and the other B for a land station, the two antenna being the same size; λ_0 indicates the natural wave length of the antennæ. The form of this curve is due to the various components which change in different ways as the wave length is increased. The *radiation resistance* varies inversely with λ^2, and the principle resistance causing loss of power, due to dielectric hysteresis in the earth's crust and objects in the electric field of the antenna, varies directly with the wave length. This wasteful resistance exists to a greater extent in a land station than in a ship station, hence the higher slope of the curve at the longer wave lengths.

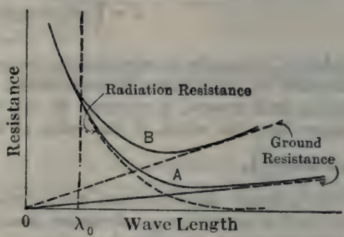

Fig. 9.

Importance of Good Ground. — With a well-grounded antenna at short wave lengths, the greater part of the energy supplied to the antenna is radiated, as indicated by the relative values of the ground resistance and radiation resistance in Fig. 9; at the longer wave lengths, however, the radiation resistance decreases very rapidly, and the ground resistance increases, so that most of the power supplied to the antenna is used up as heat and so wasted. It will thus be realized that the ground resistance must be kept low if the antenna is to be efficient. Elaborate networks of copper wire are often buried in the ground underneath an antenna, and heavy wires carried down to the permanently moist earth to minimize part of the ground resistance effect. From measurements made by J. M. Miller it would appear that wooden posts, trees, etc., close to an antenna absorb a great deal of power.

Counterpoises. — Where a good ground connection is not possible, it is found that an insulated network of wires, suspended on insulators a few feet above the ground, serve well as a ground connection. The network should have a spread considerably greater in area than the antenna itself, if possible.

Radiation Resistance. — The radiation resistance of an antenna can be obtained from experimental observation, or it can be calculated from the dimensions of the antenna. Various formulæ have been given which differ somewhat among themselves, generally due to different assumptions as to what the height of the antenna is; some formulæ use the "effective" height and others use the actual height. The following formulæ have been given by J. H. Dellinger. For a flat top antenna, of height h,

$$R = \left(39.7\,\frac{h}{\lambda}\right)^2 \text{ ohms.}$$

For a square coil of N turns, length of side, a,

$$R = \left(13.3\,\frac{a}{\lambda}\right)^4 N^2 \text{ ohms.}$$

In these formulæ the dimensions must of course all be expressed in the same units.

From the above it is readily seen that a coil is not a good radiator unless its dimensions approach those of the ordinary antenna; it has been used successfully as a transmitter only for very short waves. The above formulæ also give an idea of the comparative amounts of energy the two types of aerial will pick up when used at a receiving station; they do not give an idea of the actual relative merits of the two, however, because the coil aerial may be made with a much lower resistance than the ordinary type, and hence will produce a greater relative response in the detecting device than would be indicated by the above formulæ.

The formulæ show that the maximum radiation resistance which a straight vertical grounded wire can have is about 50 ohms (the value of h/λ for quarter wave length oscillation is about 0.17) and this is the same for all vertical wire antennæ, no matter how high they may be.

VACUUM TUBES. — When a metal is heated to a high temperature in a vacuum it gives off free electrons, i.e., particles of negative electricity (*see article on Electron Theory*). When the heated metal is made one electrode, and a second unheated electrode is provided, it is found that the space between the two electrodes becomes a relative good conductor to a current flowing through the tube in the direction from the cold electrode to the hot electrode, but that practically no current will flow in the reverse direction. Such a device is therefore an almost perfect rectifier for alternating currents. The Fleming valve and G. E. kenotron are typical rectifiers of this type.

The electrode to be heated is usually made in the form of a filament, which is heated by sending a current through it from a storage battery or other source of e.m.f. The heated electrode is therefore usually referred to as the " filament " and the other electrode as the " plate." The current from the plate to the filament is called the plate current (and must not be confused with the current passed through the filament to heat it).

Space Charge. — Experiment justifies the assumption that for a given temperature of the filament, there is a definite rate at which electrons are given off, or evaporated, from it. When there is no potential difference established between the filament and plate (the plate and filament being insulated from each other), a condition of equilibrium is reached when the number of electrons

evaporated in a given time interval is exactly equal to the number of electrons which are forced back into the filament by the mutual repulsion between the electrons which fill the space in the tube. That is, the tube may be thought of as filled with a negative charge, exerting a pressure tending to prevent a further evaporation of electrons, just as the vapor in a closed vessel partially filled with water exerts a pressure on the water tending to prevent further evaporation of the water. This negative charge is conveniently referred to as the "space charge."

Two-Electrode Tubes. — When a difference of potential is established between the plate and the filament, the plate being the anode (positive), the electrons forming this space charge are attracted to the plate, pass through the external circuit and re-enter the filament, and are again evaporated. However, since for a given temperature there is a limit to the rate at which the electrons can be evaporated, it follows that when the potential difference between the plate and the filament is increased to such a value that the plate current is equal to the rate at which negative electricity is evaporated from the filament, then any further increase in this potential difference cannot produce an increase in the value of the current. This limitation to the value of the current is said to be caused by "saturation," and this limiting current is called the "saturation current" for the given filament temperature.

When the applied potential difference is small, and the resulting current is small relative to the rate of evaporation of the electrons, it can be deduced from fundamental principles that the resultant effect of the applied potential difference and the repulsive action of electrons forming the space charge is such as to give a current which depends solely on the applied potential difference and which is independent of the temperature of the filament. Under these conditions the current is said to be limited by the space charge.

Consequently, for high filament temperatures and low potential differences the plate current is the same irrespective of the temperature of the filament

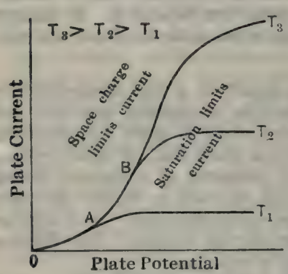

Fig. 10.

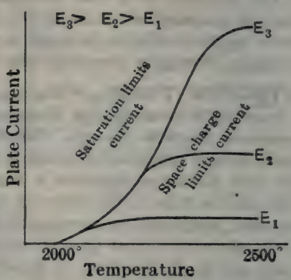

Fig. 11.

(current limited by space charge), but for higher potential differences the plate current reaches a maximum value dependent upon the filament temperature only (current limited by saturation); see Fig. 10. Or, looked at from another point of view, for high potential differences and low temperatures the plate current is the same irrespective of the potential difference (current limited by saturation), but for higher temperatures the plate current reaches a maximum value dependent upon the potential difference only (current limited by space charge); see Fig. 11.

The two electrode tube is not used to a great extent to-day; it may be used

as a rectifier in such installations as the Cottrell process for dust precipitation, and during the war was used to some extent as a voltage regulator for small self-excited, variable-speed generators. It seems likely that it may find further application in this field.

Three Electrode Tube, or Triode. — The thermionic tube becomes a much more effective device if a third electrode is interposed between the plate and filament, this electrode being in the form of a mesh or grid, so that electrons may flow through it on their way to the plate. Fig. 12 shows diagrammatically the construction of a triode. Such a tube was first put out by DeForest under the trade name of "Audion"; to-day many types of three electrode tubes are in use in both wire and radio comunication. Eccles has suggested the name "triode" for a three electrode functioning as a pure electron device, and such a name seems better than the various trade names now used. According to the service for which they are intended these triodes vary in size from a peanut to a bulb 8 inches in diameter or more; the plates and grids are flat in some and cylindrical in others; molybdenum, tungsten and nickel are all used for the construction of grids and plates.

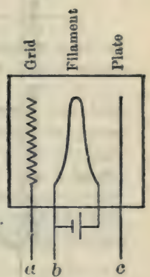

Fig. 12.

Uses of the Triode. — The triode is the most important factor in any radio system; it is only due to this device that radio has made such progeess during the last few years. It performs the functions of *rectifier*, or *detector*, in every modern receiving station; it is used singly, or several in cascade, as an *amplifier* of radio and other signals; it is used in small transmitting stations as a *converter* or generator of high-frequency power from a continuous current source; and in radio telephone outfits it is used extensively as a *modulator* of the radiated high-frequency power.

Action of the Grid. — The feature of fundamental importance in the triode is that under proper conditions a variation in the voltage between grid and filament will produce variations in the plate current *many times greater* than would be produced by equal variations in the voltage between the plate and filament.

The action of the triode is most easily explained by noting that the grid, situated in the space between the filament and the plate, may assist the space charge in limiting the plate current or may nullify the space charge effect and thereby increase the plate current, according as its potential is negative or positive. (In the literature on electron stream devices it is customary to speak of the plate and grid potentials with respect to the negative end of the filament.) The controlling action of the grid depends upon the fineness of its meshes, and upon how completely it surrounds the filament. According to Miller, the grid exerts more control the closer it is to the plate, other things being equal.

Characteristic Curves of the Triode. — In Fig. 13 are given typical curves of a triode showing how the plate and grid currents vary as the grid potential is varied, for several values E_1, E_2 and E_3 of plate potential. The source of the plate current is connected to *c* and *b* in Fig. 12, and the source of the grid current to *a* and *b*. For small detector tubes the plate current would be less than one milliampere and the grid current a few microamperes; for telephone repeater tubes, or for small power tubes for radio sets, the plate current is about 50 milliamperes; while for the largest power tubes the plate current

may be an ampere or more, the grid current being perhaps one hundredth as much.

It will be noticed in Fig. 13 that for a given grid potential the grid current increases as the plate potential decreases; this is due to the relatively greater

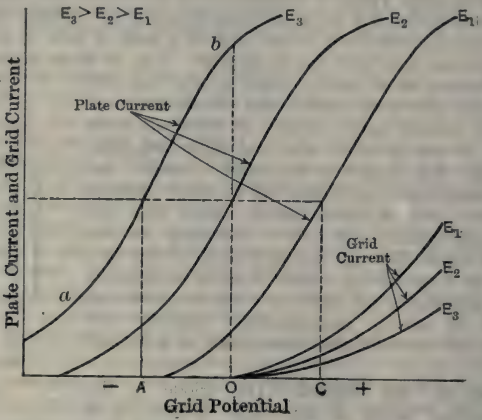

Fig. 13.

attractive force of the grid on the electrons passing through it, as the plate potential is lowered. For negative potentials the grid of an ordinary triode takes so little current that it requires a sensitive galvanometer to measure it.

The maximum plate current is nearly the same for all plate voltages, it requiring a higher positive grid to reach this maximum current, however, when the plate voltage is low. For very low plate voltage this maximum current is never reached; as the grid is made more and more positive it takes more current, thus depriving the plate of part of the electrons, and it may do this to such an extent that the plate current actually decreases with increasing grid potential, for the highest grid voltages.

Tungsten Filament and Oxide Coated Filaments. — In the U. S. practically all the triodes are manufactured either by the G. E. Co. or the W. E. Co. The G. E. tubes go by the trade names of kenotron, pliotron, dynatron, pliodynatron, etc. All of these G. E. tubes use a pure tungsten filament as the electron-emitting element, and tungsten or molybdenum for the grids and plates; the tungsten filament must be at a dazzling white heat before much electron emission occurs and so, of course, considerable power must be used in heating the filament.

For the tungsten filaments Dushman has given the following table of safe operating temperatures, etc., the life of the filament being supposedly 2000 hours.

To get the emission stated in this table the surface of the tungsten must be very clean; there must be no adsorbed gas on the surface.

The tubes made by the W. E. Co. use a platinum ribbon filament coated with certain oxides, a particular cement being used to hold the oxide on; to assist in holding the coating the ribbon is twisted on itself several times per centimeter. The type of emitting surface is frequently called a Wehnelt cathode, as Wehnelt first pointed out the use of the oxide to get more emission

Diameter of filament, cm.	Safe temperature, absolute °C.	Emission in amperes per cm. length	Power to heat filament in watts per cm. length
0.0125	2475	0.03	3.1
0.0175	2500	0.05	4.6
0.0250	2550	0.10	7.2
0.0375	2575	0.20	11.3

at a given temperature. Ordinarily about one milligram of coating is used per square centimeter, but recent experiments seem to indicate that this is much more than necessary.

The advantage of the oxide coated filament is the low temperature at which liberal emission occurs; for a given emission per square centimeter the power required to heat an oxide filament is only about one-half as much as for the pure tungsten filament. An oxide filament is never operated hotter than a bright yellow color; quite appreciable emission occurs when the filament is scarcely visibly hot. The only apparent disadvantage of the oxide filament is the difficulty with which very high vacuum is obtained in the tube without spoiling the filament.

Effect of Gas in a Tube. — An electron tube must be freed from gas sufficiently well to make its characteristic curves permanent and reproducible; if appreciable gas is present in the tube this is not so. All sorts of eccentricities were noted in the early, poorly evacuated audions, so that their inventor, DeForest, regarded their action as very mysterious. It is now known that all these peculiarities were due to the kind and amount of residual gas in the tube, and its effect upon the emission of electrons, as well as certain effects produced by ionization of these gases. Certain gases, such as argon, mercury vapor, hydrogen, etc., have very little effect on the electron emission from tungsten, but oxygen or water vapor (in pressures as low as 10^{-6} mm. of Hg) have a marked effect, much reducing the emission. Water vapor may be absorbed in the glass walls of the tube; heating the glass slightly will release this water vapor and it attacks the surface of the filament, changing the emission as much as 10 times when heating the walls only 10 degrees. Mercury vapor as high as 10^{-1} mm. has scarcely any effect on the emission. The best work on the effect of various problems connected with electron emission has been done by Langmuir, who was the first to produce the tube with very pure tungsten parts and very high vacua.

In detector tubes the effect of a slight amount of gas is unimportant; if ionization occurs to an extent sufficient to show the characteristic blue glow, the plate voltage should be reduced until the glow disappears and the tube will generally function as well as before.

In a modern tube there is not sufficient gas to make it erratic in its action. The presence of a gas may be detected by determining the grid current vs. grid potential curve. If such a curve shows an appreciable grid current with reversed grid potential (grid negative with respect to the negative end of filament), it may be concluded that a gas is present.

Amplification Factor of a Triode. — As shown in Fig. 13 the plate current depends upon both the plate and grid potentials. For the working range of the tube, the plate current is a function of the sum of the plate voltage E_p and the product of the grid voltage E_g and a factor greater than unity, i.e., the plate current is a function of $(E_p + \mu_0 E_g)$, where μ_0 is a factor greater

than 1. This factor μ_0, which expresses the relative value of E_g and E_p in controlling the plate current, is called the voltage amplification factor of the tube; it is nearly constant throughout the working range of the tube but does depend to some extent upon the filament current and plate and grid potentials. It is greater the finer the grid mesh and in actual tubes may have a value between 1 to 2 (old DeForest audions) to as much as 200. The ordinary value of μ_0 lies between 3 and 40, the latter value being infrequent. The ordinary detecting and amplifying tube used so much at present has a μ_0 of about 7.

Langmuir has expressed the relation between the plate current and the plate and grid potentials by the equation,

$$I_p = A(E_p + \mu_0 E_g)^{3/2},$$

where A is a constant. Van der Bijl, to whom is largely due the development of the W. E. tubes, gives the relation,

$$I_p = A(E_p + \mu_0 E_g + e)^2,$$

where e is a small quantity determined by the surface characteristics of the three electrodes in the tube (called the contact difference of potential), and other factors. Actually, the exponent of the term in the parentheses is neither 3/2 nor 2, but is a variable depending upon the grid and plate voltages.

For small changes in plate current, as occur in the ordinary amplifier circuit the relations of current and potentials may be written,

$$\Delta I_p = K(\Delta E_p + \mu_0 \Delta E_g),$$

where K is a constant. This relation was first given by Latour. This equation is the one on which practically all tube theory is based; it is not correct for detection, as it neglects the factor upon which detection depends. It also leads to certain vague conclusions when dealing with triodes as power converters.

Internal Resistance of Tube. — The circuit to which a tube is connected should always be designed to fit the tube resistance. There are two internal resistances which control the design of the connected circuits, viz., the resistance from the grid to the filament (input circuit), and the resistance from the plate to the filament (output circuit). The input circuit resistance is in nearly all cases very high; if the grid is maintained at some negative potential, as is generally the case, the input circuit may be treated as though it had infinite resistance, except when used with very high frequency, in which case the capacity of the input circuit may be of importance.

The resistance of the output circuit varies with the type of triode and with the voltage used in the plate and grid circuits; it generally lies between 5000 and 100,000 ohms for tubes used to-day. The continuous-current resistance is seldom of importance, the alternating-current resistance being the factor, knowledge of which is necessary. This a-c. resistance is about one-half the c-c. resistance. The c-c. resistance is given by the relation,

$$R_c = \frac{E_p + \mu_0 E_g}{I_p}.$$

The a-c. resistance is,

$$R_a = \frac{\Delta E_p}{\Delta I_p},$$

when E_g is held constant.

Triode as a Converter from C-c. to A-c. — Innumerable circuits have been devised for making the triode function as a converter of c-c. power

to a-c. power, one of the simplest being that shown in Fig. 14. The filament is heated by battery A and the plate voltage furnished by generator B: a condenser C serves as a by-pass for the high-frequency fluctuations in the plate current which occur as soon as the triode starts to function. The coil L_1 and condenser C_1 form an oscillatory circuit of resistance R. The grid is connected to the negative end of the filament through an inductance L_2, which is coupled magnetically to L_1, the mutual induction being indicated by M.

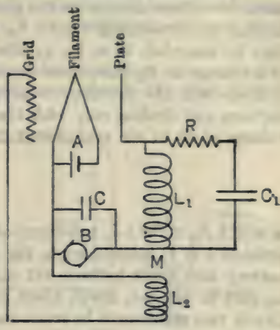

Fig. 14.

When certain conditions have been satisfied the combination will generate alternating current in circuit $L_1 - C_1$, the frequency of the current being fixed by the natural period of this circuit. The plate current itself fluctuates from zero to twice its normal value and in the $L_1 - C_1$ circuit the magnitude of current is limited (in a suitably designed circuit) only by the resistance R. It is to be noted that C_1 and R might be the capacity and resistance of an ordinary antenna.

The required conditions for producing oscillations in the oscillatory circuit may be found by assuming a damped oscillatory current to flow in the circuit, and solve for the condition which gives a damping factor of zero; such a condition means of course the continuous generation of alternating current in this circuit. It may be proved in this manner that the condition for oscillations being maintained by the triode is given by the inequality:

$$M \text{ negative and greater in magnitude than } \frac{1}{\mu_0}(L_1 + CRR_p),$$

where R_p is the alternating current resistance of the plate-filament circuit. The required negative value of M means that the coils L_1 and L must be so related that when the plate voltage rises the action of L_1 on L_2 is to make the grid potential fall, in other words, that as the circuit is in oscillation the plate and grid voltages must have opposite polarities with respect to the filament.

Power Obtainable from a Triode Converter. — A triode is rated in terms of the power safely dissipated by the plate; if this is too much the plate overheats, drives gas out from itself, and so the vacuum is spoiled. With ordinary adjustments of circuits the amount of power obtainable in the form of alternating-current power is about the same as the safe rating of the plate. For this condition the efficiency of the converter is 50 per cent, neglecting the amount of power required to heat the filament. With suitably designed circuits, however, and with sufficiently high voltage on the plate (that is, high voltage generator in the plate circuit), it is possible to get from the tube about four times as much power as the plate is rated to stand. For such an efficiency the plate circuit generator must be a 2500 volt machine or more.

Triode as Amplifier. — The equation for the plate current of a triode shows that the grid is μ_0 times as effective in controlling this current as is the plate voltage itself. For small changes in grid and plate potentials the change in plate current may be written,

$$\Delta I_p = \frac{1}{R_p}(\Delta E_p + \mu_0 \Delta E_g).$$

From this it is seen that the change in plate current for an increment in grid potential is as large as if the grid potential had been held constant and an increment of voltage change in the plate voltage μ_0 times as large were introduced, so that an alternating e.m.f., E_g, introduced in the grid circuit (input circuit) may be regarded, in so far as effect on plate current is concerned, as the equal of a voltage $\mu_0 E_g$ introduced in the plate circuit.

Neglecting the continuous components of voltages and currents in the tube circuits, and dealing only with the alternating components (r.m.s. values), for a voltage of E_g in the grid circuit there is produced in the plate circuit a current

$$I_p = \frac{\mu_0 E_g}{R_p},$$

in which R_p is the a-c. resistance of the plate circuit of the triode. If an external resistance R is introduced in the plate circuit as indicated in Fig. 15 (heating battery and plate battery left out as they play no part in the a-c. laws), there are in the plate circuit two resistances in series and the equation for plate current becomes,

$$I_p = \frac{\mu_0 E_g}{R_p + R}.$$

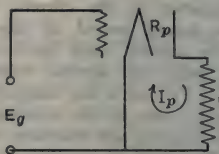

Fig. 15.

I_p will of course be an alternating current of the same form as E_g (if the distorting effects of the variable R_p are neglected) so that the drop across the external resistance R is of the same form as E_g and equal to

$$E_g \times \frac{\mu_0 R}{R_p + R}.$$

Evidently if the fraction $\mu_c R/(R_p+R)$ is greater than unity the triode and its attached circuit constitute a voltage amplifier, giving across R a voltage similar in form to E_g, and in practice from 2 to 20 times as large.

In case a reactance is used in place of R, the feasible amplification is somewhat more. Neglecting the resistance of the reactance introduced in the plate circuit, the amplified voltage across the reactance is,

$$E_g \times \frac{\mu_0 X}{\sqrt{X^2 + R_p^2}}.$$

If X is two or three times as large as R_p the amplified voltage will be sensibly μ_0 times as large as the input voltage.

Instead of inserting a reactance coil in the plate circuit the primary of a suitably designed transformer may be used. Such a transformer is generally built with a step-up ratio of about 5, so that the possible amplification is thereby increased. These repeating transformers must be built with a very large number of turns to match the resistance of the ordinary amplifying tube; and further, the natural period of the secondary must be kept much lower than that of any frequency to be amplified. It is this fact which keeps down the useful step-up ratio. If the transformer has a ratio of 5 the maximum obtainable amplification of the triode and attached transformer is equal to $5/2\ \mu_0$.

The output voltage of one triode may be impressed on the input circuit of another triode similarly arranged and the output of this triode impressed on the input of a third, etc. Thus if each stage of amplification (triode and attached circuits) gives a voltage amplification of A, and x stages are used, the theoretical

amplification is A^x times in voltage. In practice a voltage amplification of 5000 is possible but seldom used; the amplifier tends to become unstable, and special precautions must be observed in using it if the amplification is more than about 500.

In Fig. 16 is shown a three-stage resistance coupled amplifier using common batteries for all filaments and plates; in this type of circuit the grid of each

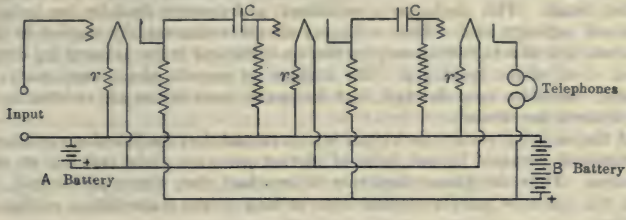

Fig. 16.

triode must be insulated from the plate circuit of the preceding one to prevent the high plate voltage from being impressed on the grid. This is the function of the condensers C, C, etc. The small resistances r, r, inserted in the negative leg of the filament serve to keep the average potential of the grids negative with respect to all parts of the filament, a necessary condition for efficient amplification.

In Fig. 17 is shown a three-stage transformer coupled amplifier. With tubes having μ_0 of 7 and transformers having a 5 : 1 step-up ratio three stages is all

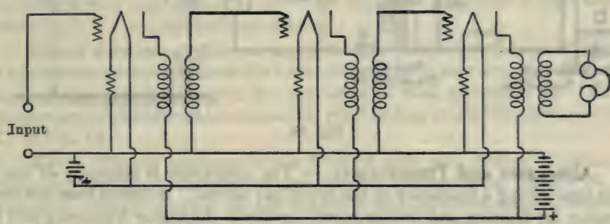

Fig. 17.

that could be used without the amplifier being very noisy and possibly not at all workable.

A regenerative coupling is possible and sometimes used with resistance coupled amplifiers; a small variable condenser serves to connect (electrostatically) the last plate to either the grid or plate of the first tube, thus tending to make the effective resistance of the amplifier as a whole zero, that is, tending towards infinite amplification.

DAMPED-WAVE TELEGRAPHY OR SPARK TELEGRAPHY.—Up to about 1915, spark telegraphy was the nearly universal method of carrying on radio communication; it is rapidly being superseded by continuous, or undamped, wave telegraphy, because of greater distance obtainable, more freedom from interference, etc. If seems likely that within a few years spark telegraphy will be done away with entirely. At present most of the radio

stations in the world still use the damped wave system; especially is this true of the smaller stations such as those on board ship. A knowledge of the function and operation of a spark station is therefore still essential.

In brief, this system comprises a source of power (generally a motor-driven alternator and step-up transformer to get a power supply of about 15,000 volts; suitable condensers are charged to this high voltage and then are discharged through a few microhenries of inductance, by the breaking down of a spark-gap in the circuit. This discharge is oscillatory, of frequency fixed by the inductance and capacity used. This oscillatory circuit is magnetically coupled to the antenna, and so a high-frequency oscillation is induced in the antenna for every break down of the spark-gap. The average set permits about 1000 sparks a second; therefore 1000 damped, high-frequency wave trains are emitted from the antenna per second.

At the receiving station the circuit is so arranged that the listening operator's telephone is actuated by one pulse of current per wave train arriving at the antenna. Thus the phone diaphram is impulsed 1000 times per second, so that the operator hears a musical note as long as the transmitting station is sending.

Typical Transmitting Circuit and Function of Parts. — In Fig. 18 is shown the general layout of a spark transmitter.

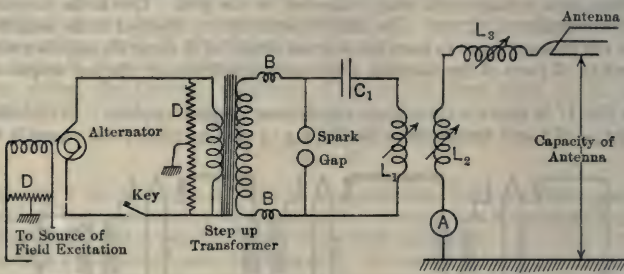

Fig. 18.

Alternator and Transformer. — The alternator is generally of the inductor type giving a frequency of 500 cycles per second. It differs from the small alternator for lighting and similar power service in the higher frequency and the very high internal impedance of its armature windings; the short-circuit current of a radio alternator is only slightly in excess of the full-load current. The step-up transformer is of the open or closed core type, more modern sets having the closed core; it steps the voltage up from perhaps 125 to 15,000 volts. *B–B* are high frequency choke coils (generally built in with the transformer) to keep the high-frequency currents from penetrating the secondary winding and breaking down the insulation.

Spark Gap. — The spark gap may consist of a couple of discs of non-arcing metal, such as zinc, but, because of the noise and difficulty of maintaining the adjustment of this type of gap, as well as the bad quality of signal radiated, such a gap is seldom used except in emergency sets.

A rotating disc, having studs mounted on its periphery which come close to a couple of stationary studs as the disc revolves, is more often used; it is called a rotating gap. If the rotating studs come in proximity to the stationary studs (so that a spark may pass) once per alternation of the power supply it is called a rotating synchronous gap; otherwise it is called a rotating non-synchronous

gap. The synchronous gap is used more often than the non-synchronous, the rotating disc being mounted on the end of the alternator shaft, thus ensuring synchronism. The disc is so shaped that as it rotates it plays a blast of air on the stationary studs to keep them cool. The U. S. Government station at Arlington from which standard time signals are sent out, uses this type of gap.

Condensers. — C consists of either copper plated glass jars or mica condensers, the latter gradually replacing the jars because of less dielectric loss, more compact structure, and less likelihood of breaking. The copper-plated jars were made of standard size, having 0.002 microfarad capacity. A 1000-cycle spark set requires four of these jars per kw. rated capacity. The mica a-c. condensers are made more generally in units of 0.004 microfarad.

In order to obtain uniform sparking of a set, the reactance of the alternator armature plus the leakage reactance of the transformer must constitute, with condenser C, a resonant circuit the frequency of which is about 20 per cent lower than the frequency generated by the alternator. In calculating this effect the transformer inductance and condenser capacity must be suitably changed to reduce the circuit to a 1–1 transformer condition.

Oscillation Transformer and Loading Coil. — The two coils L_1 and L_2 placed in proximity to each other constitute the oscillation transformer; the coil L_1 in combination with condenser C and spark gap make up the closed oscillatory circuit and fix the frequency sent out. The extra coil in the antenna circuit L_3 is called the antenna loading coil. All of these coils L_1, L_2, and L_3, as well as the mutual inductance between L_1 and L_2 are generally variable either by sliding contacts or taps.

Ammeter. — In the base of the antenna, just where it connects to ground, is placed a hot wire ammeter A to indicate the amount of current flowing in the antenna circuit. For measuring these high-frequency currents in radio circuits only two types of ammeter are available, the hot-wire type and the thermocouple and c-c. millivoltmeter type.

Safety Devices. — Various safety devices consisting of high resistances and condensers are placed across the low-voltage lines to keep any high-frequency induced voltages from damaging the insulation of field, armature, etc. A high resistance across the alternator terminals, grounded at its middle point, is shown at D, in Fig. 18.

Key. — Signals are sent in the form of dots and dashes (Continental Morse; see article on *Telegraphy*) by suitably manipulating the key in the transformer primary circuit. In the case of a high-powered station, or station using distant control, this key is of the relay type. Sometimes multiple relays are used putting several gaps in series. The spark gap and relay key are the two parts of a spark transmitter which require the most attention to maintain.

Coefficient of Coupling. — The closed oscillating circuit L_1–C is coupled to the open oscillating circuit (the antenna) by the mutual inductance between L_1 and L_2. The coefficient of coupling of the two circuits is given by

$$k=\frac{M}{\sqrt{L_1(L_2+L_3)}}.$$

The permissible value of k varies from perhaps 10 per cent in a rotating gap to 20–25 per cent with a quenched gap (described below).

Wave-Length Radiated. — Were the circuit L_1–C of Fig. 18, not influenced by the circuit L_2–L_3, it would oscillate at a frequency fixed by the formula,

$$f=\frac{1}{2\pi\sqrt{L_1C}},$$

where L_1 and C are expressed in henries and farads. When L_1 and C are expressed in microhenries and microfarads, this formula becomes

$$f = \frac{10^6}{2\pi\sqrt{L_1C}}.$$

The corresponding wave-length is then

$$\lambda = 1885\sqrt{L_1C} \qquad \text{meters,}$$

when L_1 and C are expressed in microhenries and microfarads.

In the operation of a set it is necessary to make the natural period of the antenna circuit the same as that of the closed circuit; this is called *tuning* the two circuits. If this has been done it might be supposed that the wave-length radiated from the antenna would be given by the formula $\lambda = 1885\sqrt{L_1C}$, but actually such is not the case. By use of a wave meter (described later) it is found that two separate wave-lengths are radiated simultaneously and neither one of them is the same as that expected, one being higher and the other lower. Investigation shows that the values of these two wave-lengths depends upon the coupling, and exact analysis predicts that the wave-lengths should be given by the relations,

$$\lambda' = \lambda\sqrt{1+k},$$
$$\lambda'' = \lambda\sqrt{1-k},$$

in which λ' and λ'' are the two radiated waves and λ is the wave-length for which each of the circuits is turned, and k is the coefficient of coupling. Thus a set tuned for 600 meters in both the closed and in the open oscillating circuits, and having a coupling coefficient of 20 per cent, would radiate simultaneously 657 meters and 536 meters.

Quenched Gap.—Elimination of the Double-Wave Radiation.—The two waves are generated because the transmitting system comprises two more or less independent oscillatory circuits; such a combination will always have two natural periods, and will oscillate at both of them at the same time. The high-frequency energy is first resident in the closed circuit, then by coupling in the oscillation transformer it is transferred to the open circuit, and then again flows back to the closed circuit, and continues this to and fro flow until all the energy is used up, as heat or otherwise. The forms of currents in the two circuits for such a state of affairs are shown in Fig. 19; such a "beating" current is actually made up of two currents of different frequencies.

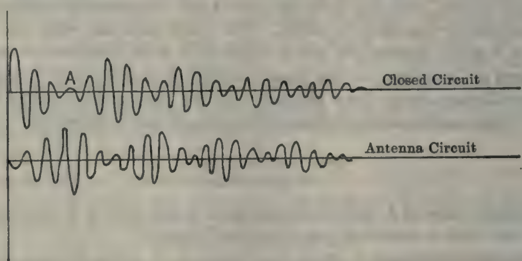

Fig. 19.

At time A, Fig. 19, all the energy has left the closed circuit and is in the antenna circuit; if, at this time the circuit L_1–C should be opened the energy

could not get back into the closed circuit and the antenna would be left free to oscillate at its own natural frequency, and but one wave will be radiated; such an occurrence is shown in Fig. 20. The time available for opening the circuit L_1–C is so short as to make a mechanical break impossible; by suitably designing the spark gap, however, the deionization of the gap, which can take place

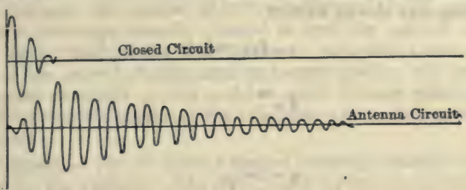

Fig. 20.

in perhaps 10^{-6} seconds, serves as a switch to open the circuit at the proper time, and so perform the required operation of preventing the two waves being formed.

A gap built suitably to accomplish this result is called a "quenched" gap; it consists of an assembly of very short gaps connected in series, each gap not more than about 0.02 cm. long. It is generally made up of a series of copper discs, separated by suitable insulating washers, or gaskets, so that the spark takes place in an air tight cavity. Two sections of such a gap are illustrated in Fig. 21; the sparking surface is of electrolytic copper or silver, let into the faces of the cast copper discs. Ten or fifteen of these discs are clamped together in a frame work of insulated construction to form the ordinary quenched gap. As many of the gaps as desired can be used by clip connections on the flanges. An air blast from a small fan serves to keep the gap cool, the air circulating around the flanges. Small gaps may have liberally designed cooling flanges and so be self-cooled; such a type of gap is used in ½ kw. sets and smaller.

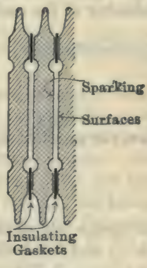

Fig. 21.

To maintain the quenching action the insulating gaskets must be made air tight, so after the oxygen, originally in the sparking cavity when the gap is assembled, is once burned out, no more oxygen can get in. If it does, an oxide layer of increasing thickness forms on the sparking surfaces and the gap ceases to quench.

Decrement. — Number of Waves in a Train. — After the circuit L_1C opens, the current in the antenna (Fig. 20) dies away, as the energy is radiated and wasted as heat, the decay being in accordance with the exponential law. The antenna current, after the gap has opened the closed oscillating circuit is of the form,

$$i = I_0 \epsilon^{-\alpha t} \sin \omega t,$$

in which I_0 is slightly larger than the highest current actually occurring in the antenna; it is equal to the actual maximum current multiplied by $\epsilon^{\frac{\alpha T}{4}}$, where T is the natural period of the antenna circuit. The constant α, called the damping constant, has the value,

$$\alpha = \frac{R}{2L}.$$

The constant ω, called the angular frequency, has the value

$$\omega = 2\pi f = \frac{1}{\sqrt{(L_1+L_2)C}},$$

where C is the capacity of the antenna.

The current may also be written,

$$i = I_0 \epsilon^{-n\delta} \sin \omega t,$$

where

$$\delta = \frac{\alpha}{f},$$

and

$$n = ft = \frac{t}{T}.$$

The quantity n is the number of complete periods or oscillations in the time t. The constant δ is called the "decrement" of the wave train, and is equal to the logarithm (to the base ϵ) of two successive maxima of current, in the same direction, the ratio being taken greater than unity.

The decrement of a spark station is a very important characteristic; if it is too high the transmitting station causes too much interference with other stations in the vicinity. The upper permissible value of the decrement is fixed by law as 0.2; in a good average spark station it is about 0.1, and in the very large stations it may be as low as 0.02–0.03.

In radio theory it is practice to regard a wave train as ended when the current has decayed to 1 per cent of its original value. Let I_N be the current at the beginning of the oscillation for which the current has a maximum value equal to $\frac{I_0}{100}$. Then,

$$I_N = \frac{I_0}{100} = I_0 \epsilon^{-(N-1)\delta}.$$

Then the number of waves in the train is,

$$N = \frac{4.60+\delta}{\delta}.$$

The larger the decrement the fewer the waves in the train; e.g., for $\delta = 0.2$, $N = 24$, and for $\delta = 0.02$, $N = 231$.

Adjustment of a Transmitting Set. — With the spark gap set to discharge at about one-half normal voltage, and with the antenna circuit open, L_1 is varied until the L_1–C circuit is generating the desired wave-length, as indicated by a wave meter (described later) held near L_1. Then with the coupling between L_1 and L_2 very weak the antenna circuit is closed and, with medium value of L_2 connected in the circuit, L_3 is varied until the antenna ammeter indicates maximum current; the antenna is then tuned to the closed circuit, which is generating the desired wave-length. The coupling between L_1 and L_2 should be now increased and the indication of the antenna ammeter noted; this reading will increase rapidly at first (with increasing coupling) and then will increase more slowly. The coupling should not be increased past the point where the rapid increase in antenna current ceases. In case a quenched gap is used the coupling may be increased until, after the current has increased with coupling, it shows a slight decrease with further increase in coupling; the coupling should be left slightly less than that value which shows the decrease in antenna current.

If this procedure is followed, but one wave will be radiated. whereas if tighter

coupling is used the ammeter in the antenna will (or may) show more current yet *the radiation at the desired wave-length will actually be less.* If too loose coupling is used the set is inefficient, too much of the high-frequency power being used up in heating the closed oscillating circuit, especially the spark gap.

To vary the power output of the set the gap should be changed in length (or, if a quenched gap, the number of active sections reduced), and the field excitation of the alternator suitably reduced to keep the musical note of the set constant. If too much or too little voltage is used with a given length of gap, the note of the set will change from its normal quality, too little voltage dropping the note an octave or two and too much voltage making the note impure, due to several discharges of the condenser per cycle, instead of two, the proper number.

Receiving Circuit. — Fig. 22 shows a typical receiving circuit; the antenna circuit is tuned by the variation of L_2 and L_3 and the closed oscillating circuit by varying the condenser C_1. It will be noted that there are the same elements in this receiving circuit as there are in the spark-transmitting circuit; the receiving coils and condensers, however, are of much lighter construction than the same parts of the transmitting set, because of the very small currents in the receiving system. In the transmitting station the currents are measured in amperes and potentials in kilovolts, whereas in the receiving system the potentials are less than a volt and the currents measurable in microamperes.

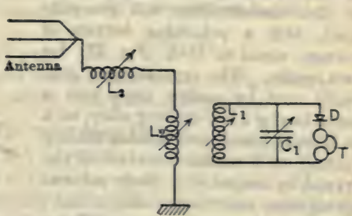

Fig. 22.

Across the tuning condenser C_1 is shunted a pair of head phones in series with some sort of rectifying device. When the two circuits are properly tuned to an incoming signal a musical note is heard in the phones, the pitch being fixed by the number of sparks per second of the transmitting station. The following procedure is followed in tuning the receiving circuit. With the condenser C_1 set at a low value and tight coupling between L_2 and L_1, the tuning of the antenna circuit is varied until the signal is heard; then, with decreased coupling between L_1 and L_2, C_1 is varied until maximum signal is heard. Because of the effect of one circuit on the other, as the coupling is weakened, both circuits must be slightly retuned. Having both circuits properly tuned it will be found that a certain value of coupling gives maximum signal strength; this will generally be quite loose coupling, depending somewhat upon the type of rectifier used and the decrement of the incoming wave.

The sharpness of tuning depends upon three factors, the decrement of the transmitting station, decrement of the receiving station, and the amount of coupling used in the receiving circuit, an increase of any one of these factors generally increasing the broadness of the resonance curve. As coupling is decreased from a high value the tuning increases in sharpness until a certain critical coupling is reached; further decrease in the coupling decrease the signal strength but does not increase the sharpness of tuning.

Rectifier or Detector. — The incoming wave train sets up oscillatory currents, of radio frequency, in circuit L_1–C_1; if the telephone were connected directly across C_1 nothing would be heard, because neither the phones nor the ear can respond to a radio-frequency current. By interposing the detector D,

which permits the passage of current more readily in one direction than in the other, the telephone diaphragm is impulsed in one direction (either in or out) for *each wave train in the receiving circuit*, thus giving a note in the telephone fixed by the number of sparks at the transmitting station; this note, it must be noticed, is entirely independent of the frequency of the radio current, that is, of the wave length used.

Crystal Rectifiers were used almost exclusively as detectors up to a few years ago, but since then the three-electrode vacuum tube (triode) has gradually displaced them, until to-day crystal detectors are used only in second-class outfits. Typical curves of carborundum and galena, two of the widely used crystals, are shown in Fig. 23. The rectifying effect depends upon the sharpness of curvature of the volt-ampere characteristic, so that carborundum was generally used, with a polarizing battery of voltage equal to O–A (Fig. 23) for best results. The crystals were generally used by casting them into a metal box by a low-melting alloy, such as Wood's metal, and making the top contact to a sensitive surface of the crystal by means of a finely pointed spring wire, sometimes called a "cat's whisker." Sometimes the rectifying contact was obtained by using two crystals touching one another, such as zincite and chalco-pyrite. In general the more sensitive the crystal rectifier the more easily it gets out of adjustment, or is spoiled at that one contact point by a very intense signal or atmospheric discharge.

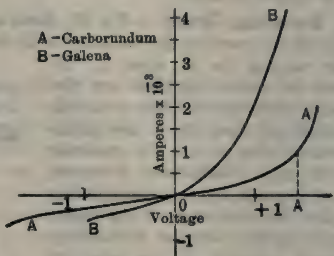

Fig. 23.

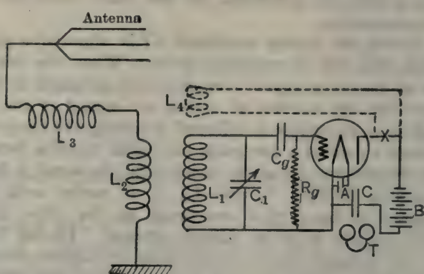

Fig. 24.

Triode as Detector. — The triode is immeasurably superior to the crystal detector, due to its reliability and still more to the possibility of suitably arranging the circuits attached to it to magnify the signal strength. The normal connection of the triode as a detector is shown in Fig. 24; with the connection as given by the full lines the triode is about 5 times as sensitive as an ordinary crystal. But if the plate circuit is opened at X and the coil L_4 inserted as indicated by the dotted lines, the signal strength may be amplified hundreds of times by the so called "regenerative action" (due to Armstrong). As the coupling between L_1 and L_4 is increased, the signal increases in strength about as indicated in Fig. 25. Beyond a certain value of coupling, the musical quality of the signal disappears and the signal becomes "mushy" or scratchy in sound, although very loud; this region is indicated in Fig. 25 by the dotted portion of the curve. With the critical value of coupling (O–A in Fig. 25), signals

which are entirely inaudible with slightly less coupling become quite loud and readable, although not of musical quality.

The coupling between L_1 and L_4 (if of right polarity), tends to produce oscillations in the L_1–C_1 circuit, as pointed out in the section on triodes; in the region shown by the dotted line in Fig. 25 the tube is oscillating and the scratchy quality of the note is due to the irregular interfering action of the continuous oscillations in the L_1–C_1 circuit, and the damped-wave signals induced in this circuit by the signals received by the antenna.

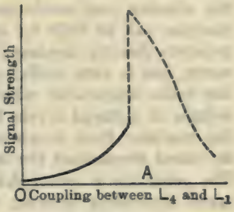

Fig. 25.

With slightly less than the critical coupling between L_1 and L_4 the decrement of the complete receiving circuit approaches zero, thus very much increasing the selectivity, or sharpness of resonance of the set; the tuning in this condition is limited in sharpness only by the decrement of the transmitting set.

A normal value of the grid condenser C_g of Fig. 24 is about 200 micro-microfarads, of the grid leak resistance R_g about 1 megohm, and for the by-pass condenser C about 5000 micro-microfarads.

CONTINUOUS, OR UNDAMPED TELEGRAPHY. — Continuous-wave telegraphy (often abbreviated C. W.) is rapidly superseding the older spark-wave system; it is the logical form of power to use for radio communication and the only reason that spark telegraphy gained the field at first was the difficulty of building a high-frequency generator which was reliable and could give several hundred kw. of power. Instead of sending out a group of discontinuous wave trains, as is done in spark telegraphy, the continuous-wave transmitter excites the antenna continuously with high-frequency power as long as the sending key is closed. This is indicated in Fig. 26; at *A* is shown the antenna excitation for a dot by spark telegraphy and at *B* the antenna excitation for a dot by continuous-wave telegraphy. For slow sending a dot might last 0.1 second; this would take 100 wave trains from a 1000-spark transmitter and would require 10,000 cycles of current, of fixed amplitude, for a 3000-meter continuous-wave transmitter.

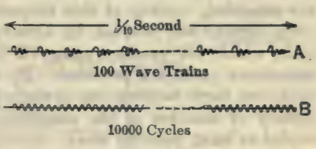

Fig. 26.

Advantages of Continuous-Wave Communication. — For a given radiated power the insulation of an antenna for continuous wave is much simpler than for spark waves, the voltage for which the insulators are built being perhaps 1/10 as great; a given amount of power in the form of continuous wave will permit communication over much greater distance than the same amount of damped-wave power, due principally to the receiving circuit used; the interference between continuous-wave stations is much less than that between spark stations.

Types of Generators Used. — There are now available three types of generators which will furnish 100 kw. or more of high-frequency power, at frequencies of 40,000 or more; namely, the inductor type of generator, the reflector type, and the Poulsen arc.

The Inductor Type of Alternator, first built by Fessenden, and now called the Alexanderson alternator, consists of stationary armature and field

coils, the revolving member being a disc of high-grade steel, carefully machined and balanced. Near its periphery the disc is cut through with closely placed radial slots, the slots being filled in with some non-magnetic alloy to make the disc smooth to cut down the windage loss. The armatures are placed one on either side of the revolving disc, these armatures being built of very finely laminated iron; they are ring shaped, having radial slots in their inner faces, in which the winding (one conductor per slot) is fitted. At the lower frequencies this machine can be built for an output of 100 kw. or more; it has been built to generate directly 200,000 cycles per second with an output of a few hundred watts, but is very inefficient (10 per cent) at these excessive frequencies. At the radio frequencies which these machines generate the iron loss per unit volume of material is very high, so that special means have to be taken to keep the machine from overheating. In the Alexanderson generator water pipes are fitted throughout the structure to keep the material at a safe temperature. The generator is motor driven, the motor being equipped with very sensitive speed controlling devices; this is necessary not only for the sake of efficient reception, but because all these high-frequency alternators require that the attached circuit be accurately tuned if much output is to be obtained, and any speed variations would spoil the adjustment of tuning.

The Reflector Type of Alternator was first suggested as possible by Boucherot in 1893, but no machine of this type proved commercially practicable until the advent of Goldschmidt's generator of this type put out in 1907. The reflector type utilizes the effect of armature reaction in a single-phase generator in inducing in the field winding a current of double the frequency of the armature current. Suppose a single-phase, 60-cycle alternator is loaded, thereby producing armature reaction on the field. It will be found that a 120-cycle e.m.f. has been generated in the field winding and, if a suitable path is provided for it, a 120-cycle current will flow in the field coils. Now an alternator turning at 60-cycle speed and having a field coil excited with 120-cycle current will generate in its armature a 180-cycle e.m.f., and if suitable paths are provided, current of this frequency will flow in the armature. This 180-cycle current will produce, by armature reaction, a 240-cycle e.m.f. in the field coils, which will produce 240-cycle current in the field circuit if suitable low impedance paths are provided. By using this idea of reflecting the frequency back and forth between the armature and the field, each time gaining the frequency of rotation of the machine, the frequency may be effectively quadrupled at least, possibly more.

In Fig. 27 the circuit arrangement of a Goldschmidt alternator connected to an antenna is shown. The key serves to vary the continuous current excitation, through coil L_1. In the armature coil L_2 is generated a frequency of 10,000, corresponding to the rotational speed of the machine. The combinations L_2–C_2 and L_4–C_4 are each resonant to this frequency. Due to this armature current of 10,000 cycles an e.m.f. of 20,000 is generated in L_1 and as the combinations L_1–C_1 and L_3–C_3 are each resonant to this frequency, a current of 20,000 circulates in the field coil L_1. This induces an e.m.f. of 30,000 in L_2, and as circuit L_2–C_2–C_6 is resonant to this frequency, 30,000 cycle current flows in the armature coils. This finally induces a 40,000 e.m.f. in the field coil, and as the circuit L_1–C_1 in series with the antenna capacity is resonant at 40,000 cycles, this frequency flows in the antenna and is used for radiating the signal. The tuning of each of these circuits is somewhat dependent on the tuning of the others, so the final tuning of the various branches has to be done experimentally after the machine is installed. Choke coils in the field-supply circuit prevent this path from short-circuiting the high-frequency currents.

The construction of this machine is a very difficult piece of work; the lami-

nated cores of field and armature are built of plates 0.002 inch thick with 0.001 inch paper between laminations; the rotor is 3 feet in diameter and weighs

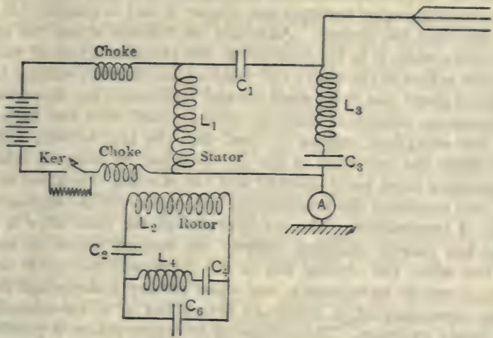

Fig. 27.

5 tons, turns 4000 r.p.m. with an airgap of about 0.03 inch. In this machine, as in the Alexanderson machine, it is very essential that the speed be kept constant; to accomplish this a resistance is automatically inserted in the field of the driving motor whenever load is put on the generator.

Although reliable figures are not available, it seems likely that the efficiency of both machines is of the order of 50 per cent at full load.

The Poulsen Arc is an outgrowth of Duddell's singing arc (*see Arc, Electric*). It is shown connected to an antenna in Fig. 28. The commercial arcs have been made in sizes up to 1000 kw. input; the efficiency being somewhat less than 50 per cent, the output of these large arcs are of the order of 400 kw. of high-frequency power. The c-c. generator of 600–800 volts e.m.f. is connected to the arc through a starting resistance and choke coils. After the arc is struck and starts to oscillate, the starting resistance is gradually cut out. The current through the arc is unstable, and when the arc is operating properly pulsates between nearly zero and about twice that supplied by the c-c. generator. The current from the generator is held nearly constant by the choke coils, so that the variations of the current from the average value flow in the antenna as high-frequency current, the frequency being determined by the natural period of the antenna circuit.

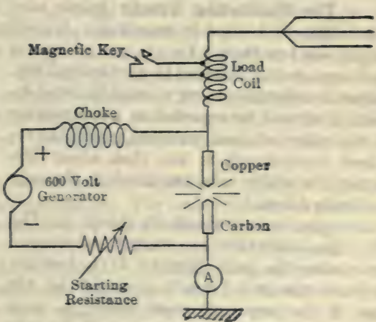

Fig. 28.

To function properly the arc must burn in a light gas, such as hydrogen or illuminating gas; the hollow copper anode must be water cooled, as must the

whole chamber in which the arc burns; the carbon cathode must be rotated slowly, the gap length must be proper (about ⅛ inch), and the arc must burn in a transverse magnetic field of proper intensity. The action of the arc becomes poorer as the natural period becomes less, the arc in fact is inapplicable for wave-lengths of less than 800 meters. Although the voltage across the arc may be only 50–60 volts when it is not oscillating, the peculiar action of the arc combined with the action of the choke coil results in an average voltage (as read on a c-c. meter) of about 500 volts when normal oscillations are occurring.

The interruption scheme of sending, such as used with the high-frequency generators just described, cannot be used with the arc, because when once it starts to oscillate the operation of the arc must not be stopped, otherwise the starting resistance must be re-inserted and the starting operation be repeated. Because of this condition the transmitting scheme of the arc station is peculiar to this type of transmitter. The arc operates all the time after it is started, the sending key merely changing slightly the natural period of the antenna circuit. Thus when the sending key is not depressed the arc is sending out radiation on a certain wave-length, and when the key is depressed the antenna radiates the same amount of power as before but at a wave-length perhaps 2 per cent higher or lower. The scheme of reception makes it possible to read the signal satisfactorily.

It will be seen that the arc station thus "uses up" two wave-lengths, whereas the two types of machines just described use but that one on which their signal is transmitted; this is one objection against the arc transmitter. Another arises from the fact that, owing to the continuous full load output of the arc (whether sending or not), its operating efficiency may be much lower than that of a generator.

Saturated Iron Cores have been used as frequency doublers and triplers to some extent, and the Marconi *multiple timed spark* scheme of generating continuous oscillations are used somewhat by the Marconi Co.; neither of these schemes has much engineering merit as a commercial device.

The Oscillating Triode (triode used as a converter; see section on *Triode* above) is used in practically all continuous wave transmitters of small output, say less than 1 kw.; in special tests it has been used in batteries to give several kw. output, but in the present state of its development it cannot be regarded as a competitor of the other schemes outlined, unless the required wave-length is less than about 800 meters, in which case the triode is the only available scheme.

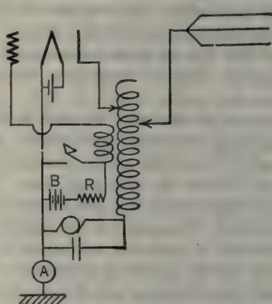

Fig. 29.

A typical triode circuit is shown in Fig. 29. The oscillations are started and stopped by opening the grid circuit. It is best not to leave the grid "free," as would be the case if the key only were used; a good scheme is to have a battery of small dry cells connected across the key through a high resistance *R*. When the key is open this battery forces the grid to such a high negative potential that the plate current is brought to zero; the resistance *R* prevents short-circuiting the battery when the key is closed.

Sometimes a triode transmitter is used to send *modulated* continuous-wave signals. When the key is pressed it not only starts the triode to oscillate, but closes a buzzer circuit which opens and closes the antenna circuit at buzzer

frequency, that is, at an audible frequency. The signals sent out by this method can be received by the ordinary crystal detector set, the transmission being practically the same as a spark station with very low decrement.

Reception of Continuous-Wave Signals. — Beat Reception. — As already pointed out, a continuous-wave signal arriving at a spark station receiving set will give no audible response in the telephones because there is no "wave-train frequency" or "spark frequency" to give the audible note. The scheme used universally to make a continuous wave signal audible is one due to Fessenden; it depends upon the creation of a "group," or "wave-train frequency" right at the receiving station, and is known as "beat reception." If a frequency of 100,000 is impressed on the receiving set by the incoming signal, and if there is already in the receiving set a continuous oscillation of 99,000 (or 101,000), these two frequencies together result in the actual current in the receiving circuit having variations in its amplitude, these variations having a frequency equal to the difference in the two frequencies present, in the above case 1000 variations or beats per second.

Originally Fessenden used a small oscillating arc to continually excite the receiving circuit, but to-day an oscillating triode is always used. The circuit is arranged as shown in Fig. 24, the coupling between L_1 and L_4 being adjusted to produce oscillations. The frequency of these oscillations will be fixed by the natural frequency of the L_1–C_1 circuit, and can be varied at will by changing C_1. Thus the beat frequency can be made just what the individual operator desires.

The triode rectifies this beat frequency just as it does the wave-train frequency from a spark station, and the operator hears the musical note of the beats; the musical note of such a signal is thus not independent of the wave-length of the transmitting station, as it is for a spark station.

This beat method of reception gives a wonderful degree of selectivity, thereby tending to prevent interference between different stations. Suppose one continuous-wave station is sending on 600 meters and another in the vicinity is sending on 605 meters. If the receiving station set is oscillating at 501,000 cycles per second the 600-meter signal will give a note in the phones of 1000, whereas the 605-meter signal will give a muscial note of about 5000 per second, practically out of audibility, whereas the desired signal note (1000) is in the most sensitive region of the ear and telephone. From what has already been said of tuning, it is evident that two spark stations sending on 600 meters and 605 meters would interfere very much, one could not be tuned out with tuning-out the other also.

Even if the two continuous-wave stations differed by only one meter in 600 they would still be easily distinguishable. The undesired signal may be made to produce with the local oscillations a beat frequency of zero; that is, the local receiving circuit may be made to oscillate at exactly the same frequency as the transmitter of the undesired signal. The desired signal would then produce a beat frequency of about 1000 per second and the interfering signal be entirely inaudible, no matter with what intensity it is arriving at the receiving antenna.

Autodyne and Heterodyne Reception. — When the detecting tube itself is used to produce the local oscillations the scheme is called "autodyne reception," and when another triode, separate from the detecting tube, is used to produce the local oscillations it is called "heterodyne reception." This distinction in names is not widely used in the literature, however, the term heterodyne being used for both methods very frequently. It will be noticed that in autodyne reception the local circuit L_1–C_1 cannot be tuned exactly to the incoming frequency, as the beat frequency for this adjustment would have

zero value and so the signal be inaudible. In the heterodyne scheme the local circuit L_1–C_1, as well as the antenna circuit, can be exactly tuned to the signal frequency, and furthermore the amount of local oscillation impressed on the detector circuit (generally by mutual induction) can be regulated, and thus maximum response in the detector be obtained.

The amplitude of the local frequency impressed in the detector circuit should be slightly less than that existing when the tube is used for autodyne reception with the minimum value of M which will sustain oscillations. In other words, the local frequency should carry the plate current variation (Fig. 13) as far as possible without forcing it over the upper and lower knees of the curve, say from a to b of curve E_3.

RADIO TELEPHONY. — Radio-telephone conversation may be carried on by means of a suitably *modulated* high-frequency radiation. Some sort of continuous-wave, high-frequency generator is connected to the antenna, and thus radiates continuous-wave power and a receiving station, tuned to this frequency will be excited by it. If now the amplitude of this high-frequency wave is varied by some scheme, in accordance with voice waves, the envelope of the high-frequency radiation will resemble the sound waves of the voice. At the receiving station the ordinary crystal rectifier will give in the phones a sound corresponding to the envelope of the arriving radiation, and so will reproduce the voice actuating the transmitting station. Thus the only additional feature required by a radio-telephone transmitter over those required for a radio-telegraph transmitter, is a scheme which varies the radiated power according to voice sounds, instead of breaking it up into dots and dashes as does the ordinary sending key.

Microphone in Antenna Circuit. — The simplest scheme of sending out voice modulated waves is that shown in Fig. 30; the high-frequency generator

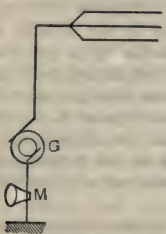

Fig. 30.

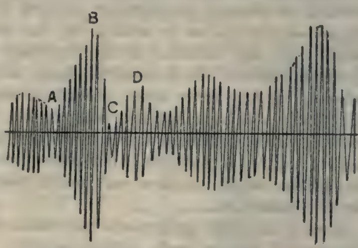

Fig. 31.

G excites the antenna in series with which is the microphone M. When spoken into the microphone varies its resistance above and below normal, and so varies the amplitude of current flowing in the antenna, the antenna current thus having a form as shown in Fig. 31. Such a wave arriving at a receiving circuit will give a noise in the phones corresponding to the envelope of the current amplitude, A–B–C–D, which represents the form of a voice sound wave. This method of modulation is far from perfect, is very inefficient (because of high microphone resistance), and is good for only small amounts of power, at most a few watts. If too much current is allowed to flow through the microphone it overheats and ceases to function.

Variable Induction Modulation. — A scheme used in a large radio-telephone station is shown conventionally in Fig. 32; it is due to Alexanderson

The radio frequency generator G is shunted by a path in which there is included an iron core inductance; wound on this core is an extra coil, the current from which is obtained through the plate circuit of a battery of high-powered triodes (G. E. pliotrons). The grid circuit of the triodes is actuated through a transformer and microphone M so that the plate current of the triodes is controlled by the microphone. The variation in the flux density of the iron core acts so that the shunt path around the generator C–L–C more or less short-circuits the generator, thus varying the amount of power applied to the antenna more or less in accordance with the voice sounds actuating the microphone. This scheme has been used to modulate outputs of perhaps 50 kw. The actual circuit is more complicated than the one shown, which is designed to give the elementary idea only.

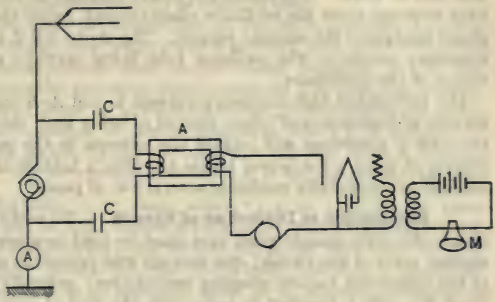

Fig. 32.

Modulation by Triode Directly. — A scheme of modulation due to Heising is shown in Fig. 33. It is by far the best scheme so far used for low-powered sets, say less than a few kw. The oscillator tube is connected to the antenna as shown, the plate current being supplied by generator G through the iron choke coil K and the high-frequency choke coil L. In small telephone outfits (300-volt tubes with an output of 3–5 watts) K is about 2 henries and L about 2 millihenries. The grid excitation for producing oscillation is controlled by the condenser C which is about twice the antenna capacity, for tubes having an amplification factor of 4. The wave-length generated is affected somewhat by the value of C, but is controlled primarily by the position of tap A. To get fine variation of the wave-length a small variable capacity condenser is usually placed in parallel with the antenna capacity. The position of tap B must be suitably chosen to get maximum current in the antenna circuit, this position varying with the resistance of the antenna.

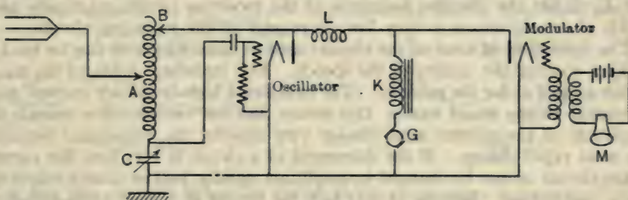

Fig. 33.

The modulator tube draws its plate current through the same iron-core choke coil as does the oscillator, and the amount of current taken by the modulator is controlled by the microphone M operating on the grid of the modulator tube. Because of the high reactance of coil K the current furnished to the two plate

circuits in parallel will be essentially constant, so that as the microphone makes the modulator take more or less current, the oscillator plate circuit is supplied with less or more. The amount of power supplied to the antenna by the oscillator depends upon the oscillator plate current, hence the action of the microphone modulates the antenna output. The choke coil L is to prevent the high-frequency output of the oscillator tube being partially absorbed by the plate circuit of the modulator.

The modulated high-frequency current in coil A–B may be used to excite a battery of high-powered tubes, instead of being itself the radiation current; in this case a condenser will replace the antenna in the oscillator circuit. This scheme of using a low-power triode to excite a battery of larger ones makes it possible to successfully modulate several kw. of power.

Elimination of Distortion of Speech. — To make the modulation follow closely the voice sounds great care must be used in properly proportioning the various parts of the circuit; the average grid potentials of the two tubes is one of the important factors affecting modulation; generally a suitable battery of small dry cells is inserted in series with the grid of each tube to maintain the average grid potential at its right value. The modulation with a well-designed Heising circuit is so perfect that the voice of the speaking operator is easily distinguishable at the receiving station, the articulation being much better than it is over the ordinary wire line.

Receiving Circuit for Radio Telephony. — At the receiving station a triode detector is used, connected as in Fig. 24. The coupling of L_1 and L_4 must not be adjusted as critically (for high amplication of signal) as is possible without having the tube oscillate, because the speech becomes drummy, the consonants being obliterated. As shown previously, the critical coupling of the coils L_1 and L_4 makes the effective resistance of the receiving circuit practically zero, which of course means a very low decrement of the receiving circuit.

The decrement of none of the circuits used in radio telephony can be too low without spoiling the quality of the speech. This statement holds for the transmitter as well as for the receiver. The modulated high-frequency current must be similar to the sound waves; this means that for certain voice sounds the amplitude of the current must change very rapidly, the consonants requiring the most rapid change. If the decrement of a circuit is very low, the current in the circuit cannot be changed in amplitude rapidly, and so cannot follow the voice modulation. Because of this fact the tuning of a good radio-telephone receiver is not as sharp as one designed for the reception of continuous-wave telegraph signals.

Two-Way Conversation. — Although the common small radio-telephone outfits require the operator to throw a switch to change from sending to receiving position, it is possible to have this done automatically by a sort of bridge scheme; it is also possible to have two separate antenna systems, one for sending and one for receiving, so interconnected that two-way conversation is obtained as in ordinary wire telephony.

ATMOSPHERIC DISTURBANCES AND THEIR ELIMINATION. — Space is continually agitated by electric disturbances, caused by ionization due to winds or the sun's rays, or by thunder storms with their electric discharges. Especially in the tropical regions are these "atmospherics" or "static" or "strays," as they are variously called, prevalent, probably because of the trade winds. Judging from the effect these electric disturbances have on an antenna it seems that they are radio pulses of very short duration, probably for the most part non-oscillatory. Due to their action all kinds of crackling and hissing noises are heard in the phones of a receiving set. These atmospheric disturb-

ances are the present limit on the utility of radio communication; were it not for these extraneous undesired signals coming into the receiving set, it would be possible to carry on transoceanic communication with one kilowatt of power, or less, instead of several hundred kilowatts as must be used.

Many investigators have put forth solutions of the static problem, but it is still to be solved. Even the very high-powered stations with their wonderful transmitting and receiving circuits have to send their messages slowly, and repeat continually because of the static interference in the receiving station. The partial elimination of static is possible by using a coil antenna, properly combined with a ground antenna, or similar scheme in which it is endeavored to balance out the static picked by the two antennæ and still leave a little of the signal unbalanced; this is then amplified until it is intense enough to read.

DIRECTIONAL RADIO. — As radio waves are of exactly the same nature as light waves, and as light waves are directive, can be focussed, bent, etc., it would seem that radio waves could be sent out in one direction only, like the beam of a searchlight. Such is the case theoretically, but it must be remembered that a mirror, or lens, to be effective, must be of dimensions hundreds of times the wave-length to be reflected or focussed. With radio waves thousands of meters long the use of mirrors, etc., is evidently out of the question.

The form of antenna used does give a certain amount of directional effect, as was first pointed out by Marconi. The type of aerial shown in Fig. 6 sends out considerably more power in the direction of its length than it does sideways. The coil antenna is more directive than any other; it sends and receives no power at all in a direction perpendicular to its plane. The distribution of radiated power from an inverted L antenna is as shown in Fig. 34 and that of a

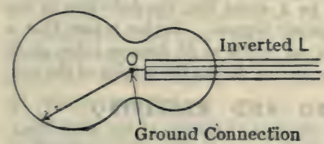

Fig. 34.

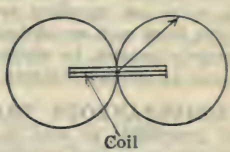

Fig. 35.

coil in Fig. 35; in each figure the radius to the locus gives a measure of the radiated power in that direction.

By suitable combinations of two or more antennæ it is possible to get somewhat more directive effects than those shown here. Practically all of the later attempts in this field follow very closely the original work of Bellini and Tosi, who were the pioneers in this branch of radio communication.

RADIO COMPASS. — It is now practicable for a ship at sea to get her bearings by radio, an especially useful operation for a ship approaching a harbor in a fog. Suppose the harbor is at *O*, Fig. 36, and the ship at *A*. The ship calls up station *O* and asks for bearings. *A* then sends out a standard signal of some sort for a short time, and by suitable coil antennae the stations at *C*, *O*, and *B* get her direction simultaneously. The direction as obtained at *C* and *B* are sent in to the central station at *O;* the operator at *O* is thus able to plot accurately the ship's position on his chart, using one of

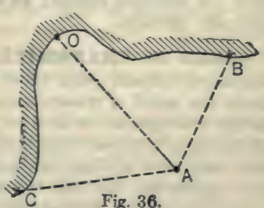

Fig. 36.

the directions as a check on the other two. Operator at O then radios back to the ship her exact position.

A scheme used by aeroplanes for getting their bearings at night or above the clouds utilizes the directive effect of a coil antenna, in this case the coil being on board the plane. A series of land stations send out characteristic signals (each station a different one, of course) and the operator on the plane, by orienting his coil, gets his bearings from two of the stations and so can plot his position. To make the directive effect as sharp as possible, two coils at right angles to each other are used, Fig. 37; they can be connected in series direct or reversed by the double-pole double-throw switch. When the plane of one of the coils is in the direction from which the signals are coming, reversing the switch will have no effect on the signal strength, as the other coil should be getting nothing. By this scheme a plane can get the direction of the waves to within one degree.

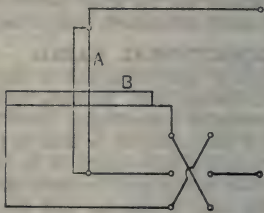

Fig. 37.

Inaccuracy of the Radio Compass. — Although the setting of such an antenna as that pictured in Fig. 37 can be carried out to such a wonderful degree of precision, the observation is not as valuable as the precision would seem to justify. Radio waves are subject to the same laws of refraction as light, and so may bend considerably, depending upon the condition of the atmosphere through which they are travelling. This effect has been noticed to a very great extent when the waves are travelling nearly parallel to a shore line, the deflection of the waves from such an obstruction being many degrees. Taylor states that a change of bearing, by wireless compass, of as much as 68 degrees takes place in only 14 minutes, this very large effect occurring during the setting of the sun.

SIMULTANEOUS TRANSMITTING AND RECEIVING. — It is not practicable to send and receive on the same antenna at the same time, although at present developments are taking place in that direction. It is however, practicable to send from one antenna and receive on another in the vicinity. This is generally accomplished by having at the receiving station two antennæ, one of them being used to balance out the intense signal induced in the real receiving antenna by the nearby transmitting antenna. The extra antenna is generally a directive one so oriented that it picks up a maximum signal from the transmitter. By suitably combining, in the coils of the detector set, the signals from this compensating antenna with those from the receiving antenna, the signal from the nearby transmitter may be completely neutralized, leaving the desired signal from the distant transmitter.

In most such stations the transmitter station is operated by remote control from the receiving station; all keys, etc., at the transmitter station being magnetically operated.

SPEED OF TRANSMITTING. — A good operator can read 25 to 30 words per minute, when there is not too much interference. This evidently limits the speed of transmission, unless some type of automatic receiver, such as the phonograph, be used at the receiving station. Up to the present none of the many schemes of mechanical reception has proved as efficient as an operator, and they are but infrequently used. Successful experiments have often been conducted, but the fact that practically all of the high-powered stations to-day are sending at slow speed shows that they are not dependable.

When high-speed sending does come into practice another effect, which cannot be analyzed in detail here, must be considered. It has been stated that the tuning of continuous waves is very sharp, so that interference between different stations is reduced to a minimum. This statement does not hold good if the transmitting is done at high speed, the higher the speed the more interference will different stations produce on one another. An antenna does not start to radiate full power the moment the transmitting key is closed, it taking an appreciable time for the antenna to build up to the steady state. With high-speed sending the time for the steady state to be set up becomes comparable with the time the key is closed, and under such conditions the conclusions one reaches from the steady state condition are of no value.

WAVEMETERS. — The wavemeter, although a very simple device, is the most important the radio engineer has at his command. It is essentially nothing but a coil and condenser in series, one or both of them variable, with means for telling when the current in the circuit is a maximum. As it is simpler to construct a variable condenser, of wide capacity range, than it is to do the same thing for an inductance, it is the condenser which is generally variable.

Evidently this circuit, coupled to another in which there is a high-frequency current, will have maximum current when it is in resonance with the e.m.f. induced by that circuit, and if the values of the capacity and inductance of the coil are known for all settings, the frequency of the impressed e.m.f. is obtained by adjusting the wavemeter for resonance, and then noting the natural frequency of the circuit for this setting. As the radio engineer generally thinks in terms of wave-length rather than frequency, the meter is calibrated, on the condenser scale, to give wave-length in meters.

As a variable condenser for this purpose is usable from its maximum value to about 1/10 of its maximum value, the wave-length range of a meter with a single coil would be only about 3 to 1. To extend the range of the meter several coils, of different inductance, are generally furnished with it. In one type of wavemeter, for example, having a wave-length range from 100 meters to 6000 meters six coils are supplied.

The methods for detecting resonance differ according to the amount of power available. If the test is being made on a transmitting set (plenty of power), a hot-wire ammeter, which is permanently connected in series with the coil and condenser (*A* in Fig. 38) is used to show maximum current. Another scheme is to shunt the condenser with a small glass tube filled with neon (or other easily ionizable gas) at low pressure, as indicated in Fig. 38 by dotted lines; when the wavemeter is in resonance with the circuit being tested, there will be maximum voltage across the condenser and so maximum glow in the tube.

In still another scheme there is a small incandescent lamp connected in series with the coil and condenser, where the ammeter is in Fig. 38; this lamp is brought almost to incandescence by a dry cell in the wavemeter, the circuit to this cell having choke coils to keep out the high frequency. When the wavemeter is in resonance its current flows through the lamp in addition to that already flowing from the cell, and the lamp glows more brightly. This scheme is better than would seem likely, for only a slight increase in current is required to give a perceptible change in the color of the lamp, so that the resonance point can be located quite accurately.

When testing on a low-power set, such as a received signal, there is not enough power available to actuate either a hot-wire meter or a neon tube, so some more delicate means of detection must be used. For such a case the telephone and rectifier are shunted across the condenser as shown in Fig. 39. Sometimes the circuit is opened at the point indicated by *X* in the figure, and an additional

connection made as shown by the dotted line. Such a connection draws less power from the wavemeter, and so does not increase its decrement as much

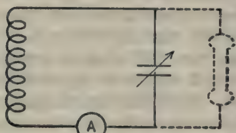

Fig. 38.

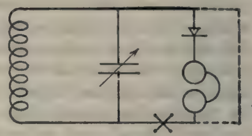

Fig. 39.

as does the other connection, thereby giving sharper resonance. This connection is often called a unilateral connection.

Uses of the Wavemeter. — The wavemeter is used principally to measure the wave-length radiated from a transmitter, as the name indicates. In doing this the coil of the wavemeter is held in proximity to the oscillating circuit, and the condenser varied until maximum indication is obtained in the hot-wire meter, phones, or other detector used. To get an accurate indication it is advisable to have the wavemeter coupled to the oscillating circuit as loosely as possible and still get a positive reading on the indicating device. This will make the indication sharp, and also, with loose coupling, there is no danger of the wavemeter changing the frequency of the oscillating circuit.

The wavemeter is also used to measure the decrement of a transmitting set. To do this the meter is brought in proximity to the antenna and adjusted for resonance as indicated by the hot-wire meter; the coupling of the meter to the antenna is then varied so as to get a reading well up on the scale of the ammeter and this reading is noted. The wavemeter is then detuned until the ammeter deflection indicates 0.707 of the current which flows at the resonance setting, and the wave-length for this reduced reading noted. Similarly the wavemeter is detuned on the other side of the resonance point until ammeter again indicates 0.707 of its resonance value and this wave-length noted. It frequently happens that the hot-wire meter used is calibrated in terms of current-squared; thus many German meters are calibrated in current-squared, and the scale values are marked off in watts used in the meter itself. Such meters are labelled "wattmeters," although evidently they are not wattmeters in the ordinary meaning of that term. In getting the two-detuned points with such an indicating instrument, the wavemeter setting is varied until the scale reading is just ½ of its value at resonance, this being the proper setting for 0.707 of the resonance current.

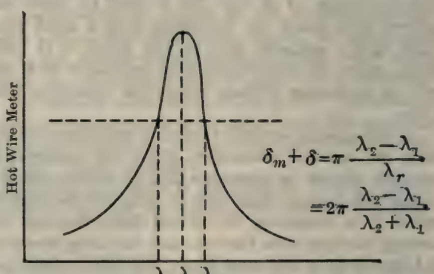

Fig. 40.

As the setting of the wavemeter condenser is varied the readings of the hot-wire ammeter will follow a curve as indicated in Fig. 40. λ_r corresponds to the resonance setting, λ_1 to the detuned condition above resonance, and λ_2 to the detuned condition

below resonance. The value of the decrement is then obtained from the relation (closely approximate),

$$\delta_m + \delta = 2\pi \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \pi \frac{\lambda_1 - \lambda_2}{\lambda_r}.$$

In this equation δ is the decrement desired, and δ_m is the decrement of the meter itself, generally given with the meter. If this is not known, it may be obtained by performing the same test with a continuous-wave transmitter; as the decrement of the set is in this case zero, the decrement obtained from the relation above is the decrement of the meter itself. An oscillating triode is a convenient power source for making this test.

Decremeter.— Kolster has designed a wavemeter with a special scale attached and having a condenser with plates properly shaped so that it is possible to actually read the decrement of the set (plus that of the meter) from the setting of the special scale, after carrying out a simple operation; this wavemeter is called a decremeter.

POWER REQUIRED IN RADIO TRANSMISSION. — The transmission of radio waves is an irregular phenomenon, as previously mentioned, varying throughout the year as well as during the day. The strength of signal required at the receiving station for successful transmission is not a constant, but varies greatly with the amount of atmospheric disturbance present; this variation is probably 100 to 1. Thus it is evident that exact figures cannot be given, but the values given in the table below are average values for a spark station having a high musical note (1000 sparks per second). The power rating is that of the input to the primary of the power transformer.

Power supplied	0.3 kw.	1.0 kw.	5 kw.	30 kw.
Range in miles in day time..	50–100	125–250	250–500	500–1200

For continuous-wave transmitters the distances successfully covered are much greater than those given above, perhaps two to three times as great. The larger stations carrying on transoceanic communication all use between 100 kw. and 500 kw. of power, in the antenna itself, the power supplied to the prime mover being nearly twice this figure. Even with this amount of power the communication is not certain, as they transmit very slowly, often repeating a signal, during the summer time. During the winter months the transmission is much more certain, and in many cases it is feasible to communicate half way around the world.

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RAILS, TRACK AND THIRD. — (*See also Bonds, Railway Track; Railways, Electric, Traction Systems for; Third-rail Systems; Trolley Systems.*) The A. E. R. A. Specification says that, unless otherwise specified, the lengths shall be 60 and 62 feet.

The American Electric Railway Association recommends that plain girder rails be used in paved streets except in large cities where the vehicular traffic is largely confined to the pavement area maintained by the railway. In the latter case, grooved rails should be used. The association has standardized the following rail sections which are described fully in its Engineering Manual.

Plain girder rails		Grooved rails	
Height, in.	Weight, lb. per yd.	Height, in.	Weight, lb. per yd.
5⅛	80	7	122
5⅝	90	9	134
7	80 or 91		
6	100		

RESISTANCE AND CHEMICAL COMPOSITION. — The chemical composition of steel rails with respect to the impurities or elements other than iron (chiefly carbon, manganese, phosphorus, sulphur and silicon) varies over a considerable range, depending upon the process of manufacture. According to J. A. Capp, the specific resistance of an ordinary * steel rail may be taken as a rough indication of the total impurities present. Fig. 1 shows the results of tests on a number of samples of different makes, ranging in total impurities

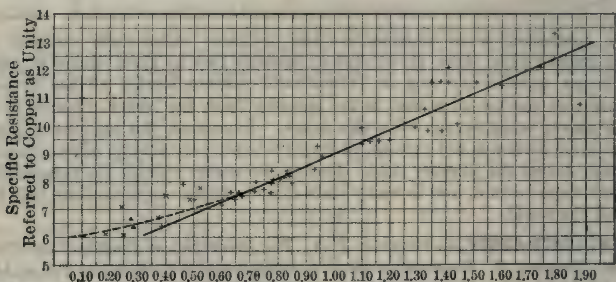

Fig. 1. (From Trans. A.S.M.E.)

from 0.1 per cent to 1.9 per cent, the specific resistance referred to copper as unity ranging from 6.1 to 13.3. The greater hardness caused by the presence of impurities is an advantage in the case of track rails which offsets the disadvantage of low conductivity, but it is usually economical to employ for the third or contact rail a rail of fairly high conductivity.

* This does not apply to special forms of rails submitted in the process of rolling to extra heavy pressures.

The compositions given in the following tables are recommended by the authorities quoted.

PER CENT IMPURITIES AND SPECIFIC RESISTANCE OF STEEL RAILS

Item	Track rails			Third rails		
	Am. El. Ry. Assoc.			J. A. Capp		A. H. Armstrong
	Lower limit	Recommended	Upper limit	Recommended	Upper limit	Recommended
Carbon	0.60	0.68	0.75	0.15	0.20	0.12
Manganese	0.60	0.80	0.80	0.30	0.40	0.40
Phosphorus	....	....	0.04	0.06	0.06	0.10
Sulphur	....	....	....	0.06	0.06	....
Silicon	....	....	0.20	0.05	0.05	0.05
Approx. spec. res. referred to copper as unity	12.5			8.0		

PER CENT IMPURITIES AND SPECIFIC RESISTANCE OF SOME TYPICAL THIRD RAILS

Railroad	Per cent of impurities					Spec. res. referred to copper as unity
	Carbon	Manganese	Sulphur	Phosphorus	Silicon	
Long Island	0.080	0.022	0.029	0.074	0.074	7.55
District & Met.Ry., London	0.05	0.19	0.06	0.05	0.03	6.4
Manhattan Ry	0.098	0.485	0.158	0.085	0.022	8.98
I. R. T. Subway, N. Y.	0.161	0.561	0.055	0.091	trace	8.56
New York Central	0.10	0.40	<0.08	<0.10	<0.05	7.85
Detroit River Tunnel	0.10	0.40	<0.08	<0.10	<0.05	7.85

Allowance for Wear in Resistance Calculations. — Resistance calculations should be made for rails worn down to the weight at which they will be scrapped. It is usual to scrap rails when they have lost from 10 per cent to 20 per cent of the original weight, depending upon the importance of the line. The

resistances in the following table should therefore be increased from 10 per cent to 20 per cent.

RESISTANCE OF T-RAILS, A.S.C.E. STANDARD SECTION (1913)

(*Full Cross section*)

Weight, lb. per yard	Cross section, sq. in.	Area, millions of circular mils	Spec. res. 12.5 times that of copper		Spec. res. 8 times that of copper *	
			Ohms per 1000 ft.	Ohms per mile	Ohms per 1000 ft.	Ohms per mile
40	3.90	4.95	0.0261	0.138	0.0167	0.0882
45	4.40	5.60	0.0231	0.122	0.0148	0.0782
50	4.90	6.23	0.0208	0.110	0.0133	0.0702
55	5.40	6.86	0.0189	0.0996	0.0121	0.0637
60	5.90	7.50	0.0173	0.0911	0.0110	0.0583
65	6.40	8.14	0.0159	0.0840	0.0102	0.0538
70	6.90	8.77	0.0148	0.0779	0.00944	0.0499
75	7.437	9.45	0.0138	0.0729	0.00884	0.0467
80	7.80	9.9	0.0131	0.0689	0.00835	0.0441
85	8.34	10.5	0.0122	0.0645	0.00781	0.0413
90	8.83	11.2	0.0115	0.0609	0.00738	0.0390
95	9.30	11.8	0.0109	0.0570	0.00701	0.0370
100	9.82	12.5	0.0104	0.0547	0.00664	0.0350

* To find the resistance of rails of any specific resistance x referred to copper as unity multiply these resistances by x and divide by 8.

A-C. Resistance and Reactance. — See articles on *Trolley Systems, Overhead; Signaling, Railway.*

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RAILWAYS, ELECTRIC, TRACTION SYSTEMS FOR. — (*See also Automobiles, Electric; Bonds, Railway Track; Car Barns and Inspection Sheds; Cars, Electric; Conduits and Conduit Lines; Control Systems for Railway Motors; Depreciation; Electrolysis; Locomotives, Electric; Locomotives, Steam; Motors; Power Stations; Rails, Track and Third; Rectifiers; Signaling, Railway; Specifications; Substations, Railway; Third-rail Systems; Railways, Energy Requirements for; Railways, Location and Permanent Way for; Transmission Lines; Trolley Systems; Wires and Cables.*) The component parts of a railway system are the following: Generating station, usually three-phase alternating current; high-tension transmission line; substations with transformers and synchronous converters; low-tension distribution system, trolley and feeders; motive power or motor equipments; bonding and feeders for return circuit.

HISTORICAL DEVELOPMENT. — The history of electric traction is given briefly by a few salient epoch-making events.

1835. T. Davenport of Vermont built and exhibited a small model electric vehicle operating on a circular track.

1842. Davidson, Edinburgh, Scotland built a seven-ton car operated by primary batteries and electro-magnets.

1851. G. G. Page operated a small locomotive with primary batteries on the Baltimore and Ohio Railroad at Washington.

1877. S. D. Field in San Francisco operated a small car driven from a stationary remote generator.

1882. J. R. Finney first used the over-running trolley at Allegheny, Pa.

1883. Leo Daft equipped a full-sized passenger train running from Saratoga to Mt. McGregor, together with the over-running-trolley construction.

1886. Bently and Knight introduced the first conduit system in Cleveland, O.

1887–8. Several small installations.

1888. Frank Sprague equipped the Richmond Street Railway and demonstrated the practicability of the system. Bentley and Knight equipped a road at Allegheny City. Thomson-Houston system installed at Washington, D. C. Van Depoele brought out the under-running trolley. The directors of the Boston West End System decided to adopt electric traction.

1896. Baltimore and Ohio tunnel at Baltimore electrified; the first electrification of a steam road.

For references to the original papers describing these early installations, see Burch, *Electric Traction for Railway Trains*, N. Y., 1911.

APPLICATIONS OF ELECTRIC TRACTION. — There are certain well defined fields in which electric traction is superior to other methods, the most important of which are the following:

1. Where the frequency of stops is so great as to require a high rate of acceleration in order to make a good schedule speed. If the service requires trains of several cars, any desired number of these cars can be made motor cars and thus a sufficient weight on the driving wheels can be economically secured to give the required adhesion and tractive effort.

2. Where local conditions prohibit the nuisance of the smoke, exhaust gases and noise of steam locomotives, as in cities and tunnels.

3. In heavy trunk-line service where the density of traffic is so great that a high load factor can be obtained with respect to both the power house and distribution system. The *operating* cost of an electric train is always less than the operating cost of a steam train but the *fixed* charges (interest on investment and depreciation) of the electric system are high, for the first cost for the electric equipment, viz., locomotives, motor-car equipment, distribution

system and power house, is much greater than the first cost of steam locomotives, and their accessory equipment. If there are sufficient trains in a system so that the pro rata share of the fixed charges for each train is less than the difference between the operating cost of an equivalent steam equipment and the operating cost of the total electric equipment then electric traction is advantageous.

SYSTEMS OF ELECTRIC TRACTION. — Three different types of motors are in use for electric traction, the direct-current motor, the single-phase commutator motor, and the three-phase induction motor; see articles on *Motors*. The trolley or third-rail voltages (i.e., volts between trolley and track rails or between third rail and track rails) and the motor voltages employed are as follows:

Direct-current Systems. — Trolley or Third-Rail voltages of 600, 1200, 1500, 2400 and 3000 are in use. The motors for the 600-volt system are designed for operation at full trolley voltage. For all the other systems two motors are permanently connected in series electrically, each designed for half the trolley voltage.

Single-phase Systems. — Trolley voltages (third rails are not used) of from 3000 to 11,000 volts and frequencies of 15 and 25 cycles per second are in use. The trolley voltage is stepped down to from 200 to 500 volts, for which voltage the motors are designed.

Three-phase Systems. — Two trolley wires for each track are required, the two wires and the track forming the necessary three conductors for the three-phase distribution. The usual frequency is 25 cycles per second. The voltages used between the two trolleys and between each trolley and track range from 6000 to 11,000 volts. Transformers are employed to step this three-phase voltage down to from 400 to 600 volts between motor terminals.

Comparison of the Various Systems. — For ordinary street railway service the 600-volt d-c. system is almost universally employed, but for interurban and trunk-line service there is a great difference of opinion as to which of the various systems is the most economical when all the factors are taken into account. The factors which must be considered in comparing the three systems in any particular case are the following:

1. For a given weight and length of trolley or third rail the per cent power loss for a given amount of power transmitted varies inversely as the square of the trolley or third-rail voltage.

2. The higher the trolley or third-rail voltage the fewer are the number of substations required for the same efficiency of distribution and weight of conductor.

3. The higher the trolley or third-rail voltage the more costly is the insulation and supporting structure, and also the greater is the cost of maintenance of the distribution system.

4. Both the first cost and the annual expense of the substations are less for the a-c. systems than for the d-c. systems, since for the former static transformers only are required whereas for the latter rotary converters must be used.

5. The relatively low power factor of a-c. motors (80 to 90 per cent) as well as the relatively low power factor of the line (due to the reactance of the trolley wire and track return) gives rise to a greater power loss in the a-c. distribution system for the same power delivered than in the case of the d-c. system, and this great loss and lower power factor makes necessary the employment of generating apparatus of greater kv-a. capacity.

6. The 600-volt d-c. motor, for the same horse-power rating and speed,

costs less, weighs less and occupies less space than either type of a-c. motor. The high-voltage d-c. motors cost more, weigh more and occupy more space than the 600-volt type.

7. With the a-c. motors transformers are required on the locomotive, which adds to the cost and weight of the locomotive equipment.

8. The 600-volt d-c. motor costs less to maintain and is liable to fewer operating troubles than any of the other motors.

9. With the commutating type of a-c. motor the power lost in the control equipment is practically negligible, since the "potential" type of control can be used. For both the d-c. motor and the induction motor a resistance control is necessary, with consequent loss in power (*see Control Systems for Railway Motors*).

10. The induction motor is inherently a constant-speed machine, and consequently the power input varies directly as the opposing resistance. The d-c. motor and the a-c. commutator motor are inherently variable-speed machines, and the power input varies approximately as the square root of the opposing resistance, the speed at the same time falling off.

11. The three-phase induction motor, when kept connected electrically to the source of power, automatically operates as a generator when the train is going down grade at a speed greater than the synchronous speed of the motor, the motor thus returning power to the line and at the same time acting as a brake preventing any considerable increase in speed. "Regeneration," as this action is called, can also be obtained with the other types of motors but only at increased expense for the additional control equipment required.

Examples of the Use of the Various Systems. — In the following tables are listed (1) the most notable examples of high voltage d-c. interurban roads in the U. S. A.; (2) the most notable examples of electrified steam railroads in the U. S. A.; and (3) a complete list of single-phase railways in the U. S. A. and Canada.

ANALYSIS OF AN ELECTRIC RAILWAY PROJECT. — In the analysis of a particular problem the following general line of procedure is followed:

1. Determine the number and capacity of cars to supply the service desired.

2. Determine the power and energy required to propel these cars at the schedule speed desired; see *Railways, Energy Requirements for.*

3. Select the motors to correspond to the power determined in 2; see *Railways, Energy Requirements for.*

4. Lay out distribution of cars, by train diagrams if necessary; see *Railways, Energy Requirements for.*

5. Calculate the capacity of the low-potential distribution system and of the substations; see *Substations, Railway; Railways, Energy Requirements for; Third-rail Systems; Trolley Systems.*

6. Determine the capacity of the generating stations and transmission system; see *Power Stations; Railways, Energy Requirements for; Transmission Lines.*

7. Estimate the first cost of the system; see articles on the various items involved.

8. Estimate the cost of operation; see articles on the various items involved.

9. Estimate the earning power of the system.

A. In the case of a new road the earning power must exceed the sum of the operating cost and fixed charges by an amount sufficient to pay dividends.

B. In the case of the electrification of a steam road it must be possible to show either: (1) that the result of electrification has been to reduce the operating charges by an amount more than sufficient to pay the fixed charges on the electrical apparatus, or, (2) that the result of electrification will increase the capacity of the road or attract sufficient new business so that the increased earning power will more than balance the increased fixed charges.

THE MOST NOTABLE EXAMPLES OF HIGH-VOLTAGE D.-C. INTERURBAN ROADS IN THE U. S. A. (1921)

Name of road	Date installed	Miles of single track	Trolley voltage	Motor cars	Locomotives	Trolley or third rail
Indianapolis & Louisville	1907	42	1200	10	0	T.
Central California Tr.	1908	71	1200	8	3	3d R.
Pittsburgh, Harmony & B.	1908	82	1200	31	0	T.
Milwaukee El. Ry.	1910	135	1200	26	0	T.
Washington, Baltimore & A.	1910	103	1200	53	3	T.
Ft. Dodge, Des Moines & S.	1911	145	1200	10	11	T.
So. Pacific, Oakland	1911	138	1200	91	0	T.
Oregon El., Portland	1912	180	1200	62	10	T.
Piedmont Ry.	1912	125	1500	33	12	T.
Kansas City, Clay & J.	1913	74	1200	27	[illegible]	T.
Oakland, Antioch & E.	1913	118	1200	18	6	T.
Texas El. Ry	1913	158	1200	28	2	T.
Pacific El., Los Angeles.	1914	92	1200	75	10	T.
Salt Lake & Utah	1914	77	1500	18	[illegible]	T.
Utah-Idaho Central	1915	97	1500	19	6	T.
Michigan Ry. Co. (G. R. to Kalamazoo & Battle Creek)	1915	95	1200	14	4	3d R.
Michigan Ry., Grand Rapids	1915	77	1200	10	0	T.

THE MOST NOTABLE EXAMPLES OF ELECTRIFIED STEAM ROADS IN THE U. S. A. (1921)

Name of road	Date installed	Miles of single track	Main line, tunnel or terminal	Trolley or third rail	Trolley voltage	System	Locomotive or motor cars
Baltimore & Ohio	1896	7.95	Tun.	3d R.	650	D. C.	L.
West Shore	1905	106	M. L.	3d R.	620	D. C.	M. C.
N. Y. Central	1906	268	Term.	3d R.	650	D. C.	L. & M. C.
West Jersey & S. S.	1906	150	M. L.	3d R.	600	D. C.	M. C.
Grand Trunk	1908	12	Tun.	T.	3,300	S. P.	L.
N. Y., N. H. & H.	1907	552	Term.	T.	11,000	S. P.	L. & M. C.
Great Northern	1909	6	Tun.	T.	6,600	3 P.	L.
Michigan Central	1910	26	Tun.	3d R.	600	D. C.	L.
Penn. & L. I. R. R	1910	316	Term.	3d R.	600	D. C.	L. & M. C.
S. Pacific, Oakland	1911	138	M. L.	T.	1,200	D. C.	M. C.
Boston & Maine	1911	21.5	Tun.	T.	11,000	S. P.	L.
Butte, Anaconda & P.	1912	114	M. L.	T.	2,400	D. C.	L.
Spokane Inland	1912	130	M. L.	T.	6,600	S. P.	L. & M. C.
S. Pacific, Portland	1914	162	M. L.	T.	1,500	D. C.	M. C.
C. M. & St. Paul	1915	650	M. L.	T.	3,000	D. C.	L.
Pa. R. R. Paoli & Chestnut Hill	1915	116	M. L.	T.	11,000	S. P.	M. C.
Norfolk & Western	1915	98	M. L.	T.	11,000	S.P. 3 P.	L.

COMPLETE LIST OF THE SINGLE-PHASE RAILWAY INSTALLATIONS IN UNITED STATES AND CANADA. (1921)

Name of road	Year started	Track miles	Trolley voltage	Manufacturer	Frequency
Indianapolis & Cincinnati	1904	107	3,300	West	25
*Atlanta Northern Railway	1905	15	2,200	West	25
*Illinois Traction Company	1905	95	3,300	G. E.	25
Long Island R. R., Sea Cliff Div	1905	5	2,200	West	25
San Francisco, Vallejo & Napa Valley	1905	34	3,300	West	25
*Warren & Jamestown	1905	26	3,300	West	25
Westmoreland Co. Traction Company	1905	7	1,200	West	25
Spokane & Inland Empire R. R	1906	162	6,600	West	25
*Toledo & Chicago Railway	1906	43	3,300	G. E.	25
*Anderson Traction Company	1907	20	3,300	G. E.	25
Erie Railroad—Rochester	1907	40	11,000	West	25
*Fort Wayne & Springfield	1907	22	6,600	West	25
*Milwaukee Elec. Ry. & Lt. Co	1907	68	3,300	G. E.	25
New York, New Haven & Hartford R. R.,	1907	500	11,000	West	25
*Pittsburgh & Butler	1907	39	6,600	West	25
Richmond & Chesapeake Bay	1907	16	6,600	G. E.	25
Windsor, Essex & Lake Shore	1907	40	6,600	West	25
*Maryland Elec. Rys. (Short Line)	1908	37	6,600	West	25
Chicago, Lake Shore & South Bend	1908	90	6,600	West	25
Denver & Interurban R.R	1908	54	11,000	West	25
Grand Trunk Ry.—St. Clair Tunnel	1908	12	3,300	West	25
*Hanover & York Railway	1908	21	6,600	West	25
Shawinigan Ry.—Quebec	1908	1	6,600	G. E.	30 & 15
Visalia Elec. Railway	1908	36	3,300	West	15
*Washington, Baltimore & Annapolis	1908	87	6,600	G. E.	25
Rock Island Southern Railway	1910	52	11,000	West	25
New York, Westchester & Boston	1911	63	11,000	West	25
Boston & Maine Railroad (Hoosac Tunnel)	1911	22	11,000	West	25
Pennsylvania R. R.—Paoli Div	1915	116	11,000	West	25
†Norfolk & Western Railway	1915	90	11,000	West	25
Total, 30 roads	......	......	1920	miles	
Changed to D-C., 11 roads	......	......	473	miles	

* Changed to direct current.

† Single-phase trolley. Single, 3-phase locomotives.

SELECTED EXAMPLES OF FOREIGN RAILROAD ELECTRIFICATIONS HAVING SPECIALLY INTERESTING FEATURES
(1921)

Name of road	Date installed	Miles of single track	Main line, tunnel or terminal	Trolley or 3d rail	Voltage	D. C. single phase or 3 phase	Locomotives or motor cars
Canadian Northern....	1915	30	Term.	T.	2,400	D. C.	Both
Hershey Ry., Cuba....	1920	80	M. L.	T.	1,200	D. C.	Both
Paulista Ry., Brazil....	1920	76	M. L.	T.	3,000	D. C.	L.
Bethlehem, Chili......	1918	24	M. L.	T.	2,400	D. C.	L.
Central Argentine Ry.,	1915	100	Term.	3d R.	800	D. C.	M. C.
England:							
London, Brighton & S. C..............	1909	70	M. L.	T.	6,700	S. P.	M. C.
N. E. Ry., Middlesboro..............	1915	50	M. L.	T.	1,200	D. C.	L.
Manchester & Bury...	1917	27	M. L.	3d R.	1,200	D. C.	M. C.
Rjukan Ry., Norway..	1918	286	M. L.	T.	15,000	S. P.	L.
Midi Ry., France......	1911	75	M. L.	T.	12,000	S. P.	Both
Germany:							
Dessau-Bittersfeld....	1910	33	M. L.	T.	10,000	S. P.	L.
Magdeburg-Halle......	1910	80	M. L.	T.	10,000	S. P.	L.
Switzerland:							
Simplon	1907	26	Tun.	T.	3,000	3 P.	L.
St. Gotthard.........	1918	30	Tun.	T.	15,000	S. P.	L.
Lotschburg..........	1910	46	M. L.	T.	15,000	S. P.	L.
Italy:							
Valtelina............	1904	70	M. L.	T.	3,000	3 P.	L.
Genoa-Giovi-Milan...	1904	26	M. L.	T.	3,000	3 P.	L.
Australia:							
Victorian Rys. Melbrn	1919	335	M. L.	T.	1,500	D. C.	M. C.
Imperial Rys., Japan ..	1915	40	M. L.	T.	1,200	D. C.	M. C.
S. Manchurian Ry., China..............	1914	43	M. L.	T.	1,200	D. C.	Both

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RAILWAYS, ENERGY REQUIREMENTS AND MOTOR EQUIPMENT FOR. — (*See also Locomotives, Electric; Motors; Power Stations; Railways, Electric, Traction Systems for; Railways, Location and Permanent Way for; Substations, Railway.*) From a consideration of the forces acting on a moving train it is possible to determine the motor capacity and energy required to operate it when the profile and contour of the road, the time table and the characteristics of the available motors are known. The various items are treated in the following sequence: -

UNITS AND ABBREVIATIONS. — Throughout this article the various quantities employed will be expressed in the following units unless specifically stated otherwise: distances in feet, weights in tons of 2000 pounds, forces in pounds, speeds in miles per hour (abbreviated mph.), accelerations in miles per hour per second (abbreviated mphps.), mechanical power in horse power, energy in watt-hours.

FORCES ACTING ON A TRAIN. — The forces tending to accelerate a train are the tractive effort developed by the motors and the component of the weight along the track on down grades. The forces which retard the motion of the train are the various frictional forces and the component of the weight along the track on up-grades; also in braking, the frictional force due to the brakes. All the various frictional forces, except the braking resistance, such as track friction, journal friction, air friction, etc., which oppose the motion of a train on a straight track are usually considered together and are referred to as the "train resistance." The extra friction due to track curvature is usually considered as an equivalent up-grade.

Tractive Effort and Draw-bar Pull. — The tractive effort of a motor is the force exerted by the motor at the rim of the driving wheel to which the motor is geared. The tractive effort of a locomotive is the force exerted by the locomotive at the rim of the drivers. The draw-bar pull of a locomotive is the force transmitted through the draw-bar of the locomotive,* and is less than its tractive effort by an amount equal to the resistance due to the rolling friction of the locomotive wheels on the track and the air resistance of the locomotive.

Direct-current motors deliver a uniform tractive effort for a given current; the tractive effort of alternating-current motors pulsates to some extent with the alternations of the current; the tractive effort of a steam locomotive varies

* In the case of a steam locomotive, the draw-bar pull usually refers to the force transmitted through the coupling between the tender and train; i.e., the tender is considered as a part of the locomotive.

from 28 to 50 per cent above and below its average value during each revolution of the drivers.

Train Resistance. — The total train resistance includes the following:

Rolling friction, which depends on the stiffness of the rail and the rigidity of the roadbed, and upon the speed and the weight of the train.

Journal friction, which depends on the speed, weight of the train, temperature and condition of lubrication.

Effect of oscillation and concussion, which depends upon the square of the speed, the weight of the train, and upon the condition of the track and roadbed.

Air resistance, which depends upon the square (approximately only) of the speed, the end area and length of train (number of cars), upon the shape of the end surface, and upon the kind of platforms or vestibules, but is independent of the weight of the train.

In view of the numerous factors which affect the train resistance, it is practically impossible to devise a formula for the resistance per ton of train weight which will be applicable to all types of cars and track construction. Numerous formulas have been suggested, most of which are of the form

$$r = A + Bv + Cv^2,$$

where r = the resistance in pounds per ton, v = speed in miles per hour and A, B and C are constants determined by experiment. The values of these constants naturally depend upon the type of cars, nature of roadbed, number of cars in the train, etc.; consequently the values of these constants as obtained from tests on various types of equipment and track construction differ considerably.

Burch (*Electric Traction for Railway Trains, New York, 1911*) gives the following values for these constants under normal conditions of track and weather. In winter the total train resistance may be as much as 60 per cent greater than the values calculated from the above formula, when these values of the constants are used. An electric locomotive is to be treated as a single car, its weight averaged with the weight of the cars. In the case of a steam locomotive and tender, the tender is to be treated as a car, but the locomotive resistance is to be added as a separate item (*see below*).

Value of *A*. — This constant depends chiefly upon the average total weight of car and load. Let w = this average total weight *per car* in tons; then for

$w =$	15	20	25–30	35	40–45	50	70
$A =$	6.0	5.5	5.0	4.5	4.0	3.5	3.0

Value of *B*. — This constant depends primarily upon the nature of the track and roadbed and also to some extent upon the weight and type of the car. Burch gives the following values:

Passenger cars on excellent track	0.06–0.11
Passenger cars on ordinary track	0.10–0.15
Freight cars on ordinary track	0.05–0.06

The heavier the car the higher the value of this coefficient.

Value of *C*. — This constant depends primarily upon the air resistance at the head and rear ends of the train and along the sides of the train. It may be expressed approximately by the formula

$$C = \frac{Ka\,(0.9 + 0.1\,N)}{Nw},$$

where a = the cross section of car in square feet, N = number of cars, w = average weight of car and load in tons, and K = 0.0010 for parabolic ends,

0.0020 for wedge-shaped ends, 0.0028 for vestibule cars, 0.0030 for open platforms, 0.0033 for freight cars and 0.0040 for flat fronts.

Tests made on three-car trains in the Boston-Cambridge subway gave a value of approximately 0.0050 for the constant K, the higher value being due to the extra air resistance in the tunnel. (*See Elec. Ry. Jour., 1912, Vol. 40, p. 280.*)

Burch's Table of Train Resistance. — Using the above values of the constants Burch has calculated the following table for train resistance.

TRAIN RESISTANCE FOR ELECTRIC TRAINS

N = number of cars (including electric locomotive, if any);
w = average weight of car loaded, in tons (= total weight of train divided by N);
r = train resistance in pounds per ton;
v = speed in miles per hour;
a = cross section of car in square feet;
A and B constants in the formula; K taken as 0.0030 throughout.

$$r = A + Bv + \frac{Ka\,(0.9 + 0.1\,N)\,v^2}{Nw}. \qquad (1)$$

TABLE I. — VALUES OF TRAIN RESISTANCE

	N	w	A	B	a	Speed in miles per hour					
						10	20	30	40	50	60
Passenger Trains	1	15	6.0	0.11	100	9.1	16.2	27.3	42.4	61.5	84.6
	1	20	5.5	0.12	100	8.2	13.9	22.6	34.3	49.0	66.7
	1	25	5.0	0.13	100	7.5	12.4	19.7	29.4	41.5	....
	1	35	4.5	0.13	100	6.7	10.5	16.1	23.4	32.4	....
	1	45	4.0	0.13	110	6.0	9.6	14.5	21.2	28.8	....
	2	15	6.0	0.11	100	8.2	12.6	19.2	28.0	39.0	52.2
	2	20	5.5	0.12	100	7.5	11.2	16.5	23.5	32.1	42.4
	2	25	5.0	0.13	100	7.0	10.2	14.8	20.8	28.0	36.5
	2	35	4.5	0.13	100	6.3	9.0	12.6	17.2	22.8	29.3
	2	45	4.0	0.13	110	5.7	8.2	11.5	15.6	20.5	26.3
	3	15	6.0	0.11	100	7.9	11.4	16.5	23.2	31.5	41.4
	3	20	5.5	0.12	100	7.3	10.3	14.5	19.9	26.5	34.3
	3	25	5.0	0.13	100	6.8	9.5	13.2	17.9	23.7	30.1
	3	30	4.5	0.13	100	6.2	8.7	12.0	16.1	21.0	26.7
	3	35	4.5	0.13	100	6.1	8.5	11.4	15.2	19.6	24.6
	3	45	4.0	0.13	110	5.6	7.8	10.5	13.9	17.8	22.4
	4	25	5.0	0.13	100	6.7	9.2	12.4	16.4	21.3	26.8
	4	30	4.5	0.13	100	6.1	8.4	11.3	14.9	19.1	24.0
	4	35	4.5	0.13	100	6.1	8.2	10.9	14.1	18.0	22.3
	4	45	4.0	0.13	110	5.5	7.6	10.0	13.0	16.5	20.4
	6	25	5.0	0.13	100	6.6	8.8	11.6	15.0	19.0	23.6
	6	35	4.5	0.13	100	6.0	8.0	10.3	13.1	16.4	20.0
	6	45	4.0	0.13	110	5.5	7.3	9.5	12.1	15.1	18.4
	8	35	4.5	0.13	100	6.0	7.8	10.0	12.5	15.5	....
	8	45	4.0	0.13	110	5.4	7.2	9.2	11.7	14.4	....
	12	45	4.0	0.13	110	5.4	7.0	9.0	11.2	13.7	16.4

TABLE I.—VALUES OF TRAIN RESISTANCE—*Continued*

	N	w	A	B	a	Speed in miles per hour					
						10	20	30	40	50	60
Freight Trains	10	30	5.0	0.06	110	5.8	7.0	8.7	10.7	....	....
	20	30	5.0	0.06	110	5.7	6.8	8.2	9.9	....	....
	30	40	4.0	0.06	110	4.7	5.6	6.7	8.1	....	....
	50	40	4.0	0.06	110	4.7	5.6	6.7	8.0	....	....
	40	50	3.5	0.06	110	4.2	5.0	6.0	7.2	....	....

Other Formulas for Train Resistance.—Of the numerous other formulas which have been proposed for train resistance the two most commonly employed are Armstrong's formula

$$r = \frac{50}{\sqrt{Nw}} + 0.03\,v + \frac{0.002\,a\,(0.9 + 0.1\,N)\,v^2}{Nw}$$

and Mailloux's formula

$$r = 3.5 + 0.15\,v + \frac{(0.25 + 0.02\,N)\,v^2}{Nw}.$$

The symbols have the same meaning as above. Armstrong's formula gives considerably lower values than those given by Burch, except for heavy trains; Mailloux's formula gives lower values than those given by Burch except for heavy trains at low speeds, but the difference in most cases is not very great. For other formulas proposed from time to time see Burch, *Electric Traction for Railway Trains.* See also an excellent paper on the resistance of freight trains by E. C. Schmidt in Trans. A.S.M.E., May, 1910.

Resistance of Steam Locomotives.—The American Railway Engineering Association recommend the formula for the resistance of a steam locomotive

$$r_1 = 18.7 + \frac{80\,X}{w_1},$$

where r_1 is the resistance (between cylinder and rim of drivers) per ton weight on the drivers, X is the number of driving axles and w_1 the total weight in tons on all the drivers. Tests by the American Locomotive Company showed that the resistance per ton weight on the drivers is 22.2 pounds.

Train Resistance at Starting.—The formulas given above are not applicable to speeds below about 10 mph. The New York Central tests on electric trains show that the train resistance decreases with decrease in speed to 10 mph., but that as the speed still further decreases the resistance per ton increases. The resistance at starting may be from 6 to 18 pounds per ton, depending upon the condition of the bearings, track, etc., and upon the duration of the stop preceding the starting. These figures also apply to freight trains. Tests on the Rock Island system showed that in the case of a train which had stood overnight in cold weather (i.e., which had become "frozen up"), the starting resistance was 30 pounds per ton. The slack in the car couplings, however, renders it unnecessary for a locomotive to exert sufficient effort to start all the cars at once.

Grades and Curvature.—(*See Railways, Location and Permanent Way for.*) An actual up-grade of G per cent produces a retarding force of 20 G pounds per

ton, and a down-grade of G per cent produces an accelerating force of $20\,G$ pounds per ton. A curve gives rise to a retarding force which may be represented by the formula:

$$r = 0.058\,SC,$$

where r = (excess) curve resistance in lb. per ton,
S = speed, m.p.h.,
C = degree of curvature.

(E. C. Schmidt and H. H. Dunn, Univ. of Illinois, Bulletin, No. 92.) Note that for angles of curvature up to 12 degrees the angle in degrees may be taken equal to $5730 \div R$ where R is the radius of curvature in feet.

ACCELERATION AND BRAKING. — The permissible rate of acceleration depends upon a number of factors.

1. The rating of the motors; the larger the motors the higher the tractive effort they can develop and therefore the greater the acceleration.
2. The weight on the driving wheels of the car or locomotive; the maximum tractive effort that a motor car can exert without slipping the wheels is from 15 to 20 per cent of the weight on the drivers (*see below under Adhesion Coefficient*).
3. The comfort of the passengers; the higher the acceleration rate the more difficult is it for a passenger to maintain his equilibrium. This also depends to some extent upon the uniformity of the acceleration.
4. To make a given schedule speed with the least amount of energy the acceleration rate should be as high as possible. Very high rates of acceleration, however, are not in general justified on this score, as the increase in the size of the motors required may more than offset the saving of energy.

The following rates of acceleration represent common practice:

TABLE II. — ACCELERATION RATES

Service	Miles per hour per second
Steam locomotive, freight service	0.1 to 0.2
Steam locomotive, passenger service	0.2 to 0.5
Electric locomotive, passenger service	0.3 to 0.6
Electric motor cars, interurban service	0.8 to 1.3
Electric motor cars, city service	1.5 to 2.0
Electric motor cars, rapid transit service	1.5 to 2.0
Highest practical rate	2.0 to 2.5

Braking. — The maximum retardation in braking is limited by the comfort of the passengers, and injury to equipment, a retardation of 1.5 miles per hour per second being the usual practical limit for electric or steam passenger trains, although 2.5 miles per hour per second is sometimes attained. For freight trains the braking retardation is from 0.7 to 0.8 miles per hour per second. The higher the rate of braking the less the energy consumption for a given schedule speed.

"Acceleration Constant." — The tractive effort required to give to one ton (2000 pounds) a linear acceleration of one mphps. is 91.2 pounds. To accelerate a train of W tons requires a tractive effort of $91.2\,aW$ pounds to produce a linear acceleration of a mphps., but on account of the accompanying angular acceleration of the rotating parts an additional force is required. This

additional force is proportional to the linear acceleration a and also depends upon the radius of gyration (*see Mechanics, Principles of*) of all rotating parts, and upon the gear ratio of the motors (i.e., ratio of number of teeth in gear to number of teeth in pinion). The effect of the moment of inertia may be looked upon either as increasing the effective weight W or as increasing the acceleration constant, the acceleration constant being defined as the quotient of total accelerating force divided by the product of weight and linear acceleration.

The increase in effective weight (in tons) due to any rotating axle or wheel is $\frac{M}{2000}\left(\frac{K}{r}\right)^2$, where M is the weight in pounds of the part in question, K its radius of gyration (*see Inertia, Moment of*) and r its actual radius, both in feet. Each motor armature adds to the effective weight $\frac{M}{2000}\left(\frac{\rho K}{r}\right)^2$ tons, where M is the weight of the armature in pounds, K the radius of gyration of the armature in feet, r the radius in feet of the wheel to which it is geared and ρ is the gear ratio. The total additional weight W_r is the sum of the above items for all the rotating parts. The total force in pounds required to produce the acceleration of a mphps. is then CaW, where W is the actual weight of the train in tons and $C = 91.2\left(1+\frac{W_r}{W}\right)$. This quantity C is the corrected acceleration constant, and this corrected value should be used in all calculations.

The acceleration constant is raised by the flywheel effect discussed above by about 5 per cent (i.e., $W_r/W = 0.05$) for heavy cars and locomotives, and between 5 per cent and 10 per cent for light low-speed cars, 8 per cent being an average figure. However, C is usually taken as 100, corresponding to an increase in effective weight of about 10 per cent. A given linear acceleration of a mphps. then requires an accelerating force of $100a$ pounds per ton.

TRACTIVE EFFORT REQUIRED. — Let

F = tractive effort, in pounds per ton, exerted by motors,
G = per cent actual grade (+ for up-grade),
g = degrees of curvature,
r = train resistance, in pounds per ton,
a = acceleration in mphps (− for retardation).

Then the tractive effort required per ton of total train weight is

$$F = 100a + r + 20G + g. \qquad (2)$$

Example. — Given a train of three 45-ton cars moving with a speed of 20 mph. and accelerating at a rate of 1.5 mphps. up a 1 per cent grade on a straight track; what is the total tractive effort required?

Answer: $(100 \times 1.5 + 7.8 + 20 \times 1)\, 3 \times 45 = 24{,}000$ pounds.

GEAR RATIO AND SPEED. — By gear ratio is meant the ratio of the number of teeth in the gear on the wheel axle to the number of teeth in the pinion on the motor shaft. A gear ratio greater than 6:1 is seldom used for railway motors. For a given torque developed by the driving motor, the tractive effort at the wheel rim and the linear speed for a given current depend upon the gear ratio and wheel diameter. Let D = the diameter of the wheel in inches, K = the gear ratio, F = the tractive effort for a given current input; then the tractive effort F_1 for this same current input but for a wheel diameter D_1 and gear ratio K_1 is

$$F_1 = \frac{DK_1}{D_1K}F.$$

If V is the speed corresponding to the tractive effort F for a given motor voltage,

then the speed V_1 corresponding to the tractive effort F_1 for the same motor voltage and current is

$$V_1 = \frac{D_1 K_1}{DK} V.$$

If the gear ratio is low, the maximum speed will be high and the rate of acceleration low; if the gear ratio is high, the maximum speed will be low and the rate of acceleration high.

For a given motor equipment, train weight, schedule, and profile the energy consumption and temperature rise of the motors depend upon the gear ratio selected, since this in turn determines the amount of coasting; see section below on *Importance of Coasting*. The proper gear ratio can be found only by trial calculation, plotting speed-time and distance-time curves from motor curves based upon different gear ratios, and calculating the energy consumption and temperature rise in the motors as described below.

MAXIMUM POSSIBLE TRACTIVE EFFORT — ADHESION COEFFICIENT. — The adhesion or "tractive" coefficient is the quotient (expressed usually as per cent) of the tractive effort in pounds which will slip the drivers, divided by the weight in pounds on the drivers. Burch gives the values in the following table. The maximum possible tractive effort is the product of the adhesion coefficient (as a decimal fraction) by the weight (in pounds) on the drivers.

TABLE III. — ADHESION COEFFICIENTS

Condition of track	Without sand	With sand
Most favorable condition	35	40
Clean, dry rail	28	30
Thoroughly wet rail	18	24
Greasy moist rail	15	25
Sleet-covered rail	15	20
Dry-snow-covered rail	11	15

Maximum Grade Train can Ascend. — Let

W = total weight of train in tons,
W_d = total weight on all drivers in tons,
p = adhesion coefficient in per cent,
r = train resistance in pounds per ton of total weight,
G = per cent grade,
g = degree of curvature,
a = acceleration in miles per hour per second.

Then the maximum tractive effort which the drivers can exert is $20\,pW_d$ pounds, and therefore $(r + 20\,G + g + 100\,a)\,W$ must be less than $20\,pW_d$, or the maximum per cent grade which the train can ascend is *

$$G = \frac{pW_d}{W} - \frac{(r + g + 100\,a)}{20}. \qquad (3)$$

* To be exact W_d should be multiplied by $\sqrt{1 - (G/100)^2}$, but except for very heavy grades this correction is negligible.

This grade is greater the less the acceleration, or the greater the retardation. The greater the speed before the train strikes the grade, the greater may the retardation be without bringing the train to rest on the grade, and therefore the steeper the grade it may ascend.

Example. — Assume no acceleration or retardation and no curvature, a train resistance of 8 pounds per ton, an adhesive coefficient of 15 per cent, and 25 per cent of total weight of train on drivers. Then the maximum grade the train can ascend is $G = 15 \times 0.25 - 8/20 = 3.35$ per cent.

The highest permissible grade is when all the weight is on the drivers, e.g., single cars or trains of motor cars with all axles equipped with motors. On steam freight roads the maximum grade seldom exceeds 2 per cent, and is usually considerably less, except in very mountainous country.

Weight of Locomotive. — The weight of locomotive required to accelerate a train weighing W tons at the rate of a miles per hour per second up a grade of G per cent on a g degree curve against a frictional resistance of r pounds per ton, when the q per cent of the weight is on the drivers and the coefficient of adhesion is p per cent, is given by the following formula:

$$\text{Weight of locomotive} = \frac{5W}{pq}(100\,a + r + 20\,G + g).$$

Example: What weight of locomotive is required to accelerate a 400-ton train at the rate of 0.5 mile per hour per second up a 0.1% grade against a frictional resistance of 8 lb. per ton, when 80% of the weight is on the drivers and the coefficient of adhesion is 20%?

$$\text{Weight of locomotive} = \frac{5 \times 400}{20 \times 80}(50 + 8 + 2) = 75 \text{ tons.}$$

POWER REQUIRED AT GIVEN SPEED. — Let

r = train resistance in pounds per ton of total train weight,
G = per cent grade,
g = degree of curvature,
a = acceleration in mphps.,
v = speed in mph.,
W = total weight of train in tons.

Then the power required *at the rims of the drivers* is $1.99\,v\,(r + 20\,G + g + 100\,a)$ watts per ton, or $p_0 = 2.67 \times 10^{-3}\,vW\,(r + 20\,G + g + 100\,a)$ horse power, total. (4)

The *power input* p_i to the car or locomotive is equal to the power at the rims of the drivers divided by the over-all efficiency ϵ of the controller, motors and gears, i.e.,

$$p_i = \frac{1.99\,Wv\,(r + 20\,G + g + 100\,a)}{1000\,\epsilon} \quad \text{kilowatts} \qquad (5)$$

Efficiency of Motors. — The over-all efficiency of the motors and gears when the motor is operating at full line voltage does not vary considerably for loads ranging from 50 to 150 per cent of rated load. The maximum efficiency is usually at about rated load, and has the values given in Table IV. At 50 and 150 per cent load, the efficiency may be from 3 to 10 per cent less, depending upon the design and the type of motor. The variation in efficiency with load is usually greater with alternating-current series than with direct-current motors.

Average Over-all Efficiency During Controller Period. — The over-all efficiency of the motors and controller depends upon the type of control employed and upon the resistance inserted or the connections made by the controller. Most modern controllers for interurban cars are provided with a

TABLE IV. — MAXIMUM OVER-ALL EFFICIENCY OF MOTORS AND GEARS AT RATED VOLTAGE.

Horse power, 1-hour rating	Kind of motor	Max. eff., per cent
30–100	D.C. geared	83–88
100–250	D.C. geared	88–89
250–500	D.C. gearless	91–93
50–200	A.C. series geared	70–80*
200–500	3-phase induction geared	85–89

* Including step-down transformers.

current-limiting device which limits the current to a given value, usually the value of the current corresponding to the 1-hour rating of the motor. Under these conditions the *average* over-all efficiency of the motors and controller over the whole of the *controller period* (i.e., from time of starting until full line voltage is established across each motor or until all resistance is cut out) may be expressed as follows, on the assumption (which is very nearly exact) that the speed of the motor varies directly as the voltage impressed across its terminals:

Let ϵ = the over-all efficiency of the motors and gears at end of controller period. Then for straight resistance control with direct-current or induction motors the average efficiency during the controller period is $\epsilon/2$; for series-parallel control with 2 direct-current motors the average efficiency during the controller period is $2\epsilon/3$; with 4 direct-current motors first all in series, then 2 series sets in parallel, and then all 4 motors in parallel the average efficiency during the controller period is $8\epsilon/11$; for alternating-current series motors with voltage control the average efficiency during the controller period (including losses in step-down transformers) is about 0.90ϵ. Hence for a motor having an 85 per cent efficiency at the 1-hour rating and the starting current limited to the corresponding current rating, the average over-all efficiency of the controller, motors and gear during the controller period is

For straight resistance control	43 per cent
For series-parallel control, direct-current motors	57 per cent
For series-series-parallel control, direct-current motors	62 per cent

Example. — Given a train of three 45-ton cars each car equipped with two 200-horse-power direct-current motors. What is the input to each motor if the speed is 30 mph. and the train is accelerating at 1.0 mphps. up a 2-per-cent grade, the track being straight, and full line voltage being across each motor?

Answer: From Table I the train resistance is $r = 10.5$, and from equation (4) the total power output is

$$p_0 = 2.67 \times 10^{-3} \times 30 \times 135\,(10.5 + 20 \times 2 + 100 \times 1)$$
$$= 1627 \text{ horse-power,}$$

or $1627/6 = 271$ horse power per motor. Assuming an efficiency of 87 per cent, the input to each motor is $271 \times 0.746/0.87 = 232$ kilowatts, and the total input to the train $6 \times 232 = 1392$ kilowatts.

If the train accelerates at this same rate from rest to a speed of 30 mph. at the end of the controller period (series multiple control) the grade being 2 per cent throughout, the average output of the motor would be the output corresponding to half speed or 15 mph., or 136 horse power per motor, requiring an

average input of 136 × 0.746/0.57 = 178 kilowatts, and the average input to the train during the controller period would be 6 × 178 = 1068 kilowatts.

SPEED-TIME AND DISTANCE-TIME CURVES. — To determine the energy required to propel a car or train a given distance over a given track in a given time requires the consideration of a number of factors which can best be taken into account by the construction of various kinds of time curves. Such curves may be constructed with practically any degree of accuracy desired when the profile of the track, the weight of the train, the various resistances, schedule speed, time of stops, etc., are accurately known. Such data are, however, seldom known with any great precision, and consequently elaborate methods of plotting and calculation are seldom justified. Below will be given (1) some results of actual tests, (2) a rough but simple method of approximating the energy requirements, (3) a step-by-step method, which, though tedious in application, is susceptible of any degree of accuracy desired, provided the given data are accurately known, and (4) an analytical method by which the effect of changes in the time of coasting, rates of acceleration and braking, etc., may be predetermined.

The following terminology will be employed:

Speed-time Curve. — A curve showing the speed (in mph.) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve *ABCEF* in Fig. 1. A speed-time curve may be conveniently divided into four parts, namely:

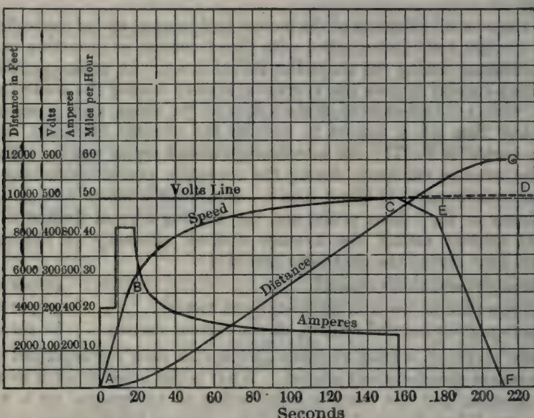


Fig. 1.

Controller Period. — The period from starting until full line voltage is established across each motor; i.e., the portion *AB* of the curve in Fig. 1.

Motor-Curve Period. — The period during which the motor is operating on full line voltage; i.e., the portion *BC* in Fig. 1. The relation between speed and tractive effort during this portion of the run is fixed by the motor characteristics; specifically by the speed-torque curve of the motor and the gear ratio.

Coasting Period. — The period during which the car or train is coasting; i.e., the portion *CE* in Fig. 1.

Braking Period. — The period during which the brakes are applied; i.e., the portion *EF* in Fig. 1.

Distance-time Curve. — A curve showing the distance covered (in feet) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve AG in Fig. 1.

Current-time Curve. — A curve showing the line current (in amperes) plotted as ordinates against elapsed time (in seconds) as abscissas; e.g., the curve marked "Amperes" in Fig. 1.

Voltage-time and Power-time Curves. — Curves showing respectively the voltage per motor (or the line voltage) and the power input to the motor (or train) plotted as ordinates against elapsed time (in seconds) as abscissas.

Average Speed and Schedule Speed. — The average speed V is the total distance run L' (in miles) divided by the time (in hours) the train is actually running. The schedule speed S is the total distance run (in miles) divided by the total time (in hours) of the run from one end of the road to the other, including time of all stops at intermediate stations. If the *total* time of all the *intermediate* stops is T_s' seconds, then

$$V = \frac{S}{1 - \dfrac{T_s'S}{3600\,L'}}, \tag{6}$$

where V and S are in miles per hour and L' is the *total* length of route in *miles*.

Duration and Frequency of Station Stops. — The duration of each stop for surface cars ranges from 5 to 10 seconds, for elevated and subway trains from 10 to 30 seconds, for interurban trains from 10 to 40 seconds. The stops per mile are the reciprocal of the average distance in miles between stops. For the six most important elevated and subway lines in Europe and America, the stops per mile average 2.5.

Average Equivalent Grade (G).—Grades may be taken into consideration by calculating the sum H_1 of all the rises on upgrades and the sum H_2 of all the drops on down grades, and taking for the average "equivalent" upgrade in per cent

$$G = \frac{100(H_1 - 0.5\,H_2)}{L}, \tag{7}$$

where H_1 and H_2 are in feet and L is the total length of the route in feet. On a round trip $H_1 = H_2$, and the "equivalent" grade in per cent is

$$G = \frac{50H_1}{L}.$$

This method of dealing with grades is equivalent to assuming that half the kinetic energy stored in the train or down grades is utilized in taking the train up the following upgrade. The amount of energy thus rendered available of course will depend upon the amount of braking necessary on down grades to prevent excessive speeds and also upon the location of the stops with respect to the grades. The figure ½ is taken as an approximate average; this figure may be varied as seems reasonable in view of the actual profile.

Average Angle of Curvature (g). — Curves may be taken into account by finding the average curvature, i.e., finding for each curve the product of the degree of curvature by the length of the curve, adding all these products and dividing by the length of the route.

ENERGY CONSUMPTION FROM TESTS. — The table on p. 1302 gives the energy consumption, as found by tests, for a number of typical services. This table will be found useful as a rough check on any calculations made for a specific service. Methods of making such calculations are given below.

TABLE V. — WATT-HOURS PER TON-MILE AT CARS, FROM ACTUAL TESTS

	N.Y.N.H.& H.R.R.		N.Y.C.&H.R.R.		N.Y.N.H.& H.R.R.		N.Y.C.&H.R.R.		N.Y. N.H. &H. R.R.	Interborough Subway, N.Y.		Northern Texas Trac. Co.		Urban trolley lines.
										Express	Local			
Service	P	P	P	P	P	P	P	P	F	P	P	P	P	P
A. C. or D. C.	A. C.	A. C.	D. C.	D. C.	A. C.	*A. C.*	D. C.	D. C.	A. C.	D. C.	D. C.	D. C.	D. C.	D. C.
Locomotive or cars	*L.*	*L.*	*L.*	*L.*	*L.*	*L.*	L.	L.	L.	C.	C.	C.	C.	C.
Train weight, tons	316	285	266	250	477	493	432	434	1438	310.6	310.6	25.4	25.4	4-12
Length of run, miles	20.6	20.6	23.9	23.9	20.6	20.6	17.1	17.1	16.8	12.9	6.6	28.4	28.4	.....
Average grade, per cent	+0.053	−0.053	+0.129	−0.129	+0.053	−0.053	+0.129	−0.129	−0.52	0	0	0	0	.....
Schedule speed, mph	22.1	22.1	23.2	22.8	44.7	49.0	44.1	43.9	35.9	20.2	12.4	28.6	29.6	.....
Stops per mile	0.63	0.68	0.84	0.75	0.048	0.048	0.059	0.059	0.06	0.62	1.76	0.32	0.27	2-10
Watt-hours per ton-mile.	85.4	74.2	81.6	59.4	35.0	30.0	29.2	17.6	25.9	58.2	78.6	75.5	72.5	100 to 160
References	1	1	2	2	1	1	2	2	1	3	3	4	4	

NOTE. — While single-phase and direct-current service are given above in adjacent columns, the operating conditions are not identical and these figures are not intended to bring the two systems into comparison.

Abbreviations. — P = passenger; F = freight; L = locomotive; C = cars;

References. — 1 = W. S. Murray, *Trans. A.I.E.E. 1911, Vol. 30, p. 1497,*
2 = E. B. Katte, *Trans. A.I.E.E., 1911, Vol. 30, p 1497,*
3 = L. B. Stillwell, *Elec. Ry. Jour. 1908, Vol. 32, p. 6.*
4 = E. R. Roberts and I. H. Sherwood, *Trans. A.I.E.E. 1903, Vol. 20, p. 1080.*

APPROXIMATE METHOD OF CALCULATING ENERGY CONSUMPTION. — The following method is based upon simple kinetic principles, and, if certain characteristics of the run are known, gives the actual energy output at the wheel rims. This fact makes the method useful, not only for rough calculations, but also to check calculations made by the step-by-step method.

When the method is applied to checking purposes, the column of Table VI. headed "Actual energy output" should be used, and the input calculated from the known efficiencies. When applied to rough calculations, the column headed "Approximate electrical energy input" should be used. In the latter case the maximum speed and length of run with power on are not known, but it is possible to assume certain values, based upon experience, which will give a rough approximation to the energy required. Let

V = average running speed in miles per hour,
V_m = maximum speed in miles per hour,
L = length of run in miles,
L_p = distance traveled, with power on, in miles,
$n = 1/L$ = number of stops per mile including one terminus,
r = average train resistance, in lb. per ton (Say that corresponding to a speed from 10 to 20 per cent greater than the average speed),
G = average equivalent grade, in per cent,
g = average curvature in degrees,
$K = \frac{V_m}{V}$ = ratio of maximum to average speed; see Table VII,
$Q = \frac{L}{L_p}$ = ratio of length of run to distance traveled with power on; see Table VII.

TABLE VI. — OUTPUT AT WHEEL RIM AND INPUT TO CAR IN WATT-HOURS PER TON-MILE

Energy for	Actual energy output at wheel rims of cars	Approx. electrical energy input to cars
Acceleration	$\frac{V_m^2}{36.2L}$	$\frac{K^2nV^2}{25}$
Train resistance	$\frac{1.99\,rL_p}{L}$	$\frac{2.9\,r}{Q}$
Grades	$\frac{39.8\,GL_p}{L}$	$\frac{57\,G}{Q}$
Curves	$\frac{1.99\,gL_p}{L}$	$\frac{2.9\,g}{Q}$
Total	Sum	Sum

NOTE. — $25 = 36.2\,\epsilon$, $57 = 39.8 \div \epsilon$, and $2.9 = 1.99 \div \epsilon$, where ϵ is the efficiency, taken as 0.7. The formula for energy due to curves assumes each degree of curvature to be equivalent to a train resistance of one pound per ton, which is probably high.

TABLE VII.—VALUES OF K AND Q

Stops per mile n	K		Q
	Locomotive Passenger Trains	Single cars, multiple-unit trains and freight trains	All trains
0	1.00	1.00	1.00
0.1	1.18	1.10	1.11
0.2	1.35	1.18	1.24
0.3	1.48	1.25	1.38
0.4	1.60	1.31	1.52
0.5	1.68	1.36	1.67
0.6	1.75	1.40	1.78
0.7	1.82	1.44	1.89
0.8	1.86	1.47	1.99
0.9	1.90	1.50	2.07
1.0	1.93	1.52	2.15
1.2	1.93	1.56	2.24
1.4	1.93	1.59	2.34
1.6	1.94	1.62	2.44
1.8	1.94	1.65	2.52
2.0	1.95	1.68	2.58
2.5	1.95	1.75	2.71
3.0	1.96	1.80	2.81
3.5	1.96	1.85	2.87
4.0	1.97	1.90	2.91
4.5	1.97	1.94	2.95
5.0	1.98	1.97	3.00
over 5.0	2.00	2.00	3.00

(The above method was worked out by D. C. Woodbury and W. A. Del Mar, the table being based upon a large number of actual and calculated runs.)

Example.—A multiple-unit train has a speed (excluding stops) of 25 miles per hour and makes 0.8 stops per mile. It ascends an average grade of 0.143 per cent. What will be its energy consumption in watt-hours per ton-mile?

From Table VII we find for $n = 0.8$, that $K = 1.47$ and $Q = 1.99$. Then using the formulas in Table VI, the results in the table on the next page are obtained, assuming 6.5 pounds per ton for friction.

Efficiency of Run.—The formulas given above enable one to judge the effect upon the energy consumption, of altering any of the principal physical elements upon which the run is based.

From the formulas for kinetic energy it is obvious that a low value of K means a low energy consumption. A low value of K, however, means a "square" speed-time curve; i.e, for low energy consumption the controller, acceleration and braking periods should be as short as possible, or in other words, the rate of acceleration and braking should be as great as practicable.

The quantitative effect of changing any of these variables may be estimated by the analytical method given further on.

Energy for	Watt-hours per ton-mile	
Acceleration	$\frac{1.47^2 \times 0.9 \times 25^2}{25} =$	48.5
Train resistance	$\frac{2.9 \times 6.5}{1.99} =$	9.5
Grades	$\frac{57 \times 0.143}{1.99} =$	4.1
Total		62.1

This example worked out by the step-by-step method gave 60.5 watt-hours per ton-mile.

STEP-BY-STEP METHOD OF PLOTTING SPEED-TIME CURVES. — There is no way of exactly predetermining a speed-time curve except by a number of successive trials. That is to say, the time the current is kept on, the time of coasting and the time of braking must each be guessed and it is usually necessary to make a number of trials, by varying the proportion of motor run, coasting and braking, before the given distance is traversed in the desired time.

If the characteristics of the train and its equipment are expressed numerically, the principles of mechanics enable such trial runs to be plotted on paper and the proper proportion of motor run, coasting and braking, selected to make the train travel the desired distance in the given time. For a given motor equipment on a given route it is possible to plot by a step-by-step method these speed-time curves, and then from these curves and the characteristic curves of the motors the various characteristics, such as energy consumption, root-mean-square current, etc., may be determined. The accuracy of this method depends solely upon the accuracy with which the assumed data are known. The necessary data are:

Profile and alignment of road.
Characteristic curves of the motors.
W = total weight of train in tons,
T = the time of run in seconds between successive stops,
I_0 = the permissible starting current, or
a = the acceleration in miles per hour per second during the controller period,
b = the braking rate in miles per hour per second,
r = the train resistance in pounds per ton at any speed.

Determination of Acceleration and Retardation Rates. — From the motor characteristics, which are usually given in the form indicated in Fig. 2, determine the tractive effort and speed corresponding to the permissible starting current. This is usually taken as the current corresponding to the 1-hour rating; in the case of the motor whose characteristics are shown in Fig. 2, this current is 315 amperes, the speed 19.6 mph. and the tractive effort 4300 pounds. If each car is equipped with two motors, the weight corresponding to each motor is half the weight of the car. If the cars weigh 44 tons each, then the tractive effort per ton developed by the motors at this speed is 4300/22 = 195 pounds per ton. At the point where the motors are changed from series to parallel the speed

will be approximately one-half the speed at rated voltage or 9.8 mph. If the average line voltage is less than the rated voltage, these speeds should be reduced in proportion to the ratio of the actual to the rated voltage. In the case selected the average line voltage is 571 volts; hence the speeds corresponding to the parallel and series points are 18.6 and 9.3 mph respectively. These speeds and the corresponding tractive efforts are entered into Table VIII.

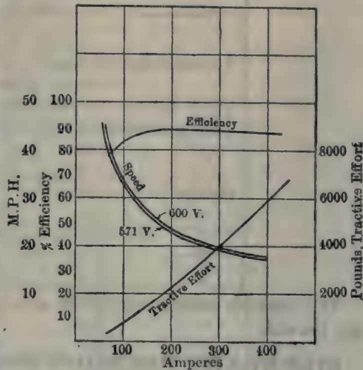


Fig. 2.

From the motor characteristics determine the tractive effort (f) in pounds per ton and the corresponding speeds (v), corrected for line voltage, up to the maximum permissible speed, and enter these in Table VIII. Also enter into this table the train resistance corresponding to the various speeds. In the table given, the train resistance is calculated from Burch's formula for three 44-ton cars, the cross section of each car being 120 square feet, and the constant K is taken equal to 0.0050 (subway service). The formula is then

$$r = 4.0 + 0.13\,v + 0.0055\,v^2,$$

where v is the speed in miles per hour.

The available tractive effort in pounds per ton for acceleration on any grade is then $f - r - 20\,G'$, and the acceleration rate is (assuming an acceleration constant of 100)

$$a = \frac{f - r - 20\,G'}{100},$$

where G' is the "equivalent" grade (including curvature, see above), in per cent. The value of a is calculated for grades of 0, 1, 2, 3, etc., per cent, both up and down, including the largest up and down grade. These values are entered into Table VIII. The maximum speeds are the speeds for which the acceleration becomes zero. These can be obtained by plotting the accelerations against speed, and finding the speed at which the curves cross the speed axis.

Similar calculations should also be made for the retardation on up-grades when the train strikes such a grade, with power on, at a higher speed than the free running speed on the grade, and also for the coasting period when no power is on. When power is on, the friction of the gears and motors is allowed for in the motor tractive effort curve, the tractive effort curve being the gross tractive effort less these losses. The friction of the gears and motors may be taken as approximately 5 per cent of the rated tractive effort, which, in the special case under consideration is $0.05 \times 195 = 10$ pounds per ton. The total train resistance in coasting is then the normal train resistance plus the resistance of gears and motors.

In the Tables a + acceleration signifies an actual increase in velocity, a − acceleration a retardation, or decrease in velocity.

The braking rate is assumed constant, usually $b = 1.5$ mphps.

TABLE VIII. — ACCELERATION RATES

Speed, mph.		Motor tractive effort, pounds per ton	Train resistance, pounds per ton	Acceleration rates, mphps., = a — Per cent equivalent grade given in first line						
	v	f	r	+3	+2	+1	0	−1	−2	−3
Accelerating, power on	9.3	195	6	1.29	1.49	1.69	1.89	2.09	2.29	2.49
	18.6	195	8	1.27	1.47	1.67	1.87	2.07	2.27	2.47
	20	159	9	0.90	1.10	1.30	1.50	1.70	1.90	2.10
	22	112	10	0.42	0.62	0.82	1.02	1.22	1.42	1.62
	25	74	11	0.03	0.23	0.43	0.63	0.82	1.03	1.23
	30	45	13	......	......	0.12	0.32	0.52	0.72	0.92
	35	27	15	......	......	......	0.12	0.32	0.52	0.72
	40	18	18	......	......	......	0.00	0.20	0.40	0.60
Retarding power on	40	18	18	−0.60	−0.40	−0.20	......	......	......	......
	35	27	15	−0.48	−0.28	−0.08	......	......	......	......
	30	45	13	−0.28	−0.08	......	......	......	......	......
Coasting, no power	50	0	34	−0.94	−0.74	−0.54	−0.34	−0.14	+0.06	+0.26
	45	0	31	−0.91	−0.71	−0.51	−0.31	−0.11	+0.09	+0.29
	40	0	28	−0.88	−0.68	−0.48	−0.28	−0.08	+0.12	+0.32
	35	0	25	−0.85	−0.65	−0.45	−0.25	−0.05	+0.15	+0.35
	30	0	23	−0.83	−0.63	−0.43	−0.23	−0.03	+0.17	+0.37
	25	0	21	−0.81	−0.61	−0.41	−0.21	−0.01	+0.19	+0.39
	20	0	19	−0.79	−0.59	−0.39	−0.19	+0.01	+0.21	+0.41

NOTE. — The train resistance is assumed to be 6 lb. per ton from zero speed to 9.3 miles per hour.

Construction of Acceleration and Retardation Time Curves. — The next step is to construct a set of acceleration speed-time and distance-time curves, a set of coasting speed-time and distance-time curves, and a braking speed-time and distance-time curve. The construction of these curves is based on the following relations:

Let

v_1 = the speed in miles per hour at time t_1,

v_2 = the speed in mph. at time t_2,

$v = \frac{v_1 + v_2}{2}$ = average speed during the interval $t_2 - t_1$,

a_1 = the acceleration in miles per hour per second at time t_1,

a_2 = the acceleration in miles per hour per second at time t_2,

$a = \frac{a_1 + a_2}{2}$ = average acceleration during the interval $t_2 - t_1$; (for the first step take a speed corresponding to half the speed at end of controller period),

x_1 = the distance in feet from the starting point at time t_1,

x_2 = the distance in feet from the starting point at time t_2.

Then, for a small change in speed,

$$t_2 - t_1 = \frac{v_2 - v_1}{a} \quad \text{seconds} \tag{8}$$

and the distance covered in this interval is

$$x_2 - x_1 = 1.466\, v\, (t_2 - t_1) \qquad \text{feet} \tag{9}$$

or, when the speed is plotted against time,

$$x_2 - x_1 = 1.466 \times (\text{Area of speed-time curve between } t_2 \text{ and } t_1). \tag{10}$$

From equation (8), using the values of a given in Table VIII, the time at which any speed is reached may be calculated. The results of such calculations for the special case under consideration are given in Table IX, and are plotted in Figs. 3, 4 and 5. The distance-time curves are found by planimetering the speed-time curves and multiplying by 1.466 (*see equation 9*), and are also plotted in Figs. 3, 4 and 5.

TABLE IX.—DATA FOR ACCELERATING, COASTING AND RETARDING SPEED TIME CURVES

	Speed v	Total time in seconds to accelerate from rest — Per cent equivalent grade: +3	+2	+1	0	−1	−2	−3
Accelerating, power on	0.0	0	0	0	0	0	0	0
	9.3	7	6	6	5	4	4	4
	18.6	14	12	11	10	9	8	7
	20	17	13	12	11	10	9	8
	22	20	16	14	12	11	10	9
	25	33	23	19	16	14	12	11
	30	Max. speed =25.3	Max. speed =27.5	37	26	21	18	16
	35	..	..	Max. speed =32	49	33	26	22
	40	..	..	..	132	52	37	29
		Total time in seconds to retard from 40 mph.						
Retarding, power on	40	0	0	0	..	..	..	..
	35	9	15	36	..	..	..	..
	30	22	43	Min. speed =37	..	..	..	..
	....	Min. speed =25.3	Min. speed =27.5	..	..	..	..	..
		Time in seconds required for speed to decrease 5 mph. to speed given in first column.					**To increase 5 mph. from from given speed**	
Coasting, power off	45	5	7	10	15	40	67	18
	40	6	7	10	17	53	48	16
	35	6	8	11	19	77	37	15
	30	6	8	11	21	125	31	14
	25	6	8	12	23	250	28	13
	20	6	8	13	25	..	25	13

The speed during the braking period t seconds *before* the train stops is $v = bt$ and the distance to travel to come to rest is $x = \frac{1}{2} bt^2$.

In the special case of a braking rate of 1.5 mphps.,

$$v = 1.5\, t \text{ miles per hour,}$$

and the distance to travel to come to rest is

$$x = \frac{1.466 \times 1.5\, t^2}{2}.$$

The corresponding curves are given in Fig. 6.

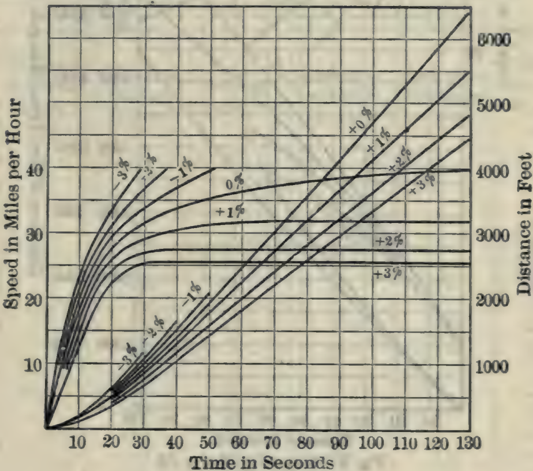


Fig. 3. Accelerating

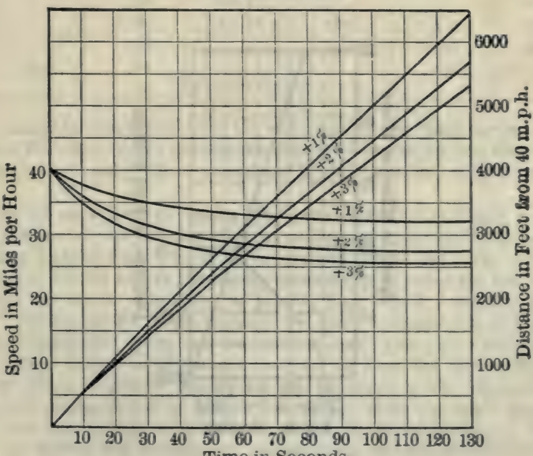


Fig. 4. Retarding, Power On

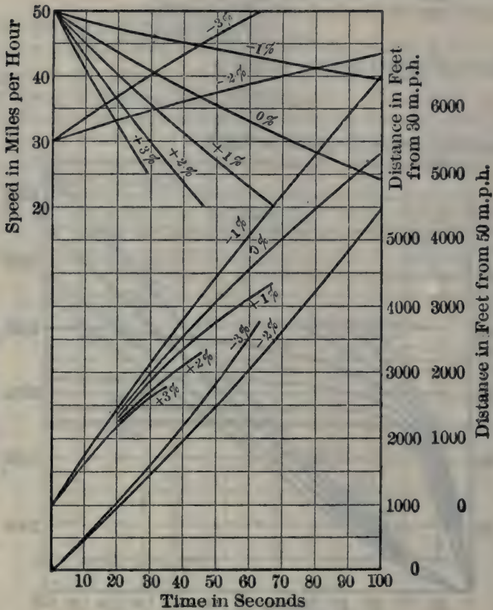


Fig. 5. Retarding, Power Off

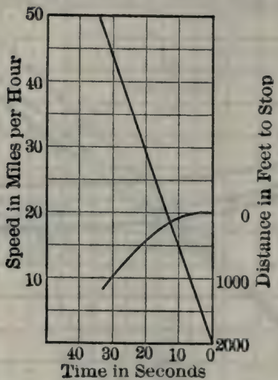


Fig. 6. Braking

Time Curves for Given Profile and Alignment. — With these four sets of curves the speed-time curve for any given run with this particular equipment may be rapidly constructed. For intermediate grades interpolation may be readily made. In Fig. 7 is given a profile and alignment between two stops. The first step is to make up a table like Table X, dividing the route into sections such that the "equivalent" grade (= actual grade plus, say, 0.05 per cent for each degree of curvature, assuming a resistance of 1 pound per ton per degree of curvature) is the same throughout each section.

TABLE X. — "EQUIVALENT" GRADES

Stop	Distance between stops, feet	Length of section in feet	Per cent grade = G	Radius of curvature in feet	Degree of curvature = g	"Equivalent" grade $G' = G + 0.05\ g$.
A	....	505	......	....	...	......
	....	145	......	1130	5.0	+0.25
	....	908	+0.70	1130	5.0	+0.95
	....	77	+0.50	1130	5.0	+0.75
	....	633	+0.50	....	...	+0.50
	....	273	+3.00	....	...	+3.00
	....	273	+3.00	800	7.2	+3.36
	....	247	......	800	7.2	+0.36
	....	94	......	....	...	......
	....	308	−2.95	....	...	−2.95
	....	800	+3.00	....	...	+3.00
	....	1862	−3.00	....	...	−3.00
	....	192	−3.00	5000	1.2	−2.94
	....	234	+0.12	5000	1.2	+0.18
	....	178	+0.12	....	...	+0.12
	....	69	+0.12	5000	1.2	+0.18
	....	358	......	5000	1.2	+0.06
B	7321	165	......	....	...	......

For a complete round trip over the entire route the time curves must be plotted for the entire route in both directions. The run in one direction between two stations only will be considered in the numerical calculations given below.

Next lay off on a piece of tracing cloth, see upper part of Fig. 7, a distance equal to the time of run between the two stations (156 seconds in the example), to the same scale as Figs. 3 to 6. The braking speed-time curve can be laid off directly at the far end of the run by placing Fig. 6 under the tracing cloth and tracing the curve. Similarly, by placing Figs. 3 and 4 under the tracing cloth and tracing for the proper distance the curve corresponding to the proper grade, an acceleration curve can be built up until this curve intersects the braking curve. If the total distance, as read off from the corresponding distance-time curve, is greater than the actual distance, it will be necessary to introduce a coasting period of proper duration to make the total distance as read off from the distance-time curves equal to the actual distance. This can be done by placing Fig. 5 under the tracing cloth, and by cut and try finding the proper amount of coasting.

In case there are curves or crossovers, requiring a reduction in speed at certain points, these reductions should be allowed for in plotting the speed-time

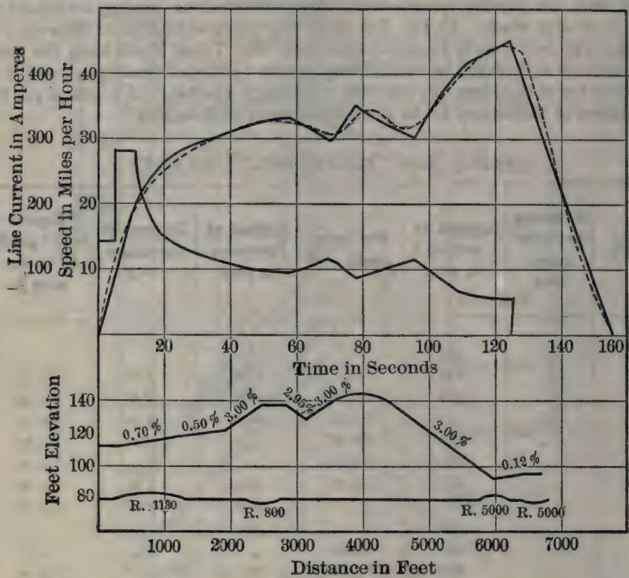


Fig. 7.

curve. A rough rule for the permissible speeds on properly constructed curves is that

$$\text{Speed on curve} = \sqrt{\text{Radius of Curve}},$$

where the radius is in feet and the speed in miles per hour.

The dotted curve in Fig. 7 was obtained from test.

Current-time Curve. — The motor current for any speed may be taken directly from the speed-time curve and motor characteristics. From the motor currents the line current is readily found by multiplying by *half* the number of motors during the *series* portion of the controller period and by the total number of motors during the rest of the time power is on. Current-time curves (line current) for the various grades may also be drawn once for all in the same manner as the speed-time curves in Fig. 3. The curve with the square shoulder in Fig. 7 is the current-time curve for the example considered.

Watt-hours per Ton-Mile. — During the first half of the controller period the input per motor is equal to the product of the current per motor by approximately one-half the line voltage (series-parallel control assumed); during the rest of the time that power is on, the input per motor is equal to the product of the motor current by the line voltage. Hence, calling A the area of the current-time curve from the start until power is shut off, and I_0 the starting current, E the average line voltage, T_1 the seconds duration of the controller period, L the length of the run in miles, M the number of motors and W the weight of the train in tons, then to a fair approximation,

$$\text{Watt-hours per ton-mile} = \frac{EM(A - 0.25 I_0 T_1)}{3600\, WL}.$$

A more accurate, but tedious method, is to plot a power-time curve by multiplying the total current per train at successive intervals of time by the average line voltage during this interval, and then integrating this curve to find the total watt-seconds. This area, in watt-seconds, divided by ($3600\ WL$) will give the watt-hours per ton-mile. In making such a calculation note that during the series part of the controller period the current per train is equal to the current per motor multiplied by *half* the number of motors.

Root-mean-square Current per Motor. — This may be found by squaring the ordinates of the current-time curve (current per *motor*), plotted as described above, and dividing by the time of run including stops and taking the square root of the quotient. Or, the current-time curve may be plotted in polar co-ordinates, taking time in seconds as the angle in degrees and current in amperes as the radius vector. See Fig. 8. Calling B the area of this curve, the unit of area being the square whose side has a length corresponding to 1 ampere, then the root-mean-square current is

$$I_e = 10.7 \sqrt{\frac{B}{T+T_s}},$$

where T is the time that the train is moving and T_s the standing time, both in seconds.

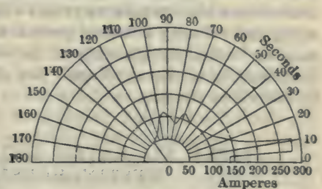


Fig. 8.

Average Motor Voltage. — For a simple run, such as shown in Fig. 1, average motor voltage during the controller period is equal to approximately 55 per cent of the line voltage, assuming 10 per cent of line voltage across the motors at the instant of starting. Let

E = average line voltage,
T_1 = time of controller period, from speed-time curve,
T_2 = time motor is running on full line voltage,
T = total time that the train is moving,
T_s = total standing time.

Then average motor voltage for the entire run is

$$E_m = \frac{0.55\, ET_1 + ET_2}{T + T_s}.$$

For a complex run, where the controller is shut off and put on again during the run, a voltage-time curve may be plotted and the average ordinate obtained by integration.

MOTOR CAPACITY. — A railway motor is usually rated in terms of the output in kilowatts or horse-power which it will give when run for 1 hour at rated voltage with a temperature rise above the surrounding air not exceeding 75° C. in any part of the motor, other than the commutator (*see Standards of the A. I. E. E.*).

The size of motor required for any particular service (i.e. for a given route, schedule, weight of car, line voltage and per cent coasting) depends upon two factors, (1) the motor must be of such a size that the maximum current required will not produce harmful sparking at the brushes or dangerous mechanical stresses in any part of the motor, and (2) the temperature must not rise to a value which will cause the insulation to deteriorate.

Size of Motor Limited by Commutation and Mechanical Stresses. — The maximum current is usually that required at starting, and since the start-

ing current remains practically constant up to the point where full line voltage is impressed across the motor, the corresponding maximum horse-power output of the motor can be calculated directly from equation (5), when v is taken as the speed at the end of the controller period and W as the weight of the train per motor, i.e., W is taken equal to the total weight of train divided by the number of motors. A safe rule for non-interpole motors in single-car or multiple-unit service is to limit the starting current to a value equal to the rated current. For interpole motors in like service the starting current may safely be 25 to 50 per cent in excess of the rated current.

In locomotive work a heavier starting current is sometimes demanded, and due to the low acceleration rate during the starting period the motor must carry this current for a longer interval than in the case of single-car or multiple-unit service. In selecting a motor for such service, information should be obtained from the manufacturer as to the maximum current which the motor can safely carry during a limited period, say for 5 minutes. This maximum current may be limited by sparking at the commutator, by mechanical stresses, or by local heating of the windings. See also section below on *Size of Motor Limited by Short-time Heating.*

TABLE XI.—RATED AND CONTINUOUS CAPACITY OF WESTINGHOUSE RAILWAY MOTORS

	Type, Westinghouse	Rated H.P.	Rated voltage	Rated amperes	Continuous current capacity		Weight in pounds
					At 300 volts	At 450 volts	
Self-ventilated	508A	25	600	37	32	35	1035
	514	40	600	60	35	36	1650
	532A	50	600	74	44	45	2300
	306CV4	65	600	94	58	60	2750
	548C8	100	600	145	90	93	3175
	333V8	125	600	180	110	115	3850
	557A8	140	600	202	130	135	4050
	577A6	190	600	268	175	180	5650
Non-ventilated	323A	32	500	58	28	26	1890
	307	50	600	73	37	35	2850
	306	50	500	87	44	40	2661
	310	60	500	107	50	46	3510
	305	60	500	107	50	46	3550
	304	75	500	130	60	55	3550
	303	100	550	158	70	65	3950
	302	125	550	195	95	85	4685
	301	160	550	246	120	110	5510
	300	200	550	310	150	130	6475

Size of Motor Limited by Heating.— The heat developed in a railway motor is carried partly by conduction through the several parts and partly by

convection through the air to the motor frame, whence it is distributed to the outside air. As the temperature of the several parts is thus dependent not only upon their own internal losses, but also upon the temperature of the neighboring parts, it becomes necessary to determine the actual value and distribution of losses in a railway motor for a given service in order to determine with precision what the temperature rise will be; or, *vice versa*, to determine what size of motor will be required to avoid too great a temperature rise.

For ordinary electric railway calculations, however, in view of the other uncertain elements which enter, it is usually sufficiently accurate to assume that the *relative* temperatures in the different parts of the motor are independent of the *relative* values of the copper and core losses. The copper loss, i.e., the rate at which heat is developed in the windings, is proportional to the square of the current, and the core loss, i.e., the rate at which heat is developed in the core, due to hysteresis and eddy currents, may be taken as proportional to the first power of the voltage across the motor terminals (this latter relation being approximate only). When the motor reaches a constant temperature under a constant load, the temperature of any part will then be proportional (approximately) to the total power (kilowatts) lost in the windings and core. Similarly, under a fluctuating load continuing over a long period during which there are no excessively long breaks or excessive overloads, the temperature becomes fairly constant and the rise is proportional to the average power (kilowatts) lost during this period. There will be times at which the temperature rise will exceed this average and times at which it will be less, but on account of the heat storage capacity (or thermal capacity) of the materials of which the motor is made the fluctuations in temperature will be very much less than the fluctuations in the load.

Size of Motor Limited by Average Temperature Rise. — The manufacturers supply information as to the current which any motor will carry continuously (on stand test) without overheating, at various voltages from one-half to full voltage; see Table XI. From this information, making the assumptions noted in the preceding paragraph, it is possible to determine the approximate temperature of the motor for any given run or series of runs. The process is to calculate the root-mean-square current per motor and the average motor voltage for the particular service contemplated, using the methods given above in the paragraphs headed *Root-Mean-Square Currrent per Motor* and *Average Motor Voltage.* Call these values of the r.m.s. service current and average motor voltage I_e and E_m respectively. Let I_c be the continuous-current capacity at a given voltage E_c, as given by the manufacturer (see Table XI), and let T_c be temperature rise corresponding to this continuous rating. (Motors having ordinary fibrous insulation are rated on the basis of a 75° C. temperature rise on stand test, which corresponds to about 65° C. rise in actual service, due to better ventilation; see *Standardization Rules of the A.I.E.E.;* hence for ordinary motors T_c is 65° C.). Let J_c be the corresponding core loss and K_c the corresponding copper loss and $L_c = J_c + K_c$ be the corresponding total electrical losses. Then the total electrical losses corresponding to the average load are

$$L_a = \frac{E_m}{E_c} J_c + \left(\frac{I_e}{I_c}\right)^2 K_c,$$

and the average temperature attained by the motor in service will be approximately

$$T_a = \frac{L_a}{L_c} \cdot T_c.$$

For safe operation the average temperature rise T_a should never exceed the value T_c, which for motors with ordinary fibrous insulation is 65° C.

Approximate Values of J_c and K_c. — When the core loss and copper loss are not given separately, a rough estimate of J_c and K_c may be made by assuming that at rated load (one-hour rating and line voltage) the core loss is, say, ¼th of the total electrical losses. The total electrical losses L_r in kw. at rated load may be found from the characteristic curves of the motor by using the formula

$$L_r = P\left(\frac{0.97}{\varepsilon} - 1\right),$$

where P is the one-hour rating in kilowatts, ε the efficiency of the motor with gears, and the 0.97 takes into account the frictional losses in the motor and the gears. Let I_r be the rated current and E_r the rated voltage; then

$$J_c = \frac{L_r}{4}\frac{E_c}{E_r} \qquad \text{and} \qquad K_c = \frac{3\,L_r}{4}\left(\frac{I_c}{I_r}\right)^2.$$

Size of Motor Limited by Short-time Temperature Rise. — When the service is such that the motor must take a heavy current for a comparatively long interval (e.g., a long starting period or a heavy grade for a considerable distance) followed by a like period of light load or no load, the average temperature for the run, as calculated above, may be within the required limits, but the short-time temperature rise may be excessive. This short-time temperature rise depends upon the heat-storage capacity of the motor, i.e., upon the *energy* loss (number of *kilowatt-hours* of heat developed in it) required to raise its temperature one degree, say, assuming no radiation of heat from its surface. The one-hour rating of a motor is an indirect measure of this heat-storage or thermal capacity.

The temperature-time curve during the first hour's application of a load is practically a straight line whose slope is proportional to the load. The rise in temperature of the motor due to a short-time load may then be assumed to be proportional to the *energy* (kilowatt-hour) input during this time, and the factor of proportionality may be obtained from the one-hour rating as follows: Let T_r = the temperature rise at the end of one hour due to a load equal to the one-hour rating of the motor (rated current and rated voltage), L_r = the total electrical losses in kilowatts corresponding to the rated load, L_a = the total electrical losses in kilowatts corresponding to the average load in service (L_r and L_a may be estimated by the method given in the preceding section), L_p = the total electrical losses in kilowatts corresponding to any given short-time or peak load, t_p = the number of minutes' duration of this peak load. Then during this interval t_p the rise of temperature above the average value T_a, is

$$T_p - T_a = \frac{t_p\,T_r}{60\,L_r}\cdot(L_p - L_a).$$

T_p as calculated from this formula gives approximately the maximum temperature rise during the run. For safe operation this maximum temperature rise T_p should not exceed the safe limit stated by the manufacturer. For motors with ordinary fibrous insulation T_p should not exceed 75° C.

Final Choice of Motor. — No motor should be employed for a given service which does not meet the above requirements regarding the maximum current and heating limits. A larger motor than that fixed by these requirements may prove the cheaper in the long run, if by using such a motor the energy consumption can be materially reduced by increasing the amount of coasting during the run. In any event the motor should be of sufficient capacity to permit of a reasonable amount of coasting under normal conditions, so that there will be a sufficient margin in which to make up for lost time, due to unexpected slowdowns or extra stops.

ANALYTICAL METHOD OF PREDETERMINING ENERGY AND MOTOR EQUIPMENT. — The following method was suggested by that developed by Cary T. Hutchinson, in two papers in the *A.I.E.E. Trans., Vol. 19, p. 129, 1902*, and *Vol. 22, p. 657, 1903*. In this method a speed-time curve similar in shape to the curve *ABCEF* in Fig. 1 is assumed. That is, the acceleration during the controller period, the train resistance and the braking retardation are all assumed constant, but a "motor-curve" period (*BC* in Fig. 1) is also taken into account, this latter constituting the essential difference between this method and the "straight-line" speed-time curve method frequently employed for approximate calculations. The introduction of this motor-curve period in the calculations enables one to approximate much more closely actual working conditions, and the results are much more accurate. In addition this method enables one to predetermine, without choosing any particular equipment, the effect of rate of acceleration, rate of braking, per cent of coasting, etc.

TABLE XII. — AVERAGE MOTOR CHARACTERISTICS

y = ratio of any given speed to speed at rated input	f = ratio of tractive effort at this speed to rated tractive effort	p = ratio of power input at this speed to rated input
0.80	2.06	1.82
0.85	1.67	1.54
0.90	1.38	1.30
0.95	1.19	1.14
1.00	1.00	1.00
1.05	0.83	0.87
1.10	0.71	0.78
1.15	0.61	0.70
1.20	0.53	0.625
1.25	0.46	0.57
1.30	0.41	0.525
1.35	0.365	0.48
1.40	0.32	0.45
1.45	0.295	0.425
1.50	0.27	0.395
1.55	0.23	0.375
1.60	0.215	0.35
1.65	0.20	0.33
1.70	0.18	0.315
1.75	0.17	0.30
1.80	0.16	0.29
1.85	0.14	0.28
1.90	0.13	0.265
1.95	0.125	0.26
2.00	0.12	0.25

Average Motor Characteristics. — To take into account the motor-curve portion of the speed-time curve it is necessary to consider the speed,

tractive effort and current input characteristics of the motors. However, instead of using the motor characteristics for any specific motor, the average characteristic curves of direct-current motors* given by Mr. Hutchinson are employed, see Table XII. These average characteristics were calculated by plotting the characteristic curves for the various sizes of direct-current motors manufactured by the General Electric and Westinghouse Companies, expressing the various quantities (current, speed, and tractive effort) as fractions of their values at rated load. Such curves are found to lie very close together, which justifies the use of a single set of curves representing the averages for the various motors. Mr. Hutchinson's curves were calculated from the characteristics of non-interpole motors, but have been found to check also quite closely with the curves for interpole motors. It is also found that these curves have practically the same shape irrespective of what point, between 75% and 125% of rated load, on the curves is taken as unity.

Method of Calculation. — The following symbols are employed:

n = number of stops per mile = total number of stops including one terminus divided by the distance between termini in miles,

V = average running speed in miles per hour,

$T = \dfrac{3600}{nV}$ = average running time between stops in seconds,

a = acceleration in miles per hour per second,†

b = braking rate in miles per hour per second,

r = train resistance in pounds per ton corresponding to a speed from 10 to 20% greater than the average speed V,

G = "equivalent" grade in per cent (*see above*),

g = average curvature, in degrees (*see above*),

$c = \dfrac{r + 20G + g}{100}$ = average "effective" coasting retardation in miles per hour per second,

ϵ = over-all efficiency (expressed as a fraction) of motors and gears at rated load; ϵ is about 0.85 for direct-current motors and about 0.75 for alternating-current motors, the latter figure including step-down transformer losses,

s = ratio of total standing time (including stops and lay overs) to total time that the train is moving,

W = total weight of train in tons,

M = total number of motors for the entire train,

E = average line voltage.

* The method is also applicable to alternating-current motors, but average characteristic curves for such motors are not available. They may, however, be readily constructed, or the characteristic curves given for direct-current motors may be used as an approximation.

† When the starting tractive effort is given, then a is to be calculated from the formula $a = (F - r) \div 100$, where F is the tractive effort in pounds per ton and r the train resistance corresponding to a speed from 10 to 20% greater than V. The actual starting acceleration will be greater than this, due to the lower train resistance at low speeds. Vice versa, if the starting acceleration is given, then the calculated tractive effort (and therefore the starting current and horse-power) at end of the controller period will produce a starting acceleration in excess of the assumed value. As far as the energy consumption and heating are concerned, however, these differences in the actual and assumed accelerations will be balanced by the higher train resistance at the higher speeds.

Calculate

$$\beta = \frac{a}{c},$$

$$A = \frac{V}{T} = \frac{nV^2}{3600},$$

$$m = 2A\left(\frac{1}{c} - \frac{1}{b}\right),$$

$$q = \frac{a}{A}(1 + m).$$

Select a value of the ratio

$$x = \frac{\text{time power is applied}}{\text{time of controller period}}.$$

A run of specified distance can be made in a given time with various values of this ratio x, since this ratio depends upon the proportion of the time that the train coasts; i.e., the greater the coasting time the smaller the value of x, see Fig. 1. The less the value of this ratio, however, the greater will be the starting current required, and therefore the larger the motor capacity in order to avoid sparking. By carrying through the calculations for several values of x one can determine the relation between energy consumption, time of coasting and starting current, and, by plotting, find the minimum value of the energy consumption corresponding to the data assumed, see Fig. 9. In typical rapid-transit service (1 or more stops per mile) minimum energy consumption usually corresponds to a value of x between 2 and 5, i.e., for the controller period lasting from 50 to 20 per cent of the total time power is on.

From Tables A to D, pp. 1196 and 1197, find the values of y, λ, u and i* corresponding to the selected value of x and the ratio β. Using these values calculate

$$J = \frac{x + \beta y}{q}, \quad \text{and} \quad H = \frac{\lambda + \beta y^2}{2q}.$$

Then calculate the quantities listed in the following table.

---

* The analytical expressions for x, λ, u and i in terms of y, f and p (see column headings of Table XII for definitions) are

$$x = 1 + \int_1^y \frac{\beta\, dy}{(1+\beta)f - 1}$$

$$\lambda = 1 + 2\int_1^x y\, dx$$

$$u = \frac{1+\beta}{18.1\beta}\left[0.75 + \int_1^x p\, dx\right]$$

$$i = 199\left(1 + \frac{1}{\beta}\right)\sqrt{1 + \int_1^x p^2\, dx}.$$

In the expression for u the constant 0.75 is for series-parallel control. For straight resistance control change 0.75 to 1.00 and for series-series-parallel control change 0.75 to 0.69. For a-c. commutator motors change 0.75 to 0.56. Tables A to D were made up by calculating the value of the above expressions graphically, using for y, f and p the values given in Table XII. Similar tables may readily be calculated for a-c. commutator motors, or for any other type of motor whose "percentage" characteristics differ from Table XII.

Speed at end of controller period in miles per hour....	$V_1 =$	$\dfrac{V}{J + \sqrt{m(H - J^2)}}$
Maximum speed in miles per hour..............	$V_m =$	yV_1
Watt-hours per ton-mile..........................	$U =$	$\dfrac{unV_1^2}{\epsilon}$
Ratio of total running time to time on controller.....	$X =$	$\dfrac{aV}{AV_1}$
Ratio of coasting time to running time..............	$C =$	$\dfrac{b(X - x) - ay}{(b - c)X}$
Horsepower output at end of controller period......	$P_0 =$	$\dfrac{(a + c)V_1W}{3.74M}$
Starting current per motor *........................	$I_0 =$	$\dfrac{199(a + c)V_1W}{ME\epsilon}$
Root-mean-square current per motor (including stops) *....................................	$I_e =$	$\dfrac{aiV_1W}{ME\epsilon\sqrt{X(1 + s)}}$
Average voltage across motor terminals (including stops)..	$E_m =$	$\dfrac{(x - 0.45)E}{X(1 + s)}$

* For alternating-current series motors insert the power factor (as a decimal) in the denominator of the formula.

Example. — Consider the following example:

Number of stops per mile..........................	$n = 0.938$
Average running speed..........................	$V = 28.7$ mph.
Acceleration rate..............................	$a = 1.84$ mphps.
Braking rate..................................	$b = 1.60$ mphps.
Train resistance at 34.4 mph..................	$r = 15$ lb. per ton.
Average equivalent grade......................	$G = 0.22$ per cent
Average curvature..........................	$g = 1.2$ per cent
Average effective coasting retardation..........	$c = 0.21$
Efficiency of motors and gears at rated load.....	$\epsilon = 0.88$
Ratio of standing to running time (20 sec. stops).	$s = 0.15$
Total weight of train in tons..................	$W = 132$ tons
Total number of motors......................	$M = 6$
Average line voltage..........................	$E = 571$

Then

$$\beta = \frac{1.84}{0.21} = 8.76,$$

$$A = \frac{0.938\,(28.7)^2}{3600} = 0.214,$$

$$m = 2 \times 0.214\left(\frac{1}{0.21} - \frac{1}{1.60}\right) = 1.77,$$

$$q = \frac{1.84\,(1 + 1.77)}{0.214} = 23.8.$$

Then (*see above for meaning of symbols*)

$x =$		1	2	4	6	10
$y =$	....	1.00	1.43	1.69	1.83	1.96
$\lambda =$	....	1.00	3.52	9.88	17.0	32.1
$u =$	....	0.0462	0.0822	0.126	0.163	0.228
$i =$	....	222	260	281	301	321
$J =$	$\dfrac{x + 8.76\,y}{23.8}$	0.410	0.608	0.788	0.925	1.143
$H =$	$\dfrac{\lambda + 8.76\,y^2}{47.6}$	0.205	0.452	0.735	0.973	1.380
$V_1 =$	$\dfrac{28.7}{J + \sqrt{1.77\,(H - J^2)}}$	43.1	28.8	23.1	20.9	19.2
$U =$	$1.07\,u V_1^2$	92	73	72	76	90
$X =$	$\dfrac{247}{V_1}$	5.73	8.57	10.7	11.8	12.9
$C =$	$1.15\left(1 - \dfrac{x}{X}\right) - \dfrac{1.32\,y}{X}$	0.72	0.67	0.51	0.36	0.06
$P_0 =$	$12.07\,V_1$	520	348	279	253	232
$I_0 =$	$17.8\,V_1$	770	512	411	371	342
$I_e =$	$\dfrac{0.076\,iV_1}{\sqrt{X}}$	304	195	151	139	131
$E_m =$	$\dfrac{497\,(x - 0.45)}{X}$	47	90	165	233	367

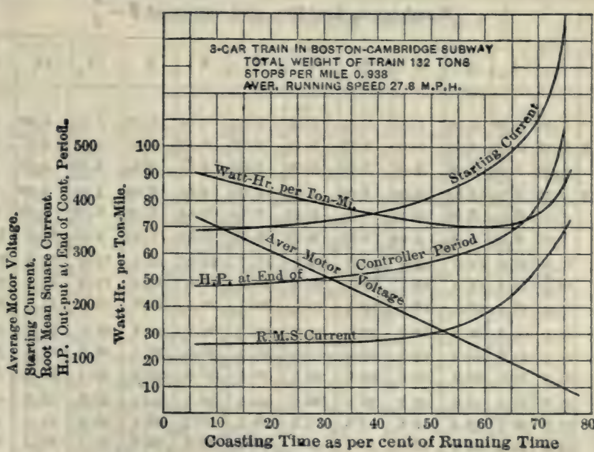


Fig. 9.

These results are plotted in Fig. 9. An actual test on this road was made, showing an average coasting time of 10 per cent. The table on p. 1320 gives the

TABLE A. — VALUES OF y IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	1.17	1.22	1.24	1.26	1.27	1.28	1.28	1.28	1.29	1.29	1.29	1.29
2	1.20	1.30	1.35	1.38	1.40	1.42	1.42	1.43	1.43	1.43	1.45	1.45
2.5	1.22	1.34	1.41	1.45	1.47	1.49	1.51	1.51	1.52	1.53	1.55	1.56
3	1.23	1.37	1.44	1.49	1.53	1.56	1.58	1.59	1.60	1.61	1.63	1.64
4	1.24	1.37	1.48	1.55	1.60	1.63	1.66	1.68	1.70	1.72	1.76	1.79
5	1.24	1.38	1.50	1.58	1.65	1.70	1.73	1.76	1.79	1.81	1.86	1.89
6	1.24	1.38	1.51	1.60	1.67	1.73	1.77	1.81	1.84	1.87	1.94	1.98
7	1.24	1.38	1.52	1.61	1.68	1.75	1.81	1.85	1.88	1.92	2.00	2.05
8	1.24	1.39	1.52	1.62	1.70	1.77	1.83	1.88	1.91	1.95	2.05	2.11
9	1.24	1.39	1.53	1.63	1.71	1.78	1.84	1.90	1.94	1.98	2.10	2.16
10	1.24	1.39	1.53	1.63	1.72	1.80	1.86	1.92	1.97	2.00	2.13	2.20
12	1.24	1.39	1.53	1.64	1.73	1.81	1.88	1.95	2.00	2.05	2.20	2.28
14	1.24	1.39	1.53	1.64	1.74	1.83	1.90	1.97	2.02	2.07	2.24	2.35
16	1.24	1.39	1.53	1.64	1.75	1.84	1.92	1.98	2.05	2.10	2.27	2.40
18	1.24	1.39	1.53	1.65	1.75	1.85	1.93	2.00	2.07	2.12	2.31	2.44
20	1.24	1.39	1.53	1.65	1.76	1.86	1.94	2.02	2.09	2.15	2.34	2.48

Example. — For $x = 2$ and $\beta = 3$, $y = 1.35$.

TABLE B. — VALUES OF λ IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	2.12	2.14	2.15	2.16	2.16	2.17	2.17	2.17	2.17	2.17	2.17	2.17
2	3.30	3.40	3.46	3.48	3.50	3.50	3.52	3.52	3.52	3.52	3.52	3.52
2.5	4.50	4.74	4.84	4.90	4.92	4.94	4.96	4.98	5.00	5.00	5.02	5.04
3	5.70	6.10	6.28	6.38	6.44	6.50	6.52	6.54	6.58	6.58	6.61	6.65
4	8.16	8.84	9.30	9.44	9.60	9.70	9.80	9.83	9.90	9.90	10.0	10.1
5	10.6	11.6	12.3	12.6	12.8	13.0	13.1	13.2	13.3	13.4	13.6	13.6
6	13.0	14.4	15.4	15.7	16.1	16.4	16.6	16.8	17.0	17.2	17.4	17.7
7	15.4	17.1	18.1	18.9	19.4	19.9	20.3	20.5	20.6	20.9	21.3	21.7
8	18.0	19.8	21.2	22.2	23.0	23.7	24.1	24.5	24.8	25.0	25.5	25.9
9	20.4	22.6	24.2	25.4	26.2	27.0	27.7	28.0	28.3	28.8	29.7	30.0
10	23.0	25.6	27.4	28.6	29.6	30.5	31.2	31.9	32.2	32.7	33.8	34.3
12	27.8	31.0	33.6	35.2	36.8	38.0	39.0	39.7	40.2	40.8	42.3	43.4
14	32.6	36.6	39.4	41.8	43.8	45.8	46.5	47.3	48.1	48.9	51.0	53.0
16	37.4	42.2	45.6	48.2	50.6	52.7	54.1	55.8	56.8	57.7	60.3	62.2
18	42.2	47.6	51.6	54.8	57.4	60.0	62.0	63.3	64.8	66.0	69.6	72.0
20	47.2	53.4	58.0	61.6	64.8	67.2	69.4	71.2	73.0	74.2	79.0	82.0

TABLE C.—VALUES OF u IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	0.082	0.0620	0.0550	0.0515	0.0495	0.0481	0.0472	0.0466	0.0460	0.0456	0.0445	0.0440
1.5	0.125	0.0922	0.0813	0.0760	0.0729	0.0709	0.0695	0.0685	0.0676	0.0670	0.0642	0.0627
2	0.161	0.115	0.100	0.0940	0.0895	0.0863	0.0843	0.0830	0.0819	0.0810	0.0780	0.0769
2.5	0.195	0.134	0.118	0.109	0.104	0.101	0.0980	0.0960	0.0948	0.0938	0.0902	0.0888
3	0.230	0.155	0.133	0.123	0.117	0.113	0.110	0.107	0.106	0.104	0.100	0.0975
4	0.298	0.193	0.162	0.149	0.140	0.135	0.131	0.128	0.126	0.124	0.118	0.115
5	0.362	0.232	0.194	0.175	0.163	0.156	0.150	0.146	0.143	0.141	0.135	0.133
6	0.431	0.270	0.222	0.199	0.185	0.176	0.170	0.165	0.162	0.159	0.151	0.147
7	0.500	0.310	0.252	0.223	0.207	0.196	0.188	0.182	0.179	0.176	0.166	0.161
8	0.565	0.347	0.279	0.247	0.227	0.215	0.207	0.200	0.195	0.191	0.179	0.174
9	0.634	0.386	0.309	0.270	0.249	0.233	0.223	0.216	0.210	0.206	0.192	0.188
10	0.705	0.425	0.338	0.293	0.270	0.252	0.241	0.232	0.227	0.221	0.204	0.199
12	0.833	0.500	0.390	0.339	0.307	0.287	0.273	0.263	0.255	0.249	0.230	0.220
14	0.965	0.580	0.450	0.384	0.349	0.324	0.309	0.296	0.286	0.278	0.251	0.239
16	1.11	0.653	0.504	0.430	0.388	0.360	0.340	0.327	0.314	0.305	0.273	0.259
18	1.24	0.738	0.560	0.475	0.427	0.395	0.373	0.356	0.342	0.331	0.293	0.278
20	1.38	0.809	0.620	0.521	0.468	0.430	0.406	0.387	0.371	0.359	0.314	0.290

TABLE D.—VALUES OF i IN TERMS OF x

x	Numbers in first line give values of $\beta = \frac{a}{c}$											
	1	2	3	4	5	6	7	8	9	10	15	20
1	398	300	267	249	239	232	228	224	221	220	213	209
1.5	452	334	297	278	267	260	253	250	248	245	238	233
2	488	355	312	292	280	272	267	262	259	257	249	243
2.5	518	369	322	301	289	280	274	270	267	263	257	250
3	546	383	337	311	298	289	281	277	273	270	261	254
4	599	411	352	327	310	300	292	287	282	279	270	263
5	647	432	369	338	321	309	301	295	290	288	277	270
6	688	455	382	350	331	318	310	303	299	295	283	275
7	730	475	397	360	340	326	317	310	303	300	289	280
8	770	497	409	369	347	331	321	315	310	305	292	284
9	806	513	421	379	355	340	329	321	315	310	297	288
10	845	531	431	387	361	344	333	327	320	315	301	291
12	911	565	454	403	377	358	345	337	329	323	307	295
14	975	600	479	422	391	370	356	347	338	331	313	300
16	1030	630	500	440	404	380	365	353	344	339	318	303
18	1085	656	518	451	414	390	372	360	350	343	320	307
20	1140	681	531	462	423	399	380	368	357	350	326	309

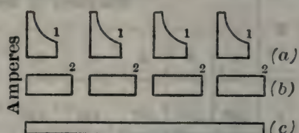
actually observed values of the various quantities and the corresponding values taken from the calculated curves in Fig. 9 for a coasting time of 10 per cent.

	Observed	Calculated
Watt-hours per ton-mile	91	88
Horse-power output at end of controller period	249	241
Starting current per motor	370	345
Root-mean-square current per motor	132	131
Average motor voltage	337	350

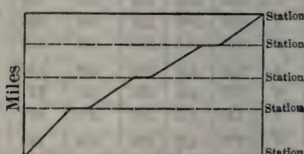
From Table XI the proper size of motor would be the interpole motor No. 300, which is rated as 200 horse-power at 550 volts. The motor actually used was a Westinghouse motor, designated as 300 – *B*, and rated at 225 horse-power at 600 volts. The temperature rise from test, after 13 hours in service, was

Armature	65° C.
Commutator	75° C.
Series field	46° C.
Interpole field	52° C.
Frame	37° C.

Importance of Coasting and Selection of Gear Ratio. — A study of Fig. 9 shows that a much less energy consumption would be required for a coasting period of 50 per cent instead of the actual coasting period of 10 per cent, namely 72 watt-hours instead of 88 watt-hours. The starting current required in order to obtain this higher coasting time is 406 amperes instead of 345 amperes, or an increase in the starting current of 18 per cent. If no change in the size of motor were made, this would require approximately the same percentage increase in the gear ratio. The root-mean-square current would increase from 131 to 150, but the average motor voltage would drop from 350 to 170. The motor could therefore probably operate at this higher gear ratio without seriously overheating, but it would be safer to use a larger size motor, particularly as the starting current of 406 amperes is also close to the safe commutating limit.



Hours
Load Diagram



Hours
Train Diagram
Fig. 10.

TRAIN AND LOAD DIAGRAMS. — The current-time curve for a train making a number of stops may be represented as shown by (*a*) in Fig. 10. On a railway line where there are several trains, the total current may be obtained by placing the current curve for each train at its proper place in the time scale, and adding the ordinates of the curves. Such a process is very tedious and unnecessary where there is a large number of trains. In such cases the high and low parts of the curves become staggered with respect to one another more or less according to the laws of chance, so that each current curve may be replaced by a rectangle of the same area but with a base extending over the entire running time as shown by (*b*) in Fig. 10.

When this is done the kilowatts and amperes per train are derived from the watt-hours per ton-mile by the following formulas:

$$\text{kilowatts} = \frac{WV}{1000} \times (\text{watt-hours per ton-mile}),$$

$$\text{amperes} = \frac{WV}{E} \times (\text{watt-hours per ton-mile}),$$

where

W = weight of train in tons,

V = average running speed (excluding stops) in miles per hour,

E = line voltage.

The time when the current is cut off is indicated by 1, and that when the train stops, by 2, in Fig. 10.

Another approximation, which is even more often used, is to replace the series of rectangles shown at *b*, by a single rectangle as shown at *c* in Fig. 10. The area of this rectangle will be equal to the sum of the areas of the smaller rectangles or current curves. Using this approximation, kilowatts or amperes may be obtained from the above formulas, taking, however, V to be the schedule speed (i.e., speed including stops). The procedure is to plot a train diagram showing when each train comes on and off the line, neglecting intermediate stops, as shown for a simple case in Fig. 11. Each time a train comes on or off, the corresponding kilowatts or amperes are added to, or subtracted from, the load diagram.

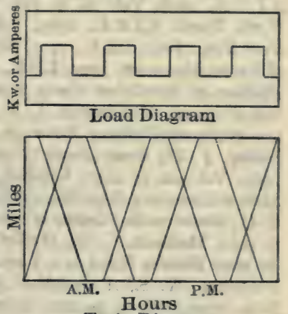


Fig. 11.

Power Required for Car Heating and Lighting. — In addition to the energy required for propelling the cars, a very appreciable amount is also required in the winter, for heating them, and a small amount at night for lighting. In making up a load diagram this energy should be included.

The average power for car heating varies, of course, with the climate and time of year. The following figures represent usual requirements in the northern parts of the United States:

TABLE XIII. — HEATING AND LIGHTING OF CARS

Length of car, feet	Average* kw. for lighting	Average† kilowatts for heating	
		Average conditions	Severe conditions
14–20	0.25	3.5	4
20–28	0.35	4.5	5.5
28–34	0.55	5.5	7.5
34–40	0.70	7.5	10.5

* During the hours lights are on, using Tungsten lamps.

† During the time car is in service.

Substation and Power-station Loads. — The load diagram obtained as described above gives the total load at the trains. To obtain the load at a sub-

station, the kilowatts or amperes must be increased by a suitable amount to allow for the losses in the distribution system. The load diagram of the power station should allow for all transmission and distribution losses between the power house and substation (*see Power Stations; Substations; Third-rail Systems; Trolley Systems*), and also for all auxiliary power, such as that required for station lighting, shop machinery, etc.

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RAILWAYS, LOCATION AND PERMANENT WAY FOR. — (*See also Railways, Energy Requirements and Motor Capacity for; Railways, Electric, Traction Systems for; Third-rail Systems; Trolley Systems.*) There are four general steps in the determination of the location of a railway, (1) the reconnoissance, which is a personal examination by the locating engineer of the country through which the railway is to run, (2) a preliminary survey and investigation more particularly of the topographic features of the proposed road, (3) office study of the data obtained in (1) and (2), and (4) the field location and the preparation of final location plans.

RECONNOISSANCE. — First a careful examination of the country should be made, with a view not only of obtaining a line which will be reasonably straight and as free from grades as possible, but also with a view of selecting a line or lines which will earn a suitable revenue. An electric railway reconnoissance calls for a much more detailed study of the country than a steam railroad location and a very much more detailed study of the amount of population, its growth, and the likelihood of the population making use of the proposed line. The adapting of an electric line to take in the various intermediate centers of population is practically a necessity if the income of the line is to be sufficient to take care of the investment. The engineer should therefore realize fully that the commercial and financial matters relating to the road are more important than the engineering problems, and should always bear in mind that the road is constructed for the purpose of selling transportation.

Electric railway locations are divided in general into two classes: those which are operated on the public highways, and those which are operated upon their own private rights of way, where they are free to make the fastest practicable schedules. Speed and comfort must be considered. Both of them are, as a rule, better obtained on private locations. It is essential in this reconnoissance for the engineer to make a careful estimate of the probable gross income of the road, as well as to choose two or three lines which will later on be investigated by preliminary surveys.

PRELIMINARY SURVEY. — After the favorable lines have been determined by the locating engineer in his reconnoissance, a party is sent into the field to make a preliminary survey. This is usually a linear traverse run by transit and tape. After this a level party determines the profile of this line, this party being followed by a topographical party which sketches in the 5-foot contours, the locations of brooks, highway crossings, the number and kinds of industries located within say a mile or so of the line, the location of existing recreation points and picnic grounds, as well as the possible location and development of those which do not now exist. The engineer running the preliminary line should keep in mind that the railroad is built for the convenience of the public and that any inconvenient detail of the road will have a material effect upon its gross income.

The topographical party should note the character and property value of buildings or other structures which the line may affect, the character of soil (whether rock or gravel), the location of possible borrow pits, etc. In some locations the topographical features should extend a considerable distance from the transit line; in other locations, where, for example, the line is passing through a narrow valley, the topographical features need not be taken very far from the transit line. While the line should be as straight as possible, some curves are necessary. In locating curves consideration should be taken of the fact that electric traction permits of very rapid acceleration and retardation, and therefore sharper curves are permissible than in steam railroad practice because of the shorter time required to pass around it, even at the

slower speed rendered necessary because of its greater curvature. The time lost in passing around the curve as well as the permissible speed are factors which should be taken into consideration. Roughly speaking, a safe rule is that the speed on curves in miles per hour can be equal to the square root of the radius in feet.

OFFICE STUDY. — On the preliminary map an accurate paper location is drawn and an estimate is made of the cost of the road and of the probable gross earnings, as well as of the fixed charges and the cost of operating and maintaining. This study is one of great importance and if the reconnoissance and preliminary survey are not thoroughly accomplished, errors in judgment may very easily creep into this part of the work. The office study shows the final alignment and the proposed grades of the road.

FIELD LOCATION. — After the completion of the office study the line is finally located in the field. Field location requires running out the line by straight lines and curves, using easement curves at the beginning and end of all changes in direction; the preparation of property maps showing the location required for the railway and all parcels of property affected by actual takings for slope easements; as well as studies of grade crossing eliminations where they are necessary. At this time soundings or borings are taken for bridges or other structures. Cross sections are made of the line, from which careful estimates are made. These cross sections are used in computing the final payment for the work of construction.

Locations in Existing Highways. — Such locations usually have the great advantage of easy grades which frequently cannot be so cheaply obtained upon private land. Locations in highways have the disadvantage of requiring much slower speed. Where such locations are made it is necessary to survey the highways as laid out by the proper officials, and not infrequently it is necessary to consider the widening of such highways by takings on either side. The electric road, for example, may be a 2-track road laid out in the center of the roadway, in which case the road would possibly occupy nearly the full width of the existing highway. In this case the electric road would be required to purchase additional property on either side and probably to build roadways for other vehicles in lieu of the portion which the electric line occupies. When this is done high speeds can sometimes be maintained. Where the electric road occupies one side of an existing highway, the speed often has to be reduced to a relatively low rate. This affects the amount of traffic which the road will attract and thereby reduces the gross earnings.

DEFINITIONS OF TERMS USED ON RAILWAY MAPS AND PROFILES. — Some of the more common terms are defined below.

Curves. — A curve is generally composed of successive arcs of circles joining two straight lines or tangents. When these arcs are of varying radii decreasing with the distance from the tangents the curve is said to be compounded or spiralled. To insure smooth riding the curve should begin with a large radius and gradually grow sharper until the circular part is reached. At the same time the outer rail should be gradually raised (see below) until it reaches its maximum elevation. A curve so built is called an easement curve. Curves, if properly eased, may be run at full speed. On high-speed lines curves sharper than 10 degrees should be avoided.

The following terms are in general use:

The point of curvature (P. C.) is the beginning of the curve, and the point of tangency (P. T.) is the end of the curve, going in the direction of the surveyor's stationing on the line.

The point of intersection (P. I.) is the point where the two tangents through P. C. and P. T. intersect.

The tangent distance is the distance between the P. C. or P. T. and P. I.

The degree of curvature is the angle subtended at the center of a curve by a chord 100 ft. long. Up to a curvature of 12 degrees the radius of curvature may be found to a close approximation by dividing 5730 by the degree of curvature. The exact relation between the degree of curvature C and the radius R is

$$R = \frac{50}{\sin\frac{C}{2}}.$$

The middle ordinate is the perpendicular distance from the center of a chord to the curve.

Elevation of Outer Rail on Curves.—The outer rail on curves is raised above the inner an amount depending upon the velocity of cars and the degree of the curve. This elevation must be gradually attained. The amount of elevation required in order to make the weight of the car just balance the centrifugal force on the curve is

$$E = \frac{DV^2}{32.2\,R} \quad \text{feet},$$

where D is the distance between center lines of the two rails (not the track gage), V the speed of car in feet per second, and R the radius of curvature in feet. For standard track gage, viz., 4 feet 8½ inches from inside edge of the *head* of one rail to the inside head of the other rail, and taking $R = 5730/C$, where C is the curvature in degrees, the above expression for the elevation becomes

$$E = 0.000325\,V^2C \text{ inches.}$$

Grades. — A railroad grade is expressed in per cent, this per cent being the number of feet vertical rise in a horizontal distance of 100 feet; i. e., a 4 per cent grade means a rise of 4 feet in a horizontal distance of 100 feet. Calling L the distance in feet along the track and H the vertical rise in feet in this distance, the per cent grade may also be expressed as

$$G = \frac{100\,H}{\sqrt{L^2 - H^2}},$$

which for a grade less than 10 per cent is equal to $100\,H/L$ with an error of less than 1 part in 100.

Grades should be as small as financially practicable. It may be cheaper to operate over a grade than to pay interest on the sum needed to reduce it. Some steam road grades are as steep as 4 per cent, but such grades are extremely costly to operate. Two per cent is about the limit for steam roads and most roads try to keep grades down to about 0.5 per cent. On electric lines operating single trolley cars grades as steep as 10 per cent exist, but no grade over 6 per cent ought to be used unless it is absolutely unavoidable.

Virtual and Momentum Grades. — A driving force or tractive effort is required to accelerate a train, and a driving force or tractive effort is required to cause a train to ascend a grade at constant speed; hence an increase of the speed of the train may be considered, as far as the motive power is concerned, as equivalent to an up-grade. *Vice-versa,* a decrease of speed may be considered as equivalent to a down-grade. The energy required to give the train a speed of v miles per hour is the same as the energy required to raise it a height of $0.0334\,v^2$ feet. Hence, if to the actual elevation of the profile at each point is added a height of $0.0334\,v^2$ feet, where v is the velocity of the train at this

point, and a line is drawn showing the sum of the actual elevation and this velocity head at each point, this line may be looked upon as the "virtual profile" of the road, and the slope of this virtual profile, at any point, i.e., the change in its elevation per hundred feet, is called the "virtual grade" at this point.

The virtual grade at any point on an actual up-grade will be less than the actual grade, if the speed of the train is decreasing as it passes this point; the grade in this case is sometimes called a "momentum grade." That is, if the train has a high speed when it strikes an up-grade, and the operating requirements of the division will permit of a slowing down of the train as it ascends the grade, then the effective or virtual grade will be less than the actual grade. Under such circumstances a given locomotive can pull a given train up a short steep grade on which it could neither start this train nor pull it at constant velocity. A momentum grade must be comparatively short.

Ruling Grade. — The limiting or ruling grade on a division is that grade which limits the weight of the train which can be hauled over this division by the regular motive-power unit. Ordinarily, it is the maximum grade on the division, but if momentum grades are relied upon, the ruling grade might be the maximum virtual grade. Reliance upon momentum grades is not always considered good practice, since a heavy train which might get over this grade if it strikes the grade with sufficient momentum, cannot start on this grade if it should have to stop on the grade due to some emergency. When steep grades occur at only one or two places in a division it is sometimes economical to use helper engines on these steep grades, thus making possible the hauling of heavier trains, the weight of train then usually being limited by maximum grade on the remainder of the division.

Switches, Frogs, Cross-overs, Etc. — A switch is an arrangement of rails, which permits a car or train to pass from one track to another. A cross-over consists of a pair of switches connected by a short piece of track in such a way as to allow a car or train to pass or cross over from one track to another parallel one. A turn-out or siding is a short length of track parallel to the main track and connected thereto by a switch at each end; when used as a means of passing cars or trains it is usually called a turn-out, and when used primarily for storing cars or loading it is called a siding.

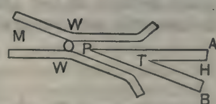

Fig. 1.

A frog, Fig. 1, is that part of the switch where the two inner rails cross each other, and is so designed as to allow the flange of the wheel to pass over without "riding up" on the rail. A guard rail is set close to the switch rail opposite the frog. The lead of a frog is the distance of a frog from the frog point P, see Fig. 1, to the point of the switch. The point of the switch is the tip end of the movable rail of the switch which is planed down to fit closely against the main rail. The number of a frog is equal to the quotient of the distance from P to the line AB divided by the length of the line AB.

Turn-outs. — Turn-outs are short sidings used on single-track roads to permit cars running in opposite directions to pass each other. The proper location of turn-outs is governed to a certain extent by topography. They can be readily located by drawing up a train diagram; see *Railways, Energy Requirements and Motor Capacity for.* To keep the number of turn-outs at a minimum the time intervals between cars (headway) should all be multiples of the shortest time interval. For accurate results the diagram should be carefully drawn to a scale large enough so that locations can be measured with an error not to exceed 200 feet for distance and one minute for time.

CONSTRUCTION OF PERMANENT WAY.—In the preparation of the roadbed for ballast, ties and track, it should be noted that rails settle more or less and allowance must be made for this when grading is in progress. The surface of the roadbed should be flat, from14 to 20 feet wide for a single-track road, and from 9 to 10 feet wider for a double-track road, with a ditch at each side for drainage. On this the ballast of broken stone, gravel or cinders is laid varying from 6 to 12 inches thick under the ties. The ballast should ultimately be brought up level with the tops of the ties.

Ballast. — Broken stone is best but most expensive in first cost and maintenance. It is clean and dustless. Gravel is cheaper and very satisfactory from a maintenance standpoint. It is apt to be very dusty. Cinders are fairly satisfactory but are very dirty.

Ties.—Ties are of the following materials in the order named: Oaks, southern pine, Douglas fir, cedar, chestnut, cypress, western pine, tamarack, hemlock, redwood, lodge-pole pine and white pine; see article on *Timber*. The average cost is about $1.00. Treated ties cost about $1.25. With the rapidly increasing cost of ties, preservative treatment is becoming important. Creosoting is best. Zinc chloride treatment is satisfactory in arid regions only. The average life of an untreated tie is 5 years for hemlock, 7 to 12 years for white oak, and 15 years for cedar ties. Treated ties may last 30 years, but must be protected from mechanical injury by rails and spikes. For this purpose tie-plates and screw spikes should be used. Ties are spaced about 2 feet between centers.

Rails.—See article on *Rails, Track and Third*.

Grade Crossings. — On high-speed lines grade crossings should be avoided wherever practicable. It costs less to avoid them in the beginning than to eliminate them afterwards. To eliminate an existing grade crossing costs from $40,000 up unless the topography is usually favorable.

COST OF PERMANENT WAY. — The following costs are based on pre-war prices. When tracks are laid in paved streets the railway usually has to pay for paving 18 inches of the street outside the rails. The total cost of such track, excluding overhead construction, and using 8- or 9-inch girder rails is about $5 per foot of single track. When the track is constructed with a light T-rail, as may be the case when laid on a private right of way or at the side of a street where there is no paving, the cost may be as low as $1 per foot, exclusive of ballast, bonding, and overhead construction. Ballast will cost from $1000 for cheap gravel to $3000 for thin rock, per mile of track. Thick rock ballasting as used on steam railroads may go as high as $5000 per mile.

Maintenance Costs. — Maintenance of way costs will depend largely on the character of the original construction and also upon the character and amount of traffic. On typical Massachusetts street and interurban railways maintenance of way expenses are reported as from $200 to over $500 per mile of track. $200 per mile is not sufficient to maintain the road and track in an adequate manner, but fom $350 to $380 is probably a fair figure.

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REACTANCE COILS OR REACTORS.—(*See also Alternating Currents; Inductance and Inductive Reactance.*) Coils designed particularly for the purpose of introducing reactance (*see Inductance and Inductive Reactance*) into a circuit, when connected therein, are called "reactance coils" or "reactors."

APPLICATIONS. — Reactance coils are used wherever (1) it is desired to produce a voltage drop in an a-c. circuit without producing a proportionate loss in power (*see Controllers*); (2) to introduce inductance into a circuit where a compounding effect is to be obtained by passing a leading current through the coils (*see Converters, Synchronous*) ; (3) in large power stations to limit the short-circuit current; and (4) as "choke coils" in connection with lightning arresters (*see Lightning Protectors*).

RATING OF REACTANCE COILS. — It is common practice to rate a reactance coil in terms of the kilo-volt-amperes absorbed by it when carrying such a current (of the given frequency) as will produce a temperature rise of 50° C., or 70° C., respectively, above the room temperature, depending upon the type or class of insulation used in accordance with the Standards of the A.I.E.E. on temperature limits. Usually the resistance is negligible compared with the reactance and under this condition the k-va. rating is I^2x.

Per cent Reactance. — A reactance coil having a reactance of x ohms is said to have a "per cent reactance" equal to $(100\,x\,I) \div V$, where I is the normal or full-load current through it and V the *total* normal voltage across the circuit in which it is placed; i.e., the percentage reactance is the percentage ratio of the reactance drop at normal current to the total voltage impressed on the circuit in which it is inserted. When the coil is to be inserted in one branch of a Y, the voltage V is the volts to neutral.

In the case of a three-phase generator, three-phase transformer or bank of transformers, or set of three single-phase reactance coils, the Y reactance, x, (*see Alternating Currents*) is related to the per cent reactance, p, by the formula

$$x = \frac{10\,pE^2}{(kv\text{-}a.)}, \tag{1}$$

where E is the ***kilovolts*** between phases (i.e., between the terminals of the Y) and (*kv-a.*) stands for the total kilovolt-ampere rating of the three phases.

DESIGN OF IRON-CORE REACTANCES. — When a high inductance is required the coils are usually wound on an iron core, which usually contains an air gap, for the inductance of a coil on a completely closed iron core is by no means a constant, due to the variation of the permeability of the iron with the magnetizing current, and a straight-line relation between voltage and current does not hold. When a relatively low inductance is required the iron core may be omitted entirely. Reactance coils for use in connection with synchronous converters have an iron core.

In iron-core reactances the air gap is usually of such a length that practically all of the reluctance of the circuit is in the air gap. Hence in calculating the reactance of the coil the reluctance of the iron portion of the path may be neglected without materially affecting the results, provided normal magnetic densities exist in the iron (60,000 to 80,000 lines per square inch, maximum values). As the gap is made adjustable such errors as result from this approximation are allowed for by adjusting the gap afterwards.

The inductance L in henries and the reactance x in ohms are given approximately (neglecting effect of iron) by the equations

$$L = \frac{3.2\,S^2A}{10^8\,l} \quad \text{and} \quad x = 2\,\pi f L, \tag{2}$$

where S = number of turns in coil; A = cross section of path in gap, in square inches; l = length of gap, in inches; f = frequency in cycles per second. This must, however, be checked to see that the magnetic density B is not too great at the maximum current to be used, thus, $B = (3.2 \times \sqrt{2}\, SI) \div l$, where I = the effective value of current; B = maximum value of flux density = 60,000 to 80,000 lines per sq. in.

The losses in such a coil are the core-loss in the iron and the RI^2 in the copper. See article on *Magnetic Properties of Materials* for the method of calculating the core-loss. The procedure in proportioning the various parts is similar to that employed in the design of transformers (q.v.).

POWER-LIMITING REACTANCES.* — (*See also Power Stations.*) In order to avoid the prohibitive expense of high-voltage insulation, power-limiting reactances are always designed to be installed in the low-tension circuits. From the standpoint of economy, this requirement prohibits the use of a magnetic core. When power-limiting reactances are installed at a distance greater than one-half their diameter from any iron or steel structure, no appreciable eddy current or hysteresis losses will be produced in such structures.

The inductance and reactance of the coils can be calculated from the formula for a short solenoid given in the article on *Inductance and Inductive Reactance.*

It is desirable to reduce the size of reactance coils as much as possible, and they are therefore now usually designed for a temperature rise of 70° C. As to the dielectric test, the A.I.E.E. rules recommend 2¼ times the line voltage plus 2000 for one minute, from conductor to ground.

Location of Reactance Coils. — As noted above the reactance coils may be inserted in the leads from the generator, between sections of the low-tension bus-bars, or in the low-tension leads of the transformers. Which one of these locations or combinations is preferable depends on a number of conditions. Modern water-wheel-driven generators are now designed for a very high inherent reactance and external power-limiting reactances are usually inserted between the bus-bar sections or in the low-tension transformer leads.

Reactances in Generator Leads. — With reactances in the generator leads the current flowing in the armature winding is limited, and this method therefore affords an excellent protection for the generator itself. An objection to generator reactances is the fact that a short-circuit on or near the bus-bars will cause a voltage drop on all the feeder circuits connected thereto.

Reactances between Bus-bar Sections. — With reactance inserted between the bus-bar sections the trouble is confined to the particular section on which short-circuit takes place. Bus-section reactances afford no protection to the generators connected to the bus to which the faulty line is connected, but they give added protection to the generators on the other sections.

Reactances in Low-tension Leads of Transformers. — Reactances in the low-tension leads of the transformer banks are of considerable value for protecting against short-circuits in the lines, where they, of course, mostly take place. They are, however, not of value if the short-circuit should occur on the low-tension bus or in the generators or in their leads. There is also a constant loss of power in the reactance coils when they are inserted in the transformer leads, as is also the case when they are installed in the generator leads. For large systems this may reach a considerable value and must not be ignored when the selection of reactances is made.

Calculation of Short-circuit Kilovolt-amperes.— The short-circuit current for any arrangement of reactances and for a short at any point can be found

* By D. B. Rushmore.

with sufficient accuracy by neglecting the resistances of the apparatus and connections and calculating the resultant reactance by applying successively the formulas for two reactances in series or in parallel, as the case may be. Note that the resultant reactance X_s of two reactances, x_1 and x_2 in series, and the resultant reactance X_p of two reactances, x_1 and x_2 in parallel, are respectively

$$X_s = x_1 + x_2, X_p = \frac{x_1 x_2}{x_1 + x_2} \qquad (3)$$

provided the resistances of the circuits are negligible. The reactances, x_1, x_2, etc., of the various circuits can be calculated from the percentage reactance by formula (1). Let X_r be the resultant reactance of all the circuits feeding into the short circuit, and let E be the kilovolt rating of the generators, then

$$\text{Total short-circuit kv-a.} = \frac{1000\, E^2}{X_r} \qquad (4)$$

For example, consider the case of four, 12,000 kv-a., three-phase, 11,000-volt generators feeding into a bus which is divided into two sections, A and B, with two generators feeding into each section, and one transmission line from each section, each line fed through a bank of transformers. A set of reactance coils, having 6 per cent reactance, connects the two bus sections. The inherent reactance of each transformer bank is 8 per cent, and the inherent reactance of each generator is 20 per cent. The reactance of each generator is then 2.0 ohms; of each transformer bank 0.4 ohm, and of each set of reactance coils 0.6 ohm. Let a short circuit occur between the three wires of the line connected to the bus section B. We then have $2.0/2 = 1.00$ ohm in series with 0.6 ohm, or a total of 1.6 ohms, which total is in parallel with the other two generators, i.e., in parallel with 1.0 ohm, giving a resultant reactance up to the transformer bank of 0.6 ohm; this is in series with the transformer bank, giving a final resultant reactance, $X_r = 0.6 + 0.4 = 1.0$ ohm. Hence the total kilovolt-amperes, from formula (4), is 121,000, or about 10 times the rating of each generator.

DIMENSIONS, WEIGHT AND COST.—Iron-core reactances with air gap occupy from 0.30 to 0.50 cubic foot per rated kv-a., weigh from 20 to 50 pounds per rated kv-a. and cost from \$3 to \$5 per rated kv-a., the first figure in each case applying to air-blast reactances and the latter figure to oil-cooled reactances. Power-limiting reactances occupy from 0.3 to 0.6 cubic foot per rated kv-a., weigh from 20 to 30 lbs. per rated kv-a. and cost from \$4 to \$12 per rated kv-a. The first figure in each case applies to reactances having a rating of approximately 500 kv-a. and the latter for ratings of approximately 50 kv-a. These figures apply to 60-cycle reactances and should be increased from 10 to 15 per cent for 25-cycle reactances. (1920 prices.)

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RECTIFIERS. — (*See also Arc, Electric; Converters; Motor-Generators.*) The term rectifier is applied to any stationary apparatus or rotating commutator for transforming alternating into direct current, or vice versa. Three types of rectifiers are in commercial use, viz., the mercury-arc rectifier, the electrolytic rectifier and the rotating commutator driven by a synchronous motor, but only the first type is at present of much importance commercially.

MERCURY-ARC OR MERCURY-VAPOR RECTIFIER. — The operation of this rectifier depends upon the fact that a tube containing mercury vapor under a low pressure and having one electrode of mercury, and the other of some other conductor, offers a very high resistance to a current tending to flow through the tube from the mercury to the other electrode, but has a very low resistance to a current flowing in the opposite direction, provided the current is once started by forming an arc in the tube, e.g., by tilting the tube so that the mercury touches for an instant the other electrode. See article on *Arc, Electric.*

Application. — Mercury-arc rectifiers find their chief application as a means of charging storage batteries and supplying a certain series type of d-c. arc lamp (*see Lamps, Electric*) from an a-c. supply. In the latter case the rectifier receives its current from a constant-current transformer and delivers a direct current of constant value. Mercury-arc rectifiers have also been tried out experimentally on electric cars and locomotives, power being supplied to the car or locomotive from a high-voltage a-c. trolley, stepped down to a lower voltage by transformers and converted into low-voltage direct current by means of the rectifiers. Although this scheme has not yet proved satisfactory commercially, it seems to be particularly promising when used in connection with high voltage (1200-volt) railway motors. The difficulty lies chiefly in producing a tube of rugged and lasting qualities.

Connections for Single-phase Operation. — The connections employed in practice are shown in Fig. 1. The complete equipment consists of a source of alternating current *HG*, the rectifier tube *AA′*, two reactances *E* and *F* and the load represented as a storage battery *J*. The rectifier tube is an exhausted glass vessel in which are two graphite electrodes (anodes *AA′*) and one mercury cathode *B*. Each anode is connected to a separate side of the a-c. supply, and also through one-half of the main reactance to the negative side of the load. The cathode *B* is connected to the positive side. There is also a small starting electrode *C* connected to one side of the a-c. circuit through a resistance and used for starting the arc. When the rectifier tube is rocked so as to form and break a bridge of mercury between the cathode *B* and starting anode *C* a small arc is formed. This produces mercury vapor in the tube and the arc immediately jumps to one or the other of the main anodes and alternates on these during regular operation.

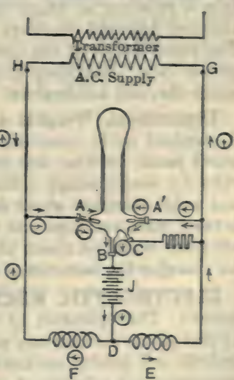

Fig. 1. Mercury-vapor Rectifier

Mode of Operation. — To analyze the operation, assume an instant when the terminal *G* is positive and *H* negative. The positive current will then flow from anode *A′* to cathode *B*, through the load *J* to *D* and through reactance *F* back to *H*. The current cannot jump from *A′* to *A* on account of the high

counter electromotive force of the arc. The small arrows surrounded by circles show the path of the current during this half cycle. During the next half cycle the terminal H is positive and the current flows to A through the tube to B, through the load J, reactance E and back to G. The small arrows show the path of the current during this half cycle. Hence during a whole cycle the cathode B is continuously negative but first one anode is active and then the other. If the voltage and current should become zero coincidentally the arc would become extinguished and operation cease. Hence the reactances E and F are introduced. At the end of the first half cycle described, when the line voltage drops to zero, the inductance of F maintains the current and a local circuit is formed through A, B and F, which maintains the arc until the voltage at G has risen to a value which will maintain the arc.

The rectifier thus makes use of both half waves, or the entire alternating current, and the result is a uniform pulsating uni-directional current. On account of the reactance in the circuit, this current in the load never falls to zero and, in fact, with sufficient reactance, may be made very nearly constant. But this extreme is not always desirable, as it distorts the current wave in the a-c. supply circuit.

Arc Rectifiers on Polyphase Circuits. — Rectifiers may be arranged on two-phase and three-phase circuits, and in fact there is an advantage in the arrangement of the reactances in these cases.

Efficiency of Mercury-arc Rectifier. — The losses in the rectifier correspond to a constant counter e.m.f. of about 14 or 15 volts, thus the efficiency of the tube is constant at all loads and at high voltages is very high; the higher the voltage the higher the efficiency. In fact, for high voltages the losses in the transformer and reactance coils form the major part of the total losses. On account of the small losses in the tube itself, there is very little heat to be dissipated, and rectifier tubes of large capacity are very small in bulk.

Kilowatt Capacity of Mercury-arc Rectifier. — Rectifiers have been built for voltages up to 6000. The tubes may be of glass or steel. The glass tubes have a capacity of about 40 amperes and the steel tubes from 200 to 300 amperes. Several rectifiers may be operated in parallel for large currents. The power factor of the combination of tube and controlling devices is about 90 per cent. Rectifiers have been built of a capacity sufficient to operate motor cars and locomotives.

Dimensions, Weights and Costs of Mercury-arc Rectifier Outfits. — A standard outfit consisting of rectifier tube, transformer, reactance, switchboard panel, switches and instruments to supply a load of 30 amperes at 110 volts d.c. would occupy a floor space 16×19 in., have a height of about 64 in., and weigh complete about 600 pounds. The first cost of the outfit would be between $200 and $250 and the cost of renewing the tube about $20. The net efficiency would be about 78 per cent. Such an equipment is frequently used for charging the batteries of electric vehicles from a-c. supply circuits.

ELECTROLYTIC RECTIFIERS. — (*See also Condensers; Lightning Protectors.*) The operation of this type of rectifier is in a general way the same as that of the mercury-arc rectifier, being based on the fact that a certain electrolytic cell having electrodes of different metals (e.g., aluminum and steel electrodes in a solution of ammonium phosphate) have the property of allowing a current to pass in only one direction. By suitably combining two of these cells both the half waves of an alternating current may be rectified. Such rectifiers have a low efficiency and due to the large losses in them heat very rapidly. They are therefore applicable only to the rectification of small currents. Examples are the electrolytic rectifier of Nodon described at the International Electric Congress of 1904.

TUNGAR RECTIFIER. — This rectifier acts on the principle of the Fleming valve (see *Radio Communication*), and consists of a heated tungsten filament for one electrode, and a tungsten plate for the other, both placed in a glass bulb containing Argon Gas. If the plate is made positive to the filament a thermionic current will flow across the intervening space in the vacuum, but if the filament is positive to the plate no current will flow. By combining two tungar rectifiers with an auto-transformer, as is done with the two halves of the mercury-vapor rectifier, both half waves may be rectified. This rectifier is limited to small current (7.5 amp.) and is used chiefly for charging small groups of storage batteries at 15 volts or less.

MECHANICAL RECTIFIERS. — These usually consist of a commutator driven by a synchronous motor. Each commutator has as many live segments as there are poles on the synchronous motor. Alternate segments are connected to the source of alternating currents, and brushes properly spaced and bearing on the commutator collect the direct current. The objections to these devices are that if there is much inductance in the d-c. circuit (e.g., a motor) there will be a great deal of sparking, or if the load has a constant counter e.m.f. (e.g., a storage battery) there will be sparking. By alternating live and dead segments, this trouble can be obviated, provided the current to be rectified is not too large and the proportions of the live and dead segments are properly chosen. Commutator rectifiers of this type capable of rectifying currents up to 50 amperes at 200 volts have been constructed. The principal losses are in the driving motor; the efficiency is poor at light loads.

Many attempts have been made to produce a satisfactory rectifier by various schemes for interchanging the connections to the a-c. circuit at the end of every half wave, but none of them have as yet proved satisfactory for rectifying currents of any considerable magnitude. The difficulty lies chiefly in providing a means of preventing the formation of a spark when the circuit is interrupted or commutated.

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REGULATORS. — (*See also Batteries, Storage, Applications of; Controllers; Rheostats; Starters, Motor; Switchgear Equipment for Power Stations.*) Devices for adjusting and controlling the voltage in an electric circuit are called regulators. They may be divided into two classes, depending on whether they operate directly on the circuit or indirectly by means of the excitation of the generator.

FEEDER OR POTENTIAL REGULATORS, as the name implies, operate to raise or lower the feeder voltage. Where a feeder circuit is operated from a constant potential bus and it is desirable to compensate for the drop in the feeder, there is usually installed a regulator which adjusts the voltage impressed upon the feeder so as to maintain proper voltage at the point of distribution.

With d-c. circuits the only way to raise the voltage of the feeder above that of the bus is to connect a booster or a storage battery (q.v.) in series. The feeder voltage may be lowered by connecting in resistance. Owing to the inefficiency of resistance control and the complication of providing boosters or batteries, feeder control of d-c. circuits is used only in special cases, such as for the distribution system of a direct-current railway.

Feeder control for a-c. circuits is accomplished by transformer action. Feeder regulation for such circuits can be made in various ways, the most common being the "induction" type and the "step" type.

Induction Regulators, also called "induction potential regulators," are used on either single-phase or three-phase circuits and are arranged for hand operation, motor operation with distant control, or for full automatic operation by means of relays. An induction regulator is a special type of transformer, built like an induction motor (q.v.) with a coil-wound secondary. The primary is permanently connected across the feeder circuit, and the secondary which is connected in series with the feeder is normally stationary, but is movable at will for the purpose of adjusting the voltage. In comparison with the step-type regulator (*see below*) the induction regulator possesses the advantage of being operated without short-circuiting any transformer coils. It has the disadvantages of a large magnetic leakage and a high value of exciting current.

Single-phase Induction Regulator. — This regulator has a secondary induced voltage whose value depends on the relative angular position of the primary and secondary coils, but which is always in time phase with the primary voltage. This induced voltage is a maximum when the axes of the coils coincide and it is zero when the coils are at right angles to one another. The resultant feeder voltage is equal to the arithmetical sum (or difference) of the primary and secondary voltages.

Polyphase Induction Regulator. — This regulator has a rotating flux of constant value, set up by the primary current. Thus the secondary induced voltage is also constant in value but differs in phase from the normal line voltage depending on the angular relation between the primary and secondary windings. The resultant delivered voltage is the vector sum of the primary and secondary voltages, and its value varies with the position of the movable member.

Step-type Regulators, also called "contact-voltage regulators," consist essentially of a stationary transformer provided with a large number of secondary taps for cutting in and out sections of the transformer windings. The taps are connected with a dial or drum so that any pair of taps can be connected to the feeder circuit according to the voltage required.

The moving arm on the dial-type regulator is usually arranged so that in passing from the position of maximum boost the number of secondary turns in series with the circuit is reduced in equal steps until the turns are all cut out. Further rotation in the same direction throws over the reversing switch and

then cuts in the same secondary turns in opposition to the main voltage until the position of maximum bucking is reached, when a stop prevents any further rotation in that direction. A similar stop prevents overtravel in the position of the maximum boost.

Automatic Operation of Feeder Regulators with either single- or three-phase circuits is obtained by the action of a voltage relay, which may or may not have a compensating device. This relay acts in conjunction with the motor on the regulator so that, as the load comes on or as the bus-voltage drops, the motor will turn the regulator in such a direction as to increase the voltage. By means of a compensator which can be set for a certain resistance and a certain inductive drop, the voltage at the point of distribution can be maintained constant independent of the amount and power factor of the load, provided the total drop is within the range of the regulator.

OUTDOOR INDUCTION REGULATORS.— During the last few years a number of induction regulators, both of the single-phase type and the polyphase type have been developed for outdoor service. The smaller regulators are arranged for mounting on a pole or pole platform while the larger ones, usually employed in connection with outdoor transforming stations, are located on the ground level. To secure satisfactory cooling the smaller size regulators are mounted in corrugated sheet-steel cases, while the larger ones are provided with radiators similar to the large outdoor self-cooling transformers.

Outdoor induction regulators have been built as large as 1750 kv-a., three-phase, 60 cycle, 13,600 volts, and larger regulators can be built whenever needed.

FIELD REGULATORS. — In order to maintain practically constant voltage on a-c. and d-c. generators, or to have these machines compound automatically to take care of feeder drop, field regulators of either non-automatic or automatic types are employed. The former are hand-controlled rheostats (see *Rheostats*), the latter are made in various forms of which the regulator originally known as the "Tirrill" type is the best known in America. These generator-voltage regulators are made in two forms, one using a d-c. magnet and the other using an a-c. vibrating magnet relay as the anti-hunting control device.

Either design of regulator depends for its operation on the rapid opening and closing of a circuit that shunts the field rheostats and thus changes the resistance in the field circuit of the generator to be regulated. For d-c. service the regulator usually works upon the main generator field and for a-c. service upon the field of the exciter. In both cases the rheostat is so adjusted that when in circuit it tends to lower the voltage considerably below normal, and when the rheostat is short circuited the generator voltage rises. The regulator automatically closes the shunt circuit and the voltage drops to a predetermined value and opens the shunt circuit when the voltage rises above that value.

The main features of the regulator with the d-c. vibrating magnet are: 1st, the method of control by shunting the rheostat; 2nd, that with the total range of regulation from no load to full load the maximum travel of the only moving part (the vibrating contact) is only $\frac{1}{32}$ inch; 3rd, the use of the exciter voltage as one of the main control circuits which prevents the generator voltage from overshooting. The vibrations are so rapid that the time factor is reduced to a minimum and there are no retarding effects due to dash pots or other damping devices.

Regulator for Direct-current Generator (Fig. 1). — This consists essentially of a main control magnet whose winding is connected across the generator terminals, and a differentially-wound relay magnet. When the effect of the potential winding increases, due to a rise in the generator voltage, the contact of the main control magnet is opened and in turn one winding of the relay

magnet is deënergized. Thus the relay contact is opened and the short-circuit removed from the generator rheostat. When the generator voltage drops, the main contact is closed and the differentially-wound relay magnet acts to short-circuit the field rheostat. The relay contacts are shunted by a condenser to reduce sparking.

In case the generator is to overcompound for line drop a current winding is added to the main control magnet and is connected across a shunt in one of the local mains. The action of this current winding opposes the action of the potential winding in the control magnet end, and thus makes the generator overcompound for line drop. Regulators of this type can be adjusted for a line drop up to 15 per cent.

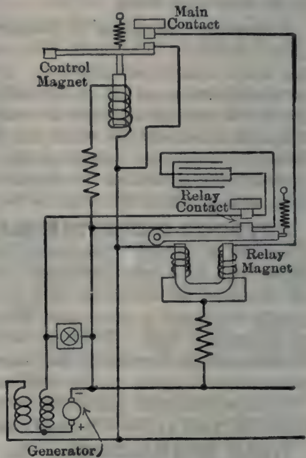

Fig. 1. Connections for Direct-current Tirrill Regulator

Where the generators are shunt wound a separate regulator is required for each machine which is operating at any time. Where several small compound machines are operating in parallel and all of the regulating is done by one machine, the others trailing after, provision is made for connecting any machine to the regulator. For field currents in excess of about 3 amperes at 250 volts, it is found that one relay contact is not sufficient, and for such cases regulators are built with as many as 10 relays, all being operated by a single main control magnet. For still larger d-c. generators whose field current could not be handled by a multiple-contact regulator, it is necessary to supply a separate exciter or exciters and the regulator then controls the field circuit of the exciter as described below for a-c. generators.

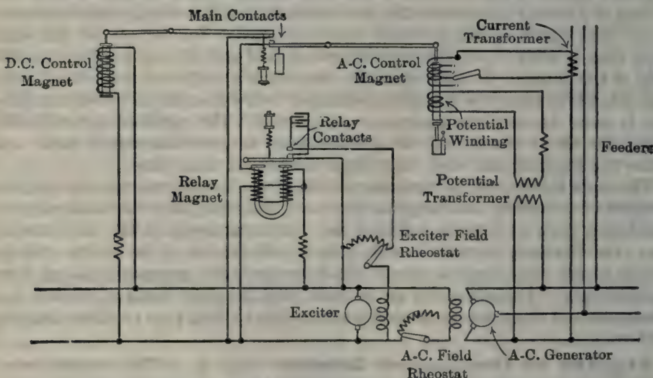

Fig. 2. Connections for Alternating-current Tirrill Regulator

Regulator with D-C. Vibrating Magnet (Fig. 2). — This works on the exciter field. The main contacts with this type of regulator are acted on by

two sets of control magnets, one connected across the exciter bus and tending to move the main contacts farther apart as the exciter voltage rises, and the other acted upon by a-c. potential and current coils. Suitable springs and counterweights allow the proper adjustment to be made. When the main contact closes, it energizes the relay magnet, thus closing the relay contact, short-circuiting the exciter rheostat and raising the exciter voltage and consequently the generator voltage. The use of the exciter voltage as one of the main control circuits prevents the generator voltage "overshooting," for as the exciter voltage rises to bring up the a-c. voltage the d-c. control tends to keep the main contacts apart and so reduce the voltage again.

The compensating current winding of the a-c. solenoid is provided with a dial switch to give any amount of compensation required for the feeder circuit

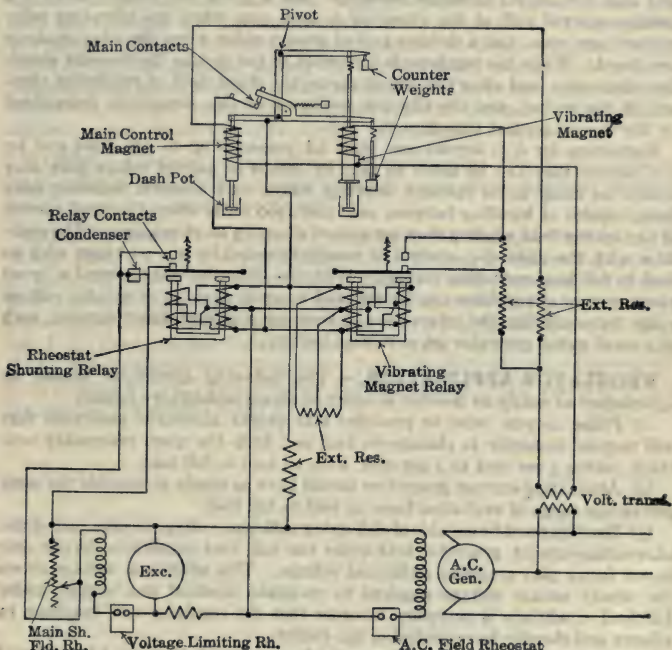

Fig. 3. Regulator with A-C. Vibrating Relay Control.

in which the current transformer is located. Where it is desired to compensate for both resistance and inductive drop under varying power factors a special compensator is provided. A modification of the regulator to take care of the large exciters has a number of relay contacts all operated at the same time from the one set of control contacts, the various relays being shunted by condensers to reduce the sparking. A single regulator may serve a number of alternators if they are operated in parallel and if all use the same exciter. If two or more exciters are operated in multiple, a single regulator will suffice; if not operated in multiple, a separate regulator is usually installed for each exciter.

Regulator with A-C. Vibrating Magnet. — This type of generator voltage regulator does not employ any d–c. main control magnets, but uses an a–c. vibrating magnet relay and its general connections are indicated in Fig. 3. The main control magnet has its core attracted upwards and its core stem connected to the floating lever which is pivoted to the bell crank of the vibrating magnet. The two magnets are energized from the same voltage transformer and actuates the main contacts into and out of engagement with the fixed contact. The opening of the main contacts open all relay contacts and inserts the full resistance into the vibrating magnet circuit, weakening the pull and closing the main contacts again. This system comprising the main control magnet, the vibrating magnet, levers, rheostats, shunting and vibrating relays, constitutes a vibrating system when the circuits are properly energized and the control elements balanced. For a given line voltage there is a definite upward pull of the vibrating magnet core when the vibrating relay contacts are open, and a definite pull of greater value when the relay contacts are closed. When the regulator is connected to the system the rheostat shunting relays open and close the circuit across the shunt field or regulating rheostat of the exciter, and the effective resistance of this rheostat is determined by the time of contact engagement.

Regulators for d–c. service are built for pressures up to 550 volts and for a–c. service they can be made so that by means of master relays they may control as many as 60 rheostat shunting relays, each rheostat shunting relay being capable of handling between 2000 and 2500 watts where the total current of any exciter field winding does not exceed about 15 or 16 amperes. The regulator with the main d–c. control is usually intended for a 2 : 1 range with no load to full load excitation current. With the vibrating relay control a broad range system of regulator can be adopted securing a 10 : 1 or greater voltage range by energizing the relays from a separate source of direct current, such as a small motor generator set or storage battery.

REGULATOR APPLICATION. — The following conditions should be approached as nearly as possible in order to obtain satisfactory results:

(1) Prime movers must be provided with proper automatic governors that will respond instantly to changes in load and keep the speed reasonably constant, within 3 per cent to 4 per cent, from no load to full load.

(2) Alternating-current generators should have as nearly as possible the same percentage range of excitation from no load to full load.

(3) Exciters must be capable of delivering sufficient voltage to take care of the alternating-current generator field under the full load conditions, 80 per cent power factor plus a certain additional voltage. This additional voltage above the steady exciter voltage required to maintain constant bus voltage under full load conditions is necessary in order that the regulator will continue to vibrate and thereby have control of the exciter.

(4) Where more than one exciter are to be considered, they must be adjusted to operate in parallel under all loads, with any point on the saturation curve.

(5) Exciters for 125-volt service should be able to build voltage up or down between the limits of 30 and 125 volts in five seconds or less under load consisting of generator field circuits. The time constant should be the same for exciters of other rated voltages over proportional changes. Exciters with greater time constants than this may not permit the regulator to maintain constant voltage with rapidly fluctuating load.

(6) Interpole exciters for 125-volt service must be able to develop at least 135 volts with series windings disconnected and should be so operated. Series windings must be out of circuit in order tc secure a satisfactory time constant. In general, the exciter must be capable of developing a voltage 10 to 15 per

cent in excess of that required by the a-c. generator at full load, 80 per cent power factor, the a-c. generator field rheostat being adjusted so that with 60 volts on a 125-volt exciter, the a-c. generator develops normal voltage at no load.

Regulators can be arranged for satisfactory parallel control where generators are in parallel and the exciters operating independently.

EXCESS VOLTAGE PROTECTION. — Unless special devices are furnished with the generator voltage regulator, when a short circuit on the system is cleared away, a dangerous voltage rise is inevitable. On the occurrence of a short circuit on a system without some protective device, the main contacts of the regulators close, causing the exciter voltage to build up to the maximum value. When the short is cleared away, high voltage results from the high exciter voltage and the consequent high generator field current which lasts until the regulator has had time to again become operative. This condition of excessive voltage can, however, be prevented by means of excess-voltage protective devices arranged so that the exciter voltage can never rise above a predetermined point, which is usually a little above the no-load excitation value required by the a-c. generators. Different types of regulators require slightly different excess-voltage protective devices to accomplish this result.

COSTS. — The cost of regulators varies considerably due to the character of installation and varies between about $1200.00 for a one-relay regulator and $2715.00 for a ten-relay regulator.

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RELAYS. — (*See also Circuit Breakers; Telegraphy; Telephony; Regulators; Signaling, Railway.*) A relay is a device which opens or closes a local circuit under predetermined electrical conditions of the main circuit. There are three general classes of relays as determined by their respective functions of (1) protecting the main circuit, (2) regulating the current in the main circuit, or (3) signaling the condition of the main circuit.

PROTECTIVE RELAYS are auxiliary devices supplied for use with circuit breakers on systems that require protection more selective and flexible than that afforded by the usual control features of automatic circuit breakers (q.v.). The closing of a local circuit through the action of the relay causes in turn the tripping of the circuit breaker.

Types of Protective Relays. — The types which are most commonly employed are given in the following table, together with the apparatus with which each type is applied and the protection furnished by the operation of the relay.

Relays are built to furnish protection against overvoltage, no voltage, overload, no load, reverse power and reverse phase. They may operate either directly

TYPES OF RELAYS

Type of relay	Application	Operations	Approximate cost per relay*
D-C. overvoltage	Storage batteries d-c. apparatus	Prevents overcharging Prevents damage from excess voltage	$25.00
D-C. reverse current	Storage batteries rotary converters	Prevents discharging into charging source Protects against reversal of flow of energy	94.00
D-C. low-voltage release	D-C. motors	Operates if voltage falls below given value	25.00
D-C. underload	Storage batteries	Disconnects on completion of charge	25.00
A-C. overload	Feeders, motors, rotary converters, transformers	Protects against overload	33.80
A-C. reverse power	Feeders, generators	Protects against reversal of flow of energy	80.00
A-C. low-voltage	Induction motors	Protects against fall in line voltage	37.60
A-C. reverse phase	Synchronous apparatus, elevators and cranes	Protects against reversal of phase progression	42.25

* Costs do not include shunts for d-c. relays or current and potential transformers for a-c. relays, as the cost of these varies greatly with current and voltage. Costs are for 1921.

or in connection with other relays, and may be made instantaneous or provided with a time limit either of definite duration (time-limit relay) or inversely proportional to the extent of overload (inverse-time-limit relay).

Definite Minimum Inverse Time-limit Overload Relays. — Where the time-limit relays are wanted for the protection of a-c. circuits, it is practically universal American practice to furnish relays that have an adjustable definite minimum time setting and an inverse time setting combined. With one type, there is a definite time setting of two seconds with the maximum adjustment so that, independent of the intensity of a short circuit, the breaker will not trip out until two seconds have elapsed. This definite time is, however, adjustable between practically any limits between 0.1 second and 2 seconds. Beyond the definite time there is an inverse time feature which gives selective action whereby the time of operation varies inversely with the load. With this type of relay the faulty line carrying the heavier load usually will have its breaker tripped out before the other breakers are affected.

Example of Use of Relays. — The various types of relays that may be used to advantage in a system are shown diagrammatically in Fig. 1, where

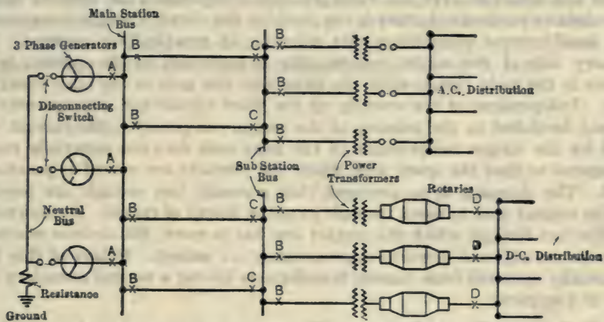

Fig. 1. Diagram Illustrating Use of Various Types of Relays]

three a-c. generators operate in parallel with their neutral points grounded through a resistance and feed a common bus supplying current to power transformers, rotaries, etc., for a-c. and d-c. distribution. Relays for the operation of circuit breakers are inserted as follows:

At A — a-c. overload and reverse power relays.
B — a-c. overload time-limit relays.
C — a-c. reverse power, time-limit relays.
D — d-c. reverse current, inverse time-limit relays.

The Principle of Operation of a protective relay depends somewhat on the duty to be performed. The actuating mechanism usually involves such a motive device as a solenoid and core, or a rotating motor. The mechanisms described below are those most frequently used for the respective types of service.

D-C. Overvoltage Relay. — One design consists of a permanent magnet which forms a base and two iron cores mounted parallel to the base and attached to it at one end with an eccentrically-pivoted armature carrying a suitable contact. With no current flowing the two cores exert an equal pull (due to the magnetic field of the permanent magnet) on the pivoted armature, which, due to its eccentric mounting, is pulled up against one core. When voltage is impressed on the coils wound on the iron cores the magnetism in one pole is strengthened and in the other weakened. At the predetermined over-

voltage the position of the armature is reversed, notwithstanding its eccentric mounting, and the contact is closed, tripping the breaker.

D-C. Reverse-current Relays for instantaneous operation are similar in construction to overvoltage relays (*see preceding paragraph*). The solenoids, however, are operated from a shunt in such a way that with the current in the normal direction the contact is held open, but when the current reverses in direction the contact is closed, thus tripping the breaker.

Reverse-current Time-limit Relays are built on the principle of a permanent-magnet d-c. ammeter operated from a shunt. In normal operation the armature of the relay tends to turn in one direction but is restrained by a stop; in the case of current reversal the armature turns in the opposite direction, its rate of movement being proportional to the strength of the current. The angle through which the armature has to turn to close the contacts is adjustable by moving the stationary contact.

A-C. Overload Relays are usually built in single pole units and are nearly always made to give a definite minimum inverse time element. These relays are of the induction type and have practically superseded the solenoid design. This definite minimum feature is obtained by the use of a torque compensator or a small current transformer that saturates at practically 5 amperes. The primary side of this torque compensator is connected to the current transformer in the main circuit while the secondary side goes to the relay operating coils. Independent of the severity of the short circuit and consequently the current furnished to the primary of the torque compensator, the current supplied by the torque compensator to the relay coils does not materially exceed 5 amperes so that the speed with which the contacts are moved is practically fixed. The damping is so arranged that normally two seconds are required for the contact arm to move through its entire length of travel. By shortening the distance through which the contact arm has to move, this definite minimum time can be adjusted down to approximately 0.1 second. Relays of this kind are usually operated from current transformers having a normal secondary current of 5 amperes.

A-C. Reverse Power Relays. — These are made in various types and one kind frequently employed practically combines the features of the definite minimum overload relay and a selective wattmeter element. The overload element closes its contacts on excess current in either direction but the selective wattmeter element does not allow it to complete the tripping circuit unless power is flowing in the reverse direction. A momentary surge which may reverse the power will cause the selective wattmeter element to close its contacts, but the relay will not operate to trip out the breaker unless the current in the reverse direction exceeds the relay setting and is maintained for two seconds or longer.

Relay Switch. — Where, as frequently happens, the tripping contacts of a relay will not carry enough current to trip the circuit breaker, a relay switch is employed. In this case the relay merely energizes the operating coil of the relay switch whose contacts can be made suitable for any reasonable amount of current.

Rating of Protective Relays for overload service is given usually in terms of maximum current in the main circuit (secondary of transformer with a-c.). For reverse-load relays the rating is expressed as the percentage of reverse load on which they will operate. For relays which are not instantaneous in their action a time rating in seconds is usually given.

REGULATING RELAYS. — See article on *Regulators*.

SIGNALING RELAYS. — See articles on *Telegraphy; Telephony; Signaling, Railway.*

COSTS. — See table above.

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RESISTANCE AND CONDUCTANCE, ELECTRIC. — (*See also Alternating Currents; Bridges for Electrical Measurements; Electricity and Magnetism, Principles of; Resistors, Standard; Skin Effect; Wires and Cables; Wires, Resistor.*) The general definition of the resistance R' of a substance between any two equipotential surfaces intersecting the path of a current I is

$$R' = \frac{P_h}{I^2}, \tag{1}$$

where P_h is the power dissipated as heat between the two equipotential surfaces and I is the *effective* value of the *total* current from one surface to the other. In the case of a varying current this dissipation of heat may occur in four different ways, viz., (1) as heat due to the conduction current through the substance, (2) as heat due to dielectric hysteresis accompanying the displacement current through the substance (when the latter is an insulator), (3) as heat due to magnetic hysteresis accompanying the varying magnetic flux produced by the current and (4) as heat due to eddy currents induced in neighboring conductors.

In the case of a continuous (non-varying) current the last three effects do not occur, and the heat is that due to the conduction current only. The resistance offered by a substance to a continuous current is called the "true," "ohmic," "continuous-current" or "direct-current" resistance, as distinguished from the "effective" resistance offered by the substance to a varying current. The effective resistance, even when there are no losses due to dielectric or magnetic hysteresis, is in general greater than the ohmic resistance, due to the skin effect; see *Skin Effect.*

Similarly, the general definition of conductance is

$$G' = \frac{P_h}{V^2}, \tag{2}$$

where P_h has the same meaning as above and V is the *effective* value of the potential difference between the two equipotential surfaces. When the voltage is non-varying the conductance is called the "true," "ohmic," "continuous-current" or "direct-current" conductance as distinguished from the "effective" conductance to a varying current.

In the case of a *continuous*, or constant current I, the above expressions for resistance and conductance reduce to

$$R = \frac{V}{I}, \qquad \text{and} \qquad G = \frac{I}{V}, \tag{3}$$

provided there is no source of e.m.f. between these two surfaces. The ohmic resistance and conductance, given by equations (3), are reciprocals of each other, but this is not true for the effective resistance and conductance.

RESISTIVITY AND CONDUCTIVITY. — When the current stream lines from one equipotential surface to the other are straight and parallel and uniformly distributed, and l is the distance between the two surfaces and A the area of each, the ohmic resistance and conductance of the prism or cylinder of the substance thus formed are respectively

$$R = \rho\,\frac{l}{A}, \text{ and } G = \gamma\,\frac{A}{l}, \tag{4}$$

where ρ and γ (which are reciprocals) are called the resistivity* and conductivity* of the substance respectively. For a given material at constant temperature throughout ρ and γ are constants.

* Also called specific resistance and specific conductance.

Units of Resistivity and Conductivity. — From (4) it is evident that ρ is the resistance of a cube of the substance of unit length on each edge when the current stream lines are uniformly distributed and parallel to four of the edges of the cube; similarly, γ is the conductivity of such a unit cube. Resistivity is therefore frequently expressed as ohms or microhms per centimeter cube or per inch cube, and conductivity as mhos or mega-mhos per centimeter or inch cube. Note that a resistivity of x microhms per centimeter (or inch) cube is equivalent to a conductivity of $\frac{1}{x}$ mega-mhos per centimeter (or inch) cube.

Ohms per Mil-foot. — It is frequently convenient in dealing with wires to express lengths in feet and cross sections in circular mils. The corresponding value of the resistivity ρ is then expressed in ohms per mil foot.

Ohms per Meter-gram. — The cross section of a bar or wire of uniform cross-section is equal to the volume of the bar divided by its length, and the volume of the bar is in turn equal to its mass divided by the density of the material of which it is made. Hence, using the same symbols as above, and in addition calling m the mass of the conductor and δ its specific gravity,

$$R = \rho\delta \frac{l^2}{m}. \tag{5}$$

For a given material at a given temperature $\rho\delta$ is also constant, hence

$$R = k \frac{l^2}{m}, \tag{5a}$$

where k is a constant (equal to $\rho\delta$) for a given material at a given temperature; this factor k is then called the specific resistance of the conductor in "ohms per meter-gram" when l is in meters and m in grams.

Pounds per Mile-ohm. — In telephone and telegraph practice the specific resistance of a conductor is also expressed as the weight w, in pounds, of a wire one mile long having a resistance of one ohm. This weight w is sometimes called the "mile-ohm equivalent" of the wire.

"Annealed Copper Standard." — Matthiessen's Standard. — In 1862 Matthiessen published the results of a number of determinations of the resistivity of copper. Recent determinations of the conductivity of a number of samples of commercial annealed copper wire at the Bureau of Standards (*Bull. Bur. Stand., 1911, Vol. 7, p. 103*) gave a mean value of the resistivity very close to Matthiessen's value. A conductivity corresponding to 0.15328 ohm per meter-gram at 20° C. has been adopted (1912) by the Bureau and by the American Institute of Electrical Engineers (1914)* as the "standard" of conductivity, the name "Annealed Copper Standard" being suggested for it. (*See Standardization Rules of the A.I.E.E.*) Matthiessen's Standard, formerly used by the American Institute of Electrical Engineers as the basis for their wire tables corresponded to 0.141729 ohm per meter-gram at 0° C.

To reduce the value of the Annealed Copper Standard to resistivity and conductivity in volume units the value of 8.89 has been adopted as the density of copper in grams per cubic centimeter at 20° C. which corresponds to 8.90 grams per cubic centimeter at 0°C.

Per cent Conductivity. — The Bureau of Standards recommends that whenever the conductivity of a sample is expressed as a percentage, the measured resistivity or conductivity be corrected to reduce it to the value it would have

* Also adopted by the national electrical engineering societies in Germany and France.

at 20° C.; see section on *Temperature Coefficient of Resistance*, below. A conductivity of P per cent is equivalent to

$15.328 \div P$ ohms per meter-gram at 20° C.
$172.41 \div P$ microhms per centimeter cube at 20° C.
$67.87 \div P$ microhms per inch cube at 20° C.
$1037.1 \div P$ ohms per mil-foot at 20° C.

For example, a conductivity of 60 per cent is equivalent to a resistance of 17.285 ohms per mil-foot.

CALCULATION OF THE OHMIC RESISTANCE OF A CONDUCTOR. — Values of the resistivity of various materials are given in the table below. Experiment shows that the resistance of a given length of wire of uniform cross section is independent of the shape into which the wire is bent, provided the diameter of the wire is small compared to the radius of curvature of the curve into which it may be bent. This condition is almost always realized in practice, and consequently formulas (4) and (5a) are in general directly applicable to the calculation of the resistance of a wire whether the wire be straight or curved or wound into a coil of any shape. These formulas are also applicable to the calculation of the resistance of a rod or bar, provided the rod or bar is not bent into a sharp curve, and the distance between its points of connection to the circuit is large compared to the linear dimensions of its cross section. Care should be taken, however, to express all quantities in the proper units; for example, the resistance in *ohms* of a wire which has a specific resistance of 1.6 microhms per centimeter cube, a length of 1000 feet and a cross section of ¼ square inch, is

$$R = 1.6 \times 10^{-6} \frac{1000 \times 12 \times 2.54}{0.25 \times (2.54)^2} = 0.0302 \text{ ohm.}$$

Resistance Formulas When the Stream Lines are Not Parallel and Uniformly Distributed. — When the stream lines of the current are not all of the same length, as, for example, when a current is established in a heavy short bar bent into a sharp curve, the resistance of the bar can be calculated only when the distribution of these stream lines is known. Again, when the stream lines of the current are not parallel, e.g., the leakage current through the insulation of a cable, these formulas are not applicable. However, when the formula for the capacity between any two conductors is known (*see Capacity and Charging Current*), the resistance of the insulation, if uniform throughout, may be found by multiplying the *reciprocal* of the capacity, viz., $\frac{1}{C}$, by $\frac{K\rho}{4\pi}$, where ρ is the resistivity of the insulation and K the specific inductive capacity in the capacity formula. See articles on *Rheostats* and *Wires and Cables, Insulated.*

TEMPERATURE COEFFICIENT OF ELECTRIC RESISTANCE. — The resistance temperature coefficient β_t of a substance *at* any temperature t is defined as the rate of change of the resistance at this temperature, viz. $\left(\frac{dR}{dt}\right)_t$, divided by the resistance R_t at this temperature, i.e.,

$$\beta_t = \frac{1}{R_t}\left(\frac{dR}{dt}\right)_t. \tag{6}$$

The *mean* temperature coefficient α_t between any two temperatures t and t_1 *referred to* the temperature t is defined as the *average* change in the resistance in this interval per degree change of temperature, divided by the resistance at the lower temperature, viz.,

$$\alpha_t = \frac{R_{t_1} - R_t}{R_t\,(t_1 - t)}. \tag{7}$$

General Expression for Change of Resistance with Temperature. — In general, the relation between the resistance R_t of a given mass of a substance at any temperature t may be expressed in terms of its resistance R_o at zero degrees as follows:

$$R_t = R_0\,(1 + at + bt^2 + \ldots), \tag{8}$$

where a, b, etc., are constant coefficients.

Linear Relation Between Resistance and Temperature. — When all the coefficients except the first one, a, are of negligible magnitude, i.e., when the relation between the resistance and temperature is a *linear* one, then, from the above definitions, the *mean* temperature coefficient (α_t) referred to a given temperature t is equal to the temperature coefficient (β_t) *at* that temperature, and the zero degree temperature coefficient α_0 ($= \beta_0 = a$) and the t' degree temperature coefficient $\alpha_t{}'$ are related as follows:

$$\alpha_t{}' = \frac{\alpha_0}{1 + \alpha_0 t'}, \tag{9}$$

and

$$R_t = R_0\,(1 + \alpha_0 t) = R_{t'}\,[1 + \alpha_{t'}\,(t - t')], \tag{10}$$

where R_0 is the resistance at zero degrees, $R_{t'}$ the resistance at t' degrees and R_t the resistance at t degrees.

For most metals the simple linear relation expressed by equation (10) represents the experimental facts within practical limits of accuracy and for ordinary temperature ranges; values of α_0 are given in the table below. For dielectrics, however, the relation between the resistance and temperature is by no means linear, and several terms in such an expression as equation (8) are needed to represent the facts; see *Insulating Materials, Properties of*, particularly the subheadings *Cambric, Paper and Rubber.*

Calculation of Change in Resistance With Temperature. — Equation (10) may be conveniently expressed in the form

$$\frac{R_t{}'}{R_t} = \frac{T_0 + t'}{T_0 + t}, \tag{11}$$

where T_0 is written for the reciprocal of α_0, viz., $T_0 = \dfrac{1}{\alpha_0}$. (11a)

Equation (11) will be found very convenient for calculating the change of resistance with temperature and the change of temperature corresponding to two measured resistances, particularly when a slide rule is used.

T_0 is approximately equal to the number expressing the absolute zero on the mercury thermometer scale (except for magnetic metals) and is sometimes called the "inferred absolute zero" for the particular metal in question.

MEASUREMENT OF RESISTANCE, RESISTIVITY AND TEMPERATURE COEFFICIENT. — The simplest method of measuring a resistance in ordinary engineering work is to send a direct current through the conductor and measure this current by means of an ammeter (q.v.) and measure the potential difference across it by means of a voltmeter (q.v.). For measuring very low resistances, however, and for high-precision measurements, a bridge method should be used; see *Bridges for Electrical Measurements.*

VALUES OF RESISTIVITY AND TEMPERATURE COEFFICIENT. — The resistivity and temperature coefficient of the more common metals and alloys are given in the tables below.

RESISTIVITY AND TEMPERATURE COEFFICIENT OF RESISTANCE*

1 microhm per centimeter cube $= \frac{1}{2.5400}$ microhms per inch cube
= 6.0153 ohms per mil-foot
= 0.01 δ ohms per meter-gram, where δ is the specific gravity.
= 57.08 δ pounds per mile-ohm.

ρ microhms per cm. cube at 20° C. $= \frac{172.4}{\rho}$ per cent conductivity at 20° C.

The zero degree Fahrenheit temperature coefficient is equal to

$$\frac{5\alpha_0}{9 - 160\alpha_0}.$$

Substance (*Numbers refer to authorities, top of p. 1356*)	Remarks	Microhms per centimeter cube at 0°C. ρ_0	Mean temperature coefficient referred to 0° C. Temperature range °C. From	To	α_0
Advance (3)	Copper-nickel	48.8	......	......	0.000018
Aluminum (1)	Pure	2.62	0	100	0.00423
Aluminum (7)	Wire, 61% cond.	2.607	......	......	0.00423
Alum. bronze (1)	97 Cu+3 Al	8.85	15	......	0.000897
Argentan (1)	61.6 Cu+15.8 Ni+22.6 Zn	28.5	0	160	0.000387
Brass (1)	90.9 Cu+9.1 Zn	3.64	0	100	0.00204
Brass (1)	65.8 Cu+34.2 Zn	6.29	0	100	0.00158
Bronze (1)	88Cu+12Sn+0.94*P*	17.8	19	92	−0.00050
Calido (8)	Ni+Cr+Fe	100.0	......	......	0.00034
Carbon (1)	Graphite	400 1150	25	387	−0.0006 −0.0012
Carbon (1)	Incand. lamp	4000	25	335	−0.0003
Climax (3)	Nickel-steel	87.1	......	......	0.00055
Constantan (1)	60 Cu+40 Ni	49.0	0	100	0.0000±
Copper (7)	**Annealed Standard**	**1.589**	**0**	**100**	**0.00427**
Copper (1)	Electrolytic	1.56	0	100	0.00428
Copper (1)	Hard-drawn	1.60	0	100	0.00408
Copper-iron (2)	0.4% Fe	4.08	0	100	0.00155
Excello (4)	....	91.4	......	......	0.00016
Ferro-nickel (3)	....	27.1	......	......	0.00216
German silver (3)	18% Ni with Cu and Zn	33.1	......	......	0.00031
Gold (1)	99.9% Au	2.20	18	100	0.00368

* See *Wires and Cables, Bare,* for the resistance of various sizes of wires; *Wires, Resistance,* for additional data on resistance wires; and *Insulating Materials, Properties of,* for the resistivity of dielectrics. See also the articles on *Aluminum, Copper,* and *Rails.*

† For resistivities at *high temperatures* see pages 1354 and 1355.

RESISTIVITY AND TEMPERATURE COEFFICIENT OF RESISTANCE — *Continued*

Substance (*Numbers refer to authorities, top of p. 1225*)	Remarks	Microhms per centimeter cube at °C. ρ_0	Mean temperature coefficient referred to 0° C. Temperature range °C. From	To	α_0
Ideal (8)	Cu+Ni	49.0			0.0000±
Ia Ia soft (4)	Copper-nickel	47.1			0.000005
Ia Ia hard (4)	Copper-nickel	50.2			−0.000011
Iron (1)	Very pure	8.85	0	100	0.00625
Iron (1)	soft steel	11.8	10	35	0.00423
	hard steel	45.6	10	35	0.00161
Iron, cast (2)	soft	74.4			
Iron, cast (2)	hard	97.8			
Krupp metal (5)	Nickel steel	85.0			0.00070
Lead (1)	Pure	19.8	0	100	0.00411
Lead-bismuth (1)	42.3 Pb+57.7 Bi	63.3			
Manganese-copper (1)	70 Cu+30 Mn	100	0	100	0.00004
Manganin (3)	Cu+Mn+Ni	41.4			0.000011
		73.8			0.000039
Mercury (1)		94.07	0	100	0.0008649†
Molybdenum (6)	Hard drawn	4.9	0	170	0.0050
Molybdenum (6)	Annealed	4.2	0	170	0.0050
Monel metal (3)	Copper nickel	40.8			0.00206
Nichrome (3)		98.7			0.00045
Nichrome II (3)		109.2			0.00016
Nickel (1)	Electrolytic	6.93	0	100	0.00618
Nickel (3)	Commercial wire	9.9			0.0039
Nickel steel (1)	4.35% Ni	29.4			
Phosphor-bronze (1)		7.75			
Platinum (1)	Drawn	11.0	0	100	0.00367
Platinum-iridium (1)	80 Pt+20 Ir	31.6	−100	100	0.002±
Platinum-rhodium (1)	90 Pt+10 Rh	21.1	15		0.00143
Rheotan (1)		44.6	0		0.00041
Rose's metal (1)	48.9 Bi+23.5 Sn+27.6 Pb	64.5	0	94.3	0.0023
Silver (1)	Electrolytic	1.47	0	100	0.00400
Steel (*see Iron*)*					
Superior (4)	Nickel-steel	87.1			0.00081
Tantalum (1)	Pure	14.6	0	100	0.0033
Therlo (3)	Cu+Mn+Al	46.7			0.0000056
Tin (1)		10.5	18	100	0.00465
Tungsten (6)	Hard drawn	5.42	0	170	0.0051
Tungsten (6)	Annealed	4.37			
Wood's metal (1)	55.7 Bi+13.7 Sn+13.7 Pb+16.2 Cd	51.8	0	69.8	0.0023
Yankee silver (3)		33			0.000154
Zinc (1)	Pure	5.38	18	100	0.00402

* See also *Rails, Track and Third*. † To be used in equation (8) with $b = 0.00000112$.

RESISTIVITIES AT HIGH TEMPERATURES. (*C. Hering, Met. & Ch. Eng.* 13, *pp.* 23 *and* 70, 1915)

At 500° C. / 932° F.	Ohms per cm.-cube	At 1000° C. / 1832° F.	Ohms per cm.-cube	At 1500° C. / 2732° F.	Ohms per cm.-cube
Graphite (b)	0.00080	Graphite (b)	0.00065	Graphite (b)	0.00058
Graphite (a)	0.00084	Graphite (a)	0.00086	Graphite (a)	0.00089
Carbon (a)	0.0027	Carbon (d)	0.0021	Carbon (d)	0.0016
Carbon (d)	0.0028	Carbon (a)	*0.0024*	Carbon (a)	*0.0022*
Carbon (c)	0.0033	Carbon (c)	0.0030	Carbon (b)	*0.0029*
Carbon (b)	0.0037	Carbon (b)	0.0034	Nernst filament, about	0.5
Carbon powder	0.22	Carbon powder	0.12	Refrax	*0.5*
Silicon	0.094	Silfrax B	0.84	Silfrax B	*0.7*
	to 0.23	Sodium chloride, fused	*0.90*	Carbon grains (b)	*0.85*
Lead chloride, fused, 520°	0.418	Glass, roughly, about	*1*	Graphite grains	*1.2*
Silver chloride, fused	0.547	Graphite grains	1.7	Kryptol	3.4
Lead chloride, solid	0.824	Carbon grains (b)	1.9	Alundum, about	750
Silfrax B	0.92	Carbon grains (a)	*2.8*		
Copper chloride, fused	2.50	Silicon powder	3.5		
Graphite grains	2.70	Refrax	3.7		
Carbon grains (b), about	4.8	Kryptol	4.8		
Carbon grains (a), about	8.5	Porcelain, about	15		
Kryptol	10	Manganese oxide powder	15.7		
Refrax	19.7	Copper oxide CuO, powder	18		
Boron, about	60	Zinc oxide powder	26.7		
Silicon powder	120	Iron oxide, Fe_2O_3, powder	31.4		
Glass, about	330	Quartz	110		
Iron oxide, Fe_2O_3, powder	1260	Magnesium oxide powder	1400		
Copper oxide, Cu_2O, powder	1570	Alundum	8000		
Manganese-oxide, MnO_2 powder	2200				
Copper oxide, CuO	5640				

RESISTIVITIES AT HIGH TEMPERATURES. (*C. Hering, Met. & Ch. Eng.* 13, *pp.* 23 *and* 70, 1915)

At 500° C. / 932° F.	Microhms per cm.-cube	At 1000° C. / 1832° F.	Microhms per cm.-cube	At 1500° C. / 2732° F.	Microhms per cm.-cube
Silver, solid	5.0	Copper, solid	9.42	Silver, fused	*23*
Copper, solid	5.1	Gold, solid	12.54	Copper, fused	*24.8*
Gold, solid	6.62	Silver, fused	17.01	Aluminium, fused	*29*
Aluminium, solid	10	Aluminium, fused	24	Gold, fused	*37*
Brass, 2–1, solid	12.5	Molybdenum, solid	*28.5*	Molybdenum, solid	*40.5*
Molybdenum, solid	*16.5*	Tungsten (a), solid	*30.5*	Tungsten, solid	*43*
Tungsten (a, b), solid	18	Tungsten (b), solid	33.4	Tungsten (b), solid	50
Platinum (b), solid	25.3	Platinum (b), solid	40.8	Platinum (b), solid	52.6
Cadmium, fused	34.12	Brass, 2–1, fused	41	Tantalum, solid (b)	74
Platinum (a), solid	34.4	Tantalum, solid	57	Tantalum, solid (a)	78
Tantalum, solid	36	Platinum (a), solid	*66*	Tin, fused	*80.5*
Zinc, fused	36.60	Tin, fused	*68*	Platinum (a), solid	*98*
Iron (a), solid, about	52	Lead-tin alloy, fused	98	Iron (a), solid, about	*131*
Tin, fused	54.62	Ferro, nickel, solid	105	Calido, solid	*136*
Lead-tin alloy, fused	81	Iron (a), solid, about	*111*	Lead, fused	*148*
Ferro, nickel, solid	94	Calido, solid	*122*	Iron (b), fused	166
Lead, fused	102.85	Lead, fused	*125*		
Calido, solid	109	Nichrome II	128		
Krupp metal, solid	115	Antimony (b), fused	136		
Nichrome II, solid	119	Bismuth, fused	*167.5*		
Bismuth, fused	139.9				
Antimony, solid	152				

Sources of Data in Tables on pages 1352 and 1353. — (1) Landolt and Börnsteins' Physical-Chemical Tables, 1912 Edition. (2) Smithsonian Physical Tables, (3) Driver Harris Wire Co. (4) Herman Boker and Co. (5) Thomas Prosser and Son. (6) Dr. Frink, Trans. Am. Electro-chem. Soc., 1910, Vol. 17. (7) *Copper Wire Tables*, Circ. No. 31, Bureau of Standards, 1914. (8) Electrical Alloy Co.

Resistivity and Temperature Coefficient of Some Common Solutions. — (*See also Electrochemistry, Principles of.*) The table on p. 1357 is based on data given by Kohlrausch and Holborn (*Leitvermögen der Elektrolyte, Leipsig,* 1898). The temperature coefficient given is that corresponding to an increase of temperature from 18° C. to 19° C., and is a negative quantity, i.e., for an increase of temperature the resistance decreases. The resistivity temperature coefficient of aqueous solutions diminishes rapidly with increase of temperature, i.e., the higher the temperature the less is the decrease in resistance for each degree increase in temperature.

Resistivity and Temperature Coefficient of Ordinary Water. — The resistivity of ordinary tap or river water ranges from 1200 to 12,000 ohms per centimeter cube, ordinarily being between the limits 2000 and 5000 ohms per centimeter cube. The change of the resistance of such water with temperature between the limits 0° C. and 100° C. may be represented to a fair degree of approximation by the formula

$$R_t = \frac{40\,R_{20}}{20 + t},$$

where R_{20} is the resistance at 20° C. and t is any other temperature between 0° C. and 100° C. (*From Tests by Applequest and McKenny, Mass. Inst. of Tech.*, 1912.)

RESISTIVITY OF SOLUTIONS AND THEIR TEMPERATURE COEFFICIENT OF RESISTANCE

$$1 \text{ ohm per centimeter cube} = \frac{1}{2.540} \text{ ohm per inch cube.}$$

By per cent solution is meant the weight of the dissolved salt or acid expressed as a percentage of the weight of the solution.

For a dilution less than 5% the resistivity is approximately inversely as the per cent of dissolved salt or acid, i.e., a 2 per cent solution of common salt has a resistivity of approximately $14.9 \times 5/2 = 37$ ohms per centimeter cube. As noted above the resistivity of ordinary tap water ranges from 1200 to 12,000 ohms per centimeter cube; a solution made from such water can not of course have a resistivity greater than that of the water.

Substance	Per cent solution	Ohms per centimeter cube at 18° C.	18° C. temperature coefficient	Substance	Per cent solution	Ohms per centimeter cube at 18° C.	18° C. temperature coefficient
HNO_3	5	3.90	−0.015	$ZnSO_4$	5	52.3	−0.022
	10	2.18	−0.014		10	31.2	−0.022
	20	1.41	−0.014		20	21.4	−0.024
	30	1.28	−0.014		30	22.5	−0.027
	40	1.37	−0.015	$CuSO_4$	5	53.0	−0.021
	50	1.59	−0.016		10	31.3	−0.022
	60	1.96	−0.016		15	23.7	−0.023
HCl	5	2.53	−0.016	Na_2SO_4	5	24.4	−0.024
	10	1.59	−0.016		10	14.6	−0.025
	20	1.31	−0.015		15	11.3	−0.026
	30	1.51	−0.015	Na_2CO_3	5	22.2	−0.025
	40	1.94	−0.015		10	14.2	−0.027
H_2SO_4	5	4.80	−0.012		15	12.0	−0.029
	10	2.55	−0.013	NaCl	5	14.9	−0.022
	20	1.53	−0.014		10	8.25	−0.021
	30	1.35	−0.016		15	6.09	−0.021
	40	1.47	−0.018		20	5.10	−0.022
	50	1.85	−0.019		25	4.68	−0.023
	60	2.68	−0.021	NH_4Cl	5	10.9	−0.020
	70	4.64	−0.026		10	5.63	−0.019
KOH	5	5.84	−0.019		15	3.86	−0.017
	10	3.19	−0.019		20	2.97	−0.016
	20	2.01	−0.020		25	2.48	−0.015
	30	1.85	−0.022				
	40	2.23	−0.027				

BIBLIOGRAPHY. — See references in text.

RESISTORS, STANDARD, AND RESISTANCE BOXES. — (*See also Bridges for Electrical Measurements; Resistance and Conductance; Wires, Resistance; Rheostats.*) The primary standard of resistance is a mercury column of certain specified dimensions (*see below*). Secondary or commercial standards are made in two forms, viz.: (1) resistance standards each consisting of a single coil, carefully calibrated, mounted in a metal case, and (2) the ordinary resistance box, which contains a group of coils of known resistance.

The ultimate or primary standard of resistance, when measured in the international units, is the resistance of a column of mercury at 0° C. having a length of 106.300 centimeters and having a mass of 14.4521 grams (*see Units, Practical Electrical*).

STANDARD MERCURY RESISTANCE. — For the practical realization of such a standard, the International Conference on Electrical Units held in London in 1908 adopted the following specifications:

The glass tubes used for mercury standards of resistance must be made of a glass such that the dimensions may remain as constant as possible. The tubes must be well annealed and straight. The bore must be as nearly as possible uniform and circular, and the area of cross-section of the bore must be approximately one square millimeter. The mercury must have a resistance of approximately one ohm.

Each of the tubes must be accurately calibrated. The correction to be applied, to allow for the area of the cross-section of the bore not being exactly the same at all parts of the tube, must not exceed 5 parts in 10,000.

The mercury filling the tube must be considered as bounded by plane surfaces placed in contact with the ends of the tube.

The length of the axis of the tube, the mass of mercury the tube contains and the electrical resistance of the mercury are to be determined at a temperature as near to 0° C. as possible. The measurements are to be corrected to 0° C.

For the purpose of the electrical measurements, end vessels carrying connections for the current and potential terminals are to be fitted on to the tube. These end vessels are to be spherical in shape (of a diameter of approximately four centimeters) and should have cylindrical pieces attached to make connections with the tubes. The outside edge of each end of the tube is to be coincident with the inner surface of the corresponding spherical end vessel. The leads which make contact with the mercury are to be of thin platinum wire fused into glass. The point of entry of the current lead and the end of the tube are to be at opposite ends of a diameter of the bulb; the potential lead is to be midway between these two points. All the leads must be so thin that no error in the resistance is introduced through conduction of heat to the mercury. The filling of the tube with mercury for the purpose of the resistance measurements must be carried out under the same conditions as the filling for the determination of the mass.

The resistance which has to be added to the resistance of the tube to allow for the effect of the end vessels is to be calculated by the formula

$$A = \frac{0.80}{1063\pi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\text{ohm},$$

where r_1 and r_2 are the radii in millimeters of the end sections of the bore of the tube.

The mean of the calculated resistances of at least five tubes shall be taken to determine the value of the unit of resistance.

For the purpose of the comparison of resistances with a mercury tube, the measurements shall be made with at least three separate fillings of the tube.

STANDARD SINGLE-COIL RESISTANCES. — The mercury standard resistance is cumbersome to work with and must be kept at constant zero temperature. Secondary standards, made of wire having a low temperature coefficient, are therefore universally employed in ordinary testing laboratories. The unit is provided with suitable heavy copper terminals so arranged that it may be hung from mercury cups and dipped into an oil bath. The object of the oil bath is to keep the temperature constant. Very low-resistance units, designed to carry large currents, are kept cool by means of water circulating in a coil of pipe within the case itself. Low-resistance units are also provided with "potential terminals," see Fig. 1, the stated resistance being the resistance between these terminals.

Reichsanstalt and N.B.S. Types. — There are at present two recognized types of resistance standards, the Reichsanstalt type and the National Bureau of Standards (or N. B. S.) type. Fig. 1 shows a 0.1-ohm standard of the first type, and Fig. 2 a 1-ohm standard of the N. B. S. type.

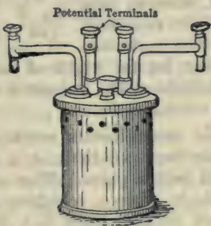

Fig. 1. Reichsanstalt Type Resistance Standard

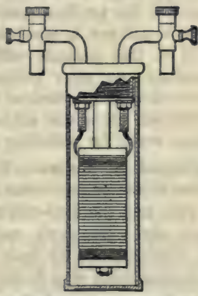

Fig. 2. N. B. S. Type Resistance Standard

The Reichsanstalt type and the N. B. S. type standards are insulated and baked in much the same manner as the coils in ordinary resistance boxes, though of course more carefully calibrated. The chief difference between the two types of standards is that the N. B. S. type is hermetically sealed in an oil-filled brass case, the oil having previously been freed from air by boiling. It is claimed that the N. B. S. units hold their calibration better than the Reichsanstalt units, the latter being subject to slight variations of resistance in climates where wide changes of humidity occur. (*See Bulletin Bureau of Standards, Vol. 5, p. 413.*)

Precision and Current-carrying Capacity of Standard Resistances. — The makers of standard units usually guarantee their accuracy as given below. The current-carrying capacity of the units is also given.

REICHSANSTALT TYPE, STANDARD RESISTANCE

Size of unit	Max. amp.	Error not greater than	Size of unit	Max. amp.	Error not greater than
0.0001	100	1/25 %	10	.3	1/100 %
0.001	30	1/25	100	.1	1/100
0.01	10	1/50	1,000	.03	1/100
0.1	3	1/50	10,000	.01	1/100
1	1	1/100	....	....	....

The National Bureau of Standards will calibrate any of these units and guarantee their accuracy of calibration to the above degree of precision, the charge being $4 to $6 per unit according to the value of the resistance.

RESISTANCE BOXES. — A brief description of the construction and arrangement of the coils in ordinary resistance boxes is given below.

Construction of Coils. — The coils are always wound non-inductively, that is, there are as many turns carrying the current in the right-handed direction as in the left-handed direction; this construction is illustrated in Fig. 3. Manganin wire (*see Wires, Resistor*) is usually employed. This wire is double-silk or enamel insulated, and is wound on wood or metal spools. Metal spools do not change in shape as wooden spools are liable to do, and since they more readily conduct the heat away, they may be safely used with larger currents.

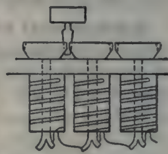

Fig. 3. Resistance Coil Construction

The wound spools are then dipped in shellac and baked from 10 to 15 hours at a temperature of 140° C. This baking removes the tension from the wire due to the winding and the resistance is rendered constant, whereas coils not treated in this manner will change their resistance to some extent long after being wound. The resistance coils are adjusted to the desired values by varying the length of the wire, and copper terminals are silver soldered to the ends of the resistance wire. The coils are then ready for soldering in place in resistance boxes.

Arrangement of Coils. — The principal arrangements employed at the present time in the construction of resistance boxes are the 1, 2, 3, 4 plan, the 1, 2, 2, 5 plan and the four-coil and five-coil decade plans. Nine- and ten-coil decades are also extensively used, particularly when a telephone receiver is used as a detector. With four- or five-coil decades there is a disagreeable sound in passing from step to step.

The 1, 2, 3, 4, and the 1, 2, 2, 5 Plans. — Resistance boxes built on the 1, 2, 3, 4 plan have coils of the following resistances: 1, 2, 3, 4, 10, 20, 30, 40. 100, 200, 300, 400, 1000, 2000, 3000, 4000 ohms. These coils are all in series and each block on the top of the box is connected to a junction between successive coils. Inserting a plug therefore short-circuits a coil, see Fig. 3. In the 1, 2, 2, 5 plan the construction is similar, but the coils of any group have resistances in the ratio 1 : 2 : 2 : 5. Any resistance from 1 to 10,000 ohms can be obtained by either of these plans. On account of the large number of plugs to be manipulated, the large number of plug contacts and the necessity of making a mental summation of the values unplugged, these arrangements are being superseded in modern resistance boxes by the decade plan described below.

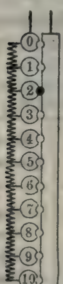

Fig. 4. 10-coil Decade

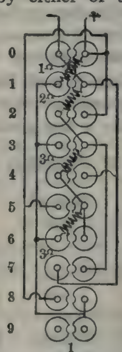

Fig. 5. 4-coil Decade

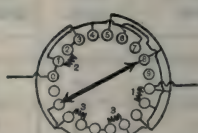

Fig. 6. 4-coil Dial Decade

Decade Plan. — The name decade arises from the use of 10 plug holes or 10 sectors in a dial arrangement, for each group of coils, each hole or sector corresponding to a given number of units, tens, hundreds, etc.

There may be either 10, 9, 5 or 4 coils per set. The 10-coil arrangement is shown in Fig. 4, a 4-coil plug decade arrangement in Fig. 5, and a 4-coil dial decade arrangement in Fig. 6, and a 5-coil decade arrangement in Fig. 7. The 9-coil decade differs from the 10-coil decade only in the omission of the tenth coil.

The only gain in having the tenth coil is to make it possible to check the total resistance of the 10 coils of one decade against any one coil of the next higher decade. The 4-coil and 5-coil arrangement possesses all the advantages of the 9-coil decade, and has the added feature of fewer coils to get out of adjustment.

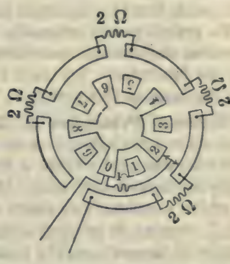

Fig. 7. 5-coil Dial Decade.

Precision of Resistance Boxes. — The precision to which the various coils in a resistance box are adjusted depends on the design and use to which the box is to be put. In the cheaper boxes the actual resistance of any coil may differ from the stated resistance by as much as ⅕ per cent, whereas higher grades of boxes can be had having an accuracy of 1/50 per cent.

Precautions to be Taken. Care of Resistance Boxes. — The directions under these same headings given in the article on *Bridges for Electrical Measurements* also apply to the use and care of resistance boxes.

COSTS (Pre-war prices). — An ordinary 10,000-ohm resistance box (with coils for obtaining all resistances from 1 to 10,000 ohms) with the coils accurate to ⅕ per cent costs about $30. A high-grade 10,000-ohm resistance box with coils accurate to 1/25 per cent costs about $90. The costs of Reichsanstalt standard resistances are approximately as follows:

0.0001 ohm	$40	1 to 1000 ohms	$20
0.001 ohm	30	10,000 ohms	25
0.01 and 0.1 ohm	30		

BIBLIOGRAPHY. — See *Bridges for Electrical Measurements.*

RHEOSTATS AND RESISTORS. — (*See also Controllers; Control Systems for Railway Motors; Motors; Regulators; Resistors, Standard; Starters, Motor.*) A rheostat is a resistance device, usually adjustable, placed in a circuit for the purpose of regulating the current in that circuit. If used in connection with a machine, it may serve for regulating the speed, input, output, voltage or power factor. The name resistor is also used synonomously with rheostat, but is sometimes limited to mean a non-adjustable resistance device. This article treats primarily of the design of the resistance units, or resistors, used in the various controlling and starting devices employed commercially and in the laboratory; for their application, connections, etc., see the articles listed at the head of this article.

General Principles of Design. — Rheostats used for starting motors or for other intermittent work are usually designed on the assumption that no heat is radiated by them during the period (usually from 15 to 30 seconds) that they are carrying current. On this assumption the maximum allowable current I which a resistor can carry for t seconds without exceeding a temperature rise of T degrees is proportional to the square root of the ratio of T to t, the factor of proportionality depending upon the specific resistance and size of the current conductor, and the specific heats, weights and volumes of the materials heated, including the insulating material in which the resistors are embedded; see *Heat and Thermal Properties*. However, since there are so many variables entering into this factor K, the usual method employed in designing a rheostat is to determine experimentally the relations between current, temperature rise and time for a series of resistance units and to plot these relations in a set of curves. See also below under *Commercial Forms of Rheostats*.

The maximum temperature rise in the case of rheostats designed to carry a continuous load will occur when the heat radiated equals the heat generated. The proper resistance units for any service are determined from capacity curves for a variety of units subject to various conditions. Capacity curves are obtained by plotting as ordinates degrees of final rise of temperature against watts dissipated as abscissas. See also below under *Commercial Forms of Rheostats*. Natural draught is usually depended on for ventilation. For very large capacities forced ventilation is occasionally applied, if the space available is limited, but little is saved on the initial investment, and the maintenance expense is increased.

COMMERCIAL RHEOSTATS. — The term "commercial rheostat" is commonly meant to include besides the resistance elements the complete switching and control mechanism which serves to vary the resistance in the circuit for any particular kind of regulation.

Forms of Resistance Elements for Commercial Rheostats. — The more common forms of resistance elements are described below.

Standard "Unit" Type of Resistance Element. — Units of this type are made in either a flat or cylindrical form. One form of flat unit consists of a moulded flat core of vitreous material on which the resistance wire is wound; the surface is then coated with a special cement and baked. Thus the resistance material is protected from injury and made proof against moisture.

Standard cylindrical units usually have a core of asbestos tubing, or sometimes of metal tube coated with a suitable insulating material, such as enamel. A wire of low temperature coefficient is used; see *Wires, Resistance*. The tube with winding is covered with a suitable insulating compound, and porcelain bushings or metal rings (the latter for the clip type) are placed on the ends. The units are then thoroughly baked and mounted rigidly on a frame or in a suitable case. The coating protects the resistance material from mechanical injury,

forms a good conductor of heat, and in case of a burn-out prevents appreciable arcing and unwinding of the wire. As a further protection, a sheet metal covering is sometimes placed around the cement.

The sizes and capacities of cylindrical units cover the following ranges approximately: power capacity, from 30 to 350 watts; current capacity, from 0.06 to 40 amperes; resistance, from 0.1 to 10,000 ohms; length, from 4 to 22 inches; diameter, from 1 to 2.5 inches. The watt capacity for these resistance units varies from approximately 1.5 to 3.5 watts per square inch of surface for continuous service, when assembled in frames affording good ventilation. This type of rheostat is rarely employed when currents of more than 50 amperes are to be handled.

Plate Type of Resistance Element. — In this type the resistance coils are attached to a circular base of insulating material. The coils are either covered with an insulating, heat-conducting cement and baked, or are enclosed in a ventilated iron case. Suitable contacts and a switch arm are provided. Currents higher than 60 amperes are rarely handled by this type of rheostat. The common sizes of plates range from 9 to 15 inches in diameter.

Grid Type of Resistance Element. — This type of rheostat consists of cast-iron grids assembled on horizontal rods bolted to pressed steel end-plates. and is the type generally used for large currents. The rods are covered with a mica insulating sleeve. If mounted on a frame affording good ventilation, the continuous capacity of the standard grids is approximately 700 amperes per square inch of cross-section for a maximum rise of 240° C.

Switching and Control Mechanism for Rheostats. — (*See also Starters, Motor.*) With those rheostats which contain distinct resistance elements, or units, the terminals of the elements are brought out to metal contacts usually arranged in a circle on an insulating, fireproof plate. The contact of the switch arm for small currents up to 25 amperes per contact arm consists of an ordinary straight finger contact brush; for currents up to 350 amperes, solid sliding plungers with evenly-faced surfaces held by springs against the contact segments are used in the switch-arms. For higher currents the laminated brush has been found more satisfactory because of the more uniform contact obtained. Most designs are based on the rule that for maximum current, 20° C. rise of temperature above surrounding air should not be exceeded for contacts and face parts. When hand-operated rheostats cannot be mounted on the panel, chain and sprocket drive is usually employed. Gear control is also occasionally used. Large rheostats are frequently operated by solenoids or motors. Motor-operated rheostats are usually employed for currents in excess of 350 amperes.

Special Forms of Commercial Rheostats. — For any particular class of service, the nature of the regulation desired, and the kind of circuit to which the rheostat is to be connected will determine the arrangement, size and number of the resistance steps necessary as well as the proper type of switching mechanism and control. See also *Motors, Industrial Application of.*

Field Rheostats. — The number of steps for a field rheostat depends on the closeness with which adjustments of field current are to be made and on the range desired. For ordinary conditions a rheostat resistance equal to the resistance of the generator field is satisfactory. Machines which are regulated by automatic voltage regulators frequently require a rheostat resistance of from 2 to 4 times that of the generator field. Field rheostats for currents up to 60 amperes are usually in form of the plate type or unit type. For higher capacities the grid type is employed. Field rheostats commonly have from 30 to 70 divisions of resistance. Double this number may be obtained in case of two or more plates, if the plates have the levers staggered. Field rheostats are either hand-controlled or solenoid- or motor-operated.

Field Discharge Resistors. — These resistances are placed across the field of motor or generator whenever the main-line switch is opened, for the purpose of providing a gradual dissipation of the electromagnetic energy stored in the field, thus limiting the arc on opening and reducing the back electromotive force due to the inductance of the field circuit. They are commonly designed for 15-second duty. In general, a resistance equivalent to that of the field is recommended. They are frequently included with the field rheostats.

Starting Rheostats. — See *Starters, Motor*.

LABORATORY RHEOSTATS AND RESISTORS. — (*See also Resistors, Standard.*) On account of the extreme variety of the requirements for rheostats in a laboratory, there are but few standard forms of rheostats for laboratory use on the market. Therefore, most laboratory rheostats are of special design.

A good form of laboratory rheostat for moderately-large currents is one consisting of a thin metal tube, on which enameled resistance wire is wound. The sliders should carry two or more laminated brushes, below which the enamel is scraped off. For fixed resistors the slider is omitted. The continuous capacity of this form of resistor of diameter from 1 inch to 2 inches ranges from 4 to 6 watts per square inch of surface, if ventilation is good.

Grooved porcelain tubes or rectangular slate slabs wound with bare resistance wire will carry on an average from 2 to 4 watts per square inch continuously.

Resistance wire (*see Wires, Resistance*) wound on open wooden frames is frequently used for laboratory rheostats.

Data on Galvanized Iron Wire. — Iron wire, although it has a much lower resistance than the various forms of resistance wires on the market, is much cheaper and is therefore sometimes used. The following data will be found useful in designing iron wire rheostats.

DATA ON GALVANIZED IRON WIRE

B. & S. gage	Circular mils	Maximum* allowable current	Feet required for 110 volts
20	1,022	2.5	594
19	1,288	2.9	626
18	1,624	3.5	673
17	2,048	4.2	710
16	2,583	5.0	750
15	3,257	6.0	790
14	4,107	7.1	840
13	5,178	8.5	886
12	6,530	10.1	941
11	8,234	12.0	990
10	10,380	14.3	1054
9	13,090	17.1	1103
8	16,510	20.3	1354

* In air with free radiation.

Carbon Rheostats. — This type of rheostat consists of a series of carbon plates stacked in a suitable frame and provided with an adjusting screw such that the pressure between the plates may be varied. Alternate plates of two different sizes are sometimes used in large rheostats to secure greater radiating

surface. These rheostats are very useful when continuous variation of resistance over relatively small ranges is desired; they are also very durable and cheap.

Data for the Design of Carbon Rheostats. — The following data is taken from an article by C. R. Moore in the *Elec. Rev. and West. Elec.*, 1912, Vol. 60, p. 672. Carbon plates, either circular or square, varying in thickness from 1/8 to 1/4 inch are satisfactory, the sizes from 1/8 to 3/16 inch being preferable because of the more uniform contact obtained between the plates. Since practically all of the resistance is in the contacts between successive plates the plate surfaces must be ground very smooth. The current-carrying capacity for such rheostats varies from 5 to 12 amperes per square inch with an average of 7 amperes per square inch. For temperature rises ranging from 40 to 85° C. the radiating coefficient is from 0.002 to 0.005 watt per square inch of radiating surface per degree centigrade rise. For pressures ranging from 7.5 to 180 lb. per sq. in., the variation in resistance per square inch per contact based on average results is shown in the following table:

RESISTANCE OF CARBON-PLATE RHEOSTATS

Pressure, lb. per sq. inch	Resistance, ohms per sq. in. per contact	Pressure, lb. per sq. inch	Resistance, ohms per sq. in. per contact	Pressure, lb. per sq. inch	Resistance, ohms per sq. in. per contact
7.5	0.0230	35	0.0063	90	0.0029
10.0	0.0185	40	0.0057	100	0.0027
12.5	0.0145	45	0.0052	120	0.0024
15.0	0.0125	50	0.0047	140	0.0021
20.0	0.0100	60	0.0041	160	0.0018
25.0	0.0082	70	0.0036	180	0.0015
30.0	0.0071	80	0.0032		

Example for Design of Carbon Rheostats. — Given a circuit of 5 ohms resistance connected to constant potential mains of 110 volts, the current is to be adjusted between the limit of 17 and 21.5 amperes by means of a series rheostat. Hence the corresponding voltage drops through the rheostat are 25 and 2.5 volts respectively, which calls for a resistance ranging from 1.47 ohms to 0.116 ohm. For a current-carrying capacity of 7 amperes per square inch, the plates must have an area of $21.5 \div 7 = 3$ sq. in. approximately; square plates 1.75 by 1.75 in. may therefore be used. A convenient thickness for plates of this size is 1/8 in. With 8 lb. per sq. in. as the minimum pressure, the resistance per plate is $0.022 \div 3 = 0.0073$ ohm (from table and area of plates as computed above). Hence the number of plates for 1.47 ohms resistance is $1.47 \div 0.0073 = 200$; thus the length of the rheostat is approximately $200 \times 0.125 = 25$ in. The minimum resistance of 0.116 ohm requires $0.116 \div 200 = 0.00058$ ohm per plate, which is equivalent to $3 \times 0.00058 = 0.00174$ ohm per square inch per contact. Hence, from the table the maximum pressure must be 165 lb. per sq. in., which is equivalent to a total pressure of $3 \times 165 = 495$ lb. acting on the frame supporting the carbon blocks.

Radiating surface must be provided for a maximum of $25 \times 17 = 425$ watts. With a radiating constant of 0.004 watts per square inch per degree centigrade rise of temperature, a maximum rise of temperature of 75° C. will require a radiation of $0.004 \times 75 = 0.30$ watts per square inch. Hence a radiating surface of $425 \div 0.30 = 1420$ sq. in. must be provided. Since the 200 plates represent a radiating surface (edges only) of only $200 \times 4 \times 0.125 \times 1.75 = 175$ sq.

in., every other plate must be of a larger size. This may be secured by using 100 small plates 1¾ by 1¾ by ⅛ inch, having a total radiating surface (edges only) of 87.5 sq. in., and 100 plates 3 by 3 by ⅛ inch, having a total edge surface of $3 \times 0.125 \times 4 \times 100 = 150$ sq. in.; the total area of their projecting sides represents $(3 \times 3 - 3) \times 2 \times 100 = 1200$ sq. in. Thus the total radiating surface is $87.5 + 150 + 1200 = 1437.5$ sq. in.

SUBMERGED WIRE RHEOSTATS. — Steady loads for temporary work may be obtained from galvanized iron resistors placed in a river or tailrace or in a barrel continuously fed with supply water. The proper diameter and length of wire for any set of requirements may be calculated from the following formulas:

$$d = KI^{2/3}, \qquad l = \frac{d^2E}{112\,I},$$

in which d = diameter of wire in mils, I = the current in amperes, E = the impressed voltage, l = length of wire in feet, 112 = average value of ohms per mil-foot of galvanized iron wire. Values of K for barrels or tanks range from 3.25 to 2.75, for tailraces or rivers of moderate flow, 2.75 to 2.25, for conditions of rapid flow, 2.25 to 2.0. Wooden sticks with fairly-sharp edges will make reliable supports for the wire. For more careful work the wires may be held by porcelain cleats screwed to wooden frames. The heating of these wires in water is so great that there must be no obstructions to a free circulation of the cooling water, otherwise failure may occur due to the formation of gaseous envelopes surrounding the wire. The water used must be clean to prevent rapid destruction by electrolysis. When tanks or barrels are used, it is preferable to raise the container above the ground (on wooden blocks about two feet high) so as to avoid seriously grounding the circuit through the stream of cooling water.

DATA FOR SUBMERGED RHEOSTATS OF GALVANIZED IRON WIRE

From *Rheostats for Dynamo Load Tests*, Am. Elec., 1903, Vol. 15, p. 512.

Size of wire, B. & S. Gage	Safe carrying capacity, amperes	Minimum length in feet for safe carrying capacity at different voltages			Feet per ohm, *hot*
		110 volts	220 volts	500 volts	
4	584	66	131	298	348.0
5	489	62	124	282	276.0
6	412	59	117	266	219.0
7	347	55	110	250	173.5
8	293	52	103	235	137.5
9	245	47	94	214	109.1
10	205	45	90	205	86.5
11	173	42	84	191	68.6
12	145	40	80	182	54.3
13	122	38	76	173	43.2
14	103	36	72	164	34.2
15	88	34	68	155	27.2
16	71	32	64	145	21.5
17	60	30	60	136	17.1
18	50	29	58	132	13.5
19	42	27	54	123	10.4
20	36	25	50	114	8.5

WATER RHEOSTATS. — A water rheostat serves as a simple, cheap and satisfactory means of dissipating large amounts of energy. It is extensively used in the commercial testing of apparatus and for other purposes where an adjustable, high-power-capacity resistance is required. Permanent installations are sometimes made in hydroelectric stations to improve hydraulic or electric regulation and to furnish an artificial load.

Resistivity of Water and Common Solutions.—The resistivity of ordinary water at 20° C. (= 68° F.), taken from streams, wells or reservoirs, usually ranges from about 800 to 2000 ohms per inch cube (2000 to 5000 ohms per centimeter cube) though extremes may be found as low as 500 and as high as 5000 ohms per inch cube (1200 and 12,000 ohms per centimeter cube respectively). In water rheostats designed for less than 1000 volts it is usually desirable to increase the conductivity of the water by adding a salt (common salt, copper sulphate or other cheap salt) or an acid (usually sulphuric). For example, the resistivity of a 5 per cent solution of common salt at 18° C. is 14.9 ohms per inch cube (38 ohms per centimeter cube). The resistivity of various solutions of different strengths is given in the article on *Resistance and Conductance.*

As noted in the article just referred to, the resistance of ordinary water and of salt and acid solutions diminishes rapidly with increase of temperature. To a close approximation the resistance of ordinary clean water between 0° C. and 100° C. may be calculated from the formulas

$$R_t = \frac{40\,R_{20}}{20 + t}, \qquad \text{or} \qquad R_t' = \frac{72\,R_{68}}{4 + t'}.$$

where R_{20} is the resistance at 20° C. = R_{68}, the resistance at 68° F., and t is any other temperature centigrade and t' any other temperature Fahrenheit.

Before designing a large high-voltage rheostat it is desirable to determine both the resistivity and temperature coefficient of a sample of the water which will be used. The sample should be tested in a miniature, low-voltage rheostat. If the large rheostat is to be operated on alternating current, the test with the miniature rheostat should also be made with alternating current, in order to avoid the effects of polarization. Determinations for a direct-current rheostat should be made with direct current, using a miniature rheostat of the same materials as are to be used in the large rheostat, and operated at the same current density and voltage. In designing a rheostat for permanent use data should also be obtained on the variations in the resistivity of the water throughout the year, for these variations are very marked in some cases.

Low-voltage Water Rheostats. — A simple form of water rheostat for direct-current or single-phase low-voltage work can be made of an oil barrel and two iron plates. One plate is placed on the bottom of the barrel. An insulated copper wire is connected to this plate and brought up along the side of the barrel and out through the top, or it may be attached to the underside of the barrel by means of a bolt passing through the plate and the bottom of the barrel. The other terminal is attached to a movable plate which is held by a window cord strung over two pulleys, the plate being held in position by a suitable counterweight at the free end of the cord. The barrel is filled with water, to which a relatively strong solution of common salt ($NaCl$), sal ammoniac (NH_4Cl) or washing soda (Na_2CO_3) is added in sufficient amount to give the required conductivity. It is essential that the substance should be dissolved before it is added to the water barrel and that the solution should be poured in very carefully, because a very small amount of a salt solution will add considerably to the conductivity of the water. A single-phase load of 100 amperes at 100 volts may be carried continuously in a 40-gallon barrel without causing the water to boil. The plates should have a surface (one side only) of at least 1 square inch per ampere.

For low-voltage three-phase work a barrel with three pipes, suitably mounted and arranged to be raised or lowered by means of a cord and pulleys, is sometimes used. The pipes should have an immersed surface (outside surface only) of at least 1 square inch per ampere.

A serious objection to the use of a water rheostat containing a salt or acid is that the resistance for a given setting of the electrodes does not remain constant, due to the evaporation of the water, rendering the solution more concentrated. The increase in the temperature of the solution also causes very appreciable changes in resistance. When direct current is used the change in the concentration of the solution at the two electrodes (polarization) also causes a variation in the resistance with time.

High-voltage Water Rheostats. — For voltages above 1000 the conductivity of the tap or river water available is usually ample, without having recourse to the use of salts or acids. The use of ordinary water also makes it possible to keep a continuous flow of water through the rheostat and thus maintain a practically constant temperature. The discussion of such high-voltage rheostats given below is adapted from an excellent paper on this subject by E. A. Ekern in the *Stone and Webster Public Service Journal* for Nov., 1911.

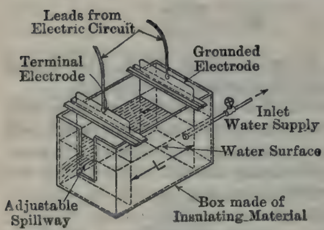

Fig. 1. Open-column Type

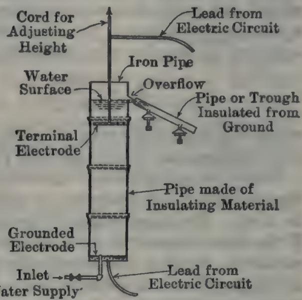

Fig. 2. Closed-column Type

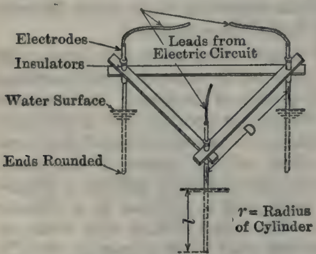

Fig. 3. Open Type, Three-phase Rheostat

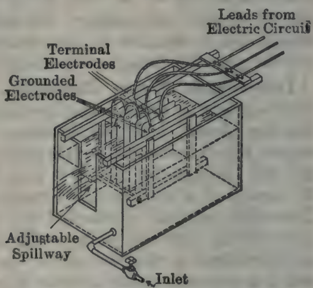

Fig. 4. Mixed Type, Three-phase

Types of High-voltage Water Rheostats. — Fig. 1 illustrates a simple form of such a rheostat, and will be referred to as the "open-column" type, and Fig. 2 illustrates another simple form which will be referred to as the "closed-column" type. The illustrations are for single-phase rheostats; three-phase rheostats are made by combining three single-phase elements with the neutral point grounded. Fig. 3 illustrates a three-phase type suitable for immersion

in an open body of water; this type will be referred to as the "open type." The type shown in Fig. 4, which is a combination of the column type and open type, will be referred to as the "mixed type;" the illustration shows a three-phase rheostat with three plates for terminal electrodes sandwiched between four neutral electrodes. The plates should preferably have rounded edges.

Calculation of Resistance. — The resistance of the column type of rheostat (Figs. 1 and 2) is computed by the formula

$$R = \rho \frac{L}{A}, \tag{1}$$

where ρ = the resistivity of the water in ohms per inch cube (as noted above ρ is usually between 800 and 2000 ohms per inch cube), L = the length of the column in inches, and A = the area of the cross section of the column in sq. in.

For the open type (Fig. 3) the resistance between any cylindrical electrode and the neutral is given by the formula

$$R = \frac{\rho}{2.73\, l} \log_{10} \left(\frac{2\,D}{d}\right), \tag{2}$$

where ρ = the resistivity of the water in ohms per inch cube, l = the wetted length of cylinder in inches, D = distance between cylinders in inches (center to center), and d = external diameter of each cylinder in inches. The equation is sufficiently accurate for all practical purposes when $2\,D \div d$ is greater than 80 and may be used with but small error when this ratio is as low as 40. (*See also the article on Resistance and Conductance.*)

The resistance to neutral of the mixed type (Fig. 4) is computed by the formula

$$R = \frac{R_1 R_2}{R_1 + R_2}, \tag{3}$$

where

$$R_1 = \rho \frac{S}{2\,A} \qquad \text{and} \qquad R_2 = \frac{\rho}{1.37\,\lambda} \log_{10} \left(\frac{4\,S}{t}\right), \tag{4}$$

R_1 is the resistance between the face of any one terminal plate and the two neutral plates between which it is placed and R_2 is the resistance between the wetted edge (assumed rounded) of the terminal plate and the two plates between which it is placed. The letters in the expressions for R_1 and R_2 have the following meanings: ρ = resistivity of the water in ohms per inch cube, S = the distance between adjacent plates (terminal plate to neutral plate) in inches. A = area in square inches (one side only) of terminal plate, t = thickness of each terminal plate in inches, λ = total length in inches of wetted edge (perimeter) of one terminal plate. The expression for R_2 is a rough approximation, but as R_2 is small compared with R_1, the error in the resultant resistance R due to an error in R_2 is relatively small.

Allowable Current Density. — A current density at the electrodes in excess of 4 amperes per square inch is accompanied by a "spitting" or arcing and there is danger of short circuit. This limiting current density applies to clean water, but as there is always a likelihood of foreign materials getting into the water, it is not advisable to use a current density between flat plates in excess of 1 ampere per square inch, but at the edges of the plates or where there is a free and large circulation of water a current density as high as 3 amperes per square inch may be safely used.

Minimum Distance between Electrodes. — Calling σ the maximum allowable current density and v the voltage to neutral, then in terms of the notation defined above, the minimum spacing of the electrode is:

For column type (Figs. 1 and 2) $$L = \frac{v}{\rho\sigma}. \tag{5}$$

For open type (Fig. 3) $$D = \frac{d}{2} \log^{-1} \left(\frac{0.869\, v}{\rho d \sigma} \right). \quad (6)$$

In the mixed type the limiting current density is usually at the edge of the plates, and the minimum spacing in this case is given by the relation:

For mixed type (Fig. 4) $$S = \frac{t}{4} \log^{-1} \left(\frac{0.869\, v}{\rho t \sigma} \right). \quad (7)$$

The logarithms are all common logarithms (i.e., to the base 10).

Minimum Size of Electrodes. — Using the same symbols as above, and in addition calling I the current per phase (terminal to neutral), the minimum area of each plate in the column type (Figs. 1 and 2) is $A = I \div \sigma$ square inches, and in the open type (Fig. 3) the minimum diameter of the electrodes is $d = I \div (\pi\, l\, \sigma)$ inches. In the mixed type the total area of the terminal plate (both sides and the edges) is approximately $(\rho S) \div R$, where R is the total required resistance. The area of the flat surface of the plate (one side only) may be taken from 10 to 15 per cent less than half of this total surface, to allow for the edges. The actual division of current between the edges and the flat surfaces will be inversely as the resistances R_1 and R_2 given by equations (4).

Example of Calculation of Dimensions. — The more complicated case of a mixed type of rheostat is chosen, to illustrate the use of both types of formulas. Power to be dissipated 3000 kw. at 2300 volts, three-phase. Then voltage to neutral is $V = 1328$ volts, current per phase $I = 753$ amperes, corresponding to a resistance of 1.762 ohms. Assume the plates to be ¼ inch thick, then $t = 0.25$. Allowing a maximum current density of 3 amperes per sq. in. at the edges, and taking $\rho = 800$, the minimum spacing is then, from equation (7), $S = 5.27$ inches.
The total surface (both sides and edges) of each terminal plate, assuming $\rho = 1200$, to allow for an increase in the assumed resistivity of the water, will be $(1200 \times 5.27) \div 1.762 = 3580$ sq. in., and the area of the plate should then be say 15 per cent less than $3580 \div 2$, or 1500 sq. in. approximately. A plate 60 by 25 inches will be a suitable size. The two resistances R_1 and R_2 are then from equation (4), $R_1 = 2.1$ ohms and $R_2 = 11.6$ ohms, and from equation (3) $R = 1.775$, which practically agrees with the required value of 1.762 ohms.

Supply of Water. — Due to the change in the resistivity of water with increase in temperature it is preferable to supply the rheostat with sufficient water to prevent much of an increase of temperature. At a hydroelectric station it is usually possible to use an open-type rheostat (Fig. 3) placed in a running stream or in a large body of water. When the water supply is limited it is sometimes necessary to allow the water to heat or even boil in extreme cases. Even in the case of limited water supply, however, a small steady overflow should be allowed, in order to prevent an accumulation in the rheostat of the salts contained in the water.

For an increase of $T°$ F. in temperature the water required to carry off P kilowatts, assuming no radiation or evaporation, is

$$\frac{0.91\, P}{T} \text{ cu. ft. per min.} = \frac{6.8\, P}{T} \text{ gallons per min.} \quad (8)$$

If only G gallons of water per minute are available and the water is allowed to boil, then calling T the difference between the boiling point and the temperature of the supply water in degrees Fahrenheit, the water evaporated when P kilowatts are supplied to the rheostat would be

$$G_e = \frac{P}{143} - \frac{GT}{970} \text{ gallons per min.} \quad (9)$$

Example. — To dissipate 3000 kw. with a 10° F. rise in the temperature of the water would require $(6.8 \times 3000) \div 10 = 2040$ gallons per min. If only 60 gallons per minute are available at 62° F., $G_e = 3000 \div 143 - 60 \times (212 - 62) \div 970 = 11.7$ gallons per min. will be evaporated, leaving 48.3 gallons per minute for the overflow.

Notes on Design of Water Rheostats. — The column type of rheostat may be used for any capacity and voltage. For voltages up to 23,000, the mixed type is usually preferable for large loads. For higher voltages, the open type may then be employed to advantage. Common iron pipe or steel plates are satisfactory materials for electrodes. Rheostats designed for permanent use should have galvanized electrodes. For voltages of 2300 volts and less, well-oiled and shellacked wooden containing vessels for the water have been found satisfactory. Open-or mixed-types of rheostats may be put in any tank or body of water where the clearance to sides or bottom is greater than one-half the spacing of electrodes. Column type rheostats up to 44,000 volts having columns built of vitrified glazed tile cemented together with Portland cement shellacked over have been operated successfully. Open-type rheostats made of iron pipes insulated and held securely above the water surface are very satisfactory. Mixed-type rheostats having plates separated by common porcelain line insulators and all clamped into a wooden frame have been used very successfully. Plates or electrodes should always be so arranged and located as to obtain free circulation of water. Where insulating material may be partially or intermittently in and out of the water, material may be deposited from the water and sufficient leakage surface must be provided. Innumerable arrangements and contrivances for the operation of rheostats may be designed but the important considerations are sufficient insulation to protect the operator, and positive movements to obtain the required degree of adjustment. Water rheostats are practically non-inductive. A slightly-leading current is obtained due to electrostatic capacity, but it is often more than offset by the inductive reactance of the conductors.

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ROOTS AND POWERS. — The symbol a^n where n is a positive whole number means a multiplied by itself n times; n is called the exponent of a. The conception of an exponent is also extended to include negative and fractional exponents as follows: Let m be a positive whole number and let $n = -m$; then the expression $a^n = a^{-m}$ is defined as equivalent to $\frac{1}{a^m}$; that is, a^{-m} is taken as equivalent to the reciprocal of a^m. Again, let m be a whole number and let $n = \frac{1}{m}$; then the expression a^n is defined as equivalent to $a^{\frac{1}{m}} = \sqrt[m]{a}$; that is, $a^{\frac{1}{m}}$ is taken to represent the m root of a. In general, then, the properties of exponents are the following, where m and n may be positive or negative whole numbers or fractions:

$$a^{-m} = \frac{1}{a^m} \qquad (a^m)^n = a^{mn}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a} \qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$a^m a^n = a^{m+n} \qquad a^0 = 1$$

$$\frac{a^m}{a^n} = a^{m-n}$$

The calculation of roots and powers can be conveniently carried out by means of logarithms (q. v.); for the highest accuracy, a seven or nine place table should be used. Note that

$$\log a^n = n \log a$$

$$\log \sqrt[n]{a} = \frac{1}{n} \log a$$

ROPES AND ROPE DRIVE. — (*See also Wires and Cables, Bare.*) The following terms relating to ropes and cordage are commonly employed:

Yarn. — Natural fibers twisted together.
Thread. — Two or more *small yarns* twisted together.
String. — Same as thread except of little larger yarns.
Strand. — Two or more *large yarns* twisted together.
Cord. — Several *threads* twisted together.
Rope. — Several *strands* twisted together.
Hawser. — A rope of three *strands*.
Shroud-laid rope. — A rope of four strands.
Cable. — Three *hawsers* twisted together.

In a strand the yarns are laid up left-handed; in a rope the strands are laid up right-handed; in a cable the hawsers are laid up left-handed.

WEIGHT AND STRENGTH OF ROPES. — The following tables give the weight per foot and breaking strength of hemp and steel ropes. Cotton ropes have approximately the same weight per foot as hemp ropes, but their breaking strength is only 60 per cent of that of hemp ropes. Iron ropes have the same weight per foot as steel ropes, but only 50 per cent of the breaking strength of cast-steel ropes.

APPROXIMATE BREAKING STRENGTH OF STEEL-WIRE ROPES

(*Trenton Iron Company*)

6 strands of 19 wires each					6 strands of 7 wires each				
Diam. rope, inches	Lb. per ft.	Approximate breaking stress, lb.			Diam. rope, inches	Lb. per ft.	Approximate breaking stress, lb.		
		Cast steel	Extra strong steel	Plow steel			Cast steel	Extra strong steel	Plow steel
2¼	8.00	312,000	364,000	416,000	¼	0.10	4,800	5,400	....
2	6.30	248,000	288,000	330,000	1½	3.55	136,000	158,000	182,000
1¾	4.85	192,000	224,000	256,000	1⅜	3.00	116,000	136,000	156,000
1⅝	4.15	168,000	194,000	222,000	1¼	2.45	96,000	112,000	128,000
1½	3.55	144,000	168,000	192,000	1⅛	2.00	80,000	92,000	106,000
1⅜	3.00	124,000	144,000	164,000	1	1.58	64,000	74,000	84,000
1¼	2.45	100,000	116,000	134,000	⅞	1.20	48,000	56,000	64,000
1⅛	2.00	84,000	98,000	112,000	¾	0.89	37,200	42,000	48,000
1	1.58	68,000	78,000	88,000	11/16	0.75	31,600	36,800	42,000
⅞	1.20	52,000	60,000	68,000	⅝	0.62	26,400	30,200	34,000
¾	0.89	38,800	44,000	50,000	9/16	0.50	21,200	24,600	28,000
⅝	0.62	27,200	31,600	36,000	½	0.39	16,800	19,400	22,000
9/16	0.50	22,000	25,400	29,000	7/16	0.30	13,200	15,000	17,100
½	0.39	17,600	20,200	22,800	⅜	0.22	9,600	11,160	12,700
7/16	0.30	13,600	15,600	17,700	5/16	0.15	6,800	7,760	......
⅜	0.22	10,000	11,500	13,100	9/32	0.125	5,600	6,440	......
5/16	0.15	6,800	8,100	......	....	......	......	......	......

MANILA ROPE

(From Blue Book of Amer. Mfg. Co., N. Y.)

Diameter of rope, inches	Approx. pounds per foot	Breaking strength, pounds	Diameter of rope, inches	Approx. pounds per foot	Breaking strength, pounds
½	0.085	1,750	1⅜	0.65	13,200
⅝	0.13	2,730	1½	0.77	15,700
¾	0.20	3,950	1⅝	0.90	18,500
⅞	0.26	5,400	1¾	1.04	21,400
1	0.34	7,000	2	1.36	28,000
1⅛	0.43	8,900	2¼	1.73	35,400
1¼	0.53	10,900	2½	2.13	43,700

Factor of Safety. — For ordinary purposes the maximum safe stress in wire ropes should be about one-third the ultimate, and for shafts and elevators about one-fourth the ultimate. In estimating the stress due to the load for shafts and elevators, allowance should be made for the additional stress due to acceleration in starting. For short inclined planes not used for passengers a factor of safety as low as 2½ is sometimes used, and for derricks in which large sheaves cannot be used and long life of the rope is not expected, the factor of safety may be as low as 2.

For power transmission by hemp ropes a factor of safety of 36 (referred to the nominal strength of the rope) is usually employed (*C. W. Hunt, Trans. A.S.M.E., Vol. 12, p. 230*).

Splicing and Knots. — See *Kent's Mechanical Engineers' Pocket-Book.*

ROPE DRIVING. — There are two methods of putting ropes on the pulleys, the multiple-rope and the single-rope systems. In the multiple-rope system several endless ropes are employed, and therefore there are as many splices as there are single ropes. In the single-rope system one endless rope is used, making several turns around the pulleys or sheaves, and therefore with but a single splice. In the first case the individual ropes are spliced in place, being made very taut at first, and less so as the rope lengthens, stretching until it slips, when it is respliced. In the second method tension pulley must be used to give the necessary adhesion and also to take up the wear.

Power Transmitted by a Rope. — The formulas given in the article on *Belts and Belting* are directly applicable to rope transmission, but may be put in a more convenient form as follows. Let

V = velocity of rope, in feet per minute,
d = diameter of rope, in inches,
m = weight, in pounds, of 1 foot of rope 1 inch in diameter,
T_1 = actual tension in driving side of rope, in pounds per circular inch (i.e., the tension in pounds in a rope 1 inch in diameter),
f = coefficient of friction between rope and pulley,
n = number of half-turns the rope makes around pulley,
$C = 1 - \epsilon^{-nf\pi}$, where the value of $\epsilon^{-nf\pi}$ is taken from the table of ϵ^{-x} in the article on *Exponential Functions*, putting $x = nf\pi$.

Then horse-power transmitted by each taut rope from one pulley to the other is

$$P = \frac{CT_1d^2V}{33{,}000}\left(1 - \frac{mV^2}{116{,}000\,T_1}\right). \qquad (1)$$

Power Transmitted by Hemp Rope. — A formula developed by C. W. Hunt (*Trans. A.S.M.E., Vol. 12, p. 230*) is frequently employed. This formula may be written

$$P = \frac{d^2V}{248}\left[1 - \left(\frac{V}{8500}\right)^2\right], \tag{2}$$

which is the same as formula (1) when $n = 1$, $f = 0.35$, $C = 0.67$, $m = 0.32$ and $T_1 = 200$. The following table is calculated from Hunt's formula. For temporary work a rope of given diameter may be used with safety to transmit twice the horse power given in the table.

HORSE POWER TRANSMITTED BY HEMP ROPE AT VARIOUS SPEEDS

Computed from formula (2) given above

Diam. of ropes, inches	Speed of the rope in feet per minute									
	1500	2000	2500	3000	3500	4000	4500	5000	6000	7000
½	1.45	1.9	2.3	2.7	3.0	3.2	3.4	3.4	3.1	2.2
⅝	2.3	3.2	3.6	4.2	4.6	5.0	5.3	5.3	4.9	3.4
¾	3.3	4.3	5.2	5.8	6.7	7.2	7.7	7.7	7.1	4.9
⅞	4.5	5.9	7.0	8.2	9.1	9.8	10.8	10.8	9.3	6.9
1	5.8	7.7	9.2	10.7	11.9	12.8	13.6	13.7	12.5	8.8
1¼	9.2	12.1	14.3	16.8	18.6	20.0	21.2	21.4	19.5	13.8
1½	13.1	17.4	20.7	23.1	26.8	28.8	30.6	30.8	28.2	19.8
1¾	18.0	23.7	28.2	32.8	36.4	39.2	41.5	41.8	37.4	27.6
2	23.2	30.8	36.8	42.8	47.6	51.2	54.4	54.8	50.0	35.2

Power Transmitted by Iron and Steel Ropes. — Wm. Hewitt of the Trenton Iron Company gives the following formula for the power transmitted by a steel rope, excluding the losses due to journal friction of the sheaves,

$$P = Bd^2v, \tag{3}$$

where v is the velocity of the rope in *feet per second* and B has the following values.

B for steel rope on	Number of half turns around sheave = n					
	1	2	3	4	5	6
Iron..........................	5.61	8.81	10.62	11.65	12.16	12.56
Wood........................	6.70	9.93	11.51	12.26	12.66	12.83
Rubber or leather...........	9.29	11.95	12.70	12.91	12.97	13.00

For iron rope take for B one-half the figures given in the above table.

Hewitt's formula (3) is equivalent to (1) when the centrifugal force of the rope is neglected and T_1 and f are as follows:

Kind of rope	Working tension T_1 in lbs. per circ. inch		Coeff. of friction $= f$
	Iron	Steel	
Iron	3600	7200	0.16
Wood	3600	7200	0.23
Rubber or leather	3600	7200	0.40

It should be noted that the coefficient of friction decreases rapidly with moisture or grease on the rope and sheaves, and consequently to transmit a given amount of power the tension must then be increased.

In the following table is given the horse-power that may be transmitted by a steel rope making a single half turn ($n = 1$) on wood-filled sheaves. This table agrees approximately with formula (3). The transmission of more than 250 h.p. by a single steel rope making a single half turn on filled sheaves is impracticable, as the filling would be rapidly cut out due to the increased tension (i.e., total tension $= T_1d^2$) and high velocities required. If the rope makes several half turns around the sheave, however, a greater amount of power can be transmitted at a given speed and tension than when only a single half turn is used.

HORSE-POWER TRANSMITTED BY A STEEL ROPE ON WOOD-FILLED SHEAVES

Diameter of rope, in.	Velocity of rope in feet per second									
	10	20	30	40	50	60	70	80	90	100
1/4	4	8	13	17	21	25	28	32	37	40
5/16	7	13	20	26	33	40	44	51	57	62
3/8	10	19	28	38	47	56	64	73	80	89
7/16	13	26	38	51	63	75	88	99	109	121
1/2	17	34	51	67	83	99	115	130	144	159
9/16	22	43	65	86	106	128	147	167	184	203
5/8	27	53	79	104	130	155	179	203	225	247
11/16	32	63	95	126	157	186	217	245	...	...
3/4	38	76	103	150	186	223	...	...	...	...
7/8	52	104	156	206	...	...	...	...	...	...
1	68	135	202	...	...	...	...	...	...	...

The horse-power that may be transmitted by iron ropes is one-half of the above.

Power Lost Due to Friction of Sheaves and Shafts. — In the above formula no allowance is made for the friction of the sheaves and shafts. Wm. Hewitt of the Trenton Iron Company gives the following expression for the horse-power lost in friction

$$6 \times 10^{-6} (W + G_1 + G_2) v,$$

where W = total weight of rope, G_1 = total weight of terminal sheaths and shafts, G_2 = total weight of intermediate sheaves and shafts, all in pounds, and v = velocity of rope in feet per second.

Diameters of Sheaves. — The following table gives the minimum diameter of sheaves recommended by Hunt for hemp ropes and by Hewitt for steel and iron ropes. The larger the diameter of the sheaves the greater the life of the rope.

MINIMUM DIAMETER OF SHEAVES

Diameter of rope, in.	Hemp	Steel			Iron		
		7-wire	12-wire	19-wire	7-wire	12-wire	19-wire
1/4	..	20	15	12	40	30	24
5/16	..	25	19	15	50	38	30
3/8	..	30	22	18	60	45	36
7/16	..	35	26	21	70	53	42
1/2	20	40	30	24	80	60	48
9/16	..	45	33	27	90	68	54
5/8	24	50	37	30	100	75	60
11/16	..	55	41	32	110	83	66
3/4	30	60	44	35	120	90	72
7/8	36	70	52	41	140	105	84
1	42	80	59	47	160	120	96
1 1/4	54	..	..	..	...	...	..
1 1/2	60	..	..	..	...	...	..
1 3/4	72	..	..	..	...	...	..
2	84	..	..	..	...	...	..

Tension Measured by Sag. — (*See also article on Transmission Lines.*) Let

w = weight of rope per foot, in pounds,

L = distance between centers of sheaves in feet,

S = sag in feet midway between sheaves.

Then the tension at the sheaves, corresponding to the sag S, is approximately

$$T = \frac{wL^2}{8S} + wS \quad \text{pounds}$$

for either the driving or the slack rope.

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RUBBER. — (*See also Gutta-Percha; Insulating Materials; Wires and Cables, Insulated.*) Rubber is derived from the milky secretion or latex of certain tropical trees, creepers and shrubs found chiefly in America, Africa, Ceylon and Malacca. When these plants are tapped, a thick milky-looking fluid or latex exudes from them. This latex is composed of very minute oil-like refractive globules, varying in size, which are in a state of rapid Brownian movement in a clear transparent liquid, called the serum. Besides these *caoutchouc* globules, or rubber-gum proper, the serum contains resins, protein, enzymes and various organic and inorganic compounds. Rubber or India rubber is the dried-up or coagulated latex. The best rubber is from a tree known as the *Hevea Brasiliensis*, and is known as *Hevea* rubber. It grows wild in Brazil and is cultivated in Ceylon and the Malay Peninsula. In Brazil coagulation is effected principally by dry heat or smoking. A wooden paddle is dipped in the latex and held over a smoky fire until the latex has coagulated. This process is repeated until the caoutchouc layers have become sufficiently thick, when the lump of raw rubber is cut off, dried for several days and despatched usually as "fine Para biscuits" to a trading center. The plantation rubber is coagulated in sheets by means of acetic acid. If subsequently smoked it is known as "*smoked sheets*." Para enterfine, Negro Heads and Sernamby are usually prepared from fine Para rubber which adheres to the tree during tapping or to the vessels containing the latex.

IMPURITIES IN COMMERCIAL RUBBER. — Commercial rubber contains, in addition to the caoutchouc, a number of foreign substances, such as sand, bark, etc., which can be removed by mechanical washing, followed by drying.

Acetone Extract. — In addition to the pure rubber gum, washed rubber contains resins and proteins which are soluble in acetone and are therefore often known as "acetone extract."

The accompanying table shows the loss in washing and the percentage of acetone extract in the best brands of raw rubber.

IMPURITIES IN COMMERCIAL RUBBER

(*Abstracted from a very comprehensive table in "Lectures on India Rubber," Edited by D. Spence.*)

Brazilian Rubber

Trade name	Geographical origin	Mean loss on washing, per cent	Acetone extract in washed dry rubber, per cent
Para, fine Island, soft cure	Brazil, the islands of	17–20	1.9–2.1
Para entrefine, Islands entrefine	the lower Amazon	18–25	Varies
Negro Heads, or Islands Coarse Sernamby	and its delta, and also other parts of the State of Para	35–40	2–6
Fine Para, upriver, hard cure	Amazon district.	15–20	1.9–2.9
Upriver entrefine, hard entrefine	Also the districts	18–25	Varies
Upriver coarse or Manaos Scrappy Negroheads	drained by its large tributaries	18–25	1.5–1.8
Cameta Negroheads	Southwestern Para	37–42	1.2–2.2

IMPURITIES IN COMMERCIAL RUBBER — *Continued*

Brazilian Rubber — *Continued*

Trade name	Geographical origin	Mean loss on washing, per cent	Acetone extract in washed dry rubber, per cent
Caucho Balls	Amazon district and	25–35	3.6–4
Caucho Slabs and Strips	its lower tributaries	35–42	4–8
Ceará Scraps	Ceará, Piauhy and	29	2.1
Maniçoba	Rio del Note		
Matto-Grosso, Virgin sheets, white Para	Matto-Grosso	15–30	2.5–3.5
Matto-Grosso, Negroheads	Matto-Grosso	25–35	2.5–6
Miscellaneous South and Central American Rubber			
Bolivian, fine medium	Bolivia	15	1.6
Virgin, coarse, entrefine	"		
Uncut Bolivian	"		
Mollendo	South Bolivia, Peru	15–25	1.9–3
Peruvian, fine, medium and scrappy	Peru	15–22	1.9–3
Peruvian Balls (also Caucho)	"	20–35	3.6–4
Orinoco, also Angostura	Venezuela	18–22	1.9–2.9
Eastern Rubber			
Plantation Rubber, as fine biscuits, sheets, fine crepe, scraps and block	Malacca and Ceylon	2–7	2–3
African Rubber			
Upper Congo, black	Congo, Angola	10	3.1
Loando Niggers, red fine and fine black.	" "	10–20	3
Kassai, fine	" "	7–10	3.3

MANUFACTURE OF RUBBER INSULATION. — The washed, dried rubber is passed between heavy rollers and flattened into thin sheets. It is then cut into small pieces and again passed through the rollers with a large proportion of fine powder consisting usually of inert mineral substances, waxy hydrocarbons and sulphur. The mixture is thus masticated until all its constituents are thoroughly mixed and a smooth homogeneous paste obtained. This process is known as compounding.

Fillers Used in Compounding. — Experience has shown that 60 or 70 per cent of mineral filler, or even a greater proportion of rubber substitute, may be added to rubber gum, before the essential qualities of the rubber

cease to predominate. The majority of commercial 30 per cent insulating compounds have compositions which fall within the following limits.

Ingredient	Per cent
Rubber	30-32
Whiting	0-30
Zinc oxide	28-67
Litharge	1-12
Ozokerite or paraffin	2-4

In addition to the above fillers, from 2 per cent to 4 per cent of sulphur is added to the compound, the greater part of which combines with the rubber in the vulcanizing process (see below).

Barium sulphate, sublimed white lead, lead carbonate, lamp-black, talc, magnesium carbonate, red lead, barium carbonate and other substances are also used in small quantities. Talc is often objected to, as making the compound porous, and lampblack, as rendering analysis difficult.

Applying the Compound to Wires. — The rubber compound is applied to the wire by "tubing" machines, or is applied in strips, and the wire thus covered with the compound is coiled up ready for vulcanizing. See article on *Wires and Cables, Insulated.*

Vulcanizing. — If exposed for a long time to air and sunlight, rubber loses its elasticity and finally oxidizes completely into resinous matter soluble in acetone. By vulcanization, however, rubber is rendered more or less immune from deterioration by weathering. Vulcanization is the chemical union of rubber gum with sulphur. It takes place at a temperature of from 248° to 302° F.

The coils of wire, covered with the compound as above described, are placed in a suitable chamber to which steam at the proper temperature is admitted. The time required for vulcanization depends upon the thickness of the insulation, the nature of the compound, the temperature and pressure of the steam, etc., ranging from 2 to 8 hours.

Sulphur Required for Vulcanization.— The amount of sulphur required to produce vulcanization varies with the brand of rubber and the nature of the adulterants with which it is mixed. The ratio of the weight of combined sulphur to the weight of caoutchouc, which is insoluble in acetone, is called the vulcanization coefficient. The highest grades of 30 per cent hevea insulation usually have a coefficient between 5 per cent and 10 per cent.

The vulcanization of some brands of rubber cannot be accomplished without either an excess of sulphur or the presence of excessive quantities of some mineral accelerator, such as red lead. Such rubber is to be avoided where permanency is an important consideration. It does not follow from this that red lead is a detrimental ingredient.

SPECIFIC RESISTANCE. — The specific resistance of 30 per cent hevea rubber compounds is exceedingly variable, depending largely upon the success with which steam is prevented from condensing in the insulation during vulcanization. The megohm-miles are usualy computed from the following formula:

$$M = K \log_{10} \frac{D}{d},$$

where

M = insulation resistance of a cable, megohm-miles,

d = diameter of cylindrical conductor,

D = diameter of cable over its insulation,

$K = 5.8 \times$ (millions of megohms per inch cube at 60° F.).

The value of K varies from 3000 to 7000, the usual specification value being 4000 for 30% hevea compounds.

The specific resistance is not an indication of the quality of rubber insulation as it depends more upon the dryness, the proportion of mineral wax, and the degree of vulcanization, than upon the quality of the ingredients. When the megohms are low they can almost invariably be raised by drying the insulation in a desiccator. The specific resistance of very poor quality rubber compound is, however, sometimes so low that when a cable lies in damp earth, sufficient leakage current may flow to permit the passage of water by endosmose, when the conductor is negative to the ground (*see article on Endosmose*).

DIELECTRIC STRENGTH. — The disruptive strength of rubber insulation is generally given as between 350 and 450 kilovolts per inch or about 140 to 180 kilovolts per centimeter effective a-c. values. The pressures which should be used in commercial testing do not depend entirely upon the dielectric strength of the rubber, as air in the rubber or between the rubber and conductor or between the rubber and ground becomes ionized at a pressure of about 30 kilovolts per centimeter with consequent generation of ozone, which rapidly oxidizes and destroys the rubber. This does not occur with paper insulation because the impregnating oil precludes the possibility of air spaces; and no harm results in the case of varnished cambric because it is not seriously attacked by ozone. Hence test pressures for rubber insulation must be based upon experience rather than upon theory and if sufficiently high to permit ionization of the air, should not last long enough to permit the formation of an appreciable amount of ozone.

SPECIFIC INDUCTIVE CAPACITY. — The specific inductive capacity of pure rubber is about 2.3 (*Floy*), but the vulcanized compounds used for insulation have specific capacities ranging between 3 and 5, the latter value being nearer the average. The specific capacities of several compounds of stated composition are given by E. Jona (*St. Louis, 1904*), but they cannot be considered as representative of American practice.

SPECIFICATIONS. — Specifications for rubber insulation for wires and cables will be found under *Wires and Cables, Insulated.*

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SECOHMMETER. — (*See also Bridges for Electrical Measurements.*) The secohmmeter is an instrument used in connection with a Wheatstone bridge for changing the direct current from a battery to an alternating current, and commutating the portion of this current that flows in the galvanometer circuit to a direct current. The secohmmeter may be used in the measurement and comparison of self- and mutual inductances and in the measurement of the resistance of electrolytes.

Fig. 1 shows the relative position of the commutators and brushes of a secohmmeter when connected in a bridge circuit: *A* is the commutator to which the galvanometer is connected and *B* the one to which the battery is connected. They are on the same shaft and turn together.

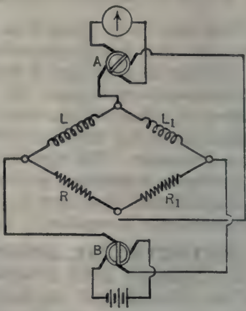

Fig. 1. Connections for Secohmmeter

The secohmmeter is not an altogether satisfactory instrument, due to the variation in the various contact resistances when used for any length of time. Whenever possible some other method of accomplishing the desired results should be employed, e.g., the use of alternating current with some sort of a-c. detector, such as a telephone receiver, electrodynamometer or a-c. galvanometer.

SELENIUM. — Selenium, an element of rare occurrence in nature, is found in sulphur and as a selenide in combination with various metals. Vitreous selenium, i.e., selenium which has been fused, is dark brown in color and has a specific gravity of 4.28. Its melting and boiling points are 217° C. and 700° C. respectively. Vitreous selenium is a very poor conductor of electricity, its resistance being about 60,000 ohms per centimeter-cube at room temperature. When vitreous selenium is annealed it assumes a crystalline form sometimes called metallic selenium. As a result of the annealing process, the electrical resistance is considerably reduced (the amount depending upon the thoroughness of the annealing) and becomes a function of the intensity of illumination, a property which is possessed to a lesser degree by tellurium and carbon.

Light-sensitive selenium is said to be light-positive or light-negative depending upon whether its resistance decreases or increases respectively when carried from dark to light. Sensitive selenium is usually light-positive. The resistance of light-negative selenium is usually less than that of light-positive selenium.

SELENIUM LIGHT-CELLS. — A selenium cell or unit is made by connecting several narrow strips of light-sensitive selenium in parallel between the edges of two brass plates. The higher the resistance the higher the sensibility of such a cell, that is, the greater the ratio of its conductivity in the light to its conductivity in the dark. The conductivity of a cell made of high-resistance selenium may show a change in conductivity of 20,000 per cent (200 times) when taken from direct sunlight to a dark room. Cells made of low-resistance selenium (light-negative) show a much smaller change in conductivity, seldom over 50 per cent for a change from direct sunlight to dark.

Effect of Intensity of Illumination. — The resistance of the cell when exposed to light depends upon the time of exposure and the intensity of the illumination. If a strong light is suddenly thrown upon a high-resistance cell placed in a dark room, the resistance of the cell decreases to its minimum value in a fraction of a second and then begins slowly to increase again. If the intensity of illumination is not so great or the cell has a comparatively low resistance the time taken to reach a minimum resistance may be several minutes. The majority of experimenters on this subject agree that the change of resistance varies as the square root of the intensity of illumination.

Effect of Wave Length of Light, Temperature, etc. — It is found that the greenish-yellow rays are the most effective. The sensibility of most cells decreases as the temperature increases. In certain selenium cells a large potential difference impressed across the terminals of the cell produces a variation of resistance in the cell similar to that caused by an exposure to light.

Uses of Selenium Cells. — The light-sensitive property of selenium has been utilized in the photophone and in connection with the electrical transmission of pictures. In the photophone a beam of light reflected from a mirror attached to the vibrating disk of a telephone transmitter is made to play across a selenium cell connected in series with a battery and a telephone receiver. In this manner, speech may be transmitted through space by means of light waves.

In the transmission of pictures a large number of selenium cells are usually made to reproduce the light and shade of the picture. The light reflected from each point on the picture is then represented at the distant point by a corresponding intensity of current, which may reproduce again electrochemically the intensity of the reflected light at each point.

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SEPARATORS, STEAM. — (*See also Boilers.*) Ordinary forms of steam boilers, without superheaters, generally deliver steam containing not more than 1.5 per cent of moisture. When the water level rises too high, however, or when the water contains substances which cause foaming, the percentage of moisture in the steam may be much higher. Condensation in long lines of steam pipes also increases the moisture, and for this reason it is customary to place in the pipe line near the engines one or more steam separators. These usually operate by suddenly changing the direction of the flowing steam, so that the particles of water are by their momentum carried out of the flowing current and projected against a baffle or bend in the pipe or separator chamber, where they are collected and removed through a trap. Separators are also frequently installed in the exhaust steam piping to eliminate the cylinder oil contained in the exhaust steam.

Although absurdly high efficiencies are claimed by manufacturers of steam separators, efficiencies which cannot be realized in commercial practice, yet no separator should be retained in service that does not remove at least 80 per cent of the water carried by the steam approaching the separator.

BIBLIOGRAPHY. — Gebhardt, G. F., *Steam Power Plant Engineering; Catalogues of Manufacturers.*

SERIES, MATHEMATICAL. — (*See also Equations, Differential; Wave Analysis.*) In calculating the numerical values of a function for given numerical values of the variable, it is frequently convenient to express the function as a sum of a series of terms each of which is a simple algebraic function of the variable. The simplest form of such a series is the

Binomial Theorem. —

$$(x+a)^n = x^n + nax^{n-1} + \frac{n(n-1)}{2!}a^2x^{n-2} + \frac{n(n-1)(n-2)}{3!}a^3x^{n-3} + \cdots a^n$$

Where $2!$ means 1×2, $3!$ means $1 \times 2 \times 3$, etc.

A particular case of this series is when $x = 1$, $a = \frac{1}{n}$ and n is taken infinitely large. The expression $\left(1+\frac{1}{n}\right)^n$ is not unity, but has the value

$$\epsilon = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

This is the base of the natural system of logarithms and is numerically equal to 2.718282 +.

Taylor's Series. — Let $f(x)$ be any function of x, and for brevity write

$$f'(x) \text{ for } \frac{df(x)}{dx},\quad f''(x) \text{ for } \frac{d^2f(x)}{dx^2}, \text{ etc.}$$

Then
$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) +, \text{ etc.}$$

This expression is known as Taylor's series. Such a series is a useful expression for $f(x+h)$ only when the terms of the higher order are negligibly small, in which case the series is said to be convergent.

Maclaurin's Series is the special case of Taylor's Series when the x in the latter is taken equal to o and the h in the latter is put equal to x. In this case

$$f(x) = f(o) + xf'(o) + \frac{x^2}{2!}f''(o) + \frac{x^3}{3!}f'''(o) +, \text{ etc.}$$

Where by $f(o)$ is meant the value of $f(x)$ for $x = o$, by $f'(o)$ is meant the value of $f'(x)$ when $x = o$. etc.

The following are useful expressions derived from the above relations:

$$\epsilon^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{etc.}$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \text{etc.}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \text{etc.}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \text{etc.}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \text{etc.}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \text{etc.}$$

SHAFTING. — (*See also Bearings; Belts and Belting.*) Ordinary shafting for mill work is usually made of solid steel rods, although hollow shafting is sometimes used.

Diameter of Shaft. — The proper size of shafting to use to transmit a given amount of power may be expressed by the formula

$$d = \sqrt[3]{\frac{CP}{N}},$$

where d is the diameter in inches, P the horse-power transmitted, N the speed in revolutions per minute, and C a constant depending upon the desired factor of safety, the number and location of pulleys or gears, the weight of pulleys or gears and belting, etc. Various authorities assign different values to C, ranging from 40 to 125, the lower value for short shafts used simply for transmitting power, a higher value for line shafts (carrying several pulleys), and the highest value for head shafts carrying the main driving pulley.

Distance Between Bearings. — The distance between bearings may be expressed by the formula

$$L = \sqrt[3]{Bd^2},$$

where L is in feet, d in inches and B is a constant dependent upon the elasticity of the shaft, the load on the shaft and the allowable deflection. The latter is usually taken as 1/100 of an inch per foot of length. The Pencoyd Iron Works take $B = 720$ for bare shafts, and $B = 140$ for shafts carrying pulleys.

Speed of Shafting. — The following represents average practice.

Machine shops..........................	120–240 revolutions per minute.
Wood-working..........................	250–300 revolutions per minute.
Cotton and woolen mills..................	300–400 revolutions per minute.

Efficiency of Shafting. — Experiments made by various engineers have shown losses between engine and machine in ordinary machine shops of from 50 to 60 per cent of the total power transmitted by the engine. This loss is greatest in shops where large machines are employed, located at some distance from each other, and it is here that other kinds of transmission can be used to advantage.

Cost of Shafting (Pre-war figures). — The cost of shafting, including the necessary hangers, couplings and pulleys will vary according to size between $2 and $6 per linear foot. A rough rule which may be used in preliminary work is to allow $1 per linear foot per inch of diameter.

BIBLIOGRAPHY. — See Kent's *Mechanical Engineers' Pocket-Book* and the various works on *Machine Design* listed in the Bibliography at the end of article on *Bearings*.

SHIPS, ELECTRIC PROPULSION OF.— The relative light weight, small size and high steam economy of high-speed steam turbines, compared with reciprocating engines, naturally suggest the use of such turbines for the propulsion of ships. The most efficient speed of a ship's propeller, however, is far below that of a steam turbine. Consequently, to obtain the best over-all efficiency, some means for speed reduction must be provided. At present there are two means available, namely, high-speed tooth-gearing and "electric gearing," i.e., high-speed generators direct-connected to the turbines and electrically connected to low-speed motors which in turn are direct-connected to the propeller shafts. By these two methods speed reductions as high as approximately 40 to 1 are being obtained. Ships equipped with electric gearing are commonly referred to as electrically propelled.

The development of the turbine and these two methods of speed reduction has made the reciprocating engine relatively undesirable for ship propulsion, except possibly for quite small boats. The electric propulsion of ships is now coming more and more into favor. There are still, however, opponents to this method among ship owners and others. The more obvious application is for large vessels, such as warships and ocean liners. The many very great advantages of electric propulsion for large warships is now fully recognized by the U. S. Navy as all the many battleships and battle cruisers which are being built, and those which have been built during the last four years, are equipped with this method of propulsion.

Two Chicago fire-boats, "Graeme Stewart" and "Joseph Medill" were equipped with electric propulsion in 1908. The first large ship equipped with electric propulsion for the United States Navy was the collier "Jupiter," which has been in continuous service since 1913. During the Great War the equipment was many times heavily overloaded. The Navy Department reports that practically no expense has been caused by the electric equipment during this whole period.

Slow-speed turbines, which in comparison with high-speed machines, are relatively inefficient, have been used for a number of years for driving ships. These turbines are either directly connected to their propellers or are connected through a single reduction gearing. In some large ships a combination of reciprocating steam engines and a low-pressure turbine have been installed.

Turbines versus Reciprocating Steam Engines. — Aside from questions of weight and cost, the higher steam economy of high-speed turbines is of itself ample in most cases to justify the adoption of the turbine for ship propulsion. The steam engine of the triple or quadruple-expansion type, due to the limitations in size of the low-pressure cylinder, cannot be made having a steam expansion ratio greater than about 16 to 1, or 20 to 1. The steam turbine has no such limitation and is capable of using efficiently steam expanded to any practicable extent. Where turbines are used, a vacuum of 28.5 inches is quite usual, and 29.5 inches is being recorded in some central stations during winter months. The economy due to large expansions is indicated by a comparison of the available energy of a pound of steam when expanded from boiler pressure to various degrees of vacuum, viz.:

200 pounds pressure to 24-inch vacuum = 220,000 foot-pounds
200 pounds pressure to 26-inch vacuum = 238,000 foot-pounds
200 pounds pressure to 28-inch vacuum = 265,000 foot-pounds
200 pounds pressure to 29-inch vacuum = 289,000 foot-pounds

These figures give an indication of the saving that can be expected by the turbine which can use efficiently steam expanded to a low degree. Little

advantage, however, is gained with a reciprocating engine when the expansion is carried below 24 inches. This not only means a saving of fuel, but also in the size of boilers, auxiliaries, etc.

Application of Internal Combustion Engines to Ship Propulsion. — Electric gearing not only serves as a means whereby high-speed steam turbines may be effectively used for ship propulsion, but also makes possible the adaptation of relatively simple internal-combustion engines for this purpose. Some years ago two or three vessels in Europe were equipped with electric transmission gear between internal-combustion engines and propellers with partial success. Recently in America some low-powered boats have been equipped in a similar manner using direct current apparatus; for example, the trawler "Mariner," which has been in successful operation for about two years. The apparatus on this boat consists of two 165 kw.-125 volt generators, each direct-coupled to an eight-cylinder, four-cycle, 350 r.p.m. Diesel engine. These two self-excited generators are normally connected in series and supply current to a 440 horsepower, 250 volt, 200 r.p.m. motor which is direct coupled to the propeller shaft. For slow speeds of the ship, one engine can be shut down and the motor supplied with current at 125 volts. Two master controllers are provided, one in the pilot house and the other in the engine room. For emergency manual operation is also provided in the engine room.

FUNDAMENTAL REQUIREMENTS OF SHIP PROPELLING MACHINERY. — A ship may be equipped with 1, 2, 3, or 4 propellers. The most favorable propeller speed is generally from 75 to 200 r.p.m., depending upon the size and speed of vessel. Destroyers and similar vessels have higher speed propellers. The following are considered desirable turbine speeds (for the impulse type of turbine) for the corresponding output:

Horsepower	R.P.M.	Poles of generator	Cycles per second
7,000	3600	2 poles	60
10,000	3000	2 poles	50
16,000	2400	2 poles	40
29,000	1800	4 poles	60
65,000	1200	4 poles	40

To secure the most favorable speeds for both propellers and turbines, a high-speed reduction is required, the amount depending upon the size of the turbine and the design of the propeller. For example, for a twin-screw ship having a propeller speed of 100 r.p.m. and a total horsepower of 18,000 equipped with two main turbines, a speed reduction of 30 to 1 should be adopted.

Loss due to Incorrect Turbine and Propeller Speeds. — In many turbine-geared equipments it has been considered advisable to depart from the most desirable propeller and turbine speeds so as to reduce the speed ratio of turbine and propeller and thus obtain a simple gearing. This sacrifice of turbine and propeller efficiency in some cases has caused an increase of fuel consumption amounting to from 25 to 30 per cent. With electric propulsion the most favorable turbine and propeller speeds can be generally adopted and a speed ratio as high as 40 to 1 can be used if desired.

Reversing. — The propelling machinery of a ship must of course, be capable of backing the ship as well as driving it forward. With reciprocating steam

engines there is of course no difficulty in this respect. A steam turbine, however, cannot be reversed. Consequently, when a ship is equipped with turbines directly connected to the propeller shafts, or connected thereto through mechanical gearing, each shaft must be equipped not only with a forward-driving turbine, but also with a turbine whose normal direction of rotation is opposite to that of the forward driving turbine. Such turbines are called "reversing turbines" or "astern turbines." When the ship is driven forward the reversing turbine is driven backward, no steam being admitted to it. To reverse the direction of motion of the ship, the steam suply is cut off from the main turbine and admitted to the reversing turbine, the main turbine then being driven backward.

With electric gearing, reversing turbines are of course unnecessary, since the direction of rotation of the motors connected to the propeller shafts can be reversed merely by operating suitable switches. This is a decided advantage of electric propulsion over direct-connected or mechanically geared turbines.

Losses in Reversing Turbines. — Experiment has shown that a turbine forced in a direction opposite to its normal direction of rotation has about 10 times as much friction loss as when driven in its normal direction. This friction loss in a reversing turbine, which is always present when the ship is in motion, may amount to more than 1 per cent of the shaft horsepower of the propeller.

Two Efficient Speeds. — The propelling machinery of a war vessel should be capable of driving the ship efficiently not only at full speed, but also at cruising speeds, which are usually from 50 to 70 per cent of full speed. The same is also desirable in certain passenger ships. As the power required to propel a ship varies approximately as the cube of the speed, this means that at cruising speeds the propelling machinery will be running at less than one-third of its maximum load. Since a steam turbine is most efficient at or near maximum load and at full speed, it follows that when directly connected or mechanically geared turbines are employed, there must of necessity be a sacrifice in efficiency at cruising speed, which, in the case of a warship, is the speed at which the ship is driven by far the greater part of the time.

Cruising Turbines. — To avoid this difficulty, it is possible to provide, in addition to the main turbines, auxiliary high-speed turbines geared to the propeller shafts, the speed reduction being such as to give the desired cruising speed. Provision has to be made in this case for disconnecting the auxiliary turbines when the ship is driven at full speed by the main turbines. These auxiliary turbines are usually referred to as "cruising turbines." The two U. S. battleships "Idaho" and "Mississippi" are equipped in this manner. The full speed of these ships is 21 knots. The cruising turbines are used for speeds up to 17 knots for the "Idaho" and up to 15 knots for the "Mississippi."

With electric propulsion two efficient speeds are readily obtained without any such complication. All that is necessary is to change the ratio of the number of poles of the motors to the number of poles of the generators, which is readily accomplished by a simple switching operation. Since the propeller torque required at half speed is only about one-quarter that required at full speed, the voltage applied to the motors at cruising speed can also be reduced, thus keeping the motors operating at relatively high efficiency at reduced speed. The equipment of the U. S. battleship "New Mexico," which was the first electrically propelled battleship, being put into commission in 1918, is arranged in this manner. This ship is a sister ship to turbine-driven ships "Idaho" and "Mississippi."

Comparison of the "New Mexico" and the "Idaho" and "Mississippi." — During the period from 1918 to 1920 the relative maneuvering

qualities and the relative fuel economy of these three ships were carefully observed when operating together. The electrically propelled "Mexico" has proven decidedly superior with regard to maneuvering qualities, and is also superior in fuel economy. The fuel consumption of the "Idaho" and of the "Mississippi" exceeds that of the "New Mexico" by the following amounts:

At 10 knots	20.0 per cent
At 13 knots	42.7
At 16 knots	47.8
At 19 knots	40.1
At 21 knots (full speed)	32.2

At 19 knots, and also at full power, the "New Mexico" uses about 0.975 pound of oil per shaft horsepower per hour, and at 15 knots 1.1 pounds of oil per shaft horsepower per hour. These figures include the total oil used for all purposes.

With reference to the "New Mexico," Commander S. M. Robinson, Fleet Engineer of the Pacific Fleet, states:

"In conclusion, it may be said that the performance of the 'New Mexico' since commissioning has been entirely satisfactory in every way and that the expectations of those who were responsible for its installation have been more than realized."

Propeller Characteristics. — Fig. 1 shows the relation between the speed and torque of a propeller when the ship is being driven at constant speeds in smooth water, and also when the ship is held stationary in the water. It will be noted that the running propeller torque increases approximately as the square of the speed. The power to drive a ship therefore increases approximately as the cube of the speed.

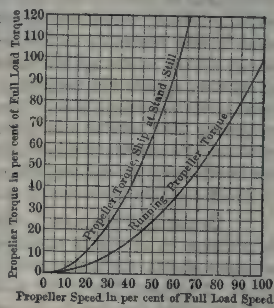

Fig. 1.

Torque Required to Stop and Reverse. — When the driving power is removed from a propeller with the ship traveling at a high speed, the propeller will continue to revolve in the same direction, being driven by the water. In order to stop the ship in the shortest time, or to maneuver quickly, it is necessary, not only to stop the propeller against the action of the water, but also to drive it in the reverse direction.

An experimental determination of the relation between the speed and torque of the propeller while being stopped and reversed with the ship traveling at constant speeds was made in the model tank at Washington, and later these data were found to approximately agree with tests taken on the U. S. S. "Jupiter" and the U. S. S. "New Mexico." Figure 2 shows such a propeller characteristic as obtained in the model tank.

Directly the power is removed, the propeller begins to slow down to approximately 73 per cent of its original speed and, if power is now applied to the shaft, tending to stop the propeller, the torque of the propeller mounts up to approximately 100 per cent of the forward driving torque.

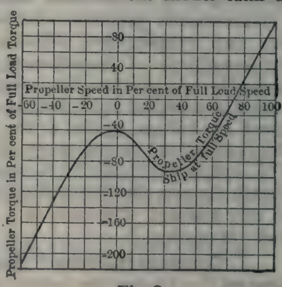

Fig. 2.

This maximum torque is attained at between 35 per cent and 40 per cent of normal propeller speed.

From this point on less torque is needed to bring the propeller to rest. If the propeller is now reversed, with the ship at full speed, 100 per cent torque is attained at a speed of about 30 per cent of normal. From this curve, therefore, it will be noted that it takes approximately 100 per cent torque to break the propeller away from the water, but only requires about 40 per cent torque to hold the propeller stationary after being stopped.

From experience obtained to date, it would seem that machinery for high-speed ships, having two or more propellers, should give 100 per cent reversing torque to quickly bring the propellers to rest when reversing from full speed. On lower speed ships, indications are that the maximum stopping torque is considerably less than 100 per cent. On single screw cargo boats, the stopping torque is probably quite low.

Variation of Propeller Torque with Pitching of Ship. — The General Electric Company has taken a number of elaborate tests with reliable apparatus to determine the variation of the propeller torque when the ship is in a sea-way. An illustration of the variation which may be expected in moderately rough sea is shown on Fig. 3. Although the ship was only pitching 4°, the increase in torque varied from no load to 175 per cent of normal load. These results were obtained on a cargo ship equipped with double reduction gearing. This turbine equipment did not have a governor and the variation in torque may, in part, be due to the variation in speed of the propelling machinery, but was mostly due to the different conditions of wake at different depths below the surface of the water.

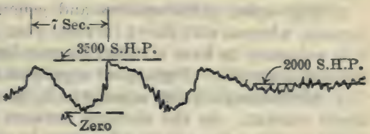

Fig. 3.

Under worse conditions of sea, it may be expected that this variation in torque would be considerably increased.

It was observed that, when the propeller approached the surface of the water, the load was quickly released and was again gradually increased as the propeller went deeper into the water. The record was taken with the ship in ballast and it was observed that the propeller did not break water. This variable condition of torque has been confirmed on electrically driven ships by observing the variation of load by electrical instruments.

The electric motor is an ideal piece of machinery for absorbing these shocks without deterioration. As there is no mechanical connection between the turbine and the propeller, this shock is absorbed in the air gap of the motor; it being necessary only to give the electrical machinery a sufficiently strong magnetic bond to hold the generator and motor together. This bond is increased by simply increasing the degree of excitation. If the motors drop out of step with the generators, then to bring them in step again it is necessary only to reduce the turbine speed and increase the excitation. Then the turbine speed can again be increased to normal. No harm is done if these motors are left out of step for short periods as the generator is so proportioned that the increase in current cannot greatly exceed the normal.

EQUIPMENT FOR ELECTRIC DRIVE. — The essential elements required for electric transmission between the turbines and the propellers of a ship are the generators, the motors, the cables connecting these two, and the switching and control devices.

Relative Weights of Electric Drive and Gear Drive. — When properly designed, the weight of the machinery required for electric drive is but little,

if any, in excess of the weight required for gear drive, except for such small high-speed boats as destroyers, scout cruisers and pleasure yachts. In certain cases the weight of the electric drive may be actually less than that of the gear drive. For example, in the case of a 3000 horsepower single-screw vessel, having a propeller speed of 100 r.p.m., assuming that the boilers are in the center of the ship and, in both cases, the turbine is installed near the boilers, and in the case of the electric drive the motor is installed in the stern of the boat, the electric propelling machinery, including the propeller shaft, weighed 10 per cent less than the geared propelling machinery.

Alternating versus Direct Current. — Alternating-current apparatus is usually adopted for ship propulsion in preference to direct current. There are, however, a few cases where direct current is used when small powers are involved, and also when Diesel engines are used as prime movers.

Alternating current has many advantages over direct current, the chief of which may be stated as follows:

1. The combined transmission losses of the generator and motor, when alternating-current apparatus is used, is from 5 to 8 per cent less than direct-current apparatus.

2. High-speed prime movers and generators can be used with alternating current, whereas with direct current, when a high-speed steam turbine is used, it is necessary to have a reduction gear between this and the low-speed direct current generator. High-speed apparatus is cheaper and lighter.

3. Alternating-current apparatus is, generally speaking, more reliable than direct current. As the distance between the generator and motor on shipboard is short, there is no advantage in using high-voltage transmission, as is the case on land. Lower voltages, therefore, are usually adopted. The range of voltage on cargo boats is from 1100 to 2300. The General Electric Company's records, over long periods of years and covering such electrical apparatus as is used on cargo ships, show that only about 1/10 of 1 per cent of machines of such voltage give any electrical trouble in a period of ten years. This electrical apparatus includes all motors and generators installed in all kinds of service.

In warships, machines designed for 5000 volts are being used.

Types of Motors Suitable for Electric Drive. — Five types of alternating-current motors have been used for ship propulsion:

1. Induction motors equipped with slip rings and external resistors.

2. Induction motors without slip rings, but with two squirrel-cage rotor windings. A high-resistance rotor winding is placed in the slot near the periphery of the rotor and beneath this a low-resistance winding. During normal running, when the current alternates slowly in the rotor, the low-resistance winding carries the current. When, however, there is a high-frequency current flowing in the rotor, as is the case during reversals of the propeller, the self-induction of the low-resistance winding forces current into the high-resistance winding, and this produces a high torque.

3. Induction motors without slip rings but with one squirrel cage rotor winding, consisting of deep, narrow bars. The action of this motor during reversal is similar to the double squirrel-cage motor.

4. Induction motors equipped with slip rings but no external resistors. The rotor is wound with a low-resistance winding near the periphery of the rotor and beneath this a high-resistance squirrel cage winding. The low-resistance winding is connected to the slip rings. To make the high-resistance winding carry the current during reversal of propeller, the low-resistance rotor winding is opened by means of contactor switches connected across the slip rings. If this motor is wound to give two different speeds,

the slip rings can be made neutral for high-speed running, and thus need not carry heavy current. All reversals are accomplished with low-speed connection.

5. Synchronous motors, the periphery of the poles being equipped with a low-resistance bar winding. With this type of motor the propeller is brought to rest during reversal by reversing the phases with the circuit "dead," and then applying a strong field on the motor, leaving the field off the generator. The propeller thus drives the synchronous motor as a generator, being loaded on the turbine generator. Sufficient torque is developed to bring the propeller approximately to rest. Field is now applied to the main generator and removed from the synchronous motor. If the turbine governor has previously been set for a slow speed, the synchronous motor will now reverse and quickly reach approximate synchronous speed at which time the field is again applied. A small and simple air cooled resistor can be switched in multiple with the generator during the stopping period. If this is done a higher torque can be developed with less excitation and with lower current flowing between the machines.

The following advantages are claimed for the synchronous motor over the induction motor:

1. A higher transmission efficiency due to operating at unity power factor.
2. Larger air gaps can be provided.
3. The generator and motor are lighter and cheaper.

The induction motor is more suited to cases where two different numbers of poles are desired for running at different speeds, such as is desired for warships.

Efficiency of Electric Transmission. — Opponents of electric propulsion have made claims that the transmission efficiency of this type of drive is low compared to other forms of speed-reducing machinery. However, when all the losses incident to the transmission by mechanical gearing are taken into account, the difference in efficiency of the two types of drive is by no means so great as is frequently stated, the advantage in favor of the mechanical gearing being less than 2 per cent. The following comparison for a low-powered cargo ship of 2500 shaft horsepower, driven by a single screw running at 100 r.p.m., is quoted from a paper by Mr. W. L. R. Emmet. (Read as a paper before the *Society of Naval Architects and Marine Engineers*, New York City, Nov. 13, 1919; also published by the *General Electric Review*, Jan., 1920).

"The selection for comparison of a ship of low power is unfavorable to electric drive in the matter of transmission efficiency, the conditions being better for this method in ships of higher power. The generator designed for this case has an efficiency of 95.6 per cent and the motor 95.9 per cent, making the transmission efficiency, including cable loss, etc., 91.6 per cent. In machinery designed for certain high-power ships, the efficiency is as high as 94 per cent.

"To determine the efficiency of gear transmission as compared with the figures just given, very careful tests have been made at Schenectady. A 2400-horsepower ship turbine was connected through two sets of double-reduction gearing to a generator, and the steam consumption was tested at various degrees of load and speed; then the same turbine was connected to the same generator without gearing, and tests were run with the same conditions and the same degree of steam flow. All this was done on a testing stand where conditions are uniform and accurate, the gears ran with perfect smoothness and all conditions were favorable. Since the comparison gives the loss of two gears, the differences are considerable and the determination should be very close to the correct value. This test showed that the performance of a single gear is as follows:

Shaft horsepower	R.P.M.	Loss of gearing, horsepower	Efficiency of gears, per cent
2400	87	125	95.0
1420	77	80	94.7

"In addition to these gear losses, we must also consider the loss in friction of the reversing turbine, which is estimated from reliable data to be 28 horsepower, and we must also consider the bearing losses on about 100 feet of shaft, which in perfect alignment will be 8.5 horsepower. These additional losses reduce the transmission efficiency to 93.5 per cent, leaving only 1.9 per cent advantage to the gearing. With the shaft more or less out of line and the gears operating under sea conditions, it is probable that the losses given would be greatly increased. Noise is an indication of loss, and most marine gears are at times noisy, while the gears in this test were almost silent. The gears tested in this case were of the General Electric Alquist type, and it might be claimed that other kinds of gears would be more efficient but it is obvious that, under fixed load and with similar gear speeds and diameters, there could be no advantage in any other type even if it ran with equal smoothness."

Gain in Efficiency at Low Speeds by Operating at Reduced Voltage. — Electric motors for shore purposes usually operate at approximately constant voltage regardless of the load carried. At light loads, therefore, they are relatively inefficient due to the iron losses remaining approximately constant. Motors, however, used for propelling ships usually obtain their power from a generator whose voltage can be varied to suit the motor's load. At reduced speeds, therefore, in an electrically driven ship, the voltage can be reduced; this reduction has the effect of materially reducing the losses in the iron laminations of both motor and generator. Thus, at reduced loads, the percentage of power lost is nearly the same as at full load. Whereas with gear, the transmission loss remains quite high at reduced speeds. Also the percentage loss of power in the gears increases when the gears have become worn.

Location of Propelling Machinery. — When vessels are equipped with turbine electric propulsion, the driving motors can be placed as far aft as convenient, thus shortening the propeller shafting. This also gives a larger and better disposition of cargo space and eliminates the long shaft tunnels. The turbine-generator units can be placed near the boilers at any desired level, reducing to a minimum the length of steam piping and other piping systems. The controlling mechanism, as in an electric locomotive, can be placed at one or more convenient positions.

Ventilation of Electrical Apparatus. — It is very important to ventilate the electrical apparatus in such a way that it will not collect moisture when shut down or running under light loads. To prevent this the temperature of the apparatus must not be lower than that of the surrounding air. The ventilating air should, therefore, be taken from the same room in which the apparatus is placed. This air, after passing through the machines, can be used under the boilers. Suitable dampers can be provided to prevent the engine room from becoming uncomfortably cold during the winter weather.

In some cases the air after leaving the generator and motors is passed between cooling tubes through which water is circulated. After passing between the tubes, the air enters the room in which the apparatus is placed and again cir-

culates through the generators or motors. Thus the same air is used continually.

This method eliminates the large ducts leading from the upper decks of the ship and also ensures clean air being always used. These coolers can be made quite small for this purpose.

Electric Drive for Auxiliaries. — A source of considerable loss in the operation of many ships is due to the use of steam driven auxiliaries. With electric drive it is necessary to maintain a direct current electrical supply independent of the main generators for purposes of excitation and lighting. This same source can be used for driving electrical auxiliaries. If turbine units are chosen for this auxiliary power, the steam from these units can be exhausted into a suitable stage of the main turbine and, by automatic means, exhausted into the main condenser when the load is removed from the main turbine. A more simple method is to design the auxiliary turbines as condensing units and connect their exhaust directly into the main condenser. An auxiliary condenser should, however, be provided for use in port. In certain cases it may be desirable to use a few steam-driven auxiliaries, such as boiler-feed pumps and the exhaust from these can be utilized for feed heating. If necessary, this steam can be supplemented by either "bleeding" the main turbine or using part of the exhaust from the auxiliary units.

In some cases these d-c. auxiliaries can be equipped with slip rings from which alternating current could be taken for driving the main motor in cases of emergency at reduced speed.

Some European cargo ships are equipped with auxiliaries driven by alternating-current motors from the main generating unit. In such cases reduced speeds of the ship are obtained by running the turbine generator at approximately full speed and reducing the motor speed by means of an external resistance. The fuel consumption during these reduced speeds is greater with this method than when the turbine speed can be reduced as there is a large loss of power in the external resistance.

Operating Force. — The following is quoted from a paper by Mr. W. L. R. Emmet. (Read as a paper before the *Society of Naval Architects and Marine Engineers*, New York City, Nov. 13, 1919; also published by the *General Electric Review*, Jan., 1920.)

"The history of the electrical industry has repeatedly shown that persons who have not used electrical apparatus assume that its operation requires a high order of skill and expert knowledge, and of this assumption we have already heard much in connection with electric drive for ships. A vast amount of experience has repeatedly shown that this assumption is the direct reverse of the truth, and a little thought as to the conditions in electrical apparatus should make the reason obvious. Conductor circuits are much simpler mechanically than pipes and mechanical motions, and electrical machinery is simply a combination of electric circuits with motion of rotation. The connections are easily shown by diagrams, and little mechanical skill is required to make them. The work of insulation can be so done that, under such conditions as exist in ship installations troubles which might involve difficulty of repair by unskilled persons are very improbable. In all the extensive uses of electricity in mills, mines, railways, and other industries, it has seldom failed to become popular immediately with the operating forces. In no case has this been more marked than in the ships which have been driven electrically. Large electrical apparatus is generally simpler than small, and the machinery used to propel a ship is in many respects simpler than that which is used to light it. Instead of introducing difficulties to the operating force, the adoption of electric drive will eliminate them and make ships much less dependent upon the skill and resourcefulness in their crews."

ADVANTAGES OF ELECTRIC PROPULSION. — Some of the advantages of electric propulsion have already been noted. This subject has been very fully treated by Mr. W. L. R. Emmet in numerous papers (*see Bibliography*), from which much of the following is quoted.

Ease of Disconnection and Repair of Propelling Machinery. — The following is quoted from Mr. Emmet: "In a geared equipment, each shaft has a system of turbines, gears, bearings, thrust-balancing devices and lubricating systems all mechanically locked together. With high-speed machinery any kind of trouble with any of these parts will almost certainly necessitate the immediate stoppage of the whole system. To keep a high-speed turbine running out of balance or with bearing trouble is impossible, and the gearing part would present almost equal difficulty. In the event of mechanical trouble of such character a ship would have to be stopped until the wreck could be cleared. The work necessary to uncouple and disconnect any part of such very heavy apparatus would be a serious matter involving much time, including that required to stop the ship.

"If it was found impracticable to make this disconnection, and the damage was such that the shaft could not be allowed to revolve, it would be necessary to lock the shaft to hold the propeller. A dragging propeller acts as a very serious drag on a vessel travelling at even moderate speeds. In fact, such a dragging screw may add 20 per cent to the horsepower and would, in addition, materially reduce the maneuvering qualities of the vessel.

"In the electrically-driven ship there is no mechanical connection of the shaft to anything but the rotors of motors. These are self-contained, iron-clad structures and cannot by any possibility be subject to mechanical interference. The shafts are subject to the same possibilities of bearing or thrust trouble as shafts in other ships, but the presence of the motors does not increase this danger, and the speed being low it is remote in any case. With this equipment any motor, generator or turbine, if in any kind of trouble, can be instantly disconnected without stopping the ship and with only a small loss from the highest speed capacity. Such a disconnection is made by simple switches."

Referring specifically to 180,000-horsepower, 33.5-knot battle cruisers now (1920) being built for the U. S. Navy, Mr. Emmet continues: "Such interchangeability as this power of disconnection gives constitutes one of the most important advantages of electric drive in such a ship. With one motor out of eight in trouble, only one-eighth of the maximum capacity is lost and the ship's maximum speed is impaired by only about 1 knot. If a generator or turbine is in trouble, the maximum speed is reduced only about 2 knots. With two generating units and four motors out the ship can make 25 knots, and with three generators and four motors out she can make 19 knots. If parts give trouble, they are simply cut out and repaired at leisure or as opportunity offers. The value of this interchangeability in a warship can hardly be overestimated. It largely overcomes the military danger incident to accidents to the driving power, a danger which has always been of primary importance."

Mud in Condensers. — In addition to the foregoing there is a great advantage when maneuvering an electrically propelled vessel in shallow or muddy water in and around harbors, provided the ship is equipped with more than one turbine and condenser. Under these conditions only one turbine generator is needed to drive the ship and, in case the condenser of this unit is plugged with mud, it can be immediately switched off and another turbine switched on to the driving motors.

In cases where turbines are mechanically connected to their respective propellers, under similar conditions of operation, it would be necessary to use all the turbines together with their condensers. If such a ship ran into mud, all

condensers might become plugged at once but, if only one became plugged, the maneuvering qualities of the ship would be dangerously impaired. Commander Robinson states:

"As an actual experience the 'New Mexico' while entering New York harbor had to shift main generators twice owing to the plugging of her condensers with mud, and these shifts were made so quickly that they did not affect the operation of the ship at all."

Backing Power. — Again quoting Mr. Emmet: "In turbine-driven ships, not electric, it is necessary to provide backing turbines which must run idle in the reverse direction when the ship is going ahead. These backing turbines involve complications which are very objectionable, and if these are reduced to a desirable minimum the backing power will be greatly inferior to that easily provided with electric drive. Experiment has shown that a turbine forced in an opposite direction involves about ten times as much friction loss as when driven in its normal direction. This loss, therefore, is very appreciable in the backing turbines of ships. There are also serious difficulties and dangers in high-speed apparatus incident to the abrupt and wide changes of temperature where steam is suddenly admitted to a cold reversing or ahead turbine. With electric drive the turbine never need be stopped when the ship is under way, and its operation is easier than in a power station because the load is steadier; with modern high-speed turbines superheat affords large gains in fuel economy, and the ability to safely use high degrees of superheat may constitute one of the important reasons for adopting electric drive.

Efficiency of Transmission. — "Various statements and estimates have been published concerning losses in high-speed gearing for which the authority is not known by the writer. Tests at Schenectady have shown losses about double those generally estimated, with rapid diminution of efficiency as load falls.

"With the electric drive in these battle cruisers (the 180,000-horsepower vessels now building for the U. S. Navy) the losses from turbines to propeller shaft at full speed will amount to 6 per cent, and by adjustment of voltage the transmission efficiency can be kept nearly as good at all speeds. Considering gearing losses, reversing turbine losses, losses incident to necessary subdivisions of the turbine and additional packings, it is believed that the actual transmission from turbine blades to shaft at full speed cannot be made more than 2 per cent better in the geared equipment than it is in the electric. At reduced loads the geared equipment will be much less efficient. The normal cruising speed of these ships will require less than one-tenth of full load, and the drag of gearing, bearings, reversing turbines and idle parts of main turbines will be very serious."

Cruising Economy. — "With the electric design, the number of motors and turbines used can be adapted to the demand for power, while with the other all parts must be kept running. This gives very important gain in economy at all speeds below the maximum. At 19 knots (in the case of the 180,000-horsepower battle cruisers for the U. S. Navy), only one turbine is required to drive the ship, and electrical arrangements are made by which it can be run at full speed instead of running at half speed, as it would if the ratio was fixed as by gearing. Thus the steam efficiency at 19 knots, a desirable cruising speed, is equal to the best attainable at any speed. Assertions which have been made by ship builders and others that equal cruising results can be obtained by the use of cruising turbines are obviously absurd. The cruising turbines themselves would be greatly inferior to the single main turbine so used, and they would be burdened with all the drags which are mentioned above. Cruising economy gives increased cruising radius without renewal of fuel supply. This

has always been considered a matter of the greatest importance in warships; it is a feature in respect to which electric drive can have no rival."

Flexibility of Installation. — A military advantage of great importance in connection with electric propulsion is the "flexibility of installation." The following is quoted from Commander Robinson (*General Electric Review*, April, 1919):

"The tendency in building modern capital ships is to provide for more and more torpedo protection and it becomes necessary to crowd the machinery away from the sides of the ship as much as possible. This arrangement is also desirable from the point of protection against gunfire for a similar reason. In this respect, electric drive has an enormous advantage over any other type of machinery in which the prime mover is mechanically connected to the propelling shaft. The main turbine-generators may be placed in any part of the ship that is most desirable; they may be placed in compartments forward of each other and they may be raised up enough to place the main condenser underneath them—in fact, there is practically no limit, other than the head room, as to the position of the main turbine-generator in the ship. This gives an enormous advantage to electric drive over all other types of machinery and enables the Naval Constructor to give far more adequate protection to the ship and machinery against damage by torpedo and gunfire. Those parts of the machinery—the main motors—which it is necessary to connect mechanically to the shafts, are comparatively small and take up only a small space so that they can be placed in small isolated compartments which will not menace the ship in case of flooding; since no main auxiliaries are required for the motors, the flooding of a motor room will not entail any loss in that respect. Also, the motors may be placed very much farther aft than can steam-driven turbines and therefore the length of the main shafting can be very materially reduced. This constitutes a big advanatge; both on account of less liability to derangement of the shafting itself, due to injury to the ship, and also of less danger to the ship itself because of the shafting not having to pierce a number of watertight bulkheads. These advantages of installation constitute the real and main reason for the adoption of electric drive for capital ships and any other advantages are minor compared with them. Utilizing these advantages to the fullest extent makes it possible to build capital ships which are far superior to any others fitted with any other form of machinery. In addition to advantages from the point of view of protection, there are also the advantages from an engineering standpoint. The shorter lengths of shafting make it easier to keep the shafts in line; the grouping of boilers around the machinery makes short and direct steam pipes with a consequent reduction in weight and complication and a smaller drop in steam pressure. The same may be said of practically all the other piping systems of the ship, such as feed lines, oil lines, exhaust lines, etc."

Use of Superheated Steam. — As is well known, superheat greatly improves the efficiency of steam turbines by reducing the friction and windage loss of turbine blades and disks. There is also a considerable gain due to increase in the available energy of the steam incident to increase in temperature. It is generally conceded that the steam consumption is reduced at least 1 per cent for every 12.5° F. of superheat. With some installations there is also a gain in boiler efficiency caused by increased heating surface exposed to the flue gases. Superheated steam also increases the life of the turbine blades and nozzles.

In Europe superheat for ships has been used for some years and great saving in fuel has been realized. In 1912, Mr. Ljungstrom in Sweden equipped the "Mjolner," a small coastwise vessel with electric drive. This ship is only 225 feet long, 56 feet beam, and 15 feet draught, requiring 950 SHP. The

steam conditions are 218 pounds pressure, 200° F. superheat. This ship uses 42.3 per cent less fuel than a sister-ship, the "Mimer," which is equipped with as triple-expansion engine. Since that date larger cargo ships have been equipped in a similar manner, using still higher degrees of superheat.

Superheat has also been used successfully for a number of years in Europe with engine-driven ships. The following extract from a letter from Van Nievelt, Goudriaan & Co., Rotterdam, Holland, illustrates the superheat possibilities in engine-driven ships:

"We are using during the last five years, in our multi-tubular boilers 3½-inch tubes, very high funnels and Diamond blowers, and have no trouble at all in getting sufficient steam. We have practically no leakage at the connections of the pipes and boxes. The original pipes are still in use. The capacity of a 20-ton evaporator is sufficient for supplying feed water.

"Three of our steamers have been running half a year without superheaters with a coal consumption of 24 to 25 tons. After fitting superheaters the consumption was about 22 tons, making a saving of at least 10 per cent."

In America superheat has, for a number of years, been extensively used in central stations but has not been applied to any extent on ships.

Superheat and Reversing Turbines. — Whereas high superheat is beneficial to the operation and life of turbines, it has caused serious trouble in reversing turbines due to expansion strains set up by rapid changes in temperature, and reversing turbines are necessary when mechanical connections between turbine and propeller are adopted. In the case of electrically driven ships, no reversing turbine is needed, the turbine-generator unit revolving continuously in the one direction regardless of the direction of rotation of the propellers.

When steam expands without doing work, its temperature remains nearly constant and, if work is done upon it by a reversed turbine, this temperature is considerably increased. With superheated steam these temperatures may be extreme and might be injurious to the main turbine as well as the reverse turbine, since it also must be used for reversal of direction and since the two often occupy the same casing.

Fig. 4 illustrates the possible steam temperatures which may be reached when maneuvering a ship equipped with reversing turbines. The turbine in this test was first run at full speed in the extremely high vacuum given on the record until a constant temperature was shown by the pyrometer. A little steam was then admitted and a constant temperature was again reached at 825° F. when more steam was admitted and the temperature then rose so quickly that the steam had to be shut off. In the General Electric Company's shops it has been discovered that the reversing wheels of marine turbines turn blue with heat when operated at normal speed in a vacuum of 20 inches.

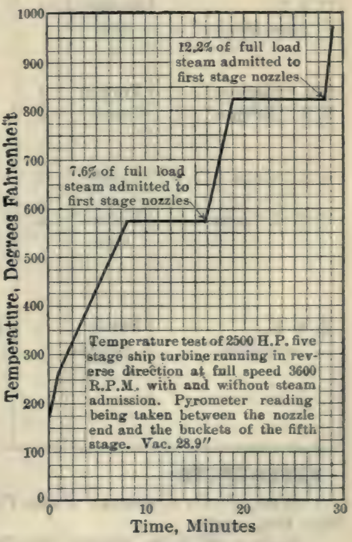

Fig. 4

When high superheat is used in combination with a reversing turbine, precautions can be adopted to preserve the turbine from damage. Various arrangements have been proposed. One is to reduce the steam temperature before it enters the reversing turbine by injecting a spray of water, or by passing the steam through a cooler. Another method is to install a steam pipe connected between the boiler drum and the superheater — this also requires special maneuvering valves. Precautions must also be taken to prevent the superheater from burning during maneuvering operations. These complications and dangers are, however, entirely eliminated by the adoption of electric drive which abolishes the astern turbine.

TYPICAL ELECTRICALLY PROPELLED SHIPS. — The following is a list of some of the more important types of electrically propelled ships:

Name	Type	No. of screws	Total S.H.P. at full speed	Type of motors	No. of motors	Reference
"Graeme Stewart" and "Joseph Medill"	Fire-boats	2	500	Direct current	2	1
U. S. S. "Jupiter"	Collier	2	5,600	Induction with external resistors	2	2
The "Mjolner"	Cargo	1	900	Induction with external resistors	2	3
U. S. S. "New Mexico"	Battleship	4	31,197	Induction double-squirrel cage type	4	4
The "Mariner"	Trawler	1	400	Direct current	1	5
S. S. "Eclipse"	Cargo	1	3,000	Induction with external resistors	1	None to date
S. S. "Cuba"	Cargo	1	3,000	Synchronous	1	None to date
U. S. S. "Iowa"	Battleship	4	60,000	Induction with high and low resistance Rotor windings	4	None to date
U. S. S. "Lexington"	Battle Cruiser	4	180,000	Induction with high and low resistance Rotor windings	8	6

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SHOCK, ELECTRIC. — The physiological effects of a shock by lightning or by artificially-produced electricity may be classified into two groups: the major effects, cessation of respiration or heart action and the minor effects, fractures and internal injuries due to falls, and burns due to contact with electric arcs.

Removal of Victim from Circuit. — If a switch or circuit breaker in the circuit is close at hand, open the circuit at once or cut away the conductors with a wooden-handled hatchet. If the circuit cannot be opened immediately, break the contact of the victim with the conductor by moving the conductor or the victim, using any available dry non-conductor such as a board, rope or clothing. *Do not touch the victim with the bare hands under any circumstances unless insulated thoroughly from the ground.*

First Aid for Slight Shock. — If the victim is seen to be breathing, administer heart stimulants such as alcohol, ether or ammonia either by the mouth or hyperdermically, produce external warmth by rubbing the body or by application of hot substance, loosen constricting clothing about the neck and chest and present aromatic spirits to the nostrils to excite consciousness. Give plenty of fresh air.

Artificial Respiration. — If the victim does not breathe, send for the nearest doctor, remove any foreign body (tobacco, gum, false teeth, etc.) from his mouth and throat and begin artificial respiration at once, even if the victim appears to be dead. Proceed as follows: *

(*a*) Lay the subject on his belly, with arms extended as straightforward as possible and with face to one side, so that nose and mouth are free for breathing; see Fig. 1. Let an assistant draw forward the subject's tongue.

Fig. 1.

(*b*) Kneel, straddling the subject's thighs and facing his head: rest the palms of your hands on the loins (on the muscles of the small of the back), with fingers spread over the lowest ribs, as in Fig. 1.

(*c*) With arms held straight swing forward slowly so that the weight of your body is gradually but not violently brought to bear upon the subject, see Fig. 2. This act should take from two to three seconds. Immediately swing backward so as to remove the pressure, thus returning to the position shown in Fig. 1.

* Report of Commission on Resuscitation from Electric Shock issued by National Electric Light Association.

(*d*) Repeat deliberately twelve to fifteen times a minute the swinging forward and back — a complete respiration in four or five seconds.

(*e*) As soon as this artificial respiration has been started, and while it is being

Fig. 2.

continued, an assistant should loosen any tight clothing about the subject's neck, chest or waist.

(*f*) *Continue the artificial respiration, if necessary at least an hour, without interruption*, until natural breathing is restored, or until a physician arrives. If natural breathing stops after being restored, use artificial respiration again.

(*g*) Do not give any liquid by mouth until the subject is fully conscious.

Other Methods of Inducing Respiration are: (1) withdraw the tongue and allow it to recede periodically, the whole manipulation occurring about fifteen times in a minute; (2) dash cold water upon the face; (3) rub spine with ice; (4) massage chest in region of heart; and (5) administer oxygen gas (obtained at drug stores) by placing cone over mouth and nose.

After Treatment. — After the victim breathes again, treatment may be administered as outlined above in the case of a slight shock. If any bones have been fractured or if the victim appears to have received internal injuries, do not move the victim any more than is necessary and prepare for removal to the nearest hospital.

Treatment of Burns and Blisters. — If the victim has received burns, raw or blistered surfaces should be protected from the air. Blisters should not be opened. Cut around any clothing that sticks and saturate the adhering cloth or cotton dressing with picric acid (0.5 per cent) or a solution of baking soda (one teaspoonful to a pint of water). Wounds may be coated with a paste of flour and water or may be protected with machine, transformer, linseed or olive oil or vaseline. The dressing should be covered with cotton, gauze, lint, clean waste or clean handkerchiefs held tightly in place by bandages. Oil should not be applied to dry charred burns, a light, dry bandage being preferable.

Indication of Death. — Efforts to revive the victim should not cease until death is indicated by the following appearances:* "complete cessation of breathing and heart action, eyelids half-closed and pupils dilated, jaws clenched, tongue appearing between teeth with frothy mucus about mouth and nostrils, fingers semi-contracted with increasing coldness and pallor of surface."

* J. A. Austin, M.D., *Manual of First Aid.*

SHOVELS, ELECTRICAL OPERATION OF. — (*See also Cranes; Motors, Industrial Applications of.*) The operating cycle of an electrically-operated shovel is about 20 seconds, the component times being: hoist 8 to 10 seconds, thrust 10 seconds and swing 10 to 12 seconds, the thrust being in operation at the same time as the hoist and swing are operating. The motors to meet these requirements must have a sufficiently low armature inertia to permit of rapid acceleration under small power, and are therefore generally of the crane or mill-type construction.

Hoist Motors. — In the case of the hoist, considerable advantage may be gained in this respect by using two motors of one-half the capacity each instead of one motor of the full capacity, as the power required for accelerating is much less. For example: Assume a shovel that requires a 225-horse-power, 514 r.p.m. motor for the hoisting operation. Such a motor requires 144 horse-power seconds for bringing up to full speed, whereas if two 115-horse-power, 600 r.p.m. motors are substituted in its place, each motor requiring 63.5 horse-power seconds or both 127 horse-power seconds to bring up to speed, there would be a saving of about 12 per cent in the power required to accelerate the motor alone. On the other hand, such an arrangement may require the use of bevel gears and additional shafting.

Swing Motor. — This motor, although not subject to the severe overloads and shocks encountered on the hoist motors, is subject to frequent reversals, and, as rapid acceleration is required, a motor of similar design as for the hoist should be used.

Thrust Motor. — This motor differs somewhat in its operation in that it is practically stalled during the digging operation, although it may revolve or overhaul according to conditions, and is operated at full speed only after the hoist operation is completed. Its duty is to keep the dipper against the bank, and it must therefore stand still and exert torque most of the time. For this reason its design should be very rugged, and the motor should be able to develop a heavy torque for short intervals of time while standing still, or rotating very slowly.

Location of Motors. — The hoist and swinging motors are as a rule located on the car and are geared to the drums through suitable reducing gears, while the thrust motor as a rule is mounted directly on the boom and communicates its motion to the bucket staff through reducing gears connected to a pinion engaging a rack on the staff.

Type and Size of Motors. — It is possible to obtain successful operation with a-c. induction motors but it can only be done at the expense of larger capacities and increased power consumption, owing to the characteristic of the motor, which radically differs from that of the steam engine ordinarily used on shovels and from the d-c. series motor.

A careful analysis of the typical steam-shovel engine reveals a characteristics which is not unlike that of the series direct-current motor in that it speeds up under light loads, slows down under heavy loads and possesses a certain elasticity in operation which minimizes mechanical strains on the shovel. Because of these facts it is not only desirable but advisable to adhere to the direct current series motor on the electric shovel. Practically all large power supplies are alternating current, but the smaller capacities in d-c. series motors permit the use of a synchronous motor generator set mounted on the shovel at practically the same cost as the a-c. induction motors with transformers, with the advantages in operating characteristics and power consumption all favorable to the direct-current equipment.

The most recent development in electric shovels has been in connection

with large strippers, which are now being built with differential wound generators and shunt-field control.

The approximate size of motors required for the various operations of a number of different size shovels may be obtained from the following table.

Weight of shovel in tons	Size of dipper in cubic yards	Average cycles of operation per minute	Horsepower of motors			Kw.-hr. per shift
			Hoist	Swing	Crowd	
135	6	2	210	46	40	890
120	5	2	210	46	40	790
105	4	2	150	46	40	725
95	3¼	2¼	150	37	30	580
80	2½	2½	86	27.5	30	482
65	2	2¾	86	27.5	20	381
45	1½	3	55	15	13	292
25	¾	3½	55	9	9	193

Control Equipment. — The magnetic contactor control is preferable for all the motors on the shovel. (*See Controllers.*) The hoist and thrust motors are then provided with a notching-back arrangement which will automatically cut resistance into the circuit when the current exceeds a certain limiting value due to the motors becoming stalled by digging too deep or striking rocks or other obstructions, thus protecting the motors from damage. Sometimes the hoist and swing motors are provided with automatic magnet control while the thrust motor is hand controlled, an ordinary drum controller being provided for this motor.

Brakes for the different shovel motions are generally of the air-brake type, a small air compressor being provided on the shovel for this purpose.

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SHUNTS. — (*See also Ammeters; Galvanometers; Voltmeters; Wattmeters.*) When heavy currents are to be measured it is impracticable in many cases to pass the entire current through the coils of the instrument proper. In such cases it is necessary therefore to pass the bulk of the current through a parallel circuit or shunt and measure only a known fraction of the total current.

CONSTRUCTION. — For laboratory work standard resistance units (*see Resistors, Standard*) are commonly employed as shunts when a high degree of precision is required. Special forms of resistance boxes, usually called shunt boxes, are also used. For switchboard work shunts made of sheets of manganin or other high-resistance metal of low temperature coefficient (*see Wires, Resistance*) are usually employed. Several sheets of metal are used in order to provide a large radiating surface. These sheets are brazed into heavy copper terminals for clamping to the bus-bars or other heavy conductors; heavy terminals are necessary to insure a uniform distribution of current in the sheets. The contact surfaces must be kept bright and the bolts tight.

THEORY OF SHUNTS. — Fig. 1 shows diagrammatically a simple shunt such as used with an ammeter or galvanometer. The galvanometer (or ammeter) current is

$$I_G = \frac{S}{G+S} \cdot I_B,$$

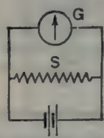

Fig. 1. Galvanometer and Shunt

where G and S are the resistances of galvanometer and shunt respectively and I_B is the total current from the battery or other source.

For example, if the galvanometer has a resistance of 9900 ohms and the shunt a resistance of 100 ohms, the galvanometer current will be 1/100 of the battery current. Shunt boxes made on this principle are usually provided with resistance coils giving a ratio of currents of 1/10, 1/100 and 1/1000 when used with a galvanometer having a given resistance. The corresponding readings of the ammeter or galvanometer, multiplied by 10, 100, 1000, or 10,000 will then give the current in the battery circuit.

"Universal" Galvanometer Shunts. — Simple shunts such as above described give even ratios of currents only when used with a galvanometer of a certain definite resistance. A set of coils arranged as shown in Fig. 2, when used as described below, will give even ratios when used with a galvanometer of any resistance. A shunt of this kind is known as a universal shunt. The Ayrton and Mather shunt is arranged on this principle.

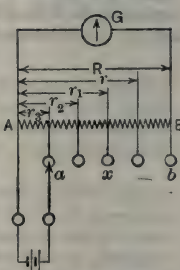

Fig. 2. Diagram of Ayrton and Mather Universal Shunt

A study of the circuits will show that if the galvanometer is calibrated (*see Galvanometers*) with the terminal a at b, that is, with all the resistance R in parallel with the galvanometer, the same calibration may be used for a connected to any other terminal x, provided the current values from the calibration curve are multiplied by the ratio R/r_x, where r_x is the resistance in the box between that terminal and A. The various resistances are usually arranged so that these ratios or "multiplying powers" of 10, 100, 1000, and 10,000 may be obtained. The galvanometer should have a relatively low resistance compared with the total resistance of the shunt.

Effect of Temperature Changes. — The resistivity temperature coefficient of a shunt made of manganin is very much lower than the temperature coefficient of the copper circuit of the ammeter or galvanometer. Consequently changes

in temperature will produce a change in the relative resistances of the shunt and ammeter or galvanometer, thus changing the multiplying power. For accurate measurements a correction should therefore be made in the nominal multiplying power of the shunt. With a universal shunt and a relatively low-resistance galvanometer the effect of temperature changes is less marked, since the resistances r_1 and R are both in the same box and are usually made of manganin or other metal having a low temperature coefficient.

COSTS (Pre-war prices).—A 4-value laboratory shunt box for use with an ordinary galvanometer and giving multiplying ratios of 10, 100, 1000 and 10,000 costs about $20. Switchboard shunts cost from $3 to $90 depending upon their current-carrying capacity, the lower figure applying to a 10-ampere shunt and the higher figure to a 10,000-ampere shunt.

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SIGNALING, RAILWAY. — Railway signaling is the art of conveying information to the person or persons in immediate charge of the movement of a train. The means for conveying the information are various, including simple movements of the hands and arms, positions or combinations of positions of semaphore arms by day, or colored lights by night, positions or combinations of positions of rows of lights visible both day and night, and colored lights or combinations of colored lights visible both day and night.

CARDINAL PRINCIPLE OF RAILWAY SIGNALING. — The cardinal principle of design of all signal circuits, apparatus and systems, is that any failure of any part, such as breaks, open or short circuits, grounds, etc., if having any effect on the signal whatever, shall result in a stop indication.

CLASSIFICATION OF SIGNALING SYSTEMS. — The two main divisions of railway signaling are block signaling and interlocking signaling. The former has to do with keeping trains which are running on the same track properly spaced. The latter has to do with the handling of trains over tracks which intersect at points of crossing or divergence, and has for its object the prevention of conflicting movements, the proper routing of trains, and the insurance that the movable parts of the track are in their right positions before the signals governing movement over them can be made to give a proceed indication.

Block signals may be divided into two main classes: non-automatic and automatic. In the United States there are 76,466 miles of track protected by the former as against 60,992 miles of track protected by the latter. Of the non-automatic blocks, those on 72,952 miles of track are operated by telegraph or telephone.*

NON-AUTOMATIC BLOCK SIGNALING. — Systems in this class are of two kinds, non-controlled and controlled.

Non-controlled Manually Operated System. — The manually-operated signals used in connection with the telephone or telegraph blocks are located at passenger stations, junctions or other convenient points where operators are available. They are put in the proceed position to permit a train to enter the next block provided information has been received by the operator from the next station in the direction of traffic that the preceding train has passed out of the block. They are placed in the stop position on the passage of the train.

The operators have blanks on which they enter the designation numbers of each train they admit to a block, together with the time of entrance. On the same sheet is checked off the departure of the train from the block as advised by the operator at the leaving end of the block. The train despatcher, located at some central point, is also kept informed of all train movements, so that in case the schedule is deranged for any cause he can give the necessary orders to expedite traffic, such as changing orders for passing points and giving superior rights to certain trains.

Defects of Non-controlled Manually Operated System. — The defects of the system are obvious, as misunderstandings may arise between operators, trains may be checked off by mistake as having left a block when such is not the case, or trains may part and the fact not be noted by the block operator because of his failing to note the absence of the tail lights carried by the rear car of a train when the first division passes his station.

Controlled Manually Operated System. — These defects are partially overcome by the use of controlled manual block signals. In such systems electric locks are applied to the levers operating the manual signals. The locks are included in circuits running between block stations, and so arranged that when an operator wishes to place a signal in the proceed position he has

* From tables compiled by the Interstate Commerce Commission, January 1, 1920.

first to ask (by bell signal or otherwise) for an unlock from the next station in the direction of traffic, and coöperate with the operator at that station in the proper manipulation of the circuits to get his unlock. A further check on the operators is sometimes obtained by the use of track circuits at the stations to effect a certain degree of control of the apparatus locking the signal levers by the passage of the trains themselves. These systems, however, are all inferior, in the degree of protection given, to the automatic blocks with signals controlled by continuous track circuits, which are described below.

Staff System. — Another form of controlled manual block signal is the staff system in which the possession of a small metal cylinder, or "staff," gives the engineer permission to run through a block. These staffs are normally in one or the other of a pair of instruments called staff instruments, one of a pair being at each end of a block. Only one staff can be taken from a pair of instruments at a time because of their locking features, controlled by circuits between the instruments, requiring the coöperation of two people, one at each instrument, to abstract a staff. Until this staff is replaced in one or the other instrument, no other staff can be withdrawn. Following movements with more than one train in the block can be accomplished by using a divisible staff, made in several sections which must be screwed together in a specified order to permit the insertion of the staff in an instrument when the last train has brought the remainder of the staff through the block.

There is also an automatic staff system in which the coöperation of the second person is not required.

There are about 511 miles of track equipped with the staff system in the United States.*

AUTOMATIC BLOCK SIGNALING. — This subject will be treated under three main headings, in the order stated: Track Circuit, Signals and Their Mechanisms, and Location of Block Signals.

TRACK CIRCUIT FOR AUTOMATIC BLOCK SIGNALS. — The standard means of control of automatic block signals is the track circuit. It is the safest means known because the control is continuous and reliable.

Steam-road Track Circuit (Fig. 1). — The figure shows the elements of a steam-road track circuit. They are a source of electrical energy, means for limiting the flow of current from the source, a section of the rails of a track

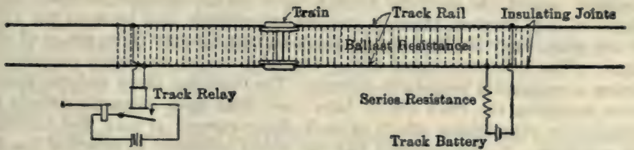

Fig. 1. Elements of a Steam-road Track Circuit

insulated by special rail joints from adjoining rails, the leakage path constituted by the ties and ballast, and an electromagnetic type of relay. The contacts of the relay open and close the circuit which effects the operation of the signal. When there is no train on the track circuit, the contacts of the relay are closed due to the energizing of the relay by current flowing over the rails, which act as conductors from the source of electrical energy. The signal is made to indicate the absence of a train on the track circuit under these conditions. The

* From tables compiled by the Interstate Commerce Commission, January 1, 1920.

presence of a train on the track circuit, or any other cause depriving the relay of energy, causes the relay to open its contacts, which results in the signal taking the stop position. The use of means for limiting the flow of current is to prevent exceeding the capacity of the source when a train shunts the rails, and at the same time to cause such drop of potential across the rails as will insure the opening of the relay.

Electric-road Track Circuit. — The track circuit for a road using electric propulsion, if a direct-current track relay is used, must, in order to avoid false clear signals, have the voltage point at which the relay closes its contacts higher than any voltage which may exist across its terminals, with a train on the track circuit, due to the flow of the propulsion current through the rails. Standard practice, to avoid the possibility of such false closing of the relay contacts, is the use of an alternating-current relay which is not operative to close its contacts under the influence of the propulsion current.

Single-rail Track Circuit (Fig. 2). — Fig. 2 shows an arrangement for d-c. propulsion roads where only one rail is used as the main return for the

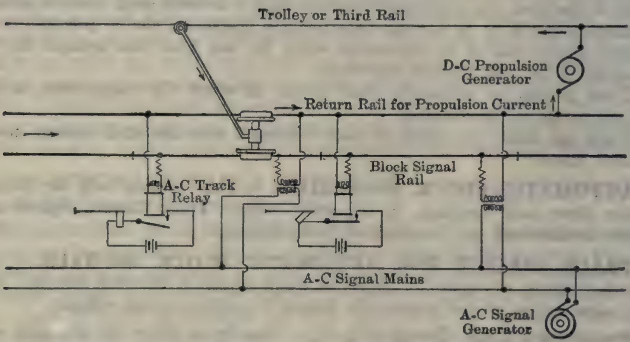

Fig. 2. A-C. Track Circuit, Single-rail System, D-C. Propulsion

propulsion current, the other rail being given up to the signaling current. The resistances used between the rails and the transformer, and between the rails and the relay, limit the flow of propulsion current through those pieces of apparatus. The inductive shunt across the relay serves the same purpose in forming a by-pass for propulsion current. The inductive shunt and the transformer supplying the track circuit have cores with open magnetic circuits to minimize the magnetizing effects of such direct current as may flow through their windings. An arrangement like the above is limited in practical use to those situations where the drop in voltage due to the propulsion current, along the length of continuous rail opposite any section of block rail, is under 50 volts. It finds its greatest application in subway and terminal or interlocking work. Where the number of tracks and shortness of track circuits bring the maximum voltage across the terminals of the relay, due to propulsion current, down to about 5 volts, a relay may be used with resistance enough in its winding to render unnecessary the use of the inductive shunt and resistance between it and the rails.

With a signaling current of higher frequency than the propulsion current and a selective relay, the single-rail track circuit is applicable to a-c. propulsion roads.

Double-rail Track Circuit (Fig. 3). — Where the conductivity of the return propulsion circuit is not sufficient to permit giving up one rail to the signaling current, the arrangement shown in Fig. 3 is used. Both rails are sectioned by insulating joints and the return propulsion current is carried around the insulating joints by means of impedance bonds (i.e., coils having a high inductance and low resistance) which are joined electrically at the middle points of their windings. The flow of the propulsion current is opposed

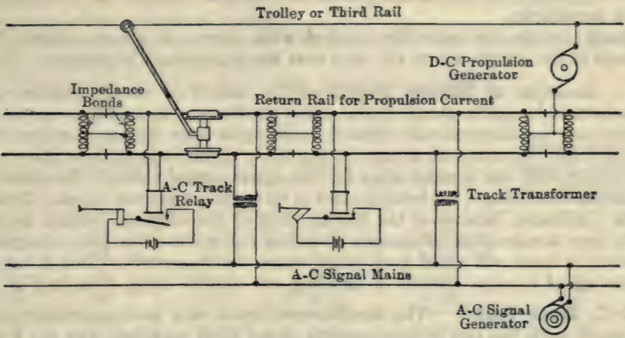

Fig. 3. A-C. Track Circuits, Double-rail System, D-C. Propulsion

only by the ohmic resistance of the bonds and their connections. The full impedance is, however, offered to the flow of the alternating signaling current from rail to rail, and it is, therefore, possible to maintain a difference in potential across the rails sufficient to operate the track relay.

Where the propulsion current is alternating, a higher frequency is used for the track circuit and the relay made correspondingly selective. 25-cycle current is ordinarily used for the signals on direct-current propulsion roads and 60-cycle current on alternating-current propulsion roads.

Source of Energy for Track Circuit. — The source of electrical energy for supplying the track circuit is generally in steam-road practice a primary battery of the caustic potash type with an external resistance. Generally three cells are used in multiple, but where the leakage between rails is high, larger numbers are used in series-multiple combinations. The storage battery is used to a limited extent and is especially advantageous on two- or four-track systems where the number of track circuits and amount of apparatus per unit distance is relatively large.

Use of Alternating Current. — Alternating current is rapdily coming into use for signaling on steam roads. Where alternating current is used for track circuits, it is almost universally used to operate the signals and supply the lamps for the night indications. 25- or 60-cycle current is used, the former generally resulting in slightly lower power consumption and being generally favored where the current has to be generated for the signalling system alone and no commercial emergency sources are available. Where such commercial emergency sources *are* available, as is generally the case in thickly-settled districts, the frequency of such sources being ordinarily 60 cycles, that frequency is often chosen for the signaling system.

Transformers for Track Circuits. — The source of alternating current for each block is either a low-voltage (5 to 20 volts) secondary of a transformer whose primary is connected to the transmission line, or a similar secondary

of a small special air-cooled transformer with a 110-volt primary. In the latter case the special transformer may be housed with other signal apparatus and its primary supplied from the 110-volt wires brought from the commercial transformer, whose primary is connected to the transmission line. The same 110-volt wires may be used also to supply the motors, line relays and "hold clear" devices, and carbon lamps used for lighting the signals. If Tungsten lamps are used, they may be supplied from suitable taps on the 110-volt primary of the special transformer or from taps on the low-voltage track secondaries.

Track circuits are generally supplied with current at about one-half the voltage of the source, due to the drop over the intermediate impedance.

Location of Energy Supply. — The source of energy (battery or transformer) is usually placed at the end of the block. If, however, the block is long and impedance bonds are used, resulting in large leakage from one rail to the other, the source of energy for the track circuit is sometimes placed at the center, and the control wires for the signal carried through the contacts of the two track relays, one being located at each end of the track circuit. Under these conditions, because of the voltage drop of the signaling current in the rails, there is only a limited distance extending either side of the point of supply in which a train will shunt both track relays simultaneously. This is taken advantage of in some systems of signaling, an example being given in Fig. 7.

D-C. Track Relay. — The direct-current track relay most commonly used on steam roads has two or four contacts closed when energized, and has a resistance of four ohms. The working voltage across its terminals is generally 0.4 or 0.5 volt under good weather conditions, but may be as low as 0.35 volt during wet weather.

A-C. Track Relay. — There are two types of a-c. track relays called, respectively, "single-energizing-circuit type" and "two-energizing circuit type," according to whether they take all of the energy to operate them from the track, or part from the track and part from a local source. These types are built on either one or both of two distinct principles: (1) on the principle of the electrodynamometer (q.v.) and (2) on the principle of the induction motor; (see *Motors, Polyphase Induction* and *Watthour Meters*).

The induction-motor type has the advantage over the electrodynamometer type, in that, its angular motion being unrestricted, a given pressure at the contact points can be produced with a less expenditure of energy. There is a limitation to this, however, due to the friction of the reduction gearing, which must not be so great that it cannot be overcome by the counterweight provided to open the contacts when the relay is deënergized.

With all types of railway signal relays, the addition of "back" contacts to be closed when the relay is deënergized, means the expenditure of more energy to satisfactorily close the "front" contacts, because of the extra counterweighting necessary to develop the necessary pressure on the back contacts.

Single-energizing-circuit Type of Track Relay. — Relays of this type are made on the induction motor principle in order to make them selective between the signal and propulsion currents. The following forms of construction are typical:

(1) Segment of sheet aluminum set in motion by the shifting magnetic field due to the "shaded" pole pieces of a laminated "C"-shaped core which carries the winding. This form is very largely used on short track circuits on d-c. propulsion roads.

(2) Double segment of sheet aluminum acted upon differentially on either side of its axis by the two sets of "shaded" poles of a double "C"-shaped

core. The core is so designed that one or the other set of poles is stronger, depending on the respective amounts of propulsion or signaling current traversing the winding. This form is used on a-c. propulsion roads, being made selective between 25 and 60 cycles, the latter frequency being used for the signaling current.

(3) Centrifuge driven by split-phase induction motor. This form is used on a-c. propulsion roads. It is selective by virtue of the centrifuge being designed not to close the contacts when driven at a speed corresponding to the lower frequency of the propulsion current.

(4) Split-phase induction motor with gearing or link connection from rotor shaft to contact supporting member.

Typical power figures at the working points are as follows:

Form	Frequency	Volt-amperes	Volts *	Power factor	Number of contacts closed when energized
(1)	25	5.0	3.0	0.55	4
......	60	8.5	5.0	0.55	4
(2)	60	20.0	3.1	0.55	2
(3)	60	6.0	2.3	0.70	10
(4)	25 } 60 }	10.0	5.0	0.70	6

* Track windings of relays.

Two-energizing-circuit Type of Track Relay. — The following forms of construction are typical: (1) Electrodynamometer; used on d-c. propulsion roads and to some extent on steam roads. (2) Induction motor with gearing; the rotor is a cylindrical shell of copper or aluminum with a stationary laminated core; used on d-c. propulsion roads and very largely on steam roads. (3) Centrifuge driven by induction motor; used on a-c. propulsion roads; see also preceding section.

The local windings of all two-energizing-circuit relays are generally supplied at from 10 to 110 volts. The objection to the higher voltage is the small size of wire and consequently greater liability to damage by lightning.

The following typical power figures are for "two-position" relays, i.e., relays

Form	Frequency	Track			Local		Number of contacts closed when energized
		Volt-amperes	Power factor	Volts	Volt-amperes	Power factor	
(1)	25	0.70	0.90	0.35	36	0.70	4
......	60	1.60	0.80	1.60	33	0.34	4
(2)	25	0.038	0.65	0.15	2.4	0.70	10
......	60	0.024	0.65	0.15	11.0	0.70	10
(3)	60	1.30	0.55	0.3	20	0.95	6
(4)	25	0.13	0.80	0.35	45	0.35	6
......	60	0.16	1.00	0.4	44	0.25	6

with one energized and one deënergized position. Relays which are caused to assume two energized positions by reversals of the current in one of the windings are termed "three-position" relays and generally require twice as much power in one of the windings as two-position relays.

The adjustment of the phase relations of the currents in two-energizing circuit relays has to be carefully cared-for in the choice of the impedance used between transformers and rails, and between local windings and the source of energy supply.

Single- Versus Two-circuit Types. — As a track circuit is, from an engineering point of view, nothing but a very inefficient transmission circuit,* track circuits above a certain length can be worked most economically with a two-energizing-circuit type of relay, as the amount of energy transmitted over the rails can be reduced to a certain point by increasing the amount of energy supplied economically to the second winding from a local source. For track circuits below a certain length, however, the energy supplied to the track circuit for the single-energizing-circuit type of relay is less than the total supplied to the track circuit and the local winding of the two-energizing-circuit type.

The above consideration, taken in conjunction with the fact that a relay with one energizing circuit usually is cheaper than one with two energizing circuits, and does not require the installation of a source of energy supply for a local winding, generally points to the choice of the single-energizing-circuit type for track circuits less than 1500 feet in length. On the other hand, as the single-energizing-circuit type of relay may be falsely energized by current from an adjacent track circuit, due to broken-down insulating joints, it does not give the same degree of protection against this failure as the two-energizing-circuit type, with the source of current supply to adjacent track circuits of opposite instantaneous polarity.

Impedance Bonds. — Impedance bonds are wound with strap copper varying from 57,000 cir. mils for 22,000-volt, 25-cycle propulsion to 220,000 cir. mils for ordinary d-c. 600-volt interurban work, and 800,000 cir. mils for heavy 600-volt d-c. traction work. The corresponding resistances across rails are respectively about 0.014, 0.0014 and 0.00045 ohm. The respective continuous propulsion current per rail ratings are 50, 500 and 1500 amperes. Typical impedance values from rail to rail are respectively 10 ohms at 2 volts and 16 ohms at 9.2 volts, 60 cycles; and 0.3 ohm at 1.5 volts, and 0.18 ohm at 1.5 volts, 25 cycles.

Though the resistance of the connection around an insulating joint formed by an impedance bond may be relatively high as compared with the resistance of the ordinary rail joint bond, the small number of impedance bonds in a system compared to the number of rail joint bonds results in but a very slight percentage increase in the total resistance of the return propulsion circuits.

The impedance bonds for direct-current propulsion roads are provided with an air gap in the magnetic circuit to limit the change in impedance due to un-

* The inefficiency of the track circuit as a transmission line is due to the drop through the current limiting means between the source of energy and the rails (and in the single-rail system between the relay and the rails), the high reactive drop in the rails, the distributed leakage of ballast and ties, and, in the two-rail system, the concentrated leakage through the impedance bonds. The voltage at which the track winding of a relay is designed to operate is selected first, to obtain safe and reliable operation, and second, to obtain as high a degree of efficiency of the track circuit as is permitted by the characteristics of the track circuit. This generally results, with relays taking a comparatively large amount of energy from the track, in higher voltages for track windings of relays used on steam roads than for those used on electric roads, though the total energy put into the track circuit is less in the former than in the latter case.

IMPEDANCE OF BONDED RAILS TO SIGNAL CURRENTS

Ohms per 1000 feet of track

Weight of rail, lb. per yard	Bonding*	27.5-ft. rails				30-ft. rails				33-ft. rails			
		25~		60~		25~		60~		25~		60~	
		z	P.F.	z	P.F.	z	P.F.	z	P.F.	z	P.F.	z	P.F.
100	To capacity	0.10	0.40	0.25	0.40	0.10	0.40	0.25	0.40	0.10	0.40	0.25	0.40
	2 No. 6 copper	0.13	0.72	0.28	0.56	0.13	0.70	0.28	0.56	0.13	0.69	0.27	0.54
	1 No. 8 iron, 1 No. 6 copper	0.17	0.83	0.30	0.65	0.16	0.82	0.30	0.63	0.15	0.79	0.29	0.62
	2 No. 6 c.c.—40%	0.19	0.87	0.32	0.69	0.19	0.86	0.32	0.69	0.17	0.84	0.31	0.68
	2 No. 6 c.c.—30%	0.25	0.91	0.36	0.75	0.22	0.91	0.35	0.74	0.20	0.88	0.34	0.73
	2 No. 8 iron	0.40	0.97	0.50	0.88	0.36	0.96	0.47	0.87	0.34	0.96	0.44	0.85
90	To capacity	0.10	0.43	0.26	0.43	0.10	0.43	0.26	0.43	0.10	0.43	0.26	0.43
	2 No. 6 copper	0.14	0.73	0.29	0.58	0.13	0.72	0.28	0.58	0.13	0.70	0.27	0.54
	1 No. 8 iron, 1 No. 6 copper	0.17	0.83	0.31	0.67	0.16	0.82	0.31	0.64	0.16	0.80	0.29	0.62
	2 No. 6 c.c.—40%	0.19	0.87	0.33	0.71	0.19	0.87	0.33	0.70	0.17	0.84	0.31	0.68
	2 No. 6 c.c.—30%	0.23	0.91	0.36	0.76	0.26	0.91	0.36	0.76	0.20	0.89	0.34	0.73
	2 No. 8 iron	0.40	0.97	0.51	0.89	0.37	0.97	0.48	0.88	0.35	0.96	0.45	0.86
85	To capacity	0.10	0.46	0.26	0.46	0.10	0.46	0.26	0.46	0.10	0.46	0.26	0.46
	2 No. 6 copper	0.14	0.74	0.29	0.60	0.13	0.73	0.29	0.59	0.13	0.71	0.28	0.58
	1 No. 8 iron, 1 No. 6 copper	0.17	0.84	0.32	0.68	0.17	0.83	0.31	0.67	0.16	0.81	0.30	0.65
	2 No. 6 c.c.—40%	0.19	0.88	0.33	0.72	0.19	0.87	0.33	0.69	0.18	0.85	0.32	0.70
	2 No. 6 c.c.—30%	0.23	0.91	0.37	0.77	0.23	0.91	0.36	0.77	0.21	0.89	0.35	0.76
	2 No. 8 iron	0.41	0.97	0.52	0.89	0.37	0.97	0.49	0.88	0.35	0.96	0.46	0.84
80	To capacity	0.11	0.48	0.26	0.48	0.10	0.48	0.26	0.48	0.11	0.48	0.26	0.48
	2 No. 6 copper	0.14	0.75	0.29	0.62	0.14	0.73	0.29	0.60	0.13	0.72	0.29	0.60
	1 No. 8 iron, 1 No. 6 copper	0.17	0.84	0.32	0.69	0.17	0.84	0.31	0.68	0.16	0.82	0.31	0.67
	2 No. 6 c.c.—40%	0.20	0.88	0.34	0.73	0.20	0.88	0.34	0.73	0.18	0.85	0.33	0.71
	2 No. 6 c.c.—30%	0.23	0.91	0.38	0.78	0.23	0.91	[illegible]	0.78	0.21	0.89	0.36	0.76
	2 No. 8 iron	0.41	0.97	0.53	0.89	0.37	0.97	0.49	0.88	0.35	0.96	0.47	0.87
70	To capacity	0.11	0.52	0.27	0.52	0.11	0.52	0.27	0.52	0.11	0.52	0.27	0.52
	2 No. 6 copper	0.15	0.77	0.30	0.65	0.14	0.76	0.30	0.65	0.14	0.75	0.30	0.64
	1 No. 8 iron, 1 No. 6 copper	0.18	0.86	0.33	0.72	0.17	0.85	0.33	0.71	0.17	0.82	0.32	0.70
	2 No. 6 c.c.—40%	0.20	0.89	0.36	0.75	0.20	0.89	0.35	0.75	0.18	0.86	0.34	0.74
	2 No. 6 c.c.—30%	0.24	0.92	0.39	0.80	0.24	0.92	0.38	0.81	0.22	0.90	0.37	0.78
	2 No. 8 iron	0.42	0.97	0.54	0.90	0.38	0.97	0.51	0.89	0.36	0.96	0.48	0.87

* c.c. = copper clad.

RESISTANCE OF BONDS TO SIGNAL CURRENTS

Ohms per 1000 feet of track

Bond wires per joint	27.5-ft. rails	30-ft. rails	33-ft. rails	
2 No. 6 copper	0.057	0.052	0.048	Bond wires 48 in. long. No allowance is made for conductance by the splice bars.
1 No. 6 copper and 1 No. 8 iron	0.098	0.089	0.082	
2 No. 6 copper clad—40%	0.124	0.112	0.103	
2 No. 6 copper clad—30%	0.166	0.150	0.138	
2 No. 8 iron	0.348	0.315	0.291	

equal amounts of propulsion current flowing in the two halves of the winding. Good practice is to provide for satisfactory operation with a maximum difference equal to 20 per cent of twice the rated capacity per rail, e.g., a 1500-ampere per rail bond should operate satisfactorily with a difference of 600 amperes between the currents in the two rails.

Impedance of Track Circuits. — The table on page 1415 gives the impedance per 1000 feet of track under various conditions of bonding in common practice and for the values of current commonly used for the energization of the track elements of relays. Where propulsion current is flowing in the rails, the power factor corresponding to the value of the propulsion current should be used to determine the most adverse conditions of drop of potential of signaling current between the point of supply to the track circuit and the relay.

Resistance of Leakage Path. — The resistance of the leakage path between rails in ohms per thousand feet of track varies with the nature of the ballast, the condition of the ties and the weather conditions. Two ohms per thousand feet is a low wet weather value for track with gravel ballast.

In connection with the calculations involving the values of rail impedance given in the table on p. 1415 the following values for resistance of ballast and ties between rails may be used. They are given for ballast clear of the rails.

	Ohms per 1000 feet of track
Wet gravel	3
Dry gravel	6
Wet broken stone	6
Dry broken stone	16

Transmission-Line Voltages. — Transmission-line (between generator and signal transformers) voltages in standard practice vary from 1100 to 4400 volts inclusive, though in some special cases of subway and elevated road work, or where the mains are carried on existing telegraph poles, they may be lower.

SIGNALS AND THEIR MECHANISMS. — Signal indications for block and interlocking work are given in standard railroad practice by semaphores. The semaphore consists of a wood, or enameled steel, blade or arm, fastened at its inner end to a "spectacle." The spectacle is a casting, or combination casting and metal stamping, embodying the hub for the shaft on which the semaphore rotates, and the colored glass roundels which change position before a lamp fitted with a clear lens, to give the night indications corresponding with the positions of the semaphore. The lamp is provided with a long-time oil burner (usually seven days) or incandescent bulbs.

Weight and Torque of Semaphores. — The weight of the semaphore is

distributed to give a maximum torque (tending to return the semaphore to the stop position) varying with different types of operating mechanisms from about 30 to 100 pound-feet. As the torque curve is necessarily a sine curve, a spectacle moving through 90° is generally designed to have maximum torque at about 55°.

Spectacles operated manually, which can be pulled to the stop position, may be lighter than those which, in returning to the stop position, have to return parts of the mechanism which cause them to assume the proceed position.

In automatic signals gravity is depended on almost universally to return the signal to stop.

Semaphore Positions and Colors of Lights. — The three fundamental signal indications are:

1. Stop,* 2. Proceed with caution, 3. Proceed. The standard method of giving these indications is in the upper right-hand quadrant as indicated in Fig. 4, the respective positions of the semaphore arm being respectively at 0°, 45°, and 90°. The corresponding colors of the night lights differ with different

Fig. 4. Standard Semaphore Positions

roads; a common practice is to use red for stop, green for proceed with caution and white for proceed, but on account of the danger of a white light being falsely displayed because of the breakage of the colored roundels it is considered best practice to use red, yellow and green, for the respective indications enumerated above.

It is also good (but less up to date) practice to give these indications in the lower right-hand quadrant, the horizontal position of the semaphore arm always indicating stop. Where the view to the right of the signal mast is obscured, as by the poles along a trolley road, or where clearances are small, due to high walls close to the right-of-way, as through a city, the semaphore is sometimes operated in the upper left-hand quadrant.

Two-position semaphores, indicating stop and proceed only, are also largely used, the proceed indication generally being given at from 60° to 90° depending on the individual road.

Power Mechanisms. — Power signals are operated by "bottom" or "top post" mechanisms. Electricity, compressed air and gas are used as motive power. Of the track mileage protected by automatic semaphores approximately 90 per cent has electrically-operated mechanisms. Compressed gas and air for automatic blocks are limited in use to very few roads, though these include some of the most important ones. The gas is supplied in tanks. The air is distributed in pipe mains.

Electrically-operated Bottom-post Mechanism. — Electric bottom-post mechanisms generally consist of a motor which, by gearing and levers, transmits the movement necessary to clear the semaphore through a vertical rod extending inside the mast between the mechanism and a crank arm on the

* An automatic signal in the "stop" position may be passed "according to rule" after bringing train to stop. An interlocking signal in the "stop" position may be passed only after bringing train to stop and receiving authority to pass the stop signal from an authorized person.

semaphore shaft. The type most generally used has interposed in the movement transmission system, as near the connection to the vertical rod as possible, an electric latching device called a *slot*. The operation of the semaphore to, and maintenance in, a proceed position depends on this latch being energized, the motor cutting out automatically after performing its work. The location of the *slot* in the movement system close to the vertical rod reduces to a minimum the number of parts moved by the return of the semaphore to the stop position, under the influence of gravity, and, consequently, reduces the possibility of the signal sticking in the proceed position due to the development of any friction.

The control of the circuit supplying the motor and slot is effected by the contacts of the track relay.

Electrically-operated Top-post Mechanism. — In top post mechanisms the motor acts directly on the semaphore shaft through a train of gears. The semaphore in its return to the stop position has to drive the gear train and usually the motor. The ratio of gearing in top-post mechanisms is made comparatively low (about 120 : 1) to diminish friction when going to the stop position, and the motors are consequently very slow speed. The latching of the semaphore in the clear position is accomplished by electromechanical means or by magnetic induction. The latter method obviates the danger of the signal sticking in the proceed position due to the adhesion of contacting surfaces in the latching device.

Power Required for Electrically-operated Signal Mechanisms. — D-c. motors and latching devices are generally supplied with current from local primary or storage batteries at from 8 to 10 volts. With a maximum semaphore torque of between 30 to 50 pound-feet and clearing 90° in about 10 seconds, the motors will consume between 2 and 2½ amperes. The latching device consumes from 0.008 amperes to 0.02 amperes, depending on the voltage and winding resistance.

Under the same conditions as given for d-c. motors, a-c. commutator motors will consume slightly less than 1 ampere at 110 volts at about 0.8 power factor, and induction motors will consume between 2.8 and 3.5 amperes at the same voltage but at about 0.5 power factor.

The electric latching device consumes approximately from 5 to 10 watts in the a-c. mechanisms, the power factor varying widely with the design.

Pneumatically-operated Signal Mechanisms. — The pneumatic mechanism is the simplest of those used for operating signals, consisting of merely a 3-inch by 4-inch cylinder with a metal-packed brass piston, a magnet valve for controlling the admission of air and suitable mechanical connections to the semaphore shaft. The electromagnet valve controlling the admission of air is used as the "slot," as its deënergization shuts off the air and exhausts the cylinder to atmosphere. The pneumatic mechanisms are readily adaptable for bottom- or top-post operation. The air pressure carried varies from 40 to 100 pounds. Ninety pounds is a common pressure.

The advantages of the pneumatic mechanisms are their simplicity, reliability and over-load capacity.

Gas-operated Signal Mechanisms. — The gas-operated mechanisms are similar in principle to those operated by air, but are complicated by the apparatus incidental to the tank supply being at a very much greater pressure than that at which the gas is used in the signal cylinder.

Day-light Signals. — Powerful light signals are sometimes used to give the day, as well as night, indications in place of the semaphore. Notable examples of this are the position light signal installations on the Pennsylvania

system, especially in the neighborhood of Philadelphia, and the installation of color day-light light signals on the Chicago, Milwaukee & St. Paul, to say nothing of numerous installations on electric interurban roads.

These light signals are generally hooded to make them more visible in sunlight. They are provided with concentrating lenses, and sometimes with specially-constructed reflectors arranged so that light entering from the exterior through the lens will not be reflected out again and give a false indication when the lamp behind the lens is not lighted.

The candle-power behind the lens varies from 5 to 120, depending upon the type of lamp, the efficiency of the lenses and reflectors, and the color density of the glass. The indication given is sufficiently powerful even in bright sunlight to permit of the satisfactory operation of the highest speed trains.

LOCATION OF BLOCK SIGNALS. — Automatic block-signal systems have to take care of such varied fundamental conditions as are found, on the one hand, on single-track roads where the distances between passing sidings may be as great as four miles or more, and the traffic relatively infrequent and, on the other hand, on roads of four tracks, or more, where the traffic is dense, the headway short, and trains of both high- and low-speeds have at times to be handled over parts of the same tracks.

Home and Distant Signals. — The signal at the entrance to a block indicating the presence or absence of a train in that block is called the "home signal." In order that a train may be given warning in time to stop at the entrance to an occupied block, a second signal, called the "distant signal," indicating the position of the home signal at the entrance to the block, is located at a point sufficiently distant to enable the runner to act on the indication and bring his train to a standstill with a reasonable factor of safety before passing the stop signal. This distance will vary with the conditions of grade, curvature of track, and character of train equipment and may be from 3500 to 4200 feet or more.

In three-position signaling the semaphore at the beginning of a block is used to give the home signal for that block and the distant signal for the next block ahead. In two-position signaling the position of the home signal is repeated by a semaphore of distinctive shape, displaying a distinctively-colored light at night. This distant signal may, where the blocks are short, be located on the same mast with the preceding home signal. If the blocks are long and the rules of the road require a train to be immediately brought under control when advance information is received that the home signal is at stop, it is best to avoid loss of time by placing the signal giving the advance information on a separate mast at the proper distance from the home signal. This applies also to three-position signaling.

With distant signals it is, therefore, evident that in order to have following trains run continually under clear signals, they must be separated by a distance equal, at least, to the distance run by the preceding train during the time occupied by the clearing of the home and distant signals plus the length of the intervening block plus the distance between the home and distant signals.

The relations stated above give the basis for the theoretical location of automatic block signals to safely obtain maximum traffic capacity. In practice one must consider character of traffic, congested conditions at approaches to terminals, busy passenger stations and junction points, and local conditions such as view and opportunity for suitable foundations, in their effect on signal location.

Arrangement of Signals on Single-track Road. — Fig. 5 shows one arrangement of single-track signaling in both directions used by a large western

system. The horizontal lines terminating at one end opposite signals and at the other end opposite insulating joints (the latter indicated by the short marks

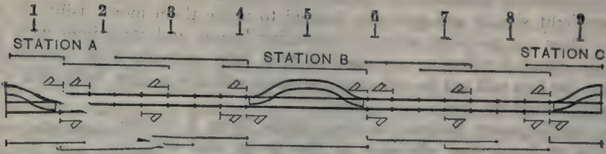

Fig. 5. Arrangement of Single-track Signaling in Both Directions

across the lines representing the rails) show the extent of track governing each signal by virtue of the track circuit.

Even where the distance between stations (or sidings) is less than a mile, it is important that intermediate home signals be placed between stations (or sidings) in order to prevent a train on the main track at a station from causing possible detention to an incoming train. It will be noticed by reference to the figure that eight signals are used to protect traffic, six between stations, and two within station limits. This number of signals is necessary to adequately protect all traffic and at the same time to insure proper flexibility, since, with this arrangement, trains can move up to the station limits under full protection from both directions, even though the main line may be occupied within these limits. This is because a train in a station affects only the entrance signals, and consequently does not affect approaching traffic on either side of the station, neither does a train on one side of a station influence traffic on the other side.

Fig. 6 shows the circuits for the system shown in Fig. 5.

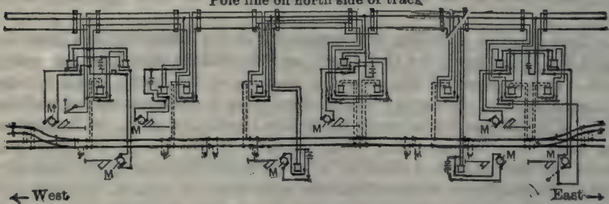

Fig. 6. Circuits for System Shown in Fig. 5

Another very typical system of single-track signaling is that which gives station to station protection for opposing moves and signal to signal protection for following moves. It is called the Absolute Permissive Block System or Traffic Direction System, generally abbreviated to A. P. B. System.

Special Arrangement for Single-track Interurban Road.— Fig. 7 shows a system of signaling which is particularly applicable to interurban single-track signaling where special conditions sometimes render it extremely desirable to permit more than one car, moving in the same direction, between sidings, and yet give both head-on and rear-end protection.

Because of the impedance of the rails and the characteristics of the voltage supply to the track circuit, track relay at signal "A" is deënergized only by a train between "A" and "X." Similarly, track relay at signal "B" is deënergized only by a train between "B" and "Y." Signals "A" and "B" are

connected so that both will go to "stop" on the entrance of a car at either end of the block, but for following cars the signal at the entering end will clear up after

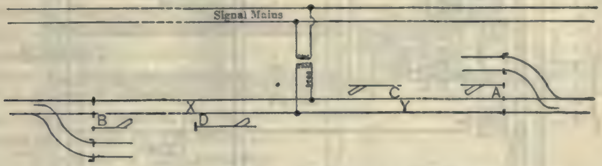

Fig. 7. System for Single-track Interurban

any preceding car has gone half the distance between sidings plus half the distance *X–Y*. *X–Y* is generally about 3000 feet where distance *A–B* is about 2 miles. If, by any misunderstanding, cars going in opposite directions should enter a stretch of track between sidings, they would be stopped by signals "C" and "D," located sufficiently far apart to allow for a proper factor of safety in stopping.

Trolley Wheel Operated Signals. — Block signals controlled from a contactor operated by the trolley wheel do not meet the requirements of high-speed signaling as to reliability, and are of value principally in preventing the loss of time due to two cars moving in opposite directions entering the same stretch of track, where conditions of vision and speed are such that there is small chance of harm resulting from a collision.

Arrangement of Signals on Double-track Road. — Fig. 8 shows a typical a-c. automatic block-signaling system with circuits for a two-track road. It will be noted that the *control* of the third position of the signals is obtained by the use of *line wires* running between the signals and supplied with current by circuit controllers operated by the signals.

Fig. 8a. Key to Fig. 8

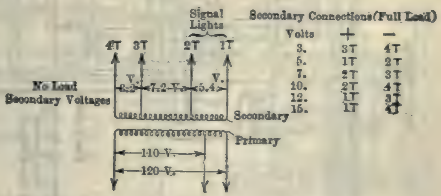

Fig. 8b. Details of Transformer Shown in Fig. 8

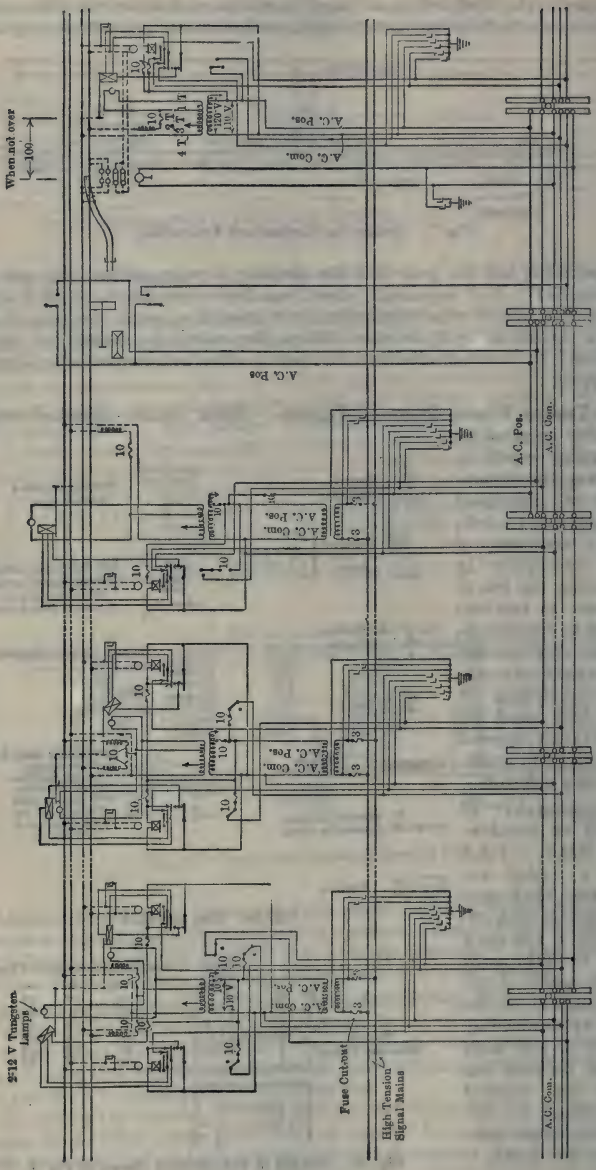

Fig. 8. A-C. Automatic Block System for Double-track Steam Road

The use of two-energizing-circuit track relays as *polarized* relays permits *dispensing* with *control line wires,* in obtaining the control of the third position of three-position signals. The polarized relays close contacts in two positions according to direction of current in track winding and open them when deënergized, as by the presence of a train on the track circuit. The polarity of the current supplied to the track circuit in the rear of any signal depends on the position of that signal and the circuit controllers attached to it. This method is known as *polarized wireless control* and has its counterpart in direct-current signaling.

Maximum Capacity Arrangement. — Fig. 9 shows the latest development of signaling to give maximum capacity where high- and low-speed trains are run on close headway. The main idea is to make possible proper *speed control.* It will be noted that the high-speed trains receive sufficient warning to stop at the proper point in spite of the spacing of signals being made close to get maximum capacity with low-speed trains, which may run for short distances on the high-speed tracks.

Beginning with the first signal behind the occupied block at the left, the indications are respectively "Stop, then proceed according to rule," "Proceed, prepared to stop at next signal," "Proceed, prepared to pass next signal at medium speed," "Proceed."

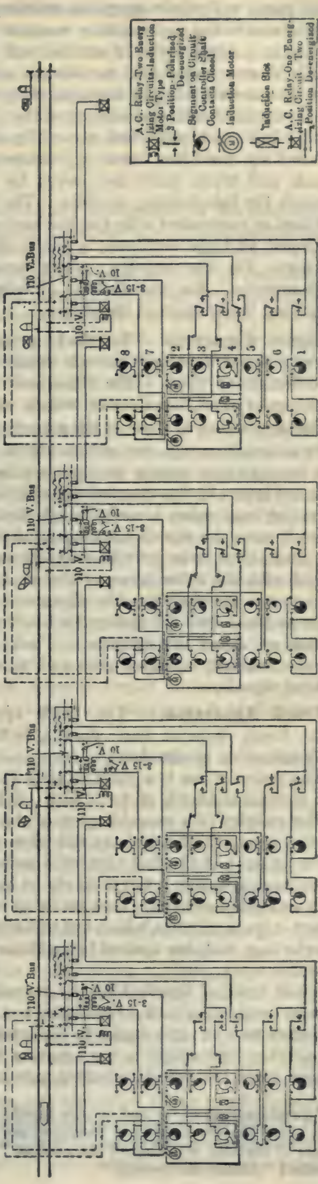

Fig. 9. Circuits of Arrangement to give Maximum Track Capacity

INTERLOCKINGS AT CROSSINGS, JUNCTIONS AND TERMINALS. — All interlockings are designed with the view of insuring that signals governing traffic over any movable track must be in the stop position before the track parts can be shifted, that the track parts cannot be moved under a train, that they must be locked in the proper position before the signals governing over them can be placed in the proceed position, and that conflicting signals cannot be given.

The apparatus at an "interlocking" consists of the signals and movable

track parts with their operating mechanisms and intermediate apparatus, the central-controlling interlocking machine (sometimes called an "interlocker") located in a signal tower or station, and auxiliary apparatus. Three types * of interlockings are used: (1) mechanical interlocking, in which movement is transmitted to the signals and track parts by means of wire and pipe connections to the levers in the interlocking machine; (2) electric interlocking in which the signals and track parts are moved by electric motors controlled through the interlocking machine; and (3), electro-pneumatic interlocking in which the signals and track parts are moved by compressed air controlled electrically through the interlocking machine. The last two types are called power interlockings as contrasted with the mechanical interlockings.

Almost all large terminals in the United States use electro-pneumatic interlockings. On the smallest power interlocking installations, "all electric" predominates. For installations of intermediate size there are many factors governing the wisdom of the choice.

Signal Apparatus. — Interlocking signals differ from automatic signals in external appearance only in such details as are necessary to enable the runner to differentiate them. The differences are generally confined to shapes and markings of arms, locations on masts, etc., and call attention to modifications in the significance of some of the aspects of interlocking signals as distinguished from automatics, e.g., see section above on *Semaphore Positions*. In mechanical interlockings signals are operated by wire or pipe connections to the mechanical levers in the interlocking machine. Sometimes a low voltage electric operating mechanism is used, though the switches remain mechanically operated. In such cases the signals are controlled through contacts operated directly from the mechanical lever or from contacts on a small auxiliary lever properly interlocked with the other levers of the machine.

In power interlockings the signal-operating mechanisms are essentially the same as those used for automatic block signals and are described above in the section on *Power Mechanisms*. Their control and operating circuits replace the mechanically operated pipe and wire connections of the mechanical interlocking signals.

Track Apparatus. — The track apparatus comprises switches, movable point frogs, detector bars, derails, bolt locks, drawbridge locks, etc., with their operating mechanisms.

The detector bar is a piece of steel ⅜ inch or ½ inch thick, 2¼ inches wide and varying in length up to 55 feet. It is supported beside the head of the rail, in the vicinity of a movable part of the track, so that it is capable of vertical motion above the head of the rail when no car is present over it.

On account of the fact that with power interlocking and a wide head of rail a mechanical detector bar is likely to be forced up outside the wheels of a car over a switch, mechanical bars are being supplanted by short track circuits, called detector track circuits, which are utilized to lock the switch-control lever in the interlocking machine against being thrown under a train while a train is over a movable track part.

In all types of interlocking the movable part of the track is locked in position by connecting to it a horizontal bar, called the lock rod, which moves with the track part at right angles to the track. The lock rod is provided with holes or notches into which a bar, moving in guides parallel to the track, may be moved when the switch, or other moving part, is in its correct position. This

* Combinations of (1) and (2) are rapidly coming into use, and are of great advantage when the tower space is restricted and a change of track layout calls for an increase of capacity over that of the existing "mechanical" machine. The combination is called "Electro-mechanical."

latter bar, which engages the lock rod, receives its motion from the connection which operates the detector bar. The sequence of movements at the switch is: (1) the lifting of the detector bars; (2) the withdrawal of the bar engaging the lock rod (the latter operation not being capable of completion unless it has been possible to raise the detector bar to the full extent of its travel); (3) the movement of the switch, or other track part; (4) the lowering of the detector bar; (5) the entrance of the locking bar into the hole or notch in the lock rod, provided the moving part has made its full travel and is in its correct position.

Intermediate Apparatus. — The movements enumerated in the above paragraph are generally effected in mechanical interlockings by pipe connections from track parts and their locking devices to corresponding levers in the mechanical interlocking machine. Bell cranks are used in the pipe connections to change the direction of motion. Sometimes all the movements enumerated are effected in proper order by a device located beside the track, called a "switch and lock movement," which may be operated by one pipe connection and machine lever. In power interlockings switch and lock movements are almost always used, power apparatus at the track and electric circuits replacing the mechanically-operated pipe connections.

Power Apparatus for Electric Interlockings. — The power supply for electric interlockings is derived from storage batteries, used in duplicate sets to provide for charging, or in single sets charged while in service or floated on the charging circuit. The nominal voltage is 110, but exceeds this figure if the batteries are charged in operation, or floated, or if, as is sometimes the case, more than 55 cells are used. A 120-ampere-hour cell is a common size for isolated plants. At such plants the charging is generally done two or three times a week by a gas-driven generator. A switch motor operating a single switch without mechanical detector bars in two seconds takes an average current of about 3½ amperes. With about 200 feet of detector bar the corresponding current is about 4½ amperes. A motor signal operating in the same time takes about 2½ amperes to clear and one-tenth ampere to hold. Solenoid dwarf signals, which are quite commonly used, operate almost instantaneously and have a resistance of about 12 ohms and require 0.3 ampere to hold, resistance being cut in when the signal clears.

Power Apparatus for Electro-pneumatic Interlocking. — Each signal cylinder in electro-pneumatic interlocking is provided with an electromagnetic pin valve fastened directly to the cylinder. The piston is metal-packed, single-acting and self-sealing at the end of its stroke under pressure. The cylinder bore is 3 inches and the stroke 4 inches except where two cylinders act jointly to obtain three positions of the semaphore. Admission of air is obtained by the energization of the magnet valve, and exhaust by the deënergization, the piston returning under the influence of the counterweight of the semaphore.

The switch cylinders are double acting, and vary from four inches to seven and one-half inches in diameter, depending on the character of the load, and have a stroke varying from a length equal to the throw of the switch up to 10 inches. The smaller cylinders are used to act directly on the switch points without bars, the larger ones to operate (through the medium of a switch and lock movement) the heavier movable-point frogs in connection with one end of a double slip with their complement of bars. The stroke of the larger cylinders is generally 8 inches. Air is admitted to the cylinder by a slide valve. The latter is shifted by two small single-acting pistons, the admission of air to which is controlled by electromagnetic pin valves similar to those used on the signal cylinders. The slide valve is locked in its extreme positions by a pin actuated by a small piston controlled by a third electromagnet valve whose energiza-

tion must precede the operation of either of the pistons which shift the slide valve.

The control of the various magnet valves is effected by contacts operated by the interlocking machine levers. Storage battery to give about fourteen volts is generally used and the individual currents may vary from 1/40 to 1/20 of an ampere. From 75 to 100 pounds air pressure is generally carried.

Interlocking Machine for Mechanical Interlocking. — The interlocking machine comprises the levers, or controller handles, corresponding to the apparatus governed, and the mechanical, or mechanical and electrical, interlocking devices between the levers themselves, and between the levers and the apparatus governed by them.

The mechanical locking between levers is effected in the "locking bed." This is an iron plate, with two sets of parallel grooves. One set of grooves contains cold-rolled steel bars of rectangular cross section connected to and moved by the levers. The other set of grooves, at right angles to the first, contains shorter bars with ends shaped to engage with projections or depressions on the bars connected to the levers. The cross bars engage with the lever bars in such a manner that a lever cannot be moved unless the lever bar affecting the other end of the cross bar is in a definite position.

In a mechanical interlocking the operation of the detector bar and locking bar operating in conjunction with a movable track part, is generally effected by a separate lever in the interlocking machine, called the lock lever. This lever is interlocked with the proper signal levers so that the latter must have been operated to put the corresponding signals to stop before the lock lever can be operated to raise the detector bar and unlock the switch. The switch lever is, in turn, interlocked with the lock lever so that if the lock lever cannot make its full stroke, due, for example, to a train being over the detector bars, the switch lever cannot be thrown. If, after the switch lever is thrown, the lock lever cannot be thrown fully back to its original position, due, for example, to the switch not having made full travel, and the locking bar not being able to properly engage the lock rod, the interlocking between the lock and signal levers prevents the latter from being operated to put the signals in the proceed position. In mechanical interlocking plants rods connected to the signals are also often made to engage with the lock rod of the switch.

Further protection is often obtained in mechanical machines by attaching to moving parts connected to the levers circuit controllers and electromagnetic locks. The electric locks act by gravity when deënergized to engage with a moving part connected to the lever. The circuit which energizes them may be carried through circuit controllers attached to the signal or track mechanisms, or through the circuit controllers on other levers.

Interlocking Machine for Power Interlockings. — In power interlockings the levers are small as they have to operate only the mechanical locking in the machine and contacts for controlling the various circuits. The strain which can be brought on the mechanical locking is thereby reduced, and it is made correspondingly smaller than in mechanical interlocking machines. In these types of machines where the levers give a rotary motion to the circuit controllers it is possible, by the use of the two extreme positions of the levers, to control signals governing over the same route in opposing directions by the same lever. Only circuits for signals in one direction are closed in an extreme position and the indication referred to below being received before the levers can be returned to the middle position, where it releases the mechanical locking, insures against signals being placed in the proceed position if opposing signals controlled by the same lever have not returned to stop.

Depending on the type, a power interlocking machine may occupy less than one-quarter of the space taken by a mechanical machine, and require but one-quarter of the operators. Space is also saved on the ground, as the pipe connections and foundations are replaced by wires.

In power interlockings the completion of the stroke of a movable-track part and its proper locking is "indicated" on the machine by the energization of an electromagnetic lock which allows the lever to complete its full stroke, thereby releasing the proper mechanical locking in the machine. The circuit for the lock is carried through contacts on the moving mechanism on the ground. The return of a signal to the stop position is indicated in a similar manner.

Control of Current for Electric Interlockings. — The supply of current to the signals, switches, etc., may be effected directly by the controller in the interlocking machine, or by simple or polarized secondary controllers located at the various signals and switches, the windings of the secondary controllers being in turn supplied in whole or in part with current from the machine's lever contacts. The use of the secondary controllers permits the wires between them and the interlocking machine to be small, the power for operating the signals, switches, etc., being supplied through contacts operated by the secondary controller from mains running through the plant.

On account of the small amount of current used to operate the secondary controllers, compared to the amount used by the apparatus they supply, the contacts in the interlocking machine may be as small and compact as in an electro-pneumatic interlocking machine, and therefore offer the same excellent opportunity for intercontrol of circuits in the machine itself. This is particularly important, for instance, where it is desired to effect *route locking* (see below), where it may be necessary to have certain signals and switches enter into a great variety of combinations.

Indication Current for Power Interlockings. — Among the various methods of providing the current for "indication" may be mentioned: (1) the use of the same current as is used for control and operation, called "battery indication;" (2) the use of polarized apparatus in connection with the preceding, or in connection with current of a different character from the operating current; (3) utilization of the momentum of the operating electric motor to generate the indication current after the motor has completed the movement of the apparatus; (4) the operation of the electric motor as a motor generator after it has completed the movement of the apparatus, the generated current being distinctive in character, viz., alternating or of greater voltage than the operating current and of such direction as to selectively operate polarized magnets.

Auxiliary Apparatus. — Various relays, indicators, annunciaters, emergency time lever releases and special circuits are used in connection with or in addition to the essential interlocking apparatus to expedite the handling of traffic and to meet special conditions.

Approach Locking. — Where electric locking is effective while a train is approaching a signal which has been set for it to proceed, to prevent manipulation of levers or devices which would endanger that train, it is termed *approach locking*.

Route Locking. — When electric locking is so arranged that it takes effect when a train passes a signal to prevent manipulation of levers which would endanger the train while it is within the limits of the route entered, it is termed *route locking*.

Sectional Route Locking. — To minimize the limitation of trackage capacity which results from route locking, especially when the route lies across the through tracks, *sectional route locking* is used. This is route locking so

arranged that a train in clearing each section of a route releases the locking affecting that section. The detector track circuits are used to accomplish this type of locking. When *route locking* is not used it is necessary, in order to safely obtain full trackage capacity, to space the signals so close that no considerable amount of track is locked by the clearing of any one signal. Where the track layout and traffic conditions are such that short switching moves predominate over through routing this latter method is superior in giving greater capacity and flexibility.

AUTOMATIC STOPS. — Automatic stops have been in use for a number of years in connection with elevated and subway roads. They have been operated by direct mechanical connection to the signals or by separate mechanisms working in conjunction with the signals, and have consisted of arms located on the roadbed so that when elevated they were in position to engage with and operate a connection to the brake system on the car. It is evident that as a train may travel a considerable distance after the brakes are applied, and as the engineman or runner is dependent on the signals for the knowledge necessary to the proper control of his train, the relation of signal and stop locations must be such that in case the signals are properly observed the brakes will not be automatically applied, while if the contrary is the case, the space intervening between the train causing the display of the danger signal and the automatic stop protecting it must be sufficient to allow a following train being brought to rest without a collision.

This may be accomplished in two ways: (1) by using an overlap (i.e., extending the control of a signal to a point sufficiently beyond the next succeeding signal), which increases the spacing of trains and diminishes the capacity of the road; (2) by giving the runner an advance indication of the position of the signal at which it may be necessary to make a stop, so that if necessary he may bring his train under control in time to stop at the proper point. In the latter case the automatic stop is located to bring the train to rest at the stop signal, in case the speed has not been properly reduced. Up to the present (1921) automatic stop installations have been almost universally on the overlap system.

On account of clearances and atmospheric conditions, the automatic stop problem for steam roads is very difficult of solution. The problem is further complicated by the fact that there are involved the different conditions affecting freight and passenger traffic, a differentiation between the absolute stop indication of an interlocking signal and the permissive stop indication of an automatic block signal, and a differentiation between the speeds permissable over different routes, as indicated by an interlocking signal, with two or more arms, located at a point of divergence.

The above considerations indicate that the satisfactory solution of the automatic-stop problem for the majority of roads must follow the trend of modern signaling, and be based on speed control.

BIBLIOGRAPHY. — *Railway Signaling,* Elec. Jour., Pittsburg; *Report on the block signal systems and appliances for the automatic control of railway trains,* U. S. Interstate Com. Com., 1907; Elliott, W. H., *Blocks and Interlocking Signals and A. B. C. of Railroad Signals;* Latimer, J. B., *Railway Signaling in Theory and Practice, Chicago;* Lavarack, F. C., *Locking;* Numerous papers in *Proc. Ry. Signal Assoc.*

SKIN EFFECT. — (*See also Electricity and Magnetism, Principles of; Resistance and Conductance, Electric.*) A conductor of finite cross-section may be looked upon as made up of separate filaments, just as a beam may be looked upon as made up of separate fibers. The inductances of the various filaments which make up such a conductor are different, due to the fact that the exterior filaments are linked by fewer flux lines than the interior filaments; see *Electricity and Magnetism, Principles of*, and *Inductance and Inductive Reactance.* Consequently, when the same potential gradient (varying with time, however) is established through all the filaments of such a conductor, by connecting it to some external source of alternating or varying e.m.f., the self-induced back e.m.f. in the interior filaments will be greater than in those filaments nearer the surface, and therefore the resistance drop in the interior filaments must be less than in the surface filaments. This can be brought about only by the current distributing itself over the cross-section of the conductor in such a manner that the current density in the interior of the wire will be less than at the surface, i.e., the current is forced toward the surface filaments or "skin" of the wire; hence the term "skin-effect" for this phenomenon.

Factors upon which the Skin Effect Depends. — The self-induced e.m.f. depends not only upon the amount of flux set up but also upon the rapidity of its variation; hence the skin effect becomes more pronounced the greater the frequency of the impressed e.m.f. It is also greater the larger the cross section of the conductor, the greater the conductivity of the conductor and the greater its magnetic permeability. It also depends slightly upon temperature since the conductivity changes with temperature.

Change in Resistance and Inductance due to Skin Effect. — As a consequence of the skin-effect the effective resistance of a conductor to alternating currents is greater than to direct currents, but the *internal* inductance *decreases* with the frequency; the external inductance is not altered; see *Inductance and Inductive Reactance.* Whereas, however, the internal inductance with increasing frequency approaches a limiting value, the resistance increases indefinitely as the frequency approaches an infinite value. The change of resistance is always relatively much larger than the change in the total inductance.

The effects just described are, for the most part, negligible at low frequencies, except in the case of heavy conductors and in coils wound with stout wire in several layers. In the latter case, however, the diminution of the inductance, due to the irregular distribution of the current, is masked, to a greater or less degree, by the effect of the capacity between the windings of the coil, which gives rise to an *increase* of the inductance with the frequency. For the same reason the resistance is increased more than it would be by the eddy currents alone.

Unfortunately, the rigorous or approximate solution of the problem at high frequencies for the various cases which arise in practice is in many instances very difficult, if not impossible.

SKIN EFFECT IN STRAIGHT ROUND WIRES. — An accurate solution for the case of straight *solid** wires of circular cross-section has been given in a number of different forms by various scientists. A summary of the formulas is given in a paper by Rosa and Grover, *Bull. Bur. Stds., 1912, Vol.*

* Tests at the Massachusetts Institute of Technology (*Proc. A.I.E.E., Sept., 1915*), indicate that the formulas for solid wires also apply to round *stranded* wires of the *same cross-section of metal*, not the same over-all diameter, for frequencies up to 1200, but above this frequency there is an increase in the skin effect, due to the spiraling of the component wires of the cable.

8, p. 172. The calculations are most conveniently made by the use of the tables given in Rosa and Grover's paper, which are given in a condensed form below. Let

f = frequency in cycles per second,
μ = permeability of the wire, *assumed constant,*
R = direct-current resistance, in ohms, of 1000 feet of the wire; see tables in the article on *Wires and Cables, Bare.*
L = direct-current inductance, in millihenries per 1000 feet, of a *non-magnetic* wire of the same *cross section* as that of the given wire, and at the given spacing between wires, taken from the tables in the article on *Inductance and Inductive Reactance.*

Calculate the quantity

$$x = 0.02768 \sqrt{\frac{\mu f}{R}}, \tag{1}$$

and take from the following table the corresponding values of K_1 and K_2. Then the alternating-current resistance at the frequency f is

$$R' = K_1 R \quad \text{ohms per 1000 ft.} \tag{2}$$

and the alternating-current inductance of the given wire at the frequency f is

$$L' = L + 0.01524\,(\mu K_2 - 1) \quad \text{millihenries per 1000 ft.} \tag{3}$$

For x greater than 7 the following relations hold to within less than 1 per cent, the error being less the greater the value of x:

$$R' = \left(\frac{x}{2.828} + 0.25\right) R, \tag{4}$$

$$L' = L + 0.01524\left(\frac{2.828\,\mu}{x} - 1\right). \tag{5}$$

Example.—Take the case of a 1,000,000-circular-mil copper cable (assumed equivalent to a solid wire of the same cross section; see footnote on preceding page), frequency 60 cycles, and return wire 10 ft. away. Then at 25° C., R = 0.0108 ohm per 1000 ft. of wire, L = 0.3494 millihenry per 1000 ft., for direct current. The value of x is

$$x = 0.02768 \sqrt{\frac{1 \times 60}{0.0108}} = 2.06.$$

From the table below K_1 = 1.088 and K_2 = 0.957, and the alternating-current resistance at 25° C. is therefore $R' = 1.088 \times 0.0108 = 0.0118$ ohm per 1000 ft, and the alternating-current inductance is $L' = 0.3494 + 0.01524\,(1 \times 0.957 - 1) = 0.3487$ millihenry per 1000 ft.

SKIN-EFFECT FACTORS FOR SOLID ROUND WIRES

x	K_1	K_2	x	K_1	K_2	x	K_1	K_2
0.0	1.00000	1.00000	3.8	1.60314	0.71729	10.5	3.97477	0.26832
0.1	1.00000	1.00000	3.9	1.64051	0.70165	11.0	4.15100	0.25622
0.2	1.00001	1.00000	4.0	1.67787	0.68632	11.5	4.32727	0.24516
0.3	1.00004	0.99998	4.1	1.71516	0.67135	12.0	4.50358	0.23501
0.4	1.00013	0.99993	4.2	1.75233	0.65677	12.5	4.67993	0.22567
0.5	1.00032	0.99984	4.3	1.78933	0.64262	13.0	4.85631	0.21703
0.6	1.00067	0.99966	4.4	1.82614	0.62890	13.5	5.03272	0.20903
0.7	1.00124	0.99937	4.5	1.86275	0.61563	14.0	5.20915	0.20160
0.8	1.00212	0.99894	4.6	1.89914	0.60281	14.5	5.38560	0.19468
0.9	1.00340	0.99830	4.7	1.93533	0.59044	15.0	5.56208	0.18822
1.0	1.00519	0.99741	4.8	1.97131	0.57852	16.0	5.91509	0.17649
1.1	1.00758	0.99621	4.9	2.00710	0.56703	17.0	6.26817	0.16614
1.2	1.01071	0.99465	5.0	2.04272	0.55597	18.0	6.62129	0.15694
1.3	1.01470	0.99266	5.2	2.11353	0.53506	19.0	6.97446	0.14870
1.4	1.01969	0.99017	5.4	2.18389	0.51566	20.0	7.32767	0.14128
1.5	1.02582	0.98711	5.6	2.25393	0.49764	21.0	7.68091	0.13456
1.6	1.03323	0.98342	5.8	2.32380	0.48086	22.0	8.03418	0.12846
1.7	1.04205	0.97904	6.0	2.39359	0.46521	23.0	8.38748	0.12288
1.8	1.05240	0.97390	6.2	2.46338	0.45056	24.0	8.74079	0.11777
1.9	1.06440	0.96795	6.4	2.53321	0.43682	25.0	9.09412	0.11307
2.0	1.07816	0.96113	6.6	2.60313	0.42389	26.0	9.44748	0.10872
2.1	1.09375	0.95343	6.8	2.67312	0.41171	28.0	10.15422	0.10096
2.2	1.11126	0.94482	7.0	2.74319	0.40021	30.0	10.86101	0.09424
2.3	1.13069	0.93527	7.2	2.81334	0.38933	32.0	11.56785	0.08835
2.4	1.15207	0.92482	7.4	2.88355	0.37902	34.0	12.27471	0.08316
2.5	1.17538	0.91347	7.6	2.95380	0.36923	36.0	12.98160	0.07854
2.6	1.20056	0.90126	7.8	3.02411	0.35992	38.0	13.68852	0.07441
2.7	1.22753	0.88825	8.0	3.09445	0.35107	40.0	14.39545	0.07069
2.8	1.25620	0.87451	8.2	3.16480	0.34263	42.0	15.10240	0.06733
2.9	1.28644	0.86012	8.4	3.23518	0.33460	44.0	15.80936	0.06427
3.0	1.31809	0.84517	8.6	3.30557	0.32692	46.0	16.51634	0.06148
3.1	1.35102	0.82975	8.8	3.37597	0.31958	48.0	17.22333	0.05892
3.2	1.38504	0.81397	9.0	3.44638	0.31257	50.0	17.93032	0.05656
3.3	1.41999	0.79794	9.2	3.51680	0.30585	60.0	21.46541	0.04713
3.4	1.45570	0.78175	9.4	3.58723	0.29941	70.0	25.00063	0.04040
3.5	1.49202	0.76550	9.6	3.65766	0.29324	80.0	28.53593	0.03535
3.6	1.52879	0.74929	9.8	3.72812	0.28731	90.0	32.07127	0.03142
3.7	1.56587	0.73320	10.0	3.79857	0.28162	100.0	35.60666	0.02828

SKIN EFFECT IN THIN STRIPS AND TUBES. — The following formulas are exact for a flat strip of **infinite width** and at an infinite distance from any other conductor carrying a current; they also apply with a close degree of approximation to a tube which has a circumference 10 or more times its thickness, provided no other conductor carrying a current is closer than a distance of 10 times the thickness of the strip or tube.

Let t = thickness of strip, or *twice* the thickness of the wall of a tube, in centimeters,

w = width of strip or *half* the mean circumference of tube, in centimeters,
l = length of strip or tube in centimeters,
ρ = specific resistance of conductor, in microhms per centimeter cube, at the given temperature,
μ = magnetic permeability of conductor in absolute units,
f = frequency in cycles per second,
$x = 0.1987\, t \sqrt{\frac{\mu f}{\rho}}$.

Then d-c. resistance is

$$R = \frac{10^{-6}\rho l}{wt} \text{ ohms.}$$

The d-c. internal * inductance is

$$L_i = 1.047 \times 10^{-6}\, l \frac{\mu t}{w} \text{ millihenries.}$$

The ratio of the a-c. to the d-c. resistance is

$$\frac{R'}{R} = \frac{x}{2}\left(\frac{\sinh x + \sin (57.3\, x)^0}{\cosh x - \cos (57.3\, x)^0}\right)$$

and the ratio of the a-c. to the d-c. internal inductance is

$$\frac{L_i'}{L_i} = \frac{3}{x}\left(\frac{\sinh x - \sin (57.3\, x)^0}{\cosh x - \cos (57.3\, x)^0}\right)$$

For x less than unity these ratios are unity to within 0.6% and 0.2% respectively, i.e., the a-c. resistance is practically equal to the d-c. resistance and the a-c. internal inductance is practically equal to the d-c. internal inductance. For x greater than 6 the following formulas are accurate to within 0.5%:

$$\frac{R'}{R} = \frac{x}{2} \quad \text{and} \quad \frac{L_i'}{L_i} = \frac{3}{x}.$$

To a very rough degree of approximation the skin effect in a conductor of any shaped cross section may be approximated by using the above formulas for a strip, taking for the effective width w one-half the perimeter of the section (in centimeters) and for its effective thickness twice the area of the section (in square centimeters) divided by its perimeter. In general, then, for the same area the skin effect will be less the greater the perimeter of the section. If the section approaches more nearly that of a solid circle than that of an elongated rectangle, the formulas for a solid round wire will give more accurate results.

Skin Effect in Bus-bars (Strips of Finite Width).—The formulas given above are for strips of infinite width; the skin effect in narrow strips, such as used for bus-bars, is much greater than the theoretical value for infinite strips, due to the crowding of the current to the edge of the strip. This latter effect has been called the "edge effect." Experimental results obtained by Kennelly, Laws

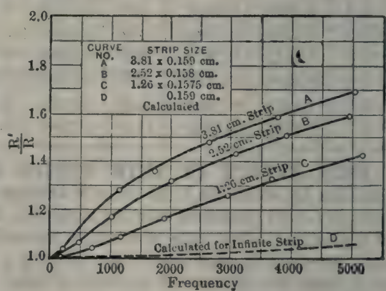

Fig. 1

and Pierce (Proc. A.I.E.E., Sept., 1915), for strip conductors, outgoing and return conductor 60 cm. apart, are given in Fig. 1. For other spacings the original paper should be consulted.

In the *Journal of the Franklin Institute* for March, 1918, are reported tests on copper bus-bars 4 inches by ¼-inch, showing a ratio of a-c. to d-c. resistance of 1.3 at 60 cycles and 1.1 at 24 cycles. It is stated that this increase in resistance is due chiefly to the crowding of the current toward the edge of the strip, and not to the sides. Three such bars in parallel with an air space of 0.25 inch between them gave a ratio of a-c. to d-c. resistance of 2.2 at 60 cycles.

SKIN EFFECT IN WIRES, STRIPS AND TUBES AT HIGH FREQUENCY. — Kennelly and Afel, from a series of tests at frequencies up to 100,000 cycles per second, reported in the *Proceedings of the Institute of Radio Engineers* (*May, 1916*), draw the following conclusions:

1. The skin-effect resistance ratio of round copper wires has been found to conform to the standard Heaviside-Kelvin Bessel-function formulas, within the limits of experimental error.

2. The skin-effect resistance ratio of a copper conductor formed of seven equal and parallel round bare strands (six surrounding a central seventh), was found to be the same as that of the equisectional solid round wire up to 100,000 cycles per second, within the limits of observational error. It is therefore inferred that the subdivision of a round wire into a round cable of uninsulated contacting strands does not alter the skin effect.

3. The effect of simple spiraling of strands in the same direction increased the resistance ratio.

4. The resistance ratio of a subdivided wire, with parallel insulated strands, fell below that of the equisectional solid wire, and diminished rapidly with the spacing, or degree of strand separation.

5. The braiding of strands, so as to effect their transposition in the cross-section at frequent intervals, was found to diminish the skin effect.

6. The skin effect in copper strips was found to be usually, but not invariably, less than that of equisectional round wires.

7. Increasing the width of a copper strip within the limits reported was found to increase the skin-effect resistance ratio; owing to what may be called "edge effect."

8. A pair of parallel going and returning flat strips, separated by a thin insulating layer, was found to have a skin effect depending only on the strip thickness; that is, without perceptible edge effect.

9. Rolling a flat strip into the form of a slit tube destroyed the edge effect, and reduced the resistance ratio to a minimum dependent only on the strip thickness.

10. Cutting longitudinal slits with a sharp knife in a thin flat copper strip, was found not appreciably to affect its skin effect resistance ratio.

11. In order to employ a stranded wire of minimum skin effect at radio-frequency, it seems desirable to employ thick insulation on the strands, such as double cotton, to transpose the strands by braiding, and to avoid spirality in one and the same direction.

12. In order to employ a flat strip conductor most effectively, it is necessary to stop rolling it out laterally when the increasing edge effect more than offsets the reduction in skin effect.

13. The hollow tubular form seems to be the most efficient type of radio-frequency metallic conductor, since, with proper mechanical internal support, the skin effect can be indefinitely diminished by diminishing the wall thickness, and the edge effect is absent.

SKIN EFFECT IN IRON AND STEEL CONDUCTORS. — (*See also Trolley Systems, Overhead*). Kennelly, Achard and Dana, from a series of tests reported in the *Journal of the Franklin Institute* (Aug., 1916) draw the following conclusions:

1. The maximum observed skin-effect resistance ratio among ten track rails, at 25 cycles per second, varied between 5.35 and 10.1, and in two contact rails between 10.92 and 13.4.

2. The maximum observed skin-effect resistance ratio was found to vary substantially as the square root of the impressed frequency between the limits of 25 and 60 cycles per second.

3. The effective skin depth of alternating-current penetration, at 25 cycles per second, among all the twelve rails tested, varied between 0.76 mm. and 1.8 mm.

4 The values of the mean superficial r.m.s. magnetic intensity H at which the maximum skin effect developed, were between the limits of 3.3 and 16.4 gilberts per centimeter. These were but little influenced by frequency, and were in good agreement with the values of H for maximum μ, as obtained by direct-current permeameter.

5. The best workshop method of measuring the skin effect in track rails was, in this case, found to be based on the use of a particular form of dynamometer.

6. The effective alternating-current conductance of a rail is, to a first approximation, inversely as the square root of the frequency, and, at a given frequency, is directly proportional to the perimeter, and to the square root of the ratio $\frac{\gamma}{\mu}$.

7. It is therefore, in general, useless to increase the conductivity γ of the steel in a rail if the permeability μ is thereby increased in the same or a greater ratio.

8. From the results here reported, it would seem that the skin-effect resistance ratio of a rail, to a given alternating-current frequency and current strength, can be approximately predetermined from measurements of the conductivity γ and permeability μ of the steel, by applying an experimentally determined factor to cover edge effect and other discrepancies. This factor, which may be called the "edge-effect coefficient," appeared to be not more than 1.3 at the maximum skin-effect ratio.

9. The best form of rail for current-carrying capacity should be one in which the effective perimeter is a maximum while allowing sufficient depth of surface. Among the worst forms are probably a thick solid prism and a cylinder. A hollow cylinder may be greatly improved by slitting it, so as to admit current to its interior surface.

Skin Effect in Copper-clad Steel Wires. — The data given below are from tests made in the Electrical Engineering Research Laboratory of the Massachusetts Institute of Technology, on samples furnished by the Duplex Metals Co. For a comprehensive treatment of effective resistance and inductance of bimetallic and also of iron wires, see Miller, J. M., Bulletin of the Bureau of Standards, 12, p. 207, 1915.

Size of wire, B. & S. gage, or diameter in inches	Solid 12	Solid 8	Solid 4	Stranded 3/16″	Stranded 1/4″	Stranded 1/2″
Per cent conductivity, manufacturer's rating..........	30	30	40	30	30	30
Total metal cross section in circular mils...............	6530	16,510	41,740	28,750	45,710	183,750
Ratio of A-C. to D-C. resistance at 20° C.:						
At 25 cycles per sec.......	1.00	1.00	1.00	1.00	1.00	1.01
At 60 cycles per sec.......	1.00	1.00	1.00	1.00	1.00	1.06
At 500 cycles per sec.......	1.02	1.07	1.11	1.14	1.15	1.59
At 1000 cycles per sec.......	1.05	1.14	1.16	1.31	1.31	2.05
At 5000 cycles per sec.......	1.24	1.29	1.27	2.01	2.22	3.16
Increase in inductance,† ΔL, millihenries per 1000 ft.:						
At 25 cycles per sec.......	0.18*	0.100	0.031	0.119	0.089	0.07*
At 60 cycles per sec.......	0.18*	0.099	0.030	0.117	0.088	0.07*
At 500 cycles per sec.......	0.15*	0.068	0.007	0.086	0.059	0.023
At 1000 cycles per sec.......	0.085	0.038	−0.002	0.057	0.038	0.007
At 5000 cycles per sec.......	0.011	−0.002	−0.009	0.010	0.003	−0.014

* Approximate.

† Calling L the inductance of a solid round, non-magnetic wire of the same cross section and on the given spacing, taken from the tables in the article on *Inductance and Inductive Reactance*, the alternating-current inductance of the copper-clad wire on this spacing, is

$$L' = L + \Delta L \qquad \text{millihenries per 1000 ft.}$$

BIBLIOGRAPHY. — See references in text. Complete bibliography will be found in the papers by Kennelly, et alia.

SMOKE PREVENTION. — The direct cause of smoke from boiler furnaces is that the gases distilled from the coal are not completely burned in the furnace before coming in contact with the surface of the boiler, which chills them below the temperature of ignition.

Smoke may be prevented from forming if each particle of gas, as it is made by distillation from coal, is immediately mixed thoroughly with hot air. Even if smoke is formed by the absence of conditions for preventing it, it may afterwards be burned if it is thoroughly mixed with air at a sufficiently-high temperature. It is easy to burn smoke when it is made in small quantities, but when made in great volumes it is difficult to get the hot air mixed with it unless special apparatus is used. In boiler firing the formation of smoke must be prevented, as the conditions do not usually permit of its being burned.

Essential Conditions. — The essential conditions for preventing smoke in boiler fires may be enumerated as follows:

1. The gases must be distilled from the coal at a uniform rate.
2. The gases, when distilled, must be brought into intimate mixture with sufficient hot air to burn them completely.
3. The mixing should be done in a fire-brick chamber.
4. The gases should not be allowed to touch the comparatively-cold surfaces of the boiler until they are completely burned. This means that the gases shall have sufficient space and time in which to burn before they are allowed to come in contact with the boiler surface.

Every one of these four conditions is violated in the ordinary method of burning coal under a steam boiler. (1) The coal is fired intermittently and often in large quantities at a time, and the distillation proceeds at so rapid a rate that enough air cannot be introduced into the furnace to burn the gas. (2) The piling of fresh coal on the grate in itself chokes the air supply. (3) The roof of the furnace, or tubes of the boiler, is a cold shell instead of a fire-brick arch, as it should be, and the furnace is not of a sufficient size to allow the gases time and space in which to be thoroughly mixed with the air supply.

Methods of Prevention. — In order to obtain the conditions for preventing smoke it is necessary: (1) That the coal be delivered into the furnace in small quantities at a time. (2) That the draft be sufficient to carry enough air into the furnace to burn the gases as fast as they are distilled. (3) That the air itself be thoroughly heated either by passing through a bed of white-hot coke or by passing through channels in hot brickwork, or by contact with hot fire-brick surfaces. (4) That the gas and the air be brought into the most complete and intimate mixture, so that each particle of carbon in the gas meets before it escapes from the furnace its necessary supply of air. (5) That the flame produced by the burning shall be completely extinguished by the burning of every particle of the carbon into invisible carbon dioxide.

If a white flame touches the surface of a boiler, it is apt to deposit soot and to produce smoke. A white flame itself is the visible evidence of incomplete combustion.

Anthracite Coal. — The first remedy for smoke is to obtain anthracite coal. If this is not commercially practicable, then obtain, if possible, coal with the smallest amount of volatile matter. Coal of from 15 to 25 per cent of volatile matter makes much less smoke than coals containing higher percentages. Provide a proper furnace for burning coal. Any furnace is a proper furnace which secures the conditions named in the preceding paragraphs. Next, compel the firemen to follow instructions concerning the method of firing.

Bituminous Coal. — It is impossible with coal containing over 30 per cent of volatile matter and with a water-tube boiler, with tubes set close to the grate

and vertical gas passages, as in an anthracite setting, to prevent smoke even by the most skillful firing. This style of setting for a water-tube boiler should be absolutely condemned. A Dutch oven setting, or a longitudinal setting with fire-brick baffle walls, is highly recommended as a smoke-preventing furnace but with such a furnace it is necessary to use considerable skill in firing.

Steam Jets and Mechanical Stokers. — Mechanical mixing of the gases and the air by steam jets is sometimes successful in preventing smoke, but it is not a universal preventive, especially when the coal is very high in volatile matter, when the firing is done unskillfully, or when the boiler is being driven beyond its normal capacity. It is essential to have sufficient draft to burn the coal properly and this draft may be obtained either from a chimney or a fan. There is no especial merit in forced draft, except that it enables a larger quantity of coal to be burned and the boiler to be driven harder in case of emergency, and usually the harder the boiler is driven, the more difficult it is to suppress smoke.

Down-draft furnaces and mechanical stokers (q.v.) of many different kinds are successfully used for smoke prevention, and when properly designed and installed and handled skillfully, and usually at a rate not beyond that for which they are designed, prevent all smoke. If these appliances are found giving smoke, it is always due either to overdriving or to unskillful handling. It is necessary, however, that the design of these stokers be suited to the quality of the coal and the quantity to be burned, and great care should be taken to provide a sufficient size of furnace with a fire-brick roof and means of introducing air to make them completely successful.

BIBLIOGRAPHY. — Kent, Wm., *Steam Boiler Economy*; Kent's *Mechanical Engineers' Pocket-Book*. For a comprehensive, authoritative treatment of this subject see Report of the Chicago Association of Commerce, *Smoke Abatement and Electrification of Railway Terminals in Chicago*.

SPARK GAP FOR MEASURING HIGH VOLTAGES. — One of the simplest methods of obtaining a measure of the value (maximum value in case of an alternating voltage) of a high voltage is to determine the length of air gap between two electrodes across which the given voltage will just cause a spark to pass. The voltage (maximum value) required to break down such a gap depends upon: 1. the shape and size of the electrodes; 2. the presence of other conductors in the vicinity of the gap; 3. the time of application of the voltage; and 4. upon the temperature and pressure of the air. The dependence of the break-down voltage upon the shape and size of the electrode is due not only to the effect of these factors upon the potential gradient in the gap but also to the fact that the maximum potential gradient at which air breaks down is dependent upon the distribution of the electrostatic field in the gap (*see article on Corona*) and is greater for very short gaps (0.5 cm. or less) than for long gaps.

For the same *maximum* value of the voltage it has been found that the striking distance is independent of the wave shape and frequency, and is the same for direct as for alternating voltages.

Time of Application of Voltage. — Air (and other gases) will stand for a short period of time (i.e., a few seconds) a much larger potential than it will stand for an indefinitely long period of time; this phenomenon is known as dilatation. After a voltage has been applied to a gap for about one minute, the apparent dielectric strength becomes sensibly constant. When measurements must be made with great rapidity, dilatation may be prevented by illuminating the gap by an arc-lamp. This procedure reduces the value of the sparking voltage, but only to a slight extent. (The theory of the action is that the ultra-violet light from the arc-lamp ionizes the air in the gap, thereby enabling the spark to pass at once.)

NEEDLE SPARK GAP. — Steinmetz in 1898 (*Trans. A.I.E.E., Vol. 15, p. 281*) made a careful determination of the voltages required to produce a spark between spherical, cylindrical and pointed (needles) electrodes. Steinmetz concluded that the sparking voltage between needle points (a sine wave of potential being used) was especially constant for any given set of conditions and that after the curve connecting spark potential and distance between needle points had been carefully ascertained, the needle spark gap offered a very valuable means for measuring high voltages. Although the needle spark gap is less trustworthy than the sphere gap, the Standardization Rules of the A.I.E.E. state that the needle gap may be used for voltages between 10 and 50 kilovolts. At higher voltages it has been found difficult to duplicate conditions nearly enough for different observers to obtain results which are in agreement.

Needle Spark-gap and Spark-over Voltages. — The needle spark-gap shall be between new sewing needles, supported axially at the ends of linear conductors, which are at least twice the length of the gap. There must be a clear space around the gap for a radius at least twice the gap length.

The sparking distances in air between No. 00 double long sewing-needle points for various root-mean-square sinusoidal voltages shall be assumed to be as shown in Table I.

SPHERE SPARK-GAP. — In 1913, Farnsworth and Fortescue (*Proc. A.I.E.E., Feb., 1913*) showed that a spark-gap between two spheres is a much more reliable measure of high voltage than the gap between needle points. The sphere spark-gap is now (1922) generally recognized as standard.

Besides giving more consistent results the sphere spark-gap occupies much less space for high voltages than the needle-gap. In addition to this advantage the sphere spark-gap breaks down with a sharply-defined spark discharge

TABLE I.—NEEDLE-GAP SPARK-OVER VOLTAGES

(At 25° C. and 760 mm. barometer)

R. M. S. Kilovolts	Millimeters	R. M. S. Kilovolts	Millimeters
10	11.9	35	51
15	18.4	40	62
20	25.4	45	75
25	33	50	90
30	41		

The values in this table refer to a relative humidity of 80 per cent. Variations from this humidity may involve appreciable variations in the sparking distance.

without previous formation of corona, provided the distance of separation is less than the diameter of either sphere. For greater distances of separation the corona forms first and the sparking voltage for a given length of gap becomes more or less variable. Another advantage of the sphere spark-gap is that the terminals do not have to be renewed after each discharge, as is the case with the needle-gap.

Construction of Sphere Spark-gap. — Fig. 1 shows the arrangement recommended by Farnsworth and Fortescue. The top sphere is stationary but slightly adjustable in height so as just to make contact with the lower sphere when it is set for zero separation. The lower sphere is mounted on a piece of brass tubing which carries a threaded bushing on its lower end. This bushing works on a carefully-threaded rod having a pitch of two per centimeter. The bushing being graduated to fiftieths on its circumference, separation may be measured to the nearest 1/100 cm. directly, thus providing a micrometer adjustment. This arrangement is mechanically strong and the spheres are kept in constant alignment. The frame work may be conveniently mounted on wheels, to render it easily moved.

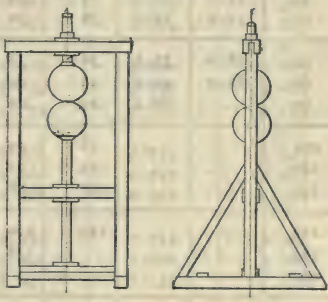

Fig. 1. Sphere Spark Gap.

A.I.E.E. Sphere Spark-gap and Spark-over Voltages. — The standard sphere spark gap shall be between two suitably mounted spheres. No extraneous body, or external part of the circuit, shall be nearer the spheres than twice their diameter. The shanks shall be not greater in diameter than 1/5 the sphere diameter. Metal collars, etc., through which the shanks extend, shall be as small as practicable and shall not, during any measurement, come closer to the sphere than the maximum gap length used in the measurement. The sphere diameter should not vary more than 0.1 per cent, and the curvature, measured by a spherometer, should not vary more than 1 per cent from that

of a true sphere of the required diameter. When used as specified, the accuracy obtainable should be approximately 2 per cent.

The sparking distance between spheres for various r.m.s. sinusoidal voltages shall be assumed to be as shown in Table II, when the air density is that corresponding to a barometric pressure of 760 mm. mercury and a temperature

TABLE II.—SPHERE-GAP SPARK-OVER VOLTAGES

(At 25° C. and 760 mm. barometric pressure)

Kilovolts	Sparking distance in millimeters							
	62.5 mm. spheres		125 mm. spheres		250 mm. spheres		500 mm. spheres	
	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated
10	4.2	4.2	.....	.....	.....	.....	.....	.....
20	8.6	8.6	.....	.....	.....	.....	.....	.....
30	14.1	14.1	14.1	14.1	.....	.....	.....	.....
40	19.2	19.2	19.1	19.1	.....	.....	.....	.....
50	25.5	25.0	24.4	24.4	.....	.....	.....	.....
60	34.5	32.0	30	30	29	29	.....	.....
70	46.0	39.5	36	36	35	35	.....	.....
80	62.0	49.0	42	42	41	41	41	41
90	.....	60.5	49	49	46	45	46	45
100	.....	.....	56	55	52	51	52	51
120	.....	.....	79.7	71	64	63	63	62
140	.....	.....	108	88	78	77	74	73
160	.....	.....	150	110	92	90	85	83
180	.....	.....	.....	138	109	106	97	95
200	.....	.....	.....	.....	128	123	108	106
220	.....	.....	.....	.....	150	141	120	117
240	.....	.....	.....	.....	177	160	133	130
260	.....	.....	.....	.....	210	180	148	144
280	.....	.....	.....	.....	250	203	163	158
300	.....	.....	.....	.....	.....	231	177	171
320	.....	.....	.....	.....	.....	265	194	187
340	.....	.....	.....	.....	.....	.....	214	204
360	.....	.....	.....	.....	.....	.....	234	221
380	.....	.....	.....	.....	.....	.....	255	239
400	.....	.....	.....	.....	.....	.....	276	257

of 25° C. For any other air density the voltage at which the gap breaks down, for any given spacing, is equal to the break-down voltage for this spacing at standard pressure and temperature multiplied by the correction factor given in Table III. The relative air density is found from the formula:

$$\text{Relative air density} = \frac{0.392b}{273+t}$$

where b = barometric pressure in mm. and t = temperature in degrees C.

It is important that the surface of the spheres be kept perfectly clean, i.e., free from dust and moisture (*Marvin, R. H., Elec. W., 67, p. 649, 1916*).

The sphere-gap is more sensitive than the needle-gap to momentary rises of voltage and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage, or by slowly raising the voltage with a fixed setting of the gap. Open arcs should not be permitted in proximity to the gap during its operation, as they may affect its calibration.

TABLE III.—AIR DENSITY CORRECTION FACTORS FOR SPHERE-GAPS

Relative air density	Diameter of standard spheres in mm.			
	62.5	**125**	**250**	**500**
0.50	0.547	0.535	0.527	0.519
0.55	0.594	0.583	0.575	0.567
0.60	0.640	0.630	0.623	0.615
0.65	0.686	0.677	0.670	0.663
0.70	0.732	0.724	0.718	0.711
0.75	0.777	0.771	0.766	0.759
0.80	0.821	0.816	0.812	0.807
0.85	0.866	0.862	0.859	0.855
0.90	0.910	0.908	0.906	0.904
0.95	0.956	0.955	0.954	0.952
1.00	1.000	1.000	1.000	1.000
1.05	1.044	1.045	1.046	1.048
1.10	1.090	1.092	1.094	1.096

CORONA VOLTMETER. — The corona voltmeter, described in detail in Vol. 39 of the Trans. A.I.E.E., may often be used to advantage for measuring high voltages, in preference to a sphere-gap. See the article on *Corona*.

BIBLIOGRAPHY. — In addition to the references in the text see the following: Peek, F. W., *Sphere Gap for Measuring High Voltages*, Proc. A.I.E.E., 33, p. 889, 1914; Clark and Ryan, *Sphere-gap Discharge Voltages at High Frequencies*, Proc. A.I.E.E., 33, p. 937, 1914; Work, W. R., *Measurement of High Voltages*, Proc. A.I.E.E., 35, p. 203, 1916.

SPECIFICATIONS AND CONTRACTS. — (*See also Standardization Rules and Standard Specifications; also under name of apparatus.*) Webster defines a specification as a written statement containing a minute description or enumeration of particulars, and a contract as a formal writing which contains the agreement of parties, with the terms and conditions, and which serves as a proof of the obligation. A "preliminary" specification is a description or enumeration of a purchaser's requirements when calling for bids. A "contract" specification is a description or enumeration of labor and material to be supplied under a contract of which the specification is an integral part. A "manufacturer's" specification is a description or enumeration of secondary details peculiar to the work of the individual manufacturer. This also is often made an integral part of a contract.

POLICY TO BE FOLLOWED. — When writing a specification the engineer should bear the following points in mind:

1. A preliminary specification is a commercial instrument designed to describe labor, material or results desired by a purchaser, with the object of enabling competitive bidders to estimate with equal facility upon the amount of a contract.
2. As a rule, contractors are as honest as the struggle for commercial existence permits them to be, and every unnecessary or unfair clause in a specification has its part in limiting competition and lowering the standard of honesty among contractors. A similar remark applies to requirements of which the purchaser cannot ascertain or enforce the fulfillment.
3. The standards of the national engineering societies should be followed unless local conditions prohibit them, and manufacturer's standards should be followed if low bids are to be expected.
4. No specification should contain a blanket clause covering the furnishing of unnamed contingencies unless all bidders are to have an equal opportunity to ascertain what such contingencies are likely to be. If such a clause is made very broad, it is unlikely that the courts would hold it valid.
5. Before drawing the specifications, determine to what extent the contractor is to be made responsible for the final results.
6. Avoid specifying proprietary articles or material as far as possible in order not to restrict competition.
7. Quality is an important factor in cost, hence unless the very best materials and workmanship are required, in spite of the higher cost, it is necessary to adopt some fair commercial standard.
8. There are many minor items in specifications which cannot be minutely described without making the specifications unduly long. In such cases it is good practice to specify that these items shall be made "to the reasonable satisfaction of the inspector," the word "reasonable" saving the contractor from arbitrary and unjustifiable actions of the inspector by enabling him to refer the reasonableness of such actions to the arbitrament of the courts.

FORM OF SPECIFICATIONS. — All specifications should be written in clear, concise language, free from ambiguity, and should be in convenient form for reference purposes.

It is desirable to number the paragraphs or clauses of all specifications to facilitate reference, and every item of the work should be alloted a separate clause.

There are two general classes of specifications. In one class results only are specified and all requirements should be rigorously exact. In the other class, details of construction are specified and requirements can be stated in approximate terms only. Wherever practicable the former class of specification should be used. A combination of the two classes is also commonly adopted, but in

such cases an agreement as to results does not bind the contractor if the methods of arriving at these results are also specified. (*See J. C. Wait, Eng. News, June 8, 1905*). In either case considerable study should be given to securing a happy medium between brevity and elaboration of details. The degree of detail should be governed very largely by the magnitude and importance of the work.

The following is suggested as a suitable plan to follow in drawing up specifications for materials. It is the result of a study of this subject by a special committee of the American Society for Testing Materials.

Whatever form is adopted should be closely adhered to in writing all the specifications for a given job.

1. The specification shall be divided into eight "Parts."

2. The titles of these Parts shall be in large upper-case type centered over the text and preceded by a Roman numeral.

3. The titles of these Parts shall be as follows unless conditions necessitate some variation. In case a Part is omitted, the Roman numerals shall be continued in an unbroken sequence.

I. Manufacture.
II. Chemical Properties and Tests.
III. Physical Properties and Tests: Mechanical, Electrical, Magnetic, Thermal, etc.
IV. Standard Sizes, Dimensions, Weights, Gauges, etc.
V. Workmanship and Finish.
VI. Packing, Marking and Shipping.
VII. Inspection and Rejection.
VIII. Definition of Terms. (If a specification contains numerous terms that admit of ambiguity, they shall be defined under this sub-title.)

4. Each "Part" of a specification shall be divided into "Clauses" or "Sections," which shall be numbered continuously throughout the specification in Arabic numerals. Every Section or Clause of printed specifications shall have a marginal heading in bold-face type, briefly indicative of its content. If specifications are on one side of the paper, the clause titles shall be in the left-hand margin; if printed on both sides, as in a book, the clause titles shall be in the outside margins.

5. Each Section or Clause may be subdivided into paragraphs distinguished by lower-case italics in parenthesis.

6. Directly after the title of the specification insert sections of an introductory, descriptive or general character. No sub-title shall precede this matter, which shall precede Part I.

7. Desired values, rather than permissible limits, shall be given, followed by a statement with respect to permissible variations.

8. In so far as practicable, specified values shall be expressed in tabular form.

9. The Style sheets of the A.I.E.E., A.S.T.M., etc., should be followed in all matters pertaining to typography, standard terms, abbreviations, spelling, etc.

The words "shall" and "will" are used in the following sense by the American Society for Testing Materials. Use "shall" wherever the specifications are to be made binding on parties of the first or second part. Use "will" wherever the specifications are intended to express a declaration of purpose not mandatory upon the parties of the first or second part. Many engineers use "shall" to express a command binding on the Contractor and "will" to express a declaration of purpose binding the Purchaser.

Specifications for machinery, apparatus or construction work may be written in similar form using different Part headings. In the case of apparatus and machinery, the following sequence of Parts has been found very practical by the author of this article.

The object of the specification and general conditions to be met having been stated, the following parts follow:

I. Summarized Description of principal Characteristics and Conditions of Service.
II. Style and Description of Apparatus; Details of Construction.
III. Dimensions, Weights, Drawings and Schedules.
IV. Work to be done by other Contractors.
V. Performance and Tests.
  (a) Performance which may be checked by mere observation.
  (b) Factory Tests.
  (c) Performance and Tests after Erection.
VI. Workmanship and Finish.
VII. Packing, Marking, Shipping and Delivery.
VIII. Inspection and Rejection.
IX. Guarantees.
X. Conditional Payments. (The details of conditional payments depending upon results of inspection and tests.)
XI. Definition of Terms. (If a specification contains numerous terms that admit of ambiguity, they shall be defined under this sub-title.)

It will be observed that the sequence of topics in the above schedule approximately follows the order of the life history of the machine. The form may be extended upon this principle to cover any kind of work.

POINTS TO BE COVERED IN SPECIFICATIONS. — A specification should often cover the following points.

Items 1 to 3 refer to preliminary specifications only. Items 4 and 5 are frequently included in the contract proper.

1. These specifications are intended to furnish such information to the Bidders as will enable them to prepare detail plans upon which to give prices. Should any Bidder consider the requirements of these specifications prohibitive to the free exercise of his skill, any suggestions made by him will be duly considered.
2. Before a contract is awarded, final specifications will be prepared by the Company.
3. In comparing proposals due consideration will be given by the Company to availability, reliability, simplicity, cost of maintenance and quickness of delivery.
4. The Engineer agrees to consider all drawings submitted by Bidders as confidential, and that he will not show any such drawings to other manufacturers, whether they are tendering under this specification or not.
5. Definition of words describing the parties to the contract, such as "Contractor," "Purchaser," "Company," "Engineer."
6. Person or persons concerned and their respective powers.
7. Where work is to be done or material delivered.
8. The point where the work of two contracts meet should be carefully designated in both contracts, not merely in a general way, but specifically to the minutest detail.
9. The Contractor shall be responsible for the correctness of all drawings even after they are approved by the Engineer.
10. Materials ordered or work commenced prior to the approval of the drawings will be at the Contractor's risk.
11. No approved drawing may be changed without the approval of the Engineer.
12. The Contractor shall inspect the work of other contractors whose work affects his and shall notify the Engineer of anything which injuriously affects

his work. The Contractor shall give these other contractors the privilege of inspecting his work in so far as it affects the acceptance of their work.

13. Contractor shall not drill or in any way impair the strength of buildings or structures except with the written consent of the Engineer.

14. All work shall conform to the requirements of the "National Board of Fire Underwriters," and to all Government regulations.

15. The Contractor shall obtain all necessary permits from the City or County.

16. All like parts shall be interchangeable as far as practicable.

17. The Contractor shall install and maintain in his offices and at the site of the work such telephones as may be required by the Engineer, and shall place them at the disposal of the Engineer or his representative for any purpose relating to the execution of the contract.

18. The Standardization Rules of the A.I.E.E. and other similar documents should be followed wherever possible.

19. The provisions of the preliminary specification and those subsequently agreed upon between Company and Bidder should be incorporated as part of the final or contract specification.

POINTS TO BE COVERED IN CONTRACT. — In addition to items 5 and 6 above, the Engineer should make sure that the following points are included in the contract. The actual preparation of the contract, at least where the amount of money involved is large, should be left to a competent lawyer.

1. Description and value of bond and indemnity to be delivered by Contractor to Company.

2. Statement of fire insurance to be secured by Contractor for Company.

3. Protection of Company by Contractor against losses resulting from letters patent.

4. The work to be performed in such a way as not to interfere with safety and continuity of service.

5. Responsibility for damage to work from fire, floods, storm, earthquake, or any other cause whatsoever.

6. Contiguous work by other contractors shall not be delayed.

7. Statement of liquidated damages to be paid in the event of work not being completed on the date agreed.

8. The Contractor shall not transfer the contract without permission.

9. Time of commencement of work, rate of progress and date of completion.

10. The character of the methods and appliances to be used and grade of workmen to be employed.

11. Method of payment, time of settlement and basis thereof.

12. Arbitration and settlement of disputes.

13. Extras and claims therefor.

(The subject of Engineering contracts is more fully covered in the books by J. B. Johnson and J. I. Tucker, cited in the Bibliography.)

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STANDARDIZATION RULES AND STANDARD SPECIFICATIONS. — (*The A. I. E. E. Standards are given in full at the end of this book.*) Numerous national and international societies issue standardization rules and standard specifications dealing with testing and construction of machinery, apparatus and materials. Brief outlines of specifications and standards of these engineering bodies, with direction for securing copies of them, are given in this article. Additional information can be secured from the American Engineering Standards Committee. Brief statements of the functions of the publications of the American Engineering Standards Committee, the Bureau of Standards, and the International Electrotechnical Commission are included.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS. — (*Secretary, F. L. Hutchinson, 29 West 39th St., New York City.*) This Institute issues a pamphlet (172 pages) entitled "Standards of the American Institute of Electrical Engineers" (price $2.00). These standards define the terms and conditions which characterize the rating and behavior of electrical apparatus, with special reference to the conditions of acceptance tests. These standards are generally accepted by the manufacturers and users of electrical machinery and apparatus in the United States. Standards for dimensions or details of construction are not given.

The standards of the A.I.E.E. comprise sixteen chapters, as follows:

1. **General Principles.** — Heating, Mechanical and Commutation Limitation, Wave Shape, Dielectric Strength and Insulation Resistance, Efficiency, Rating.

2. **General Rules.** — Operation, Rating, Tests, Construction.

3. **General Definitions.**

4. **Standards for Rotating Machinery (General).** — Definitions, Operation, Rating, Tests.

5. **Standards for Electric Railways and for Automobile Propulsion Machines.** — Definitions, Operation, Rating, Tests, Characteristic Curves of Railway Motors, Selection of Railway Motor for Specified Service, Construction.

6. **Standards for Transformers and Other Stationary Induction Apparatus.** — Definitions, Rating, Tests, Construction.

7. **Standards for Switching Control and Protective Apparatus.** — Definitions, Operation, Rating, Tests.

8. **Standards for Meters, Instruments and Instrument Transformers.** — Definitions, Operation, Rating, Tests, Characteristics.

9. **Standards for Wires and Cables.** — Definitions, Annealed Copper Standard, Operation, Designation, Tests, Construction.

10. **Standards for Storage Batteries.**

11. **Standards for Illumination.** — General, Surfaces and Media Modifying Luminous Flux, Illumination, Illuminants, Lamp Accessories, Photometry.

12. **Standards for Telephony and Telegraphy.** — Definitions.

13. **Standards for Radio Communication.**

14. **Standards for Prime Movers and Generator Units.**

15. **Standards for Transmission and Distribution Lines.**

16. **Miscellaneous Devices.** — Heating Devices.

AMERICAN ENGINEERING STANDARDS COMMITTEE. — (*Secrtary, P. G. Agnew, 29 West 39th St., New York City.*) This organization

serves as a national clearing house for engineering and industrial standardization. The Committee is made up at present (1922) of representatives of, and is maintained by, nine national engineering societies, five government departments, and fourteen national industrial associations. Its functions are: to unify methods of arriving at standards; to secure nationally recognized standards, through cooperation between various interested organizations, thus preventing duplication of work and promulgation of conflicting standards; and to act as a general bureau of information regarding standardization.

Standards covering eighteen different subjects have already (1922) been adopted and active work is now in progress on about sixty projects, including a number of industrial safety codes. Information on the organization and the work will be sent free upon request.

AMERICAN ELECTRIC RAILWAY ENGINEERING ASSOCIATION. — (*Secretary, E. B. Burritt, 8 West 40th St., New York City.*) This Association issues an *Engineering Manual*, which contains all standards set by the Association, all practices recommended by it, and those practices which, while they have not been formally adopted either as standards or recommended practice, have been discussed and put forward by the Association's various committees. The *Manual* is in loose-leaf form, and is revised each year by the Committee on Standards. The contents are grouped under the headings: Buildings and Structures; Power Distribution; Equipment; Power Generation; Block Signals; Way Matters. The price of the *Manual*, including all sections in a binder, is $5.00 to non-members.

AMERICAN RAILWAY ASSOCIATION. — (*General Secretary, J. E. Fairbanks, 75 Church St., New York City.*) This is a general association representing the steam railroads of the country. There are now incorporated in it several technical associations which were formerly distinct.

Mechanical Division (Master Car Builders; Master Mechanics). — (*Secretary, V. R. Hawthorne, 431 South Dearborn St., Chicago, Ill.*) The Division, with its predecessor organizations, has numerous specifications covering parts and details of cars and locomotives, practices, etc., which are now (1922) being unified for publication in a loose-leaf manual.

Signal Section. — (*Secretary, H. S. Balliet, 75 Church St., New York City.*) The Section publishes a loose-leaf *Manual* containing specifications for primary batteries, storage batteries, bells, insulated wires and cables for various kinds of signal use, conduit, interlocking devices, lightning protection devices, relays, and various other signal apparatus and supplies; and standard designs for a large number of devices, and parts used in railway signal work.

Telegraph and Telephone Division. — (*Secretary, W. A. Fairbanks, 75 Church St., New York City.*) The present specifications of the division, which include specifications for various kinds of conduits and conduit fittings, telephone and telegraph equipment, protection against electrolysis and against lightning, transpositions of circuits are published in a *Manual* (1922).

AMERICAN RAILWAY ENGINEERING ASSOCIATION. — (*Secretary, E. H. Fritch, 431 So. Dearborn St., Chicago, Ill.*) This Association, which is closely allied to the American Railway Association, issues a *Manual* containing the various committee reports which have been formally adopted by the Association. The following items are covered: Roadway; Ballast; Ties; Rail; Track; Buildings; Wooden Bridges and Trestles; Masonry; Signs, Fences and Crossings; Signals and Interlockings; Records and Accounts; Rules and Organization; Water Service; Yards and Terminals; Iron and Steel Structures; Economics of Railway Location; Wood Preservation; Electricity.

The last edition of the *Manual* (1915) is now out of print, but a new edition is in preparation (1922).

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. — (*Secretary, Calvin W. Rice, 29 West 39th St., New York City.*) The Boiler Code of this Society contains standard specifications for the construction, equipment, and use of steam boilers ($1 to non-members). Official interpretations of the Code by the Boiler Code Committee are published separately. The Power Test Codes ($2 to non-members), are rules for the testing of boilers, engines, turbines, compressors, fans, power plants, locomotives, water-wheels, etc. The Standard Pipe Thread has been approved by the American Engineering Standards Committee as an "American Standard," the Society acting with the American Gas Association as joint sponsors. The work of committees of the Society on Shafting and on Pipe Flanges and Pipe Fittings is now being extended, under the auspices of the American Engineering Standards Committee. A list of the publications of the Society will be sent free upon request to the Society.

AMERICAN SOCIETY FOR TESTING MATERIALS. — (*Secretary, C. L. Warwick, 1315 Spruce St., Philadelphia, Pa.*) This Society issues a triennial volume of "A.S.T.M. Standards," and annual supplements in intervening years. The last triennial volume was issued in September, 1921. The specifications and methods of tests cover a wide range, including Steel Rails, Structural Steel, Various Steel Objects, Steel Castings, Wrought Iron, Pig Iron, Iron Castings, Locomotive Material, Magnetic Tests of Iron and Steel (*see Magnetic Testing*), Various Kinds of Copper Wire and Wire Bars (*see Wires and Cables, Bare*), Spelter. Manganese Bronze Ingots, Cement, Lime, Clay Products, Preservative Coatings, Road Materials, Timber, Brass Paints, Oils, and various other materials. The price of the 1921 Book of Standards is $11.00 to non-members. Single standards may be purchased for 25 cents.

BUREAU OF STANDARDS. — The Bureau of Standards at Washington is a branch of the U. S. Department of Commerce. It is the official custodian of the fundamental standards of measurement. In addition to making tests and comparisons of measuring apparatus, the Bureau carries on numerous researches related to the establishment and maintenance of the various standards and units of measurement, the development of measuring instruments and methods of measurement, the determination of physical constants and the properties of matter. The results of these investigations are published in pamphlet form issued in two separate series: (1) *Scientific Papers* and (2) *Technological Papers*. In addition, the Bureau issues a third series, *Circulars*, giving useful technical data, standard specifications for apparatus, description of the nature of the standard tests carried out at the Bureau, etc.

A complete list of the publications of the Bureau will be furnished by the Bureau upon request and free of charge; this list is contained in *Circular No. 24*, which contains an index and a brief summary of each of the *Scientific* and *Technological Papers*, and *Circulars* most of which can be furnished by the Bureau upon request and free of charge. Others may be purchased from the Superintendent of Documents, Washington. D. C.

The *Bulletin of the Bureau of Standards* is a serial publication in which all *Scientific Papers* are published collectively. Each number of the *Bulletin* contains about 150 pages, the separate numbers being issued as material accumulates, at intervals of about three months. Four numbers constitute a volume. The bound volume is now (1921) issued under the title "Scientific Papers." Sixteen volumes had been issued at the beginning of 1921. The complete *Bulletin* is furnished free to educational and scientific institutions. Individuals may obtain separate numbers in paper covers (current or back num-

bers) at 25 cents per copy, complete volumes bound in cloth at $1.50 per volume, and may subscribe in advance at the rate of $1.00 per volume to receive the four separate numbers as issued. Orders and payments for the *Bulletin* should be addressed to the Superintendent of Documents, Washington, D. C., and *not* to the Bureau of Standards.

The principal publications of the Bureau dealing primarily with specifications and standards of interest to the electrical engineer are: Electrical Units and Standards (Circular 60); Specifications for Incandescent Lamps (Circular 13); Copper Wire Tables (Circular 31); Wire and Cable Terminology (Circular 37); and the "National Electrical Safety Code" (Circular 54), which is a general code dealing with safety, covering electrical supply stations, electrical line construction, electrical utilization equipment, and electrical operation.

ELECTRIC POWER CLUB. — (*Secretary, C. H. Roth, 1410 West Adams St., Chicago, Ill.*) This Club is an association of companies engaged in the manufacture of electric power apparatus. Its standardization rules, which are printed in the Handbook of the Club, deal with the industrial standardization of motors, generators, transformers, and electric power control apparatus. The pamphlet referred to may be obtained from the Secretary of the Club (price, 50 cents).

ILLUMINATING ENGINEERING SOCIETY. — (*General Secretary, Clarence L. Law, 29 West 39th St., New York City.*) The Committee on Nomenclature and Standards of this Society published a report in 1918. The definitions in this report are substantially the same as those incorporated in the *Standardization Rules of the A.I.E.E.* (1921 edition). The Society has also published Automobile Headlighting Specifications, and a Code of Lighting for Factories, Mills and other Work Places.

INSTITUTE OF RADIO ENGINEERS. — (*Secretary, Alfred N. Goldsmith, College of the City of New York, New York City.*) See the reports of the Standards Committee of this Institute. The standard nomenclature adopted by this institute is included in the *Standardization Rules of the A.I.E.E.* bearing on radio matter.

NATIONAL ELECTRIC LIGHT ASSOCIATION. — (*Executive Manager, M. H. Aylesworth, 29 West 39th St., New York City.*) The Apparatus Committee of this Association has published Transformer Standards; Methods of Testing Insulating Oils; and Rules for the Installation and use of Motors on Central Station Distribution Systems. The Underground Systems Committee has published Specifications for Lead Covered Paper Cables. Other committee reports deal with Railway Rates, Transportation, Accounting, Prime Movers, Overhead Line Construction, Lamps, Electrical Apparatus, Meters, Terminology, Grounding Secondaries, Underground Construction, Electrical Measurements and Values and Street Lighting.

The N.E.L.A. issues the *Electrical Meterman's Handbook*, prepared by the Committee on Meters. This book is intended primarily for practical meter men. The *Meterman's Handbook* deals with the subject of metering in great detail, containing over 1000 pages. It may be had from the Secretary of the Association for $5.00.

NATIONAL FIRE PROTECTION ASSOCIATION. — This Association issues the *National Electric Code* which contains the rules and requirements of the National Board of Fire Underwriters for electric wiring and apparatus. The *Code* is revised every two years, the next revision being in 1922. A *List of Electrical Fittings* is also issued by the National Board of Fire Underwriters. This is a list of fittings which have been examined and are suitable

for the use intended; this list is revised semi-annually. Other publications set forth suggestions, rules and regulations of the National Board in regard to special installations of various kinds which involve fire hazards, and in regard to apparatus and equipment for fire protection. Single copies of the *Code* and *List of Electrical Fittings* may be obtained free of charge from the National Board of Fire Underwriters, 135 William St., New York City.

The Underwriters' Laboratories. — (*President, W. H. Merrill, 207 E. Ohio St., Chicago, Ill.*) This organization issues detailed rules and regulations covering inspection and approval of electrical fittings and devices by the Laboratories.

The Associated Mutual Fire Insurance Companies. — (*Secretary, H. O. Lacount, 31 Milk St., Boston, Mass.*) This organization issues lists of electrical fittings and of fire-protection devices approved by these companies under the general rules of the National Fire Protection Association.

SOCIETY OF AUTOMOTIVE ENGINEERS. — (*General Manager, C. F. Clarkson, 29 West 39th St., New York City.*) The Society publishes its standards, recommended practices, and data sheets in loose-leaf form in its Handbook. These include the automobile, aeronautic, motorboat, and tractor fields, and are classified as follows: Miscellaneous Fittings, Tires and Wheels, Materials, Bearings, Engine Fittings, Lighting, Aeronautic Standards, Truck Standards, Electrical Installations, Springs, Electrical Vehicle, Motorboat, Tractor, and Military Motorcycle. The price of the *Manual* is $10.00, and that of individual sheets, 6 cents each.

OTHER ENGINEERING BODIES. — The Bodies listed below, through committee reports or otherwise, have published specifications and standards or matter relating thereto. The Bureau of Standards is now (1922) preparing a publication on bodies interested in engineering and industrial standardization.

American Electrochemical Society, *Secretary*, J. W. Richards, Lehigh University, South Bethlehem, Pa.

American Gas Association, *Secretary*, Oscar H. Fogg, 130 East 15th St., New York City.

American Society of Heating and Ventilating Engineers, *Secretary*, C. W. Obert, 29 West 39th St., New York City.

American Society for Municipal Improvements, *Secretary*, Charles C. Brown, 803 Lincoln Ave., Valparaiso, Indiana.

Associated Manufacturers of Electrical Supplies, *Secretary*, C. E. Dustin, 30 East 42nd St., New York City.

Association of Railway Electrical Engineers, *Secretary*, J. A. Andreucetti, C. & N. W. Terminal Bldg., Chicago, Ill.

Association of Iron and Steel Electrical Engineers, *Secretary*, J. F. Kelly, Empire Building, Pittsburgh, Pa.

Bureau of Mines, *Acting Director*, E. A. Holbrook, Washington, D. C.

National Association of Electrical Contractors and Dealers, *General Manager*, W. H. Morton, 110 West 40th St., New York City.

National Screw Thread Commission, *Secretary*, H. W. Bearce, Bureau of Standards, Washington, D. C. This is a government body. A progress report (1921) has been issued.

INTERNATIONAL ELECTROTECHNICAL COMMISSION. — (*General Secretary's Office, 28 Victoria St., Westminster, London, S. W., England.*) This Commission, which was organized in London in 1906, is made up of National Committees in about 25 countries. The United States National Committee, (*Secretary, Dr. A. E. Kennelly, Harvard University, Cambridge, Mass.*) is

appointed by the President of the American Institute of Electrical Engineers, and keeps in close touch with the A.I.E.E. Standards Committee. There are at present Special Committees on Prime Movers, Rating, Nomenclature, Symbols, Units, and Standard Pressures.

The Commission meets in full session about every two years. The last meeting was in 1921. These meetings are for the purpose of discussing and acting upon the reports and recommendations of the Special Committees.

The Central Office has, from time to time, published bulletins giving reports of the meetings of the different committees of the Commission. The list of *International Symbols* (*see Symbols and Abbreviations*), and *the Rules for the Rating of Electrical Machinery*, published by the Commission, are printed as appendices to the *Standardization Rules of the American Institute of Electrical Engineers*,

FOREIGN STANDARDIZING BODIES. — There are now national standardizing bodies in Austria, Belgium, Canada, Czecho-Slovakia, France. Germany, Holland, Hungary, Italy, Japan, Sweden and Switzerland. These organizations serve as clearing houses and bureaus of information on engineering and industrial standardization in their respective countries. Information about the work of these organizations can be obtained from the American Engineering Standards Committee, 29 West 39th St., New York City, where copies of all specifications and standards approved by the various national bodies are on file.

STARTERS, MOTOR. — (*See also Controllers; Motors, Alternating-Current; Motors, Direct-Current; Rheostats*) An auxiliary resistance which is used with a motor, either d-c. or a-c., during acceleration is called a starter, or a starting rheostat. A compensator, or induction starter, consists of an auto-transformer and a switch, by the operation of which a reduced voltage is supplied to the terminals of an induction motor for starting.

STARTERS FOR DIRECT-CURRENT MOTORS. — The starters for use with constant speed d-c motors consist usually of a resistance to be inserted in the armature circuit, this resistance being gradually cut out by the movement of a contact arm over a face plate as the motor comes up to speed. Such rheostats connect the motor field in circuit at the first step and are provided with various safeguards such as low-voltage release, overload release, etc.

Resistance Steps for D-C. Starters. — The resistance is designed to limit the starting current to about 1½ times full load current with enough steps to limit the voltage between contacts. Motors of 10 horsepower or less usually are brought up to speed in about 15 seconds and larger motors in about 30 seconds. Resistors should carry full load current for two minutes starting cold with an average temperature rise not exceeding 250° C. Resistors will withstand 500° C. without deterioration.

Fig. 1 shows the connections of a typical d-c. starter with low-voltage release. If the rheostat arm is moved from the off position shown in the cut to the contact *R*–1. current will flow from *L*+ to the arm; from this to contact *R*–1. through the regulating resistance to *R*–11, thence through the armature and series field of the motor to *L*–. The shunt field is connected from *R*–1 to *L*–. As the rheostat arm is being moved from *R*–1 to *R*–11 there is a small drop in voltage across the shunt-field circuit due to the field current flowing through the starting resistor but this is so small that it may be neglected and the field can be considered as having full voltage impressed upon it. The rheostat arm is provided with a spring which returns it to the off position when the handle is released during the starting of the motor. After the motor has been brought up to speed and the rheostat arm rests upon contact *R*–11 the low-voltage release magnet holds the arm in this position. Brush *B* bridges between the terminals *M* and *N* so that in the running position the current passes from *L*+ to terminal *M*, through the brush *B* to the terminal *N*, thence to the armature of the motor and through the series field to *L*–.

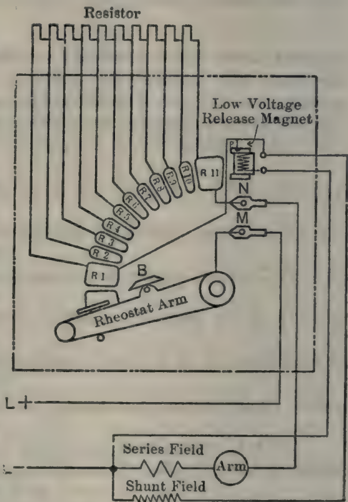


Fig. 1.

Overload Release. — An overload release is often furnished, especially with motors of large capacity. This device consists essentially of a solenoid which is connected in series with the motor armature and which, in case of a predeter-

mined overload, opens the circuit of the low-voltage coil. This action releases the contact arm and thus stops the motor.

Mounting of Starting Boxes. — Starting rheostats are occasionally mounted on a small panel, together with the line switches and fuses or circuit breaker required for the complete protection of the motor circuit.

Multiple Switch Starters. — With the larger d-c. motors, where the starting conditions are severe, the face type of starter is not found satisfactory; recourse is therefore had to multiple switch starters, drum controllers, or contactors. Multiple switch starters consist essentially of a number of switches mounted on a panel and a separate resistor usually of cast-iron grids. The switches are mechanically interlocked so that it is necessary to close them in sequence.

The type of starter shown diagrammatically in Fig. 2 is used with 110-volt motors up to 300 h.p. and with 220- and 500-volt motors up to 600 h.p. The first switch on the right of the diagram is provided with magnetic blow-out-coils and acts as a circuit breaker. It also has voltage release device. Protection against inadvertently leaving part of the switches open when starting is provided by means of an auxiliary push-button switch that must be held closed until the last switch on the left is closed, thus energizing the coil of the low-voltage release.

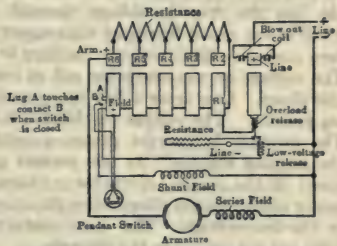


Fig. 2. Connections of Multiple-switch Starter

Starters with Automatically-operated Contactors. — In starting a motor the energy required to overcome the inertia of the motor and the apparatus it is driving must be admitted gradually, which is accomplished by introducing resistance into the armature circuit. If the time taken in starting is too long the resistor may be injured by overheating; if the time is too short the motor may be damaged, the supply circuits seriously disturbed, or gears stripped on the driven machine. For this reason a motor starter that will automatically take care of the proper rate of acceleration presents many advantages. Such a starter can be devised by means of contactors and suitable relays, which are operated either by the variation of the voltage drop across a resistor as the motor speeds up, or by the decrease in current that permits a solenoid to drop its core, or by other means of acceleration.

Contactor Starter with Series Relay Control (Fig. 3). — Acceleration by series relay control is satisfactory where the voltage does not vary more than 12½ per cent either way from a constant value. Fig. 3 shows the connections of a shunt motor controlled by means of two contactors, a push button station, and a series relay. When the push button is closed in the starting position, the coil of contactor No. 1 is energized, closing the contactor and completing the circuit through the series relay, the starting resistor,

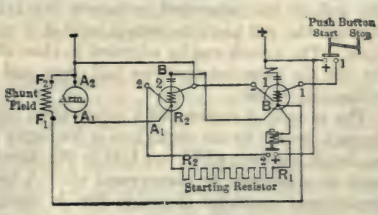


Fig. 3.

and the armature of the motor. When the starting current has dropped to a certain value the current in the series relay is no longer sufficient to hold open the contacts so that connection is automatically made to the operating coil of contactor No. 2 closing that contactor and short circuiting the starting resistors. With large motors several contactors with series relays are provided.

A Contactor with Counter E.M.F. Control is shown in Fig. 4. With this method of starting, the operating coils of the three contactors have one side connected to the motor brush farthest away from the starting resistor. The other sides of the operating coils are connected to the taps on the starting resistor, the coil on contactor 1, being connected to R_2 on the resistor. The voltage on this coil is equal to the line voltage less the drop in voltage through the first section of the resistor. As the speed of the motor increases the counter e.m.f., causes a decrease in the armature current. This reduces the drop to the first section of the starting resistor. The voltage on the operating coil of contactor 1, is gradually increased until this closes. Contactor 2 has its operating coil connected to R_3 on the starting resistor. The voltage on this coil is increased by the closure of contactor 1. The increase in current causes a considerable drop in the second section of the starting resistor. As this current gradually decreases, due to the increased speed of the motor, contactor 2 closes. Contactor 3 is connected across the motor armature and closes when the counter e.m.f. of the motor is nearly equal to the line voltage. The main contactors for closing the main + and − circuits are not shown in the diagram.

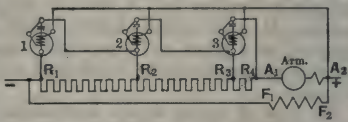


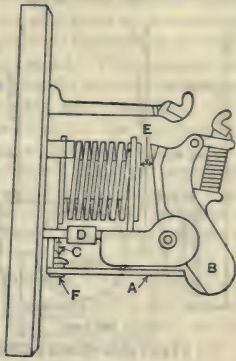
Fig. 4.

Series Lockout Contactor Method. — Another scheme of acceleration frequently used is the series lockout method. With this arrangement the magnetic contactor is provided with a series coil and does not require a separate relay for controlling it. The closing of the magnetic contactor depends upon the saturation of the iron in one portion of the magnetic circuit. This can be understood from the diagram of a contactor of this design shown in Fig. 5. The flux or magnetism in the iron is caused by the current flowing through the operating coil. This flux passes through the air gaps in the armature of the contactors. Part of this flux passes from the armature through the armature brackets to the magnetic yoke and thence to the magnet core. Another part of the flux passes from the armature through the tail piece of the magnet yoke. The flux through this last circuit exerts a pull which prevents the contactor from closing. The magnetic path through the armature brackets is of small cross-section so that when the current flowing through the operating coil exceeds a certain value it becomes saturated and forces the balance of the flux through the tail piece holding the contactor open. As the current decreases the flux in the saturated armature bracket remains constant and the flux through the tail piece decreases until it is not sufficient to hold the contactor open. The switch can be adjusted to close at a predetermined value by changing the air gaps between the tail piece and the magnet yoke by means of a calibrating screw.

The success of starters with acceleration control such as previously described depends largely on the contactors or contactor switches. Contactors are switches or circuit breakers which are held in the closed position by some auxiliary power such as a solenoid or compressed air. A typical contactor is very similar to the lockout contactor shown in Fig. 5, using a shunt coil instead of the series coil and omitting the damping coil and similar features so that as soon as the shunt coil is energized the contactor is closed against the pressure of a spring.

Contactors are built both for d-c. and a-c. operation and are made single pole and multiple, single throw and double throw, in various capacities. The main arcing contacts are usually protected by means of a magnetic blowout.

These contactors are ordinarily used in connection with a master switch or controller and protective relays of various kinds to insure the performance of various functions, such as the automatic cutting in and out of resistance in the secondary of an induction motor to maintain constant input to a flywheel set or any similar feature that may be desired such as overload, low voltage and field protection relays, limit and sequence switches, etc.



A is tailpiece
B is armature
C is lockout air gap
D is damping coil
E is closing air gap
F is calibrating screw

Fig. 5.

STARTERS FOR INDUCTION MOTORS may be divided into two classes: (1) those used with motors having a squirrel cage or short-circuited secondary, and (2) those for motors having a wound secondary. In the former case the starting is done by impressing on the primary a voltage sufficient to induce in the short-circuited secondary the current required to develop the proper starting torque, and then transferring the primary connections to full voltage. With induction motors having phase-wound secondaries the method of starting is to connect the primary circuit directly to the line, with the secondary winding short-circuited through a resistance which is cut out in one or more steps.

With squirrel-cage motors up to about 7½ horsepower it is usually feasible to connect the primary immediately to the full line voltage without drawing an abnormal current from the line. The limiting feature is the power disturbance and its effect on other loads, particularly lighting. With larger squirrel cage motors this is not feasible and consequently there have been developed various means of reducing the voltage impressed on the motor. Motors with special windings and all leads brought out, can be arranged for series parallel grouping of coils, delta-star connections for three phase, and other similar schemes but these are not used to any appreciable extent in America.

Starting Compensators. — Under normal conditions the most satisfactory means of obtaining the reduced voltage for starting induction motors with squirrel-cage secondaries is by the use of auto-transformers or compensators (q.v.). The auto-transformers supplied for starting induction motors are provided with taps permitting the choice of any one of several voltages. The auto-transformers are designed for starting service only and are not intended to be left permanently in circuit.

Connections of Starting Compensator (Fig. 6). — The connections of the starting compensator for a two-phase motor are shown diagrammatically in Fig. 6, the switching mechanism being omitted for the sake of simplicity. Two auto-transformers are used with a two-phase motor; with a three-phase motor there would be used either three Y-connected or two V-connected auto-transformers. In the starting position the voltage at the motor primary which is connected to the auto-transformer is cut down by the auto-transformers from 200 to 130 in this particular case.

The auto-transformers are provided with taps giving 50, 65 and 80 per cent of line voltage for starting, though it is usually found that the 65 per cent tap gives the proper conditions for average service. With the switch in the running

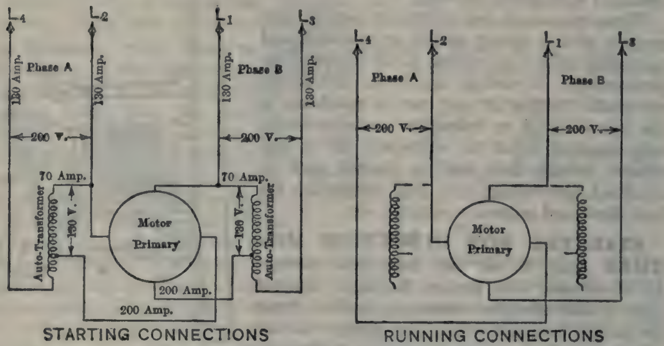


Fig. 6. Connections of Starting Compensator

position the auto-trasformers are disconnected from the circuit and the motor connected to the full line voltage.

Switch Mechanism. — The auto-starter switches or circuit breakers are made of various types, with contacts suitable for the current to be handled. The equivalent of a double throw switch is usually furnished, one throw energizing auto-transformers and connecting the motor to low-voltage taps for starting and the second throw connecting the motor directly to full voltage for running. The second throw always disconnects enough circuits so that no current flows through any part of the winding of the transformer and sometimes completely disconnects the auto-transformers from the circuit. The handle of the switch has three positions and locks automatically in the off and running positions but has to be held in the starting position. In starting a motor the handle is first moved very quickly to the starting position and held there until the motor gets up to speed; the handle is then moved quickly past the off position to the running position where it becomes locked. A mechanical device prevents throwing the handle to the switch from the off position to the running position without moving first to the starting position.

Automatic protection with auto-starters is usually secured by means of overload trip coils, either connected directly in the circuit or operated from current transformers. Usually the overload protection is only in the running position. With certain types of auto-starter switches the overload release device consists of two solenoids with plungers in two different phases and an oil-filled dash pot on each solenoid plunger gives an inverse time element feature. The switch contacts usually trip independently of the handle so that the switch cannot be held closed on an overload.

Resistance Starters for Induction Motors. — With motors having a wound secondary it is customary to connect the primary to full line voltage at starting and to short-circuit the secondary through a resistor. As the motor speeds up, this secondary resistance is cut out in one or more steps until at full speed the secondary is short-circuited. By properly designing the resistors for continuous service instead of for intermittent service, this type of control can be used for speed regulation as well as for starting purposes (*see Controllers*).

With constant-speed induction motors up to 200 h.p. output it is sometimes possible to mount the starting resistor upon the rotor spider, and to control it by butt-contact switches operated by a rod which passes through the hollow rotor shaft. By moving this rod out or in by means of a knob at its outer end the resistance may be connected into the secondary circuit for starting or be disconnected therefrom after the motor has come up to speed. With large motors the terminals of the secondary windings are connected by means of slip rings and a drum controller to the resistor, which in this case consists of three resistors mounted separately from the motor. Each resistor has one terminal connected with one of the three phases of the motor secondary.

The other terminals of the resistors are usually connected in star but sometimes in delta, depending on the current in the motor secondary, the capacity of the resistors, etc. When in star, the neutral is usually grounded to the frame of the controller. The drum of the controller short-circuits the various sections of the resistor by steps.

Resistance Steps for Induction Motor Starters. — Wound secondary motors at standstill act as transformers and the resistance in the secondary circuit has to absorb practically the entire input to the motor; as the motor speeds up, the secondary voltage drops off and resistance is cut out of circuit at such a rate that the motor draws from the line about 50 per cent more than full load primary current. Grid resistors for starting service will rise about 250° F. in two minutes. Grid resistors for speed regulation are designed for the same rise on continuous service.

Starters for "Self-starting" Synchronous Motors. — Where self-starting synchronous motors are used they are provided with a squirrel-cage winding on the rotor in addition to the usual field poles and field coils. Owing to this squirrel-cage winding they are started up as induction motors and controlled by the same type of starting devices. Suitable auxiliary apparatus, such as field rheostats and field switches, must be provided.

CALCULATION OF STEPS FOR RESISTANCE STARTERS.* — (*See also Rheostats.*) Resistance starters usually have from four steps for small motors to seven or ten for large sizes. The number of steps used is commonly somewhat larger than the computed values as a safeguard against excessive starting currents due to improper handling by inexperienced operators. Starters for motors starting under heavy loads are frequently designed with two or three extra steps by which it is permitted to switch the loaded machine on the line with a low current and to raise the current step by step. On the other hand, in stations where experienced operators are available, large motors and synchronous converters are often started on three or four steps.

Calculation for Shunt-motor Starters. — The design is usually based on a fixed maximum current on each step of the starter, as well as on a fixed minimum current for each step. The minimum current must not be taken as less than the load current required for the maximum load with which the motor is to be started.

Let

I_m = the maximum current on each step of the starting rheostat,
I_0 = the lowest current for which the steps are to be designed,
E = the fixed line voltage,
$C = I_m \div I_0$.

Let $K - 1$ be the number of steps required in the rheostat, and let $R_1, R_2, \ldots R_k$ be the total resistances in the circuit, including the resistance of the motor, for the successive positions of the contact arm. K is the number of working

* By O. R. Schurig.

contacts over which the contact arm moves, the first contact being when all the resistance of the rheostat is in the circuit and the Kth contact corresponding to all the resistance of the rheostat cut out; whence $R_K = r =$ the motor resistance and $R_1 = E \div I_m$. This equation does not take into account the inductance of the motor resistor.

It can be shown that the successive resistances form a geometric series with the ratio C, whence

$$K = 1 + \frac{\log\left(\frac{E}{rI_m}\right)}{\log C}, \qquad \text{and} \qquad R_1 = \frac{E}{I_m}, \quad R_2 = \frac{R_1}{C}, \quad R_3 = \frac{R_2}{C}, \quad \text{etc.}$$

The value of C must be so chosen that K is a whole number. For most motors which are likely to start under load, the value of C is taken about 1.5. For machines which are started with no load, a larger value for C is usually taken, the limiting value being determined by the ratio of maximum allowable current at starting to current at no load. The maximum allowable current is commonly limited to 1.5 times full-load current. Occasionally twice full-load current is taken, but the lower value is preferable, particularly if both power and lighting services are connected to the same feeders.

Design of Series Motor Starters. — An algebraic method for the computation of the various steps for series-motor starters is rather complicated because the field flux varies with the line current. A graphic method based on equal fluctuations of current on each step of the starting rheostat is as follows.

Let $E =$ the fixed line voltage, I_m the maximum allowable current during starting, and $I_0 =$ the minimum allowable current during starting.

Construct the rectangle AI_mOB as in Fig. 7, in which (AI_m) is the maximum allowable current during starting, and $(OR_1) = E \div I_m$ is the entire resistance of the circuit when the contact arm of the starter is on the first operating contact. Lay off (AI_0) equal to the minimum current during starting. Determine from the magnetization curve of the machine, at some definite speed the armature voltage E_0 for the current I_0, and the armature voltage E_m for the current I_m at this same speed (*see Motors, Direct-current*). Lay off (OB_0) equal to $(OB) \times \frac{E_0}{E_m}$, and draw the two straight lines (I_mB_0) and (I_0B). Lay off (Or) equal to the resistance of the motor between terminals, and draw (rs) parallel to (OB), and find the point D where (rs) intersects (I_mB_0). Then draw the zig-zag line $I_0v_1Cv_3Fv_5$ . . . If this line does not meet the point D, then the ratio $I_0 \div I_m$ and the corresponding ratio $E_0 \div E_m$ and the distances (AI_0) and (OB_0) must be altered until the new zig-zag line meets the new point D. Then extend the horizontal line Cv_1, Fv_3, . . . until they cut the vertical axis, at R_2, R_3, . . . The resistances of the successive steps (resistances *between* successive contacts) are then equal to the distances (R_1R_2), (R_2R_3), . . .

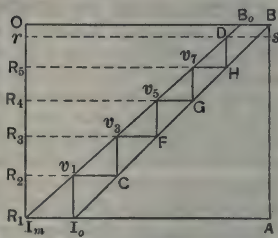


Fig. 7. Graphic Method of Design of Series Motor Starters

For machines working on a straight line magnetization curve, the steps for the starter would be equal, as the line (I_0B) would be parallel to the line (I_mB_0).

Graphic Method for Shunt-motor Starters.—The above method may also be applied to the determination of the steps of a shunt-motor starter, in which case the point B_0 coincides with B.

COST OF MOTOR STARTERS. — The following figures are approximate only and are intended merely to give a rough idea of the cost of some of the common types of starters in 1921.

COST OF STARTERS FOR 220-VOLT MOTORS *

Horsepower of Motor	1	5	25	100	250
Face plate type	....	$12	$38	$140	.....
Multiple switch type	....	....	.....	500	$600
Contactor type	$45	53	105	450	800
Compensators (auto starters)	....	80	90	200	590

* Starters for 110 or 550-volt motors cost more than 220-volt motors for most sizes.

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STARTING AND LIGHTING SYSTEMS FOR AUTOMOBILES.

— (*See also Batteries, Storage; Generators, Direct-current; Ignition, Electric; Lamps, Incandescent; Motors, Direct-current.*) The items comprising the electric starting and lighting equipment of a gasoline automobile are (1) the generator, (2) storage battery, (3) starting motor, (4) lamps, (5) wiring, (6) switches, cut-outs, and fuses. In the so-called "single unit" systems the generator and starting motor are combined in a single unit. The battery is used not only to supply energy for starting and lighting but also for ignition (*see Ignition, Electric*).

GENERATOR. — The generator employed is almost invariably shunt wound, and is driven directly by the engine through suitable gears or chains. The purpose of the generator is to charge the storage battery, which is connected across its terminals. In order to keep the terminal voltage of the generator substantially constant, which condition is required for satisfactory charging of the battery, some auxiliary controlling device is employed, as otherwise the terminal voltage of the generator would vary directly as the speed of the engine.

Battery Cut-out. — The generator must of course not be connected to the battery until its voltage is equal to that of the battery. This is usually taken care of by means of an automatic switch which closes when the engine reaches a speed corresponding to a car speed of about 10 miles per hour, and opens when the speed falls below this value.

Common Types of Voltage Control. — Numerous devices have been proposed and tried out for keeping the generator voltage substantially constant after the battery cut-out closes, but at the present time only three types of control are in common use, viz., (1) outside regulation, (2) third brush regulation, (3) reverse series-field regulation.

Outside Regulation. — A simple form of outside regulation is shown in Fig. 1, which embodies the principle of the Bijur regulator. An external resistance is included in the field circuit, which resistance is short circuited when the vibrator is in contact with the contact *C*. When the vibrator is pulled away from the contact *C* by the electromagnet the resistance is inserted in the field circuit, and the field current is thereby reduced. As long as the vibrator is operating, the field current will vary between maximum and minimum values, and its average value will depend upon the rate of vibration. The rate of vibration in turn will depend upon the voltage across the winding of the electromagnet, and a very small increase in this voltage will cause a relatively large increase in the rate of vibration, and therefore make a substantial decrease in the average value of the field current. Consequently, as the speed of the engine varies, thereby tending to produce a varying voltage at the generator terminals, this tendency is counteracted by the reduction in the field current, so that the terminal voltage of the generator remains substantially constant for all values of the speed.

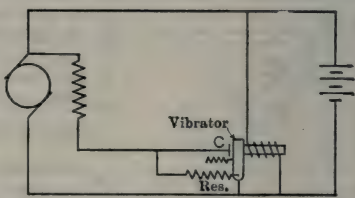

Fig. 1.

A modification of this type of control is used by the Westinghouse Company. In the Westinghouse regulator, the position of the contact *C* is controlled by an electromagnet connected across the terminals of the generator, and the vibrator, is actuated by an eccentric coupled to the armature shaft. As the generator

voltage increases, the contact C is drawn away from the vibrator, with the result that the resistance is inserted in the field circuit for a greater length of time.

Another form of outside regulation which is coming into favor on vehicles is similar to that of the train lighting systems (*see Lighting of Trains*). The plain shunt machine is used and regulation is accomplished by passing the armature current through a solenoid which in turn controls a plunger linked to a movable arm of a small rheostat.

Third Brush Regulation. — In this system use is made of the distorting effect of armature reaction (*see Generators, Direct-current*). When current flows through the armature, the resultant flux is skewed around in the direction of rotation (see Fig. 2), with the result that the number of lines of force entering the armature from the leading pole-tip is reduced and a corresponding increase in the number of lines of force takes place at the trailing pole-tip. Consequently, referring to Fig. 2, it is evident that the electromotive force generated between the brush A and the third brush C, located as shown, will decrease as the current through the armature increases. If the shunt field is connected between the brushes A and C, the current through the shunt field will therefore decrease as the current in the main circuit increases, which in turn will reduce the electromotive force between A and C still further. Consequently, if the battery is connected across the main brushes A and B, any tendency for the current to the battery to increase when the speed of the generator increases is counteracted by the decrease in the field current. Such a generator when connected to a storage battery will tend to act as a constant current machine, irrespective of the speed. As a matter of fact, due to the effect of the short-circuit current in the coil passing under the brush C, which current is practically proportional to the speed and which produces a demagnetizing action, the current to the battery actually reaches a maximum value at a definite speed and then falls off.

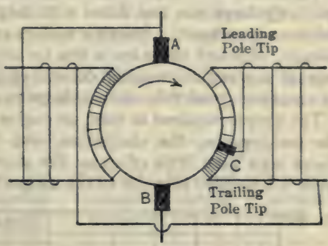

Fig. 2

This system of regulation has come into extensive use, due to its simplicity. The maximum current supplied to the generator is readily controlled merely by shifting the position of the intermediate brush. For a full treatment of the theory of third brush regulation, see Langsdorf's *Principles of Direct-current Machinery*.

Reverse Series-field Regulation. — In this system of regulation a compound wound generator is employed, with the series field connected so as to *oppose* the shunt field, i.e., differentially compounded. Up to the present this system has found but little favor, on account of the inability to control its output. It gives neither a constant voltage nor a constant current characteristic, the current output rising rather rapidly to a maximum and then gradually dropping off, due chiefly to the cross magnetization of the armature current.

BATTERY. — The lead-acid type of storage battery is almost invariably used (*see Batteries, Storage*). Batteries of from 6 to 24 volts are employed, although the 6-volt (3 cell) battery is rapidly becoming the standard. To take care of starting requirements, an ampere-hour capacity of from 60 to 120 is required, depending upon the size of car. A 60-ampere-hour, 6-volt bat-

tery weighs about 45 pounds, and a 120-ampere-hour, 6-volt battery about 70 pounds.

STARTING MOTOR. — The conditions under which the starting motor operates are widely different from those encountered in other fields, in that an extremely high initial torque is required to overcome the inertia of the engine. There is also a constant variation in load, due to the alternate compression and suction of the engine. As the service periods are of short duration, the motor is designed to give its maximum output for only a very short time, and is therefore relatively small compared with a motor designed to give the same maximum torque continuously.

The motor is compact and rugged to withstand vibrations. Shafts are of relatively large size and fitted with large bearings, to withstand the strains of sudden engagement. Starting motors are usually designed for 6 volts terminal voltage, although there are a few 12- and 24-volt systems in use. To start the average size engine an initial current of about 200 amperes at 6 volts is required. At cranking speed this current drops to about 80 amperes.

The torque required to start an engine is dependent upon the bore, the stroke, the compression, the clearances, and the lubricant used. Air temperatures above zero have little effect on the starting torque, but at lower temperatures there is a decided increase in the torque required. Fig. 3 shows, for an average

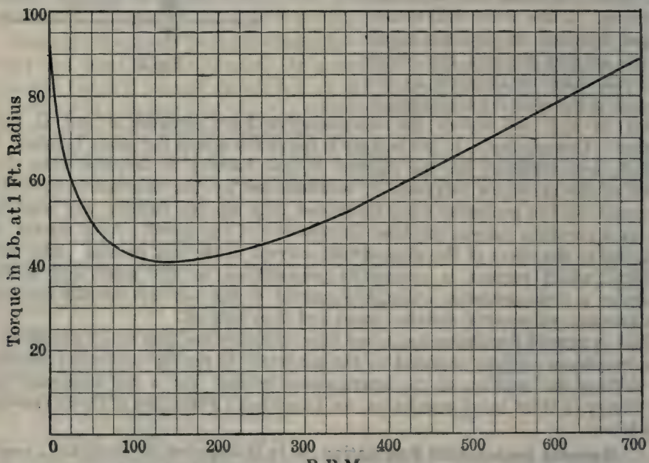

Fig. 3

engine at normal temperature, the variation with speed of the torque required to spin the engine.

The torque required to start an engine, under normal conditions, is usually about 1 pound-foot for each 5 cubic inches of piston displacement. The starting motor is usually designed so that under normal conditions it will spin the engine at about 100 r.p.m., at which speed the torque required is about one-half the starting torque, or 1 pound-foot for each 10 cubic inches of piston displacement. The speed at which the motor will keep the engine spinning at constant speed is called the "cranking" speed, and the corresponding torque the "cranking torque," as distinguished from the "starting torque," which is the initial torque required to start the engine turning over.

The speed of the starting motor when developing its cranking torque ranges from 1800 to 3000 r.p.m., depending upon the design. The gear reduction employed ranges from 8 to 1 to upwards of 25 to 1.

LAMPS. — There are two common types of lamps, namely, focusing and non-focusing. In the focusing type, used for head and spot lights, the filament is wound into a compact spiral, which brings the source of light as near a point as possible. The non-focusing lamps are usually constructed with a looped type filament. Lamps of this type are used for tail lights, cowl lights, and side lights.

Tungsten filament lamps are now used almost exclusively. Both the vacuum type and the nitrogen filled type are employed, the former consuming about 1 watt per candle-power and the latter about ¾ watts per candle-power. The life of these lamps ranges from 100 to 300 hours under ordinary service conditions.

The following table, taken from a paper by H. Schroeder (*Trans. Soc. Aut. Eng., 1916*), shows the variation of the characteristics of tungsten lamps with the condition of the battery.

Battery condition	Per cent, normal volts	Per cent, normal amps.	Per cent, normal watts	Per cent, normal C. P.	Per cent, normal W. P. C.
Fully discharged (1.8 volts per cell)..	68	79	54	23	233
Fully charged (2.6 volts per cell)..	120	111	133	190	70

STARTING SWITCH AND FUSES.— Switches used on starting systems are of two types, viz., (1) those completing the circuit directly and (2) those using a solenoid to complete the circuit.

Of the first, the so-called harpoon type is most generally used, consisting of terminal connections to laminated copper sections at either side of the switch box. A movable member extends through the cover and is held extended by a spring below. Depressing this plunger completes the circuit. The solenoid type consists of a plunger closing the main circuit to the motor after the coil has been excited.

Automobile fuses are now made of standard capacity for maximum currents of 10, 20, 30 amperes. Two sizes of fuses are used, namely,

1¼ in. length overall with ferrules ¼ in. in diameter,
1½ in. length overall with ferrules 13/32 in. in diameter.

Automobile fuses are usually of the sight type.

WIRING. — There are two common systems of wiring in use, i.e., the single-wire and the double-wire systems. The single-wire system is the more common, having the advantage of cheapness and simplicity. In this system, the metal portions of the chassis are used to complete the circuit.

The double-wire system is used when leads are not located adjacent to the metal parts of the chassis. It has the disadvantages of being more expensive, and the danger from open circuits is twice as great as in the single wire.

For lighting circuits and connections between battery and switch, wires of from No. 14 to No. 10 B and S. Gauge are generally used. It is well to use heavy wire, as it costs but little more, and there is a decided saving in electrical energy. No. 14 stranded copper wire, rubber covered and cotton wrapped

is generally used from switches to side lights, tail lights, cowl lights, etc. No. 10 stranded copper wire, rubber covered and cotton wrapped, is generally used for the lines where heavier currents are carried, e.g., from generator to battery, switch to head lights, horn, etc.

The size of starting motor cable in general use ranges from No. 4 to No. 0 B. and S. Gauge. The length should always be kept as short as possible, to overcome excessive losses in voltage. A good rule is to use such a size of cable that for the length required the voltage drop in the cable when carrying 500 amperes will not exceed 0.3 volt.

SPECIFICATIONS FOR INSTALLING ELECTRIC EQUIPMENT. — The following specification for electrical installations is abstracted from the S.A.E. specification.

Insulation Requirements. — Electrical apparatus from 6 to 25 volts shall be capable after installation of withstanding 500 volts, the test being applied between conducting circuit and ground. Tests of this nature must cover entire circuit.

Exception. — Batteries will not be subjected to any insulation test above their working potential.

Use of Conduit. — Insulated conductors must be protected by metallic or non-metallic conduit outside of insulation, except where otherwise protected from the elements or where out of contact with metal surfaces.

Metallic conduit ends must be provided with ferrule having rounded edges and having a fit sufficiently tight to exclude free entrance of oil and water.

Where wires are not protected by conduit, they are to be cleated at intervals not exceeding 10 inches. Cleats may be of metal with insulating material interposed between cleats and wire.

Clearance. — Where no conduit is used, as on wooden dash board, wire shall be at least two inches from exhaust pipe.

Connectors. — All connectors shall be so made that when disconnected the section which is alive will have all live parts recessed 1/16 inch or more below end of shell of connector.

Springs used in connectors shall not be relied upon to carry current.

Protection against Accidental Short-Circuits. — All binding posts on fuse and junction block or on instruments, generators, motors, and switches which must have live parts exposed, shall be so constructed, recessed, or installed that an accidental short-circuit cannot be effected by metallic tools while making minor repairs or adjustments.

Bulb Sockets. — Bulb sockets must be so made that the continuity of the return path is not impaired when lamp is subjected to vibration.

In lamps, hinges shall not be relied upon to carry current. Train of connections must be either permanent or so spring held that contact is maintained tight under vibration.

Protective Devices. — The current to all lighting and signal circuits must be passed through protective devices.

Protective devices shall be on battery side of switches and junction boxes.

Temperature Test. — All insulating material used in connection plugs, sockets, and similar devices for electrical apparatus for use on gasoline automobiles shall be capable of withstanding for 30 minutes a temperature of 300° F.

BIBLIOGRAPHY. — Moyer, J. A., *Gasoline Automobiles;* Dyke, A. L., *Automobile and Gasoline Engine Encyclopedia;* Heldt, P. M., *Gasoline Automobile*, Vol. 3; Hobbs and Elliot, *Gasoline Automobile.*

STEAM. — (*See also Boilers; Calorimeters, Steam; Pipes and Piping; Steam Engines; Steam Turbines; etc.*) Steam at a given pressure and at a temperature such that any decrease in temperature, the pressure being kept constant, will cause a condensation of water, is said to be "saturated." When the temperature is such that a decrease of temperature, the pressure being kept constant, can take place without the formation of water, the steam is said to be "superheated," and the number of degrees that the temperature can be lowered before the formation of water takes place is called the "degrees of superheat." Saturated steam may contain fine particles of water in the form of spray or mist, in which case the steam is said to be "wet," while if there is no such moisture present the steam is said to be "dry." (When the steam produced by a boiler is wet the steam is also said to be "primed," and the amount of moisture is called the amount of "priming.") Superheated steam is always dry. The weight of the actual (dry) steam in a pound of wet steam (steam and moisture) is called the "quality of the steam." The temperature of saturated steam at a given pressure is the same whether the steam be dry or wet, and is the same as the boiling point of water at that pressure. The temperature of saturated steam, or the boiling point, depends solely upon the pressure of the steam.

TOTAL HEAT OF STEAM. — The amount of heat required to change one pound of water at 32° F. into steam at any pressure, p, is called the "total heat" of the saturated steam at this pressure. The number of B.t.u. required to raise the temperature of one pound of water from 32° to the temperature of the saturated steam at the given pressure is called the "heat of the liquid." The difference between the total heat and the heat of the liquid is called the "heat of evaporation."

In the case of superheated steam, heat is also required to raise the temperature of the steam from the temperature corresponding to saturation to the temperature of the superheated steam. The total number of B.t.u. required to change one pound of water at 32° F. into superheated steam is given in the second table below.

ENTROPY OF STEAM. — (*See also Thermodynamics, Principles of.*) In dealing with steam the change in entropy resulting from adding to one pound of the water at 32° F. an amount of heat necessary to raise its temperature to the boiling point is called the "entropy of the water," the change in entropy during evaporation, i.e., the heat of evaporation divided by the absolute temperature of the boiling point, is called the "entropy of evaporation," and the entropy of the water plus the entropy of evaporation is called the "entropy of the steam." The entropy of the water is approximately the quotient of the heat added to 1 lb. from 32° to the boiling point divided by the average of these two temperatures above absolute zero.

STEAM TABLES. — Tables giving the values of the various properties of steam for the range of temperatures and pressures met with in practice have recently been recomputed by Marks and Davis, using the latest experimental results as the basis for their calculations. The following tables are condensed from those given in their *Steam Tables and Diagrams* (N.Y., 1909).

Saturated Steam. — Using the symbols at top of the table:

Gage pressure in lb. per square in. $= p - 14.7$
Vacuum, inches of mercury, $= 29.92 - 2.036\,p$
Entropy of evaporation $= N - n$

Pounds per cubic foot $= \frac{1}{v}$.

SATURATED STEAM

Inches mercury	Abs. Press., lb. per sq. in. p	Temp., °F. t	Spec. Vol., cu. ft. per lb. v	Heat of Liq., B.t.u. h	Heat of Evap., B.t.u. L	Total Heat, B.t.u. H	Entropy of Liq. n	Total Entropy N
29.72	0.1	35.0	2935	3.02	1071.7	1074.7	0.0062	2.1727
29.51	0.2	53.1	1524	21.18	1061.6	1082.8	0.0423	2.1127
29.31	0.3	64.5	1042	32.57	1055.2	1087.8	0.0640	2.0776
29.11	0.4	72.9	794	40.95	1050.6	1091.6	0.0800	2.0531
28.90	0.5	79.7	643	47.74	1046.9	1094.6	0.0926	2.0339
28.70	0.6	85.3	540.7	53.32	1043.8	1097.1	0.1029	2.0183
28.49	0.7	90.2	466.8	58.20	1041.1	1099.3	0.1118	2.0053
28.29	0.8	94.5	411.5	62.49	1038.7	1101.2	0.1195	1.9942
28.09	0.9	98.3	367.9	66.28	1036.6	1102.9	0.1265	1.9843
27.88	1	101.8	333.0	69.8	1034.6	1104.4	0.1327	1.9754
25.85	2	126.2	173.5	94.0	1021.0	1115.0	0.1749	1.9180
23.81	3	141.5	118.5	109.4	1012.3	1121.6	0.2008	1.8848
21.78	4	153.0	90.5	120.9	1005.7	1126.5	0.2198	1.8614
19.74	5	162.3	73.3	130.1	1000.3	1130.5	0.2348	1.8432
17.70	6	170.1	61.9	137.9	995.8	1133.7	0.2471	1.8285
15.67	7	176.9	53.6	144.7	991.8	1136.5	0.2579	1.8161
13.63	8	182.9	47.27	150.8	988.2	1139.0	0.2673	1.8053
11.60	9	188.3	42.36	156.2	985.0	1141.1	0.2756	1.7958
9.56	10	193.2	38.4	161.1	982.0	1143.1	0.2832	1.7874
7.52	11	197.8	35.1	165.7	979.2	1144.9	0.2902	1.7797
5.49	12	202.0	32.4	169.9	976.6	1146.5	0.2967	1.7727
3.45	13	205.9	30.0	173.8	974.2	1148.0	0.3025	1.7664
1.42	14	209.6	28.0	177.5	971.9	1149.4	0.3081	1.7604
0.00	14.7	212.0	26.8	180.0	970.4	1150.4	0.3118	1.7565
Pounds gage								
0.3	15	213.0	26.3	181.0	969.7	1150.7	0.3133	1.7549
5.3	20	228.0	20.1	196.1	960.0	1156.2	0.3355	1.7320
10.3	25	240.1	16.3	208.4	952.0	1160.4	0.3532	1.7136
15.3	30	250.3	13.7	218.8	945.1	1163.9	0.3680	1.6991
20.3	35	259.3	11.9	227.9	938.9	1166.8	0.3808	1.6868
25.3	40	267.3	10.5	236.1	933.3	1169.4	0.3920	1.6761
30.3	45	274.5	9.39	243.4	928.2	1171.6	0.4021	1.6665
35.3	50	281.0	8.51	250.1	923.5	1173.6	0.4113	1.6581
40.3	55	287.1	7.78	256.3	919.0	1175.4	0.4196	1.6505
45.3	60	292.7	7.17	262.1	914.9	1177.0	0.4272	1.6432
50.3	65	298.0	6.65	267.5	911.0	1178.5	0.4344	1.6368
55.3	70	302.9	6.20	272.6	907.2	1179.8	0.4411	1.6307
60.3	75	307.6	5.81	277.4	903.7	1181.1	0.4474	1.6252
65.3	80	312.0	5.47	282.0	900.3	1182.3	0.4535	1.6200
70.3	85	316.3	5.16	286.3	897.1	1183.4	0.4590	1.6151

SATURATED STEAM (*Continued*)

Pounds gage	Abs. Press., lb. per sq. in.	Temp., °F.	Spec. Vol., cu. ft. per lb.	Heat of Liq., B.t.u.	Heat of Evap., B.t.u.	Total Heat, B.t.u.	Entropy of Liq.	Total Entropy
	p	t	v	h	L	H	n	N
75.3	90	320.3	4.89	290.5	893.9	1184.4	0.4644	1.6105
80.3	95	324.1	4.65	294.5	890.9	1185.4	0.4694	1.6061
85.3	100	327.8	4.43	298.3	888.0	1186.3	0.4743	1.6020
90.3	105	331.4	4.23	302.0	885.2	1187.2	0.4789	1.5980
95.3	110	334.8	4.05	305.5	882.5	1188.0	0.4834	1.5942
100.3	115	338.1	3.88	309.0	879.8	1188.8	0.4877	1.5907
105.3	120	341.3	3.73	312.3	877.2	1189.6	0.4919	1.5873
110.3	125	344.4	3.58	315.5	874.7	1190.3	0.4959	1.5839
115.3	130	347.4	3.45	318.6	872.3	1191.0	0.4998	1.5807
120.3	135	350.3	3.33	321.7	869.9	1191.6	0.5035	1.5777
125.3	140	353.1	3.22	324.6	867.6	1192.2	0.5072	1.5747
130.3	145	355.8	3.11	327.4	865.4	1192.8	0.5107	1.5719
135.3	150	358.5	3.01	330.2	863.2	1193.4	0.5142	1.5692
140.3	155	361.0	2.92	332.9	861.0	1194.0	0.5175	1.5664
145.3	160	363.6	2.83	335.6	858.8	1194.5	0.5208	1.5639
150.3	165	366.0	2.75	338.2	856.8	1195.0	0.5239	1.5615
155.3	170	368.5	2.68	340.7	854.7	1195.4	0.5269	1.5590
160.3	175	370.8	2.60	343.2	852.7	1195.9	0.5299	1.5567
165.3	180	373.1	2.53	345.6	850.8	1196.4	0.5328	1.5543
170.3	185	375.4	2.47	348.0	848.8	1196.8	0.5356	1.5520
175.3	190	377.6	2.41	350.4	846.9	1197.3	0.5384	1.5498
180.3	195	379.8	2.35	352.7	845.0	1197.7	0.5410	1.5476
185.3	200	381.9	2.29	354.9	843.2	1198.1	0.5437	1.5456
190.3	205	384.0	2.24	357.1	841.4	1198.5	0.5463	1.5436
195.3	210	386.0	2.19	359.2	839.6	1198.8	0.5488	1.5416
200.3	215	388.0	2.14	361.4	837.9	1199.2	0.5513	1.5398
205.3	220	389.9	2.09	363.4	836.2	1199.6	0.5538	1.5379
210.3	225	391.9	2.05	365.5	834.4	1199.9	0.5562	1.5361
215.3	230	393.8	2.00	367.5	832.8	1200.2	0.5586	1.5344
220.3	235	395.6	1.96	369.4	831.1	1200.6	0.5610	1.5327
225.3	240	397.4	1.92	371.4	829.5	1200.9	0.5633	1.5309
230.3	245	399.3	1.89	373.3	827.9	1201.2	0.5655	1.5293
235.3	250	401.1	1.85	375.2	826.3	1201.5	0.5676	1.5276
245.3	260	404.5	1.78	378.9	823.1	1202.1	0.5719	1.5244
255.3	270	407.9	1.72	382.5	820.1	1202.6	0.5760	1.5214
265.3	280	411.2	1.66	386.0	817.1	1203.1	0.5800	1.5185
285.3	300	417.5	1.55	392.7	811.3	1204.1	0.5878	1.5129
385.3	400	444.8	1.17	422	786	1208	0.621	1.489
485.3	500	467.3	0.93	448	762	1210	0.648	1.470
585.3	600	486.6	0.76	469	741	1210	0.670	1.453

SUPERHEATED STEAM

v = specific volume in cubic feet per pound, **h** = total heat, from water at 32° F. in B.t.u. per pound, **n** = total entropy, from water at 32°.

Abs. pres., lb. per sq. in.	Temp. sat. steam, °F.		Degrees of superheat, Fahrenheit							
			20	50	100	150	200	250	300	400
20	228.0	v	20.73	21.69	23.25	24.80	26.33	27.85	29.37	32.39
		h	1165.7	1179.9	1203.5	1227.1	1250.6	1274.1	1297.6	1344.8
		n	1.7456	1.7652	1.7961	1.8251	1.8524	1.8781	1.9026	1.9479
40	267.3	v	10.83	11.33	12.13	12.93	13.70	14.48	15.25	16.78
		h	1179.3	1194.0	1218.4	1242.4	1266.4	1290.3	1314.1	1361 6
		n	1.6895	1.7089	1.7392	1.7674	1.7940	1.8189	1.8427	1.8867
60	292.7	v	7.40	7.75	8.30	8.84	9.36	9.89	10.41	11.43
		h	1187.3	1202.6	1227.6	1252.1	1276.4	1300.4	1324.3	1372.2
		n	1.6568	1.6761	1.7062	1.7342	1.7603	1.7849	1.8081	1.8511
80	312.0	v	5.65	5.92	6.34	6.75	7.17	7.56	7.95	8.72
		h	1193.0	1208.8	1234.3	1259.0	1283.6	1307.8	1331.9	1379.8
		n	1.6338	1.6532	1.6833	1.7110	1.7368	1.7612	1.7840	1.8265
100	327.8	v	4.58	4.79	5.14	5.47	5.80	6.12	6.44	7.07
		h	1197.5	1213.8	1239.7	1264.7	1289.4	1313.6	1337.8	1385.9
		n	1.6160	1.6358	1.6658	1.6933	1.7188	1.7428	1.7656	1.8079
120	341.3	v	3.85	4.04	4.33	4.62	4.89	5.17	5.44	5.96
		h	1201 1	1217.9	1244.1	1269.3	1294.1	1318.4	1342.7	1391.0
		n	1.6016	1.6216	1.6517	1.6789	1.7041	1.7280	1.7505	1.7924
140	353.1	v	3.32	3.49	3.75	4.00	4.24	4.48	4.71	5.16
		h	1204.3	1221.4	1248.0	1273.3	1298.2	1322.6	1346.9	1395.4
		n	1.5894	1.6096	1.6395	1.6666	1.6916	1.7152	1.7376	1.7792
160	363.6	v	2.93	3.07	3.30	3.53	3.74	3.95	4.15	4.56
		h	1207.0	1224.5	1251.3	1276.8	1301.7	1326.2	1350.6	1399.3
		n	1.5789	1.5993	1.6292	1.6561	1.6810	1.7043	1.7266	1.7680
180	373.1	v	2.62	2.75	2.96	3.16	3.35	3.54	3.72	4.09
		h	1209.4	1227.2	1254.3	1279.9	1304.8	1329.5	1353.9	1402.7
		n	1.5697	1.5904	1.6201	1.6468	1.6716	1.6948	1.7169	1.7581
200	381.9	v	2.37	2.49	2.68	2.86	3.04	3.21	3.38	3.71
		h	1211.6	1229.8	1257.1	1282.6	1307.7	1332.4	1357.0	1405.9
		n	1.5614	1.5823	1.6120	1.6385	1.6632	1.6862	1.7082	1.7493
220	389.9	v	2.16	2.28	2.45	2.62	2.78	2.94	3.10	3.40
		h	1213.6	1232.2	1259.6	1285.2	1310.3	1335.1	1359.8	1408.8
		n	1.5541	1.5753	1.6049	1.6312	1.6558	1.6787	1.7005	1.7415
240	397.4	v	1.99	2.09	2.26	2.42	2.57	2.71	2.85	3.13
		h	1215.4	1234.3	1261.9	1287.6	1312.8	1337.6	1362.3	1411.5
		n	1.5476	1.5690	1.5985	1.6246	1.6492	1.6720	1.6937	1.7344
260	404.5	v	1.84	1.94	2.10	2.24	2.39	2.52	2.65	2.91
		h	1217.1	1236.4	1264.1	1289.9	1315.1	1340.0	1364.7	1414.0
		n	1.5416	1.5631	1.5926	1.6186	1.6430	1.6658	1.6874	1.7280
280	411.2	v	1.72	1.81	1.95	2.09	2.22	2.35	2.48	2.72
		h	1218.7	1238.4	1266.2	1291.9	1317.2	1342.2	1367.0	1416.4
		n	1.5362	1.5580	1.5873	1.6133	1.6375	1.6603	1.6818	1.7223

FLOW OF STEAM THROUGH A NOZZLE. — The rate of flow of steam through a nozzle increases as the difference in the pressures on the two sides of the nozzle increases, until the absolute pressure p_0 of the atmosphere into which the nozzle discharges reaches 58 per cent of the initial absolute pressure p of the steam. For greater differences in pressure, i.e., for $\frac{p_0}{p} < 0.58$, the rate of flow through a given nozzle depends only upon the initial absolute pressure of the steam. For $\frac{p_0}{p} < 0.58$ the percentage change in the volume of the steam as it passes through the nozzle also remains constant, the ratio of initial volume to expanded volume being 1.624. The following formulas have been given by the authorities stated for the flow of steam through a nozzle when $\frac{p_0}{p} < 0.58$. The notation is P = initial absolute pressure in pounds per square inch; A = smallest cross section of nozzle in square inches; W = flow in pounds per minute; x = quality of steam = $\frac{100 - y}{100}$ where y is the per cent of moisture; D = superheat in degrees F.

	Dry saturated	Moist	Superheated
Napier	$W = 0.857AP$		
Grashoff	$W = AP^{0.97}$	$\frac{AP^{0.97}}{x}$	$AP^{0.97}(1 + 0.00065D)$

FLOW OF STEAM THROUGH PIPES, VALVES, AND BENDS. — (*See Pipes and Piping.*)

BIBLIOGRAPHY. — Marks and Davis, *Steam Tables and Diagrams:* *Peabody's Steam Tables;* Ennis, W. D., *Applied Thermodynamics.* See also bibliographies in the articles on *Boilers*, *Steam*, and *Steam Engines.*

STEAM ENGINES. — (*See also Condensers; Power Stations; Steam; Steam Turbines.*) A steam engine is a machine in which the energy of heat is converted into mechanical energy by means of the pressure of steam upon one or more parts of the machine. This article treats of the reciprocating engine only; steam turbines (q.v.) are treated in a separate article.

CLASSIFICATION. — The principal classifications of reciprocating engines are the following:

Single- and Double-acting Engines. — If the steam acts on only one side of the piston it is a single-acting engine; if on both sides, a double-acting.

Throttling and Automatic Engines. — A throttling engine is one in which the speed of the engine is governed by a throttle valve in the steam pipe, used to vary the pressure of the steam admitted to the engine. An automatic cut-off engine is one in which steam is always admitted at full pressure, but is cut off by a valve or valves at different points in the stroke according to the load, the point at which the cut-off takes place being automatically controlled by a governor.

Throttling as a means of governing is seldom employed in other than small engines. It affords a simple and reliable means of speed control for pumps, etc. Automatic regulation of cut-off is capable of closer speed regulation and permits higher ratios of expansion.

Condensing and Noncondensing Engines. — A condensing engine is one in which the steam is exhausted into a condenser, by means of which its pressure is reduced usually to within 1 to 3 pounds per square inch above a perfect vacuum. A non-condensing engine is one whose exhaust steam is discharged at or above atmospheric pressure. Any engine can be operated condensing or noncondensing, but certain minor modifications are usually made in the design of an engine when it is to be operated normally condensing.

Binary-vapor Engine. — This is a very special form of condensing engine. Sulphur dioxide, instead of water, is used as a cooling medium in a surface condenser. The sulphur dioxide in condensing the exhaust steam is itself vaporized and this vapor, under a pressure of about 175 pounds per square inch, is used expansively in a secondary reciprocating engine. The exhausted sulphur dioxide is discharged into a surface condenser in which it is liquefied by cooling water, much the same as in refrigerating practice, and used over and over again. This type of engine has never come into extensive use.

High- and Low-speed Engines. — This classification refers to rotative (fly wheel) speed only; engines having a rotative speed of 150 revolutions per minute or less are usually classified as low-speed engines; when the fly-wheel speed is greater than 150 revolutions per minute the engine is called a high-speed engine. A high-speed engine may have a lower *piston speed* than a low-speed engine; the relation between piston speed and rotative speed depends solely on the length of stroke.

Compound and Multiple-expansion Engines. — A compound engine is one in which the steam is partially expanded in a smaller cylinder and then carried to one or more larger cylinders in which it is further expanded. If the cylinders are in line with each other, using a common piston rod, it is called a tandem compound; if the smaller and larger, or high-pressure and low-pressure, cylinders are side by side it is called a cross-compound. The tandem type is simpler, lighter, cheaper and more compact, and serves well where exact balancing and uniform crank effort are not essential. The cylinders may be vertical or horizontal. Vertical engines take up less floor space, but are more costly than horizontal engines.

An engine in which the total expansion of the steam is divided into three stages, high-, intermediate- and low-pressure, is a triple-expansion engine; if into four stages, a quadruple-expansion engine. Multiple-expansion engines (other than compound) are seldom used for driving electric generators.

Classification According to Valve Gear. — Engines are also classified according to their valve gear (*see Valve Gear, below*), as Corliss engines, slide-valve engines, piston-valve engines and poppet-valve engines.

DESIGN AND CONSTRUCTION. — For a more complete treatment of the design and construction of steam engines see Kent's *Mechanical Engineers' Pocket-Book.* A brief discussion of valves and governors is given below.

Valves and Valve Gear. — Three types of valves are used to control the admission and exhaust of the steam to and from the cylinder, namely the slide valve, Corliss valve and poppet valve.

Slide Valves. — Fig. 1 shows a cylinder with a plain slide valve of the ordinary type. The valve rests in a V-shaped groove in the bottom of the steam chest, and is held up against its seat by pressure of the steam. The valve spindle passes through a channel in the back of the valve, as shown by the transverse section of the cylinder and steam chest, which allows the valve to press against its seat without springing the valve spindle.

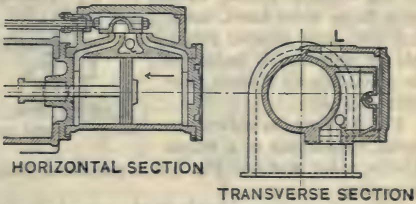

Fig. 1. Plain Slide Valve

In the drawing, the valve is shown moved over to the left so as to allow steam from the steam chest to pass into the head end of the cylinder and force the piston toward the left. Steam from the crank end of the cylinder can flow through a cavity in the valve to the exhaust space Q. Steam enters by the

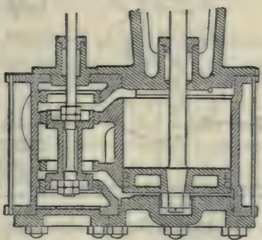

Fig. 2. Piston Valve

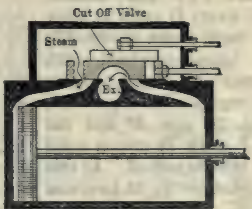

Fig. 3. Riding Cut-off Slide Valve

opening L (transverse section) and escapes through the exhaust space Q. As the piston moves toward the end of the forward stroke, the valve is moved by the eccentric to the right, and it first shuts off the supply of steam from the head end and the exhaust from the crank end of the cylinder, and then opens the supply of steam for the crank end and the exhaust for the head end just before the return stroke is begun.

Other types of slide valves are the piston valve and the riding cut-off valve. In the former, Fig. 2, is used a piston filling an auxiliary cylinder which takes

the place of the steam chest used with the common type. The riding cut-off slide valve, Fig. 3, is essentially a double slide valve. The riding cut-off controls the point of cut-off only, the points of admission, release and compression being controlled by the main valve. (*See Fig. 8 below.*)

Any adjustment of the simple slide valve changes all four events of the stroke, namely admission, cut-off, release and compression. With the riding cut-off slide valve the cut-off can be varied independent of the other events.

Corliss Valve. — To make all four events of the stroke independently adjustable, four separately controlled valves are necessary. One of the first successful four-valve engines was invented by George Corliss in 1848. Various modifications of the original Corliss valve gear have since been made, but the same general principle of operation is embodied in practically all modern four-valve engines.

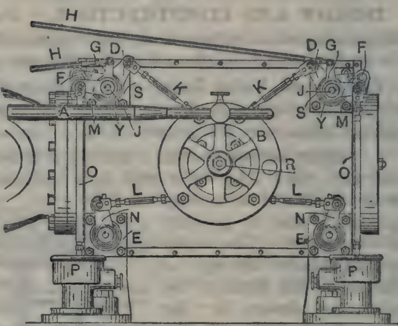

Fig. 4. Corliss Valve Gear

Fig. 4 shows the general external appearance of a simple Corliss valve gear and Fig. 5 is a simplified diagram showing the principle of operation.

"The steam valves work in the chambers *SS*, and the exhaust valves in the chambers *EE*. The double-armed levers *DD* work loosely on the hubs of the steam bonnets; they are connected to the wrist plate *B* by the rods *KK*; the levers *MM* are keyed to the valve stems *JJ*, and are also connected by the rods *OO* to the dashpots *PP*. The double-armed levers *D* carry at their outer ends steam hooks *FF*, these being provided with hardened-steel catch plates which engage with arms *MM*, making the arm *M* and the hook *F* work in unison until steam is to be cut off. At this point another set of levers or

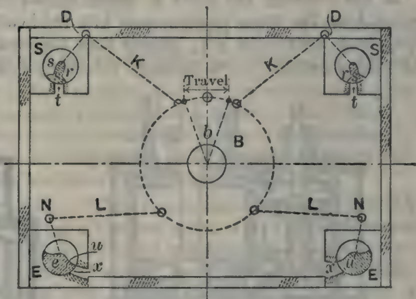

Fig. 5. Diagram of Corliss Valve Gear

cams *GG*, connected by the cam rods *HH* to the governor, come into play, causing the catch plates on the hooks *F* to release the arms *MM*, the outer ends of which are then pulled downwards by the dashpot plunger, causing the steam valves to rotate on their axis and thus cut off steam. The exhaust valve arms *N* are connected to the wrist plate by the rods *LL*, and it is seen that all the valves receive their motion from the wrist plate *B*; the latter receives its motion from the hook rod *A*. This rod is attached to a rocker arm; to this arm the eccentric rod is also attached.

"In order to obtain a greater range of cut-off in Corliss engines, a separate steam and exhaust eccentric is used. With two eccentrics the admission and

exhaust valves can be adjusted independently, and steam may be cut off anywhere, nearly to the end of the stroke." (*From Types of Modern Engines and Their Valve Setting, by M. C. Myers, Boston, 1910.*)

Poppet Valves. — This type of valve for engine cylinders is largely used in Europe, but only to a limited extent in this country. Fig. 6 illustrates one type of poppet valve and valve gear, known as the Sulzer valve gear.

Comparison of Different Types of Valves. — Multi-valve engines are more expensive than single-valve types, but give better economy, due to the independent regulation of cut-off and other events of the stroke, and to the reduced requirements of clearance and port space. Rotary valves are more difficult to make and keep steam-tight when used with superheated steam than the poppet and ordinary types. The erosive action and the severe temperature strains caused by superheat add much to the difficulty of maintaining a good valve fit, but the difficulties are minimized in the best designs.

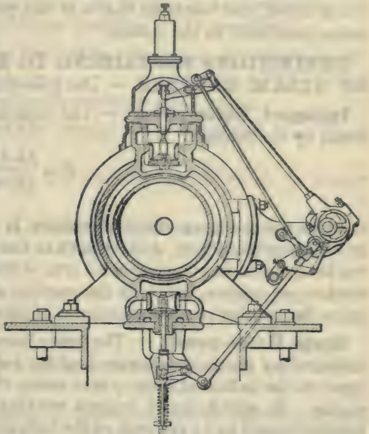

Fig. 6. Sulzer Valve Gear

Governors. — Two types of governors are used, viz., the pendulum or flyball and the flywheel types.

Pendulum Governor.—The construction of the pendulum governor is too well known to need description. This type of governor can be made to control the quantity of steam admitted to the cylinder either by opening or closing a throttle valve, or by varying the point of cut-off. In the latter case a suitable link motion (*see below*) must be provided.

Flywheel or Shaft Governor. — This type of governor is now largely used, especially for high-speed automatic engines

The Rites governor, Fig. 7, one of the commonest forms of flywheel governors, is of the single-weight or inertia type. The center of gravity of the weight is located approximately at *G*. The governor is shown in position for latest cut-off; the rotation is in the direction of the arrow. Any increase in speed tends to make the center of gravity of the weight seek a position further away from the center of the shaft and causes the weight to swing on its pivot in a direction opposite to that of the wheel; the eccentric is attached to the weight, and this movement brings the center of the eccentric nearer the center of the shaft, and increases its angular advance, thus effecting an earlier cut-off.

Fig. 7. Flywheel Governor

This and other types of shaft governors are described in detail in a small book on *Shaft Governors* (*Hill Pub. Co., 1908*).

Link Motion. — Link motions, of which the Stephenson link is the most commonly used, are designed for two purposes: first, for reversing the motion of the engine and second, for varying the point of cut-off by varying the travel

of the valve. The Stephenson link motion is a combination of two eccentrics, called forward and back eccentrics, with a link connecting the extremities of the eccentric rods, so that by varying the position of the link the valve rod may be put in direct connection with either eccentric or may be given a movement controlled in part by one and in part by the other eccentric. When the link is moved by the reversing lever into a position such that the block to which the valve rod is attached is at either end of the link, the valve receives its maximum travel, and when the link is in mid-gear the travel is the least and cut-off takes place early in the stroke.

DEFINITIONS PERTAINING TO RATING AND PERFORMANCE OF STEAM ENGINES. — The following terms are commonly employed.

Indicated Horse-power. — The indicated horse-power P of an engine is found by the formula

$$P = \frac{pLAN}{33,000},$$

in which p is the mean effective pressure, in pounds per square inch, as found by an indicator (*see below*), L the length of the stroke in feet, A the area of the piston in square inches (corrected for area of the piston rod), N the number of single strokes per minute, equal to the number of revolutions of a single-acting, or twice the number of revolutions of a double-acting engine.

Brake Horse-power. — The brake horse-power of an engine is the power delivered by its shaft as determined by a dynamometer or Prony brake. It is equal to the indicated horse-power minus the power absorbed in friction of the engine. In well-designed engines working under normal loads the friction is usually from 8 to 12 per cent of the indicated power; the total power absorbed in friction is nearly a constant at all loads, so that its percentage increases to 100 as the load decreases to zero, when the whole of the indicated horse-power is absorbed in overcoming friction.

Rated Horse-power. — When an engine is commercially rated at a certain horse-power it is understood that it will deliver that power when run under those conditions for which it is designed, such as speed, steam pressure, back pressure, etc., and cutting off at that fraction of the stroke which will give its best economy. Its rated overload capacity is the power it will deliver with the same speed, steam pressure and back pressure, and cutting off at the latest point which the design of the valve motion will permit.

Indicator Diagram. — An indicator diagram is a diagram (see Fig. 8), showing the steam pressure in the engine cylinder at each point of the stroke. Such a diagram may be actually obtained by means of a steam-engine indicator, which is an instrument which causes a pencil to record on paper the pressure in the cylinder at every point of the stroke. The diagram drawn by the pencil shows whether the valves are properly adjusted, and it is also used in figuring the power developed in the cylinder, and approximately the steam consumption. A diagram of a noncondensing engine in which the steam is cut off at about one-quarter of the stroke is shown in Fig. 8.

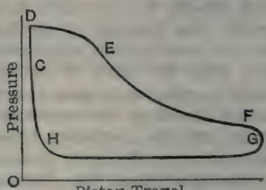

Fig. 8. Indicator Diagram of Simple Engine

The lines and points have the following significance.

Point of Admission, C, is the point at which the steam valve opens.

Admission Line, *CD*, shows the rise of pressure due to the admission of steam to the cylinder by opening the steam valve.

Steam Line, *DE*, is drawn when the steam valve is open and steam is being admitted tc the cylinder.

Point of Cut-off, *E*, is the point where the admission of steam is stopped by the closing of the valve. It is often difficult to determine the exact point at which the cut-off takes place. It is usually located where the outline of the diagram changes its curvature from convex to concave.

Expansion Curve, *EF*, shows the fall in pressure as the steam in the cylinder expands doing work.

Point of Release, *F*, shows when the exhaust valve opens.

Exhaust Line, *FG*, represents the change in pressure that takes place when the exhaust valve opens.

Back-pressure Line, *GH*, shows the pressure against which the piston acts during its return stroke.

Point of Exhaust Closure, *H*, is the point where the exhaust valve closes. It cannot be located definitely, as the change in pressure is at first due to the gradual closing of the valve.

Compression Curve, *HC*, shows the rise in pressure due to the compression of the steam remaining in the cylinder after the exhaust valve has closed.

Initial Pressure is the pressure acting on the piston at the beginning of the stroke.

Terminal Pressure is the pressure above the line of perfect vacuum that would exist at the end of the stroke if the steam had not been released earlier. It is found by continuing the expansion curve to the end of the diagram.

Other Definitions. — In addition to the terms defined above, the following are commonly employed:

Throttle Pressure is the pressure in the steam pipe at the entrance to the throttle valve.

Mean Effective Pressure is that equivalent constant pressure which will do the same amount of work on the piston per stroke as is done by the varying pressure shown by the indicator card. This may be calculated by dividing the area of the card by the length and multiplying by the scale of the spring used in the indicator.

Clearance. — The portion of the cylinder volume, including the steam ports, not swept through by the piston but which is nevertheless filled with steam when admission occurs is called the clearance volume. It ranges from 1 per cent of the piston displacement in very large engines to 10 per cent or more in small high-speed engines.

Ratio of Expansion is the ratio of the piston displacement (in low-pressure cylinder in case of a multiple-expansion engine) to the volume of the steam admitted through the throttle valve at each stroke (or half stroke in case of a double-acting engine), the volume being that corresponding to the pressure on the engine side of the throttle valve.

Wire Drawing, as applied to steam, is the reducing of its pressure, due to its flowing through restricted pipes and passages.

EFFICIENCY OF A STEAM ENGINE. — By the thermal efficiency of a steam engine is meant the quotient of the B.t.u. per hour equivalent to the

indicated horse-power divided by the total number of B.t.u. in the steam supplied per hour; or, putting

W = pounds of steam supplied per hour per indicated horse-power,

x = quality of the steam supplied (= lb. of dry steam per lb. of wet steam; see article on *Steam*),

L = heat of evaporation per pound of steam at throttle pressure, in B.t.u. per pound,

h = heat of the liquid at throttle pressure, B.t.u. per pound,

h_e = heat of the liquid of feed water, B.t.u. per pound, taken at the temperature corresponding to the pressure in the exhaust pipe near the engine. (*See A.S.M.E. code, 1913.*)

Then the thermal efficiency for saturated steam is

$$E_t = \frac{2546.5}{W\,(xL + h - h_e)}.$$

If the steam is superheated, let

H = total heat of superheated steam at throttle pressure and degree of superheat, in B.t.u. per pound.

Then the thermal efficiency is

$$E_t = \frac{2546.5}{W\,(H - h_e)}.$$

The above formulas are also applicable to the calculation of the over-all efficiency, including friction losses, if W is taken as the pounds of steam supplied per hour per brake horse-power.

Efficiency of Rankine Cycle. — The maximum possible thermal efficiency is that of an engine performing the ideal Carnot's cycle (*see Thermodynamics, Principles of*), but this cycle is not very closely simulated by ordinary engines, even when the losses are neglected. Instead the efficiency of the "Rankine cycle" is employed as a standard of reference. This is an ideal cycle which assumes that the work done by the engine is equal to the "maximum work" (*see Thermodynamics*) corresponding to an adiabatic expansion from throttle pressure to exhaust pressure and an isothermal condensation of the exhaust steam and a compression of the condensed water to throttle pressure.

Rankine Efficiency for Dry Saturated Steam. — Let

H_1 = total heat, B.t.u. per pound, of dry saturated steam at throttle pressure,

H_2 = total heat, B.t.u. per pound, of dry saturated steam at exhaust pressure,

h_2 = heat of the liquid, B.t.u. per pound, at exhaust temperature,

T_2 = temperature of exhaust steam, in °F.,

N_1 = total entropy of 1 pound of dry saturated steam at throttle pressure,

N_2 = total entropy of 1 pound of dry saturated steam at exhaust pressure.

Then the efficiency is

$$E_r = \frac{H_1 - H_2 + T_2\,(N_2 - N_1)}{H_1 - h_2}.$$

Rankine Efficiency for Wet Steam. — In addition to the above symbols let

H_w = total heat, B.t.u. per pound, of wet steam at throttle pressure,

N_w = total entropy of 1 pound of wet steam at throttle pressure,

T_1 = temperature of steam at throttle pressure, in °F.

Then the efficiency is

$$E_r = \frac{H_w - H_2 + T_2\,(N_2 - N_w)}{H_w - h_2}$$

Rankine Efficiency for Superheated Steam.—In addition to the above symbols let

H_s = total heat, B.t.u. per pound, of superheated steam at throttle pressure,
N_s = total entropy of 1 pound of superheated steam at the given degree of superheat at throttle.

Then the efficiency is

$$E_r = \frac{H_s - H_2 - T_2 (N_s - N_2)}{H_s - h_2}.$$

Efficiency Ratio Referred to the Rankine Cycle.—This is the ratio of the thermal efficiency of the actual engine (referred to indicated or brake horse-power as the case may be) to the efficiency of the Rankine cycle between the same limits of temperature and pressure as those observed in the actual engine. In the case of steam turbines the thermal efficiency is referred to the brake horse-power. In the case of reciprocating engines it is usually referred to the indicated horse-power.

STEAM ENGINE EFFICIENCIES—SATURATED STEAM.
(*From Gebhardt's Steam Power Plant Engineering.*)

Gage pressure	Noncondensing				Condensing — 1 lb. absolute pressure			
	Carnot cycle	Rankine cycle	Best actual (1907)	Eff. ratio %	Carnot cycle	Rankine cycle	Best actual	Eff. ratio %
25	7.5	7.3	5.5	76	22.6	21.0	11.6	55
50	11.2	10.7	8.5	80	25.7	23.5	13.5	60
75	13.7	13.0	10.4	80	27.8	25.3	15.9	61
100	15.7	14.8	12.0	81	29.5	26.7	20.2	76
125	17.3	16.3	13.5	83	30.8	27.8	20.3	74
150	18.7	17.5	14.3	82	32.0	28.8	21.6	75
175	19.8	18.5	14.8	80	32.9	29.6	21.9	74
200	20.8	19.3	15.2	79	33.7	30.2	22.6	75
225	21.6	19.9	15.5	78	34.5	30.6	22.6	74
250	22.4	20.5	....	..	35.1	31.0	....	..
275	23.0	21.0	....	..	35.6	31.3	....	..
300	23.6	21.4	....	..	36.0	31.5	....	..

The actual thermal efficiencies of multiple expansion engines using superheated steam range from about 19 to 23 per cent.

Engine Losses. — The actual efficiencies of steam engines are necessarily much lower than the efficiency of the Rankine cycle due to the thermal and mechanical losses. These losses consist chiefly of (1) cylinder condensation, (2) steam leakage, (3) incomplete expansion and compression on return stroke, (4) friction, (5) clearance loss, (6) radiation and (7) the admission of wet steam.

Cylinder Condensation is due to the chilling of steam by the cooler cylinder walls during admission and early expansion, thus reducing the active heat during expansion. Condensation increases with the range of expansion per cylinder and with the duration of the cycle. The condensation loss is

augmented by the formation of a water film on the cylinder walls. It is successfully reduced by expanding the steam in several stages, thus reducing the range of temperature in each cylinder and rendering the heat carried to the exhaust in that cylinder available in succeeding cylinders. Other remedies for condensation are: increasing the rotative speed; steam jackets about the cylinders; and reheaters to dry the steam between cylinders. The most positive and effective remedy is superheat sufficient to keep the steam dry at least during admission.

Steam Leakage past valves, pistons and packing increases with the pressure difference. It is an important element of loss in all piston engines and tends to increase with the wear of moving parts. Leakage is especially serious in cylinders employing a wide range of pressures. Flow of steam through orifices, in pounds, increases directly as the pressure (Napier's rule), and through pipes as $\sqrt{\text{density}}$. (*p. 845* Kent's *Mechanical Engineers' Pocket-Book.*)

Incomplete Expansion and Compression. — As a rule the release occurs before the piston reaches the end of the stroke and the point of closure comes before the piston reaches the end of the back stroke. There is a consequent loss of expansion on the forward stroke. The piston also has to do work in compressing the steam remaining in the cylinder on the reverse stroke. These two features are, as a rule, necessary; the first to reduce cylinder condensation, and the second for its cushioning effect. The first always, and the second usually, results in a net loss.

Friction varies with the type and condition of the engine, is greater in compound engines than in simple engines, and is nearly independent of the load, increasing but slightly as the load increases. It ranges from 4 to 20 per cent of the rated output.

Losses Due to Radiation, Clearance and Moisture. — With wall-lagged cylinders radiation losses are small. Some clearance is necessary in every engine, but the amount is much greater for high speeds than for low speeds. The loss due to it is trifling in slow-speed engines of long stroke. Moisture dilutes the steam admitted but has little effect on the consumption of *dry* steam; the consumption of the fluid (steam and moisture) is of course increased in proportion to the percentage of moisture present.

ECONOMY OF STEAM ENGINES.* — The performance of a steam engine is frequently expressed in terms of the number of pounds of steam per hour required per indicated horse-power, per brake horse-power or per kilowatt (if used to drive an electric generator). The number of pounds of steam per unit of output is called the "economy" of the engine or of the combined engine and generator unit.

Such data may be very misleading unless comparisons are based on identical conditions of steam pressure, superheat, vacuum, etc. A true measure of economy applicable to all conditions may be expressed in net heat units consumed per unit of output, as B.t.u. per kilowatt-hour. In such determinations of economy all heat returned from the exhaust to the boiler should be credited to the engine or turbine.

The commonest means of gaining good engine economy are: raising the initial pressure, compounding, condensing and superheating.

Effect of Steam Pressure on Economy and Capacity. — There can be no universal rule connecting initial pressure and working economy. Compound

* Adapted from lecture notes of Prof. W. E. Wickenden.

engines are better adapted to high pressures than simple engines. Experience indicates that the following gauge pressures are desirable:

Simple, condensing engine without steam jackets	60 lb.
Simple, condensing engine with steam jackets........	80 lb.
Simple, non-condensing engine without steam jackets .	100 lb.
Compound, non-condensing engines	175 lb.
Compound, condensing engines	150 lb.
Triple-expansion, condensing engines	175 to 200 lb.
Quadruple-expansion condensing engines	200 lb.

An important advantage from the use of high pressures is the greater capacity which can be developed from a given engine. The ratio of pressure and capacity is nearly a direct one. High pressures tend to improve economy but not to the extent theoretically available, due to the counteracting effects of condensation and steam leakage with increasing pressure ranges. High steam pressures, however, usually add to the expense of piping and its maintenance. From an examination of a large number of engine performances Stevens and Hobart give the general relation of pressure and economy as shown in Fig. 9.

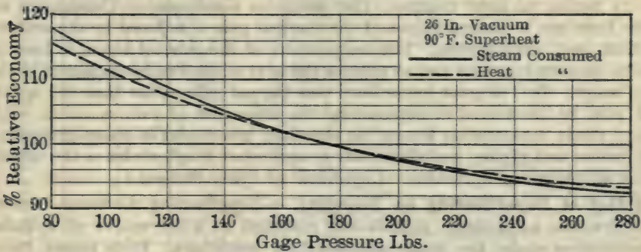

Fig. 9. Effect of Steam Pressure on Economy of Compound Piston Engines

Effect of Multiple Expansion on Economy. — The loss due to cylinder condensation increases with an increase of the difference of temperature of the steam during admission and exhaust, and therefore tends to become greater as the initial pressure is higher. By dividing the total range of expansion into two or more stages, the range of temperature in any one cylinder is decreased, and this lessens the total loss due to cylinder condensation.

Triple and quadruple expansion afford substantial economies with constant load as in pumping and marine service, but have little advantage with the variable loads of electric service to compensate for their great weight, complexity, bulk and cost. Compound engines with high cylinder ratios permit more complete expansion, but have less overload capacity than engines of smaller ratios. For electric service more than 18 expansions is seldom profitable. A cylinder ratio of 1 : 2.5 is well adapted to a pressure of 100 pounds; 1 : 3 to 125 pounds; 1 : 3.5 to 150 pounds and 1 : 4 to 175 pounds or above. Compound engines are relatively uneconomical at light loads, but have a greater economical range of overloads than simple engines. Compound engines, with a moderate cylinder ratio, usually give their best economy at ¼ cut-off in the high-pressure cylinder and will safely carry an overload of 50 per cent.

Effect of Condensing on Economy. — Condensing serves to increase both the capacity and economy of engines. The theoretical gain from condensing to various back pressures is indicated in Fig. 10. Actual engines can avail themselves of but a part of this gain, the proportion falling as the vacuum is increased due to the added condensation and leakage. Expansion to very

low pressures requires cylinders of excessive volume. Condensing equipment, especially that for high vacuum, adds much to the first cost of a plant and hence

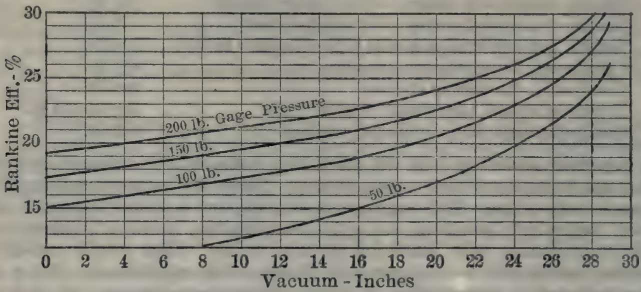

Fig. 10. Effect of Vacuum on Theoretical Efficiency

to its burden of fixed charges. (*See article on Condensers.*) Condensers require from one to three pumps and the thermal gain is reduced by the amount of their heat consumption. The temperature of the exhaust steam is lowered as the vacuum increases and so reduces the heat which can be reclaimed in the feed water. It is of great importance that the net gain rather than the apparent gain be considered in determining the economics of various vacua. Reduced steam consumption lessens the necessary investment in steam-generating equipment and piping, and this tends to somewhat counterbalance the investment in condensing apparatus. (*See article on Power Stations.*)

Effect of Superheating on Economy. — Superheating the steam is more effective than using steam jackets and reheaters as a preventive of cylinder condensation under the usual central-station conditions. With proper valve structures there is a decided net saving in heat consumption as the superheat is increased within reasonable limits, but this procedure usually adds to the

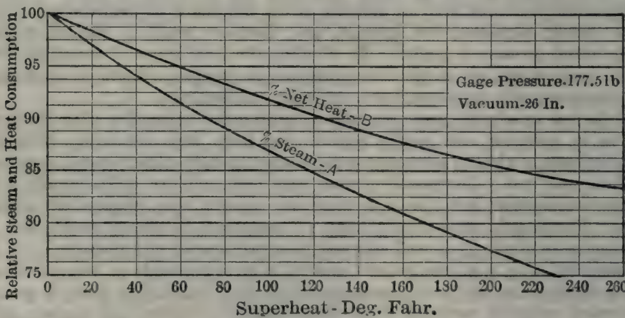

Fig. 11. Effect of Superheat on Piston-engine Economy

cost of piping, fittings and their upkeep. Stevens and Hobart report the mean relationship between superheat and economy in compound and multiple expansion engines, as shown in Fig. 11. It is especially important that economic comparison be based on the heat unit consumption as in Curve B. Additional advantages from superheating are the reduction of condensation, radiation losses and pressure drop in steam pipes.

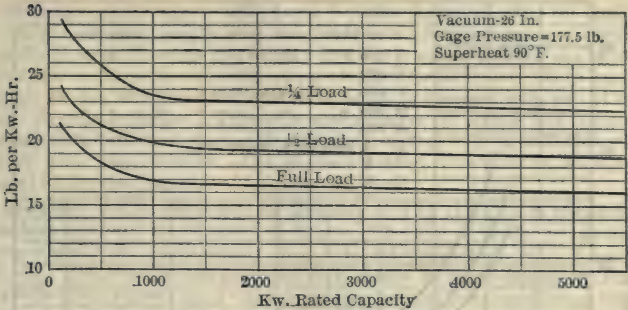

Fig. 12. Steam Consumption of Representative Compound Engine

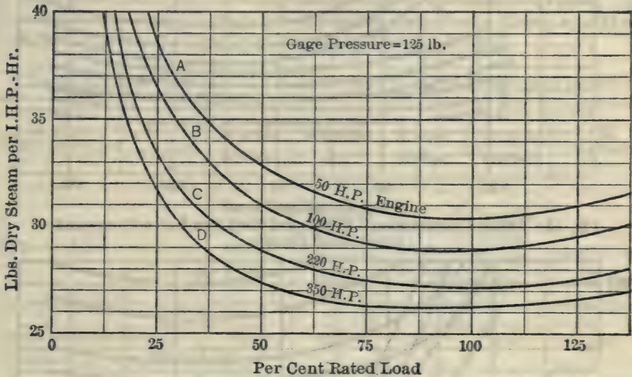

Fig. 13. Steam Consumption of Good Piston Valve Simple High-speed, Non-condensing Engines

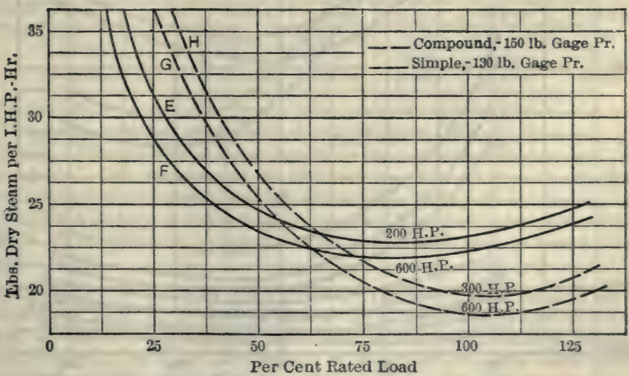

Fig. 14. Steam Consumption of Good 4-Valve Non-condensing Engines

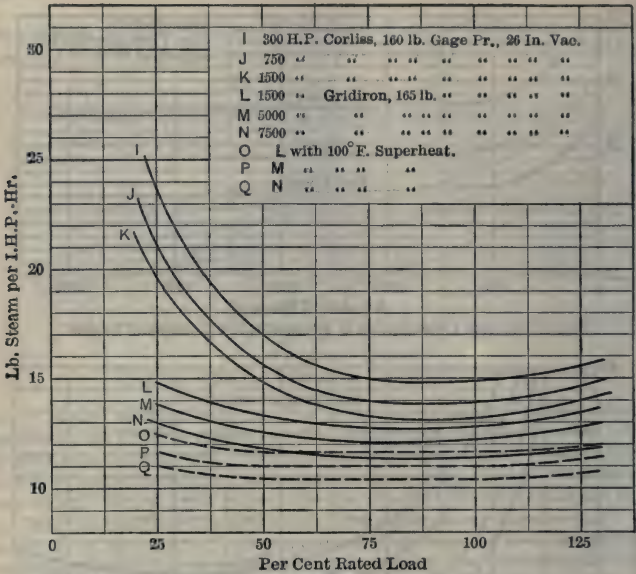

Fig. 15. Steam Consumption 4 Corliss and Grid-iron Valve Compound Condensing Engines

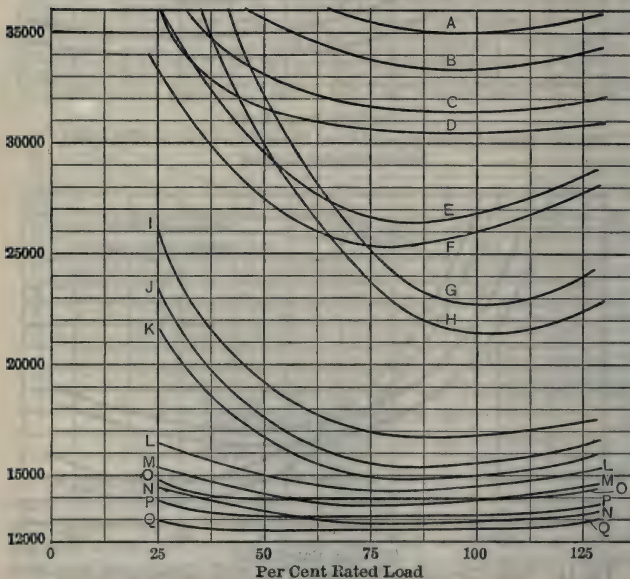

Fig. 16. Engine Performance in B.t.u. per Indicated Horse-power-hour

Variation of Economy with Load. — Other factors affecting the economy of engines are the size of the unit, the speed, the mode of governing and the uniformity of load. Stevens and Hobart give the relations between engine sizes, loads and economy shown in Fig. 12 as averages for compound piston engines. Simple engines as a rule show smaller variation in economy with partial loads than compound engines. Typical economy curves for simple and compound engines with saturated steam and compound engines with superheated steam are shown in Figs. 13 to 16. The curves in Fig. 16 correspond to like lettered curves in the other figures.

Economy of the Binary-vapor Engine. — With this type of engine there has been obtained an economy of only 8.36 pounds of steam per indicated horse-power-hour, corresponding to a heat consumption of 158.3 B.t.u. per minute. This was the best recorded performance in steam-engine practice at the time of the report (1907) and has not been noticeably exceeded since (1913).

DIMENSIONS AND WEIGHT OF ENGINES depend so largely upon the type, speed, etc., that it is impossible to give representative figures in the space available. The following table from the *Elec. World*, (*Sept., 1902*), gives the dimensions of some very large engines in New York City.

DIMENSIONS OF SOME LARGE RECIPROCATING ENGINES

Name of station	Metropolitan	Manhattan	Kingsbridge	Rapid Transit	Edison
Type of engine	Vert. cross-comp.	Double 2 hor. 2 vert. cyls.	Vert. cross-comp.	Double, 2 hor. 2 vert. cyls.	3-cyl. vert.
Rated horse-power	4500	8000	4500	8900	5200
Cylinders, all 60-in. stroke, in	46, 86	44, 88	46, 86	42, 86	43½, 2–75½
Piston rods, diam., in	9, 10	8	9, 10	8, 10	9
Crank pins, in	14×14	18×18	14×14	20×18	22 & 16×14
Wrist pins, in	14×14	12×12	14×14	12×12	14×14
Shaft length	27 ft. 4 in.	25 ft. 3 in.	27 ft.	25 ft. 3 in.	35 ft.
max. diam	37 in.	37 in.	39 in.	37 in.	29⅝ in.
bearings, in	34×60	34×60	34×60	34×60	26×60

The shafts are hollow, with a 16-inch hole, except the Edison which have a 10-inch hole. The speed of all the engines is 75 revolutions per minute, or 750 feet per minute. The crank pins of the Manhattan and Rapid Transit engines are each attached to two connecting rods, side by side, horizontal and vertical, each rod having a bearing 9 inches long on the pin. The crank pins of the Edison engine are 16 inches in diameter for the side cranks, and 22 inches for the center crank.

TESTING OF STEAM ENGINES. — The actual work done by an engine may be tested by a brake or friction dynamometer, or in the case of engines driving electric generators by measuring the electric energy delivered, correction being made for the efficiency of the generator. The work done in the cylinder is obtained by taking numerous indicator diagrams and calculating

from them the mean effective pressure and the horse-power. The steam consumption is tested by condensing all the steam discharged by the engine and weighing the water of condensation. When a condenser is not available the feed water delivered to the boilers may be weighed instead, precautions being taken to insure that all of the steam made by the boiler is used by the engine. For directions for making steam-engine tests see A.S.M.E. Codes.

Plant Tests. — In the introduction to the report above referred to the Committee says:

The heat consumption of a steam-engine plant is ascertained by measuring the quantity of steam consumed by the plant, calculating the total heat of the entire quantity, and crediting this total with that portion of the heat rejected by the plant which is utilized and returned to the boiler. The term "engine plant" as here used should include the entire equipment of the steam plant which is concerned in the production of the power, embracing the main cylinder or cylinders; the jackets and reheaters; the air, circulating, and boiler-feed pumps, if steam driven; and any other steam-driven mechanism or auxiliaries necessary to the working of the engine. It is obligatory to thus charge the engine with the steam used by necessary auxiliaries in determining the plant economy, for the reason that it is itself finally benefited, or should be so benefited, by the heat which they return, it being generally agreed that exhaust steam from such auxiliaries should be passed through a feed-water heater, and the heat thereby carried back to the boiler and saved.

SPECIFICATIONS. — (*See also article on Specifications.*) In obtaining bids for steam engines the prospective purchaser or his engineer usually furnishes only general specifications, stating the kind of work to be done by the engine, such as driving a cotton mill, or an electric power plant, and the following requirements: (1) Indicated horse-power required, (*a*) Maximum, (*b*) Average; (2) Steam pressure available at engine; (3) Maximum absolute back pressure; (4) Number of revolutions; (5) Required steam consumption.

The bidders then furnish with their bids complete detailed specifications with guarantees of the engines they propose to supply, with drawings, when these are required. The bidders are usually prepared to contract for the installation, erection and starting of the engine, especially if it is of a large size.

FOUNDATIONS FOR STEAM ENGINES. — Engine foundations may be built of brick or concrete; the latter is more commonly employed in modern practice. The concrete is made with proportions of one barrel of Portland cement, three of sand, and five of crushed stone.

If the foundation is to be of concrete a wooden box is built in the ground, the inside of the box being the shape of the foundation. Over the top of this box are a number of pieces of joist, from which the anchor bolts or holding-down bolts are hung. Most engine builders furnish drawings giving location of foundation bolts and size of foundation required. The nuts at the top of these bolts are blocked up to the height they are to be when the engine is in place.

The heads of the bolts are at the bottom. A cast-iron plate eight inches by eight inches by one inch, with a square recess for the bolt head, is at the bottom of each rod. Around each bolt a piece of four-inch galvanized-iron gutter pipe is placed, and the top end of the pipe stuffed with waste to keep the mason from dropping cement into the space between the pipe and the bolt. The concrete is now dumped in and tamped down. After it has set, the wooden joists holding the bolts may be taken awav.

The engine is next put over the foundation and supported on iron bars one-half inch square or perhaps larger, these bars being placed near the bolts. By means of wedges, etc., the engine is leveled; the main shaft is placed parallel

with the shafting which it is to drive. The bolts are next tightened fairly tight. As it is impossible to core the holes in the engine bed exactly as called for on the drawing, the leeway given by the space between the bolts and the iron pipe surrounding it takes care of variations of from an inch to an inch and a half.

A wall of putty, sand, or cement about one inch high is built about one inch from the engine bed. A thin, neat Portland cement, mixed to the consistency of a thick cream, is poured under the engine bed and serves to fill the space around the bolts and to give the bottom of the bed a perfect bearing over the entire foundation. Before the cement becomes hard, the edge may be trimmed up nicely by means of a trowel; the bolts are then tightened again.

Sometimes type metal instead of cement is poured under the engine bed.

OPERATION. — Full directions for setting the valves of a steam engine will be found in a little book by M. C. Myers, entitled *Types of Modern Engines and Their Valve Setting* (Boston, 1910). Other useful data on the operation and performance of steam engines will be found in a book published by the Crosby Steam Gauge and Valve Co., entitled *Steam Engine Indicator.* See also the more elaborate treatises listed in the bibliography below.

Speed Control. — The speed of an engine can always be controlled by changing the opening of the throttle valve, but this method is usually wasteful. A more efficient method is to change the point of cut-off. This can be done while the engine is running by using a suitable link motion (*see above*), or by changing the setting of the governor spring if the governor is of the "automatic" type. On engines driving alternators in parallel this is usually accomplished by having an electric motor arranged to act directly on the governor spring so that the tension of the spring may be controlled from the switch-board.

Equalizing Variable Load by Storing Heat in Hot Water. — There is no satisfactory method for equalizing the load on the engines and boilers in electric-light stations. Storage batteries have been used, but they are expensive in first cost, repairs and attention. Mr. Halpin, of London, proposes to store heat during the day in specially constructed reservoirs. As the water in the boilers is raised to 250 pounds pressure, it is conducted to cylindrical reservoirs resembling English horizontal boilers, and stored there for use when wanted. In this way a comparatively small boiler-plant can be used for heating the water to 250 pounds pressure all through the twenty-four hours of the day, and the stored water may be drawn on at any time, according to the magnitude of the demand. The steam engines are to be worked by the steam generated by the release of pressure from this water, and the valves are to be arranged in such a way that the steam shall work at 130 pounds pressure. A reservoir 8 feet in diameter and 30 feet long, containing 84,000 pounds of heated water at 250 pounds pressure, would supply 5150 pounds of steam at 130 pounds pressure. At a steam consumption of 18 pounds per horse-power-hour, such a reservoir would supply 286 effective horse-power-hours.

COST OF STEAM ENGINES (Pre-war figures).—The price of steam engines varies not only with their size and weight, but also with their style, design and workmanship. When builders are asked to submit bids for the same rated power of engine, with the same steam pressure and number of revolutions and of the same general type, such as cross-compound condensing, it is not uncommon to receive prices of which the highest is double that of the lowest, the range being say from $10 to $20 per horse-power for an engine of 500 horse-power. For specially designed engines requiring new patterns to be made the price may be considerably above $20. The price also varies with the condition of the market, discounts from regular prices of as much as 20 per cent being

sometimes quoted in periods of dull business. The catalogues of builders usually give the weights of standard sizes of engines and a rough approximation to the probable price may be made by taking it at from 5.5 to 10 cents per pound, the lower price being for engines with heavy bed-plates and of the more simple forms.

From a number of tables and sets of curves kindly furnished by Mr. Jay M. Whitham, Consulting Engineer, Philadelphia, containing the actual bids he has received for different sizes and styles of engines, the following approximate expressions have been deduced, where P is the rated horse-power of the engine.

Type of Engine	Cost in dollars
Corliss engines:	
Single cylinder, noncondensing	$500+9P$
Single cylinder, condensing	$500+11P$
Compound, noncondensing	$1200+10P$
Compound, condensing	$1200+12P$
Compound slide-valve engine:	
Portable (locomotive) boiler, noncondensing engine and stack	$300+15P$

The cost of condensing engines is exclusive of condenser.

Professor Wickenden gives the following values for engines used to drive electric generators, these values being based on the actual costs of a large number of installations.

Type of Engine	Cost in dollars
Simple high-speed engines and settings	$350+9P$
Compound high-speed engines and settings	$1000+16P$
Simple low-speed engines and settings	$1400+11.5P$
Compound low-speed engines and settings	$2500+14.5P$

Cost of Operation. — See articles on *Power Stations* and *Depreciation*.

BIBLIOGRAPHY. — Richardson, J., *Modern Steam Engines;* Ripper, W., *Steam Engine Theory and Practice;* Sennett and Oran, *Marine Steam Engines;* Rankine, *Steam Engine and Other Prime Movers;* Whitham, J. M., *Steam Engine Design;* Stumpf, J., *Una-flow Steam Engines;* Kent's *Mechanical Engineers' Pocket-Book;* Marks, *Mechanical Engineers' Handbook;* Gebhardt, G. F., *Steam Power Plant Engineering;* Low, F. R., *The Compound Engine.*

STEAM TURBINES. — (*See also Condensers, Steam; Power Stations, Steam; Steam Engines.*) Steam turbines may be classified as impulse, reaction, and composite; single-stage and multi-stage; nozzle expansion and blade expansion; vertical and horizontal. A set of nozzles or stationary guide blades and the rotating vanes through which the steam passes immediately upon leaving the nozzles or stationary blades constitute a single "stage." If there are several sets of stationary nozzles or guide blades and rotating vanes through which the steam passes successively, the turbine is called a "multi-stage" turbine.

APPLICATIONS OF STEAM TURBINES. — The steam turbine, when designed to use steam economically, is characterized by extremely high speed of rotation. The special field of application of the steam turbine is therefore the driving of electric generators, centrifugal blowers and pumps, and other machinery in which a high speed of the motor shaft is desirable. When lower speeds are desired in the motor shaft a reduction gear of some kind is placed between it and the turbine shaft. See also articles on *Generators; Power Stations; Pumps.*

RATING OF STEAM TURBINES. — Turbine ratings are usually based on maximum sustained load. Momentary overload capacity is very large and moderate overloads of considerable duration can be carried but may require the admission of high-pressure steam to low-pressure stages by means of a secondary valve. Small turbines for driving pumps, blowers, etc., are rated in horse-power. Turbines used to drive electric generators are usually rated in connection with the generator, the combined unit or turbo-generator being rated in kilowatts. Turbo-alternators of 62,500 kilovolt ampere capacity at 80 per cent power-factor are in use at the present time (1925).

DESIGN AND CONSTRUCTION OF STEAM TURBINES. — Below are noted briefly some of the chief features in the design and construction of steam turbines. For further details see the treatises by Stevens and Hobart, Moyer, Stodola, Foster, Thomas, etc.

Impulse Turbines. — An impulse steam turbine of the simplest form is a wheel similar to a water wheel, which is moved by a jet of steam impinging at high velocity on its blades. By expansion of the steam its pressure energy is converted into kinetic energy, and the wheel is propelled by the impact of the steam moving at high velocity.

De Laval Turbine. — The distinguishing features of this turbine are the diverging nozzles, in which the steam expands down to the atmospheric pressure in non-condensing, and to the vacuum pressure in condensing wheels; a single forged-steel disk carrying the blades on its periphery; a slender, flexible shaft on which the wheel is mounted and which rotates about its center of gravity; and a set of reducing gears, usually 10 to 1 reduction, to change the very high speed of the turbine to a moderate speed for driving machinery.

The number and size of nozzles vary with the size of the turbine. The nozzles are provided with valves, so that for light loads some of them may be closed, and a relatively high efficiency is obtained at light loads. The taper of the nozzles differs for condensing and noncondensing turbines. Some turbines are provided with two sets of nozzles, one for condensing and the other for non-condensing operation.

Zolley or Rateau Turbine. — The Zolley or Rateau turbines are developments of the De Laval and consist of a number of De Laval elements in series, each succeeding element utilizing the exhaust steam from the preceding. The steam is partly expanded in the first row of nozzles, strikes the first row of

buckets and leaves them with practically zero velocity. It is then further expanded through the second row of nozzles, strikes a second row of moving buckets and again leaves them with zero velocity. This process is repeated until the steam is completely expanded.

Curtis Turbine, made by the General Electric Company, is an impulse wheel of several stages. Steam is expanded in nozzles and enters a set of three or more blades, at least one of which is stationary. The blades are all non-expanding, and the pressure is practically the same on both sides of any row of blades. In smaller sizes of turbines, only one set of stationary and movable blades is used, but in large sizes there are from two to five sets, each forming a pressure stage, separated by diaphragms containing additional sets of nozzles. The smaller sizes and the more recent (1914) larger sizes have horizontal shafts. Earlier large sizes have vertical shafts supported on a step bearing supplied with oil or water under a pressure sufficient to support the whole weight of the shaft and its attached rotating disks.

Reaction and Composite Turbines.— In a simple reaction turbine steam, expanding through a set of blades or nozzles capable of rotating about a fixed axis, produces a reacting force and drives the rotating part in the opposite direction to the motion of the steam. Hero's engine is the prototype of the reaction turbine. The term "reaction" turbine, however, as ordinarily used, refers to a turbine in which the reaction principle is combined with the impulse principle, i.e., to turbines in which jets of steam striking blades or buckets inserted in the rim of a wheel give it a forward impulse, and escaping from it in a reverse direction react upon it.

Parsons Turbine. — This is a reaction turbine in which there are a large number of rows of blades, mounted on a rotor or revolving drum. Between each pair of rows there is a row of stationary blades attached to the casing, which take the place of nozzles. A set of stationary blades and the following set of moving blades constitute what is known as a stage. The steam expands and loses pressure in both sets. The speed of rotation, the peripheral speed of the blades and the velocity of the steam through the blades are very much lower than in the De Laval turbine. The rotor, or drum, on which the moving blades are carried, is usually made in three sections of different diameters, the smallest at the high-pressure end, where steam is admitted, and the largest at the exhaust end. In each section the radial length of the blades and also their width increase from one end to the other, to correspond with the increased volume of steam. The Parsons turbine is built in the United States by the Westinghouse Machine Co. and by the Allis-Chalmers Co.

The Westinghouse Double-flow Turbine. — For sizes above 5000 kw. a turbine is built in which the impulse and reaction types are combined. It has a set of non-expanding nozzles, an impulse wheel with two velocity stages (that is, two wheels with a set of stationary non-expanding blades between), one intermediate section and two low-pressure sections with Parsons blading. After steam has passed through the impulse wheel and the intermediate section, it is divided into two parts, one going to the right- and the other to the left-hand low-pressure section. There is an exhaust pipe at each end. In this turbine, the end thrust, which has to be balanced in reaction turbines of the usual type, is almost entirely avoided. Other advantages are the reduction in size and weight, due to higher permissible speed; blades and casing are not exposed to high temperatures; reduction of size of exhaust pipes and of length of shaft; avoidance of large balance pistons.

Comparison of Impulse and Reaction Turbines. — Moyer gives the following comparison of impulse and reaction turbines:

Impulse	*Reaction*
1. Few stages.	1. Many stages.
2. Expansion in nozzles.	2. No nozzles.
3. Large drop in pressure in a stage.	3. Small drop in pressure in a stage.
4. Initial steam velocities 1000 to 4000 feet per second.	4. All steam velocities low, 300 to 600 feet per second.
5. Blade velocities 400 to 1200 feet per second.	5. Blade velocities 150 to 400 feet per second.
6. Best efficiency when the blade velocity is nearly half the initial velocity of steam.	6. Best efficiency when the blade velocity is nearly equal to the highest velocity of the steam.

Experience has not made manifest any marked and consistent difference in economy and reliability between the impulse and reaction types.

Vertical Versus Horizontal Turbines. —There is also no marked difference in economy and reliability of the vertical and horizontal types. The former is more economical of floor space, especially when the base type of condenser is employed, but requires more head room, as sufficient overhead space must be allowed to permit the lifting clear, and removal of, the shaft from the casing.

Reduction Gear for Steam Turbines. — Double spiral reduction gears, usually of a ratio of 1 to 10, are used with the De Laval turbine to obtain a velocity of rotation suitable for dynamos, centrifugal pumps, etc. G. W. Melville and J. H. McAlpine have designed a similar gear, with the pinion carried in a floating frame supported at a single point between the bearings to equalize the strain on the gear teeth, for reducing the speed of large horizontal turbines to suitable speeds for marine propellers. A 6000-horse-power gear with reduction from 1500 to 300 revolutions per minute has been tested, giving an efficiency of 98.5 per cent (*Eng'g, Sept. 17; Eng. News, Oct. 21 and Dec. 30, 1909*).

TURBINES VERSUS RECIPROCATING ENGINES. — The inherent advantages of the turbine are: (1) The perfect continuity and uniformity of effort. This permits the working parts to be made light and gives the uniform angular velocity which is desired for the parallel running of alternators. (2) The high velocity of rotation and of steam flow. This tends to great compactness, permits light weights, provides for the full expansion to high vacuum without enormous volume of expansive space, cheapens the cost of turbine and alternator, and is favorable to the efficiency of the latter. The effect of items (1) and (2) on weights, sizes and costs may be best appreciated by comparing turbine and gas-engine sets of equal maximum capacity. (3) The reduction to a minimum of condensation, radiation and leakage. (4) The absence of all lubrication from the steam space. Difficulties of lubrication with high superheat are thus obviated and the condensed steam is free from oil. (5) The maintenance of good economy over a wide range of fractional loads. (6) The great momentary overload capacity obtained from the high inertia of the rapidly rotating parts. To match this advantage in slow-speed reciprocating engines requires very heavy working parts and the addition of an expensive flywheel. The reciprocating engine can be built to equal the turbine in economy over a wide range of loads but the refinements necessary to this end are costly. For equal heat consumption the costs of large engine-alternator units are from 35 to 50 per cent greater than those of turbine sets.

Turbines versus Engines in Units of Small Capacity. — Under this title, K. S. Barstow presented the following summary before the A.S.M.E. in December, 1915, for units of less than 500 horse-power capacity.

Applicability of Turbines. — 1. Direct-connected units, operating condensing. 60-cycle generators in all sizes, also 25-cycle generators above 1000 kw. capacity. (This paper is, however, not intended to deal with units of this size.) Direct-current generators in sizes up to 1000 kw. capacity, including exciter units of all sizes.

Centrifugal pumping machinery operating under substantially constant head and quantity conditions, and at moderately high head, say from 100 feet up, depending upon the size of the unit.

Fans and blowers for delivering air at pressures from 1½ inch water column to 30 pounds per square inch.

2. Direct-connected units, operating non-condensing for all the above purposes, in those cases wherein steam economy is not the prime factor or where the exhaust steam can be completely utilized, and, in the latter case, particularly where oil-free exhaust steam is desirable or essential.

3. Geared units, operating either condensing or non-condensing for all the above-mentioned applications, and in addition, many others which would otherwise fall in the category of the steam engine, on account of the relatively low speed of the apparatus to be driven.

Applicability of Engines. — 1. Non-condensing units, direct-connected or belted and used for driving: Electric generators of all classes excepting exciter sets of small capacity, unless belted from the main engine.

Centrifugal pumping machinery, operating under variable head and quality conditions and at relatively low heads, say up to 100 feet, depending on the capacity of the unit. Pumps and compressors for delivering water or gases in relatively small quantities and at relatively high pressures in the case of pumps at pressures above 100 pounds per square inch, and in the case of compressors at pressures from 1 pound per square inch and above.

Fans and blowers (including induced draft fans) for handling air in variable quantities and at relatively low pressures, say not over 5 inches water column. Line shaft of mills, where the driven apparatus is closely grouped and the load factor is good.

All apparatus requiring reversal in direction of rotation, as in hoisting engines and engines for traction purposes.

2. Condensing units direct-connected or belted, for all the above purposes particularly where the condensing water supply is limited, and where the water must be recooled and recirculated.

Low-pressure Turbines compounded with high-pressure reciprocating engines are coming into extensive use where it is necessary to increase the capacity or improve the efficiency of existing engine plants. By combining the superior efficiency of the engine in the pressure range above the atmosphere with that of the turbine below the atmospheric range a resultant superior to the efficiency of either single type may be secured. Standard piston engines are able to sustain full-rated load when run non-condensing and often will carry from 25 to 50 per cent above rated load without danger or excessive wear. Under such conditions the water rate per kilowatt-hour is high but all the heat rejected in the exhaust is available to a low-pressure turbine; hence the net economy of the combined engine and turbine may be considerably superior to that of the engine when run condensing. By proportioning the turbine to efficiently utilize the exhaust steam and by connecting to it a high-vacuum condenser, the initial capacity of the unit may be doubled or even trebled, and if the engine is in good condition, the resultant efficiency may be superior to that obtainable from a new complete-expansion turbine of equal capacity.

Structurally, the low-pressure turbine is similar to the full-pressure type with the high-pressure stages omitted. It is, therefore, somewhat shorter but

the steam spaces are necessarily greater than in a full-pressure type of equal rating. It is highly desirable that an efficient separator be installed between the engine and the turbine as the presence of moisture in the steam adds greatly to the friction against the turbine blades. When the exhaust is to be returned to the boiler it is also necessary to extract the oil from the steam as it leaves the engine.

EFFICIENCY AND LOSSES. — The maximum theoretical efficiency of a steam turbine is the efficiency of the Rankine ideal engine between the temperatures of admission and exhaust (*see Steam Engines*).

The several losses which tend to reduce the efficiency of turbines below the theoretical maximum are: (1) residual velocity, or the kinetic energy due to the velocity of the steam escaping from the turbine; (2) friction and imperfect expansion in the nozzles; (3) windage, or friction due to rotation of the wheel in steam; (4) friction of the steam traveling through the blades; (5) shocks, impacts, eddies, etc., due to imperfect shape or roughness of blades; (6) leakage around the ends of the blades or through clearance spaces; (7) shaft friction; (8) radiation. The sum of all these losses amounts to about 25 per cent of the available energy in the largest and best designs and to 50 per cent or more in small sizes or poor designs.

Condensation, leakage and radiation are of small magnitude. Windage and friction are especially marked in the high-pressure stages and the turbine is most efficient in its low-pressure stages, in direct contrast to the piston engine. The mechanical friction is relatively small, due to the small loads on wearing surfaces and the effectiveness with which they may be lubricated. Steam friction, windage and condensation are much increased by moisture in the steam. Superheated steam is particularly advantageous in turbines because of its low heat conductivity, dryness and high fluidity.

STEAM ECONOMY OF TURBINES. — Like all heat engines the efficiency of the turbine depends on the range of expansion and the reduction of losses. Increasing the expansion range by raising the admission pressure affords a relatively small gain in efficiency. The chief advantage resulting from high initial pressure is in the decreased size of the turbine of a given capacity. The relation of initial pressure to the steam and heat consumption per unit of output varies somewhat with the individual turbine, but the general relation with fixed conditions of vacuum and superheat is shown in Fig. 1.

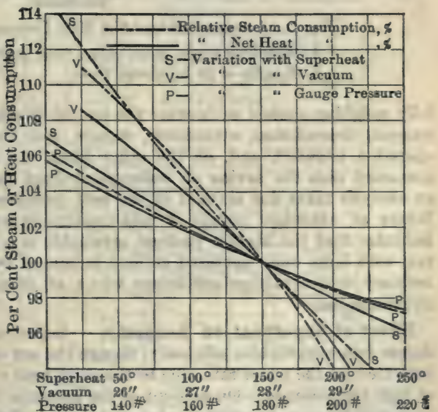

Fig. 1. Relations of Turbine Efficiency to Steam Conditions and Vacuum

The advantage gained by increasing the range of expansion at the lower end greatly exceeds that gained by an equal pressure increment at the upper end of the cycle. In the utilization of high vacua the turbine greatly surpasses the engine. Fig. 1 shows the relation of vacuum to turbine efficiency for typical units. This relation depends to a marked degree

on the design of the particular turbine and is therefore not identical in all units of a given general type.

Effect of Vacuum on Economy.,— The economical limit of vacuum in turbines is much higher than in engines. The attainment of very high vacua involves a very heavy investment in condensers and auxiliaries and a considerable consumption of heat for their operation. The ultimate effect on plant economy is therefore decidedly less than the percentage gain indicated in Fig. 1. Factors unfavorable to high vacuum are a poor load factor for the unit; costly, warm or corrosive cooling water; the necessity of artificially cooling condensing water when the supply is limited; and very cheap fuel.

The question of the most economical vacuum must be determined by care-

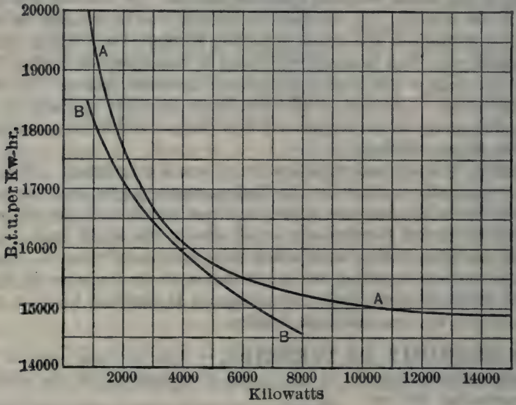

Fig. 2. Average Full-load Net Heat Consumption of Turbines

A. American types, mean values from guarantees and tests
B. Results of 50 European Tests

fully weighing against each other the annual charges on the condenser system, interest, depreciation, attendance, repairs, and the net cost of water for its operation included, and, on the other hand, the annual saving in total fuel consumed plus the saving on boiler-plant fixed charges and operation. Such an analysis takes due account of the heat consumed in driving the auxiliaries. Where an abundant supply of cold condensing water is available, experience indicates that the highest vacuum attainable is a good investment in connection with large units operated at load factors of 60 per cent, or above. With ordinary central-station conditions vacua of from 27 to 28 inches are most advantageous.

Effect of Superheat on Economy. — Superheat of almost any practicable degree improves turbine efficiency, though the net saving is much less than that indicated by the water rate, due to the extra heat content of superheated steam of high temperature. On account of the influence of superheat on the first cost and maintenance expense of piping, valves and fittings, also because of the difficulties in providing for expansion of turbine parts at very high temperatures, the tendency of American practice is to exceed a steam temperature of 500° F. only in very large units, though the tendency in Europe is toward a higher limit. Fig. 1 shows the effect of superheat on steam consumption and net heat consumption of steam turbines, the relation being practically uniform for all types.

Effect of Size and Type on Economy. — The efficiency of the turbine, like that of the engine, improves as the size of the unit increases, see Fig. 2. Large units are relatively cheaper than small, and afford better economy in piping, condensing equipment, space, attendance and maintenance.

A turbo-generator is rated as a single unit. The momentary and sustained overload capacity of the turbine proper is very large, but the generator is much restricted by its ability to dissipate the heat developed in its parts. The usual ratings are based either upon maximum sustained capacity (*M* in Fig. 3) or upon normal sustained capacity with an allowable overload of 25 per cent for a two-hour period (*N* in Fig. 3).

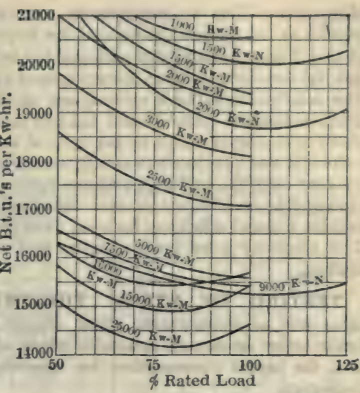

Fig. 3. Net Heat Consumption of Modern Turbines

M. Maximum Rating. N. "Normal" Rating

Experience has not made manifest any marked or consistent differences in economy and reliability between the several commercial types. The vertical type is not well suited to the highest speeds, but is especially economical of floor space in connection with the base type of condenser. It requires greater head-room than the horizontal type. Fig. 4 shows the steam consumption of a number of small non-condensing turbines (*see paper by G. A. Orrok, Trans. A.S.M.E., 1909*). All curves are test results.

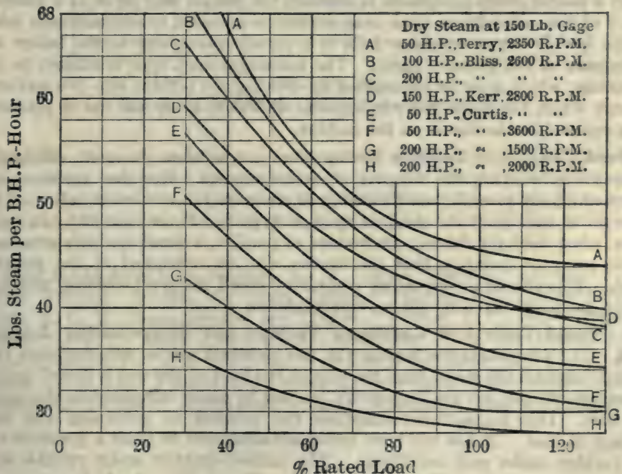

Fig. 4. Steam Consumption of Small Non-condensing Steam Turbines

ECONOMY OF COMBINED TURBINE AND ENGINE UNIT. — The following is reported by H. G. Stott and R. J. S. Pigott in *Jour. A.S.M.E.*,

Mar., 1910. The steam-engine is one of the 7500-kilowatt Manhattan type engines at the 59th St. Station of the Rapid Transit Co., New York, with two 42-inch horizontal high-pressure and two 86-inch vertical low-pressure cylinders, and the turbine, also 7500 kilowatt, is of the vertical three-stage impulse type. The principal results are summarized as follows: An increase of 100 per cent in the maximum capacity and 146 per cent in the economical capacity of the plant; a saving of about 85 per cent of the condensed steam for return to the boilers [it was previously wasted]; an average improvement in economy of 13 per cent over the best high-pressure turbine results and of 2.5 per cent (between 7500 and 15,000 kilowatts) over the results obtained by the engine alone; an average thermal efficiency between 6500 and 15,500 kilowatts of 20.6 per cent. [This efficiency is not quite equal to that reached by triple-expansion pumping engines.]

TESTING OF STEAM TURBINES. — Turbines are tested in the same way as reciprocating steam engines, measuring the power by a Prony brake or by an electric generator whose efficiency at different loads is known and the steam consumption by weighing the water discharged from the condenser.

SPECIFICATIONS. — See *Specifications* in article on *Steam Engines*.

OPERATION. — Several important features are noted below. For additional information see article on *Power Stations*.

Oil-pressure System for Step Bearing of Vertical Turbine. — When running the end of the shaft is lifted slightly from its seat and supported by a forced circulation of water or oil under a pressure of from 500 to 1200 pounds per square inch. This pressure is produced by a special step-bearing pump which may be of the steam-driven, direct-acting type or of the motor-driven, triplex type, and is reinforced by a heavily weighted plunger accumulator. When a double pump equipment is installed the second may serve as a reserve and, if of the motor-driven type, may readily be arranged to automatically take up the load if the accumulator falls to a certain point, due to the failure or inadequacy of the primary pump. The horizontal-shaft type is subject to an end thrust, which must be neutralized by dividing the turbine into two sections through which the steam flows in opposite directions or by the use of pressure rings opposed to the thrust on the turbine.

Speed Control. — Turbines having nozzle expansion are most efficiently governed by varying the number of active nozzles. Reaction turbines are governed by the intermittent throttling of steam at the admission valve. The governor is usually of the shaft or flywheel type (*see Steam Engines*). In both types, when used for driving alternators in parallel, a small motor is attached to the governor spring so that the tension may be controlled at the switchboard. This affords a convenient control of speed for synchronizing and adjusting the loads taken by the several machines.

Speed Control of Combined Turbine and Reciprocating Engine Unit. — In central-station service the two elements are so adjusted that the turbine takes all the steam from the engine, the load on the two is varied simultaneously and the two are electrically coupled by being connected to the same bus bars. This arrangement simplifies the problem of governing and the division of load. It has not been found advantageous to place a governor on the turbine under such conditions. The turbine operates under variable admission pressure, depending on the load. The turbine should be equipped with a speed-limiting relay, which opens the atmospheric exhaust from the engine should the speed of the turbine become unsafe.

COST OF STEAM TURBINES (Pre-war figures). — Small turbines for driving pumps, blowers, etc., cost from \$20 to \$40 per horsepower.

In Fig. 5 are given the total costs of various turbo-generator sets in 1912. These costs include both the turbine and generator but do not include condensers and auxiliaries. The curves were kindly supplied by Prof. W. E. Wickenden and represent the average of the actual costs of a number of units of various

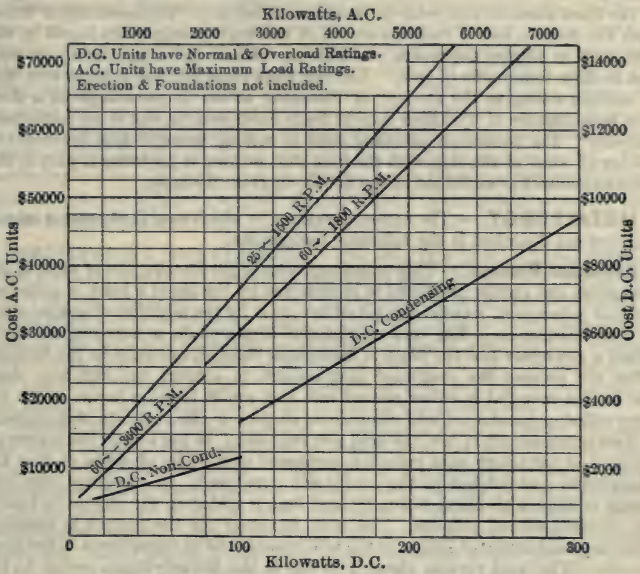

Fig. 5. Approximate Cost of Turbo-generator Sets

capacities. Impulse and reaction turbines cost approximately the same. Turbine costs are subject to considerable variation, the tendency being a decided decrease in the cost per kilowatt from year to year. The costs given in the curves should therefore be checked against actual quotations, even when used in preliminary estimates.

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STEEL. — (*See also Iron, Pig and Cast; Castings, Iron and Steel; Rails, Track and Third.*) Carbon steel (the ordinary commercial steel) may be defined as an alloy of iron and carbon produced by any one of the methods described in the following article. It differs from wrought iron in not being slag bearing and from cast iron in being malleable at certain temperatures.

Slag is the substance which collects on the surface of molten iron or steel in the blast furnace, converter or open-hearth furnace. Its constitution varies with the process but it includes oxides of the various impurities found in the iron such as silicon, aluminum, phosphorus, calcium and magnesium. It may also include some oxide of iron. The slag from blast furnaces is often very rich in calcium owing to the amount of limestone added to the charge as a flux. The manufacture of Portland cement from blast-furnace slag is an important industry. The process of puddling used in making wrought iron involves the mixing of some of the slag with the iron, thus making it an integral part of the finished product as explained in the article on *Iron, Wrought.*

METALLURGY. — The various processes by which steel is at present manufactured are described in the following paragraphs:

Bessemer Process. — This process consists of the burning out of most of the impurities and carbon contained in molten iron by the aid of a blast of unheated air, and subsequent recarburization of the mass by the addition of spiegel-eisen or ferro-manganese. The molten material is placed in a pear-shaped vessel called a converter and the blast is introduced through openings in the bottom. After the process has continued long enough to convert the iron into steel, the molten material is either poured into iron molds forming ingots which are later rolled or hammered into commercial shapes, or else cast into *steel castings*.

Two processes are in use, the acid and the basic; in the former the converter lining is of refractory acid materials composed principally of silica, so that the phosphorus and sulphur in the charge are unreduced; in the latter, the converter is lined with burnt dolomite, and lime is added to the charge to reduce the phosphorus and sulphur. The heat necessary for the process is obtained from the combustion of silicon in the acid process and of phosphorus in the basic process. The acid process alone is in use in the United States owing to the prevalence of low phosphorus and sulphur ores. Steel of fair quality is produced by this process at low cost and with great rapidity, but Bessemer steel is gradually being replaced for important structures by the more reliable open-hearth steel.

Open-hearth Process. — This process consists of the reduction of the carbon and other impurities contained in a mixture of pig iron, scrap iron or scrap steel, and iron ore by exposure for seven or eight hours to an intense heat obtained from a mixture of producer gas and air, the process being stopped when the bath is found to have the right proportions as determined by physical and chemical analyses. The steel is then withdrawn from the furnace and either poured into ingots or cast into usable forms. Steel of excellent quality for structural purposes is thus produced in large quantities, a single furnace often having a capacity of 50 tons.

Duplex Process. — This process consists of a combination of the Bessemer and open hearth; pig iron is first blown into the converter to remove the silicon and a portion of the carbon; it is then transferred to the open hearth, where it is dephosphorized and the carbon reduced to the proper percentage. The process permits the use of a poorer grade of material than the Bessemer, and gives a higher grade of finished product, while the speed of production is superior to that of the open hearth.

Crucible Process. — This process consists of melting down, in covered clay or graphite crucibles, wrought iron (sometimes diluted with steel scraps) with charcoal, ferro-manganese and various so-called physics, such as salt, oxide of manganese, etc. The crucibles hold from 50 to 100 lb. each. Crucible steel is high in carbon and low in sulphur and phosphorus. It is superior to Bessemer and open-hearth steel and is used for tool making.

Cementation Process. — This process consists of converting wrought iron into steel by piling iron bars and charcoal together, bringing to a yellow heat and keeping in this condition for a period extending from ten days to two weeks. The steel thus obtained is known as blister steel. If the blister steel is cut, piled, heated and forged, *shear* steel is produced; if the operation is repeated on shear steel, *double shear* steel is obtained. Blister steel is frequently melted in the crucible to free from slag; the resulting product is of the highest quality and is used in making the finest grades of cutting tools, such as razors. Case hardening is a special form of cementation by which the outer surfaces of articles forged from wrought iron or steel are hardened. Harveyizing of armor plate is also a cementation process.

MECHANICAL TREATMENT. — (*See also Steel Mills, Electric Drive of.*) Steel made by any of the preceding processes may be put into usable form either by casting, or by forging while hot under the hammer, or usually by rolling while hot between heavy grooved rollers, the latter process being used for rods, plates, rails, and structural shapes such as angles, channels and I-beams. The rolling process is usually divided into two parts: (*a*) the reduction of the ingot into *blooms* or *billets* by the *blooming mill;* and (*b*) the reduction of the blooms or billets into commercial shapes by the *shaping mill.*

The blooming mill consists of a pair of horizontal grooved rollers, one above the other, between which the ingot is passed back and forth, the rolls being gradually forced closer together. These rolls are actuated by a reversing engine. This process reduces the ingot from a block of steel perhaps 16 inches square by 6 feet in length to a bar of small cross-section, usually rectangular and of considerable length. This is then cut into sections several feet in length which are called blooms if the cross-section contains more than 36 square inches and billets if less. In England this mill is called a cogging mill.

The shaping mill is similar to the blooming mill, but there are usually three non-reversing rolls in a vertical plane, the bloom passing forward between the lower and middle rolls, and returning between the middle and upper roll.

HEAT TREATMENT; CRITICAL TEMPERATURE. — When iron or steel cools from a high temperature, say 1000° C., the rate of cooling is not uniform, but is marked by certain periods of retardation accompanied by spontaneous evolution of heat, the opposite phenomena occurring when the temperature is increased. There are in general three of these critical points which are designated by metallurgists by the letters A_1, A_2 and A_3, A_1 being at the lowest temperature. Owing to *lag* the changes occur at slightly lower temperatures on cooling than on heating, and to distinguish between them, the cooling points are designated Ar_1, Ar_2 and Ar_3, and the heating points Ac_1, Ac_2 and Ac_3. These points vary with the percentage of carbon, and for certain grades of steel the three cooling points may be merged into one and also the three heating points. With favorable conditions when the temperature Ar_1 is reached an increase in temperature of the metal may be detected by its change of color. This point is called the point of recalescence. At this temperature also a softening of the metal occurs. An interesting feature of the point A_2 is that at a higher temperature steel is *non-magnetic*, but in passing through Ar_2 it becomes suddenly *magnetic*. *See Magnetic Properties of Iron and Other Metals.*

The approximate values of these critical points in degrees Centigrade as given by Sauveur are shown in the following table:

CRITICAL TEMPERATURES IN DEGREES CENTIGRADE

Grade of steel	Low carbon	Medium-high carbon	High carbon
Carbon content, per cent	0.1	0.45	Above 0.85
Ar_1	675	650 to 700	675
Ar_2	750	725	675
Ar_3	850	725	675
Ac_1	700	...	...
Ac_2	750	...	...
Ac_3	875	...	...

ANNEALING. — Steel with a coarse structure due to rapid cooling may be improved in grain by heating to a temperature varying with the carbon content and cooling slowly. The accompanying table gives the annealing temperatures which are commonly specified for this purpose.

Range of carbon content, per cent	Range of annealing temperature	
	° C.	° F.
Less than 0.12	875 to 925	1607 to 1697
0.12 to 0.29	840 to 870	1544 to 1598
0.30 to 0.49	815 to 840	1499 to 1544
0.50 to 1.00	790 to 815	1454 to 1499

The object should be kept at the annealing temperature long enough to insure a uniform temperature throughout and then cooled. The higher the carbon, or the greater the ductility and softness required, the slower should be the cooling. It is usually sufficient to remove the object from the furnace and to allow it to cool in air protected from rain, snow and drafts. Steel containing more than 0.50 per cent carbon should cool more slowly until the color dies out before removing from the furnace; it may then be removed and cooled in air. To remove the effects of cold working the object should be heated to about 775° C. and cooled with a slowness depending upon the thickness.

For detailed specifications see *Amer. Soc. for Test. Mat. Standards.*

CARBON STEEL. — The alloy of carbon and iron of which ordinary carbon steel is composed is divided into the five following groups by Sauveur:

Grade of steel	Hardness	Carbon content, per cent
Very low carbon	Very soft	Not over 0.10
Low carbon	Soft	Not over 0.25
Medium-high carbon	Half hard	0.26 to 0.60
High carbon	Hard	Over 0.60
Very high carbon	Hard, extra hard	Over 1.25

The properties of these different groups when made under normal conditions depend upon the relative amounts of the following constituents:

Ferrite, or iron free from carbon, this being the same as the ferrite in wrought iron (q.v.).

Pearlite, consisting of a mixture of iron carbide and ferrite.

Cementite, or iron carbide, with the formula Fe_3C, combined with a varying amount of carbide of manganese.

Austenite and Martensite. — Steel that is suddenly cooled from a high temperature contains also certain other compounds of iron and carbon, the two of most importance being known as *austenite* and *martensite.* The former is generally considered as a solid solution of carbon, or of the carbide Fe_3C, in iron in the allotropic condition in which the latter exists at a high temperature (above 875° C.). It is not a constituent of constant composition, and is seldom found in ordinary steels since it is rapidly transformed in cooling into an aggregate of ferrite and cementite. To obtain it highly carburized steel may be suddenly cooled from a temperature of 1000° C. or more by a bath such as ice cold water. It may also be obtained by slow cooling if the steel contains a considerable portion of manganese or nickel. Martensite is generally believed to be an early stage in the transformation of austenite.

Phosphorus and Sulphur. — The common injurious impurities in carbon steel are phosphorus and sulphur, both of which cause brittleness and should be allowed only in small quantities. Specifications (*see section below on this topic*) frequently forbid more than 0.05 per cent of sulphur and 0.04 per cent of phosphorus. Other impurities commonly occuring in small quantities in steel are silicon and manganese, which in small quantities help to reduce blow holes by their deoxidizing action. These elements while often present in higher percentages than the carbon have comparatively little influence upon the strength of ordinary carbon steel.

Composition of Low-carbon Steel. — Low-carbon steel consists chiefly of a mass of ferrite with pearlite occuring at the junction of the grains of ferrite, the percentage of the former decreasing in medium and hard steels until in steel having 0.8 per cent to 0.9 per cent carbon, it disappears entirely, the steel then consisting exclusively of pearlite. In still higher carbon steels some of the pearlite is replaced by free cementite, the substance which gives steel its hardening property.

Strength and Weight of Carbon Steel. — The tensile strength of steel varies with its composition. For ordinary carbon structural steel the working values given in the following table may be adopted with safety:

Nature of stress	Working strength, pounds per square inch
Tension on net section, and extreme fiber stress in bending	16,000
Compression in columns*	$16{,}000 - \frac{70\,l}{r}$, with a maximum of 14,000
Shear on net section of plate-girder webs and on machine-driven shop rivets	12,000
Bending on extreme fiber of pins	24,000
Bearing on pins and shop-driven rivets	24,000
Bearing on hand-driven rivets	18,000
Shear on hand-driven rivets	9,000
Modulus of elasticity	28,000,000 lb. per sq. in.

* In the expression for compression in columns, $\frac{l}{r}$ = maximum value of ratio of the unsupported length of column to radius of gyration, both values being expressed in inches. This ratio should be restricted by the form of the column so that it will not exceed 100 for main members and 120 for lateral and other secondary members.

Steel weighs approximately 490 lb. per cu. ft., or 2 per cent more than wrought iron.

CHEMICAL COMPOSITION

Elements considered	Structural steel	Rivet steel	Steel castings	
			Class A	Class B
Carbon, max. per cent	....	....	0.30	....
Phosphorus, max., per cent:				
Acid	0.06	0.04	0.07	0.06
Basic	0.04	0.04	0.06	0.05
Sulphur, max., per cent	0.05	0.045	....	0.06

TENSILE PROPERTIES

Properties considered	Structural steel	Rivet steel	Steel castings, Class B, medium
Tensile strength, pound per square inch	55,000 to 65,000	46,000 to 56,000	70,000
Yield point, min.	0.5 Ten. str.	0.5 Ten. str.	0.45 Ten. str.
Elongation in 8 in., min., per cent	$\frac{1{,}500{,}000}{\text{Tensile strength}}$	$\frac{1{,}500{,}000}{\text{Tensile strength}}$	....
Elongation in 2 in., min., per cent	22	....	18
Reduction of area	....	....	25

Specifications for Structural Steel and Steel Castings. — The American Society for Testing Materials has adopted standard specifications for steel to be used in various classes of structures. (*See A.S.T.M. Standards.*) Among the items covered are the chemical and physical properties of the material, the specimens and methods of tensile and bend tests and the permissible variation in weight and gauge. The preceding tables give the requirements as to chemical composition and tensile properties as given in the Society's specifications for structural steel for bridges and ordinary steel castings (*A.S.T.M. Standards, 1918*).

SPECIAL OR ALLOY STEELS. — The more usual forms of special steels and their uses, composition and general characteristics are as follows:

Nickel Steel. — Steel containing a considerable percentage of nickel, but generally not more than $3\frac{1}{2}$ per cent, is called nickel steel. This steel has a greater strength and a higher elastic limit than carbon steel, and practically the same modulus of elasticity; it is more ductile in proportion to its strength and somewhat harder than carbon steel of like properties, it is more expensive because of the cost of the nickel, and the additional cost of working. Nickel steel has been used considerably in recent years for bridges of large size, for steamer shafts, automobile parts, etc.

Invar Steel. — This is a nickel steel containing about 36 per cent of nickel. Its coefficient of expansion is practically zero. It is used in making steel tapes for base line measurements in geodetic surveying.

Manganese Steel. — The steel commonly known as manganese steel contains from 12 per cent to 13 per cent of manganese and generally from 1.25 per cent to 2.00 per cent of carbon for castings; for forgings or rolled steel, less carbon is used. It is austenitic in composition and is very hard without brittleness, making it well fitted for railroad frogs, railroad rails or curves and burglar-proof safes. It is adapted principally to parts that can be cast, since working it with tools is practically impossible and forging is difficult. It is not used for structural work as its elastic limit is so low as to offset the advantage of its great strength. Like nickel, the manganese interferes with the transformation of austenite to pearlite during cooling. It is often called "Hadfield" steel from the name of its inventor.

Chrome Steels. — Steels containing from 1 per cent to 2 per cent of chromium are very hard and have a high elastic limit. They are well adapted for armor-piercing projectiles, steel balls, files and automobile gears. Krupp armor plate is said to contain 3.25 per cent nickel, 1.5 per cent chromium and 0.25 per cent carbon.

Vanadium Steels. — The addition to steel of a small amount of vanadium, usually less than 0.3 per cent, adds greatly to the elastic limit, ductility and resilience. Vanadium is especially advantageous in increasing strength, toughness and temper of nickel and chromium steels. This steel is particularly useful for springs, axles and automobile parts.

Silicon Steels. — The only silicon steels in use contain less than 5 per cent of silicon. The alloy recommended by Hadfield, its patentor, contains 2.75 per cent silicon with the smallest possible amounts of carbon, manganese and other impurities. Such steel has a greater magnetic permeability than the purest iron, and also a high electrical resistance. Its magnetic hysteresis is low. It is a valuable material for use in electromagnetic and in electrical generating machinery. See *Magnetic Properties of Iron.*

Tungsten Steels. — These steels have the property of becoming martensitic upon heating to a high temperature and air cooling and are said to be "self-

hardening." Such steels are used for springs, magnets and with manganese added for self-hardening tools. See also *High-speed Steels and Magnetic Steels, below.*

Self-hardening Steels. — Steel which is hard without tempering and cannot be annealed by known processes is called self-hardening steel. All such steels are non-magnetic. Any steel which is in an austenitic condition at atmospheric temperature is self-hardening. (*See Mushet Steel, below.*)

Ternary and Quartenary Steels. — These are steels consisting of three or four essential components, viz., iron, carbon and one or two alloys. Some of the more important ternary steels are nickel steel, manganese steel, chrome steel, tungsten steel and silicon steel. Among quartenary steels are nickel-chromium, nickel-manganese, nickel-vanadium, tungsten-chromium, etc. The effect of these various alloys upon carbon steels is very striking and their possibilities are very great.

Chrome-nickel Steel. — The chrome-nickel steels in use are pearlitic and contain moderate amounts of carbon, nickel and chromium. Such steel has a high elastic limit combined with high ductility and has greater hardness, resilience and wearing power than carbon steels. It is used in automobile construction and for the manufacture of armor plate.

Chrome-tungsten Steel. — Tungsten and chromium have the property of producing a finely martensitic structure in steel when heated to a high temperature and cooled in air. The martensite in this case is so stable that the steel may be heated to a red heat without losing hardness. Such steels are known as "high speed steels" (q.v.).

High-speed Steels. — These steels have a finely martensitic structure which is stable up to red heat. They may contain from 0.25 per cent to 1.00 per cent of carbon, from 5 per cent to 25 per cent tungsten, 2 per cent to 10 per cent of chromium and seldom over 0.40 per cent of manganese. Tungsten may be replaced wholly or in part by molybdenum. Steels of this character are of great importance in machine work as tools made from them can be driven without injury until the cutting edge is red hot.

Mushet Steel. — This steel contains both tungsten and manganese. It is a self-hardening steel and has been used for many years for tools which make very heavy or deep cuts such as is required for armor plates. The speed allowable is a little greater than for carbon steel, but it lasts a long time without grinding.

Magnetic Steels. — Steels containing 4 per cent to 5 per cent of tungsten and from 0.5 per cent to 0.7 per cent of carbon if heated to red heat and quenched in water will retain their magnetism better than carbon steel. The addition of chromium increases the permanency but decreases the magnetic force.

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STEEL MILLS, ELECTRIC DRIVE OF. — (*See also Flywheels for Load Equalization; Motors, Industrial Applications of.*) The application of electric motors to steel mill service may be divided in two general classes; 1. that dealing with the application of motor drive to the main rolls, that is to those rolls in which the ingot or billet is reduced in section, and 2. that dealing with the problems of electrifying the numerous other auxiliary machines and devices, such as tables, screw-downs, charging machines, etc.

MAIN-ROLL APPLICATION. — The term "mill" is sometimes used to designate a single stand or group of stands, and sometimes to include the main rolls and all auxiliaries involved in the production of a given class of materials. The stands are generally classified according to the arrangement of rolls and method of operation, that is two-high or three-high, the two-high being either reversing or non-reversing, see Fig. 1.

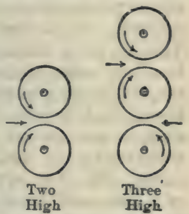

Fig. 1. Diagram of Two-high and Three-high Rolls

Reversing and Non-reversing Mills. — The great majority of rolling mills in this country are of the non-reversing type where the rolls run continuously in one direction. Flywheels are used to equalize the input to the motor. The ideal combination would be to use a motor of sufficient capacity to carry the average load and a flywheel which would take care of all the peaks. Such a wheel would be too large and a compromise between the motor and flywheel must generally be made. As opposed to the heavy flywheel effect required by the two-high or three-high non-reversing mill, the minimum flywheel effect consistent with the necessary speed and torque is desirable for reversing mills. In order to avoid the reflection of peak loads of these motors back upon the system, it is customary to interpose a flywheel motor-generator set with one or more generators between the power station and the mill. In order to minimize the flywheel effect of the reversing motor, two or more direct-current motors are often mounted on the same shaft and their armatures connected in series. This arrangement renders possible the use of smaller armature diameters with consequent reduction of acceleration losses and also permits the use of a high-voltage generator on the flywheel set. The mill motor is separately excited and perfect speed control is obtained by varying the excitation of the generator on the flywheel set. Direct-current generators can be designed for 2400 volts to give excellent results when operating two 1200-volt motors in series.

Power Required. — Rolling mill loads are irregular in the extreme due to the intermittent character of the process. For any particular ingot the motor load consists of periods of heavy duty increasing in length with the length of the metal in the pass, interspersed with periods of friction load only. With a given mill the load varies widely with the difference in the section rolled, differences in temperatures of metal, personal equation of the operator, etc. The practical determination of the power required to roll steel is a matter of elaborate and extensive tests under widely varying conditions. For methods and results see papers in *Bibliography*. The information for predetermining the sizes of motor and flywheel for an installation must cover the following points:

Type of mill; rail, plate, etc.	Time between passes.
Diameter and speed of rolls.	Elongation in each pass and total.
Weight of ingot.	Initial length.
Initial and final section.	Average and maximum rate of rolling.
Number of passes.	Temperature of metal.

Load Curves. — From these data and a thorough understanding of the many variable factors encountered in steel-mill practice, it is possible to lay out load curves similar to the ones shown in Figs. 2, 3 and 4. From such curves it is then possible to determine the proper size of the motor and the flywheel.

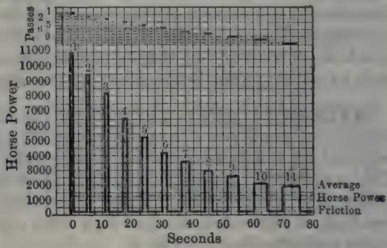

Fig. 2.

The typical curve shown in Fig. 2 represents a load curve calculated for a two-high single-stand reversing blooming mill in which a 3000-pound ingot is reduced from initial to final section in eleven consecutive passes occurring at 5-second intervals. This curve is characteristic of blooming mills, roughing mills, etc., which in general give the lowest load factor. The work done in horse-power seconds is indicated by the area under the curve. Fig. 3 is representative of a large and mixed class with medium load factor, such as small merchant mills, bar mills, sheet mills, etc. Fig. 4 shows a load factor so high as to be scarcely approximated except in merchant mills with a large number of stands working at full capacity.

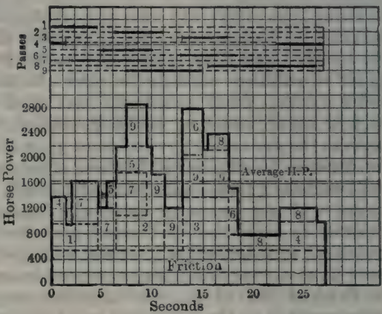

Fig. 3.

In determining the most suitable size of motor for the work indicated by the load curve full consideration must be given to the fact that excessive loads may be encountered at times due to the low temperatures of the metal in the pass, too heavy draft or other causes, and ample margin allowed. Assuming for the moment that by properly combining simultaneous passes on the several stands and the application of a suitable flywheel the load curve has been flattened out as much as appears possible, the question of continuity of service and duration of peaks must be considered to determine whether the limiting feature will be maximum torque or heating. With rolling-mill motors the limiting feature of design is usually torque rather than heating. The motor rating is determined by a consideration of the root-mean-square value of the current for a single cycle, and the frequency and duration of successive cycles.

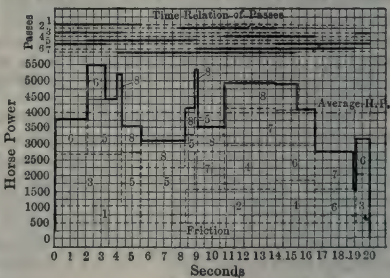

Fig. 4.

Use of Induction Motors. — The choice of alternating- or direct-current motors for main rolls must be determined by a careful consideration of several factors, chief of which are the capacity of individual units, constant, variable or adjustable speeds, reversing or non-reversing, transmission distance, existing power-station equipment, etc.

It is in general conceded that for large units and constant speed, constant-load service induction motors with phase-wound rotors possess many advantages over direct-current motors. This is due to the ruggedness of construction and simplicity of operation of the induction motor, and to the facility with which alternating current may be stepped up to any desired voltage, thus rendering the problem of economic power distribution relatively simple.

For those main-roll drives where it is essential that close regulation be maintained for a large number of speeds, each constant under varying loads, a direct-current motor with shunt characteristics has until recently been practically necessary. It is now possible to obtain these characteristics by using standard induction motors with phase-wound rotors and speed regulating sets.

The method of obtaining speed control consists in replacing the non-inductive external resistance by a compensated commutator machine which may be mounted either on the shaft of the main motor, or more commonly as one element of a two-unit motor-generator set, in which latter case, the second element is a squirrel-cage induction motor. The speed of the motor driving the mill may be varied both above and below synchronism. When operating below synchronism, the commutator machine acts as a motor and drives the induction machine slightly above synchronism, thus returning to the line energy proportional to the slip of the main motor minus the losses in the regulating set itself. When the main motor is operated above synchronism, the induction machine acts as a motor and drives the commutator machine as a generator which delivers power to the slip-ring circuit of the mill motor.

Control Equipment for Induction Motors. — The control for large steel mill motors is important because of the size of the apparatus involved and the extreme importance of continuous operation. Most motors for main-roll drive are started up at the beginning of a shift, immediately brought up to full speed, and left running until the end of the shift. Provision must be made for running a short time at reduced speed when desired, for making adjustments to the mill, and for reversing, in order to back out ingots which have stuck in the mill. If a flywheel is used, there should also be provision for quick stopping, so that if a spindle is broken, the motor can be brought to rest quickly and a new spindle substituted without delay. Current-limiting relays are desirable in order to avoid severe fluctuations in input during starting and to maintain an approximately even torque without depending upon the intelligence of the operator, for which service magnetically-operated contactors are necessary. It is always desirable to have some device by which the motor can be shut down from a remote point in case of accident.

Resistors for Starting Rheostats. — The resistance should be liberal, because the control may often be held on the first point for some time when adjustments are being made and the starting torque may sometimes be heavy, particularly when an ingot has stuck in the rolls. Also, where a flywheel is used, the acceleration is necessarily slow, and some mills, particularly cold rolls, require a very heavy torque during starting and until they have warmed up. Resistances should always be furnished having at least enough capacity to provide for full-load torque for two minutes. In special cases more will be required.

Contactors and Oil Switches. — Where 440- or 550-volt alternating current is used, contactors can be employed for both primary and secondary, and remote control is an easy matter. 2200- or 6600-volt motors require oil

switches in the primary, which cannot be operated readily by means of alternating current. They can be operated with direct current, but this complicates the system and makes the operation of the alternating-current motor dependent upon the direct-current supply. As remote control is not essential in the large majority of cases, hand-operated oil switches can be used.

Control Systems. — Figs. 5 and 6 show two typical connection diagrams of double-range speed-regulating equipments. The connections in Fig. 5 show the main mill motor when used with a speed regulating set. This set consists of an induction machine and a polyphase, commutator regulating machine with shunt-field excitation. The slip rings of the main induction motor are connected to the shunt winding of this regulating machine so that its counter electromotive force opposes the voltage of the main motor. The slip-ring voltage of the induction motor follows the variation in counter electromotive force of the regulating machine produced by changes in its shunt excitation. The shunt characteristics of the regulating machine are reflected in the speed-torque characteristics of the induction motor which is similar to that of the well-known shunt direct-current motor. The slip energy of the main-roll motor is conserved by converting it through the induction generator into electrical power and returning it to the power lines. Another arrangement (not shown in the diagram) consists of mounting the polyphase regulating machine on the main-roll shaft and thus convert this slip energy into mechanical power.

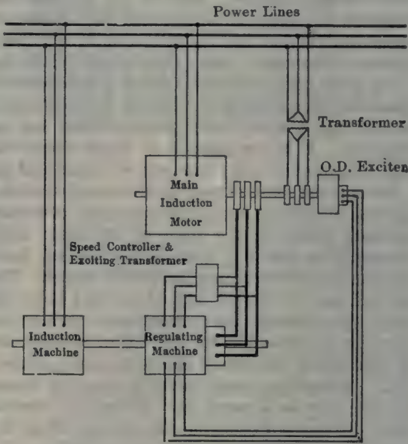

Fig. 5. Double-range Speed Regulating Equipment.

In addition to the regulating set there is required a small machine known as an ohmic-drop exciter. Its armature is mounted on the main motor shaft so that its speed is always the same as that of the main motor. It is the function of this ohmic-drop exciter to supply the ohmic-drop component to the main field of the regulating machine when operating at or near synchronism. The main field of the regulating machine also received excitation from the exciting transformer. This transformer supplies the necessary reactive component, the effect of which is greatest at speeds remote from synchronism. The use of this form of excitation has made possible the development of a method of operation of the main motor that retains all the desirable characteristics of the normal induction motor operated with short-circuited slip rings, not only at speeds considerably below synchronism but equally well at speeds near synchronism, in synchronism and above synchronism.

Fig. 6 shows the arrangement in the synchronous converter system where the slip energy is converted into direct current through a rotary converter, and in turn, into mechanical power by a direct-current motor mounted on the main motor shaft. The arrangement shown in Fig. 5 has certain advantages over the rotary converter method, such as greater stability when the main-roll motor is operating near synchronism, higher efficiency, lower first cost (except

in special cases) and ability to operate non-regulating at an intermediate speed, i.e., practically at synchronism.

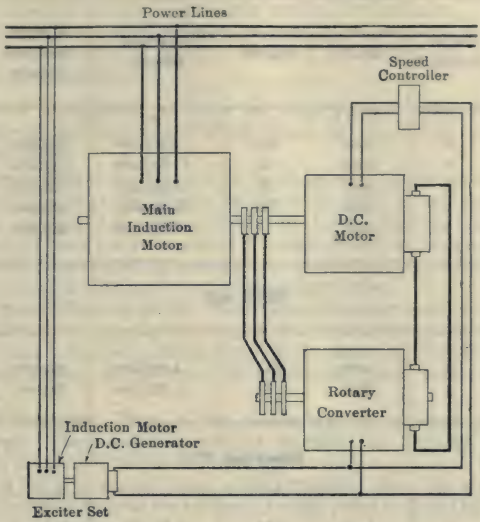

Fig. 6. Rotary Converter Speed Regulating Equipment.

Direct-current Motors are used only on low-voltage circuits, usually 250 volts, and can be handled by contactors. Quick stops can be accomplished with d-c. motors by dynamic braking, by which the armature is disconnected from the line and connected across a resistance so that the motor becomes a generator. Where flywheels are used, compound motors are necessary in order to give automatic reduction in speed as the load increases.

Characteristics of Gary Equipment. — The characteristics of the main-roll motor equipment for the rail, billet, and sheet-bar mills of the Indiana Steel Company, Gary, Indiana, are given in the table on page 1508.

AUXILIARY MOTOR APPLICATIONS.—The application of electric motors for auxiliary steel-mill machinery has been used for a long time. The nature of this service is unusually severe and has led to the development of the mill-type motor for both d-c. and a-c. service. These motors are designed to withstand heavy overloads and abnormally rapid acceleration, and their construction is such that various parts are interchangeable, which greatly facilitates repairs and minimizes the delays incident to the same.

All A-C. versus Mixed System. — Whether a-c. or d-c. motors should be used for driving the auxiliary machinery in a steel mill is a problem which has been very seriously discussed, and has led to the use of two recognized systems. These are known as the all a-c. system, where no direct current is used, and the mixed system, where both a-c. and d-c. motors are used.

A great many factors must be considered in comparing the two systems. In the first place, it is to be assumed that power will primarily be alternating current, as the transmission distances ordinarily preclude the use of direct-current

CHARACTERISTICS OF MAIN ROLL MOTORS

Rail mill

No. of motors*	H.P.	R.p.m.	Flywheel effect†	Weight complete, pounds
2	2000	214	4,312,000	392,000
1	6000	83	11,600,000	749,000
1	2000	68	8,950,000	578,000
1	6000	88	11,600,000	749,000
1	6000	75	14,100,000	783,000
Billet mill				
2	2000	214	4,312,000	392,000
3	6000	83	11,600,000	624,000
Sheet-bar Mill				
1	6000	83	11,600,000	624,000

* All motors 25 cycles, 6600 volts.
† Weight of flywheel in pounds times square of radius of gyration in feet.

generators. It would therefore seem to be simplest and most efficient to step down to a suitable voltage through static transformers and use alternating-current motors. On the other hand, with certain of the auxiliaries, such as screw-downs, live-roll tables, etc., the advantages in favor of d-c. motors are so great that they are almost universally adopted. The mixed system involves additional expense for motor-generator sets and entails considerable power loss due to the low efficiency of conversion. On the other hand, direct-current motors are lower in first cost than induction motors and a higher power factor is maintained on the entire system where they are used. In the mixed system an increase in power factor is effected by eliminating the lagging current of the induction motors, and, in addition, the motor-generator sets can be equipped with synchronous motors which will take a leading current from the line and offset part of the lagging current on the rest of the system. The increase in power factor enables a reduction to be made in the size and cost of transformers and generators, and also increases their efficiency due to the lower currents which they are required to handle and to the decreased excitation required by the generators.

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STOKERS, MECHANICAL. — (*See also Boilers.*) Mechanical stokers serve a twofold purpose: they make possible a more uniform and a more complete combustion of the fuel and thereby prevent the formation of smoke, and they also reduce the amount of labor required in handling and firing the fuel. To insure the prevention of smoke a fire-brick arch should also be used over the front part of the furnace.

TYPES OF STOKERS. — There are three general types of mechanical stokers although there are several which cannot be conveniently placed in any one of these classes.

Chain-grate Stoker. — The first is the chain-grate stoker consisting of an endless chain placed in the furnace of the boiler with its top side revolving slowly from the front of the furnace toward the back. The coal is fed onto this moving grate in front and is burned as it passes toward the bridge wall, where, when the grate is moving at its proper speed, the coal will have been completely burned to ash which will drop down into the ashpit below.

The one great advantage of the chain-grate stoker over most of the others is that it is suitable for burning with natural draft bituminous coals of very high volatile content. In plants where the load is reasonably steady and where sudden peaks are not thrown upon the boilers this stoker makes a first-class installation.

Inclined Overfeed Type. — The second type of stoker is the inclined overfeed type. These stokers consist of movable bars forming an inclined grate with a mechanism for moving these bars in such a manner as to cause the coal which is fed in at the top to be pushed gradually down the grates until it reaches the dumping grate at the bottom, at which point it should be completely burned.

Underfeed Type. — The third type is the underfeed. In these stokers the coal is fed onto the grate or up under the grate in such a manner that the fresh coal is always close to the grate while the coal which is being burned is at the top. The air is also introduced at the bottom and while passing up through the bed of coal is heated and thoroughly mixed with the volatile gases distilled from the coal and when passing through the incandescent layer at the top reaches its temperature of combustion, so that generally no arches are necessary.

Other Types. — There are several other stokers which are used to a greater or lesser extent and work on somewhat different principles, perhaps the most notable being the so-called "finger" stoker, which has several oscillating paddles which pick up the coal and throw it into the furnace. There is another stoker somewhat of this order in which the coal is mechanically shovelled into the furnace. These stokers, however, have not reached any extensive application as yet and in general are only applicable to particular conditions.

SAVING BY USE OF MECHANICAL STOKERS. — The difference between good and bad firing may easily amount to from 5 to 20 per cent of the amount of fuel fired; hence, there is no investment around a steam plant which will pay better than the extra amount paid to secure good boiler practice.

Automatic stokers are now developed to a remarkable degree of perfection, and when suited to the fuel, have an advantage over hand-firing in that under all conditions they are reliable, can be adjusted to the minimum of the air and the maximum of load, and can be depended upon to operate continuously with the minimum amount of skilled labor. The economic saving will depend on the basis of comparison and the method of operation. Compared with an ordinary or poor fireman, they should show a large saving. Whether a stoker will

save labor in the fire room depends upon the size of the plant. As a rule, mechanical stokers are not labor-saving devices in plants containing less than six to eight boilers (1500 to 4800 h.p.).

One man can handle the coal and ashes, fire the boilers and attend to the water level of 200 h.p. of boilers equipped with the common hand-fired furnace. With shaking or dumping grates 300 h.p. may be controlled by one man. With large boilers equipped with dumping grates one man will fire around 1000 boiler h.p., when using the steam sizes of anthracite coal, but the coal must be delivered in front of the boiler and a water tender is usually provided for every 24 boilers. With soft coal about 700 boiler h.p. may be fired by one man under similar circumstances. In a large plant containing twelve 650 B. & W. boilers, equipped with stokers, a water tender, one fireman and one helper are required per watch for their efficient operation. In stations of this size the ash men are in the basement, and the change from hand to stoker firing would make no difference in their number.

One authority states that stokers save 30 to 40 per cent of the boiler labor in plants using over 200 tons of coal per week; 20 to 30 per cent in plants using from 50 to 200 tons of coal per week, and no saving in plants below 50 tons.

It should be remembered that unless the type of stoker is suited to the kind of fuel obtainable, the maintenance of the stoker plant is likely to be extremely high, running in some cases twice or three times as high as fire-room labor under hand-fired conditions.

COST OF MECHANICAL STOKERS (Pre-war figures). — In general, mechanical stokers cost from $3.50 to $6.50 per boiler horsepower, but the cost depends more on the width of the stoker than on the horsepower of the boiler. Chain-grate stokers cost in the neighborhood of from $180 to $250 per foot of width. Inclined-grate stokers from $140 to $225 per foot of width. Underfeed stokers from $200 to $300 per foot of width. The length of the stokers is usually standard and depends on the type of coal to be burned. These prices differ considerably with the amount of auxiliary material furnished with the stoker, such as fronts, air boxes, coking arches, stoker drives and speed-changing devices, but are based on labor and material costs current in New York prior to the European war.

R. J. S. Pigott (*Proceedings Am. Elec. Ry Assoc., 1914*) gives the following data for mechanical stokers:

AVERAGE DATA FOR STOKERS

Type of stoker	Step and slope overfeed	V overfeed	Chain overfeed	Gravity underfeed	Horizontal retort underfeed
Average price per rated b.h.p. . . .	\$3.60	\$3.60	\$3.50–\$6.55	\$5.65	\$4.44
Normal forcing ability in per cent of rating. . .	190	175	260	300–350	300
Price per max. h.p. developable	\$1.90	\$2.06	\$2.52	\$1.62–\$1.88	\$1.48
Maintenance per ton coal fired, in cents.	10–12	11–14	6–10	2.5–4	4–6
Attendance in man-hours per active hour. . . .	0.45	0.45–0.50	0.20–0.30	0.08–0.10	0.30–0.40
Pounds coal per sq. ft. grate surface (max.). .	35–38	35–42	45–48	60–75	50–65

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STRENGTH AND ELASTICITY. — (*See also Buildings, Allowable Unit Stresses in; Mechanics, Principles of; Structures, Simple.*) When opposing external forces are applied to a body the latter is in general deformed more or less, and, in the case of solids, when the applied forces are increased sufficiently the body ruptures.

STRESSES. — A stress is the internal resisting force set up within a body opposing the external forces which tend to deform it. When the external forces cause a stretching of the body the stress is called a tension; when the external forces compress the body the stress is called a compression; when the external forces cause a relative slipping of the particles in two contiguous parallel planes in the body the stress is called a shear.

Unit Stress. — By unit stress is meant the stress per unit area; in the case of tension or compression the area is taken perpendicular to the line of action of the forces producing the stress; in the case of shear the area is taken parallel to the forces producing the stress. Unless otherwise stated the stress is assumed uniformly distributed over the area upon which it acts.

Ultimate Stress or Ultimate Strength. — The ultimate stress which a body will stand, or the ultimate strength of the body, is the greatest stress which can be produced in the body without rupturing it. Ultimate strength for tension and compression are usually quite different; the former is usually called tensile strength and the latter compressional strength. The ultimate strength of materials is usually specified in terms of the stress per unit area; see *Units and Conversion Factors.*

DEFORMATIONS OR STRAINS. — The deformation accompanying any stress is usually called a strain. The strain corresponding to a tension is called an elongation, the strain corresponding to a compression is a shortening, the strain corresponding to a shear is called a detrusion. An elongation is usually specified as the ratio (or percentage) of the increase in length to the original length of the specimen. For example, if a rod 10 feet long is stretched by an applied force to a length of 10.2 feet, the elongation is $(10.2-10)/10 = 0.02$ or 2 per cent. The term elongation is frequently used to designate specifically the *maximum* elongation just before rupture.

Hooke's Law. — Experiment shows that when the unit stress in a body does not exceed a certain value it bears a constant ratio to the resulting unit strain (ratio of change in size to original size). This fact is known as Hooke's Law. In the case of a tension the change in size is usually taken as the change in length, no attention being paid to the decrease in cross-section, which is usually negligible, except just before rupture.

ELASTICITY. — A body deformed under stress will return to its original shape when the stress is removed, provided it has not been strained beyond the point at which the proportionality between stress and strain ceases. This ability to return to its original form after deformation is called elasticity.

Elastic Limit. — The stress per unit area corresponding to the point at which the proportionality between stress and strain ceases is called the elastic limit of the material.

Permanent Set. — When a body is stressed beyond its elastic limit, the deformation per unit increase in stress becomes greater than it was before this point was reached, and the material takes a permanent "set," i.e., when the stress is removed the body does not return to exactly its original form. As a measure of the "set" is taken the ratio of the permanent change in size (after the stress is removed) to the original size.

Modulus of Elasticity. — Let ΔF denote the change in the total tension producing a change Δl in a rod of length l and cross section A; then the quotient of the increase in the unit stress by the increase in the elongation per unit length, is called the modulus of elasticity (Young's modulus) of the material, and may be designated by the symbol M, i.e.,

$$M = \frac{\Delta F}{\Delta l} \cdot \frac{l}{A}.$$

This modulus has the dimensions of pounds per square inch.

VALUES OF TENSILE STRENGTH AND MODULUS OF ELASTICITY.— Values of these properties given by different authorities are extremely variable, since the measured values depend largely upon the chemical composition, heat treatment, age, size and shape of the test specimen.

Alloys, Miscellaneous.— See articles on *Alloys* and *Wires, Resistance.*

Aluminum. — See article on *Aluminum.*

Belting. — Tensile strength, in pounds per square inch: Cotton, 4500 to 8900; Single leather, 3200 to 5900; Double leather, 2200 to 5400. See also article on *Belts and Belting.*

Brass Wire. — Tensile strength ranges from about 50,000 to 150,000 pounds per square inch; modulus of elasticity is about 14×10^6 pounds per square inch.

Brick. — See article on *Bricks and Brick Masonry.*

Bronze Wire. — The tensile strength of phosphor-bronze wire ranges from about 44,000 to 140,000 pounds per square inch; of silicon-bronze wire, from 95,000 to 115,000 pounds per square inch.

Cement. — See article on *Cement.*

Concrete. — See article on *Concrete.*

Copper. — See articles on *Copper* and *Wires and Cables, Bare.*

Earth. — See *Soils,* below.

Glass.—Tensile strength of common green or of flint glass ranges from about 2500 to 5000 pounds per square inch. Modulus of elasticity ranges from about 8.5×10^6 to 11.5×10^6.

The crushing or compressive strength of glass ranges from about 13,000 to 40,000 pounds per square inch.

Granite. — Crushing or compressive strength ranges from about 9700 to 34,000 pounds per square inch.

Iron. — See articles on *Iron, Pig and Cast,* and *Iron, Wrought.*

Lead. — Tensile strength ranges from about 2600 to 3300 pounds per square inch.

Limestone. — Crushing or compressive strength ranges from about 6000 to 25,000 pounds per square inch.

Marble. — Crushing or compressive strength ranges from about 7500 to 21,000 pounds per square inch.

Nickel. — Tensile strength of cast nickel ranges from about 40,000 to 85,000 pounds per square inch, and tensile strength of annealed nickel ranges from about 70,000 to 95,000 pounds per square inch. Modulus of elasticity ranges from about 24×10^6 to 27×10^6 pounds per square inch.

Rope. — See *Ropes and Rope Drive.*

Soils. — See articles on *Power Stations, Hydroelectric,* and *Buildings, Allowable Unit Stresses in.*

Steel. — Tensile strength in pounds per square inch:

Bessemer: 56,000 to 74,000, Open-hearth: 50,000 to 69,000,
Cast: 67,000 to 106,000, Wire: 50,000 to 450,000.

Modulus of elasticity of various kinds of steels usually ranges between 27×10^6 and 35×10^6 pounds per square inch. See also articles on *Steel* and *Wires and Cables, Bare.*

Timber. — See articles on *Timber* and *Poles for Overhead Lines.*

Tin. — Tensile strength ranges from about 4000 to 5000 pounds per square inch; modulus of elasticity from about 2.5×10^6 to 6×10^6 pounds per square inch.

Zinc. — Tensile strength ranges from about 7000 to 30,000 pounds per square inch; modulus of elasticity is about 12×10^6 pounds per square inch.

Wood. — See *Timber,* above.

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STRUCTURES, SIMPLE. — (*See also Building Laws; Cement; Concrete; Iron; Mechanics, Principles of; Steel; Timber; Towers for Transmission Lines.*) A structure in the sense used by engineers is a member or combination of members constructed to hold forces in equilibrium. The common structures are beams, girders, columns and trusses.

A brief outline of the contents of this article is as follows:

DEFINITIONS AND FUNDAMENTAL RELATIONS. — The more common terms used in the treatment of structures are defined below.

Center of Gravity; Moment of Inertia; Radius of Gyration; Stress and Strain. — See *Mechanics, Principles of.*

Neutral Axis. — The neutral axis of the cross-section of a stressed bar is the locus of those points in the plane of the cross-section at which the direct fiber stress equals zero. If the bar is of homogeneous material and subjected to flexure (*see Bending Moment, below*) only, the neutral axis passes through the center of gravity of the cross-section, and if the loads causing flexure are applied in the plane of one of the principal axes, the neutral axis coincides with the other principal axis. This is the condition commonly occurring in wooden and steel beams. Table I which follows gives the location of the neutral axis for such cases only. If the cross-section is subjected to combined flexure and direct stress, the neutral axis will not pass through its center of gravity, but will deviate from it by an amount depending upon the ratio of bending moment to direct stress, reaching infinity when the stress is direct tension or compression only, as in the case of a tie rod or column centrally loaded.

If the beam is of unhomogeneous material the position of the neutral axis must be determined by calculation. The reinforced concrete beam is the only common example of such a case. For formulas for this case, see *Concrete, Reinforced.*

Section Modulus. — If c is the normal distance from the neutral axis of a given plane surface to the most remote portion of its perimeter, and I the moment of inertia of the surface about its neutral axis the expression I/c is called the section modulus. For symmetrical beams exposed to pure flexure only, c = one-half the depth of the beam. Tabular values of this quantity for plane surfaces of varying shapes are given in Table I and for the ordinary structural steel beams in Tables III to VI, which follow.

Moment of Resistance. — If s is the allowable unit stress per square inch upon the extreme fiber of a beam or girder, the expression $\frac{sI}{c}$ is called the moment of resistance since its value equals the maximum bending moment which the member may carry without causing a stress greater than the allowable unit stress.

Statical Moment. — The statical moment of a plane surface about a given axis lying in the plane is the summation of the products of each elementary area,

dA, of which the surface is composed, times the distance of its center of gravity from the given axis, distances above the axis being considered positive, and below negative. It may be expressed mathematically as follows: (See Fig. 1.)

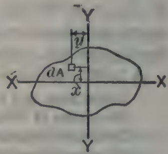


Fig. 1.

Let Q = statical moment of a given area about an axis Y-Y lying in the plane of the area,

dA = an elementary area,

y = distance of center of gravity of the area dA from the axis,

then $Q = \int y dA$.

The statical moment of an area about an axis of symmetry evidently equals zero since the positive lever arm of any elementary area will be balanced by a corresponding negative lever arm.

Equations of Equilibrium. — (*See also Mechanics, Principles of.*) The following laws apply to a structure lying in a plane and acted upon by forces lying in the same plane:

1st. The algebraic sum of the components of all the forces acting parallel to any axis in the plane of the forces must equal zero.

2nd. The algebraic sum of the moments of all the forces about any axis at right angles to the plane of the forces must equal zero.

If we resolve the applied forces into components parallel to two rectangular axes, OX and OY, and let ΣX and ΣY equal the algebraic sum of the forces parallel respectively to OX and OY, and ΣM the algebraic sum of their moment about any axis, normal to the plane of the forces, these two laws will be fully comprehended by the three equations:

$$\Sigma X = 0, \ \Sigma Y = 0, \ \Sigma M = 0.$$

If OX and OY are respectively horizontal and vertical these equations take the generally used form:

$$\Sigma H = 0, \ \Sigma V = 0, \ \Sigma M = 0.$$

Unless a structure satisfies all these conditions it cannot be in equilibrium.

Reactions. — The forces which the abutments or piers exert upon structures, such as beams, girders or trusses, are called the reactions. Each reaction may, in general, have three unknown factors, viz., direction, magnitude and point of application. In order to simplify computations it is customary for girders or trusses to eliminate three of these unknowns by the method of construction; e.g., in end-supported trusses and long girders, the ends are usually supported on pins of comparatively small diameter thereby fixing the points of application of both reactions. Moreover, the pin at one end is in turn supported on a set of rollers thereby making that reaction normal, or nearly so, to its supporting surface, thus fixing the direction of one of the reactions. These refinements are not generally applied to short girders the ends of which are supported on steel plates or castings. Even in such cases, however, the point of application is fixed within comparatively small limits, and the base plate at one end is usually planed thus making the reaction at that end approximately normal to the supporting surface. For ordinary beams no special provision is made to fix any of the reaction conditions but each reaction is assumed to be normal to the supporting surface and to act at its center.

When the unknown reaction factors are thus reduced to three, their values may be determined by the three equations of equilibrium previously given, this being the usual procedure in the case of beams on two supports as illustrated later. For beams supported at more than two points, the three-moment equation (*see below*) may be applied.

Three Moment Equation. — This is an equation connecting the moments at three adjoining points of support of a continuous structure. It is strictly applicable only to structures having a constant moment of inertia and on level supports but is applied approximately to structures the moment of inertia of which is not constant.

The formula for concentrated loads is as follows:

Let M_c = moment at any support,
M_a = moment at adjoining support on left,
M_b = moment at adjoining support on right,
L_1 = length of span to left of support at which M_c acts,
L_2 = length of span to right of support at which M_c acts,
P_1 = any load in span L_1 and k_1L_1 its distance from left support,
P_2 = any load in span L_2 and k_2L_2 its distance from support at left of that span.

Then

$$M_aL_1 + 2\,M_c(L_1 + L_2) + M_bL_2 = \Sigma P_1L_1^2(k_1^3 - k_1) + \Sigma P_2L_2^2\,(3\,k_2^2 - k_2^3 - 2\,k_2).$$

For uniform load of w_1 lb. per ft. over span L_1 and w_2 lb. per ft. over span L_2 the preceding formula becomes

$$M_aL_1 + 2\,M_c(L_1 + L_2) + M_bL_2 = -\frac{1}{4}w_1L_1^3 - \frac{1}{4}w_2L_2^3.$$

The application of these formulas to successive series of spans enables the moments at all supports to be computed and from these the reactions and shears may also be determined, and the structure completely solved; see section below on *Girders, Design of.*

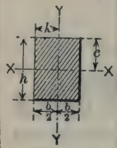
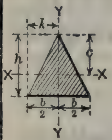
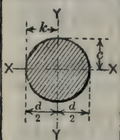
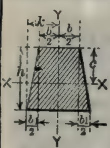
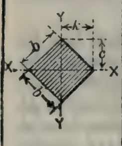
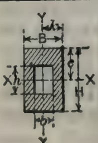
Shear. — The shearing force or shear at any section of a body is the force which tends to produce slipping along the given section. Shearing failure may be due either to transverse fracture or to slipping of the fibers on each other. Of the ordinary structural materials wood is the only one of fibrous character and shearing failure in this material frequently occurs on planes parallel to the fibers.

Bending Moment; Flexure. — The bending moment at any section of a body due to a set of coplanar forces is the resultant moment about an axis passing through the center of gravity of the section of all the forces on either side of the section, it being understood that the section and the axis are perpendicular to the plane of the forces. Fractures due to excessive bending moment occur through longitudinal failure of the fibers either by tension or crushing. The bending which results from the application of a bending moment is termed flexure.

Dead and Live Loads. — The loads acting upon a given structure may be divided into two distinct types: viz., quiescent loads which are known as *dead loads* and moving or intermittent loads which are known as *live loads.* The first class includes all loads which are fixed in magnitude and position such as the weight of the structure itself and such superimposed loads as the floor of a building or bridge; the second class includes such loads as crowds of people, merchandise, snow, wind and vehicles of all sorts. The live load through its rapidity and irregularity of application is more injurious in its effect than the dead load, hence in a structure subjected to live loads, either the value of the live load should be increased as explained below under *Allowances for Impact,* or the allowable unit stress in the material should be reduced. The former method seems to be more logical and is commonly adopted at the present time by leading structural engineers.

(*Text continued on p. 1522.*)

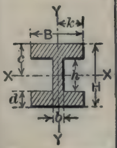
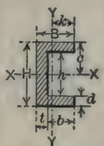
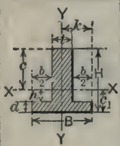
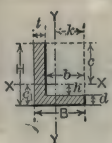
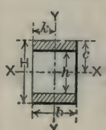
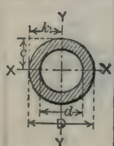
TABLE I.—PROPERTIES OF

Section. Axes $X-X$ and $Y-Y$ pass through center of gravity	Distance from $X-X$ to extreme edge of section	Moment of inertia with respect to axis $X-X$	Section modulus with respect to axis $X-X$	Radius of gyration with respect to axis $X-X$
	c	I_x	$\frac{I_x}{c}$	ρ_x
	$\frac{h}{2}$	$\frac{1}{12}bh^3$	$\frac{1}{6}bh^2$	$\frac{h}{\sqrt{12}}=0.289h$
	$\frac{2}{3}h$	$\frac{1}{36}bh^3$	$\frac{1}{24}bh^2$	$\frac{h}{\sqrt{18}}=0.236h$
	$\frac{d}{2}$	$\frac{\pi d^4}{64}=0.0491d^4$	$\frac{\pi d^3}{32}=0.0982d^3$	$\frac{d}{4}$
	$\frac{h}{3}\left(\frac{3b+2b_1}{2b+b_1}\right)$	$\frac{h^3}{36}\left(\frac{6b^2+6bb_1+b_1^2}{2b+b_1}\right)$	$\frac{h^2}{12}\left(\frac{6b^2+6bb_1+b_1^2}{3b+2b_1}\right)$	$\frac{h}{2b+b_1}\sqrt{\frac{6b^2+6bb_1+b_1^2}{18}}$
	$\frac{b}{\sqrt{2}}=0.707b$	$\frac{b^4}{12}$	$\frac{\sqrt{2}}{12}b^3=0.118b^3$	$\frac{b}{\sqrt{12}}=0.289b$
	$\frac{H}{2}$	$\frac{BH^3-bh^3}{12}$	$\frac{BH^3-bh^3}{6H}$	$\sqrt{\frac{BH^3-bh^3}{12(BH-bh)}}$

PLANE SECTIONS

Distance from $Y-Y$ to extreme edge of section	Moment of inertia with respect to axis $Y-Y$	Section modulus with respect to axis $Y-Y$	Radius of gyration with respect to axis $Y-Y$
k	I_y	$\frac{I_y}{k}$	ρ_y
$\frac{b}{2}$	$\frac{1}{12}hb^3$	$\frac{1}{6}hb^2$	$\frac{b}{\sqrt{12}}=0.289b$
$\frac{b}{2}$	$\frac{1}{48}hb^3$	$\frac{1}{24}hb^2$	$\frac{b}{\sqrt{24}}=0.204b$
$\frac{d}{2}$	$\frac{\pi d^4}{64}=0.0491d^4$	$\frac{\pi d^3}{32}=0.0982d^3$	$\frac{d}{4}$
$\frac{b+b_1}{2}$	$\frac{hb^3}{12}+\frac{hb_1^3}{48}+\frac{bb_1h(3b+2b_1)}{24}$	$\dfrac{\frac{hb^3}{6}+\frac{hb_1^3}{24}+\frac{bb_1h(3b+2b_1)}{12}}{b+b_1}$	$\frac{1}{12}\sqrt{\frac{24b^3+6b_1^3+12bb_1(3b+2b_1)}{2b+b_1}}$
$\frac{b}{\sqrt{2}}=0.707b$	$\frac{b^4}{12}$	$\frac{\sqrt{2}}{12}b^3=0.118b^3$	$\frac{b}{\sqrt{12}}=0.289b$
$\frac{B}{2}$	$\frac{HB^3-hb^3}{12}$	$\frac{HB^3-hb^3}{6B}$	$\sqrt{\frac{HB^3-hb^3}{12(BH-bh)}}$

TABLE I.—PROPERTIES OF

Section. Axes X–X and Y–Y pass through center of gravity	Distance from X–X to extreme edge of section c	Moment of inertia with respect to axis X–X I_x	Section modulus with respect to axis X–X $\frac{I_x}{c}$	Radius of gyration with respect to axis X–X ρ_x
	$\frac{H}{2}$	$\frac{BH^3-h^3(B-b)}{12}$	$\frac{BH^3-h^3(B-b)}{6H}$	$\sqrt{\frac{BH^3-h^3(B-b)}{12[BH-h(B-b)]}}$
	$\frac{H}{2}$	$\frac{BH^3-h^3b}{12}$	$\frac{BH^3-h^3b}{6H}$	$\sqrt{\frac{BH^3-h^3b}{12(BH-hb)}}$
	$H-\frac{1}{2}\left(\frac{tH^2+bd^2}{tH+bd}\right)$	$\frac{1}{3}(Bc_1^3-bh^3+tc^3)$	$\frac{1}{3}\frac{Bc_1^3-bh^3+tc^3}{H-\frac{1}{2}\left(\frac{tH^2+bd^2}{tH+bd}\right)}$	$\sqrt{\frac{Bc_1^3-bh^3+tc^3}{3[BH-b(c+h)]}}$
	$H-\frac{1}{2}\left(\frac{tH^2+bd^2}{tH+bd}\right)$	$\frac{1}{3}(Bc_1^3-bh^3+tc^3)$	$\frac{1}{3}\left(\frac{Bc_1^3-bh^3+tc^3}{H-\frac{1}{2}\frac{tH^2+bd^2}{tH+bd}}\right)$	$\sqrt{\frac{Bc_1^3-bh^3+tc^3}{3[BH-b(c+h)]}}$
	$\frac{H}{2}$	$\frac{b}{12}(H^3-h^3)$	$\frac{b}{6}\frac{H^3-h^3}{H}$	$\sqrt{\frac{H^2+Hh+h^2}{12}}$
	$\frac{D}{2}$	$\frac{\pi}{64}(D^4-d^4)$	$\frac{\pi}{32}\frac{D^4-d^4}{D}$	$\frac{1}{4}\sqrt{D^2+d^2}$

PLANE SECTIONS — (*Continued*)

Distance from $Y-Y$ to extreme edge of section	Moment of inertia with respect to axis $Y-Y$	Section modulus with respect to axis $Y-Y$	Radius of gyration with respect to axis $Y-Y$
k	I_y	$\frac{I_y}{k}$	ρ_y
$\frac{B}{2}$	$\frac{2dB^3+hb^3}{12}$	$\frac{2dB^3+hb^3}{6B}$	$\sqrt{\frac{2dB^3+hb^3}{12(2dB+hb)}}$
$\frac{HB^2-hb^2}{2(BH-hb)}$	$\frac{Ht^3+2db^3}{3}-(Ht+2db)(b-k)^2$	$\frac{Ht^3+2db^3}{3k}-\frac{(Ht+2db)(b-k)^2}{k}$	$\sqrt{\frac{Ht^3+2db^3}{3(Ht+2db)}-(b-k)^2}$
$\frac{B}{2}$	$\frac{(H-d)t^3+dB^3}{12}$	$\frac{(H-d)t^3+dB^3}{6B}$	$\sqrt{\frac{(H-d)t^3+dB^3}{12(H-d)t+12dB}}$
$\frac{HB^2-(H-d)b^2}{2(Ht+bd)}$	$\frac{Ht^3+db^3}{3}-(Ht+db)(b-k)^2$	$\frac{Ht^3+db^3}{3k}-\frac{(Ht+db)(b-k)^2}{k}$	$\sqrt{\frac{Ht^3+db^3}{3(Ht+db)}-(b-k)^2}$
$\frac{b}{2}$	$\frac{(H-h)b^3}{12}$	$\frac{(H-h)b^2}{6}$	$\frac{b}{\sqrt{12}}=0.289b$
$\frac{D}{2}$	$\frac{\pi}{64}(D^4-d^4)$	$\frac{\pi}{32}\frac{(D^4-d^4)}{D}$	$\frac{1}{4}\sqrt{D^2+d^2}$

Impact is the name given to the dynamic force which is added to the forces due to live and dead loads to obtain the true forces acting upon structures; see section below on *Allowances for Impact*.

Beam or Girder. — A beam is a bar of wood, metal, concrete or other stress-resisting substance supported at certain definite points along its length and loaded transversely to its longitudinal axis, which is usually but not always, placed in a horizontal position. The name girder is commonly applied to large beams built up of structural members; for example, a plate girder is a steel or iron structure made up of plates and angles, or other structural shapes, riveted together.

Floor Beam; Panel.—Frequently girders and trusses receive all their live load and much of their dead load through other members called floor beams. Fig. 2 illustrates such construction. The distance between adjacent floor beams is the panel length.

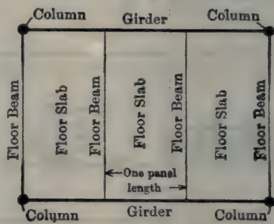


Fig. 2.

Column. — A column is a member intended primarily to resist direct compression, although it may also be subjected to bending stresses due either to transverse loads or to eccentric application of the direct loads.

Truss. — A truss is a structure consisting of separate bars constructed to carry either direct tension or direct compression. These bars are connected at their ends and occasionally at intermediate points, the points of connection being called joints. The connections are sometimes made by riveting the bars directly together and sometimes by riveting them to a common steel plate, the truss in either case being called a *riveted truss*. The connections may also be made by fastening together with a large steel pin all the members meeting at a joint; such a truss is called a *pin truss*. The outer forces should be applied at the joints only, since the members are not intended to carry bending. This is accomplished by the use of floor beams in a bridge and purlins in a roof.

PROPERTIES OF PLANE SECTIONS. — In Table I are given the location of the center of gravity, and values of the moment of inertia, section modulus and radii of gyration of the more common plane sections.

TABLES OF STRUCTURAL SHAPES. — Tables II to VI give the dimensions and properties of the commonly used shapes of structural steel; namely, sheared plates, angles with either equal or unequal legs, I-beams, and channels. For information upon other metallic structural material see the publications of the various steel manufacturing companies.

TABLE II.—SHEARED PLATES *

Width in inches	Thickness in inches																		
	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1	1 1/8	1 1/4	1 1/2	1 3/4	2
	Maximum length in inches																		
24	400	525	575	600	600	600	600	600	....	...	....	...	....	...	...	...	....	...	...
25– 30	375	525	500	600	600	625	625	625	....	...	....	...	....	...	...	...	....	...	...
31– 36	375	475	525	550	550	575	575	575	575	550	525	500	475	475	450	425	400	375	350
37– 42	450	525	550	575	610	600	600	600	575	575	525	500	500	500	475	425	400	375	350
43– 48	450	525	575	600	600	600	600	600	600	575	550	550	525	525	500	450	400	375	350
49– 54	450	525	550	600	600	625	625	625	600	575	550	550	525	525	500	450	400	375	350
55– 60	400	525	550	600	600	625	625	625	600	575	550	550	525	525	475	425	400	375	325
61– 66	350	475	500	575	575	600	600	600	600	575	550	550	525	525	475	425	375	350	325
67– 72	325	450	500	540	550	575	575	575	575	575	550	525	500	500	475	425	375	350	300
73– 78	...	425	475	440	540	540	540	540	540	525	500	475	450	450	425	375	325	300	280
79– 84	...	400	475	440	540	540	540	540	540	500	450	450	425	425	375	350	325	300	280
85– 90	...	350	375	400	450	450	450	450	450	425	400	400	375	375	350	325	280	270	260
91– 96	...	300	325	350	400	400	400	400	400	400	375	375	350	325	300	275	260	260	250
97–102	...	275	300	325	375	375	375	375	375	375	350	350	325	300	275	250	250	240	240
103–108	...	250	275	300	350	350	350	350	350	350	325	325	300	275	250	250	180	175	160
109–114	...	175	200	225	275	275	275	300	300	300	275	275	250	250	225	200	175	160	150
115–120	...	...	175	200	250	250	250	250	250	275	250	250	225	225	200	200	175	160	150
121–126	...	...	...	180	180	180	180	180	180	180	200	200	175	175	160	160	150	144	144
Diam. of Head, in.	72	115	124	127	127	127	127	127	127	127	126	126	126	126	126	125	125	125	125
Minimum diameter of heads = 30 inches.																			

From *Cambria Steel*, George E. Thackray, Engineer, (1919).

* Edges trimmed by shearing. Narrower plates are rolled to dimensions and can be obtained in practically any width; even inches should be used.

TABLE III. — STANDARD ANGLES WITH EQUAL LEGS

Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of leg	Moment of inertia axis 1-1	Section modulus axis 1-1 $=\frac{I}{a-x}$
$a \times a$	t		A	x	I	S
Inches	Inch	Pounds	Sq. in.	Inches	Inches ^{4}	Inches ^{3}
1½×1½	⅛	1.23	0.36	0.42	0.08	0.072
"	3/16	1.80	0.53	0.44	0.11	0.104
"	¼	2.34	0.69	0.47	0.14	0.134
"	5/16	2.86	0.84	0.49	0.16	0.162
"	⅜	3.35	0.98	0.51	0.19	0.188
2 ×2	⅛	1.65	0.48	0.55	0.19	0.13
"	3/16	2.44	0.72	0.57	0.27	0.19
"	¼	3.19	0.94	0.59	0.35	0.25
"	5/16	3.92	1.15	0.61	0.42	0.30
"	⅜	4.7	1.36	0.64	0.48	0.35
"	7/16	5.3	1.56	0.66	0.54	0.40
"	½	6.0	1.75	0.68	0.59	0.45
2½×2½	⅛	2.08	0.61	0.67	0.38	0.20
"	3/16	3.07	0.90	0.69	0.55	0.30
"	¼	4.1	1.19	0.72	0.70	0.39
"	5/16	5.0	1.47	0.74	0.85	0.48
"	⅜	5.9	1.73	0.76	0.98	0.57
"	7/16	6.8	2.00	0.78	1.11	0.65
"	½	7.7	2.25	0.81	1.23	0.72
3 ×3	¼	4.9	1.44	0.84	1.24	0.58
"	5/16	6.1	1.78	0.87	1.51	0.71
"	⅜	7.2	2.11	0.89	1.76	0.83
"	7/16	8.3	2.43	0.91	1.99	0.95
"	½	9.4	2.75	0.93	2.22	1.07
"	9/16	10.4	3.06	0.95	2.43	1.19
3½×3½	¼	5.8	1.69	0.97	2.01	0.79
"	5/16	7.2	2.09	0.99	2.45	0.98
"	⅜	8.5	2.48	1.01	2.87	1.15
"	7/16	9.8	2.87	1.04	3.26	1.32
"	½	11.1	3.25	1.06	3.64	1.49
"	9/16	12.4	3.62	1.08	3.99	1.65
"	⅝	13.6	3.98	1.10	4.33	1.81
"	11/16	14.8	4.34	1.12	4.65	1.96

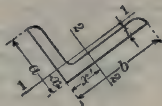
From *Cambria Steel*, George E. Thackray, Engineer, 1919.

TABLE III. — STANDARD ANGLES WITH EQUAL LEGS — (*Continued*)

Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of leg	Moment of inertia axis 1–1	Section modulus axis 1–1 $= \frac{I}{a-x}$
$a \times a$	t		A	x	I	S
Inches	Inch	Pounds	Sq. in.	Inches	Inches⁴	Inches³
3½ × 3½	¾	16.0	4.69	1.15	4.96	2.11
"	13/16	17.1	5.03	1.17	5.25	2.25
"	⅞	18.3	5.36	1.19	5.53	2.39
4 × 4	5/16	8.2	2.40	1.12	3.71	1.29
"	⅜	9.8	2.86	1.14	4.36	1.52
"	7/16	11.3	3.31	1.16	4.97	1.75
"	½	12.8	3.75	1.18	5.56	1.97
"	9/16	14.3	4.18	1.21	6.12	2.19
"	⅝	15.7	4.61	1.23	6.66	2.40
"	11/16	17.1	5.03	1.25	7.17	2.61
"	¾	18.5	5.44	1.27	7.66	2.81
"	13/16	19.9	5.84	1.29	8.14	3.01
"	⅞	21.2	6.23	1.31	8.59	3.20
6 × 6	⅜	14.9	4.36	1.64	15.39	3.53
"	7/16	17.2	5.06	1.66	17.68	4.07
"	½	19.6	5.75	1.68	19.91	4.61
"	9/16	21.9	6.43	1.71	22.07	5.14
"	⅝	24.2	7.11	1.73	24.16	5.66
"	11/16	26.5	7.78	1.75	26.19	6.17
"	¾	28.7	8.44	1.78	28.15	6.66
"	13/16	31.0	9.09	1.80	30.06	7.15
"	⅞	33.1	9.73	1.82	31.92	7.63
"	15/16	35.3	10.37	1.84	33.72	8.11
"	1	37.4	11.00	1.86	35.46	8.57
8 × 8	½	26.4	7.75	2.19	48.65	8.37
"	9/16	29.6	8.68	2.21	54.09	9.34
"	⅝	32.7	9.61	2.23	59.43	10.30
"	11/16	35.8	10.53	2.25	64.64	11.25
"	¾	38.9	11.44	2.28	69.74	12.18
"	13/16	42.0	12.34	2.30	74.72	13.11
"	⅞	45.0	13.23	2.32	79.58	14.02
"	15/16	48.1	14.12	2.34	84.34	14.91
"	1	51.0	15.00	2.37	88.98	15.80
"	1 1/16	54.0	15.87	2.39	93.53	16.67
"	1⅛	56.9	16.73	2.41	97.97	17.53

From *Cambria Steel*, George E. Thackray, Engineer, 1919.

TABLE IV. — STANDARD ANGLES WITH UNEQUAL LEGS



Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of longer leg	Moment of inertia axis 1–1	Distance of center of gravity from back of shorter leg	Moment of inertia axis 2–2
$b \times a$	t		A	x	I	x'	I'
Inches	Inch	Pounds	Sq. in.	Inch	Inches⁴	Inches	Inches⁴
2½×2	3/16	2.75	0.81	0.51	0.29	0.76	0.51
"	¼	3.62	1.06	0.54	0.37	0.79	0.65
"	5/16	4.5	1.31	0.56	0.45	0.81	0.79
"	⅜	5.3	1.55	0.58	0.51	0.83	0.91
"	7/16	6.1	1.78	0.60	0.58	0.85	1.03
"	½	6.8	2.00	0.63	0.64	0.88	1.14
3 ×2½	¼	4.5	1.31	0.66	0.74	0.91	1.17
"	5/16	5.6	1.62	0.68	0.90	0.93	1.42
"	⅜	6.6	1.92	0.71	1.04	0.96	1.66
"	7/16	7.6	2.22	0.73	1.18	0.98	1.88
"	½	8.5	2.50	0.75	1.30	1.00	2.08
"	9/16	9.5	2.78	0.77	1.42	1.02	2.28
3½×2½	¼	4.9	1.44	0.61	0.78	1.11	1.80
"	5/16	6.1	1.78	0.64	0.94	1.14	2.19
"	⅜	7.2	2.11	0.66	1.09	1.16	2.56
"	7/16	8.3	2.43	0.68	1.23	1.18	2.91
"	½	9.4	2.75	0.70	1.36	1.20	3.24
"	9/16	10.4	3.06	0.73	1.49	1.23	3.55
3½×3	¼	5.4	1.56	0.79	1.30	1.04	1.91
"	5/16	6.6	1.93	0.81	1.58	1.06	2.33
"	⅜	7.9	2.30	0.83	1.85	1.08	2.72
"	7/16	9.1	2.65	0.85	2.09	1.10	3.10
"	½	10.2	3.00	0.88	2.33	1.13	3.45
"	9/16	11.4	3.34	0.90	2.55	1.15	3.79
"	⅝	12.5	3.67	0.92	2.76	1.17	4.11
"	11/16	13.6	4.00	0.94	2.96	1.19	4.41
"	¾	14.7	4.31	0.96	3.15	1.21	4.70
"	13/16	15.8	4.62	0.98	3.33	1.23	4.98
"	⅞	16.8	4.92	1.00	3.50	1.25	5.24
4 ×3	5/16	7.2	2.09	0.76	1.65	1.26	3.38
"	⅜	8.5	2.48	0.78	1.92	1.28	3.96
"	7/16	9.8	2.87	0.80	2.18	1.30	4.52
"	½	11.1	3.25	0.83	2.42	1.33	5.05
"	9/16	12.4	3.62	0.85	2.66	1.35	5.55

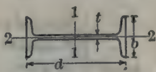
From *Cambria Steel*, George E. Thackray, Engineer, 1919

TABLE IV.—STANDARD ANGLES WITH UNEQUAL LEGS—(Continued)

Dimensions	Thickness	Weight per foot	Area of section	Distance of center of gravity from back of longer leg	Moment of inertia axis 1–1	Distance of center of gravity from back of shorter leg	Moment of inertia axis 2–2
$b \times a$	t		A	x	I	x'	I'
Inches	Inch	Pounds	Sq. in.	Inches	Inches⁴	Inches	Inches⁴
4 ×3	5/8	13.6	3.98	0.87	2.87	1.37	6.03
"	11/16	14.8	4.34	0.89	3.08	1.39	6.49
"	3/4	16.0	4.69	0.92	3.28	1.42	6.93
"	13/16	17.1	5.03	0.94	3.47	1.44	7.35
"	7/8	18.3	5.36	0.96	3.66	1.46	7.75
5 ×3	5/16	8.2	2.40	0.68	1.75	1.68	6.26
"	3/8	9.8	2.86	0.70	2.04	1.70	7.37
"	7/16	11.3	3.31	0.73	2.32	1.73	8.43
"	1/2	12.8	3.75	0.75	2.58	1.75	9.45
"	9/16	14.3	4.18	0.77	2.83	1.77	10.43
"	5/8	15.7	4.61	0.80	3.06	1.80	11.37
"	11/16	17.1	5.03	0.82	3.29	1.82	12.28
"	3/4	18.5	5.44	0.84	3.51	1.84	13.15
"	13/16	19.9	5.84	0.86	3.71	1.86	13.98
"	7/8	21.2	6.23	0.88	3.91	1.88	14.78
5 ×3½	5/16	8.7	2.56	0.84	2.72	1.59	6.60
"	3/8	10.4	3.05	0.86	3.18	1.61	7.78
"	7/16	12.0	3.53	0.88	3.63	1.63	8.90
"	1/2	13.6	4.00	0.91	4.05	1.66	9.99
"	9/16	15.2	4.47	0.93	4.45	1.68	11.03
"	5/8	16.8	4.92	0.95	4.83	1.70	12.03
"	11/16	18.3	5.37	0.97	5.20	1.72	12.99
"	3/4	19.8	5.81	1.00	5.55	1.75	13.92
"	13/16	21.3	6.25	1.02	5.89	1.77	14.81
"	7/8	22.7	6.67	1.04	6.21	1.79	15.67
"	15/16	24.2	7.09	1.06	6.52	1.81	16.49
6 ×3½	3/8	11.7	3.42	0.79	3.34	2.04	12.86
"	1/2	15.3	4.50	0.83	4.25	2.08	16.59
"	5/8	18.9	5.55	0.88	5.08	2.13	20.08
"	3/4	22.4	6.56	0.93	5.84	2.18	23.34
"	7/8	25.7	7.55	0.97	6.55	2.22	26.39
"	15/16	27.3	8.03	0.99	6.88	2.24	27.84
"	1	28.9	8.50	1.01	7.21	2.26	29.15
6 ×4	3/8	12.3	3.61	0.94	4.90	1.94	13.47
"	1/2	16.2	4.75	0.99	6.27	1.99	17.40
"	5/8	20.0	5.86	1.03	7.52	2.03	21.07
"	3/4	23.6	6.94	1.08	8.68	2.08	24.51
"	7/8	27.2	7.98	1.12	9.75	2.12	27.73
"	15/16	28.9	8.50	1.14	10.26	2.14	29.26
"	1	30.6	9.00	1.17	10.75	2.17	30.75

From *Cambria Steel*. George E. Thackray, Engineer, 1919

TABLE V. — STANDARD I-BEAMS



Depth of beam	Weight per foot	Area of section	Thickness of web	Width of flange	Moment of inertia axis 1-1	Section modulus axis 1-1	Radius of gyration axis 1-1	Moment of inertia axis 2-2	Radius of gyration axis 2-2	Coefficient of strength*
d	(a)	*A*	*t*	*b*	*I*	*S*	*r*	*I'*	*r'*	
In.	Lb.	Sq.in.	In.	In.	In.⁴	In.³	In.	In.⁴	In.	
3	5.70	1.63	0.17	2.33	2.5	1.7	1.23	0.46	0.53	17,650
3	6.50	1.91	0.26	2.42	2.7	1.8	1.19	0.53	0.52	19,140
3	7.50	2.21	0.36	2.52	2.9	1.9	1.15	0.60	0.52	20,710
4	7.7	2.21	0.19	2.66	6.0	3.0	1.64	0.77	0.59	31,810
4	8.50	2.50	0.26	2.73	6.4	3.2	1.59	0.85	0.58	33,890
4	9.50	2.79	0.34	2.81	6.7	3.4	1.54	0.93	0.58	35,980
4	10.50	3.09	0.41	2.88	7.1	3.6	1.52	1.01	0.57	38,070
5	10.00	2.87	0.21	3.00	12.1	4.8	2.05	1.23	0.65	51,590
5	12.25	3.60	0.36	3.15	13.6	5.4	1.94	1.45	0.63	58,100
5	14.75	4.34	0.50	3.29	15.1	6.1	1.87	1.70	0.63	64,630
6	12.50	3.61	0.23	3.33	21.8	7.3	2.46	1.85	0.72	77,460
6	14.75	4.34	0.35	3.45	24.0	8.0	2.35	2.09	0.69	85,270
6	17.25	5.07	0.47	3.57	26.2	8.7	2.27	2.36	0.68	93,110
7	15.30	4.42	0.25	3.66	36.2	10.4	2.86	2.67	0.78	110,410
7	17.50	5.15	0.35	3.76	39.2	11.2	2.76	2.94	0.76	119,400
7	20.00	5.88	0.46	3.87	42.2	12.1	2.68	3.24	0.74	128,560
8	18.4	5.33	0.27	4.00	56.9	14.2	3.27	3.78	0.84	151,660
8	20.25	5.96	0.35	4.08	60.2	15.0	3.18	4.04	0.82	160,510
8	22.75	6.69	0.44	4.17	64.1	16.0	3.10	4.36	0.81	170,970
8	25.25	7.43	0.53	4.26	68.0	17.0	3.03	4.71	0.80	181,430
9	21.8	6.31	0.29	4.33	84.9	18.9	3.67	5.16	0.90	201,300
9	25.00	7.35	0.41	4.45	91.9	20.4	3.54	5.65	0.88	217,930
9	30.00	8.82	0.57	4.61	101.9	22.6	3.40	6.42	0.85	241,460
9	35.00	10.29	0.73	4.77	111.8	24.8	3.30	7.31	0.84	264,990

From *Cambria Steel*, George E. Thackray, Engineer, 1919.

* Divide by span in feet to determine total allowable dead load in pounds, including weight of beam, uniformly distributed over length of beam. Table is based upon a fiber stress of 16,000 lb. per sq. in. For allowable live and dead load reduce load as determined from table by amount of impact. Compression flange should be supported laterally at intervals not greater than 20 times its width. If this cannot be done reduce coefficients to one-half of above value where unsupported length divided by width = 70, and proportionally for intermediate values between 20 and 70.

(*a*) Weights revised Sept. 1, 1920.

TABLE V. — STANDARD I-BEAMS — (*Continued*)

Depth of beam	Weight per foot	Area of section	Thickness of web	Width of flange	Moment of inertia axis 1–1	Section modulus axis 1–1	Radius of gyration axis 1–1	Moment of inertia axis 2–2	Radius of gyration axis 2–2	Coefficient of strength*
d	(a)	*A*	*t*	*b*	*I*	*S*	*r*	*I'*	*r'*	
In.	Lb.	Sq.in.	In.	In.	In.⁴	In.³	In.	In.⁴	In.	
10	25.4	7.37	0.31	4.66	122.1	24.4	4.07	6.89	0.97	260,470
10	30.00	8.82	0.45	4.80	134.2	26.8	3.90	7.65	0.93	286,250
10	35.00	10.29	0.60	4.95	146.4	29.3	3.77	8.52	0.91	312,390
10	40.00	11.76	0.75	5.10	158.7	31.7	3.67	9.50	0.90	338,530
12	31.8	9.26	0.35	5.00	215.8	36.0	4.83	9.50	1.01	383,670
12	35.00	10.29	0.44	5.09	228.3	38.0	4.71	10.07	0.99	405,800
12	40.8	11.76	0.56	5.21	245.9	41.0	4.57	10.95	0.96	437,170
15	42.9	12.48	0.41	5.50	441.8	58.9	5.95	14.62	1.08	628,270
15	45.00	13.24	0.46	5.55	455.8	60.8	5.87	15.09	1.07	648,310
15	50.00	14.71	0.56	5.65	483.4	64.5	5.73	16.04	1.04	687,530
15	55.00	16.18	0.66	5.75	511.0	68.1	5.62	17.06	1.03	726,740
15	60.8	17.65	0.75	5.84	538.6	71.8	5.52	18.17	1.01	765,960
18	54.7	15.93	0.46	6.00	795.6	88.4	7.07	21.19	1.15	942,880
18	60.0	17.65	0.56	6.10	841.8	93.5	6.91	22.38	1.13	997,680
18	65.0	19.12	0.64	6.18	881.5	97.9	6.79	23.47	1.11	1,044,740
18	70.0	20.59	0.72	6.26	921.2	102.4	6.69	24.62	1.09	1,091,800
20	65.4	19.08	0.50	6.25	1169.5	117.0	7.83	27.86	1.21	1,247,490
20	70.0	20.59	0.58	6.33	1219.8	122.0	7.70	29.04	1.19	1,301,110
20	75.0	22.06	0.65	6.40	1268.8	126.9	7.58	30.25	1.17	1,353,400
24	79.9	23.32	0.50	7.00	2087.2	173.9	9.46	42.86	1.36	1,855,310
24	85.0	25.00	0.57	7.07	2167.8	180.7	9.31	44.35	1.33	1,926,950
24	90.0	26.47	0.63	7.13	2238.4	186.5	9.20	45.70	1.31	1,989,700
24	95.0	27.94	0.69	7.19	2309.0	192.4	9.09	47.10	1.30	2,052,440
24	100.0	29.41	0.75	7.25	2379.6	198.3	8.99	48.55	1.28	2,115,190

From *Cambria Steel*, George E. Thackray, Engineer.

* Divide by span in feet to determine total allowable dead load in pounds, including weight of beam, uniformly distributed over lengrh of beam. Table is based upon a fiber stress of 16 000 lb. per sq. in. For allowable live and dead load reduce load as determined from table by amount of impact. Compression flange should be supported laterally at intervals not greater than 20 times its width. If this cannot be done reduce coefficients to one-half of above value where unsupported length divided by width = 70, and proportionally for intermediate values between 20 and 70.

(*a*) Weights revised Sept. 1, 1920.

ALLOWANCES FOR IMPACT. — The force applied to a structure by a moving load such as a locomotive or electric car is a function of its weight and method of application. The value of the former can usually be determined with a reasonable degree of accuracy; the effect of the latter cannot in general

TABLE VI.—STANDARD CHANNELS

Depth of Channel	Weight per foot (a)	Area of section	Thickness of web	Width of flange	Moment of inertia axis 1-1	Section modulus axis 1-1	Radius of gyration axis 1-1	Moment of inertia axis 2-2	Section modulus axis 2-2	Radius of gyration axis 2-2	Distance of center of gravity from outside of web
d		A	t	b	I	S	r	I'	S'	r'	x
In.	Lb.	Sq. in.	In.	In.	In.^4	In.^3	In.	In.^4	In.^3	In.	In.
5	6.70	1.95	0.19	1.75	7.4	3.0	1.95	0.48	0.38	0.50	0.49
5	9.00	2.65	0.33	1.89	8.9	3.5	1.83	0.64	0.45	0.49	0.48
5	11.50	3.38	0.48	2.04	10.4	4.2	1.75	0.82	0.54	0.49	0.51
6	8.20	2.38	0.20	1.92	13.0	4.3	2.34	0.70	0.50	0.54	0.52
6	10.50	3.09	0.32	2.04	15.1	5.0	2.21	0.88	0.57	0.53	0.50
6	13.00	3.82	0.44	2.16	17.3	5.8	2.13	1.07	0.65	0.53	0.52
6	15.50	4.56	0.56	2.28	19.5	6.5	2.07	1.28	0.74	0.53	0.55
7	9.80	2.85	0.21	2.09	21.1	6.0	2.72	0.98	0.63	0.59	0.55
7	12.25	3.60	0.32	2.20	24.2	6.9	2.59	1.19	0.71	0.57	0.53
7	14.75	4.34	0.42	2.30	27.2	7.8	2.50	1.40	0.79	0.57	0.53
7	17.25	5.07	0.53	2.41	30.2	8.6	2.44	1.62	0.87	0.56	0.55
7	19.75	5.81	0.63	2.51	33.2	9.5	2.39	1.85	0.96	0.56	0.58
8	11.50	3.35	0.22	2.26	32.3	8.1	3.10	1.33	0.79	0.63	0.58
8	13.75	4.04	0.31	2.35	36.0	9.0	2.98	1.55	0.87	0.62	0.56
8	16.25	4.78	0.40	2.44	39.9	10.0	2.89	1.78	0.95	0.61	0.56
8	18.75	5.51	0.49	2.53	43.8	11.0	2.82	2.01	1.02	0.60	0.57
8	21.25	6.25	0.58	2.62	47.8	11.9	2.76	2.25	1.11	0.60	0.59
9	13.40	3.89	0.23	2.43	47.3	10.5	3.49	1.77	0.97	0.67	0.61
9	15.00	4.41	0.29	2.49	50.9	11.3	3.40	1.95	1.03	0.66	0.59
9	20.00	5.88	0.45	2.65	60.8	13.5	3.21	2.45	1.19	0.65	0.58
9	25.00	7.35	0.61	2.81	70.7	15.7	3.10	2.98	1.36	0.64	0.62
10	15.30	4.46	0.24	2.60	66.9	13.4	3.87	2.30	1.17	0.72	0.64
10	20.00	5.88	0.38	2.74	78.7	15.7	3.66	2.85	1.34	0.70	0.61
10	25.00	7.35	0.53	2.89	91.0	18.2	3.52	3.40	1.50	0.68	0.62
10	30.00	8.82	0.68	3.04	103.2	20.6	3.42	3.99	1.67	0.67	0.65
10	35.00	10.29	0.82	3.18	115.5	23.1	3.35	4.66	1.87	0.67	0.69
12	20.70	6.03	0.28	2.94	128.1	21.4	4.61	3.91	1.75	0.81	0.70
12	25.00	7.35	0.39	3.05	144.0	24.0	4.43	4.53	1.91	0.78	0.68
12	30.00	8.82	0.51	3.17	161.6	26.9	4.28	5.21	2.09	0.77	0.68
12	35.00	10.29	0.64	3.30	179.3	29.9	4.17	5.90	2.27	0.76	0.69
12	40.00	11.76	0.76	3.42	196.9	32.8	4.09	6.63	2.46	0.75	0.72
15	33.90	9.90	0.40	3.40	312.6	41.7	5.62	8.23	3.16	0.91	0.79
15	35.00	10.29	0.43	3.43	319.9	42.7	5.57	8.48	3.22	0.91	0.79
15	40.00	11.76	0.52	3.52	347.5	46.3	5.44	9.39	3.43	0.89	0.78
15	45.00	13.24	0.62	3.62	375.1	50.0	5.32	10.29	3.63	0.88	0.79
15	50.00	14.71	0.72	3.72	402.7	53.7	5.23	11.22	3.85	0.87	0.80
15	55.00	16.18	0.82	3.82	430.2	57.4	5.16	12.19	4.07	0.87	0.82

From *Cambria Steel*, George E. Thackray, Engineer, 1919.
(a) Weights revised Sept. 1, 1920.

be so determined since it depends upon such uncertain factors as rapidity of application, irregularity of track, improper counterbalancing of locomotive driving wheels, swaying action of crowds of people and similar causes. The allowance for impact (so-called) ranges from 0 to 100 per cent of the live load and is determined by empirical rules as indicated in the formulas below. An exception to this rule is made in the case of timber structures where the effect of impact is commonly allowed for by using a low unit stress. (*See Timber.*)

Impact on Buildings. — No allowance is generally made for impact on buildings except with special loadings such as moving machinery, swinging cranes, etc., for which the designer should use his judgment. The building laws of the larger cities fix arbitrary loads and unit stresses by which the designer must be governed; see article on *Buildings, Allowable Unit Stresses in.*

Impact on Highway Bridges Carrying Electric Railways. — The following extract from the *Specifications for Bridges Carrying Electric Railways* adopted by the Massachusetts Public Service Commission (revised March, 1915), may be safely used for such cases:

"The total maximum stress in any piece shall be computed by adding together the dead and live stresses, the live loads being placed in the most unfavorable position, together with a percentage of the live stress to allow for impact and vibration. This added percentage shall be as follows:

For auto truck.	
For stringers, floor beams hangers and truss members receiving their whole load from one panel point only	50 per cent
For all other live loads:	
For wood flooring and wood stringers	no impact
For floor beams and stringers	25 per cent
For floor beam hangers	40 per cent
For all counters	40 per cent

For other members in trusses, and for main girders, the percentage shall be $26\frac{2}{3}$ minus one-twelfth the loaded length in feet with a maximum of 25 and a minimum of 10 per cent."

Impact on Steam Railroad Bridges. — The following rule of the American Railway Engineering and Maintenance of Way Association is commonly used in the United States:

"The dynamic increment of the live load shall be added to the maximum computed live-load strains* and shall be determined by the formula $I = S \frac{300}{L + 300}$,

where I = impact or dynamic increment to be added to live-load strains,
S = computed maximum live-load strain,
L = loaded length of track in feet producing the maximum strain in the member.

For bridges carrying more than one track, the *aggregate length of all tracks* producing the strain shall be used. Impact shall not be added to strains produced by longitudinal, centrifugal and lateral or wind forces."

CALCULATION OF REACTIONS. — The reactions upon a truss, girder or beam lying in a plane and acted upon by forces lying in the same plane may be determined by the application of the equations of equilibrium (*see preceding section*) provided there are but two reactions which together have three unknown components; such a structure is said to be statically determined. If more than three unknown reaction components exist the reactions cannot be computed by statics, and the structure, although stable, is statically undetermined; if less than

* Strains as here used means stresses in the more modern meaning of the words.

three exist the structure is unstable. If the structure is supported at more than two points it cannot be made statically determined except by the insertion in the structure of special devices, such as hinged joints. The tower shown in Fig. 3 is statically determined with respect to the outer forces since the points of application of the reactions at a and b are fixed in position, and the reaction at b is also fixed in direction. If neither of the reactions were fixed in direction the structure would be indeterminate; if both of the reactions were fixed in direction the tower would be unstable, e.g., if both ends were on rollers and neither end bolted to the masonry.

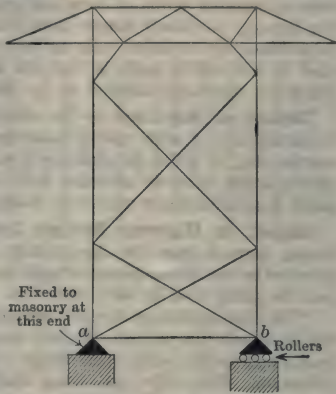


Fig. 3.

Such towers when used for transmission lines are seldom built with rollers or even planed plates, and frequently the bottoms of the columns are imbedded in a concrete base, the latter condition making it impossible to accurately compute the reactions. Similar towers for railroad and highway bridges are usually made determinate.

Method of Computation. — The following example illustrates the analytical method of computing reactions on statically determinate structures. The method may be applied equally well to uniformly distributed loads provided the resultants of such loads are used in place of concentrated loads.

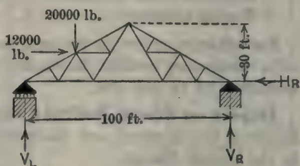


Fig. 4.

To Compute the Reactions on Truss Shown in Fig. 4. — Let V_L = left reaction, and V_R and H_R = the components of the right reaction, the direction of which is unknown. V_L will be vertical since it is supported on rollers bearing on a horizontal surface. V_R and V_H may be assumed to act in either direction. A negative sign in the final result indicates that the force acts in the opposite direction to that assumed. The determination of the reactions by the application of the equations of equilibrium (*see above*) may be carried out as follows:

1st. Apply $\Sigma M = 0$ about right end. This gives $100\,V_L + 12{,}000 \times 15 - 20{,}000 \times 75 = 0$. Hence $V_L = 13{,}200$ lb.

2nd. Apply $\Sigma H = 0$. This gives $H_R - 12{,}000 = 0$. Hence $H_R = +\,12{,}000$ lb.

3rd. Apply $\Sigma V = 0$. This gives $V_R + 13{,}200 - 20{,}000 = 0$. Hence $V_R = +\,6800$ lb.

In these results a positive sign shows that the reaction acts as indicated in Fig. 4.

Reactions on Continuous Girders. — See sections which follow on *Continuous Girders, Beams, Slabs* and *Trusses*.

CALCULATION OF SHEAR. — The nomenclature used in the discussion of shear is as follows:

V = the total external shear on any section in pounds,
P = magnitude of a single concentrated load in pounds,

w = a uniform load in pounds per foot,
L = span of beam in feet (distance center to center of supports),
p = length of a panel in feet,
x = distance in feet from a given section to one of the points of support,
n = total number of equal panels into which a girder is divided,
z = number of panels between a given panel and the more remote abutment.

Method of Computation. — The magnitude of the shear at any section of a body due to a set of coplanar forces may be readily computed in the following manner: Resolve each force into two components, parallel and perpendicular respectively to the given section; the algebraic sum of the components parallel to the section of all the forces on either side of the section equals the shear. The shear is generally considered positive when the resultant force is *upward* on the *left* of the section.

Curve of Shear. — A curve of shear is a line the ordinate to which at any point equals the shear on the given body at the section where the ordinate is measured. Fig. 5 shows typical curves of this sort.

Maximum Shear with Single Load or Uniform Load. — In a simple end-supported beam, girder or truss a concentrated load causes maximum shear at a given section when placed an infinitesimal distance from the section on the side toward the more distant point of support; the magnitude of this shear equals that of the nearer reaction. A concentrated load causes a maximum shear on the beam when placed an infinitesimal distance to one side of either point of support; this shear equals the magnitude of the load itself. A uniformly distributed live load causes maximum shear at a given section when placed over the entire distance from the section to the more distant point of support; its value equals that of the nearer reaction and is given by the equation $V = wx^2/2L$. Its maximum value equals $wL/2$ at either end of the beam when the latter is fully loaded.

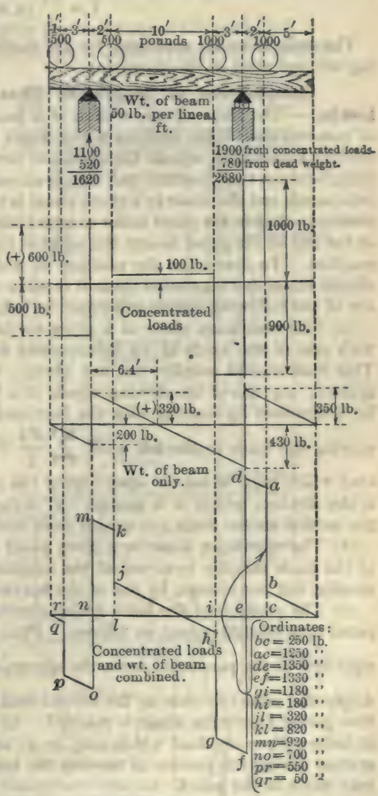


Fig. 5. Curves of Shear. Positive results are shown above the axis. Curve shown by full line

End-supported Girders or Trusses with Loads Applied by Floor-Beams. — A concentrated load causes maximum shear in any panel when

placed at the end of the panel nearer the more remote reaction. The value for equal panels is given by the equation $V = Pz/n$. The maximum shear occurs in the end panel and its value for equal panels is $V = P\ (n-1)/n$.

Maximum positive shear in any panel due to uniform load occurs when all panel points to the right are assumed to be loaded with full panel loads, and all panel points to the left are unloaded; its value equals the left reaction. This method is approximate but is on the safe side and is commonly used.

The maximum positive shear occurs in the left panel and for girders with equal panels is given by the formula:

$$V = \frac{wp}{2}(n-1).$$

The same rules are applicable for the maximum negative shear by interchanging left and right in the discussion.

Computation of Maximum Shear for System of Concentrated Loads. — The shear at any section of a beam, girder or truss, due to a system of concentrated loads such as wheel loads of a moving crane, electric car or electric locomotive, equals the algebraic sum of either reaction and the loads lying between that reaction and the given section. In case the girder is divided into panels and the shear in a given panel is to be computed, only that portion of the load acting in the panel under consideration which is carried by the floor-beam at the end of the panel nearer the selected reaction should be deducted from the reaction. To determine the position in which a system of concentrated loads should lie to give maximum shear it is often necessary to proceed by trial. The use of the following simple rules may be helpful.

(*a*) The maximum shear at a given section of a simple beam always occurs with one of the loads at an infinitesimal distance to one side of the section. This load should usually, but not always, be one of the heavier loads of the system; e.g. one of the driving wheels of a locomotive. The proper load may be determined by starting with the first load just to the left of the section and moving the loads to the left until $\sum \frac{Pa}{L} \leqq P'$. In this expression P = any load which may be on the span during the process of moving the loads, and a is the distance which it is moved. P' = the load which passes to the left of the section as the loads are moved from one position to another.

(*b*) The maximum shear in a given panel of a girder always occurs with one of the loads directly over one of the adjoining floor-beams. This load should usually, but not always, be one of the heavier loads.

(*c*) The position of the loads for maximum positive shear in an intermediate panel may be determined as follows. Place the first load of the system at the right end of the panel and move the system to the left until $\sum \frac{Pa}{L} \leqq \sum \frac{P_1 a_1}{p}$. Apply the same criterion to the second load and following loads until the position giving maximum shear is reached. In the above expression, P and a are as before, P_1 = any load which may be at any time in the panel under consideration during the process of moving the loads, and a_1 = the distance which P_1 may move in the panel.

If no load comes on or goes off the span and if no load passes out of the panel, $a = a_1$ and we may write

$$\sum \frac{P}{L} \leqq \sum \frac{P_1}{p}.$$

It follows that for this case the first load should lie at the panel point unless the average load per foot on the entire span is greater than the first load divided by a panel length, in which case the second load should be tried at the panel point and so on until the position for maximum shear is determined.

(*d*) The maximum shear in the end panel of a girder equals the maximum moment at the first panel point divided by the length of the panel. (*See section on Bending Moment, below.*)

CALCULATION OF BENDING MOMENT. — The nomenclature used in the discussion of bending moment is as follows:

M = external bending moment at any section in foot pounds,
P = magnitude of a single concentrated load in pounds,
w = a uniform load in pounds per foot,
L = span of beam in feet (distance center to center of support),
p = length of a panel in feet,
x = distance in feet from a given section to one of the supports,
n = total number of equal panels into which a given girder is divided.

To determine the magnitude of the bending moment at a given section it is necessary to obtain the algebraic sum of the products of every force by its distance from the neutral axis of the section. A mistake commonly made is the failure to consider all the forces, particularly horizontal forces. The moment is considered positive if it is clockwise on the *left* of the section; it follows that the moment is also positive if it is counter-clockwise on the *right* of the section.

Curve of Moments. — A curve of moments is a line whose ordinate at any point equals the moment on the given body at the section where the ordinate is measured. Fig. 6 shows a typical set of moment curves.

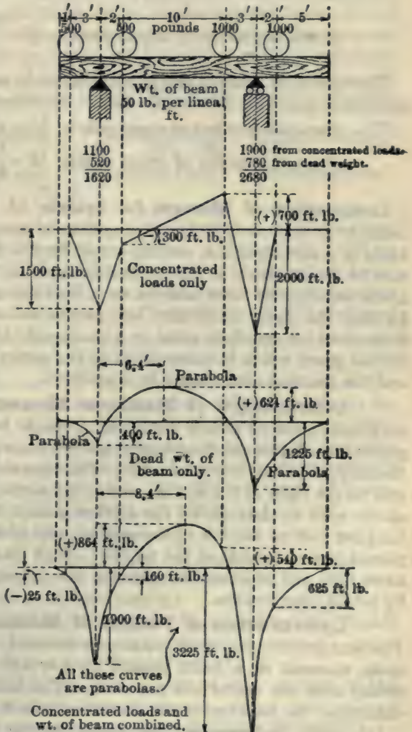


Fig. 6. Curves of Moments

Maximum Moment with Single Load or Uniform Load. — For simple end-supported beams, girders and trusses a single concentrated load causes maximum moment at a given section when placed at the section. The following formula gives its value:

$$M = \frac{P(L-x)}{L}x.$$

The maximum possible moment occurs when $x = L/2$ and has the value $M = PL/4$.

For simple end-supported structures a uniform load causes maximum moment at any section when it covers the entire beam. Its value at any section is

$$M = \frac{wx}{2}(L-x).$$

The maximum possible moment occurs at the center of the span and equals $wL^2/8$.

End-supported Girders or Trusses with Loads Applied by Floor-Beams. — Moments at panel points are exactly the same as moments at corresponding points on simple girders. It is seldom necessary to consider moments between panel points for concentrated loads. For a uniform load the moment between floor-beams varies uniformly; i.e., the moment due to that portion of the load applied through the floor-beams.

The maximum moment due to a concentrated load occurs at the panel point nearest the center with the load at that point. Its value is given by the following equations:

For even number of equal panels, $M = \dfrac{Ppn}{4}$.

For odd number of equal panels, $M = \dfrac{Pp}{4n}(n^2 - 1)$.

The maximum moment due to a uniform load occurs at the panel point nearest the center with load over entire span. Its value is:

For even number of equal panels, $M = wL^2/8$.

For odd number of equal panels, $M = \dfrac{1}{8}\left[wL^2\left(1 - \dfrac{1}{n^2}\right)\right]$.

Computation of Moment for System of Concentrated Loads. — The moment at any section due to a system of concentrated loads, such as wheel loads of a moving crane, electric car or electric locomotive equals the algebraic sum of the moments of either reaction and the loads lying between it and the given section, the lever arm for each load being its distance from the section. In case the girder is divided into panels and the section under consideration is at some intermediate point in a panel only that portion of the load applied in this panel which is transmitted to the girder by the floor-beams between it and the reaction used should be considered.

Determination of Maximum Moment at a Given Section. — The maximum moment at any section of a simple beam or at any panel point of a girder loaded through floor-beams always occurs with some load at the section (usually one of the heavier loads). This load should be one which when located just to the right of the section makes the average load per foot of all the loads on the span to the right of the section greater than the corresponding average load per foot on the left of the section, and which reverses this condition when placed just to the left of the section. Such loads may be selected by trial; if more than one satisfies this criterion the largest moment must be determined by actual computation.

Determination of Position of Maximum Moment on a Beam. — Previous articles deal with maximum moment at a given section. The maximum moment on a beam caused by a system of concentrated loads usually occurs near the center but not at it. The following method may be used to determine the location of the section at which the maximum moment occurs:

(*a*) Select, by inspection, one of the heavier loads which would probably be located at the center to give the maximum moment at the center.

(*b*) Determine by inspection the loads which would be on the span when this load is near the center and compute the position of their resultant.

(*c*) Place the loads on the beam in a position such that the center of the span lies half way between this resultant and the assumed load. If the loads on the span in this position correspond to those assumed under *b*, the moment at the load should be computed; otherwise, another trial may be made. If more

than one set of loads corresponds to this condition the moment at the load should be computed for both sets of loads.

(*d*) Apply the same method to any other load which would possibly give maximum moment at the center.

Since the maximum moment at any section always occurs with some load at the section this method if applied to enough loads will give the absolute maximum moment. It is often unnecessary to try more than one load, and seldom more than two. It should be noted that with two equal loads such a distance apart that with one at the center the other will not be on the span, but that with the center of the span midway between the resultant and one of the loads both will be on the span, the maximum moment may possibly occur with either condition, both of which should be investigated.

BEAMS; FORMULAS AND COSTS. — The effect of the external forces upon any cross-section of a simple beam is to cause bending moment and shear, the former usually being the predominant factor in determining the size of the beam. If the beam is subjected to axial forces as well as transverse forces, it becomes a combination of beam and column, or beam and tie, and can no longer be considered as a simple beam. If a beam has an end which

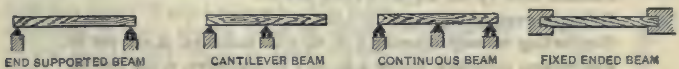


Fig. 7.

projects beyond its adjoining point of support, it is called a *cantilever beam,* the over-hanging end forming the cantilever portion. A beam supported at more than two points of support is a *continuous beam.* A beam having one or both ends rigidly fixed either by being built into a wall, or by being fastened to another member is called a *fixed ended beam.* Fig. 7 represents all of these types.

Beam Formulas for Ordinary Cases. — As a preliminary step in the design of beams, it is necessary to determine the maximum value of the external bending moments, shears and reactions on the beam by the methods given in the sections above which deal with these subjects inclusive. With these items determined, the following formulas of mechanics (*see Mechanics of Materials, Merriman, 10th Edition, Arts. 41 and 108*) may be employed for the design of beams of *symmetrical section* and *homogeneous material* loaded with *transverse loads* applied in the *plane* of one of the *principal axes.* For the treatment of beams of unhomogeneous material (reinforced concrete) see article on *Concrete.*

Let M = maximum external bending moment on beam in inch-pounds,
I = moment of inertia in $(\text{inches})^4$ about the neutral axis lying perpendicular to the plane of the external loads. (This axis in a symmetrical beam lies at mid-height.)
c = distance in inches from neutral axis to extreme fiber, equals one-half the depth for symmetrical beams,
s = allowable value of fiber stress (working value),
Q = statical moment about the neutral axis of that portion of the cross-section lying above the neutral axis,
v = allowable intensity of longitudinal shear in lb. per sq. inch,
V = maximum external shear on beam in pounds,
b = thickness of beam at neutral axis.

Then,
$$M = \frac{sI}{c} \qquad \frac{I}{c} = \frac{M}{s},$$
and
$$v = \frac{VQ}{bI} \qquad \frac{Q}{bI} = \frac{v}{V},$$

Steel Beams. — Such beams are made in the shape of the letter I in order to obtain maximum strength for a given amount of material. They are rolled to desired shape and size while hot. For dimensions, allowable loads and properties of the standard sizes made in the United States see Table V above. In addition to the I-beams listed in the table of standard I-beams which are manufactured by all the leading structural steel makers, the Bethlehem Steel Co. manufacture certain special beams of the same general shape but either of greater depth or with wider flange, and the Carnegie Steel Co. also manufacture certain special types of beams. These beams may often be used to great advantage.

Cost of Steel Beams (Pre-war prices). — The cost of steel beams in place depends upon the cost of transportation, punching, riveting and other necessary shop work and the cost of erection and painting. The base price at the mills is quoted each week in the *Iron Age;* the figures which follow came from the issue of June 19, 1913, and are representative of average prices prior to the World War.

I-beams under 15 in., 1.45¢ to 1.50¢ per lb.
I-beams over 15 in., 1.55¢ to 1.60¢ per lb.
H-beams over 18 in., 1.55¢ to 1.60¢ per lb.
For cutting to length, under 3 ft. to 2 ft. inclusive, 0.25¢ per lb.
For cutting to length, under 2 ft. to 1 ft. inclusive, 0.50¢ per lb.
For cutting to length, under 1 ft., 1.55¢ per lb.
No charge for cutting to lengths 3 ft. and over.

For total cost in place an estimate of 3 cents per pound is ordinarily safe.

PLATE GIRDERS; FORMULAS AND COSTS. — A typical plate girder with the various parts is shown in Fig. 8. Plate girders are used for openings

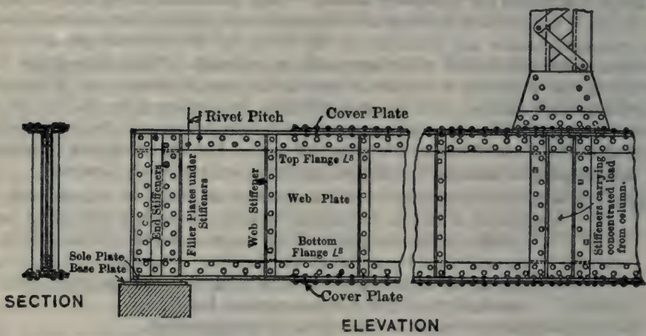


Fig. 8. Typical Plate Girder

where either the load to be carried is too heavy or the clear span too great to be spanned with safety by rolled beams. The maximum length and depth are limited by the possibility of transportation. Probably the largest girder yet made was designed for the Boston & Albany Railroad for use at Worcester, Mass. It is 122 feet 6 inches long, 10 feet 11½ inches deep, and weighs 170 tons. The ordinary and economical depth of plate girders is from ⅙ to 1/12 the span; ⅛ the span is a common value. The maximum depth is usually restricted to 10 feet 6 inches or less.

Essential Points in Plate-Girder Design are as follows:

a. The determination of area of web required to carry the maximum external shear.

b. The determination of area of flange required to resist the maximum external bending moment.

c. The determination of the flange rivet spacing necessary to transmit the flange stress from web to flange.

d. The determination of spacing and size of stiffeners required to prevent the web plate from buckling sidewise, and of size of stiffeners at all points of application of concentrated loads together with number of rivets required therein.

e. The determination of length of flange cover plates.

f. The design of splices for web, angles and cover plates.

g. The design of bearing plates to transmit end reactions into masonry.

It is impossible to give in the limited space available here the necessary formulas and data for the design of plate girders. For further information see Spofford's *Theory of Structures*, N. Y., 1915.

Design of Box Girders. — Box girders resemble plate girders, but have two or more webs as indicated in Fig. 9. They are used where conditions require a shallow girder of great strength. The rules applicable to plate girders apply in general to box girders, but owing to their shallow depth it is usually advisable to determine their strength against bending by the general beam formula $M = \frac{sI}{c}$, the moment of inertia being computed with due allowance for rivet holes (*see Spofford's Theory of Structures, N. Y., 1915, Arts. 62 and 63*). The shear in a two-web girder may be considered as equally divided between the two webs, and in a three-web girder as carried half by the center web and half by the two side webs combined. The flange rivets, stiffeners, etc., should be computed on the same basis.

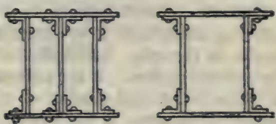


Fig. 9.

Cost of Girders (Pre-war prices). — The cost of girders is usually computed on a cent-per-pound basis. Price of girders erected in place and painted usually ranges in the Eastern states from 2½ to 4 cents per pound, depending upon the state of the market, location, amount and complexity of the work, rigidity of specifications and inspection, cost of maintaining traffic (in railroad work), etc. The price of 3½ cents per pound may be considered a fair average allowance for approximate estimates. In the West this figure should be materially increased to cover the additional freight rate from Pittsburgh or Chicago.

For basic prices of material see below under *Price of Structural Material*. For freight rates to various points from Pittsburgh, see below under *Freight Rates*.

COLUMNS; FORMULAS AND COSTS. — Steel columns may be made of single rolled sections such as I-beams or channels, or of compound sections consisting of single sections riveted together. A typical steel column is shown in Fig. 10. Single sections of steel columns should be limited in length for convenience in erection and economy in shipment to 60 feet over all or even less. Lengths up to 120 feet may, however, be shipped in one piece if necessary.

Cast-iron columns are made by pouring liquid iron into molds. They may be of various shapes, but are always of comparatively short lengths seldom exceeding 15 or 20 feet. Common shapes are shown in Fig. 11. To obtain freedom from initial stresses due to unequal rates of cooling and from stresses due to

eccentricity with centrally applied loads, the shells of hollow cast-iron columns should be of uniform thickness throughout. This result may best be secured by pouring the column in an upright position; if cast in a horizontal position the core must be carefully restrained against flotation.

Timber columns should consist of single sticks and may be either circular or rectangular in cross-section. Two lengths are sometimes connected by means of cast-iron caps which also furnish seats for beams.

Concrete columns are made by pouring concrete into wooden or metal molds; they may be made of any desired shape or length. Such columns are generally reinforced by longitudinal steel rods extending from end to end of the column and held in place at intervals by transverse steel rings. The transverse reinforcement may be made to add materially to the strength of the column by placing the rings at frequent intervals or by making it in the form of a spiral extending from end to end of the column, the column then being called a hooped column. The strengthening effect of the transverse reinforcement is due to its influence in preventing the bursting tendency of the concrete when subjected to compression.

Fig. 10.

Splices. — Steel columns are commonly spliced by bringing the abutting ends into close contact after first planing them at right angles to the axis. Splice plates are used on webs and flanges to hold the sections in line. If the column carries no bending moment the size of splice plates and number of rivets may be determined by the designer's judgment; if the column carries flexure as well as direct stress the splice must be sufficient to transmit the flexure.

Changes of cross-sections in steel columns may be accomplished by changing the thickness of the material or by adding additional material. An advantageous location of a splice is at a section where the cross-section area is to be changed. If the column carries only direct compression and is loaded at intervals throughout its length such a section should be near the point of application of one of the loads and on that side of it in which the smaller stress occurs, e.g., in a high building column the section should be changed just *above* one of the floors. For columns carrying flexure as well as direct stress the splices should be located where the bending moment is small, as for example at a point of contraflexure. Cast-iron columns should


Fig. 11. Cross Sections of Typical Cast-iron Columns

not be used for positions where a considerable amount of flexure may occur or where they may be subjected to severe shock or vibration. Their use in construction is practically confined to buildings of moderate height. They may be spliced by bolting the flanges. In interior building columns the load is sometimes transferred from one section to another by means of a pintle extending between the beams which are supported on the cap of the lower column. Such a joint depends for its lateral rigidity upon the friction between the beams and column cap and can evidently transmit no bending moment.

Ratio of Unsupported Length of Column to Radius of Gyration. — The controlling factor in column formulas is the value of the ratio between the unsupported length of the column and the radius of gyration of its cross-section. If the column is of constant cross-section and restrained against lateral deflection at

both ends only, then l equals the total length of the column and r its least radius of gyration. If the column is held against lateral deflection in every direction at one or several intermediate points, the value of $\frac{l}{r}$ equals the largest value existing between any two adjoining lateral supports. If the column is held at an intermediate point in one direction only, the value of $\frac{l}{r}$ equals the maximum obtainable by using for l the length of either section, or the total length of the column, and for r in each case the corresponding least radius of gyration referred to any axis about which the column is free to bend. If the column is of variable cross-section the designer must use his judgment in determining the value of r to be used, but generally the value should be that at the middle of the unsupported length of the particular portion of the column that is under consideration.

Condition of Ends.—It was formerly assumed that the strength of columns used in practice depended very largely upon the condition of the ends. At the present time, however, no difference is made between round-ended, pin-ended and square-ended columns as used in ordinary practice. For columns of length such that Euler's formula should be applied, a distinction should be made between round-ended and square-ended columns (*see following section*). Columns with one free end should receive special consideration.

Recommended Formulas for Steel Columns.—

For Columns of Ordinary Length:

$$\frac{P}{A} = 16{,}000 - 70\frac{l}{r}, \tag{1}$$

l/r not to exceed 120 for principal members,
l/r not to exceed 150 for secondary members,
P/A not to exceed 14,000 lbs.

For meaning of symbols, see list following equation (4).

For Long Columns having values of l/r greater than allowed in formula (1), the column may fail by collapsing rather than crushing; in this case the following formulas should be used:

$$\text{Fixed Ends} \quad \frac{P}{A} = \frac{4\pi^2E}{c}\left(\frac{r}{l}\right)^2, \tag{2}$$

$$\text{Round Ends} \quad \frac{P}{A} = \frac{\pi^2E}{c}\left(\frac{r}{l}\right)^2, \tag{3}$$

$$\text{One Free End} \quad \frac{P}{A} = \frac{\pi^2E}{4c}\left(\frac{r}{l}\right)^2, \tag{4}$$

where P = total allowable centrally applied load on column in pounds including proper allowance for impact,
A = minimum area of cross-section in square inches,
$\frac{l}{r}$ = maximum ratio of length to radius of gyration for any laterally unsupported section of column,
c = suitable factor of safety, usually 5 to 6,
E = modulus of elasticity in pounds per square inch.

Recommended Formulas for Cast-iron Columns. —

$$\frac{P}{A} = 6100 - 32\frac{l}{d}, \quad \frac{l}{d} \text{ not to exceed } 40, \tag{5}$$

where P = total allowable centrally applied load on column in pounds including proper allowance for impact,
A = minimum area of cross-section in square inches,
d = diameter of circular column or shorter side of rectangular column in inches.

Recommended Formulas for Timber Columns. —

For longleaf yellow pine, $$\frac{P}{A} = 1300\left(1 - \frac{l}{60\,d}\right), \tag{6}$$

For shortleaf pine and spruce, $$\frac{P}{A} = 1100\left(1 - \frac{l}{60\,d}\right), \tag{6a}$$

where P = total allowable centrally applied load on column in pounds (disregarding impact),
A = area of cross-section in square inches,
$\frac{l}{d}$ = unsupported length divided by least radius of gyration.

The values in (6) and (6a) are for railroad bridges. For highway bridges increase these values by 25 per cent. For buildings protected from the weather and reasonably free from impact, increase these values by 50 per cent. For other timbers, multiply value for longleaf yellow pine by ratio between working compressive stress of timber in question and of longleaf yellow pine. (*See also section on Unit Stresses in article on Timber.*)

Steel Column Details. — Columns composed of separate parts should be so constructed that the various parts will resist buckling as a unit; otherwise, the strength of the column will be no greater than that of the individual pieces combined. As an illustration of this, the column shown in Fig. 10 may be considered. If the two channels are not connected, the strength of this column will be only double that of the two separate channels, and the radius of gyration will have the low value corresponding to a single channel. Similarly a wooden column composed of thin planks bolted loosely together has little if any greater strength than the same number of separate planks standing alone. To fasten together the ribs of a steel column, plates or diaphragms may be used throughout its length, the plates forming a part of the main or stress bearing cross-section, or plates and lattice bars may be used as shown in Fig. 10. The width of the lattice bars is usually taken as the minimum value allowable for the rivets used. For columns consisting of two channels, the minimum sizes of lattice bars given in the accompanying table may be used. In no case should single lattice bars have a thickness less than 1/40 the distance between rivets connecting them to the channels; but for double lattice bars riveted at their intersection the thickness may be 1/60 of the distance.

Size of channel, in.	Lattice bar section, in.
8 and 9	2¼ by 5/16
10	2¼ by 5/16
12	2¼ by 5/16
15	2½ by 5/16

For columns of unusual size, the lattice bars should be carefully designed. (*See Spofford's Theory of Structures, N. Y., 1915, Art. 145.*)

Rivet Pitch in Built Steel Columns. — For columns carrying direct stress only, the rivets should not be farther apart than 6 inches or 16 times the thinnest plate connected and at each end or at each point of application of concentrated loads they should not be farther apart than four diameters of the

rivet. See also sections below on *Design of Tension Members* and *Design of Riveted Joints.*

Eccentrically Loaded Columns. — If the resultant force acting at any cross-section of a column is not applied at its center of gravity the column is said to be eccentrically loaded. The effect of an eccentric load is to subject the column to a combination of direct stress and flexure, and thereby to increase the maximum fiber stress over what it would otherwise be. A similar condition arises if the resultant force on the cross-section is the resultant of an axial force acting at the center of gravity of the section and a couple causing flexure at the section. The maximum fiber stress at such a section may be determined by the following closely approximate formula for the usual case where the resultant force acts in the plane of one of the principal axes and the neutral axis for flexure coincides with the other principal axis.

$$s = \frac{P}{A} + \frac{My}{I - \frac{Pl^2}{10E}}, \qquad (7)$$

where s = maximum fiber stress at any section in pounds per square inch,
P = resultant axial force on section in pounds,
l = unsupported length of column in inches,
A = area of cross-section in square inches,
M = bending moment on section in inch-pounds,
I = moment of inertia of cross-section about neutral axis in (inches)^4,
E = modulus of elasticity in pounds = 29,000,000 for steel,
y = distance in inches from neutral axis to most remote fiber. (Neutral axis as used in this formula is the neutral axis for flexure.)

Flexure in a Building Column Carrying an Eccentric Load. — The case of a building column supporting a track for a traveling crane is a common one. The curve of moments for such a case is shown by the dashed line in Fig. 12 in which P is the applied load acting on the track at a distance x from the center of the column. The maximum bending moment always occurs at the load and depends upon the height of application of the latter. Its maximum value is Px and occurs when the load is at the top of the column.

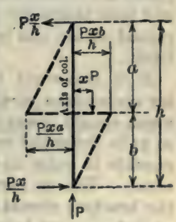


Fig. 12.

Proportions of Columns. —The following specifications should be closely followed:

In compression members the metal shall be concentrated as much as possible in webs and flanges. The thickness of each web shall be not less than one-thirtieth of the distance between its connections to the flanges. Cover plates shall have a thickness not less than one-fortieth of the distance between rivet lines.

The open sides of compression members shall be provided with lattice bars and shall have tie plates as near each end as practicable. Tie plates shall be provided at intermediate points where the lattice is interrupted. In main members the end tie plates shall have a length not less than the distance between the lines of rivets connecting them to the flanges, and intermediate ones not less than one-half this distance. Their thickness shall not be less than one-fiftieth of the same distance.

Abutting joints in compression members when faced for bearing shall be spliced on four sides sufficiently to hold the connecting members accurately in place. All other joints in riveted work, whether in tension or compression, shall be fully spliced.

Where splice plates are not in direct contact with the parts which they con-

nect, rivets shall be used on each side of the joint in excess of the number theoretically required to the extent of one-third of the number for each intervening plate.

Column Caps, Bases and Brackets. — The proper application of the applied loads to the column, and the distribution of the column load over the footing, is a matter of great importance. The three following rules should be applied in designing column caps:

a. Provide sufficient bearing area so that the crushing strength of the column, or of the loading beam, girder or column will not be exceeded.

b. Arrange connections to eliminate eccentric application of the load, or if this is impossible to reduce the eccentricity to the smallest possible amount.

c. Arrange connections so that the stress-bearing portions of the column shall be directly under the stress-transmitting portion of the loading member, e.g., if a column receives load from a girder, the outstanding legs of the end-stiffeners on the girder should be located directly over the webs of the columns.

Typical column bases are shown in Fig. 13. The same general rules should be observed in the design of column bases as in the design of column caps. The advantage of a separate base is that it can be set up on the foundation and leveled to receive the column with less difficulty than would be the case with the finished column. Such bases should always be planed on top and the base of the column should also be planed. Steel columns to which cap and base plates are shop riveted should be planed at each end before these plates are riveted on, and the cap and base plates are also generally planed on top and bottom respectively.

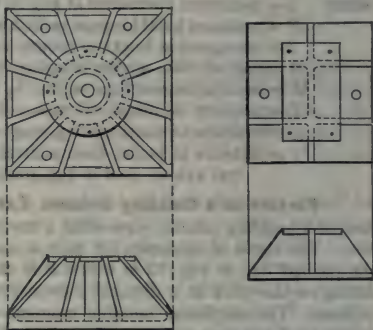


Fig. 13. Typical Column Bases

It is important that brackets for connections of girders to columns should always be designed so as to throw the resultant beam reaction as near the center of the column as possible in order to reduce the eccentricity of loading which would otherwise occur. This is particularly important for cast-iron columns, and for these the brackets should be slightly beveled downward so that the loading beam or girder cannot be supported at or near the edge of the bracket.

I-Beam Column Footings. — On poor soils it is often necessary to distribute heavy column loads over an area so large as to require a special footing which may be economically constructed of steel beams, in concrete. The beams should not be closer together than 3 inches to permit concrete to be easily placed between and around them. The method of design of an I-beam footing for the ordinary case, that of a column carrying direct load only, is as follows: (1) Determine the number of square feet required in the footing; (2) assume the number of tiers of I-beams; (3) assume the entire load above any given layer of beams (weight of over-lying footing courses and column load) to be distributed equally over all the beams in that layer and assume that each beam is uniformly loaded; (4) determine the maximum moment on the beam, which will occur at the center and equal approximately

$$M = \frac{W}{8n}(l - h) \tag{8}$$

where W = load on layer of beams under consideration, n = number of beams, l_1 = portion of beam loaded from above, in feet, and l = length of beam loaded from below, in feet; (5) determine in the usual manner the size of beam required to carry this bending moment; (6) test the web thicknesses of the beams selected to see if they are sufficient to transmit the applied load without buckling, applying formula (1).

Pile Footings. — If the ground is soft, piles may be necessary to provide sufficient bearing area. For the allowable bearing value for these see Baker's *A Treatise on Masonry Construction*, N. Y., 1909.

Cost of Columns and Footings. — Steel columns cost about the same as steel girders, values for which are given in the section on *Cost of Girders*, above. In determining the weight of columns it should be remembered that the details will add materially to the weight of the stress-bearing section, especially in the case of latticed columns, where the excess may amount to as much as 50 per cent. For the cost of footings, the I-beams may be computed on a pound price basis from the weekly quotations given in *The Iron Age* with a reasonable allowance for transportation and delivery, or more accurate figures can be obtained from a local dealer. For cost of cast-iron columns see section on *Cost* in article on *Iron, Pig and Cast*. For the cost of concrete and excavation, see articles on these respective subjects.

Tables for Steel Columns. — The allowable load for a number of simple steel columns and the important properties of the columns are given in Tables VII to X. To determine the allowable load for built-up columns not given in the table the radius of gyration may be interpolated for columns having the same general dimensions and the unit stress computed by substitution in the column formula. The areas and other properties of the simple structural shapes, such as beams, angle irons, and channels, are given in Tables II to VI above.

These tables are all based on the formula

$$\frac{P}{A} = 16{,}000 - 70\frac{l}{p}$$

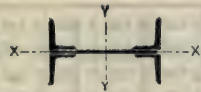
with a maximum value of 14,000 and are calculated for Cambria Steel Co.'s sections. The load is assumed to be applied at the center of gravity of the column and to act parallel to its axis.

TABLE VII. — STEEL I-BEAM COLUMNS

ρ_x = radius of gyration about axis X-X
ρ_y = " " " " " Y-Y

Depth of beam, in.	Width of flange, in.	Weight per foot, lb.	Thickness of web, in.	Area of column, sq. in.	Least radius of gyration ρ_x, in.	ρ_y, in.	Max. allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_x.											
							$\frac{l}{\rho} = 10$	20	30	40	50	60	70	80	90	100	110	120
5	3.00	9.75	0.21	2.87	0.65	2.05	40.2	40.2	39.9	37.9	35.9	33.9	31.9	29.8	27.8	25.8	23.8	21.8
6	3.33	12.25	0.23	3.61	0.72	2.46	50.5	50.5	50.2	47.7	45.1	42.6	40.1	37.5	35.0	32.5	30.0	27.4
7	3.66	15.00	0.25	4.42	0.78	2.86	61.9	61.9	61.4	58.3	55.3	52.2	49.1	46.0	42.9	39.8	36.7	33.6
8	4.00	18.00	0.27	5.33	0.84	3.27	74.6	74.6	74.1	70.4	66.6	62.9	59.2	55.4	51.7	48.0	44.2	40.5
9	4.33	21.00	0.29	6.31	0.90	3.67	88.3	88.3	87.7	83.3	78.9	74.5	70.0	65.6	61.2	56.8	52.4	48.0
10	4.66	25.00	0.31	7.37	0.97	4.07	103.2	103.2	102.4	97.3	92.1	87.0	81.8	76.6	71.5	66.3	61.2	56.0
10	5.10	40.00	0.75	11.76	0.90	3.67	164.6	164.6	163.5	155.2	147.0	138.8	130.5	122.3	114.1	105.8	97.6	89.4
12	5.00	31.50	0.35	9.26	1.01	4.83	129.6	129.6	128.7	122.2	115.8	109.3	102.8	96.3	89.8	83.3	76.9	70.4
12	5.21	40.00	0.56	11.76	0.96	4.57	164.6	164.6	163.5	155.2	147.0	138.8	130.5	122.3	114.1	105.8	97.6	89.4
15	5.50	42.00	0.41	12.48	1.08	5.95	174.7	174.7	173.5	164.7	156.0	147.3	138.5	129.8	121.1	112.3	103.6	94.8
15	5.84	60.00	0.75	17.65	1.01	5.52	247.1	247.1	245.3	233.0	220.6	208.3	195.9	183.6	171.2	158.8	146.5	134.1
18	6.00	55.00	0.46	15.93	1.15	7.07	223.0	223.0	221.4	210.3	199.1	188.0	176.8	165.7	154.5	143.4	132.2	121.1
18	6.26	70.00	0.72	20.59	1.09	6.69	288.3	288.3	286.2	271.8	257.4	243.0	228.5	214.1	199.7	185.3	170.9	156.5
20	6.25	65.00	0.50	19.08	1.21	7.83	267.1	267.1	265.2	251.9	238.5	225.1	211.8	198.4	185.1	171.7	158.4	145.0
20	6.40	75.00	0.65	22.06	1.17	7.58	308.8	308.8	306.6	291.2	275.8	260.3	244.9	229.4	214.0	198.5	183.1	167.7
24	7.00	80.00	0.50	23.32	1.36	9.46	326.5	326.5	324.2	307.8	291.5	275.2	258.9	242.5	226.2	209.9	193.6	177.2
24	7.25	100.00	0.75	29.41	1.28	8.99	411.7	411.7	408.8	388.2	367.6	347.0	326.5	305.9	285.3	264.7	244.1	223.5

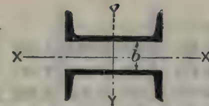
TABLE VIII. — STEEL PLATE AND ANGLE COLUMNS



ρ_x = radius of gyration about axis X-X
ρ_y = " " " " Y-Y

Depth base to base of angles, in.	Size of web, in.	Size of angle, in.	Width of flange, in.	Area of column, sq. in.	Weight per foot of column exclusive of details, lb.	Least radius of gyration ρ_x, in.	ρ_y, in.	Max. allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_x.											
								$\frac{l}{\rho}$ = 10	20	30	40	50	60	70	80	90	100	110	120
6½	6 by ¼	3 by 2½ by ¼	6¼	6.79	23.1	1.24	2.41	95.1	95.1	94.4	89.6	84.9	80.1	75.4	70.6	65.9	61.1	56.4	51.6
6½	6 by ⅝	3 by 2½ by ⅝	6⅝	15.94	54.4	1.43	2.29	223.2	223.2	221.6	210.4	199.3	188.1	176.9	165.8	154.6	143.5	132.3	121.1
7½	7 by ¼	3½ by 2½ by ¼	7¼	7.50	25.6	1.46	2.88	105.0	105.0	104.3	99.0	93.8	88.5	83.3	78.0	72.8	67.5	62.3	57.0
7½	7 by ⅝	3½ by 2½ by ⅝	7⅝	17.82	60.9	1.65	2.76	249.5	249.5	247.7	235.2	222.8	210.3	197.8	185.3	172.8	160.4	147.9	135.4
8½	8 by ¼	3 by 2½ by ¼	6¼	7.29	24.8	1.19	3.25	102.1	102.1	101.3	96.2	91.1	86.0	80.9	75.8	70.7	65.6	60.5	55.4
8½	8 by ⅝	4 by 3 by ⅝	8⅝	20.96	71.4	1.82	3.14	293.4	293.4	291.3	276.7	262.0	247.4	232.7	218.0	203.3	188.6	173.9	159.2
10½	10 by ¼	3 by 2½ by ¼	6¼	7.79	26.5	1.16	4.07	109.1	109.1	108.3	102.8	97.4	91.9	86.5	81.0	75.6	70.1	64.7	59.2
10½	10 by ⅝	4 by 3 by ⅝	8⅝	22.21	75.7	1.77	3.98	310.9	310.9	308.7	293.2	277.6	262.1	246.6	231.0	215.4	199.9	184.3	168.8
12½	12 by ¼	3 by 2½ by ¼	6¼	8.29	28.2	1.12	4.87	116.1	116.1	115.2	109.4	103.6	97.8	92.0	86.2	80.4	74.6	68.8	63.0
12½	12 by ⅝	6 by 3½ by ⅝	12⅝	29.69	101.1	2.68	4.93	415.7	415.7	412.7	391.9	371.1	350.4	329.6	308.8	288.0	267.2	246.4	225.6
14½	14 by 7/16	7 by 3½ by 7/16	14 7/16	23.76	80.8	3.05	5.92	332.6	332.6	330.2	313.6	297.0	280.4	263.8	247.1	230.5	213.8	197.2	180.6
14½	14 by ⅝	7 by 3½ by ⅝	14⅝	33.48	113.7	3.13	5.85	468.7	468.7	465.4	442.0	418.5	395.1	371.6	348.2	324.7	301.3	277.9	254.4
16½	16 by 5/16	5 by 3½ by 5/16	10 5/16	15.23	51.8	1.94	6.59	213.2	213.2	211.7	201.0	190.4	179.7	169.1	158.4	147.7	137.1	126.4	115.7
16½	16 by ⅝	5 by 3½ by ⅝	10⅝	29.73	101.2	2.08	6.48	416.2	416.2	413.3	392.5	371.6	350.8	330.0	309.2	288.4	267.6	246.8	225.9
16½	16 by 7/16	7 by 3½ by 7/16	14 7/16	24.65	83.8	3.00	6.75	345.1	345.1	342.7	325.4	308.1	290.9	273.6	256.4	239.1	221.8	204.6	187.3
16½	16 by ⅝	7 by 3½ by ⅝	14⅝	34.72	118.0	3.08	6.69	486.1	486.1	482.6	458.3	434.0	409.7	385.4	361.1	336.8	312.5	288.2	263.9

TABLE IX.—STEEL LATTICED CHANNEL COLUMNS



ρ_y = radius of gyration about axis Y-Y
b = minimum allowable distance apart of channels

Depth of channel, in.	Weight per foot of each channel, lb.	Width of flange, in.	Thickness of web, in.	Area of section, sq. in.	Weight per foot exclusive of details, lb.	Radius of gyration ρ_y, in.	b in.	Max. allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_y.											
								$\frac{l}{\rho}=10$	20	30	40	50	60	70	80	90	100	110	120
6	8.0	1.92	0.20	4.76	16.00	2.34	3.52	66.6	66.6	66.2	62.8	59.5	56.2	52.8	49.5	46.2	42.8	39.5	36.2
6	15.5	2.28	0.56	9.12	31.00	2.07	2.90	127.7	127.7	126.8	120.4	114.0	107.6	101.2	94.8	88.5	82.1	75.7	69.3
7	9.75	2.09	0.21	5.70	19.50	2.72	4.22	79.8	79.8	79.2	75.2	71.3	67.3	63.3	59.3	55.3	51.3	47.3	43.3
7	19.75	2.51	0.63	11.62	39.50	2.39	3.48	162.7	162.7	161.5	153.4	145.3	137.1	129.0	120.8	112.7	104.6	96.4	88.3
8	11.25	2.26	0.22	6.70	22.50	3.10	4.90	93.8	93.8	93.1	88.4	83.8	79.1	74.4	69.7	65.0	60.3	55.6	50.9
8	21.25	2.62	0.58	12.50	42.50	2.76	4.20	175.0	175.0	173.8	165.0	156.3	147.5	138.8	130.0	121.3	112.5	103.8	95.0
9	13.25	2.43	0.23	7.78	26.50	3.49	5.64	108.9	108.9	108.1	102.7	97.3	91.8	86.4	80.9	75.5	70.0	64.6	59.1
9	25.0	2.81	0.61	14.70	50.00	3.10	4.82	205.8	205.8	204.3	194.0	183.8	173.5	163.2	152.9	142.6	132.3	122.0	111.7
10	15.0	2.60	0.24	8.92	30.00	3.87	6.32	124.9	124.9	124.0	117.7	111.5	105.3	99.0	92.8	86.5	80.3	74.0	67.8
10	25.0	2.89	0.53	14.70	50.00	3.52	5.66	205.8	205.8	204.3	194.0	183.8	173.5	163.2	152.9	142.6	132.3	122.0	111.7
10	35.0	3.18	0.82	20.58	70.00	3.35	5.18	288.1	288.1	286.1	271.7	257.3	242.8	228.4	214.0	199.6	185.2	170.8	156.4
12	20.5	2.94	0.28	12.06	41.00	4.61	7.68	168.8	168.8	167.6	159.2	150.8	142.3	133.9	125.4	117.0	108.5	100.1	91.7
12	30.0	3.17	0.51	17.64	60.00	4.28	7.06	247.0	247.0	245.2	232.9	220.5	208.2	195.8	183.5	171.1	158.8	146.4	134.1
12	40.0	3.42	0.76	23.52	80.00	4.09	6.60	329.3	329.3	326.9	310.5	294.0	277.5	261.1	244.6	228.1	211.7	195.2	178.8
15	33.0	3.40	0.40	19.80	66.00	5.62	9.52	277.2	277.2	275.2	261.4	247.5	233.6	219.8	205.9	192.1	178.2	164.3	150.5
15	40.0	3.52	0.52	23.52	80.00	5.44	9.16	329.3	329.3	326.9	310.5	294.0	277.5	261.1	244.6	228.1	211.7	195.2	178.8
15	55.0	3.82	0.82	32.36	110.00	5.16	8.54	453.1	453.1	449.8	427.2	404.5	381.9	359.2	336.5	313.9	291.2	268.6	245.9

TABLE X.—STEEL H-COLUMNS

ρ_x = radius of gyration about axis X-X
ρ_y = " " " " Y-Y

Depth of column, in.	Width of flange, in.	Weight per foot, lb.	Thickness of web, in.	Area of section sq. in.	Least radius of gyration, ρ_x in.	ρ_y, in.	Maximum allowable load P in thousands of pounds for various values of $l \div \rho$. To determine strength of column use maximum possible value of $l \div \rho$. If column is unsupported laterally use for ρ the value ρ_x. $\frac{l}{\rho}$ = 10	20	30	40	50	60	70	80	90	100	110	120
8	8.00	34.5	0.31	10.17	2.01	3.46	142.4	142.4	141.4	134.2	127.1	120.0	112.9	105.8	98.6	91.5	84.4	77.3
8½	8.16	53.0	0.47	15.53	2.07	3.57	217.4	217.4	215.9	205.0	194.1	183.3	172.4	161.5	150.6	139.8	128.9	118.0
9	8.32	71.5	0.63	21.05	2.12	3.68	294.7	294.7	292.6	277.9	263.1	248.4	233.7	218.9	204.2	189.4	174.7	160.0
9½	8.47	90.5	0.78	26.64	2.17	3.80	373.0	373.0	370.3	351.7	333.0	314.4	295.7	277.1	258.4	239.8	221.1	202.5
10	10.00	54.0	0.39	15.91	2.51	4.32	222.7	222.7	221.1	210.0	198.9	187.7	176.6	165.5	154.3	143.2	132.1	120.9
10½	10.16	77.0	0.55	22.59	2.57	4.43	316.3	316.3	314.0	298.2	282.4	266.6	250.7	234.9	219.1	203.3	187.5	171.7
11	10.31	99.5	0.70	29.32	2.62	4.55	410.5	410.5	407.6	387.0	366.5	346.0	325.5	304.9	284.4	263.9	243.4	222.8
11½	10.47	123.5	0.86	36.32	2.67	4.67	508.5	508.5	504.8	479.4	454.0	428.6	403.2	377.7	352.3	326.9	301.4	276.0
12	12.00	78.0	0.47	22.94	3.01	5.18	321.2	321.2	318.9	302.8	286.8	270.7	254.6	238.6	222.5	206.5	190.4	174.3
12½	12.16	105.0	0.63	30.94	3.07	5.30	433.2	433.2	430.1	408.4	386.8	365.1	343.4	321.8	300.1	278.4	256.8	235.1
13	12.31	132.5	0.78	38.97	3.13	5.41	545.6	545.6	541.7	514.4	487.1	459.9	432.6	405.3	378.0	350.7	323.5	296.2
13½	12.47	161.0	0.94	47.28	3.18	5.53	661.9	661.9	657.2	624.1	591.0	557.9	524.8	491.7	458.6	425.5	392.4	359.3
14	14.00	99.0	0.51	29.06	3.50	6.07	406.8	406.8	403.9	383.6	363.3	342.9	322.6	302.2	281.9	261.5	241.2	220.9
14½	14.16	130.5	0.67	38.38	3.56	6.18	537.3	537.3	533.5	506.6	479.8	452.9	426.0	399.2	372.3	345.4	318.6	291.7
15	14.31	162.0	0.82	47.71	3.62	6.30	667.9	667.9	663.2	629.8	596.4	563.0	529.6	496.2	462.8	429.4	396.0	362.6
15½	14.47	195.0	0.98	57.35	3.67	6.41	802.9	802.9	797.2	757.0	716.9	676.7	636.6	596.4	556.3	516.2	476.0	435.9
16	14.62	227.5	1.13	66.98	3.73	6.53	937.7	937.7	931.0	884.1	837.2	790.4	743.5	696.6	649.7	602.8	555.9	509.0
16½	14.78	261.5	1.29	76.93	3.77	6.65	1077.0	1077.0	1069.3	1015.5	961.6	907.8	853.9	800.1	746.2	692.4	638.5	584.7

TRUSSES, DESIGN OF. — All trusses may be divided into two general classes based upon the methods necessary to determine the bar stresses; if these stresses can be determined by the application of the three equations of equilibrium (*see paragraph on Equations of Equilibrium, above*), they are statically determined; otherwise they are statically undetermined. It should be noted that a truss may be statically undetermined with respect to the outer forces, i.e., the reactions cannot be determined by statics; or it may be statically undetermined with respect to the inner forces, i.e., the bar stresses cannot be determined by statics.

Trusses that are statically indeterminate through having superfluous bars or reactions are frequently constructed because of their supposed rigidity and economy, but it is doubtful if such trusses are desirable.

Statically determined trusses may be distinguished from statically undetermined trusses by comparing the number of unknown reaction components plus

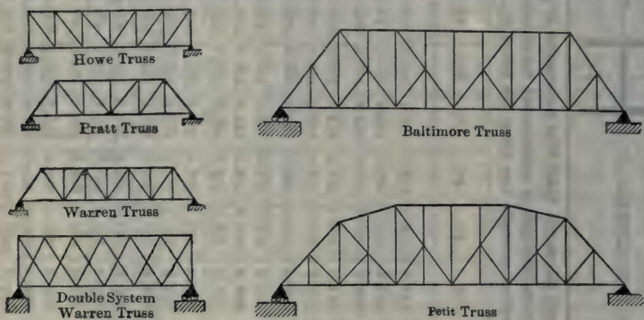


Fig. 14. Bridge Trusses

the number of bars with the number of joints. In general it may be stated that any planar truss supported at two points will be statically determined provided the number of bars equals $(2n-3)$, where n is the number of joints. If the number of unknown bars and reaction components combined exceed twice the number of joints the truss is statically indeterminate but may usually be computed by methods based upon its elasticity. If the number of unknowns is less than twice the number of joints the truss is unstable and may collapse; such structures should never be constructed.

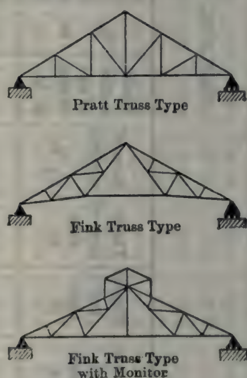


Fig. 15. Roof Trusses

The preceding test should not be used alone to determine whether a truss is or is not stable. The framework should also be so put together as to make it consist of a series of triangles.

Common Types. — All the trusses shown in Figs. 14 and 15, except the double-system Warren truss and the roof truss with monitor, are statically determined; the former may be made statically determined by omitting one of the bars, say a diagonal; the latter by omitting one properly chosen bar.

Theory of Trusses. — The common theory of trusses is based upon the following assumption:

That the members are connected at the intersections of their center of gravity lines by frictionless pins, and that in consequence the stresses in the various members are all direct stresses. The assumption of frictionless pins is approximate particularly in the case of riveted trusses; however, the results obtained by this method are sufficiently accurate for trusses of ordinary proportions and are in common use. Great care should be used in design to insure that center of gravity lines meet at the joints, otherwise the connection will be eccentric and the truss exposed to bending as well as direct stress.

The methods of computing stresses in ordinary trusses involve only the correct application of the three equations of statics (*see section on Equations of Equilibrium, above*). Three methods are in common use: viz., method of joints, method of moments, and method of shear. All of these are methods of sections and are applied by passing a section through the bar under consideration, cutting the truss apart and applying the equations of equilibrium to the outer forces on one of these sections and the stress in the bar under consideration. The first two of these methods are described below.

Method of Joints. — The method of joints is the most general of the three methods of computing the stresses in trusses; it may be applied either analytically or graphically.

Analytical Method. — The mode of procedure for this method is as follows: (1) Computation of reactions. (2) Selection of a joint at which only two bars meet. (3) Application of the two equations of equilibrium, $\Sigma H = O$ and $\Sigma V = O$, to the outer forces and unknown bar stresses acting at this point. The bar stresses should be assumed as tension, that is, as acting away from the joint; a negative result in any case would indicate compression. (4) Consideration of any other joint at which only two unknown bars meet and determination of the stresses in these bars in the same manner, treating known bar stresses as outer forces.

Graphical Method. — This method consists of drawing polygons of forces for each joint in succession. It may be employed readily by draftsmen and others who are not familiar with truss theories. It is also self-checking and reasonably rapid. It is less accurate than the analytical method, but if the diagrams are carefully constructed it is accurate enough for ordinary structures.

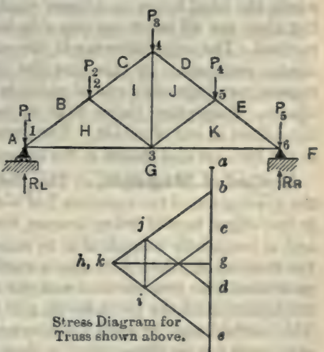


Fig. 16.

The mode of procedure is: (1) A sketch of the structure is drawn to any suitable scale and on it are shown all the outer forces including reactions. (2) All the forces and bars are designated by letters so located that each force and each bar will lie between two letters and only two, as illustrated by Fig. 16. (3) A polygon of outer forces is drawn. This should be drawn to a scale of sufficient size to give the desired accuracy. The forces should be plotted in the order in which they are reached by going around the figure in a clockwise direction, and should be lettered at the ends by the letters in the order obtained by this clockwise rotation. This polygon should close if the reactions have been determined

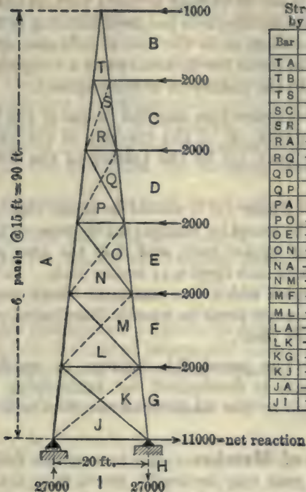
correctly. (4) A triangle of forces for each joint is drawn, beginning at any joint where an outer force and two bars only meet, and proceeding thence, joint by joint, selecting the joints in such an order that at no joint will there be more than two undetermined forces to consider. The sides of these triangles representing the outer forces are the sides of the force polygon. The sides representing bar stresses should be lettered at the ends by the letters obtained by going around the joints in a clockwise direction. The diagram thus drawn should form a closed figure. (5) The magnitude and character of the bar stresses are determined from the diagram. The magnitude of the stress in any member equals the length of the line of the diagram parallel to the bar in question measured to the scale of the force polygon; its character is determined by the order in which the letters are reached in going about any joint in a clockwise direction. For example, to determine the character of the stress in bar *CI* of Fig. 16, note that *ci* in the stress diagram acts downward to the left, as determined by the order in which the letters are reached in going around joint 2; hence the stress in *CI* also acts downward to the left, or toward the joint, since the bar is above the joint, and is therefore compression. A similar result is obtained by considering joint 4. For this joint clockwise reading gives the designation of the bar as *IC*, and *ic* in the stress diagram acts upward to the right, that is toward joint 4, since the bar is below this joint.

Solutions by Graphical Method. — Fig. 16 shows a small truss drawn to scale and with all the outer forces represented in direction and point of application. The force polygon is *abcdefga;* this is a straight line, since all the forces are vertical. In it $ab = P_1$, $bc = P_2$, etc. The reactions R_L and R_R are represented by *ga* and *fg*. The triangle of forces for joint 1 is *gabhg* (*gb* is the resultant of R_L and P_1). Reading around joint 1 in a clockwise direction shows that the magnitude and direction of the stress in bar *BH* is determined by the length and direction of *bh* in the force polygon; as this acts downward to the left, it acts toward joint 1 and hence is compression. In the same manner the stress in *HG* is found to be tension. The stresses in the other bars may be determined similarly.

Fig. 17 shows a simple tower with wind forces acting on one side only. Ordinarily the wind forces on a tower would be equally divided between the two sides but they are all taken on one side here in order to more clearly illustrate the method. The only difference in the result would occur in the stresses in the horizontal members. In this case the diagonals in all panels are assumed to be tension members, only one of which in any one panel is in action under the forces shown. Inspection shows that in each case the bar shown dotted should be considered as out of action. The stress diagram and the scaled values of stresses in the bars are also shown in the figure.

Method of Moments. — This method of finding truss stresses is based upon the application of the equation $\Sigma M = 0$. It is very useful for determining stresses in individual bars of many trusses, but is not so general as the method of joints and is frequently inapplicable to some bars even in the simplest trusses. Like the method of joints, it is also a method of sections, the truss being considered as divided into two portions and the equilibrium of one of these portions under the influence of the outer forces and the stress in the cut bar being considered. The method can be used to determine the stress in a given bar when all the other bars cut by the section, or their prolongations, meet at a point not on the line of action of the given bar. This point should be selected as the origin of moments.

Design of Tension Members. — The design of tension members involves little more than the selection of bars with sufficient net area to carry the total stress without exceeding the allowable unit stress. Steel or iron tension



Stresses by scale

Bar	Stress pounds
T A	− 4500
T B	+ 4500
T S	− 2000
S C	+ 4500
S R	+ 4750
R A	− 9000
R Q	− 3000
Q D	+ 9000
Q P	+ 5000
P A	−13500
P O	− 4000
O E	+13500
O N	+ 5700
N A	−18000
N M	− 5000
M F	+18000
M L	+ 6500
L A	− 22500
L K	− 6200
K G	+22500
K J	+ 7200
J A	−27000
J I	+ 3000

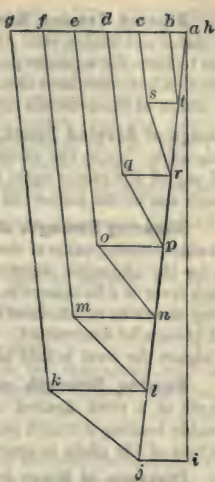


Fig. 17.

members may be divided into two general types: viz., solid bars rectangular or circular in cross-section, and built-up members composed of structural shapes riveted together. Solid bars are used generally in pin trusses for diagonals and bottom chord members, and in Howe trusses for verticals. Built-up members are generally employed for tension members in riveted trusses and for the end hangers in pin-trusses.

Solid Tension Pieces. — The eye bar shown in Fig. 18 is commonly used for the solid bar type of tension members. Such bars are made by most of the large steel manufacturers and are fully described in their handbooks. The heads of these bars are designed so that the bar if tested to destruction will fail in the body rather than in the head, and the engineer should specify that full-sized tests should give this result and not attempt to proportion the heads. A good rule to observe in selecting bars is to keep the thickness between

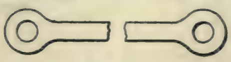


Fig. 18.

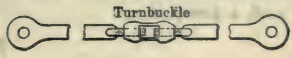


Fig. 19.

one-sixth and one-third the width. Eye-bars are generally used in pairs, since an odd number of bars would give a poor arrangement on the pin. For counters adjustable eye-bars, such as those shown in Fig. 19, may be used, the two parts being connected by a turnbuckle or sleeve nut; iron rods with loops formed by welding may be used if the stresses are small. In proportioning adjustable members allowance must be made for the decrease in section due to the screw threads. It is usually advisable to upset the screw end, so as to give sufficient area at the root of the thread to make the bar as strong there as elsewhere. For short rods, however, the labor cost involved in this process may be greater than the saving of material would warrant.

Riveted Tension Bars may be made of various sections such as channels, plates and angles, etc. Although these members do not need latticing or tie-plates to keep the separate parts from buckling, as in the case of columns, some connection between them should be used to make the different parts act together. The design of these details must be left to the judgment of the engineer.

Design of Riveted Joints.— A riveted connection may fail in one of the following ways: (*a*) by the shearing of the rivets, (*b*) by the crushing of the rivets or of one of the pieces upon which they bear, (*c*) by the tearing of the rivets through one of the connected pieces.

Shearing Strength of Rivet. — Under (*a*) it should be noted that the allowable shearing value of the rivet may be found by multiplying its cross-section area by the allowable shearing stress per square inch, and that the area of a ⅞-inch rivet is 0.60 square inch, and of a ¾-inch rivet 0.44 square inch. In designing rivets to resist shear the plane upon which the maximum shear occurs must always be determined. If the maximum shear be equally distributed over two planes the rivet is said to be in *double shear*.

Bearing Strength of Rivet. — The permissible bearing or crushing strength of a rivet against a given plate is determined by multiplying the allowable bearing strength per square inch by the diameter of the rivet and the thickness of the plate in question.

Distance of Rivet from Edge of Member. — To prevent the failure noted under (*c*) the following empirical rule may be used: rivets may not be spaced closer than three times the diameter, and the distance of a rivet from the edge or end of a piece may not be less than 1¼ inches for a ⅞-inch rivet if the edge in question be rolled or planed, or 1½ inches if it be sheared, though where possible this distance should be at least twice the diameter of the rivet. For other sizes of rivets proportional allowances should be made. These values refer to the center of the rivet in each case.

Riveted Joints Carrying Torsion. — For cases where torsion as well as direct stress has to be transferred by rivets, as is sometimes the case where brackets carrying cranes are riveted to the sides of columns or where girders are riveted to columns, the conditions shown by Figs. 20 and 21 may occur. For the case shown in Fig. 20 the rivets carry torsion only, and each rivet may be assumed to offer a resistance to torsion proportional to its distance from the center of gravity of the group. The resistance to torsion of such a group is

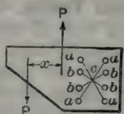


Fig. 20.

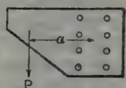


Fig. 21.

$$R = \frac{rI}{d},$$

where r = the allowable working value of the most stressed rivet in pounds,
I = summation of the squares of the distances in inches from the center of gravity of the group of rivets to each rivet,
d = distance in inches from center of gravity of group of rivets to the most stressed rivet,
R = resistance to torsion of the group of rivets in pounds.

In applying the formula to the case shown in Fig. 20 $d = ac$ and the stress in the rivets at a is r. For the case shown in Fig. 21 this method must be modified to allow for the effect of the vertical load. To make this correction it is only necessary to determine the allowable resistance to torsion consistent with the rivet also carrying its share of the vertical load.

CONTINUOUS GIRDERS, BEAMS, SLABS AND TRUSSES. — These are structures which are continuous over several points of support. They are statically indeterminate (*see section on Trusses, above*) with respect to the outer forces and hence the moments and shears should be determined by the "three-moment equation" (*see above*). Continuous girders are commonly used in reinforced-concrete construction, in which case the continuity is allowed for by the following simple rules recommended by the Joint Committee on Concrete and Reinforced Concrete. (*See article on Concrete.*)

(*a*) For floor-slabs, the bending moments at center and at support should be taken at $\frac{wl^2}{12}$ for both dead and live loads, where w represents the load per linear unit and l the span length.

(*b*) For beams, the bending moment at center and at support for interior spans should be taken at $\frac{wl^2}{12}$ and for end spans it should be taken at $\frac{wl^2}{10}$ for center and interior support, for both dead and live loads.

(*c*) In the case of beams and slabs continuous for two spans only, with their ends restrained, the bending moment both at the central support and near the middle of the span should be taken as $\frac{wl^2}{10}$.

(*d*) At the ends of continuous beams, the amount of negative moment which will be developed in the beam will depend on the condition of restraint or fixedness, and this will depend on the form of construction used. In the ordinary cases a moment of $\frac{wl^2}{16}$ may be taken; for small beams running into heavy columns this should be increased, but not to exceed $\frac{wl^2}{12}$.

For spans of unusual length, or for spans of materially unequal length, more exact calculations should be made. Special consideration is also required in the case of concentrated loads.

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SUBSTATIONS, LIGHTING. — (*See also Batteries, Storage, Applications of; Bus-bars and Bus-bar Structures; Circuit Breakers; Converters, Synchronous; Distribution Lines; Distribution Systems; Lighting Plants; Motor-Generators; Regulators; Relays; Substations, Railway; Switchboards; Switchgear Equipment for Power Stations; Transformers.*) Three types of substations are employed in lighting systems viz.:

Substations for Voltage Conversion and Regulation on A-C. Systems. — These substations receive power from high-tension transmission lines or primary feeders designed for economical cross section of conductor rather than for small voltage drop. The standard equipment of such substations includes bus-bars and oil circuit breakers for high-tension lines, step-down transformers, bus-bars, oil switches and voltage regulators for delivery lines, and often in addition constant-current transformers serving local series street-lighting circuits. The construction of bus-bar and switch structures is similar to that in power stations for the control of circuits of equal power. Transformers are often of the air-blast type, and are usually grouped in banks of three single-phase units. The constant-current transformers supplying d-c. arc circuits are associated with mercury-arc rectifiers. The equipment is usually arranged to make the current path through the substation as short and direct as possible. Each outgoing line has its individual equipment of voltage regulators in the best practice.

Substations for the Conversion of Frequency. — Such substations are usually combined with the conversion of voltage. These substations receive power from 3-phase, 25-cycle primary feeders, convert it to 60 cycles and transform its voltage for the supply of local 60-cycle lighting feeders. The type of frequency-changer most used is a pair of synchronous machines of the proper frequency ratio running on a common shaft. When space is restricted the vertical-shaft type is generally used. By proper regulation of the field excitation of the motor element it is possible to control the power factor and terminal voltage of the primary feeders to the most advantageous values.

Substations for the Conversion of Alternating Current to Direct Current. — The alternating current is received from high-tension, 3-phase, primary feeders, and is converted to low-voltage direct current for the supply of local d-c. lighting circuits. Substations of this type are largely used in connection with the 3-wire lighting networks in large cities. The standard equipment comprises bus-bars and oil switches for incoming lines, either (1) synchronous converters associated with lowering transformers and voltage regulating accessories, or (2) motor-generators, and direct-current switchboard equipment. In very many cases the above equipment is supplemented by a large floating storage battery to insure the continuity of service in case of breakdowns. A converter must be associated with step-down transformers while the motor element of the motor-generator may be wound for voltages up to 20,000. The converter equipment is somewhat more efficient and in most cases slightly less expensive. The motor-generator equipment affords the greater range of power-factor compensation.

For methods of voltage control see *Converters, Synchronous*.

In converter substations the step-down transformers are usually in banks of three single-phase units. Transformer banks supply converters individually and are not connected in parallel on the low-tension side. Converters of 300 kw. and higher are usually wound 6-phase. The neutral point of a well balanced 3-wire system may be derived from the neutral of the transformer bank. For storage battery arrangements see *Batteries, Storage, Applications of*.

BIBLIOGRAPHY. — See articles on *Power Stations*.

SUBSTATIONS, RAILWAY. — (*See also Converters, Synchronous; Railways, Energy Requirements for; Switchboards; Switchgear Equipment; Transformers.*) Substations are used for electric railways when the length of the road is so great that the whole road cannot be supplied with one power station at the voltage required by the motors without either an excessive drop in voltage or a prohibitive amount of copper or both. Practically all electric railways have developed to such an extent that substations are required for successful operation.

ECONOMIC CONSIDERATIONS. — The location, capacity and number of substations is a matter for careful study and calculation, involving not only the cost of the copper required for distribution and the cost of the substations, but also the distribution of traffic and special local conditions. The fundamental economics of the subject are expressed by the general theorem that the cost of operating the substations plus the cost of interest and fixed charges on investment in substations and line copper shall be a minimum. This is explained by the fact that if to any given arrangement an additional substation be added with the proper rearrangement of the spacing, the amount of copper used in the distributing system may be considerably decreased on account of the lesser distance between substations. If the saving in interest on the value of the copper is greater than the cost of operation of this new substation plus the interest and fixed charges on the first cost of the substation, the change is warranted.

Allowable Voltage Drop. — The distance between substations depends directly upon the allowable loss in voltage, the amount of copper in the trolley and inversely as the load. The allowable loss of voltage is a matter of the special conditions of each road. Roads may be logically divided into two general classes: City and Interurban. In City roads a drop in voltage of 8 per cent with average load and 15 per cent with maximum load is frequently observed. In Interurban roads a drop of 12 per cent with average load and 30 per cent with maximum load is customary.

The two conditions which determine the allowable drop in voltage are the effect on the lights and the effect on the control circuit of the car, each of which factors requires a lesser drop in voltage than the actual operating characteristics of the motors.

Distance between Substations. — For the service usually found in practice the following tabulation shows the average or customary distance between substations.

Where the length of track receives its power from only one direction the allowable distance for a given drop in voltage is one-quarter the distance allowable between substations.

Capacity Required. — The capacity of the substation depends upon the traffic and the size of the cars, since if there are many cars operating it is probable that only a few will be starting simultaneously and it is during starting that the cars demand the maximum amount of power. It is, therefore, necessary to determine the number of cars which will be located on the section of track supplied by one substation and the frequency of starting. In interurban service the number of cars on a given section is given by the formula:

$$X = \frac{2 \times (\text{length of section}) \times (\text{number of cars per hour})}{\text{schedule speed in miles per hour}}$$

The average load on the substation is equal to the average power demand of each car times the number of cars on the section divided by the efficiency of the distribution system. In a complex system the local conditions must be

Volts	Type	Miles between substations	
		Single-track road	Double-track road
600	Direct-current, Trolley	10	15
600	Direct-current, 3d Rail	13	19
3,300	Single-phase, Trolley	17	23
1,200	Direct-current, Trolley	19	25
1,200	Direct-current, 3d Rail	38	43
6,600	Single-phase, Trolley	45	50
11,000	Single-phase, Trolley	..	70

studied to determine how many of these cars are liable to be starting at once. In other than city roads a load factor of from 0.3 to 0.5 may be used to determine the maximum load on a substation, and in single-track interurban roads it is customary to assume one car starting and one car running. See article on *Railways, Energy Requirements for.*

TYPES OF SUBSTATIONS. — In practice there are two types of substations: 1. for transforming from alternating to direct current, and 2. for transforming from high-voltage alternating current to low-voltage alternating current.

In standard direct-current railways it is customary to generate power in the power stations at about 2200-volt three-phase alternating current, step up by means of transformers to 33,000 volts 3-phase, transmit over any distance to the substations at this voltage, transform from 33,000 volts to approximately 400 volts 3 phase and then to convert to direct current at 600 volts in which form the power is used by the motors of the cars. This is the standard a-c.-d-c. substation. In certain single-phase a-c. railways it is found necessary to transmit the power over long distances at a higher voltage than can be collected from the trolley, in which case the a-c.-a-c. transformer substations are installed to convert the alternating current at the high transmission voltage to alternating current at a voltage convenient for the trolley and collector devices.

SUBSTATIONS FOR TRANSFORMING FROM ALTERNATING TO DIRECT CURRENT. — In these stations it is almost universally the custom to employ synchronous converters to convert from a-c. to d-c., rather than motor-generator sets, as the converters have a higher efficiency and may be compounded to give the desired regulation. Thus the converter makes possible a saving in first cost, space and energy as compared to the motor-generator set. A standard a-c.-d-c. railway substation usually contains the following pieces of apparatus: converters, transformers, reactances, blowers, cables, switching apparatus.

Arrangement of Apparatus. — (*See also article on Switchgear Equipment.*) It is customary to arrange the apparatus in a substation so that the current travels in as nearly as possible a straight line across the station. Thus, with the incoming line on one side there are in the order mentioned: the lightning arresters, oil switches, transformers, a-c. switchboard, reactance coils, con-

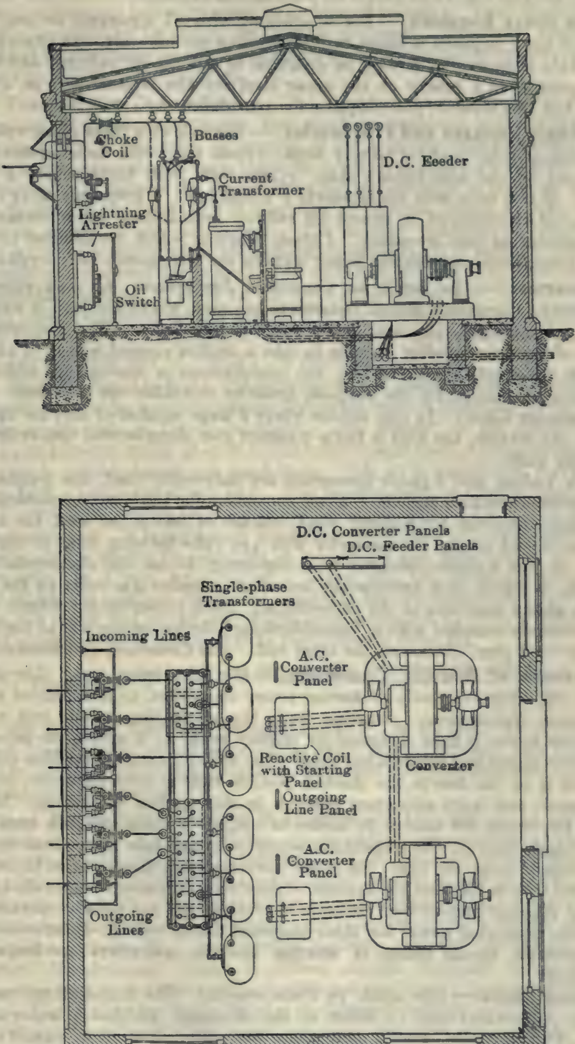

Fig. 1. Plan and Elevation of Typical Railway A-C.-D-C. Substation

verters, d-c. line panel. A typical arangement for a small station is shown in Fig. 1; see also Fig. 16 in article on *Switchgear Equipment.*

Floor Space Required. — For substations with all apparatus on one level the floor space required is about 0.20 square foot per kilowatt. In cities where real estate is expensive, it may be desirable to put the oil switches and lightning arresters in a gallery on the upper level and the transformers on a floor above, but this involves more expense for attendance.

Standard Voltages and Frequencies. — The following three-phase voltages have been standardized for railway work: 11,000 volts with transformers delta connected on the high-tension side; 33,000 volts with transformers *Y* connected; 66,000 volts with transformers *Y* connected. A frequency of 25 cycles per second is employed in almost all railway installations, as converters are more reliable at this frequency. Certain stations which are obliged to take power from previously existing power stations use a frequency of 60 cycles.

Converters. — (*See article on Converters, Synchronous, for data regarding converters.*) Compound-wound converters are used where the load is variable to provide automatically any desired voltage regulation. The converter by means of its series winding is made to take a leading current which, in passing through the reactance of the line, the transformers or through an additional reactance introduced for the purpose, tends to neutralize the line drop. (*See Transmission Lines.*) In city service where a large number of cars are operating on one section, the load is fairly constant and shunt-wound converters are generally used.

Both 3-phase and 6-phase converters are universally used, the 3-phase for the smaller capacities and the 6-phase converters for the larger capacities. In a 3-phase converter for a direct e.m.f. voltage of 600 at no load, the transformers should supply the converter with 370 volts between rings or between lines. In the 6-phase converter it is customary to use the diametrical connection, in which each transformer secondary supplies 430 volts to the converter, giving 600 volts at the commutator. Each converter for railway work is customarily supplied with a "speed-limiting" device on one end of the shaft and an "end-play" device on the other end. (*See Converters, Synchronous.*)

Methods of Starting Converters. — (*See also below under Operation.*) There are several methods of starting rotary converters, as is explained under *Converters, Synchronous.* Starting from the alternating-current end as an induction motor is most desirable for railway work, as it avoids the necessity of synchronizing and requires less time. The ability to start a machine quickly and get it on the line in the shortest possible time is very important in railway work, and is an advantage inherent in this method of starting. Three-phase converters are started, by means of suitable starting switches, from one-half voltage taps on the transformer secondaries and take approximately full-load current from the line. Six-phase converters are started from ⅓ voltage taps and take ¾ full-load line current. Since 60-cycle converters usually take a greater starting current than 25-cycle converters and are usually operated on a system supplying power for other purposes, where voltage disturbances are objectionable, special means of starting 60-cycle converters are frequently employed.

Transformers. — (*See article on Transformers.*) The transformers used in railway substations may be either of the oil-cooled, air-blast or water-cooled type. The oil-cooled type is used where the units are small in capacity, i.e., less than 400 kilowatts, and where the expense of the complications for air blast or water cooling are not warranted. Air-blast transformers are used in all except the smallest sizes, and for voltages up to and including 33,000; the objections to their use are the necessity of providing a pit, air ducts and blower

to supply the ventilation. Water-cooled transformers (which are oil-insulated transformers with water circulating in a special coil submerged in the oil) are built in sizes from 500 kilowatts upwards and for all voltages. Their use depends upon the availability of water for cooling purposes. The usual aggregate capacity of transformers for the various sizes of synchronous converters is about the same as the converter capacity.

Transformer Connections. — The transformers may be either of the single-phase or three-phase type. For small or moderate installations the single-phase type is preferable on account of the economy of maintaining only one single-phase transformer as a spare for a whole station. In railway work it is customary to connect the secondary of the transformers in delta for three-phase, because of the possibility of operating at reduced output on open delta in case of failure of one transformer. The primary windings of the transformer are usually connected *Y* with grounded neutral. It is common practice to provide transformers for railway work with four 2½ per cent taps on the high-potential winding, in order to use similar transformers in all substations and yet make allowances for the difference in the line drop between the power station and the various substations. Either ⅓- or ½-voltage taps are provided on the low-potential side for starting the converters.

Blowers for Air-blast Transformers (*see Blowers and Compressors*) are usually driven by 25-cycle 3-phase induction motors receiving power from the low-tension side of the transformers. The amount of air required per minute per kilowatt rating of each transformer ranges from 3 cubic feet in the large sizes to 5 cubic feet in the small sizes. This air is supplied at a pressure of from ⅜ to 1 ounce per square inch. The blowers must be capable of supplying this amount of air with an allowance of about 10 per cent for leakage in the air-blast chamber. It is customary to provide two blower sets each capable of supplying air for all the transformers in the station and maintaining one as a reserve unit. In a very large station 3 blower sets are sometimes provided, two of which are together capable of supplying the service and the third is a reserve. A rough idea of the size of the motor necessary to drive a blower for a given purpose may be obtained from the following formula:

$$\text{Horse-power} = \frac{(\text{cu. ft. air per minute}) \times (\text{pressure in ounces})}{1200}.$$

Air Duct for Air-blast Transformers. — The pit or trench over which air-blast transformers are placed must be made water-tight and well drained, as any moisture accumulating is liable to be carried into the transformers where it will injure the insulation.

Voltage Regulation. — Use of Reactances. — The voltage at the d-c. bus-bars is regulated automatically by means of line compounding which consists in adjusting the shunt-field excitation of each converter so that the converter takes lagging current at no load, operates at unity power factor at about ¾ load and takes leading current at all loads greater than ¾ load. To accomplish this compounding, it is necessary that there should be a certain amount of reactance between the power-station bus-bars and the converters. (*See Converters, Synchronous.*) There is seldom enough reactance in the transmission line for this purpose, so that additional reactance is inserted either by the use of special transformers having considerable leakage reactance, or by means of reactance coils. The usual reactance coil has a capacity in kv-a. equal to 15 per cent of the kilowatt rating of the converter, i.e. with full-load current passing through the reactance the voltage measured across its terminals would be 15 per cent of the voltage to neutral on the a-c. side of the converter.

Reactance coils are either oil cooled or air blast depending upon the transformers used and are usually built with the three circuits in one unit, with the starting switches for the converters mounted upon the frame. For 6-phase converters a 3-phase reactance coil is used, but since the current per wire is one-half that of the current of a 3-phase converter of the same rating, the reactance per line must be twice as great for the 6-phase converter.

Switchboards. — (*See also article on Switchboards.*) The following sections of switchboards are standard for converter substations.

1. Incoming a-c. line panel.
2. Outgoing a-c. line panel.
3. High-tension a-c. converter panel.
4. D-c. converter panel.
5. D-c. feeder panel.
6. Equalizer and negative panel (on the converter).

Where a substation is tapped off a transmission line at an intermediate point it is good practice to bring the transmission line into the substation, interpose control switches, and then carry the circuit out of the substation on to the next substation. For this reason, in all but terminal substations, it is customary to provide both an incoming and an outgoing a-c. line panel. In connection with the a-c. panel of the switchboard there are a line switch, lightning arrester, choke coil, current transformers and main oil switch, by means of which potential may be removed from all transformers. Between the transformers and the converter are the starting switches, reactance coil and possibly measuring devices.

Direct-current Panels. — Single-pole switchboard panels are used, the positive main bus-bar being the only one on the board. The negative terminals of the converters are connected with switches to the negative or ground return bus-bar, which is frequently located beneath the converter. The series field is connected on the negative side, and the equalizer, series-field-shunt and field break-up switches are frequently placed on the machine itself. The equipment of a standard d-c. converter panel comprises one of each of the following:

Carbon break circuit breaker with overload and low-voltage release, the latter interconnected to the speed-limit device.
Illuminated dial ammeter with shunt.
Field rheostat.
Two-point receptacle.
Single-pole main switch.
Single-pole double-throw station lighting switch.
Watt-hour meter.

Storage Battery. — In certain installations, where reliability of service is of utmost importance, a storage battery is installed, operating in parallel with the converters, and preferably regulated by a series booster. Such a storage battery will not only take care of the load for a considerable period of time in case either the transmission line or converters are out of action, but it will also improve the load factor of the system and thereby the line regulation. In other cases, where the service does not warrant the outlay for such a storage-battery system, a small storage battery is sometimes installed to take care of the lighting of the substation in case the converters should shut down accidentally. See articles on *Batteries, Storage.*

Ventilation. — As the transformers, reactance coils and converters all dissipate a considerable amount of energy in the form of heat, a substation must be well ventilated in order that the temperature of the air surrounding the apparatus

is not so great as to cause an injurious temperature in the apparatus itself. This is particularly important where oil-cooled transformers are used.

Crane. — Where ground space is limited it is good economy to provide a crane in order that the various pieces of apparatus may be lifted over each other when they are taken apart for repairs, as otherwise considerable space must be left to move them about to and from the entrance. See article on *Cranes*.

Operation of Converter Substations. — The only apparatus in a converter substation that requires any great amount of attention are the synchronous converters. As noted above these are usually started from the a-c. supply.

Instructions for Starting a Synchronous Converter from A-C. Supply. —

1. Open all switches except main negative (on machine).
2. Close a-c. line switch feeding busses.
3. Close H.T. transformer switch.
4. Close starting switch on low-voltage taps.
5. When converter reaches synchronism as shown by low frequency of swings of d-c. voltmeter, close equalizer switch.
6. Close switch between series field and the shunt to series field.
7. Correct polarity if necessary by throwing shunt-field break-up switch to reverse position, leaving it closed only momentarily, then throw to running position.
8. Connect a-c. terminals of converter to full voltage of transformers by throwing starting switch to running position.
9. Close d-c. circuit breaker.
10. Adjust field rheostat.
11. Close main d-c. switch.
12. Adjust for correct division of load, power factor and voltage.

Load Factor and Efficiency. — The load factor (q.v.) of a converter substation is usually low, from 30 per cent to 50 per cent, i.e., the load on the station is relatively light except during the morning and afternoon rush-hours, 7 to 9 A.M., and 5 to 7 P.M respectively (*see Train Dynamics*). The all-day efficiency of the station itself, or the ratio of the kilowatt-hours output to the kilowatt-hours input, is less than the efficiency at maximum or rated load; but the over-all efficiency between the a-c. generators in the power house and the cars is about constant throughout the day, since the efficiency of a transmission line increases with decrease of load, thus offsetting the low light-load efficiencies of the transformers and converters. In general practice in interurban 600-volt railways the maximum and all-day efficiencies of the various apparatus are approximately as given in the following table:

Apparatus	Full-load eff., per cent	All-day, eff., per cent
Step-up transformers	98	97
High-tension line	95	98
Step-down transformers	97	94
Converters	90	88
Low-tension distribution	85	88
Over-all, a-c. generator to motors	69	69

Automatic Substations. — To decrease the cost of operation, by eliminating the need of attendants, substations are now equipped so that the entire control and operation are accomplished mechanically and automatically. When the voltage on the adjacent section of the trolley falls below a specified value (450 volts) a contact-making voltmeter closes a control circuit which starts a motor-driven controller operating a large gang of switches and contactors. These switches automatically and consecutively accomplish the various operations listed under "Operation." The converters are started from the a-c. side so there is no synchronizing, but the polarity of the d-c. end is regulated, the field excitation adjusted at the proper time and the several converters connected in parallel. Protection against overload, low-voltage and over-speed are provided.

When the load falls below a specified value a contact-making ammeter closes a circuit which opens all switches and shuts the converters down. A large number of these automatic substations have been installed containing up to three converters per station and in sizes up to 2000 kw. units of converters.

First Cost of Converter Substations (1921). — Synchronous converters cost from $10 to $16 per kw. The transformers cost from $6 to $12 per kw. and the switch-gear from $6 to $20 per kw. rating of substation.

All the electrical apparatus makes up about 67 per cent of the total cost of the station including the building. The floor space required is from .20 square foot per kw. in large sizes to one square foot per kw. in small sizes.

In an approximate way the cost of substations including building, erection and installation, for 33,000-volt transmission, 25-cycle converters, 600 volts on trolley is as follows:

Station capacity, kw.	No. Converters	Capacity of each converter, kw.	Approximate cost of station
300	1	300	$25,000
600	2	300	40,000
800	2	400	50,000
1000	2	500	60,000

Cost of Operation of Converter Substations. — Labor or wages of the attendants is the principal item in the cost of operation of a substation. Supplies and maintenance form a very small percentage of the cost. In general only one man per shift is necessary, as the principal duties are to start up the machines in the morning and occasionally during the day if they drop out of step due to a short-circuit. Frequently the duties of the attendant can be combined with that of ticket seller or express agent. Where the duty is only attending to the substation, $2500 to $3000 per year may be taken as the cost of operation of a substation, allowing for the two shifts a day.

PORTABLE SUBSTATIONS. — Circumstances frequently occur under which the traffic on a certain branch of a railway is very heavy for only a few weeks in the year, and possibly for only two or three days in the year. Moreover traffic may be exceptionally heavy on one branch for one short period and on another branch at another period. In such cases as these it would be very expensive to provide on each branch substations having a capacity to meet the heavy demand. To meet such conditions "portable" substations are used. These consist of what are practically large steel furniture cars, in

which are installed one synchronous converter and the necessary transformers and control devices. Side tracks are provided at points on the branch lines and when needed this portable substation is hauled to the place desired and its high-tension terminals are connected to the high-tension transmission line, provision for which must be made in building the line, and its direct-current terminals are connected to the trolley and the rail respectively. Such a substation may be of great convenience and value for interurban roads where summer parks, circuses and athletic games occasionally render traffic conditions difficult. The car is limited in its size by the standard clearance outlines of the roads over which it must pass. It usually weighs, with all its equipment, about 75 tons. Apparatus having a capacity of 500 kw. may be installed in such a car and the primary voltage may be as high as 33,000 volts (*see article on Switchgear Equipment*).

HIGH-VOLTAGE DIRECT-CURRENT SUBSTATIONS. — Many electrical railways are operated at 1200, 1500, 2400 and 3000 volts d-c. between trolley and ground. For this purpose power is transformed from alternating current by means of either motor-generator sets or synchronous converters. At first the motor-generator set was considered more fitted to the purpose as the design of the commutator is not as restricted in a generator as in a converter and the interpole made it possible to operate at a high potential between brushes. Subsequently, synchronous converters with interpoles and operating at 25 cycles were developed, and are now in successful operation. At first two 600-volt machines were connected in series to give 1200 volts. Later, machines were developed which would give 1200 volts on each machine and these became available for 2400 volts by connecting two in series. For 3000 volts three-unit motor-generator sets are used, one a-c. motor driving two 1500 volt interpole generators in series.

ALTERNATING-CURRENT SINGLE-PHASE SUBSTATIONS. — For a single-phase railway operating at a high voltage on the trolley (11,000 usually) it is not necessary to place substations as near together as in the case of d-c. systems. However, even with 11,000 volts on the trolley, there comes a limit to the length of road which may be supplied from one feeding point. In this case it becomes necessary to step up the voltage at the power station to 33,000 or some higher voltage and step down again to the trolley voltage at distant substations. A substation for this purpose need contain only transformers and control devices. It is much simpler and less expensive than the converter substation. It has even been a matter of discussion as to whether such a substation would require continual attendance to maintain proper operation. This is, however, a matter which must be decided by local conditions. If oil-cooled transformers are used there is no moving apparatus in the station, and the only need of an attendant is in case of accident or to reset a circuit breaker in case it goes out as a result of an overload.

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SWITCHBOARDS. — (*See also Bus-Bars; Circuit Breakers; Meters; Switches; Switchgear Equipment for Power Stations.*) The term switchboard formerly comprised all apparatus in that portion of an installation devoted to collecting, measuring and distributing the electric energy. In small plants all of the switchgear is mounted on a single structure which is then called the switchboard. In large modern plants the control and measuring pieces of apparatus are frequently mounted on one structure, and the busbars and switches on another. In such cases the term switchboard is confined to the structure, whether panel, pedestal, or desk type, which supports the meters, controlling handles and similar devices. The following article discusses the arrangements of apparatus on representative switchboards selected to illustrate the general trend of switchboard design. For a description of the respective structural arrangements required by direct control and distant control, see the article on *Switchgear Equipment in Power Stations.*

CONSTRUCTION OF PANEL SWITCHBOARDS. — The earliest so-called "panel" boards were made of wooden panels. The various switches, instruments, etc., each on its own base, were attached to the wooden panel, with the wiring on either the front or the rear. The next step in advance was the elimination of the wooden panel and framework. Each piece of apparatus was mounted on a marble slab, which was arranged for placing in an angle-iron framework, and switchboards were made by combining the necessary ammeter, voltmeter, switch and rheostat slabs to make the panels for the different generators, feeders, etc. This form of construction was entirely fireproof but various disadvantages ultimately lead to its being superseded by the modern design of panel switchboards with the apparatus grouped on panels made of one or more comparatively large pieces of marble, or slate.

The design of modern panel switchboards has undergone the standardizing process that has been applied to all electrical apparatus. By careful study of requirements the vast majority of plants of moderate size can be equipped with control switchboards that are made up by assembling standard panels.

Framework of Panel Boards. — In general the framework of the standard switchboard may be divided into two types. The cheaper and smaller boards are mounted on a framework of gas pipe, and usually comprise panels about 4 feet high with a space between them and the floor. The larger and more expensive boards have a total height of about 7 feet 6 inches, extend down to the floor, and are provided with an angle iron or pipe frame.

Gas-pipe Frames. — Although the gas-pipe construction is considerably lighter than the angle-iron construction it has been found amply secure for these smaller switchboards and in fact some manufacturers use gas-pipe construction for most of their larger switchboard installations. Where the number of panels does not exceed four or five the complete board can sometimes be shipped with the panels attached to the framework and most of the small wiring, etc., undisturbed, but if the boards are large ones the panels and frame are shipped separately.

Fig. 1 shows the pipe framework for switchboard panels 90 inches high. This is made of 1¼″ wrought-iron pipe with one wrought-iron top strap ½″ × ⅜″ with cast-iron foot nuts and panel supports. The various panel supports are attached to the vertical pipes by "J" bolts or "U" bolts so that no drilling or fitting is required.

Angle-iron Frames.—Fig. 2 shows a typical angle-iron framework for switchboard panels 90 inches high. Each panel of a total height of 90 inches is provided with 2 by 3 by ¼ inch or some similar section angle irons with the narrow web bolted to the panel. These angle irons extend from the bottom

of the panel to within ½ inch of the top. The vertical angles on adjacent panels are bolted together through the 3-inch web. They are provided with

Fig. 1.—Pipe Framework

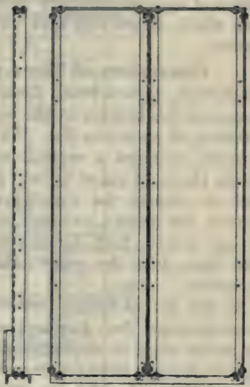

Fig. 2.—Angle Framework

corner angles for bolting at the bottom to a 6 by 2-inch channel iron forming the base of the frame and at the top to two ½ by 1⅜-in. iron strips. The channel iron and the top irons are made continuous the entire length of the board, if this is not over 16 feet. Each panel is shipped bolted to the two angle irons that form its individual frame, which obviates any necessity of disconnecting the wiring between the various slabs making up the panel. This practice of shipping the framework with the panels reduces to a large extent the breakage due to rough handling and facilitates the erection of the board at its destination.

Switchboard Panels. — After trying various materials practically all switchboard builders have adopted either slate or marble, although in a few instances soapstone, brick or steel has been used. The marble used in switchboards is usually of the grade known as "Blue Vermont," although occasionally "White Italian," or "Pink Tennessee" is used.

Comparison of Slate and Marble. — Where switchboard panels are to be given a black finish the choice of slate or marble is largely a question of cost and insulation. Slate is considerably cheaper and somewhat stronger than marble and where the voltage of live metal parts mounted on the panels does not exceed 750 volts it answers just as well.

Finish Used on Panels. — The marble is sometimes polished on the front face and bevels, and occasionally on the edges and back. The present standard finish for marble is a dull black marine finish applied to honed panels. Ordinary slate, owing to its irregular color and marking, is usually given an enamel or marine finish and natural black slate is given an oil finish.

Dimensions of Two-piece Panels. — The total height of standard switchboard panels is 90 inches from the channel iron. The division resulting in a 25-inch lower slab as furnished by one company is due to the fact that these particular dimensions were best adapted to the line of switches, circuit-breakers meters, etc., which were in use at the time the standard railway switchboard panels were first brought out. The lower 25-inch slab was used for rheostat face

plates having the contacts and contact mechanism on the front of the panel. In order to correspond with the old d-c. panels the a-c. panels were brought out having a lower slab 25 inches high and a main slab 65 inches high. The similar design of another company provides panels with slabs 62 inches and 28 inches high which dimensions were suited to their apparatus at the time of standardization.

Dimensions of Three-piece Panels. — When the present standard brush-type, carbon-break circuit-breaker was designed it was found advisable to mount this breaker at the top of the panel in order to take advantage of the tendency of an arc to rise. As all these standard breakers up to 3000 amperes capacity required a vertical space of less than 20 inches, it was decided to divide the main upper 65-inch panel into two slabs, one portion being 20 inches high to contain the circuit-breaker and the other portion 45 inches high to contain the meters, switches, etc. For this reason the standard d-c. panels of one maker, both for railway and for light and power work, were divided into three slabs, the upper 20 inches high, the middle 45 inches and the lower 25 inches.

As the size of rotary converters has increased to such a point that circuit-breakers larger than 3000 amperes are often required the top slabs are now made 25 inches high to accommodate breakers up to 10,000-amperes capacity with middle slabs 45 inches high and lower slabs 20 inches, retaining the total height of 90 inches. Another manufacturer uses sections 31 inches, 31 inches and 28 inches having the same total height of 90 inches. With this design the upper slab usually contains the instruments as well as the circuit-breaker. Nearly all switchboard builders have adopted the three-section panels for boards with heavy capacity carbon circuit-breakers.

TYPICAL DIRECT-CURRENT SWITCHBOARDS. — A few standard equipments described in the subsequent paragraphs have been selected to illustrate standard practice.

Small Isolated Plant Switchboards. — Where there is only one d-c. generator of small capacity with one or two feeders and there is little likelihood of any additional equipment being needed, a board like Fig. 3 can be used with a main slab 36 inches high, 16 inches wide, provided with 2 polarized ammeter and voltmeter, single-pole carbon breaker, 2-pole main switch, field rheostat in the generator circuit, and switches with fuses in the feeder circuits. This type of board uses subpanels for the feeder switches and is usually limited to 10 kw. at 125 volts or 20 kw. at 220 volts.

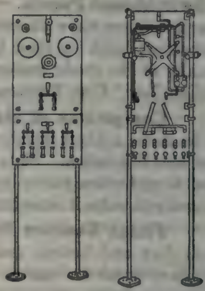

Fig. 3. Direct-currrent Isolated Plant Switchboard.

2-Wire Switchboards. — For the control of small capacity, d-c. generators operating in parallel on a 2-wire, d-c. circuit the connections are made as indicated in Fig. 4. Each generator is provided with a 3-pole, single-throw switch, a single-pole circuit-breaker, an ammeter, and rheostat and a 4-point voltmeter receptacle used for connecting the voltmeter to any machine. Each feeder is provided with a 2-pole switch with inclosed fuses, and a lamp ground detector is connected across the bus-bars.

3-Wire Switchboards. — Fig. 5 shows the diagram of connections for a typical d-c., 3-wire installation and indicates the method of deriving the neutral

connection from the middle point of auto-transformers; the use of series fields in the positive and negative circuits; the placing of the ammeter shunts on the

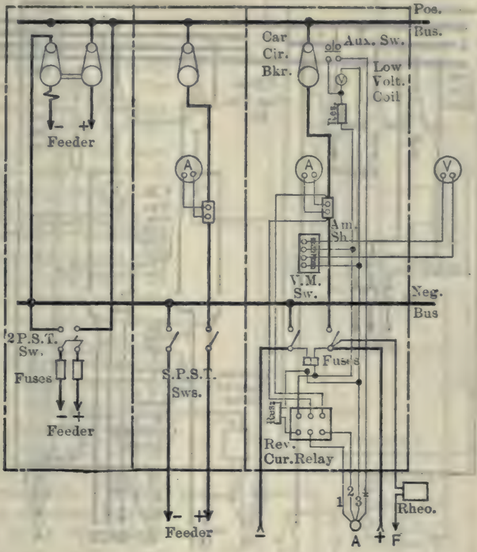

Fig. 4.

terminal boards of the machines so as to be connected inside the series fields; and the 4-pole circuit-breakers or the equivalent two single-pole breakers with equalizer contacts to open the positive, positive equalizer, negative and negative equalizer circuits.

Double-bus Railway Switchboards. — For direct-current railway or tramway work at about 600 volts it was for a long time the standard practice to bring both the positive and the negative leads from the generator to the switchboard and to locate both the positive and negative bus-bars on the switchboards, such boards being called "double-bus" boards. With these boards it was customary to use compound-wound d-c. generators with the series fields in the armature circuit which connected to the overhead trolley. The equalizer connection was usually made through a switch at the machine and the equalizer connection ran from one machine to the other without going to the panel board.

The double-bus scheme is still usually adopted for 250-volt, 2-wire industrial switchboards or any d-c. installation that does not use the ground return.

Single-bus Railway Switchboards. — Modifications in this class of switchboards came from placing the series fields in the grounded circuits instead of in the trolley circuits, and running only one polarity, usually the positive, to the switchboard. The equalizer switch and the negative switch, if the latter is used, are mounted at the machine. Fig. 6 is a typical diagram of a d-c. generator installation arranged for single-bus operation with only one polarity, namely the positive, on the board. The advantages

of this single-bus d-c. system are greater security against short circuit, narrowing of the d-c. generator or synchronous converter panel, consequent

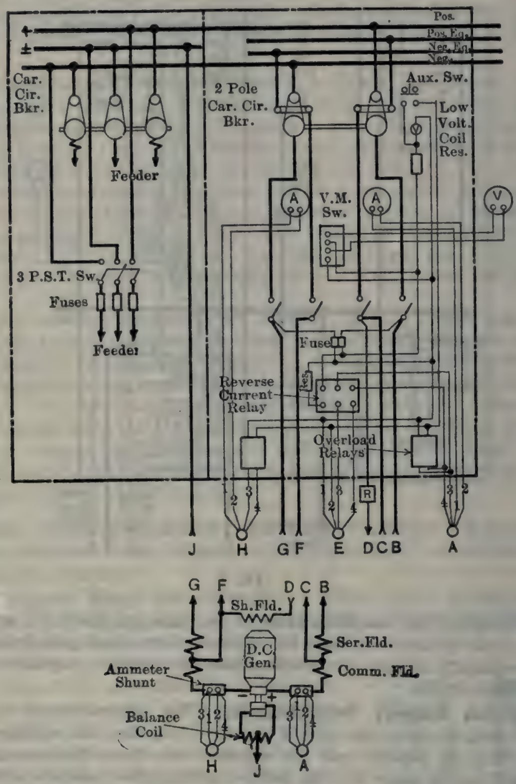

Fig. 5.

reduction in length of switchboard, reduction in amount of cable required and the use of bare copper strap or cables between machines for the negative and equalizer buses.

For converters or motor-generator sets, the panel switchboard if such is used, takes care of both the d-c. and a-c. circuits, which are usually controlled by separate panels. The d-c. panels are practically the same as those used for d-c. generators. The a-c. panels usually correspond with a-c. feeder panels and control the high-tension side of the step-down transformers used with the converter where the latter is made self-starting from the a-c. end. With such machines, in addition to the panels on the high-tension side of the transformers, it is customary to furnish a small slab mounted near the step-down transformers, for connecting the converter, first to low-voltage taps for starting, then to full voltage taps for running.

It is customary to locate the 3-pole, double-throw switch used for starting the synchronous converter, on a panel with the 2-pole double-throw field reversing switch, so that if the converter should build up with its polarity reversed, it is a very simple matter to make the machine slip a pole and build up with the correct polarity. For railway service, the equalizer and negative switches are frequently mounted on the same panel as the starting switch and field-reversing switch.

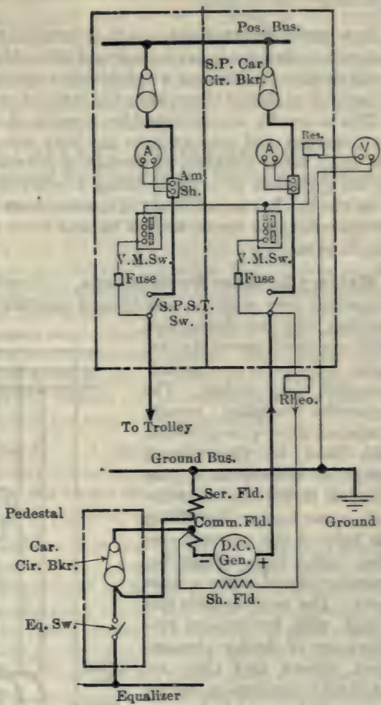

Fig. 6

It is customary practice to provide automatic protection on the alternating-current side of synchronous converters by making the breaker on the high-tension side of the step-down transformer automatic with inverse time element relays, or with inverse time element attachments on the trip coils of the breaker. A low voltage and instantaneous over-load carbon circuit-breaker is provided for the direct-current side. The low-voltage trip of the direct breaker is actuated by a speed-limit switch furnished with and mounted on the converter, the operation of which opens the breaker. It is also actuated by an auxiliary switch that is provided on the oil circuit-breaker so that when this breaker opens the direct-current breaker also opens in this manner providing against the motoring from direct-current and eliminating the liability of reversal in polarity on compound wound machines. For 600-volt railway service where there is an independent source of direct-current power such as an adjacent substation, d-c. reverse current relays are provided, these relays being arranged to trip the alternating-current breaker upon reversal of direct-current power and this in turn opens the direct-current breaker.

High Voltage Direct-current Railway Switchboard. — For 1200-volt d-c. railway service the switchboard panels differ from those used for 600 volts, in the increased height of panel and in the fact that the circuit-breaker and knife switch are mounted out of reach and are mechanically operated by an insulated mechanism.

For 2400–3000 volt d-c. service special precautions have to be taken to carefully insulate the circuit-breakers and switches. Owing to the greater likelihood of flash-over occurring on the high-voltage d-c. converters or motor-generators, high-speed circuit breakers have been designed to trip on overload before the current in the machine rises to a dangerous amount. The same

result is obtained by a high-speed device that places an a-c. short circuit on the converter or generator before the direct-current has an opportunity to rise to a dangerous amount.

Automatic Substations. — On interurban railways operating cars or trains on hourly schedules it has been found advisable in certain cases to install automatic devices for starting a synchronous converter on the approach of a train or car and to shut it down after the train or car has passed.

This is usually accomplished by a series of contactor switches that start up the converter and connect it to the lines whenever the voltage on the trolley line drops below a certain percentage of normal, this drop usually being caused by the approach of a car drawing a relatively heavy current from the line. After the car has passed and the current from the substation has dropped to a certain point, the converter will be shut down after the expiration of a definite time interval.

Mercury Rectifier Switchboards. — Another combination of a-c. and d-c. equipment met with in switchboard practice is that furnished for mercury rectifiers used in conjunction with constant-current d-c. arc lights fed from a constant-potential a-c. system. The apparatus supplied usually comprises a two-pole oil switch with fuses for the primary circuit of the regulator, plug switches in the secondary circuit, a mechanism for tilting the bulb of the mercury rectifier, and a high-voltage d-c. ammeter placed under a glass cover. The switchboards furnished with low-tension rectifiers are of similar arrangement, except that they are fitted with low-tension meters and switches, and have a single-pole starting switch which automatically applies a rheostat load for starting.

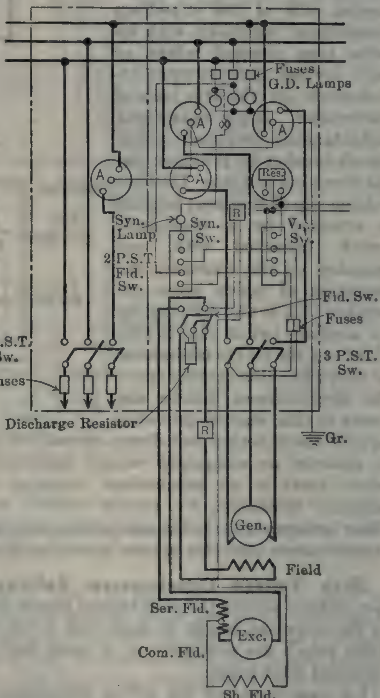

Fig. 7.

Other Arc-light Switchboards. — For use with d-c. series arc generators, switchboards are frequently supplied with a multiplicity of plug switches to permit any generator or feeder circuit to be connected in series with any other circuit to circuits. (*See Switches.*)

Exciter Panels, although d-c., are almost invariably combined with and form part of a-c. switchboards. The d-c. sections of the board are naturally considered with the a-c. sections and made to correspond with them in general design.

Exciter panels are either single or double, depending on whether they control one or two machines. Usually no fuse equipment or other automatic protection is provided with exciter panels, as the sudden opening of the field circuit due to the blowing of an exciter fuse or the tripping of an exciter breaker is apt to injure the insulation of the generator and cause far greater damage to the plant than the overloading or short circuiting of an exciter. Where there are several large exciters operating in parallel with each other or a large storage battery, it is occasionally considered good practice to furnish reverse-current circuit-breakers in the exciter circuits.

TYPICAL ALTERNATING-CURRENT SWITCHBOARDS. — Some standard equipments are described and illustrated in the following paragraphs.

Small Low-voltage A-C. Boards (Fig. 7), made for circuits of not over 600 amperes capacity at 500-volts, are usually built of panels 48 inches high on pipe frames. Each a-c. generator may have its own exciter as shown in Fig. 7, or the exciters may be operated in parallel and controlled from exciter panels. As shown on this diagram the d-c. circuits are taken care of by an exciter rheostat, a generator rheostat and a 2-pole field switch with discharge resistance. Each a-c. generator is provided with a 3-pole single-throw knife switch, three ammeters, a 3-way voltmeter switch, and a synchrononizing switch. Each feeder has a 3-pole switch and an ammeter. A lamp ground detector and a voltmeter are connected in such a manner that the voltage can be read across any phase of any machine. In this diagram lamps are used for synchronizing.

Small High-voltage A-C. Boards. — A similar arrangement of panels may be used for small-capacity high-voltage switchboards, using oil switches in place of knife switches in the generator and feeder circuits, and operating the voltmeter and lamp ground detector from potential transformers.

Large Low-voltage A-C. Boards. — For low-voltage a-c. circuits of greater capacity than can be readily handled by the 48-inch panels, larger panels made in two or three sections with a total height of 90 inches are usually employed. For a-c. service where the voltage does not exceed 550 volts, knife switches are frequently used but the practice is rapidly growing, of utilizing oil circuit-breakers even for moderate voltages, owing to the greater safety, greater convenience, and other advantages. Where the voltage does not exceed 2400, or the amount of power does not exceed 3000 kv-a., it is customary to use oil circuit-breakers mounted directly on the switchboard panels. Fig. 8 shows a typical switchboard with panel-mounted oil breakers arranged for the control of two exciters, three generators, and two feeders.

The bracket devices comprise two voltmeters, one connected through transformers to the bus, and the other arranged to plug on any phase of any of the three generators through suitable transformers, one synchronoscope with two synchronizing lamps, and one voltage regulator. The first panel on the left controls two exciters and is provided with two d-c. ammeters, one d-c. voltmeter, two rheostats, two 2-pole switches, and one equalizing switch.

The next three panels are generator panels, each being provided with an a-c. ammeter, a polyphase indicating wattmeter, field ammeter, field rheostat, field switch, voltmeter and synchronizing receptacle, ammeter switch and a non-automatic breaker.

The last two panels on the right are feeder panels and are each provided with three ammeters, a watthour meter, two overload relays, and an automatic breaker.

Knife-type disconnecting switches are furnished for disconnecting the oil circuit-breakers from the busses, to facilitate the inspection of the breakers.

Where the voltage exceeds 2400, or the amount of power to be controlled is

in excess of about 3000 kv-a., it is preferable to locate the oil breakers apart from the board and to control them either electrically or mechanically from the panel.

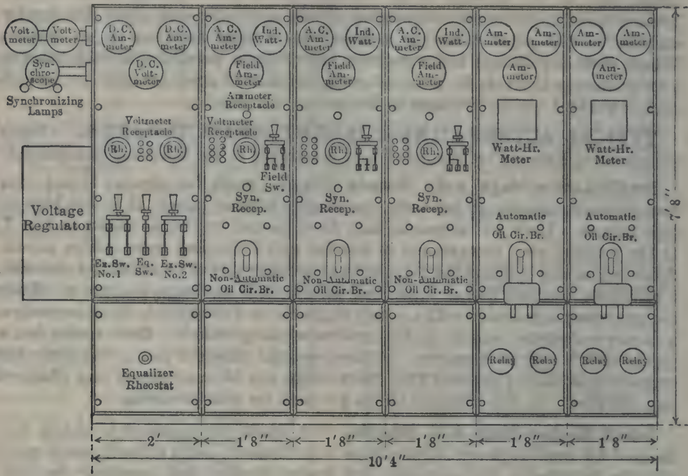

Fig. 8.

Large Distant-control A-C. Switchboards. — Where the amount of power to be controlled is large or the voltage is above 3300 it is customary to mount the oil circuit-breakers apart from the board and to operate them either mechanically or electrically. On these boards the usual equipment of instruments is mounted at the top of the panel, the switch-control handles are in the middle and the time-limit relays at the bottom. Targets and signal lamps to indicate the condition of the feeder are furnished with electrically operated boards.

Fig. 9 shows diagrammatically the various standard arrangements for electrically controlled equipment. *a* illustrates a typical vertical panel board with the instruments, control switches, relays, and similar devices mounted on the face of the panel.

b shows an arrangement of a control desk with the control switches placed on the desk and the instruments mounted on a wall in front of the operator. *c* shows an arrangement of control desk where there are comparatively few meters, these being set flush in the face of the desk. *d* shows a modification of the desk arrangement with the meters on a small slab or bracket extending up from the horizontal slab of the desk. *e* shows the control desk arrangement with vertical panels forming the back of the desk, the vertical panels containing the indicating meters. *f* is a further modification of the control desk arrangement with vertical panels containing the indicating meters and a complete switchboard at the rear to contain the recording meters, relays, and similar devices. With this arrangement a self-supporting control desk is provided. *g* shows the so-called gallery type of desk with the meters located on a framework supported above the horizontal slab of the desk at such a height that the operators standing at the control desk can look above the edge of the desk and

below the meter panels to observe from the switchboard gallery the machine which he is controlling. *h* is a modification of the gallery type of control desk. *i* is a modified arrangement of control desk using a separate instrument

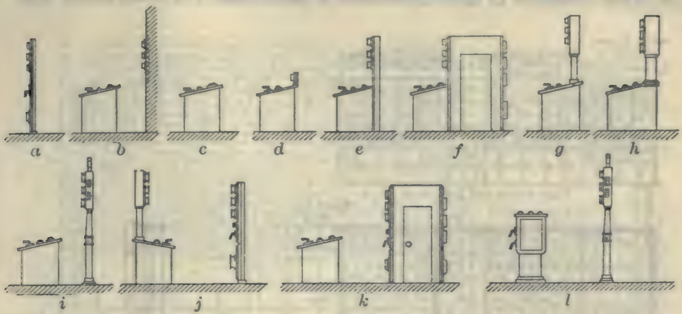

Fig. 9.

frame supported on ornamental pillars, these pillars as a rule, being arranged to form the supports of a gallery railing.

j shows the combination of a gallery type control desk for the generators and vertical panels switchboard for the feeder.

k shows a combination control desk and panel board, the generator breakers being controlled from the desk, the generator instruments being on the vertical panels and all of the feeders being controlled from the vertical panels. The recording meters, graphic meters, and relays are placed on an auxiliary board back to back with the feeder board.

l shows an arrangement of control pedestals and instrument posts corresponding with the arrangement adopted by the Ontario Power Company at Niagara Falls., Ont. In this control room pedestals and posts are provided for a total of 16 water-wheel generators, 12,000 volts, the units averaging about 8770 kv a capacity, and each generator being provided with its own control pedestal and instrument post.

CONTROL DESKS (Figs. 10 and 11).—Where it is desired to have a very compact arrangement and to control the generators and feeders from the same switchboard, the control desk has many advantages; particularly where a group system of circuits is used and it is desirable to have a miniature bus-bar to show the general scheme of connections and the arrangement of circuits in use. The desk has mounted on it the field switches, field-rheostat handles and small switches for operating electrically the various controllers, circuit-breakers etc. The field switches and rheostats may also be operated electrically.

The relative arrangement of the control devices and the meters is indicated on Fig. 10, and as may be noted,-various combinations can be readily secured. For most cases where a control desk is placed in a gallery, a so-called gallery type of desk proves to be the most advantageous. With this type of desk the control switches and indicating lamps for the electrically operated breakers in the main circuits are placed on the nearly horizontal portion of the desk. The corresponding control switches for the field and excited circuits are on the front section of the desk. The indicating instruments are placed on vertical panels coming down to the horizontal part of the desk and graphic meters, watthour meters, relays, and calibrating jacks are placed on separate panel boards, back to back with the main desk.

On the horizontal section of the desk a miniature bus is often placed showing the connections made by the main breakers.

Each generator is provided with three ammeters, one per phase, a polyphase indicating wattmeter to show the amount of power delivered to the main bus, a

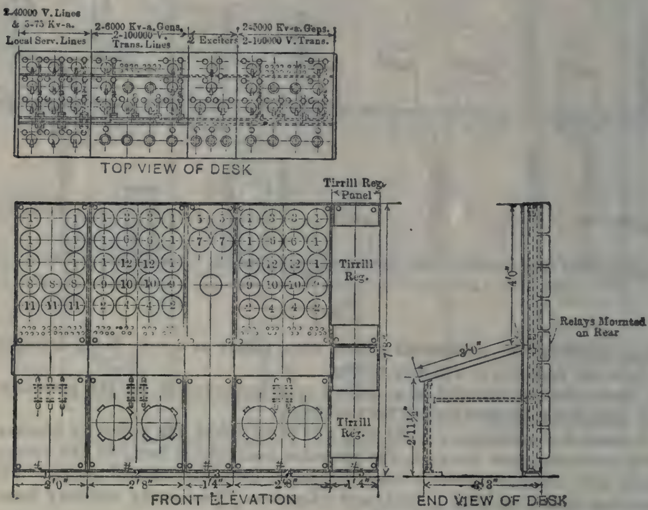

Fig. 10. Contral Desk with Instrument Panels.

polyphase indicating wattmeter to show the amount of power going over the transfer bus, a synchronoscope, power factor indicator, field ammeter, ammeter in the circuit of the synchronizing bus, a frequency meter, and an indicating temperature meter. In addition to the indicating temperature meter a potentiometer is sometimes provided with a dial switch for other temperature readings.

Control Desk with Separate Instrument Frame. — Fig. 11 shows the plan view, elevation and side view of a control desk provided with 7-inch diameter round pattern meters located on an instrument frame back of and above the desk. The height of the instrument frame is 7 feet 8 inches, which corresponds to the height of standard 90-inch switchboard panels mounted on the usual 2-inch channel iron base. This control desk with a total length of 12 feet, when fully equipped, is to take care of six 10,000-kv-a., 3-phase, 6600-volt generators, six banks each of three 3333 kv-a. step-up transformers, and four 110,000-volt transmission lines.

As shown by the miniature bus system placed on the top of the control desk, each of the six sections intended for a generator and its bank of transformers is provided with a generator ammeter, transformer ammeter, generator indicating wattmeter, power-factor meter, voltmeter, and field ammeter, as well as four circuit-breaker controllers. Each generator, with the low-tension side of its bank of step-up transformers, is provided with three breakers, one in the main generator circuit connecting the generator to a generator bus, one for connecting this generator bus to a main bus, and the

third for connecting the generator bus to the low-tension side of the step-up transformers. With this arrangement under normal conditions each generator will supply current to its own bank of step-up transformers, but in emergency any generator or any transformer can be connected to the low-tension bus,

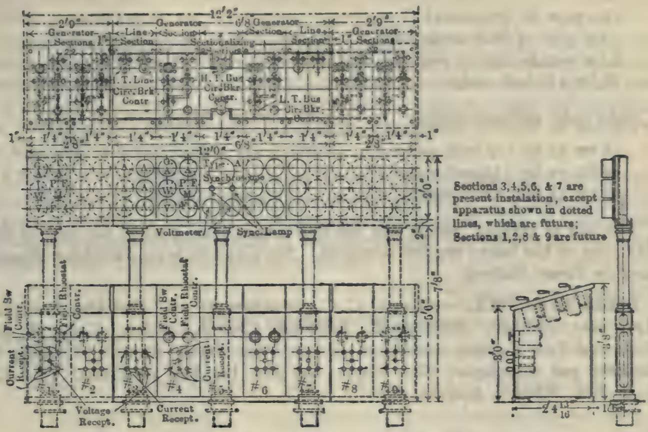

Fig. 11. Control Desk with Separate Frame for Instruments.

which is sectionalized in the middle by means of an electrically operated breaker. On the high-tension side each bank of transformers connects through a single breaker to a high-tension bus that is sectionalized in the middle by means of an electrically operated breaker. Two high-tension lines are fed from each half of this sectioned bar.

Dimensions of Panels. — The smaller panels intended for use with gas-pipe framework are made either in single slabs and as a rule have a height of 48 inches and a width of either 22 inches or 32 inches, although some of the panels are smaller. As few of the smaller d-c. panels have to be made wider than 22 inches or 24 inches and as 1¼ inches is ample thickness for a 24 by 48-inch slab, the thickness of 1¼ inches has been adopted as a standard for most of the small d-c. panels. In order to secure sufficient mechanical strength the 32-inch panels are made 1½ inches thick. As these wide panels are usually required for alternating-current generator panels this thickness of 1½ inches has been usually adopted as standard for a-c. panels. For heavier panels 2-inch marble and slate have been adopted as standards for mechanical reasons. This thickness is required for the heavy switches, circuit-breakers, etc., often furnished on these switchboards. When a board has both a-c. and d-c. panels the thickness of all panels is made the same.

Beveled Edges. — The edges of all panels are usually beveled to improve their appearance and to insure against chipping, as it is almost impossible to handle square-edged marble or slate slabs without injury. The front edges are ordinarily beveled with a 45-degree bevel of either ⅜ inch or ½ inch.

SPECIFICATIONS AND TESTS. — See article on *Switchgear Equipment for Power Stations* and *Standards of the A. I. E. E.*

COSTS. — The cost of any particular type of switchboard is arrived at by summing up the costs of the various instruments, switches, etc., and adding to this the cost of the framework, slabs and wiring. The following (1921) figures are approximate only:

Gas pipe for framework	$0.20 per linear foot
2 by 3 by ¼-inch angle iron	0.30 per linear foot
6 by 2-inch channel iron	1.00 per linear foot
½ by 1⅜-inch strips	0.40 per linear foot

Slabs only, without drilling	Marble	Slate
48 by 24 by 1¼-inch panel	$16.00	$15.00
48 by 32 by 1½-inch panel	21.00	18.00
90 by 24 by 2-inch panel	50.00	48.00

The cost of drilling, wiring and erection ranges from 5 to 25 per cent of the total cost of the board erected complete. See also article on *Switchgear Equipment for Power Stations*.

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SWITCHES. — *(See also Circuit Breakers; Switchboards; Switchgear Equipment for Power Stations, Wiring of Buildings.)* A switch is a device for making breaking or changing connections in an electric circuit. Various types of knife, drum, plug and oil switches are used; some representative types are described below.

Fundamental Requirements are (1) a switch must, when closed, carry the rated current without excessive drop, usually from 5 to 25 millivolts, or excessive heating, usually 30° C. temperature rise; (2) it must take care of overloads met in practice; (3) it must be designed to prevent or render harmless any arcs that are formed when being opened; (4) it must, when open, insulate all live parts for the maximum potential of the circuit.

KNIFE SWITCHES.— The rules of the National Board of Fire Underwriters relative to knife switches advise for pure copper blades a current density at contact surfaces of not over 75 amperes per square inch. The recommended minimum spacings between points of opposite polarity for various currents and voltages are given in the accompanying table.

Full-load current, amperes	Inches between points of opposite polarity		
	125 v. d-c.	250 v. d-c. 500 v. a-c.	600 v. d-c.
100	1¼	2¼	4½
200–300	2¼	2½	4½
400–600	2¾	2¾	4½
800–1000	3	3	4½

Throw and Poles of Switches. — A switch closing a circuit only when thrown in one position is called a single-throw switch; a switch closing a circuit when thrown in either of two positions (e.g., up or down) is called a double-throw switch. A switch closing only one side of a circuit (one blade) is called a single pole switch; one closing both sides (two blades) is called a double-pole switch. Evident abbreviations are used, such as S.P.S.T. for single-pole single-throw; D.P.D.T. for double-pole double-throw.

Single- and Multi-blade Knife Switches. — Up to 1000 amperes in capacity knife switches are usually made with single blades. For larger capacity two or more blades per pole are supplied in order to secure sufficient contact surface without making the blades and jaws of abnormal width.

Switch Studs. — The smaller capacity switches are made both rear connected and front connected, while the larger switches are almost invariably made for rear connection. Up to approximately 1000 amperes the standard studs for rear connected switches are circular and the switch studs are attached to their bases by nuts screwed on these circular studs. Additional nuts are provided for clamping strap connections or terminals.

For the larger capacity switches employed on low-voltage boards, strap connections are almost invariably used and to facilitate the employment of the strap connections the switch studs are frequently made laminated. A modification of the laminated studs employs copper studs cast under high pressure by means of which the conductivity of the cast studs is approximately 90 per cent that of rolled copper. With the laminated stud switches the laminations can be arranged for the horizontal or vertical plane as is best adapted to the wiring.

Knife Switches for A-C. Service. — Up to 800 amperes there is no appreciable difference in the heating of knife switches on direct or alternating-current. For larger capacities it is found that for the same temperature rise it is necessary to rate lower for alternating-current. The constants vary with different designs and different capacities, a typical case being the switch that has a rating of 3000 amperes (d-c.), 30 degrees; 2500 amperes, 25 cycles, 30 degrees; 2200 amperes, 60 cycles, 30 degrees; 2400 amperes (d-c.), 20 degrees; 2000 amperes, 25 cycles, 20 degrees; 1800 amperes, 60 cycles, 20 degrees. The 30-degree rise is that covered both by the N. E. Code and A.I.E.E. Rules.

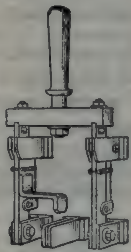

Fig. 1. Quick-break Field Switches

Field Switches (Fig. 1). — A modification of the standard knife switch (q. v.) as shown in Fig. 2 is furnished for use in connection with field circuits. These field switches, whether single-pole or double-pole, single-throw or double-throw, are provided with an auxiliary contact and extra jaw which are used for connecting a resistor across the field terminals before the field is disconnected from the source of supply. The field discharges through this resistor without any inductive kick such as will occur on the sudden opening of a highly inductive circuit without such a device.

Auxiliary Breaks and Quick-break Attachments are furnished with knife switches in many cases so as to make it impossible to draw a dangerous arc by opening the switch slowly. Fig. 2 shows a typical quick-break knife switch. The main-switch blade here carries auxiliary blades, which are attached to the main blades by a hinge and spring. When the switch is opened, the auxiliary blades are held in the jaws by friction until they are suddenly jerked out by the spring tension. A similar attachment-is used on the field switch shown in Fig. 1.

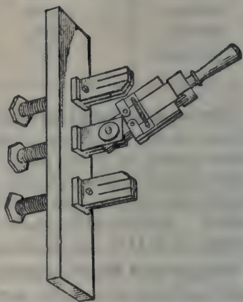

Fig. 2. Quick-break, Single-pole, Double-throw Switch

Starting Switches. — A modification of the standard knife switch is the multi-point starting switch used occasionally for starting d-c. motors, or synchronous converters or motor-generator sets from the d-c. end. These multi-point starting switches consist usually of single-pole, single-throw switches with several break jaws connected to various steps of the starting resistor.

Disconnecting Switches. — In all high-tension circuits it is customary to install knife-type disconnecting switches for isolating feeders, oil circuit-breakers, etc., or for making various connections that do not have to be opened under load. In American practice the knife switches for 2500 volts or less are usually mounted directly on a base of soapstone, marble or similar material; for higher voltages insulators of various kinds are used to support the switch jaws. Up to 33,000 volts these disconnecting switches are made for either front connection, rear connection, or both, but for higher voltages they are almost invariably made for front connection only.

Fig. 3 shows a disconnecting switch built in capacities of 400 up to 4000 amperes at 7500 and 15,000 volts, and up to 600 amperes for higher voltages up to and including 66,000. In this switch a corrugated conical pillar type insulator is used with the switch part attached to the top of the insulator and the bottom of the insulator attached to a metal base in such a manner that if

an insulator proves defective, it can readily be replaced without the necessity of replacing the balance of the switch. Owing to the severe mechanical stresses set up at the instant of short circuit on systems of large capacity, latches are provided on these disconnecting switches to prevent them being blown open.

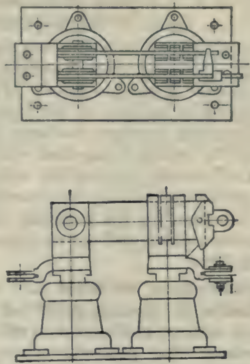

Fig. 3

Disconnecting Switches on Post Type Insulators. — For voltages of 66,000 and above, it is customary to employ disconnecting switches mounted on porcelain posts of the built-up type employing a sufficient number of sections or units to secure the voltage test desired, either for indoor or for outdoor service. With this type of switch if an insulator becomes damaged or defective, the units can be readily unbolted from the built-up pillar and replaced by a new section.

DRUM SWITCHES are often used with electrically-operated devices such as rheostats, field switches and circuit-breakers. In the type shown in Fig. 4 the operating handle and direction dial are mounted on the front of the switchboard, while the contacts are back of the board and are made part of a drum-type controller. This construction removes even the d-c. operating voltage from the front of the board and readily

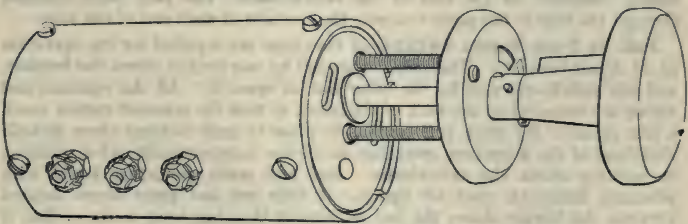

Fig. 4. Drum Control Switch

adapts itself for use on 550-volt control circuits. The handle is so designed that after turning to the trip position it can be pulled out in such a manner as to disconnect the control switch and extinguish the indicator lamps. For use with this control switch lamp indicators with colored prisms are furnished.

PLUG SWITCHES are commonly used for interrupting small currents at low voltages. They are particularly applicable for such uses as reading the voltages across various phases or circuits. By placing suitable receptacles on generator and feeder panels and using only one voltmeter plug switch, one instrument in a suitable location can be used for reading the voltage across any phase of any circuit without the possibility of trouble from an attempt to connect two or more circuits to the voltmeter at the same time. Other types of receptacles and plugs are used in connection with current transformers for connecting instruments into any phase of any circuit. By using suitable current and potential receptacles it is feasible to connect in testing meters for calibrating the switchboard instruments without removing them from the board.

The latest designs of instrument switches are built of the drum type with attached or removable handles, depending on their function. The ammeter switch usually has an attached handle and the switch is turned to connect an ammeter in any phase of the circuit. The voltmeter switch usually has a removable handle of the key type so that by having a switch on each panel and a voltmeter on a bracket the instrument can be connected to read the voltage across any phase of any circuit With the synchronizing switch a removable handle is also provided. When synchronizing to the bus this key-type plug must be inserted in the proper receptacle and the switch turned. When synchronizing between machines the escutcheon plate on the front of the synchronizing switch is provided with two slots, one permitting the insertion of a key that permits turning the switch in one direction for the running machine, and another permitting the insertion of a key for turning the switch in the opposite direction, for the incoming machine. Proper location of the slots in the escutcheon plates and the corresponding pins on the plug prevent the use of the wrong keys.

Synchronizer Switches may be fitted with auxiliary contacts so that it is impossible to close an electrically-operated breaker in a generator or similar circuit without putting the synchronizing plug in the receptacle used for synchronizing that particular circuit. The tripping circuit, however, is kept independent of the synchronizing receptacle so that the breaker may be tripped out independently of the position of the plug.

Plug Switches for Arc Service are built in capacities of 10 amperes or more suitable for use on circuits up to 10,000 volts. Each switch comprises a tube of fiber or similar material, with socket contacts at each end, and a plug consisting of a metallic rod or tube with an insulating handle. The fiber tubes are usually mounted on the rear of the switchboard. The plug when inserted through the hole in the panel connects the contacts at each end of the tube.

Pull- or Push-Button Switches of twin type are supplied for the operation of oil circuit-breakers. The switch operated by one button closes the breaker, and the switch operated by the other button opens it. All the contacts and wiring are mounted on the back of the panel, so that the operator cannot touch a live circuit. By using pull buttons in place of push buttons there is little likelihood of the attendant operating the device unintentionally when cleaning or working about the switchboard. Red and green indicating lamps with prismatic lenses are used for signals. A little red and green target located between the button shows the last movement that has been made so that if the target shows one color and the indicating lamps another it is known that the breaker has been tripped automatically.

Push-Button Switches with Signal Devices are sometimes used for transformer-type ground detectors, engine-room signals and similar equipment. These are frequently arranged as the equivalent of double-throw switches normally maintained in one position by a spring to make one set of connections, and making other connections when pushed in by hand or some mechanism.

Button and Snap Switches of various designs are used for interior lighting circuit (*see Wiring of Buildings*).

OIL SWITCH. — (*See Circuit Breakers.*) There is usually no distinction drawn between the terms "oil switch" and "oil circuit-breaker," although in some cases the term "oil switch" is applied to a device with jaw or similar contacts that tend to remain closed, and the term "oil circuit-breaker" is applied to a device with butt, brush, cone wedge or similar contacts that tend to come open and must be held closed by a toggle, latch or similar mechanism. "Oil circuit-breaker" is the term usually employed.

END-CELL SWITCHES. — (*See Batteries, Storage.*)

COSTS. — Switches vary so much in design that it is impossible to give a comprehensive table of costs in a limited space. The following table gives the approximate range of (1922) prices of a few typical switches:

S. P. S. T.,	100–3000 amps., 600 volts.....	\$3.50 to \$130.00
S. P. D. T.,	100–3000 amps., 600 volts.....	5.00 to 200.00
D. P. S. T.,	100–3000 amps., 600 volts.....	6.50 to 235.00
D. P. D. T.,	100–3000 amps., 600 volts.....	9.00 to 360.00
S. P. D. T. Q. B.,	100–3000 amps., 600 volts.....	9.50 to 375.00
Disconnecting Switch,	15,000 V., 400 A........	30.00
Disconnecting Switch,	37,500 V., 400 A........	40.00
Disconnecting Switch,	73,000 V., 600 A........	75.00
Disconnecting Switch,	115,000 V., 400 A........	150.00
Simple Plug Switch	....	3.00
Simple Snap Switch	....	.50

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SWITCHGEAR EQUIPMENT FOR POWER STATIONS. — (*See also Bus-bars; Circuit Breakers; Fuses; Lightning Protectors; Power Stations; Substations; Switches; Switchboards.*) This article is a summary of modern practice in regard to the arrangement of the control and switching apparatus in generating stations, transforming stations and converting stations. The special building arrangements to suit the switching apparatus are also discussed.

In the earliest plants the switchgear, which then comprised only knife switches, plugs, fuses and lamps, was scattered around the station in a more or less haphazard way, or possibly assembled on one of the walls, so that practically no space was allotted to it that could possibly be used for any other purpose. The next step with increasing amount of auxiliary apparatus involved the placing of the switchgear on a panel switchboard near the wall where very little room was taken up by it. As stations grew still larger, proportionately more space had to be allotted to the switchgear, until in the modern high-voltage, large-capacity plant, one portion of the building, or in some cases a special building, is assigned to the switchgear and designed especially for its proper housing.

As regards switchgear equipment power stations may be classified as follows:

The switching requirements of the several classes are treated in the sections following.

DIRECT-CONTROL SWITCHGEAR IN GENERATING STATIONS. — Direct-control panel switchboards are usually installed for a-c. or d-c. lighting, power and railway service of low voltage and moderate size. The switchboards for such plants are of the panel type and may be located on the station floor, on a platform or in a gallery. All the switching appliances are mounted directly on the panels of the switchboard. With such equipment it is fairly simple to locate the switchboard in such a manner as to reduce to a minimum the amounts of connecting cables and similar material that depend on the relative position of the switchboards and generators.

The amount of space required and the amount of cables, bus-bars and wiring needed in a direct-current railway generating station can be reduced to a minimum by arranging to have only one polarity, usually the positive, on the switchboard, and to place the negative and equalizer busses in the basement or in a conduit near the machine, locating equalizer and negative switches at the machine.

DISTANT-CONTROL SWITCHGEAR IN A-C. GENERATING STATIONS DISTRIBUTING AT GENERATOR VOLTAGE. — In stations that distribute current at the generator voltage there are three usual locations for the bus-bars, oil circuit breakers and control apparatus, depending principally on the amount of space needed for this portion of the installation. These locations are: 1. at the end of the building; 2. at the side of the building and 3. in a separate switch-house.

Location of Switchgear. — The end of the building is a favorite location for the switchgear when the number of feeders is such that this location provides sufficient space for the breakers and the bus-bars, making due allowance for probable future additions. With this arrangement it is customary in large plants to provide a number of galleries for the switching equipment. The switchboard is usually placed on one of the upper galleries so that the station attendant can readily watch the operation of the machines which he is controlling.

Where the end of the building does not provide sufficient space the switching equipment is frequently located along one of the side walls, usually the side remote from the boiler room in a steam station, or from the incoming penstock in a hydraulic station. The switching equipment when arranged in one or more galleries along the side of the building can easily be extended, as the lengthening of the building provides for the switchgear proportionately with the space available for the generating equipment. With this arrangement it is usually customary to locate the generator breakers directly opposite the individual machines and to run the bus-bars the length of the station; the length of the generator leads will then be reduced to a minimum and it is sometimes possible to use bare conductors for these leads. The switchboard itself, if electrical operation is provided, may be located either on one of the side galleries or at the end of the building in such a position that the switchboard attendant can readily watch the operation of the machines which he is controlling.

An extension of this scheme of utilizing the side walls is to provide a separate switch-house and to control all of the apparatus electrically from a switchboard in the main building or from a switchboard in the switch-house as preferred.

Structures for Bus-bars and Circuit Breakers. — The arrangement of the supporting structures for bus-bars and circuit breakers is influenced chiefly by the voltage and capacity of the circuits. For circuits of moderate capacity and 2500 volts or less, mechanical control is usually employed, though electrical control is sometimes used; for circuits of large capacity or of higher voltage than 2500 electrical control is practically universal.

Structures for Mechanical Control. — Fig. 1 shows a typical arrangement of distant mechanical control applied to circuit-breakers mounted on the wall or on a metal framework. The view shows a section through a 2300-volt, 3-phase generator circuit. Each circuit is provided with a 3-pole oil circuit-breaker which can be connected to the three-phase bus-bars supported on the wall brackets. Sprocket-operated face plates with suitable resistors are located in the basement to allow for the regulation of the voltage on the generators. The current and voltage transformers for the instruments are located back of the board on a suitable framework or on the wall.

Structures for Electrical Control of Moderate Capacity. — Fig. 2 shows a section and the front and rear elevations of a part of the structure for use with the breakers and bus-bars for the control of four 500-kw., 2300-volt, 3-phase generators, nine 3-phase feeders and six single-phase feeders. Though the capacity and voltage of these machines would have permitted the use of hand-operated oil circuit-breakers, other considerations caused the adoption of electrical operation for the breakers and of inclosed bus-bars. The relative locations of the circuit-breakers, disconnecting switches, series transformers, shunt transformers, bus-bars, etc., are indicated in the cut. The bus-bars of laminated copper strap are supported on petticoat insulators, and porcelain floor tubes are used for insulating the leads where they pass through the back wall of the structure. The breakers here used are self-contained and all three poles are in the same compartment. The disconnecting switches are mounted directly on soapstone bases.

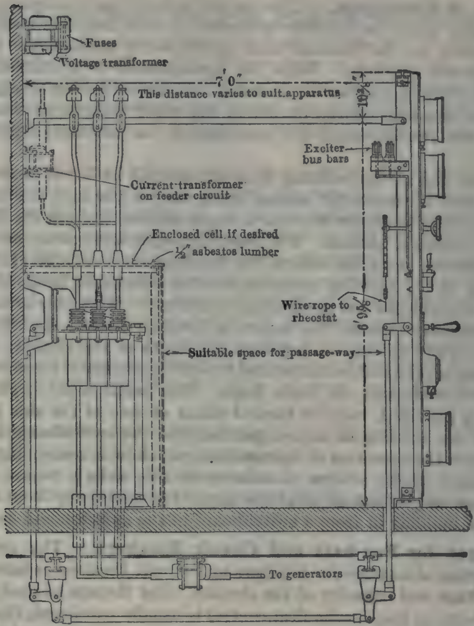

Fig. 1. Structure for Distant Control of 2300-volt, 3-phase Circuit

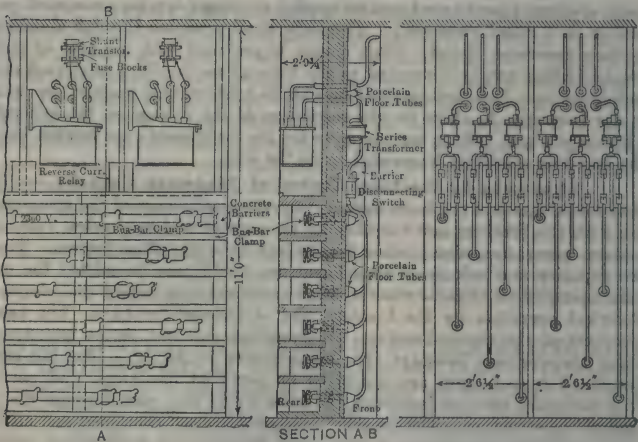

Fig. 2. Structure for Electrical Control of 2300-volt, 3-phase Circuit

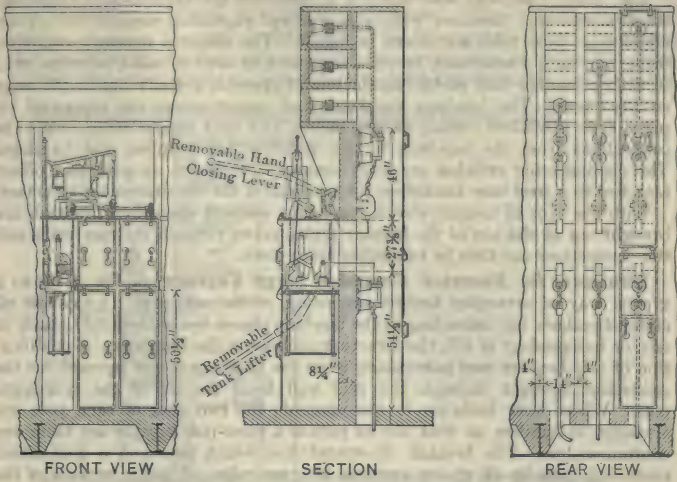

Fig. 3.—Structure for Top-connected 6600-volt Breakers

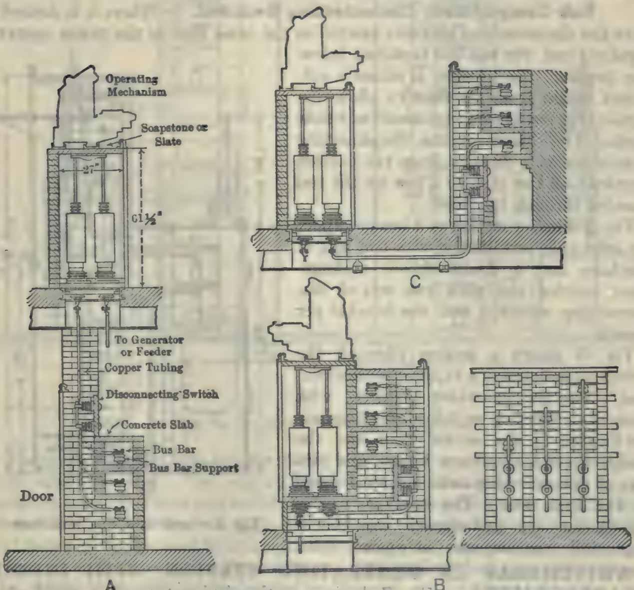

Fig. 4. Structure for Bottom-connected 13,200-volt Circuit Breakers

The front, rear, and side views of a structure for three-phase, 6600-volt, solenoid operated breakers with a guaranteed rupturing capacity of 23,000 amperes at 7500 volts are shown in Fig. 3. The fireproof masonry compartment, bus-bars, connections, etc., are separated by shelves, walls, septums, etc., in such a manner that no two conductors of opposite polarity are in the same compartment. The bus-bars and laminated copper straps are supported on pillar type insulators resting on the shelves and bent copper strap forms the connections from the bus-bars to the disconnecting switches and breakers. The disconnecting switches are front connected, mounted on porcelain pillars attached to a steel base located on the rear wall of the circuit-breaker structure.

With the type of breaker shown, employing solenoid operation, the leads are brought out at the top of the breaker tanks, taken through the rear walls and the connections can then be run either up or down.

Structures for Electrical Control of Large Capacity. — Fig. 4 shows a typical way of arranging bottom connected, motor operated, 13,200-volt oil circuit-breakers for connecting to the bus-bars placed below them (*A*), back of (*B*), or independent of (*C*), the structure containing the breakers. A modification of this breaker used particularly when the leads are to run upward has the connection brought through the rear wall from the top of the cylindrical pots. Where both leads are to run upwards the two pots of each pole are arranged in tandem so that the six pots of a three-pole breaker are all in one continuous line. The breaker illustrated has 8-inch pots and a guaranteed rupturing capacity of 23,000 amperes at 7500 volts. A larger breaker of the same type with 10-inch pots and a guaranteed rupturing capacity of 41,400 amperes, and a still larger breaker with 12-inch pots has a guaranteed rupturing capacity of 52,900 amperes at 7500 volts.

Sub Compartment Disconnecting Switches. — Where it is desired to have the disconnecting switches located on the same floor as the motor operated breaker, and the bus-bar is placed below the breaker, the breaker is raised a sufficient height from the floor to allow disconnecting switches to be mounted below them. The blade of the disconnecting switch connects directly from the bottom terminal of the pots to a jaw on the rod running through the floor.

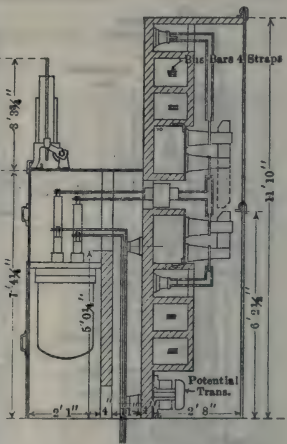

Fig. 5.—4000-volt Bus Structure

Fig. 5 shows a section of a heavy capacity 4000-volt installation with two sets of bus-bars and a double-throw arrangement carried out with two sets of disconnecting switches and one breaker per circuit.

Fig. 6 shows a section through the switching galleries of a 22,000-volt installation where the generator breakers with their disconnecting switches and bus-bars are located on the second floor and the feeder circuits, duplicate buses, and two sets of disconnecting switches are located on the first floor. The breakers in this installation are guaranteed capable of rupturing 5350 amperes at 25,000 volts.

SWITCHGEAR IN GENERATING STATION WITH INDOOR TRANSFORMERS. — Fig. 7 shows a sectional view through a plant which

contains four 16,000 h.p. vertical shaft water wheels driving 12,000 kv-a. 6600 volt generators, and four three-phase transformers. The leads from the generators are taken through the breakers to the 6600-volt main bus, back from that bus through similar breakers to the low-tension side of the transformers. A cross connection allows the generators to feed directly to the transformers.

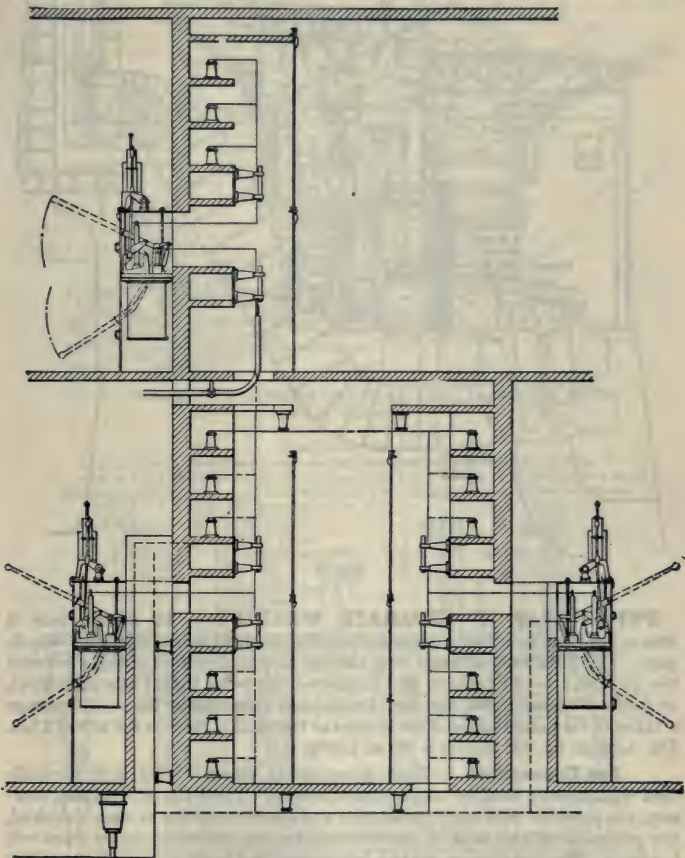

Fig. 6.—22,000-volt Switching Galleries

The leads from the high-tension side of the transformers are taken up through floor openings where they are attached between insulators, thence through choke coils and disconnecting switches to the high-tension breakers and to the high-tension bus. From the bus the current passes through disconnecting switches and breakers to the roof bushings and so out to the lines. A grounding switch is located near these roof outlet bushings for grounding the high-tension circuit. The electrolytic arresters are located on the roof.

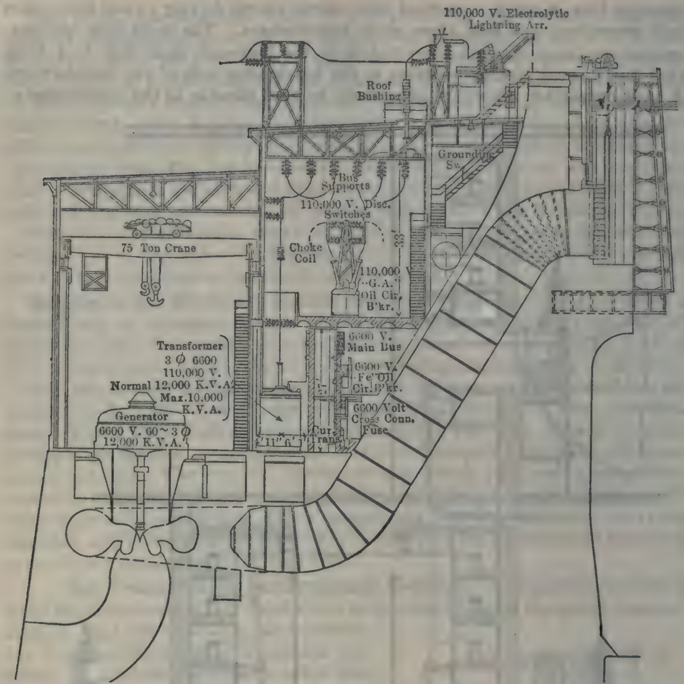

Fig. 7

SWITCHGEAR IN SEPARATE BUILDING. — In many cases it proves desirable to utilize a separate building for the transformers and switchgear. Fig. 8 shows a sectional view through a typical station of this kind with two 500-kw. 250-volt exciters, six 12,000-kv-a. 6600-volt three phase generators, six banks of transformers, and four 110,000-volt transmission lines. Advantage is taken of the natural slope of the ground as the plant is built on the side of a hill. The diagram for this station is shown in Fig. 9.

Bus Connections. — Each generator as shown in Fig. 9 is normally used with its own bank of step-up transformers. Three circuit-breakers, however, are provided with each transformer and generator group so that, if desired, any generator or any bank of transformers may be connected to the 6600-volt bus-bars. The low-tension neutral bus is provided with a grounding resistor and each generator is furnished with a single-pole, single-throw knife switch for connecting it to this grounding bus. In a similar manner the high-tension neutral bus is provided with a grounding resistor, and each bank of transformers is furnished with a single-pole, single-throw knife switch for connecting its neutral point to this ground bus.

Ventilation of Generators and Field Rheostats. — It should be noted that it is intended to locate directly opposite each machine a field-regulating panel containing electrically operated field rheostats and field switches, the

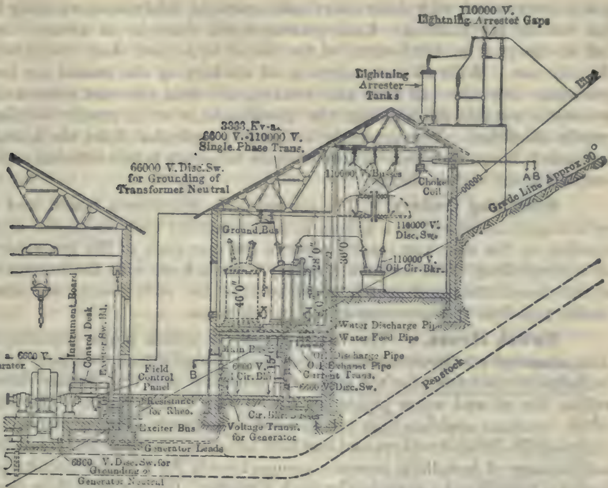

Fig. 8. Switchgear Arrangement for a 60,000-kv-a., 110,000-volt Hydro-electric Station

Plant Comprises

2-500 K.W. 250 V. Water Wheel Exciters, each large enough for exciting 6-10,000 K.V.A. Gens.

6-10,000 K.V.A. 6600 V. 514 R.P.M. 60 Cycle Gens. 13,000 K.V.A. 80% P.F. Continuous Maximum Rating

6-Banks each of 3-3333 K.V.A. 6600-110,000 V. O.I.W.C. 1 Ph. Trans.-delta-low tension, star high tension

4-20,000 K.V.A. 110,000 3 Ph. Transmission Lines

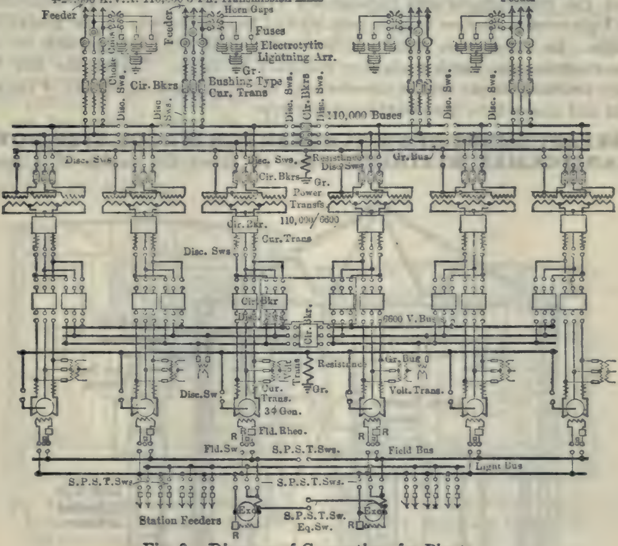

Fig. 9. Diagram of Connections for Plants

grid resistors used with these electrically operated field rheostats being located just outside the building. Each generator will draw in air around the shaft and discharge it at the bottom of the stator to a short duct connecting with the tail race. With this arrangement of discharging the heated air from the generators into the tail race and locating the field rheostat resistors of the generators outside the building, the question of securing proper ventilation is greatly simplified. The generator building is separated from the transformer and switch house by about 20 feet in order to secure better lighting and better ventilation for both buildings.

Switching Gallery. — The control desk, instrument board, and exciter switchboard are placed on a gallery in such a manner that the station attendant standing at the desk can readily observe the operation of the generator which he is controlling. The switching gallery is on a level with the basement floor of the transformer and switch house so that the switchboard attendant can readily pass into the basement of the switch house.

Switch House. — The basement of the switch house contains electrically operated oil circuit-breakers, bus-bars, disconnecting switches, series and potential transformers for the 6600-volt generator and transformer circuits, as well as the oil and water piping for the various transformers. The 6600-volt circuit breakers and bus-bars are inclosed in masonry compartments, but the 110,000-volt circuit-breakers and bus-bars are open, owing to the great difficulty and expense of installing masonry compartments for 110,000 circuits and the doubtful benefit to be obtained by such a course. On the main floor the transformers are arranged in one row and are mounted on cast-iron bases provided with wheels in such a manner that any transformer can be rolled out onto a low truck and then pulled outside the building, after the various water pipes, oil pipes, low-tension and high-tension connections have been opened.

In many cases, even for generating stations, it has been found advisable to place the transformers and high-tension switchgear out of doors controlling the high-tension devices from a switchboard located indoors in the generating station. In other cases due to local conditions it may be advisable to use indoor apparatus, particularly for moderate voltages. During the last few years, probably 80 per cent of transformers and switchgear for service of 100,000 volts and above have been of the outdoor type.

ARRANGEMENT OF SWITCHGEAR FOR INDOOR STEP-DOWN TRANSFORMER STATIONS. — Fig. 10 shows the section of a trans-

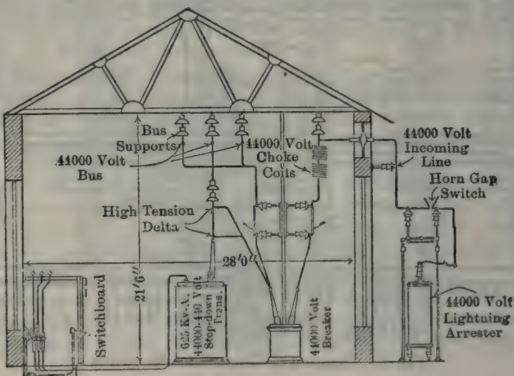

Fig. 10. 44,000-volt Step-down Transformer Substation

former station for the control of two 44,000-volt, 3-phase incoming lines, four banks, each of three 625-kv-a. single-phase step-down transformers, and a number of 6600-volt feeder circuits. The 44,000-volt lightning arresters with their horn-gap disconnecting switches are placed out of doors, and the circuit breakers, transformers, disconnecting switches, and busses are located indoors as shown. The incoming leads pass through porcelain bushings in the walls to the choke coils and through disconnecting switches to the 44,000-volt breakers. From these breakers the current passes through other disconnecting switches to the 44,000-volt bus-bar that is hung from the ceiling by means of suspension insulators. From this bus the current passes through other disconnecting switches to the circuit-breakers, and thence to the high-tension side of the step-down transformers.

Comparison of Top- and Bottom-connected Breakers and Closed and Open Wiring (Fig. 11). — (*See also Bus-bars; Circuit Breakers.*) In order to illustrate the difference in the design of the station made necessary by the use of bottom-connected breakers and inclosed bus-bars for the high-tension circuits, Figs. 11A, 11B, and 11C show three different designs of switching equipment for the control of 66,000-volt, 3-phase step-down transformers (each 10,000 kv-a.) supplying current to the two sets of 13,200-volt bus-bars.

Fig. 11A shows the general arrangement of the circuit breakers, bus-bars, connections, etc., using bottom-connected breakers of standard design and arranging to locate the 66,000-volt bus-bars with their disconnecting switches on the lower floor. In order to provide sufficient headroom for lifting the coils and iron out of a transformer case, it is necessary to slide the transformer into the passageway and run it along to the central portion of the building where the floor has been raised under the control desk in such a manner as to provide the necessary headroom. With this arrangement the total height of the building from the floor line to the roof girders is 47 feet 6 inches.

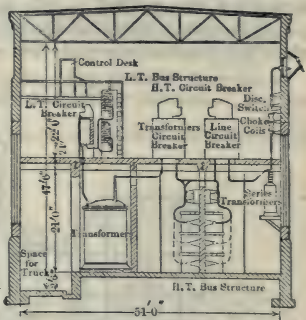

Fig. 11A. Bottom-connected Breakers, Inclosed H.T. Bus-bars, 66,000-volt Substation

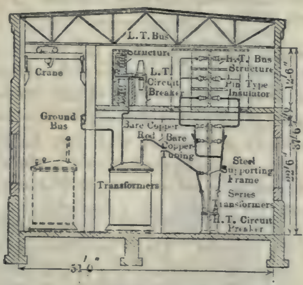

Fig. 11B. Top-connected Breakers, Inclosed H.T. Bus-bars, 66,000-volt Substation

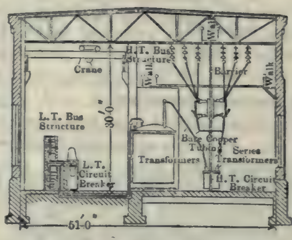

Fig. 11C. Top-connected Breakers, Open H.T. Bus-bars, 66,000-volt Substation

Fig. 11B shows the arrangement necessary if it is desired to use top-connected breakers and still inclose the 66,000-volt bus-bars. With this arrangement the 66,000-volt bus-bars, as well as the 13,200-volt circuit-breakers with their bus-bars, and the control desk, are placed on the upper floor, while the 66,000-volt breakers themselves with their disconnecting switches are located on the main floor near the transformers. With this arrangement any transformer can have its coils or iron removed as soon as it is slid into the passageway. The building arranged in this manner requires a height of 37 feet 6 inches from the floor line to the roof girders and requires a second floor the same as shown in Fig. 11A.

Fig. 11C shows the arrangement of this same station with top-connected breakers and open bus-bars and wiring for the 66,000-volt circuits. With this arrangement there is no necessity of a second floor. The height of the building is greatly reduced as the distance from the floor line to the bottom of the roof girders is only 30 feet.

ARRANGEMENT OF SWITCHGEAR FOR OUTDOOR TRANSFORMER STATIONS. — The use of outdoor transformers, switchgear and protective devices should be considered when designing high-voltage installations where it is essential to keep the first cost of the installation down to a minimum.

11,000-volt Station. — In Fig. 12 is shown an installation of three 50-kv-a., 11,000-volt, single-phase transformers with outdoor type expulsion fuse

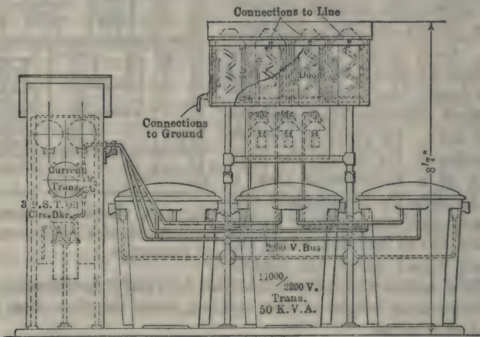

Fig. 12. Switchgear Arrangement for 11,000-volt, 150-kv-a. Outdoor Transformer Station

switches and non-arcing lightning arresters. These arresters are placed in a wooden box with a swinging door. The low-tension switchboard is placed in a similar wooden box with a glass door so that the readings of the ammeter, voltmeter and watthour meter can be taken without opening the switch box.

33,000-volt Installation. — A 33,000-volt installation with two banks of single-phase radiator type transformers and one spare transformer is shown in Fig. 13. All of the apparatus furnished is suitable for use later at 66,000-volts. Each transformer bank connects through its breaker to a high-tension bus sectioned between the two transformers, and each section of this high-tension bus supplies current to two outgoing feeder circuits. These transformers with their switching equipment are located immediately outside of a steam generating station and the low-tension breakers and control equipment are inside the station.

When installing outdoor equipment of this character the steel work has to be

regularly painted and it is advisable to obtain samples of the oil from the bottom and the top of the circuit-breaker tanks at regular intervals, and if any evidence

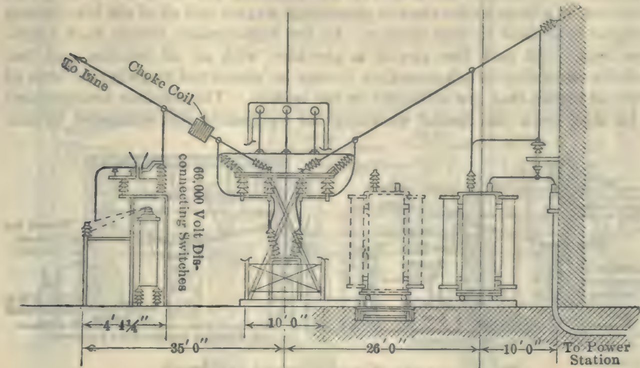

Fig. 13.—33-kv. Outdoor Switchyard

of an excessive amount of carbon appears in the oil, the oil should be immediately filtered or changed.

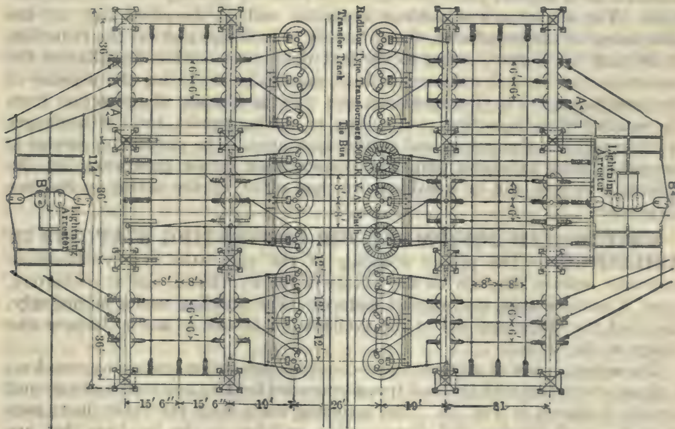

Fig. 14.—110-kv. Outdoor Switchyard

110,000-volt Installation. — Fig. 14 shows in plan view a typical 110-kv., outdoor transformer and switching station with four 110-kv. lines, six banks of transformers each of three 5,000 kv-a. capacity of the radiator type. At this station the low-tension switchgear is located in a switch-house but all of the high-tension apparatus is outdoor. The high tension leads from the transformers pass through disconnects and breakers to either of two sets of bus-bars.

154,000-volt Installation. — Fig. 15 shows the transformer and switch yard for an installation of six banks of step-up transformers each comprising three 7500 kv-a. units with four 154 kv. outgoing transmission lines. This yard is close to a large hydro-electric generating station and all of the low-tension switchgear is located in that station. In the high-tension circuit each transformer bank, and each line circuit is provided with an oil breaker and two sets of disconnecting switches so that it may be connected to either or both of two sets of busses that are sectioned in the middle. The outdoor arrangement with its steel work and special weatherproof equipment was estimated to cost

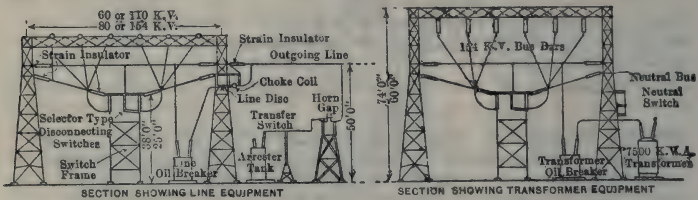

Fig. 15.—154-kv. Transformer and Switchyard

about 13 per cent less than the corresponding indoor equipment with the building to house it.

Advantages and Disadvantages of Outdoor Stations. — The foregoing illustrations of outdoor stations indicate the tendency of design for such installations. The advantages of outdoor apparatus such as described are: (1) the cheapening of the installation due to saving in building, and (2) the reduction in life and fire hazard, owing to the fact that in an outdoor installation the apparatus can be well scattered without materially increasing the expense of the installation. The disadvantages are: (1) the absence of protection from the weather when inspecting, overhauling, or making repairs, and (2) the danger of trespassers to themselves and to the apparatus. The outdoor *apparatus* is somewhat more expensive than the corresponding indoor apparatus, but the difference in cost will be usually more than offset by the saving in *building* investment.

SWITCHGEAR EQUIPMENT FOR CONVERTING STATIONS IN BUILDINGS. — The proper grouping of the apparatus in a synchronous-converter station depends on the voltage of the a-c. circuit, size of converters, type of transformers, and similar features. The building varies accordingly, provided the shape and size of the available lot is such as not to hamper the design of the station.

The sectional view of a synchronous-converter substation containing 1000-kw., 6-phase converters with air-blast transformers fed from 13,200-volt, underground circuits is shown in Fig. 16. The incoming leads from the cable ducts pass through an oil breaker and disconnecting switches to the bus-bars that are located on a gallery. Provision is made for an additional set of bus-bars and an additional set of disconnecting switches to be installed at a later date so that any breaker may be connected to either of the two sets of buses.

The circuits from these bus-bars pass back through other disconnecting switches and breakers to the high-tension terminals located at the bottom of the air-blast transformers. The low-tension leads from the transformers go to a starting panel provided with double-throw switches that permit low voltages to be impressed on the converter for the purpose of starting and full voltage

for running. The converters are provided with series fields on the negative side.

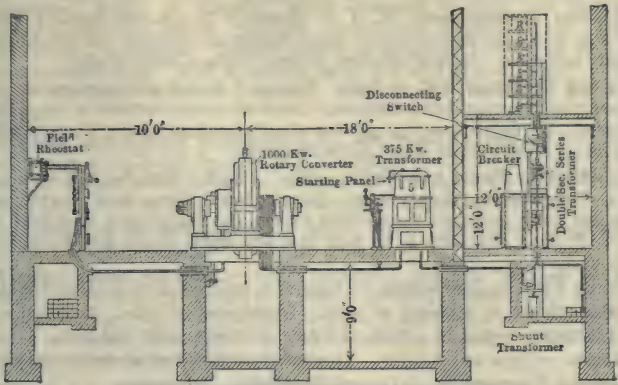

Fig. 16. Arrangement of Switchgear Equipment in Rotary Converter Station

The negative and equalizer switches are placed on a pedestal at the machine, and the negative and equalizer busses run on a bracket in the basement. The positive leads run to the panel board near the left-hand wall and the positive bus is located on the back of this board. The railway feeders are run out through underground ducts. All of the high-tension a-c. circuits are provided with electrically operated breakers controlled from the main switchboards. It will be noted that the entire design of this station hinges on the proper arrangement of the switching equipment.

SWITCHGEAR EQUIPMENT FOR PORTABLE CONVERTING STATIONS. — Many interurban electric railways have portable substations located in freight cars and arranged for ready transportation to whatever point requires their temporary service. A typical installation of this kind consists of a synchronous converter; oil-insulated, self-cooling transformers; and the necessary panel switchboard, high-tension oil circuit breaker, and lightning-protective devices. The apparatus may be so arranged in the car that the operator is convenient to the handle of the high-tension oil breaker, the panel switchboard, and the commutator end of the synchronous converter.

SPECIFICATIONS FOR SWITCHGEAR EQUIPMENT. — The great variety of conditions to be met and the numerous ways of meeting each condition in switchboard work make it impossible to write specifications in the standard form suitable for general use. This specification is therefore in the form of a series of memoranda to assist in writing a more specific one. (*See also general article on Specifications.*) The items are ararnged in alphabetical order.

Barriers. — Purpose. Material. Method of support. Dimensions.

Battery for Control. — Number of cells. Discharge rate. Style of rack. Protection of exposed copper from acid fumes. Condition of cells when handed over by contractor (whether charged or not).

Benchboard (*see Switchboard*).

Bus-bars. — Material. Conductivity. Style of joints. Minimum length of section. Position to be supported in. Protection, if required, in case of positive or exposed H. T. bus.

Bus-bar Insulators, H. T. — Type. Material. Style of clamps for bus-bars.

Bus-bars, Insulators and Supports, L. T. — Type. Materials.

Bus-bar Compartment, H. T. — Material. If concrete, whether monolithic or in blocks as described. State quality of concrete. If brick, state quality and color. Style of windows, if any. Style of doors, if any.

Circuit Breakers, D-C. — Style. Voltage. Rating and temperature rise. Description of current-break features. Overload reverse-current or low-voltage features. Device to sound gong when circuit breakers open. Current which may be ruptured without injury to the contacts.

Conduits in Floors and Walls. — Size. Finish. Shall be laid when contractor is notified that floor is ready. Junction boxes, if any, shall have interiors of stated finish and shall be provided with a neat metal cover flush with the floor. Style of pipe joint to be such that any single length may be removed without disturbing others.

Crane Service Connection. — Apparatus required and location. Wiring.

False Floors. — Material of slabs. Description of slabs. Description of piers for supporting slabs. Style of fastening between slabs and piers.

Gongs. — Type and location. To what apparatus they are to be connected.

Ground Connections. — General description. Dimensions.

High-potential Tests. — After installation, the apparatus and wiring shall be subjected to the potential tests specified in the following table, the tests being performed in accordance with the Standardization Rules of the American Institute of Electrical Engineers. (Give table of test voltages for different classes of apparatus and wiring, preferably from Section of the *Standardization Rules*, q.v.)

Instruments. — Type. Finish. Rating and description of scale. Accuracy at various parts of the scale to be within stated limits of accuracy. Location and style of terminals. Style, rating and temperature rises of shunts. Magnetic shielding.

Insulation (*see Insulating Materials*).

Lightning Arrester Equipment. — Type and purpose. Accessibility of all parts. Removability of all parts which may be injured by a stroke of lightning. Switches for disconnecting arresters from line. Description of ground plates and ground connections. Choke coils. Barriers.

Name Plates. — Description. Dimensions.

Relays. — Style. Location. Function.

Rotary-converter Starting Accessories. Method. Description of apparatus pertaining thereto.

Station Shunt. — Style. Location. Whether in positive or negative bus. Rating and temperature rise.

Switchboards. — Purpose. Material and color. General dimensions. Bench or upright. Number of sections per panel. Style of support. Sills, quality and finish (should not be painted if they are to be set in concrete). Size of bevel. Barriers: material, size and finish. Illumination.

Switches, H. T., Electrically Operated. — Style. Number of phases. Voltage. Rating and temperature rise. Accessibility of all parts. Description of concrete or brick compartment. Description of barriers between phases. Style of doors. Structure in base or elsewhere for disconnection switches. Voltage limits between which control apparatus shall operate successfully. Device to indicate positively at the controlling board whether the oil switch is open or closed. Device to sound gong when oil switch opens. Amount of oil to be supplied.

Switches, Oil, Hand-operated. — Style. Number of phases. Voltage. Rating and temperature rise. Location.

Switches, Knife or Toggle, Hand-operated. — Style and material. Number of poles and throws. Voltage. Rating and temperature rise. Quick break features, if required. Current density in metal and at contacts shall not exceed a stated value. With or without base, and style of base, if any. Front or rear terminals. Style of terminals. Long-handled hooks for h.t. knife switches.

Switches and Circuit Breakers, L.T., Electrically Operated. — Circuit breakers (*see also Circuit Breakers, D.C.*). Switches (*see also Switches, H.T.*). Device to indicate positively at control board whether switch is open or closed. Voltage limits between which control apparatus will operate sucessfully.

Transformers for Instruments. — Style. Ratio. Rating and temperature rises. Maximum permissible percentage error in ratio.

Wiring, Power Circuits. — Use. Location. Shall be clamped so that wires will not be dislodged should any joint or terminal become loose. Description of protection where cables pass through floors or walls. Tagging with number of circuit. Conductivity. Size or carrying capacity. Conductivity of clamps or terminals. Style of insulation.

Wiring for Control and Instruments. — Use. Location. Accordance with city ordinances and rules of "National Board of Fire Underwriters." Ends of each conductor shall be tagged with numbered tags of stated material and design fastened with brass wire. Wires which it is inadvisable to keep together shall be in different pipes as directed by the Engineer. Size. Style of insulation.

COSTS.— It is impossible to give reasonably accurate figures upon the cost per kv-a. of switch-gear equipment because of the large number of independent factors entering each case. such as type of station, voltage, number of generators, feeders, transformers and lines, single-throw or double-throw switching arrangements, etc.

BIBLIOGRAPHY. — See bibliography under *Switches*.

SYNCHRONIZERS AND SYNCHROSCOPES. — (*See also Alternating Currents; Converters, Synchronous; Generators, Alternating-Current; Power Stations; Substations; Switchgear Equipment for Power Stations.*) The two principal functions of any synchronizing device are to indicate (1) when the two circuits to which it is connected are operating at the same frequency, and (2) when the voltages of the two circuits are in phase with each other. The voltages of the two circuits must also be equal in effective value; this is usually determined by means of voltmeters connected to the two circuits. The process of synchronizing is simplified if the synchronizer indicates whether the machine to be synchronized is operating at a frequency lower or higher than that of the system to which it is to be connected.

The name "synchroscope" is usually applied to any device which, in addition to indicating synchronism, shows whether the machine to be synchronized is fast or slow (*see Standardization Rules of the A.I.E.E.*). The name "synchronizer" is used in a broader sense for any kind of synchronizing device.

Synchronizing Lamps. — The simplest form of synchronizer consists of two incandescent lamps shunted around the main switch which connects the machine to the bus-bars; see Fig. 12A, p. 669. Equality in frequency, voltage and phase between the bus-bars and the machine to be synchronized is then indicated by continued darkness of the lamps. For three-phase circuits a similar arrangement may be used, if the lamps are connected between corresponding phases of the two machines which are to be brought into synchronism with each other. If the machines are not in synchronism the frequency of pulsations of brightness of the lamps will be proportional to the difference of the frequencies of the two machines. The lamps do not show whether the machine is running too fast or too slow, nor is the point of exact synchronsim very definitely indicated, as it requires an appreciable voltage to cause the lamps to glow. The latter difficulty may be avoided in the case of single-phase machines by employing the connections shown in Fig. 12B, p. 669; when the lamps are thus connected they indicate synchronism when at maximum brightness.

It has been shown by Robbins (Elec. World, Vol. 70, p. 813, 1917) that synchronizing "dark" is to be preferred to synchronizing "light" in spite of the fact that incandescent lamps require an appreciable fraction of rated voltage in order to glow perceptibly. Tungsten lamps respond to a much smaller percentage of rated voltage than carbon lamps, and are therefore to be preferred for synchronizing dark, while carbon lamps are preferable for synchronizing light.

Use of Transformers with Synchronizing Lamps. — Transformers are frequently used in connection with the lamp. The arrangement with two transformers is to have the primary of one connected across the terminals of the machine, and the primary of the other connected across the bus. The secondary windings are connected in series with the lamp. The secondary connections may be arranged so as to have either darkness or maximum brightness at the instant of zero phase displacement between the bus-bar voltage and the voltage of the machine. Instead of using two separate transformers a single transformer with two primary windings and one secondary winding may be used; such a transformer is called a "synchronizing transformer." The primary windings are usually connected to give full voltage across the lamps at synchronism.

Siemens and Halske Three-phase Synchronizing Lamps. — Three similar lamps L_1, L_2 and L_3 are connected between the three terminals A_1, A_2 and A_3 of the machine and the bus-bars B_1, B_2 and B_3. Lamp L_1 is connected between corresponding phases, that is, between A_1 and B_1. Lamp L_2 is connected between A_2 and B_3 and lamp L_3 is connected between A_3 and B_2. Synchronism will be reached when L_1 remains dark; L_2 and L_3 will then be equally bright.

If the machine falls out of step, the lamps will successively grow bright and dark in rotation, no two lamps being in darkness or at maximum brightness at the same time, i.e., the light will appear to travel from one lamp to another at a definite speed and in a definite direction. The speed of rotation of the light is proportional to the difference of frequency between the voltage of the bus-bars and that of the machine. The direction in which the light appears to travel indicates whether the frequency of the machine is greater or less than that of the bus-bars. For high-tension circuits, potential transformers are required.

The electrostatic synchronism indicator differs from the preceding arrangement in using electrostatic "glowers" instead of lamps. These glowers are connected to the lines to be synchronized through the electrostatic capacity of suitable insulators. This arrangement dispenses with the use of potential transformers.

Synchronizing with a Voltmeter. — Instead of lamps, a dead-beat voltmeter is sometimes used for synchronizing. When connections are made for the voltmeter such as would be made for synchronizing "dark" with lamps, there is the difficulty that a-c. voltmeters are very insensitive at the lower part of the scale. To overcome this, it has been suggested by Keinath (E. T. Z., 1918, Vol. 39, p. 455), to replace the wire resistor of an a.c. voltmeter by a metal-filament lamp. Such a lamp when cold has only about one-tenth of the resistance when hot, and thus the sensitivity of the voltmeter for small voltages is greatly increased. When the machines are in opposition, so that the voltmeter is subjected to the sum of their voltages, the greatly increased resistance of the lamp limits the voltmeter current to a safe value.

Dial Synchroscopes. — Dial synchroscopes are similar in construction to power-factor indicators (q.v.), but are connected between the two circuits which are to be brought into synchronism. They are designed to indicate by a movable pointer whether the machine to be synchronized is running too fast or too slow, or whether it is in synchronism with the circuit to which it is to be connected.

Moving-coil Type. — A fixed coil is connected through a resistor across the bus; in polyphase circuits across one phase. The movable coil is connected through a condenser to the terminals of the machine to be synchronized, and is normally held in a position at right angles to the fixed coil by means of a spring. There will be no torque acting on the movable coil when the machine and bus voltages are either exactly in phase or 180° out of phase with respect to each other, because in either case the currents in the fixed and movable coils are in quadrature. When the two circuits are not in synchronism, the pointer attached to the movable element will swing back and forth over the scale. In order to distinguish the point of synchronism from that of 180° phase displacement, a translucent glass scale is placed in front of the pointer, the scale being illuminated by a lamp connected to the low-tension side of a "synchronizing transformer" which causes the lamp to be lighted when coincidence of phase between machine and bus occurs. Hence the pointer is visible only during every other swing, i.e., it will appear to rotate in one direction; the direction of rotation indicates whether the machine is fast or slow.

This instrument is applicable to single-phase as well as polyphase circuits, as it is usually unnecessary to indicate synchronism of more than one phase on any machine. The moving-coil synchroscope is ordinarily used with transformers, stepping the line voltage down to 110 volts.

Rotating Iron-vane Type. — This instrument is similar to the moving-vane type of power-factor meter (q.v.). The movable iron vane is magnetized by a stationary coil connected across the machine to be synchronized. A rotating field is produced by passing current from the bus-bars through a split-phase winding and two fixed coils placed nearly 90° apart. The pointer indicates

at any instant the phase displacement between the voltages of bus-bars and machine. The speed and direction of its rotation depends on the difference in frequency of the two circuits. When exact synchronism is reached the pointer will remain at rest in a vertical position. The instrument is usually connected through potential transformers; on polyphase circuits only one phase is used.

Lincoln Synchroscope. — This instrument has a rotating iron core carrying two coils placed nearly 90° apart; one coil is connected through a resistance, and the other through an inductance, both being supplied from the machine to be synchronized. A bipolar, laminated iron field is magnetized by means of a coil connected to the bus-bars. The operation of this instrument is similar to that of the moving-coil type of single-phase power-factor meter. A pointer attached to the moving system indicates at every instant the difference of phase angle, as well as the approximate difference of frequency, between the voltage of the bus-bars and that of the machine to be synchronized. The current has to be led into the armature coils through slip rings as the armature is free to revolve. These instruments are furnished for direct connection to circuits of 110 or 220 volts; other circuits require transformers.

Weston Synchroscope. — This consists of an electrodynamometer movement having the fixed coils connected to the buses through a non-inductive resistance and the moving coil to the incoming machine through a condenser. When the voltage of the machine is either in phase with or in opposition to the bus voltage no torque is exerted on the moving coil, and the pointer is vertical. For any other phase relation the pointer will be deflected to right or to left, when the incoming machine is ahead or behind. The pointer is back of a translucent screen, and only its shadow, cast by a synchronizing lamp, can be seen. Thus when the shadow is seen to be vertical, the incoming machine is in phase, and when the machine is too fast or too slow the shadow appears to rotate in the clockwise or counter-clockwise direction respectively.

Use of Reactance in Synchronizing. — Synchronous machines may be connected to the bus through suitable current-limiting reactors, and brought up to speed by an auxiliary motor. They will synchronize after a few oscillations. With large machines, considerable current rushes will occur when the reactors are short-circuited. Osnos has described (Elektrotechnik und Maschinenbau, Vol. 19, p. 29, 1917), the use of reactors having an auxiliary d-c. winding which may be connected in the field circuit. The field current is very small at starting, and as it increases the d-c. flux in the reactors reduces their reactance gradually to a value permitting the machine to synchronize as it approaches normal voltage. The reactors may then cut out of circuit without disturbance.

COSTS. — A moving-coil synchroscope costs approximately $80, a 9-inch moving-vane synchroscope costs approximately $70; a Lincoln synchroscope for voltages of from 110 to 220 costs from $55 to $65; transformers and accessories are extra in all cases. Only one synchroscope or synchronizer is required in a station, although a spare instrument is usually installed. (1922 prices.)

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TELEGRAPH INSTRUMENTS AND APPARATUS. — (*See also Radio Communication; Telegraph Lines; Telegraph Systems.*) The more important instruments and apparatus used in the various systems of telegraphy are described below.

KEYS. — A telegraph key is essentially a switch for opening and closing a circuit. Various types of keys are in use.

Standard Morse Key (Fig. 1). — The knob end of the lever is normally held raised so as to open the key by an adjustable coiled spring. The auxiliary switch arm constitutes the "circuit closer" by which the circuit of the line is kept closed, through the key, while the key is not in use.

Fig. 1. Standard Morse Key

Double-acting Key. — While the key of the general type illustrated in Fig. 1 is still standard, several later forms, having a sidewise rather than an up and down motion of the lever, have come into some vogue. One type of these, known as the double-acting key, has its operating lever arranged to swing in a horizontal arc between two stationary contact points. The key lever normally stands clear of both points, leaving the circuit open. A movement either to the right or left will close the circuit.

Vibroplex. — Another type of key is represented by such instruments as the Vibroplex, which are more properly semi-automatic transmitters. The motion of the key lever in this is similar to that of the double-acting key just described, the lever normally standing in the middle or open position. When the key is pushed to the left, it closes the circuit in the ordinary way to make a dash. When pushed to the right, a vibratory reed is set in motion, which opens and closes the circuit rapidly to form a succession of dots, the number of dots being controlled by the length of time the key is held in this position. By this means the operator is relieved of the muscular effort of rapidly vibrating the key to form dots.

Operator's Paralysis. — The objects of the double-acting key and of the Vibroplex and other similar transmitters are, in the main, twofold: To relieve or prevent the malady known as "operator's paralysis" or "loss of grip," which sometimes follows long continued use of the ordinary key; and to increase the speed of transmission.

AUTOMATIC TRANSMITTERS. — (*See also Telegraph Systems.*) The transmitters in automatic systems usually employ a paper tape, perforated in accordance with the code to be sent. This tape, so prepared, is caused, by means of revolving rollers, to pass rapidly through the transmitter, the perforations serving as they pass to permit electrical contacts to be made which send the proper impulses to the line.

SOUNDERS (Fig. 2). — A standard form of sounder is shown in Fig. 2. The winding of the electromagnet is connected directly between the binding posts shown on the base of the instruments. The armature is attached to a lever mounted on trunnions in a fixed support. The adjustable stops provided for the lever permit a slight up and down movement of its free end, the downward movement being caused by the pull of the electromagnet, and the upward one by the force of an adjustable retractile spring. The rugged construction of the armature lever and its co-operating parts is required to produce the necessary loudness and quality of sound.

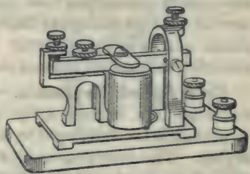


Fig. 2. Morse Sounder

This, with the heavy retractile spring necessary for the prompt return movement of the lever, makes the instrument require more energy for its operation than can usually be delivered to it directly over the line wire. On this account the sounder magnet is usually placed in a local circuit, which is in turn controlled by the more sensitive relay, the magnet of which is placed directly in the line.

When used in local circuits sounders usually have a magnet resistance of about 4 ohms, and require about ¼ ampere for their proper operation. Where the sounder magnet is placed directly in the line, it is termed a main line sounder, and differs in no respect from the local sounder except that its magnet is wound to about 20 ohms.

RELAYS. — Two kinds of telegraph relays are employed; the ordinary or Morse relay, and the polarized relay. The armature of a Morse relay always moves in the same direction irrespective of the direction of the current impulse, while the armature of a polarized relay moves in one direction for a positive impulse, and in the opposite direction for a negative impulse.

Morse Relay (Fig. 3). — The Morse relay has the same essential parts as the sounder, with the addition of a pair of contacts closed by the armature lever when in its attracted position. The armature lever is usually mounted vertically, and is made light in construction so as to be capable of the necessary rapid to and fro movements with a minimum expenditure of energy. The retractile spring is light and delicately adjustable. An extra pair of binding posts is added to form the terminals of the local circuit, these binding posts being wired to the armature and the magnet frame, respectively, so that the circuit between them will be closed upon the attraction of the armature. The back contact of the relay is non-conducting, so as to avoid closing the circuit when the armature is released. A typical form of relay, known as the "pony relay," is shown in Fig. 3.

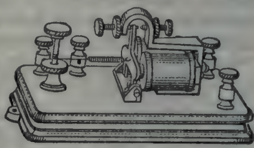


Fig. 3. Morse Relay

Polarized Relay. — A polarized relay is similar to a Morse relay except that the end of the armature which plays between the soft-iron pole pieces is permanently magnetized. For example, if this end of the armature is a north magnetic pole, then, obviously, the armature will be moved in one direction or the other, according as the magnetizing force of the coil tends to set up a south pole in one or the other of the poles. The polarity of the poles, of course, changes with the direction of the current in the coil.

RECORDERS OR REGISTERS. — (*See also Telegraph Systems.*) Recorders or registers are used only where for some reason it is desired automatically to record a message. Simple forms of comparatively slow speed registers are used in connection with district telegraph messenger and fire alarm telegraph service. More elaborate high speed recorders are used in connection with automatic and printing systems. A special form of recorder, known as a siphon recorder, is used on submarine lines.

Slow Speed Registers. — The paper upon which the message is to be recorded is a narrow strip of tape, carried on a reel. A clock-work motor is provided for moving the tape at uniform rate under the recording pen or stylus. The receiving magnet of the register is placed in a local circuit controlled by a relay in the line, and this magnet, when attracted, serves either to move the pen or stylus against the tape or the tape against the pen or stylus, in either case resulting in a mark upon the tape. Some registers record by merely embossing the tape; others by marking thereon with a pen or pencil; and still others by actually punching holes in the tape.

High Speed Recorders. — In very high speed automatic and printing systems receiving devices have been developed giving a record in ink on ordinary paper, or a record on a sensitized paper, or in ordinary print. For a description of the more important printing recorders see below under *Telegraph Systems*.

Chemical Recorders.—For very high speed work with non-printing telegraphs the chemical recorder has been generally found the most satisfactory. In this the tape is impregnated with a solution of some unstable chemical compound which will easily be decomposed upon the passage through it of an electric current, and which, either by changes within itself or by combination with the material of the pen through which the current passes, will cause a definite discoloration of the tape upon the passage of the current.

A common sensitizing solution is formed of 5 parts of prussiate of potash, 150 parts of ammonic nitrate, and 100 parts of water. Under electric decomposition, this solution sets free an acid which attacks the iron of the pen, leaving a blue mark on the paper.

Siphon Recorder. — The high electrostatic capacity of long submarine cables makes necessary some modification of the methods ordinarily employed in overland work. This high capacity results in the variations in the signaling current being less sharply defined than on land lines. The receiving apparatus for long cables must, therefore, be responsive to minute variations in current to such an extent as to preclude the use of the responsive devices employed on land lines.

The siphon recorder is merely a special form of recording galvanometer. The galvanometer proper is of the D'Arsonval or swinging coil type. The motion of the coil is imparted to a very light glass tube bent to form a siphon and suspended with its long end opposite the receiving tape and its short end dipping into a small tank of ink. The siphon moves across the width of the tape, the contact with the tape being agitated so as to be nearly frictionless. Displacements in this record line to the left of an imaginary zero line represent dots, those to the right, dashes. The Continental code is universally employed when a siphon recorder is used.

TELEGRAPH REPEATERS. — When, for any reason, it is desired to transmit messages from one line to another line without manual retransmission, the telegraph repeater is used. By its use lines of excessive length may be avoided, and a consequent increase of speed may be attained. Also, the repeater is often useful in automatically transmitting messages from a through circuit to a side circuit.

Non-Automatic or "Button" Repeater (Figs. 4 and 5). — A fundamental conception of the repeater may be gained from Fig. 4, where a relay magnet

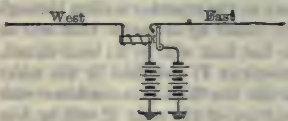


Fig. 4.

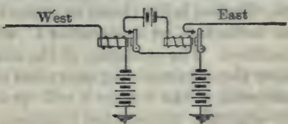


Fig. 5.

placed in one line causes an armature to act as a key to make and break the other line. This conception may be carried a step farther by considering Fig. 5, where a relay magnet in the West line receives impulses from the distant West station, and controls a local circuit containing a battery and a second relay magnet, which latter causes its armature to act as a key in transmitting signals

to the East line. From these two figures it is obvious that such a repeating scheme would transmit in one direction only, providing no facilities for transmitting from East to West.

The "button repeater" of early telegraph history consisted practically of two such sets of relays as are shown in Fig. 5, these sets being arranged for transmission in opposite directions. Only one set of relays could be kept connected to the line, for when both are connected in, the impulses sent out from the repeating station on, say, the West line react on the line relay of that line at the repeating station in such way as to act through its local circuit to send impulses back on the East line. This difficulty was overcome in the button repeater by providing a button switch, operated by an attendant at the repeater station, this switch being thrown manually in one direction or the other, according to the direction of transmission desired.

Automatic Repeaters. — The necessity of an attendant for the purpose of working the switch of a repeater has long since been overcome by the production of the so-called automatic repeaters, of which there are many forms. In these, the arrangement is such as to automatically prevent the transmitter of the line that is being repeated into from reacting to make and break the line over which the signals are initially sent.

The Milliken Repeater (Fig. 6). — This is one of the most common of the automatic repeaters for the simple Morse system. Its action may be

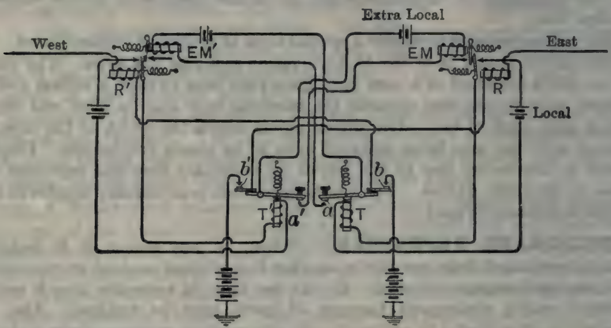


Fig. 6. Milliken Repeater

understood by reference to Fig. 6. If the line East is sending, the relay *R* will respond to the make and break impulses of the key at the distant station on that line, this line being closed to ground and battery at the repeater station through the contacts *b′* on transmitter *T′*, which is held closed by an action that will be described. The operation of the relay *R* of the line East will, by its local circuit, cause the corresponding operation of the transmitter *T*, and this, by the make and break at the tongue *b*, will open and close the line West in accordance with the originally transmitted impulses. The armature of the relay *R* will be free to work, because as long as the magnet of transmitter *T′* remains energized it will hold the circuit of the extra magnet *EM* closed at the point *a′*, and thus will hold the armature of that magnet out of the way of the armature of the relay *R*. The extra magnet *EM′*, associated with the relay *R′* of the line West, prevents this relay from breaking the West line in response to the repeated signals, for the following reason. When the line West is closed by the transmitter *T*, the current through the relay *R′* holds its armature attracted. When

the line West is opened, however, by the transmitter T, the circuit of the extra magnet EM' is also opened by this transmitter, and the retractile spring of the extra magnet prevents the armature of the relay R' from falling back. In this way the line relay of the West line cannot react on its own transmitter to cause makes and breaks in the transmitting line. When the operator at the distant station on the West line breaks, the relay R' is then permitted to open and thus cause the transmitter T' to open the line East.

Duplex and Quadruplex Repeaters. — Automatic repeaters are more simple for duplex and quadruplex systems than those already described for "single" wire working. The reason for this is that in the duplex or quadruplex the transmission from a given sending operator is always to a corresponding receiving operator, and — as between these operators — it is never in the reverse direction. Hence, a one-way repeater only is required for any one of the several simultaneous transmissions. It therefore suffices to place the transmitter at the repeating station under the control of the neutral or polar relays of the duplex or quadruplex, the repeating operation for each set being in one direction only.

CURRENT SUPPLY. — Either a battery or a dynamo may be used to supply the current required for telegraph transmission.

Gravity Battery. — The gravity battery was once almost universally used for supplying current in telegraph systems. It is still largely used on unimportant lines and in small offices where it is not economical to employ machine generators or secondary batteries.

Storage Battery. — The storage battery, on account of its extremely low internal resistance, is capable of handling any desired number of lines in parallel, and this led to the use of the so-called "universal battery system," in which a single battery, with one pole grounded, serves the line wires which are tapped off from the other pole. Suitable resistances are included in the lines of lower resistance, if desired, to compensate for the variation in resistance among the lines. One battery, thus used, would not suffice for an office, however, since with the various systems of duplex and quadruplex working it is necessary to send currents of alternate polarity to line, thus making necessary the use of another battery with its opposite pole grounded. Usually also a reserve is provided for each of these batteries, so that one can be used while another is being charged.

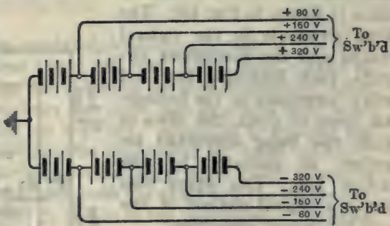


Fig. 7. Storage Battery for Current Supply

In Fig. 7 is shown an arrangement of storage battery supply that provides for gradation of voltage of either polarity in an obvious manner. By using a duplicate of this arrangement, one set may be used while the other is being charged.

The Dynamo. — The dynamo has generally supplanted both the primary and secondary batteries in all offices large enough to warrant its use. In Fig. 8 a dynamo arrangement employed by the Western Union Telegraph Company is shown. In this five machines constitute a set, their armatures being connected in series, taps being made as indicated to give the required voltages. All of these machines are separately excited except the fifth, which

latter is a shunt machine and furnishes current not only for line supply but also for exciting the fields of all the other machines. Each field circuit is provided with a separate rheostat *R* for varying the field strength. Another set like this is employed to furnish currents of the opposite polarity, and still another set of five machines is employed as a reserve, this being so arranged that

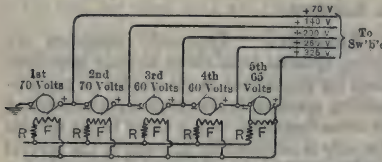


Fig. 8. Western Union Dynamo Arrangement

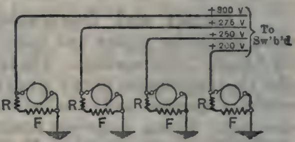


Fig. 9. Postal Dynamo Arrangement

it may be made to generate either positive or negative according to which of the regular sets it is to replace. The dynamo arrangement employed by the Postal Telegraph-Cable Company is shown in Fig. 9.

TELEGRAPH SWITCHBOARDS (Fig. 10). — Telegraph switchboards were the precursors of the vastly more complicated telephone switchboards. The telegraph switchboards which have been most commonly used in America are of two types — the "strap and disc" and the "cross-bar."

A simple "strap and disc switchboard" with its connections is shown in Fig. 10. The lines terminate in the vertical straps and the instrument circuits, each including key and relay, terminate in the horizontal rows of discs, the discs in each row being connected together, as indicated by the dotted line. This figure also shows a typical method of line and instrument protection.

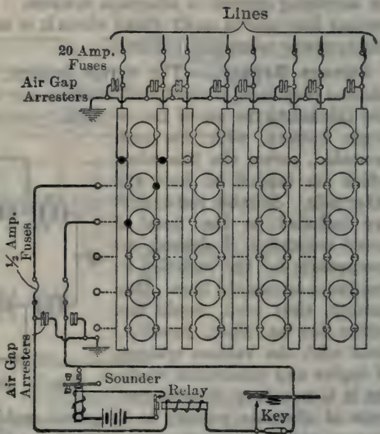


Fig. 10. Strap and Disc Switchboard

COSTS (Pre-war figures).— The following prices were current prior to the World War, standard Morse key, $2.20; double-acting key, $10.00; Vibroplex, $12.00; standard Morse sounder, $3.00; standard Morse relay, $7.00; polarized relay, $20.00; ink recorder, $35.00; punching recorder, $75.00. See also *Batteries*, *Generators*, etc.

BIBLIOGRAPHY. — See Bibliography in article on *Telegraph Systems.*

TELEGRAPH LINES. — (*See also Poles and Cross-Arms; Telephone Lines; Transmission Lines; Wires and Cables.*) The conductor over which transmission is effected is commonly referred to as the line. These conductors may exist as open wires, or they may be disposed in aerial, underground and submarine cables.

OPEN-WIRE LINES. — Most of the open-wire telegraph lines are of hard-drawn copper or of galvanized iron. Recently a wire known as copper-clad steel, consisting of a core of steel wire with an outer shell of copper welded thereto, has appeared as a competitor. For the properties of these wires see *Wires and Cables, Bare.*

Hard-drawn copper wire is vastly superior to iron in point of conductivity and in its ability to withstand the action of the elements. Galvanized-iron wire is advantageous in point of first cost, it has a somewhat greater strength, but has a life ranging from 4 to 20 years, according to atmospheric conditions. Copper-clad steel has a conductivity superior to that of iron, but inferior to that of copper. It is stronger and somewhat cheaper than hard-drawn copper, and should possess the same ability as copper to indefinitely withstand the action of the elements.

The mile-ohm values (i.e., the weight in pounds of a wire one mile long having a resistance of one ohm) of the various grades of wire used for telegraphic purposes are as follows: E.B.B. iron, 4700 pounds; B.B. iron, 5500 pounds; steel, 6500 pounds; hard-drawn copper, 895 pounds; copper-clad steel (having conductivity 40 per cent of solid copper), 2010 pounds.

No. 9 B. & S. gauge hard-drawn copper wire, weighing 209 pounds per mile, is more used than any other size of hard-drawn copper wire for telegraphic purposes; and No. 8 B. W. G. galvanized-iron wire, B.B. grade, weighing 378 pounds per mile, is the principal standard in iron wire.

AERIAL CABLES. — The paper-insulated, lead-covered cable of the type employed in telephony (*see Telephone Lines*) but with single wires instead of twisted pairs, is rapidly supplanting the old form of rubber-insulated cable formerly used in telegraphy. Sometimes the cores are saturated with insulating compound, and sometimes they are left "dry," according to whether the consideration for greater safety outweighs the desirability for lower capacity. For aerial work such cables are supported from messenger wires, either by means of clips or by a wrapping of marline around both the cable and the messenger, the latter form of support being applied by means of the well-known "Spinning Jenny."

UNDERGROUND CABLES. — (*See also Wires and Cables, Insulated.*) Underground telegraph cables are of the same type as those used for aerial. The practice of placing them in ducts extending from manhole to manhole, so as to facilitate their withdrawal, has largely replaced the older practice of burying the cable in such manner as to necessitate its being dug up for replacement or repairs.

SUBMARINE CABLES. — For comparatively short lengths, in crossing rivers or bays where the current is not swift and the bottom is of such a nature as naturally to embed the cable, ordinary multiple-conductor, lead-covered cables, with either paper or rubber insulation, provided with a heavy braiding over the sheath, are often employed. Where paper insulation is used, it is customary in telegraphic work to thoroughly impregnate the core so as to minimize the bad effect of moisture; but this increases the electrostatic capacity of the conductors.

Where great tensile stresses or abrasion are likely to occur, the core, after being provided with its lead sheath, is served with a thick cushoning layer of tarred jute, and then with an armor of closely wrapped galvanized-steel wires, over which an additional protecting layer of tarred jute is sometimes placed.

Long Submarine Cables. — For long submarine cables, such as those across the Atlantic Ocean, a single conductor is employed, this being in the form of stranded copper, weighing from 70 to 650 pounds per nautical mile. This conductor is then encased in several layers of gutta-percha. The insulated core, so formed, is protected with a thick layer of jute, outside of which the armoring of galvanized-steel wires is placed. After being sheathed, the whole cable is sometimes covered with a thick layer of tarred tape. The shore ends of cables, where there is greater liability to mechanical injury, are armored with a layer of very heavy galvanized-steel wires.

LIMITING TRANSMISSION DISTANCE. — The length of the line over which a signal can be transmitted depends upon the strength of the current impulses sent out, the sensitiveness of the receiving instruments, the resistance and capacity of the line and upon the number of stations connected to the line.

Open Wire Lines. — The following table, compiled by Mr. F. F. Fowle, shows the result of his calculations as to the limiting lengths for through working (no intermediate stations) of simplex, duplex and quadruplex Morse transmission with standard apparatus over open wire lines with ground return having different resistances per mile, and also the maximum line distances for way circuits equipped with 35-ohm line relays.

Resistance per mile in ohms	Limiting length of line for Through Working, miles			Limiting length of Way Circuits, miles		
	Duplex	Simplex	Quadruplex	A*	B*	C*
2	783	597	531	597	391	313
3	658	510	442	510	363	299
4	580	450	386	450	341	286
6	485	376	313	376	305	265
8	425	331	268	331	280	248
10	384	299	236	299	260	234
15	318	248	186	248	225	208
20	278	217	156	217	201	188
25	250	195	135	195	183	174
30	229	179	120	179	170	162
40	200	156	98.7	156	150	144
50	180	140	84.3	140	...	...

* *A*, stations at ends of line only, *B*, stations every 10 miles; *C*, stations every 3 miles.

CONSTRUCTION OF LINES. — (*See Telephone Lines.*)

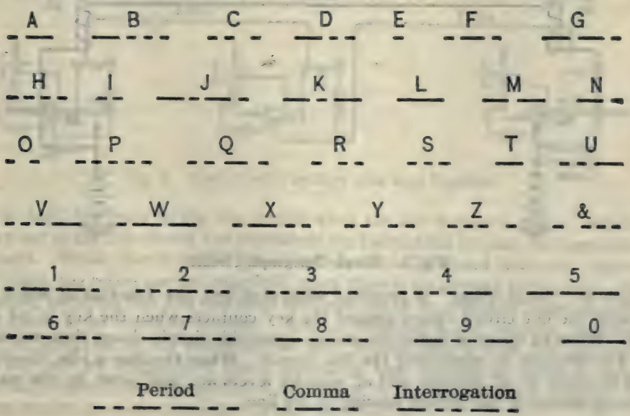
BIBLIOGRAPHY. — See Bibliography in article on *Telegraph Systems*.

TELEGRAPH SYSTEMS. — (*See also Telegraph Instruments and Apparatus; Telegraph Lines.*) The following is a brief table of contents of this article:

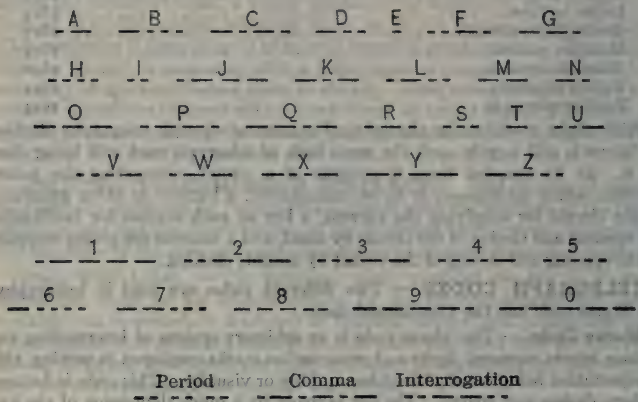
With slight modification the original system of electric telegraphy devised by Morse is still largely used, the great bulk of telegraph work still being done by it. In its simplest form it consists of a circuit, connecting the points between which intelligence is to be transmitted, a battery or other generator in the circuit for supplying the current, a key at each station for facilitating the opening and closing of the circuit by hand, and a sounder for giving response to the opening and closing of the circuit by audible clicking.

TELEGRAPH CODES. — Two different codes are used in telegraphy, the Morse code and the Continental code.

Morse Code. — The Morse code is an arbitrary system of interpreting the various letters, numeral digits and punctuation marks employed in writing, this interpretation being intelligible either audibly or visually. This code is formed of dots, dashes and spaces. The dot is made by a quick depression of the key followed by its immediate release; the dash by a depression of the key with a delayed release; and the space by permitting a short interval to elapse between the completion of two dots. Thus, the letter *a* is represented by a dot and a dash, and it is formed by depressing the key, immediately releasing it, depressing the key again and holding it a slight interval before release. Similarly, the letter *i* is represented by two dots, and it is formed by depressing and releasing the key twice in rapid succession. The letter *o* is an example of a so-called space letter, and is represented by dot-space-dot. It differs from the letter *i* only in that an interval of time is allowed to elapse between the two dots. The Morse code, as it has been employed practically without modification since the time of Morse, is as follows:



Continental Code. — The so-called Continental Code differs from the Morse code in that it is composed entirely of dots and dashes, employing no spaces. It is almost universally used for submarine lines, for land lines in nearly all countries except the United States, and is the world's standard for radio telegraphy. This Continental or universal code is as follows:



SIMPLEX OR MORSE SYSTEM. — There are two types of circuits used, the so-called "open-circuit" system and the "closed-circuit" system. In both cases but one line wire is employed, the earth forming the return circuit.

Closed-circuit System (Fig. 1 and 2). — This system is mainly used in the United States. In its simplest form the keys and the sounder magnets are placed in series in the line with a source of current, as shown in Fig. 1. A

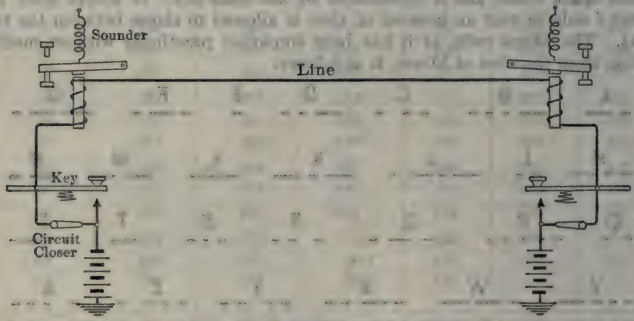


Fig. 1. Simple Telegraph Circuit

switch or "circuit closer" is provided with each key, the function of which is to keep the line circuit closed around the key contacts when the key is not in use. When an operator begins to send he opens this circuit closer, thus making his key effective in controlling the line circuit. When the line is idle, the circuit is closed at all stations, and all sounder levers are held down by the magnetizing effects of the current flowing in the line.

In Fig. 1 but two stations are shown, one at each end of the line. Obviously intermediate or way stations could be added by merely placing the necessary keys and sounders in the line circuit at the desired points.

A more common practice is to employ a line relay at each station as the device directly responsive to the line currents, using this line relay to control a local circuit containing a local battery and a sounder. The reason for the choice of this slightly more complicated system for commercial practice is that the sounder, in order to make the necessary amount of noise, is required to be of a rather sturdy nature and is not as readily responsive to the comparatively small line currents as is the relay, which, having no other function than to be responsive,

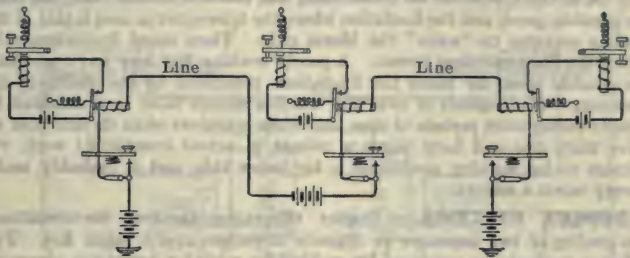


Fig. 2. Closed Circuit System with Way Stations

is readily made of the required sensitiveness. A line equipped for three such stations is shown in Fig. 2. While the constant flow of current in the closed-circuit system results in the use of a greater amount of current than would otherwise be necessary, it has the advantage of keeping the line under test at all times and of not requiring a line battery at each of the stations, as is the case with the open-circuit system.

Open-circuit System (Fig. 3). — This is largely used abroad. A battery is required at each station. Each key has a front and back contact so arranged

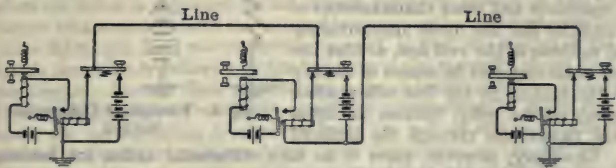


Fig. 3. Open Circuit System with Way Stations

as to hold the relay in the line circuit when the key is raised, and to cut the relay out of the line circuit and substitute the battery for it when the key is depressed. While, therefore, the line circuit is normally closed from one end to the other, the relays are not energized, since there is no current in the line. As a result, both the line and the local batteries stand in normally open circuits, and there is no waste of current. Besides requiring a line battery, sufficient for the operation of the entire line, at each station, this system does not hold the line and its instruments under automatic test conditions, which is automatically done by the constant flow of current in the closed-circuit system. A three-station open-circuit system is shown in Fig. 3.

Sound Reading.—In the first use of Morse telegraphy the received characters were recorded on a moving tape and subsequently translated. It was found in practice, however, that the operators soon learned to read what was coming over the wire by the clicking of the instruments, and, although this method of sound reading was condemned at first as being dangerous, it has survived to such an extent that the major portion of the telegraph work of the world is to-day received in this way.

Speed of Handling Messages by Morse System. — The speed of handling messages by Morse was limited by the speed of the receiving operator in writing down the message, as long as this was done in ordinary handwriting. The use of the typewriter for transcribing has, however, considerably increased the possible speed, and the limitation where the typewriter is used is that of the sending operator to "write" the Morse code. These speed limitations exist where the line conditions permit. A poorly working line may serve to reduce the possible speed far below the limitations prescribed by the ability of the operators. While speeds of over fifty words a minute have been attained by experts during short periods of time, and while operators often do send and receive thirty-five and forty words a minute in actual commercial work, the average under working conditions is far below this, and is probably under twenty words a minute.

DUPLEX SYSTEMS. — Duplex telegraphy involves the simultaneous sending of two messages in opposite directions over a single line. Two different methods are employed for making the receiving apparatus unresponsive to the home key. These two methods are termed the "differential" and "bridge" methods, respectively. There are also two different methods of sending impulses over the line, one of which depends on variations in current strength and the other on changes in current direction.

Differential Method (Fig. 4). — The differential method is readily understood from a consideration of Fig. 4, which shows a line relay having two windings in opposite directions. Current from the battery is supplied through one of these windings to the line, and through the other of these windings to the artificial line represented by a resistance at the left. The resistance and other characteristics of the artificial line are made approximately equal to those of the real line, so that impulses of current sent by means of the key from the battery to the line will produce no effect on the relay, because an equal current passes also through the artificial line. Obviously, however, there is no such differential action for incoming currents, and, therefore, the relay is responsive to currents from the distant station.

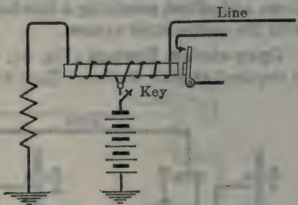


Fig. 4. Principle of Differential Method

The differential method is ordinarily employed in land telegraphy.

Bridge Method (Fig. 5). — The bridge method of rendering the home relay unresponsive to the home key and responsive to the distant key is illustrated in principle in Fig. 5. In this the resistances shown are so proportioned with respect to the resistance of the line that the line relay will receive no current upon closure of the key, for the

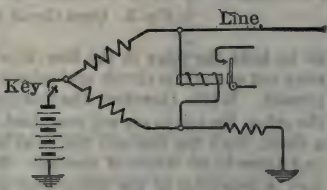


Fig. 5. Principle of Bridge Duplex

same reason that the galvanometer in a Wheatstone bridge does not respond when the bridge is balanced.

The bridge method finds its chief use in overland and in submarine telegraphy.

Stearns Duplex for Land Lines (Fig. 6). — The Stearns duplex for land lines operates on the increase and decrease in current strength, without changing the direction of current flow; and for the reason that it employs the differential method of making the home relay unresponsive to the home key, it is commonly referred to as the Stearns differential duplex system.

A simplified diagram of the Stearns differential duplex is shown in **Fig. 6.** It is to be noted that the batteries at opposite ends of the line have their opposite poles grounded, so that whether one or the other, or both, of the batteries are

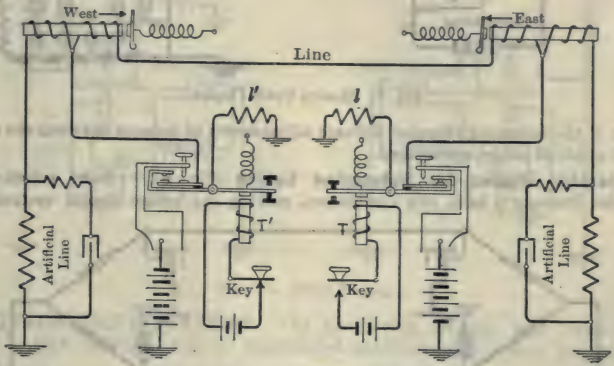


Fig. 6. Stearns Differential Duplex

in the circuit the current flows in the same direction. The key controls a local circuit of an electromagnetic transmitter T or T'. The tongue of each transmitter normally engages the contact on the under side of the bend in the lever; when the latter is attracted by its magnet upon the closure of the key, the tongue makes contact with the adjustable screw on the standard, and this prevents the tongue from moving farther upward, so that the making of this contact is immediately followed by the breaking of the normally closed contact of the tongue. Because these transmitters make one contact before breaking the other, they are called "continuity-preserving transmitters." When the key is open, the corresponding transmitter keeps its end of the line connected to ground through the resistance l, but upon the closure of the key the transmitter causes the substitution of the battery at that end of the line for this resistance. This resistance l sometimes is called the "spark coil," since its use tends to prevent sparking at the transmitter contacts, and its function is to compensate for the internal resistance of the line battery.

The artificial line shown at each station is composed of resistances and condensers arranged for ready adjustment, so as to balance not only for the resistance of the line, but for its capacity also.

Stearns Duplex for Submarine Cables. — Fig. 7 shows the Stearns duplex arrangement of terminal apparatus for cable working. This is a bridge method. The normal position of the keys grounds the cable conductor, thus discharging it. Alternately depressing the two keys gives the cable alternate negative and positive charges, thus sending dots and dashes. By moving the

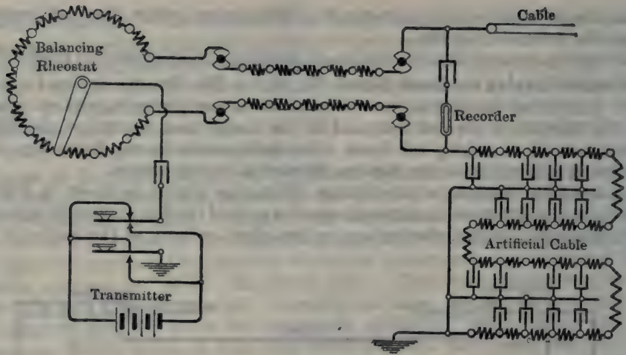


Fig. 7. Stearns Cable Duplex

arm of the balancing rheostat, delicate adjustments of balance between the real and artificial cables are secured.

Jacobs' Duplex for Submarine Cables (Fig. 8). — Short submarine cables frequently have two conductors, making the bridge duplex system of

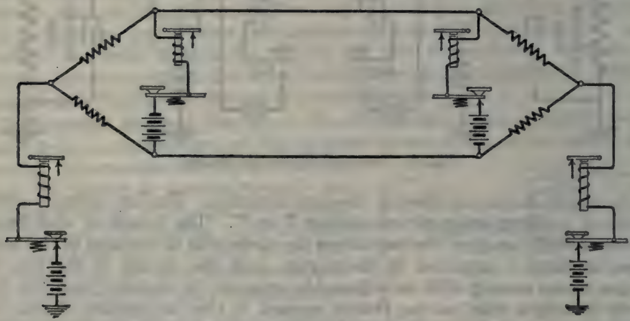


Fig. 8. Jacobs' Cable Duplex

Fig. 8 available. This is similar to simplex working in combined telephony and telegraphy. This method is found preferable to using each conductor separately for the two transmissions, since it avoids inductive interference between the two wires.

Polar Duplex (Fig. 9 and 10). — The polar duplex depends for the principal feature of its operation on the fact that a polarized relay will move its armature oppositely for opposite directions of current. The key used in sending operates to close and open a local circuit which controls a pole-changing transmitter. The action of this is made clear in Fig. 9. With the key closed, as shown in that figure, the negative side of battery will be connected to line and the positive side to ground.

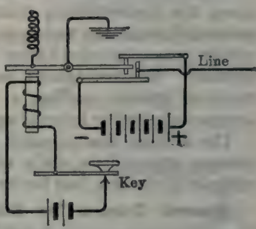


Fig. 9. Pole-changing Transmitter—Polar Duplex

With the key open, the reverse will be true. The circuit of the polar duplex is shown in Fig. 10.

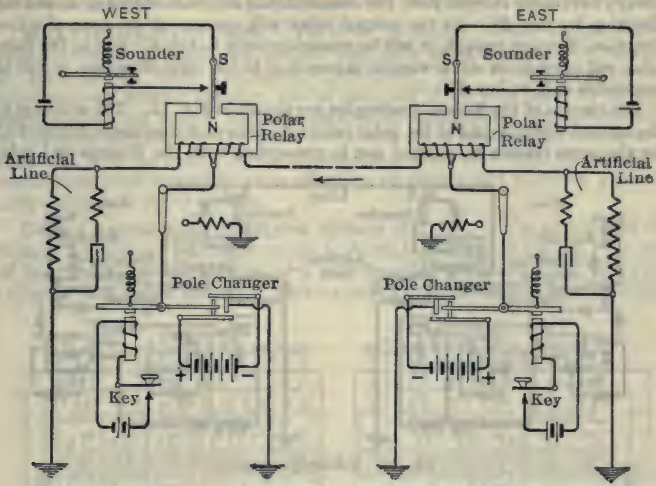


Fig. 10. Polar Duplex

QUADRUPLEX SYSTEMS. — In quadruplex systems four messages, two in each direction, may be transmitted simultaneously over a single line wire.

Edison Quadruplex (Fig. 11 and 12). — The Edison quadruplex depends for its principle of operation on a combination of the two duplex systems, the Stearns and the polar just described. At each station two relays are provided, one, the neutral relay, responsive to currents in either direction, and the other, the polar relay, responsive only to currents in one direction. At each

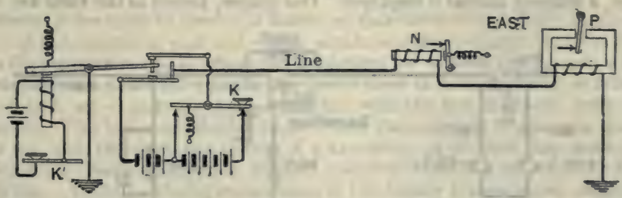


Fig. 11. Principle of Edison Quadruplex

station also two key-controlled transmitters are provided, one of which operates, as in the polar duplex, to change the direction of current in the line and the other, as in the Stearns duplex, to increase and diminish the strength of current flow.

The principle of tr..smitting in one direction by means of the Edison quadruplex is indicated in Fig. 11, where *N* is the neutral relay and *P* the polar relay at the East station. *K* and *K′* are transmitting keys at the West station, key

K operating to alternately weaken and strengthen the line current, and K' operating as a pole changer to reverse the line current. As shown, the line will always have some current flow, the amount being determined by key K and the direction by key K'. As the neutral relay will operate on currents of either direction, if strong enough, it will be responsive only to key K; and as the polar relay will operate on either current strength, if in the right direction, it will be responsive only to key K'.

The circuits of the Edison quadruplex are shown in Fig. 12. The windings of the neutral relays N and of the polar relays P are in each case made differential, for the same reasons as pointed out in connection with the Stearns and polar

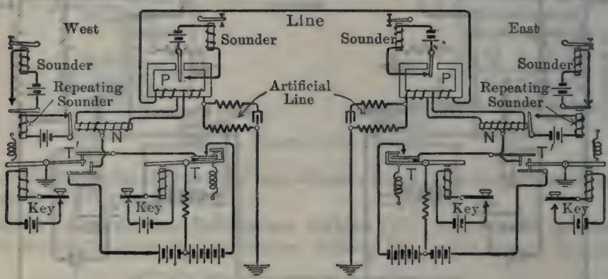


Fig. 12. Edison Quadruplex

duplex systems. At each station of the "quad" system one winding of each relay is in the line, and the other in the artificial line. As will be seen from Fig. 12, the two transmitters at each station modify the main battery current, as pointed out in connection with Fig. 11, the continuity-preserving transmitter T controlling the current strength and the pole-changing transmitter T' controlling its direction.

Key Systems for Quadruplex Working. — In the quadruplex so far described, the current was supplied by batteries, and the strength-varying transmitter operated to accomplish its purpose by alternately cutting a portion of the line battery out of and into the circuit. Where dynamos furnish the current, a different scheme is employed. One system, known as the Field key sys-

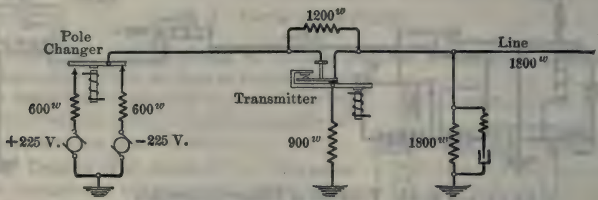


Fig. 13. Field Key System

tem, is illustrated in principle in Fig. 13. Two dynamos are employed, having opposite poles grounded. The pole changer merely chooses between the two live poles for securing its reversals in polarity. The continuity-preserving transmitter varies the current strength in the line by changing the resistance relations as shown. Thus, when the key is closed, the 1200-ohm resistance is short-circuited and the 900-ohm "leak" resistance to ground is cut off. The

current under these circumstances flows to line at full strength. When the key is opened, the 1200-ohm resistance is introduced in the line and the 900-ohm leak is made effective. The resistances shown are those for an 1800-ohm line.

MULTIPLEX SYSTEMS. — The term "multiplex" should include duplex and quadruplex systems, but practice has been otherwise, and has resulted in applying the term "multiplex" principally to a system wherein two synchronously moving switches, one at each end of the line, serve to connect the corresponding sets of telegraph instruments momentarily and successively in operative relation with the line. While each set of instruments, therefore, is connected with the line for only a small fraction of the total time, the successive connections recur so rapidly as to afford, to all intents and purposes, a practically continuous use of the line.

Delany Synchronous Multiplex System (Fig. 14). — The principle of the Delany synchronous multiplex system is shown in diagram in Fig. 14. Six sets of instruments, each consisting of a pole-changing key and a polar relay, are shown at each end of the line. The two switch arms of the synchronous rotary

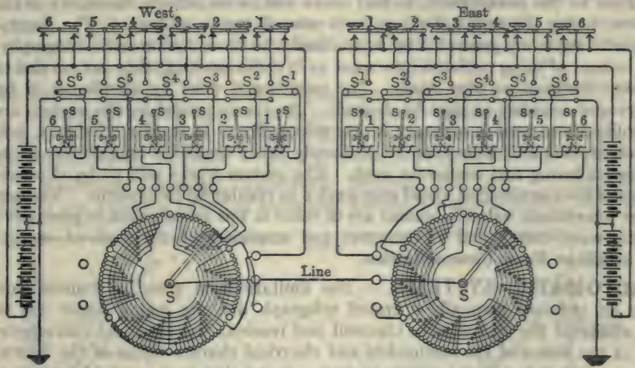


Fig. 14. Delany Synchronous Multiplex

switches S are made to revolve in unison, so that the correspondingly numbered sets of instruments at each end will be simultaneously connected with the line. As the switch arms make about 180 revolutions per minute, and as each set of instruments is connected with the line twelve times at each revolution, it follows that each set of instruments is connected with the line thirty-six times per second. This is a much shorter interval than that required to make the shortest dot, and, therefore, any closure of a key which an operator may make will necessarily cover a time during which at least one connection will be made with the line. In the particular embodiment of the Delany system shown polarized relays and pole-changing keys are used, the advantage of this being that the relay armature will remain in the position to which it was last brought by a current impulse, in spite of the rapid interruptions caused by the synchronous switch. A small switch lever, S^1, S^2, etc., is associated with each key and relay and is moved by the operator to one or the other of its positions, according to whether he is sending or receiving.

The synchronizing mechanism for the switch movements is ingenious and interesting. The two arms are each driven by step-by-step motors, the driving impulses being regulated by means of tuning forks, as accurately tuned to each

other as possible. Obviously, complete synchronism could not be obtained by this means alone, and, therefore, the two driving devices are each brought into association with the line by the rotary switch arms six times during each revolution, the arrangement being such that if one switch arm arrives on one of the dead or synchronizing contacts slightly in advance of the other, it will apply a corrective measure to its motor device, which will tend to slow it down. This corrective measure is in the form of an electromagnet, in the field of which the tuning fork vibrates, and the energization of this magnet exerts a retarding influence on the rate of vibration, and, therefore, on the rate of rotation of the synchronous switch.

This system has been used to some extent commercially with as high as six simultaneous transmissions on comparatively short lines, and four or fewer on long lines. Obviously, the time constant of the line enters as a limiting factor to a greater extent than in the ordinary systems of Morse telegraphy.

Squier or "Wired Wireless" System. — General George O. Squier has developed a system of multiplex telegraphy in which the signals are transmitted by electromagnetic waves directed by wires. Patents on the system have been assigned to the government, so that there will be no private monopoly of the system during the terms of the patents nor of course afterward. The electromagnetic waves are generated by means similar to those used in radio telegraphy, but instead of being radiated into space as in radio working, they are carried to definite destinations over wires. The receiving is done by telephones or by relays and sounders as may be desired. The currents used in this system are so unlike those used in ordinary Morse telegraphy or in regular telephony, and the electromagnetic waves can be so sharply tuned that wide opportunity exists for multiplexing and composite work. The system works well under derangements of line circuits that would stop service on regular Morse systems. Vacuum tubes (thermionic valves, audions) are of value in this system both as generators of the wave-energy and as detectors and amplifiers of the received energy. See article on *Radio Communication.*

AUTOMATIC SYSTEMS. — The limitations in speed of transmission over an ordinary manually operated telegraph line are mainly those due to the ability of the operators to transmit and receive, the electrical characteristics of the line, and the mechanical and electrical characteristics of the instruments which it has been possible to devise, permitting very much higher rates. On this account, the so-called "automatic systems," wherein the transmitting and the receiving of the message are automatically accomplished, have come into being.

Wheatstone Automatic System. — This is one of the most highly developed and widely used of the ink recording automatic systems. In Fig. 15 is shown a piece of the tape and a plan of the die for perforating it. The perforator has three levers usually adapted to be struck by mallets in the hands of the operator, these levers punching the necessary holes to form dots, spaces and dashes, respectively. Striking the left-hand lever punches three holes, 1, 2, 3, directly across the paper, this being the combination required for a dot. Striking the center lever punches a single small hole, 4, in the center of the tape, thus making a space, while the third or right-hand lever punches four holes, arranged as 5, 6, 7 and 8, thus setting up a combination for a dash. The function of the small holes along the center line of the tape is to effect the proper driving of the tape, these being engaged by the points of a star wheel in con-

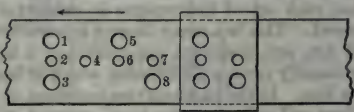


Fig. 15. Wheatstone Tape

nection with the feeding mechanism. As this tape is driven through the transmitter, one row of larger holes determines the intervals at which the positive pole of the battery shall be put to line, and the other row of large holes the intervals during which the negative pole is connected. As the two members which engage these holes are arranged so that one is slightly in advance of the other, it follows that the holes 1 and 3 will cause first a negative impulse, immediately followed by a positive impulse, this being the combination for a dot. The two holes, 5 and 8, will likewise cause a negative, followed by a positive, impulse, but the two will be separated by an interval of time, this being the combination for a dash. The Wheatstone is thus a so-called "double-current" system, depending for its operation on the actual reversal of the line current.

Wheatstone Receiver. — The receiver of the Wheatstone system is in the form of a polarized relay, the armature of which will stay in either of its extreme positions even though the current which moved it there ceases. The inking is done by a very small ink wheel which is constantly rotated in close proximity to the moving paper tape. The shaft on which this ink wheel revolves is so mounted as to be deflected by the movements of the relay armature. The ink wheel receives its ink on the back stroke from another wheel which revolves in a tank of ink. On the forward stroke the ink wheel is brought close enough to the moving paper tape to make a mark, although there is no actual contact between the wheel and the tape.

Chemical Systems. — These systems, once employed in commercial work to a considerable extent, have been in large measure superseded either by the ink recording systems or by ordinary Morse. They present possibilities for future development, however, which give them more than an historic interest.

Anderson Chemical Automatic System (Fig. 16). — The salient points of the Anderson chemical automatic system are shown in Fig. 16. The

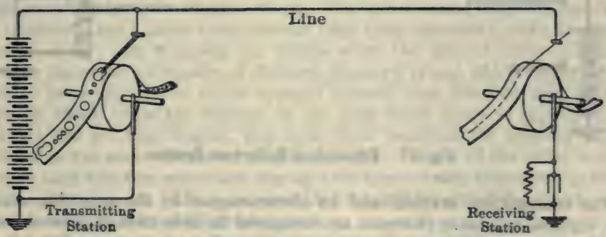


Fig. 16. Principle of Chemical Automatic System

passage of a hole in the transmitting tape under the contact pen serves to short-circuit the transmitting battery and thus cause a cessation of current in the line. The condenser at the receiving end, previously charged by the flow of current from the battery, then discharges into the line, this being intended to cause an actual reversal of current flow. This is an interesting example of so-called "double-current" working with a single battery. At the receiver the chemically saturated tape passes under a contact pen, so that the current is compelled to flow through the paper. Its flow decomposes the solution and marks the paper, as already described. The reverse flow of current from the condenser tends to prevent the occurrence of "tailings."

Speed of Anderson System. — This system, like the Wheatstone, records in dots and dashes, and, according to Maver, it has been found capable of transmitting 600 words per minute over a 1000-mile line having a resistance

of 2500 ohms, under all sorts of weather conditions. On a shorter line, 360 miles in length, having a resistance of about 700 ohms, the astonishing rate of 3000 words per minute has been accomplished.

WRITING TELEGRAPHS OR AUTOGRAPHIC SYSTEMS. — Writing or autographic telegraphs are to be distinguished from printing telegraphs in that they write the received messages in script, while the printing systems actually print them as from type.

In these systems the message to be transmitted is written down by the sender as with a pen or stylus, while at the receiving end a recording pen simultaneously executes the same movements and thus writes the message in the same or very similar handwriting. Nearly all autographic systems depend on resolving the movements of the transmitting pen or stylus into two component straight-line movements at right angles to each other, electromagnetically reproducing these component rectilinear motions at the receiving station, and recombining them there into the resultant movement, which, therefore, traces a path closely approximating that originally made by the sending stylus.

Robertson System. — Fig. 17 is a schematic diagram of this system and clearly illustrates the principle of autographic systems. Two line wires are employed, each including a battery and a variable-resistance element

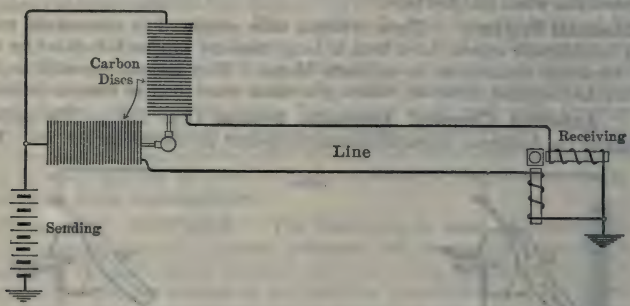


Fig. 17. Principle of Robertson System

at the transmitting station, and an electromagnet at the receiving station. The variable-resistance elements are composed in this case of piles of carbon discs arranged at right angles to each other, these being so related to the transmitting stylus that its movements in writing will cause, due to variations in pressure on the carbon discs, corresponding variations in resistance in the paths through the discs. The receiving pen is actuated by the armature common to the two magnets at the receiving end. As the respective pulls of these two magnets are at right angles and vary as the strength of the currents flowing, the two components of the transmitting stylus are reproduced and these are recombined to produce the corresponding movements of the receiving pen.

Telautograph. — This is another autographic system, due largely to Elisha Gray. It employs two wires, and like the system just described resolves the complex motion of the transmitting pencil into two rectilinear components at right angles, reproduces these at the distant end, and recombines them to obtain the facsimile writing. The transmitting stylus operates two bell-crank arms carrying rollers which operate over rheostat contacts to cause the variations in resistance and current strength. At the receiving end the pen is attached to the arms of two bell cranks which are operated on by solenoids,

which lie in uniform magnetic fields and which, under the influence of the varying currents passing through them, cause the required movements of the bell cranks.

Pollak-Virag System. — In this the message is prepared by perforating a tape, the passage of which through the transmitter results in currents being sent over the two line wires in proper sequence, direction and strength to cause the receiving instrument to record the message in a style very similar to longhand writing. Two circuits are obtained from the two wires; one the metallic circuit and the other the two wires in parallel with earth return. At the receiving end two instruments closely resembling telephone receivers are affected by the currents received over the two circuits. The diaphragms of these receivers control the movements, on horizontal and vertical axes, respectively, of a small mirror. A ray of light reflected from this mirror is caused to move upon a rapidly moving strip of light-sensitized paper, and the co-ordinated movements on the two axes of the mirror under the influence of the currents received result in writing. The photographic paper is automatically passed through the necessary developing and fixing baths, the time required in this process being only about twelve seconds.

PRINTING TELEGRAPHS. — These are systems in which the received messages are automatically recorded in print. Most printing telegraph systems depend upon synchronism being maintained between the two wheels at the transmitting and receiving stations; although in some systems, such as the Murray, for instance, no such requirement exists.

Ticker Systems. — In the ticker systems, which are widely employed for transmitting stock-market quotations, the type wheels are moved synchronously a step at a time, either by causing the same set of line impulses to affect each, or by causing the wheel at the sending station, in its revolution, to transmit impulses which turn the distant wheel. The wheels must not only revolve at the same rate, but a given letter on each must come opposite a given fixed point simultaneously. By stopping the wheels when the desired letter is brought opposite the paper, an impulse of different character is sent which causes the imprint to be made. Some ticker systems operate over one line wire, and the impulses which cause the rotation of the type wheel occur so rapidly as not to affect the printing magnet, but upon their cessation the printing magnet included in the same circuit is brought into play. Many of the ticker systems operate over two line wires and employ two type wheels, one carrying letters and the other figures. Means are provided for shifting either the wheels or the tape, so as to print from either as desired.

The Hughes System. — This is largely employed in Europe, and is a synchronous type-wheel system in which the type wheels revolve continuously rather than by step-by-step movements. Electric motors furnish the driving power. The wheels are adjusted to revolve at as nearly the same rates as possible and suitable corrective means are brought into play over the line wire to maintain complete synchronism. The transmitting apparatus resembles the keyboard of a piano, a key being assigned to each character. In connection with this keyboard there is a rotating contact device revolving in synchronism with the type wheels, and the arrangement is such that the depression of any key will cause a contact to be made at the exact instant when the letter corresponding to that key is in printing position at both instruments.

Speed of Hughes System. — The Hughes system is capable of operating with great accuracy at a speed in the neighborhood of from thirty to forty words per minute.

Phelps System. — The Phelps system is a modification of the Hughes and

operates on the same general principles. It has superseded the Hughes system in the United States.

The Buckingham System. — One difficulty with the ordinary step-by-step system is that the great number of impulses required per letter necessarily slows down the speed. The Buckingham printing system is designed to operate with a maximum of six current impulses per letter, the necessary choice between letters being effected by various combinations of short and long current impulses in either direction, with short and long intervening spaces. The Buckingham system resembles the Wheatstone, in that the spaces are made by negative currents and the dots and dashes by positive. As in the Wheatstone system, also, the message is prepared by perforating the tape in such manner as automatically to send the required combinations of dots, dashes and spaces, but a different code is used, in which each letter and other character is composed of three positive impulses. dashes or dots, or both, variously combined with long or short negative impulses, corresponding to spaces. By a most ingenious mechanism at the receiving end these positive and negative currents, instead of being received in dots and dashes on a moving tape, are made to effect proper selection of the various printed characters, and to print them in desired order in page form. The type wheels are four in number, all mounted side by side on the same shaft and rotating together. Each type wheel carries eight letters or characters, making thirty-two in all. By means of levers, operated by electromagnets, the type wheels are, under the action of the code impulses, given any desired lateral or rotary motion to bring the required type face into printing position. The space between the letters is utilized to give the printing impulse.

Tape-punching Machine for Buckingham System. — The tape-punching machine of the Buckingham system employs a standard typewriter keyboard, one depression of the key effecting the tape punching for a complete character. The maximum speed possible is about 80 words per minute, as against 20 to 40 words per minute in the mallet method of punching Wheatstone tape.

Speed of Buckingham System. — This system has a possible average speed of transmission of about 100 words per minute, and is reported to have worked on a duplex circuit between New York and Chicago with an average speed of 80 words per minute in each direction.

The Barclay System. — This has been employed extensively by the Western Union Telegraph Company, and resembles in many respects the Buckingham system. It uses the same code alphabet and the same general method of transmitting, but the impulses received result in the operation of a group of magnets which in turn affect the operating levers of a Blickenderfer typewriter, which does the actual printing. The printing is done on ordinary message blanks.

Speed of Barclay System. — The rate of transmission possible is approximately 100 words per minute in each direction when operated as a duplex.

The Rowland System. — This is one of the most highly developed of the printing systems, and was for a time used to considerable extent by the Postal Telegraph Cable Company between New York and Chicago, and on other circuits. Essentially the system is a one-way quadruplex, but as it is capable of being worked duplex it becomes in effect an "octoplex," transmitting four simultaneous messages in each direction over one line wire.

The line current employed is alternating, generated by a dynamo at one end, which runs a synchronous motor at the other, thus effecting the required synchronism. The necessary current combinations to effect the selection and

printing of a given letter are brought about by reversing certain of the half waves in each group of line current waves. The simultaneous transmission of four messages in one direction is provided for by an adaptation of the multiplex system, a distributing switch giving each set of four transmitting and receiving instruments successive and simultaneous control of the line. Since each letter is represented by a specific combination of positive and negative currents, it is possible by means of these combinations to effect the closing of the proper local circuits at the receiving end to cause the paper to be pressed against the type wheel at the time when the letter required is opposite it.

Speed of Rowland System. — Each transmitter and receiver has a speed of about 30 words per minute, making a total transmission of 240 words per minute when the octoplex method of working is employed.

The Murray Automatic System. — This is a non-synchronous printing system. Messages are first prepared by perforating a paper tape. This tape is then fed into the transmitting device and permits currents to flow over the line in such a manner as to cause the receiving device, which in itself is a tape-perforating machine, to perforate another tape there. This receiving tape is a replica of the transmitting tape, and when passed through an automatic typewriter the perforations are translated into Roman type letters, the message being in page form.

Speed of Murray System. — This system is capable of transmission speeds upward of 200 words per minute, but speeds of from 100 to 125 words per minute have been found to be the best for practical working.

The Morkram System and Modifications. — This system is in wide use in the United States. The Western Union Telegraph Company uses it in a form somewhat different from the original. It is capable of being quadruplexed and in that case the method described under *Delany Synchronous Multiplex System* divides the use of the line among the four sets in regular succession. The transmitter receives tape perforated manually before transmitting begins; the receiving is done on electrically operated typewriting machines which write the messages in page form. The paper is fed to the machines from a roll. Only upper-case (capital) letters are used and these are of the simplest and most legible form, known to printers as Gothic. The use of the system has reduced the amount of manual key-sending and ear-receiving between the principal cities very greatly.

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TELEPHONE INSTRUMENTS AND CIRCUITS. — (*See also Radio Communication; Standardization Rules of the A.I.E.E.; Telephone Lines; Telephone Traffic.*)

The chief subjects treated in this article are:

The telephone transmits speech and other sounds by this cycle of action: sound waves in air vibrate a diaphragm; this vibration produces fluctuating current in a line joining the transmitter to a distant receiver; the fluctuating current produces vibration in the diaphragm of the receiver; these vibrations produce sound waves in air.

Bell's original telephone utilized one device as both transmitter and receiver. It survives in its original form as a receiver, but seldom is used as a transmitter, more powerful devices having supplanted it.

RECEIVERS. — The original telephone consisted of a magnet, a coil of fine wire and a diaphragm, arranged as in Fig. 1. *d* is a diaphragm, *m* is a magnet,

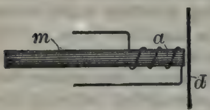

Fig. 1. Principle of Permanent Magnet Receiver

Fig. 2. Principle of Bipolar Receiver

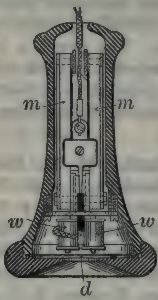

Fig. 3. Section of Bipolar Receiver

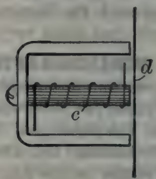

Fig. 4. Principle of Direct Current Receiver

and *a* is a coil. The magnet was first made of soft iron, getting its magnetism from direct currents in the line; a permanently magnetized bar of hard steel with soft iron poles (Figs. 2 and 3) soon was substituted for this soft iron core.

Bipolar Receiver. — The bipolar receiver is the present standard form, as in Fig. 2, in which *d*, *m* and *c* represent, as before, the diaphragm, magnet and coil. Both poles of the permanent magnet are presented to the diaphragm, being extended by soft iron pole pieces, each carrying a bobbin wound with fine, silk-covered or enameled wire. The wires of the two bobbins are connected in series. When used to transmit speech, such an arrangement is called a magneto transmitter.

The magnet, winding and diaphragm of a receiver are assembled in a case of hard rubber or an imitation thereof, adapted at one end to fit the ear and at the other to hang on a hook switch. Fig. 3 is a section of a typical assembled receiver. *m*, *m* are permanent magnets, arranged to produce opposite poles at the ends of the soft iron cores carrying the windings *w*, *w*; *d* is the diaphragm. In good types of receivers the entire mechanical arrangement of parts is supported from the ear end of the shell and not from the other end. This precaution is necessary in order to prevent the adjustment of the pole pieces to the diaphragm being upset by the difference in expansion of the shell and magnet.

Direct-current Receiver (Fig. 4). — For use in lines carrying direct current during conversation, as in common-battery systems, the iron-core type of receiver recently has been revived. They are extremely simple and cost little to make. It is not yet established that they are as efficient as permanent-magnet receivers, but the difference seems to be slight. They have the virtue that the direction of flow of direct current through them is not important.

The Monarch Telephone Manufacturing Company uses a receiver which has two pairs of windings, as in Fig. 5. One pair is of large impedance, serving to magnetize the cores, while the other, of smaller impedance, serves to actuate the diaphragm. Direct currents pass through both pairs of windings, their magnetizing effects being additive, but the direct current through the coils farther from the diaphragm is the larger. Voice currents pass through both pairs of windings and their effects are additive, but the voice currents through the windings nearer to the diaphragm are the larger.

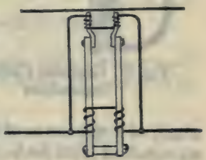

Fig. 5. Monarch Receiver

Head Receiver. — A head receiver is a compact one carried by a band fitting the operator's head, leaving her hands free. The compactness is gained by making the magnets small and placing them parallel with the diaphragm.

TRANSMITTERS (Figs. 6 and 7). — Present standard carbon transmitters are many times more powerful than the best magneto transmitters. Carbon transmitters and their circuits take many forms, but in all the function is to produce current fluctuation as a consequence of diaphragm vibration caused by sound.

The elements of a transmitter are a diaphragm, two polished solid carbon electrodes, and a charge of polished carbon granules. Vibration of the diaphragm moves one electrode to compress and release the granules between the electrodes. Fig. 6 illustrates the elements; *d* is the diaphragm, *m* the movable and *f* the fixed electrode. Fig. 7 is a section of a typical granular carbon transmitter.

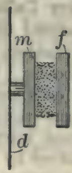

Fig. 6. Principle of Transmitter

All carbon transmitters operate by varying the resistance of some material proportionally to movement of the diaphragm. Receivers, on the other hand, are caused to operate by a varying magnetic flux. The reason for the excellent action of carbon in varying its contact resistance under pressure is not known.

RINGERS OR BELLS (Fig. 8). — The present standard telephone signal is a two-gong bell, actuated by alternating current. Its principle is that of the polarized telegraph relay. Its construction, as in Fig. 8, is of two coils, a permanent magnet *N*, two gongs and an armature *S*.

Magneto Generator (Fig. 9). — The bells of telephones in all exchanges are rung by alternating current sent out from the central office power plant. In magneto systems the subscriber calls the central office by operating a magneto generator, which is a part of his telephone, and with this generator

magneto generator, which is a part of his telephone, and with this generator the subscriber also rings other bells on the same line. The magneto generator is a standard part of subscribers' equipment for rural and other party lines, being the best way for a subscriber to do signaling between various telephones on one line. The armature has many turns of fine wire (Nos. 32 to 36 B. & S.

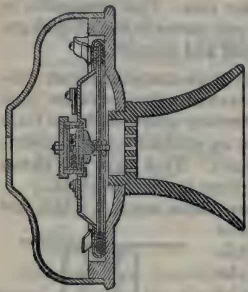

Fig. 7. Granular Carbon Transmitter

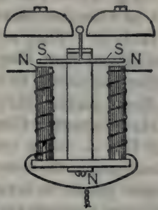

Fig. 8. Alternating Current Ringer

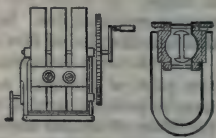

Fig. 9. Magneto Generator

gage), of a resistance of from 300 to 1000 ohms. A frequency of 20 cycles and an e.m.f. of from 60 to 80 volts can be produced by turning the crank briskly by hand.

BATTERIES. — (*See articles on Batteries.*) The transmitter of a telephone set requires direct current and produces from it the alternating (or otherwise fluctuating) voice currents, which pass over the line to the distant receiver. A central storage battery furnishes the direct current over the line in "common battery systems," or "central energy systems." A primary battery furnishes the direct current in systems not having a central storage battery for the purpose.

Dry Cells. — The primary battery usually is of the dry cell type and is associated directly with the telephone set. Such an arrangement is called a local battery set. Two or three cells are used in series. The cells have an e.m.f. of about 1.5 volts, and can give a current on short circuit as high as 15 amperes. Dry cells, designed particularly for local battery telephone service, have higher internal resistance than those intended for gas engine ignition and other heavy current intermittent work. Longer life, with only slightly less efficiency, is obtained by increasing the internal resistance. Dry cells may be used for but a few minutes at a time.

Gravity Cells. — Other types of primary cells are used to energize transmitters. Gravity cells are suitable for operators' telephones in small exchanges, and can furnish current steadily with no periods of rest. Gravity cells must not stand long on open circuit. They give about 1 volt per cell. Three or four in series are suitable for a transmitter.

Fuller Cells are suitable for heavy service (many long conversations per day), and also can stand on open circuit without harm. They approach nearest the storage battery in e.m.f. (about 2 volts) and output. They are sloppy and costly to maintain and are used less widely now than heretofore.

Storage Batteries give good results in local battery sets; they require charging by direct current from outside themselves, however, and suffer from the lack of frequent charges. They are suitable for but few conditions of local battery use for those reasons. The most widely used method of charging

storage batteries is by motor-generator sets, the motor being adapted to the source of energy and the generator being shunt wound. Shunt-wound generators driven by internal-combustion engines are used as emergency charging equipments.

POLE CHANGERS. — Ringing current in central offices is usually produced by alternating current generators or by pole changers. The latter are vibrating contact-makers having a natural frequency corresponding to the frequency of the current desired. They are operated by direct current from a storage battery or other source. The Warner pole changer, Fig. 10, is widely used for ringing ordinary telephone bells at a frequency of from 16 to 20 cycles per second. Pole changers of multiple frequency are also used to ring harmonic bells on party lines.

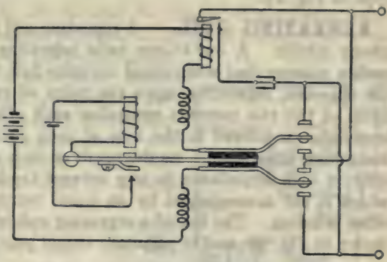

Fig. 10. Warner Pole Changer

IMPEDANCE COIL. — An impedance coil is a single coil of wire wound on an iron core. In telephony the iron core usually is of wires, or occasionally of sheets.

INDUCTION COIL. — A step-up transformer used in a telephone circuit is called an induction coil in telephone parlance. Fig. 11 shows the use of such a transformer in a local battery set. The battery and the carbon button are connected in series with its primary winding; the line and the receiver are connected in series with its secondary winding. The function of the induction coil is to convert the local variations of direct current of low e.m.f. into alternating currents of higher e.m.f. in order that it may pass over the line with less loss.

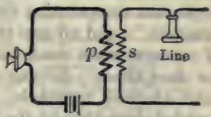

Fig. 11. Induction Coil in Local Battery Set

Local battery induction coils have a large ratio between the two windings. The primary winding is of low resistance and of fewer turns than the secondary winding. Single silk insulated wire is used for both windings; practice varies as to wire sizes and turns; a representative example has 350 turns of No. 20 B. & S. gage wire in the primary and 2400 turns of No. 30 B. & S. gage wire in the secondary. Other examples use from 200 to 700 turns in the primary, 700 to 5500 turns in the secondary, No. 20 to No. 28 B. & S. gage wire in the primary and No. 26 to No. 36 B. & S. gage wire in the secondary.

The induction coils in common battery sets have different functions from those in local battery sets. Figs. 12 and 13 show two forms of common battery telephones using them. In most cases one function of the induction coil is to in

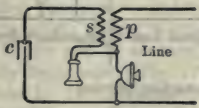

Fig. 12.

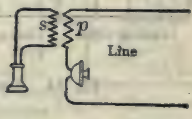

Fig. 13.

sulate the receiver from the direct current of the line, so that its permanent magnetism never shall be opposed by direct current flowing in its coil. In Fig.

12 the induction coil has the further function that, co-operating with the condenser *c* it assists the transmitter to convert the direct current of the line into an alternating voice current. The condenser alternately receives and gives up charges, responsive to the state of the varying resistance of the transmitter. This particular transforming function is lacking in all circuits having the general type of Fig. 13.

REPEATING COIL. — A repeating coil is any transformer used in a telephone circuit. A one-to-one ratio repeating coil is frequently connected in such a manner that when equal currents flow through each winding in the same (or opposite, depending upon the point of reference) direction, the magnetic fields of the two currents neutralize each other, and, therefore, practically no impedance is offered to the flow of such currents, but each winding by itself has a high impedance. As the result of such an arrangement the repeating coil offers a high impedance to any current tending to flow through one winding which is not accompanied by an equal current in the proper direction in the other winding. The use of a repeating coil in a widely adopted cord circuit is shown in Fig. 29, and in phantom circuit practice in Fig. 32.

HOOK-SWITCH. — The time during which a subscriber's telephone is idle much exceeds the time in which it is in use. When idle, it must be ready to receive or to make a call. When in use, its transmitter must be able to receive direct currents and its receiver to receive voice currents. The hook-switch makes these changes automatically.

PROTECTIVE DEVICES. — Telephone apparatus and lines are protected from damage due to abnormal currents by means of fuses supplemented by two important co-operating elements. The latter are heat coils (or "sneak current arresters"), and air-gap arresters. Heat coils operate by short circuiting a pair of contacts when small currents flow for long times or when large currents flow for short times. Air-gap arresters operate when potentials of 350 volts or more exist at their terminals. The operation of either a heat coil or an air-gap arrester short circuits the line, grounds the wires in trouble, and blows the fuse. On account of the high resistance of the telephone instruments, an abnormal current in the line may not be large enough to blow a fuse unless there is a dead short circuit. The heat coil insures such a short circuit whenever the amount or duration of the current exceeds a safe value.

The relation of the three elements to each other and to lines and apparatus is shown in Fig. 14. The fuses should be located at the point of junction between wires which are "exposed" and wires which are "not exposed" to possible contact with a source of dangerous current. All aerial wires and aerial cables are

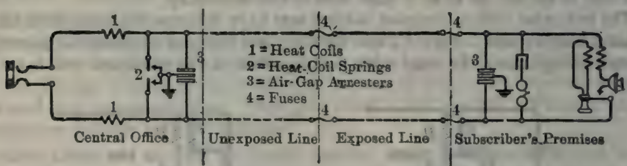

Fig. 14. Arrangement of Protective Devices

exposed. Underground and indoor wires are to be considered unexposed, if the indoor wires are run in accordance with good rules, such as those of the National Electrical Code.

TELEPHONE SETS. — The several elements necessary at a subscriber's station are associated to form series magneto sets, bridging magneto sets and common battery sets.

Series Magneto Sets (Fig. 15) are suitable for use on single party lines, and are widely so used in exchange practice, and, unfortunately, to some extent on lines having more than one telephone (party lines). Fig. 15 is the circuit of a series set. When the receiver is on the hook, the bell is connected between the terminals of the set, the battery and receiving circuits both are open, and if the generator is not turned its armature winding is shunted. When the generator is turned in calling, the shunt is broken automatically.

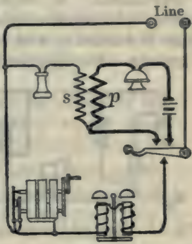

Fig. 15. Series Local Battery Magneto Set

In series sets the voice currents must pass in series *through* all but one of the ringers of a line. The impedances of the ringers, therefore, must be as *low* as possible, yet high enough to respond to ringing current of reasonable e.m.f. 80 to 100 ohms is the usual resistance for both coils of a series ringer. The cores usually are shorter than in bridging ringers.

Bridging Magneto Sets (Fig. 16) are better than series magneto sets for party lines. The ringer is bridged across the line permanently. The generator is bridged across the line when the crank is turned. The receiver and transmitter circuits are closed when the hook is up. Three of the four circuits of the set are open when it is not in use, as in Fig. 16.

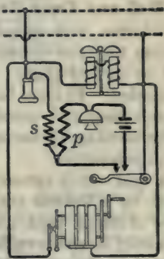

Fig. 16. Bridging Magneto Set

In bridging sets the voice currents must pass *by* all of the ringers of a line. The impedance of the ringers, therefore, must be as *high* as possible, yet low enough to respond to ringing current of reasonable e.m.f. 1000 to 1800 ohms is a usual resistance for the two coils of a bridging ringer. The cores are usually longer than in series ringers.

Common Battery Sets (Figs. 17–19) have no magneto or local battery and can, therefore, be used only on lines leading to a central office. A storage battery at the central office, common to all the lines, and operation of the instrument. Common battery telephones cannot send out current to ring bells, as can magneto telephones; when used on a party line they can, therefore, call each other only indirectly, by asking the central office operator to send the necessary ringing current over the line. Common battery sets are nearly always bridging sets.

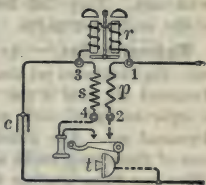

Fig. 17. Bell Common Battery Desk Set

Figs. 17, 18 and 19 are typical circuits of common battery telephones. Fig. 17 is that used in the United States by the Associated Bell Telephone Companies and in Western Electric Company's apparatus throughout the world. As the condenser is "opaque" to direct current, but "translucent" to alternating current in a degree dependent upon the frequency, no direct current from the central office normally will flow

through the set when the hook switch is down, but alternating current from the central office can actuate the ringer. When the switch-hook is up, direct current from the central office flows through the transmitter t and the primary p of the induction coil.

In the circuit of Fig. 18, used by the Kellogg Switchboard & Supply Company, the ringer, in series with a condenser c_2, normally is bridged across the line. When the switch-hook is up, the transmitter is in series with the line and can vary its current. The receiver is in a path "opaque" to direct current, but "translucent" to voice current, because of the condenser c_1. The receiver path, including the condenser c_1 is shunted by a coil i of low resistance but of high inductance, which is opaque to voice currents but translucent to direct currents. No direct currents can pass through the receiver.

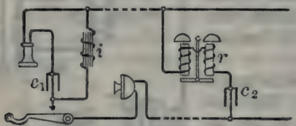

Fig. 18. Kellogg Common Battery Set

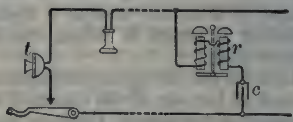

Fig. 19. Desk Set with Direct Current Receiver

The circuit of Fig. 19 is one in which all of the direct current intentionally is allowed to pass through the receiver. The latter has no permanent magnet (i.e., is of the "direct current" type, Fig. 4 or 5), and depends on the direct line current from the central office to magnetize its core.

Wall and Desk Sets. — Telephone instruments are assembled for use in two forms, wall sets and desk sets. The latter also are called portable sets, having the receiver, transmitter and hook switch combined into a unit which can be carried about. Wall sets are self-contained, having space for batteries if needed. In local battery and common battery desk sets the ringers are separately mounted, usually upon a wall. Local battery desk sets in addition require housing for batteries, and magneto desk sets require separately mounted hand generators.

A desk set is connected to line and ringer, and to battery and generator, if used, by a flexible cord. The number of strands in the cord depends on the type of circuit used, and on whether the induction or impedance coil, if used, is mounted in the portable set itself or on the wall with the ringer. In Figs. 17, 18 and 19 the parts of the circuit carried by cords between the portable and fixed portions of the set are shown by dotted lines.

Desk sets are more convenient for use but are more costly to maintain than are wall sets. They fall frequently, breaking transmitter mouthpieces, receiver shells, and other parts. Supporting arms for desk sets minimize this breakage.

Extension Set. — An extension set is a set used as an auxiliary to a regular set; the extension set usually has no bell; signals are received on the bell of the terminal set, and if the user of the extension set is wanted, he is notified by a separate signal, such as a buzzer, operated by the person who answered the terminal set. Any number of extension sets may be bridged to a line. Any one of them may make a call. None can receive a call direct from the central office nor from another station on the line unless equipped with a ringer, and not then from other stations on the line unless the latter are magneto sets.

PARTY LINE CIRCUITS. — Party line sets differ from individual line sets only in methods of signaling. The apparatus used and the method of connecting up depends upon whether the party lines are to be non-selective or selective.

Non-Selective Party Lines. — On such lines a call made for any party on

the line operates the ringers at all stations. Each station has a particular group of periods of bell ringing, which group only it is meant to answer. Up to five stations from one to five short rings per station serve well. Beyond five, long and short rings are combined. Magneto sets, both series and bridging, are used on non-selective party lines. The generator of the magneto set at any station can ring all the bells of the line at once, and operate the central office signal also, if the line leads to a central office. The bridging system is so used on many private lines, farmers' co-operative lines, rural toll lines, and, to a degree, on exchange party lines in towns.

Non-selective stations should not answer other calls than their own; this extra listening on lines is an extra drain on local batteries in sets having them; it also destroys privacy. A receiver off the hook impairs signals unless circuits are specialized; for example, as follows:

If a condenser of low capacity, say one-half microfarad, be placed in series with the receiver, it will obstruct ringing current (frequency 20 per second) considerably, but voice currents (mean frequency 800 per second) very little.

Selective Party Lines are equipped with ringers arranged to respond each to its own call, the others remaining silent. There are three such methods in use in the United States, and still a fourth method, which has certain very decided advantages, is coming into use.

Metallic Circuit Two Party Lines (Fig. 20.) — On metallic circuits selective two party working is simple. Fig. 20 is the standard system of the

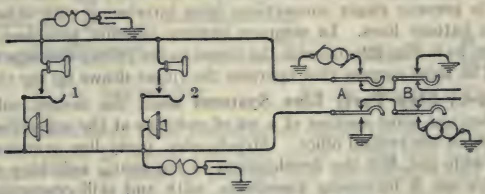

Fig. 20. Two Party System

Associated Bell Telephone Companies in the United States. Each wire of the metallic circuit is used as a private grounded ringing circuit to one station, and the pair of wires is used as a talking circuit by each station. Key *A* connects the ringing generator to the upper wire, ringing station No. 1; key *B* rings station No. 2. If a station answers while being rung, the other station is not rung falsely, the ground on the non-ringing side of the used key preventing that.

Harmonic Selective Systems (Fig. 21) operate on the principle that a ringer having a spring-supported armature will ring for one frequency of current but not for any other frequency. Frequencies of 16, 33, 50 and 66 per

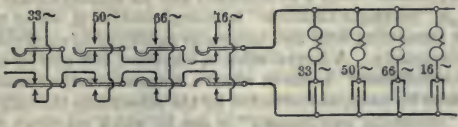

Fig. 21. Four-party Harmonic System

second are used in the Dean four-party harmonic system. Each calling cord in the central office switchboard can be rung upon by one of four keys, as in

Fig. 21. The ringers are alike except as to the sizes of tappers, which are cylinders of metal, heaviest for the lowest frequency and graduating to lightest for the highest frequency.

The number of selective stations may be made twice the number of frequencies by connecting harmonic ringers from each wire to ground as in Fig. 20, instead of across the line as in Fig. 21. This enables eight stations to be placed on a line leading to a four-frequency central office.

Biased Ringer Systems (Fig. 22).—The Hibbard party line system uses unidirectional pulsating current for the selection of biased ringers in four-party lines. An improvement on that system (Fig. 22) has been used by the

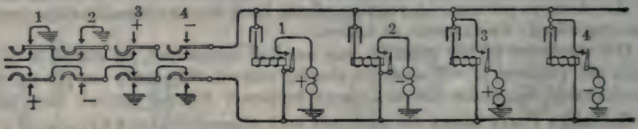

Fig. 22. Four-party Biased Bell System

American Telephone & Telegraph Company. A biased ringer is an ordinary ringer which has its armature drawn to one side by a spring. Impulses of current in one direction only will operate the ringer. Condensers permanently in series with such ringers would interfere with the selective action; but condensers are necessary to prevent ringer connections from interfering with other functions of common battery lines. In common battery systems, therefore, a relay at each station, as in Fig. 22, connects the ringer to the proper line wire only during ringing. The same current that operates the ringer draws up the ringer relay.

Step-by-Step Party Line Systems select the desired subscriber by the operation one after another of a set of switches at the subscriber's station, controlled from the central office. Step-by-step party line systems have not come into wide use, but the development of automatic switching systems for central office use is increasing knowledge, faith and skill concerning step-by-step mechanism. (*See article on Dispatching of Trains by Telephone.*)

Semi-Selective Party Lines.—Semi-selective ringing is done for more than two stations on a line, as in Fig. 20, by connecting more than one ringer to each wire, ringing code signals on key *A* for the ringers on the upper wire and ringing code signals on key *B* for ringers on the lower wire of the figure.

CENTRAL OFFICE EQUIPMENT.—Telephones are used in two general ways: (1) On a line permanently connecting two or more telephones, this line not being connectible to another, and, (2) On a line carrying one or more telephones, this line being connectible at will to any other of a group of lines. In the second case a switchboard and the necessary adjuncts for producing the various kinds of current required must be provided for connecting any line to any other. Such a switchboard and its adjuncts make up a "central office." A "telephone exchange" is an organization of one or more central offices and the connecting lines and apparatus employed in supplying telephone service to a community. A central office is, therefore, but part of an exchange.

A "manual switchboard" is one in which a human operator connects and disconnects the lines by hand, as instructed by the subscribers through signals and speech. An "automatic switchboard" is one in which machines connect and disconnect the lines in response to electrical changes in the calling line. The functions of manual and automatic switchboards sometimes are combined and named "semi-automatic systems" or "automanual systems."

A switchboard designed to interconnect telephone sets equipped with local

batteries and magnetos is called a "magneto switchboard," while a board designed to interconnect telephone sets supplied with current from a single battery in the central office is called a "common battery switchboard." When each of the lines entering a central office is permanently connected to a single "jack" or terminal, the board is called a "simple switchboard." When each line is permanently connected to a plurality of jacks, located at different parts of the board, the board is called a "multiple switchboard." A multiple switchboard is divided into sections, each section accommodating three operators, and containing a jack for each line entering the central office. The group of jacks in each section is referred to as the "multiple." "Simple switchboards" can be used only in case the number of lines connected to the central office is small (less than about 300), unless local trunking in the central office is resorted to. It should be noted that the cost per subscriber of central office switchboard equipment increases as the number of subscribers increases when the total number of subscribers is sufficient to require the use of multiple switchboards.

Cords, Plugs, Jacks and Drops. — As principally used, manual switchboards connect lines by means of flexible conducting "cords" tipped with plugs which fit line-terminal-sockets called jacks. The plug contains some number of parts, electrically insulated from each other but connected to conducting strands of the cord terminating in the plug. There are two strands in the cords of simple magneto switchboards, and therefore two parts in the plug to connect to two conductors of the jack. Fig. 23 shows a jack and

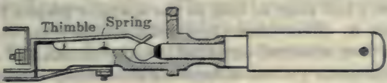

Fig. 23. Jack and Plug


Fig. 24. Diagram of Plug and Jack

plug for a two-strand cord. Few switchboards require circuits calling for more than three conductors in regular switching cords. Plugs and jacks for that many conductors can be made in compact and serviceable forms. Fig. 24 shows diagrammatically a three-conductor jack and plug, in which it is intended that the tip of the plug shall engage the short spring, the ring of the plug the long spring, and the sleeve of the plug the tubular sleeve or thimble of the jack.

The cords and plugs are associated with keys, signals, wiring and current sources, and are links which the operator uses to do her work upon the lines made accessible by the jacks. At *D* in Fig. 25 is shown diagrammatically the type of signal employed on magneto manual switchboards. This signal, called a "gravity drop," is bridged across the switchboard end of the line in the same manner as the ringer at the subscriber's station. When the core is energized by current from the magneto, at the subscriber's station, the armature is attracted, lifting the catch and releasing the shutter *D* allowing it to fall forward by gravity. These signals are called gravity drops for this reason.

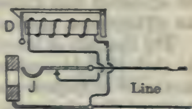

Fig. 25. Magneto Drop and Jack

Power Plants. — The power plant is an organization of devices in a central office to furnish to the lines connected to that office the several kinds of currents required. The principal elements of power plants are storage batteries, charging generators and sources of alternating current for ringing.

In the common battery system one storage battery usually supplies current to all transmitters, whether of subscribers or operators, and actuates all principal relays and lamp signals. In magneto offices storage batteries need only be

large enough to supply operators' transmitters and a few auxiliary signals, as magneto equipments usually have primary batteries in subscribers' telephones.

Magneto Manual Switchboards (Figs. 25 and 26). — Fig. 25 represents diagrammatically the central office end of a subscriber's line and Fig. 26 includes the essential elements of a magneto cord circuit. Normally the tip of one plug is connected through the cord to the tip of the other, and the same is true of the plug sleeves. When a subscriber rings, the drop D in Fig. 25 falls. The central operator then inserts the plug P_1 (Fig. 26) of an idle cord circuit in the jack of this subscriber's line; this disconnects the line drop, the shutter of which is returned to its normal position either by hand or automatically. She then connects her telephone set to this cord by closing the switch A and gets the number of the subscriber wanted, and inserts the plug P_2 in the corresponding jack. By closing the switch B she connects the called subscriber's bell to the central office source of ringing current, which usually is running continuously. The called subscriber's bell rings as long as the operator presses the ringing key. When she releases it, direct connection between the two lines results. When the called subscriber answers, conversation may ensue, and at the close of it the subscribers are expected to signal for disconnection by ringing. The gravity drop D (Fig. 25), in this case called a clearing-out drop, is bridged across the cord circuit, and in response to this signal the operator removes both plugs. The line signals of both lines thus are reconnected in readiness to receive calling signals.

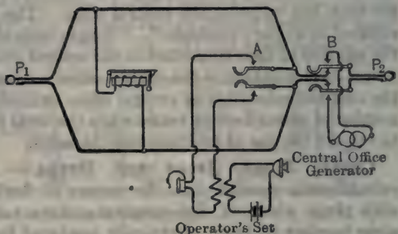

Fig. 26. Magneto Cord Circuit

The operator's set is connectible to the cord circuit by means of a key in each cord circuit, enabling her to answer the calling subscriber by telephone, to listen to the conversation, and, by releasing one listening key and pressing another, to shift her telephone set from cord circuit to cord circuit. Depending upon a variety of conditions, an operator can utilize from ten to twenty cord circuits, transferring her telephone set from one to another as required.

In Fig. 26, ringing current can be sent out only from the right-hand plug. Calls, therefore, are answered by using the left-hand plug of the figure, called lines being taken up by the right-hand plug. Some users require both plugs to be equipped with calling keys, in which case the left-hand plug would be given an arrangement symmetrical with that of the right-hand one.

The shutters of gravity drops used as line or clearing-out signals are restored to their upright position by hand in most simple magneto switchboards. Formerly the drops were placed in one group and the jacks in another, the drops being restored directly by the operator's fingers. Latterly the drop and jack of a line are associated together, so that the plug as inserted into the jack restores the shutter by some thrusting movement on the part of the jack. A simple way is to form a tongue upon the jack spring, this tongue thrusting the shutter directly into its latch.

Large magneto switchboards have been equipped with drops grouped at a distance from the jacks and out of reach of the operators. Such drops are restored after operation by electro-mechanical means, the act of placing the plug in the jack of a line closing circuits to restore the shutter.

Jacks are adapted to be mounted in insulating strips in whatever arrangement

is desired, and any jack of a group may be dismounted. When more compact arrangement is desired, jacks are made up in strips of ten or twenty, an entire strip of which must be removed from the framework of the switchboard to enable one jack to be inspected or worked upon.

During hours of light telephone traffic, as at night, it is desirable to use as few operators as can handle the traffic satisfactorily, and, in small exchanges, even to allow the operator to sleep between calls. On a long switchboard a drop may fall many feet away from the night operator and its falling be unheard. A night alarm signal is provided by equipping each drop with a contact closed by the falling of the shutter. These contacts of all the drops of a group are wired in multiple to a battery, bell and switch which is closed at night. When any drop of that group falls, the bell will ring if the switch is closed.

Switchboard plugs, when not in use, stand vertically in a plug shelf, the cords being housed within the switchboard framework and kept reasonably taut by means of weighted pulleys. Weights should be heavy enough to enable cords to return plugs to position after disconnection, and not so heavy as to shorten unduly the lives of the flexible conductors within the cords.

Where grounded circuits are connected to metallic circuit switchboards and require interconnection with metallic circuits, a ground connection must be provided to furnish the second limb of the line, as switchboards now are built entirely for metallic circuits. Such grounds must be connected to jacks in one unvarying way or two grounded circuits may be connected so as not to be able to talk to each other. The usual practice is to connect the line wire to the spring of the jack and the ground to the sleeve.

Simple Common Battery Switchboard (Fig. 27).—The circuits of a representative simple common battery switchboard are shown in Fig. 27 and

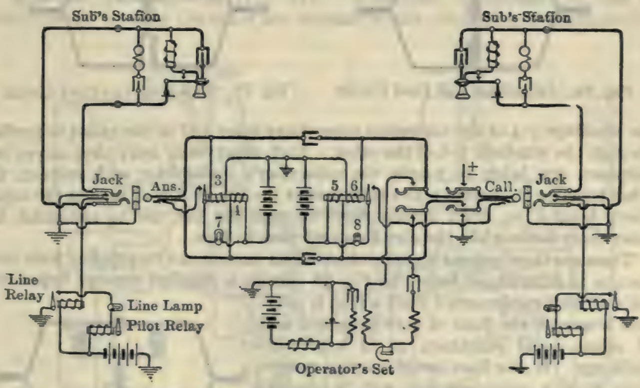

Fig. 27. Circuits of Simple Common Battery Switchboard

are those of the Kellogg Switchboard & Supply Company. They are a type of many other makes. The subscribers' lines terminate in cut-off jacks. There is only one jack per line. The calling plugs only are equipped with ringing keys.

The cycle of operations of such a system is as follows: The subscriber removes the receiver from the hook and as there is no plug in the jack of his line at the central office, the line relay is actuated, lighting the line lamp and operating the pilot relay. The pilot relay, in turn, lights a pilot lamp, of which there is one

for a considerable number of line lamps, and with which a bell or other night alarm signal can be associated. Responsive to the lighting of the line lamp, which in practice is mounted close to the line jack, the operator inserts an answering plug in the jack, breaking the line relay circuit in two places, and so extinguishing the line lamp. Operating her listening key, the operator connects her talking set with the cord circuit, asks for the subscriber's order and executes it by inserting the calling plug in the jack of the line called for, unless that jack already is occupied by a plug. If it is occupied, she tells the calling subscriber that the called line is busy. If it is not occupied, she plugs into it, as stated, and rings. At this stage, the supervisory lamp 7, adjacent to the answering plug, is dark and the supervisory lamp 8, adjacent to the calling plug, is lighted, indicating that the calling subscriber's receiver is off the hook and the called subscriber's receiver is on the hook. When the called subscriber answers in response to the ring, lamp 8 is extinguished. At the close of the conversation, both lamps will re-light as the respective receivers are hung up. In this way the operator can tell at all times the state of the connection and conversation and, responsive to the final lighting of both supervisory lamps, she removes both plugs with the positive knowledge that the conversation has been finished.

The heavy lines in Fig. 27 are those over which the subscriber's conversation takes place. In the cord circuit the two conductors which carry voice current are continuous from tip to tip and from sleeve to sleeve of the plugs, except that a condenser is interposed in each of the two conductors. Direct current to actuate subscribers' transmitters is furnished to the calling subscriber through

Fig. 28. Impedance Coil Cord Circuit

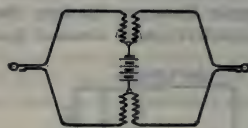

Fig. 29. Repeating Coil Cord Circuit

the windings 3–4 of the answering supervisory relay and to the called subscriber through the windings 5–6 of the other. Each supervisory relay thus serves two purposes, one being to supply the subscriber's transmitter with current, the other being to co-operate with the ground at the line jack so as to light the supervisory lamp if no direct current flows through the line, and to extinguish the supervisory lamp while the direct current does so flow.

There are three general ways in which direct current can be supplied from cord circuits to subscribers' lines. In Fig. 28 current is supplied through impedance coils without condensers. Such a plan is suitable where all the lines of a switchboard are of about equal resistance, as in a private exchange. If the lines are of varying resistance, a high resistance line will be denied proper current when connected through such a cord circuit to a low resistance line (low compared to the resistance of the impedance coil). The circuit of Fig. 29, using a repeating coil, is free from this objection and is standard with the American Telephone & Telegraph Company. The circuit of Fig. 30, using impedance coils and condensers, is as efficient as that of Fig. 28, though entirely different in principle. It is widely used in manual and automatic switchboards other than those of the American Telephone & Telegraph Company.

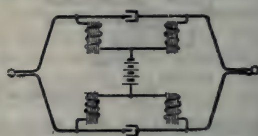

Fig. 30. Condenser Cord Circuit

Comparison of Magneto and Common Battery Systems.—The mag-

neta system is best in small towns and country exchanges. Magneto systems are best for exchanges under 500 lines and common battery systems best above that number. No positive rule can be laid down. For example, the magneto system may be arranged in exchanges which start with little equipment and do not grow beyond 500 lines while the equipment still is in good condition. Conversely, common battery systems may be warranted in exchanges as small as 200 lines, wherein the conditions are favorable and the need of the highest grade of service is great.

Transfer Systems or Local Trunking. — When it is desired to use a simple magneto or common battery system in an office requiring more than three operators, so that there will be lines beyond the reach of each operator, it is possible to provide local trunks or transfer circuits to interconnect lines which cannot be reached directly. These are called transfer systems or local trunking systems. If an operator has a call for a line which she cannot reach directly, she inserts the calling plug in a jack of a trunk which leads to the section carrying the jack of the called line. At that section an operator connects the chosen trunk with the called line and assists in disconnecting it at the end of conversation. These systems have service disadvantages which in most cases more than outweigh the savings, and the best practice is to use multiple switchboards where simple switchboards are outgrown. An important exception is the case of large cities where the majority of calls would have to be completed through a second central office in any event. Under such circumstances multiple jacks for the trunks only are provided, the subscriber's line being provided with but a single jack and practically all calls are trunked irrespective of their designation.

Multiple Switchboards (Fig. 31). — A multiple switchboard is one in which all the lines of the office are brought within reach of each operator by

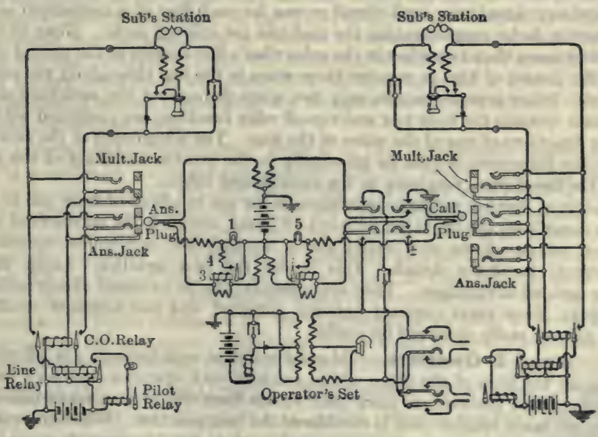

Fig. 31. Circuits of Multiple Battery Switchboard

equipping each line with a number of jacks, distributed over the board in such a manner that each operator can reach at least one of these jacks. The usual practice is to equip each line with one answering jack, at which all its calls will

be answered, and in addition with a plurality of line jacks into any one of which a plug may be inserted when that line is called. As there is a plurality of places where the called line may be taken some guard is required to prevent an operator taking the line when it is in use at another part of the switchboard. This guard is called the " busy test " and is provided by changing the electrical state of a metal thimble of each jack of a line whenever a plug is inserted. When idle, all the thimbles of all the jacks of a line are at the same potential as all the tips of all idle plugs. Touching the tip of a plug to the sleeve of a jack of an idle line produces no electrical result. But when an operator touches the tip of a plug to the sleeve of a line which is busy, difference of potential between tip and sleeve causes current to flow and a click to be audible in her receiver. She then tells the calling subscriber the called line is busy.

Fig. 31 shows the complete circuits of a calling line, a called line, and the connecting cords of the Western Electric Company's relay switchboard No. 1, which is the standard of the American Telephone & Telegraph Company. The cycle of operations is exactly as described with reference to the simple common battery switchboard with the addition of testing the jack of the called line. Current supply to the cord circuit is of the form shown in Fig. 29. The jacks have fewer parts than in the simple switchboard before described. The line relay is disconnected from the line by a cut-off relay instead of by a cut-off jack. The cut-off relay is operated by current from the cord circuit when a plug is placed in any jack of the line. This current also furnishes the busy test potential to all jacks of a line so long as a plug is in any jack. Direct current which actuates the subscriber's transmitter also energizes the relay 3, placing the shunt 4 around the supervisory lamp 1. It results that the lamp 1 or 5 will be lighted or dark, depending upon whether the switchhook of its line is down or up.

All principal common battery multiple switchboard systems now in general use have the common features of an audible busy test, line lamp signals, two supervisory signals per cord circuit — one for each line of a connection — and answering jacks supplementing the multiple (or calling) jacks. None of them requires more than two wires in the subscriber's line outside of the central office, but the system of Fig. 31 requires three wires in the line throughout the switchboard. Other systems require only two wires throughout the switchboard and in these systems the busy test and cut-off relay functions are combined with the other functions of one of the wires of the line. The advantages of the two-wire over the three-wire system lie principally in the greater simplicity of the jacks and the fewer wires necessary in the switchboard cables.

Common battery multiple switchboards for use in offices of from 500 to 1200 lines sometimes are provided with cut-off jacks, avoiding the necessity of using cut-off relays. Some such switchboards use magnetic mechanical signals as a substitute for lamp signals. This is not the best practice. Lamp signals have decided advantages. They are compact, can be mounted closely adjacent to jacks, show great contrast between actuated and non-actuated condition, and are not easily obscured by cords.

A and B Operators. — Manual multiple switchboards are adapted to bring all the lines of the office within reach of each operator, but such switchboards cannot concentrate more than 10,000 lines within such reach unless the jacks are made smaller than is considered the best practice. When an exchange contains more than 10,000 lines, it is customary to provide more than one office. This makes possible a saving in lines and things relative to them, but requires calls to be trunked from one office to another. In an exchange of 100,000 lines, for example, there would be ten switchboards of 10,000 lines each, each switchboard serving a district of the exchange.

Calls *originating* in one district will be *for* lines in all the districts and only

those for lines in the originating district can be completed in the switchboard of that district. Calls for subscribers in all the nine other districts must be trunked. To accomplish this, the operators in a multi-office exchange have equipment enabling them not only to complete calls directly in the multiple before them but to connect subscribers with trunk lines leading to the other offices, and in each office furthermore there are operators and equipments adapted to receive and complete calls trunked to them from the offices where they originated.

Operators who answer subscribers are called *A* operators; operators who serve trunk lines incoming from other offices are called *B* operators. The cycle of operations in a call for a line not in the office first called is as follows: subscriber lifts his receiver and his line lamp lights; operator answers and asks number; learning by prefix that call is for a subscriber in a distant office, operator presses a key marked with that prefix, so connecting her telephone set with a line leading directly to a receiver of a *B* operator in the distant office; *A* operator speaks the number desired, following with the *prefix of her own office; B* operator in distant office names back a trunk number; *A* operator inserts calling plug in a multiple jack of that trunk and simultaneously distant *B* operator inserts the plug of that trunk in the called line; method of ringing depends on type of apparatus; at close of conversation *A* operator disconnects, which act lights disconnect lamp before *B* operator, who disconnects in response, extinguishing signal.

Main Distributing Frame. — The main distributing frame is a device upon which are terminated, usually upon opposite sides, the lines from subscribers and from switchboard apparatus. Between these terminals connecting links of wire, called "jumpers," are soldered. The purpose of the main distributing frame is to enable a subscriber's line to be connected semipermanently with a given switchboard line. A subscriber may move anywhere within the district of his central office and may utilize any pair of wires entering that office without relinquishing his particular telephone number, as his switchboard circuit may be connected by jumper to whatever entering cable wires his line may use. Protective apparatus, consisting of sneak current arresters and carbon air-gap arresters, is associated with all lines entering a central office. These protective devices customarily are mounted on the switchboard side of the main distributing frame.

Intermediate Distributing Frame. — The intermediate distributing frame is a device on which lines to multiple jacks and lines to answering jacks respectively are accessible for inter-connection. Its object is to enable a given line to terminate on an answering jack at any part of the switchboard and this connection also is semi-permanent. By this is meant that when a subscriber's line has been connected by jumper to an answering jack on a certain operating position, all the calls of that line will be answered at that position until the jumper in the intermediate distributing frame has been changed. The purpose is to enable the traffic originated by the subscribers of an office to be divided equally among the operators. This equalization is not automatically done by the intermediate distributing frame, as is the intent in the automanual system.

AUTOMATIC OR MACHINE-SWITCHING SYSTEMS. — There are two methods of switching lines in a central office by means of machines. One of them, generally known as the automatic system, connects and disconnects the calling and called lines by the agency of central office machines under the control of the calling subscriber. The other utilizes machines similar in function to those used in the automatic system, but these are controlled by skilled operators in the central office, and the subscriber gives his call orally to these operators as in the common battery relay manual system.

The fundamental merit of machine systems is in the greater accuracy of machines over human beings in doing certain things. The routine of telephone-line switching has become so fully standardized that it lends itself with advantage to machine methods, with diminution of error and increase of speed of the same order as resulted from the adoption of typewriters and adding machines as substitute for script and mental addition.

Such systems are in successful use in many exchanges in the United States and to a growing degree abroad. Their use latterly has been stimulated by rising labor costs and by the difficulty of securing and holding sufficiently-skilled women operators for the manual system. The opinion of telephone-system managers throughout the world recently has crystallized favorably to machine switching, principally for economic reasons and partly because of a fuller but perhaps belated recognition of what machine switching systems can do.

Machine switching systems so far most widely used utilize the step-by-step principle. In the system of the Automatic Electric Company, which now is in wide use, selector and connector switches take as many steps per movement as there are units in the digit called.

In manual multiple switchboards, all the lines of an office are accessible to each operator. At the time of making a connection the operator and the calling plug of a cord circuit may be considered as a terminal device attached to the calling line and seeking the called line in the multiple of all the lines. In a manual transfer switchboard or in the trunking of a call from one office to another in an exchange having several offices with multiple switchboards, a first and a second operator must co-operate to complete certain connections.

A machine system represents a third case in which two or more devices must co-operate to complete a connection. In a word, as many separate machines are engaged in making a connection as there are digits in the called number, less one.

Success in machine telephony came with the abandonment of the complete multiple principle and the adoption of the complete trunking principle, which had been abandoned in manual telephony some time before.

Selector and Connector Switches. — The underlying feature of this type of machine trunking system is that the calling subscriber directs the first "selector switch" to seek an idle trunk from a certain group of trunks, all of which lead in a direction determined by the first digit of the called number. This first switch makes as many vertical steps as there are units in the first digit called, and immediately thereafter seeks out and appropriates an idle trunk of that group. On the calling of the next digit, a switch at the end of this selected trunk takes as many vertical steps as there are units in the second digit, and immediately seeks out and appropriates an idle trunk in that group. This process is repeated until all but two digits have been called, which carries the connection to the last switch which will be used. This switch is called a "connector" and has vertical and rotary motions like the preceding switches, but, unlike them, both vertical and rotary motions are under the subscriber's control. As he calls the next to the last (tens) digit, the switch takes as many vertical steps as there are units in that digit but does not follow with an automatic rotation. It rotates in response to the calling of the last digit, taking as many steps as there are units in that digit and stopping with its wipers in contact with terminals of the called line. Ringing of the called subscriber then is caused by automatic means or by relays responsive to a push button in the calling subscriber's telephone. Disconnection ensues when the calling subscriber hangs up his receiver.

Automanual System. — What has just been said relates to machine systems under subscriber control, in which all the digit-movements of the selector and

connector switches are dictated by the subscriber's turning of a disk attached to his telephone. In the second type of machine-switching apparatus, of later origin than that formerly called automatic, there are in the central office operators who receive by telephone the orders of subscribers as in the manual system, but who execute those orders wholly by machines. The latter are controlled by sets of keys like those of an adding machine. Disconnection is instantaneous on the hanging up of subscribers' receivers. Very high efficiencies are attained by this apparatus, and operators' labor costs are low, as few operators are required and their work is done under most favorable conditions.

The two systems share the advantages of the use of machines; one requires the subscriber to operate a dial, the other requires him to state his wants orally; it is not yet determined which shall be properly called the "natural" method of operating. In both systems, however, a higher quality of service is attainable than in any manual system, and operating costs are somewhat reduced.

Relay Automatic System. — The subscriber-controlled system first described above operates on the step-by-step method within the central office. The relay automatic system contains no such devices, but consists almost wholly of simple relays, much like those used for telegraphy or for relay manual switchboards. Current impulses resulting from the operation of the subscriber's dial actuate these relays in the central office, connecting the lines. Disconnection follows hanging up the receiver of the calling station. Furthermore, relay systems of this type may be used in connection with dial-less telephones by the addition of trunk lines leading to operators in automanual central offices, and meet the requirements of many small rural exchanges adjacent to city exchanges. Calls from the rural subscribers are answered by city operators, who actuate the relay equipment in the rural exchange over signal trunks. As soon as the connections are made the trunks are released for other use. Disconnection of the subscribers' lines follows hanging up the receiver of the calling line. Many rural exchanges thus can be operated from one city office, in conjunction with its other work, the rural offices requiring no operators or technical attendance.

PRIVATE EXCHANGES. — A private exchange is a group of telephone lines and switching apparatus subordinate to a central office, and usually on the premises and for the uses of a single business. Private exchanges are widely used in hotels, apartment houses, factories and suites of offices.

Private exchanges are of three types: those using simple switchboards which are attended by operators through whom all local and outside calls pass; those requiring keys at each telephone whereby each user may select the line of any local telephone or a trunk line to a central office, and those having a local equipment of automatic switches of the step-by-step or of the relay type.

In all cases the object of the private exchange is to enable the telephones within them to communicate with each other directly without using lines through the central office at all. A second object is to enable communication from telephones of the private exchange through the central office by the use of fewer trunk lines than there are telephones in the private group. A serviceable ratio of trunk lines to local telephones may be from one-tenth to one-fifth at a considerable saving in cost over the providing of a line to the central office from each local telephone.

PHANTOM CIRCUITS (Fig. 32.) — In Fig. 32, four wires join two offices. *RR* are transformers. They are designed to be efficient in transforming both talking and ringing currents. Currents from telephones connected to either physical pair of wires pass at any instant in opposite directions in the two wires of the pair. The phantom circuit uses one of the physical pairs as a wire of its line. It does this by tapping the middle point of the line side of each of the

transformers. The currents of the phantom circuit are not heard in the physical circuit because they pass outwardly from the middle points of the secondaries in equal and simultaneous amounts and, therefore, produce no resultant magnetization in the core. The currents of the physical circuits are not heard in the phantom circuit because the former can produce no differences of potential in the phantom circuits, provided all four wires are equal in resistance and insulation; under these conditions no difference of potential can exist between the middle points of the repeating coils at the two ends of the line.

Phantom circuits also are arranged with transformers inserted between the line and the phantom jack at each end of the circuit as in Fig. 32.

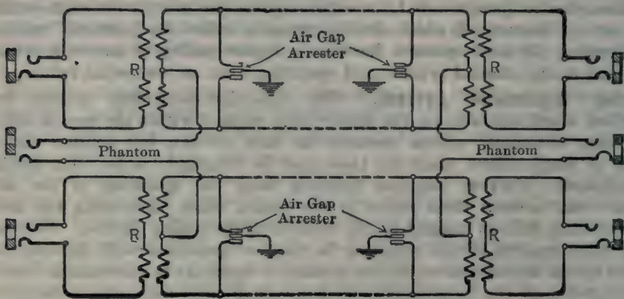

Fig. 32. Phantom Circuits

It is now possible to phantom loaded circuits and to load phantom circuits. The physical circuits, however, must be carefully designed and constructed and well maintained, in order to get good results on both physical and phantom circuits.

JOINT TELEPHONE AND TELEGRAPH CIRCUITS. — Telephone lines may be equipped so as to permit speech and telegraphy to go on over the same wires at the same time without interference with each other. There are two ways of accomplishing this, being known respectively as the simplex system and the composite system.

Simplex Circuits (Fig. 33) are made from metallic circuit telephone lines as in Fig. 33. The principle is the same as that of a phantom telephone cir-

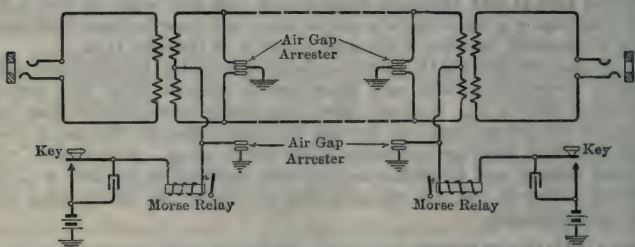

Fig. 33. Simplex Telephone and Telegraph Circuit

cuit. Nearly the same results can be obtained by using a simple impedance coil (i.e., a single winding with tap brought out from the center) bridged across the telephone line instead of a transformer inserted in it, but in ringing on such a line with a grounded generator the telegraph relays will chatter.

Composite Circuits (Fig. 34) depend upon a different principle. Impedance coils are inserted to oppose alternating currents, and condensers to oppose direct currents. In Fig. 34 one telephone circuit forms two Morse circuits, so that two wires form one telephone circuit and (with the earth) two telegraph circuits. Each Morse circuit includes in series two 50-ohm impedance coils and condensers are shunted to ground between the Morse sets and the impedance coils. The 50-ohm impedance coils are connected differentially and so offer low impedance to the Morse impulses, whose frequency is not high. The coils, however, offer great impedance to voice currents, since they are not differentially connected with respect to the latter. Voice currents can pass through the con-

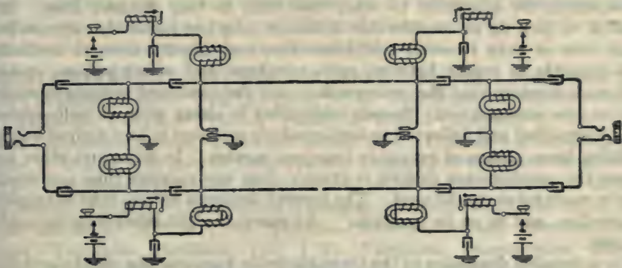

Fig. 34. Composite Telephone and Telegraph Circuit

densers in the telephone line, but direct currents cannot. Impulses due to discharge of coils and capacities in the Morse circuit would make sounds in the telephone but that they are choked off or led off by the 30-ohm impedance coils and the large capacities connected to the Morse sets.

Railway Composite Systems. — The principles of the respective impedances of inductances and capacities to direct and alternating currents, upon which the composite system depends, are applied to the conversion of series telegraph lines into combined telegraph and telephone lines. It sometimes is of advantage to modify a single-wire telegraph line so that telephones may be worked upon it, without abandoning its use as a regular telegraph circuit.

Such an arrangement is known as a railway composite system. Calls between telephone stations on such a circuit are sent by means of high-frequency alternating currents, such as are developed, for example, in an automobile spark coil. The signals are received at the telephone stations on "howlers," which are merely telephone receivers with special horn-shaped mouthpieces. These calling currents are of such high frequency as not to affect the telegraph relays. The paths for the higher-frequency currents in general are supplied by condensers and the paths for the lower-frequency currents for telegraphy in general are supplied by inductances.

TELEPHONE REPEATERS. — A telephone repeater is an apparatus which amplifies the voice currents flowing in the line. Repeaters are not needed in lines within a city, but are useful in long-distance working. They must be able to amplify speech in both directions in the line, without requiring switching of elements of the apparatus, so that the conversation may proceed the same as on a line having no repeaters. Amplifications sufficient for around-the-globe transmission are now possible, assuming a line to be available.

Two methods of amplification exist. That of Shreeve uses an electromagnetic receiver to vibrate the movable electrode of a carbon-granule transmitter.

The other method uses a three-electrode vacuum tube, and has no moving parts whatever, unless electrons be called moving parts. In both methods there are associated transformers and condensers to accomplish the two-way requirements of the problem.

In the vacuum-tube method one of the tube's electrodes is a filament, glowing as in an incandescent lamp; the other two electrodes are metal surfaces insulated from each other and from the filament within the highly evacuated tube. Variations of potential on one of the insulated surfaces cause variations of current in a circuit between the filament and the second insulated surface. The current so varied may have many times the amplitude of variation possessed by the current causing the varying.

Repeaters are adaptable to other uses than in long distance lines. It is possible to pick up at considerable distances stray but inaudible currents from single wire or even two-wire telephones line, and amplify them to audibility. It is possible to repeat between radio-telephone and wire-telephone systems with such freedom that the conversation over the combined systems is equivalent to wire conversation. Speech originating in either wire- or radio-systems can be amplified and delivered to loud-speaking receivers in any desired volume, enabling large groups of people to hear one speaker. In these and other uses, where the amplification desired is greater than one vacuum tube can supply, two or more tubes can be used. The current delivered by one tube is amplified by another, and so on in "cascade." As many as eight tubes may be used in cascade.

For a detailed discussion of the three-electrode vacuum tube see the article on *Radio Communication*.

MULTIPLEX TELEPHONY. — Vacuum tubes can change direct currents into high-frequency alternating currents, capable of being propagated as electromagnetic waves through space or along conductors. In the latter form of propagation a number of different frequencies can be carried along the same wires, each frequency being a carrier for one conversation. Inductances and capacitances properly related to the frequencies prevent interference among the several conversations. The currents used in such multiplex telephony are believed to be carried through space adjacent to the wires utilized and not through the metal of the wires. This multiplex telephony can be carried on over wires that at the same time are being used for ordinary telephony and for Morse telegraphy. See article on *Telegraph Systems*.

COSTS (Pre-war figures). — The following prices were current in 1913; they are all f.o.b. factory. These costs are given merely as a rough guide for preliminary estimates. For accurate estimates quotations should be obtained from the manufacturers.

Magneto wall set complete:	
Series (80-ohm ringer)	$9.00
Bridging (1000- to 2500-ohm ringer)	10.50–12.25
Magneto desk set complete:	
Add 75 cents to above prices.	
Common battery wall set complete:	
Oak or walnut, 500-ohm ringer	8.50
Oak or walnut, 1000-ohm ringer	8.75
Hotel type, 1000-ohm ringer	7.25
Common battery desk set complete with cord:	
500-ohm ringer	9.25
1000-ohm ringer	9.50
Automatic wall or desk set	13.00

Magneto switchboards:	
50–200 lines, 1-operator equipment	175–575
250–300 lines, 2-operator equipment	730–865
Simple common battery switchboards:	
100–600 lines	550–2500
Corresponding cost per line	5.50–4.17

Multiple common battery switchboards, formerly standard for offices in large city exchanges, are so largely superseded by machine-switching equipments in present practice that prices will fluctuate widely among different types. Machine-switching equipment prices vary with the type; the amount of equipment depends on the amount of traffic and on the number of telephones in the system. In general, however, machine-switching equipments cost more to install and less to operate than do manual equipments.

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TELEPHONE LINES. — (*See also Distribution Lines; Transmission Lines; Wires and Cables.*) The line always consists of two conductors, though one of these may be the earth. The standard telephone line consists of two wires of the same size and of the same material. If only one wire is used and the earth forms the return the line is called a "grounded circuit." A line of two wires without earth return is called a "metallic circuit."

LINE CONSTANTS. —(*See also Transmission Lines.*) The transmitting efficiency of a telephone line depends upon its resistance, electrostatic capacity, leakage and inductance.

Leakage. — High insulation, i.e., small leakage, between the two wires of a line and from each wire to earth is possible in cables, and reasonably high insulation is possible in open wire lines.

Resistance.— The resistance of the conductors of a line attenuates the telephone current, but does not distort it appreciably, because the losses due to resistance are practically independent of the frequency of the current.

Electrostatic Capacity. — The electrostatic capacity of a line attenuates the current and distorts it also, because the charging current is not independent of the frequency. As the voice current is an alternating one of composite frequency, the higher frequencies are reduced more than the lower.

There are two ways of expresssing the important and controlling quantity of capacity in cables, i.e., in terms of the "mutual capacity" of the two wires or in terms of the "grounded" or "regular capacity." (*See article on Capacity and Charging Current.*) The mutual expression is the better because it is the amount of mutual capacity which determines the capacity losses. Mutual capacity in telephone cables is about two-thirds the regular capacity of the same wires. A given cable has higher capacity at high temperatures and cable capacities should be referred to a standard temperature in specifying.

Inductance. — The inductance of telephone lines is usually comparatively small, since the wires are usually close together. Insulated wires twisted together for use as lines in cables have very small inductance. (*See article on Inductance and Inductive Reactance.*)

Pupin Coils. — In 1887 Oliver Heaviside pointed out that an increase of inductance would decrease the harmful effects of capacity. In 1900 Dr. M. I. Pupin made public (*Trans. A.I.E.E., Vol. XVII, p. 445*) his method of reducing attenuation and distortion by inserting additional inductance into lines and showed how to determine how much inductance to insert and where. In an ideal method. distributed inductance would offset or neutralize the distributed capacity. In the Pupin method the inductance is inserted in the form of coils at predetermined intervals. The coils ordinarily are a mile or more apart. They are called "loading coils" because of physical analogies fully described in the publication cited.

CHARACTERISTICS OF OPEN WIRE LINES. — For open wire aerial lines hard drawn copper is now almost universally used, except for short unimportant lines, where iron wire may be employed. The sizes of copper wire employed range from No. 16 B. & S. to No. 8 B. & S., depending upon the length and importance of the line. Where the tensile strength of the wires is unimportant, as when the wires are fastened along walls or fences, with supports close together, smaller sizes may be employed, and soft drawn wire used instead of hard drawn. Aerial wires are usually spaced about a foot apart, horizontally, and from 18 to 24 inches vertically, depending upon the spacing of the cross arms. For the resistance, capacity, and inductance of aerial wires see *Wires and Cables, Bare.* The insulation resistance between a pair of aerial wires depends upon the types of insulator, pin and cross arm, and upon the

condition of the weather. The following values are representative of good practice: —

Very dry, 500 megohms per mile.
Average, 25 megohms per mile.
Very wet, 2 megohms per mile.

Three different gages are employed for specifying the size of telephone wire: the B. & S., the B. W. G., and the N. B. S. (*See article on Gages, Wire.*) The following table gives the diameter, weight and resistance of the various sizes employed in telephone practice.

TABLE I

Number	Diameter in mils	Weight per mile in pounds	Resistance per mile of wire in ohms, 60° F.	Resistance per mile of circuit in ohms, 60° F.
8 B. W. G.	165	435	1.97	3.95
6 B. & S. G.	162	419	2.05	4.10
8 N. B. S. G.	160	409	2.10	4.20
9 B. W. G.	148	350	2.45	4.91
7 B. & S. G.	144.3	331	2.59	5.19
9 N. B. S. G.	144	331	2.59	5.19
10 B. W. G.	134	287	2.98	5.97
8 B. & S. G.	128.5	262	3.28	6.56
10 N. B. S. G.	128	262	3.28	6.56
11 B. W. G.	120	230	3.73	7.47
11 N. B. S. G.	116	215	3.99	7.99
9 B. & S. G.	114.4	208	4.14	8.27
12 B. W. G.	109	190	4.52	9.05
12 N. B. S. G.	104	173	4.97	9.94
10 B. & S. G.	101.9	166	5.17	10.33
13 B. W. G.	95	144	5.96	11.91
13 N. B. S. G.	92	135	6.35	12.70
11 B. & S. G.	90.74	132	6.49	12.98
14 B. W. G.	83	110	7.80	15.61
12 B. & S. G.	80.81	105	8.19	16.39
14 N. B. S. G.	80	102	8.40	16.80

CHARACTERISTICS OF TELEPHONE CABLES. — Telephone cables are groups of pairs of paper insulated wire twisted together and enclosed in a lead sheath. Soft drawn copper is invariably used for the conductors. The size wire usually employed ranges from No. 19 to No. 22 B. & S. For long distance underground lines individual wires as large as No. 10 B. & S. have been employed. For the resistance of the different sizes of conductors see *Wires and Cables*. The capacity of any pair of wires and the capacity of any one wire to ground depends largely upon the way in which the cable is made up. When the wires are twisted tightly together the capacity is greater than when they are loosely assembled. The mutual capacity ranges from 0.067 to 0.090 microfarad per mile, and the regular or grounded capacity from 0.10 to 0.12

microfarad per mile for tightly twisted wires, as against 0.054 to 0.067 and 0.080 to 0.10 respectively for loosely twisted wires. The former range of capacity is usually referred to as "high" and the latter as "low" capacity.

The effective insulation resistance between the two wires of a pair is about ½ megohm per mile at 800 cycles per second in a well-constructed cable. This figure includes the effect of dielectric hysteresis. The corresponding insulation resistance measured by direct current would be about 500 to 1000 megohms per mile.

The inductance of a pair of wires in a telephone cable is practically negligible, being less than 1 millihenry per mile of cable (2 miles of wire).

DISTORTION AND ATTENUATION. — (*See also Transmission Lines.*) Corresponding to any vowel, syllable or word spoken into a transmitter a current wave of a definite shape is sent over the line. This wave is made up of a number of simple sine waves of different frequencies, called harmonics, displaced with reference to one another, i.e., reaching their zero values at different times. Each of the simple sine waves as it progresses along the line decreases in amplitude or is "attenuated." Moreover, the shorter waves are in general attenuated more than the longer ones, with the result that the wave which reaches the receiver is made up of harmonics of relatively different magnitudes than the wave sent out from the transmitter, that is, the resultant wave at the receiver is "distorted." The sound produced by the receiver, consequently, differs from the word spoken into the transmitter, since the quality of the sound depends upon the relative magnitude of the constituent harmonics. There is also a displacement of the harmonics with reference to each other, which displacement, although it changes the shape of the resultant wave, has but little, if any, effect upon the quality of the sound.

Attenuation of a wave without distortion merely changes its amplitude, but since the wave form remains unchanged, only the loudness of the sound produced thereby, were it converted into a sound wave, is affected. Resistance alone, since it produces the same relative attenuation of all wave lengths, is therefore the least troublesome of the line characteristics. In most cases, the capacity of the line is the controlling characteristic, since the attenuation produced thereby depends upon the wave length, and therefore distortion results.

In addition to the distortion produced by the line itself, there is also a distortion in the transformation at the transmitter of the sound wave into a current wave, and again another distortion at the receiver where the current wave is reconverted into a sound wave.

DISTURBANCES DUE TO NEIGHBORING POWER TRANSMISSION LINES.* — (*See also Distribution Lines; Transmission Lines.*) A very small voltage between the wires of the telephone circuit is sufficient to produce noise in the telephone apparatus comparable in volume with the sound produced by the voice currents. The noise is due almost entirely to the harmonics of the power system, especially to those between 150 and 1200 cycles. At these frequencies induced currents in the telephone line equal to a few millionths of an ampere are sufficient to make conversation difficult.

Balanced versus Unbalanced Power Circuits.—In considering the effect of neighboring power circuits on telephone circuits the difference between balanced and unbalanced power circuits must be clearly kept in mind. A *completely balanced* power circuit consists of two or more wires energized in such a way that the vector sum of the currents in all the wires of the circuit is practically zero, and the vector sum of the voltages between the several wires and the ground is practically zero. A *completely unbalanced* power circuit consists of one or more wires with a ground return, the currents in all the wires being practi-

* By H. Pender.

cally in phase and the voltages between the several wires and the ground being practically equal and in phase. In practice power circuits are frequently neither completely balanced nor completely unbalanced, but the currents and voltages may be resolved into completely balanced and completely unbalanced components. Examples of such circuits are a three-phase circuit having one wire grounded and a three-phase circuit with grounded neutral; in the latter case the neutral current may be of fundamental frequency due to an unbalanced load on the system, or the neutral current may be of triple frequency due to a third harmonic in the voltage wave (*see Generators, Alternating Current*).

The inductive effects arising from unbalanced voltages or currents are very much larger than the inductive effects arising from balanced voltages or currents of the same magnitude, because with balanced voltages and currents the effect of one wire is largely neutralized by the near presence of other wires of opposite polarity.

Voltages Induced between Wires and between Wires and Ground. — The power circuits affect the telephone circuits by producing: (1) a voltage between the two wires of the telephone circuit; (2) a voltage between telephone wires and ground.

The voltage between telephone wires and ground is usually large compared with the voltage between wires. Even though the telephone circuit is transposed so that equal voltages are induced between the two wires and ground, the voltage to ground produces a voltage between wires because of the unavoidable minute inequalities in the constants of the two sides of the circuit. If the voltage to ground is high it endangers the users and operators of telephones connected to the circuits and puts the circuits out of commission by operating the protective devices.

Remedies for Inductive Disturbances in Telephone Lines. — It is in practice impossible to perfectly realize any of the following remedies, and no one of them would be sufficient even though perfectly realized. In order to minimize inductive disturbances it is therefore necessary to carry out each of the remedies as far as possible.

1. **Transposition of Power and Telephone Circuits.** — The way in which inductive effects are affected by the transposition of the power and telephone circuits is indicated in the following table:

EFFECT OF TRANSPOSITIONS

Power Line	Voltages Induced in Telephone Line	Affected by Transpositions in	
		Power Circuit	Telephone Circuit
Balanced	To ground	Yes	No
	Between wires	Yes	Yes
Unbalanced	To ground	No	No
	Between wires	No	Yes

In order to balance as far as possible the inductive effects between a three-phase power circuit and a telephone circuit it is necessary to divide each uniform section of the exposure into six or a multiple of six equal sections in which equal voltages of the six possible phase angles are induced. The methods of making transpositions of a telephone line are described below in the section on *Line Construction;* methods of transposing power lines are described in the article on *Transmission Lines.*

2. **Balancing the Power Circuit.** — The voltages induced between the telephone circuits and ground by small unbalanced components of the power voltages and currents are relatively large and are not affected by transpositions. In order to reduce the inductive interference, it is therefore important to construct and operate the power circuit in such a way that it is as far as possible balanced with respect both to voltage and to current.

3. **Balancing the Telephone Circuit.** — In order that the noise caused by the voltage between the telephone wires and ground may be minimized, it is necessary that the telephone circuits be carefully balanced, that is, that the two sides of the circuit be of the same resistance and have as nearly as possible the same insulation resistance and the same capacity to ground. The unbalancing due to unequal insulation and to unequal capacities between cable conductors and ground is frequently important and every effort should be made to minimize it. All apparatus connected to the circuit must be such that the impedances inserted in the two sides of the circuit, or the impedances connected between the two sides of the circuit and ground, are as nearly as possible exactly equal.

A Joint Committee on Inductive Interference of the Railroad Commission of the State of California made a thorough study of the facts of inductive interference between electric power and electric communication circuits between 1911 and 1919, and its findings, reports and recommendations, together with the Commission's rules and orders, are a source of full information on this subject. The matter is available in a volume issued by the Commission, as noted in bibliography.

COMMERCIAL TRANSMISSION. — LIMITING TRANSMISSION DISTANCES. — Transmission is said to be "commercial" when two persons with normal ears and voices, using standard transmitters and receivers, can converse with reasonable ease. The length of the line over which such a conversation can take place is called the "limiting transmission distance." The limiting transmission distance of a pair of No. 19 B. & S. copper wires in a paper insulated cable is about 30 miles, the resistance per mile of cable or "loop mile" (2 miles of wire) being 88 ohms and the mutual capacity of the two wires 0.054 microfarad per mile. Such a line is taken as a standard to which all other lines may be referred.

For ease in comparing various lines, the qualities of the standard line just described are very closely imitated by assembling resistance coils and condensers in a portable case, the number of coils and condensers being such that the artificial line thus made is equivalent to many miles of the standard line. This artificial line is usually referred to as a standard cable set.

To determine the qualities of an unknown line in terms of standard cable, an observer listens to a distant speaker alternately through the unknown line and the standard cable set. He adjusts the latter until he hears the speech equally and similarly through both. The number of miles of the standard set then gives the limiting transmission distance of the line tested. The quotient of the limiting transmission distance by the actual length of the line is a measure of its transmitting ability.

The limiting transmission distances of various types of lines determined in the manner explained above are given in Table II, below. This table includes no allowance for switchboard and connection losses.

Effect of Bridging Telephone Set Across Line. — The effect of bridging a standard local telephone set across a non-loaded open-wire line is to diminish the limiting transmission distance by approximately 12 per cent. The loss due to bridging such a set across a cable circuit is considerably less, namely, about

7 per cent. These figures also apply roughly to each of several sets bridged across the line, provided these sets are widely separated.

LINE CONSTRUCTION.—In open country and small towns aerial lines with bare wires are almost invariably used. In large cities cables containing from 5 to 600 pairs of wires are usually employed. Cables may be run overhead or underground. Aerial telephone cables, on account of their low tensile strength, are hung from steel messenger wires, the latter being supported from the cross

TABLE II

Gage of wire	Theoretical limiting distance, with no allowance for switchboard losses and without Pupin coils
	Miles
No. 8 B. W. G. copper, open wire line	900
10 B. W. G. copper, open wire line	700
10 B. & S. copper, open wire line	400
12 N. B. S. copper, open wire line	400
12 B.& S. copper, open wire line	240
14 N. B. S. copper, open wire line	240
8 B. W. G. iron, open wire line	135
10 B. W. G. iron, open wire line	120
12 B. W. G., iron, open wire line	90
16 B. & S. cable, copper	40
19 B. & S. cable, copper	30
22 B. & S. cable, copper	20

Wires smaller than those given in this table should not be used on pole lines, on account of their lack of mechanical strength.

arms or fastened directly to the poles. Underground cables are usually run in tile or other type of ducts. (*See Conduits and Conduit Lines, Underground; Wires and Cables.*)

Open Wire Lines.—With the exception of the smaller insulators, usually glass, and the larger number of wires carried on a single pole line (as many as 100), the construction of open wire telephone lines differs but slightly from that of pole lines for power transmission. (*See Cross Arms; Distribution Circuits; Poles for Overhead Lines; Transmission Lines.*)

The size of poles and their spacing usually employed for various classes of lines through level country are given in the following table:

Type of line	Ultimate number of wires	Height, feet	Diameter of top, inches	Number per mile
Short, local	6	22	5 to 6	30
Important routes	20	25	6 to 7	40
Long distance	30 to 40	30 to 35	7 to 8	40

Where special conditions arise the height and size of poles must be selected accordingly. The depth to which a pole is set in the ground ranges from one-

fifth to one-eighth of its total height, the larger figure applying to short poles (25 feet), the latter to tall poles (65 feet).

Hard-drawn copper wire is now used on all important lines, though galvanized-iron wire is still employed on short unimportant lines. Recently copper-clad steel wire has been proposed for telephone lines. See *Wires and Cables* for the properties of wires. The particular size to be employed in any case will depend upon the length of the line; see Table II above. In open-wire lines the wires are usually spaced 1 foot apart horizontally and the cross arms are spaced 18 to 24 inches apart.

Transpositions to Prevent Cross-Talk. — Any varying magnetic flux which may thread the space between the two wires of a telephone line, or any varying electrostatic field about the line, will set up a varying current in the line, and if the frequency is such as to produce in the receiver a sound of audible pitch, this sound will interfere with the proper function of the receiver. Any noise produced in this manner by one telephone line on another is called "cross-talk." Noise may be produced in a similar manner by the induction from other sources, such as railway or lighting (alternating) circuits; see above.

To prevent such cross-talk or noise the wires of a telephone line are usually transposed every quarter of a mile. Fig. 1 shows a common transposition scheme. The vertical lines represent the cross arms of the poles where the

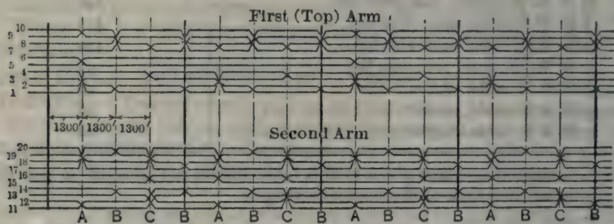

Fig. 1. Transposition Scheme

transpositions are made. The ordinary method of making a transposition is to cut each wire at each transposition point for that wire and to fasten the two ends either to separate insulators or to an insulator with two grooves, and then make the cross-overs with short wires. Transpositions can also be made without cutting the wire by using a single insulator with two grooves one well above the other. This scheme, known as the "single pin transposition," is also cheaper than the ordinary scheme.

The transpositions for the first and second cross arms are not alike, as shown in Fig. 1; when there are more than two cross arms, the wires on the odd numbered arms are usually transposed in the same manner as the first or upper arm, and the wires on the even numbered arms in the same manner as those on the second arm. When phantom circuits are used the wires on the several arms are not transposed alike.

Transposition systems become more comprehensive and more complicated with the growth of phantom circuit working and the development of improved technique in neutralizing inductive disturbances. Fig. 1 is merely an example of the principle of transpositions.

Telephone Cables for either aerial or underground use are made as follows: Soft copper wires are insulated with dry paper laid on spirally; two such insulated wires then are twisted into a pair, the two wires having differ-

TABLE III. PROPERTIES OF STRANDED TELEPHONE CABLES

No. pairs	Gage B. & S.	Electro-static capacity	Thickness of sheath, inches	Approximate external diameter, inches	Approximate weight per foot, pounds
5	22	High	1/12	0.48	0.55
10	22	High	1/12	0.59	0.71
15	22	High	1/12	0.66	0.83
20	22	High	1/12	0.72	0.93
25	22	High	1/12	0.77	1.02
50	22	High	1/12	0.97	1.45
50	20	High	3/32	1.10	1.88
100	22	High	3/32	1.32	2.35
100	22	Low	3/32	1.50	2.63
100	20	High	1/8	1.57	3.60
100	20	Low	1/8	1.81	4.11
200	22	High	1/8	1.84	[illegible].43
200	22	Low	1/8	2.11	4.99
200	20	High	1/8	2.11	5.47
200	20	Low	1/8	2.46	6.19
200	19	High	1/8	2.24	6.08
200	19	Low	1/8	2.65	6.94
300	22	High	1/8	2.21	5.71
300	22	Low	1/8	2.51	6.32
300	20	High	1/8	2.53	7.09
300	20	Low	1/8	2.96	7.94
300	19	High	1/8	2.69	7.95
300	19	Low	1/8	3.20	9.04
400	22	High	1/8	2.51	6.84
400	22	Low	1/8	2.86	7.56
400	20	High	1/8	2.89	8.56
400	20	Low	1/8	3.43	9.37
600	22	High	1/8	3.20	9.21
90	16	Low	1/8	2.88	7.2
43	13	Low	1/8	2.88	7.17
50	10	High	1/8	2.88	8.95

Note. — High capacity, 0.067–0.090 mutual, 0.10–0.12 grounded. Low capacity, 0.054–0.067 mutual, 0.080–0.10 grounded.

ent colored papers; a number of pairs are laid up spirally to form a cylindrical core. The core then is wrapped spirally with paper tape, is dried thoroughly and a lead sheath molded on it in a lead press. Many users require three per cent of tin in the lead sheath.

The external diameter and weight of a cable containing a given number of pairs depend upon how closely the wires are wrapped together, and this in turn determines the capacity of the wires, the capacity being greater the closer the wires are wrapped together. Table III gives the dimensions and weights of standard cables.

Telephone cables, if used in connecting subscribers' telephones to central offices, usually are formed of 22 B. & S. gauge wires, except for long loops, when No. 19 B. & S. is generally used. For trunk lines between offices in large exchanges, cables are usually formed of No. 19 B. & S. gauge wires. Loading coils are used in trunk lines, and successful operation of the largest exchanges would be impossible without loading coils.

Cables for long distance purposes have large wires. Where conditions permit the use of loaded cables between cities, the operating conditions are most uniform. Chicago and New York are connected to cities within a radius of two hundred miles by loaded underground cables, and that practice is extending. The principal advantages are that the cables are not disturbed by storms and their insulation does not vary. The insulation resistance affects the attenuation and distortion of the waves sent over the wire. Lines equipped with loading coils are particularly sensitive to changes in insulation resistance. If the leakage in wet weather greatly exceeeds the normal leakage the loading coils may do more harm than good.

Installation of Cables. — (*See also Conduits and Conduit Lines, Underground; Wires and Cables.*) It is of particular importance in all operations connected with telephone cables to keep the core dry. The conductors are insulated only by paper, which is useless if moist and is of value only because it incloses dry air. As soon as the sheath is removed from a cable for splicing or terminating, the paper absorbs moisture from the air and from the hands; therefore expose the core as little as possible and boil out all moisture by pouring hot paraffine over and through the conductors before finally closing the cable.

Splicing. — To splice a telephone cable, a lead sleeve is slipped over one of the ends and the sheath cut off of each of the ends, after scraping bright the part at which the sleeve later is to be soldered with wiped joints. The core now is bound tight with dry muslin just at the end of the sheath and the muslin packed under slightly. The cable ends are boiled in hot paraffine by pouring or immersion. If white fumes arise from the paraffine it is too hot. The wires then are joined by stripping the paper from the ends, twisting the bared parts together so as to include a little of the insulation, then sliding down a paper sleeve previously placed over one wire.

When all the pairs are spliced, the conductors are again boiled out with paraffine. The lead sleeve is slipped into place, its ends dressed down and a wipe joint made at each end. This gives the splice a sheath continuous with the cable itself. A *Y-splice* is one in which one cable branches into two.

Cable Terminals are devices in which the paper insulated wires of a telephone cable are made accessible to wires outside the cable. The fundamental requirement is that the insulation of the conductors shall be maintained, both by keeping moisture from the cable insulation and by insulating the points to which the wires outside the cable attach.

A standard form of cable terminal consists of a box of porcelain, or partly iron and partly porcelain, wherein the end of the cable is sealed by bituminous

compound. Binding posts in the porcelain receive the cable wires inside the sealed part and the outer wires on the outside.

Potheads are splices between rubber-insulated wires and paper-insulated cable wires. They are filled with bituminous sealing compound, so that moisture shall not lead to the paper insulation along the rubber-insulated wires.

Potheads are less widely used than before the development of the porcelain terminal. They are necessary for places where more than 50 pairs of wires are to be terminated. At the junction between aerial and underground cables, fuses require to be inserted to complete the protective system. Potheads then are placed on both cables, and the rubber-covered wires led to the fuse terminals. It is good practice to house the fuses and pothead ends in wood or metal protecting boxes.

Drop Wires. — The wires used to connect open wire lines or aerial cables to subscribers' premises are called drop wires and are of hard drawn copper insulated with rubber, covered with a braid, then twisted into pairs. Standard sizes are No. 16 B. & S. gage in regions where ice does not form, and No. 14 B. & S. gage in other regions. No. 17 B. & S. copper-clad steel wire is successfully used as a substitute for either.

ELECTROLYSIS of the sheaths of underground telephone cables takes place when the sheaths carry stray direct current from one region to another if the current leaves the sheath in the presence of moisture (*see Electrolysis of Grounded Structures*).

COSTS.—The cost of building a telephone line depends on so many variables that no data of any value can be given in the space here available. Useful cost data on telephone lines will be found in J. C. Slippy's *Telephone Cables* and in Clark's *Telephone Construction Methods and Costs*, Appendix A. Approximate costs of the various constituent items will be found in the articles on *Conduits and Conduit Lines, Cross Arms, Insulators, Poles for Overhead Lines, Wire and Cables, etc.*

BIBLIOGRAPHY.—See *Bibliography* in article on *Telephone Instruments and Circuits.*

[S. G. McMeen.]

TELEPHONE TRAFFIC AND RATES.— (*See also Telephone Instruments and Circuits; Telephone Lines.*) For the purpose of charges against telephone users, the unit of telephone traffic is the "conversation." For the purpose of designing and using telephone equipment the telephone "call" often is the unit. A call does not always result in a conversation. Telephone traffic is subject to general variations closely linked with variations in human activities. Obviously, telephone traffic relative to business is lower on holidays than on working days; on all days telephone traffic varies with the hours, in a way fairly uniform from day to day. It is possible to plot this variation, and Fig. 1 is a load curve representative of what happens each working day in most regular exchanges. The number of calls which subscribers originate depends on the kind of service rendered, whether residence, business, etc., and on the method of charging. Subscribers will call about twice as often under flat rates as under measured rates.

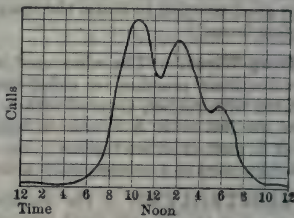

Fig. 1. Load Curve

A knowledge of the amount of traffic in a system, of its distribution as to time and as to divisions of the exchange, is important. By that knowledge the equipment must be designed, modified from time to time and the load distributed upon it as changing circumstances shall require.

Methods of Counting Calls. — Amounts of traffic in manual equipment are observed in three general ways. A peg-count is a record made by the operator actuating some counting device for each call answered or completed. A second way is to determine a ratio existing, for the particular time and place, between the calls in a given period and the average number of cord circuits in use in that period. Knowing these, the probable total can be computed from the cord circuit as counted. The third method is applicable to offices having service meters on all lines and is to associate one master meter per position with all the meters of that position, so that it will count one each time any service meter of the position is operated.

In machine switching systems, meters can be arranged to count, record and tabulate any desired part of the traffic automatically, whether or not the lines themselves are equipped with service meters for the message-rate system of charging for the service.

Operator's Speed.— The number of calls an operator can complete in an hour depends on the percentage of total calls which she must trunk to other offices. With standard manual equipment, for example, she can complete 240 calls in an hour if none have to be trunked, and 165 in an hour if 90 per cent of them have to be trunked. In machine switching systems of the operator controlled type, an operator completes a much larger number of connections per hour. All calls being handled alike, whatever the location of the called station within the exchange system, the work of the operator is uniform, and may be as many as 1200 connections per busy hour. Loads of 800 per hour are average and usual performance, as compared with loads in manual offices of 165 per hour.

Prompt disconnection is of great importance. A rule should be that disconnect signals shall be given prompt attention by some operator and shall take precedence over a call for connection. A flashing keyboard lamp indicating a recall should be given precedence over all other calls. In all machine-switching systems disconnection is automatic and instantaneous.

TRUNKING. — Traffic studies enable the determination of the number of trunks required for an anticipated traffic. The number of trunks required, between two central offices, or between a central office and a private exchange depends upon the number of calls which must be trunked per maximum or busy hour. The element of the probable coincidence in the time at which the calls will be made must be considered, and for this reason the more trunks installed the greater will be the number of calls which each trunk may be expected to handle. The following illustrate current manual practice:

Trunked calls per busy hour	35	90	1080
Number of trunks required	5	10	60
Calls per trunk	7	9	18

TELEPHONE RATES. — Rates for telephone service are of two kinds, "flat rates" and "measured service rates." Under flat rates, a fixed sum per month is charged and the subscriber may use his telephone as much as he pleases. Under measured service rates, the subscriber is charged a certain sum for each conversation he originates. Ordinarily also the subscriber guarantees to pay not less than a certain minimum amount per month. Rates for private exchanges often are made up by charging a rate per month for each local telephone, another rate per month for each section of local switchboard, another rate for each trunk line, and a still further rate for each outward call over the trunk lines leading from the private exchange, the monthly charge being the sum of all.

The cost of telephone service depends on the amount of use, though there is a fundamental cost of being prepared to furnish service. There is a tendency to change from the flat-rate to the measured-rate system. The latter is in operation in most large cities.

Flat-rate Systems. — In flat-rate systems there is no continuous effort to record the number of conversations. Counting is done from time to time for statistical purposes.

Measured-rate Systems. — In measured-rate systems it is necessary to record the conversations as they occur, so that each subscriber may be charged for the service he receives. Sometimes this is done by hand on tickets, but that method interferes with the operator's regular work, losing not only her service in operating but wasting switchboard investment by not using it efficiently. The best way is to equip each line with a meter under electrical control which causes it to count one unit for each completed conversation. Such meters are in extensive use in the larger cities. In manual systems, when a conversation takes place, the operator presses a meter key associated with the answering cord, and this makes the record upon the meter. In automatic systems the answering of the called station operates the meter.

In certain offices in London usual calls are charged at a penny each and certain special calls at two-pence each. In trunking calls of the latter kind, the operator is reminded of the higher charge by the lighting of a specially colored lamp as she uses the trunk order key. Being so reminded, she presses the meter key twice when such a conversation occurs, the result being that the calling subscriber pays two-pence for each such special call, because his meter records two units for each.

LONG-DISTANCE CALLS are those which occur between cities, as distinct from those within cities, no matter how large the latter may be. Long-distance calls usually are handled by special operators at special switchboards connected by trunks to the switchboards which handle city service. Long-distance calls usually require that a particular person be found in the called city, and the record of such calls includes the name as well as the number of the called person. On this ticket is recorded the duration of the conversation in minutes

and fractions so that the charge may be computed. This elapsed time is best recorded by a time-computing machine which prints a record.

Two-number Method. — Where calls between cities reach a sufficient number, it is found economical to provide for connecting by number only the stations of the subscribers who call frequently, and this is called the "two-number" method. Such connections generally are established without trunking to and from special long-distance switchboards. Operators at regular multiple-switchboard positions make and complete these calls. They are ticketed only at the originating end.

COIN-COLLECTING DEVICES. — Measured service calls, either local or long distance, may be made over telephones equipped with coin-collecting devices. These are so arranged that the operator making the connection knows in advance of conversation that the proper amount of money has been placed in the machine. Machines of one type require the deposit of the coin before the central office can be signaled at all; the coin is returned if the call is not completed. Machines of the other type require that the coins be deposited after the order has been given but before the conversation occurs.

DEVELOPMENT STUDIES — FACTORS AFFECTING COSTS. — A development study is an inquiry to determine what kind and what amount of construction or reconstruction is warranted by the conditions and probabilities of a region. Some form of development study always precedes such work. Economical construction and efficient operation require constant improvement in development-study methods and their use.

Forecasting Requirements. — In cities a first step is to forecast the future development through a period of from 15 to 20 years. This is done by examining statistics of growth. Ratios of telephones to population then are tabulated for the past, and future ratios estimated. A house count then is undertaken. This means the counting of existing buildings and the estimating of the telephone-using abilities of the occupants. At the same time a forecast is made of the probable future development of each region as it is studied.

Determination of Number of Central Offices. — If the exchange will require switchboards for more than 10,000 lines there will be more than one central office. The more switchboards there are in a town, other things being equal, the shorter will be the average length of subscribers' lines, and the greater will be the mileage of trunk lines. The more offices there are, the greater the cost of owning and maintaining the buildings for them, and the greater the cost of operating the equipment in them. A balance for these conditions must be found.

The economical number of districts having been determined, the office of each district can be located so as to attain the greatest wire economy. The conduit runs and pole lines then are laid out for the subscribers' and trunk lines. Speaking generally, it is good practice to provide a main conduit passing the central office and having cross routes extending from it at right angles, one on each alternate street or in alleys if they exist.

In machine-switching practice, it is possible further to subdivide exchange districts and to place some of the apparatus in small sub-district offices having trunks leading to the district office, still other trunks interconnecting all the district offices. This lowers still further the cost of subscribers' lines and increases still further the cost of trunk lines.

BIBLIOGRAPHY. — See Bibliography in article on *Telephone Instruments and Circuits*.

TEMPERATURE AND THERMOMETERS. — (*See also Heat and Thermal Properties; Pyrometers; Thermodynamics, Principles of; Units and Conversion Factors.*) The temperature of a body may be defined as its relative hotness or coolness referred to some standard substance under standard conditions. The change in some physical property of a standard substance must be utilized in order to give a number to temperature. Any device which serves this purpose is called a thermometer; if the device is applicable to the measurement of very high temperatures it is also called a pyrometer, q.v.

TEMPERATURE SCALES. — The standard* temperature-measuring device is the constant volume hydrogen thermometer, which consists essentially of a suitable receptacle containing a constant mass of hydrogen gas kept at constant volume, viz., the volume it would have at a pressure of 1000 millimeters of mercury and at the temperature of melting ice, with means provided for measuring any variation that may be caused to take place in the pressure of the gas.

Centigrade Scale. — The temperature of melting ice at a pressure of 760 millimeters of mercury is arbitrarily taken as zero degrees, and the temperature of saturated steam at a pressure of 760 millimeters of mercury is taken as 100 degrees. Calling p_0 the pressure of a constant volume of hydrogen gas when the receptacle is immersed in the melting ice and p_{100} its pressure when immersed in the saturated steam, and p_t its pressure when immersed in any given substance (the pressure in each case being measured after the lapse of a sufficient time for it to reach a constant value), the numerical value of the temperature of the given substance is defined as

$$t = \frac{p_t - p_0}{p_{100} - p_0} \times 100.$$

A degree centigrade is abbreviated deg. cent. or °C.

Fahrenheit Scale. — The Fahrenheit scale of temperature is derived in the same manner, except that the temperature of the melting ice is taken as 32° and the boiling point of water (at 760 millimeters mercury) as 212°. A temperature of t_f degrees Fahrenheit is then equal to

$$t_c = \tfrac{5}{9}(t_f - 32) \quad \text{degrees centigrade.}$$

Vice versa, a temperature of t_c degrees centigrade is equal to

$$t_f = \tfrac{9}{5}t_c + 32 \quad \text{degrees Fahrenheit.}$$

A degree Fahrenheit is abbreviated deg. fahr. or °F.

Réaumur Scale. — This scale, which is used to some extent in Europe and in breweries in this country, is defined in the same manner as the centigrade scale, except that the boiling point of water (at 760 millimeters mercury) is taken as 80°. A temperature of t_r degrees Réaumur is then equal to

$$t_c = 1.25\,t_r \quad \text{degrees centigrade.}$$

Platinum Scale. — See article on *Pyrometers*.

ABSOLUTE TEMPERATURE. — Let p_0 = absolute pressure of a given mass of gas at 0° C., p = absolute pressure of this same mass of gas at any temperature t° C. as above defined. For all values of t between about − 150° and + 1000° C., i.e., for all values of t within the experimental range, it is found

* Adopted by the Bureau International des Poids et Mesures.

DEGREES FAHRENHEIT CORRESPONDING TO DEGREES CENTIGRADE

°C.	0	−1	−2	−3	−4	−5	−6	−7	−8	−9
−40	−40.0	−41.8	−43.6	−45.4	−47.2	−49.0	−50.8	52.6	−54.4	−56.2
−30	−22.0	−23.8	−25.6	−27.4	−29.2	−31.0	−32.8	34.6	−36.4	−38.2
−20	− 4.0	− 5.8	− 7.6	− 9.4	−11.2	−13.0	−14.8	−16.6	−18.4	−20.2
−10	14.0	12.2	10.4	8.6	6.8	5.0	3.2	1.4	− 0.4	− 2.2
0	32.0	30.2	28.4	26.6	24.8	23.0	21.2	19.4	17.6	15.8

°C.	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
0	32.0	33.8	35.6	37.4	39.2	41.0	42.8	44.6	46.4	48.2
10	50.0	51.8	53.6	55.4	57.2	59.0	60.8	62.6	64.4	66.2
20	68.0	69.8	71.6	73.4	75.2	77.0	78.8	80.6	82.4	84.2
30	86.0	87.8	89.6	91.4	93.2	95.0	96.8	98.6	100.4	102.2
40	104.0	105.8	107.6	109.4	111.2	113.0	114.8	116.6	118.4	120.2
50	122.0	123.8	125.6	127.4	129.2	131.0	132.8	134.6	136.4	138.2
60	140.0	141.8	143.6	145.4	147.2	149.0	150.8	152.6	154.4	156.2
70	158.0	159.8	161.6	163.4	165.2	167.0	168.8	170.6	172.4	174.2
80	176.0	177.8	179.6	181.4	183.2	185.0	186.8	188.6	190.4	192.2
90	194.0	195.8	197.6	199.4	201.2	203.0	204.8	206.6	208.4	210.2
100	212.0	213.8	215.6	217.4	219.2	221.0	222.8	224.6	226.4	228.2
110	230.0	231.8	233.6	235.4	237.2	239.0	240.8	242.6	244.4	246.2
120	248.0	249.8	251.6	253.4	255.2	257.0	258.8	260.6	262.4	264.2
130	266.0	267.8	269.6	271.4	273.2	275.0	276.8	278.6	280.4	282.2
140	284.0	285.8	287.6	289.4	291.2	293.0	294.8	296.6	298.4	300.2
150	302.0	303.8	305.6	307.4	309.2	311.0	312.8	314.6	316.4	318.2
160	320.0	321.8	323.6	325.4	327.2	329.0	330.8	332.6	334.4	336.2
170	338.0	339.8	341.6	343.4	345.2	347.0	348.8	350.6	352.4	354.2
180	356.0	357.8	359.6	361.4	363.2	365.0	366.8	368.6	370.4	372.2
190	374.0	375.8	377.6	379.4	381.2	383.0	384.8	386.6	388.4	390.2
200	392.0	393.8	395.6	397.4	399.2	401.0	402.8	404.6	406.4	408.2

Example: − 12° C. = 10.4° F.; − 33° C. = − 27.4° F.; 13° C. = 55.4° F.

that, in the case of the so-called permanent gases (i.e., those which are not readily liquefied, such as air, hydrogen and nitrogen), the following relation exists between p_0, p and t provided there is no change in volume

$$p = p_0 (K + t),$$

where K is a constant, approximately equal to 273 to within less than 1 part in 300.* Consequently, assuming this relation to hold when the pressure p becomes zero, the value of t which would correspond to *zero* absolute pressure is

$$t_0 = -273° \text{ C.}$$

This temperature is called the absolute zero. That is, the zero of the centigrade scale is 273° C. above the absolute zero.* Similarly, the zero of the Fahrenheit scale is approximately 460° F. above the absolute zero.

* For hydrogen Callender gives the value 273.10.

The temperature measured above the absolute zero is called the absolute temperature. Let t_c = temperature in ° C. above the centigrade zero, t_f = temperature in ° F. above the Fahrenheit zero, then the absolute temperature in ° C. corresponding to t_c is

$$T_c = 273 + t_c,$$

and the absolute temperature in ° F. corresponding to t_f is

$$T_f = 460 + t_f.$$

Absolute Thermodynamic Temperature. — (*See article on Thermodynamics, Principles of.*) The difference between the temperature scale as defined by the constant-volume hydrogen thermometer and the absolute thermodynamic scale is less than 0.1° C. at ordinary temperatures, and is less than 1° C. throughout the range from − 150° to 1000° C.

THERMOMETERS. — The standard constant-volume hydrogen thermometer is seldom used except for standardizing purposes, and then only for temperatures up to about 500° C. The constant-volume nitrogen thermometer is used for standardizing purposes for temperatures up to about 1500°. For higher temperatures radiation pyrometers are used as standards (*see Pyrometers*).

Mercury-in-Glass Thermometers. — For ordinary temperature measurements, between about − 35° C. and 350° C., the ordinary mercury-in-glass thermometer is almost universally employed. The temperature may be read directly from the position of the end of the mercury column as given by a uniformly divided scale on the stem, provided the 0° and 100° points have been properly located and the stem has a uniform bore.

For accurate measurements the scale on the thermometer should be checked to determine whether the 0° and 100° points are properly located and whether the bore is uniform. Unless the proper quality of glass is used the bulb does not return to exactly the same volume after successive heatings. Even when a high-grade thermometer is employed, the zero should be frequently checked if the thermometer is used for high-temperature measurements.

Methods of calibrating mercury thermometers are described in text books on heat, but at the present day it is more convenient to compare the thermometer with a secondary standard mercury thermometer which has been calibrated by a central standardizing bureau, such as the Bureau of Standards at Washington.

High-range Mercury Thermometers. — The mercury thermometer when made of very hard glass and filled under pressure with the space above the mercury column containing some inert gas, like nitrogen, may be used to measure temperatures up to about 550° C. If a considerable length of stem emerges into the air, a very considerable error, 25° C. or so, may be introduced at high temperatures. This "stem correction" varies slightly with the kind of glass but may be represented very nearly by the formula

$$\text{Correction to be added to reading} = 0.00016\, n(t_b - t),$$

where the temperatures are all in degrees centigrade and n = number of degrees emergent from bath or furnace, t_b = temperature of bath and t = mean temperature of emergent mercury column.

COSTS. — Ordinary mercury thermometers cost from 50 cents to $5.00 apiece, depending upon the accuracy of their calibration. A 100° C. mercury thermometer sufficiently accurate (error not over ½ per cent) for ordinary engineering work costs about $4.00. A 400° C. mercury thermometer costs about $6.00 and a 550° C. mercury thermometer about $12.00.

BIBLIOGRAPHY. — See Bibliography in articles on *Heat* and *Pyrometers.*

THERMODYNAMICS, PRINCIPLES OF. — (*See also Electrochemistry, Principles of; Heat and Thermal Properties; Steam; Steam Engines; Temperature and Thermometers.*) "Thermodynamics" is a general name employed to include all problems involving the transfer of energy from one body to another, and the transformations of energy within a body. The word "energetics" is a better general term to designate such problems, but the term "thermodynamics" is the one ordinarily employed, since heat (Greek, "thermos") is developed in practicall; every case of transfer or transformation of energy. The transfer of energy from one body to another in every known instance is found to be consistent with the following fundamental principles or laws:

FIRST PRINCIPLE OF THERMODYNAMICS. — All known experimental facts are in accord with the principle that, when a body changes from a given state or condition 1 to any other given state or condition 2, the total energy given out by the body is always the same, independent of how the change takes place.

Intrinsic Energy. — Any given body in any given state or condition may, therefore, be considered as having associated with it a definite amount of energy, which, in general, changes when the state or condition of the body changes. This energy is called the "intrinsic energy" of the body, and depends solely upon the state or condition of the body. If the intrinsic energy in a given state or condition 1 is U_1, and in some other state or condition 2 is U_2, then the total energy given out by the body when it changes from the state 1 to the state 2 is $U_1 - U_2$, and this difference depends only upon the initial and final states of the system and is independent of how the body passes from one state to the other.

Heat and Work. — The energy given out by a body may be either heat, mechanical work, electrical, magnetic or other forms of energy. For convenience, all other forms of energy given out by the body than heat, may be called the "external work" done by the body. Hence, calling Q_e the heat *evolved* and, W the external work *done by* a body when it changes from a state 1 to a state 2, the total energy given out by the body may be expressed as $Q_e + W$. In general the amount of heat evolved and the external work done by a body when it changes from a state 1 to a state 2 depend respectively upon the manner in which the change takes place, but the sum of the heat evolved and the work done is always the same for given initial and final states, irrespective of how the change takes place.

Mathematical Expression of First Law. — Equating the two expressions given above for the total energy transferred from a body when it changes from a state 1 to a state 2, gives the relation

$$U_1 - U_2 = Q_e + W,$$

which is the usual mathematical expression of the first law of thermodynamics. If, in the change from state 1 to state 2, heat is actually evolved, then Q_e is positive, while if the heat is absorbed Q_e is negative. It is usually more convenient to consider the heat absorbed by the body as a positive quantity. Hence, putting $Q = -Q_e$ the above expression may be written

$$W = U_1 - U_2 + Q, \tag{1}$$

in which U_1 is the intrinsic energy of the body in any state 1, U_2 the intrinsic energy in any other state 2, Q is the heat *absorbed* and W is the *work done by* the body when it changes from the state 1 to the state 2.

PATH OF CHANGE. — Consider a body which changes from a state or condition 1 to another state or condition 2. In changing from the initial

state to the final state the body passes through a series of successive states, each state differing but infinitesimally from the preceding. The given series of states through which the body passes is called the "path" from state 1 to state 2. In general, a body may pass from a state 1 to a state 2 by an infinite number of such paths; therefore, a change can be completely specified only by stating the "path" of the change as well as the initial and final states.

The *sum* of the work done and the heat given by a body when it changes from a state 1 to a state 2 depends only upon these states, i.e., upon the ends of the path, but the proportions of the energy given out as work and heat respectively depend not only upon the series of successive states through which the body passes (i.e., the "shape" of the path), but also upon the relation of the given body to any external bodies which may in any way affect it.

ADIABATIC PROCESS. — Any process by which a change can be produced in a body under conditions such that the body neither absorbs nor gives out heat to any other body is called an "adiabatic" process. Adiabatic processes can never be completely realized, since no known substance is a perfect heat insulator, but such processes can be closely approximated, e.g., the expansion or compression of a gas in a cylinder with well-insulated walls when the expansion or compression is so rapid that no heat is conducted through the walls.

ISOTHERMAL PROCESS. — Any process by which a change can be produced in a body without changing its temperature is called an "isothermal" process. For example, the melting of ice, when the ice and water are kept well stirred, is an isothermal process.

REVERSIBLE PROCESS. — Whenever a change takes place in a body A, a change also takes place in some other body or bodies B. If the changes in the system formed by A and B are such that the path of each change may be reversed in direction without changing by an appreciable amount the total energy of the system, then the process by which the change takes place is called a "reversible" process.

Since heat can pass only from a hot to a cold body and never in the reverse direction (definition of heat), it follows that, if during any step of a process there is a transfer of heat between bodies whose temperatures differ by a finite amount, then the process is irreversible. Hence, a reversible process can take place under two conditions only: either there must be no transfer of heat (adiabatic process) or the transfer of heat must be between bodies which differ in temperature only by an infinitesimal amount.

Strictly speaking, the last type of process is absolutely reversible only in the limiting case when the bodies between which the transfer of heat takes place are at exactly the same temperature. This condition can be only approximated since there must always be a difference in temperature, though this difference may be infinitely small, in order that a transfer of heat may take place.

SECOND LAW OF THERMODYNAMICS. — The so-called second law of thermodynamics involves three distinct principles, which may be stated as follows:

1. When a body changes from a state 1 to a state 2 by any *reversible* process, and then back from 2 to 1 over the same path, but in the reverse order, then the heat absorbed by the body during the change from 1 to 2 is exactly equal to the heat evolved by this body during the reverse change from 2 to 1.

2. When a body changes from a state 1 to a state 2 by any irreversible process, the heat *absorbed* during this change can never be greater than the heat which it would absorb were the change from 1 to 2 over the same path produced under conditions which would render the process reversible. Similarly, if the body gives out heat during an irreversible change, the heat given out can never be

less than the heat which it would give out were the change over the same path produced under conditions which would render the process reversible.

3. The heat (Q) absorbed by a body during any reversible isothermal process bears the following relation to the temperature (t) of the body during the process,

$$Q = MK\,(t + T_0), \tag{2}$$

where M is the mass of the body, T_0 is a constant which depends *solely upon the scale on which the temperature is measured*, and K is a constant which depends *solely upon the nature of the body and its initial and final states*.

Absolute Thermodynamic Temperature. — It is found impossible by any known means to produce a negative temperature, as measured on any temperature scale, numerically greater than the corresponding value of constant T_0 in the above expression (equation (2)). Hence that temperature below the zero of any given scale equal to this constant T_0 is called the "absolute thermodynamic zero" of this scale, and the temperatures measured from this point are called "absolute thermodynamic temperatures." Equation (2) may therefore be written

$$Q = MKT, \tag{2a}$$

where $T = (t + T_0)$ is the absolute temperature corresponding to the temperature t. The value of the absolute thermodynamic zero is practically the same as that temperature, as measured on the constant-volume hydrogen-gas thermometer, at which the pressure of the gas would be zero, assuming the decrease in pressure per degree ($= 1/273$ of the pressure at 0° C.) to remain constant at all temperatures.

On the centigrade scale the absolute zero is, therefore, $-273°$ (approximately), and on the Fahrenheit scale $-460°$ (approximately).

Entropy. — Since the factor K, in equation (2) above, depends solely upon the nature of the body and its initial and final state, this factor may be looked upon as representing a change in a property of the body. That is, calling this property of the body for the initial state N_1 and for the final state N_2, then K may be put equal to $N_2 - N_1$, and equation (2) may be written

$$Q = M\,(N_2 - N_1)\,T. \tag{2b}$$

This equation, which represents the relation between the heat absorbed and the temperature, for a reversible isothermal process, may also be applied to an adiabatic process, if the property of the body represented by the symbol N is assumed to remain unchanged during such a process. For an adiabatic process the heat absorbed is zero, by definition, and this, on the assumption of no change in N during such a process, is consistent with equation (2b).

In general, the temperature of a body during any process is not constant, but the process may be considered as made up of a series of reversible isothermal and adiabatic steps, and the above definition of the property N may be applied to each step. That is, the change in N for each step composed of a reversible isothermal "tread" and an adiabatic "rise" is

$$dN = \frac{dQ}{MT},$$

where M is the mass of the body, dQ is the heat absorbed, and T is the absolute temperature of the body during this step.

This property N, whose increase during any step in a reversible process is equal to the heat absorbed per unit mass divided by the absolute temperature of the body during this step, is called the "entropy" of the body per unit mass. Since entropy is defined in terms of its change, its absolute value for any given

standard state of a body may be arbitrarily taken as zero. In steam tables, the entropy of water at 32° F. and at atmospheric pressure is usually taken as zero. The entropy of a body can in many cases be calculated from the other properties of the body. (*See article on Steam.*)

In accordance with the above definition of entropy the heat absorbed by a body when it changes from a state 1 to a state 2 by any *reversible* process is then

$$Q = M \int_1^2 T dN, \tag{3}$$

where M is the mass of the body, T its absolute temperature during any step of the process, and dN the increase in its entropy per unit mass during this step.

Principle of the Increase of Entropy. — It can be shown, from the principles above stated, that the only possible changes which can take place in a system of bodies to which no energy is added or subtracted are changes which involve an *increase* in the total entropy of the system. Reversible changes may theoretically take place without increasing the total entropy, but reversible changes never take place in nature nor can they be realized *absolutely* by any known experimental means.

MAXIMUM WORK. — The maximum external work which a body can do when it changes from a state 1 to a state 2 along a given path (i.e., by passing through a given series of states) is equal to the decrease in its intrinsic energy plus the maximum amount of heat it can absorb when it changes along this path. The maximum external work which a body can do in changing from a state 1 to a state 2 along a given path is therefore

$$W_{\max} = U_1 - U_2 + M \int_1^2 T dN. \tag{4}$$

The value of the last term in this expression depends upon the temperature at each step in the change, and this temperature depends upon the "path" along which the change takes place. If the path is such that *all* the heat is absorbed at one given temperature T, then

$$W_{\max} = U_1 - U_2 - TM (N_1 - N_2). \tag{4a}$$

This may also be written

$$W_{\max} = U_1 - U_2 + T \frac{dW_{\max}}{dT}, \tag{5}$$

where $\frac{dW_{\max}}{dT}$ is the rate of increase of the maximum work done by the body with the temperature at which the heat is absorbed.

Free Energy. — The maximum work which a body can do in changing from a state 1 to a state 2 is sometimes referred to as the "free energy" of the body corresponding to this change. This free energy depends not only upon the initial and final states of the body itself but also upon the temperature of the hottest external body available as a source of heat; this is apparent from equation (4).

CYCLIC PROCESSES. — In the theory of the steam and other heat engines the question arises as to what is the maximum amount of external work which can be obtained from a body by alternately heating it to a high temperature and then letting it do work (e.g., by expanding), thereby cooling to a lower temperature, this cycle of operations being repeated over and over again. That is, the body or "working substance" changes from a state 1 to a state 2, then back again to 1, then changes again to 2, and so on for any number of cycles. The intrinsic energy of the body at the beginning and end of any cycle is then the same, and therefore, the net work done by the body during any cycle is

the difference between the heat actually absorbed and that actually given out during the cycle.

Thermal Efficiency. — The ratio of the net work done to the heat absorbed during the cycle gives the proportion of the heat added which is converted into external work, and is called the "thermal efficiency" of the cycle.

Carnot's Cycle. — From the principles stated above it follows that the heat absorbed will be a maximum when this heat is absorbed in a reversible manner under conditions such that the temperature of the working substance is the same as that of the hottest body available as a source of heat; and the heat given out is a minimum when this heat is given out in a reversible manner under conditions such that the temperature of the working substance is the same as that of the coldest body available as an absorber of heat. The "path" along which the body changes (*see* Fig. 1) will then consist of an isothermal change at temperature T_1, an adiabatic change to a lower temperature T_2, an isothermal change at this temperature T_2, and then a second adiabatic change to T_1. Such a path is called a "Carnot's cycle." The figure represents such a path for a perfect gas, the ordinates being pressure and volume.

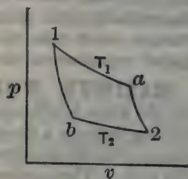

Fig. 1.

From the general expression for maximum work, equation (4), it follows that the efficiency of this cycle is

$$\frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}.$$

For a given amount of heat added to a working substance, during a cyclic change, the maximum possible work which the working substance can do is that done when the cycle is a Carnot's cycle, and this maximum work is directly proportional to the difference between the temperature of the body from which the working substance absorbs heat and the temperature of the body to which it gives out heat, and is inversely proportional to the absolute temperature of the body from which it absorbs heat.

APPLICATIONS OF THE LAWS OF THERMODYNAMICS. — The laws of thermodynamics serve as the basis for the mathematical treatment of the performance of the steam engine, steam turbine, internal-combustion engines, refrigerating machines, air compressors, etc.; they are also involved in many important electrochemical relations (*see Electrochemistry, Principles of*). It is beyond the scope of this book to go into these matters here; see the treatises listed in the following bibliography.

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THIRD-RAIL, OR CONTACT-RAIL, SYSTEMS. — (*See also Bonds and Bonding; Rails, Track and Third; Standardization Rules of the A.I.E.E.; Trolley Systems, Overhead; Trolley Systems, Underground.*) The following is a brief table of contents of this article:

The contact rail, or third rail, is a conductor supported on insulators near the ground and presenting a continuous contact surface to a collector or shoe attached to the rolling stock. In its commonest form it is a rail of standard section supported at intervals of a few feet by substantial insulators, electrical continuity between adjacent lengths being obtained by copper bonds across the joints.

TERMINOLOGY. — The following terms and definitions are used in connection with third-rail systems. See also Standards of the A.I.E.E.

Contact Shoe. — A third-rail contact shoe is a conductor, fastened to the rolling stock, which is designed to make electrical contact with the third rail. This is hereinafter referred to as the "shoe."

Contact Surface. — The contact surface of a third rail is the surface against which the shoe presses.

Gage of Track. — The minimum clearance between the inside surface of the heads of the two track rails, i.e., the distance *A* in Fig. 17. The track gage in the U. S. A. is 4 feet 8½ inches.

Third-rail Gage. — The distance measured parallel to the plane of the running rails, between the gage line of the nearer track rail and the inside gage line of the contact surface of the third rail, i.e., the distance *B* in Fig. 17 of this article.

Location.— Third rail locations will be described in terms of the third-rail gage and the elevation or vertical distance (*C* in Fig. 17) between the normal contact surface and the normal top of the track rail. See also Table II below.

Top-contact Rail. — A top-contact third rail is one on which the contact surface is on the upper side of the rail.

Under-contact Rail. — An under-contact third rail is one on which the contact surface is on the under side of the rail.

Third-rail Insulator. — A third-rail insulator is that portion of the third-rail support which forms the principal electrical insulation.

Insulator Base or Bracket. — A third-rail insulator base or bracket is a device used to support the third-rail insulator.

Incline. — An incline is a portion of third rail sloped to gradually bring the shoe from its free position into contact with the normal surface of the third rail.

End Incline. — An end incline is an incline at the end of a run of third rail and is made to receive shoes moving in line with the third rail.

Offset-end Incline. — An offset-end incline is an incline *at the end* of a run of third rail and is made to receive shoes moving laterally towards the third rail.

Cross Incline. — A cross incline is a combination in one piece of two end inclines from adjacent third rails which meet at a turn-out.

Side Incline. — A side incline is an incline at *the side of* a third rail and is made to receive shoes moving laterally towards the third rail.

Offset-side Incline. — An offset-side incline is an incline offset from the standard third rail in order to gain clearance between the third rail and the maximum equipment line of the rolling stock.

Maximum Equipment Line. — The maximum equipment line is the boundary which encloses the cross-sectional outlines of all the rolling stock, under all normal operating conditions.

Third-rail Protection. — A third-rail protection is a partial covering of the third rail which affords some degree of protection from the weather or from accidental contact with foreign bodies.

Third-rail Anchorage. — A third-rail anchorage is a device to prevent the third-rail creeping longitudinally.

Tie Motion. — Unless otherwise specified, tie motion will be understood to mean the vertical play of the ties caused by the passage of trains.

TYPES OF CONSTRUCTION. — Third-rail construction may be classified into the top-contact and under-contact types, each of which is susceptible of important variations in design, especially with reference to the type of protection.

Interborough Top-contact Type (Fig. 1). — One of the most commonly used types is illustrated in Fig. 1. It is often called the "Interborough Type" on account of its use in the subways of the Interborough Rapid Transit Co., of New York. The rail is a standard T-section and rests on reconstructed granite insulators. A board protection is attached to the rail itself by means of clamps and uprights and is thereby kept in perfect alignment.

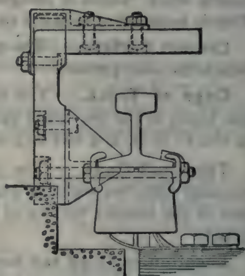


Fig. 1. Interborough Top-contact Type

Pennsylvania Top-contact Type (Fig. 2). — Another type has the protection supported on separate brackets independent of the third rail itself. It is claimed that this reduces the amount of labor which has to be done on the live rail, when repairs are being made, but it cannot be relied upon as well as the Interborough type to keep the rail and protection in perfect alignment.

London Tube Type (Fig. 3). — The mounting and location of the third rail and the negative contact rail of the London Tube Railways is illustrated in Fig. 3.

Under-contact Types (Figs. 4, 5, 6 and 7). — While the top-contact types have given first-class service, they are considered to have certain disadvantages for exposed locations as they cannot be wholly protected from snow, ice and sleet. The lower part is only a few inches from the ties, while holding clips generally reduce this clearance, increasing the danger of grounding from accumulation of wet snow and ashes and from flooding. The occasional suspension of traffic during sleet and snowstorms and floods, on railroads using the top-contact type of third rail, led to the idea of an under-contact third rail loosely clasped

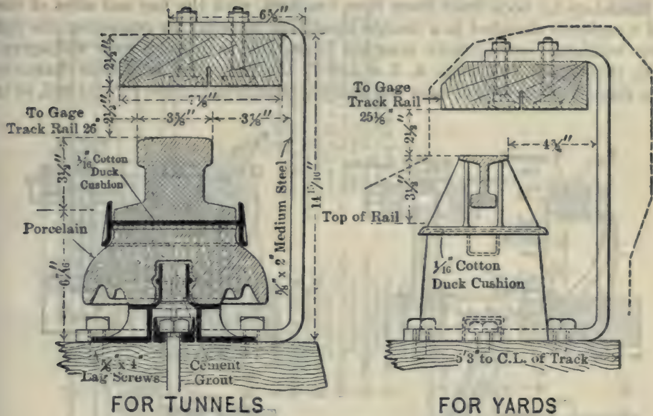


Fig. 2. Pennsylvania Top-contact Type

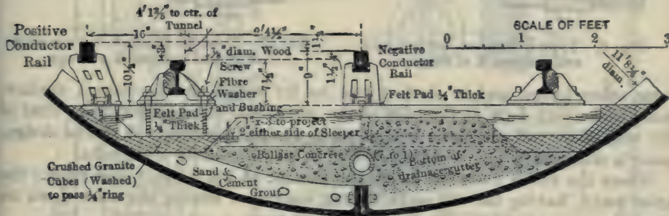


Fig. 3. London Tube Type

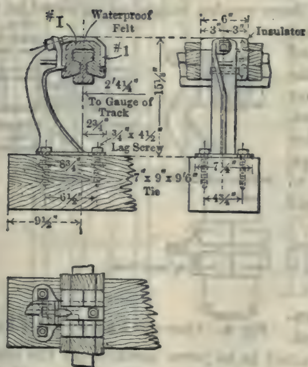


Fig. 4. Type X Bracket, for use on Straight Work

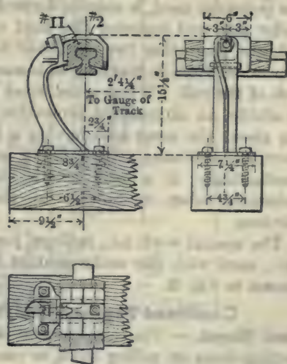


Fig. 5. Type Y Bracket, Applied to End Incline

in insulators by hook bolts hung from brackets, with the top and sides of the rail completely sheathed in a flexible insulating material for protecting the rail from accidental contact with man and beast, and from sleet, snow and spray. With this type of rail (Figs. 4, 5, 6 and 7) the protection is of such character that there is no packing of snow between the sheathing and the contact rail, as in

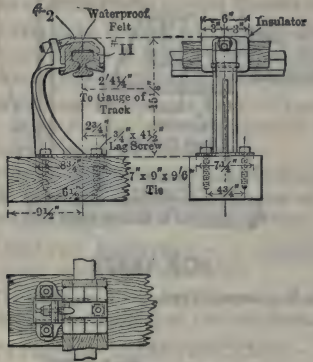


Fig. 6. Type Z Bracket, for use with end inclines A at frogs

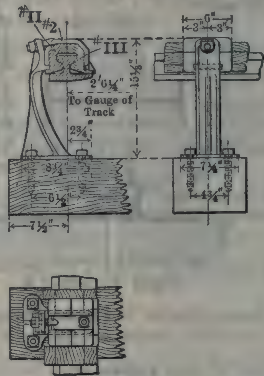


Fig. 7. Type W Bracket, for use with offset end inclines B at frogs

some other forms, and in sleet storms no ice forms on the contact surface; some icicles may form at the edge of the petticoats, but hanging down clear of the edge of the rail, are easily broken off by the passing shoe. A special design, using a standard T-rail, is shown in Fig. 8, and is used for 1200 volts.

Where the rail is buried in snow, the passage of the contact shoe breaks the snow away, leaving the rail surface clear, instead of ironing the snow down on the rail, as may happen with the top-contact type.

Sheathing and Special Work. — The sheathing between the insulator blocks, depending upon local conditions and the price of materials, as well as the potential used, is usually formed of three wooden strips, one grooved on the under side and inclosing the head of the rail, and the other two, attached to and dependent from it, reaching in toward the web of the rail. Where good wood is not available, an alternative protection costing about the same and having a higher electrical resistance, although not so good mechanically, is a semi-flexible shell of indurated fiber conformed to the rail sections.

The special work, i.e., inclines, jumpers, etc., used with the under-contact rail, is shown in Fig. 9.

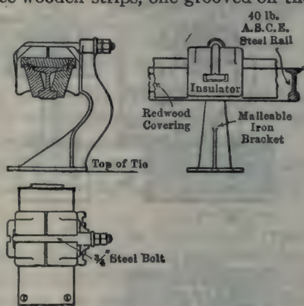


Fig. 8. 1200-volt Under-contact Rail.

Combined Top- and Under-contact Shoes. — The employment of collecting shoes on rolling stock so constructed as to press upwards on the under-contact rail and downwards on the top-contact type solves the question of interchange between railroads not using the same type.

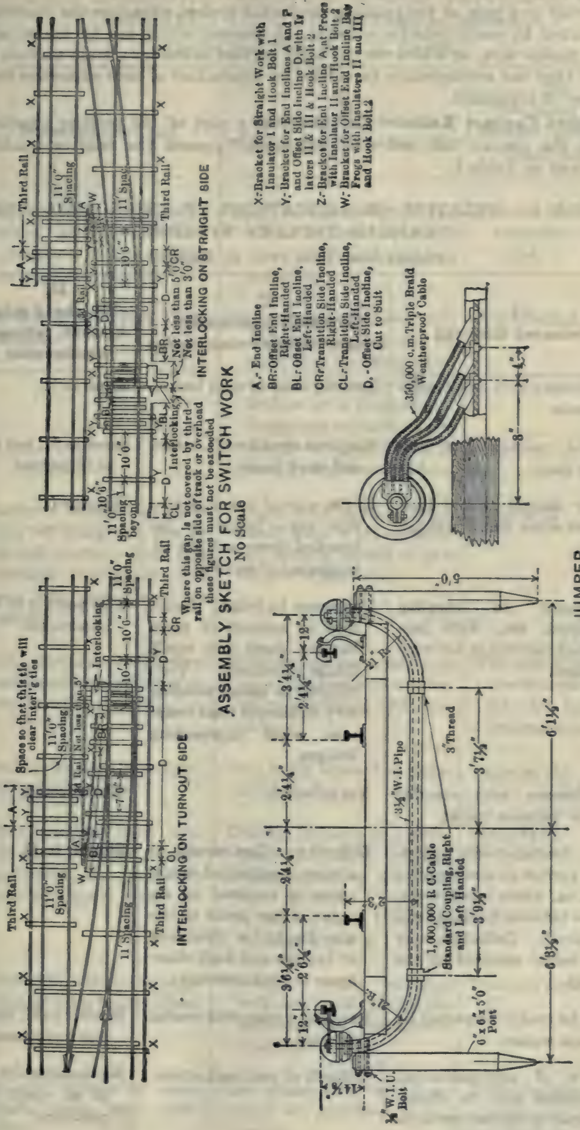


Fig. 9. Special Work used with Under-contact Rail

CONTACT RAILS VERSUS OVERHEAD TROLLEY. — The contact rail is used as a part of the positive conductor system whenever the current to be collected by each collector exceeds the amount which can be taken safely from a trolley wire, or whenever the total current taken by a train exceeds the amount that can economically be carried by conductors of such expensive metal as copper or aluminum.

Positive Contact Rails. — Considered as a part of the positive conductor system, the contact rail and overhead trolley possess the relative qualifications given in Table I.

TABLE I. — RELATIVE QUALIFICATIONS OF THIRD RAIL AND OVERHEAD TROLLEY SYSTEMS.

(*Adapted from Table by C. E. Eveleth.*)

I Protected third rail	II Overhead high-tension bridge, catenary construction	III Overhead side bracket, catenary construction
Interference with track maintenance.	Entirely clear of road bed.	Same as II.
Can be maintained by section gang.	Requires special tools, crews and work trains.	Same as II, but not as important.
Easily cleared up and insulated when derailment occurs.	In the way of boom of derrick car — liable to be knocked down and put all tracks out of service.	Same as II.
Hindrance to coupling freights, etc. With protected rail this is not very serious.	Dangerous to freight brakemen on account of parts hanging down and small bridge clearance. Very difficult to install satisfactory ticklers to warn trainmen when approaching bridges.	This point is of less importance.
Interference with clearing snow between tracks.	Not affected.	Not affected.
Ease of satisfactorily collecting current on account of location, where relative motion between track and rail is small. Collectors may be safely replaced on the road.	Difficult to collect current, as a more complicated mechanism is required on account of the grade of the wire due to low clearances at bridges and high clearances at road crossings.	Similar to II.
May be readily inspected by track walker.	Requires a man with special training.	Similar to II, but of less importance.
Ease of sectionalization. Jumpers may be disconnected at the nearest adjacent road crossings.	Difficult of sectionalization.	Sectionalization not of so much importance.

TABLE I. — RELATIVE QUALIFICATIONS OF THIRD-RAIL AND OVERHEAD TROLLEY SYSTEMS — *Continued*

(*Adapted from Table by C. E. Eveleth.*)

I Protected third rail	II Overhead high-tension bridge, catenary construction	III Overhead side bracket, catenary construction
May be worked on while alive to make track changes or repairs, making system very flexible.	Requires that current be shut off no matter how slight repairs are, making system inflexible.	Similar to II, but not of such importance.
No interference with visual signals.	Signals located and seen with difficulty, as they must be lower than the bridges and even then have the distant bridges as a background. Dangerous to maintain signals in this location, as it must be done from ladders.	Not affected.
Danger of wreck from burning off track rail due to arcing current. This is a possible contingency, but one not very likely to occur.	Danger from dangling overhead work when messenger cable is burned off at a defective insulator.	Same as II
Little interference with fire-extinguishing apparatus in the car storage yard.	Difficulty in removing cars on account of high-tension wires interfering with firemen.	Will probably have low-tension wires for this type of construction.
Absolute freedom from lightning disturbances.	Very much exposed to lightning.	Same as II.
Entire freedom from telephone and telegraph disturbances, also inductive effects on signal wires.	Difficult problem in connection with these interferences, affecting not only the railroad company's wires, but those belonging to other interests.	Same as II.
No trouble at grade crossings with crossing trolley wires.	Probably trolley crossings will have to be avoided by overhead or undergrade crossings.	Troubles similar to II.
Can add sidings or more tracks with little difficulty.	Can make such additions only at considerable expense.	Difficulty not very great.

Negative Contact Rails. — Negative contact rails are used as part of the negative-feeder system when the drop of potential in the track rails is limited to such a small amount that it is cheaper to use an insulated rail than to reinforce the track rails with feeders. Considered as part of the negative feeder system, the contact rail possesses the following advantages over the track-rail return system. If used in connection with a positive contact rail, the negative rail is placed between the track rails.

(I) When properly insulated, it eliminates every possibility of electrolysis.

(II) It gives the block-signal system complete independence from the electric-traction system. Among the advantages which this entails are, no unbalancing of signal circuits, saving the cost of reactance bonds (*see Signaling, Railway*) and increased economy in signal circuits.

(III) It gives greater safety to passengers.

(IV) It reduces the probability of short circuits.

(V) It decreases the cost of track-rail maintenance by the elimination of bonds.

(VI) It halves the first cost of bonds, as one high-conductivity contact rail usually replaces two track rails.

(VII) It reduces the wear of bonds by saving them from the shock of trains.

These advantages are usually offset by the first cost of the negative contact-rail, by the complications it introduces at special track work, and by the impossibility of protecting a central rail from snow and ice on account of insufficient clearance for any kind of covering.

ELECTRICAL DESIGN. — The calculations of potential drop, network, resistance, etc., are treated under *Trolley Systems, Overhead*. The composition, weight, dimensions, resistance and reactance of rails will be found in the article on *Rails, Track and Third*.

Selection of Suitable Rail. — When the various types of rail are under consideration it is well to arrange a table with the following headings, in order to compare the relative economy of the different types. (1) Circular mils of copper equivalent to rail; (2) Additional circular mils of copper required to equal the rail of highest conductivity; (3) Cost of rail for the entire railroad; (4) Cost of additional copper for entire railroad; (5) Total cost of rail and additional copper for entire railroad.

MECHANICAL DESIGN. — The general design of the rail itself having been settled, the next step is to secure a set of track plans on which to lay out the special work. When an entirely new railway is being projected, the track designer and the third-rail designer can work together, but when an existing line is being converted, the general track plans cannot be used as they are seldom sufficiently accurate for the electrical engineer's purpose. In this case, the contact-rail engineer has to take measurements of the track work in order to make drawings of the cross-overs and other complications.

Location and Weight of Third Rails. — There is no standard gage for contact rails, corresponding to the standard track gage. This unfortunate condition arises from lack of uniformity in the clearance lines of the right-of-way and in the maximum equipment lines of various railroads. The following standard clearances have been recommended (1915) by the American Electric Railway Engineering Association:

The clearance lines for third-rail and permanent-way structures and rolling equipment to be as shown in Fig. 10, thus reserving the space within lines *AT*, *BT*, *CT*, *DT*, *ET*, *FT* and *AT*, *JT*, *KT*, *LT*, *MT* for third-rail structures; rolling equipment not to encroach upon the third-rail space under conditions of

maximum wear and deflection beyond the line *AE, BE, CE, DE, EE, FE, GE*, and permanent-way structures not to encroach upon the third-rail space beyond the line *AS, JS, KS, LS, MS*; this leaves a clearance space or neutral zone of one inch both horizontally and vertically upon which neither the third-rail structures nor equipment shall encroach. On curves of less radius than 800 feet, the third rail must be moved back and the

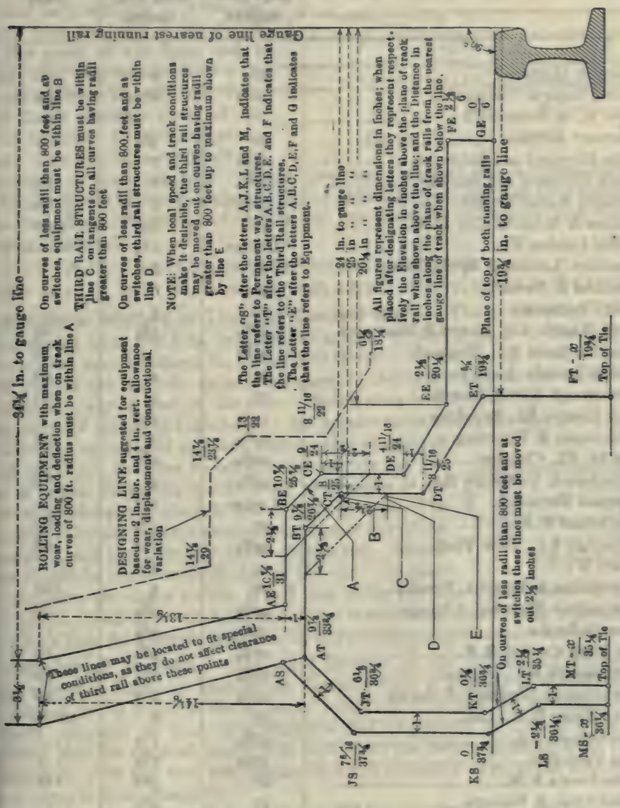


Fig. 10. Recommended Standard Design for Limiting Clearance Lines for Third-Rail and Permanent Way Structures and Rolling Equipment A. E. R. A.

equipment may be allowed to swing outward as indicated by the lines and notes on the diagram.

In the design and construction of new rolling stock to be used in interchange service allowance must be made for such horizontal and vertical variations as may in any reasonable probability occur in combination at one time as well as allowance for deflections on curves. Encroachments on normal clearance due to deflection of springs and wear vary with the type of construction used, and the practice of the respective roads as regards permissible wear and deflection before repairs and adjustments are made. For the general guidance of the roads, in designing equipment, but not as a standard, there-

TABLE II. — LOCATION AND WEIGHT OF THIRD RAIL

Name of railroad	Center of third rail to near gage line, inches	Contact face above top of running rail, inches	Weight, lb. per yard
Albany & Hudson	27	6	....
Aurora, Elgin & Chicago	20⅛	6 5/16	100
Baker St. & Waterloo Ry.	....	....	90
Baltimore & Ohio	30	3½	...
Berlin Elevated and Subway	14⅝	7	....
Boston Elevated and Subway	20⅜	6	85
Brooklyn Rapid Transit	21¾	6	70
Camden & Atlantic City R.R.	26	3½	100
Central London Ry.	Center	1½	....
Columbus, London & Springfield	27	6	....
Columbus & Newark	27	6	....
Fayet-Chamounix	23	9	....
Grand Rapids, Gd. Haven & Muskegon	20⅝	5¾	60
Great Northern Ry., England	19¼	....	80
Hudson Tunnels, New York	26	4	75
Interborough Rapid Transit	26	4	153
Kings County El., Brooklyn	19½	5¼	....
Lackawanna & Wyoming	20⅜	3	75
Lake St. El., Chicago	20⅛	6½	....
Lancashire & Yorkshire Ry.	19¼	3	70
Liverpool Elevated	Center	1½	....
Long Island R.R.	27	3½	100
Manhattan Ry., New York	20¾	7½	100
Mersey Ry.	22	4½	....
Metropolitan & District, London	16	3	100
Metropolitan Elevated, Chicago	20⅛	6¼	48
Milan Gallerate	26⅝	7½	....
*New York Central R.R.	28¼	2¾	70
Northeastern Ry., England	19¼	3¼	80
Northwestern Elevated, Chicago	20⅛	6½	48
North Shore R.R., Cal.	27	6	50–60
Paris Orleans Ry.	25⅝	7⅞	....
Paris Versailles	25⅝	7⅞	....
Pennsylvania R.R.	27	3½	150
*Philadelphia & Western	27	3⅝	....
*Philadelphia Rapid Transit	27	6	70
Seattle & Tacoma R.R.	20	7½	....
South Side Elevated, Chicago	20⅛	6½	....
Wamseebahn (Berlin)	33½	12⅝	....
Waterloo & City Ry.	28¼	0	....
West Jersey & Seashore	26 †	3½	100
*West Shore R.R.	32	2¾	70
Wilkesbarre & Hazleton	28	5	80

* Bottom contact surface. All others have top-contact surface.
† Gage, as defined under Terminology.

fore, a dotted line is shown on the diagram located 2 inches distant horizontally and 4 inches distant vertically from the limiting clearance line for equipment.

Turnouts. — (*See also Railways, Location and Permanent Way for.*) A typical turnout is shown diagrammatically in Fig. 11 which shows the lengths which should be measured; F is the distance from the point of the switch to the point of the frog, and is called the "frog distance"; D is the length between the adjacent frogs, and Z the distance apart of the track centers. In such a diagrammatic view the lines represent the gage lines of the track rails. When, however, a contact rail is represented by a line, it is the center line that is given. Having measured the four distances specified above, the radius of the turnout curve may be calculated from the following formulas:

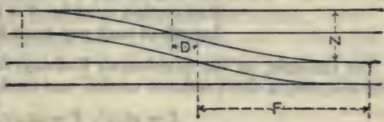


Fig. 11. Typical Turnout

$R = 0.127\ F^2$, when the turnout is from a straight track.

$R = 0.271\ F^2$, when the turnout is from a track of equal radius to the turnout.

Graphical Method of Locating Contact Rail. — The track plans having been drawn from the field sketches, to a scale of say ¼ inch to the foot, the contact rail may be drawn in, as outlined below. An aid in this work is an outline plan of the electric car or locomotive to the same scale as the track work drawings — say ¼ inch to the foot. Such a plan is shown in Fig. 12, in which the points W represent the wheels, S the contact shoes and K the king-pins. This had better be made of celluloid with a large pinhole at each point S. In order to lay out the contact rail the car is drawn along the tracks with the points W on the rails and a pencil stuck in one of the pinholes. The line traced by the pencil represents the center line of the shoe path, and therefore that of the contact rail. Such a line should be made on each side of the car. In cases where there are sharp curves, it will not do to run the wheel points W along the tracks, as an error will arise due to the truck rotation not being represented. In such a case the king-pin points must be run along a track center line.

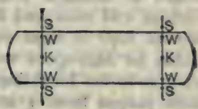
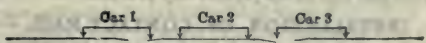


Fig. 12.

Location of Inclines. — The exact location of the contact-rail inclines on each side of a track-rail intersection depends upon a number of conditions, an important one of which is the extent to which the equalizer bar and journal box project outward. If there is a train-bus line connecting all the cars, the end inclines may be situated many feet back from the switch point or frog, but if there is no such bus line, the contact rail will probably have to be terminated in a cross incline extending as near as possible to the switch point or frog.



Isolated Section
Fig. 13.

Sectionalizing the Contact Rail. — Where a train-bus line is not used, it is customary to break the third rail in front of substations and use an isolated section of third rail between the two main sections, as shown diagrammatically in Fig. 13. The length of this isolated section and of the gaps which bound it should be such that both gaps are never spanned by cars at the same time.

Let L = car length, feet,

T = distance between shoe centers (usually the same as distance between truck centers),

S = shoe length.

Then,

Total space must be $> L + T + S$.
Rail section must be $< 2L - T - S$.
Length of each gap must be $< T - S$.

For example, on the Manhattan Railway, New York, before a train-bus line was adopted, a short isolated section was used. The lengths were as follows:

$$L = 46.37,\ T = 32.27,\ S = 1.$$

Total space $>$ 79.64, actually 83 feet.
Section of rail $<$ 59.47, actually 57 feet.
Gap length $<$ 31.27, actually 13 feet.

Cross and End Inclines. — Inclines serve the purpose of assisting the contact shoes to rise (or fall) from their free position to the position of contact with the rail. They should, therefore, have sufficient slope to prevent any undue shock either to the rail or to the shoes. Low-speed lines and sidings may therefore have shorter inclines than high-speed lines. The usual length of straight slope (i.e., without end nose and flat surface at top) is from 60 to 70 inches for high-speed work and 30 or 40 inches for sidings. The slope ranges from 1 in 45 for high-speed work to 1 in 30 for urban railways. Inclines are almost invariably made of cast iron. A typical design is shown in Fig. 14.

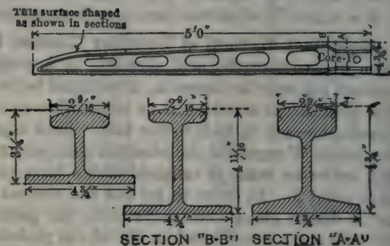


Fig. 14. Typical Incline

Third-rail Insulators. — Third-rail insulators should have the following qualifications. Strength to withstand weight and vibration; surface impervious to moisture; resistance wet shall not be less than one megohm; shall have a drip edge; shall allow free motion of rail laterally, longitudinally and vertically to allow for expansion, contraction and tie motion; and must be capable of easy and quick removal without disturbing the rail.

Spacing of Insulators. — A spacing of 10 feet between insulators was recommended by the American Street & Interurban Assn., Oct., 1908, for 30-foot rails and represents present day practice.

INSTALLATION OF CONTACT RAIL. — In the case of an existing railway being electrified, the first step is to replace standard ties at proper intervals by long ones. A man then proceeds along the line with a template and marks on them the location of the screw holes of the insulator bracket. He is followed by an augur gang which drills the holes. Meanwhile the brackets (in the case of an under-contact rail) or the stool and insulators (in the case of a top-contact rail) are distributed along the line, and a gang following the augur gang screws the brackets or stools to the ties.

Where the rails rest on tie plates which have not yet penetrated the ties, it is not unusual to place shims under the brackets or stool until the tie plates have reached their normal position.

In the case of under-contact rails the brackets must be accurately checked for principal dimensions before being distributed.

The rails and fish-plates, etc., are next distributed along the line on that side of the track where they are to be installed and a gang follows which installs the rail, and, in the case of an under-contact rail, attaches the insulators and hook bolts; in the case of a top-contact rail, another gang follows, attaching the clips or whatever is used to fasten the rail to the insulators. Another gang follows to install the fish plates, expansion joints and anchorages, if any. If there are no anchorages or expansion joints, expansion and contraction are provided against by making an allowance at all joints, depending on the temperature at which the rails are installed. This is done by inserting an "expansion shim" while the rails are being bolted together. The thickness of shim for different temperatures on a 30-foot rail is given in Fig. 15.

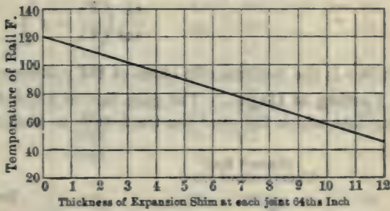


Fig. 15. Thickness of Expansion Shims for 30-ft. Rail

While the rails are being installed, the materials for the protection are distributed along the line and a gang follows installing them. A last gang paints the protection first with a priming and then with a finishing paint.

Inclines have to be carefully located to make sure that the shoes will ride smoothly upon them.

OPERATION AND MAINTENANCE OF CONTACT RAIL. — The wear of the third rails is negligible and even without painting the deterioration due to rust is very slow. The principal items of maintenance are the insulators, protection and bonds. Unless the supports are carefully designed, insulators are shattered by "tie motion," i.e., by the depression and rebound of the ties as the trains pass over them. Every time a wheel passes over a tie carrying an insulator, the latter is pounded against the rail and is likely to be broken unless sufficient play is provided.

Inductive effects due to large currents sometimes causes the third-rail voltage to rise to 2 or 3 times its normal value. (D. D. Ewing, E. R. J., 1917, Vol. 56, p. 1072.)

Removal of Sleet. — The best protection against sleet is undoubtedly the under-contact third rail and the next best is a top-contact third rail with a wide protecting board. Where these are impracticable the deposit of ice on the rail cannot be prevented and means for its removal have to be adopted. The two principal means are scrapers and hot water. Electrical heating of the rail has also been suggested but found to require too much energy. When the hot-water method is used, some salt such as calcium chloride, or some other substance, which lowers the freezing point of water must be added to the water. There are serious objections to the hot-water method, especially on elevated railways, as the saline solution not only rusts the metal of the structure but when calcium chloride is used in the water, damage also occurs to the roofs of street cars, etc., underneath.

TESTING OF CONTACT RAILS. — Contact rails have to be tested for (1) electrical continuity at the joints, (2) conductivity, (3) insulation from ground and (4) gage.

Test of Continuity at Joints are described in the article on *Bonds, Railway Track.*

The Conductivity Test is best performed by passing a considerable current through a length, measuring the drop with a milli-voltmeter and calculating the resistance by Ohm's law.

The Insulation Test is best performed by means of a pair of voltmeters, as shown in Fig. 16. The rail A to be tested is connected to a live rail B through a voltmeter, and the voltage V_1 between the two noted. At the same instant the voltage V_2 between the rail B and ground is also read. Let L be the length in miles of the rail A. Then the insulation resistance of the rail A in megohms per mile is

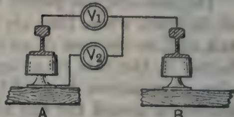


Fig. 16.

$$\frac{rL}{10^6}\left(\frac{V_2}{V_1} - 1\right),$$

where r is the resistance in ohms of the voltmeter on which V_1 is read.

Testing of Gage. — The gage line of a third rail is measured by means of a template which fits over the track rails as shown in Fig. 17.

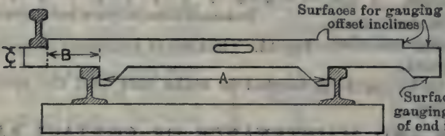


Fig. 17.

COST OF CONTACT-RAIL CONSTRUCTION. — The estimates in Table III include the cost of (1) handling and distributing the material from the storehouse to the place where it is used; (2) the solder, gasoline, etc., used in bonding contact rail; (3) putting three coats of paint on the protection; (4) bending rai s on curves; (5) 5 per cent for breakage; (6) foremen's and engineers' salaries. They do not include the cost of tools or of jumpers.

These estimates are approximately correct where existing traffic does not materially impede the work. Under less favorable conditions, the cost may rise 50 per cent or more over the figures given.

The estimate on the top-contact type is based upon the Interborough Rapid Transit Co's. construction, New York (Stillwell-Slater patent), the weight of rail, however, being slightly less than on that railway. The estimate of the under-contact type is based upon construction similar to that used by the New York Central Railroad (Wilgus-Sprague patent). Little third-rail cost data having been published since the Great War, the data from the 1914 edition are reproduced.

TABLE III. — COST PER MILE OF CONTACT-RAIL CONSTRUCTION

Item	Top contact		Under contact	
	Amount	Cost	Amount	Cost
Material:				
Rail, 70 lb.	55 tons	$1815	55 tons	$1815
Rail, special	....	....	1.2	40
Inclines	11	47	11	47
Insulators, standard	511	92	1000	165
Insulators, special	....	....	25	13
Brackets or pedestals	515	62	500	250
Brackets, special	....	....	15	7
Bolts	515	10	515	90
Lag screws	1030	20	1515	30
Clips	1030	41	......	....
Drive screws	....	....	80 gross	24
Soldered bonds	350	168	350	168
Splice plates and bolts	350	53	180	31
Protection	....	793	....	642
Paint	....	49	....	82
Felt separator	....	....	....	2
Long ties, excess only *	505	177	505	177
Total material	....	$3327	....	$3583
Labor:				
Installing, bonding and protection of third rail	....	$ 800	....	$1000
Installing long ties	....	101	....	101
Total labor	....	$ 901	....	$1101
Grand total	....	$4228	....	$4684

* This item includes only the *difference* in cost between the long ties which carry the insulators and the cost of the same number of standard ties. The cost of 132 miles of third rail on the West Jersey and Seashore R. R. is given by Duer in 1915 as $4, 235 per mile, and the average cost of maintenance, as $82 per mile year.

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TIMBER. — (*See also Structures, Simple; Poles for Overhead Lines.*) The standard names for structural timbers as adopted by the Am. Soc. for Test. Mat and the Am. Ry. Eng. Assoc. are reprinted below by permission from the present (1913) board of the Am. Soc. for Test. Mat.

Fir, Douglas. — The term "Douglas Fir" is to cover the timber known likewise as yellow fir, red fir, western fir, Washington fir, Oregon or Puget Sound fir or pine, norwest and west coast fir.

Hemlock, to cover Southern or Eastern hemlock; that is, hemlock from all states east of and including Minnesota.

Hemlock, Western, to cover hemlock from the Pacific coast.

Larch, Western, to cover the species of larch or tamarack from the Rocky Mountain and Pacific coast regions.

Pine, Norway, to cover what is known also as "Red Pine."

Pine, Southern Yellow. — This term includes the species of yellow pine growing in the southern states from Virginia to Texas, that is, the pines hitherto known as longleaf pine (*Pinus palustris*), shortleaf pine (*Pinus echinata*), loblolly pine (*Pinus taeda*), Cuban pine (*Pinus heterophylla*), and pond pine (*Pinus serotina*).

Under this heading, two classes of timber are designated: (*a*) dense southern yellow pine and (*b*) sound southern yellow pine. It is understood that these two terms are descriptive of quality rather than of botanical species.

Pine, Western, to cover the timber sold as white pine coming from Arizona, California, New Mexico, Colorado, Oregon and Washington. This is the timber sometimes known as "Western Yellow Pine," or "Ponderosa Pine," or "California White Pine," or "Western White Pine."

Pine, White, to cover the timber which has hitherto been known as white pine, from Maine, Michigan, Wisconsin and Minnesota.

Pine, Idaho White, the variety of white pine from western Montana, northern Idaho and eastern Washington.

Redwood, to include the California wood usually known by that name.

Spruce, to cover Eastern spruce; that is, the spruce timber coming from points east of and including Minnesota.

Spruce, Western, to cover the spruce timber from the Pacific coast.

Tamarack, to cover the timber known as "Tamarack," or "Eastern Tamarack," from states east of and including Minnesota.

A fuller description of the timbers used for transmission line poles is given in the article on *Poles for Overhead Lines*.

DEFECTS IN TIMBER. — The standard names for defects in structural timber as adopted by the above association are the following:

Encased Knot. — An encased knot is one whose growth rings are not intergrown and homogeneous with the growth rings of the piece it is in. The encasement may be partial or complete; if intergrown partially or so fixed by growth or position that it will retain its place in the piece, it shall be considered a sound knot; if completely intergrown on one face, it is a watertight knot.

Large Knot. — A large knot is a sound knot, more than 1 ½ inches in diameter.

Loose Knot. — A loose knot is one not firmly held in place by growth or position.

Pin Knot. — A pin knot is a sound knot not over ½ inch in diameter.

Pith Knot. — A pith knot is a sound knot with a pith hole not more than ¼ inch in diameter in the center.

Rotten Knot. — A rotten knot is one not as hard as the wood it is in.

Round Knot. — A round knot is one which is oval or circular in form.

Sound Knot. — A sound knot is one which is solid across its face and which is as hard as the wood surrounding it; it may be either red or black, and is so fixed by growth or position that it will retain its place in the piece.

Spike Knot. — A spike knot is one sawn in a lengthwise direction; the mean or average width shall be considered in measuring these knots.

Standard Knot. — A standard knot is a sound knot not over 1½ inches in diameter.

Pitch Pockets. — Pitch pockets are openings between the grain of the wood containing more or less pitch or bark. These shall be classified as *small*, *standard* and *large* pitch pockets.

(*a*) *Small Pitch Pocket.* A small pitch pocket is one not over ⅛ inch wide.

(*b*) *Standard Pitch Pocket.* A standard pitch pocket is one not over ⅜ inch wide, or 3 inches in length.

(*c*) *Large Pitch Pocket.* A large pitch pocket is one over ⅜ inch wide, or over 3 inches in length.

Pitch Streak. — A pitch streak is a well-defined accumulation of pitch at one point in the piece. When not sufficient to develop a well-defined streak, or where the fiber between grains, that is, the coarse-grained fiber, usually termed "Spring wood," is not saturated with pitch, it shall not be considered a defect.

Shakes. — Shakes are splits or checks in timbers which usually cause a separation of the wood between annual rings.

Ring Shake. — An opening between the annual rings.

Through Shake. — A shake which extends between two faces of a timber.

Rot, Dote and Red Heart. — Any form of decay which may be evident either as a dark red discoloration not found in the sound wood, or the presence of white or red rotten spots, shall be considered as a defect.

Wane. — Wane is bark, or the lack of wood from any cause on edges of timbers.

DECAY AND PRESERVATION. — Decay is due usually to a fungus growth. It is prevented by cutting off the air by immersing the timber in water, which so far as is known is a sure preventive. Decay is retarded by thorough seasoning either naturally or in the kiln, or by poisoning the food supply of the fungus by chemical treatment. The latter method usually consists in impregnating the wood with creosote or zinc chloride, the former being more commonly employed. The methods used for preserving timber poles are described in the article on *Poles for Overhead Lines.*

UNIT STRESSES FOR STRUCTURAL TIMBER. — The values of unit stresses in structural timber given in the following table are recommended by the Committee on Wooden Bridges and Trestles of the American Railway Engineering Association. See *Manual* of the Association, Chicago, 1911. For unit values allowable in various cities of the United States see article on *Buildings, Allowable Unit Stresses in.*

UNIT STRESSES FOR STRUCTURAL TIMBER IN POUNDS PER SQUARE INCH

(*See Note under table*)

Kind of timber	Compression						
	Perpendicular to grain		Parallel to grain		Working strength of columns		Ratio of length of stringer to depth
	Elastic limit	Working strength	Average ultimate	Working strength	Length under 15 diameters	Length over 15 diameters	
Cedar, Red	470	230	2800	900	680	$900\left(1-\frac{L}{60\,D}\right)$	..
Cypress, Bald	340	170	3900	1100	830	$1100\left(1-\frac{L}{60\,D}\right)$	..
Fir, Douglas	630	310	3600	1200	900	$1200\left(1-\frac{L}{60\,D}\right)$	10
Hemlock, Western	440	220	3500	1200	900	$1200\left(1-\frac{L}{60\,D}\right)$	..
Oak, White	920	450	3500	1300	980	$1300\left(1-\frac{L}{60\,D}\right)$	12
Pine, Longleaf	520	260	3800	1300	980	$1300\left(1-\frac{L}{60\,D}\right)$	10
Pine, Norway	....	150	2600*	800	600	$800\left(1-\frac{L}{60\,D}\right)$	..
Pine, Shortleaf	340	170	3400	1100	830	$1100\left(1-\frac{L}{60\,D}\right)$	10
Pine, White	290	150	3000	1000	750	$1000\left(1-\frac{L}{60\,D}\right)$	10
Redwood	400	150	3300	900	680	$900\left(1-\frac{L}{60\,D}\right)$	..
Spruce	370	180	3200	1100	830	$1100\left(1-\frac{L}{60\,D}\right)$	..
Tamarack	....	220	3200*	1000	750	$1000\left(1-\frac{L}{60\,D}\right)$	..

* Partially air dry.

L = unsupported length in inches. D = least side in inches.

NOTE. — These unit stresses are for a green condition of timber and are to be used without increasing the live-load stresses for impact.

The working units given in these tables are intended for railroad bridges and trestles. For highway bridges and trestles the unit stresses may be increased twenty-five (25) per cent. For buildings and similar structures in which the timber is protected from the weather and practically free from impact, the unit stresses may be increased fifty (50) per cent. To compute the deflection of a beam under long-continued loading instead of that when the load is first applied, only fifty (50) per cent of the corresponding modulus of elasticity given in the table is to be employed.

UNIT STRESSES FOR STRUCTURAL TIMBER IN POUNDS PER SQUARE INCH

(*See Note bottom of p. 1589*)

Kind of timber	Bending			Shearing			
	Extreme fiber stress		Modulus of elasticity	Parallel to grain		Longitudinal shear in beams	
	Average ultimate	Working strength	Average	Average ultimate	Working strength	Average ultimate	Working strength
Cedar, Red	4200	800	800,000	....	....	....	....
Cypress, Bald	4800	900	1,150,000	500	120	....	....
Fir, Douglas	6100	1200	1,510,000	690	170	270	110
Hemlock, Western	5800	1100	1,480,000	630	160	270*	100
Oak, White	5700	1100	1,150,000	840	210	270	110
Pine, Longleaf	6500	1300	1,610,000	720	180	300	120
Pine, Norway	4200	800	1,190,000	590*	130	250	100
Pine, Shortleaf	5600	1100	1,480,000	710	170	330	130
Pine, White	4400	900	1,130,000	400	100	180	70
Redwood	5000	900	800,000	300	80	....	....
Spruce	4800	1000	1,310,000	600	150	170	70
Tamarack	4600	900	1,220,000	670	170	260	100

* Partially air dry.

STANDARD SIZES. — The commercial sizes of spruce and yellow pine in the eastern part of the United States are as follows:

Spruce. — Cross-sectional dimensions in inches are:

2 by 3, 2 by 4, 2 by 5, 2 by 6, 2 by 7, 2 by 8, 2 by 10, 2 by 12
3 by 4, 3 by 6, 3 by 8, 3 by 10, 3 by 12
4 by 4, 4 by 6, 4 by 8, 4 by 10, 4 by 12
6 by 6, 6 by 8, 6 by 10, 6 by 12
8 by 8, 8 by 10, 8 by 12.

12 ft. to 22 ft. are ordinary lengths.
23 ft. to 26 ft. are less common.
27 ft. to 32 ft. are obtained with difficulty.

Yellow Pine. — Same cross-sectional dimensions as spruce and also the following (dimensions in inches):

2 by 14, 2 by 16	6 by 14, 6 by 16	12 by 14, 12 by 16
3 by 14, 3 by 16	8 by 14, 8 by 16	14 by 14, 14 by 16
4 by 14, 4 by 16	10 by 14, 10 by 16	16 by 16

Yellow pine sticks are commonly longer than spruce sticks, frequently exceeding 40 ft.

DESIGN OF TIMBER BEAMS. — Timber beams may be designed by the application of the ordinary beam formulas (*see Design of Beams in article on Structures, Simple*). Timber is especially weak in longitudinal shear which is the determining factor in many cases as shown by the preceding tables. It should, however, be noted that the projection of the end of a beam over the end supports often gives the timber a greater resistance to shear than would be allowed by the strict application of theory and the designer must often use considerable discretion in applying the shear formula.

ALLOWABLE LOADS (POUNDS) ON BEAMS OF LONGLEAF YELLOW PINE

(See Note below table)

Allowable uniformly distributed load in pounds per inch width per lineal foot, in excess of weight of beam, for end-supported yellow pine beams, supported laterally at intervals of 12 times their thickness or less. *Tabular values to be multiplied by width of beam in inches to get carrying capacity of beams per lineal ft.*

Span in ft. = distance c. to c. end bearings	Depth of beam in inches									Coefficient of deflection
	4	5	6	7	8	10	12	14	16	
5	91	142	190	221	253	316	380	443	506	0.65
6	63	98	142	184	210	263	315	368	421	0.94
7	46	72	104	142	180	224	270	315	360	1.27
8	35	54	79	108	141	196	235	275	314	1.66
9	27	43	62	85	111	177	209	244	279	2.11
10	22	34	50	68	90	141	188	219	250	2.60
11	18	28	41	56	73	116	167	198	227	3.15
12	15	23	34	47	61	96	140	181	207	3.74
13	12	19	28	39	52	82	119	162	191	4.39
14	10	16	24	33	44	70	102	139	177	5.10
15	9	14	21	29	38	60	88	120	158	5.85
16	....	12	18	25	33	53	77	105	138	6.66
17	....	11	16	22	29	46	67	93	122	7.51
18	....	....	14	19	25	41	59	82	108	8.43
19	....	....	....	17	23	36	53	73	96	9.39
20	....	....	....	....	20	32	47	65	86	10.40
21	....	....	....	....	....	29	43	59	78	11.47
22	....	....	....	....	....	26	38	53	70	12.59
23	....	....	....	....	....	23	35	48	64	13.76
24	....	....	....	....	....	21	32	44	58	14.98
25	....	....	....	....	....	19	29	39	53	16.25
26	....	....	....	....	....	18	26	36	49	17.58
27	....	....	....	....	....	16	24	33	45	18.96
28	....	....	....	....	....	15	22	31	41	20.39
29	....	....	....	....	....	....	20	29	38	21.87
30	....	....	....	....	....	....	19	26	35	23.41

NOTE.—The tables pp. 1688 and 1689 are for railroad structures of green timber and require no allowance for impact.

For highway structures add 25 per cent to tabular values.

For buildings and other structures where timber is protected from weather add 50 per cent to tabular values.

Bold-faced type is used for lengths where shear limits.

Values below zigzag line cause a deflection under intermittent load in excess of 1/360 of span.

For actual deflection in inches, except where strength is limited by shear, divide deflection coefficient by depth of beam in inches. Deflection under permanent load will ultimately equal double this value.

ALLOWABLE LOADS (POUNDS) ON SPRUCE BEAMS

(*See Note, bottom p. 1688*)

Allowable uniformly distributed load in pounds per inch width per lineal foot, in excess of weight of beam, for end-supported spruce beams, supported laterally at intervals of 12 times their thickness or less. *Tabular values to be multiplied by width of beam in inches to get carrying capacity of beams per lineal ft.*

Span in ft. = distance c. to c. end bearings	Depth of beam in inches									Coefficient of deflection
	4	5	6	7	8	10	12	14	16	
5	70	92	111	130	147	185	221	258	296	0.57
6	48	76	92	108	122	154	184	215	246	0.82
7	35	56	79	92	105	131	157	184	210	1.12
8	27	42	62	81	91	115	137	160	184	1.47
9	21	33	48	66	81	102	121	142	163	1.85
10	17	27	39	54	69	91	109	128	146	2.29
11	14	22	32	44	57	83	99	116	133	2.77
12	11	18	27	37	47	75	90	106	121	3.30
13	10	15	23	31	40	64	83	98	112	3.87
14	8	13	19	27	34	55	77	90	104	4.49
15	7	11	17	23	30	47	68	84	97	5.15
16	....	10	15	20	26	41	60	79	90	5.86
17	....	9	13	18	23	36	52	72	85	6.62
18	....	....	11	16	20	32	46	64	80	7.42
19	....	....	....	14	18	29	41	57	76	8.27
20	....	....	....	....	16	26	37	51	68	9.16
21	....	....	....	....	....	23	33	46	62	10.10
22	....	....	....	....	....	21	30	42	56	11.08
23	....	....	....	....	....	19	27	38	50	12.12
24	....	....	....	....	....	17	25	35	46	13.19
25	....	....	....	....	....	16	23	32	43	14.31
26	....	....	....	....	....	14	21	29	39	15.48
27	....	....	....	....	....	13	19	27	36	16.70
28	....	....	....	....	....	12	17	25	33	17.96
29	....	....	....	....	....	....	16	23	31	19.26
30	....	....	....	....	....	....	15	21	29	20.61

The unit values used in the tables are:

Yellow Pine	Lb. per sq. in.	*Spruce*	Lb. per sq. in.
Fiber stress:		Fiber stress:	
Bending	1,300	Bending	1,000
Longitudinal shear	120	Longitudinal shear	70
Modulus of elasticity	1,500,000	Modulus of elasticity	1,310,000

These unit values used are the same as given in the table on pp. 1686 and 1687, except the modulus of elasticity for Yellow Pine, which is taken as 1,500,000 lb. per sq. in. For other values of stress modify tabular values in proportion to allowable unit stresses.

WEIGHT OF TIMBER. — This is variable, depending upon the moisture in the timber and whether it has been chemically treated or not. It is common to assume yellow pine at 4½ lb. per board foot and spruce at 4 lb. per board foot in estimating the strength of beams. See also section on *Volume and Weight of Poles* in article on *Poles for Overhead Lines*.

COST OF TIMBER (Pre-war figures). — The price of timber is variable and depends for ordinary stock upon the size and length. Yard quotations in New York in June, 1913, gave the price of spruce as from $35 to $37 per 1000 board feet. Similar quotations on longleaf yellow pine were as follows:

COST OF YELLOW PINE IN DOLLARS PER 1000 BOARD FEET AT YARD IN NEW YORK

Cross section	Length of stick			
	20 feet and under	21 to 25 feet	26 to 30 feet	31 to 35 feet
2 in. thick by 8 in. wide, and under..	$36	$36	$36	$38
10 in. thick by 10 in. wide, and under..	42	42	42	45
12 in. thick by 12 in. wide, and under..	42	42	42	45
14 in. thick by 14 in. wide, and under..	47.50	48	50	50
16 in. thick by 16 in. wide, and under..	50	52	54	60

For regular quotations on timber see the first issue of each month of the *Engineering-News Record*.

The cost of hauling timber and putting in place may be roughly estimated at $10 per 1000 board-feet for fairly heavy yellow pine timbers which require little labor.

BIBLIOGRAPHY. — U. S. Dept. of Agriculture, *Forest Service Bulletins*. (These give much valuable information concerning strength, preservation, classification, etc., of timber.) Snow, *The Principal Species of Wood;* Hartig, *Diseases of Trees;* Johnson, *Materials of Construction;* Betts, H. S., *Timber; Its Strength, Seasoning and Grading; Am. Soc. of Test. Mat. Standards; Manual of Am. Ry. Association; Yellow Pine Manual*, published by the Yellow Pine Manufacturers' Association, St. Louis.

TOWERS FOR TRANSMISSION LINES. — (*See also Distribution Lines; Insulators for Transmission Lines; Poles for Distribution Lines; Structures, Simple; Transmission Lines.*) Steel towers are often used instead of wooden poles for supporting electrical circuits. They have the advantages over poles that they can be of as large size as desired and are much more permanent. Steel towers are especially advantageous for high voltages (over 50,000) where wide wire spacing is necessary; for long spans, such as river crossings, where the mechanical stresses are exceptionally heavy; and in localities where grass, brush or forest fires occur.

GENERAL FEATURES OF DESIGN. — Towers are composed of two parts, the tower proper and the foundation. Towers are usually built of standard structural steel shapes. Angles are used for most members. Channels are used for the larger members of some towers, usually the cross arms, or for posts of flexible towers. Flat pieces are sometimes used for the minimum-sized bracing of light towers. Round rods are used for tension members in some types. The principal members of a tower are the corner posts or legs, which are vertical or approximately vertical, and are usually the heaviest members of the tower proper, and the horizontal and diagonal web members which connect the posts together in vertical planes which constitute the sides or faces of the tower.

The spread of a tower at the base is generally between one-fourth and one-fifth of the height. The greatest economy in cost of tower plus foundation usually requires a little wider base than that which gives the least cost for the tower taken alone.

Towers are usually designed for either one or two three-wire circuits, usually with one or two ground wires above and sometimes with a telephone circuit below. Where two circuits are on one tower they are usually located on opposite sides to reduce the hazard of repairing the line. The three wires of a circuit are occasionally arranged to form an equilateral triangular prism, but frequently lie in a single plane which is usually horizontal for single-circuit towers, and vertical for two-circuit towers.

Types of Towers. — There are two general types of towers, viz., (1) the flexible, having only two legs and (2) the rigid, having three or more legs.

Flexible Towers (Fig. 4). — The flexible tower is set with face at right angles to the line so as to rigidly resist any transverse force, such as a wind blowing across the line, while bending if necessary to relieve any stress in the direction of the line. The stability of the tower in the direction of the line is maintained by the conductors which it supports acting as guys. Where conductors cannot perform this function, due to being attached by suspension insulators, the ground wires are used as the guys. The flexible tower is especially economical in the smaller sizes, where the line conditions approximate those where wooden poles would otherwise be used. On lines of flexible towers, rigid anchor towers are set at intervals.

Rigid Towers. — These are usually triangular, square (Fig. 2) or rectangular (Fig. 3). The square tower is probably the most common. Square towers usually have the four faces framed with the same size members (even though the stresses in the longitudinal and lateral faces rarely figure the same), because of the economy of manufacture and erection which results from the simplicity. This feature has an advantage in design in that the torsional stresses are more simply determined. Rectangular (including square) towers have the disadvantage that the unequal settlement of the foundations may produce high internal stresses not allowed for in the design. Triangular towers avoid internal stresses from unequal foundation settlement, but present diffi-

culties in the joining of standard structural shapes, and stresses in them are difficult to calculate.

Connections of Members. — The members of a tower are usually connected by bolts. By using no rivets the members may be compactly bundled, easily handled even in rough country, erected by less skillful labor and the galvanizing can be done after all shop work is completed. All the bolts of a tower should be of one diameter (5/8 inch is suitable for the members generally used) and of as few different lengths as possible. Bolt holes should be slightly larger than the bolts (1/16 inch is a usual amount). By designing bolted connections so that friction between the surfaces develops the full compressive strength of members, the play in the bolt holes with changing compression and tension in the members is eliminated.

Clearance Between Conductors and Tower. — The clearance between conductor and tower should be sufficient so that the current will not jump to the tower at a lower voltage than is required to arc over the insulator. This is usually determined approximately by making the clearance equal or exceed the length of the string of suspension insulators. Allowance must also be made for change of position of conductor with swing of suspension insulator. The amount of swing will not ordinarily exceed 45 degrees from the vertical, but should be determined for each case with and without ice loading; see *Transmission Lines*. The angle of swing will be greater for small than for large conductors and will be more for aluminum than for copper of the same diameter. Clearance to the cross arm will require special consideration where the angle of swing exceeds 45 degrees from the vertical and also for lesser angles where insulators are used which do not have an arc-over voltage very high compared to the working voltage.

Foundations for Towers (Figs. 5 and 6). — Structural steel, mass (unreënforced) concrete, reënforced concrete or piles may be used for tower foundations. Rock footings are also used in special locations.

Structural Steel Foundations are cheap and easily transported. While their durability has been questioned, they have been widely used without much trouble on this account yet apparent.

Concrete Foundations have an advantage over structural steel in that they can more easily be varied in depth, spread, etc., to accommodate themselves to local conditions of soil. This is especially advantageous where boulders or irregular ledges interfere with the use of a standard-sized foundation.

Mass-concrete foundations are advantageous in those cases where it is necessary or desirable to have a foundation of such weight as to withstand much uplift with little reliance on the holding power of the earth. The towers may be conveniently attached to anchor bolts imbedded in the foundation. To avoid tension in the concrete the bolts must extend to the bottom, with proper plates for distributing the stress. The anchor bolts and plates then become a crude system of reënforcement.

Reënforced concrete foundations are durable and require less material than mass concrete, thereby facilitating transportation.

Piles are used under or for foundations in very marshy ground where the holding power of other foundations is unreliable.

Rock Footings for towers standing on ledges may consist of anchor bolts grouted into holes drilled in rock and extending through level bearing plates grouted to the rough rock surface at the proper elevation.

FORCES ACTING ON TOWERS. — The stresses in towers are caused by: (1) The weight of tower, insulators, clamps, cables (conductors, ground wires,

telephone wires) and ice loads on them; (2) The wind pressure on above; * (3) The unbalanced tension in cables when dead ended or broken on one side; (4) The unbalanced resultant due to cable tension at angles in the line; (5) The loads imposed when erecting towers, stringing wire or repairing line.

A careful study should be made of all the combinations of these loads which are possible or probable. Often no single combination can be found which will produce the maximum stress in all tower members, and therefore several combinations must be used to determine the design.

In a square anchor tower carrying six wires, three on each side, the maximum stress may be expected in the corner posts when all six wires are pulling in the same direction; the maximum stress in the web members will probably be produced by three wires pulling on one side in one direction and the three on the other side in the opposite direction. In the first case the tower is subject to a bending stress and in the second to a torsional stress. In each case the stresses due to weight and wind are to be superimposed. The wind may act as a force along the line or across the line, but generally its longitudinal effect is negligible while its lateral effect is important.

Stresses in Tower Members. — The stresses in the several members of a tower are usually determined graphically from the assumed loadings by means of stress diagrams; see section on *Trusses* in the article on *Structures, Simple.* In most designs the distribution of stress is not fully determinate.

Fundamental Assumptions. — Certain assumptions are, however, commonly made which give a determinate distribution for the purposes of design. Among these assumptions are:

(1) An unbalanced stress on the tower (say a broken wire pulling on one side) can be resolved into an equal stress at the axis of the tower and a torsional moment.

(2) The equivalent stress at the axis of a rectangular tower can be considered as balanced between the two faces parallel with it, each face taking one-half the stress.

(3) The torsional moment can be considered as divided between all four faces of a rectangular tower.

(4) If the tower is square each face takes one-fourth of the torsion.

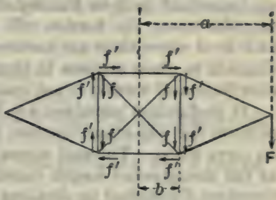


Fig. 1.

The above relations may be expressed as follows, (see Fig. 1):

Let F = unbalanced force in pounds applied at end of cross arm,

a = distance, in feet, from *end of cross-arm* to axis of tower,

b = distance, in feet, from *side of tower body* to axis of tower;

then

$f = \frac{F}{4}$ = balanced force in pounds applied at each corner post equivalent to F in bending effect on body of tower,

* The wind pressure on a tower is assumed to be uniformly distributed per square foot of surface against which the wind blows, one-half of it consequently being on the windward side of the windward face and the other half on the windward side (inside) of the leeward face. For simplicity in calculation this uniformly-distributed force is replaced by a series of concentrated forces, one at each panel point equivalent to the total distributed force extending over a half panel above and a half panel below the panel point. By panel point is meant a point of intersection of principal members, for example, of horizontal members with vertical members in Fig 4; and by panel is meant the section of a side between panel points, a panel usually being bounded by two vertical and two horizontal members as in Fig 4.

$f' = \frac{Fa}{8b}$ = torsional force in pounds applied at each corner post equivalent to F in twisting effect on body of tower.

(5) In a tower framed with a double system (i.e., diagonals in duplicate and suitable for compression as well as tension) each system may be considered as taking one-half of the stress as far as possible.

Approximations Made in Calculating Tower Stresses.— The stress diagrams are usually simplified by employing certain approximations:

(a) Faces of towers are usually battered so that they deviate slightly from a true vertical plane, but the stress diagrams usually neglect this inclination and are based on the vertical projection of the face.

(b) Where the face of a tower does not lie in one plane (i.e., has a change of batter as occurs frequently at bottom cross arm where a prismatic cage joins a pyramidal base) the change of inclination is neglected and the diagrams are based on a single vertical projection as before.

Subject to the limitations of the assumptions and approximations given above, the four faces of the tower can be regarded as four cantilevers, supported at the base and loaded at the top, which are independent except that the four corner posts are each common to two faces and must contain the resultant of both stresses.

Where a face of a tower or any part of a face has any considerable inclination the above approximations may not be used without danger of serious error.

Unstressed Members. — A tower usually contains members no stress in which is shown by the stress diagram, viz.:

(1) Diagonal members in a horizontal plane do not usually appear in the stress diagram when located below the lowest cross arm. These members play an important part in the distribution of torsion among the faces. In a rectangular tower the torsion will usually redistribute between the four faces at each level where there are horizontal diagonals, therefore the failure of the stress diagram to show stresses in them may be taken to indicate that the assumed distribution of torsion is not quite correct rather than a true absence of stress.

(2) Redundant members are braces which carry no determinate stress but perform the important function of supporting the compression members which do carry stress. The unit stress allowable in a compression member diminishes as the unsupported length increases. The weight of compression members is therefore diminished by dividing their unsupported length by braces applied at one or more intermediate points.

Unit Stresses in Towers. — The factors of safety and unit stresses used in tower design have not been standardized. In some cases it has been assumed that the combination of loading taken for tower design represented conditions which would happen so rarely that a tower which would stand these loadings in test without failure would be sufficient. This is equivalent to taking a severe combination of loads and designing for a factor of safety of but one. The result, however, may be as good a tower as one designed for the same conditions individually (but not in combination) with a factor of safety of two or four. It is probable that few towers are designed for the combination of all loadings, each taken at the most severe condition reasonably possible with an actual factor of safety exceeding two.

For a factor of safety of from 2½ to 1½ (as determined by test of completed tower, not of individual members) the following unit stresses have been used:

Tension: 12,000 to 20,000 lb. per sq. in.
Shear: 12,000 to 20,000 lb. per sq. in.
Bearing: 16,000 to 30,000 lb. per sq. in.
Compression: $12{,}000 \div (1 + K)$ to $20{,}000 \div (1 + K)$ lb. per sq. in.

Where

$$K = \frac{L^2}{36{,}000\, R^2},$$

L = unsupported length of member, in inches,

R = least radius of gyration of section, in inches,

L and R are so limited that for main members $L \div R$ does not exceed 125 to 180 and for secondary members $L \div R$ does not exceed 150 to 220.

Eccentricity in Stresses at Joints. — As tower members are ordinarily connected together the stresses in them are slightly eccentric, thereby preventing the full strength of the members being developed. The eccentricity should be eliminated or reduced as much as possible by having the center of gravity of the several members at each connection meet at one point as exactly as possible.

FORCES ACTING ON TOWER FOUNDATIONS.—Foundation stresses are of two classes: first, the foundations resist the tendency of the tower to slide and overturn due to the external forces on it considering the tower as a self-contained structure; second, the foundations resist certain stresses which would be internal tower stresses were the tower framed as a complete self-contained structure, but which become external stresses because the ground is depended on for the function of certain omitted members. The weight of the tower can evidently be reduced by thus substituting the ground for certain members, but the size of foundation is thereby increased. The amount of these latter stresses depends on the outline and framing of the tower and their effect should not be overlooked in determining loadings on foundations.

The magnitude and direction of the forces acting on the tower foundations may be illustrated by taking the case of a rectangular tower, and considering a transmission line which runs north and south.

Let

a = width (feet) of base of tower (east and west);

b = length (feet) of base of tower (north and south);

W = total weight (pounds) of tower, insulators, fittings and one span of all the wires, including ice load, if any;

W' = total weight (pounds) of any unbalanced load, such as a wire c feet off center;

F = resultant force (pounds) of the wind on the tower and a complete span of all the wires (with ice coat, if any), acting at a distance of d feet above the foundation, wind assumed blowing across line from west to east,

P = pull (pounds) of any unbalanced force toward the south applied at a distance of e feet above the foundation and f feet to the west of axis of tower, as for example, a dead-ended wire or when a wire on the north side of the tower is broken.

Then, assuming that the forces divide equally among the four foundations and that the torsional forces are in a circumferential direction, the relations given in the following table hold. These assumptions are reasonably correct for a tower with the four legs joined at the bottom with a horizontal strut in each of the four faces and with horizontal ties across the diagonals, unless the framing which is usually provided in the other faces is inadequate. Probably few towers in use fully meet these requirements. Therefore, there are usually additional stresses of large magnitude due to the foundations performing the function of missing or inadequate members, as pointed out in the notes appended to the table.

FORCES ON TOWER FOUNDATIONS

Magnitude of force on each of the four foundations	Direction at each foundation			
	N. E. corner	S. E. corner	S. W. corner	N. W. corner
$\frac{W}{4}$	Down	Down	Down	Down
$\frac{cW'}{a}$	Down	Down	Up	Up
$\frac{F}{4}$ (Note 1)	East	East	East	East
$\frac{dF}{a}$	Down	Down	Up	Up
$\frac{P}{4}$ (Note 2)	South	South	South	South
$\frac{eP}{b}$	Up	Down	Down	Up
$\frac{afP}{4(a^2+b^2)}$ (Note 3)	North	North	South	South
$\frac{bfP}{4(a^2+b^2)}$ (Note 3)	West	East	East	West

NOTES. — Where there are no struts between the bottoms of the legs, and especially where the bottom panel is framed on the single system, both of which conditions are usual:

(1) The force of the wind F will give a greater force than $F/4$ in an easterly direction on the two west foundations and a correspondingly less force on the other two, and will in addition produce four new forces tending to force the legs apart on the compression side and draw them together on the tension side.

(2) Similarly the pull of the wire P will give a greater force than $P/4$ in a southerly direction on the two north foundations and a correspondingly less force on the other two, and will in addition produce a westerly force at the N. E. and S. W. corners and an easterly force at the S. E. and N. W. corners.

(3) Similarly, the torsional forces due to P will increase in magnitude and change in direction, and the unbalanced pull P may develop new forces tending to raise two diagonally-opposite legs and depress the other two.

Resultant Forces on Foundations. — From the above relations the resultant force on each of the four foundations may be found. In general there will be on each foundation: (1) a downward pressure, (2) a direct uplift, and (3) a horizontal overturning force, producing a tendency to slide and an uplift on one side of the foundation and a downward pressure on the other side.

Downward Pressure. — The downward pressure usually is of little importance in determining the size of the foundation as a foundation large enough for uplift and overturning is unnecessarily safe against downward pressure.

Direct Uplift. — The uplift is very important, as the weight of tower and foundation is rarely sufficient to provide more than a small fraction of the holding-down power required. The excess uplift is usually resisted by the earth in which the foundation is buried. Not only is the weight of the earth directly over the foundation effective but there is an additional resistance due to friction or cohesion of the earth which may be several times greater. These forces are usually computed on the assumption that they are equivalent to the weight of the earth in a frustrum of an inverted cone or pyramid covering the founda-

tion and extending to the surface of the earth. The face of this cone is usually taken as making an angle of 30° with the vertical.

Horizontal Overturning Force, Sliding and Indirect Uplift. — The horizontal overturning force is also important. Its effect on the base may be resolved into two components, one a horizontal force tending to slide the foundation and the other a moment tending to rotate the base about a horizontal axis. The resistance of the earth to these forces is an obscure subject, especially if the foundation is of irregular shape. The following discussion which neglects several favorable elements may be considered as a conservative view of the earth resistance.

The resistance to sliding may be considered as due to the friction of the bottom of the base on the earth, and it may further be assumed that any base large enough to resist uplift will also furnish sufficient friction to prevent sliding. The arm of the overturning force is then the same as the height of the foundation (bottom of base to top where tower is attached), and the overturning moment is equal to the horizontal force multiplied by this arm. The resisting moment may be considered as due entirely to the vertical reaction of the earth on the top and bottom of the foundation. These vertical pressures may be taken as varying uniformly from zero at a horizontal neutral axis through the middle of the base, to a maximum at the edges of the base most remote from the axis. The moment of resistance is calculated as for a beam subject to bending and having a cross-section identical with the area of the base. It may be assumed that any unit pressure allowable on the uplift edge will be amply safe on the opposite edge where the pressure is downward, so that calculations for uplift only are necessary. The maximum allowable unit stress on the uplift edge may be taken as equal to the average unit resistance to uplift of the whole foundation, determined as described above under *Direct Uplift*.

Limiting Conditions. — Usually the most severe condition that a tower foundation is required to meet consists of a combination of uplift and overturning. For this condition the unit stress of uplift proper must be added to the maximum unit stress of uplift due to overturning and the sum must be within the allowable average unit resistance to uplift.

Strength of Foundations. — (*See also article on Strength and Elasticity.*) A foundation is subject to stresses from the tower tending to move it and from the resistance of the earth preventing motion. The foundation should of course be strong enough not to break when subject to these opposing forces. As the points of application of the resistance of the earth and the magnitude of the unit stresses transmitted by the earth at any point are subject to great uncertainty, the foundation should be designed for strength for the distribution of earth resistance which is most severe, considering for example that while the holding power is calculated on a uniformly-distributed earth resistance, it may be developed in practice by concentrated pressure from stones or timber located near the outer edge of the base.

Important Points Regarding Design of Foundations. — The following are important conclusions which follow from the above discussion:

(1) The inverted cone theory of resistance to uplift gives a calculated resistance which increases at a rapid rate with the depth (eventually increasing approximately as the cube of the depth). It would however be unsafe to apply the theory for foundations differing much from those of usual dimensions, say for depths much exceeding six feet and for foundations where the spread of base was much less than the depth to which it is buried.

(2) The foot of the tower (top of foundation) should be brought as close to the surface of the ground as possible to reduce the overturning moment.

(3) The tower diagonals of the bottom panel of the tower should intersect the corner posts as low as possible, because this is the actual point of application of the overturning force to the foundation and if this intersection is above the foundation this extra length must be added to the arm of the overturning moment.

(4) By inclining the axis of the foundation approximately in line with the inclined tower leg (i.e., to bring it as near as possible into line with the resultant of the horizontal overturning force and the vertical pressure or uplift) a more economical use of material may be made to resist the combined uplift and overturning.

TESTING OF TOWERS.—Since towers are generally indeterminate structures, new designs are usually tested. Test loads proportional to the loads specified for the design are applied until the required factor of safety has been proven or failure occurs. Towers are usually mounted on a rigid base for testing. This gives the strength which the tower would develop on an "ideal" foundation. In practice the tower must be expected to develop somewhat less strength, as unequal movement of the foundations will ordinarily overstrain certain members. Test loads may be applied by means of weights suspended directly at proper points for vertical loads and applied by means of pulleys attached to a tower-testing structure ("test tower") for horizontal loads. This method of application makes the determination of the test loads easy, but it is inconvenient to have the weights fall any considerable distance if the tower is tested to failure.

Testing of Foundations. — Foundations are occasionally tested, but the test is usually more for determining the holding power of the soil than the strength of the foundation. For the former purpose the test result will depend largely on the character of the soil and its condition (dry, wet or frozen). Tests for holding power are only necessary for uplift and overturning forces. In testing it is important that the testing machine should not press down on the surface of the soil near the foundation. The machine should rest on the ground outside of the base of an inverted cone of angle of 45° from the vertical and enveloping the base of the foundation under test.

SPECIFICATIONS, CONTRACTS AND PROPOSALS. — Different manufacturers of towers use different details of construction so that specifications written for the purpose of obtaining proposals should contain only conditions and requirements. These should state the loadings, factors of safety, maximum allowable stresses, and show outline dimensions of the tower as determined from clearances required.

The specifications should state whether towers are bolted or riveted, galvanized or painted, shipped assembled or partly assembled, tested or not tested, etc., besides containing the usual structural steel specifications; see *Steel*.

The proposals should show the arrangement and sizes of members and typical details of connections.

Contracts are oftentimes let on the basis of furnishing an approximate number of towers when the exact number required cannot be determined in advance, and a unit price per pound is included to cover extensions, special foundations and modifications that may be required.

The galvanizing and sherardizing of towers and parts are usually specified to pass the American Telephone and Telegraph Co's. Specification for galvanized iron wire (*see Galvanizing for Iron or Steel*). All members of towers that are not too heavy, so that there is danger of their buckling in the process, may be specified to be galvanized by the hot dip process. All bolts, nuts, threaded rods and turnbuckles may be specified to be sherardized.

INSTALLATION AND ERECTION. — Towers are generally shipped entirely disassembled, all of the members for one tower being bundled together. The tower is usually assembled lying on its face on the ground adjacent to the foundation. When assembled it is temporarily braced when necessary (usually by struts between the legs at the base of the tower) and up-ended onto the foundations. For the latter operation temporary hinges connecting the two legs lying on the ground to the two adjacent foundations are convenient. Towers too heavy to up-end and towers in inaccessible places are built up in place.

With bolted connections the nuts should be prevented from working loose by checking threads on bolts, riveting over the end of the bolt, or by lock-nut or washer.

Each tower member should have a number which should be shown on the erection drawings and marked on the corresponding member of each tower.

The members should be cut, bent and punched to template so that the parts will be interchangeable and the assembled tower will fit the foundation prepared for it.

Preparation of Foundations. — Foundations should be set with their bases below the frost line. The hole should not be excavated deeper than necessary, so that the foundation may rest on undisturbed earth. If any of the several foundations of a tower rest on loose backfill, unequal settlement may be expected which may greatly weaken the tower. The hole should not be larger than necessary, as the backfill will be less effective than undisturbed earth in resisting uplift. The resistance of concrete foundations can sometimes be increased by digging a hole smaller in diameter than the base and undercutting it at the bottom, while with structural steel foundations large stones can sometimes be placed against the steel to increase its resistance to motion, either vertical or lateral. The backfill should be well tamped in place and especial care should be used if it is probable that the foundation will be subject to heavy stress before the earth has had time to settle. On sloping ground filling may be required on the low side to give the designed weight of earth against uplift. In water or mud the floating power of hydrostatic pressure beneath the foundation must be allowed for. Where towers are raised on extensions the increased foundation stresses due to increased moment must not be overlooked.

The several foundations of a tower should be set accurately by template both as regards spacing and elevation so that towers stand trully vertical and have no initial stresses due to distortion.

MAINTENANCE OF TOWERS. — All bolted connections on towers should be carefully watched and kept tight, and the galvanizing or paint should be inspected regularly and towers must be repainted before any deterioration from rusting occurs. Foundations are the most likely source of trouble in operation. These should be kept properly backfilled and should be watched for unequal settlement of legs.

DIMENSIONS, WEIGHTS, COSTS. — Data on these items for a number of towers are given in the following table. Galvanized towers cost from 2½ to 4 cents per pound, f.o.b. factory. Galvanizing costs from ½ to 1 cent per pound; the cost of galvanizing is included in the figures just given.

DATA ON TYPICAL STEEL TOWERS

Use (a)	Conductors (b)		Number and diam. of ground wires, inches	Kilovolts between wires	Pin or suspension insulators	Height, feet		Base, feet		Weight, pounds
	Number	Size, cir. mils or B. & S.				To lowest cross arm	Over-all	Across line	Along line	
L (2)	6	300,000	1-½	110	*S*	55	79	20	20	6,800
A	6	300,000	1-½	110	*S*	50	75	24	24	10,500
L (3)	3	683,000 (*c*)	1-½	150	*S*	43	47	18	20	4,335
A	3	683,000 (*c*)	1-½	150	*S*	37	41	24	24	6,985
L (4)	3	0000	1-⅜	50	*P*	42	53	9.5	(*d*)	2,150 (*e*)
A	3	0000	1-⅜	50	*P*	42	53	9	9	3,750
L	3	0	2-⅜	102	*S*	43	45	14	13	1,800
L	6	000	1-⅜	32	*S*	51	76	17	17	4,560 (*f*)
L	6	000	1-⅜	100	*S*	51	77	17	17	....

(*a*) *L* = straight-line or suspension tower, *A* = anchor or angle tower; numbers in brackets refer to accompanying cuts, Figs. 2, 3 and 4 respectively. (*b*) All conductors stranded. (*c*) 605,000 circular mils of aluminum with a 78,000 circular-mil steel core. (*d*) Flexible tower. (*e*) Including foundation. (*f*) Average for 197 towers, including anchor towers and hardware.

DATA ON TYPICAL TOWER FOUNDATIONS

(*Used in connection with the first four towers listed in above table, respectively*)

Type	Fig. No.	Number per tower	Height, in.		Base, in.	Lb. of steel	Cu. yd. of concrete
			Over-all	Projection above earth			
Reënf. concrete.	5	4	78	6	60 by 60	500	4.4
Reënf. concrete.	..	4	96	6	96 by 96	1584	14.4
Steel............	6	4	90	6	44 by 45	1285	......
Steel............	..	4	88	6	52 by 52	1865	......

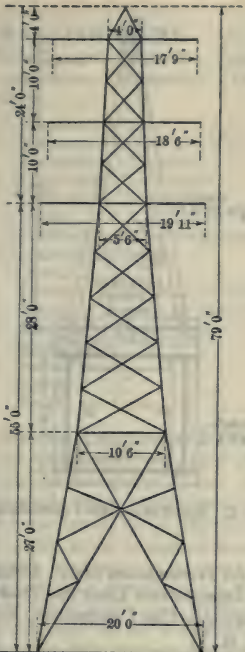


Fig. 2. Square Two-circuit Tower

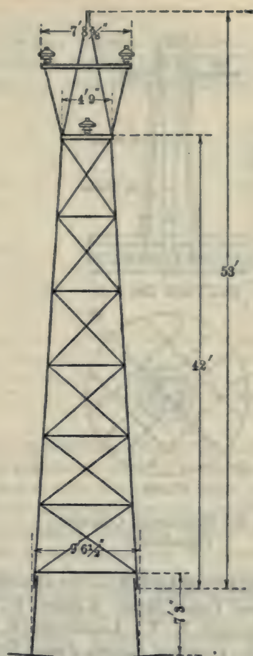
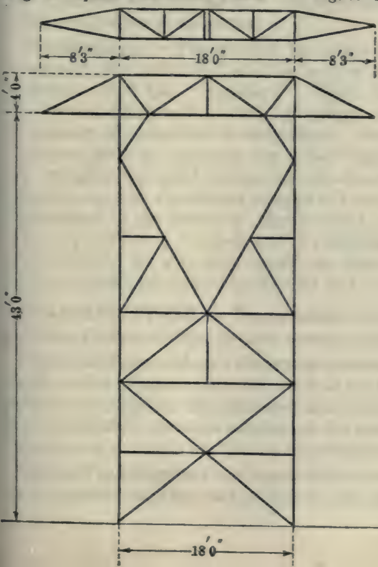
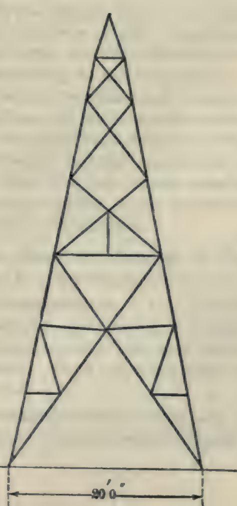


Fig. 4. Flexible Tower with Steel Foundations





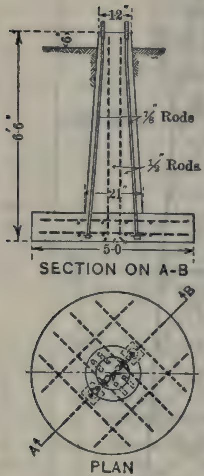


Fig. 5. Reënforced Concrete Foundation

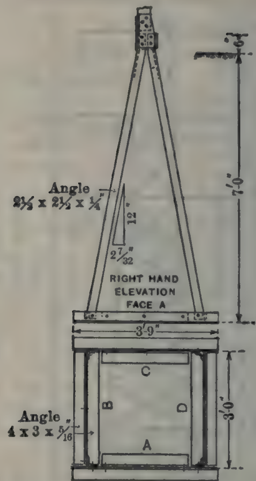


Fig. 6. Structural Steel Foundation

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TRANSFORMERS. — (*See also Alternating Currents; Auto-transformers; Electricity and Magnetism, Principles of; Standardization Rules of the A.I.E.E.; Transformers, Instrument.*) The following is a brief table of contents of this article:

General Description. — The electrical transformer, commonly called the static transformer, is a piece of stationary apparatus used to transform alternating-current energy at one voltage to some other higher or lower voltage. The single-phase transformer consists of two electrical circuits, usually of a large number of turns, interlinked with a common magnetic circuit of iron. Since the power is approximately the same in both windings, the currents in the two windings are inversely proportional to the voltages in the windings. The polyphase transformer is essentially two or more single-phase transformers made into a single piece of apparatus, but so designed that at least a part of the magnetic circuit is common to all the phases. A three-phase transformer has three high-tension and three low-tension windings arranged on a single iron core; see Figs. 1 and 2. The auto-transformer is treated in a separate article, viz., *Auto-Transformers.*

Terminology. — The winding by which the energy enters the transformer is logically the "primary" and the one by which the energy leaves the transformer is called the "secondary." Since either winding of the transformer may be connected to the source of energy, these terms are not definite unless the manner of connection is also stated. When referring to the transformer as a separate piece of apparatus the terms "high-tension" winding and "low-tension" winding are used to distinguish the two windings, the high-tension winding being the one with the greater number of turns. When the high-tension winding is connected to the source of supply it is the primary, and the transformer is said to be used as a "step-down" transformer; when the low-tension winding is connected to the source of supply, the high-tension winding is the secondary, and the transformer is said to be used as a "step-up" transformer.

CLASSIFICATION. — Transformers may be classified according to their operating characteristics, to their construction, or to the method of cooling.

Constant Potential and Constant-current Transformers.— Transformers may be either constant potential, such as are intended to give an approximately constant potential on the secondary side, or constant current, intended to give an approximately constant current on the secondary. Both types are intended to operate on a supply circuit of a constant potential.

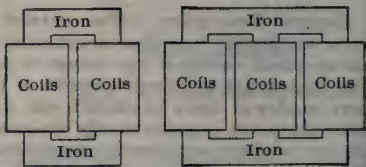
Series Transformers are connected in series with the main circuit and receive a variable voltage and current in the primary. The secondary circuit is

closed through a path of low impedance and thus the secondary current will be proportional by the ratio of turns to the load current flowing in the primary or supply circuit. They are usually used to supply low-reading ammeters and wattmeters from circuits carrying very heavy currents.

Auto-Transformers or Compensators, sometimes called single-circuit transformers, consist of one electric circuit interlinked with the magnetic circuit and a tap brought off from some part of the winding. The voltage between this tap and either terminal of the electric circuit will be a fraction of the total voltage and thus a fractional voltage may be secured from this piece of apparatus. It is customary to proportion the windings on each side of the tap in accordance with the current to be carried. Auto-transformers are generally used where the ratio of voltages is quite near to unity as in this case they can be constructed with much less copper than the regular transformer. See article on *Auto-transformers.*

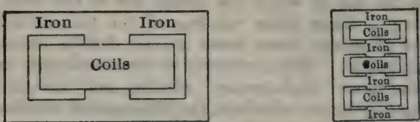
Potential Regulators are a form of transformer in which the voltage of one member may be varied from zero to a fixed maximum either by changing the direction of the magnetic flux or by changing the phase of the electromotive force of the secondary with respect to the electromotive force of the primary.

Core and Shell Types.—Two methods of arranging the electric and magnetic circuits are in use, the corresponding construction being designated as the "core type" or the "shell type." At present a large number of lighting transformers of small and medium capacity are constructed in a manner which is a composite of the core and shell types.



Single Phase Three Phase
Fig. 1. Core-type Transformers

Core Type (Fig. 1).—The single-phase core-type transformer consists of a single magnetic circuit interlinked with two electric circuits, each consisting of a group of coils as shown in Fig. 1. The three-phase core-type transformer is also shown in Fig. 1.



Single Phase Three Phase
Fig. 2. Shell-type Transformers

Fig. 2*a*. Distributed Core Type

Shell Type (Fig. 2).—In the shell type of transformer each electric circuit is interlinked with two magnetic circuits having a common path inside the coils but branching outside of the coils as shown in Fig. 2.

Distributed Core Type (Fig. 2*a*) is a compromise between shell and core type and is much used for small lighting transformers because it economizes in iron and distributes the heat. It consists of the two electric circuits wound concentrically about one central core which divides at the top and bottom into four radiating magnetic circuits surrounding the coils on four sides. It is shown in Fig. 2*a* in plan view.

Comparison of Core and Shell Type.—The core type of construction is best adapted for high-voltage low-capacity transformers and the shell

type for low-voltage high-capacity transformers. This arises from the fact that the most economical disposition of material in the core type demands a large number of turns and small cross-section of iron, while in the shell type a large cross-section of iron and small number of turns may be used to advantage. In the shell type the coils are usually wound in flat "pan-cakes," this type of construction being particularly well suited to the use of heavy copper ribbon or straps. In the core type the coils usually consist of two or more spools, long in comparison with their diameter. The use of the core type is increasing both in number and size of transformers as it is found possible to arrange the coils so they will better withstand the mechanical strain due to short circuits.

Classification According to Method of Cooling. — (*See also below.*) Transformers may be subdivided into classes in accordance with the method used for dissipating the heat due to their internal losses. As a transformer is a very compact piece of apparatus the problem of carrying away the heat is very important and various ingenious means have been devised for the purpose.

Naturally Cooled Transformers. — As this name implies this type of transformer has no special means of cooling but relies upon the ordinary circulation of the air. It is only used in transformers of very small sizes, such as those intended to supply meters.

Oil-cooled Transformers. — In this type of transformer the core and windings are submerged completely in oil in a tank and the windings and core are subdivided by ducts in order that the oil may circulate and carry off the heat from the internal parts. The heat is carried to the surface of the tank which contains the transformer and from there carried off by the surrounding air. The tank is specially designed to provide large air-cooling surfaces, e.g., provided with deep corrugations, or with projecting vanes or tubes.

Air-blast Transformers are designed with special passages through which a current of air is forced by means of a blower, the heat being carried off to the atmosphere in this manner.

Water-cooled Transformers consist of a construction similar to that of the oil-cooled type and in addition a coil of pipe carrying running water is submerged in the oil.

Forced-oil Transformers. — Transformers artificially cooled by circulation of oil are used when the size is too great for the self-cooling oil type and no cooling water is available. The oil circulates through external coils or tanks which give a greater cooling surface.

METHODS OF RATING. — The *Standardization Rules of the A.I.E.E.* specify that the continuous rating of a transformer is that load in kilo-volt-amperes which when applied continuously will cause a rise in temperature of 50° C. as measured by a thermometer, or 55° as measured by resistance of the windings. The iron of transformers takes a long time to reach a constant temperature, that of oil-cooled transformers in particular requiring from 10 to 12 hours. Thus a transformer will stand a considerable overload for a short time without overheating, the principal danger being excessive mechanical stresses.

TRANSFORMER PRINCIPLES. — (*See also Electricity and Magnetism, Principles of.*) The essential features of a transformer consist of a primary winding having a number of turns interlinked with a magnetic circuit, and a secondary winding also interlinked with the same magnetic circuit, as shown in Fig. 3.

Simple Theory, Neglecting Leakage Reactance. — In the following discussion the assumption will first be made that all the flux links both primary and secondary windings; the effect of the leakage flux will be discussed later.

In Fig. 3, S_1 is the primary winding, M the magnetic circuit (of iron) and S_2 the secondary winding. If an alternating current is caused to flow in S_1 it will set up a flux in M which at any instant is proportional to the current i_1. Thus

$$\phi = \frac{4\pi S_1 i_1}{10 R},$$

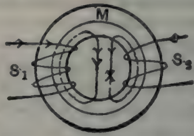


Fig. 3. Elementary Transformer

where S_1 is the number of turns, i_1 the instantaneous value of the current and R the magnetic reluctance of the path in M.

Thus ϕ alternates with i_1 and since it interlinks with S_2 it will induce a voltage in S_2 at any instant equal to

$$e_2 = - S_2 \frac{d\phi}{dt} 10^{-8} \quad \text{volts}$$

in which S_2 is the number of turns. The negative sign means that any current due to e_2 will tend to diminish ϕ.

No-load Conditions. — When there is no current in the secondary, the vector relations are as shown in Fig. 4. Let E_1 be the voltage impressed upon S_1, then the maximum value of the alternating flux will be

$$\phi = \frac{10^8 E_1}{4.44 f S_1},$$

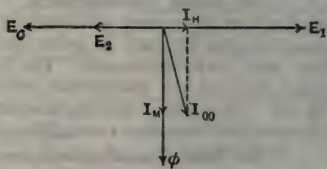


Fig. 4. No-load Relations

where f is the frequency of alternation of E_1. Strictly, the numerator is $10^8(E_1 - r_1 I_{00})$, where I_{00} is the exciting current and r_1 the resistance of the primary, but the term $r_1 I_{00}$ is practically negligible. This flux will lag 90° behind E_1 or 90° ahead of the counter electromotive force E_c which it induces in S_1. The true magnetizing current I_m will be in phase with the flux and the hysteresis component of current I_H will be in phase with E_1. Hence the total no-load current I_{00} will assume some phase a little less than 90° behind E_1.

The flux ϕ will induce in the secondary turns S_2 an e.m.f. 90° behind the flux, hence 180° behind E_1 or in phase with the primary counter e.m.f. The effective value of this e.m.f. is

$$E_2 = 4.44 f S_2 \phi \, 10^{-8}.$$

Load on Secondary. — When the secondary is closed through a non-inductive external circuit a current will flow. This current will tend to demagnetize the iron, that is, to reduce the flux and hence the counter e.m.f. of the primary. This action, however, allows the current in the primary to increase until the secondary m.m.f. is balanced and there is left an excess m.m.f. in the primary to give sufficient flux to induce the counter e.m.f. E_1'. The vector relations are as shown in Fig. 5, again neglecting the leakage flux.

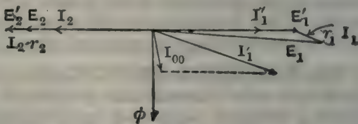


Fig. 5. Full Load, No Leakage

Leakage Reactance.— Due to the fact that a part of the lines of induction set up by the currents in the two windings pass through the air space (X in Fig. 3), the m.m.f.'s set up by the secondary current and the load current in

the primary (the current I_1' in Fig. 5) cannot neutralize each other, since the leakage fluxes established by the two currents are in the same direction, and not in opposition as are the fluxes in the iron. The primary leakage flux is in phase with the total primary current and the secondary leakage flux is in phase with the secondary current; these leakage fluxes are therefore not in phase with the useful flux (i.e., the flux which links both primary and secondary). The result is that the leakage fluxes cause a decrease in the secondary voltage and also a shifting of its phase with respect to the primary voltage.

Since the two leakage fluxes are in phase with the total currents in the primary and secondary respectively the voltages induced by the alternation of these fluxes are in quadrature with the currents. The quotient of the voltage induced in the primary by the alternating primary leakage flux divided by the primary current is called the "primary leakage reactance," and the quotient of the voltage induced in the secondary by the alternating secondary leakage flux divided by the secondary current is called the "secondary leakage reactance." These reactances are practically constants, since the major portion of the leakage path is in the air.

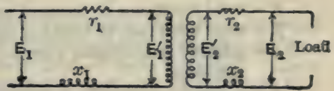


Fig. 6. Diagram of Circuits

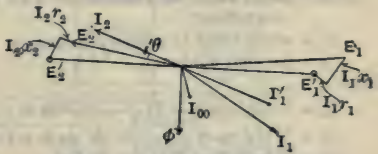


Fig. 7. Complete Vector Diagram

Complete Vector Diagram of Transformer.—Fig. 6 shows diagrammatically the primary and secondary windings of the transformer and Fig. 7 the vector diagram.

E_2 = the secondary terminal e.m.f.;
$\cos\theta$ = power factor of load;
I_2 = secondary current, lagging by angle θ behind E_2;
I_2r_2 = secondary resistance drop in volts; in phase with I_2;
I_2x_2 = secondary reactance drop in volts due to ½ of leakage flux, at 90° to I_2;
E_2' = induced e.m.f. in the secondary (hypothetical);
E_1' = e.m.f. necessary to overcome counter e.m.f. in primary; opposed and proportional to E_2' by ratio u;

u = ratio of transformation $= \dfrac{S_1}{S_2}$;

I_1' = primary load current; opposed and proportional to I_2 by ratio $\dfrac{1}{u}$;

ϕ = mutual or useful flux;
I_{00} = primary no-load current;
I_1 = total or resultant primary current;
I_1r_1 = primary resistance drop; parallel to I_1;
I_1x_1 = primary reactance drop, at 90° to I_1;
E_1 = the required voltage on the primary, that is, the impressed e.m.f.

"Equivalent" Circuit of Transformer. — In practice it is not possible to measure separately the primary and secondary reactances. They must be measured together and treated as one quantity, called the total reactance x_t of the transformer, which may be expressed in terms of either the primary turns or the secondary turns.

A simple approximate method which is sufficiently accurate for all practical purposes is based on the equivalent circuits shown in Fig. 8. This method is

used in calculating the characteristics of a transformer from constants obtained from tests.

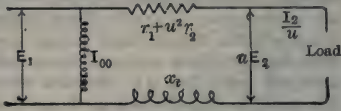


Fig. 8. Equivalent Circuit

Let E_2 = terminal e.m.f. of secondary;
$\cos\theta_2$ = power factor of load;
I_2 = secondary current;
u = ratio of transformation $= \frac{S_1}{S_2}$;
$E_2' = uE_2$;
$I_2' = \frac{I_2}{u}$;
R = total resistance in terms of primary $= r_1 + u^2 r_2$;
X = total reactance in terms of primary;
I_M = primary magnetizing current;
$I_H = \frac{\text{core-loss}}{uE_2}$, to a close approximation;
E_1 = primary impressed e.m.f.;
I_1 = total primary current;
$\cos\theta_1$ = power factor of transformer with load, i.e., power factor at primary terminals;
P_1 = watts input at primary terminals;
η = per cent efficiency.

Then

$$E_1 = \sqrt{(E_2'\cos\theta_2 + RI_1)^2 + (E_2'\sin\theta_2 + XI_1)^2};$$
$$I_1 = \sqrt{(I_2'\cos\theta_2 + I_H)^2 + (I_2'\sin\theta_2 + I_M)^2};$$
$$P_1 = E_2 I_2 \cos\theta_2 + R(I_2')^2 + E_2' I_H;$$
$$\cos\theta_1 = \frac{P_1}{E_1 I_1};$$
$$\eta = \frac{100\,P_2}{P_1};$$
$$P_2 = E_2 I_2 \cos\theta_2;$$
$$\text{Per cent regulation} = \frac{100\,(E_1 - uE_2)}{uE_2}.$$

Constant-current Transformers are used for supplying current to arc lamps connected in a series circuit which requires from 4 to 10 amperes and about 2000 volts. The principle of operation for this type of transformer is based on the existence of the leakage or useless flux which causes a repelling force between the primary and secondary windings. One winding is arranged so that it may move with respect to the other. As the current in the windings increases the repelling force increases and the windings move apart. This allows more flux to leak or pass between the windings and less to thread the secondary. Therefore the voltage induced in the secondary decreases as the current tends to increase. A counterweight is attached to the movable winding to regulate the amount of movement. The current in the primary remains fairly constant, but the power factor decreases with decreasing load on the secondary so the power in the primary is approximately proportional to the output of the secondary.

Series Transformers operate under conditions similar to those in a constant-potential transformer with short-circuited secondary. Under operating conditions the flux is small as the only voltage to be induced in the secondary is that required to overcome the impedance drop in that winding and the instrument which it supplies. The only counter e.m.f. in the primary is that due to impedance drop. The series transformer must be operated with the secondary

short circuited by a low impedance. If the secondary circuit is open there is no counter m.m.f. to balance the primary turns, which, being in series with the main line, carry a current irrespective of the secondary circuit. The primary ampere-turns would then set up a magnetic flux of considerable magnitude. As the transformer is not designed for this condition, the density would become very high in the magnetic circuit of small cross-section, and the transformer is liable to burn up from the heat due to core-loss.

TRANSFORMER CONNECTIONS. — The various transformer connections which are commonly used in lighting and power services are described below; see also section on *Operation* below.

Single-phase System with Three-wire Secondary (Fig. 9). — Standard practice in residence lighting with the a-c. system involves grounding the neutral wire on the low-tension side, the primary side not being grounded. Lamps or motors operating at 110 volts are connected between the neutral and either

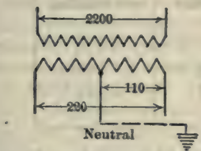


Fig. 9. Single-phase, Three-wire

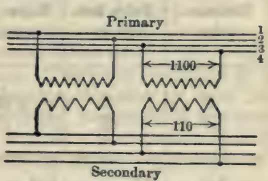


Fig. 10. Two-phase, Four-wire

side. The maximum potential between any secondary and ground is 110 volts but if either of the outside wires becomes grounded it constitutes a short-circuit on that half of the transformer.

Two-phase or Quarter-phase Four-wire System (Fig. 10).—The standard two-phase or quarter-phase system is essentially two independent single-phase systems which are usually independent electrically throughout. When the two phases are not electrically connected inside the generator, either wire of one phase may be connected to either wire of the other phase, without any flow of current resulting. In certain two-phase generators, however, the windings are interconnected, in which case any interconnection of the wires coming from the generator will cause a flow of current through this connection.

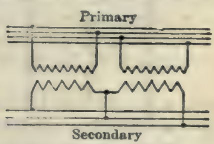


Fig. 11. Two-phase, Three-wire

Two-phase Three-wire System (Fig. 11). — This connection is occasionally used for the distribution of power in small systems. There is a possibility of a slight saving in copper, but the chances of unbalanced voltage and bad regulation, particularly with an inductive load, render it objectionable.

Three-phase Y and Δ Connections (Figs. 12 and 13). — Transformation in a three-phase system with either three independent single-phase transformers or with a three-phase transformer, having three primary coils and three secondary coils on one iron core, is accomplished by connecting the primary either in Y or in Δ and the secondary either in Y or in Δ. With Y or Δ connections there are the following relations between voltage per transformer winding and voltage between lines, and between current in transformer windings and current in lines. The power in the three transformers in any case is $3 \times 0.58 \times EI = 1.73\,EI$.

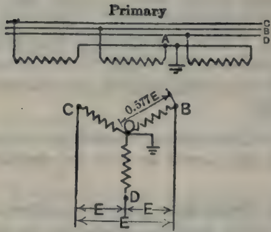


Fig. 12. Three-phase Y

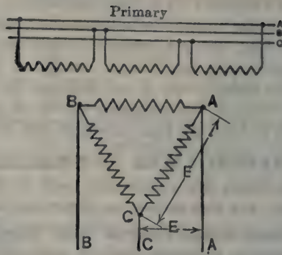


Fig. 13. Three-phase Delta

Connection	Volts bet. lines	Volts per winding	Current per line	Current per winding
Y	E	0.58 E	I	I
Δ	E	E	I	0.58 I

There are four combinations of these connections which may be used on any bank of transformers. These connections are given in the following table, and also the ratio of the voltage between lines for the low-tension and high-tension sides for a given ratio (u) of transformation in each individual transformer connection.

Connection		Low-potential volts bet. lines	High-potential volts bet. lines
Low pot.	High pot.		
Δ	Δ	E	uE
Y	Y	E	uE
Δ	Y	E	1.73 uE
Y	Δ	E	0.58 uE

Parallel Connection of Three-Phase Banks. — If there are several banks of transformers in the same system connected in parallel on one side then to connect the other sides in parallel the connections must be such that the voltage between any two lines on this side will have the same phase in all the banks. From this relation result the following rules:

With $\Delta\Delta$ on one bank, the other bank must be $\Delta\Delta$ or YY.
With YY on one bank, the other bank must be $\Delta\Delta$ or YY.
With ΔY on one bank, the other bank must be ΔY or $Y\Delta$.
With $Y\Delta$ on one bank, the other bank must be $Y\Delta$ or ΔY.

Even when these relations are satisfied a short-circuit will result unless the three phases of each bank are connected in the proper sequence. This can be

readily determined by the polarity test described below in the section on *Testing*.

Relative Advantages of Y and Δ Connections. — There has been much discussion as to the relative advantages of Y- and Δ-connected transformers for high-tension transmission and as to the value of a grounded neutral. The general and fundamental arguments may be summed up as follows:

With a $\Delta\Delta$-system the advantages are that if one transformer becomes disabled the system may be operated from the other two, operating on open delta (*see below*). If the load is unbalanced the voltages do not become unduly unbalanced. Resonance cannot occur. On the other hand, each transformer must be insulated for full line voltage as there is no neutral to ground, and if one line becomes grounded the voltage strain on the rest of the system becomes 1.73 times the normal strain, and this strain may extend to the low-tension winding and the generator which is connected to the transformers.

With a YY-connection there is a very unstable neutral and a possible excessive voltage strain on one phase, unless the neutral is grounded.

With a ΔY arrangement for step-up transformers, the neutral on the high-tension side may be grounded. The voltage strain on any transformer is then limited to 58 per cent of the line voltage. There is a possibility of operating the remaining two transformers, if one becomes damaged, by using the neutral as a third conductor. However, there is the possibility of resonance under certain circumstances and the danger of causing disturbances in nearby telephone and telegraph circuits. Any accidental ground on the system makes a definite short-circuit on one phase.

The arrangement of $Y\Delta$ for step-up transformers is not desirable on account of the unstable neutral with unbalanced load, but is permissible with a balanced load or with a good connection from neutral of transformers to neutral of generator. This connection, however, is frequently used for step-down transformers particularly when connected to a balanced load.

Grounded Neutral vs. Ungrounded Neutral. — In any system without a grounded neutral there are numerous possibilities of disturbances resulting in a high voltage strain on the various parts of the insulation of the system. With the high voltages now in use for transmission systems these disturbances may give a great deal of trouble by breaking down the insulation, as it is not always possible to employ a large margin of safety and lightning arresters do not always protect from these disturbances. On the other hand, if the neutral is grounded, most of these disturbances will merely result in an excessive current, and if the circuit breakers are installed in the proper places they will open the circuit, so that the only adverse result will be a temporary interruption of service.

The choice is then between a system with ungrounded neutral and a large margin of safety in the insulation, and a system with grounded neutral, moderate insulation and the possibility of occasional interruption of service.

Effect of Higher Harmonics. — Many alternators generate an e.m.f. whose wave shape contains higher harmonics (*see Wave Analysis*) and the magnetizing current of transformers operating at high magnetic densities has a distorted wave shape which contains a prominent third harmonic. If these harmonics are present in a Y-connected machine they cause the voltage between each line and the neutral to vary so that there exists an unstable neutral, and the voltage strain on any one phase is indeterminate. If the neutrals of two pieces of apparatus in which these harmonics exist are grounded or joined together, a high-frequency current will flow in the neutral connection. If on the other hand these pieces of apparatus are Δ-connected a third harmonic current will circulate inside the apparatus. This may be measured by con-

necting an ammeter in the Δ between two phases. This current heats the apparatus, does no profitable work, and is therefore to be avoided. High frequency current in the line will cause high voltages to occur at certain points if a large amount of capacity is present, as is the case in any underground line, even of moderate length, and also in long overhead lines. All possible means both in the design of the generators and the connection of the apparatus should be taken to prevent the occurrence of, or to diminish the effect of, these higher harmonics.

Three-phase Open Δ or V Connection (Fig. 14). — This system consists in omitting one transformer from the delta connection, and is used to save expense, particularly in temporary installations or in new installations where the load is not great at first but is expected to increase in time. Thus the

Fig. 14. Three-phase, Open Delta

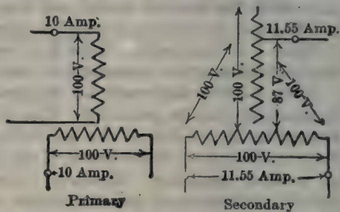


Fig. 15. Two-phase to Three-phase

purchase and installation of the third transformer is postponed until the load requires it. This connection can only be recommended for low voltages, such as 2300, as it is liable to produce dangerous potentials due to electrostatic unbalancing. The regulation and efficiency are also poor, as one phase of the load receives its power from two transformers in series. The aggregate capacity of the two transformers should be 15 per cent greater than the load.

Two-phase to Three-phase Transformation (Fig. 15). — The Scott or *T*-connection, the standard method of transforming from two-phase to three-phase, consists of two transformers which on the two-phase side may be connected in the normal two-phase manner either independently or interlinked. On the three-phase side one transformer has a tap at the middle point and the other a tap giving 87 per cent of full-transformer voltage. Fig. 15 shows the method of connecting and the currents in primary and secondary with balanced loads. Since the total power is $2\,E_2I_2 = \sqrt{3}\,E_3I_3$, it follows that $I_3 = \frac{2}{\sqrt{3}}\frac{E_2}{E_3}I_2$, or the three-phase current is 16 per cent greater than it would be for straight single- or two-phase transformation. Thus one of the windings of one transformer must carry 16 per cent more than its share of the volt-amperes of the load and is overloaded in that proportion.

In commercial practice it is customary for the sake of interchangeability to make the capacity of both transformers 16 per cent greater than the load and to put both a 50 per cent and 87 per cent tap on both transformers. For this connection of transformers each half of the main transformer winding must be distributed over both legs of the core in order to prevent flux (and therefore voltage) unbalancing. If this precaution is taken both the primary and secondary voltages are balanced if the load is balanced. Any unbalancing of the secondary causes a like amount of unbalancing on the primary; thus if this connection is used to transform from 3-phase to 2-phase and all the load comes on one phase of the secondary it will also come on one phase of the primary.

Transformer Connection for Synchronous Converters.—For a two-phase converter the connections are simple as shown under *Converters*. For a three-phase converter there is a choice between Δ or *Y* secondaries, which is usually decided in deference to the conditions in the high-tension line. The delta connection is customary in railway substations. A diagram of the connections is given in the article on *Converters, Synchronous*. When the converter is to supply a three-wire d-c. lighting system, the "distributed" *Y* connection for the secondary should be used as shown in Fig. 16. By this arrangement the unbalanced current from the d-c. neutral is divided, and sent through the two halves of each transformer in opposite directions. Thus it does not increase the magnetic densities in the transformers, and avoids the excessive core-loss which would occur with a simple *Y* connection.

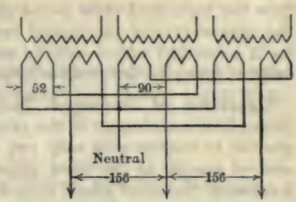


Fig. 16. Distributed Y for Three-wire Converter

For six-phase converters there is the choice between the "diametrical" and "double delta" connections as shown in Figs. 17 and 18. The diametrical is common with railway converters, or where the conditions are such that the delta is not needed to prevent an unstable neutral.

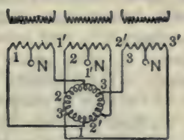


Fig. 17. Six-phase Diametrical

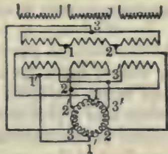


Fig. 18. Six-phase Delta

For regulating pole or split-pole converters certain special conditions must be borne in mind in choosing between the diametrical and the double-Δ connections. With the variation of the direct voltage of the converter the wave shape of the alternating e.m.f. is distorted and this introduces harmonics in either the counter e.m.f. or current waves, among which the third is quite prominent. If the delta connection is used a local harmonic current circulates in the converter armature and transformer secondaries, which increases the losses and heating and lowers the efficiency. If the diametrical connection is used the higher harmonic voltage gives an unstable neutral, and introduces extra potential strains in the high-tension windings and line if *Y*-connected. This may be obviated by grounding the neutral, but then a considerable current may flow in the neutral connection, which may interfere with neighboring circuits. The best solution is to use the diametrical connection in the secondary and ungrounded *Y* in primary, and use transformers having a good margin of safety in the insulation of the high-tension side.

THREE-PHASE TRANSFORMERS. — Considerable space, wiring and first cost may be saved by the use of a three-phase transformer in place of three single-phase transformers. This saving is warranted in large installations, but not in small installations where the convenience of having one interchangeable single-phase transformer as a spare for several banks of three transformers each is an important item.

Core of Three-Phase Transformer. — Three-phase transformers may be of the core type as shown in Fig. 1, in which case all the magnetic paths are of the same cross-section, or of the shell type shown in Fig. 2. In the latter type the horizontal cross pieces and outside vertical pieces may have one-half the cross-section of the central vertical core, providing the coil on the central leg is connected with a definite polarity with respect to the other two coils in order to have the fluxes differ in phase by 60 degrees.

Operation of Damaged Three-phase Transformer. — In a shell-type three-phase transformer if the primary and secondary are delta-connected it is possible to operate with only two phases in open delta at reduced capacity in case of trouble in the third phase of the system. This is accomplished by separating the third phase entirely from the system and short-circuiting both the primary and secondary windings.

DESIGN. — (*See also section below on Cooling of Transformers.*) The methods of design for single-phase and polyphase transformers are similar, the chief difference being that of the magnetic circuit. In the design of polyphase transformers each leg is treated as an independent single-phase transformer. The number of turns is adjusted to the voltage per phase and the cross-section of the conductors to the current per phase. All legs are wound alike and the phases are connected up as described in the preceding section.

The alternating-current transformer is the most efficient piece of electric apparatus. Its efficiency at full load is usually better than 95 per cent and frequently better than 98 per cent. For this reason it is very small in volume and light in weight for a given output compared to other pieces of electrical apparatus and hence the chief difficulty in design is the necessity of providing sufficient surface and a proper means to carry off the heat developed.

The difference in mechanical construction between the core and shell type is that the winding of the core type usually consists of two or more long spools (in which the length is considerably greater than the diameter) surrounding each leg of the transformer. The shell type consists usually of large flat "pancake" coils, laid alongside of each other on the central core with proper separating devices to permit ventilation. See Figs. 1 and 2.

The following discussion applies alike to both core and shell types except where specific mention to one or the other is made.

Preliminary Calculation of Main Dimensions. — Let

E_1 = primary voltage, effective value;
I_1 = primary full-load current, effective value;
E_2 = secondary voltage, effective value;
I_2 = secondary full-load current, effective value;
f = frequency in cycles per second;
S_1 = number of primary turns;
S_2 = number of secondary turns;
ϕ = maximum value of flux;
$C = \dfrac{\phi}{S_1 I_1}$ = ratio of total flux to full-load primary ampere-turns;
e = volts per turn, effective value;
B = maximum value of magnetic flux density in lines per square inch;
A = cross-section of core in square inches;
f_1 = space factor of winding space;
D = total section of copper in square inches;
U = current density in amperes per square inch;
σ = ampere-conductors per inch length of core.

Determination of Flux (ϕ) and Number of Primary Turns (S_1).— In order to estimate a desirable value of the flux to be used there is employed a factor C which is defined by the relation

$$C = \frac{\phi}{S_1 I_1}. \tag{1}$$

The proper value of this factor C depends upon the type, capacity and voltage of the transformer, and the relative weights of copper and iron.* Usual values are given in the accompanying table.

VALUES OF C

Form of transformer	Voltage	Core type Value of C	Shell type Value of C
Natural draft	0–6000	55–70	500–700
	6000 up	70–75	500–700
Oil-cooled	0–10,000	75–100	
	10,000 up	100–150	
Air-blast or water-cooled	0–10,000	110–160	600–1000
	10,000 up	160–240	600–1000

The relation between the effective value of the primary voltage and the maximum value of the flux is

$$E_1 = 4.44 f S_1 \phi \times 10^{-8}. \tag{2}$$

Inserting in equations (1) and (2) the proper values of the primary voltage E_1, the frequency f, the constant C and the primary full-load current I_1, and solving the two equations for ϕ and S_1, reasonable values for the flux ϕ and the number of primary turns S_1 are obtained.

In the preliminary estimation of the full-load primary current I_1 it is sufficiently accurate to ignore the efficiency and power factor and to take the current as the watts output divided by primary voltage.

Volts per Turn (e). — The above value of ϕ should be checked by finding the volts per turn,

$$e = 4.44 f \phi \times 10^{-8},$$

and comparing it with values used in practice. In large transformers a greater voltage between turns is permissible than in small transformers. The accompanying table for core-type transformers is based on the assumption that double-cotton-covered wire is used for the windings. With special insulation higher values may be used. For the shell type values three times as great are customary.

Kilowatt rating of transformer	Allowable volts per turn for core type
10	2.5
20	3.5
50	5.5
100	7
200	9
500	13
1000	18

Number of Secondary Turns (S_2). — Having determined in a preliminary way the number of primary turns, the number of turns in the secondary is $S_2 = \frac{E_2}{E_1} S_1$. It will considerably simplify the mechanical arangement of the coils if S_2 is divisible by 4. The nearest multiple of 4 is taken as the final value of S_2 and then S_1 and ϕ are readjusted to correspond to this value of S_2.

* A discussion of this factor C will be found in Arnold's *Transformers* and in S. P. Thompson's *Dynamo Electric Machinery*, Vol. II.

Cross-section of Core (A) **and Magnetic Flux Density** (B). — The core section is determined by the magnetic density which it is desirable to use, which in turn is determined by the core-loss. It is found that in average practice a loss of one watt per pound of iron can be dissipated without excessive rise in temperature. The corresponding flux densities in iron having a thickness of 14 mils are given in the following table. These values are approximate only, since the quality of the steel used is exceedingly variable. It should also be noted for a transformer intended for supplying a load of low power factor that the iron loss should be less than 1 watt per pound for the best distribution of material.

VALUES OF FLUX DENSITY

Size of transformer	Kind of steel	Lines per sq. in.	
		25 cycles	60 cycles
Small	Ordinary transformer sheet	50,000	40,000
Small	Special; silicon-steel	70,000	60,000
Large	Ordinary transformer sheet	75,000	65,000
Large	Special; silicon-steel	90,000	75,000

The proper cross-section of core in square inches is then

$$A = \frac{\phi}{B}.$$

In the core type this cross-section may be assumed as a square whose side is d. Since the core is made up of laminations whose effective length one way is 0.9 d, the value of d is

$$d = \sqrt{\frac{A}{0.9}}.$$

In the shell type this cross-section is a rectangle whose length is from 2 to 3 times its width.

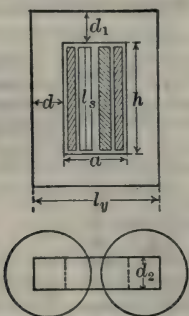


Fig. 19. Dimensions, Core Type

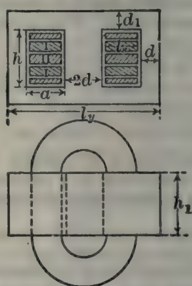


Fig. 20. Dimensions, Shell Type

Dimensions of Winding Space (Figs. 19 and 20). — The area or cross-section of the "window" or winding space depends upon the amount of copper,

insulation and ventilating space. The first can be determined accurately by the chosen current density in the copper. The second and third are allowed for by a space factor (f_1) which gives the ratio of the actual total cross-section of copper (D) to the cross-section of the window. Usual values of f_1 are given in the following table.

SPACE FACTOR IN TRANSFORMERS (f_1)

Size of transformer, Kw.	Core type		Shell type		
	Up to 10,000 volts	33,000 volts	Up to 2000 volts	2000 to 10,000 volts	33,000 volts
0–50	0.22	0.15	....	....	....
50–1000	0.33	0.25	0.40	0.33	0.18
Above 1000	0.38	0.33	0.45	0.36	0.21

In the core type there is one window and it contains $S_1I_1 + S_2I_2$ ampere conductors; in the shell type there are two windows and each contains $S_1I_1 + S_2I_2$ ampere conductors. The cross-section of copper in one window in either case is

$$D = \frac{S_1I_1 + S_2I_2}{U} = \frac{2\,S_1I_1}{U},$$

where U, the current density in amperes per square inch, has the following values:

Condition of transformer	U = amperes per sq. in.
Poorly cooled	850–1200
Ordinary oil cooled, air blast, etc.	1100–1600
Large, well cooled	1500–1900

Then the height h and width a of the window, see Figs. 19 and 20, must be such that

$$ha = \frac{D}{f_1}.$$

Height of Core. — In the core type this depends upon the ampere-turns or ampere-conductors per inch length of core, which in turn is related to the heating. The ampere-conductors per inch length of core is

$$\sigma = \frac{S_1I_1 + S_2I_2}{2\,h} = \frac{S_1I_1}{h},$$

where h is the height of core.

For a maximum rise in temperature of 50° C. σ will have values as given in the accompanying table. A reasonable value of h can then be found.

Form of transformer	σ = amp. conductors per inch of core
Natural cooled	500–750
Oil cooled or air blast	750–1250
Water cooled	1250–2000

In the shell-type transformer the practice is to make the height (h) from 1.5 to 3 times the width of window (a) and the width of window from 0.75 to 1.25 times the width of the central iron core, $2d$ in Fig. 20. See also paragraph above on *Cross-section of Core*.

Details of the Winding.—

Cross-section of primary conductor in square inches $= \dfrac{I_1}{U}$.

Cross-section of secondary conductor in square inches $= \dfrac{I_2}{U}$.

For minimum loss the copper density should be the same in both members, but to save space the copper density in the high-potential winding is sometimes greater than that in the low-potential winding.

The low-potential winding is usually placed between the high-potential winding and the iron.

Each coil is now laid out in detail, placing as many turns in a layer as the height and insulation will allow. The voltage per layer must not exceed 150 to 200 volts. A space of 10 mils is to be allowed for insulation between layers when insulated wire is used and 8 mils when insulated strip is used. The space between coils is from 0.04 to 0.30 inch, depending upon the voltage per coil. Space for the air or oil ducts is also provided. Each duct is from ⅛ to ⅜ inch wide and there should be one on each side of each coil. No part of the winding should be more than ⅜ inch from the surface of a duct.

Insulation of Windings.— The insulation of each turn usually consists of the cotton covering if the coils are wound with wire, or mica paper if the coils are wound with strip copper. This is proportioned to withstand the potential between turns as given above. The voltage between conductors on adjacent layers may be equal to twice the voltage per layer. To prevent breaking down between layers, a layer of Fuller board is used as a separator and this should project beyond the windings at the ends to prevent creepage. The maximum voltage between layers should be kept below 400 and to accomplish this it is customary to limit the voltage per coil to about 5000 volts. Between the windings and the core a layer of pressboard and sometimes wood is placed, while between primary and secondary windings a layer of pressboard and micanite may be used. For very high voltages the end turns of the high-tension winding for about 75 feet from the terminals is given a special insulation to withstand the sudden high potentials which occur when there is a sudden change in the potential applied to the transformer. These high potentials result from the distributed capacity of the transformer, between the high-tension winding and the core, frame and other winding.

Terminal Bushings. — At the point where the high potential leads pass through the case there is a very great dielectric stress which must be taken care of by the use of a proper kind of insulation and a proper disposition of the

insulation to prevent a concentration of the dielectric flux at a few points. For voltages below 40,000 a porcelain bushing is customarily used.

For higher voltages it is necessary to supply a large creepage distance and to have the surface submerged in oil to prevent corona effect. This is accomplished in one form of bushing, known as the "condenser type," by surrounding the terminal with layers of insulation and putting sheets of tinfoil between the layers. This arrangement is equivalent to a series of condensers. By properly proportioning the area of the successive layers of tinfoil the drop in potential across the insulation is kept uniform. The whole terminal is inclosed in an oil-filled casing. Another form of bushing known as the "oil-filled type" consists of a long cylinder of composition insulation which surrounds the lead and is filled with oil. The cylinder is divided into compartments to keep the oil properly distributed, and disks or collars project outward from the outside to increase the creepage distance.

End Coils of Shell Type. — With the flat coils customarily used in the shell-type transformer the subdivision of the windings is usually such that there is a half coil of the low-potential winding at each end of the winding space as in Fig. 20. This is to reduce the leakage flux. In order to further reduce the leakage flux all coils may be divided into halves with a ventilating duct between halves. The space between a primary and secondary coil is then reduced to that necessary for the insulation.

Adjustment of Core Dimensions. — After the final details of the windings are arranged the cross-section and length of core is finally settled. Sometimes the cross-section of core in the core type is made cruciform instead of square in order to use more effectively the area inside of circular coils.

PREDETERMINATION OF THE PERFORMANCE OF A TRANSFORMER. — From the above calculations a drawing to scale of the transformer may be laid out. The next step is to calculate its performance, i.e., to predetermine the values of the efficiency, regulation and temperature rise. This last feature is treated in the following section on the *Cooling of Transformers*.

Magnetizing Current (I_M). — The final flux density will probably differ a few per cent from the value assumed earlier in the calculation. The cross-section of core is usually proportioned to give the same magnetic density in all parts, as this condition gives minimum core-loss for a given weight of iron.

The mean length of path in the iron is measured or calculated. If H (found from the magnetization curve of the iron, see article on *Magnetic Properties of Iron*) is the ampere-turns per inch for the given density the magnetizing current of the transformer will be

$$I_M = \frac{H \times \text{(length path)}}{\sqrt{2}\, S_1}.$$

Sometimes it is desired to allow for the minute air gaps at the joints of the punchings. Arnold finds that under practical conditions each joint represents a gap of 0.002 inch. Thus if there are n joints (usually 4) there should be added to I_M an amount

$$\frac{0.313 \times 0.002\, Bn}{\sqrt{2}\, S_1}.$$

Core-loss. — The core-loss consists of hysteresis and eddy losses in the steel punchings. These losses have been considerably reduced in recent years

by improvements in the manufacture and quality of the steel (*see article on Magnetic Properties of Iron*).

$$\text{Core-loss} = k_1 f V \left(\frac{B}{1000}\right)^{1.6} 10^{-6} + k_2 V \left(t f \frac{B}{1000}\right)^2 10^{-5} \text{ watts,}$$

where f = frequency in cycles per second;
V = volume of iron in cubic inches;
B = magnetic density in lines per square inch;
t = thickness of laminations in inches;
k_1 = a constant, ranging from 8 in ordinary transformer steel to 4 in silicon-steel;
k_2 = a constant, ranging from 4 in ordinary transformer steel to 1.5 in silicon-steel.

The core-loss may be calculated more easily by the method and curves given in the article on *Magnetic Properties of Iron.*

Copper Loss. — The mean length of turn of both primary and secondary windings is obtained from a sketch to scale. The direct-current or ohmic resistance of each member at 60° C. is obtained by substituting the proper values in the formula

$$R = \frac{0.0093\, lS}{12{,}000\, an},$$

where l = mean length of turn of primary or secondary respectively in inches,
S = number of turns in series,
a = cross-section of conductor in square inches,
n = number of conductors or coils in multiple.

To allow for eddy currents caused by the leakage flux, the above resistance is increased by 15 per cent to give the effective resistances r_1 and r_2. The total copper loss is then $I_1^2 r_1 + I_2^2 r_2$.

Transformer Reactance. — As explained in the section above on *Transformer Principles*, the leakage of flux between the primary and secondary windings of a transformer causes a component of voltage in each member which is out of phase with the current; if the current lags very much this voltage may have a considerable component opposed to the useful voltage and cause a loss of voltage or poor regulation. It is, therefore, necessary in designing to estimate the amount of this flux and calculate the voltage it would produce. In practice empirical formulæ are usually used, but a logically deduced formula is desirable as it is more easily adapted to unusual cases.

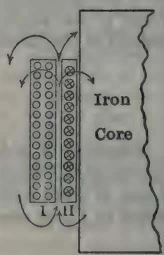


Fig. 21. Leakage Flux

The phase of the current in the secondary coil is nearly opposed to that in the primary and may be assumed to be exactly opposed without any great error. The result as shown in Fig. 21 is that all these ampere-conductors, both primary and secondary, tend to set up a flux in the same direction in the space between the windings and to a lesser degree in the windings themselves. A part of the flux in the intervening space interlinks with the primary turns and another part with the secondary turns. In addition the flux in the windings themselves interlinks with some of the turns. The flux in each part is proportional to the ampere turns producing it and to the permeance of the path. In this case since the path is in air, the permeance is the area divided by the length, where the length is the average length of the flux lines.

The leakage flux passes between the windings and one part closes its path in the iron inside the inner coil and the other part in air outside the outer coil (in the core type). The reluctance of the path between the two coils is large compared to that of the other two portions, because the inside path has a high permeability and the outside path has a large cross-section. The reluctance of the path between the coils is therefore accurately (and easily) calculated and a constant used to allow for the rest of the path.

In order to calculate the inductance it is necessary to have a cross-section of the windings showing their thickness, length and arrangement, as in Figs. 19 and 20.

Two cases must be considered:

(A) Where there are as many primary coils as secondary coils, and all coils are full size (usual core type).

(B) Where there is a "half" secondary coil at each extremity of the group of windings (the usual arrangement in the shell-type transformers).

Arnold's Method of Calculating Reactance. — Arnold calculates the inductance of a single primary coil and its secondary mate or mates, and multiplies this by the number of primary coils in series or divides by the number in multiple. This gives the total "short-circuit" inductance of the transformer in terms of the primary voltage or turns.

Referring to Figs. 19 and 20 for the core and the shell type respectively, let the various quantities be represented as follows, all dimensions being in inches:

q = number of primary coils in series,
p = number of primary coils in parallel,
s_1 = number of turns in one primary coil,
s_2 = number of turns in one whole secondary coil,
l_s = height of coils in cylinder type,
l_s = depth of coils in flat type,
t_1 = thickness of a whole primary coil,
t_2 = thickness of a whole secondary coil,
t = distance between coils,
U_1 and U_2 = mean length of primary and secondary turns respectively,
U_m = average of U_1 and U_2,
k = an empirical constant, varying from 0.95 with flat coils to 1.06 with cylinder coils.

Reactance of Core Type (Fig. 19). — In a core-type transformer with cylindrical coils and an equal number of full-sized primary and secondary coils, the permeance of the path of the flux in the duct which interlinks with a primary coil is $\frac{tU_1}{2\,l_s}$ and the permeance of the path in the winding itself is $\frac{t_1 U_1}{3\,l_s}$.

The inductance of one primary coil is

$$L_1 = \frac{3.2\,k s_1^2 U_1}{l_s}\left(\frac{t_1}{3} + \frac{t}{2}\right) 10^{-8}.$$

The inductance of one secondary coil is

$$L_2 = \frac{3.2\,k s_2^2 U_2}{l_s}\left(\frac{t_2}{3} + \frac{t}{2}\right) 10^{-8}.$$

Reducing the latter to terms of the primary turns by multiplying by $\frac{s_1^2}{s_2^2}$, adding together and multiplying by $2\pi f$, the total reactance of the transformer in terms of the primary is

$$X = 2\pi f\,\frac{3.2\,q k s_1^2 U_m}{p l_s}\left(\frac{t_1 + t_2}{3} + t\right) 10^{-8}.$$

Reactance of Shell Type. — If, as in the shell-type transformer shown in Fig. 20, there is a half secondary coil at each end to give an increased intermixing, then

$$X = 2\pi f \frac{3.2\, qks_1^2 U_m}{2\, pl_s}\left(\frac{t_1 + t_2}{6} + t\right) 10^{-8}.$$

Efficiency and Regulation. — The efficiency and regulation may be calculated as described above under "*Equivalent*" *Circuit of Transformer*, p. 1707, or by the following method, which is sufficiently accurate for most practical purposes.

Let

E_2 = secondary terminal voltage,
I_2 = secondary current,
$\cos\theta_2$ = power factor of load on secondary,
$P_2 = E_2 I_2 \cos\theta_2$ = secondary output in watts,
A = core-loss in watts = input for no load on secondary, approximately,
u = ratio of number of primary to number of secondary turns,
$R_2 = \frac{r_1}{u^2} + r_2$ = total resistance in terms of secondary, r_1 and r_2 being the actual resistances of primary and secondary respectively,
$X_2 = \frac{X}{u^2}$ = total reactance in terms of secondary, where X is the total reactance in terms of primary.

Then the per cent efficiency is

$$\eta = \frac{100\, P_2}{P_2 + A + R_2 I_2^2}.$$

Secondary voltage at no load is

$$E_{20} = \sqrt{(E_2\cos\theta_2 + R_2 I_2)^2 + (E_2 \sin\theta_2 + X_2 I_2)^2}$$

and the regulation is then

$$\frac{100\,(E_{20} - E_2)}{E_2} \text{ per cent.}$$

All-day Efficiency. — The all-day efficiency is the ratio that would exist between the readings of a watt-hour meter connected on the secondary and a similar meter on the primary. It is of importance because a great many transformers, particularly for lighting, operate at full load for only a few hours each day, but the core-loss or iron losses are just as great when the load is very light or when there is no load at all. Hence these transformers may waste a great deal of energy, although their efficiency at full load is very good.

To calculate the all-day efficiency multiply each quantity in the usual formula for efficiency by the number of hours per day that this factor occurs; thus

$$\text{All-day eff.} = \frac{100\, P_2 \times h}{P_2 \times h + h_1 A + h R_2 I_2^2} \text{ per cent,}$$

where h = hours per day of secondary load; h_1 = hours per day that transformer is on line, and the other symbols as above. Since the core-loss occurs 24 hours a day and P_2 only 2 or 3 hours, the importance of making the core-loss low in a transformer for this class of service is apparent.

However, there is another side to the question, since energy may not be worth as much during the day when there is little demand on the central station, as in the evening when the demand is great.

EXAMPLES OF DESIGN. — In the following table are given the chief design characteristics of four different transformers.

EXAMPLES OF SINGLE-PHASE TRANSFORMER DESIGN

American or foreign	Foreign	Foreign	American	American
Form (cooling)	Oil	Air	Oil	Air
Type	Core	Shell	Distributed	Shell
Frequency	50	50	60	25
Kv-a. rating	40	100	20	75
High-tension voltage	3120	2200	2200	2500
Low-tension voltage	230	110	220	320
C (design constant)	56	670	222	1550
High-tension Winding:				
Current at rating, amperes	12.8	45.5	9.1	30
Total turns in series	1408	180	640	219
Coils in series	8	4	2	3
Coils in multiple	1	1	1	1
Size of conductor, inches	d=0.183	0.79×0.059	0.105×0.09	0.34×0.08
Resistance at 25° C., ohms	1.13	0.24	1.43	0.492
Low-tension Winding:				
Total turns in series	104	9	64	28
Coils in series	4	1	4	4
Coils in multiple	1	3	1	1
Size of conductor, inches	0.59×0.13	0.87×0.1	2, each 0.23×0.155	6, each 0.34×0.09
Resistance at 25° C., ohms	0.009	0.00059	0.0204	0.0095
Flux density in core, kilolines per square inch	38.6	52	68	67.5
Core dimens., inches, length	(26 sq. in.)	5.1	4.6	8
Core dimens., inches, width	......	22.8	4.6	21
Window dimens., inches, height	35.5	5.9	9.2	5.5
Window dimens., inches, width	5.45	9.9	3.1	8
Magnetizing current, amperes	0.21	2.12	0.432	1.82
Core-loss, watts	490	1320	164	1520
No-load current, per cent	2.04	4.85	4.8	6.9
Core-loss, per cent	1.22	1.30	0.80	1.96
Primary, RI^2, per cent	0.50	0.52	0.59	0.57
Secondary, RI^2, per cent	0.68	0.48	0.81	0.67
Efficiency, per cent	97.6	97.7	97.8	96.8
Total XI drop, per cent	1	2	2.84	1.36

COOLING OF TRANSFORMERS. — It is necessary to keep the temperature of the various parts of a transformer within such limits that the materials of which it is constructed do not become damaged and deteriorate too rapidly. When subjected to too high a temperature the insulating materials disintegrate and lose their mechanical and dielectric strength, and the iron deteriorates in its magnetic qualities so that the core-loss becomes greater for a given density. This so-called ageing of the iron may cause an increase of as much as 50 per cent in the loss in the iron. The effect varies with the character of the iron and is practically negligible in silicon-steel.

Maximum Rise in Temperature.—The maximum rise in temperature above a room temperature of 25° C. at which the various materials of a transformer should be operated are given in the table.

Material	°C
Iron	70–75
Cotton	60
Paper	70
Mica, asbestos	90

To guard against this possibility of damage the Standardization Rules of the A.I.E.E. recommend that the windings shall not increase in temperature more than 50° C., as measured by resistance, above the surrounding air, and the other parts shall not increase more than 50° C. as measured by thermometer.

Means of Dissipating Heat.—The problem is to provide a path of low thermal resistance by which the heat energy may pass to the surrounding air. To accomplish this it is necessary, first, to provide sufficient surface in the subdivided transformer to transfer the heat to the cooling agent, air or oil, without too great a difference in temperature; second, to so subdivide the transformer that no part of the iron is more than one inch, and no part of the copper more than ⅜ inch, from a cooling surface; third, to provide a sufficient quantity of the cooling agent, air, oil or water to carry away the heat at the same rate as it is generated; and fourth, to provide sufficient surface on the containing case or tank to transfer the heat from the internal oil to the external air without too great a difference in temperature.

Calculation of Exposed Surface of Transformer.—The practical method of estimating the rise in temperature consists in calculating the drop in temperature in the successive media through which the heat must pass to be dissipated. The first step is to calculate the exposed surface of the transformer core, core ducts, coils and coil ducts. In accordance with design practice the surface in the ducts is only rated at half the effectiveness of the outside surface (*see Generators, Alternating-current*), and therefore only half the duct surface is included in the figure for "Effective Radiating Surface, A_t." Having found a value for A_t in square inches, the following criteria apply to the respective types.

Naturally-cooled Transformers should have from 4.75 to 5.3 square inches of surface (A_t) for each watt loss for 50° C. rise in temperature.

Oil-cooled Transformers.—(*See also Oil, Transformer.*) The transformer is submerged in oil in a tank so that the level of the oil is from 2 to 3 inches above the top of the transformer. The quantity of oil is from 6 to 10 pounds per kv-a. rating, or in small sizes 500 pounds per kiolwatt loss. There should be from 1.5 to 2.3 square inches of transformer surface (A_t) per watt loss. The oil ducts should be ¼ inch wide and run vertically. If the tank is smooth there should be from 4 to 8 square inches of tank surface (not including top or bottom) for each watt loss. For sizes greater than 25 kilowatts it is customary to use fluted or corrugated sides to the tank. In this case there should be from 6 to 10 square inches of radiating surface per watt loss because the air does not circulate as rapidly in the grooves as over a smooth surface and consequently radiation is poor.

For a rise of 50° C. of the transformer there is an average rise of 30° of the oil. The maximum rise of the oil is 1.3 to 1.5 times the average.

The rise in temperature of the windings by resistance or of the iron by thermometer is

$$T = \frac{2W}{A_t} + \frac{tW}{S} + t_1,$$

where W = total watts lost;

A_t = radiating surface of transformer in square inches;

S = radiating surface of tank in square inches;

t = from 160 to 200 with smooth tanks.

= 200 to 270 with corrugated tanks;

t_1 = maximum difference of temperature in oil, usually 40 to 45° C., depending upon circulation.

Air-blast Transformers. — This type of transformer is usually set over a large air duct in the floor which is supplied by a blower with air at a pressure of from ½ to 1 ounce (¾ to 1 ½ inches of water). The air enters the transformer at the bottom and is divided into two streams, one passing vertically through the windings and the other transversely through the iron. There is a damper or valve for each stream so that the proper amount of air is provided to each part independently. Ducts ½ inch wide are provided every 3 to 4 inches. For 50° rise the cooling surface of the transformer (A_t) should be from 1.5 to 2.3 square inches per watt loss. A liberal amount of air for 50° C. rise is 150 cubic feet per minute per kilowatt loss. The air is expected to rise from 15° to 20° C. in its passage through the transformer. The theoretical quantity of air in cubic feet per minute is

$$Q = \frac{1.65W}{T_a},$$

where T_a is the rise in temperature of the air, which is usually about half as great as the rise of the transformer.

Water-cooled Transformers. — The cooling coils are suspended in the tank in the oil near the top, usually above the transformer, and carry a continually circulating stream of water. It is customary to provide ⅙ gallon of water per minute for each kilowatt of loss and to allow the outgoing water to rise 25° C. above the incoming. Other requirements are from 1 to 1.5 square inches of surface (A_t) per watt loss for 50° C. rise and 1 square inch surface of water coils per watt loss. This assumes a temperature drop of 20 to 25° C. in the insulation and oil, 2 to 3° between oil and water and 20 to 25° in the water.

TESTING SINGLE-PHASE TRANSFORMERS. — The customary commercial tests on transformers and the best order of making them are: cold resistance, polarity, ratio and checking of taps, impedance, core-loss and exciting current, parallel run, heat run, insulation tests. The efficiency and regulation are calculated from the results of these tests.

Oil-cooled transformers should never be tested or subjected to potential unless they have been filled with oil from which all moisture has been removed.

Examples of test results are given below.

Cold-resistance Measurement. — The cold resistance must be very carefully made as it is used as a basis of calculating the temperature after the heat run. After the transformer has been standing in one place long enough for all of its parts to have reached the same temperature as the surrounding air, a direct current of from 10 to 15 per cent of the rated current of the coils is sent through the windings and the drop measured with a voltmeter. At the same time the temperature of the windings is measured by a thermometer.

Test of Polarity. — This test gives information necessary for connecting several tranformers in a bank and have them operate in parallel or on polyphase cicruits. Direct current is sent through one winding of a transformer and a d-c. voltmeter connected across the other winding. If the current is

stopped, the voltmeter will give a deflection either positive or negative. For similar connection and direction of current on all the transformers of a bank the deflection should be of the same sign. A small current should be used as otherwise the throw of the voltmeter needle may be sufficient to bend it.

Ratio of Turns. — With no load on a transformer the ratio of voltages is the same as the ratio of the turns. Thus, with a known voltage applied to the low-tension winding, the ratio of turns of the two main windings and of the sections between taps can be checked up by connecting across the other terminals another voltmeter of proper range (using a potential transformer if necessary).

Impedance Test. — This test is important in order to calculate the regulation and in order to determine whether transformers will run in parallel with each other. One winding of a transformer is short circuited and a voltmeter, ammeter and wattmeter are connected in the circuit of the other winding. A voltage of from 1 to 8 per cent of the rated voltage of this winding and of the proper frequency is impressed. The voltage is regulated so that readings are taken at values of current from 50 to 125 per cent of the rated current of the winding. The wattmeter reading will be in the neighborhood of 2 per cent of the rating of the transformer. The total impedance of the transformer will be $Z = \frac{E}{I}$. The total effective resistance will be R = Watts/I^2, which will include the effect of eddy currents, and the total reactance will be $X = \sqrt{Z^2 - R^2}$. This reactance cannot be separated into primary and secondary reactance. The results of this test are usually plotted in two curves, one between volts and amperes and the other between volts and watts.

Core-loss and Exciting Current Test.— The alternator supplying the power for this test should give a sinusoidal e.m.f. wave, as any distortion in the shape may cause a variation of from 5 to 10 per cent in the core-loss. A peaked wave gives a lower core-loss than a sine wave. For this test the high-tension winding is left open and rated voltage at the proper frequency is impressed on the low-tension winding. A voltmeter, ammeter and wattmeter are connected in the low-tension circuit, the voltmeter having a range including the rated voltage of the transformer and the ammeter a range of approximately 15 per cent of the rated current of the machine. Readings are taken with a voltage of from 50 per cent to 125 per cent of the rated voltage of the transformer. Both ampere-volt and watt-volt curves are then plotted. If extreme accuracy is desired, the RI^2 loss should be subtracted to give true core-loss.

Separation of Eddy and Hysteresis Losses. — In certain special cases it is desired to investigate the iron of a transformer by separating the hysteresis from the eddy loss. Let

W_1 = the core-loss at normal voltage and frequency.
W_2 = the core-loss at half voltage and half frequency.
W_e = eddy-current loss at normal voltage and frequency.
W_h = hysteresis loss at normal voltage and frequency.

Then

$$W_e = 2\,W_1 - 4\,W_2,$$
$$W_h = 4\,W_2 - W_1.$$

Parallel Run Test. — The test for polarity and ratio having shown nothing wrong in the transformers, they are connected two at a time, the

low-tension windings being connected in parallel to a generator and the high-tension windings in parallel with each other with an ammeter of about 10 per cent the capacity of the transformer connected in one lead between them. The voltage of the alternator is gradually increased from zero and the current in the ammeter noted. This current should not be greater than 5 per cent of the rated current of that winding at rated voltage. If a transformer has double windings in either member the same test should be made on these windings in parallel.

Heat Run. — The heat run is made at the rated load of the transformer and sometimes at an overload of 25 per cent or 50 per cent, depending upon the guarantees. The run may be made by connecting the transformer to a load such as a water rheostat (*q.v.*) but as there are other equally good methods, avoiding the waste of so much energy, this dead load is seldom used. The other methods require two or three transformers to be tested simultaneously.

Two Transformers "Bucking" (Fig. 22). — The low-tension windings of two transformers are connected in multiple to a voltage of rated value and frequency. The high-tension windings are connected up so that their e.m.f.'s oppose or "buck" each other, and are in series with an adjustable source of e.m.f., of rated frequency having a value of from 2 to 5 per cent of the rated voltage of the windings. This "loss-supply" may be either an alternator and transformer, or a potential regulator. The voltage of this source is adjusted so that full-load current circulates in the high-tension windings and thereby induces full-load current in the low-tension windings. It should be realized that although only 2 to 5 per cent of the rated voltage is needed to send the current through the primaries, yet each primary is generating its rated voltage and if there should be a ground anywhere a dangerous potential strain or shock might result.

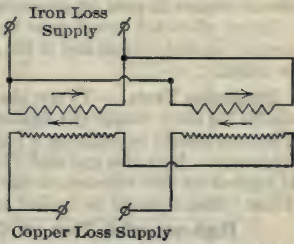


Fig. 22. Connections for Heat Run by Bucking

Three-transformer Arrangement (Fig. 23).— Three single-phase transformers may be connected with their low-tension windings in delta to a three-phase source of supply of proper voltage and with the high-tension windings also connected in delta with one corner open into which the "loss supply" is connected preferably by means of an auxiliary transformer to isolate the dangerous potential of the primaries. The "loss-supply" voltage must be adjusted so that the desired current flows in the primary winding.

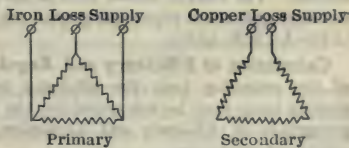


Fig. 23. Heat Run by Open-delta Method

Time for Heat Run. — The time required for a heat run may be considerably shortened by operating at an overload for a short while, or, if the transformer is an air-blast transformer, by operating without the blast until a reasonably high temperature has been attained. Then the proper load is adjusted and the run continued until all temperatures remain practically constant, that is, do not rise more than 1° C. in two hours. The voltage should be cut off once every hour and a resistance measurement quickly made to determine the temperature of the windings.

Temperature Rise by Resistance is calculated from the formula

$$T = (234.5 + t)\left(\frac{R_h}{R_c} - 1\right),$$

where T = temperature rise in ° C.;
t = temperature in ° C., by thermometer, at which the cold resistance is measured;
R_c = resistance in ohms at temperature t;
R_h = "hot" resistance in ohms at end of heat run.

The number 234.5 is the reciprocal of the resistivity temperature coefficient (referred to 0° C.), of 100 per cent conductivity copper, i.e., the reciprocal of 0.00427. For any other conductivity, C per cent say, divide 234.5 by $C/100$. (*See article on Resistance and Conductance.*)

Temperature Rise by Thermometer. — In an air-blast transformer it is desirable to measure by thermometer the temperature of the incoming air, of the outgoing air (from iron and coils), of the primary windings, and of the secondary windings. Spirit thermometers and not mercury thermometers should always be used for measuring the temperature of transformer windings.

In an oil-cooled transformer the temperature, by the thermometer, of the tank at top and bottom and of the oil in two or three places near the top should be taken.

Insulation Tests. — To test the insulation between turns and sections of coils double the rated voltage per section is impressed on each section for one minute. This test should be made at a high frequency, preferably at double the rated frequency. After this one and a half times normal voltage at rated frequency is applied for five minutes, to discover the effects of the double voltage test. These voltages should be applied and removed gradually.

High-potential Test on Complete Transformer. — To test the dielectric strength of the insulation as a whole the following high-potential test is made. Connect both terminals of the high-tension winding to one terminal of the high-potential transformer. Ground both ends of the secondary winding to the core and frame and connect to the other terminal of the high-potential transformer. Adjust a needle gap to arc at the desired test voltage and increase the voltage gradually until the gap arcs over. Decrease the voltage till the arcing ceases and hold the voltage as near as possible to the arcing point for one minute. The voltage should then be decreased gradually. The proper testing voltages and spark-gap adjustments are given in the Standardization Rules of the A.I.E.E. (q.v.).

Calculation of Efficiency and Regulation from Test. — From the results of the preceding tests the efficiency and regulation of the transformer at various loads may be calculated by the methods given above, p. 1625 in the paragraph on *Efficiency and Regulation*, using the test data instead of the calculated quantities.

TESTING OF THREE-PHASE TRANSFORMERS. — In testing three-phase transformers the same methods are followed as with single-phase units. The only difference is in testing for polarity; owing to the mixing of the magnetic circuits special care must be exercised. The direct current must be sent in one direction through one primary phase and in the opposite direction through the other two, so that they will not neutralize one another. With a voltmeter similarly connected on the primary and secondary of each phase in turn, break the direct current; if the connections are right the voltmeter on the secondary will deflect in the same direction as the steady deflection on the primary.

Parallel Run. — For the parallel run of two three-phase transformers, connect their low-potential sides in multiple to a source of three-phase potential. Connect together the primary terminals No. 1 of both transformers and bring the pairs of terminals No. 2 close together so they may be connected by a small fuse wire. If no spark is noticeable when the fuse wire spans the connection, then the No. 2 terminals may be permanently connected. The same procedure is followed with the No. 3 terminals.

EXAMPLES OF PERFORMANCE. — Usual values of the performance characteristics of transformers are given below.

Exciting Current ranges from 2 to 6 per cent of full-load current in lighting transformers of the core type and from 5 to 10 per cent in power transformers.

Core-loss ranges from 0.5 per cent to 1.25 per cent of the rated output.

Total I^2R ranges from 0.75 per cent in large sizes to 2 per cent in small sizes of the rated output.

Total Reactance drop ranges from 1.25 to 5 per cent of the voltage, being less for the shell type than for the core type. The value depends largely on the purpose of the transformer, methods of construction and opinion of the designer.

Efficiency and Regulation. — The variation of the efficiency with the load of two small 60-cycle transformers for lighting purposes is shown in Fig. 24. The

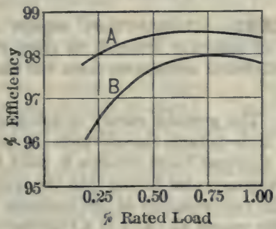


Fig. 24. Efficiency of 60-Cycle Transformers. A = 50 Kv-a.; B = 10 Kv-a.

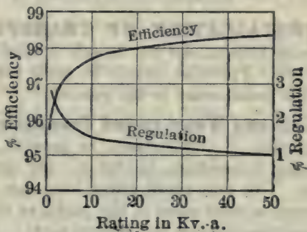


Fig. 25. Efficiency of a Line of 60-Cycle Transformers

efficiency and regulation at full load of a line of these transformers is shown in Fig. 25. These transformers are small and of the core type. Larger transformers would have even better characteristics.

See also above under *Examples of Design.*

SPECIFICATION FOR TRANSFORMERS.* — The following memoranda are intended to assist in writing specifications. See also article on *Specifications.*

Principal Characteristics and Conditions of Service. — Service for which transformer is to be used, e. g., operating synchronous converters, induction motors, lights, etc. High- and low-tension voltages at normal load. Taps for obtaining different voltages. Rating in kilovolt-amperes and in kilowatts at stated power factor. Frequency.

Style and Description: Details of Construction. — Whether oil-, air- or water-cooled. Style and location of terminals. Where line surges are likely to occur, it is usual to specify, for large transformers, that the end turns, say 10 per cent of the total turns at each end, shall have extra heavy insulation.

Work to be Done by Other Contractors. — Who is to supply and install floor framing and supports; high- and low-tension wiring, wiring and supports for delta- or star-connections, if to be used three-phase.

* By W. A. Del Mar.

Performance and Tests. — (*See Standardization Rules of the A.I.E.E.*) Temperature rises upon which ratings are to be based. Details of overload capacity. Efficiency at 25, 50, 75, 100, and 125 per cent load with stated transformation ratio. High-potential tests of insulation. Requirements regarding effect of moisture and heat on insulation, such as the following: The transformers shall contain no material which will be permanently injured by moisture or by an occasional temperature of 95° C., provided that this temperature is not maintained at any one time for a period greater than 3 hours. The transformers shall be capable of operating continuously at 80° C., without the insulation being damaged thereby. Regulation with rated non-inductive load. Regulation with load of rated kilovolt-amperes at stated power factors, say 100 per cent and 90 per cent. State formula by which regulation is to be calculated. Reactance between primary bus and secondary terminals when transformers are to operate compound-wound synchronous converters. (The required reactance is usually specified by the manufacturers of the synchronous converter.) Amount of air or water for cooling, in cubic feet per minute at stated pressure. After the transformer has been in service for one year, its efficiency at full load shall be not lower than the above guaranteed amount by more than a stated percentage and after two years its efficiency shall be not lower than the guaranteed amount by not more than a stated percentage.

INSTALLATION OF TRANSFORMERS. — Transformers require no special foundations but merely a level floor of sufficient strength to carry the weight. Provision should be made for an electrical connection from the tank of the transformer to the ground.

Oil- and Water-cooled Transformers. —The greatest enemy to successful operation of transformers in general, and high-voltage transformers in particular, is moisture, in that 0.1 per cent of moisture in oil renders it unfit for use. This may result either from rain or dripping water falling into the transformer or onto some of the parts, or from the condensation of the moisture in the atmosphere on the various parts. For this reason all parts of a transformer must be very carefully inspected, cleaned and dried before the transformer is put into service. If the operating potential is 15,000 volts or less an inspection will tell whether the transformer should be dried, but for voltages greater than 15,000 the drying operation should be carried out in every case.

Methods of Drying. — The best method of drying is to send a current of dry air at 90° C. through and around the transformer. This should continue for 24 hours in all transformers, for 72 hours in high-voltage transformers of reasonable capacity and longer in special cases. Another method of drying is to short-circuit one winding of the transformer and to apply to the other member a voltage of from 1 to 2 per cent of the rated voltage of that winding so that a current of from ⅕ to ⅓ of the rated value flows through the winding. This current should be adjusted so that a spirit thermometer placed on the low-tension coils shows a temperature of 80° C. and not greater.

Preparation of Oil. — No potential should be applied to any oil-cooled transformer unless it is supplied with a proper amount of oil as the oil forms the essential insulating medium as well as cooling medium. The oil should be tested before using. Oil is considered in good condition when it will withstand 40,000 volts between disks ½ inch in diameter and $\frac{2}{10}$ inch apart. Transformers for 40,000 volts or less will operate satisfactorily when this dielectric strength has dropped to 25,000 volts, but when this condition has been reached the oil should be dried, purified and strained. Transformers for a voltage greater than 40,000 should not be used with oil which breaks down under 35,000 volts in the above described test apparatus. A special apparatus for

drying and purifying the oil is on the market (*see G. E. Bulletin No. 4134*). See also article on *Oil, Transformer*.

The transformer tanks should be filled by pouring the oil through a fine-cloth strainer and allowing the oil to settle 12 hours before using. Rubber tubing should not be used for carrying the oil as the sulphur in the rubber will eventually cause trouble. The cover of the transformer should prevent any water dripping into the transformer but there should be a free exit allowed for gases which may gather. If the transformer stands in a moist atmosphere a special "breather" containing calcium chloride should be employed. In all transformers the level of the oil should be well above the top of the transformer proper and this should be noted 2 or 3 days after the original filling to see whether the transformer windings have absorbed sufficient of the oil to lower the level a dangerous amount.

Certain small sizes of oil-cooled transformers are designed to be suspended from cross arms on poles or the sides of buildings or to be installed in manholes in a subway. These transformers are especially protected against the weather.

Precaution Against Overheating. — It is most important to be assured of the proper circulation of the cooling medium as the temperature of the transformer will rise quickly to an excessive value in case of a failure of the cooling medium. In this case the load must be reduced and kept at such a value that the temperature of the oil at the top does not exceed 80° C. If at any time the oil reaches an excessive temperature there will be a tendency for a deposit to form on the transformer and coils, which will interfere with the proper cooling. An inspection should be made occasionally for this purpose and the deposit removed. The oil should be sampled and tested each week for the first month and every six months thereafter. In taking samples of the oil great care should be exercised that the vessel in which the sample is contained is perfectly dried. The temperature of the oil in a self-cooled transformer should never exceed 80° C. and in a water-cooled transformer 65° C. Oil-cooled transformers must be in a well-ventilated compartment in which the air should not be more than 5° above the outside atmosphere. An inspection should be made from time to time to see that there is no condensation on the inside walls of the tank.

Precaution for Multiple Operation. — Transformers should never be connected in multiple on both the primary and secondary sides unless it is known that the polarity is correct and that the ratio and regulation of the two transformers are the same.

Air-blast Transformers; Ducts and Blower Set. — Air-blast transformers should be placed over an air duct of sufficient size to permit the required current of air to flow at a velocity of less than 500 feet per minute. The duct should be of non-combustible material and should have smooth sides in order to offer very little resistance to the current of air. Air is supplied to this duct by means of motor-driven blowers of capacity to supply the proper quantity of air for a group of transformers at pressures from ½ to 1½ ounces (*see Blowers and Compressors; Fans*). Roughly the rating of the blower motor in horse-power is equal to

$$\frac{(\text{Cubic feet of air per minute}) \times (\text{pressure in ounces})}{1200}.$$

The air enters the transformer at the bottom, flows vertically through the coils and passes out at the top, and also flows transversely through the iron, passing out at the side. Separate dampers at each outlet are provided in order to regulate the two currents independently. The dampers are regulated so that the outgoing air has a temperature 20° above the incoming air.

Since the transformers are so dependent on the air blast for their operation and safety, it is customary to provide a reserve blower set.

Location. — Care must be exercised to protect the transformers from moisture and dirt and particularly, since they are open at the top, to place them where water and dirt cannot drop in the open top. All terminals of the air-blast type of transformer are usually brought out below so that the primary and secondary cables may be laid in the air duct and all connections and inspection may be made from below. It is therefore desirable that the air duct be sufficiently large to enable a man to move about therein.

Drying and Cleaning. — In putting the transformer in operation all moisture must be removed before applying potential. This may be accomplished by running the blower set and forcing air through the transformers, which will be sufficient if the air is dry. Another method is to short-circuit one winding of the transformer and apply a low voltage of from 1 to 2 per cent of the rated voltage to the other winding, so that 75 per cent of full-load current flows through the windings. This is maintained for from 24 to 36 hours. All transformers of this type should be cleaned once a month by means of compressed air at 20 pounds pressure.

Measurement of Temperature. — In determining the temperature of the windings only that calculated from a resistance measurement is dependable, as the coils are so thickly wrapped in insulation that a thermometer will not show the true temperature.

OPERATION OF TRANSFORMERS. — Single-phase transformers may be used singly or in parallel on single-phase circuits, and in various combinations on polyphase circuits as described above. The polarity should be determined before making the connections.

Parallel Operation on Single-phase Circuits. — Single-phase transformers are very generally operated with the primaries in parallel on the supply circuit and their secondaries in parallel on the load circuit. In order to successfully operate in this manner two or more transformers, the transformers must have: (1) the same ratio of transformation; (2) the same regulation; and (3) the same value of X/R, where R and X are the total resistance and reactance respectively of any one of the transformers. Transformers divide the load inversely proportional to their reactances, provided the ratio of reactance to resistance is the same in both cases. If transformers are purchased to operate in parallel with others, proper provision must be made in the design of the transformers, and this fact should be stated in the specifications.

In general, a 50-kilowatt transformer will have about twice the resistance and twice the reactance of a 100-kilowatt transformer of the same commercial line, and therefore these transformers will divide the load in proportion to their capacity. But this is not likely to be true of transformers of different lines or made at widely different times.

If transformers not satisfying the above conditions are to be run in parallel, an impedance coil having a proper resistance and reactance may be connected in the circuit of one of them, thus establishing the necessary conditions for the proper distribution of load, provided the ratios of transformation are the same.

Transformers on Polyphase Circuits. — See section above on *Transformer Connections*.

WEIGHTS AND COSTS (Pre-war prices).— The curves of Figs. 26 to 29 inclusive give approximate values of the weights and costs of several representative lines of single-phase transformers of the most usual common capacities, voltages and frequencies. These values are sufficiently accurate to be used in

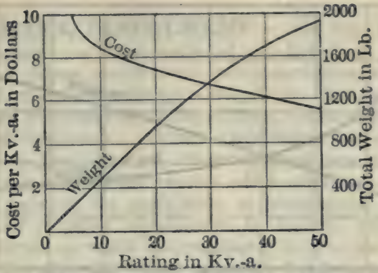


Fig. 26. Oil-cooled, 60-cycle, 2200-volt Lighting Transformers

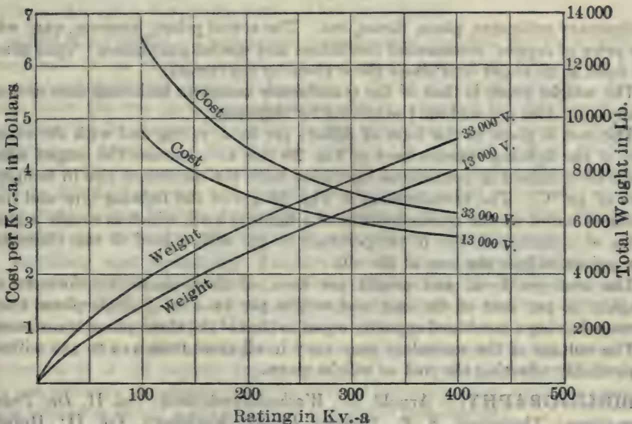


Fig. 27. Oil-cooled, 60-cycle, Single-phase Transformers

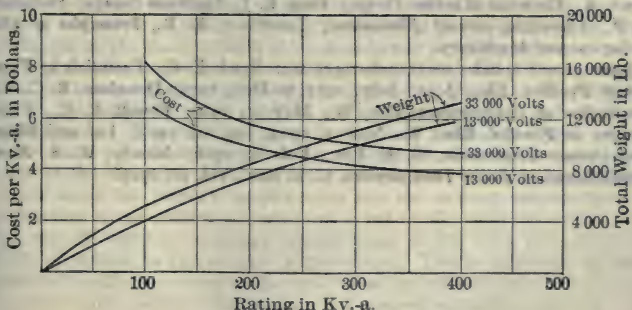


Fig. 28. Oil-cooled, 25-cycle, Single-phase Transformers

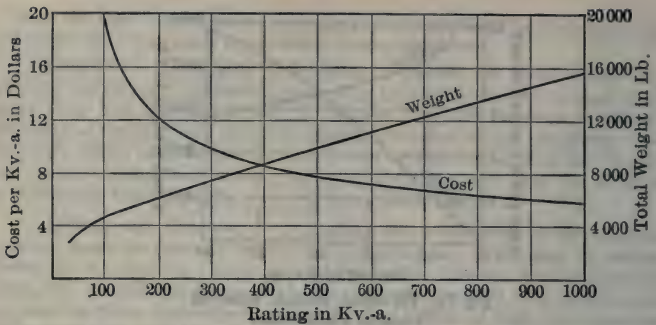


Fig. 29. Air-blast, 25-cycle, 33000-volt, Single-phase Transformers

preliminary estimates, plans, theses, etc. The actual prices, however, vary with the price of copper, commercial conditions and special guarantees. 1922 prices are from 40 to 50 per cent above those given by the curves.

The weight given is that of the transformer complete including case and oil, if any, but does not include the boxing for shipping.

The cost is given in the form of dollars per kv-a. rating and with the exception of the lighting transformers of Fig. 26 this kv-a. means the output which could be obtained in continuous operation with the maximum rise in temperature of 50° C. The transformers of Fig. 26 are of the lighting type and their rating is based on the characteristics of a lighting load, which is rougly a guarantee of a maximum rise in temperature of 50° with a load of this character. The prices include the cost of the oil.

The cost per kv-a. and weight per kv-a. of three-phase transformers are roughly 90 per cent of the cost and weight per kv-a. of 3 single-phase transformers having a combined capacity equal to that of the three-phase transformer.

The voltage of the secondary may vary in all cases from 110 to 440 without appreciably affecting the cost or weight given.

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Some recent articles on transformers are: Reed E. G. *Essentials of Transformer Practice*, Elec. Jour., July, 1917, to Dec., 1919; Dennison, B. C., *Cost comparison of Transformers*, Elec. W., May 31, 1919; Powell, R. C., *Transformers in Parallel*, Elec. W., Nov. 7, 1914, Jan. 16, 1915; Lof and Blume, *Transformer Connections*, G. E. Rev., Vol. 19, 1916; Moody, W. S., *Recent Developments in Power Transformers*, G. E. Rev., Vol. 22, 1919.

TRANSFORMERS, INSTRUMENT. — (*See also Transformers.*) Instrument transformers are either potential transformers or current transformers. Potential or voltage transformers are transformers of comparatively small output arranged for shunt connection to the primary lines, designed to produce a secondary voltage which represents the primary voltage with sufficient accuracy for application to instruments. Current transformers are transformers of comparatively small output arranged for series connection in the primary lines, designed to produce a secondary current which represents the primary current with sufficient accuracy for application to instruments. The insulation of both current and potential transformers is designed with reference to the voltage of the circuit on which the transformer is to be used. The most common voltage for the secondary of the potential transformer is 115 volts. The most common current for secondary full load on the current transformer is 5 amperes. The secondary voltage of the current transformer is usually very low under operating conditions, being only sufficient to force the secondary current through a few instruments of low impedance.

APPLICATIONS. — Instrument transformers are used for three principal purposes: (1) to supply current and voltage to measuring apparatus; (2) to operate regulating devices; (3) to operate circuit protective devices. In each case there are two principal advantages to be attained by the use of transformers: (1) to protect the devices in the secondary circuit against the inconvenience or danger of a direct application of the primary voltage or current; (2) to permit the use of measuring, regulating and protective devices designed for one standard current and voltage for the entire range of currents and voltages used under various operating conditions, thus simplifying design and manufacture, increasing reliability and accuracy and lowering cost.

Accuracy and Reliability Required. — Of the three applications given above, that involving measuring instruments, requires the highest accuracy in the transformer; in the operation of regulating devices the certainty of continuous operation is more important; in the operation of circuit protective devices, a very moderate degree of accuracy is satisfactory, but certainty of operation is of the highest importance. It is, therefore frequently desirable to use different transformers for the several purposes, even where convenience and cheapness would suggest the use of a single transformer.

THEORY OF CURRENT TRANSFORMER. — The current transformer consists of a primary winding in which the line current flows, a magnetic circuit, and a secondary winding for connection to a load of instruments and other devices connected in series. The current flowing in the primary is usually unaffected by the characteristics of the transformer or by the amount of secondary load. With the secondary winding short-circuited, the secondary ampere-turns are nearly equal to the primary ampere-turns, the slight difference being due to the relatively small exiciting current (*see Electricity and Magnetism, Principles of*). This exciting current is always of low power factor, while the power factor of the secondary current depends on the resistance and inductance of the secondary circuit. While the reversed secondary ampere-turns always equal the primary ampere-turns less the exciting ampere-turns, the subtraction of the exciting ampere-turns is not usually arithmetical but vectorial, and the resulting secondary ampere-turns are less than and not in exact phase opposition to the primary ampere-turns. This is expressed by stating that the transformer current ratio varies from the ratio of turns, and that there is a phase angle between the primary and reversed secondary currents. Since these errors depend on the relative amount of the exciting current and its phase position with respect to the secondary current, they will vary with the imped-

ance of the secondary load and the current flowing through it. When the secondary load impedance increases, the voltage across the secondary winding, and consequently the flux density in the magnetic circuit, grows larger. When the impedance becomes so great that the transformer magnetic circuit approaches saturation, the exciting current becomes so large that the secondary current is no longer proportional to the primary current and also differs from it appreciably both in wave form and phase position.

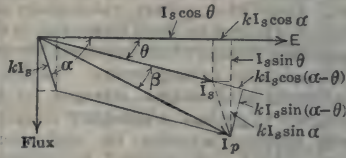

Fig. 1

Ratio and Phase Angle. — For accurate current measurements by means of a current transformer the exact ratio of the primary to the secondary current must be known. For power measurements the phase angle between the two currents must also be accurately known. Refer to Fig. 1 and let

u = ratio of the number of secondary to the number of primary turns,
I_p = primary current,
I_s = secondary current,
k = ratio of exciting current (referred to secondary) to the secondary current,
α = phase angle between the exciting current and primary induced e.m.f.
θ = phase angle between the secondary current and secondary induced e.m.f. (positive for lagging current).
β = the "phase angle" of the transformer, i.e., the angle between the primary current and the secondary current reversed. If the power factor of the secondary impedance load is higher than that of the exciting current, the secondary current (reversed) leads the primary current and β is positive; if the power factor of the secondary impedance load is lower than that of the exciting current, the secondary current (reversed) lags behind the primary current and β is negative.

Then the ratio of transformation is

$$\frac{I_p}{I_s} = u\sqrt{(\cos\theta + k\cos\alpha)^2 + (\sin\theta + k\sin\alpha)^2}$$

and the phase angle is

$$\beta = \tan^{-1}\frac{k\sin(\alpha-\theta)}{1 + k\cos(\alpha-\theta)}.$$

The value of the ratio k for a given primary current depends upon the impedance of the load connected to the secondary; the higher this impedance the greater the value of k and therefore increasing the impedance of the secondary connected load tends to increase the difference between the ratio of turns and the true ratio of the primary and secondary currents; increasing the impedance of the secondary also tends to increase the phase angle β of the transformer. The true ratio $\frac{I_p}{I_s}$ and the phase angle β also depend upon the power factor of the load.

In order to use these formulas for numerical results it is necessary to know the amount and power factor of the exciting current under the particular conditions of load. Since this is difficult to predetermine, it is customary to use curves similar to Figs. 2 to 5 below, obtained by plotting actual test results.

THEORY OF POTENTIAL TRANSFORMER. — The potential transformer consists of a primary winding, a magnetic circuit and a secondary winding.

The primary winding is placed across the line, and the secondary winding is connected to instruments or other devices connected in multiple. The voltage across the primary is usually unaffected by the characteristics of the transformer or of the secondary load. With the secondary open the secondary terminal voltage is approximately equal to the primary impressed voltage divided by the ratio of the number of turns in the primary to the number of turns in the secondary, the difference being due to the impedance drop produced in the primary by the exciting current. The impedance drop in the primary due

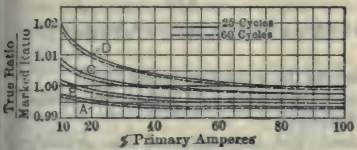

Fig. 2. Ratio

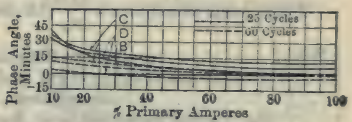

Fig. 3. Phase Angle

Current Transformer No. 1

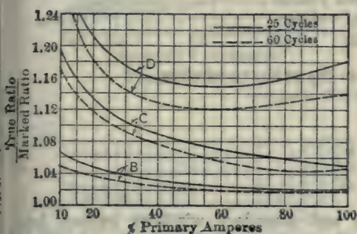

Fig. 4. Ratio

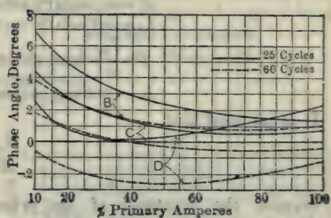

Fig. 5. Phase Angle

Current Transformer No. 2

to exciting current is usually not in phase with the primary voltage; hence its subtraction in a vector relation from the primary voltage results both in a variation of the voltage ratio from the ratio of turns and in a difference of phase between the primary and reversed secondary voltages. This no-load ratio and phase angle are modified under load conditions by the additional impedance drop in the primary and secondary windings due to the load current. The less the impedance of the load the greater will be the current through the two windings of the transformer, and therefore the greater the impedance drops in these windings; hence the greater will be the discrepancy between the actual ratio of the terminal voltages and the ratio of turns. As the impedance

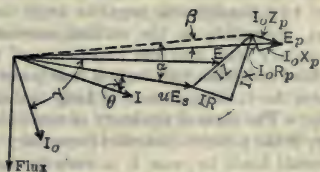

Fig. 6

drop due to the load current may be in almost any phase, whereas the drop due to exciting current bears a constant relation to the induced voltage. the phase angle with loads of various power factors may be either less than or greater than that at no load.

Ratio and Phase Angle. — For accurate voltage measurements by means of a potential transformer the exact ratio of the primary to the secondary

terminal voltages must be known. For power measurements the phase angle between the two voltages must also be accurately known.

The relations of the quantities involved are shown in the vector diagram, Fig. 6.

$$\text{Ratio} = \frac{E_p}{E_s} = u\left[1 + \frac{IR\cos\theta + IX\sin\theta}{uE_s} + \frac{(IR\sin\theta - IX\cos\theta)^2}{2u^2E_s^2} + \frac{I_0R_p\cos\gamma + I_0X_p\sin\gamma}{uE_s}\right].$$

$$\text{Phase angle in minutes} = \frac{3438}{E_p}[IR\sin\theta - IX\cos\theta + I_0R_p\sin\gamma - I_0X_p\cos\gamma].$$

Where u = ratio of the primary to the secondary turns;
R, X and Z = equivalent resistance, reactance and impedance of the transformer, respectively;
R_p, X_p and Z_p = resistance, reactance and impedance of the primary winding;
E_p = primary impressed voltage;
E_s = secondary terminal voltage;
E = secondary induced voltage;
I_s = secondary current;
$I = \frac{I_s}{u}$ = load current in the primary winding;
I_0 = exciting current.

It is assumed that,

$$\frac{R_p}{R} = \frac{X_p}{X}.$$

because the reactance of the primary and secondary windings cannot be directly determined. The third term in the expression for the ratio, within the brackets, is very small and can be omitted with very little error. For actual variations in the true ratio $\frac{E_p}{E_s}$ with the power factor, voltage and volt-amperes of the load on the secondary, see Figs. 7 and 9.

The phase angle ϕ of a potential transformer, i.e., the angle between the primary terminal voltage and the secondary terminal voltage reversed, may range from a positive angle (secondary voltage lagging behind primary voltage) to a considerable negative angle (secondary voltage leading primary voltage), depending on the exciting current, power factor and impedance of the secondary load and the impedance of the transformer windings. Under the no-load condition this angle is nearly always negative, and at high core densities where the exciting current is large and of very low power factor, it may be very large. The general tendency of non-inductive secondary loads is to cause ϕ to vary in the positive direction (secondary voltage to lag) and of inductive (lagging current) loads to cause it to vary in the negative direction (secondary voltage to lead). It is frequently possible to bring ϕ practically to zero for a single voltage and load by adding a non-inductive load in suitable amounts. See Figs. 8 and 10 for actual variations in the phase angle ϕ with the voltage, power factor and volt-amperes of the load on the secondary.

DESIGN. — The general principles of design of instrument transformers are the same as for distribution or power transformers. The special uses of the instrument transformers, however, change the details of design to a considerable degree.

Design of Current Transformers.—A very important application of current

transformers is the operation of protective devices. When so used the failure of a current transformer may cause destruction of expensive apparatus, and they are therefore designed to stand a higher insulation test than other types of transformers. Again, the exciting current of the current transformer has direct relation to the phase angle and errors of ratio. The flux density in the magnetic circuit is therefore kept very low for the ordinary metering conditions, in order to keep errors within satisfactory limits. The resistance and leakage reactance of the primary winding do not affect the ratio of the transformer nor the phase angle between primary and secondary currents; their only effect is on the total impedance in the primary line. Resistance and leakage reactance

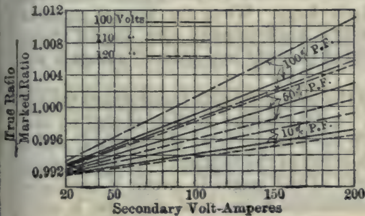

Fig. 7. Ratio

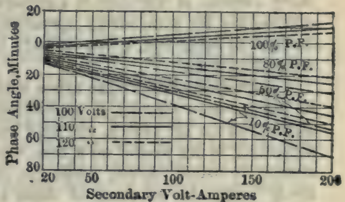

Fig. 8. Phase Angle

Potential Transformer No. 1

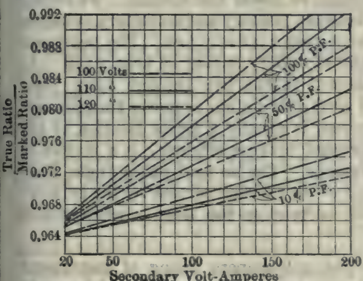

Fig. 9. Ratio

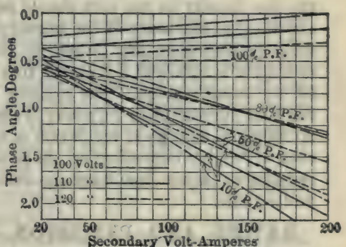

Fig. 10. Phase Angle

Potential Transformer No. 2

in the secondary or leakage between primary and secondary, constitute definite additions to the secondary load, and affect the phase angle and ratio of the transformer. As the current rating of a current transformer is always greater than the maximum continuous current to be carried by the lines of which it is a part, no considerable overload capacity is required.

A suitable design for current transformers, therefore, should include:

- High factor of safety in insulation,
- Low density of flux under the conditions of metering service (this is best secured by a moderate cross-section of magnetic circuit with a relatively large number of turns),
- Low resistance and leakage reactance in the secondary winding,
- Resistance and reactance of the primary winding low enough so that the impedance will not cause an objectionable drop in voltage in the primary lines, when the transformer is to be used in low-voltage circuits.

Special Designs of Current Transformers. — For certain special purposes special designs are useful. In stations where very large power is on the bus-bars while small power circuits, such as station auxiliaries, are drawn from the same bus-bars, current transformers must be used which are adapted to operate on small current, while under enormous extremes of current the transformers must at least remain in service long enough to operate the protective devices. Where the currents are not too low, transformers with a single copper bar as primary may be used. These resist mechanical injury from short-circuit, but the number of ampere-turns under the ordinary load conditions is limited to the number of amperes flowing in the primary line. Hence these transformers have an accuracy suitable for metering purposes only where the rated current of the line is comparatively large. When the normal currents are too low for satisfactory operation with a single primary turn, special transformers of large size, great primary current capacity and reduced accuracy are sometimes used.

A somewhat similar case arises where the cost of current transformers for use in high-tension circuits is prohibitive. A transformer is then placed on the bushing of an oil switch, the lead inside the bushing being the primary, and a secondary being wound on a core which encircles the bushing. As the primary current is usually small, while the length of the magnetic circuit is comparatively large, the accuracy of these transformers is low, and their use is ordinarily confined to tripping devices or specially calibrated ammeters.

Design of Potential Transformers. — The design of a potential transformer is very similar to that of a small distribution or power transformer. To obtain accuracy of ratio and small phase angle between primary and secondary voltages, the flux density in the core should be kept comparatively low. The resistance and leakage reactance of the primary and secondary should be kept small. As these transformers are ordinarily connected to the line through fuses, their insulation is sufficient if it equals that of good distribution or power transformers for the same voltage. The exciting current is usually a larger fraction of the full load current than in power transformers, because efficiency is a matter of no moment, and good regulation is necessary. The no-load phase angle, however, is roughly dependent on the product of the exciting current by the primary resistance; hence too large an exciting current or too large a resistance is undesirable.

RATING AND PRECISION. — The volt-ampere rating of instrument transformers as given by many manufacturers is purely formal, and has only the most distant relation to the characteristics of the transformer. There is no general correspondence between accuracies of transformers produced by various manufacturers for the same load. Where transformers are designed with oil immersion for high-voltage circuits, their quality is less likely to be poor than in the small types without oil, because the proportionate difference in cost of manufacture between good and poor transformers is then comparatively small.

Standard Voltage Ratings. — The following are the normal circuit voltages for which the manufacturers usually list separate lines of current transformers: 2300, 4600, 6900, 11,500, 23,00, 33,000, 44,000, 66,000.

The following are the voltages for which potential transformers are usually listed: 230, 460, 575, 2300, 4600, 6900, 11,500, 13 800, 23,000, 33,000, 44,000 66,000.

A transformer listed for one of these nominal voltage circuits, will ordinarily operate satisfactorily on lines ten per cent above the class voltage

Standard Current Ratios. — The current transformers are usually listed by manufacturers in the following current ratios, for each voltage class:

5 : 5	50 : 5	200 : 5	750 : 5	2500 : 5	7,500 : 5
10 : 5	75 : 5	300 : 5	1000 : 5	3000 : 5	10,000 : 5
15 : 5	100 : 5	400 : 5	1500 : 5	4000 : 5	
25 : 5	150 : 5	500 : 5	2000 : 5	5000 : 5	

Secondary Volt-Ampere Rating. — Current transformers intended for use with a single measuring instrument usually have a secondary rating of 10 volt-amperes, and for use with more than one measuring instrument a rating of 50 volt-amperes, at rated secondary current

Voltage transformers usually have a rating of 200 volt-amperes at rated secondary voltage.

Compensation. — By properly proportioning the number of turns in the winding of a current transformer, it is possible to raise the secondary current to overcome the ratio error, with a given condition of load. It is usual to compensate a current transformer for one-half its rated secondary capacity, with secondary power factor of 80 per cent and frequency of 60 cycles.

The actual ratio of turns in a voltage transformer differ from the marked ratio, by an amount sufficient to make up the voltage drop in the transformer at a specified load. Voltage transformers are usually compensated for one-fifth their volt-ampere rating, with a secondary power factor of 80 per cent, and at rated frequency.

The effect of phase displacement, however, cannot be compensated for. as it depends not only on the constants of the transformer itself, but on the power factor of the load on the voltage transformer, and on the power factor of the line current in the current transformer.

Precision of Current Transformer. — The variation of ratio and phase angle with the load on the secondary of two typical 2300-volt, 20 : 1 ratio, current transformers is shown in Figs. 2 to 5. Figs. 2 and 3 show curves of ratio and phase angle obtained from tests on a current transformer (No. 1) of low flux density, high ampere turns, and comparatively low secondary resistance and leakage reactance. Figs. 4 and 5 show the results of similar tests on a current transformer (No. 2) where these values are not so strictly limited. It should be noted that the scales of $\frac{\text{true ratio}}{\text{marked ratio}}$ and of phase angle in Figs. 4 and 5 are four times those used in Figs. 2 and 3.

Each pair of curves was made with a different load on the secondary, loads consisting of a series of combinations of instruments, watt-hour meters, and switchboard devices representing conditions in ordinary practice. Table I gives the volt-amperes and power factor of the loads, designated by the letters

TABLE I

LOADS REFERRED TO IN FIGS. 1 TO 4

Load	Volt-amperes, load at 60 cycles and 5 amperes	Power factor at 60 cycles and 5 amperes, per cent	Equivalent resistance, ohms	Inductance, milli-henries
A	4.8	99.8	0.192	0.032
B	10.9	90	0.392	0.504
C	45.1	54	0.972	4.03
D	132.4	38	2.012	13.0

A to *D*, at 60 cycles and 5 amperes and also the corresponding inductance and equivalent resistance components.

Precision of Potential Transformers. — The variation of ratio and phase angle with the power factor of the load on the secondary and with the secondary volt amperes of two typical 2300-volt, 23-cycle, 200 volt-ampere, 20 : 1 ratio, potential transformers is shown in Figs. 7 to 10. Figs. 7 and 8 show the curves of ratio and phase angle obtained in test on a potential transformer (No. 1) in which the flux density is low, and the resistances and leakage reactances of the windings are within moderate limits. Figs. 9 and 10 show the results of similar tests on a transformer (No. 2) of the same rating in which these quantities reach larger limits, resulting in inferior regulation and larger phase angle.

GENERAL TESTS DURING MANUFACTURE. — As the last step in manufacture the insulation of every current or potential transformer should be tested at a voltage greater than that at which it is to operate, the actual voltage selected being determined by the A.I.E.E. Standardization Rules. Some manufacturers find it desirable to apply considerably higher test voltages than these to insure additional safety. A sufficient number of each type should be given heating tests to assure safety of operation for the entire group. A check on accuracy should be made on all transformers to protect against errors in counting turns, short-circuits, etc.

TESTING OF CURRENT TRANSFORMERS FOR RATIO AND PHASE ANGLE. — Where large numbers of current transformers are to be tested to a high degree of accuracy for ratio and phase angle, those methods which use primary and secondary shunts, balancing the voltage drop through a zero reading instrument, are most rapid and satisfactory. The care and expense of such an outfit is justified only where a large number of transformers must be carefully tested. The phase angle may also be determined by the use of two wattmeters or electrodynamometers. Where transformers are in use and require an occasional check to determine that they have not changed in characteristics, a simple method which covers most cases is to compare the ratio with that of a standardized portable transformer. These methods are described below.

The uniformity among transformers of a certain type, make and size is usually very good; in the case of a large lot of transformers detailed tests need be made only on a few representative ones.

Demagnetization of Core Before Testing. — The cores of current transformers should be demagnetized before the test for ratio or phase angle is made. With small low-voltage current transformers demagnetization may be carried out by putting at least one-half-load primary current through the transformer with 10 ohms or more connected to the secondary in series with the instruments to be used. With large high-voltage current transformers having massive cores the resistance should be several times greater to assure perfect demagnetization. This resistance should then be gradually reduced to zero by steps of one ohm or less.

When however current transformers are subjected to test in order to use the results for the correction of observations already taken, the tests should be made in such a way as to avoid changing the magnetic condition of the core before the test results are secured. The secondary should have no greater load than that used in the working condition, and especial care should be taken that current is not allowed to flow in the primary or the secondary while the other winding is open circuited. The current should be brought up to the lowest current point first, and raised to the higher points only after the lower readings have been made.

Frequency of Test Current. — In all current-transformer work where results of test are to be used for correction, the frequency used in test should be the same as that of the circuit in which the transformer is to be employed. When the intention is only to check the condition of the transformer, any commercial frequency may be used.

Shunt Methods of Testing Current Transformers. — The following is a description of the details of one of these methods which has been found satisfactory in use.

Apparatus. — Referring to Fig. 11 a three-phase supply is used to excite a phase-shifting transformer with a single-phase secondary. The supply of current to the transformer under test is obtained from one phase of the same source through a control resistance R_1 and a step-up current transformer. for small ratios the step-up transformer is omitted. Its input side is wound to use up to the voltage of the supply (125 volts). The high-current winding consists of heavy loose cable, of which one or more turns may be wound through the large core opening. By suitably selecting the number of turns, currents from 10 to 4000 amperes are obtained. The primary shunt has a resistance such that full rated current gives a drop of 0.5 volt. It is arranged to carry

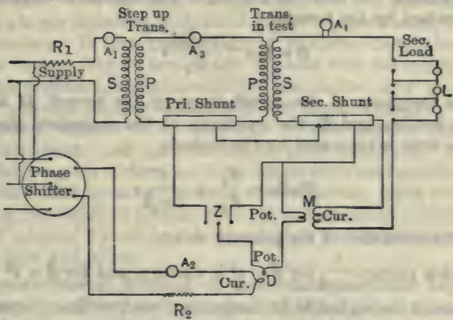

Fig. 11. Connections for Shunt Method

50 per cent overload current for a short time for testing purposes. The secondary shunt is adjustable, and is marked in percentage of a normal resistance. By the use of several interchangeable scales the normal resistance corresponding to 100 per cent may be made 0.06, 0.075, 0.08, 0.12, or 0.125 ohm. These allow the use of a primary 100-ampere shunt of 0.005 ohm resistance for transformers of 12: 1, 15: 1, 16: 1, 20: 1, 24: 1 and 25: 1 rated ratios, while still obtaining a direct reading on the secondary scale of the $\frac{\text{true ratio}}{\text{marked ratio}}$ of the transformer. A set of primary shunts rated 5, 10, 20, 50, 100, 200, 500, 1000, 2000 and 5000 amperes will thus cover all probable ratios from 1 : 1 to 1250 : 1. *M* is an adjustable calibrated mutual inductance whose current coil is in series with the secondary of the transformer under test, and whose potential coil is in series with the potential element o the sensitive electrodynamometer *D*, whose current element in turn is supplied from the secondary of the phase-shifting transformer mentioned above. *Z* is a double-throw single-pole switch. *L* is a series of secondary loads suitable to represent the instrument combinations usually found in practice.

Procedure. — The proper load is connected to the secondary of the transformer, and the primary current is adjusted to the proper point by means of the resistance R_1. This current may be read by an ammeter at A_3, or by an ammeter at A_4 which is short-circuited after reading. The switch Z is closed to the right. The mutual inductance then acts as an air-core transformer, supplying to the dynamometer potential element a voltage in quadrature with the secondary current of the transformer under test. The phase of the field current of the dynamometer is then adjusted to the point where the dynamometer reads zero for all positions of the mutual inductance. This is the position of maximum sensibility to changes in the resistance of the secondary shunt. The switch Z is then thrown to the left, and the resistance drop of the secondary shunt is adjusted by moving the drop contact until the dynamometer reads zero. The phase of the dynamometer field current is then shifted through 90 electrical degrees by moving the handle of the phase-shifting transformer a definite distance. This is the position of maximum sensibility to phase-angle variation. The mutual inductance is then adjusted until the dynamometer reads zero. The phase of the field current is then shifted back to the previous position, and a slight readjustment of the secondary shunt resistance made if necessary.

Calculations. — From the reading of the mutual inductance the phase angle is determined by the formula

$$\beta = \tan^{-1} \frac{2\pi f M}{R_s},$$

where f is the frequency of the circuit in cycles per second, M is the mutual inductance in henries, and R_s the resistance of the secondary shunt in ohms. Then the true ratio is expressed by the formula

$$\text{Ratio} = \frac{R_s}{R_p} \times \frac{1}{\cos\beta},$$

where R_p is the resistance of the primary shunt.

The correction factor $\frac{1}{\cos\beta}$ for $\beta = 1^0$ is 1.00015, and for $\beta = 2^0$ it is 1.0006. At these values it is negligible in comparison with ordinary errors arising in measurement work involving transformers and instruments. If the phase angle is so great that this correction becomes important, the transformer is so inferior that it should not be used for any purpose involving accurate measurement.

Speed of Operation. — By this means a single operator can take about 30 points of ratio and the same number of phase angle per hour. With a helper to perform necessary calculations and a suitable means of rapidly replacing transformers in the circuit, it is quite practicable to make from 15 to 30 tests of transformers including from 6 to 12 points each of ratio and the same number of determinations of phase angle in an eight-hour day. Checks of single points can be made practically as fast as the transformers can be connected and the current and secondary load adjusted.

Standardized Transformer Method for Determining Ratio of Current Transformer. — A portable transformer with four primary coils arranged for series-multiple connection, giving ratios of 5 : 1, 10 : 1, and 20 : 1 will cover practically everything up to 100 amperes. A second portable transformer without primary, so arranged that one turn through the core gives a rated ratio of 200: 1, and more turns a correspondingly lower ratio, will cover from 100 to 1000 amperes. In testing, the primary windings of the transformer

under test and the standardized portable transformer are connected in series. Either ammeters, wattmeters or watthour meters may be used as indicating instruments, the last being susceptible of giving the higher precision.

Use of Ammeters with Standardized Transformer. — The secondary of the standardized transformer is connected to a standardized 5-ampere ammeter, and that of the transformer under test to a similar ammeter and a suitable secondary load. With a sufficient current flowing to give satisfactory readings on the ammeters, readings are made; the ammeters are interchanged, and readings are repeated, to eliminate difference between the instruments. From the mean of the results and the ratio of the standardized transformer, the ratio of the transformer under test is obtained.

Use of Wattmeters with Standardized Transformer. — The two transformer secondaries are connected to the current coils of the wattmeters and the potential elements are placed in multiple and supplied with a definite voltage from a phase-shifting transformer. The phase shifter is adjusted at each point to cause the wattmeter connected to the standardized transformer to read a maximum. The instruments then are read as ammeters, using a calibration made with the same voltage applied to the potential element. By this means a larger deflection is obtained for low-current values, and consequently greater accuracy. If only transformers of moderate-current rating are tested, so that the current may be kept nearly in phase with the supply voltage by the use of series resistance, the phase-shifting transformer may be omitted and the wattmeter potential circuits excited from the supply voltage.

This method requires little apparatus or care beyond that necessary for keeping instruments and transformers in good condition. It does not test phase angle, but gives a check on ratio, whose error may readily be kept within 0.3 per cent. This includes the error of comparison, using ordinary care, and the error of the standard transformer. It is reasonably convenient and rapid, and is well adapted for use in the laboratories maintained by most companies supplying light and power.

Two-dynamometer Method of Determining Phase Angle of Current Transformers. — Phase angle may be determined (where primary currents are not too great) by the use of similar types of dynamometer wattmeters whose current elements are connected in the primary and secondary circuits, and whose potential elements are supplied in multiple by a phase-shifting transformer. The phase of the voltage is shifted until the primary wattmeter indicates zero, showing a quadrature relation of its current and voltage. If W is the watt reading of the secondary wattmeter, E the voltage applied to its potential element, and I the current in its current coil

$$\beta = \sin^{-1}\frac{W}{EI}.$$

The sensibility of this method is very low with ordinary portable wattmeters, but quite satisfactory with sensitive reflecting dynamometers. It is the best method in use for the accurate determination of phase angle.

Two Watt-hour Meter Method of Determining Ratio and Phase Angle of Current Transformers. — This method is used by many operating companies in checking their current transformers. It consists of comparing the transformer under test with another of known characteristics. The advantages of the method are:

1. Little or no special apparatus required.
2. All apparatus is simple and rugged.

3. No special skill required to operate.
4. Transformers may be tested where they are installed.
5. Ordinary variations of voltage, frequency and load during the test do not affect the results.

Apparatus.—The primaries of the standard transformer (S) and the transformer (T) being tested, are connected in series with a variable resistance, as shown in Fig. 12. The secondaries are connected through the desired load

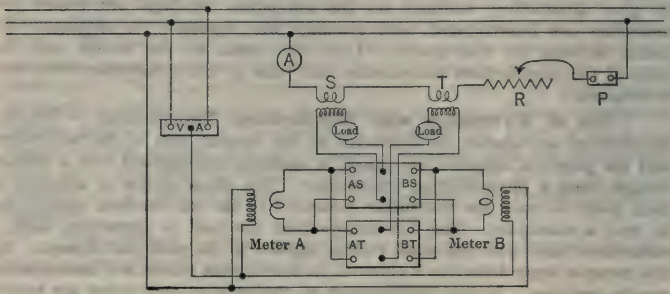

Fig. 12

to the current coils of the two watt-hour meters which are calibrated so that they can be read to 1/100 of a revolution. Ordinary portable standard watt-hour meters are generally used. In making the ratio test the voltage coils of the watt-hour meters are excited from the same phase which supplies the primaries of the current transformers, and in making the phase angle test, from one of the other two phases.

Procedure.—The double pole double throw switches are thrown to AS and BT and loads of the desired value are connected in the secondary circuits of the two transformers. Switch P is then closed and the primary current adjusted to the desired value. The switch V is now closed and the meter allowed to run until a reading is obtained. Then the switches V and P are opened, and the other switches thrown to AT and BS. Another reading is taken with switches V and P closed. For convenience in calculation, the meters should be stopped when the one connected to T shows an even reading, say 10,000 divisions. These readings give the data for calculating the ratio. For the phase angle measurement the same procedure is followed, except that the voltage switch is thrown to A instead of V, and the meters should be stopped when the one connected to S shows an even reading.

Calculations.—

Let R_S, R_T = ratio of the standard transformer and the transformer under test respectively;

α_S, α_T = the respective phase angles (positive for reversed secondary leading primary);

a_S, a_T = reading on meter A when connected to transformer S and T respectively;

b_S, b_T = ditto for meter B;

$\cos\theta$ = effective power factor on the meter,

= 0.866 for the connection shown, with the voltage switch thrown to A.

Then

$$R_T = R_s\sqrt{\frac{a_s\,b_s}{a_T b_T}},$$

$$\frac{R_T - R_s}{R_s} = \frac{1}{2}\left[\frac{a_s - a_T}{a_T} + \frac{b_s - b_T}{b_T}\right] \text{ (approximately)},$$

where the a's and b's are readings taken with the voltage switch thrown to V.

Also,

$$\tan\alpha_T - \tan\alpha_s = \frac{1}{2\tan\theta}\left[1 - \frac{a_T b_T R_T^2}{a_s b_s R_s^2}\right],$$

or

$$\alpha_T \text{ (in minutes)} = \alpha_s \pm \frac{3438}{\tan\theta}\left[\frac{a_s - a_T}{2a_s} + \frac{b_s - b_T}{2b_s} - \frac{R_T - R_s}{R_s}\right] \text{ (approximately)},$$

where the a's and b's are readings taken with the voltage switch thrown to A.

In the last expression the + sign before the bracket should be used when the meters, with the voltage switch thrown to A, are working on leading current, and the − sign when the current is lagging. Or the sign may be determined experimentally from the fact that adding non-inductive resistance to the secondary of a current transformer tends to advance the phase of the secondary current.

Speed of Operation. — An observer who is accustomed to this method can take one point of ratio and one of phase angle in about ten minutes. If the check is being made where the transformer is installed, additional time is required to cut the apparatus into the circuit and out again.

Other Methods of Testing Current Transformers. — Many modifications of the above methods and other entirely different methods are in more or less satisfactory use in various laboratories.

TESTING POTENTIAL TRANSFORMERS FOR RATIO AND PHASE ANGLE. — Where a large number of potential transformers are to be tested, the potentiometer method of testing ratio is very satisfactory. In this method the low-tension voltage is balanced against a part of the drop through a large resistance in which current is maintained by the high-tension voltage. This may be combined with the two-dynamometer method of testing phase angle between primary and secondary voltages. Where the number of transformers to be tested is not large, and where only a few ratios are to be tested they may be compared with standardized transformers.

Potentiometer Method of Testing Potential Transformers. — The following is a description of the apparatus and method of procedure which has given satisfactory results.

Apparatus. — The general connections of such an outfit are shown in Fig. 13. The supply is 3-phase for the excitation of the phase-shifting transformer P. One phase is connected through a voltage regulator Q to the low-tension side of the step-up transformer T_1, which has a sufficient number of

connections to provide the range of voltages required for testing. This supply excites the high-tension side of the transformer T_2 which is under test. r_2 and r_1 are placed across the high-tension circuit in multiple with the high-ten-

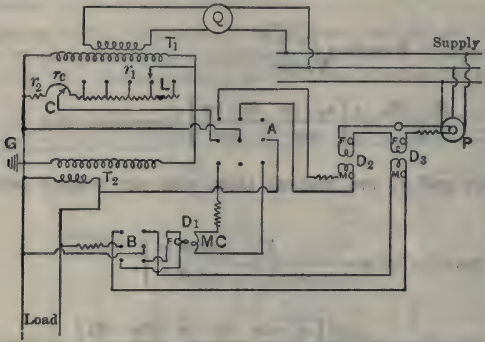

Fig. 13. Connections for Potentiometer Method

sion side of the transformer under test. They consist of non-inductive resistances, containing a non-adjustable portion constituting the greater part of r_2, a middle portion r_c subdivided into equal steps on which a contactor C travels, and a third part constituting the greater part of r_1 on which taps are brought out at points suited to the various voltages to be used in test. One side of the secondary winding is connected to the end of r_2 and the ground G; the other is connected (for ratio test) through switch A, which is in the downward position, through a resistance and the moving element of the dynamometer D_1 to the contactor C. By suitably proportioning the resistances r_1, r_2 and the steps of r_c, the outfit is made direct reading in terms of $\frac{\text{true ratio}}{\text{marked ratio}}$ of the transformer.

Precaution. — Balance can only be secured by a proper connection with regard to polarity of the secondary of the transformer under test to the primary. If the reversed connection is used, double the secondary voltage is applied to the circuit through contactor and dynamometer. A sufficient resistance to protect the dynamometer should therefore be inserted when trying out each transformer to be tested. As soon as an approximate balance is obtained, this resistance may be reduced to secure sensitiveness.

Determination of Ratio. — The fixed coil of the dynamometer D_1 is excited from the low-tension terminals of the transformer under test through the switch B in the downward position. Other secondary load may also be applied to the secondary terminals. With this load and the voltage and frequency suitably adjusted, and with the high-tension lead L connected to the proper tap on r_1, the contactor C is moved until the dynamometer D_1 indicates nearly zero. If the resistance at which the dynamometer indicates zero lies between two steps of the contactor, the nearest may be taken or the readings of dynamometer on both steps may be noted and the result obtained by interpolation. If r_1 and r_2 are respectively the resistances from the contactor C to the high-tension and to the grounded lines, then

$$\text{True ratio} = \frac{r_1 + r_2}{r_2},$$

subject to a small error due to phase angle between primary and secondary voltages, which is negligible for a phase angle of 2 degrees or less, provided

the resistance of the circuit from the contactor C through the dynamometer is approximately equal to r_2.

Determination of Phase Angle. — To obtain the phase angle the switches A and B are thrown upward. The drop of primary voltage across r_2 is thus applied to the potential element of dynamometer D_2, while the secondary voltage is applied to the potential element of dynamometer D_3, both through large non-inductive resistances. The current elements of the dynamometers are placed in series and excited from the phase-shifting transformer P. When the phase of the excitation is shifted so that dynamometer D_2 indicates zero, the phase angle between primary and secondary voltages is

$$\gamma = \sin^{-1} \frac{W}{VA},$$

where W is the watts indicated by the dynamometer D_3, V is the voltage and A the current applied to its potential and current elements respectively.

Speed of Operation. — Ratio points may be taken at the rate of one minute each while operating, and phase-angle points at a somewhat slower rate. Single-point checks can be made practically as rapidly as the transformers can be connected and the voltage, load and frequency adjusted. One observer is required for ratio test, two for phase-angle test.

Standardized Transformer Method for Determining Ratio of Potential Transformer. — The transformer to be tested should be placed in multiple with the standard transformer of the same rated ratio on the high-tension side, and standardized portable voltmeters should be connected to the two secondaries. These should be read, interchanged and read a second time to eliminate errors in calibration of the voltmeters. It is not difficult to maintain an accuracy of about 0.2 per cent on ratio test by this method with carefully standardized apparatus near the rated voltages of the transformer. More accurate determinations of ratio and determinations of phase angle if desired should be made in a thoroughly equipped laboratory, and the check results used simply to verify the unchanged condition of the transformer.

Voltmeter and Resistance Method of Determining Ratio. — Where two transformers of the same rated ratio and a voltmeter with suitable multipliers are available, good checking may be done according to the method shown in Fig. 11. T_1 is a step-up transformer to furnish the voltage for use in test. T_2 and T_3 are potential transformers of similar ratio, T_3 being the transformer under test. V_2 is a portable voltmeter. V_1 is a good portable voltmeter or a laboratory standard instrument. M is a multiplier of such resistance R_m that closing the switch P causes the voltmeter and multiplier to read nearly the same from the primary supply as the voltmeter alone reads from the secondary with the switch S closed. In operation the switch P is closed, and readings made on V_1 and V_2. Then the switch P is opened and S is closed. V_2 is held at the same reading and V_1 is read. If the resistance of V_1 is R_v, its first reading E_1 and its second reading E_2,

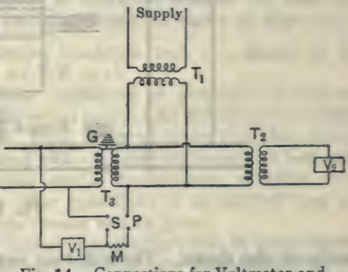

Fig. 14. Connections for Voltmeter and Resistance Method

$$\text{True ratio} = \frac{E_1 (R_m + R_v)}{E_2 R_v}.$$

If the two readings are close together in the scale of V_1, no specially standardized instrument is necessary. The reading of V_2 does not enter into the result, as it is only used to hold the primary voltage at the same point for the two readings. The accuracy of the method depends on the accuracy of reading of the instruments used, although the absolute values of the readings do not affect it. With care the error should not exceed 0.2 per cent.

Mutual Inductance Method of Determining Phase Angle of Potential Transformer. — The phase angle may be determined by the addition to the ratio-testing outfit described above of a mutual inductance whose primary is placed in series with r_1 or r_2 and whose secondary is in series with the circuit from the contactor C through the dynamometer D_1 in a way similar to that described under current transformers. The primaries of two similar transformers may be placed in multiple, and the difference in their secondary voltages read on a low reading instrument. Where the phase angle is the same in both the transformers, this is an excellent method of comparison if the current drawn in the voltmeter is kept small by the use of a high-resistance instrument.

Two Watt-hour Meter Method of Determining Ratio and Phase Angle of Potential Transformers. — This is similar to the method used for current transformers (see page 1745) and consists in comparing the transformer to be tested with one whose characteristics are known. It has advantages similar to the method for current transformers and in addition all instruments are in the low-voltage circuit.

Apparatus. — In Fig. 15 the standard transformer is shown at S and the one under test at T. The connections to the two watt-hour meters and the

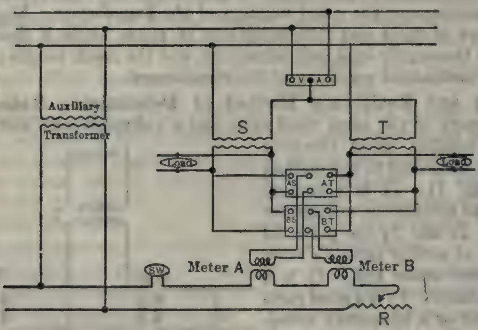

Fig. 15

loads are also shown. The watt-hour meters can be read to 1/100 of a revolution, portable standard watt-hour meters being most frequently used. The auxiliary transformer supplies current for the current coils of the watt-hour meters, and may be one of the main power transformers. The switch SW is a convenient means of starting and stopping both meters simultaneously. If ratio only is desired, the whole power supply may be taken from a 115-volt circuit, the auxiliary transformer being used to step up the voltage for the primaries of the instrument transformers.

Procedure. — The primary switch is thrown to V and the loads on the secondaries are adjusted to the desired values. The current in the current

coils of the meters is adjusted to about 5 amperes by means of the resistance R. The double-throw switches are now thrown to AS and BT and a reading is taken. Then the double-throw switches are thrown to BS and AT and another reading is taken. For convenience in calculation it is well to stop these readings when the meter connected to T shows an even number of divisions, say 10,000. These readings give the data for calculating the ratio. For the phase-angle measurements the same procedure is followed except that the primary switch is thrown to A instead of to V, and the meters should be stopped when the one connected to S shows an even reading.

Calculation. — The equations are the same as given for current transformers on page 1747, namely,

$$R_T = R_S \sqrt{\frac{a_S b_S}{a_T b_T}},$$

or

$$\frac{R_T - R_S}{R_S} = \frac{1}{2}\left[\frac{a_S - a_T}{a_T} + \frac{b_S - b_T}{b_T}\right] \text{ (approximately)},$$

where the a's and b's are readings taken with the primary switch thrown to V, and

$$\tan \alpha_T - \tan \alpha_S = \frac{1}{2 \tan \theta}\left[1 - \frac{a_T b_T R_T^2}{a_S b_S R_S^2}\right]$$

or

$$\alpha_T \text{ (in minutes)} = \alpha_S \pm \frac{3438}{\tan \theta}\left[\frac{a_S - a_T}{2a_S} + \frac{b_S - b_T}{2b_S} - \frac{R_T - R_S}{R_S}\right] \text{ (approximately)},$$

where the a's and b's are readings taken with the primary switch thrown to A. The letters have the same meaning as given on page 1746. In the last expression, the + sign before the bracket should be used when the meters, with the primary switch thrown to A, are working on lagging current, and the − sign when the current is leading. Or the sign may be determined experimentally from the fact that adding a non-inductive load to a voltage transformer always tends to lag the secondary voltage.

Speed of Operation. — The speed of operation is about the same as when testing current transformers by the same method as shown on page 1747.

SPECIFICATIONS. — Heating limits, insulation test, etc., should be in accordance with the Standards of the A.I.E.E. (*q.v.*).

Ratio and phase angle under definite conditions of secondary connected load may, for the highest class of commercial product, be defined substantially in accordance with Figs. 2, 3, 7 and 8. Even better accuracy than that shown in these figures can sometimes be secured with considerable increase in cost and amount of material. For less exacting service transformers of lower cost and less accuracy can be procured. The conditions to be fulfilled vary so much that definite general specifications cannot well be given.

For accurate work, especially with current transformers and in cases where much depends on the result, abridged tests covering the performance under operating conditions may be required in connection with each individual transformer. Unless very careful tests are made, however, the information from a curve representing the average of many determinations is apt to be more reliable than the results of one determination.

INSTALLATION. — Instrument transformers previous to installation should be kept in a cool, dry place. All handling should be done in such a manner as to avoid damage to the insulation, particular care being used on those types where the insulation is exposed. Oil-type transformers are usually shipped

without oil, and temporary wooden blocking is often placed in the tank. This blocking should be removed before installation. If the transformers have been stored for a considerable time, or if they have been exposed to moisture, they should be thoroughly dried out before installing. (*See Transformers.*) Those of the oil type should be filled with a good quality of dry oil.

Grounding of Transformers. — Both current and potential transformers should be installed in a clean, dry place, preferably at the back of the switchboard in the case of low-voltage transformers, so that all parts, except those that are grounded, will clear all conducting or semi-conducting material by at least twice and preferably by three times the sparking distance of the normal line voltage. The casings and frames should be grounded. The secondary wiring should be grounded at such a point as will not interfere with proper operation of instruments connected to the transformers. See article on *Ground Connections*.

Resistance of Leads. — Secondary leads should be of low resistance and special attention should be given to making all contacts perfect in the wiring connected to the secondary of current transformers. A resistance not exceeding 0.2 ohm (200 feet of No. 10 wire) is usually satisfactory for the secondary circuit of a current transformer operating watt-hour meters or wattmeters. Considerably greater length of leads may be used where ammeters and protective devices only are used because of the lower accuracy required. With voltage transformers, the error due to drop in leads is proportional to the amount of current passing through them, which is determined by the number and kind of devices operated. For ordinary circumstances the drop should be less than 0.5 per cent; where special accuracy is desired, the lead resistance should be negligible, or the accuracy test of the transformer should be made with a resistance equivalent to the actual leads placed in series with the testing load. Leads should always be close together, enclosing as little area as possible between them.

Fuses. — Potential transformers should be fused on the primary side. The fuse should be designed to protect the line from disturbance due to failure of the transformer, rather than to protect the transformer against secondary overloads. The fuse should be of such resistance as will not cause appreciable errors in the ratio of transformation. A combination of a relatively high current capacity fuse and resistor is sometimes used. The resistor limits the current to 20 or 40 amperes, while the fuse is designed to open the circuit with such a current.

OPERATION. — After the current transformer has been installed it should need no other care than being kept clean. The secondary of a current transformer must never be left open-circuited. If it becomes desirable to open the secondary winding to insert or remove instruments, the secondary of the current transformer should first be carefully short-circuited. If the secondary has been open even momentarily while current flowed in the primary, the transformer should be demagnetized; see above under *Testing*. Current transformers should always be considered as part of the line circuit and should never be handled except to change connections. Even then only the secondary leads should be touched. When it becomes necessary to change secondary connections the ground connection should be inspected to assure that it is in good condition and is so made that it will not be disconnected in handling the secondary leads. Grounded protective cases should be used where necessary to prevent attendants from coming in accidental contact with current transformers.

Potential transformers should receive the same care in operation as power transformers. No power should be drawn from them except that required for a suitable load of instruments and the usual switchboard devices. The same precautions should be used in handling as with power transformers.

REPAIRS. — In general, replacement or repair by the maker is to be preferred to repairs made by the user where anything more than broken leads or simple external injuries are to be remedied. If repairs to magnetic circuit or windings are attempted, care should be exercised to duplicate as exactly as possible the original construction especially in regard to number of turns and resistance of windings and exact assembly of punchings. After repairs are completed, the transformer should be carefully tested both for accuracy and for insulation strength.

WEIGHTS, DIMENSIONS AND COSTS. — Current transformers in ordinary use for circuits from 115 to 110,000 volts range from 4 pounds to 1500 pounds weight with oil, and from a maximum dimension of 6 inches to a height of 8 feet with a diameter of 30 inches, depending on the quality of the transformer and the voltage of the circuit on which it is to operate. Potential transformers have nearly the same range of sizes.

The cost of instrument transformers depends on the circuit voltage for which they are designed, and is also affected in the case of current transformers by the ratio of the transformer. There is a considerable range between the prices offered by various manufacturers for transformers of the same rating; the lower prices are, however, associated with less accuracy or with less insulation strength or both.

The approximate cost of an instrument transformer of high grade representing accuracies as per Figs. 2 and 3 for current transformers and Figs. 7 and 8 for potential transformers is as follows:

COST OF VOLTAGE AND CURRENT TRANSFORMERS (1922)

Tyye of transformer	Voltage of circuit	Insulation test voltage	Price in dollars	
			Operated dry	Operated with oil
Voltage	230	5,000	20 to 25	......
"	460	5,000	23 to 27	......
"	575	10,000	25 to 30	......
"	2,300	10,000	26 to 32	39 to 44
"	4,600	10,000	40 to 48	61 to 70
"	6,900	15,000	52 to 61	70 to 78
"	11,500	25,000	......	97 to 117
"	13,800	30,000	......	100 to 122
"	23,000	50,000	......	325
"	34,500	70,000	......	435
"	44,000	95,000	......	610
"	66,000	140,000	......	870 to 1050
Current	2,500	10,000	......	17 to 26
"	6,900	20,000	......	32 to 45
"	13,800	35,000	......	44 to 56
"	23,000	56,000	......	87
"	34,500	81,000	......	195
"	44,000	108,000	......	283
"	66,000	160,000	......	400

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TRANSIENT ELECTRIC PHENOMENA AND OSCILLATIONS. — (*See also Alternating Currents; Capacity and Charging Current; Electricity and Magnetism, Principles of; Inductance and Inductive Reactance; Transmission Lines.*)

Establishment of a Direct Current in a Coil. — Let E be a constant direct electromotive force, r the resistance of the coil, L the inductance of the coil (assumed constant); then t seconds after closing the circuit (placing the coil and e.m.f. in series) the current in the coil is

$$i = \frac{E}{r}\left(1 - \epsilon^{-ut}\right), \tag{1}$$

where ϵ is the base of the natural logarithms and $u = \frac{r}{L}$. See *Exponential Functions* for values of ϵ^{-x}.

Time Constant of a Coil. — The time required for the exponent $ut = \frac{rt}{L}$ to reach the value unity, and therefore for the current to reach 63.2 per cent of its final value, is called the time constant of the coil, and is equal to $\frac{L}{r}$. The larger the time constant the longer the time required for the current to reach its steady value $\frac{E}{r}$.

Decay of Current in a Coil when Coil is Short-circuited. — Using the same notation as above, the current in the coil t seconds after short-circuiting it is

$$i = I_0 \epsilon^{-ut}$$

where I_0 is the current in it at the instant the short-circuit is made.

Charging a Condenser Through a Resistance from a Source of Constant E.M.F. — Let E be a constant e.m.f., C the capacity of the condenser, r the resistance in series with it, and let the condenser be originally uncharged. Then if the conductance of the condenser and the inductance of the circuit are both negligible, the voltage across the condenser, the current in the circuit and the charge on the condenser t seconds after the circuit is closed (thereby connecting the condenser, resistance and e.m.f. in series), are respectively

$$\left.\begin{aligned} v &= E(1 - \epsilon^{-ut}), \\ i &= \frac{E}{r}\epsilon^{-ut}, \\ q &= CE(1 - \epsilon^{-ut}), \end{aligned}\right\} \tag{2}$$

where $u = \frac{1}{rC}$. For values of ϵ^{-x} see *Exponential Functions.*

Time Constant of a Condenser and Resistance in Series. — The time required for the exponent $ut = \frac{t}{rC}$ to reach the value unity, and therefore for the voltage and charge on the condenser to reach 63.2 per cent of their final values, is called the time constant of this circuit, and is equal to rC. The larger the time constant the longer the time required for the voltage across the condenser to reach the value of the e.m.f. impressed on the circuit.

Discharge of a Condenser Through a Resistance. — Using the same notation as above, and assuming negligible conductance in the condenser and negligible inductance in the circuit, the voltage across the condenser, the current

in the circuit and the charge on the condenser t seconds after short-circuiting it through a resistance r, the condenser being charged to a voltage V_0 at the instant of short-circuit (but no current flowing), are respectively

$$\left.\begin{aligned} v &= V_0 \epsilon^{-ut}, \\ i &= -\frac{V_0}{r}\epsilon^{-ut}, \\ q &= CV_0 \epsilon^{-ut}, \end{aligned}\right\} \qquad (3)$$

where $u = \dfrac{1}{rC}$.

GENERAL EQUATIONS FOR LUMPED INDUCTANCE AND CAPACITY. — Let the various quantities be as designated in Fig. 1. The differential equations for this circuit are

$$\left.\begin{aligned} i &= gv + C\frac{dv}{dt}, \\ v &= e - ri - L\frac{di}{dt}, \end{aligned}\right\} \qquad (4)$$

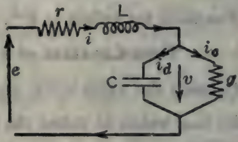

Fig. 1.

where i is the instantaneous current in the impedance coil (equal to the total displacement and conduction current through the condenser), v is the potential drop through the condenser in the direction of the current, and e is the impressed voltage, constant or varying. g represents the "leakance" of the condenser, i.e., the power dissipated in the condenser at any instant is gv^2.

At any given instant, say at $t = 0$, let the impressed e.m.f. be changed from any previous value to a new value $E \sin \theta$, and let this impressed e.m.f. from this instant vary sinusoidally, i.e., at any instant t thereafter, let $e = E \sin (\omega t + \theta)$, where $\omega = 2\pi f$ and f is the frequency of this e.m.f. in cycles per second. (The solution for a constant impressed e.m.f. E can be obtained by putting $f = 0$ and $\theta = \pi/2$ in the equations below.) Let I_0 and V_0 be respectively the values of the current i and of the potential drop v through the condenser at time $t = 0$. I_0 and V_0 may or may not be zero, depending upon the condition of the circuit previous to the change in the impressed e.m.f.

Steady State Relations. — The complete solution of equations (4) shows that the current and voltage at any instant after impressing the e.m.f. on the circuit are each composed of two terms, (1) a term which dies out with time and (2) a term, added to the first, which represents the final or steady (alternating) state of the current and voltage respectively. The "steady state" solution of (4) is readily effected by the ordinary methods of alternating-current calculation (*see Alternating Currents*), and is as follows:

$$\left.\begin{aligned} i_2 &= \frac{E}{Z}\sin(\omega t + \theta - \phi), \\ v_2 &= \frac{E}{yZ}\sin(\omega t + \theta - \phi - \eta), \end{aligned}\right\} \qquad (5)$$

where

$$\left.\begin{aligned} Z &= \sqrt{R^2 + X^2} \quad \text{and} & \phi &= \tan^{-1}\frac{X}{R}, \\ R &= r + \frac{g}{g^2 + b^2}, & X &= x - \frac{b}{g^2 + b^2}, \\ b &= \omega C, & x &= \omega L, \\ y &= \sqrt{g^2 + b^2}, & \eta &= \tan^{-1}\frac{b}{g}, \end{aligned}\right\} \qquad (6)$$

Free Oscillations. — The transient terms in the complete solution of equations (4) are as follows:

$$\left.\begin{aligned} i_1 &= \epsilon^{-ut}\left[D\cos\omega_0 t-\left(\frac{D'}{\omega_0 L}+\frac{qD}{\omega_0}\right)\sin\omega_0 t\right], \\ v_1 &= \epsilon^{-ut}\left[D'\cos\omega_0 t+\left(\frac{D}{\omega_0 C}+\frac{qD'}{\omega_0}\right)\sin\omega_0 t\right], \end{aligned}\right\} \qquad (7)$$

where

$$\left.\begin{aligned} u = \frac{1}{2}\left(\frac{r}{L}+\frac{g}{C}\right) \quad &\text{and} \quad q = \frac{1}{2}\left(\frac{r}{L}-\frac{g}{C}\right), \\ \omega_0 &= \sqrt{\frac{1}{LC}-q^2}\ . \end{aligned}\right\} \qquad (8)$$

These three constants, u, q and ω_0 are constants of the circuit, and are independent of the manner in which the transient oscillation is set up. The frequency of this "natural" oscillation is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-q^2}.$$

When q^2 is small compared with $\frac{1}{LC}$ the natural frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$. Due to the resistance of the coil and leakance of the condenser, these natural oscillations die out with time proportionally with ϵ^{-ut}, or the amplitudes of both the transient current and the transient voltage decrease by the fraction ϵ^{-u} each second.

The constants D and D', which determine the amplitudes of the transient terms (equations 7) are given by the following relations:

$$\left.\begin{aligned} D &= I_0 - \frac{E}{Z}\sin(\theta-\phi), \\ D' &= V_0 - \frac{E}{yZ}\sin(\theta-\phi-\eta), \end{aligned}\right\} \qquad (9)$$

where I_0 is the current in the coil and V_0 the voltage across the condenser at the instant ($t = o$) at which any change is made in the circuit conditions, i.e., switch opened or closed, or e.m.f. short circuited.

Solution when Damping is Great. — When q^2 is greater than $1 \div (LC)$, ω_0 becomes imaginary and the oscillation therefore has an imaginary frequency, which means that the transient terms die out without oscillating. Equations (7), (8) and (9) still hold in this case and will give the *real* solution *provided* that in equation (7) "cos" is changed to "cosh" (hyperbolic cosine), and "sin" is changed to "sinh" (hyperbolic sine) and for ω_0 is taken the value $\sqrt{q^2-\frac{1}{LC}}$. See *Hyperbolic Functions* for tables of "sinh" and "cosh." The expressions for u, q, D and D' remain unaltered.

Critical Damping. — Where $\omega_0 = o$, that is, when $q^2 = 1 \div (LC)$, equations (7) for the transient current and voltage become

$$\left.\begin{aligned} i_1 &= \left[D\left(1-qt\right)-\frac{D't}{L}\right]\epsilon^{-ut} \\ v_1 &= \left[\frac{Dt}{C}+D'\left(1+qt\right)\right]\epsilon^{-ut} \end{aligned}\right\} \qquad (10)$$

where D and D' are given by equations (9).

Complete Solution. — The complete expressions for the current and voltage at any instant are in all cases

$$\left.\begin{aligned} i &= i_1 + i_2, \\ v &= v_1 + v_2, \end{aligned}\right\} \tag{11}$$

where i_2 and v_2 are given by equations (5) and i_1 and v_1 are given by equations (7), the latter changed as noted above when q^2 is greater than $1 \div (LC)$ or reducing to equation (10) when $q^2 = 1 \div (LC)$.

Discharge of a Condenser through an Impedance Coil. — The above equations may be used to obtain the solution of the special case of a condenser charged to a voltage V_0 and short-circuited at time $t = 0$ through a coil having both resistance and inductance. Assume that at time $t = 0$ the current through the condenser is zero. The "steady state" terms in this case are zero, and only the transient terms appear, viz.,

$$\left.\begin{aligned} i &= -\frac{V_0}{\omega_0 L}\epsilon^{-ut}\sin\omega_0 t, \\ v &= V_0\epsilon^{-ut}\left(\cos\omega_0 t + \frac{q}{\omega_0}\sin\omega_0 t\right), \end{aligned}\right\} \tag{12}$$

where the values of u, q and ω_0 are as given above, equations (8). When $1 \div (LC)$ is greater than q^2, equations (12) apply directly; the current-time curve in this case is shown in Fig. 2. When $1 \div (LC)$ is less than q^2 the circular functions become hyperbolic functions and $\omega_0 = \sqrt{q^2 - \frac{1}{LC}}$ and the current-time curve is as shown in Fig. 3. When $q^2 = 1 \div (LC)$, equations (12) reduce to

$$i = -\frac{V_0}{L}t\epsilon^{-ut} \quad \text{and} \quad v = V_0\epsilon^{-ut}(1 + qt). \tag{13}$$

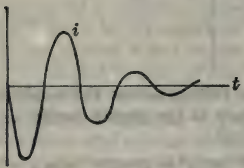

Fig. 2. $q < \frac{1}{\sqrt{LC}}$

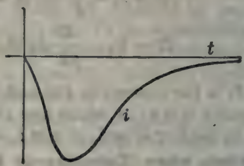

Fig. 3. $q > \frac{1}{\sqrt{LC}}$

OSCILLATIONS IN TRANSMISSION LINES. — Oscillations similar in nature to those discussed above occur in a transmission line when any change takes place in the load connections and particularly when high voltages are suddenly induced in the line by lightning discharges or when heavy currents are suddenly interrupted. The mathematical analysis of such cases is extremely involved, since account must be taken of the distributed nature of the inductance and capacity, and also of the fact that the possible periods of oscillations depend not only upon the line constants but also upon the constants of the apparatus at the two ends of the line. See article on *Transmission Lines*.

BIBLIOGRAPHY. — Steinmetz, C. P., *Transient Electric Phenomena and Oscillations; Electric Discharges, Waves and Impulses;* Pender, H., *Electricity and Magnetism for Engineers;* also *Bibliography* in article on *Radio Communication.*

TRANSMISSION LINES. — (*See also Capacity and Charging Current; Conduits and Conduit Lines, Underground; Copper; Corona, Electric; Cross Arms; Distribution Lines; Distribution and Transmission Systems; Inductance and Inductive Reactance; Insulators for Overhead Lines; Insulator Pins; Trolley Systems, Overhead; Wind Pressure; Wires and Cables, Bare; Wires and Cables, Insulated; Wiring of Buildings.*) Circuits designed for transmitting relatively large amounts of power from one fixed point to another are called transmission lines, while those for delivering small amounts at numerous points are called distribution circuits. The same type of construction is used for both kinds of lines when designed for operation at the same voltage. The formulas used in the electrical design of both distribution and transmission lines and in the mechanical design of wire spans are included in this article; see also the articles on *Trolley Systems, Overhead*, for formulas for railway distribution, and *Wiring of Buildings* for formulas for interior wiring. The construction details given in this article refer primarily to three-phase lines designed to operate at or above 13,200 volts; for the construction of low-voltage lines see *Distribution Lines*. For a discussion of the general features of the various types of distribution and transmission systems see the article on *Distribution and Transmission Systems*.

The following is a brief table of contents of this article:

DEFINITIONS AND FUNDAMENTAL RELATIONS. — The following definitions and relations apply to all types of transmission and distribution lines.

Generator End and Load End. — By the generator end of the line is meant the end which is connected to the source of power (either directly or through transformers), and by the load end is meant the end which is connected to the load or substation which is supplied with power over the line.

Per cent Power Loss (Q). — By per cent power loss as used in this article is meant the percentage ratio

$$Q = 100 \cdot \frac{\text{total power lost in the line}}{\text{total power delivered at load end}}. \tag{1}$$

Hence if P is the power delivered, then the total power supplied to the line and load is

$$P_0 = P\left(\frac{100+Q}{100}\right). \tag{2}$$

Per cent Voltage Loss (D). — By per cent voltage loss as used in this article is meant the percentage ratio*

$$D = 100 \cdot \frac{(\text{voltage at generator end}) - (\text{voltage at load end})}{\text{voltage at load end}}. \tag{3}$$

Hence calling E the voltage at the load end, the voltage at the generator end is

$$E_0 = E\left(\frac{100+D}{100}\right). \tag{4}$$

The per cent voltage loss allowed under various conditions is discussed in detail in the article on *Distribution and Transmission Systems;* the allowable voltage loss is usually between 2 and 20 per cent, the most common figure being 10 per cent.

In the case of a direct-current line with a single load at its far end, the per cent power loss and the per cent voltage loss are always equal, but in the case of an alternating-current line the per cent voltage loss may be either greater or less than the per cent power loss, depending upon the constants of the line and the power factor of the load, or may even be negative, i.e., there may be an actual rise of voltage at the load end above the voltage at the generator end; see below.

Efficiency of Transmission. — By the efficiency of transmission is meant the percentage ratio

$$100 \cdot \frac{\text{power output of line at load end}}{\text{total power input to line at generator end}}. \tag{5}$$

The per cent efficiency is related to the per cent power loss Q as follows;

$$\text{Per cent efficiency} = \frac{10{,}000}{100+Q}. \tag{6}$$

ELECTRICAL DESIGN OF DIRECT-CURRENT LINES. — Two types of problems arise: (1) given a definite line with known constants, what is the power loss and voltage loss for a given load, and (2) to transmit a given amount of power a given distance with a given allowable loss, what will be the size and weight of the conductor required? In the following paragraphs are given the necessary formulas for the several cases.

Two-wire Line; Concentrated Load at Far End. — Let

E = volts between wires at the load end of the line,

$P = \dfrac{EI}{1000}$ = kilowatts taken by load,

$I = \dfrac{1000\,P}{E}$ = amperes taken by load,

l = length of each line wire in feet,

r = ohms per 1000 feet of conductor; see tables in article on *Wires and Cables, Bare,*

$R = \dfrac{rl}{500}$ = total resistance of line (both conductors), in ohms.

* In the case of an alternating-current line the difference in the numerator of this ratio is the algebraic difference between r.m.s. values of the two voltages, not the vector difference.

The following relations then hold:

$$\text{Total kilowatts lost} = p = \frac{RI^2}{1000} = \frac{rlI^2}{500{,}000} = \frac{2\,rlP^2}{E^2}, \tag{7}$$

$$\text{Total volts lost} = v = RI = \frac{rlI}{500} = \frac{2\,rlP}{E}, \tag{8}$$

$$\text{Per cent power loss} = Q = \frac{100\,p}{P} = \frac{rlI^2}{5000\,P} = \frac{200\,rlP}{E^2}, \tag{9}$$

$$\text{Per cent voltage loss} = D = \frac{100\,v}{E} = \frac{rlI}{5\,E} = \frac{200\,rlP}{E^2}, \tag{10}$$

$$\left.\begin{matrix}\text{Resistance of each con-}\\ \text{ductor per 1000 feet}\end{matrix}\right\} = r = \frac{500\,v}{lI} = \frac{QE^2}{200\,lP} = \frac{DE^2}{200\,lP}. \tag{11}$$

Calculation of Size and Weight of Conductor for Concentrated Load. — From the value of r calculated from any one of the relations given in equation (11), the size of wire may be found from the tables in the article on *Wires and Cables, Bare;* the next larger size of wire (next smaller gauge number) should usually be chosen when the calculated resistance lies between that of two commercial sizes. The wire selected must also have sufficient current-carrying capacity; see *Wires and Cables, Bare*; *Wires and Cables, Insulated;* and *Wiring of Buildings*. For outside lines, however, the current-carrying capacity will in general be ample unless the allowable voltage loss is excessive. For an outside overhead line a wire smaller than No. 6 A.W.G. (or B. & S.) gauge is seldom used, chiefly on account of its lack of mechanical strength.

Let w = weight per 1000 feet of the wire finally selected, then

$$\text{Total weight of conductor in pounds} = W = \frac{wl}{500}. \tag{12}$$

Direct Calculation of Total Weight of Conductor (W); Two-wire Line. — For preliminary estimates it is sometimes convenient to calculate the total weight of conductor directly, without reference to a wire table. The total weight of conductor for a two-wire line with concentrated load at its end is given by the formula

$$W = \frac{KP}{Q}\left(\frac{l}{E}\right)^2 \text{ pounds,} \tag{13}$$

where P is the power taken by the load, l the length of the line (length of each wire), E the voltage at the load, Q the per cent power loss (= per cent voltage drop for 2-wire d-c. line) and K a constant depending upon the material of the conductor and the units in which P, l and E are expressed, viz.,

VALUES OF K IN FORMULA 13

Material	E in volts, l in feet, P in kilowatts	E in kilovolts, l in miles, P in kilowatts
Copper (98 per cent conductivity).......	13.5	380
Aluminum (61 per cent conductivity)...	6.5	185
Any material of specific gravity δ having a conductivity of c per cent at 20° C...	$\frac{141\,\delta}{c}$	$\frac{3940\,\delta}{c}$

NOTE. — The values of K given for copper and aluminum are about 5 per cent greater than their theoretical values to allow for stranding, higher working temperature, etc.; 100 per cent conductivity corresponds to 1.724 microhms per centimeter cube at 20° C.

Example: Two-wire D-C. Line, Concentrated Load. — A load of 100 kw. is to be transmitted over a two-wire line to a motor operating at 230 volts, the motor being 1000 feet from the power-house switchboard. For a 10 per cent power loss or voltage drop in the line, the approximate total weight of copper required is, from equation (13),

$$W = \frac{13.5 \times 100}{10}\left(\frac{1000}{230}\right)^2 = 2550 \text{ pounds.}$$

From equation (11) the resistance per 1000 feet is

$$r = \frac{10 \times (230)^2}{200 \times 1000 \times 100} = 0.0264 \text{ ohm per 1000 feet.}$$

The nearest even circular mil size is 400,000 circular mils (stranded), which has a resistance of 0.0270 ohm per 1000 feet at 77° F. (*see Wires and Cables, Bare*), and a weight of 1240 pounds per 1000 feet. From equation (12) the total weight of conductor is then

$$W = \frac{1240 \times 1000}{500} = 2480 \text{ pounds.}$$

This wire, if bare, weather-proofed, or insulated with paper or varnished cambric, will safely carry the required current of 100,000/230 = 435 amperes, but if rubber insulated and mounted indoors, a larger wire should be required, viz., 600,000 circular mils, according to the National Electric Code (*see Wires and Cables, Bare*).

Calculation of Two-wire Direct-current Line in Terms of Voltage at Generator End. — When the volts E_0 at the generator end are given instead of the volts E at the load end, the calculations for a line of given total resistance of R ohms with a concentrated load of P kilowatts at the load end may be made in the same manner as above by first finding the volts E at the load by the formula

$$E = \frac{E_0}{2}\left[1 + \sqrt{1 - \frac{4000\,RP}{E_0^2}}\right]. \qquad (14)$$

For an efficiency of transmission of less than 50 per cent, the sign before the radical should be − instead of +, but an efficiency of less than 50 per cent practically never occurs in power transmission. It is of interest to note that for an efficiency of 50 per cent $P = E_0^2 \div (4000\,R)$ which is the maximum power which can be delivered at the far end of the line for a given impressed voltage at the generator end.

When E has been calculated by this formula (14), the formulas (7) to (13) above may be applied directly.

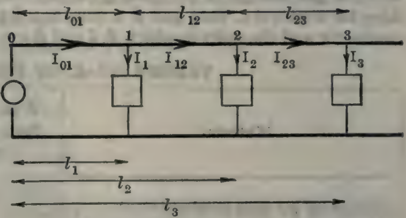

Fig. 1.

Two-wire Line; Distributed Load. — When a line supplies a number of loads at different distances from the generator end, the voltage loss to the far end of the line is the same as would be produced by a load, concentrated at the "center of gravity" of the line and taking a current equal to the total current taken by all the loads. The center of gravity of the line is defined as follows: Let I_1, I_2, I_3, etc., be the currents taken from the line by the various loads, Fig. 1, and let R_1, R_2, R_3, etc., be the total line resistances (both wires) from the generator end to the respective loads, and put

$$I = I_1 + I_2 + I_3 + \ldots \qquad (15)$$

Then the center of gravity is that point between which and the generator end of the line the total line resistance is

$$R_g = \frac{R_1 I_1 + R_2 I_2 + R_3 I_3 + \ldots}{I}. \tag{16}$$

When the line conductor has the same cross-section throughout its length, then the center of gravity is at the distance,

$$l_g = \frac{l_1 I_1 + l_2 I_2 + l_3 I_3 + \ldots}{I} \tag{16a}$$

from the generator end, where l_1, l_2, l_3, etc., are the distances of the respective loads from the generator end.

The total voltage loss to the far end of the line is then

$$v = R_g I = \frac{r l_g I}{500}, \tag{17}$$

where the distance l_g is in feet and r is the resistance of the conductor per 1000 feet, the second relation in (17) holding only when the conductor has the same cross-section throughout its length.

The total kilowatts lost in the line are

$$p = \frac{1}{1000}\left(R_{01} I_{01}^2 + R_{12} I_{12}^2 + R_{23} I_{23}^2 + \ldots\right), \tag{18}$$

where, referring to Fig. 1, R_{01} = total resistance (both wires) from 0 to 1, R_{12} = total resistance (both wires) from 1 to 2, etc., and $I_{01} = I_1 + I_2 + I_3 + \ldots$ = the current in the line from 0 to 1, $I_{12} = I_2 + I_3 + \ldots$ = the current in the line from 1 to 2, etc. When the cross-section of the line conductor is the same throughout its length equation (18) may be also written

$$p = \frac{r}{500{,}000}\left(l_{01} I_{01}^2 + l_{12} I_{12}^2 + l_{23} I_{23}^2 + \ldots\right), \tag{18a}$$

where the distances l_{01}, l_{12}, l_{23}, etc., are as shown in Fig. 1, and are measured in feet, and r is the resistance of the line conductor per 1000 feet.

Calculation of Size and Weight of Conductor for Distributed Load. — For a conductor of the same cross-section throughout, the required resistance per 1000 feet for a given voltage loss of v volts to the end of the line may be calculated from the formula

$$r = \frac{500\,v}{l_g I}, \tag{19}$$

where l_g, expressed in feet, and I are given by formulas (16a) and (15). The size and weight of conductor can then be found by reference to the wire tables in the article on *Wires and Cables, Bare.*

When the loads are far apart and the smaller loads are farthest from the generator, it is sometimes advisable to use different sizes of conductors for the various portions of the line. For a given voltage loss v to the end of the line, the minimum weight of conductor is obtained when the volts lost per unit length of conductor in each section of the line is proportional to the square root of the current in this portion of the line. For minimum total weight of conductor then, referring to Fig. 1, the resistance per 1000 feet *of wire* for the section between 1 and 2, say, must be

$$r_{12} = \frac{1}{\sqrt{I_{12}}} \cdot \frac{500\,v}{l_{01}\sqrt{I_{01}} + l_{12}\sqrt{I_{12}} + l_{23}\sqrt{I_{23}} + \ldots}, \tag{20}$$

where the lengths are in feet; and similarly for the other sections. The weight of wire for each section may then be found by reference to the wire tables in the article on *Wires and Cables, Bare*, and the total weight W can then be computed. Vice versa, for a line proportioned in this manner the voltage loss to the end of the line for a given total weight of copper will be a minimum.

A line proportioned in this manner, however, does not give minimum power loss for the total weight of conductor used. For a given total weight of conductor the total power loss will be a minimum when the power loss per unit length of conductor in each section of the line is directly proportional to the current in this section, i.e., when the

voltage loss per unit length in each section is the same, which means that the weight per unit length of each section must be proportional to the line current in this section. Whence letting W be the total weight of the conductor in pounds, then the power loss will be a minimum when the section from 1 to 2, say, has a weight in pounds per 1000 feet of

$$w_{12} = I_{12} \frac{500\, W}{l_{01}I_{01} + l_{12}I_{12} + l_{23}I_{23} + \ldots}, \tag{21}$$

where the lengths are in feet; and similarly for the other sections. The size of wire for each section may be obtained from the wire tables in the article on *Wires and Cables, Bare.* When the sizes as found by the two formulas (20) and (21) differ considerably, the choice will depend upon which is the more important, minimum power loss or minimum voltage loss.

Three-wire Direct-current Line. — When a three-wire circuit is exactly balanced, i.e., when the loads between each of the two outer wires and the neutral are the same and are connected to the neutral at the same point or points, no current flows in the neutral wire. The formulas given above for a two-wire line then apply directly to the case of a balanced three-wire line, noting however that the E (= volts between wires) in these formulas is to be taken as the volts between the *outer* wires, and that the weight as calculated by the above formulas is the weight of the two outer wires. The neutral wire is usually made equal in size to each outer wire, but when only slight unbalancing is expected it is sometimes made smaller. When the neutral is made equal in size to the outer wire the total weight of the three conductors will be 50 per cent more than that given by formula (13), when E in this formula is taken equal to the volts between the outer wires.

The exact calculation of the voltage loss and power loss when the loads on the two sides of the system are different and are connected at different points is somewhat complicated, but can always be effected by the application of Kirchhoff's Laws for an electrical network; see *Electricity and Magnetism, Principles of.*

SIZE AND WEIGHT OF CONDUCTORS FOR ALTERNATING-CURRENT LINES. — As a rough guide in fixing upon a preliminary design, the following facts should be noted; complete formulas for the various calculations required are given later.

1. A power loss of approximately 10 per cent of the delivered power is usually allowed.

2. A line voltage of approximately 1000 volts per mile of line is common practice for long-distance lines not over 150 miles in length; that is, for a 10-mile line a line voltage of 10,000 volts would be employed; for a 100-mile line a line voltage of 100,000 volts would be used. The maximum line voltage at present (1914) employed is 150,000 volts and the maximum distance of transmission is 240 miles.

3. On the basis of 1000 volts per mile of line, unity power factor at the load, a 10 per cent power loss, and copper at 15 cents per pound, the cost of the copper required for a three-phase line is $4.00 per kilowatt delivered, and for a single-phase or two-phase four-wire line $5.33 per kilowatt delivered.

Calculation of Total Weight of Conductor for A-C. Lines. — The size and total weight of the conductor required for any conditions * of length, power delivered, power factor, line voltage and power loss may be calculated as follows:

* These formulas are based on the assumption that the charging current is negligible in comparison with the load current, which condition is practically realized in all but the longest high-voltage lines; formulas for power loss taking the charging current into account are given later.

Let

E = voltage between wires at the load end of the line,
P = total power taken by all phases of the load,
$\cos\phi$ = power factor of load, as a decimal,
l = length of line (= length of each line wire),
Q = allowable total power loss in per cent of delivered power.

Then the total weight of all conductors is given by the formula

$$W = \frac{KP}{Q}\left(\frac{l}{E\cos\phi}\right)^2 \quad \text{pounds}, \qquad (22)$$

where K is a constant depending upon the number of phases and wires, the material of the conductor and the units in which the various quantities are expressed, viz.,

VALUES OF K IN FORMULA 22

Material and units	Single-phase or balanced 4-wire 2-phase*	Balanced 3-wire 3-phase
Copper (98 per cent conductivity):		
E in volts, l in feet, P in kilowatts ...	13.5	10
E in kilovolts, l in miles, P in kilowatts	380	280
Aluminum (61 per cent conductivity):		
E in volts, l in feet, P in kilovolts....	6.5	4.9
E in kilovolts, l in miles, P in kilowatts	185	140
Any material of specific gravity δ having a conductivity of c per cent at 20° C.:		
E in volts, l in feet, P in kilowatts....	$\frac{141\,\delta}{c}$	$\frac{106\,\delta}{c}$
E in kilovolts, l in miles, P in kilowatts	$\frac{3940\,\delta}{c}$	$\frac{2950\,\delta}{c}$

NOTE. — The values of K given for copper and aluminum are taken about 5 per cent greater than their theoretical values to allow for stranding, higher working temperatures, etc. 100 per cent conductivity corresponds to 1.724 microhms per centimeter cube at 20° C.

* For a 3-wire 2-phase system with the middle conductor having a cross-section equal to $\sqrt{2}$ times the cross-section of either outer wire multiply these constants by 0.85, taking for E the voltage (volts or kilovolts) between the middle wire and either outer wire.

Calculation of Commercial Size of Conductor and Corresponding Total Weight. — Formula (22) takes no account of the available commercial sizes of wire. These sizes differ successively by approximately 25 per cent in cross-section. The weight can also be determined by calculating the resistance of the required conductor per 1000 feet or per mile, and taking from the wire tables in the article on *Wires and Cables, Bare,* the nearest commercial size. Neglecting the charging current, the required resistance per unit length of wire is

$$r = \frac{K_1 Q\,(E\cos\phi)^2}{lP} \quad \text{ohms}, \qquad (23)$$

where K_1 is a constant depending on the number of phases and wires and the units in which the other quantities are expressed, as given in the table below.

From the wire table the corresponding weight (w) per unit length of conductor having a resistance nearest to that calculated by formula (23) is obtained; the total weight of conductor, including all wires, is then

$$W = K_2 wl \quad \text{pounds,} \qquad (24)$$

where K_2 is a constant depending upon the number of phases and wires and the units in which w and l are expressed, as given in the table below.

VALUES OF K_1 AND K_2 IN FORMULAS 23 AND 24

Units	Single-phase	Balanced 4-wire 2-phase*	Balanced 3-wire 3-phase
E in volts, l in feet, P in kilowatts, r in ohms per 1000 feet and w in pounds per 1000 feet.	$K_1 = 0.005$ $K_2 = 0.002$	$K_1 = 0.01$ $K_2 = 0.004$	$K_1 = 0.01$ $K_2 = 0.003$
E in kilovolts, l in miles, P in kilowatts, r in ohms per mile and w in pounds per mile.	$K_1 = 5$ $K_2 = 2$	$K_1 = 10$ $K_2 = 4$	$K_1 = 10$ $K_2 = 3$

* The values of K_1 given in this column when used in formula (23) will give the resistance per 1000 feet or per mile of either outer wire in a 3-wire 2-phase system; the middle wire should, for the same energy loss per pound, have a cross-section 41 per cent greater than either outer, but when commercial sizes (B. & S. gage) are used, either a wire one gage number smaller (25 per cent greater cross-section) or two gage numbers smaller (60 per cent greater cross-section) may be used; in the first case the corresponding value of K_2 is 0.81 times the values given in this column and in the second case 0.90 times the values given in this column.

Current per Wire; Heating of Line Conductors. — The size of wire as determined from formula (23) must be ample to carry the required current without overheating. Heating of the line conductors is seldom a limitation in outside overhead lines, but for inside wiring or underground cables the temperature rise may set a limit to the size of wire which may be used. It is therefore always wise to determine the current which the conductor must carry, and make sure that the wire is sufficiently large not to overheat; see articles on *Wires and Cables, Bare; Wires and Cables, Insulated;* and *Wiring of Buildings,* for tables of current-carrying capacity under various conditions.

The current per line wire in amperes may be calculated from the following formulas, in which E *is the kilovolts* between wires at the load end, P *the total kilowatts* (all phases) delivered to the load, and $\cos\phi$ the power factor of the load as a fraction.

$$\left.\begin{array}{ll} \text{Single-phase:} & I = \dfrac{P}{E\cos\phi}. \\[2ex] \text{Two-phase,* 4-wire, balanced:} & I = \dfrac{P}{2E\cos\phi}. \\[2ex] \text{Three-phase, 3-wire, balanced:} & I = \dfrac{P}{\sqrt{3}E\cos\phi}. \end{array}\right\} \qquad (25)$$

Example: Calculation of Weight and Size of Conductor for a Three-phase Line. — A load of 20,000 kilowatts is to be transmitted by means

* E is here the volts between the two wires of the same phase. This formula also gives the current in each outer of a balanced 2-phase 3-wire line, E being the kilovolts between either outer and the middle wire; the current in the middle is $\sqrt{2}$ times the current in each outer.

of an overhead three-phase line of copper wire to a substation 50 miles away operating at 60,000 volts between wires, the frequency being 25 cycles per second, and the power factor of the load 80 per cent with the current lagging; a power loss of 10 per cent of the delivered power to be allowed. From formula (22) the required total weight of copper is

$$W = \frac{285 \times 20{,}000}{10}\left(\frac{50}{60 \times 0.8}\right)^2 = 607{,}000 \text{ pounds}.$$

From formula (23) the required resistance per mile of conductor is

$$r = \frac{10 \times 10(60 \times 0.8)^2}{50 \times 20{,}000} = 0.231 \text{ ohm per mile.}$$

The nearest commercial size is 250,000 circular mils (stranded), which has a resistance of 0.228 ohm per mile at 77° F. (*see Wires and Cables, Bare*), and a weight of 4080 pounds per mile. From formula (24) the total weight is then

$$W = 3 \times 4080 \times 50 = 612{,}000 \text{ pounds.}$$

The current corresponding to the given load is, from formula (25),

$$I = \frac{20{,}000}{\sqrt{3} \times 60 \times 0.8} = 241 \text{ amperes,}$$

which will give a negligible temperature rise in the wire. See section on *Current-carrying Capacity* in article on *Wires and Cables, Bare.*

Calculation of Size and Weight of Conductors for a Given Per cent Voltage Loss. — The voltage loss in an alternating-current line depends not only upon the resistance of the line, but also upon the line reactance, and in the case of long lines upon the electrostatic capacity of the line. It is therefore impossible to express directly in a simple formula the size or weight of the wire in terms of the voltage loss. The most practical method of making such calculations is to assume first that the per cent power loss is equal to the given per cent voltage loss, and calculate the size by formula (23); then using this size of wire calculate the per cent voltage loss by the formulas given below. If this calculated voltage loss differs appreciably from the given voltage loss, choose the next larger or smaller size of wire (accordingly as the calculated loss is greater or less than the given loss) and recalculate the voltage loss, and so on, until the proper size of wire has been found.

FACTORS WHICH AFFECT THE VOLTAGE AND POWER LOSS IN A-C. LINES. — Due to the inductance and electrostatic capacity the per cent voltage loss in an alternating-current line is not so easily calculated as the voltage loss in a direct-current line and is in general different from the per cent power loss, by an amount dependent upon the inductance and capacity of the line, the frequency and the power factor of the load.

Determination of Line Constants. — The four fundamental line constants are the resistance (r) and inductance (L) of the line conductors per unit length and the capacity (C) and leakage conductance (G) per unit length. For all but the shortest transmission lines the mile is usually the most convenient unit of length, and this unit will be used throughout the remainder of this article unless distinctly stated otherwise. Tables of resistance, inductance and capacity both per mile and per 1000 feet are given respectively in the articles on *Wires and Cables, Bare; Inductance and Inductive Reactance;* and *Capacity and Charging Current*. From the inductance and capacity per mile or per 1000 feet may be calculated for any given frequency the reactance x ($= 2\pi fL$) and the capacity

susceptance* b ($= 2\pi fC$) for the corresponding unit of length. In the last two articles just mentioned are given full tables of reactance and of capacity susceptance per mile for frequencies of 60 and 25 cycles per second; dividing the numerical values given in these tables by 5.28 will give the corresponding quantity per 1000 feet. For any other frequency of f cycles per second, multiply the numerical values given in the tables for 25 cycles by the ratio $f/25$; i.e., for 40 cycles the reactance is 1.6 times the reactance for 25 cycles.

Allowance for Skin Effect in Conductors. — For non-magnetic wires the increase in the conductor resistance due to the so-called skin effect is equal to 2 per cent when the quotient

$$\text{(cycles per sec.)} \div \text{(ohms per mile of conductor)} = 485. \tag{26}$$

For smaller values of this quotient the increase of resistance due to skin effect diminishes very rapidly; see article on *Skin Effect*. The skin effect is therefore practically negligible at 25 cycles for all copper conductors smaller than 1,000,000 circular mils, and at 60 cycles for all copper conductors smaller than 450,000 circular mils. The corresponding limiting sizes for aluminum are about 30 per cent larger. The skin effect is quite appreciable in copper or aluminum cables with a steel core; it is usual to neglect the conductivity of the steel core entirely in calculating the resistance of such cables.

Apparent Resistance and Reactance in Unsymmetrical Arrangements of Wires. — When the three wires of a three-phase line are so arranged that they form the three edges of an equilateral prism the reactance of each wire is the same as for one wire of a two-wire line. However, when the wires are arranged all in one plane, as is frequently done, the unequal mutual induction sets up a reactive electromotive force in each outer wire which is not in quadrature with the current in this wire; see equation (20) in the article on *Inductance and Inductive Reactance.* As a result, both the apparent resistance and the apparent reactance of each outer wire is different from its true resistance and reactance. Let r = the true resistance per mile of each wire in ohms, x = the reactance per mile of each wire in ohms, as given in the tables in the article on *Inductance and Inductive Reactance;* and f = the frequency in cycles per second, then the apparent resistances and reactances per mile of the three wires, No. 2 being the middle wire, are:

$$\left.\begin{aligned} r_1 &= r + 0.00121f, & r_2 &= r, & r_3 &= r - 0.00121f, \\ x_1 &= x + 0.00070f, & x_2 &= x, & x_3 &= x + 0.00070f. \end{aligned}\right\} \tag{27}$$

The changes in the apparent resistances do not indicate any change in the power dissipated as heat in the wires but a transfer of energy from one wire to the other by the magnetic field surrounding the wires. These relations are deduced from equation (20) in the article on *Inductance and Inductive Reactance*, assuming sine-wave currents equal in effective value and differing in phase by exactly 120°. The assumption that the currents are exactly balanced cannot be strictly true, since the inequality in the apparent resistances and reactances of the three wires tends to unbalance the system, but the values just given may be taken as a fair approximation when the voltage loss in the line is not over 10 per cent, say. When the line wires are transposed these mutual inductance effects are eliminated from the line as a whole, though the apparent impedances of the three wires in any one "exposure" of the transposition will be different; the transpositions, however, keep the currents balanced.

Similar effects take place in a two-phase three-wire line, see p. 1773.

Leakage Conductance. — The leakage current, even at very high voltages, is usually negligible in power transmission lines, but for telephone lines, the leakage is much greater, due to the large number of small insulators used, and has a very appreciable effect on both the attenuation and distortion of the voice currents. McMeen (*see article on Telephone Lines*) gives the fol-

* The capacity susceptance to neutral as given in the tables (i.e., in micromhos per mile) is equal to the charging current in amperes per mile per million volts between wire and neutral; the charging current for any other voltage to neutral is in proportion.

lowing values of the leakage conductance of telephone lines, the conductance being from one wire to neutral,

$$\left.\begin{array}{ll} \text{Very dry} & g = 0.004 \text{ micromhos per mile.} \\ \text{Average} & g = 0.08 \text{ micromhos per mile.} \\ \text{Very wet} & g = 1.0 \text{ micromhos per mile.} \end{array}\right\} \quad (28)$$

The leakage conductance from one wire to the other is one-half these values. Note that A megohms per mile equals $1/A$ micromhos per mile.

When the voltage is sufficiently high on a power line to cause the formation of corona (*see article on Corona*), an appreciable leakage current passes from one wire to the other. Even for a sine-wave voltage this leakage current is by no means sinusoidal, since its instantaneous values are practically zero except during the peak of the voltage wave, and consequently the corona loss cannot be accurately represented by a constant leakage conductance. Roughly, however, calling p_c the average value of the corona loss from each wire in watts per mile, corresponding to the given line voltage, the leakage conductance to neutral in micromhos per mile due to the corona may be taken equal to

$$g_c = \frac{p_c}{V^2}, \qquad (29)$$

where V is the effective (r.m.s.) kilovolts to neutral.

Rise of Voltage at Load End of Line on Open Circuit. — In every alternating-current transmission line the voltage at the load end when this end is open is higher than at the generator end, although in short low-frequency lines this rise is inappreciable. In overhead lines for which the product

$$(\text{cycles per sec.}) \times (\text{length of line* in miles}) < 10{,}000, \qquad (30)$$

this no-load rise as a percentage of the delivered voltage is, to a close approximation when the resistance of the wire is less than that of a No. 0 B. & S. copper wire, equal to $\left(\dfrac{fl}{4000}\right)^2$, where f is the frequency in cycles per second, and l the length of the line in miles. For example, in a 25-cycle line 160 miles long this no-load rise is 1 per cent of the delivered voltage, in a 60-cycle line of the same length the no-load rise in voltage is 5.8 per cent of the delivered voltage. The relation expressed by the above formula under the conditions stated is independent of the value of the delivered voltage and of the size and spacing of the wires, at least for all practical cases.

This rise, which is due to the charging current taken by the line, may be looked upon as present at all loads, but when the load is appreciable the voltage drop, due to the load current, unless leading, more than offsets this voltage rise. A leading current may increase the rise in voltage at the load end as the load comes on.

Use of Synchronous Condensers (or Phase Modifiers) to Maintain Constant Voltage at Load End. — By making the line current at the load end of the line lead the line voltage by the proper phase angle, it is possible to compensate entirely for the change in the load voltage which normally takes place as the load current increases. An overexcited synchronous motor connected in parallel with the load is sometimes used for this purpose, as described in detail in the article on *Motors, Synchronous*. A synchronous motor so used is commonly called a synchronous condenser, since the current taken by it leads the voltage impressed on its terminals. A striking example of the use of synchronous motors for this purpose is in connection with the 150,000-volt, 240-mile

* Length of each conductor.

line from Big Creek, Cal., to Los Angeles, constructed in 1913. Two separate lines are used, the total generator output is 70,000 kilovolt-amperes, and the voltage at the load end is controlled by two 15,000-kilovolt-ampere overexcited synchronous motors.

METHODS OF CALCULATING VOLTAGE AND POWER LOSS IN A-C. LINES. — The absolutely rigorous calculation of an alternating-current line requires that the distributed nature of the inductance and capacity be considered, i.e., that the line be considered as made up of an infinite number of sections such as shown in Fig. **7**. However, simpler approximate methods may be employed for nearly all power lines such as are now used, and give results sufficiently accurate for all practical purposes. The accuracy of these approximate methods depends primarily upon the frequency and length of the line; the less the value of the product of these two quantities the simpler is the method which may be employed. Accurate calculation of even a short a-c. line with distributed load can be effected only by rather complicated network equations; see under *Kirchhoff's Laws* in the article on *Alternating Currents*.

Limitations of Approximate Methods of A-C. Line Calculations. — The approximate methods of calculating a-c. lines, in the order of their simplicity, may be designated as (1) the simple impedance method; (2) the single end-condenser method; (3) the middle condenser or "T" method and (4) the split condenser or "π" method; the "π" method is also called the "U" method. For short low-voltage lines, the simple impedance method, which neglects entirely the charging current, is usually sufficiently accurate. The single end-condenser method takes the charging current into account in a manner usually sufficiently accurate for all but the longest high-voltage lines. The "T" and "π" methods are still more accurate, but for exact calculations the rigorous method given on p. 1781ff. should be used. In fact, it is always well to check an approximate solution by this exact method.

Fundamental Assumptions on which Formulas Given Below are Based. — All the formulas given below are based on the assumptions of pure sine-wave currents and voltages and a perfectly balanced * system. By using the voltage to neutral instead of the voltage between wires, the formulas are also put in such shape that they may be applied directly to either a single-phase, a two-phase four-wire line or to a three-phase three-wire line, a two-phase four-wire line being considered as two separate single-phase lines. The fundamental idea in this method of treatment is that each line wire is considered as a separate circuit. The return wire shown in the various diagrams is therefore to be considered as having no impedance. This method of treatment of *balanced* polyphase circuits is strictly accurate (*see Alternating Currents*), but when the system is not balanced the circuit must be treated by the more general methods of network calculations; see section on *Kirchhoff's Laws in Symbolic Notation* in the article on *Alternating Currents*. Also, when the voltage and current waves are not sinusoidal, each harmonic must be treated separately as described in the article on *Alternating Currents*. The calculation of a two-phase three-wire line is treated separately, p. 1773.

SIMPLE IMPEDANCE METHOD.—This method is based upon the assumption that the electrostatic capacity of the line may be neglected entirely, and that the line may be considered simply as an impedance in series with the load, this impedance having a resistance equal to the total resistance of the line conductor and a reactance equal to the total inductive reactance of the line

* The meaning of a "balanced" system is fully explained in the article on *Alternating Currents* (q.v.).

conductor. Fig. 2 is a diagram of the circuit, and Fig. 2a is a complete vector diagram of the current and voltage. When the wires are unsymmetrically arranged transpositions are assumed.

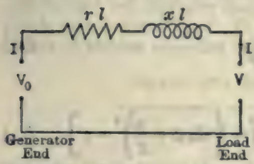

Fig. 2.

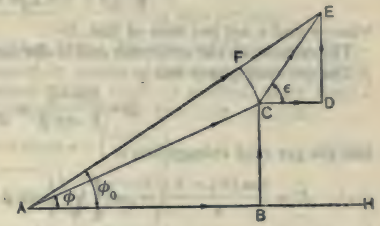

Fig. 2a.

$\overline{AH} = I$, $\quad \overline{BC} = V \sin \phi$, $\quad \overline{CE} = zlI$,
$\overline{AC} = V$, $\quad \overline{CD} = rlI$, $\quad \overline{AE} = V_0$,
$\overline{AB} = V \cos \phi$, $\quad \overline{DE} = xlI$, $\quad \overline{FE} = V_0 - V$

Let V = volts to neutral at load end of the line (= volts between wires divided by 2 in case of a single-phase line, and volts between wires divided by $\sqrt{3}$ in the case of a three-phase line),

I = amperes per wire; see formula (25) above,

l = length of each line wire in miles,

$Z = \dfrac{V}{lI}$ = "equivalent" impedance of the load per mile of line,

$\cos \phi$ = power factor of the load at end of line,

$\sin \phi = \sqrt{1 - \cos^2 \phi}$ = the reactive factor of the load; $\sin \phi$ is to be taken positive for a lagging and negative for a leading current,

V_0 = volts to neutral at the generator end of the line,

r = conductor resistance per mile of line in ohms; see tables in article on *Wires and Cables, Bare,*

x = conductor reactance per mile of line, in ohms; see tables in article on *Inductance and Inductive Reactance,*

$z = \sqrt{r^2 + x^2}$ = conductor impedance * per mile of line, in ohms,

Q = per cent power loss, as a percentage of delivered power,

D = per cent voltage loss, as a percentage of delivered voltage.

From the vector diagram it is evident that the voltage at the generator end is

$$V_0 = \sqrt{(V \cos \phi + rlI)^2 + (V \sin \phi + xlI)^2}, \tag{31}$$

---

* A convenient way of calculating an expression of the form $\sqrt{a^2 + b^2}$ is to write it $a\sqrt{1 + \left(\frac{b}{a}\right)^2}$ or $b\sqrt{1 + \left(\frac{a}{b}\right)^2}$ accordingly as a is greater or less than b; the expression under the radical will then always lie between the numbers 1 and 2, and no difficulty will be experienced with decimal points. When b/a is less than 0.3, then the expression $\sqrt{a^2 + b^2} = a + \dfrac{b^2}{2a}$ with an error of less than 0.1 per cent, and when a/b is less than 0.3, then $\sqrt{a^2 + b^2} = b + \dfrac{a^2}{2b}$ with an error of less than 0.1 per cent. The error in the approximate expressions, diminishes very rapidly as the ratio of b/a or a/b, as the case may be, decreases, being only 0.02 per cent when the ratio is 0.2.

which may also be written

$$V_0 = V\sqrt{\left(\cos\phi + \frac{r}{Z}\right)^2 + \left(\sin\phi + \frac{x}{Z}\right)^2}, \tag{32}$$

where r and x are per mile of line.

The current at the generator end is the same as at the load end.

The per cent power loss is

$$Q = \frac{100\, rlI}{V\cos\phi} = \frac{100\, r}{Z\cos\phi}, \tag{33}$$

and the per cent voltage loss is

$$D = \frac{100\,(V_0 - V)}{V} = 100\left[\sqrt{\left(\cos\phi + \frac{r}{Z}\right)^2 + \left(\sin\phi + \frac{x}{Z}\right)^2} - 1\right]. \tag{34}$$

The power factor at the generator end is

$$\cos\phi_0 = \left(\frac{100 + Q}{100 + D}\right)\cos\phi. \tag{35}$$

Relation between Impedance Drop and Voltage Loss. — The total impedance drop, which is zlI volts, should be carefully distinguished from the voltage loss, which is $v = V_0 - V$ volts. The vector diagram, Fig. 2a, will make the difference clear. For a given impedance drop of, say, A per cent, the voltage at the load end of the line may be anything from A per cent less than the voltage at the generator end to A per cent greater than the voltage at the generator end. The determining factor is the difference between the power-factor angle (ϕ) of the load and the power-factor angle $\left(\epsilon = \tan^{-1}\frac{x}{r}\right)$ of the line; only when $\epsilon - \phi = 0$ are the voltage loss and impedance drop the same. When $\epsilon - \phi$ is greater than 90° (which may occur for a leading current, since ϕ is then negative) the voltage at the load end will in general be higher than at the generator end although the impedance drop in the line may be very large. As a fair approximation, when the impedance drop is less than 20 per cent, that is, when z/Z is less than 0.2, the percentage voltage loss may be written

$$D = \frac{100\, z}{Z}\cos(\epsilon - \phi), \tag{36}$$

z and Z being the impedances of the line and load respectively, and ϵ and ϕ the power-factor angles of the line and load respectively.

Example of Calculation by Simple Impedance Method. — Take the case of a three-phase, 60-cycle line 50 miles long, the wires being No. 0000 A.W.G. (or B. & S.) stranded copper spaced symmetrically with 6 feet between centers, and let the load be 15,000 kilowatts at 60,000 volts between wires and at a power factor of 80 per cent with lagging current. The voltage to neutral is then $60{,}000 \div \sqrt{3} = 34{,}600$ and the current per wire $15{,}000 \div (\sqrt{3} \times 60 \times 0.8) = 180$ amperes. The resistance of each wire per mile is 0.269 ohm at 77° F., and the reactance 0.728 ohm. The equivalent impedance of the load per mile of line is $Z = 34{,}600 \div (50 \times 180) = 3.84$. $\cos\phi = 0.8$ and $\sin\phi = \sqrt{1 - \overline{0.8}^2} = 0.6$. Whence

$$\text{Per cent power loss} = Q = \frac{100 \times 0.269}{3.84 \times 0.8} = 8.76 \text{ per cent,}$$

$$\text{Per cent voltage loss} = D = 100\left[\sqrt{\left(0.8 + \frac{0.269}{3.84}\right)^2 + \left(0.6 + \frac{0.728}{3.84}\right)^2} - 1\right] = 17.5\%,$$

Power factor at generator end $= \cos\phi_0 = \left(\frac{100 + 8.76}{100 + 17.5}\right) \times 0.80 = 74.1$ per cent,

$$\text{Per cent impedance drop} = \frac{100\sqrt{\overline{0.269}^2 + \overline{0.728}^2}}{3.84} = 20.2 \text{ per cent.}$$

Graphical Determination of Voltage Loss; Mershon's Chart (Fig. 3). — The voltage loss may also be determined by means of the chart shown in Fig. 3, which was devised by R. D. Mershon. This chart is nothing more than a means of solving equation (32) graphically. To use the chart calculate

$$\text{the per cent resistance drop} = r\left(\frac{100\,lI}{V}\right), \tag{37}$$

$$\text{the per cent reactance drop} = x\left(\frac{100\,lI}{V}\right), \tag{38}$$

and from the point on the curve marked "0" where this curve cuts the vertical line corresponding to the load power factor lay off horizontally the per cent resistance drop, and from the end of this horizontal line lay off vertically upward the per cent reactance drop; the per cent voltage loss is then given by the number on the circle through the end of this vertical line.

Taking the same example as given in the preceding section, $(100\,lI) \div V = (100 \times 50 \times 180) \div 34{,}600 = 26.0$, whence the per cent resistance drop is $0.269 \times 26.0 = 7.0$ per cent and the per cent reactance drop is $0.728 \times 26.0 = 18.9$ and therefore from the chart the per cent voltage loss is 17.6 per cent.

Calculations in Terms of Voltage at Generator End. — When the volts to neutral V_0 at the generator end are given instead of the volts to neutral V at the load end, the calculations for a line of total resistance of R ohms per wire, and total reactance of X ohms per wire for a load of P kilowatts per wire* at a power factor of $\cos\phi$ at the load end may be made in the same manner as above by first finding V by the formula,

$$V = A\sqrt{1 \pm \sqrt{1 - \frac{(R^2 + X^2)\,P^2 \times 10^6}{A^4\cos^2\phi}}}, \tag{39}$$

where

$$A = V_0\sqrt{\frac{1}{2} - \frac{1000\,P\,(R\cos\phi + X\sin\phi)}{V_0^2\cos\phi}}. \tag{40}$$

The plus or minus sign in the formula for V arises from the fact that for a given amount of delivered power and given voltage at the generator end two different voltages at the load end are possible; for an inductive load the voltage corresponding to the minus sign will in general be less than 50 per cent of the voltage at the generator end, and the "minus sign" solution is therefore usually of no practical importance. The maximum power which can be delivered at the load end for a given voltage at the generator end is

$$P_m = \frac{V_0\cos\phi}{2000\,(\sqrt{R^2 + X^2} + R\cos\phi + X\sin\phi)}. \tag{41}$$

Voltage and Power Loss in Two-phase Three-wire Line. — Take the usual case of the three wires in the same plane, with the common wire midway between the two outers. Let r be the resistance per mile of each outer, and r_1 the resistance per mile of the common wire, taken from the tables in the article on *Wires and Cables, Bare*, and x and x_1 the corresponding reactances per mile taken from the tables in the article on *Inductance and Inductive Reactance*, for a spacing equal to the distance between the middle wire and each outer. Let E be the voltage between each outer and the middle wire, I the amperes in each outer, $\cos\phi$ the power factor of the load, and l the length of the line. Assuming a balanced load, then the voltage between the two outers is $\sqrt{2}\,E$ and the current in the middle wire is $\sqrt{2}\,I$, and the total power delivered to the load is $(2\,EI\cos\phi) \div 1000$ kilowatts. The per cent power loss is then

$$Q = \frac{100\,(r + r_1)\,lI}{E\cos\phi}.$$

* Calling P_t the total kilowatts delivered to the load, then $P = P_t/2$ for a single phase line. $P = P_t/3$ for a three-phase line and $P = P_t/4$ for a two-phase four-wire line

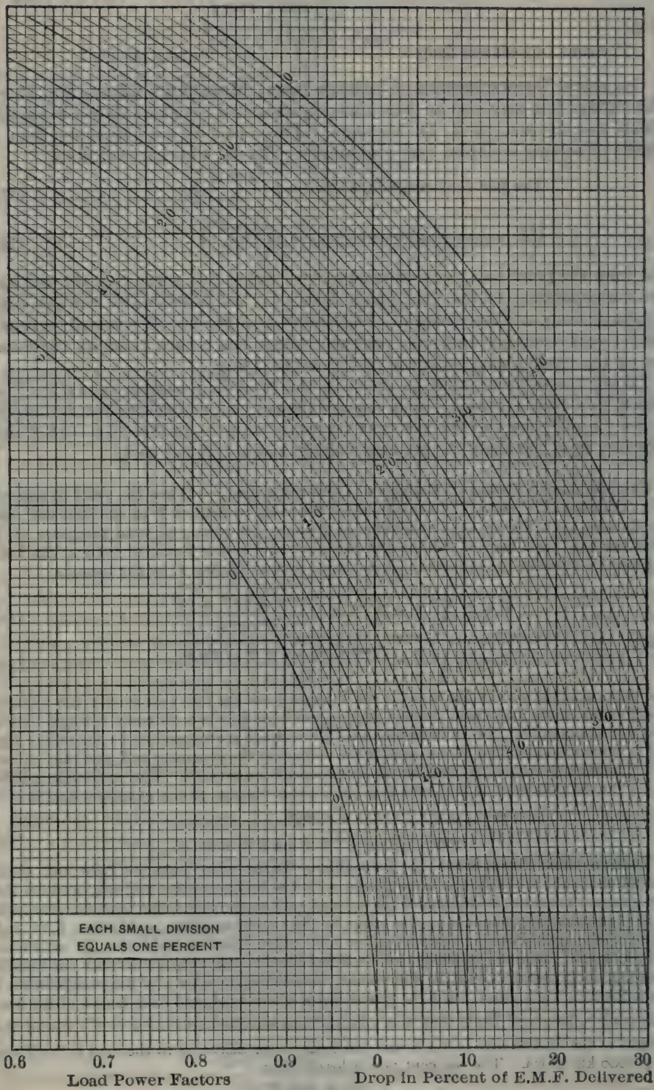

Fig. 3. Mershon's Chart for Calculating Drop in A-C. Line

The equivalent resistances and reactances of the line for the two phases, assuming the currents in the two outer wires to be exactly equal in effective value and to differ in phase by exactly 90°, are

$$R_1 = (r + r_1 - x_1 + 0.0014 f)\, l, \qquad X_1 = (x + x_1 + r_1)\, l,$$
$$R_2 = (r + r_1 + x_1 - 0.0014 f)\, l, \qquad X_2 = (x + x_1 - r_1)\, l,$$

where f is the frequency in cycles per second. When the wires are transposed only the term $0.0014\,f$ in the expressions for R_1 and R_2 go out, the other terms in the R's and X's remaining unaltered. The voltages at the generator end for the two phases are then

$$E_1 = E\sqrt{\left(\cos\phi + \frac{R_1 I}{E}\right)^2 + \left(\sin\phi + \frac{X_1 I}{E}\right)^2},$$
$$E_2 = E\sqrt{\left(\cos\phi + \frac{R_2 I}{E}\right)^2 + \left(\sin\phi + \frac{X_2 I}{E}\right)^2}.$$

Of course, if the voltages E_1 and E_2 are maintained equal at the generator end, which is usually the case, the assumptions of equal currents and equal voltages at the load end are incorrect, but the above formulas may be used to obtain an approximate value of the voltage loss in the two phases. The unbalancing due to the unequal equivalent impedances of the two phases renders the two-phase three-wire system undesirable except for lines in which the voltage loss is a small percentage of the voltage at the load, and the unbalancing therefore small.

DWIGHT'S TRANSMISSION LINE CHART.— The chart given in Fig. 3A, though approximate only, gives a simple and ordinarily sufficiently accurate means for calculating the voltage drop or size of wire for any alternating-current, overhead transmission line which is not over 100 miles long. When the resistance drop and the reactance drop are each under 15 per cent of the line voltage, the error in the voltage drop as calculated by this chart is less than 0.5 per cent of the line voltage.

To use the chart, lay a straight edge across it from the proper spacing point to the proper size of wire, and read on the vertical scale corresponding to the given power factor the volts per mile per ampere; call this V. Or, vice versa, when V the volts drop to neutral per mile per ampere is known, lay the straight edge across the chart from the proper spacing point to the given value of V as read on the vertical scale corresponding to the given power factor, and read the size of wire on the right-hand vertical scale. V represents a drop in voltage when it is read on the same side of the zero line as the resistance point used, and a rise in voltage when it is on the opposite side. Let V = volts drop per mile per ampere, E = volts between wires at the receiver end, U = kilovolt-amperes delivered to the receiver, l = length of line, in miles. Then

$$\text{Total drop, in per cent of } E = \frac{100{,}000\, U\, L\, V}{E^2} - 100\, K\left(\frac{l}{1{,}000}\right)^3$$

$$\text{Total drop, in volts} = \frac{1{,}000\, U\, l\, V}{E} - E\, K\left(\frac{l}{1{,}000}\right)^3$$

where K = 2.16 for 60 cycles, and 0.375 for 25 cycles. The last term in each of these two formulas is a correction factor which takes into account the effect of the line capacity, and need not be considered when the line is less than 30 miles in length.

When the three wires of a three-phase line are separated by distances a, b and c, use as the effective spacing $s = \sqrt[3]{a\, b\, c}$. When the three wires are all in the same plane, and the middle wire is at the distance a from each of the others, use as the effective spacing $s = 1.26\, a$.

The above formulas are for two-phase and three-phase lines. For a single-phase line use $2\,V$ in place of V.

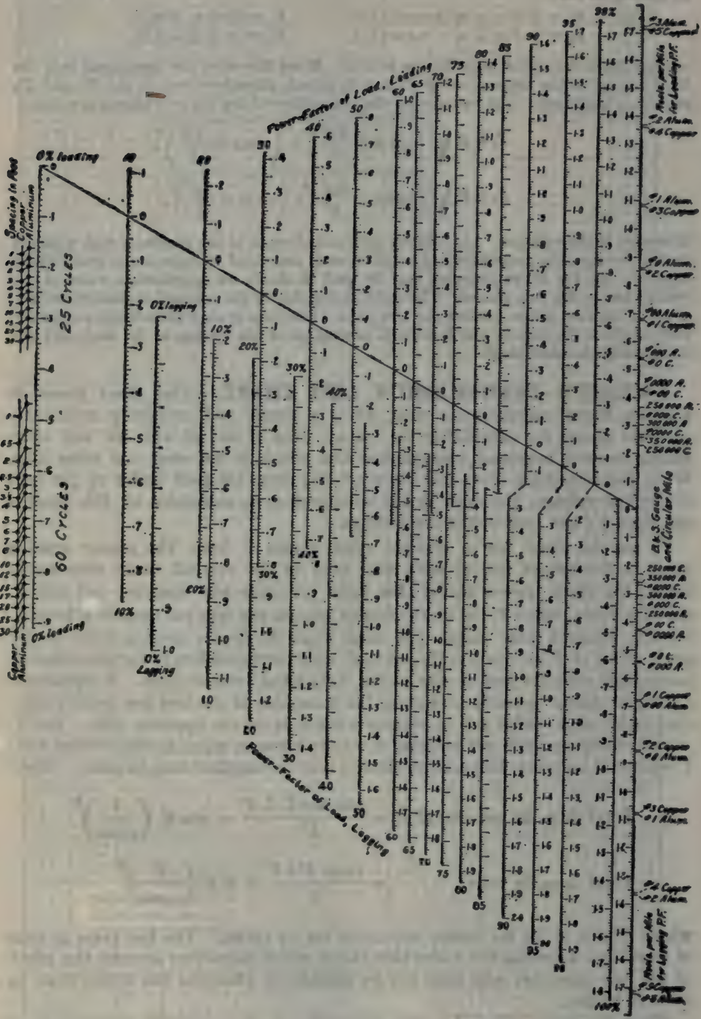

Fig. 3A. Dwight's Transmission Line Chart.

Example.—Length of line, 100 miles; effective spacing, 8 feet; No. 3 copper, B. & S. gauge; load, measured at the receiver end, 3,000 kv-a., 66,000 volts, 90 per cent power-factor lagging, 3-phase, 60 cycles.

Lay a straight edge from the 8-ft. spacing point for copper conductor, 60 cycles, to the point for No. 3 copper on the right-hand vertical scale. It crosses the 90 per cent power-factor line at $V = 1.34$.

$$\text{Per cent drop} = \frac{100{,}000 \times 3{,}000 \times 100 \times 1.34}{66{,}000 \times 66{,}000} - \frac{100 \times 2.16}{100} = 7.07\%$$

The voltage rise at no load is 2.16 per cent.

The regulation at the receiver end, or change in voltage from full load to no load, is 9.23 per cent of E. The exact values of the voltage drop and regulation as calculated by the formulas on page 1786 are 7.08 per cent and 9.40 per cent. If the power factor had been leading instead of lagging, V would have been 0.70, and the drop would have been 2.66 per cent, as against the exact value of 2.51 per cent. In the above example, the errors due to the chart are less than 0.2 per cent of line voltage.

SINGLE END-CONDENSER METHOD. — This method assumes that the total current at the load end is equal to the actual load current plus (vectorially) the charging current which would be taken by a single condenser shunted across the line at the load end, the capacity of this condenser being taken equal to the total capacity of the line. This method gives too low a voltage at the generator end by approximately the same amount that the straight impedance

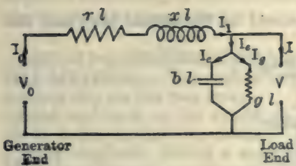

Fig. 4.

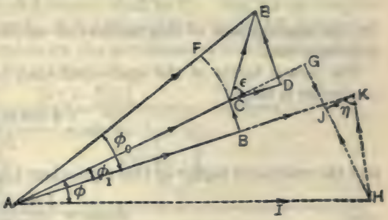

Fig. 4a.

$\overline{AH} = I$,
$\overline{AC} = V$,
$\overline{AG} = I \cos \phi$,
$\overline{GH} = I \sin \phi$,
$\overline{HJ} = blV \times 10^{-6}$,
$\overline{JK} = glV \times 10^{-6}$,
$\overline{HK} = ylV \times 10^{-6}$,
$\overline{AK} = I_1 = I_0$,

$\overline{AB} = V \cos \phi_1$,
$\overline{BC} = V \sin \phi_1$,
$\overline{CD} = rlI_1$,
$\overline{DE} = xlI_1$,
$\overline{CE} = zlI_1$,
$\overline{AE} = V_0$,
$\overline{FE} = e$,

method gives it too high, and also gives the power loss too low by approximately the same amount that the straight impedance method gives it too high. By averaging the losses obtained by the two methods a close approximation to their true values is obtained.

Fig. 4 is a diagram of the circuit and Fig. 4a is a complete vector diagram of the voltage and current; voltages are shown by full lines and currents by dotted lines. The diagrams and formulas are for the general case of a line with leakage, but for nearly all practical cases the leakage may be neglected.

The effect of the electrostatic capacity of the line is to change both the numerical value and the phase angle of the line current. Or, the condenser and the load may be looked upon as forming together an equivalent load taking a current I_1 at a power factor $\cos\phi_1$ differing from the actual current and power factor of the load. Let

V = volts to neutral at the load end of the line,
I = actual amperes per wire at the load end,
$\cos\phi$ = actual power factor at the load end,
$\sin\phi = \sqrt{1-\cos^2\phi}$ = actual reactive factor at the load end,
l = length of each line wire in miles,
b = capacity susceptance to neutral per mile of line, in micromhos, see tables in the article on *Capacity and Charging Current*,
g = leakage conductance to neutral per mile of line in micromhos, usually taken equal to zero in power lines, as explained above,
$y = \sqrt{g^2+b^2}$ = dielectric admittance to neutral per mile of conductor, in micromhos. Note that for no leakage $y = b$.

The total leakage current, total charging current and total exciting current of the line are then respectively

$$I_g = glV \times 10^{-6}, \qquad I_c = blV \times 10^{-6}, \qquad I_e = \sqrt{I_g^2 + I_c^2}. \tag{42}$$

The total line current, i.e., the resultant of the actual load current and the exciting current, is

$$I_1 = I\sqrt{\left(\cos\phi + \frac{I_g}{I}\right)^2 + \left(\sin\phi - \frac{I_c}{I}\right)^2}. \tag{43}$$

On the assumptions of this method of calculation I_1 is also the current at the generator end.

The power factor of the equivalent load formed by the actual load and the condenser is then

$$\cos\phi_1 = \frac{I\cos\phi + I_g}{I_1}, \tag{44}$$

and the reactance factor of this equivalent load is

$$\sin\phi_1 = \frac{I\sin\phi - I_c}{I_1}. \tag{45}$$

The formulas given above for the straight impedance method are then directly applicable, using for I, $\cos\phi$ and $\sin\phi$ in those formulas the values of I_1, $\cos\phi_1$, and $\sin\phi_1$ just calculated; i.e., the straight impedance method is to be applied not to the actual load but to the equivalent load formed by the actual load and a condenser having an admittance equal to the total admittance of the line.

Example of Calculation by Single End-Condenser Method. — Three-phase line, 50 miles long, No. 0000 A.W.G. stranded copper, 6 feet between centers, frequency 60 cycles, load 15,000 kilowatts, 60,000 volts between wires at load end, 80 per cent power factor at load. This is the same example as used above for the straight impedance method. Then $V = 34{,}600$, $I = 180$, $\cos\phi = 0.8$, $\sin\phi = 0.6$, $l = 50$, $b = 6.03$, $g = 0$, $r = 0.269$, $x = 0.728$. Then,

Charging current $= I_c = 6.03 \times 50 \times 34{,}600 \times 10^{-6} = 10.4$ amperes,

$$\text{Resultant current} = I_1 = 180\sqrt{(0.8)^2 + \left(0.6 - \frac{10.4}{180}\right)^2} = 174 \text{ amperes,}$$

$$\text{Power factor of equivalent load} = \cos\phi_1 = \frac{180 \times 0.8}{174} = 0.828.$$

$$\text{Reactive factor of equivalent load} = \sin\phi_1 = \frac{180 \times 0.6 - 10.4}{174} = 0.561,$$

$$\text{Impedance of equivalent load per mile of line} = Z_1 = \frac{34{,}600}{50 \times 174} = 3.98,$$

$$\text{Per cent power loss} = Q = \frac{100 \times 0.269}{3.98 \times 0.828} = 8.16 \text{ per cent},$$

Per cent voltage loss $= D$

$$= 100\left[\sqrt{\left(0.828 + \frac{0.269}{3.98}\right)^2 + \left(0.561 + \frac{0.728}{3.98}\right)^2} - 1\right] = 16.30 \text{ per cent},$$

$$\text{Power factor at generator end} = \cos\phi_0 = \left(\frac{100 + 8.16}{100 + 16.3}\right) \times 0.822 = 76.4 \text{ per cent.}$$

The per cent power loss and voltage loss obtained by the straight impedance method neglecting the line capacity are respectively 8.76 and 17.6 per cent. As noted above the single end-condenser method gives these losses too low (for inductive loads) by approximately the same amount that the straight impedance method gives them too high, whence closer approximations to the true losses are: per cent power loss $= (8.16 + 8.76) \div 2 = 8.46$ and per cent voltage loss $= (16.3 + 17.6) \div 2 = 17.0$.

Calculation of Effect of Synchronous Condenser. — Formulas (43) to (45) apply directly to the calculation of the effect of a synchronous condenser at the end of the line taking a current having an in-phase or energy component equal to I_g and a quadrature leading component equal to I_c. Fig. 4a then represents the vector relations of the currents and voltages, the vector JK being the in-phase component of the current taken by the synchronous condenser and HJ the quadrature component.

MIDDLE-CONDENSER OR "T" METHOD. — This method assumes that the line may be considered equivalent to the circuit shown in Fig. 5, which represents a single condenser, having a capacity and leakage conductance equal respectively to the total capacity and leakage conductance of the line, shunted across the line at its middle point, the resistance and inductance on each side of this condenser being equal respectively to half the total conductor resistance and inductance.

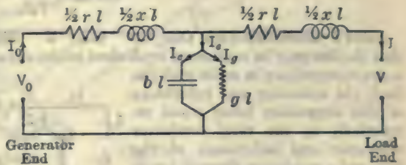

Fig. 5.

An inspection of Fig. 5 will show that the half of the line nearest the load is represented by a straight impedance in series with the load. Hence the voltage, current and power factor at the middle point of the line may be figured by the straight impedance method given above. Then, considering the voltage, current and power factor thus calculated as forming a load at the middle point of the line, the second half of the line may be calculated by the single end-condenser method just described.

By assigning proper values (which are somewhat complicated functions of the actual line constants and the frequency) to the constants of the "T" circuit shown in Fig. 5, this circuit may be made *exactly equivalent*, as far as the relations between the voltage, current and power at the two ends are concerned, *to a line of any length and for any frequency;* the "corrected" constants are entirely independent of the voltage, current and power taken by the load; see p. 1786.

Solution of the "T" Circuit in Symbolic Notation. — The solution of the circuit shown in Fig. 5 is also given below in terms of complex quantities (*see under Symbolic Notation in the article on Alternating Currents*). Let

V = numerical value of volts to neutral at the load end of the line,
I = amperes per wire at the load end of the line, taken as the vector of reference,
$\cos\phi$ = power factor at the load end of the line,
$V = V\cos\phi + jV\sin\phi$ = volts to neutral at the load end of the line, referred to the current at the load end,
$z = r + jx$ = conductor impedance in ohms per mile of wire, r being the resistance per mile and x the reactance per mile,
$y = g + jb$ = dielectric admittance, one wire to neutral, in micromhos per mile of line, where g is the leakage conductance per mile and b the capacity susceptance per mile,
l = length of each wire in miles.

Put

$$M = 10^{-6} yzl^2 = [(gr - bx) + j(br + gx)]\, 10^{-6} l^2. \qquad (46)$$

Then the current at the generator end, in symbolic notation, is

$$I_0 = (1 + \tfrac{1}{2} M)\, I + 10^{-6} ylV, \qquad (47)$$

and the volts to neutral at the generator end, in symbolic notation, are

$$V_0 = (1 + \tfrac{1}{2} M)\, V + zl\,(1 + \tfrac{1}{4} M) I. \qquad (48)$$

Calling I_0' and I_0'' the real and "j" components respectively of the current I_0, and V_0' and V_0'' the real and "j" components of the voltage V_0, then the numerical values of the current and voltage at the generator end are

$$I_0 = \sqrt{(I_0')^2 + (I_0'')^2}, \quad \text{and} \quad V_0 = \sqrt{(V_0')^2 + (V_0'')^2}, \qquad (49)$$

and the power input per wire at the generator end is

$$P_0 = V_0' I_0' + V_0'' I_0''. \qquad (50)$$

In applying this last formula particular attention must be paid to the signs of the quantities involved, e.g., if $V_0 = 1000 + j\,300$ and $I_0 = 100 - j\,20$, then $V_0' = 1000$, $V_0'' = 300$, $I_0' = 100$ and $I_0'' = -20$, and $P_0 = 1000 \times 100 - 300 \times 20$. The power factor at the generator end is

$$\cos\phi_0 = \frac{P_0}{V_0 I_0}. \qquad (51)$$

SPLIT-CONDENSER OR "π" OR "U" METHOD. — This method assumes that the line may be considered equivalent to the circuit shown in Fig. 6, which represents a single impedance in series with the load, the resistance and inductance of this impedance being equal respectively to the total conductor resistance and inductance of the line, and a condenser shunted across the line at each end, each condenser having a capacity and leakage conductance equal respectively to half the total capacity and half the total leakage conductance of the line.

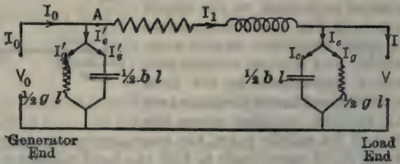

Fig. 6.

An inspection of Fig. 6 will show that the calculation of the line up to the point A at which the condenser at the generator end is connected is exactly the same as for the single end-condenser method described above, except that the capacity and leakage of the end condenser are taken respectively as half the total capacity and half the total leakage of the line. The voltage at the point A is the same as the voltage at the generator end, but to find the current I_0 at the generator end the exciting current I_e' of the condenser shunted across the line at the generator end must be added (vectorially) to the current I_1 at A. Using the same notation as in the section above on the single end-condenser method, the total generator current is then

$$I_0 = I_1 \sqrt{\left(\cos\phi_e + \frac{I_g'}{I_1}\right)^2 + \left(\sin\phi_e - \frac{I_c'}{I_1}\right)^2},$$

where

$$I_g' = \tfrac{1}{2} glV_0 \times 10^{-6} \quad \text{and} \quad I_c' = \tfrac{1}{2} blV_0 \times 10^{-6}.$$

As in the case of the "T" circuit, a "π" circuit, such as shown in Fig. 6, may be given such "corrected" constants as will make it exactly equivalent for any one frequency, as far as the terminal conditions are concerned, to a line of any length whatever no matter how high this frequency may be; see p. 1787.

Solution of the "π" Circuit in Symbolic Notation. — Using the same notation as in the section above on the *Solution of the "T" Circuit in Symbolic Notation*, and calculating the quantity M by the same formula, viz., equation (46), the current and voltage at the generator end in symbolic notation are respectively

$$I_0 = (1 + \tfrac{1}{2} M) I + 10^{-6} yl (1 + \tfrac{1}{4} M) V, \qquad (52)$$

$$V_0 = (1 + \tfrac{1}{2} M) V + zlI. \qquad (53)$$

The numerical values of these quantities, the power and the power factor are then to be calculated in the same manner as for the "T" circuit, viz., by formulas (49) to (51).

EXACT CALCULATION OF A-C. LINES OF ANY LENGTH AND FREQUENCY.* — The charging current (and also the leakage current) for any element of a transmission line passes through only that portion of the line conductor between the given element and the generator end. The exact determination of the line current, voltage and power at any point therefore requires that this fact be taken into account; in other words the capacity and leakage of the line are distributed and not lumped as assumed in the above methods of calculation. In Fig. 7 are shown three successive elements of one wire of a line, a return or neutral of zero impedance being also shown to complete the circuit. This method of treating separately each wire of a line is fully explained above, p. 1770.

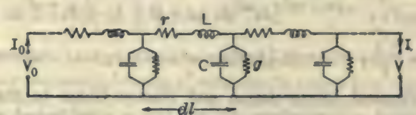

Fig. 7.

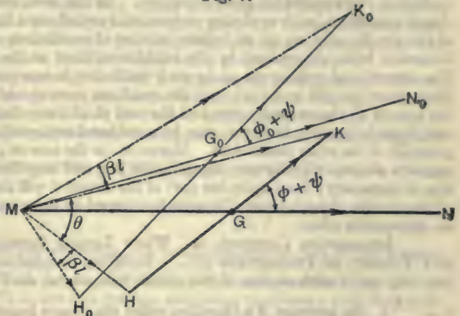

Fig. 7a.

$\overline{MN} = I,$ $\quad \overline{MK_0} = A\epsilon^{\alpha l},$

$\overline{HK} = YV,$ $\quad \overline{MH_0} = B\epsilon^{-\alpha l},$

$\overline{MK} = A,$ $\quad \overline{MN_0} = I_0,$

$\overline{MH} = B,$ $\quad \overline{H_0K_0} = YV_0.$

In order to make clear the physical meaning of the various terms employed, the general solution is first given in terms of instantaneous values. Following this are given the working formulas (1) in terms of exponentials, viz., ϵ^{x} and ϵ^{-x}, (2) in terms of real hyperbolic functions, and (3) in terms of hyperbolic functions of complex angles.

General Equations of Transmission Line. — Let l = the distance in miles of any point along a transmission line, measured from any arbitrarily chosen point (say from the load end); let i = the instantaneous current in amperes in the conductor at this point, taken positive in the direction *opposite* to that in which l is measured (i.e., positive when *toward* the load end); let v = the potential drop in volts between this point and the neutral; and let r = the conductor resistance in ohms per mile, L = the inductance of the conductor in henries per mile, C = the capacity to neutral in microfarads per mile

* This section is abstracted from the lecture notes of Dr. H. Pender.

and g = the leakage conductance to neutral in micromhos per mile. The following relations then hold at any point along the line, t being time in seconds measured from any arbitrarily chosen instant:

$$\frac{dv}{dl} = ri + L\frac{di}{dt}, \tag{54}$$

$$10^6\frac{di}{dl} = gv + C\frac{dv}{dt}. \tag{55}$$

If the circuit is composed of two or more sections of different constants (e.g., an overhead section and an underground section, or a circuit formed by a step-up transformer, transmission line and a step-down transformer) then a similar set of equations holds for each section of the circuit, the constants r, L, g, and C being in general different for the several sections.

The complete solution of any two equations of the form given by (54) and (55) consists of an infinite series of terms for both v and i, corresponding terms in the two series having the following values:

$$i = \sqrt{2}\,\epsilon^{-(u-s)t}\left[A\epsilon^{\alpha l}\sin(\omega t + \beta l + \theta_1) + B\epsilon^{-\alpha l}\sin(\omega t - \beta l + \theta_2)\right], \tag{56}$$

$$v = \frac{\sqrt{2}}{Y}\epsilon^{-(u-s)t}\left[A\epsilon^{\alpha l}\sin(\omega t + \beta l + \theta_1 - \psi) - B\epsilon^{-\alpha l}\sin(\omega t - \beta l + \theta_2 - \psi)\right]. \tag{57}$$

Physical Interpretation and Names Given to the Various Constants. — The constant ω in equations (56) and (57) is equal to $2\pi f$, where f is the frequency of the oscillation represented by these equations. In the most general case any change in the circuit conditions, such as closing or opening a switch, or a lightning stroke in the vicinity of the circuit, may set up an infinite number of oscillations of different frequencies, their frequency being determined by the initial conditions at the instant the change is made. The current and voltage set up by each oscillation is represented by a set of terms of the form given by (56) and (57), and the resultant current and voltage will be respectively the sum of all the current terms and the sum of all the voltage terms. The oscillation of any given frequency, however, may be considered separately, as it is uninfluenced by the presence of the other oscillations. Moreover, in the case of a composite circuit, consisting say of a step-up transformer, transmission line and step-down transformer, if an oscillation of frequency f is set up in one part of the circuit it will also appear in all other sections of the circuit, though it may be greatly damped in these sections and therefore produce no appreciable effect.

Attenuation Constant (α), Wave Length Constant (β), Wave Length (λ) and Velocity of Propagation (U). — Referring to equations (56) and (57) each oscillation sets up in each section of the circuit two waves, each of which has a wave length $\lambda = \frac{2\pi}{\beta}$; the constant β is therefore called the "wave length constant;" it is a function of the frequency and of the constants r, L, g and C of the circuit. The two waves travel along the line in opposite directions each with a velocity $U = \frac{\omega}{\beta}$; in a composite circuit this velocity U is in general different for each section of the circuit. One wave may be looked upon as the "incident" and the other as the "reflected" wave. The amplitude of each wave diminishes by the factor ϵ^{α} as the wave travels unit distance; the factor ϵ^{α} is called the "attenuation factor" and the constant α is called the "attenuation constant." The attenuation constant is a function of the frequency and the constants r, L, g and C of the circuit; see p. 1784.

Surge Admittance (Y) and its Power-factor Angle (Ψ). — The constant Y, which is equal to the quotient of the amplitude (or r.m.s. value) of the incident current wave by the incident voltage wave, is called the "surge admittance" and its reciprocal is called the "surge impedance"; it is a function of the frequency and the constants r, L, g and C of the circuit; see p. 1784. The constant ψ, which is equal to the angle by which the incident current wave leads the incident voltage wave, is called the "power factor angle of the surge admittance"; it is a function of the frequency and the constants r, L, g and C, see p. 1785.

Amplitude Constants (A and B) and Phase-Angle Constants (θ_1 and θ_2). — The constants are equal to the amplitudes of the incident and reflected current waves at the point from which the distance l is measured, and the constants θ_1 and θ_2 give the

phase of these two waves at this point ($l = 0$) at the instant from which time is measured ($t = 0$). Note that the incident current wave at $l = 0$ leads the reflected current wave by the angle $\theta = \theta_1 - \theta_2$. The determination of these constants for steady state conditions is given on p. 1785.

Natural Damping Constant (u), Energy Transfer Constant (s) and Composite Damping Constant ($u - s$). — In the general case of a natural oscillation in a composite circuit the amplitude of each wave diminishes in unit time by the factor $\epsilon^{(u-s)}$; this factor is called the "composite damping factor," and the constant ($u - s$) is called the "composite damping constant." The composite damping constant, like the frequency f, is the same for all sections of a composite circuit. In the case of a line of uniform constants throughout, not connected to any terminal apparatus, it can readily be shown that $s = 0$, in which case the amplitude of the oscillations diminishes in unit time by the factor ϵ^{u}; this factor is therefore called the "natural damping factor" and the constant u, which for a section having the constants r, L, g and C per unit length, is equal to

$$u = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right), \tag{58}$$

is called the "natural damping constant." Any section of a composite circuit for which the actual damping $\epsilon^{(u-s)}$ is less than the natural damping ϵ^{u} must receive energy from some other section; consequently a positive value of s for a given section means that energy is transferred into this section from some other section of the circuit. Similarly, a negative value of s for a given section means that energy is transferred from this section to some other section. The constant s may therefore be called the "energy transfer constant." Since the voltage and current in the circuit cannot increase indefinitely the energy transfer constant s can never have a positive value greater than u.

Since the composite damping constant ($u - s$) is the same for all sections of a circuit, it follows that the transfer of energy from one section to another by oscillations in a composite circuit will always be into the section in which u is the larger from the section in which u is the smaller. Neglecting the leakage conductance g, this means that energy will be transferred into section 1 from section 2, when $\frac{r_1}{L_1}$ is greater than $\frac{r_2}{L_2}$, that is, energy is transferred from the section of the larger "time constant" $\left(\frac{L_2}{r_2}\right)$ to that of the smaller "time constant" $\left(\frac{L_1}{r_1}\right)$. When the resistances are small, this means that in the limiting case all the energy ($= \frac{1}{2} L_2 I^2$) of the magnetic field of the second section may go into electrostatic energy ($= \frac{1}{2} C_1 V^2$) in the first section, producing therefore a very high voltage at the junction point when the inductance L_2 of the second section is large compared with the capacity C_1 of the first section. This accounts for the very high voltages sometimes set up during switching operations at the junction point of an overhead line with an underground line, or in a transformer connected to a long overhead line.

"Steady State" Conditions in a Transmission Line. — From the above discussion it is evident that when a sufficient time (usually a small fraction of a second) has elapsed after any change in the circuit conditions, the only terms left in the general equations of a transmission line for a given impressed sine-wave voltage of frequency f are those for which $s = u$, viz.,*

$$i = \sqrt{2}\left[A\epsilon^{\alpha l} \sin(\omega t + \beta l) + B\epsilon^{-\alpha l} \sin(\omega t - \beta l - \theta)\right] \tag{59}$$

$$v = \frac{\sqrt{2}}{Y}\left[A\epsilon^{\alpha l} \sin(\omega t + \beta l - \psi) - B\epsilon^{-\alpha l} \sin(\omega t - \beta l - \theta - \psi)\right]. \tag{60}$$

The effective value of the current at any point is then equal to the sum of two vectors having the lengths $A\epsilon^{\alpha l}$ and $B\epsilon^{-\alpha l}$, the former leading the latter by the angle $(2\beta l + \theta)$, and the effective value of the voltage is equal to the difference of these same two vectors divided by Y. The phase angle between the voltage and current is equal to the phase angle between the sum and difference of the A and B vectors less the angle ψ.

* For steady state conditions time may be counted from any arbitrarily chosen interval, i.e., θ_1 in equations (56) and (57) may be put equal to zero, and for convenience $-\theta$ may be used for θ_2.

Notation for Steady State Conditions. — These relations are clearly shown in the vector diagram, Fig. 7a, which is a complete vector diagram of a transmission line with distributed capacity and leakage. The four constants α, β, Y and ψ are constants of the line, independent of the load, and are expressed in terms of the ordinary line constants as follows: Let

f = frequency in cycles per second,

r = conductor resistance per mile, in ohms; see tables in article on *Wires and Cables, Bare,*

$x = 2\pi fL$ = conductor reactance per mile, in ohms, corresponding to the impressed frequency f; see tables in article on *Inductance and Inductive Reactance,*

$z = \sqrt{r^2 + x^2}$ = conductor impedance per mile, in ohms,

g = leakage conductance to neutral per mile of line, in micromhos; see above, p. 1667. For power lines g is usually taken equal to zero,

$b = 2\pi fC$ = capacity susceptance to neutral per mile of line, in micromhos, corresponding to the impressed frequency f; see tables in article on *Capacity and Charging Current,*

$y = \sqrt{g^2 + b^2}$ = dielectric admittance per mile, in micromhos; when $g = 0$, then $y = b$,

$\alpha = 10^{-3}\sqrt{\dfrac{yz - bx + gr}{2}}$ = the attenuation constant; for r and g small compared with x and b this reduces to $\dfrac{10^{-3}}{2}\left(r\sqrt{\dfrac{C}{L}} + g\sqrt{\dfrac{L}{C}}\right)$,

$\beta = 10^{-3}\sqrt{\dfrac{yz + bx - gr}{2}}$ = the wave length constant; for r and g small respectively compared with x and b this reduces to $2\pi f \times 10^{-3}\sqrt{LC}$, which for an overhead line equals approximately $\dfrac{2\pi f}{180{,}000}$,

$Y = 10^{-3}\sqrt{\dfrac{y}{z}}$ = surge admittance; for r and g small respectively compared with x and b, this reduces to $10^{-3}\sqrt{\dfrac{C}{L}}$. The reciprocal of the surge admittance is called the "surge impedance,"

$\psi = \tan^{-1}\sqrt{\dfrac{yz - bx - rg}{yz + bx + rg}}$ = the power-factor angle of the surge admittance, taken positive for $gx < br$ and negative for $gx > br$. For r and g small compared with x and b, then $\psi = 28.7\left(\dfrac{r}{x} - \dfrac{g}{b}\right)$ degrees,

$U = \dfrac{2\pi f}{\beta}$ = velocity of propagation in miles per second; for a frequency f sufficiently high to make r negligible compared with x, and g negligible compared with b, this reduces to $\dfrac{10^3}{\sqrt{LC}}$, which for an overhead line with wires far apart is equal to the velocity of light in air, viz., 180,000 miles per second, approximately,

$\lambda = \dfrac{2\pi}{\beta} = \dfrac{U}{f}$ = wave length of each wave in miles; for a frequency f sufficiently high to make r negligible compared with x, and g negligible

compared with b, this reduces to $\frac{10^3}{f\sqrt{LC}}$, which for an overhead line is equal approximately to $\frac{180{,}000}{f}$ miles.*

In the vector diagram and in the formulas given below, let

l = length of the line in miles,
I = effective (r.m.s.) value of the amperes per wire at the load end,
V = effective (r.m.s.) value of the volts to neutral at the load end,
ϕ = the power-factor angle at the load end, i.e., $\cos\phi$ is the power factor at the load end. ϕ is taken positive for a lagging and negative for a leading current,
I_0, V_0, ϕ_0 = corresponding quantities at the generator end.

Solution by Vector Diagram (Fig. 7a). — Having calculated the constants α, β, Y and ψ, and knowing the current I, voltage V and power-factor angle ϕ of the load,

Lay off $\overline{MN} = I$ as the base line, and bisect it at G,
At the angle $(\phi + \psi)$ ahead of $\overline{MN}$ lay off the line $\overline{HK}$ equal in length to YV, so that it is also bisected by G,
Then measure off $\overline{MK} = A$ and $\overline{MH} = B$,
Lay off at the angle $57.3\,\beta l$ degrees ahead of A the line $\overline{MK_0}$ equal in length† to $A\epsilon^{\alpha l}$,
Lay off at the angle $57.3\,\beta l$ degrees behind B the line $\overline{MH_0}$ equal in length to $B\epsilon^{-\alpha l}$.
Bisect H_0K_0 at G_0.
Then the line $\overline{MN_0} = 2\,\overline{MG_0}$ is equal to the current at the generator end,
The line $\overline{H_0K_0}$ divided by Y is equal to the voltage at the generator end,
The angle between $\overline{G_0N_0}$ and $\overline{G_0K_0}$ less the angle ψ is the power-factor angle at the generator end.

Note that the voltage at the load end if drawn in the diagram would be at the angle ψ behind the vector $\overline{GK}$, and at the generator end would be at the angle ψ behind $\overline{G_0K_0}$.

Algebraic Solution for Steady-state Conditions. —The vector diagram may be solved algebraically as follows: Calculate first the constants A, B and θ from the formulas:

$$A = \tfrac{1}{2}\sqrt{I^2 + (YV)^2 + 2\,YVI\cos(\phi+\psi)}, \tag{61}$$

$$B = \tfrac{1}{2}\sqrt{I^2 + (YV)^2 - 2\,YVI\cos(\phi+\psi)}, \tag{62}$$

$$\theta = \tan^{-1}\left[\frac{2\,YVI\sin(\phi+\psi)}{I^2 - (YV)^2}\right]. \tag{63}$$

Note that $\sin\theta$ has the same algebraic sign as the numerator of this fraction and $\cos\theta$ has the same algebraic sign as the denominator of this fraction; this fixes the quadrant in which θ lies.

Put $$A_0 = A\epsilon^{\alpha l}, \quad \text{and} \quad B_0 = B\epsilon^{-\alpha l}. \tag{64}$$

* The above formulas for $z, y, \alpha, \beta, Y, \psi, U$ and λ also hold for the transient or free oscillations in a single circuit, and also for each section of a composite circuit, provided in these formulas $r_1 = r\,(u - s)\,L$ is substituted for r and $g_1 = g + (u - s)\,C$ is substituted for g.

† See the table in the article on *Exponential Functions* for values of ϵ^x and ϵ^{-x}, where x is any number.

The current, voltage and power-factor angle at the generator end are, then, expressing all angles in degrees,

$$I_0 = \sqrt{A_0^2 + B_0^2 + 2\,AB\cos(114.6\,\beta l + \theta)}, \tag{65}$$

$$V_0 = \frac{1}{Y}\sqrt{A_0^2 + B_0^2 - 2\,AB\cos(114.6\,\beta l + \theta)}, \tag{66}$$

$$\phi_0 = \tan^{-1}\left[\frac{2\,AB\sin(114.6\,\beta l + \theta)}{A_0^2 - B_0^2}\right] - \psi. \tag{67}$$

Note that the quadrant in which $(\phi_0 + \psi)$ lies is determined by the algebraic signs of the numerator and denominator of the fraction in the brackets, just as in the case of the angle θ.

Solution of Steady State Conditions in Terms of Hyperbolic Functions.* — The above expressions for the current, voltage and power factor at the generator may also be put in the form,

$$I_0 = I\sqrt{\frac{\cosh(2\,\alpha l + \gamma) + \cos(114.6\,\beta l + \theta)}{\cosh\gamma + \cos\theta}}, \tag{68}$$

$$V_0 = V\sqrt{\frac{\cosh(2\,\alpha l + \gamma) - \cos(114.6\,\beta l + \theta)}{\cosh\gamma - \cos\theta}}, \tag{69}$$

$$\phi_0 = \tan^{-1}\left[\frac{\sin(114.6\,\beta l + \theta)}{\sinh(2\,\alpha l + \gamma)}\right] - \psi, \tag{70}$$

where θ and γ are given by the formulas

$$\gamma = \tanh^{-1}\left[\frac{2\,YVI\cos(\phi + \psi)}{I^2 + (YV)^2}\right], \tag{71}$$

$$\theta = \tan^{-1}\left[\frac{2\,YVI\sin(\phi + \psi)}{I^2 - (YV)^2}\right]. \tag{72}$$

The other quantities are as above defined. Note that θ is the same angle as given by equation (63) and the quadrant in which it lies is to be determined as described in the note under (63). Also note that the constant γ given by (71) may be expressed in terms of A and B, given by (61) and (64), by means of the formula

$$\gamma = \log_e\left(\frac{A}{B}\right) = 2.302\log_{10}\left(\frac{A}{B}\right). \tag{73}$$

Formulas for Open Circuit and Short Circuit at Load End. — When the line is open at the load end, $I = 0$, whence from equations (61) to (63), $A = \frac{1}{2}\,YV$, $B = \frac{1}{2}\,YV$, and $\theta = 180°$ (since the denominator of the fraction is negative). When the line is short-circuited at the load end, $V = 0$, and $A = \frac{1}{2}\,I$, $B = \frac{1}{2}\,I$, and $\theta = 0°$ (since the denominator of the fraction is positive). The current, voltage and power-factor angle at any point along the line may then be found in either case by substituting these values in equations (65) to (67) which reduce to the simple hyperbolic forms:

On open circuit	On short circuit
$I_0 = YV\sqrt{\sinh^2(\alpha l) + \sin^2(57.3\beta l)}$	$I_0 = I\sqrt{1 + \sinh^2(\alpha l) - \sin^2(57.3\beta l)}$
$V_0 = V\sqrt{1 + \sinh^2(\alpha l) - \sin^2(57.3\,\beta l)}$	$V_0 = \frac{I}{Y}\sqrt{\sinh^2(\alpha l) + \sin^2(57.3\beta l)}$
$\phi_0 = -\tan^{-1}\left[\frac{\sin(114.6\beta l)}{\sinh(2\,\alpha l)}\right] - \psi$	$\phi_0 = +\tan^{-1}\left[\frac{\sin(114.6\beta l)}{\sinh(2\,\alpha l)}\right] - \psi$

* Tables of hyperbolic functions are given in the article on *Hyperbolic Functions.*

Solution of Steady-state Equations in Complex Hyperbolic Functions.* — Expressing all quantities other than the length l in symbolic notation (*see Alternating Currents*), the equations of the transmission line for *steady state conditions only*, may be written

$$I_0 = I \frac{\sinh (l\sqrt{yz} + B)}{\sinh B}, \tag{74}$$

$$V_0 = V \frac{\cosh (l\sqrt{yz} + B)}{\cosh B}, \tag{75}$$

where the symbols other than B have the same meanings as above, except that they are all expressed as complex quantities and

$$B = \tanh^{-1} \left(\frac{I}{V} \sqrt{\frac{z}{y}} \right) \tag{76}$$

The real part of $\sqrt{yz}$ is the attenuation constant and the imaginary or "j" part is the wave length constant.

Rigorously Equivalent "T" and "π" Circuits. — As noted above the middle condenser or "T" circuit, Fig. 5, or the split condenser or "π" circuit, Fig. 6, is rigorously equivalent to the actual transmission line when proper values are assigned to the constants of these circuits. The corrected values z' and y' of the impedance per mile and of the admittance per mile are, in symbolic notation, as follows:

"T" circuit	"π" circuit
$z' = z \dfrac{\tanh\left(\dfrac{l\sqrt{yz}}{2}\right)}{\dfrac{l\sqrt{yz}}{2}}$	$z' = z \dfrac{\sinh (l\sqrt{yz})}{l\sqrt{yz}}$
$y' = y \dfrac{\sinh (l\sqrt{yz})}{l\sqrt{yz}}$	$y' = y \dfrac{\tanh\left(\dfrac{l\sqrt{yz}}{2}\right)}{\dfrac{l\sqrt{yz}}{2}}$

In Dr. Kennelly's tables above referred to are given tables of these correction fractions.

FACTORS AFFECTING THE MECHANICAL DESIGN. — In designing a transmission line the following factors, in addition to the line losses, must be considered: (1) right-of-way, (2) telephone circuits, (3) ground wires, (4) clearances, (5) type of supporting structure, (6) type of insulators, (7) conductors, (8) temperature range, (9) collection of ice, (10) wind velocity and wind pressure. The requirements imposed by these several conditions are noted below in the order mentioned; this is followed by the formulas and curves necessary to make the required calculations.

Location and Width of Right-of-way. — The right-of-way should be as short and as straight as practicable. Its width for a single line should be equal to the width of space over which the conductors normally hang, plus twice the side swing of conductors under maximum wind, plus twice the safe clearance from conductor to possible buildings adjacent to the right-of-way. In computing the side swing allowance must be made for swing of suspension insulators. Side swing of conductor should be based on the longest span ordinarily

* Complete tables and charts of such functions have been published by Prof. A. E. Kennelly, *Tables of Complex Hyperbolic and Circular Functions*, Harvard University Press, 1914.

used, and extra width should be secured wherever extraordinarily long spans are made. For telephone lines on separate poles extra width will be required unless the power wires are high enough to swing safely over them. Where two power lines are on one right-of-way the towers for the two lines should be located directly opposite each other, especially on long spans, since the two lines may then be placed close together with less danger of the wires of the two lines swinging together. However, it is usually desirable to have the lines far enough apart so that the towers of one line may fall without striking those of the other line. The right-of-way should be passable (or at least accessible) for patrolling as well as for construction.

When it is necessary that the right-of-way cross railroads, roads or other lines the length of crossing should be reduced to a minimum by making the crossing at as near right angles as practicable. Rights-of-way through swamps often require expensive road building and expensive tower foundations, though small swamps (up to about 1000 feet across) can often be crossed in a single span. Steep side hills require extra expense for foundations and tower extensions, and introduce a hazard of injury to tower from sliding earth, rocks, trees or snow. A right-of-way through forests requires expensive clearing.

It is usually advantageous to own and fence the right-of-way. However, when the right-of-way passes through farm lands, it is sometimes advantageous not to fence it in, but to have it cultivated and kept free from brush. Instead of purchasing a right-of-way, it is often sufficient to obtain easements covering the location of towers and suspension of wires. Easements should include the right to remove and trim trees under and adjacent to the line.

Telephone Circuits for Power Lines. — When the line voltage does not exceed 66,000 volts, the telephone circuits are usually carried on the same poles or towers as the power circuits, being placed below the power conductors. A separate line of wooden poles on the same right-of-way is usually employed when the voltage is higher than 66,000.

Where the telephone wires are on the same supporting structure as the power wires, sufficient clearance between the power and telephone circuits must be allowed to make the telephone line accessible for repairs and also to prevent the two circuits touching under abnormal conditions. On wood pole lines of short span (100 to 125 feet) the vertical clearance at the poles or towers ranges from 4 feet for 22,000 volts to 6 feet for 66,000 volts. On long span lines greater spacing is necessary to allow for safe clearance in the middle of the longest span due to the change in the sag of the power and telephone wires under all conditions of unequal ice loading and all variations of side deflection due to wind.

Telephone wires are ordinarily of copper, though copper-clad steel is sometimes used for long spans. For spans up to 125 feet No. 10 B. & S. copper may be used (though No. 8 is preferable) while for longer spans larger sizes (usually No. 8 or No. 6 or even No. 4) are necessary to allow for sleet load. A spacing of 12 inches between wires may be used for 125-foot spans, but a wider spacing is necessary for longer spans. Where inadequate spacing is used the telephone lines will frequently become crossed by the wind, unless they are strung with little sag, in which case they are overloaded and broken by sleet. With wide spacing and large sag higher poles must be used.

Ground Wires. — Grounded cables or wires are placed above the transmission line circuits to protect the latter from lightning discharges (*see Lightning Protectors*). They are usually grounded at each supporting structure except where short spans are used. The same care must be exercised to obtain clearances between conductors and ground cables or wires, as outlined above under telephone circuits. For tower lines having flexible towers and single-circuit towers having conductors arranged in a horizontal plane, two ground cables are

preferable, but for double-circuit tower lines either one or two may be used. As a general rule a line drawn through the ground cable and any conductor should not make an angle of more than 45 degrees with the vertical.

Clearances. — The following clearances require determination: conductor to ground; to edge of right-of-way; to tower; to ground wire; to telephone wire; to other conductors.

The minimum clearance from the high-voltage wires to the surface of the ground is ordinarily 20 feet or more, and from the telephone wires on towers used for high-voltage wires is 18 feet to ground. The minimum clearance from the wires to the edge of the right-of-way or to the tower, under maximum swing of insulators and cables in the wind, should exceed the striking distance corresponding to the arc-over voltage of the insulators used. The minimum clearance between conductors, under extreme conditions of unequal ice loading (*see below*), should exceed the striking distance of the normal voltage by a factor of safety of at least two. The normal clearance between conductors should be much greater than this, and is ordinarily from 10 inches to 1 foot per 10,000 volts between wires.

Type of Supporting Structure. — (*See also Poles for Overhead Lines; Cross Arms; Towers.*) Wood poles are usually the cheapest form of supporting structure for lines up to about 66,000 volts. Most lines of voltage over 66,000 and some lines of lower voltage are supported on steel towers.

A single line of poles or towers may support one, two and occasionally more circuits. Where several circuits are required two or more single-circuit tower lines with conductors arranged in a horizontal plane have the advantage that the effects of an accident will usually be confined to one circuit, and that one circuit may safely be repaired with another circuit alive, but this arrangement requires the maximum width of right-of-way. Where it is important to use the right-of-way economically two-circuit towers having the conductors of each circuit arranged in vertical planes on each side of the tower are used.

Wood poles are ordinarily placed from 100 to 200 feet apart and steel towers from 400 to 800 feet apart. The span for steel tower lines is ordinarily chosen so as to render the sum of the costs of all of the items a minimum, and is usually found to lie within the limits mentioned.

The poles or towers ordinarily used where the line is straight are designed to resist the weight of the conductors with sleet and the side pressure of wind on them, but not to resist the tension in the conductors which is assumed to be wholly or partially balanced. Such towers are called standard, straight line or suspension towers. Heavier towers designed to resist the unbalanced tension of all of the cables (which may occur if the cables are broken on the other side of the tower), called anchor or dead-end towers, are used at intervals of from five to ten spans where the line is straight. Heavier towers designed to resist the unbalanced lateral resultant of the tension in the cables are also used at angles and are known as angle towers. For economy in design and manufacture anchor and angle towers (for any angle up to about 60°) are usually made interchangeable. Where towers are not very high, economy is often secured by using flexible towers intermediate between square anchor and angle towers. On wood pole lines extra strong poles guyed or braced in all four directions, with double cross arms and double pins and insulators, are used for anchoring the line and at angles.

Type of Insulators. — (*See also Insulators.*) Pin insulators are usually used for voltages up to 50,000 and suspension insulators for higher voltages. Suspension insulators are sometimes used for lower voltage where a high factor of safety is desired. For the lower voltages the cost of the line is usually less for pin than for suspension insulators. The suspension insulator requires a higher tower

than the pin insulator, for the conductor is below the cross arm, and a longer cross arm is also required to allow for the swing of the insulators in the wind.

Conductors for Overhead Lines. — (*See also Wires and Cables.*) Copper, aluminum or steel, or combinations of the two former with steel, are used for line conductors. The use of an all-steel conductor is limited to those cases where great mechanical strength is required, although some economy is claimed for it when high voltages are used and a conductor of large diameter is required on account of corona (*see Corona*). Conductors consisting of a steel center with copper or aluminum outside are used where long spans are desirable and mechanical strength of conductor is required, and may also be used where large diameter is required to prevent corona formation. The maximum stress in wires and cables should not exceed the elastic limit. This condition is ordinarily met if a factor of safety of two, based on the ultimate strength, is used.

Distribution of Stress in Stranded Conductors. — Conductors for long spans are usually stranded when larger than No. 6 B. & S. gage, on account of the tendency to crystallize at the points of support, due to swinging in the wind. However, the maximum size of solid conductor which may be used is dependent largely upon the kind of support; if these are designed to prevent sharp bending, solid conductors, larger than No. 6, may be used. One difficulty in the use of a stranded cable is that for a given total tension in the cable the wires in the various layers are not all stressed equally. In particular, the center wire or core, if of metal, takes more than its share of the tension. On this account the actual sag in the cable when suspended may differ from that calculated, unless an allowance is made for the unequal tension in the various wires composing the cable. One method of avoiding this difficulty is to stress the cable, before stringing it, to a tension per square inch corresponding to the elastic limit of the component wires; the center wire will then be given a permanent set and when the cable is again stressed each of the component wires will tend to take the same proportion of the total tension.

Cables having centers or cores of a different material than the outside strands should be so designed that both materials share the stress over the operating range of temperatures, to such extent as is possible. The properties of steel-core aluminum cables and the method of considering stresses therein are given in the article on Wires and Cables, Bare.

The distribution of stress among the wires of which conductors may be composed applies more particularly to long spans where clearances are important and it is necessary to use small deflections compared to lengths of spans. For all ordinary construction, having spans of 500 to 600 feet or less, a very slight change in either thickness of ice or amount of wind pressure on the conductor will affect results as much as the error resulting from not taking into consideration the distribution of stresses in conductors.

Mechanical Characteristics of Conductor Materials. — For purposes of design both elastic limit and modulus of elasticity of cables should be obtained from tests on samples of the cables actually to be used, since errors in these materially affect the accuracy of the calculations of deflections in spans. The modulus of elasticity of cables varies widely with the number of wires and their pitch or lay in the different layers. Table I shows the values of these quantities and also of the other mechanical properties of conductors commonly used in design, when specific test results are not available. See also the articles on *Aluminum; Copper; Strength and Elasticity; Wires and Cables, Bare.*

Temperature Range. — The maximum and minimum air temperatures which have occurred in any locality for a period of years can be obtained from the records of the United States Weather Bureau, or from similar records for other countries. The minimum air temperature recorded (which may be as

TABLE I

MECHANICAL CHARACTERISTICS OF WIRES AND CABLES

Item	Copper	Aluminum	Steel
Ultimate strength, in lb. per sq. in.	60,000–65,000	25,000–50,000	60,000–80,000
Elastic limit, in lb. per sq. in.	30,000–35,000	11,000–14,000	35,000–40,000
Modulus of elasticity, in pound-inch units	12×10^6–16×10^6	7×10^6–10×10^6	22×10^6–28×10^6
Coefficient of linear expansion per ° F.	9.6×10^{-6}	12.8×10^{-6}	6.6×10^{-6}
Weight in pounds of a 1-ft. length having a cross section of 1,000,000 circ. mils (stranded)	3.09	0.92	2.67

low as − 40° F. in some of the northern states) will be the minimum temperature which the conductor may be expected to reach. However, since the conductors are exposed to the direct rays of the sun, they will reach a maximum temperature in the summer considerably in excess of the Weather Bureau records, which give the temperatures in the shade.

Another important temperature which should be determined is that of the wire when coated with ice. As noted below, a sleet storm is usually followed by a fall in temperature, and although the ice forms at 32° F., the wire may reach a much lower temperature while the ice is on it.

The following temperature ranges have been used in the design of certain lines:

	Maximum	Minimum
Eastern Canada	+ 120° F.	− 40° F.
Mississippi Valley	+ 120° F.	− 20° F.
Southern California	+ 140° F.	+ 10° F.

Collection of Ice on Wires. — Investigation of the records of the Weather Bureau leads to the conclusion that sleet and ice storms are generally followed by falling temperatures and high winds, and transmission lines should be designed to meet these conditions. Records indicate that under favorable conditions ice and sleet will collect on wires and cables to the same amount in any climate where freezing temperatures are obtained. Mild, moderate and cold climates differ in the frequency with which conditions are favorable. In general, sleet storms are most frequent in the moderate climates, since precipitation takes place more often at freezing temperatures. Destructive sleet storms occur in the eastern part of the United States at least as far south as Atlanta. One-half inch thickness of solid ice on wires and cables is generally assumed in designing transmission lines, but thicknesses of one-quarter and three-quarter inch are also assumed in the more favorable and unfavorable localities.

Ice and sleet generally collect quite uniformly on wires throughout their length. The collection is sometimes in the form of icicles but more often is egg-shaped in cross section, with the wire in the small end of the section. It frequently falls off non-uniformly in sections.

Clear solid ice weighs 57 pounds per cubic foot or 0.033 pound per cubic inch, but sleet or frozen snow such as often collects on wires weighs much less, sometimes as little as 8 pounds per cubic foot.

Effect of Surface and Electric Potential on Collection of Ice. — Local observations of single ice or sleet storms have shown that ice will sometimes collect on one wire or cable and not on an adjoining one. This has led to the conclusion that ice will not collect on certain kinds of wires, but more extensive observations indicate that ice will collect on any kind of wire under favorable conditions. Observations on wires carrying sufficient current to heat them and wires having potential near the critical corona voltage are not sufficiently extensive to warrant the conclusion that they will not collect ice and sleet.

Wind Velocity and Wind Pressure on Wires. — As noted in the article on *Wind Pressure*, the records of the United States Weather Bureau give nominal average velocities for five minute intervals, the true average velocities differing from the recorded velocities as follows:

Recorded velocity in miles per hour	10	20	30	40	50	60	70	80	90	100
Actual velocity in miles per hour	9.6	17.8	25.7	33.3	40.8	48.0	55.2	62.2	69.2	76.2

The Weather Bureau records give no indication of the "gust" velocities which may occur during the five minute periods, and which may greatly exceed the average velocity. Tests with a Dines pressure tube anemometer have shown that the extreme maximum is about 50 per cent greater than the average for short periods.

The extreme maximum wind velocity observed in Chicago in the whole thirty-six year period from 1873 to 1910 was 84 miles per hour (uncorrected) in February, 1894. A velocity of 76 miles per hour (uncorrected) was observed once in November, 1898, and a velocity of 72 miles per hour (uncorrected) was observed seven times. During the ten year period from 1894 to 1903 the maximum wind velocity in a few other representative localities was as follows, all velocities being the observed or uncorrected velocities: Bismark, N. D., 72; Eastport, Me., 78; Buffalo, N. Y., 90; New York City, N. Y., 78; Galveston, Tex., 84; Savannah, Ga., 76; Salt Lake City, Utah, 60. All the maxima range between 60 miles and 90 miles per hour.

The wind pressure on a cable is usually calculated from the formula

$$p = KV^2, \tag{1}$$

where p is the pressure in pounds per square foot of the projected area of the cable (including sleet and insulation, if any), and V is the actual velocity of the wind in miles per hour blowing perpendicularly across the span. The projected area, in square feet, of a one-foot length of cable is equal to the over-all diameter in inches divided by 12, where by over-all diameter is meant the diameter over the ice and insulation, if any. Buck's formula (*see article on Wind Pressure*) gives a value of 0.0025 for the constant K, whereas a value of 0.002 derived from the work of the Weather Bureau and experiments of Borda is also used.

Maximum Loading. — There is considerable difference of opinion as to what should be taken as the maximum loading in designing a transmission line. The Committee on Overhead Line Construction of the N.E.L.A. have proposed three different loadings, viz.: (A) No ice and a wind pressure of 15 pounds per square foot; (B) ice ½-inch thick and a wind pressure of 8 pounds per square foot; and (C) ice ¾-inch thick and a wind pressure of 11 pounds per square foot. Class B loading gives greater stress than Class A loading for all sizes of wire in use. The difference is greatest for small sizes.

Several important lines have been designed on the assumption of a maximum loading of ½ inch of ice and wind pressure at 6 pounds per square foot, the temperature corresponding to this loading being taken as 0° F. In these same lines the clearance under unequal loadings was calculated on the assumption of five

spans between anchor towers, with no ice on the third span of the lower conductor, and no ice on the first, second, fourth and fifth spans of the upper conductor, but with a loading of ¼ inch of ice on all other spans the temperature being taken as 32° F. and no wind assumed.

VERTICAL AND TRANSVERSE FORCES ON A SUSPENDED WIRE. — The resultant force acting on one foot of a suspended wire is in general made up of three components, viz.:

c = weight of the conductor (including insulation, if any) per foot length, in pounds;

i = weight of the ice coating per foot length of the conductor, in pounds;

h = wind pressure per foot length of the conductor, in pounds.

The weight of the conductor per foot length may be taken directly from the tables in the articles on *Wires and Cables, Bare*, as can also the diameter of the conductor (over the insulation, if any). Let d be this diameter in inches and let t be the thickness of the ice coating, then the weight of the ice coating per foot length of the conductor is

$$i = 1.24\,t\,(d+t). \tag{2}$$

Let p be the wind pressure per square foot of projected area, assumed or calculated from equation (1); then the wind pressure per foot length of the conductor, i.e., the horizontal component of the resultant force, is

$$h = \frac{p\,(d+2t)}{12}. \tag{3}$$

The vertical component of the resultant force per foot length of conductor, which is equal to the resultant force for no wind, is

$$v = c + i. \tag{4}$$

Values of v and h for various ice thicknesses and various sizes of wires are given in Table II. The values of h are given in the table for a wind pressure of 10 pounds per square foot; these values of h are designated h_0; for any other wind pressure of say p pounds multiply these values by $\frac{p}{10}$. Knowing v and h the resultant force w for any combination of wind and ice loads is readily determined by the formula

$$w = \sqrt{v^2 + h^2}. \tag{5}$$

TABLE II. VERTICAL AND HORIZONTAL LOADING FORCES*

Pounds per Foot of Conductor

The horizontal component h_0 is given for a wind pressure of 10 lb. per sq. ft.; for any other wind pressure of p lb. per sq. ft., $h = \frac{ph_0}{10}$.

The resultant force for any ice thickness and wind pressure is $w = \sqrt{v^2 + h^2}$

Wire, size, B. & S. (A. W. G.)	and circular mils	No Ice		¼-inch ice		½-inch ice		¾-inch ice	
		Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0
Aluminum, stranded									
	500,000	0.460	0.679	0.791	1.096	1.280	1.511	1.919	1.928
	450,000	0.414	0.643	0.731	1.060	1.205	1.476	1.834	1.893
	400,000	0.368	0.604	0.670	1.021	1.130	1.438	1.744	1.855
	350,000	0.322	0.566	0.610	0.983	1.055	1.399	1.655	1.815
	300,000	0.276	0.517	0.546	0.934	0.973	1.351	1.555	1.767
	250,000	0.230	0.473	0.483	0.890	0.894	1.306	1.459	1.723
0000	211,600	0.195	0.435	0.434	0.852	0.831	1.269	1.382	1.685
000	167,800	0.155	0.387	0.376	0.804	0.755	1.220	1.288	1.636
00	133,100	0.122	0.345	0.328	0.762	0.691	1.179	1.208	1.595
0	105,500	0.097	0.307	0.289	0.724	0.637	1.140	1.140	1.556
1	83,690	0.077	0.273	0.256	0.690	0.592	1.106	1.082	1.524
2	66,370	0.061	0.243	0.229	0.660	0.533	1.076	1.032	1.493
3	52,640	0.049	0.217	0.207	0.634	0.522	1.051	0.992	1.467
4	41,740	0.039	0.193	0.188	0.610	0.494	1.026	0.954	1.443
Copper, stranded									
	500,000	1.525	0.683	1.856	1.100	2.345	1.516	2.989	1.933
	450,000	1.373	0.642	1.689	1.059	2.165	1.475	2.791	1.892
	400,000	1.220	0.607	1.523	1.024	1.984	1.440	2.599	1.856
	350,000	1.068	0.566	1.356	0.983	1.801	1.399	2.401	1.815
	300,000	0.915	0.525	1.188	0.942	1.618	1.359	2.203	1.775
	250,000	0.762	0.492	1.022	0.909	1.440	1.325	2.012	1.742
0000	211,600	0.645	0.442	0.887	0.859	1.286	1.275	1.831	1.692
000	167,800	0.513	0.392	0.736	0.809	1.116	1.225	1.651	1.642
00	133,100	0.406	0.350	0.614	0.767	0.978	1.184	1.498	1.600
0	105,500	0.322	0.313	0.516	0.728	0.866	1.146	1.372	1.563
1	83,690	0.255	0.275	0.435	0.692	0.771	1.109	1.263	1.525
2	66,370	0.203	0.243	0.371	0.660	0.695	1.076	1.174	1.493
3	52,640	0.160	0.217	0.318	0.634	0.633	1.051	1.103	1.467
4	41,740	0.127	0.193	0.276	0.610	0.582	1.026	1.042	1.443
5	33,100	0.101	0.172	0.242	0.589	0.540	1.005	0.992	1.422
6	26,250	0.080	0.153	0.215	0.570	0.505	0.986	0.951	1.403

* This table is in agreement with the *Report of the Committee on Overhead Line Construction*, Proc. N.E.L.A., May, 1910, Vol. 1, p. 472; the diameters used are slightly greater than those given in the article on *Wires and Cables, Bare.*

TABLE II. — VERTICAL AND HORIZONTAL LOADING FORCES — *Continued*

Wire, size, B. & S. (A. W. G.) and circular mils		No ice		¼-inch ice		½-inch ice		¾-inch ice	
		Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0	Vertical v	Horizontal h_0
Copper, solid bare									
0000	211,600	0.641	0.383	0.911	0.800	1.238	1.216	1.770	1.634
000	167,800	0.509	0.341	0.713	0.758	1.074	1.175	1.591	1.591
00	133,100	0.403	0.304	0.594	0.724	0.940	1.138	1.443	1.554
0	107,500	0.320	0.271	0.498	0.688	0.833	1.104	1.323	1.521
1	83,690	0.253	0.241	0.420	0.658	0.744	1.075	1.223	1.491
2	66,370	0.202	0.215	0.359	0.632	0.673	1.048	1.142	1.465
3	52,640	0.159	0.191	0.308	0.608	0.613	1.025	1.073	1.441
4	41,740	0.126	0.170	0.267	0.587	0.564	1.004	1.016	1.425
5	33,100	0.100	0.151	0.234	0.568	0.524	0.985	0.969	1.402
6	26,250	0.079	0.135	0.207	0.552	0.491	0.969	0.930	1.385
Copper, solid, triple braid, weatherproof									
0000	211,600	0.767	0.533	1.043	0.950	1.476	1.366	2.064	1.783
000	167,800	0.629	0.494	0.890	0.911	1.309	1.328	1.882	1.744
00	133,100	0.502	0.429	0.739	0.846	1.133	1.263	1.682	1.679
0	105,500	0.407	0.417	0.639	0.834	1.029	1.250	1.573	1.666
1	83,690	0.316	0.378	0.534	0.795	0.909	1.210	1.438	1.627
2	66,370	0.260	0.364	0.473	0.781	0.843	1.198	1.367	1.614
3	52,640	0.199	0.338	0.403	0.755	0.763	1.171	1.278	1.588
4	41,740	0.164	0.299	0.353	0.716	0.698	1.133	1.199	1.549
5	33,100	0.135	0.287	0.319	0.704	0.660	1.120	1.146	1.536
6	26,250	0.112	0.273	0.291	0.690	0.627	1.106	1.118	1.523
Steel, stranded, galvanized									
⅞ in.	575,000	1.540	0.730	1.888	1.147	2.393	1.563	3.050	1.980
13/16	500,000	1.336	0.677	1.666	1.094	2.151	1.510	2.789	1.927
¾	425,000	1.138	0.625	1.448	1.042	1.903	1.459	2.533	1.875
11/16	357,000	0.958	0.572	1.249	0.989	1.694	1.406	2.295	1.822
⅝	295,000	0.791	0.522	1.062	0.939	1.489	1.354	2.070	1.772
9/16	250,000	0.668	0.469	0.920	0.886	1.392	1.303	1.889	1.719
½	190,000	0.510	0.417	0.742	0.834	1.132	1.250	1.672	1.667
7/16	145,000	0.415	0.364	0.628	0.781	0.998	1.198	1.519	1.614
⅜	106,000	0.295	0.312	0.489	0.779	0.839	1.146	1.341	1.562
5/16	74,000	0.210	0.260	0.384	0.677	0.715	1.094	1.200	1.510

CALCULATION OF SAG AND TENSION. — A wire or cable suspended between towers takes the form of a catenary curve. In transmission-line practice the maximum deflection or sag is always small compared to the span, that is, the curve is very flat. The shape of such a flat catenary curve does not differ appreciably from a parabola and, as the approximate parabolic formulas are much simpler than the more exact catenary formulas, they are used instead. The flatness of the curve allows of some further simplifications even in the parabolic formulas, viz., (1) the tension is considered uniform throughout the span, the slight excess of tension at the ends over that at middle being neglected; (2) the change of length of the wire due to elastic stretch or temperature expansion is taken as equal to the change of length of a wire equal in length to the horizontal distance between the points of support.

Notation Used in Sag-tension Formulas. — The following notation, listed alphabetically, will be used throughout the discussion of sag and tension:

A = cross section of the conductor (actual metal cross section) in circular inches = square of diameter in inches; 1 circular inch = 1,000,000 circular mils.

a = coefficient of linear expansion of the conductor per °F.; see Table I above.

D = deflection, in feet, of the lowest point of the conductor when suspended from two points of support at the same elevation and at a distance L apart. (D is measured in the direction of the resultant transverse force.)

e = difference in elevation of the two points of support, in feet.

F = longitudinal horizontal component of the stress in the conductor, in pounds. (The resultant stress in the wire at the insulator is equal to $\sqrt{F^2 + H^2 + V^2}$, where V is the weight of the conductor and ice from the insulator to the lowest point of the span, and H is the total wind pressure on half the length of span; H and V in this expression are usually negligible compared with F.)

h = wind pressure in pounds per foot length of conductor assumed perpendicular to the vertical plane through the two points of support; see equation (3) and Table II above.

L = length of span in feet, i.e., the horizontal distance between the two points of support in feet.

l = length in feet of the arc of the curve in which the conductor hangs, i.e., the length of stretched conductor between the two points of support.

M = modulus of elasticity of the conductor in pound-inch units; see Table I above.

$S = \dfrac{vD}{w}$ = sag of the lowest point of the conductor below the horizontal line through the lower point of support; for no wind $S = D$.

T_0 = maximum allowable tension in the conductor in pounds per square inch of its cross section; T_0 is usually taken as one-half the ultimate strength of the conductor; see Table I.

v = vertical force in pounds on a one-foot length of the conductor, including the weight of conductor and the weight of the ice, if any, on it; see equation (4) and Table II above.

$w = \sqrt{v^2 + h^2}$ = resultant load in pounds on a one-foot length of the conductor.

$Z = \dfrac{hD}{w}$ = side swing, in feet, of the middle point of the conductor, measured perpendicularly to the vertical plane through the two points of support.

The various symbols with the subscript "o" will be used to designate the values of the various quantities under the conditions of maximum assumed loading; see paragraph above on *Maximum Loading*.

Fundamental Equations of a Wire Span. — As noted above, a perfectly flexible wire suspended between two points of support hangs in a catenary. The assumption that the wire hangs in a parabola instead of in a catenary is sufficiently accurate for all practical calculations of wire spans, the error in the sag calculated on this assumption being less than 2 per cent of its true value when this sag is less than 0.06 times the length of the span (e.g., less than 60 feet in a 1000-foot span), and the error in the length of the wire calculated on this assumption being less than 0.002 per cent of its true value for the same limiting conditions. The formulas given below are all based on the assumption of a parabola.

Deflection, Sag and Side Swing. — For a given length of span L, loading w, and stress F, the deflection D for the points of support at the same elevation is given by the relation

$$D = \frac{wL^2}{8F}. \tag{6}$$

When there is no wind this is also equal to the vertical sag, that is $S = D$. When there is wind, w is greater than the vertical loading v, and the vertical sag for the points of support at the same elevation is

$$S = \frac{vD}{w} = \frac{vL^2}{8F}. \tag{7}$$

D in equation (7) has the value given by equation (6). When one point of support is at an elevation e above the other, then the vertical sag of the lowest point of the conductor below the lower point of support is

$$S' = S\left(1 - \frac{e}{4S}\right)^2, \tag{8}$$

where S is given by equation (7). The horizontal distance of the lowest point of the conductor from the lower point of support is

$$L' = \frac{L}{2}\left(1 - \frac{e}{4S}\right). \tag{9}$$

The side swing Z of the middle point of the conductor, which is the point which is deflected the maximum distance from the vertical plane through the two points of support, is

$$Z = \frac{hD}{w} = \frac{hL^2}{8F}. \tag{10}$$

D in equation (10) has the value given by equation (6).

Length of Stretched Conductor. — The length of conductor between the two points of support for a given length of span L, loading w, stress F and difference of elevation e, is

$$l = L + \frac{8}{3}\frac{D^2}{L} + \frac{e^2}{2L}, \tag{11}$$

where D has the value given by equation (6), that is, D is the deflection for the same length of span, loading and tension, but for the points of support at the same elevation.

Effect of Changes in Loading and Temperature. — When the loading or the temperature changes, the stress in the conductor will change to some new value, F_0 say, and the deflection will change to some new value, say D_0. Let the new loading be w_0 and the new temperature be t_0, the initial temperature being t; also let a be the coefficient of linear expansion, M the modulus of elasticity, and A the cross section of the conductor in millions of circular mils. Then, when the points of support remain fixed, the following relation must hold

$$\frac{8}{3L^2}(D^2 - D_0^2) = a(t - t_0) + \frac{1.273}{MA}(F - F_0). \tag{12}$$

The D's in this equation are the same as given by equation (6) for a loading of w and w_0 respectively and stresses of F and F_0 respectively. Note that equation (12) is independent of the difference in elevation of the two points of support; also that the two sets of symbols, with and without the subscripts, refer to *any* two sets of conditions.

Stress - Deflection Charts (Fig. 8). — In order to apply the above equations to the calculation of clearances under various conditions equation **(6)**, namely

$$D = \frac{wL^2}{8F}, \tag{13}$$

and equation (12), which may be written

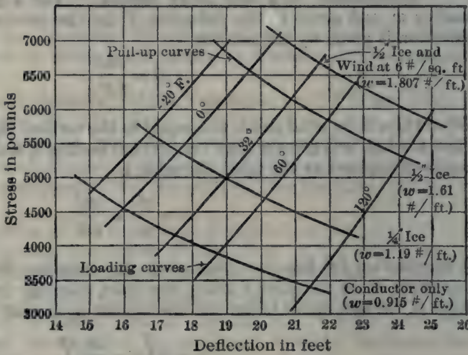

Fig. 8. Stress-Deflection Chart for a 300,000-Circular-Mil, 19-Strand Copper Conductor on an 800-ft. Span

$$F = F_0 - \frac{2.10\,MA}{L^2}D_0^2 - \frac{MAa}{1.273}(t - t_0) + \frac{2.10\,MA}{L^2}D^2, \tag{14}$$

may be plotted for any given length of span as shown in Fig. 8, with deflection D as abscissas and the stress F as ordinates.

Pull-up Curves. — The curves representing the relation between D and F in equation (13) are equilateral hyperbolas, as many of these curves being drawn as there are loadings to be considered. These hyperbolas may be called "pull-up" curves, since they give the stress to which the wire must be pulled up for any given deflection. In Fig. 8 four pull-up curves are shown, one for the conductor only (w = 0.915), conductor coated with ¼ inch of ice but no wind (w = 1.19), conductor coated with ½ inch of ice but no wind (w = 1.61), and conductor coated with ½ inch of ice and a wind pressure of 6 pounds per foot of projected area of wire (w = 1.807), this last being taken as the maximum loading.

Loading Curves. — The curves representing the relation between D and F in equation (14) are parabolas with vertical axes, as many of these curves being drawn as there are temperatures to be considered. These parabolas may be called "loading curves," since for a change in loading (at constant temperature) they give the relation which must exist between the new stress and new deflection produced. In order to plot these curves it is necessary to assume a maximum allowable tension, say T_0 pounds per square inch, a maximum load-

ing w_0, and the minimum temperature t_0 at which this loading is assumed to occur. Then

$$F_0 = \frac{AT_0}{1.273} \qquad \text{and} \qquad D_0 = \frac{w_0 L^2}{8 F_0},$$

and for any given temperature t all the terms in the right-hand member of (14) are constants except the last term, which varies as D^2. In Fig. 8 the maximum allowable tension T_0 is taken as 30,000 pounds per square inch, the maximum loading w_0 as 1.807 pounds per foot length, and the minimum temperature under maximum loading as 0° F. Then $F_0 = 7065$ pounds and $D_0 = 20.5$, which fixes one point on the 0° loading curve, and the other points on this curve are calculated directly from (14) by putting $t = 0$. For any other temperature, equation (14) gives the same shaped parabola as for 0°, but each point of the curve is shifted vertically downward a distance $\frac{MAa}{1.273}$ for each degree increase of temperature. Hence it is only necessary to calculate the 0° loading curve, and the loading curve for any other temperature can be readily plotted by the use of a pair of dividers.

Stringing Stresses and Deflections. — In stringing cable it is essential that the stress or deflection be correct for the stringing temperature, otherwise the maximum stress will be more than safe or the clearance to ground less than safe due to maximum deflection exceeding the designed amount. To insure proper stringing, curves of temperature-sag and temperature-stress are plotted from the stress-deflection curves. These curves are for no wind and no ice. Fig. 9 shows such curves for the 800-foot span plotted from Fig. 8. For example, if the cable is strung at 70° F., then it should be given a deflection (vertical sag) of 19.25 feet, provided the points of support are at the same elevation, or at a stress of 3800 pounds, this stress being independent of the difference of elevation of the two points of support. If there is a difference of elevation of 30 feet, say, between the two points of support, then from equation (8a), the cable should be drawn up until the sag of the lowest point below the lower point of support is

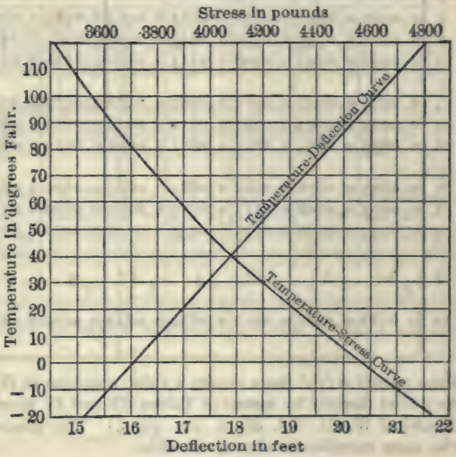

Fig. 9. Stringing Chart

$$S' = 19.25\left(1 - \frac{30}{4 \times 19.25}\right)^2 = 7.17 \text{ feet},$$

and the stress will then be 3800 pounds.

Direct Calculation of Change in Deflection and Stress with Loading and Temperature.* — When for F in equation (12) is substituted its value

* From Lecture Notes by Dr. H. Pender.

TABLE III.—VALUES OF y IN TERMS OF x

$b_1 > (b_2 + b_3)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0013	0.0025	0.0036	0.0047	0.0057	0.0067	0.0076	0.0084	0.0092
0.1	0.0099	0.0106	0.0113	0.0119	0.0125	0.0130	0.0135	0.0140	0.0145	0.0150
0.2	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0178	0.0182	0.0185	0.0188
0.3	0.0191	0.0194	0.0197	0.0200	0.0203	0.0206	0.0209	0.0212	0.0215	0.0218
0.4	0.0221	0.0224	0.0227	0.0229	0.0231	0.0233	0.0235	0.0237	0.0239	0.0241
0.5	0.0243	0.0245	0.0247	0.0249	0.0251	0.0253	0.0255	0.0257	0.0259	0.0261
0.6	0.0263	0.0265	0.0267	0.0269	0.0271	0.0273	0.0275	0.0277	0.0279	0.0281
0.7	0.0283	0.0285	0.0287	0.0289	0.0291	0.0293	0.0295	0.0297	0.0299	0.0301
0.8	0.0303	0.0304	0.0305	0.0306	0.0307	0.0308	0.0309	0.0310	0.0311	0.0312
0.9	0.0313	0.0314	0.0315	0.0316	0.0317	0.0318	0.0319	0.0320	0.0321	0.0322
x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.0327	0.0340	0.0352	0.0363	0.0373	0.0383	0.0392	0.0401	0.0410	0.0418
2	0.0426	0.0434	0.0442	0.0449	0.0456	0.0463	0.0470	0.0477	0.0484	0.0490
3	0.0496	0.0502	0.0508	0.0514	0.0520	0.0526	0.0532	0.0537	0.0542	0.0547
4	0.0552	0.0557	0.0562	0.0567	0.0572	0.0577	0.0582	0.0587	0.0591	0.0595
5	0.0599	0.0603	0.0607	0.0611	0.0615	0.0619	0.0623	0.0627	0.0631	0.0635
6	0.0639	0.0643	0.0647	0.0651	0.0655	0.0659	0.0663	0.0666	0.0669	0.0672
7	0.0675	0.0678	0.0681	0.0684	0.0687	0.0690	0.0693	0.0696	0.0699	0.0702
8	0.0705	0.0708	0.0711	0.0714	0.0717	0.0720	0.0723	0.0726	0.0729	0.0732
9	0.0735	0.0738	0.0741	0.0744	0.0747	0.0750	0.0753	0.0756	0.0759	0.0762
x	0	1	2	3	4	5	6	7	8	9
10	0.0765	0.0789	0.0812	0.0834	0.0855	0.0875	0.0894	0.0913	0.0931	0.0949

from equation (13) there results a cubic equation in D. This cubic equation can be solved directly by means of Tables III and IV given herewith. (Pender, H., *Electrical World, Vol. 66, p. 344, 1915.*) The procedure is as follows: using the same notation as above, calculate,

$$b_1 = \frac{1273\,F_0}{MA},$$

$$b_2 = \left(\frac{w_0 L}{0.155\,F_0}\right)^2,$$

$$b_3 = 1000\,a(t - t_0),$$

$$b = \frac{1273\,wL}{MA},$$

$$B = \text{numerical value of } \{b_1 - (b_2 + b_3)\},$$

$$x = \frac{b}{B\sqrt{B}}.$$

TABLE IV.—VALUES OF y IN TERMS OF x

$b_1 < (b_2 + b_3)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0194	0.0200	0.0205	0.0210	0.0215	0.0220	0.0224	0.0228	0.0232	0.0236
0.1	0.0239	0.0242	0.0245	0.0248	0.0252	0.0255	0.0258	0.0261	0.0263	0.0266
0.2	0.0269	0.0272	0.0274	0.0277	0.0279	0.0281	0.0283	0.0286	0.0288	0.0290
0.3	0.0292	0.0295	0.0297	0.0299	0.0300	0.0302	0.0304	0.0306	0.0308	0.0310
0.4	0.0312	0.0314	0.0316	0.0318	0.0320	0.0322	0.0323	0.0325	0.0327	0.0329
0.5	0.0330	0.0332	0.0333	0.0335	0.0337	0.0338	0.0340	0.0341	0.0343	0.0345
0.6	0.0346	0.0348	0.0349	0.0350	0.0352	0.0353	0.0354	0.0355	0.0358	0.0359
0.7	0.0360	0.0361	0.0362	0.0363	0.0365	0.0366	0.0368	0.0369	0.0370	0.0371
0.8	0.0372	0.0373	0.0374	0.0375	0.0377	0.0378	0.0379	0.0380	0.0381	0.0383
0.9	0.0384	0.0385	0.0386	0.0387	0.0388	0.0390	0.0391	0.0392	0.0393	0.0394
x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.0395	0.0406	0.0416	0.0425	0.0434	0.0443	0.0451	0.0460	0.0467	0.0475
2	0.0482	0.0490	0.0497	0.0504	0.0510	0.0516	0.0522	0.0528	0.0534	0.0539
3	0.0544	0.0550	0.0556	0.0561	0.0567	0.0571	0.0576	0.0581	0.0586	0.0590
4	0.0595	0.0600	0.0604	0.0609	0.0613	0.0617	0.0621	0.0625	0.0629	0.0633
5	0.0637	0.0641	0.0645	0.0649	0.0653	0.0657	0.0660	0.0663	0.0667	0.0670
6	0.0673	0.0677	0.0681	0.0685	0.0689	0.0692	0.0695	0.0698	0.0701	0.0704
7	0.0708	0.0711	0.0714	0.0717	0.0720	0.0723	0.0726	0.0729	0.0732	0.0735
8	0.0738	0.0741	0.0743	0.0747	0.0749	0.0751	0.0754	0.0757	0.0760	0.0762
9	0.0765	0.0768	0.0771	0.0773	0.0776	0.0779	0.0781	0.0784	0.0787	0.0789
x	0	1	2	3	4	5	6	7	8	9
10	0.0792	0.0817	0.0840	0.0861	0.0881	0.0900	0.0919	0.0937	0.0954	0.0970

Take y from table.
Then the deflection is

$$D = yL\sqrt{B},$$

and the stress is,

$$F = \frac{wL^2}{8D}.$$

Note that b_1 may be greater or less than (b_2+b_3); in either case B is to be taken *positive* and equal to the numerical value of $b_1-(b_2+b_3)$. Also note that

b_1 is the elongation, in feet per 1000 feet, of a straight wire when subjected to a stress of F_0 pounds.

b_2 is the number of feet per 1000 feet of wire, by which the length of the wire at t_0 degrees and loading w_0 exceeds the horizontal distance between the points of support (when these are at the same elevation).

b_3 is the elongation, in feet per 1000 feet of wire, due solely to a change in temperature from t_0 to t degrees.

b is the elongation, in feet per 1000 feet of wire, due to a stress equal to wL(= its own weight plus the total ice and wind, if any, at the temperature t).

Example. — To find the sag at which a No. 0 B. & S. stranded copper wire must be strung at 60° F. on a 400-ft. span so that the wire will have a factor of safety of two at 0° F. when loaded with ice 0.5 in. thick all around the wire and with a wind pressure of 8 lb. per square foot of projected area. The breaking strength of the wire is 4980 lb., cross-section 105,500 circ. mils, modulus of elasticity 16×10^6, and coefficient of expansion 9.6×10^{-6}.

The data and calculations are then as follows:

$$F_0 = 2490 \qquad M = 16 \times 10^6,$$

$$w = 0.323, \qquad L = 400,$$

$$w_0 = 1.26, \qquad t - t_0 = 60$$

$$A = 0.1055, \qquad a = 9.6 \times 10^{-6}.$$

Then

$$b_1 = \frac{1273 \times 2490}{16 \times 10^6 \times 0.1055} = 1.878,$$

$$b_2 = \left(\frac{1.26 \times 400}{0.155 \times 2490}\right)^2 = 1.705,$$

$$b_3 = 1000 \times 9.6 \times 10^{-6} 60 = 0.576,$$

$$b = \frac{1273 \times 0.323 \times 400}{16 \times 10^6 \times 0.1055} = 0.0975.$$

Noting that b_1 is less than $(b_2 + b_3)$,

$$B = 1.705 + 0.576 - 1.878 = 0.403,$$

$$x = \frac{0.0975}{0.403\sqrt{0.403}} = 0.382,$$

and from Table IV, $y = 0.0308$. Whence

$$D = 0.0308 \times 400 \sqrt{0.403} = 7.82 \text{ ft.}$$

This is the sag at 60° F. with no ice or wind.

The sag at 0° F. with no ice or wind is found in exactly the same way, noting that for these conditions $b_3 = 0$, and b_1 is greater than $(b_2 + b_3)$, giving for B the value

$$1.878 - 1.705 = 0.173,$$

and for x the value

$$\frac{0.0975}{0.173\sqrt{0.173}} = 1.355,$$

and y from Table III is then 0.0367, and

$$D = 0.0367 \times 400 \sqrt{0.173} = 6.10 \text{ ft.}$$

Stress and Deflection in Spans of Unequal Length.— By stringing cables according to the stress determined for each particular length of span, the maxi-

mum allowable tension will be reached in all spans under maximum loading conditions. Under other loading conditions the tension will be unequal where span lengths are not the same. This is shown in Fig. 10 which is plotted for the same conditions as stated in the title of Fig 8, but for spans of from 600 to 900 feet in length. Fig. 11 shows the corresponding deflections.

The unequal stresses on the two sides of the insulator will tend to bend the tower or insulator pin, but any motion of the point of support will tend to equalize the stresses. When suspension insulators are used, the stresses are practically equalized, since the insulator is free to move. It should be noted that the motion of the insulator necessary to equalize the stresses is small. When suspension insulators are used the cable is therefore strung at a tension corresponding to the average length of span. When the cable is thus strung the tension under minimum temperature and maximum loading will usually exceed the assumed tension in the shorter spans and be less than the assumed tension in the longer spans. Similarly, under maximum temperature conditions, the longer spans will have a sag in excess of the value calculated on the assumption of the maximum allowable tension being reached with maximum loading, and the shorter spans will have a sag less than the calculated value. The actual stresses and deflections can be calculated by the method given in the following section.

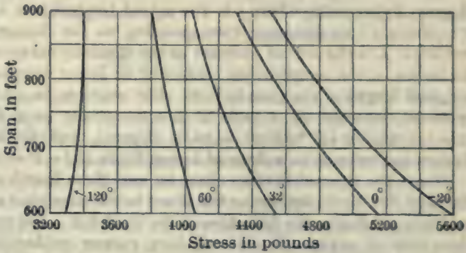

Fig. 10.

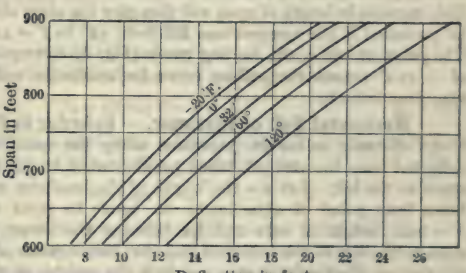

Fig. 11.

Calculation of Stresses in Unequally Loaded Spans.* — When suspension insulators are used, any tendency of the stresses in two adjacent spans to become unequal will produce such a deflection of the insulator, in the direction of the span with greater stress, as will establish equilibrium in the line. This state of affairs will occur (1) when adjacent spans carry unequal ice loads, (2) when the wire on one side of the insulator breaks, and (3) to a slight extent with changes in temperature when the adjacent spans are unequal in length, as noted above. The following method of calculating the "equilibrium" stresses in the wires and corresponding sags is applicable to all cases of initially unbalanced stresses, irrespective of their cause. The method may also be used to calculate the stress and sag in spans supported on pin insulators, provided the moment of bending of the pin and of the pole or tower is known or can be calculated.

* From lecture notes by Dr. H. Pender.

Change of Stress Due to Change in Length of Span. — When the length L of the span (i.e., distance between points of support) increases by λ inches, due to a horizontal displacement of the insulators (without slipping of the wire), the stress in the wire is increased by the same amount as would be produced by a fall in temperature of

$$t - t' = \frac{\lambda}{12\,aL} \qquad \text{or} \qquad t' = t - \frac{\lambda}{12\,aL}, \tag{15}$$

degrees Fahrenheit, where t is the actual temperature and t' may be called the "equivalent" temperature corresponding to the change in length λ. In this equation λ is the actual increase in the distance between the points of support in inches, a the temperature coefficient of linear expansion per ° F., and L the original length of the span in feet. For example, in an 800-foot span of copper wire an increase of 1 inch in L corresponds to a drop of temperature of $1 \div (12 \times 9.6 \times 10^{-6} \times 800) = 10.85$ degrees, and an increase in length of λ inches corresponds to a drop of temperature of $t - t' = 10.85\,\lambda$ degrees.

Hence the stress-deflection chart for any given length of span, see Fig. 8, may be used directly to determine the stress in the wire after any change in the length of the span, due to the deflection of the insulator. For example, consider an 800-foot span of 300,000-circular-mil, bare, copper conductor, at 32° F., without ice or wind, initially stressed to 4170 pounds, and let the length of the span be increased 4 inches as the result of the deflection of the insulators by this amount. This increase in length of span will then give the same stress in the wire as would be produced if the temperature fell from 32° to $t' = 32 - 4 \times 10.85 = -11.4°$. The point at which the loading curve (Fig. 8) corresponding to a temperature of $-11.4°$ crosses the pull-up curve for conductor only gives the new stress, viz., 4700 pounds.

Horizontal Pull of Insulator. — Referring to Fig. 12, let m = the horizontal distance in inches (measured along the span) which any insulator is deflected from the vertical, taken positive when to the right, say, and negative when to the left. Let x = the length of the insulator string in inches, i.e., distance from point of attachment to tower to point of attachment to wire; V = total weight of wire and ice between the lowest point of the wire in the span to the left of the insulator and the lowest point of the wire in the span to the right of the insulator plus one-half the weight of the insulator; H = total wind pressure on the length of wire between the middle points of the two adjacent spans, plus half the wind pressure on the insulator; and put $W = \sqrt{V^2 + H^2}$. Then the horizontal component of the pull of the insulator toward the left along the line of the span is *

$$P = \frac{m}{\sqrt{x^2 - m^2}} \cdot W. \tag{16}$$

For example, consider two adjacent spans of 300,000-circular-mil copper, each 800 feet long and with points of support at the same elevation. Let the span to the left be free of ice and let the one to the right have a ¼-inch ice coating; assume the insulator to be 60 inches long and to weigh 100 pounds. Then for no wind $H = 0$, $V = 1.19 \times 400 + 0.915 \times 400 + 100/2 = 892$ pounds. Whence for deflections of the insulator of less than 12 inches the horizontal pull of the insulator is $P = (892 \times m) \div 60 = 14.9\,m$, or 14.9 pounds per inch deflection.

* When m is less than 20 per cent of x this may be written, with an error of less than 2 per cent,

$$P = \frac{m}{x} W. \tag{16a}$$

Stresses in a Series of Spans When Points of Support are not Fixed. — Referring to Fig. 12, let the left-hand end of span No. 1 be anchored, and assume the insulator at the right-hand end to be deflected a horizontal distance of m_1 inches, due, for example, to a change in the loading on the succeeding

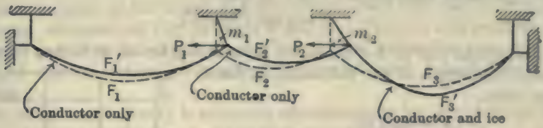

Fig. 12.

spans (or to a change in temperature when the spans are of unequal length). From equation (15) calculate the "equivalent" temperature t_1' corresponding to this change in length, and from the pull-up curve in Fig. 8 corresponding to the assumed loading w_1 of this span find the stress on this curve corresponding to a temperature of t_1' degrees; call this stress F_1'.

Next calculate the transverse and vertical loads on the insulator, viz., H_1 and V_1, and the resultant load $W_1 = \sqrt{V_1^2 + H_1^2}$, as explained in the preceding section. Then from equation (16) or (16a) calculate the horizontal pull P_1 of the insulator. The stress in the second span, assuming the value of m_1 chosen at the start is correct, must then be

$$F_2' = F_1' + P_1. \tag{17}$$

t_2' is then determined from F_2' on the pull-up curve. From equation (15) the corresponding increase in the length of span No. 2 must then be

$$\lambda_2 = 12\, aL_2\,(t - t_2'), \tag{18}$$

where L_2 is the length of the second span. The corresponding deflection of the insulator at the right-hand end of span No. 2 must then be

$$m_2 = m_1 + \lambda_2, \tag{19}$$

always taking the insulator deflection positive when to the right, say.

Using the values of λ_2 and m_2 thus found, calculate the λ_3 and m_3 in exactly the same manner as λ_2 and m_2 were calculated, and similarly for the succeeding spans until the next anchor tower is reached. For the anchor tower at the right-hand end of the n-th span, say, the deflection of the insulator must be zero, viz.,

$$m_n = 0. \tag{20}$$

If m_n as calculated comes out greater than zero, then the assumed value of m_1 is too great; if m_n comes out less than zero the assumed value of m_1 is too small. By calculating m_n for two or three assumed values of m_1, and plotting m_n as ordinates against m_1 as abscissas, the correct value of m_1 will be where this curve crosses the axis of abscissas. Using this correct value of m_1, the stresses and deflections in each span may then be accurately calculated by the process just given, using the Stress-Deflection Chart, Fig. 8. The complete process is best shown by an example.

Example. — Consider the case of three spans between anchor towers, (Fig. 12) all of the same length, 800 feet, and all supports at the same elevation, 300,000-circular-mil copper being used for the conductor. Let the temperature be 32° F., and let the middle span have a ¼-inch ice coating but the other two spans have no ice on them; also assume no wind. The Stress-Deflection Chart given in Fig. 8 then applies directly, provided the wires are strung in accordance therewith. Assume that each insulator weighs 100 pounds and has a length of

60 inches. Then for an increase of λ inches in the length of any span, the "equivalent" temperature is, from equation (15),

$$t' = 32 - 10.85\lambda$$

or if the equivalent temperature rise t' is known

$$\lambda = 0.092\,(32 - t').$$

The horizontal pull of any insulator for a deflection of m inches (small compared with the length of the insulator) is, from equation (16a),

$$P = 14.9\,m.$$

In the following table are given the calculations for assumed values of m_1 of 1, 2, 3 and 4 inches, and in Fig. 13 are plotted the corresponding calculated values of m_3 against m_1. It is seen that the relation between m_3 and m_1 is practically a straight line cutting the horizontal axis at $m_1 = 2.4$, which is therefore the correct value of m_1. The calculations for $m_1 = 2.4$ inches are given in the last column of the table. Hence the stresses and deflections in the two end spans (without ice) are $F_1' = F_3' = 4460$ pounds and $D_1' = D_3' = 16.3$ feet respectively, and the stress and deflection in the middle span loaded with ¼-inch of ice are $F_2' = 4496$ pounds and $D_2' = 21.1$ feet respectively, the deflections being read directly from Fig. 8 corresponding to the proper stresses and loadings (w).

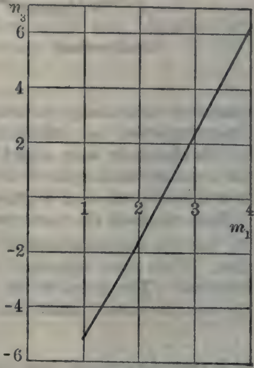

Fig. 13.

m_1	= assumed	1	2	3	4	2.4
t_1'	$= 32 - 10.85\,m_1$	21.1	10.3	−0.6	−11.4	6
F_1'	From Fig. 8 ($w = 0.915$)	4300	4410	4550	4700	4460
P_1	$= 14.9\,m_1$	15	30	45	60	36
F_2'	$= F_1' + P_1$	4315	4440	4595	4760	4496
t_2'	From Fig. 8 ($w = 1.19$)	105	90	74	55	84
λ_2	$= 0.092\,(32 - t_2')$	−6.72	−5.33	−3.86	−2.12	−4.78
m_2	$= m_1 + \lambda_2$	−5.72	−3.33	−0.86	1.88	−2.38
P_2	$= 14.9\,m_2$	−85	−50	−13	28	−36
F_3'	$= F_2' + P_2$	4230	4390	4582	4788	4460
t_3'	From Fig. 8 ($w = 0.915$)	26	13	−3	−16	6
λ_3	$= 0.092\,(32 - t_3')$	0.55	1.75	3.22	4.42	2.39
m_3	$= m_2 + \lambda_3$	−5.17	−1.58	2.36	6.30	0.01

LOCATION OF TOWERS AND DETERMINATION OF CLEARANCES.—The height of towers is determined so as to give some specified minimum clearance from conductor to ground for some length of span chosen as a nominal standard on the basis of level ground. In practice the ground is rarely level and the towers are actually located to conform to the irregularities of the ground. In locating towers of a given height the spans are made as long as possible consistent with maintaining the ground clearance. The irregularities of the ground are ordinarily advantageous and permit of slightly longer spans on the average than could be obtained with the same height of towers on level ground.

Profile and Plan of Right-of-Way. — In order to locate towers properly it is necessary to have a profile of the right-of-way. Profiles are conveniently plotted on standard ruled profile section paper to a vertical scale of 20 feet to the inch and a horizontal scale of 200 feet to the inch. Three profiles are desirable, one along the center of the tower line and one on each side, say at each edge of right-of-way, as shown on Fig. 14 at *A*, *B* and *C*. The two side profiles indicate the amount and direction of the slope of the ground across the line, which must be allowed for in determining ground clearance and foundation or tower extensions.

A plan of the right-of-way is of course also necessary for determining the construction at angles in the line, and the clearances from the conductor to the edge of the right-of-way when the conductor is deflected horizontally by the wind. Such a plan is shown at the bottom of Fig. 14.

Templates for Locating Towers. — Three templates are required, one for ground clearance with maximum sag, marked *M* in Fig. 14, one for uplift at times of minimum sag, marked *N*, and one for maximum side swing, marked *Z*. These are cut from thin celluloid and are to the same horizontal and vertical scales as used for the profile and plan of the right-of-way.

Since the curvature of the catenary or parabola in which the wire hangs depends only on the tension and loading and not on the length of the span or on the difference in elevation of the points of support, all spans having the same tension and loading can be drawn (for any one predetermined scale) from a single template, irrespective of their lengths or of the differences in elevation of the points of support. However, when the elevations of the points of support are not the same, the lowest point of the curve is shifted from the middle of the span toward the lower support, but the axis of the curve remains vertical.

Construction of Maximum Sag Template *M*. — The maximum sag is found from the Stress-Deflection Chart, Fig. 8, and may be the deflection corresponding to the maximum temperature, and conductor only, e.g., 21.4 feet in Fig. 8, or may be the deflection corresponding to 32° F. and the maximum ice loading. Wind will increase this deflection but will not increase the *vertical sag*. Call S_m this maximum sag. Then the equation of the maximum sag template, or template *M*, is

$$y = \left(\frac{4\,S_m}{L^2}\right) x^2, \tag{21}$$

where L is the length of the particular span for which the maximum sag is S_m, and the origin of the curve is its lowest point and the axis of y is vertical; all dimensions are in feet. Three parabolic curves are given by this template; the top curve represents the position of the cable and is drawn on the basis of the average span under the maximum load; the middle curve is a similar parabola below the upper curve a distance equal to the minimum allowable clearance to ground; the bottom curve is another similar parabola below the upper curve by a distance equal to the height of cable above ground at the support.

Construction of Minimum Sag Template *N*. — The minimum sag is also found from the Stress-Deflection Chart, and is usually the deflection corresponding to the minimum temperature and conductor only, e.g., 15.1 feet in Fig. 8. Call this sag S_n; then the equation of the minimum sag template, or template N, is

$$y = \left(\frac{4\,S_n}{L^2}\right) x^2. \tag{22}$$

Construction of Maximum Side-swing Template *Z*. — The maximum side swing occurs at time of maximum wind pressure and may be at

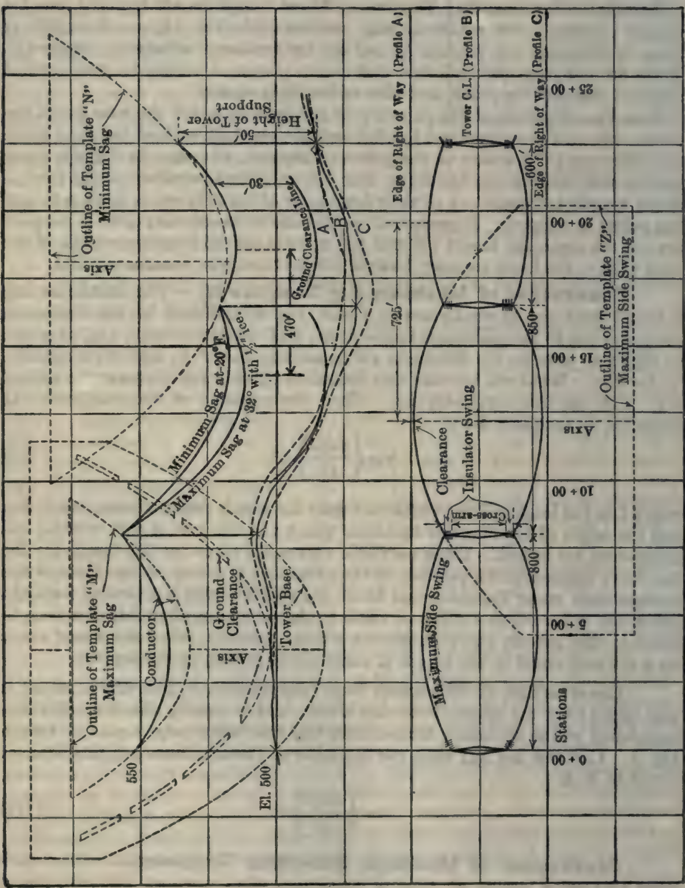

Fig. 14. Chart Showing Use of Templates in Locating Towers and Determining Clearances

maximum temperature or may be at 32° F. when covered with ice. In the latter case the side swing depends on the shape (circular or elliptical) of the ice covering and its specific gravity. For a circular covering of solid ice one particular thickness (usually but not necessarily the maximum thickness) gives the greatest side swing. For example, in Fig. 8, the maximum side swing occurs at $D = 21.7$ and $w = 1.807$, and its value is, see equation (10), $Z = (0.815 \times 21.7) \div 1.807 = 9.8$ feet, 0.815 being the wind pressure per foot of wire. Calling the maximum side swing Z_m, then the equation of the side swing template, or template Z, is

$$y = \left(\frac{4\,Z_m}{L^2}\right)x^2. \tag{23}$$

Locating Towers by Means of Template M. — Choose a starting point, as shown for example in Fig. 14, at station 0 + 00, elevation 500.0 feet for the first tower location. The template M is then placed over the profile and shifted until its axis is vertical and the lower curve is at station 0 + 00, elevation 500.0 feet, and the middle curve is tangent to the ground profile as shown. The proper location for the second tower is at the point where the lower curve again intersects the ground profile, or at station 8 + 00, elevation 506.0 feet, in the example. The operation is then repeated for the next tower. Adjustments in length of span are usually necessary to meet local conditions, in order to avoid locating towers in roads or swamps and to bring towers at angle points. Adjustments which increase the ground clearance are of course allowable.

The position of the conductors with maximum sag may be drawn on the profile from the top curve of the template.

Uplift on Insulator; Use of Template N. — An insulator sustains the weight of the lengths of conductor from the insulator to the lowest point of the span on each side. If the conductor leaves the insulator horizontally on one side, the lowest point of that span is at the insulator, which then sustains no weight due to that span. If the conductor has an upward inclination where it leaves an insulator, it is exerting an uplift equal to the weight of a length of conductor extending from the insulator along the span produced in the reverse direction to the lowest point of the parabola. Where the conductor has a downward inclination on one side and upward on the other side of the insulator, there will be a weight or uplift on the insulator equal to the difference between the weight of conductor on one side and uplift on the other.

Suspension insulators when used hanging downward to sustain weight are incapable of resisting uplift. Where uplift occurs the conductor may be dead ended or may be tied down or weighted down.

The method used for locating towers ordinarily precludes uplift under the loading which gives maximum sag, but uplift may occur when the loading is less. To determine this the minimum sag is drawn on the profile with template N. The minimum sag curve is drawn between points of support, keeping the axis of the parabola vertical as before.

Side Swing of Suspension Insulators. — Let l_1 = the length of conducter between the lowest point in the span to the left of the insulator and the lowest point in the span to the right of the insulator, and let l_2 = the distance between the middle points of these two spans, both in feet. Also let v = the weight of the conductor and ice per foot length, and h = the wind pressure per foot length (see Table II). Then the vertical pull on the insulator is vl_1 and the transverse horizontal force is hl_2. Also let v_1 = the weight of the insulator and h_1 = the total wind pressure on it. Then the insulator is deflected sidewise from the vertical by approximately the angle

$$\theta = \tan^{-1}\left[\frac{hl_2 + 0.5\,h_1}{vl_1 + 0.5\,v_1}\right]. \tag{24}$$

Usually the weight of the insulator and the wind pressure on it are negligible compared with the weight and wind pressure on the conductor, in which case

$$\theta = \tan^{-1}\left(\frac{hl_2}{vl_1}\right). \qquad (24a)$$

When the points of support are at the same elevation $l_1 = l_2$ and

$$\theta = \tan^{-1}\frac{h}{v}. \qquad (24b)$$

Calling X the length of the insulator in feet then the transverse horizontal deflection of the insulator is $X \sin \theta$ feet.

For example, consider the side swing of the third insulator (from the left) in Fig. 14. Then $l_1 = 470$, $l_2 = 850/2 + 600/2 = 725$, $v = 1.61$ (for 300,000-circular-mil conductor with ½ inch of ice), and $h = 0.82$ (for wind pressure of 6 pounds per square foot). Whence, neglecting the weight of the insulator and the wind pressure on it,

$$\theta = \tan^{-1}\frac{0.82 \times 725}{1.61 \times 470} = 38°.$$

If the insulator is 5 feet long, the transverse horizontal deflection is then $5 \sin 38° = 3.1$ feet.

If the angle of swing as thus determined is excessive the cables and insulators will be lifted up into the cross arms at times of low temperatures and high winds. The remedy is the same as in case of direct uplift.

Side Clearance; Use of Template *Z*.—Where a right-of-way of definite width is obtained it is necessary to determine whether the conductor will swing beyond the edge of the right-of-way. Therefore after the towers have been located by the use of the profile they should be marked on the plan and the side swing marked in from template Z, as shown in Fig. 14. In determining side swing, the swing of the insulator (if suspension type) must be allowed for, as well as the side swing of the conductor. Adequate margin should be allowed between the extreme position of conductor and the edge of the right-of-way, so that a safe clearance will be preserved from any structures erected adjacent thereto. Where extraordinarily long spans must be used, an adequate extra width of right-of-way should be obtained in the first place.

Loss of Clearance Between Conductors Due to Unequal Ice Loading.—Where one span is loaded with ice and the one immediately below it is not, the clearance is reduced. This condition may sometimes arise due to the ice falling off the lower wire before it falls off the upper wire. Where the wires are directly over each other the normal clearance must be great enough to prevent the crossing of wires under these conditions. For ice loading without wind, clearance under unequal loading is most easily obtained by offsetting the wires horizontally for the required clearance instead of increasing the vertical clearance. However, to prevent crossing of the unequally loaded wires when deflected by wind pressure, this horizontal offset must be considerable, as the clearance must then be obtained between the wires in their inclined positions.

If the conductors to which Fig. 8 refer are normally 10 feet apart vertically on an 800-foot span, the sag would be 17.5 feet without ice at 32° F., 19.0 feet with ¼-inch ice, and 20.9 feet with ½-inch ice at 32° F. Consequently, if two cables are used one above the other, and ice should form on the upper but not on the lower, then, assuming fixed points of support (pin insulators), the clearance would be reduced by 3.4 feet for ½-inch ice, and 1.5 feet for ¼-inch ice, making the clearances 6.6 feet and 8.5 feet respectively, instead of 10 feet.

Where suspension insulators are used the reduction of clearance from unequal ice loading is greater. If one span is loaded with ice and the adjacent spans of the same wire are not loaded, the sag of the loaded span will be increased because the insulators will swing toward that span. Similarly if one span is unloaded and adjacent spans are loaded, the sag will be decreased. The minimum clearance occurs where only one span of the upper wire is loaded and is immediately over the only unloaded span of the lower wire. The actual reduction is readily calculated by the method given above in the section on *Calculation of Stresses in Unequally Loaded Spans*, p. 1803. The amount of reduction depends on the number of spans between anchor towers, and the distance from the anchor towers at which the unbalanced loading occurs. For a 300,000-circular-mil copper cable on 800-foot spans at 32° F. with unequal loadings on sections of one, two, three and five spans (see Fig. 15) the assumed conditions of unequal loading and the loss of clearance are as follows:

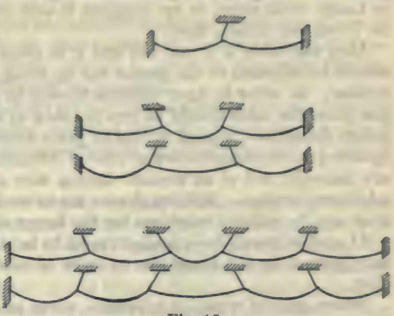

Fig. 15.

Number of spans between anchor towers	1	2	3	5
Upper conductor; ¼-inch ice on spans Nos.:*	1	1	2	3
Lower conductor; ¼-inch ice on spans Nos.:*	...	2	1, 3	1, 2, 4, 5
Sag of middle span of upper conductor, feet	19.0	20.5	21.1	21.6
Sag of middle span of lower conductor, feet	17.5	15.9	15.5	15.2
Loss of clearance in middle span, feet	1.5	4.6	5.6	6.4

* The other spans assumed to have no ice load.

Stresses and Deflections Due to Broken Conductors.—When a conductor breaks in a span supported by suspension insulators, the insulators adjacent to the broken span swing up into line with the cable, throwing increased slack into the unbroken part of the cable equal to the length of the insulator. This slack divides between the unbroken spans, increasing the deflection of each. The stresses and deflections of the unbroken spans may be determined by the method given above in the section on *Calculation of Stresses in Unequally Loaded Spans*, p. 1803, calling the span in which the break occurs span No. 1.

ERECTION OF TRANSMISSION LINES.—The preliminary work in constructing a transmission line includes the clearing of the right-of-way, moving of buildings (if necessary), building roads, bridges, fences, gates, etc., to make the right-of-way passable. If the telephone line is to be supported on separate structures, this is generally built in advance of the main line and used during the rest of the construction.

After the towers are in place and the insulators placed, the wires and cables are strung. The reels are distributed and spaced according to the length of cable on each. The reels of heavy cable are supported on reel jacks. The lighter wire and cable is usually drawn out on the ground and carried or hauled up on the supporting structures. Heavy cables are drawn over rollers or sheaves attached to the towers at approximately the final point of support. The pulling

out of wires and cables is usually done with horses, but under favorable conditions a traction engine or hoisting engine can be used. Where lines parallel railroads they have been pulled out successfully with locomotives. Care should be taken in drawing out the cable so as not to scratch or injure it by sharp bends or faulty cable grips, or by dragging it over sharp stones. See also the article on *Wires and Cables, Bare.*

After the cable is drawn out and is in place on the sheaves on the towers, it should be adjusted to the proper sag by using dynamometers for measuring the tension. In order to have uniform tension in all spans when pulling several at one time, the cable should be free to move at all points of support. Wires and cables should be clamped or tied in place while under the proper tension corresponding to the temperature at time of stringing. Wind loads on cables or wires are generally not of an amount during wire stringing to require allowance for additional tension. Splicing sleeves are twisted by hand wrenches in the smaller sizes and by splicing machines for the larger sizes.

At anchor towers loose jumpers must be bent to shape and not left so that they may ground on the tower due to twist of cable, pressure of wind or weight of sleet.

Where ground wires or telephone wires are also on the tower, it is equally important that they be strung at the proper tension; otherwise they may cross and ground the conductors.

Suspension towers (i.e., those intermediate between the dead-end towers) are ordinarily not strong enough to stand the strain of dead-ended cables during high winds and heavy sleet storms; consequently care must be used if cables are temporarily dead ended on them during construction.

Transposition of Transmission Lines. — (*See also Telephone Lines.*) Transpositions are not necessary for the proper operation of an isolated transmission line, but are used to diminish the inductive effects of the transmission line on neighboring circuits. The number of transpositions required depends principally on the sensitiveness and proximity of other circuits, especially telephone lines and on the distance that such lines parallel the transmission line, and also to a lesser degree on the current and voltage of the transmission line.

Where the voltage of transmission is 2200 or less the length of exposure (i.e., distance between two successive transposition points) is usually insufficient to require transpositions and, when the voltage is over 60,000, the lines are often at sufficient distance from nearest telephone line to make transpositions unnecessary. For voltages above 2200 and not over 60,000 a telephone line is ordinarily run the whole distance on the same poles or towers with a transmission line, thereby making transpositions necessary. Transpositions on such transmission lines are ordinarily located three miles or more apart, the telephone line also being transposed every 500 or 600 feet.

Each transposition of a three-phase line ordinarily consists of a spiral of one-third of a turn. Three transpositions give a complete spiral and bring the phases back to their original position. A line is ordinarily transposed by giving it one or more complete spirals, the transpositions being located so as to divide the line into approximately equal sections. For one complete spiral two transpositions dividing the line into thirds are sufficient, though the equivalent of a third transposition is necessary at one end if the phases are to have the same relative position in the station wiring at each end.

Special poles or towers are usually necessary at transposition points.

TESTS AND INSPECTION OF TRANSMISSION LINES. — Tests on transmission lines include testing of the several parts; conductors, insulators, pins, clamps, ties, towers, etc., and are described in the articles on *Wires and*

Cables; Insulators for Overhead Lines; Poles for Overhead Lines, Cross Arms; Towers. The efficiency and other electrical characteristics of transmission lines can be computed with such certainty that tests are not necessary to determine these features, though the calculations are occasionally checked by observations made during actual operation.

When a line is completed it should be carefully inspected to see that all joints have been made, all insulators put on, etc. Moreover, before starting continuous operation, it is advisable to apply voltage in order to make sure that the line is not open circuited, short-circuited or grounded.

Where two or more transmission lines are to be in parallel, or where a new line connects two points already connected by other lines, it is necessary to test out to find corresponding phases before connecting in multiple, especially if any of the lines have transpositions.

OPERATION. — Patrolmen are usually located between ten and twenty miles apart, depending on the character of the country. The coöperation of people living along the line must be obtained in order to prevent damage by breaking of insulators, throwing wires over the conductors, blasting rocks or stumps near the line, etc. Patrolmen should inspect the line regularly and keep weeds, brush and inflammable material away from the poles and towers. They should note the condition of the foundations, towers, poles, insulators and conductors. In addition they should make minor and emergency repairs on the line.

Section switches are often installed at each patrolman's house. By manipulating these switches any section of line may then be tested for faults.

The proper maintenance of a line includes resetting foundations that have settled; covering of foundations with earth to proper depth after heavy rains; repainting ot towers before they are affected by rust; renewal of rusted ground cables; replacement of cracked or partially defective insulators that have not failed; correcting sag of any cable where sag has changed due to stretch of cable, change of length during emergency repairs, etc.

Telephone Connections. — A good telephone line is essential and the patrolmen should report by telephone at regular intervals. As a convenience in operation, telephones are installed or connections are provided for a portable telephone set at intervals of three or four miles along the line. For the higher potential lines protection must be provided for the person using telephone. The usual protection includes insulated stools, telephone insulating transformers, drainage coils, telephone lightning arresters and fuses.

COSTS OF TRANSMISSION LINES. — To obtain an accurate estimate of the cost of a transmission line the cost of the various elements should be separately determined. The following over-all costs based on before-the-war prices are given as a rough guide in preliminary estimating.

Cost of Right-of-Way. — The cost of the right-of-way for a low-voltage line ranges from a nominal amount for a line on public roads, to $10,000 per mile or more for a private right-of-way in thickly settled districts. A right-of-way 100 feet wide requires approximately 12 acres per mile, which for farming land at $200 per acre, amounts to $2400 per mile. Land values may be expected to average considerably higher for right-of-way than for farming, especially if a strip of land is desired crossing a farm diagonally.

The right-of-way for high-voltage tower lines should be estimated liberally as such lines should have long spans and few angles, and cannot be diverted around expensive property and obstructions as readily as short-span wooden-pole lines.

Effect of Size and Cost of Conductor on Total Cost. — The size and cost per pound of conductors greatly affect the cost of a line. A circuit of 3

No. 0000 cables costs about $1500 per mile with copper at 14.4 cents per pound and $2000 per mile at 19.2 cents per pound. The cost of smaller cables is proportionally less; No. 0 being about one-half, No. 3 about one-fourth and No. 6 about one-eighth the cost of No. 0000. Thus the copper cost of a three-wire line will range from about $200 per mile for No. 6 at 15.6 cents per pound to $2000 per mile for No. 0000 at 19.2 cents per pound.

For the same power loss in the line aluminum conductors usually cost about 10 per cent less than copper.

Total Structural Cost of Wooden Pole Lines. — The structural cost of a wooden-pole transmission line ordinarily ranges from about $1500 per mile for a 11,000-volt line with 3 No. 6 wires to about $4000 per mile for a 55,000-volt line with 3 No. 0000 wires, excluding the cost of right-of-way. To the structural cost and cost of right-of-way should be added the charges for engineering, contractor's services, superintendence, tools and equipment, etc., which may amount to from 20 to 40 per cent of the structural cost.

Total Structural Cost of Tower Lines. — The cost of light tower lines for 55,000 volts with 3 No. 0000 wires may be as low as for a wooden pole line, i.e., about $4000 per mile. For 110,000 volts with three 300,000-circular-mil wires the cost may be about $8000 per mile or more. The cost of towers proper varies greatly according to the views of the designers as to what risks are proper in design, that is, the severity of assumed wind and ice loadings, and the factors of safety. The cost of foundations is very small when simple steel stubs are used in very firm ground, but foundations add greatly to the cost of a line when concrete or steel structures are used, so designed as to make the strength, rigidity and holding power of the foundations actually equal to the strength of the tower under reasonably unfavorable conditions of ground. See article on *Towers for Transmission Lines*.

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Reports: *Report of Engineering Data Committee of the A.I.E.E.*, presented at the annual convention of the A.I.E.E., June 1914 (contains a large amount of important data on all features of the transmission lines of a number of large hydroelectric plants); *Transmission Line Crossings, Joint Report of the Committee on Overhead Line Construction*, N.E.L.A., and *Report of Committee on Transmission Line Crossings*, A.R.E.A.

TRIGONOMETRIC FUNCTIONS. — (*See also Derivatives; Integrals; Series, Mathematical; Trigonometry.*) The trigonometric functions of an angle are the ratios to one another of the various sides of a right triangle having the given angle as one of its angles. Referring to Fig. 1, let B, P and H be the three sides of a triangle. Then the trigonometric functions of the angle x are

sine of x, abbreviated $\sin x = \frac{P}{H}$; cotangent of x, abbreviated $\cot x = \frac{B}{P}$;

cosine of x, abbreviated $\cos x = \frac{B}{H}$; secant of x, abbreviated $\sec x = \frac{H}{B}$;

tangent of x, abbreviated $\tan x = \frac{P}{B}$; cosecant of x, abbreviated $\csc x = \frac{H}{P}$.

When B, P and H are limited to the three sides of a right triangle, the above definitions are directly applicable only to angles lying between 0 and 90°. The definitions, however, may be extended by considering the point A (Fig. 2) as

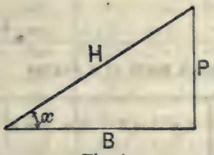


Fig. 1.

Fig. 2.

describing a circle of radius OA with the center at O. Let XX' be the horizontal diameter and YY' the vertical diameter of this circle, and call P the perpendicular distance from A to the line XX' and B the horizontal distance from A to YY'. P is to be considered positive when A lies above XX', negative when below. B is considered positive when A is to the right of YY' and negative when to the left. The four quarters of the circle are called quadrants, and are designated as the first, second, third and fourth quadrants as indicated. The angle is said to lie in the quadrant in which the point A lies. In Fig. 2 the angle x is in the second quadrant.

ALGEBRAIC SIGNS OF THE FUNCTIONS

	Sine	Cosine	Tangent
Angle in first quadrant..................	+	+	+
Angle in second quadrant..................	+	−	−
Angle in third quadrant..................	−	−	+
Angle in fourth quadrant..................	−	+	−

Period. — From the above definitions it is evident that adding 2π radians or 360° to an angle does not change the value of any of its functions, that is, these functions repeat themselves every time the angle increases by the 2π radians or 360°. They are therefore said to have a period equal to 2π radians or 360°.

Functions of Angles in Any Quadrant in Terms of Angles in First Quadrant.—

$$\sin(-x) = -\sin x, \qquad \sin(90+x) = \cos x,$$
$$\cos(-x) = \cos x, \qquad \cos(90+x) = -\sin x,$$
$$\tan(-x) = -\tan x, \qquad \tan(90+x) = -\cot x,$$

$$\sin(180-x) = \sin x, \qquad \sin(180+x) = -\sin x,$$
$$\cos(180-x) = -\cos x, \qquad \cos(180+x) = -\cos x,$$
$$\tan(180-x) = -\tan x, \qquad \tan(180+x) = \tan x,$$

$$\sin(270-x) = -\cos x, \qquad \sin(270+x) = -\cos x,$$
$$\cos(270-x) = -\sin x, \qquad \cos(270+x) = \sin x,$$
$$\tan(270-x) = \cot x, \qquad \tan(270+x) = -\cot x.$$

Table of Trigonometric Functions. — By making use of the above relations the functions of any angle may be obtained from a table giving the values of the functions for angles between 0 and 90°. Such a table is given below. For the cot, sec and csc take the reciprocals of the tan, cos and sin respectively.

Example of Use of Table. — sin 21.6° = 0.3681, cos 21.6° = 0.9298, tan 21.6° = 0.3959; sin 107° = sin (180° − 107°) = sin 73° = 0.9563, cos 107° = − cos 73° = − 0.2924, tan 107° = − tan 73° = − 3.2709.

TRIGONOMETRIC FUNCTIONS 0.0°–6.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	sin	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
	cos	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
	tan	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157
1	sin	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
	cos	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995
	tan	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332
2	sin	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506
	cos	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987
	tan	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507
3	sin	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680
	cos	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977
	tan	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682
4	sin	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854
	cos	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963
	tan	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857
5	sin	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028
	cos	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947
	tan	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033
6	sin	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201
	cos	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928
	tan	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210

TRIGONOMETRIC FUNCTIONS 7.0°–20.9°

Angle in degrees	Name of function	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374
7	cos	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905
	tan	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388
	sin	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547
8	cos	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880
	tan	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566
	sin	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719
9	cos	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851
	tan	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745
	sin	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891
10	cos	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820
	tan	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926
	sin	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062
11	cos	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785
	tan	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107
	sin	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232
12	cos	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748
	tan	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290
	sin	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402
13	cos	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707
	tan	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475
	sin	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571
14	cos	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664
	tan	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661
	sin	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740
15	cos	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617
	tan	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849
	sin	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907
16	cos	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568
	tan	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038
	sin	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074
17	cos	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516
	tan	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230
	sin	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239
18	cos	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461
	tan	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424
	sin	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404
19	cos	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403
	tan	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620
	sin	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567
20	cos	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342
	tan	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819

TRIGONOMETRIC FUNCTIONS **21.0°–34.9°**

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730
21	cos	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278
	tan	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020
	sin	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891
22	cos	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212
	tan	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224
	sin	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051
23	cos	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143
	tan	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431
	sin	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210
24	cos	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070
	tan	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642
	sin	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368
25	cos	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996
	tan	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856
	sin	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524
26	cos	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918
	tan	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073
	sin	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679
27	cos	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838
	tan	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295
	sin	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833
28	cos	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755
	tan	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520
	sin	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985
29	cos	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669
	tan	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750
	sin	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135
30	cos	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581
	tan	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985
	sin	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284
31	cos	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490
	tan	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224
	sin	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432
32	cos	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396
	tan	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469
	sin	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577
33	cos	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300
	tan	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720
	sin	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721
34	cos	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202
	tan	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976

TRIGONOMETRIC FUNCTIONS 35.0°–48.9°

Angle in degrees	Name of function	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
		Value of function for each tenth of a degree									
35	sin	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864
	cos	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100
	tan	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239
36	sin	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004
	cos	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997
	tan	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508
37	sin	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143
	cos	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891
	tan	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785
38	sin	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280
	cos	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782
	tan	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069
39	sin	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414
	cos	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672
	tan	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361
40	sin	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547
	cos	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559
	tan	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662
41	sin	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6653	0.6665	0.6678
	cos	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443
	tan	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972
42	sin	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807
	cos	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325
	tan	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293
43	sin	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934
	cos	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206
	tan	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623
44	sin	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059
	cos	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083
	tan	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965
45	sin	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181
	cos	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959
	tan	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	sin	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302
	cos	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833
	tan	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	sin	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420
	cos	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704
	tan	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	sin	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536
	cos	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574
	tan	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463

TRIGONOMETRIC FUNCTIONS 49.0°–62.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
49	sin	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649
	cos	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441
	tan	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	sin	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760
	cos	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307
	tan	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	sin	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869
	cos	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170
	tan	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	sin	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976
	cos	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032
	tan	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222
53	sin	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080
	cos	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892
	tan	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	sin	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181
	cos	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750
	tan	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229
55	sin	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281
	cos	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606
	tan	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
56	sin	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377
	cos	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461
	tan	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340
57	sin	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471
	cos	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314
	tan	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	sin	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563
	cos	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165
	tan	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577
59	sin	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652
	cos	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015
	tan	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
60	sin	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738
	cos	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863
	tan	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	sin	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821
	cos	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710
	tan	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	sin	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902
	cos	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555
	tan	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542

TRIGONOMETRIC FUNCTIONS 63.0–76.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980
63	cos	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399
	tan	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413
	sin	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056
64	cos	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242
	tan	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
	sin	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128
65	cos	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083
	tan	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355
	sin	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198
66	cos	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923
	tan	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
	sin	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265
67	cos	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762
	tan	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627
	sin	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330
68	cos	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600
	tan	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916
	sin	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391
69	cos	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437
	tan	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326
	sin	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449
70	cos	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272
	tan	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878
	sin	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505
71	cos	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107
	tan	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
	sin	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558
72	cos	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940
	tan	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
	sin	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608
73	cos	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773
	tan	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
	sin	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655
74	cos	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605
	tan	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
	sin	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699
75	cos	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436
	tan	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
	sin	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740
76	cos	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267
	tan	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972

TRIGONOMETRIC FUNCTIONS 77.0°–89.9°

Angle in degrees	Name of function	Value of function for each tenth of a degree									
		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	sin	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778
77	cos	0.2250	0.2232	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096
	tan	4.3315	4.3662	4.4015	4.4374	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
	sin	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813
78	cos	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925
	tan	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
	sin	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845
79	cos	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754
	tan	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
	sin	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874
80	cos	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582
	tan	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
	sin	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900
81	cos	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409
	tan	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
	sin	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923
82	cos	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236
	tan	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
	sin	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943
83	cos	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063
	tan	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
	sin	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960
84	cos	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889
	tan	9.5144	9.6768	9.8448	10.02	10.20	10.39	10.58	10.78	10.99	11.20
	sin	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974
85	cos	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715
	tan	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95
	sin	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985
86	cos	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541
	tan	14.30	14.67	15.06	15.46	15.89	15.35	16.83	17.34	17.89	18.46
	sin	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993
87	cos	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366
	tan	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27
	sin	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998
88	cos	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192
	tan	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08
	sin	0.9998	0.9999	0.9999	0.9999	0.9999	1.000	1.000	1.000	1.000	1.000
89	cos	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017
	tan	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0

Anti-functions. — If $a = \sin x$, then x is the angle whose sine is a; this may be expressed symbolically $x = \sin^{-1} a$, which is read "x equals the angle whose sine is a." The angle x is also called the "anti-sine" or the "inverse sine" of a. Similar notation is used for the other functions; for example, $x = \cos^{-1} b$ is used to express the relation that x is the angle whose cosine is b. At least two "anti-functions" must be known to completely determine the quadrant in which an angle lies; for example, if $x = \sin^{-1} 0.5$ then x may be either 30° or 150°, but if we also have $x = \cos^{-1} 0.866$, then x must equal 30°, while if $x = \cos^{-1}(-0.866)$, then x must equal 150°.

Anti-functions may be taken from the table given above by finding the angle in the margin corresponding to the function in the table. *Example:* $\sin^{-1} 0.319 = 18.6°$ or $180° - 18.6° = 161.4°$.

Versine. — The expression $(1 - \cos x)$ is called the "versine" of x.

Relations Among Functions of the Same Angle. —

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}, \qquad \sin^2 x + \cos^2 x = 1,$$

$$\sec x = \frac{1}{\cos x}, \qquad 1 + \tan^2 x = \frac{1}{\cos^2 x},$$

$$\csc x = \frac{1}{\sin x}, \qquad 1 + \cot^2 x = \frac{1}{\sin^2 x},$$

$$\sin(90 - x) = \cos x, \qquad \sin(-x) = -\sin x,$$

$$\cos(90 - x) = \sin x, \qquad \cos(-x) = \cos x,$$

$$\tan(90 - x) = \cot x, \qquad \tan(-x) = -\tan x.$$

Sum and Difference of Two Angles. —

$$\sin(x + y) = \sin x \cos y + \cos x \sin y,$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y,$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y},$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y,$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y,$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}.$$

Product of the Functions of Two Angles. —

$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)],$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)],$$

$$\cos x \sin y = \tfrac{1}{2}[\sin(x + y) - \sin(x - y)],$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x + y) + \cos(x - y)].$$

Functions of Twice an Angle. —

$$\sin 2x = 2 \sin x \cos x, \qquad \cos 2x = \cos^2 x - \sin^2 x,$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$

Functions of Half an Angle. —

$$\sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}, \qquad \cos \frac{x}{2} = \sqrt{\frac{1 + \cos x}{2}}, \qquad \tan \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}.$$

Functions of Three Times an Angle. —

$$\sin 3x = 3 \sin x - 4 \sin^3 x, \qquad \cos 3x = 4 \cos^3 x - 3 \cos x,$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.$$

TRIGONOMETRY. — (*See also Trigonometric Functions.*) Any triangle is completely defined when, (1) two sides and the included angle are known, (2) one side and two angles are known, (3) three sides are known. Let the sides and angles of a triangle be designated as in Fig. 1.

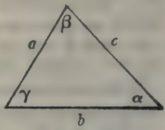


Fig. 1.

1. **Given two sides a and b, and the included angle γ.** Then

$$c = \sqrt{a^2 + b^2 - 2\,ab\cos\gamma}$$

$$\sin\alpha = \frac{a}{c}\sin\gamma$$

$$\beta = 180 - \alpha - \gamma.$$

2. **Given the side a and the two angles β and γ.** Then

$$\alpha = 180 - \beta - \gamma$$

$$b = a\frac{\sin\beta}{\sin\alpha}$$

$$c = a\frac{\sin\gamma}{\sin\alpha}.$$

3. **Given the three sides a, b and c.** Put

$$s = \tfrac{1}{2}(a + b + c)$$

Then

$$\sin\alpha = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$$

$$\sin\beta = \frac{b}{a}\sin\alpha$$

$$\gamma = 180 - \alpha - \beta.$$

Relations Between Sides and Angles. — The following relations between the sides and angles of a triangle are sometimes useful:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2\,bc}$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

and similar relations for the other two angles.

TROLLEY SYSTEMS, OVERHEAD. — (*See also Cars, Electric; Cross Arms; Locomotives, Electric; Poles for Overhead Lines; Rails, Track and Third; Railways, Electric, Traction Systems for; Railways, Location and Permanent Way for; Third-rail Systems; Transmission Lines; Trolley Systems, Underground; Wires and Cables, Bare.*) The following is a brief table of contents of this article:

The trolley wire is usually of hard-drawn copper but sometimes of steel, which is suspended from insulators some 16 to 30 feet above the ground, and presents a continuous contact surface to a trolley wheel or bow attached to the rolling stock. There are two classes of trolley construction, the span wire and the side bracket; each may have either simple or catenary suspension.

Span-wire and Side-bracket Construction. — In the simple span wire construction the trolley wire is supported by wires stretched across the tracks between poles or building walls. The side-bracket construction resembles the span wire except that instead of the supporting wire being stretched between two poles, it is stretched between two supports on the same pole. In both of these types of construction, the trolley wire is supported at intervals of 100 feet or more and sags considerably between supports, making it necessary for the trolley to be in constant vertical vibration as the cars move.

Catenary Construction. — The speed attained upon modern electric roads makes it difficult to obtain satisfactory service with a trolley wire which dips between each support and sags and sways with every impulse. The catenary construction was devised to meet this condition. In general, it consists of a grooved copper trolley wire suspended horizontally from a sagging messenger cable, which is suitably insulated and firmly held in place. The supporting structure preferably employed for interurban single- or double-track roads is of the side-bracket type, but for some conditions cross-span construction becomes necessary. The latter method of support differs only in the substitution of a catenary cross span for the bracket arm and doubling the number of poles required for single track.

APPLICATIONS OF VARIOUS TYPES OF CONSTRUCTION. — The overhead trolley system is used on urban railways, wherever the unsightliness or danger of its exposed construction is not considered objectionable. It is used on interurban and suburban lines wherever the current taken by the trains is not too great to be economically carried on copper wires. In recent years it has been used in conjunction with the alternating-current systems of electric traction on electric trunk lines.

Center-pole construction is the most sightly for double-track city railways, especially if ornamental brackets are used, side-pole construction being generally used for single-track lines. Span-wire construction is used where, for any reason, it is impracticable to have the poles near the tracks or where, as is commonly the case in Europe, the span wires are supported from the walls of buildings. Prejudice against overhead lines is often due to the excessive loading of poles, which is both unsightly and dangerous.

ELECTRICAL DESIGN OF CONDUCTORS. — In designing the trolley system careful attention must be given to both the electrical and mechanical features. In this section will be treated the electrical features, and in the following section the mechanical features. The electrical design of railway distribution systems involves the consideration of potential drop, heating of conductors and feeder economy. The heating of conductors is seldom an important factor in railway feeder design, as it is usually necessary to use a low current density to keep down the drop of potential. A discussion of the heating of conductors will be found under *Wires and Cables, Bare*. The feeder system being sufficient to meet the conditions imposed by the allowable potential drop (*see below*), it will be economical to make it greater if the saving in the cost of energy which will result, is greater than the increase in interest and other charges on the additional investment.

Allowable Potential Drop. — The total potential drop is limited by the necessity of running the cars at a certain speed and by the need of keeping the car lights brilliant. The potential drop in the ground conductors, i.e., the track rails and bare negative feeders, is further limited by the danger of electrolysis by current leaking into the earth (*see article on Electrolysis*).

The drop in the grounded conductors under maximum-load conditions is limited by law in Great Britain to 7 volts between any two points of the system. In Germany the maximum drop in the grounded conductors is limited to 1 volt per kilometer (1.61 volts per mile). In the United States the legal limit is a matter of local option and is, in general, less severe than in Europe.

Calculation of Potential Drop. — The method of calculation of potential drop depends upon the following conditions: (1) whether the current is direct or alternating; (2) whether the load is concentrated at one point, sparsely distributed or evenly distributed; (3) the distribution of metal in the feeder circuits, and (4) whether the section is being fed by one or by two or more substations. Calculations for alternating-current lines differ from those for direct-current lines only in taking into account the inductance, as described below. On city railways it is usual to assume the load to be evenly distributed, it being stated as a given number of amperes per foot (*see article on Railways, Energy Requirements and Motor Capacity for*). If the load is actually concentrated at n equidistant points, the drop will exceed that calculated on the assumption of uniform distribution by about $\frac{100}{n}$ per cent. On interurban and trunk lines, the cars are usually concentrated at one or two points between substations, making the assumption of uniform distribution impracticable. In such cases the loads should be located so as to give the worst conditions, and calculations made as for any network.

Where electrolytic damage is to be guarded against, the drop of potential in the track rails themselves has to be calculated, as well as the total drop in the rails and feeders.

Resistance of Trolley and Track. — Values of the resistance of trolley wires to direct current will be found in the article on *Wires and Cables, Bare*, and values of the resistance of rails to direct current will be found in the article on *Rails, Steel*. It should be noted that the resistances of the trolley and positive feeders are in parallel and that the track rails and negative feeders are in parallel. Also note that in the case of high-voltage systems a considerable portion of the current returns through the earth and not through the rails, and consequently the drop in the rails is due only to that part of the current which returns through them. For preliminary calculations, however, the full current may be assumed as returning through the rails.

Formulas for Direct-current Trolley Circuits. — The following formulas apply to certain typical circuits which frequently occur in practice. Let

I = total current in amperes taken by all cars on section considered,
L = total length of section in 1000 feet,
V_p = total drop in volts, in positive conductors between substation bus and far end of line,
V_n = total drop in volts in negative conductors between substation bus and far end of line,
$V = V_p + V_n$ = total drop in volts in both positive and negative conductors,
r_p = resistance in ohms of all the positive conductors in multiple per 1000 feet of line,
r_n = resistance in ohms of all the negative conductors in multiple per 1000 feet of line,
$r = r_p + r_n$ = total resistance in ohms per 1000 feet of line,
l = distance in 1000 feet, from far end of line to any point P,
v = drop to the point P, subscripts used as for V.

Uniformly Distributed Load, Uniform Conductor, Fed from One Substation. — Then

$$v = \frac{rIl^2}{2L},$$

$$V = \frac{rIL}{2}.$$

These formulas are applicable to either the positive or negative conductors considered separately or to both in series.

Uniformly Distributed Load, Conductor Tapered to give Minimum Weight of Metal, Fed from One Substation. — For minimum weight the tapering must be such that at any point P

$$r = \frac{3V}{2I\sqrt{L}\sqrt{l}},$$

i.e., the cross-section, if all the conductors are of the same metal, must increase directly as the square root of l. The drop to the point P is

$$v = \frac{V\sqrt{l^3}}{L^3}.$$

These formulas also apply to either the positive or negative conductors separately or to both in series.

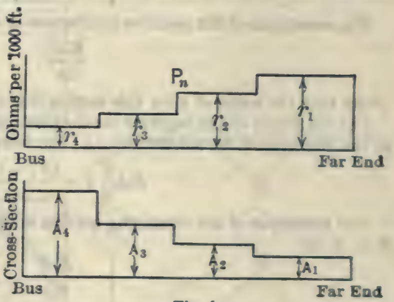


Fig. 1.

Uniformly Distributed Load, Conductor Divided into Sections (Fig. 1); Each Section of Constant Resistance, Fed from One Substation. — The drop from P_n to the far end of line is

$$\frac{I}{2L}\sum_{n=1}^{n=n} r_n[l^2_n - l^2(n-1)].$$

This formula is applicable to either the positive or negative conductors. The subscript n is here the general form of the subscript corresponding to each section, represented by 1, 2, 3 and 4 in Fig. 1.

Concentrated Load, Section Fed from One End.—Fig. 2 shows a 4-track road with 4 trolley wires and 3 feeders with the tracks cross-bonded at intervals. The solution given below is a general one, and may be applied to any case from 1 track and 1 trolley wire up. Let all distances be expressed in 1000 feet and let

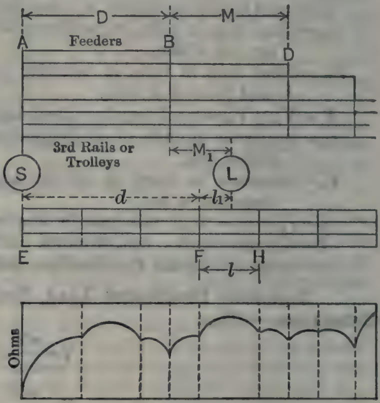


Fig. 2.

N_f = number of feeders in section considered, e.g., for the section AB, $N_f = 3$, and for the section BD, $N_f = 2$,

N_c = number of contact conductors, trolley wires or third rails in section considered, e.g., four are shown in Fig. 2,

N_t = number of tracks in section considered,

R_f = resistance of each feeder per 1000 feet,

R_c = resistance of each contact conductor per 1000 feet,

R_t = resistance of each track per 1000 feet (1 rail or 2 rails in multiple depending upon whether 1 or 2 rails are used for return conductor),

$$n = N_c + \frac{R_c}{R_f} N_f \qquad \text{for the section considered.}$$

Then for the section in which the load may be, the resistance of the positive conductors from the load to the end of that section in the direction of the substation, e.g., the resistance from L to B, is

$$R_c M_1 \left[1 - \frac{(n-1)\,M_1}{nM} \right].$$

The resistance of the positive conductors in any section such as BA is

$$\frac{R_c D}{n}.$$

(Note that the value of n for this section is not the same as for the section BD.)

The resistance of the negative conductors from the load to the first cross bond in the direction of the substation, e.g., the resistance from L to F, is

$$R_t l_1 \left[1 - \frac{(N_t - 1)\,l_1}{N_t l} \right].$$

The resistance of the remaining portion of the negative conductors, e.g., from F to E, is

$$\frac{R_t d}{N_t}.$$

(Note that if negative feeders are used, each negative feeder having a resistance of R_f' per 1000 feet, then for N_t in the last two formulas substitute $n' = N_t + \frac{R_t}{R_f'} N_f'$, where N_f' is the number of negative feeders for that section.)

The total resistance from the load to the substation is the sum of the resistances as above calculated.

Concentrated Load, Section Fed from Both Ends, Substation Voltage at the Two Ends the Same. — The most convenient method of treating such problems is to plot an "equivalent-resistance-distance" curve such as shown in Figs. 2 to 5. By "equivalent resistance" is here meant that resistance by which the total current taken by the load must be multiplied to give the total drop in voltage between the load and either substation. For example, if the substation voltage is 600 at each end, the voltage across the load is 550 and the current taken by the load is 200 amperes, then the equivalent resistance is $R = \frac{600 - 550}{200} = 0.25$. This method avoids the determination of the distribution of the current in the various parts of the network, and the resistance when once determined can be applied to any load.

1. In Fig. 3 is shown a single track and single trolley. S_1 and S_2 are substations; L is a load placed arbitrarily between corresponding points P_1 and P_2 on the positive and negative conductors respectively. Let

a = resistance of the conductors between the points $S_1P_1P_2S_1$;

b = resistance of the conductors between the points $S_2P_1P_2S_2$.

These resistances are the resistances of the transmitting conductors and do not include the internal resistances of the substations and load. Then the equivalent resistance is

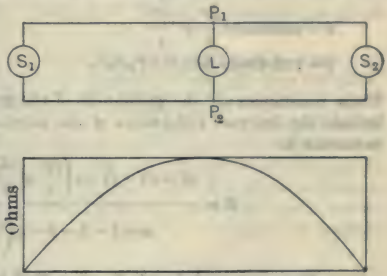


Fig. 3.

$$R = \frac{ab}{a+b}.$$

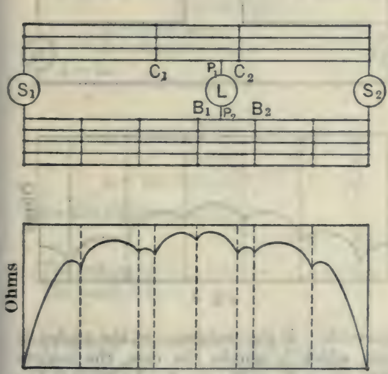


Fig. 4.

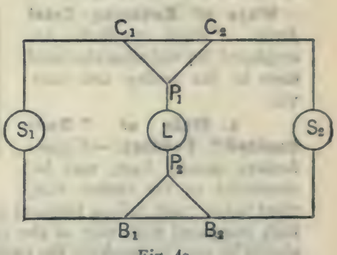


Fig. 4a.

2. In Fig. 4 is shown a 4-track road with 4 trolleys, both track and trolley cross-bonded. Fig. 4a is a simplified diagram of Fig. 4, corresponding points being designated by identical letters.

S_1 and S_2 are substations; L is a load placed arbitrarily between points P_1, on the positive system and P_2 on the negative system; C_1 and C_2 are ties between positive conductors and B_1 and B_2 ties between negative conductors.

Let

a = resistance of loop $S_1C_1P_1LP_2B_1S_1$,
b = resistance of loop $C_1P_1C_2C_1$,
c = resistance of contact conductor C_1P_1,
d = resistance of contact conductor C_2P_1,
e = resistance of loop $B_1P_2B_2B_1$,
f = resistance of track B_1P_2,
g = resistance of track B_2P_2,
h = resistance $\frac{d^2}{b} + \frac{g^2}{e}$,
i = resistance $\frac{c^2}{b} + \frac{f^2}{e}$,
j = resistance $S_2C_2P_1LP_2B_2S_2$.

These resistances are the resistances of the transmitting conductors and do not include the internal resistances of the substations and load. The equivalent resistance is

$$R = \frac{aj - ah - ji - 2\left(\frac{fg}{e} \times \frac{cd}{b}\right) + \frac{f^2d^2 + c^2g^2}{eb}}{a + j - h - i - 2\left(\frac{fg}{e} + \frac{cd}{b}\right)}.$$

Concentrated Load, Section Fed by Feeders from Both Ends. — Fig. 5 shows a simple case with feeders for the positive conductors only. No general formula is available for such a circuit. Any such network can, however, be calculated by Kirchhoff's Laws (q.v.), using the numerical values of the resistances for the various resistances. The equivalent resistance from the substations to the load is then the drop in voltage for a load of one ampere.

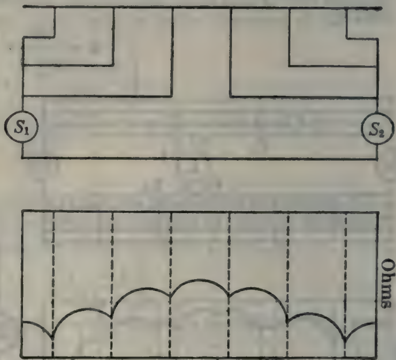


Fig. 5.

Ways of Reducing Total Drop. — Three methods are employed for reducing the total drop in the trolley and rails, viz:

1. Plain or "Non-boosted" Feeders. — Copper feeders, usually bare, may be connected to the trolley wire, third rail or track rails at frequent intervals. If the resistance of the trolley is high compared with that of the rails, which is usually the case, the feeders should be used to reënforce the trolley wire and not the track. In general, the most economical use of additional copper is to connect it in parallel with that side of the circuit having initially the higher resistance.

2. Boosted Feeders. — An insulated feeder may be tapped into the trolley wire or third rail at one point, and connected to a booster at the substa-

tion. The principle of this method is to increase the activity of the feeder, i.e., to make it carry more current and thereby reduce the current in the trolley wire. This is necessary only where the resistance of the line is so low that a moderate-sized feeder does not take its full share of current. It is not much used because if the line resistance is already low and the load is small, the drop will usually be small enough naturally, and if the load is great, the booster and feeder will usually have to be so large in order to produce any effect, as to be prohibitive in first cost and cost of operation.

3. **Floating Battery,** i.e., a battery connected across the line so as to charge when the line potential is high and discharge when it is low. This scheme is very little used on account of the high cost of operating and maintaining a battery. It is difficult to imagine under what conditions such a system would be commercially practicable, but if anywhere, it would be on a long line with light load and small feeders. Typical calculations for floating batteries are given in Lyndon's *Storage Battery Engineering*, Chap. XLIV.

Ways of Reducing Drop in the Negative Side of Circuit. — Owing to the necessity of preventing the leakage of current from track rails into the earth, it may be necessary to reduce, not the total drop in the negative system, but the drop in the grounded portion of the negative. This may be accomplished by the use of insulated feeders connected to the track rails so as to drain the current from the latter and thus reduce the drop in the rails, regardless of the drop in the feeders themselves. Such feeders may be used with or without a booster. Used with boosters, this method is probably the most economical way of reducing the drop in return rails where a large reduction is necessary to prevent electrolysis.

Negative Feeder with Booster. — In the case where the load is uniformly distributed over the line, the size of booster which must be installed in the substation may be determined as follows:

Let a = amperes entering negative feeder system per foot of line,
r = resistance of negative feeder system per foot,
R = total resistance of the feeder connecting the booster to the negative feeder system,
I = total amperes entering negative feeder system from the motors,
I_0 = amperes to be taken off by booster,
$l = \frac{I - I_0}{a}$ = distance from substation to the point at which the current in the negative feeders is to be zero,
l_1 = distance from booster tap to point in negative feeders where current is zero,
l_2 = the distance from the booster to the end of the line.

Then the booster voltage is

$$RI_0 - \tfrac{1}{2}\, ar(l^2 - l_1^2).$$

The output of the booster is, of course, the product of the current and voltage. The total drop is

$$\tfrac{1}{2}\, ar(l^2 - l_1^2 + l_2^2).$$

See Del Mar's *Electric Power Conductors*, Chap. V, for a more complete treatment.

Formulas and Constants for Alternating Current Trolley Circuits. — Little information is available on the effective or a-c. resistance (*see Alternating Currents*) and reactance of steel rails. As a rough approximation the a-c. resistance of a rail at 25 cycles per second may be taken as 4 to 5 times its d-c. resistance and the reactance of the rail as approximately the same as that of one wire of a pair of No. 0000 wires at a distance apart equal to the height of the

trolley wire above the track i.e., roughly as 0.4 ohm per mile. As a matter of fact the a-c. resistance depends upon the shape of the rail, its magnetic quality, the value of the current and the frequency; to a lesser extent the inductance, and therefore the reactance, depends upon these same items.

Data from Tests on N. Y., N. H. & H. R.R. — The following data were obtained by A. W. Copley from tests on the New York New Haven and Hartford Railroad and reported in the Trans. A.I.E.E., 1908, Vol. 27, (2) p. 1171. An analysis of these tests is also given by H. Pender in the Elec. World, 1909, Vol. 53, p. 1457. Copley's results showed that a considerable portion of the current entering the track from the load leaked to the earth. The combined resistance r of any number of trolley wires, and track rails and earth, as determined by Copley, can be represented by the following formula

$$r = \frac{p_1 r_1 + p_2 r_2}{100},$$

where r_1 = the resistance of each trolley wire separately, p_1 = the per cent of the total current in this wire, r_2 = the a-c. resistance of each rail separately and p_2 = the per cent of the total current in this rail. Similarly, the combined reactance x of any number of trolley wires and track rails and earth return can be represented approximately by the formula

$$x = \frac{p_1 x_1 + p_2 x_2}{100},$$

where x_1 = the reactance of a given trolley wire, x_2 = the reactance of a given rail and p_1 and p_2 as above.

Pender gives the following values for trolley wire and rail separately, based upon Copley's tests:

Item	Resistance, ohms per mile		Reactance,* ohms per mile	
	25 cycles	15 cycles	25 cycles	15 cycles
Single trolley wire, No. 0000 B. & S....	0.26	0.26	0.38	0.23
000 B. & S....	0.33	0.33	0.38	0.23
00 B. & S....	0.42	0.42	0.39	0.23
Single rail, 100 pounds to the yard.....	0.16	0.13	0.44	0.26

* For the trolley wire 25 ft. above the track, changing the height of the trolley wire 5 ft. up or down changes these values by less than 5 per cent.

The combined resistance and reactance respectively of the trolley wires, rails and earth are given in the following table:

For any other weight of rail the a-c. resistance of the rail itself may be taken as roughly inversely proportional to the weight per yard. The reactance of the rail, however, is practically independent of its cross-section, for the magnetic flux within the rail is relatively small compared with the total flux surrounding the rail (*see Inductance*).

Formulas for Single-phase Calculations. — The calculation of the power lost and drop in voltage between a load and the power house are calculated in the same manner as in the case of an ordinary single-phase a-c. trans-

RESISTANCE AND REACTANCE PER MILE OF ROAD

100-pound Rails

Number of tracks	....		1		2		4	
Number of trolley wires	1		1		2		4	
Number of rails	1		2		4		8	
Per cent of total current returning through rails†	25		40*		58*		75*	
Cycles per second	25	15	25	15	25	15	25	15
Combined resistance of trolley wires, track and earth per mile of road								
No. 0000 B. & S. trolley	0.30	0.29	0.29*	0.28*	0.16*	0.15*	0.086*	0.082*
000 B. & S. trolley	0.37	0.36	0.36	0.35	0.20	0.19	0.11	0.10
00 B. & S. trolley	0.46	0.45	0.45	0.44	0.24	0.23	0.13	0.12
Combined reactance of trolley wires, track and earth per mile of road, all three sizes of trolley wire	0.49	0.30	0.47*	0.28*	0.27*	0.16*	0.17*	0.10*

* The figures marked thus * are taken directly from the paper by Copley, whose tests were confined to No. 0000 trolley wires; the other values are calculated by Pender.

† The percentages of total current returning through the rails are test results on the N. Y., N. H. & H. R.R., and refer to relatively long sections (over 3 miles); a greater proportion of current flows in the rails in the immediate vicinity of the load and power house. The proportion of the total current which will return through the earth will depend upon the nature of the soil and ballast; for any other division of current the resistance and reactance can be obtained from the approximate formulas given above. The proportion of current returning through the earth on the N. Y., N. H. & H. R.R. has been greatly reduced since the above data were obtained, by the use of the three-wire system shown in Fig. 6.

mission line (*see Transmission Lines*). For a concentrated load (e.g., a single train) the calculation is as follows: Let

V = volts between trolley wire and track;
P = kilowatts at the locomotive delivered to locomotive;
$\cos\phi$ = power factor of load taken by locomotive;
l = distance in miles, between power house and locomotive;
r = combined resistance in ohms, per mile of trolley wire, rails and earth return;
x = combined reactance, in ohms, per mile of trolley wire, rails and earth returns;
$R = rl$;
$X = xl$.

Then the current taken by the locomotive is

$$I = \frac{1000\,P}{V\cos\phi} \quad \text{amperes,}$$

the power lost is

$$p = \frac{R\,I^2}{1000} \quad \text{kilowatts,}$$

the drop in voltage between the power house and locomotive is

$$v = \sqrt{(V\cos\phi + RI)^2 + (V\sin\phi + XI)^2} - V \quad \text{volts,}$$

and the power factor at the power house is

$$\cos\phi_0 = \frac{1000\,(P + p)}{I\,(V + v)}.$$

Example. — 2000 kw. at 25 cycles per second are to be supplied to a locomotive at 90 per cent power factor and 10,000 volts at a distance of 20 miles from the power house. The circuit consists of a No. 0000 trolley wire and two 100-lb. track rails. Then $V = 10{,}000$, $P = 2000$, $\cos \phi = 0.9$, $\sin \phi = 0.435$, $l = 20$, $r = 0.29$ (assuming only 40 per cent of the current returning through the rails), $x = 0.47$ (same assumption), $R = 0.29 \times 20 = 5.8$, $X = 0.47 \times 20 = 9.4$. Whence

$$I = \frac{1000 \times 2000}{10{,}000 \times 0.9} = 222 \text{ amperes,}$$

$$p = \frac{5.8 \times (222)^2}{1000} = 286 \text{ kilowatts,}$$

$$v = \sqrt{(10{,}000 \times 0.9 + 5.8 \times 222)^2 + (10{,}000 \times 0.435 + 9.4 \times 222)^2} - 10{,}000 = 2130 \text{ volts,}$$

$$\cos \phi_0 = \frac{1000\,(2000 + 286)}{222\,(10{,}000 + 2130)} = 0.848.$$

Scheme of Connections Used by N.Y., N.H. & H.R.R. — The distribution system is shown diagrammatically in Fig. 6 where A is the trolley

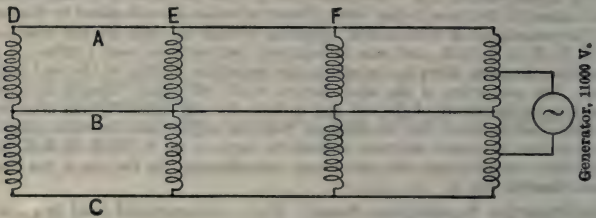


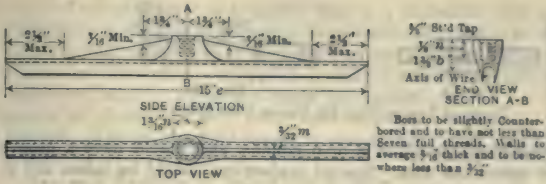
Fig. 6.

wires, B the track rails and C a feeder. These three sets of conductors are connected at intervals of about two miles through auto-transformers D, E and F. The difference of potential between A and B, and between B and C is 11,000 volts and between A and C is 22,000 volts. The effect of this arrangement is to practically give a transmission voltage of 22,000 with 11,000 volts at the trolley, and to greatly reduce the current in the grounded conductors with consequent reduction of disturbances to telephone lines.

MECHANICAL DESIGN OF OVERHEAD SYSTEM. — The various parts of the overhead system will first be described, and then the various types of construction, span wire, catenary, etc. The American Electric Railway Engineering Association has adopted specifications for overhead line material, which cover most of the mechanical details pertaining to ordinary trolley lines.

Parts of Overhead Trolley Construction. — The trolley wire is secured to an "ear" by soldering, clamps or other means; the ear is bolted to a "suspension" which may or may not be provided with an insulating portion. These suspensions are carried by span wires which in turn are fastened to the poles or brackets, or may be fastened directly to the bracket. If the "ear" is not insulated, " strain" insulators are inserted in the span wire between the ear and its point of attachment to the poles or brackets. A slightly different form of suspension, called a "pull-off," is used on curves. At turnouts "trolley frogs"

must be used to guide the trolley wheel. These various parts are illustrated in the accompanying cuts.



Permissible Variation	Over	Under
b	1/32"	1/32"
c	1/16"	1/16"
d	1/8"	1/8"
e	1/4"	1/4"
g	1/8"	1/8"
m	1/16"	0"
n	1/8"	0"
o	1/4"	0"

Fig. 7

Trolley Ears. — Figs. 7 to 10 are for round wire. Fig. 7 shows the standard straight-line ear of the A.E.R.E.A. It is made of cast bronze of approximately the following composition:

Copper	84 per cent
Zinc	10
Tin	3
Lead	3
	100 per cent

The ear shown in Fig. 8, which has a deep groove into which the wire is soldered is a type in common use. These ears may be provided with rings as in Fig. 9

Fig. 8.

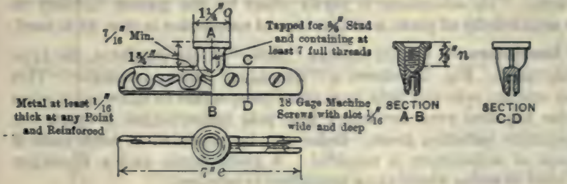
Fig. 9.

Fig. 10.

Fig. 11.

Types of Trolley Ears for Round Wire

to which guy wires are attached to relieve the strain at curves and for steadying the line at intervals. Fig. 10 shows the type of ear used at points where the



PERMISSIBLE VARIATION	OVER	UNDER
a	1/64"	1/64"
b	1/32"	1/32"
c	1/16"	1/16"
e	1/4"	1/4"
m	1/16"	0"
n	1/8"	0"
o	1/4"	0"

Fig. 12

trolley is spliced, which should always be at an ear. Fig. 11 shows an ear with a terminal for a feeder connection.

Figs. 12 to 14 are designed for grooved or figure "8" trolley wire. Special grooved or figure-8 wire affords a smoother running surface for the trolley wheel or bow as the ear only grips the upper part, leaving the lower part absolutely even. This construction is practically essential for bow trolleys. The design shown in Fig. 12 is the standard of the A.E.R.A. It is made of malleable iron which must conform with A.S.T.M. standards.

Soldering Trolley Wire to Ears. — The strain should be taken off the wire by a U-shaped clamp catching hold of it on each side of the ear. The

Fig. 13.

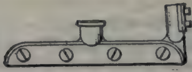


Fig. 14.

Types of Trolley Ears for Figure "8" Wires

soldering iron should weigh about 8 or 9 pounds, and should have a groove fitting half way round the wire. Special precautions should be taken not to have the iron unnecessarily hot.

Suspensions for Straight Line Work. — Fig. 15 shows an uninsulated suspension with ear attached for straight line work. Fig. 16 shows a suspension with strain insulators at each end; this is also used on curves for double-track

Fig. 15.

Fig. 16.

Fig. 17.

Fig. 18.

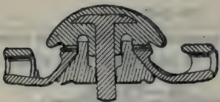


Fig. 19.

Fig. 20.

Types of Suspensions

work. Figs. 17 and 18 show solid insulated suspensions for span wire and side-bracket construction, respectively. These types are frequently used, and are quite satisfactory if the insulating material used in their construction is properly made. Fig. 19 shows a section of an assembled cap and cone suspension. Fig. 20 shows a cap and cone suspension with ear attached. The cap and cone are made separate. This type is preferred by some engineers on account of the possibility of replacing injured bolts and insulation without removing the whole suspension. This advantage is partly offset by the greater liability to trouble due to multiplicity of parts, and this type of suspension is now little used.

Strain Insulators. — Fig. 21 shows a "globe" strain insulator, the type most commonly used. Fig. 22 shows a "Brooklyn" strain insulator. This type is used on wooden pole and light iron pole construction to draw span wires taut. It is also required even for heavy iron pole construction if spans are long and temperature variation great. Bolts may be provided at both ends if an extra amount of adjustment is required. A globe strain and a Brooklyn strain may be used in series where extra insulation is required.

Fig. 21.

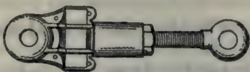


Fig. 22.

Strain Insulators

Fig. 23.

Fig. 24.

Pull-offs

Pull-offs. — Fig. 23 shows a cap and cone pull-off for single-curve construction and Fig. 24 a cap and cone pull-off for double-track-curve construc-

tion. Pull-offs of the same type as the uninsulated and solid insulated suspension shown in Figs. 15, 17 and 18 are also used.

Trolley Frogs. — A trolley frog is a malleable iron casting used at switches or crossovers where trolley wires from different tracks unite. Its function is to hold the diverging wires together and afford a smooth running path to the trolley wheel when a car passes or enters a switch. A common type is illustrated in Fig. 25. Frogs are made for various angles of divergence, and both right and left handed. The usual angles are 8, 15 and 20 degrees.

Fig. 25. Trolley Frog

Sag and Tension in Overhead System. — Particular attention must be paid to designing the overhead structure in such a manner that it will safely stand the extra tension due to the contraction of the wires at low temperatures and the extra loads due to wind and sleet, and a sufficient allowance should be made in the height of the trolley wire to take care of the extra sag which it experiences at high temperatures. With a simple trolley construction the variations in tension and sag may be calculated by a direct application of the rules given in the article on *Transmission Lines;* in the case of catenary construction, the additional load due to trolley wire, clips, etc., is treated in the same way as a sleet or wind load.

Direct Suspension. — The following discussion applies primarily to span-wire and side-bracket construction; catenary construction is treated in a separate section below. Details of standard parts are given in the A.E.R.A. Engineering Manual.

Height of Trolley Wire Above Rail. — The height of wire varies between a minimum of 16 feet and a maximum of 22 feet, the usual height being about 18 feet. It is usual to raise the wire at railroad crossings to a height of 22 feet or more above the top of rails. Catenary construction should be used for spans exceeding 150 feet.

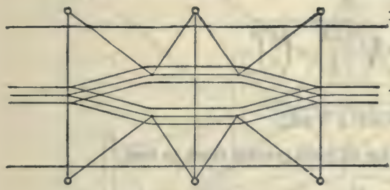


Fig. 26. Simple Pull-off Arrangement

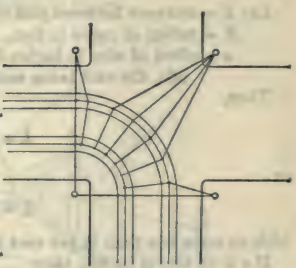


Fig. 27. Simple Pull-off Arrangement

Rake of Poles. — Bracket-arm poles on tangent construction should have a rake backwards not exceeding 3 inches, and span-wire poles in hard ground a rake of from 4 to 5 inches. In soft ground a rake of 12 inches is not uncommon. Center poles should be set vertically except at curves, where they should bend away from the curve along the perpendicular to the tangent at that point of the curve.

Anchorage. — At both ends of every grade and curve, there should be a permanent anchorage. If there are not many grades and curves, anchorages should be provided at intervals of from ¼ to ¾ of a mile. An anchorage is made

by means of a steel cable running from the trolley wire to one or more anchor poles through an anchor ear and strain insulators.

Curves. — At curves in the track the trolley wire should be made to follow the curve by means of pull-offs or wires pulling the trolley wire outward as shown in Figs. 26 and 27.

Wherever possible the pull-off wires should be radial to the trolley wire. This, however, requires a large number of poles, and is therefore impracticable in cities. In such cases, the bridle, bow-string or backbone construction shown in Figs. 28, 29, and 30 is resorted to. In this construction the pull-off wires

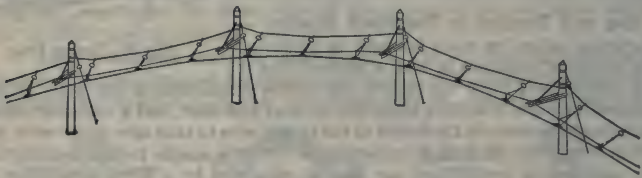


Fig. 28. Bridle Construction at Curve

instead of being anchored to individual poles are fastened to a wire which is stretched between poles. While this construction is almost universally used in cities, it is more expensive to maintain than single pull-offs, and is therefore less favored for interurban lines.

A combination of the two types of construction is shown in Fig. 31. Figs. 28 to 31 are from G. E. Co. publications.

Spacing of Pull-offs at Curves. — The number of pull-offs should be sufficient to keep the wire within about 2½ inches from the theoretical curve. This may be accomplished by spacing the successive ears in accordance with the following relations:

Let L = distance between pull-offs in feet,
R = radius of curve in feet,
a = offset of wire in inches from theoretical curve, midway between pull-offs, the ears being assumed to lie on the theoretical curve.

Then,

$$L = \sqrt{\frac{2\,aR}{3} - \left(\frac{a}{6}\right)^2},$$

or

$$L = 0.815\sqrt{aR},$$

with an error less than ¼ per cent for all radii greater than 40 feet.

If a is to be 2½ inches, then

$$L = 1.29\sqrt{R} \text{ approximately.}$$

If the ears are set exactly on the theoretical curve, the wire will depart 2½ inches from the correct position half way between the two ears; if the ears are set 1¼ inches from the curve, the mid-point of the wire will be only 1¼ inches when L is taken equal to $1.29\sqrt{R}$.

Offset of Trolley Wire at Curves. — It is usual, at curves, to offset the trolley wire from the center of the track (*a*) because the trolley wheel is tilted inward due to the elevation of the outer rail, and (*b*) because in order to keep the wheel on the wire, the latter must be so placed that the projection of the pole on the plane of the track is always tangential to the wire.

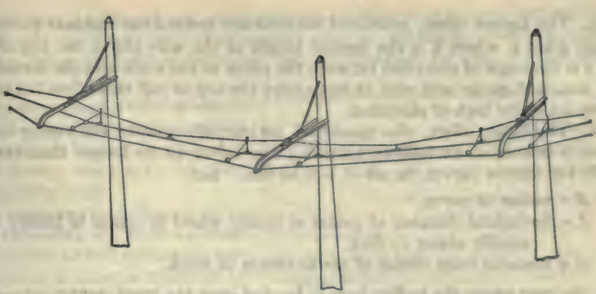


Fig. 29. Bow-string Construction at Curve

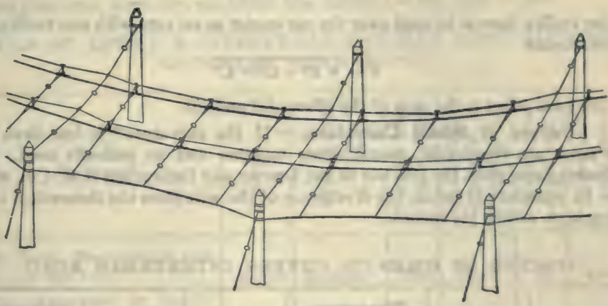


Fig. 30. Backbone Construction at Curve

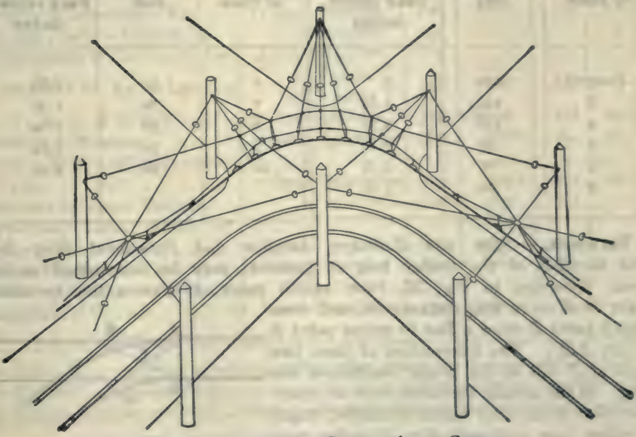


Fig. 31. Combination Construction at Curve

(*a*). The former offset, measured horizontally toward the inside of the curve, equals $h \tan \theta$, where h is the normal height of the wire above the top of rail, and θ is the angle of elevation between the plane of the track and the horizontal. For standard gauge and wire 18 feet above the top of rail this offset is about 4 inches for every inch of elevation.

(*b*). The latter offset, also measured horizontally toward the center of the curve, is calculated as follows, assuming the curve to be longer than the car itself; for shorter curves the offset will be less. Let

R = radius of curve,
L = horizontal distance of center of trolley wheel to center of trolley base, usually about 11 feet,
G = distance from center of car to center of truck.

In the case where the trolley base is located over the truck center, as on cars with two trolleys, the offset is

$$R - \sqrt{R^2 - L^2}.$$

If the trolley base is located over the car center as on cars with one trolley, the offset equals

$$R - \sqrt{R^2 - G^2 - L^2}.$$

The total offset is the sum of the offsets (*a*) and (*b*).

Curves of Small Curvature. — If the curvature* is less than 10°, the poles are frequently spaced closer together and no pull-offs used. The following table gives the practice of the Denver and Interurban R.R. Co., which may be considered typical, the divergence of the wire from the theoretical curve being kept within 3½ inches.

SPACING OF POLES ON CURVES, INTERURBAN ROAD

Degree of curvature of track	Pole spacing, feet	Divergence of trolley wire from track center, inches	Degree of curvature of track	Pole spacing, feet	Divergence of trolley wire from track center, inches
Tangent	120	0	6	60	2.87
1	120	1.87	7	50	2.34
2	110	3.37	8	50	2.64
3	90	3.06	9	50	2.94
4	80	3.37	10	50	3.24
5	70	3.06			

Turnouts. — (*See also Railways, Location and Permanent Way for.*) The location of the trolley frog at turnouts may be determined as follows. Referring to Fig. 32, from switch point A draw a line to center point D of track frog distance BC and from switch point B draw a line to center point E of arc AEC. The intersection of these two lines at F will be the proper location of the frog. While certain variables, such as superelevation of the outer rail on the curve, length of wheel base and projection of trolley pole

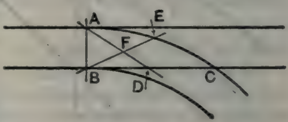


Fig. 32.

* The degree of curvature of a track is the angle which subtends a chord 100 feet long, between points on the center line of the track.

rearward from center of car, may necessitate slight variation of setting, this location will be found so nearly correct that a very small alteration, which must be determined by experiment, will compensate for the variable conditions.

The accompanying table gives the range of distance from track switch point to track frog with which each set of trolley frogs may be most satisfactorily used:

Track-frog distance	Divergence angle of trolley frog
Up to 22 feet	20°
From 20 to 30 feet	15°
Above 28 feet	8°

The minimum frog distance given in the table with which the 15° frogs may be used to best advantage corresponds to a turnout radius of 40 feet, but when suburban cars, using high-speed trolley wheels, run over city tracks it is advisable to use 15° rather than 20° frogs throughout the city construction even where the minimum frog distance is less than 20 feet.

DETAILS OF TRUNK & INTERURBAN RAILWAY CONSTRUCTION. CATENARY SUSPENSION

Railway	Voltage	Type of support	Tangent spacing of poles	Size of trolley wire, A. W. G. or B. & S.	Size of messenger wire, in.	Suspensions per span
New York, Westchester and Boston Ry. Co.	11,000	Bridges	300	0000	5/8	30
Butte, Anaconda & Pacific Ry. Co.	2,400	Span wire	110	0000	..	11
Chicago, Milwaukee & St. Paul Ry.	3,000	Bracket	150	0000	1/2	10
Denver & Interurban Ry.	11,000	Bracket	120	0000	7/16	12
Illinois Traction System	.....	Bracket	140	.....	...	3
Indianapolis & Cincinnati Co.	3,300	Bracket	120	.....	...	...
Milwaukee El. Ry. & Lgt. Co.		Center pole	110	0000	9/16	3
N. Y., N. H. & H. R.R.	11,000	Bridges	300	0000	5/8‡	30
Norfolk & Western	11,000	Span wire	300	000*	1/2	10
Penn. R.R. (Philadelphia–Paoli)	11,000	Span wire	300	000†	9/16	10
Spokane & Inland Empire	.....	Span wire	100	000	5/16	...
Syracuse, Lake Shore & Northern	6,600	Bridges	300	0000	7/16	10
Texas Traction Co.	600	Bracket	150	000	3/8	3
Washington, Baltimore & Annapolis Ry. Co.	6,000	Bracket	150	0000	3/8	9

* Also a N–0 copper or steel supporting wire above the copper contact wire.

† Also a No. 00 supporting wire above the N–000 phono-electric contact wire.

‡ Compound catenary with D inch messenger cable supporting the smaller one.

(Messenger cables composed of 7 strands in all cases.)

Catenary Construction. — Modern practice regarding catenary construction is illustrated in the preceding table. The constructions on the N. Y., N. H & H. R.R., and Pennsylvania R.R. are described in greater detail below.

Simple and Compound Catenary Construction. — Catenary construction may be divided into simple and compound. In the former the trolley wire or wires are carried by one or two messenger cables which are supported only at the poles, bents or span wires; in the latter, the horizontal wire or wires are carried by a messenger cable, which is itself suspended from another messenger cable which is supported at the poles, bents or span wires. The advantages of the compound catenary are greater flexibility, reduced stresses in the supporting wires, shorter hangers, better lightning protection and superior curve construction. (*See description of N. Y., N. H. & H. R.R., construction below.*) The simple catenary is sometimes made with two messenger cables from which the horizontal wire or wires are suspended by triangular frames.

To obtain a line which will not require frequent readjustment, the messenger cable must be installed with practically uniform tension throughout its length, making it necessary to have less sag in the shorter spans. For this reason certain definite pole spacings and corresponding hanger lengths have been standardized (*see next section below*).

TANGENT CONSTRUCTION

Number of Hangers per Span. Pantagraph or Bow Trolleys

Length pole spacing, feet	Number of points of suspension	Length of hangers, inches										
		6	6¾	8½	11	12	13½	14¾	16	17½	19¼	20½
150	11	1	2	2	2	..	..	2	..	..	2	..
125	9	..	..	..	1	2	2	..	2	..	2	..
110	8	..	..	..	..	..	2	2	..	2	..	2
95	7	..	..	..	..	..	..	..	3	2	..	2
80	6	..	..	..	..	..	..	..	..	2	2	2
70	5	..	..	..	..	..	..	..	..	..	3	2
55	4	..	..	..	..	..	..	..	..	..	..	4

Number of hangers per span. Wheel trolleys

Length pole spacing, feet	Number of points of suspension	Length of hangers, inches							
		6	11	13½	14¾	16	17½	19¼	20½
150	3	1	..	..	2	..	..	..	..
125	3	..	1	..	..	..	2	..	..
110	3	..	..	1	..	..	2	..	..
95	3	..	..	..	..	1	..	2	..
80	3	..	..	..	..	..	1	2	..
70	2	..	..	..	..	..	..	..	2
55	2	..	..	..	..	..	..	..	2

Suspensions. — (*See also p.* 1841.) The number of suspensions depends upon the speed at which the cars are to be run and upon whether a bow or wheel trolley is used. The three-point suspension in which, with 150 feet spacing, the hangers are 50 feet apart has been found ample for wheel collectors at speeds up to 65 m.p.h. With the sliding pantagraph or bow trolley an eleven-point suspension has been found sufficient, with 150 feet pole spacing.

Hangers. — (*See also p.* 1842.) Where only one horizontal wire is suspended from the messenger cable, the hangers should hang loosely from the cable and should be screwed fast to the trolley wire. This permits the trolley wire to rise slightly as the trolley passes under it, thereby making the wire equally flexible along its entire length. Where a steel contact wire is clipped to the horizontal conductor, the hangers are usually rigidly attached at both ends, as the duplication of horizontal wires assures uniform flexibility, regardless of the hangers.

Labor and Equipment or Erection. — Work should be done, as far as practicable, with the aid of a construction train at least five cars long and equipped with suitable scaffolds. With such a train the following daily results are attainable:

	Men	Number erected
Erecting poles	26	117
Erecting brackets	18	150
Stringing catenary	17*	7 miles
Stringing and splicing trolley wire	12*	7 miles
Clipping messenger and trolley	13*	3 ½ miles

* Exclusive of train crews.

TYPICAL EXAMPLES OF CATENARY CONSTRUCTION. — The following examples are given to illustrate in greater detail the methods employed in catenary construction.

New York, New Haven & Hartford R.R. — The various types of construction successively adopted by this company are all for high-speed traffic. The original construction consisted of a simple catenary system made rigid by

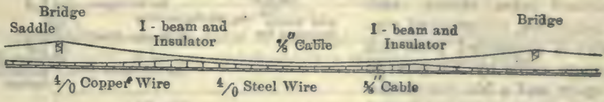


Fig. 33

the use of two steel messenger cables with equilateral triangular steel pipe frames of different lengths, joining them to the copper trolley wire. This having been found to be too rigid was modified by the addition of a steel contact wire suspended from the copper wire by clips midway between hangers. Considerations of economy and the desirability of securing better lightning protection led to the adoption of a compound catenary, that is one in which one messenger cable is supported from another by suspension insulators, the upper cable being grounded. The copper wire is supported horizontally by suspensions of suitable lengths from the lower steel cable and a steel contact wire clipped to the

copper wire midway between hangers as shown in Fig. 33. Both of the above types of construction involve the use of heavy lattice steel bents spaced 300 feet apart.

Pennsylvania R.R. — While both types remain in successful operation on the N. Y., N. H. & H. R.R., a less expensive one was developed for the New York Connecting Railroad and the Philadelphia & Paoli Branch of the Pennsylvania Railroad. This construction is especially suitable where there is opportunity to use guy poles instead of lattice bents. In this construction pairs of guyed steel poles, one on each side of the track, are spaced 300 feet apart. The two poles of a pair are joined by two messenger cables as shown in Fig. 34, making a small catenary system across the tracks with a vertical sus-

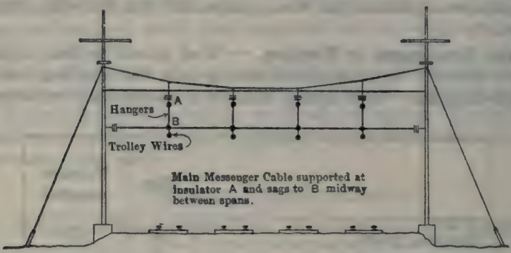


Fig. 34

pension rod over the center of each track. The upper of these cables has a diameter of 3/4 inch and the lower of 1/2 inch. The vertical rods are 3/4 inch in diameter and terminate in malleable iron clamps. An insulator consisting of three 10-inch porcelain disks is attached to each of these clamps and the main messenger cables suspended therefrom over the center of each track. These messenger cables consist of 9/16-inch double galvanized extra strong steel. The usual No. 2–0 grooved copper wire is suspended from this messenger cable by hangers 30 feet apart and a No. 3–0 phono-electric contact wire is clipped 1 3/4 inches below the copper wire.

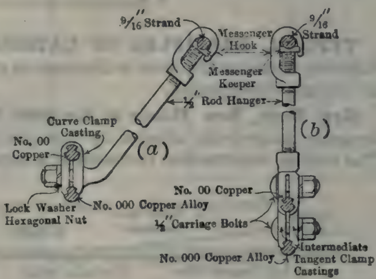


Fig. 35

The hangers used for supporting the copper wire are shown in Fig. 35. They are made of hot-galvanized steel.

To prevent side swaying of the lower wires in heavy winds a steady-brace is installed between poles and wires. These consist of steel wires insulated from the structure by strings of three porcelain insulators similar to those used to support the messenger cable. The contact wires themselves are insulated from it by single wooden strain insulators 6 inches long.

On the curves, steadies or pull-offs are used, but the hangers between the main messenger cables and the copper wires are spaced 15 feet instead of 30 feet as on tangents and the suspension rods are differently bolted as shown in Fig. 35.

The pole guys are 1 ½-inch rods attached to the poles through collars provided with turn-buckles. They are anchored in reinforced concrete blocks 8 feet or more below ground and protected from the ground by 3-inch boiler tubes.

Chicago, Milwaukee & St. Paul R.R. — The 3000-volt trolley system of the C., M. & St. P. R.R., comprising 660 miles of electrified trunk line, has two copper contact conductors suspended by alternate hangers from the same catenary. This enables 2000 amperes to be collected by a copper-shod pantograph collector at a speed of 60 m.p.h. The return circuit has an aerial conductor which is connected to the track rails every 8000 feet, preventing the attainment of dangerous voltages at rail joints in case of broken bonds.

Yard Construction. — In yards, wire cross-spans between poles, are used to support the messenger cables the insulators being suspended from the cross-span by $\frac{5}{16}$-inch soft-steel cable. A horizontal steady-wire insulated by strains slightly above the level of the contact wire supports the steady-hangers to which the contact wire is clipped. On curves, an additional insulated steady-wire is attached to the messenger wire at the poles. The messenger wire is often reduced from ⅝ inch to ⅜ inch and the hangers are spaced 15 feet on tangents and 10 feet on curves. A copper contact wire is not required.

Insulated Sections. — Main-line conductors are usually divided into sections from cross-over to cross-over. Wherever conditions permit, sections are insulated by staggering the contact wires laterally and running the two wires parallel, about 18 inches apart, each to a separate insulator. The ends are turned up to avoid catching the pantograph.

Turnouts. — Turnouts are made by means of steel deflectors or frames which keep the diverging wires at a fixed angle apart.

Stresses in Spans. — The stresses in the ⅞-inch cable of the original triangular construction of the N. Y., N. H. & H. R.R. are as follows on a tangent 300-foot span:

Condition	Temperature, ° F.	Sag, ft.	Stress, lb.
Normal	60	6.42	7,550
Normal	−10	5.71	8,550
Sleet and wind	−10	6.46	14,750

The stresses in the ⅝-inch cable of the original triangular construction of the N. Y., N. H. & H. R.R. are as follows on a tangent 140-foot span:

Condition	Temperature, ° F.	Stress, lb.
Normal	60	4930
Normal	−10	7100
Sleet and wind	−10	8400

The stresses in the copper wire and contact wire are negligible. Sleet is assumed to cover the wire to a depth of one-half inch and the wind is assumed to exert a pressure of ⅔ of 8 pounds per square foot.

Hangers. — The longest hangers of the original triangular construction of the N. Y., N. H. & H. R.R. (140 feet apart) have a length of $15\frac{7}{16}$ inches measured between centers of wires. The shortest hangers are $4\frac{1}{2}$ inches long. For main-line construction they are made of $\frac{1}{2}$-inch rods, for yards, of $\frac{3}{8}$-inch rods.

SPECIFICATIONS. — (*See also article on Specifications.*) Specifications for trolley wire are given under *Wires and Cables, Bare*, and for insulators under *Insulators for Overhead Lines*. The specifications of the American Electric Railway Engineering Association, which refer to overhead trolley systems, are as follows:

Specifications for Overhead Line Material (for 750 Volt Direct-Current, Direct Suspension).

Railroad Specifications for Electric Light, Power Supply and Trolley Lines Crossing Steam and Electric Railways.

These two specifications aggregating 87 pages contain a great deal of valuable data, including drawings of various detail parts and general layouts.

OPERATION. — The operation of overhead lines for low-speed cars presents few difficulties owing to the adaptability of the trolley pole and wheel at such speeds. With high-speed cars the case is different as the collector vibrates rapidly and the line must be kept uniformly level, smooth and elastic throughout and all special work must be maintained in first-class condition. If the collector vibrates in resonance with the loops in the contact wire, an increase in the number of supports will usually cure the trouble.

Prevention of Formation of Sleet. — Sleet may be prevented from forming on the wires by greasing the latter with petroleum jelly and if it does form it may be easily removed by any of the numerous commercial forms of sleet cutters.

Wear of Trolley Wheel, Bow and Trolley Wire. — The wear of the trolley wires is not serious either with wheel or bow collectors. On the lines of the Indianapolis & Cincinnati Traction Co., a copper trolley wire lost less than 1 per cent in weight after it had experienced 39,000 car movements, each car taking an average of about 40 amperes by an aluminum slider.

The vertical wear of the steel contact wire on the N. Y., N. H. & H. R.R. was 0.028 inch in thirty months, which is practically 4.5 per cent per year of the half diameter of the wire (one-half taken to permit wire to be held in clips) which, even on this vertical diameter basis, indicates a life of over 20 years; but as a matter of fact it will be much more than this, for the reason that as the vertical diameter lessens the breadth of contact increases throughout, thus diminishing the rate of vertical wear. There is practically no corrosion on the under side of the wire where there is much traffic, for the wire is covered with a film of grease deposited by the pantograph shoe (*W. S. Murray*). The upper part, however, rusts and rain water washes the rust on to the roofs of the cars (*Amberg*).

Change in Length of Trolley Wire. — The change in length of copper due to changes in temperature is one of the greatest difficulties in the maintenance of overhead work. A drop of 100° F. in temperature will cause a copper bar to contract approximately 1 inch for every 100 feet of length. If it be restrained at the ends this will cause an additional stress of 2500 pounds in a No. 0000 trolley. (*See Transmission Lines.*)

European catenary lines are usually maintained at constant tension by means of weights pulling on the free ends of the trolley wire at the end of every section.

Flashing. — Considerable trouble has been experienced with flashing on high-voltage d-c. lines. This has been largely overcome by placing a resonant shunt across the station buses to shunt out the harmonics set up by the slots

and teeth of the machine armatures. In addition to the above, flash suppressors are used consisting of electrically operated switches that kill the field of the separately excited d-c. generators, thereby killing the d-c. voltage instantly. Flash barriers are also used on the high-voltage commutators.

TESTING OF TROLLEY CIRCUITS. — Trolley lines have to be tested, (1) for resistance, and (2) for insulation from ground. The resistance may be measured by grounding one end of the wire to the track rails and circulating current at reduced voltage. The drop in the overhead lines may be measured using another trolley wire, a telephone wire or the earth as a potential lead. If the earth is used, the connections to ground should be sufficiently far from the track rails to be outside the zone of potential disturbance (*see Electrolysis of Underground Structures*).

REPAIRS OF TROLLEY CIRCUITS. — The repair methods of four typical railways are given below.

Philadelphia Rapid Transit Co. — The overhead line organization is in charge of the Superintendent of Overhead Lines and is divided into maintenance and construction crews, emergency crews, a pole crew and one special crew on inspections and electric switch and signal maintenance.

The maintenance and construction crews and the pole crew are under separate foremen. The emergency crews and the electric switch and signal crew report direct to the Superintendent of Overhead Lines.

The maintenance and construction foreman has four crews, each consisting of two linemen, one helper and a driver. The pole gang consists of the foreman, assistant foreman, five helpers and two drivers. There are also two men who make repairs to sidewalks and metal awnings in connection with the pole work. There are nine emergency stations, seven of which are manned for 24 hours per day and two for twelve hours per day. These stations care for approximately 70 miles of line each, taking care of line troubles, traffic blockades and a large part of the line repairs in the district. They, however, when needed, respond to calls in other districts. The movements of these emergency crews are controlled by telephone by an emergency operator at headquarters to whom all trouble is reported and who keeps a log sheet showing all movements of these crews. They do not leave their stations except on his orders.

Five of these emergency stations are equipped with two-ton auto trucks carrying telescopic towers. Four stations are each equipped with two-horse telescopic tower wagon and four horses. One of these wagon stations has also a light auto truck equipped for handling wrecks but not line troubles.

In addition to the above there are tower trolley cars for heavy work located at three of these stations.

Detroit United Railway Company. — The maintenance and construction equipment consists of three gasoline power line wagons, one three-ton horse drawn Windlass truck, one single truck line car, and one double truck line car, equipped with hoist and windlass. The gasoline power wagons and line cars are usually manned by one foreman, two line men, a driver or trolley man.

The emergency station is located in the center of the city and serves twenty-seven miles of city line. Its equipment includes three gasoline power line wagons and one gasoline wrecking and hose jumper wagon. These men are on duty for emergency work during the day time and four at night.

The emergency station is the headquarters for all equipment.

Illinois Traction System. — The line is divided, for overhead maintenance purposes, into districts averaging 90 miles in length. A line foreman and two helpers are assigned to each district and are given a tower line car with a regular crew.

Light maintenance work is handled from gasoline motor car equipped with light collapsible tower, the regular line car being used only for the heavier work.

N. Y., Westchester & Boston Ry. — This line has 54 miles of 11,000-volt contact wire. The general foreman has one day foreman, one night foreman, five linemen and one assistant lineman. They use a work train consisting of a gasoline-electric locomotive and work car, the former carrying the working platform. (Zogbaum.)

COST PER MILE OF SPAN-WIRE TROLLEY CONSTRUCTION (600 VOLTS)

(Exclusive of Track Work and Bonding—Before the War Prices)

Item	Unit price	Single track		Double track	
		Quantity	Total cost	Quantity	Total cost
Material (incl. 2 double curves):					
Yellow-pine poles, octagon..	$6.00	88	$528	....	....
Iron poles, No. 2...........	19.00	....	....	88	$1672
Iron poles, No. 4...........	36.00	....	....	4	144
Cement....................	2.35 & 2.15	22 bbl.	52	33 bbl.	71
Broken stone...............	0.95	14 cu. yd.	13	14 cu. yd.	13
Black paint................	0.90	20 gal.	18	11 gal.	10
Span wire..................	0.012	1250 ft.	15	2500 ft.	30
Pull-off wire..............	0.006	1250 ft.	8	2500 ft.	15
No. 000 copper wire, per lb..	0.18	1 mi.	483	2 mi.	966
Straight-line suspensions....	0.285	36	10	72	21
Side-feed suspensions........	0.57	8	5	16	9
Deep groove ears............	0.235	56	13	112	26
Frogs......................	3.25	2	7	4	13
Diagonals..................	3.60	....	....	2	7
Brooklyn strains...........	0.71	....	....	110	78
Frog pull-offs.............	0.36	6	2	12	4
Pole clamps................	0.12	9	1	18	2
Globe strains..............	0.31	15	5	30	9
Side-feed wire (No. 0, inches)	0.102	120 ft.	12	240 ft.	24
Double bodies..............	0.93	6	6	12	11
Single bodies..............	0.53	6	3	12	6
Miscellaneous..............	....	....	1	....	7
Total material............	....	....	$1182	....	$3138
Labor (incl. 2 double curves):					
Setting poles..............	....	....	$156	....	$138
Trucking...................	....	....	25	....	25
Painting, one coat.........	....	....	9	....	12
Running trolley wire.......	....	....	50	....	75
Building 2 double curves....	....	....	34	....	50
Putting up span wire.......	....	....	20	....	20
Total labor...............	....	....	$ 294	....	$320
Grand total per mile.........	....	....	$1476	....	$3458

COST OF TROLLEY CONSTRUCTION. — The costs given in the following tables will serve as a rough guide in making preliminary estimates. They are pre-war prices, as but little work of this character has been done since the war.

Extras for Curves. — Under ordinary conditions curves add about 10 per cent to the cost of direct-suspension construction and about 15 per cent to the cost of catenary construction.

Extras for 1200-volt Construction. — The following amounts should be added to give proper values for 1200-volt construction.

Direct suspension:

Bracket construction	$40.00
Span construction	40.00

Catenary suspension:

Bracket construction	$10.00
Span construction	10.00

COMPARATIVE COST PER MILE OF SINGLE-TRACK DIRECT SUSPENSION AND CATENARY CONSTRUCTION

600-volt Line, Tangent Track

Item	Direct suspension		Catenary, three-point	
	Bracket	Span	Bracket	Span
Material:				
Poles, 8 inches by 30 feet	$265	$530	$180	$360
Anchor, guy and span cable	45	150	21	100
Messenger cable	....	....	92	92
No. 0000 trolley wire	540	540	540	540
Other line material	145	99	144	101
Total material	995	1319	977	1193
Labor:				
Erecting poles	185	371	126	252
Mounting brackets	13	....	9	....
Installing span wire and guys	....	212	....	144
Stringing and clamping wire	75	75	200	200
Installing anchors	100	100	50	60
Total labor	373	758	385	656
Miscellaneous extras	150	150	150	150
Grand total	$1518	$2227	$1512	$1999

A 1500-volt trolley system on the Portland Division of the Southern Pacific Ry. Co., having wooden poles, bracket suspension and simple catenary, cost $1500 per mile for material and $600 for labor, a total of $2100 for tangent construction. Due to curves, etc., the average cost was about 20 per cent greater.

COST OF COMPOUND CATENARY ON TANGENT, N. Y., N. H. & H. R.R. PER MILE OF SINGLE TRACK

(E. J. Amberg, 1915)

Item	2-track	4-track	6-track
Catenary Material	$2800	$2800	$2800
Catenary labor and work train	600	600	600
Steel work erected	2300	2200	3300
Foundations installed	800	800	800
Total	$6500	$6400	$8500

The cost per mile of 2400-volt, single-track distribution system (including feeders and bonding), on the Butte, Anaconda & Pacific Ry. Co. was given by J. B. Cox as $5514 per mile of single track (1915).

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TROLLEY SYSTEMS, UNDERGROUND.— (*See also Trolley Systems, Overhead.*) This system is little used on account of its cost, the installations at Buda-Pest, New York, Washington and London being the only notable ones. The essential feature of all these systems is the underground conductor which is reached from the car by a "Plough" extending through a continuous slot parallel to the tracks.

NEW YORK SYSTEM. — The most successful construction is the latest New York type, a brief description of which is given below.

General Description. — The street is excavated and cleared of obstructions for a width of about 5 feet 6 inches and a depth of about 3 feet and cast-iron yokes (Fig. 1) set in the excavation about 5 feet apart. The track and slot rails are supported on these yokes and the whole system made solid with concrete which fills the excavation from the foundation to near the top of the rails leaving only a tunnel under the slot free from masonry. The conductors are suspended in this tunnel, by special strain insulators, no part of the electrical system being grounded. The use of two insulated conductors avoids trouble in case of accidental grounding.

Yoke. — The yoke shown in Fig. 1 is made in three parts. The lowest piece which rests on the floor of the excavation is a 6-inch steel I-beam, 4 feet 8 inches long. Two castings 6½ inches wide are riveted to this, each one serving to support one track rail and one slot rail. Fig. 1 shows both slot rails in place but only one track rail. The track rail is carried on a timber stringer which extends from yoke to yoke along the whole line. The stringers are held to the yokes by countersunk iron plates the ends of which project beyond the stringer and have holes which accommodate bolts running through the yoke. These bolts serve the additional purpose of fastening the rails to the stringers as they also pass through clips which bear upon the foot of the rail. The rails are further secured by long bolts running to the center of the yoke. The slot rails are bolted direct to the yoke and are connected to the track rails by long bolts every 30 inches.

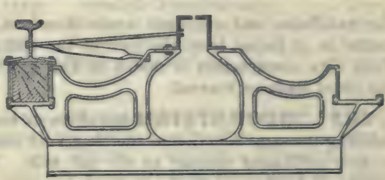
Fig. 1.

Insulator Boxes. — A pair of cast-iron boxes are laid across from the slot rails to the track rails and bolted to them every 15 feet as shown in Fig. 2. These contain the insulators which support the contact conductors. Inside each box is a pair of shelves running normal to the track; resting on these shelves and bolted to them is a cast-iron bridge which holds the insulator. The top of the box is provided with a cast-iron cover which is flush with the street surface.

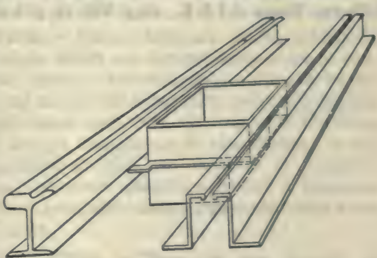
Fig. 2.

Concrete Work. — When the iron work is laid the tunnel which is to protect the conductors and do the draining is formed with collapsible sheet iron, fitting closely to the center opening of the yokes and run from yoke to yoke. The space from the bottom

of the insulator boxes to the road foundation is formed with boards. Concrete is then poured into the excavation completely filling every part but the tunnel and its offsets at the insulator boxes. The concrete is poured to within the height of a paving block of the rail tops. When the concrete is hard the sheet-iron form is collapsed and pulled along to the next section. The forms at the bottom of the insulator boxes being collapsible are drawn out through

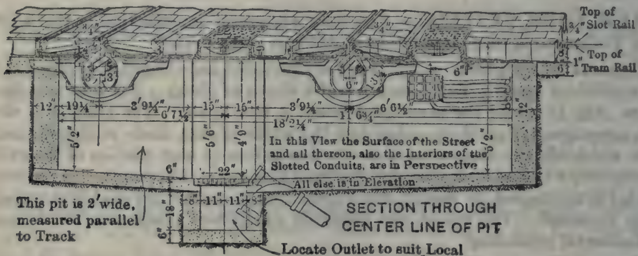

Fig. 3.

the handholes and are used again elsewhere. The depth of the completed tunnel is 18 inches from the base of the slot rails. A view of an older New York City construction is shown in Fig. 3, which is taken from an article in the *Street Railway Journal.*

THE LONDON SYSTEM has alternate long and short yokes, the long ones fulfilling the same function as the New York ones, the short ones serving merely to give additional support to the slot rails. No stringers are used, the rails resting on hard-wood blocks at the yokes only.

THE BUDA-PEST CONSTRUCTION has the slot in the track rail, thus saving considerable iron, but necessitating an excessively wide slot to accommodate the wheel flanges.

COSTS. — The type of construction in New York City cost from $60,000 to $100,000 per mile of track, exclusive of feeders, depending upon the conditions under which it was installed. Due to the high cost of this system, its use has not extended.

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UNITS AND CONVERSION FACTORS. — In this article are given the numerical interrelations, or "conversion factors" for all the commonly employed English and metric units. For the definitions of the various quantities see *Mechanics, Principles of; Heat and Thermal Properties; Temperature; Electricity and Magnetism, Principles of; Alternating Currents; Units, Practical Electrical;* and *Photometric Quantities.* For a list of the abbreviations and symbols adopted by the American Institute of Electrical Engineers see *Abbreviations and Symbols.*

The following is a list of the tables of conversion factors given in this article:

SYSTEMS OF UNITS. — By the magnitude of any quantity is meant the *relative* magnitude of this quantity as compared with some other quantity *of the same nature;* e.g., a length of 10 inches means a length which is ten times the length of one inch. To measure a quantity, then, it is necessary either (1) to adopt first some fixed magnitude of the same nature which may be used as a standard of reference, or unit, or (2) to define the quantity in such a way that the unit of measurement may be derived from the arbitrarily selected units of the quantities involved in the definition.

Fundamental and Derived Units. — Units which are chosen arbitrarily are called "fundamental" units; those which are derived from a set of fundamental units are called "derived" units. All mechanical units can be derived from a set of three fundamental units; the three units ordinarily chosen as fundamental are those of length, mass and time. In relations involving temperature changes, an arbitrary unit of temperature is also chosen; to express electrical and magnetic quantities at least one electrical or magnetic unit is arbitrarily chosen; to express photometric quantities the unit of luminous intensity is arbitrarily chosen. These arbitrarily chosen non-mechanical units are called "auxiliary fundamental" units.

English and Metric Systems of Units. — There are two systems of mechanical units in use in English speaking countries, the English and the French, or metric, systems. The latter system is used universally by physicists in all civilized countries and also forms the basis of the electrostatic and elec-

tromagnetic systems of electric units used by engineers as well as physicists. The English system of mechanical units, however, is used almost universally by English speaking peoples, in engineering, in commerce and in the arts; it is also the basis of the system of thermal units used largely by engineers. Engineers, however, are coming to use the metric system to a greater and greater extent for all purposes.

In both the English and the metric systems length, mass and time are chosen as the fundamental quantities. In the English system the fundamental units for these three quantities are respectively the foot, the pound (avoirdupois) and the second; this system is therefore also called the foot-pound-second system. In the metric system the corresponding units are the centimeter, gram and the second; this system is therefore also called the centimeter-gram-second, or c.g.s., system. The magnitudes of these fundamental units have been fixed arbitrarily by law; see below.

Absolute and Gravitational Units. — Two different units of force (and units derived therefrom) are used in both systems, viz.: (1) the unit of force is defined as that force which will give unit acceleration to unit mass, and (2) the unit of force is defined as that force which will give to unit mass an acceleration equal to the acceleration of a freely falling body under the influence of "gravity" or the pull of the earth alone (*see Mechanics, Principles of*). The first unit is called the "absolute" unit of force and the second the "gravitational" unit of force, and derived units based on these two units are called absolute and gravitational units respectively. The gravitational units are not definite unless the latitude and elevation of the point at which the acceleration due to gravity, usually represented by the symbol "g," is specified. By international agreement (*Troisème Conf. Gen. des Poids et Mes., 1901, p. 66*) the value $g_0 = 980.665$ cm. per sec. per sec. (= 32.1739 feet per sec. per sec.) has been chosen as the standard value for the acceleration due to gravity; this value of g may be called the "gravitational acceleration constant." The value 980.665 was chosen to represent the value of g at 45° latitude and sea level, but later measurements give a slightly different value of g at this latitude and elevation; the value 980.665, however, is retained as the *standard* value of g_0.

The gravitational unit of force in the English system is called the pound and in the metric system the gram, the same names being used for the unit of force as for the unit of mass. This arises from the fact that unit mass has a force exerted upon it by the earth, i.e., has a *weight*, numerically equal to its mass, when the force is expressed in terms of this gravitational unit. (This is exactly true only at the latitude and elevation for which $g = g_0$, but the variation with both latitude and elevation is usually negligible in engineering work.*) The absolute unit of force in the English system is called the poundal, but is practically never used; the absolute unit of force in the metric system is called the dyne, and is the unit of force commonly used by scientists. In engineering work the common units of force are the gravitational units, pound or gram, and their multiples and submultiples.

The relation between the absolute and gravitational units of force are:

1 pound = 32.1739 poundals,
1 gram = 980.665 dynes.

Weight and Mass. — The word weight is used in two different senses, viz.: (1) to mean the force with which the earth attracts a piece of matter,

* The force acting on a mass of 1 pound at sea level at the equator is 0.997363 pound, and at sea level at either pole of the earth is 1.002651 pound. An increase in elevation of 10,000 feet diminishes the force acting on a mass of 1 pound by only 0.00096. See Landolt, Börnstein and Roth, *Physikalish-Chemische Tabellen*.

and (2) to designate a given mass or quantity of matter; in other words, weight is used to designate both force and mass. As commonly employed the term almost invariably has its second meaning; that is, by a "weight of 10 pounds" is meant a piece of matter which has a mass of 10 pounds. A weight (2nd sense) of 10 pounds has a weight (1st sense) of 10 pounds at 45 degrees latitude and sea level (practically), but at any other latitude and elevation it has a weight (1st sense) slightly different from 10 pounds, and on the sun, for example, would have a weight (1st sense) many times 10 pounds.

In this connection it should be noted that a beam-balance when used in the ordinary way to compare two pieces of matter measures weight in the second sense (i.e., it would "read" the same whether the measurement were made on the earth or on the sun), that is, a beam-balance as ordinarily used compares masses and not forces. A beam-balance, however, is frequently used in testing (e.g., in connection with a Prony brake, see ***Index***) to compare a force produced by a machine with the pull on a standard mass (*or* "*weight*") due to gravity. If great precision is desired the reading of the balance should then be corrected for the value of "g" at the place of observation, but in ordinary measurements the inaccuracy of the balance is likely to be much greater than this small correction factor. For values of "g" at various places see reference in footnote, bottom of preceding page.

STANDARDS OF THE FUNDAMENTAL UNITS. — The physical standards upon which the c.g.s. system of units is based, and the legalized standards of the foot and pound used in Great Britain and the United States are described below.

Standard of Length. — The standard meter (100 centimeters) is the distance between two lines on a platinum-iridium bar carefully preserved at the Bureau of Weights and Measures, at Sevres, France, when the bar is kept at a uniform temperature of zero degrees centigrade throughout. In the United States the yard (3 feet) was defined by Act of Congress, July 28, 1866, as

$$1 \text{ U.S. yard} = \frac{3600}{3937} \text{ meter},$$

and similarly the British imperial yard is defined by law as

$$1 \text{ British imperial yard} = \frac{3600}{3937.079} \text{ meter}.$$

For engineering purposes the U. S. and British yards may be considered as identical.

Standard of Mass and Force. — The standard kilogram (1000 grams) as a unit of mass is a cylinder of platinum preserved at the Bureau of Weights and Measures, at Sevres, France. The U. S. pound avoirdupois is defined by law (*Act of Congress, 1866*) as $\frac{1}{2.2046}$ kilogram, but in 1893, the Superintendent of Weights and Measures, with the approval of the Secretary of the Treasury, declared* the pound to be

$$1 \text{ U. S. pound} = \frac{1}{2.204622} \text{ kilogram.}$$

The British imperial pound has the same value.

The same relations between pound and kilogram hold whether these units be taken as units of mass or as units of force, the unit of force being defined in

* Bull. Bureau Standards, 1904, Vol. 1, p. 380.

both cases as the pull of the earth on unit mass at 45 degrees latitude and sea level.

Standard of Time.—The standard second universally adopted is the $\frac{1}{86,400}$ th part of a mean solar day. The solar day is the interval of time between two successive transits of the sun across a meridian of the earth at the point of observation; this interval varies in length at different times during the year, but the average length of the interval for one year is constant as far as can be determined by any known methods of observation.

STANDARDS OF THE AUXILIARY FUNDAMENTAL UNITS.— The standards of temperature, dielectric coefficient, magnetic permeability and luminous intensity ordinarily used are defined below.

Standard of Temperature. — Two units of temperature, or temperature scales, are commonly employed, viz., the centigrade and the Fahrenheit units. The relation between these two units results solely from the manner in which they are defined.

$$1 \text{ degree centigrade} = \frac{9}{5} \text{ degrees Fahrenheit.}$$

Due to the difference in the zeros of the two scales, a temperature of t_f degrees Fahrenheit corresponds to a temperature of

$$t_c = \frac{5}{9}(t_f - 32) \text{ degrees centigrade,}$$

and vice versa,

$$t_f = \frac{9}{5} t_c + 32 \text{ degrees Fahrenheit.}$$

Standards of the Auxiliary Fundamental Electrical Units. — Three systems of electrical and magnetic units are in use, viz., (1) the c.g.s. electrostatic system, (2) the c.g.s. electromagnetic system, and (3) the practical system. In the c.g.s. electrostatic system the dielectric coefficient k of air at 0° C. and 760 mm. mercury pressure is arbitrarily chosen as unity. In the c.g.s. electromagnetic system the magnetic permeability of air under the same standard conditions is arbitrarily chosen as unity. In the practical system a concrete standard of the unit of resistance (called the ohm) is arbitrarily chosen (*see Units, Practical Electrical*); this unit resistance was originally designed to be equal to 10^9 times the unit of resistance in the c.g.s. electromagnetic system, and within the limits of ordinary experimental error this relation may still be considered exact.

Use of the Prefixes "Stat" and "Ab."— To designate the electrical and electromagnetic units in the electrostatic and electromagnetic systems of units respectively the prefixes "stat" and "ab" may be used with the name of the corresponding practical unit. For example, the c.g.s. electrostatic unit of quantity may be called the statcoulomb and the c.g.s. electromagnetic unit of quantity may be called the abcoulomb. For the names and definitions of the various electrical and magnetic units see the article on *Electricity and Magnetism, Principles of.*

Standards of Luminous Intensity. — Two standards are in use, the international candle and the hefner, the latter being used chiefly in Germany. See article on *Photometric Quantities* for the specifications for these units.

EXPERIMENTALLY DETERMINED CONVERSION FACTORS. — By conversion factor is meant the numerical factor which gives the magnitude of one unit for any given quantity in terms of any other unit for the same quantity.

For example, in the expression 1 yd. = 3600/3937 meter, the factor 3600/3937 is the conversion factor between the yard and the meter.

In the case of two units for the same quantity based on two different sets of arbitrarily chosen units which are defined independently, the conversion factor can be obtained only by experiment. The more important experimentally determined conversion factors are given in the paragraphs immediately following.

Mercury- and Water-column Pressure. — To convert pressure per unit area into height of mercury column the density of mercury must be known. Thiesen and Scheel (*Zeitsch. f. Instrk. de., 1898, Vol. 18, p. 138*) give the density of mercury at 0° C. as 13.59545 grams per (cm.)^3. Using the standard value of the gravitational acceleration constant, $g_0 = 980.665$, the relation between the dyne per sq. cm. and 1 cm. of mercury column at 0° C. is then

1 cm. mercury column at 0° C. = 13332.6 dynes per sq. cm.

Taking the density of water as 1 gram per (cm.)^3 at 4° C.,

1 cm. water column at 4° C. = 980.665 dynes per sq. cm.

Mechanical Equivalent of Heat. — Very useful units of energy are those based upon the mass of a standard substance and temperature, viz., the heat (which is energy) required to raise the temperature of a specified mass of water a specified number of degrees; see *Heat and Thermal Properties*. The experimentally determined relation between the mean small calorie (i.e., the one-hundreth part of the heat required to raise the temperature of 1 gram of water from 0° C. to 100 C. at 760 mm. mercury pressure) and the erg is

1 mean small calorie = 4.1834×10^7 ergs.

See Marks and Davis, *Steam Tables and Diagrams*, N. Y., 1912. From this relation and the relations (fixed by definition) given above between the yard and the meter, the pound and the kilogram, the value of the gravitational acceleration constant g_0, and the two temperature scales, Marks and Davis, deduce the relation

1 British thermal unit = 777.52 foot-pounds.

The British thermal unit as here used is defined as the $\frac{1}{180}$th part of the heat required to raise the temperature of 1 pound of water from 32° F. to 212° F. at 760 mm. mercury pressure. See also *Heat and Thermal Properties*.

The experimentally determined conversion factor between the energy unit based on the units of temperature and mass, and the energy unit based on the units of length, mass and time, i.e., the unit of mechanical work, is called the "mechanical equivalent of heat."

Relations between the Three Systems of Electrical Units. — The fundamental relation, experimentally determined, between the c.g.s. electrostatic and the c.g.s. electromagnetic system is that

1 abfarad = 9×10^{20} statfarads,

which as a consequence of the definitions of the various terms is equivalent to

1 abcoulomb = 3×10^{10} statcoulombs, the erg being the unit of energy in both systems.

The fundamental relations between the c.g.s. electromagnetic system and the practical system are

1 abcoulomb = 10 coulombs,

1 erg = 10^{-7} watt-seconds, or joules,

the erg being the unit of energy in the c.g.s. electromagnetic system and the watt-second or joule the unit of energy in the practical system.

Relation between the International Candle and the Hefner. — The experimentally determined relation is

1 hefner = 0.9 international candle.

MULTIPLES AND SUBMULTIPLES OF UNITS. — Multiples and submultiples of the metric units are designated by the following prefixes; the relations are *definitions* and are therefore absolutely exact.

$$\text{micro} = \frac{1}{1{,}000{,}000} \quad \text{or} \quad 10^{-6}$$

$$\text{milli} = \frac{1}{1{,}000} \quad \text{or} \quad 10^{-3}$$

$$\text{centi} = \frac{1}{100} \quad \text{or} \quad 10^{-2}$$

$$\text{deci} = \frac{1}{10} \quad \text{or} \quad 10^{-1}$$

$$\text{deka} = 10 \quad \text{or} \quad 10$$

$$\text{hecto} = 100 \quad \text{or} \quad 10^{2}$$

$$\text{kilo} = 1{,}000 \quad \text{or} \quad 10^{3}$$

$$\text{myria} = 10{,}000 \quad \text{or} \quad 10^{4}$$

$$\text{mega} = 1{,}000{,}000 \quad \text{or} \quad 10^{6}$$

The multiples and submultiples of the English units are given in bold-face type in the tables below.

CALCULATION OF CONVERSION FACTORS. — The conversion factors noted above, which are either definitions or results of experiment, form the basis for the calculation of the conversion factors for the various derived units. The method of procedure is first to express one of the pair of units, say A, in terms of its component units (length, mass, time, temperature, capacity or resistance, and luminous intensity), then express the magnitude of each of these units in terms of the corresponding component units of B; the value of the resultant numerical factor is the conversion factor. For example, let it be required to find the conversion factor between pressure in pounds per square foot and pressure in dynes per square centimeter.

$$1 \text{ lb. per sq. ft.} = \frac{(1 \text{ lb. force})}{(1 \text{ ft.})^2}.$$

$$1 \text{ lb. force} = \frac{1}{2.204622} \times (1 \text{ kilogram force}).$$

$$1 \text{ kilogram force} = 1000 \times (1 \text{ gram force}).$$

$$1 \text{ gram force} = 980.665 \times (1 \text{ dyne}).$$

Therefore

$$1 \text{ lb. force} = \frac{1000 \times 980.665}{2.204622} \times (1 \text{ dyne}).$$

Also

$$1 \text{ ft.} = \frac{1}{3} \times (1 \text{ yd}).$$

$$1 \text{ yd.} = \frac{3600}{3937} \times (1 \text{ meter}).$$

$$1 \text{ meter} = 100 \times (1 \text{ cm.}).$$

Therefore

$$1 \text{ ft.} = \frac{1}{3}\left(\frac{3600}{3937}\right) \times (100) \times (1 \text{ cm.}).$$

Whence

$$\frac{1 \text{ lb. force}}{(1 \text{ ft.})^2} = \frac{1000 \times 980.665 \times (3 \times 3937)^2}{2.204622 \times (3600 \times 100)^2} \times \frac{(1 \text{ dyne})}{(1 \text{ cm.})^2}$$

$$= 478.799 \times \frac{(1 \text{ dyne})}{(1 \text{ cm.})^2}.$$

But $\dfrac{(1 \text{ dyne})}{(1 \text{ cm.})^2} = 1$ dyne per sq. cm.

Therefore

1 pound per sq. ft. = 478.799 dynes per sq. cm.

DIMENSIONAL FORMULAS. — Instead of writing the expression for a derived unit in words, it is frequently convenient to express it in algebraic symbols, using a specific symbol for each of the fundamental or auxiliary fundamental units. For example, let L stand for a length of 1 centimeter, M for a mass of 1 gram, and T for a time interval of 1 second; then the unit of force in the c.g.s. absolute system (the dyne) may be written

$$F = MLT^{-2},$$

which is the same as 1 dyne = (1 gram) $\times \dfrac{(1 \text{ cm.})}{(1 \text{ sec.})^2}$.

A formula, such as $F = MLT^{-2}$, where each letter represents the magnitude of a *single* unit of the quantity for which it stands, is called a "dimensional formula." A pure number, or a quantity which is the ratio of two units of the same dimensions, such as an angle, has zero dimensions and does not appear in a dimensional formula.

Dimensional formulas are useful for two purposes, viz., (1) as a systematic method for calculating conversion factors and (2) as a check of algebraic formulas expressing various relations. Unless one has a large number of conversion factors to calculate it is probably simpler to use the direct method given in the preceding section. Certain quantities, for example, work and torque, may have the same dimensions although they are physically entirely different. Although such quantities are physically different their conversion factors are identical. The second use of dimensional formulas is based on the fact that every term in any set of terms connected either by equal signs, plus signs or minus signs, must have the same dimensions.

For a complete list of the dimensional formulas for the various mechanical, electrical, thermal, and photometric quantities see Hering, C., *Conversion Tables*, N. Y., 1904. In the tables below are given, in the parentheses after the titles, the dimensional formulas of the mechanical quantities in terms of length, mass and time, designated by L, M and T respectively. For the electrical and magnetic quantities the dimensions are expressed in terms of electric potential difference V, quantity of electricity Q and time T. In the *c.g.s. electromagnetic system* of units Q may be expressed in terms of L, M, and T by the formula

$$Q = L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}};$$

in the *c.g.s. electrostatic system* by the formula

$$Q = L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}k^{\frac{1}{2}}.$$

In both systems $VQ = L^2MT^{-2}$ = energy.

TABLES OF CONVERSION FACTORS.* — The following tables give the most commonly required conversion factors for mechanical, electrical and magnetic quantities. See *Heat and Thermal Properties* for special heat units (other than energy) and *Photometric Quantities* for the special photometric units.

LENGTH (L)

cm. = *centimeter.* *m.* = *meter.*
ft. = *foot.* *mi.* = *mile.*
in. = *inch.* *mm.* = *millimeter.*
km. = *kilometer.* *yd.* = *yard.*

1 centimeter	1 foot	1 inch	1 kilometer	1 knot
0.3937 in.	30.48 cm.	2.540 cm.	10^5 cm.	6080 ft.
0.01 m.	12 in.	0.02540 m.	3281 ft.	1.853 km.
393.7 mil	0.3048 m.	10^3 mil	1000 m.	1853 m.
10 mm.	304.8 mm.	25.40 mm.	0.6214 mi.	1.152 mi.
....	0.3333 yd.	....	1094 yd.	2027 yd.
1 meter	**1 mil**	**1 mile**	**1 millimeter**	**1 yard**
100 cm.	0.002540 cm.	1.609×10^5 cm.	0.1 cm.	91.44 cm.
3.281 ft.	0.001 in.	5280 ft.	0.03937 in.	3 ft.
39.37 in.	0.02540 mm.	1.609 km.	0.001 meter	36 in.
0.001 km.	....	1609 m.	39.37 mil.	0.9144 meter
1000 mm.	....	1760 yd.	....	....
1.094 yd.	....	....	....	....

* **Example.** — 7 yards equals how many meters? *Answer*, 1 yard = 0.9144 meters, therefore 7 yards = 7 × 0.9144 = 6.4008 meters.

AREA (L^2)

C.M. = *circular mil.*
hect. = *hectare.*
sq. cm. = *square centimeter.*
sq. ft. = *square foot.*
sq. in. = *square inch.*
sq. km. = *square kilometer.*
sq. m. = *square meter.*
sq. mil. = *square mil.*
sq. mi. = *square mile.*
sq. mm. = *square millimeter.*
sq. yd. = *square yard.*

1 acre	1 are	1 circular mil *	1 hectare
40.47 ares	0.02471 acre	5.067×10^{-6} sq. cm.	2.471 acres
0.4047 hect.	0.01 hect.	7.854×10^{-7} sq. in.	100 ares
43,560 sq. ft.	1076 sq. ft.	0.7854 sq. mil.	1.076×10^{5} sq. ft.
4.047×10^{-3} sq. km.	10^{-4} sq. km.	5.067×10^{-4} sq. mm.	0.01 sq. km.
4047 sq. m.	**100 sq. m.**	....	10^{4} sq. m.
1.562×10^{-3} sq. mi.	3.861×10^{-5} sq. mi.	....	3.861×10^{-3} sq. mi.
4840 sq. yd.	119.6 sq. yd.	....	1.196×10^{4} sq. yd.

1 sq. centimeter	1 square foot		1 square inch
1.973×10^{5} C.M.	2.296×10^{-5} acre	9.290×10^{-8} sq. km.	1.273×10^{6} C. M.
1.076×10^{-3} sq. ft.	9.290×10^{-4} are	0.09290 sq. m.	6.452 sq. cm.
0.1550 sq. in.	1.833×10^{8} C. M.	1.44×10^{8} sq mil.	6.944×10^{-3} sq. ft.
1.550×10^{5} sq. mil.	9.290×10^{-6} hect.	3.587×10^{-8} sq. mi.	10^{6} sq. mil.
10^{-4} sq. m.	929.0 sq. cm.	9.290×10^{4} sq. mm.	645.2 sq. mm.
100 sq. mm.	**144 sq. in.**	0.1111 sq. yd.	....

1 square kilometer	1 square meter	1 square mil †
247.1 acres	2.471×10^{-4} acre	1.273 C. M.
10^{4} ares	0.01 are	6.452×10^{-6} sq. cm.
100 hect.	10^{-4} hect.	10^{-6} sq. in.
10.76×10^{6} sq. ft.	10.76 sq. ft.	6.452×10^{-4} sq. mm.
10^{6} sq. m.	10^{-6} sq. km.	....
0.3861 sq. mi.	3.861×10^{-7} sq. mi.	....
1.196×10^{6} sq. yd.	1.196 sq. yd.	....

1 square mile	1 square millimeter	1 square yard
640 acres	1.973×10^{3} C. M.	2.066×10^{-4} acre
2.590×10^{4} ares	**0.01 sq. cm.**	8.361×10^{-3} are
2.590×10^{2} hect.	1.076×10^{-5} sq. ft.	8.361×10^{-5} hect.
27.88×10^{6} sq. ft.	1.550×10^{-3} sq. in.	**9 sq. ft.**
2.590 sq. km.	1.550×10^{3} sq. mil.	8.361×10^{-7} sq. km.
2.59×10^{6} sq. m.	....	0.8361 sq. m.
3.098×10^{6} sq. yd.	....	3.228×10^{-7} sq. mi.

* A circular mil is the area of a circle 1 mil, or 0.001 in., in diameter.
† A square mil is the area of a square 1 mil, or 0.001 in., on each side.

VOLUME (L^3)

bu. = *bushel.*
cu. cm. = *cubic centimeter.*
cu. ft. = *cubic foot.*
cu. in. = *cubic inch.*
cu. m. = *cubic meter.*
cu. yd. = *cubic yard.*
gal. = *gallon.*
lit. = *liter.*
pt. = *pint.*
qt. = *quart.*

1 bushel (dry*)	1 cu. centimeter	1 cubic foot	1 cubic inch
1.244 cu. ft.	3.531×10^{-5} cu. ft.	2.832×10^{4} cu. cm.	16.39 cu. cm.
2150 cu. in.	6.102×10^{-2} cu. in.	**1728 cu. in.**	5.787×10^{-4} cu. ft.
0.03524 cu. m.	**10^{-6} cu. m.**	0.02832 cu. m.	1.639×10^{-5} cu. m.
4 pk. (dry)	1.308×10^{-6} cu. yd.	0.03704 cu. yd.	2.143×10^{-5} cu. yd.
64 pt. (dry)	2.642×10^{-4} gal.	7.481 gal.	4.329×10^{-3} gal.
32 qt. (dry)	10^{-3} lit.	28.32* lit.	1.639×10^{-2} lit.
....	2.113×10^{-3} pt.	59.84 pt.	0.03463 pt.
....	1.057×10^{-3} qt.	29.92 qt.	0.01732 qt.

1 cubic meter	1 cubic yard	1 gallon (liq.*)
10^{6} cu. cm.	7.646×10^{5} cu. cm.	3785 cu. cm.
35.31 cu. ft.	**27 cu. ft.**	0.1337 cu. ft.
61,023 cu. in.	46,656 cu. in.	**231 cu. in.**
1.308 cu. yd.	0.7646 cu. m.	3.785×10^{-3} cu. m.
264.2 gal.	202.0 gal.	4.951×10^{-3} cu. yd.
10^{3} lit.	764.6 lit.	3.785 lit.
2113 pt.	1616 pt.	8 pt.
1057 qt.	807.9 qt.	4 qt.

1 liter	1 pint (liq.*)	1 quart (liq.*)
10^{3} cu. cm.	473.2 cu. cm.	946.4 cu. cm.
0.03531 cu. ft.	0.01671 cu. ft.	0.03342 cu. ft.
61.02 cu. in.	28.87 cu. in.	57.75 cu. in.
10^{-3} cu. m.	4.732×10^{-4} cu. m.	9.464×10^{-4} cu. m.
1.308×10^{-3} cu. yd.	6.189×10^{-4} cu. yd.	1.238×10^{-3} cu. yd.
0.2642 gal.	0.125 gal.	**0.25 gal.**
2.113 pt.	0.4732 lit.	0.9464 lit.
1.057 qt.	**0.5 qt.**	**2 pt.**

* Dry measure units = 1.164 × (liquid measure units). Quarts, pints, bushels and pecks as here used are United States measures; see Hering's *Conversion Tables* for English measures.

PLANE ANGLE (*Zero Dimensions*)

deg. or ° = *degree.* *rad.* = *radian.*
min. or ′ = *minute.* *rev.* = *revolution.*
quad. = *quadrant.* *sec.* or ″ = *second.*

1 degree	1 minute	1 quadrant
60 min.	2.909×10^{-4} rad.	**90 deg.**
0.01745 rad.	**60 sec.**	5400 min.
3600 sec.	....	1.571 rad.
....	....	324,000 sec.

1 radian *	1 revolution	1 second
57.30 deg.	**360 deg.**	4.848×10^{-6} rad.
3438 min.	21,600 min.	....
0.637 quad.	4 quad.	....
206,265 sec.	6.283 rad.	....
....	1.296×10^{6} sec.	....

* 2π radians = 360 degrees by definition.

SOLID ANGLE (*Zero dimensions*)

1 hemisphere	1 sphere *	1 sph. right angle	1 steradian †
0.5 sphere	2 hem. sp.	**0.25 hem. sph.**	0.1592 hem. sph.
4 sph. rt. ang.	8 sph. rt. ang.	0.125 sphere	0.07958 sphere
6.283 steradians	12.57 steradians	1.571 steradians	0.6366 sph. rt. ang.

* A sphere is the total solid angle about a point.
† 4π steradians = 1 sphere, by definition.

TIME (T)

hr. = *hour.* *mo.* = *month.* *wk.* = *week.*
min. = *minute.* *sec.* = *second.* *yr.* = *year.*

1 day	1 hour	1 week	1 aver. month	1 common year	1 leap year
24 hr.	60 min.	**7 days**	**1/12 civil yr.**	**365 days**	**366 days**
1,440 min.	3600 sec.	168 hr.	30.42 days	8,760 hr.	8,784 hr.
86,400 sec.	....	10,080 min.	730 hr.	525,600 min.	527,040 min.
....	....	604,800 sec.	43,800 min.	12 mo.	31,622,400 sec.
....	....	....	2,628,000 sec.	31,536,000 sec.	52.27 wk.
....	....	....	....	52.14 wk.	

LINEAR VELOCITY AND SPEED (LT^{-1})

1 ceutimeter per sec.	1 foot per minute	1 foot per second
1.969 ft. per min.	0.5080 cm. per sec.	30.48 cm. per sec.
0.03281 ft. per sec.	0.01667 ft. per sec.	1.097 km. per hr.
0.036 km. per hr.	0.01829 km. per hr.	0.5921 knot per hr.
0.6 m. per min.	0.3048 m. per min.	18.29 m. per min.
0.02237 mile per hr.	0.01136 mile per hr.	0.6818 mile per hr.
3.728×10^{-4} mile per min.	....	0.01136 mile per min.

1 kilometer per hour	1 kilometer per minute	1 knot per hour
27.78 cm. per sec.	54.68 ft. per sec.	51.48 cm. per sec.
54.68 ft. per min.	32.38 knots per hr.	1.689 ft. per sec.
0.9113 ft. per sec.	16.67 m. per hr.	1.853 km. per hr.
0.5396 knot per hr.	37.28 miles per hr.	1.152 miles per hr.
16.67 m. per min.	0.6214 mile per min.	....
0.6214 mile per hr.	....	....

1 meter per minute	1 meter per second
1.667 cm. per sec.	196.8 ft. per min.
3.281 ft. per min.	3.281 ft. per sec.
0.05468 ft. per sec.	3.6 km. per hr.
0.06 km. per hr.	0.06 km. per min.
0.03728 mile per hr.	2.237 miles per hr.
....	0.03728 mile per min.

1 mile per hour	1 mile per minute
44.70 cm. per sec.	2682 cm. per sec.
88 ft. per min.	88 ft. per sec.
1.467 ft. per sec.	1.609 km. per min.
1.609 km. per hr.	0.8684 knot per min.
0.8684 knot per hr.	60 miles per hr.
26.82 m. per min.	....

ANGULAR SPEED (T^{-1})

1 deg. per sec.	1 rad. per sec.	1 rev. per min.	1 rev. per sec.
0.01745 rad. per sec.	57.30 deg. per sec.	6 deg. per sec.	360 deg. per sec.
0.1667 rev. per min.	0.1592 rev. per sec.	0.1047 rad. per sec.	6.283 rad. per sec.
0.002778 rev. per sec.	9.549 rev. per min.	0.01667 rev. per sec.	60 rev. per min.

LINEAR ACCELERATION (LT^{-2})

1 cm. per sec. per sec.	1 foot per sec. per sec.	Gravity-g_0
0.03281 ft. per sec. per sec. 0.001020 gravity 0.036 km. per hr. per sec. 0.01 m. per sec. per sec. 0.02237 mile per hr. per sec.	30.48 cm. per sec. per sec. 0.03108 gravity 1.097 km. per hr. per sec. 0.3048 m. per sec. per sec. 0.6818 mile per hr. per sec.	980.7 cm. per sec. per sec. 32.17 ft. per sec. per sec. 35.30 km. per hr. per sec. 9.807 m. per sec. per sec. 21.94 miles per hr. per sec.
1 kilometer per hour per sec.	**1 meter per sec. per sec.**	**1 mile per hour per sec.**
27.78 cm. per sec. per sec. 0.9113 ft. per sec. per sec. 0.02833 gravity 0.2778 m. per sec. per sec. 0.6214 mile per hr. per sec.	100 cm. per sec. per sec. 3.281 ft. per sec. per sec. 0.1020 gravity 3.6 km. per hr. per sec. 2.237 miles per hr. per sec.	44.70 cm. per sec. per sec. 1.467 ft. per sec. per sec. 0.04559 gravity 1.609 km. per hr. per sec. 0.4470 m. per sec. per sec.

ANGULAR ACCELERATION (T^{-2})

rad. = *radian* *rev.* = *revolution*

1 rad. per sec. per sec.	1 rev. per min. per min.
573.0 rev. per min. per min. 9.549 rev. per min. per sec. 0.1592 rev. per sec. per sec.	1.745×10^{-3} rad. per sec. per sec. 0.01667 rev. per min. per sec. 2.778×10^{-4} rev. per sec. per sec.
1 rev. per min. per sec.	**1 rev. per sec. per sec.**
0.1047 rad. per sec. per sec. 60 rev. per min. per min. 0.01667 rev. per sec. per sec.	6.283 rad. per sec. per sec. 3600 rev. per min. per. min. 60 rev. per min. per sec.

MASS (M) AND WEIGHT *

kg. = *kilogram.* *oz.* = *ounce.*
mg. = *milligram* *lb.* = *pound* †

1 grain †	1 gram	1 kilogram	
0.06480 gram	15.43 grains	15,432 grains	2.205 lb.
64.80 mg.	10^{-3} **kg.**	10^3 grams	9.842×10^{-4} long ton
2.286×10^{-3} oz.	10^3 mg.	10^6 mg.	10^{-3} metric ton
....	0.03527 oz.	35.27 oz.	1.102×10^{-3} short ton
....	2.205×10^{-3} lb.	....	....

1 milligram	1 ounce †	1 pound †
0.01543 grain	437.5 grains	**7000 grains**
10^{-3} gram	28.35 grams	453.6 grams
10^{-6} **kg.**	0.02835 kg.	0.4536 kg.
....	28,350 mg.	4.536×10^5 mg.
....	0.06250 lb.	**16 oz.**

1 long ton	1 metric ton	1 ton (short)
1016 kg.	10^3 **kg.**	907.2 kg.
2240 lb.	2205 lb.	**2000 lb.**
1.016 metric tons	0.9842 long ton	0.8929 long ton
1.120 short tons	1.102 short tons	0.9072 metric ton

* These same conversion factors apply to the *gravitational* units of force having the corresponding names. The dimensions of these units when used as gravitational units of force are MLT^{-2}; see table for *Force* on next page.

† Avoirdupois pound and its subdivisions used throughout.

DENSITY OR MASS PER UNIT VOLUME (ML^{-3})

1 gram per cu. cm.	1 kg. per cu. meter
10^3 kg. per cu. m.	10^{-3} g. per cu. cm.
62.43 lb. per cu. ft.	0.06243 lb. per cu. ft.
0.03613 lb. per cu. in.	3.613×10^{-5} lb. per cu. in.
3.405×10^{-7} lb. per mil ft.	3.405×10^{-10} lb. per mil ft.

1 pound per cu. ft.	1 pound per cu. in.
0.01602 g. per cu. cm.	27.68 g. per cu. cm.
16.02 kg. per cu. m.	2.768×10^4 kg. per cu. m.
5.787×10^{-4} lb. per cu. in.	1728 lb. per cu. ft.
5.456×10^{-9} lb. per mil ft.	9.425×10^{-6} lb. per mil ft.

FORCE (MLT^{-2})

kg. = kilogram. *lb. = pound.*

1 dyne *	1 gram	1 kilogram
1.020×10^{-3} gram	980.7 dynes	980,665 dynes
1.020×10^{-6} kg.	10^{-3} kg.	10^{3} grams
2.248×10^{-6} lb.	2.205×10^{-3} lb.	2.205 lb.
7.233×10^{-5} poundal	0.07093 poundal	70.93 poundals

1 pound	1 poundal †
444,823 dynes	13,826 dynes
453.6 grams	14.10 grams
0.4536 kg.	0.01410 kg.
32.1739 poundals	0.03108 lb.

* Force required to give a mass of 1 gram an acceleration of 1 cm. per sec. per sec.
† Force required to give a mass of 1 pound an acceleration of 1 ft. per sec. per sec.

TORQUE OR MOMENT OF FORCE (L^2MT^{-2}) *

cm-dyne = centimeter-dyne. *m-kg. = meter-kilogram.*
cm-gram = centimeter-gram. *lb-ft. = pound-foot.*

1 centimeter-dyne	1 centimeter-gram
1.020×10^{-3} cm-gram	980.7 cm-dynes
1.020×10^{-8} m-kg.	10^{-5} m-kg.
7.376×10^{-8} lb-ft.	7.233×10^{-5} lb-ft.

1 meter-kilogram	1 pound-foot
9.807×10^{7} cm-dynes	1.356×10^{7} cm-dynes
10^{5} cm-grams	13,825 cm-grams
7.233 lb-ft.	0.1383 m-kg.

* Same dimensions as energy; see below.

PRESSURE OR FORCE PER UNIT AREA ($L^{-1}MT^{-2}$)

barie = *dyne per sq. cm.*
mercury = *column of mercury.*
ton = *2000 lb.*
water = *column of water.*

1 atmosphere	1 barie or dyne per sq. cm.	1 cm. of mercury
76.0 cm. mercury	9.870×10^{-7} atmosphere	0.01316 atmosphere
29.92 in. mercury	0.01020 kg. per sq. m.	0.4461 ft. water
33.90 ft. water	2.089×10^{-3} lb. per sq. ft.	136.0 kg. per sq. m.
10,333 kg. per sq. m.	1.450×10^{-5} lb. per sq. in.	27.85 lb. per sq. ft.
14.70 lb. per sq. in.	....	0.1934 lb. per sq. in.
1.058 tons per sq. ft.	....	....

1 inch of mercury	1 inch of water	1 foot of water
0.03342 atmosphere	0.002458 atmosphere	0.02950 atmosphere
1.133 ft. water	0.07355 in. mercury	0.8826 in. mercury
345.3 kg. per sq. m.	25.40 kg. per sq. m.	304.8 kg. per sq. m.
70.73 lb. per sq. ft.	5.204 lb. per sq. ft.	62.43 lb. per sq. ft.
0.4912 lb. per sq. in.	0.03613 lb. per sq. in.	0.4335 lb. per sq. in.

1 kilogram per square meter	1 kilogram per sq. mm.	1 pound per square foot
9.678×10^{-5} atmosphere	96.78 atmospheres	4.725×10^{-4} atmosphere
98.07 baries	98.07×10^{6} baries	478.8 baries
3.281×10^{-3} ft. water	3.281×10^{3} ft. water	0.01602 ft. water
2.896×10^{-3} in. mercury	2.048×10^{5} lb. per sq. ft.	0.01414 in. mercury
0.2048 lb. per sq. ft.	1.422×10^{3} lb. per sq. in.	4.882 kg. per sq. m.
1.422×10^{-3} lb. per sq. in.	....	6.944×10^{-3} lb. per sq. in.

1 pound per sq. inch	1 short ton per sq. ft.	1 short ton per sq. in.
0.06804 atmosphere	0.9450 atmosphere	136.1 atmospheres
2.307 ft. water	32.04 ft. water	4613 ft. water
2.036 in. mercury	28.28 in. mercury	1.406×10^{6} kg. per sq. m.
703.1 kg. per sq. m.	9765 kg. per sq. m.	2.88×10^{5} lb. per sq. ft.
144 lb. per sq. ft.	2000 lb. per sq. ft.	2000 lb. per sq. in.
....	13.89 lb. per sq. in.	....

ENERGY AND WORK (L^2MT^{-2})*

B.t.u. = *British thermal unit.*
ft-lb. = *foot-pound.*
kg-cal. = *kilogram-calorie.*
gram-cal. = *gram-calorie.*
gram-cm. = *gram-centimeter.*
hp-hr. = *horse-power-hour.*
kg-m. = *kilogram-meter.*
kw-hr. = *kilowatt-hour.*
watt-hr. = *watt-hour.*

1 British thermal unit	1 erg or dyne-centimeter	1 foot-pound
0.2520 kg-cal.	9.486×10^{-11} B.t.u.	1.286×10^{-3} B.t.u.
777.5 ft-lb.	2.390×10^{-11} kg-cal.	3.241×10^{-4} kg-cal.
3.927×10^{-4} hp-hr.	7.376×10^{-8} ft-lb.	1.356×10^{7} ergs
1054 joules	1.020×10^{-3} gram-cm.	5.050×10^{-7} hp-hr.
107.5 kg-m.	10^{-7} joule	1.356 joules
2.928×10^{-4} kw-hr.	1.020×10^{-8} kg-m.	0.1383 kg-m.
....	....	3.766×10^{-7} kw-hr.

1 kilogram-calorie †	1 gram-centimeter	1 horse-power-hour
3.968 B.t.u.	9.302×10^{-8} B.t.u.	2547 B.t.u.
3086 ft-lb.	2.344×10^{-8} kg-cal.	641.7 kg-cal.
1.558×10^{-3} hp-hr.	980.7 ergs	1.98×10^{6} ft-lb.
4183 joules	7.233×10^{-5} ft-lb.	2.684×10^{6} joules
426.6 kg-m.	9.807×10^{-5} joule	2.737×10^{5} kg-m.
1.162×10^{-3} kw-hr.	10^{-5} kg-m.	0.7457 kw-hr.

1 joule or watt-second	1 kilogram-meter
9.486×10^{-4} B.t.u.	9.302×10^{-3} B.t.u.
2.390×10^{-4} kg-cal.	2.344×10^{-3} kg-cal.
10^{7} ergs	9.807×10^{7} ergs
0.7376 ft-lb.	7.233 ft-lb.
0.1020 kg-m.	9.807 joules
2.778×10^{-4} watt-hr.	2.724×10^{-6} kw-hr.

1 kilowatt-hour	1 watt-hour
3415 B.t.u.	3.415 B.t.u.
860.5 kg-cal.	0.8605 kg-cal.
2.655×10^{6} ft-lb.	2655 ft-lb.
1.341 hp-hr.	1.341×10^{-3} hp-hr.
3.6×10^{6} joules	367.1 kg-m.
3.671×10^{5} kg-m.	10^{-3} kw-hr.

* See note at bottom of next table.

† 1 gram-calorie = 10^{-3} kilogram-calorie; 1 Ostwald calorie = 10^{-1} kilogram-calorie.

POWER OR RATE OF DOING WORK (L^2MT^{-3})*

B.t.u. = British thermal unit; met. h.p. = metric horse-power.

1 B.t.u. per minute	1 erg per second	1 foot-pound per minute
777.5 ft-lb. per min.	5.692×10^{-9} B.t.u. per min.	1.286×10^{-3} B.t.u. per min.
12.96 ft-lb. per sec.	1.434×10^{-9} kg-cal. per min.	3.241×10^{-4} kg-cal. per min.
0.02356 h.p.	4.426×10^{-6} ft-lb. per min.	0.01667 ft-lb. per sec.
0.01757 kw.	7.376×10^{-8} ft-lb. per sec.	3.030×10^{-5} h.p.
0.02389 met. h.p.	1.341×10^{-10} h.p.	2.260×10^{-5} kw.
17.57 watts	10^{-10} kw.	3.072×10^{-5} met h.p.
....	1.360×10^{-10} met. h.p.	....

1 foot-pound per second	1 horse-power	1 kg-cal. per minute
7.717×10^{-2} B.t.u. per min.	42.44 B.t.u. per min.	3086 ft-lb. per min.
1.945×10^{-2} kg-cal. per min.	10.70 kg-cal. per min.	51.43 ft-lb. per sec.
1.818×10^{-3} h.p.	33,000 ft-lb. per min.	0.09351 h.p.
1.356×10^{-3} kw.	550 ft-lb. per sec.	0.06972 kw.
1.843×10^{-3} met. h.p.	0.7457 kw.	0.09481 met. h.p.
1.356 watts	1.014 met. h.p.	69.72 watts
....	*745.7 watts	....

1 kilowatt	1 metric horse-power	1 watt
56.92 B.t.u. per min.	41.86 B.t.u. per min.	0.05692 B.t.u. per min.
14.34 kg-cal. per min.	10.55 kg-cal. per min.	0.01434 kg-cal. per min.
4.425×10^{4} ft-lb. per min.	3.255×10^{4} ft-lb. per min.	10^{7} erg per sec.
737.6 ft-lb. per sec.	542.5 ft-lb. per sec.	44.26 ft-lb. per min.
1.341 h.p.	0.9863 h.p.	0.7376 ft-lb. per sec.
1.360 met. h.p.	0.7354 kw.	1.341×10^{-3} h.p.
10^{3} watts	735.4 watts	10^{-3} kw.
....	....	1.360×10^{-3} met. h.p.

* The value 746 watts, has been adopted by the Bureau of Standards as the exact equivalent of one horse-power; this, however, is not consistent with the use of 980.665 cm. per sec. per sec. as the standard value of g, which latter is used throughout these tables.

QUANTITY OF ELECTRICITY; DIELECTRIC FLUX (Q)

Abcoul. = abcoulomb; coul. = coulomb; statcoul. = statcoulomb.

1 abcoulomb	1 coulomb	1 statcoulomb
10 coul.	1/10 abcoul.	$\frac{1}{3} \times 10^{-10}$ abcoul.
3×10^{10} statcoul.	3×10^{9} statcoul.	$\frac{1}{3} \times 10^{-9}$ coul.

CHARGE PER UNIT AREA; DIELECTRIC FLUX DENSITY (QL^{-2})

1 abcoulomb per square centimeter	1 coulomb per square centimeter
10 coul. per sq. cm.	1/10 abcoul. per sq. cm.
64.52 coul. per sq. in.	6.452 coul. per sq. in.
3×10^{10} statcoul. per sq. cm.	3×10^{9} statcoul. per sq. cm.

1 coulomb per square inch	1 statcoulomb per square centimeter
0.01550 abcoul. per sq. cm.	$\frac{1}{3}\times10^{-10}$ abcoul. per sq. cm.
0.1550 coul. per sq. cm.	$\frac{1}{3}\times10^{-9}$ coul. per sq. cm.
4.650×10^{8} statcoul. per sq. cm.	2.150×10^{-9} coul. per sq. in.

ELECTRIC CURRENT (QT^{-1})

1 abampere	1 ampere	1 statampere
10 amperes	1/10 abamp.	$\frac{1}{3}\times10^{-10}$ abamp.
3×10^{10} statamp.	3×10^{9} statamp.	$\frac{1}{3}\times10^{-9}$ ampere

CURRENT DENSITY ($QT^{-1}L^{-2}$)

1 abampere per square centimeter	1 ampere per square centimeter
10 amperes per sq. cm.	1/10 abampere per sq. cm.
64.52 amperes per sq. in.	6.452 amperes per sq. in.
3×10^{10} statamp. per sq. cm.	3×10^{9} statamp. per sq. cm.

1 ampere per square inch	1 statamp. per square centimeter
0.01550 abamp. per sq. cm.	$\frac{1}{3}\times10^{-10}$ abamp. per sq. cm.
0.1550 ampere per sq. cm.	$\frac{1}{3}\times10^{-9}$ ampere per sq. cm.
4.650×10^{8} statamp. per sq. cm.	2.150×10^{-9} ampere per sq. in.

ELECTRIC POTENTIAL DIFFERENCE; ELECTROMOTIVE FORCE (V)

1 abvolt	1 statvolt	1 volt
$\frac{1}{3}\times10^{-10}$ statvolt	3×10^{10} abvolts	10^{8} abvolts
10^{-8} volt	300 volts	1/300 statvolt

ELECTRIC POTENTIAL GRADIENT; ELECTROSTATIC FIELD INTENSITY (VL^{-1})

1 abvolt per centimeter	1 statvolt per centimeter
$\frac{1}{3}\times 10^{-10}$ statvolt per cm. 10^{-8} volt per cm. 2.540×10^{-8} volt per in.	3×10^{10} abvolts per cm. 300 volts per cm. 762.0 volts per in.
1 volt per centimeter	**1 volt per inch**
10^{8} abvolts per cm. $\frac{1}{300}$ statvolt per cm. 2.540 volts per in.	3.937×10^{7} abvolts per cm. 1.312×10^{-3} statvolt per cm. 0.3937 volt per cm.

ELECTRIC RESISTANCE ($Q^{-1}VT$)

1 abohm	1 megohm	1 microhm
10^{-15} megohm 10^{-3} microhm 10^{-9} ohm $\frac{1}{9}\times 10^{-20}$ statohm	10^{15} abohms 10^{12} microhms 10^{6} ohms $\frac{1}{9}\times 10^{-5}$ statohm	10^{3} abohms 10^{-12} megohm 10^{-6} ohm $\frac{1}{9}\times 10^{-17}$ statohm

1 ohm	1 statohm
10^{9} abohms 10^{-6} megohm 10^{6} microhms $\frac{1}{9}\times 10^{-11}$ statohm	9×10^{20} abohms 9×10^{5} megohms 9×10^{17} microhms 9×10^{11} ohms

ELECTRIC RESISTIVITY † ($Q^{-1} VTL$)

See also *Resistance and Conductance*

1 abohm per cm. cube	1 microhm per cm. cube	1 microhm per in. cube
10^{-3} microhm per cm. cube	10^{3} abohms per cm. cube	2.540×10^{3} abohms per cm. cube
3.937×10^{-4} microhm per in. cube	0.3937 microhm per in. cube	2.540 microhms per cm. cube
6.015×10^{-3} ohm per mil-ft.	6.015 ohms per mil-ft.	15.28 ohms per mil-ft.
$10^{-5}\,\delta$ ohm per meter-gram	$10^{-2}\,\delta$ ohm per meter-gram	$2.540 \times 10^{-2}\,\delta$ ohm per meter-gram

1 ohm per mil-foot	1 ohm per meter-gram *
1.662×10^{2} abohms per cm. cube	$\frac{10^5}{\delta}$ abohms per cm. cube
0.1662 microhm per cm. cube	$\frac{10^2}{\delta}$ microhms per cm. cube
0.06524 microhm per in. cube	$\frac{39.37}{\delta}$ microhms per in. cube
$1.662 \times 10^{-3}\,\delta$ ohm per meter-gram	$\frac{6.015 \times 10^2}{\delta}$ ohms per mil-ft.

* See *p. 1858 for multiples and submultiples.*

† In this table δ is density expressed as a decimal fraction.

ELECTRIC CONDUCTIVITY † ($QV^{-1}T^{-1}L^{-1}$)

1 abmho per cm. cube	1 mho per meter-gram	1 mho per mil-foot
$\frac{10^5}{\delta}$ mhos per meter-gram	$10^{-5}\,\delta$ abmho per cm. cube	6.015×10^{-3} abmho per cm. cube
1.662×10^{2} mhos per mil-ft.	$1.662 \times 10^{-3}\delta$ mho per mil-foot	$\frac{601.5}{\delta}$ mhos per meter-gram
10^{3} megmhos per cm. cube	$10^{-2}\delta$ megmho per cm. cube	6.015 megmhos per cm. cube
2.540×10^{3} megmhos per in. cube	$2.540 \times 10^{-2}\delta$ megmho per in. cube	15.28 megmhos per in. cube

1 megmho per cm. cube	1 megmho per in. cube
10^{-3} abmho per cm. cube	3.937×10^{-4} abmho per cm. cube
$\frac{10^2}{\delta}$ mhos per meter-gram	$\frac{39.37}{\delta}$ mhos per meter-gram
0.1662 mho per mil-foot	0.06524 mho per mil-foot
2.540 megmhos per in. cube	0.3937 megmho per cm. cube

† In this table δ is density expressed as a decimal fraction

CAPACITY (QV^{-1})

Abf. = abfarad; mf. = microfarad; statf. = statfarad

1 abfarad	1 farad	1 microfarad	1 statfarad
10^9 farads 10^{15} mf. 9×10^{20} statf.	10^{-9} abf. 10^6 mf. 9×10^{11} statf.	10^{-15} abf. 10^{-6} farad 9×10^5 statf.	$\frac{1}{9}\times10^{-20}$ abf. $\frac{1}{9}\times10^{-11}$ farad. $\frac{1}{9}\times10^{-5}$ mf.

INDUCTANCE $(VQ^{-1}T^2)$

Abh. = abhenry; mh. = millihenry; stath. = stathenry

1 abhenry	1 henry	1 millihenry	1 stathenry
10^{-9} henry 10^{-6} mh. $\frac{1}{9}\times10^{-20}$ stath.	10^9 abh. 10^3 mh. $\frac{1}{9}\times10^{-11}$ stath.	10^6 abh. 10^{-3} henry $\frac{1}{9}\times10^{-14}$ stath.	9×10^{20} abh. 9×10^{11} henries 9×10^{14} mh.

MAGNETIC FLUX (VT)

1 maxwell or "line"	1 kiloline	1 volt-second*
10^{-3} kiloline 10^{-8} volt-sec.*	10^3 maxwells 10^{-5} volt-sec.*	10^8 maxwells 10^5 kilolines

MAGNETIC FLUX DENSITY (VTL^{-2})

1 gauss, or line per square centimeter	1 line per square inch
6.452 lines per sq. in. 10^{-8} volt-sec.* per sq. cm. 6.452×10^{-8} volt-sec.* per sq. in.	0.1550 gauss 1.55×10^{-9} volt-sec.* per sq. cm. 10^{-8} volt-sec.* per sq. in.
1 volt-sec. per square centimeter	**1 volt-sec. per square inch**
10^8 gausses 6.452×10^8 lines per sq. in. 6.452 volt-sec.* per sq. in.	1.550×10^7 gausses 10^8 lines per sq. in. 0.1550 volt-sec.* per sq. cm.

* By volt-second is meant the unit of flux which must be used in the equation $e=\frac{d\phi}{dt}$ in order to obtain e in volts when t is in seconds; this unit is sometimes called a "weber."

MAGNETIC POTENTIAL DIFFERENCE; MAGNETOMOTIVE FORCE (QT^{-1})

Abamp-turns = abampere-turns; amp-turns = ampere-turns; statamp-turns = statampere-turns

1 abampere-turn	1 ampere-turn	1 gilbert
10 amp-turns 12.57 gilberts	1/10 abamp-turn 1.257 gilberts	0.07958 abamp-turn 0.7958 amp-turn

MAGNETIC POTENTIAL GRADIENT; MAGNETIZING FORCE ($QL^{-1}T^{-1}$)

1 abamp-turn per cm.	1 ampere-turn per cm.
10 amp-turns per cm. 25.40 amp-turns per inch 12.57 gilberts per cm.	1/10 abamp-turn per cm. 2.540 amp-turns per inch 1.257 gilberts per cm.
1 amp-turn per inch	**1 gilbert per cm.**
0.03937 abamp-turn per cm. 0.3937 amp-turn per cm. 0.4950 gilbert per cm.	0.07958 abamp-turn per cm. 0.7958 amp-turn per cm. 2.021 amp-turns per inch

BIBLIOGRAPHY. — A more complete set of conversion factors is given in a book by Hering called *Conversion Tables*. See also circular of the Bureau of Standards entitled *Tables of Equivalents*, 1913.

UNITS, PRACTICAL ELECTRIC. — (*See also Electricity and Magnetism, Principles of; Units and Conversion Factors.*) Following the recommendation of the Chicago International Electrical Congress of 1893, the following Act was passed by Congress. (Act approved July 12, 1894.) *Be it enacted, etc.* That from and after the passage of this act the legal units of electrical measure in the United States shall be as follows:

First. The unit of resistance shall be what is known as the international ohm, which is substantially equal to one thousand million units of resistance of the centimeter-gram-second system of electromagnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice fourteen and four thousand five hundred and twenty-one ten-thousandths (14.4521) grams in mass, of a constant cross-sectional area, and of the length of one hundred and six and three-tenths (106.3) centimeters.

Second. The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electromagnetic units, and is the practical equivalent of the unvarying current, which, when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths (0.001118) of a gram per second.

Third. The unit of electromotive force shall be what is known as the international volt, which is the electromotive force that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of an international ampere, and is practically equivalent to $\frac{1000^*}{1434}$ of the electromotive force between the poles or electrodes of the voltaic cell known as Clark's cell, at a temperature of fifteen degrees centigrade (15° C.), and prepared in the manner described in the standard specifications.

Fourth. The unit of quantity shall be what is known as the international coulomb, which is the quantity of electricity transferred by a current of one international ampere in one second.

Fifth. The unit of capacity shall be what is known as the international farad, which is the capacity of a condenser charged to a potential of one international volt by one international coulomb of electricity.

Sixth. The unit of work shall be the joule, which is equal to ten million units of work in the centimeter-gram-second system, and which is practically equivalent to the energy expended in one second by an international ampere in an international ohm.

Seventh. The unit of power shall be the watt, which is equal to ten million units of power in the centimeter-gram-second system, and which is practically equivalent to the work done at the rate of one joule per second.

Eighth. The unit of induction shall be the henry, which is the induction in a circuit when the electromotive force induced in this circuit is one international volt while the inducing current varies at the rate of one ampere per second.

SECTION 2. That it shall be the duty of the National Academy of Sciences to prescribe and publish, as soon as possible after the passage of this act, such specifications of detail as shall be necessary for the practical application of the definitions of the ampere and volt hereinbefore given, and such specifications shall be the standard specifications herein mentioned.

NOTES ON THE ABOVE DEFINITIONS. — The specifications mentioned above were prepared by a committee, and their report, based on the best

* Later experiments give an e.m.f. of 1.4328 volts at 15° C.

work that had been done up to that time, was accepted and adopted by the National Academy of Sciences on February 9, 1895.

Attention is called to the wording of the definitions of the three fundamental quantities:

1. The international ohm is based upon, but *not stated to be equal* to, 10^9 c.g.s. units of resistance; it is, however, *definitely defined* in terms of a column of mercury.

2. The international ampere *is* one-tenth of the c.g.s. unit of current, and is stated to be represented *sufficiently well for practical purposes* by the current which deposits a definite mass of silver per second under specified conditions.

3. The international volt is the e.m.f. which will cause an international ampere to flow through an international ohm, and is stated to be represented *sufficiently well for practical purposes* by a definite fractional part of the e.m.f. of the Clark cell.

Subsequent investigation has shown that the figures for the silver voltameter and the Clark cell are not equivalent. The e.m.f. of the Clark cell is really 1.4328 international volts at 15°.

PRACTICAL STANDARDS. — The Act of July 12, 1894, is still in force but advantage has been taken of the manner of stating how the international volt shall be practically realized to use as a standard of e.m.f. the Weston normal cell instead of the Clark cell (*see Cells, Standard*).

Standard resistance coils (*see Resistors, Standard*) are used as secondary standards of resistance. These are usually accurately adjusted by the makers. When great accuracy is required these standard resistances should be sent to the Bureau of Standards for calibration. Much more accurate results can be obtained in an ordinary laboratory by using properly calibrated secondary standards than by attempting to construct primary standards in accordance with the definitions in the above Act.

The use of a voltameter for the calibration of current-measuring instruments is also now seldom employed. A more accurate method for ordinary laboratory work is to determine, by means of a properly calibrated potentiometer (q.v.), the voltage drop produced by the current through a standard resistance.

BIBLIOGRAPHY. — *The so-called International Electric Units*, Bull. Bur. Stand., 1904, Vol. 1, p. 30; *Selection and Definition of the Fundamental Electrical Units*, Bull. Bur. Stand., 1908, Vol. 5, p. 243; *Announcement of Change in Value of the International Volt*, Circ. No. 29, Bureau of Standards.

UNLOADERS, COAL AND ORE. — (*See also Cranes; Motors, Industrial Applications of; Power Stations.*) Coal and ore unloaders may be divided into two distinct classes, namely, the stiff-leg type and the bridge type. Both are adapted to electric drive and each possesses certain advantages under the conditions for which it is designed. Unloaders range in capacity from three to fifteen tons per bucket load. The capacity in tons per hour varies within wide limits, but records have been established where under normal operating conditions four 15-ton equipments unloaded 10,000 tons in five hours. The power required naturally also varies under different conditions, but an average of from 0.4 to 0.5 kilowatt hour per ton is not uncommon.

STIFF-LEG UNLOADERS (Fig. 1) are mainly used at receiving docks for unloading ore from the boat to railway cars or to a large concrete trough from which it is in turn transferred to the storage yards by means of ore bridges. The machine, as seen from Fig. 1, consists essentially of a massive pantograph, the short leg of which forms an integral part of the carriage *C*. This leg is rigidly vertical, hence the stiff leg *L* which carries the bucket *B* must always be vertical. The weight of the moving parts of the link motion is nearly counterbalanced at *W* by the main hoist, rotation and bucket motors and their respective drums. The motor house on the carriage *C* contains the trolley motor and the magnetic control panels for all the above motors. The carriage is mounted on trucks and moves back and forth on the girder runway *G*. The girder structure supports a hopper with rotating gates which receives the ore from the bucket *B* and distributes it slowly to the weighing larry which in turn deposits it in the ore car underneath or in the concrete trough *I*. Cables from the main hoist drum at *W* are attached to the rear of the carriage. The operator rides in the bucket leg and has absolute control of lowering, opening, closing, bucket rotation, hoisting and trolleying movements. The entire girder structure is mounted on trucks and can be run along the dock at will by means of the bridge movement motor. Magnetic control is employed throughout and all motors except leg-rotation and bridge-movement motors are arranged for dynamic braking.

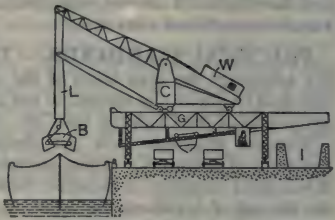

Fig. 1. Stiff-leg Unloader

Cycles of Motor Movements. — The service of the main hoist and opening and closing motor is intermittent with continuous repetition of cycle often at the rate of one round trip per minute for several consecutive hours. The leg rotation motor is used chiefly in cleaning up the bottom of the boat between hatches and involves rapid and frequent reversals at irregular intervals. The trolley-motor cycle includes acceleration, free running, retardation, reversal and repeat with every bucket load hoisted. The ore-gate motor operates intermittently whenever ore is distributed to the weighing larry. The larry travel and hopper-gate motors both operate on a continuously repeated cycle of short duration. The bridge-movement motor is used only at irregular intervals for locating the bucket over the hatches.

Motor Equipment. — The following table shows the motor equipment of a modern unloader of the stiff-leg type. The main-hoist motor has a continuous rating, but all the others are rated on an intermittent basis.

MOTOR EQUIPMENT, 15-TON STIFF-LEG UNLOADER

No. of motors	H.P.	R.p.m.	Volts, d-c.	Application	Type of motor
1	135	350	230	Main hoist	Compound wound
1	75	500	230	Trolley	Compound wound
1	75	500	230	Bucket { Opening / Closing	Compound wound
1	100	450	230	Ore gates	Compound wound
1	100	450	230	Larry	Compound wound
1	100	450	230	Bridge movement	Compound wound
1	20	750	230	Leg rotation	Series wound
1	30	750	230	Larry hopper	Series wound

BRIDGE-TYPE UNLOADERS (Figs. 2 and 3) can be divided into two general classes, viz., the "man-trolley" and the "rope-trolley." In the former all hoisting and conveying motors together with a cab containing the control equipment and operator are mounted on a carriage which runs the length of the bridge, as shown in Fig. 2. In the rope-trolley unloader the motors are located in a stationary motor house and power is transmitted to the hoisting drum and carriage by means of cables, as shown in Fig. 3. The operator is also located in a stationary cab as seen.

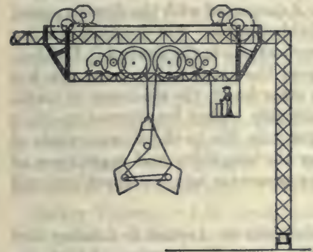

Fig. 2. "Man-trolley" Bridge Unloader

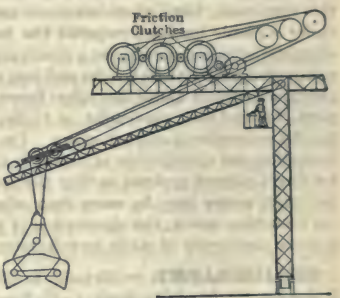

Fig. 3. "Rope-trolley" Bridge Unloader

MOTOR EQUIPMENT, 625-FOOT "MAN-TROLLEY" BRIDGE UNLOADER

No. of motors	H.P.	R.p.m	Volts	Application	Type of motor
4	125	450	230	Main hoist	Series wound
4	50	500	230	Main trolley	Series wound
4	40	460	230	Bridge movement	Series wound

MOTOR EQUIPMENT, 7-TON ROPE-OPERATED BRIDGE-TYPE UNLOADER

No. of motors	H.P.	R.p.m.	Volts	Application	Type of motor
2	55	430	250	Opening lines	Series wound
3	55	430	250	Closing and hoisting	Series wound
1	25	430	250	Travel	Series wound
1	11	...	250	Bucket twist	Series wound
1	15	...	250	Car-loading drums	Shunt wound

TYPE OF MOTOR. — The selection of the most suitable type of motor for coal and ore bridge work must be determined for each specific installation by local considerations. In general the requirements are not quite as severe as those commonly included under the term "steel mill service," and standard industrial motors in a few instances have been used for high-speed bridges with entire success. As a rule, however, especially for ore bridges, the mill-type motor is recommended, since even under the best conditions the short cycle, rapid and frequent acceleration, severe vibration, dirt and moisture, all demand the most rugged construction.

The question of alternating- or direct-current motors is usually one of comparative first costs, costs of maintenance and operation. Where adjustable speeds under variable load are required the induction motor with its constant-speed characteristics is somewhat at a disadvantage if compared with the direct-current motor equipped with interpoles and best available control. For dynamic braking the direct-current motor also offers fewer complications in the control. Furthermore, for the rack movement or trolley travel the acceleration losses are less for the series direct-current motor. On the other hand, the efficiency of transformation and distribution of power is considerably higher for alternating- than for direct-current systems so that the apparent gain due to the characteristic of the series motor may be more than offset by the simplicity and ruggedness of the induction motor, the elimination of motor-generator sets, equalizer sets and in some cases even of static transformers.

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VALVES. — (*See also Boilers; Gas Engines; Pipes; Power Stations; Steam Engines; Steam Turbines.*) The following types of valves are used to control the flow of fluids in pipes.

Gate Valve. — The opening in this valve is perpendicular to the axis of the pipe, and is closed by a disk which moves across the opening. A gate valve offers but little resistance to the flow of the fluid, since when fully opened it provides a straight passage the full diameter of the pipe (*see Pipe and Piping*).

Globe Valve. — In this valve the opening is in a partition parallel to and passing through the axis of the pipe. The opening is closed by a mushroom-shaped disk which is screwed down against the partition. The name "globe" valve arises from the globular form of the casing. The passage through a globe valve has the general shape of the letter S, and consequently offers considerable resistance to the flow (*see Pipe and Piping*). It is simpler in construction and cheaper than a gate valve, and the contact surfaces are more easily ground.

Angle Valve. — This valve has the inlet and outlet at right angles to each other; it may therefore be installed in place of an elbow. The opening is at right angles to the inlet. The closing mechanism may be either of the gate or of the globe type.

Cocks. — This type of valve consists of a conical plug with a hole through it perpendicular to its axis, mounted in a suitable casing. The valve is opened by turning the plug so that the hole is in line with the pipe and is closed by giving the plug a quarter turn. Cocks are frequently used on blow-off piping.

Stop Valves. — A stop valve is any valve that is controlled by hand or other external means, and used to stop the flow of fluid in a pipe.

Check Valves. — A check valve is any valve which opens automatically when the pressure in the normal direction reaches a predetermined value, but remains closed when the pressure is less than this amount or is in the opposite direction. Ordinary forms of check valves resemble a globe valve without a valve spindle. The closing device may be a disk, ball or cup. When the disk is arranged to swing about an axis like a hinge the valve is called a "swing check," or "flap valve," or "butterfly valve." When the disk or other device is lifted vertically, the valve is called a "lift-check valve." A check valve placed at the base of the suction pipe of a pump is called a "foot valve."

Safety Valves. — Lift check valves provided either with a helical spring or with a ball and lever mechanism to hold the valve closed under normal pressure are used as safety valves; the former type is called a pop safety valve, the latter a lever safety valve. The pop safety valve has practically supplanted the lever type, since it closes more promptly and is less liable to leak.

Relief Valves. — Check valves used to relieve excessive back pressure in atmospheric exhaust pipes are called "back-pressure valves," and when used in the piping to condensers to relieve excessive pressure are called "atmospheric relief valves."

Reducing Valves are used when it is desired to obtain steam at a pressure less than that of the boiler. In this type of valve the opening is relatively small, and the steam in expanding through this small opening has its pressure reduced. The proper size of opening for various rates of flow is maintained automatically by balancing the force produced on the closing mechanism by the pressure on the low-pressure side of the valve against the force produced on this mechanism by a helical spring. The size of the opening then adjusts itself automatically to maintain this balance irrespective of the rate of flow.

SELECTION OF TYPE OF VALVE. — For low-pressure work the standard cast-iron valves with bronze seats have been more than satisfactory. They are now made of a great many types, all of which give very good results. The solid wedge gate is perhaps the earliest and the best known. The split-wedge type and the parallel two-gate type are also well known and largely sold. Globe valves are not usually used for steam work on account of the resistance offered to the passage of the steam, but for throttle valves and for stop valves are still the standard. For high-pressure work and especially with superheated steam the use of the steel body valve with steel seats and disks has becoms standard and many varieties of valves are now on the market, some of which are doing excellent work. Nickel-bronze and nickel are also used for seats and stems with good results. In choosing a valve for high-pressure and high-temperature work, great stress should be laid on the absence of chance for unequal expansion in the body and gates. The metal should be so placed that what expansion occurs will be equable in all directions and the gates so designed that they cannot spring out of true under different degrees of heat. Such mechanism as may be used between the gates in a double-gate valve to press them up against the seats should be as carefully designed as the body of the valve, as small deflections in this part of the mechanism will prevent tightness. The most satisfactory valves for this work have been of the double-wedge type, although there are good parallel seat and solid-wedge valves on the market which have stood severe tests.

ELECTRICAL OPERATION OF VALVES. — (*See also Motors, Industrial Applications of.*) The advantages of electrical motor operation for large valves is emphasized not only by the fact that such valves must be closed in a very short time, which would be impossible with hand control, but also by the fact that remote control is very often essential, as, for example, in hydroelectric power developments where it becomes desirable to control the gate valves from the control switchboard. The service of valve motors is exceedingly intermittent and may vary from comparatively short intervals, such as once every hour, to weeks or even months. Due to the intermittent nature of the service, efficiency or power factor need not be considered in this kind of motor application, the main consideration being the most reliable system of operation.

Size of Motor. — The proper size of a motor for driving a valve will vary with the duty and conditions under which the valve operates. A small valve may only require a one-horse-power motor, whereas very large valves, such as the Stoney-Gate valves at Panama, require 40-horse-power motors. The required motor capacity also depends to a large extent on the pressure on the valve. When opening the valve, the torque is a maximum shortly after the time of unseating, that is, after the wedges have been released and the actual motion begins. The torque then drops some until the valve has opened about one-fourth, after which it takes comparatively little power to complete the opening. When closing the valve, friction only needs to be overcome in starting as there is no pressure on the valve until it has begun to close. After the valve is about three-fourths closed the pressure causes the torque to increase rapidly. At the end of the closing cycle the torque does not, however, reach the value it did during the period of starting.

Type of Motor. — Either direct- or alternating-current motors may be used. With the former the series- or compound-wound type is generally used as it gives a high starting torque. With the latter the squirrel-cage induction motor is most widely used for small- and medium-size valves, principally on account of its simplicity. To overcome the sticking when opening, the drive is sometimes provided with a "lost motion" so as to give a hammer blow. For alternating-

current motors this is furthermore of value in that it permits the motor to speed up some and gain in torque before the load comes on.

Control Equipment. — Where the size of the motor permits, the simplest method of control is to throw the motor directly on the line without the use of starting resistances. The automatic overload circuit breaker which should afford protection under normal operating conditions, must be prevented from tripping during the rush of starting current. This is readily accomplished either by short-circuiting the overload coil of the circuit breaker or by opening the connections to the coil. The former is the preferable method as this will leave some overload protection even though the coil is short-circuited. A push button is generally provided for this purpose.

Limit switches which will open the circuit when the gate has reached its limit of travel should always be provided. As a further precaution against overtravel or too high closing torque the motors should preferably be geared to the valve stems through efficient and reliable friction clutches.

When the motors are too large to be thrown directly on the line, starting resistances must be provided. With direct-current series motors a permanent resistance and a running resistance are often provided. The permanent resistance remains fixed in the circuit and should be so dimensioned as to give the maximum torque required. The running resistance is short-circuited at starting until the motor current drops to any desired value, at which time a current limit relay opens the shunt circuit of the contactor used for short-circuiting the resistance. This running resistance should be such as to prevent excessive speed at light load and to limit the torque should the valve seat itself before the motor is disconnected. The same general scheme may be used with alternating-current motors, but with these there is no danger of exceeding the synchronous speed.

Devices for Indicating Valve Positions. — When it is desired to indicate on the switchboard the position of the gate throughout its range of travel, this can readily be done by placing a number of sliding contacts on the inside of the limit-switch housing. These contacts are then connected with a row of incandescent lamps installed on the switchboard, and as the limit switch travels along, the lamps on the board will light successively.

Another system, much more comprehensive, is used in connection with the valve motors at the Panama Canal. In connection with each limit switch there is installed a "synchronizing transmitting device." This consists merely of a small generator which is mechanically connected to and revolves with the limit switch. It is electrically connected to a small motor which is mounted on the control switchboard and there mechanically connected to some sort of indicating device such as a pointer. When the generator at the valve revolves, the motor on the board will follow in synchronism thus indicating the exact position of the valve.

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VECTORS. — (*See also Complex Quantities.*) Any quantity which requires for its complete specification a magnitude and a direction is called a vector quantity. Such a quantity may be represented graphically by a line having a length equal to the magnitude of the quantity and a direction parallel to the direction of the quantity. Such a line is called a vector. Its direction is, in general, specified by the angles which it makes with three arbitrarily chosen lines or axes of reference. Vectors which lie in the same or in parallel planes are called co-planar vectors; the direction of a vector in the plane in which it lies (or parallel planes) may be specified by the angle which it makes with a single line of reference in this plane.

Quantities which are completely specified by magnitude and "sense" (i.e., whether positive or negative) are called scalar quantities. Forces, velocities, displacements, etc., are vector quantities, while time, work, mass, etc., are scalar quantities.

Vector Addition and Subtraction. — Since vector quantities cannot be completely specified by ordinary numbers, the various operations of arithmetic such as addition, subtraction, multiplication and division have no meaning when applied to such quantities. Analogous processes, however, are of great value in dealing with them. For example, the resultant of two forces which make an angle with each other is, by the ordinary parallelogram of forces, equal to the diagonal of the parallelogram formed by drawing from the free ends of the two lines representing the forces, lines parallel to these forces. That is, the resultant of OA and OB in Fig. 1 is the diagonal OC. Both the length OC and direction (the angle AOC) of the resultant is fixed by this construction. The resultant OC is called the vector sum of OA and OB. Similarly, the vector sum of OA and AB is equal to OB, or vice versa, AB may be called the vector difference of OB and OA. The angle XAB gives the direction of this vector difference when OA is subtracted vectorially from OB. When OB is subtracted vectorially from OA the vector difference also has the same length or magnitude, but is in the opposite direction, i.e., makes the angle $180° - \angle XAB$ with OA.

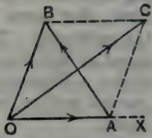

Fig. 1

Components of a Vector. — Any vector may be considered as made up of any number of vectors which when added vectorially, as described above, give a resultant vector equal to the given vector. It is frequently convenient in analyzing problems involving vectors to resolve each vector into two components, one parallel to and the other perpendicular to the axis of reference. The first component is usually referred to as the horizontal component and the second as the vertical component. It is readily proved that the horizontal component H of the resultant (or vector sum) of any number of vectors is equal to the algebraic sum of the horizontal components h_1, h_2, etc., of the individual vectors, i.e.,

$$H = h_1 + h_2 + \text{etc.}$$

and the vertical component V of the resultant is equal to the algebraic sum of the vertical components v_1, v_2, etc., of the individual vectors, i.e.,

$$V = v_1 + v_2 + \text{etc.}$$

The length of the resultant is then

$$S = \sqrt{H^2 + V^2},$$

and it makes with the axis of reference the angle σ where

$$\tan \sigma = \frac{V}{H}.$$

Analytical Representation of a Vector. — The geometrical representation of a complex quantity (*see Complex Quantities*) is a vector, i.e., the line representing a complex quantity has both magnitude and direction. Any vector quantity may, therefore, be represented by a complex quantity, and the operations of vector addition and subtraction may be carried out by the ordinary processes of algebraic addition and subtraction of complex quantities. For example, two vectors A and B may be represented by the algebraic expressions

$$\underset{\cdot}{A} = h_1 + jv_1,$$
$$\underset{\cdot}{B} = h_2 + jv_2,$$

where h_1 and h_2 and v_1 and v_2 are the horizontal and vertical components respectively, and the dots under A and B signify that $\underset{\cdot}{A}$ and $\underset{\cdot}{B}$ are vector quantities. The vector sum of $\underset{\cdot}{A}$ and $\underset{\cdot}{B}$ is then

$$\underset{\cdot}{S} = \underset{\cdot}{A} + \underset{\cdot}{B} = (h_1 + h_2) + j\,(v_1 + v_2),$$

which has the magnitude

$$S = \sqrt{(h_1 + h_2)^2 + (v_1 + v_2)^2},$$

and the angle

$$\sigma = \tan^{-1} \frac{v_1 + v_2}{h_1 + h_2}.$$

Multiplication and Division of a Vector by a Number. — Multiplying a vector by a real number n means taking a vector n times as long; dividing a vector by n means taking a vector $\frac{1}{n}$ of the length of the original vector. The result of multiplying a vector by an imaginary number jn is defined as a vector n times the length of the original vector and making a positive angle of 90° with the original vector. The result of dividing a vector by an imaginary number jn is defined as equivalent to multiplying the vector by $\frac{1}{jn} = -j\frac{1}{n}$ that is, as equivalent to a vector $\frac{1}{n}$ of the length of the original vector and making a negative angle of 90° with the given vector. Hence the multiplication or division of a vector $h + jv$ by a complex number $n + jn$ is equivalent to ordinary algebraic multiplication or division of two complex numbers (*see Complex Quantities*).

Scalar Product of Two Vectors. — Multiplication, in the ordinary sense, of one vector by another has no meaning, since a vector is not a number but involves both magnitude and direction. In the analysis of the more complicated problems of mechanics and electricity, certain expressions arise, however, which are analogous to ordinary multiplication. One of these expressions is a scalar quantity equal to the product of the magnitude of the two vectors by the cosine of the angle between them. This product is called the scalar product of the two vectors. The scalar product of the two vectors A and B is $AB \cos \theta$ where θ is the angle between them. If the vectors are expressed as complex quantities

$$\underset{\cdot}{A} = h_1 + jv_1,$$
$$\underset{\cdot}{B} = h_2 + jv_2,$$

the scalar product is

$$h_1h_2 + v_1v_2.$$

Note that this has no relation to the algebraic product of the two expressions $h_1 + jv_1$ and $h_2 + jv_2$.

Vector Product of Two Vectors. — Another expression which arises is a vector quantity at right angles to the plane of the two vectors and equal in magnitude to the product of the magnitudes of the two vectors by the sine of the angle between them. This product is called the vector product of the two vectors.

Example. — The vector product of the vector A by the vector B (Fig. 1) is $AB \sin \theta$, and is in the direction in which a right-handed screw, to the head of which A is conceived to be fixed, is advanced when A is turned through an angle of less than 180° into coincidence with B. From this definition it follows that the vector product of B times A is equal but *opposite* to the vector product of A times B. When A and B are expressed as complex quantities the vector product of A times B is

$$h_1v_2 - h_2v_1.$$

Note that the algebraic product of the expressions

$$h_1 + jv_1 \quad \text{and} \quad h_2 + jv_2 \quad \text{is} \quad h_1h_2 - v_1v_2 + j\,(h_1v_2 + h_2v_1).$$

The real and imaginary parts of this expression are *not* equal to the scalar and vector products.

VISION, LAWS OF. — (*See also Illumination, Laws of; Photometric Quantities; Photometry.*) Light enters the eye through the *cornea*, a thin, transparent, curved wall, and passes successively through the *aqueous humor*, the *pupil*, the *biconvex lens*, and the *vitreous humor* to an image on the rear wall or *retina*. The cornea and lens possess the image-forming function, and accommodation to distance is given by the varying curvature of the lens. The humors give optical contact and preserve the size and form of the eye. The contraction and dilation of the pupil gives automatic accommodation to the intensity and quantity of light in the field of view. The retina adapts itself automatically in sensibility to the flux density of the light falling upon it; while comfortable vision covers a range of brightness of ten billion to one, the sensibility of the retina varies in the ratio of one hundred million to one.

The retina comprises an elaborate structure of microscopic nerve terminals known as rods and cones. In its periphery rods predominate, but the ratio of cones to rods increases steadily toward the axial region of *fovea*, where the cones greatly predominate. The directly visualized image falls on the fovea and the sharpness of perception diminishes markedly in passing to the periphery. At very low intensities rod vision predominates. At moderate and high intensities cone vision predominates. In the former state the relative sensibility to the blue end of the spectrum is emphasized, in the latter state the relative sensibility to the red. As an optical system the eye has a chromatic aberration of approximately two diopters.

ELEMENTS OF VISION. — Visual sensations in general are characterized by intensity, extensity, quality and duration. Intensity refers to the aspects of vision measurable photometrically, such as brightness and contrast, determining flux density at the retina. Extensity involves the perception of contour, perspective, relief, detail and distance. Quality discriminates between sensations of color and non-color. Duration relates to the growth, fatigue, persistence and intermittency of visual sensations.

Intensity Relations. — For a given quantity of radiant energy, the luminous sensation (light) varies with the wave frequency of the radiation, with its intensity and with the time of exposure of the eye to it. The eye is most sensitive and the ratio of light to energy is a maximum at a wave length of about 0.000558 mm. in the green part of the spectrum. Here the ratio of light to radiation is about 0.0015 lumen per watt of radiation. From this wave length the luminous equivalent of radiation decreases rapidly toward both shorter and longer wave lengths. It is about 0.0000006 lumen per watt at wave lengths 0.00040 and 0.00073 mm. The data on the complete curve for ratio of light to radiation are given in the following table and in Fig. 1. These data are the means for about 200 subjects collected by Ives, Nutting, Coblentz and Hyde in four different laboratories. These data are applicable to all moderate and high intensities. At very low intensities a similar curve displaced toward shorter waves obtains.

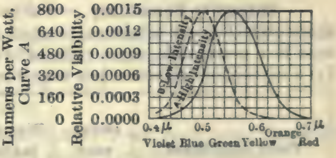

Fig. 1

Purkinje Effect. — Transition from vision at low intensity to high involves relative loss of sensibility to blue and gain of sensibility to red, so that two lights of reddish and bluish hue respectively show a ratio of intensities which varies with the brightness of the field of view. This is known as the Purkinje effect. In fields above a brightness of one lumen per square foot the

VISIBILITY OF RADIATION AVERAGE NORMAL EYE

Wave length ($\mu\mu$)	Relative to that at 556 $\mu\mu$	Absolute (Lumens per watt)	Wave length ($\mu\mu$)	Relative to that at 556 $\mu\mu$	Absolute (Lumens per watt)
400	0.0004	0.0000006	600	0.631	0.00095
10	.0012	.0000018	10	.503	.00075
20	.0040	.0000060	20	.380	.00057
30	.0116	.000017	30	.262	.00039
40	.023	.000034	40	.170	.00025
450	0.038	0.000057	650	0.103	0.000154
60	.060	.000090	60	.059	.000089
70	.091	.000136	70	.030	.000045
80	.139	.000208	80	.016	.000024
90	.208	.000312	90	.0081	.0000122
500	0.323	0.00048	700	0.0041	0.0000061
10	.484	.00073	10	.0021	.0000031
20	.670	.00100	20	.0010	.0000015
30	.836	.00125	30	.00052	.00000078
40	.942	.00142	40	.00025	.00000037
550	0.993	0.00149	750	0.00012	0.00000018
60	.996	.00149	60	.00006	.00000009
70	.952	.00143			
80	.870	.00130			
90	.757	.00114			

shift is slight. Photometric comparisons involving color differences should be made only in bright fields.

Intensity Sensibility. — For an eye viewing a field of a given brightness there exists for each brightness to which the eye is adapted (1) a minimum brightness just perceptible, (2) a ratio of brightness (contrast) just perceptible and (3) a maximum brightness just tolerable.

Field brightness	Difference fraction	Discrimination factor	Threshold limit	Glare limit
0.000001 m.l.	(1.00)	1.0	0.00000093 m.l.	20.1 m.l.
0.00001	(0.66)	1.5	0.0000042	40.7
0.0001	0.395	2.5	0.000019	89.
0 001 (E N.)	0.204	4.5	0.000087	86.
0 01	0.078	12.8	0.00039	400.
0.1 (I.N.)	0.037	27.0	0.00174	810. m.l.
1.0	0.0208	48.2	0 0081	1.66 l.
10. (I.D.)	0.0174	57.5	0.036	3.47
100.	0.0172	58.1	0.28	7.25
1,000. (E.D.)	0.0240	41.7	2.15	14.45
10,000.	(0.048)	(20.9)	(232.)	30.90

A brief summary of the data on threshold limit, discrimination and glare limit is given in the table at bottom of page 1888.

For example, an average eye adapted to a field brightness of 0.1 m.l. can just see a contrast of 1.037 : 1—the discrimination is about half (27/57) its maximum value. The deepest shadow in which it can see anything is 0.0017 m.l. in brightness and the brightest tolerable high light is 800 m.l.

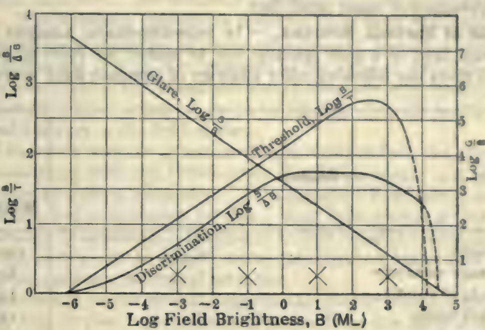

Fig. 2. Threshold, Discriminating Power and Glare Sensibility at Various Levels.

The illuminating engineer is chiefly concerned with four quite distinct brightness levels, namely:

0.001	m.l.	(E.N.)	Exteriors at night, street lighting.
0.01	m.l.	(I.N.)	Interiors at night, interior artificial lighting
1.0	m.l.	(I.D.)	Interior daylight lighting.
1.0	l.	(E.D.)	Exterior full daylight lighting.

Log **glare** sensibility (G) is a linear function of log field brightness (B) such that

$$\text{Log } G = 3.23 + 0.32 \log B,$$
$$G = 1700 B^{0.32}.$$

Log **threshold** sensibility (T) is also a linear function of log field brightness except at the highest intensities (over 0.1 lambert) such that

$$\text{Log } T = 0.66 \log B - 2.10,$$
$$T = 125 B^{0.66}.$$

Sensibility to **contrast** is nearly constant (about 2 per cent) over the wide range of intermediate intensities with which vision is chiefly concerned.

RATE OF INCREASE AND DECREASE OF THRESHOLD SENSIBILITY

Time	$B_0 = 0$, $B = 25$m.l. Sensibility decrease	$B_0 = 25$m.l., $B = 0$ Sensibility increase
1 second	2.1 times	1.6 times
2 seconds	4.2 times	2.6 times
5 seconds	16.2 times	7.6 times
10 seconds	58 times	14.4 times
10 minutes	120 times	20.9 times

Effect of Alternate Exposures. — If an eye is fully adapted to a field of a given brightness, then suddenly exposed to a dark field it gains rapidly in sensibility as shown in the table on page 1889, while if adapted to darkness and then exposed to a bright field it loses sensibility but at a lower ratio.

Since the loss of sensibility in a given interval is greater than the recovery, the net result of a succession of exposures to alternate light and darkness is a considerable *depression* of mean sensibility.

Color of Various Sources. — In monochromatic analysis white light is mixed with light of a pure spectral hue to match the color to be analyzed. L. A. Jones gives the following color analyses of common light sources.

Source	Color	
	White	Hue
1. Sunlight	100	472.0
2. Average clear sky	60	472.0
3. Standard candle	13	593.0
4. Hefner lamp	14	593.0
5. Pentane lamp	15	592.0
6. Tungsten glow lamp at 1.25 w.p.c.	35	588.0
7. Carbon glow lamp at 3.8 w.p.c.	25	591.0
8. Nernst glower at 150 w.p.c.	31	586.7
9. Nitrogen filled tungsten:		
at 1.00 w.p.c.	34	586.0
at 0.50 w.p.c.	45	584.5
at 0.35 w.p.c.	53	584.0
10. Mercury-vapor arc	70	490.0
11. Helium tube	32	598.0
12. Neon tube	6	605.0
13. Crater of carbon arc:		
at 1.8 amperes	59	584.6
at 3.2 amperes	62	584.6
at 5.0 amperes	67	583.4
14. Acetylene flame (flat)	36	585.5

Extensity Relations. — Extensity is perceived largely through differences in brightness and color, by varying visual angles and by acuity, or the power of resolving detail. Acuity follows a logarithmic relation to brightness similar to the strength of sensation. The same practical lower limit, say from one to two lumens per square foot, affords a fair normal basis of acuity. Above this the gain in acuity is relatively much less than the increase in illumination. Acuity, however, is not independent of color, due to the chromatic aberration of the eye. Retinal images formed by approximately monochromatic light are sharper than those due to a rich spectrum. Luckeish (*see Elec. W., Vol. 58, p. 1252*) gives the data in Fig. 3 for the relative acuity with various colors. For near vision blue light is more readily focussed, whereas for distant vision red light is more readily focussed.

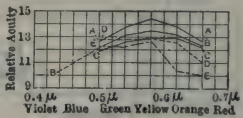

Fig. 3

Glare. — The effectiveness of illumination is depressed to a marked degree by disturbing influences grouped under the term glare, which may be approximately defined as the relative overbrightness of part of the field of view. Transient glare, caused by the slowly changing adaptation of the eye when emerging from a dim region into a brilliantly lighted one, causes only temporary discomfort. Persistent glare, in addition to the depression of visual functions, may be highly injurious. No complete quantitative analysis of glare has yet been made. The following qualitative relations have been well established:

(*a*) Glare effect, or reduction of visual effectiveness, increases as the glaring source approaches the eye. Glare increases as the ratio of the distance of the glaring source to that of the visualized object from the eye diminishes. The distance at which a light source ceases to be glaring depends on its intensity, brilliancy and position in the field of view.

(*b*) Glare increases with the quantity of light received from the source.

(*c*) Glare increases with the brilliancy of the glaring source and with its degree of contrast to objects visualized. An automobile headlight is exceedingly glaring at night, but not glaring by day. Any light source giving an after-image is excessive in brilliancy. Glare due to this cause is accentuated by the reflex tendency of the eye to wander from the object of vision and fix on the brilliant source, causing a fatigue of attention.

(*d*) Glare increases as the retinal image of the glaring source approaches the center of the field of view. Depression of vision from side light is due in part to contrast and in part to the dilution of the central image by light scattered in the eye. Glare due to bright walls and backgrounds is largely of the latter class. The glare from bright sources situated at an angle from the line of vision exceeding 26 degrees is generally negligible.

The chief practical cases of glare are due to directly visible light sources of high power and brilliancy, to scattered light from side sources and backgrounds and the direct reflection of bright images by glazed surfaces in the immediate field of view. Prevention should be sought by the proper location and shading of light sources, by creating suitable constrasts between the field and its surroundings and by avoiding the use of glazed surfaces, especially highly-sized paper.

Duration. — Below certain frequencies intermittent sensations retain a measure of distinctness and produce a flicker effect. If a critical frequency is exceeded the sensations blend into a continuous effect equal in intensity to the mean intensity over the complete cycle. Examples are the rotating sectored disk and the electric lamp operated by alternating current. The vanishing frequency of flicker increases with the degree of variation from maximum to minimum with the solid angle subtended by the object viewed and with the intensity of its brightness. The vanishing frequency of flicker is higher for white than for colored light. Within the common limits of practice the vanishing frequency is between 30 and 45 cycles per second. Electric lamps display no observable flicker in themselves or in illuminated objects at frequencies exceeding 40 cycles.

Quality or Color Relations. — Non-colored sensations are white gray, and black. Color sensations differ in brightness in hue and in tone. The hue of a light is determined by the spectral position of its predominant component. Its tone or tint depends on the degree of dilution with white or black. Thus spectral red, pink and claret may agree in hue, but differ essentially in tone. Color sensations are equal in brightness when they may be rapidly alternated in the field of view without the appearance of flicker.

Synthesis and Analysis of Colors may be accomplished by mixing red, green and blue in proper proportions, or by diluting the predominant hue with the proper amount of white or black. A color may be specified by the synthesis

necessary to reproduce it. Tri-chromatic specification gives the mixing proportions of red, green and blue. Mono-chromatic specification gives the spectral hue and the degree of dilution. The tri-chromatic analysis as determined by the colorimeter (*see Photometry*) is of greatest practical utility. In this system the proportions of red, green and blue found in average daylight, though not equal, are taken as 33.3 per cent each to fix three color scales. The mixing proportions of these colors on the scales so established which reproduce any given light serve to specify it and to compare its color composition with daylight.

Relation of Color of Light to Appearance of Objects. — Objects appear in the colors which they reflect or transmit to the eye. To produce true color effects light must possess all the components necessary to reveal objects and possess them in proper proportions. If the spectrum is deficient in parts or its components differ greatly from the proportions in daylight it will in general produce distorted color effects.

Artificial White Light. — With the exception of the carbon dioxide vacuum tube no artificial illuminant approaches closely the color value of daylight. White light may be approximated by synthesis by combining the light of two sources, one excessive in red and the other excessive in blue, as the incandescent electric lamp and the mercury arc. A closer realization may be obtained from any incandescent lamp by means of a selective absorption screen which removes the excess of red and green light. The latter method involves a large sacrifice of efficiency.

SUMMARY OF CONDITIONS FOR BEST VISION. — The surface brightness of objects viewed should not be less than 0.0015 candle per square inch nor more than 1.5 candles. In terms of the illumination of ordinary white paper these values correspond roughly to limits of 1 to 1000 foot-candles. The value best adapted to any condition varies with the individual, the closeness of application the nature of the surroundings and the general degree of diffusion. For reading by daylight 100 foot-candles is approximately the best condition

ILLUMINATION IN FOOT-CANDLES

Nature of illuminated surface	Foot-candles at floor level	
	Minimum required	Good practice
Roadways and yard thoroughfares	0.02	0.05– 0.25
Storage spaces, stairways, passageways, aisles, exits and elevator entrances	0.25	0.50– 2.00
	Foot-candles at the work	
Work not requiring discrimination of detail	0.50	1.00– 2.00
Rough manufacturing and assembling	1.00	2.00– 4.00
Machining and bench work	2.00	3.00– 6.00
Fine manufacturing and office work	3.00	4.00– 8.00
Special fine work, drafting, engraving, sewing	5.00	7.00–15.00

by artificial light approximately from 4 to 10. The degree of illumination required is strongly influenced by the contrast between objects visualized and their surroundings. The high diffusion of daylight and the flatness of the contrasts which it gives account in large measure for the higher illumination desired. For best seeing the objects viewed should be brighter than the surroundings, but the contrast should not exceed 10 : 1. The illumination provided for work requiring close application should be in inverse ratio to the reflecting power of the predominant surfaces. Directly visible light sources may generally be shaded down to a brightness of 2.5 candles per square inch with advantage to vision. Precautions outlined under *Glare* should be observed.

The accompanying table indicates the range of artificial illumination in foot-candles which has been found satisfactory under practical conditions.

Definition. — Vision is aided by the sharp definition of details down to an angular range of 1 in 10,000. Reading is easiest with type subtending an angle of about 1 in 100. For comfort it should not be larger than 1 : 20 nor less than 1 : 300.

Contrasts in intensity are best at about 1 : 10, but should not be less than 1 : 100. Contrasts as slight as 98 : 100 are readily perceivable in good light.

Uniformity of light on the working area is important. Excessively bright spots constitute an annoying glare and interfere with the fixation of vision.

Steadiness of light is essential, as flicker causes rapid fatigue.

Color. — A well-balanced spectrum is required for the true revelation of surface colors. Fatigue is most rapid in the extreme blue end of the spectrum and ultra-violet radiations may cause injury if present in unusual quantity. Light having a strongly dominant yellow-green hue gives the highest visual acuity.

Diffusion. — Complete diffusion tends to eliminate shadows, with the resulting loss of relief. The value of moderate shadows varies greatly with the circumstances, but is greatest where color distinctions are lacking.

VISUAL EFFICIENCY. — The resultant of all conditions affecting vision is included in the somewhat loose term visual efficiency. This concept must take account of the fundamental discriminations of brightness, detail and color and of the maintenance of these faculties in use. Tests of visual acuity have been largely used as a working criterion of visual efficiency. As applied to the momentary state of vision this criterion is unsatisfactory as it invites a semi-conscious mental spur to offset fatigue. An ideal criterion must take account of the maintenance of acuity in work, of the ease with which fixation of vision and attention are sustained, and of the general requirements of ocular hygiene. (*See Trans. Ill. Eng. Soc., Vol. 8, p. 40.*)

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VOLTMETERS. — (*See also Ammeters; Wattmeters.*) A voltmeter is an instrument for measuring the potential difference, or voltage, between the two points to which its two terminals are connected. Any type of ammeter (q.v.) may be used as a voltmeter, but when designed for this purpose the coil which carries the current is made of a number of turns of fine wire, so that the moving element will be deflected by a very small current (from 10 to 200 milliamperes for full-scale deflection, depending upon the type) and to limit the current to this value a high resistance, contained within the case of the instrument, is usually inserted in series with the coil. The high resistance may be provided with taps brought out to additional binding posts, so that the same instrument will give full-scale deflection for say 75, 150 and 300 volts, depending upon which binding post is used in conjunction with the terminal marked "0" or "+." The range of a voltmeter may also be indefinitely extended by the use of multipliers (*see below*) or potential transformers (*see Transformers, Instrument*).

Current Taken by, and Resistance of, Voltmeters. — The current taken by voltmeters of various types for full-scale deflection, and the resistance per volt of full-scale deflection are usually as follows:

Type of voltmeter	Milliamperes for full-scale deflection	Ohms per volt of full-scale deflection *
Iron-vane type	83 to 60	12 to 17
Moving-coil type	20 to 10	50 to 100
Electrodynamic type	83 to 40	12 to 25
Hot-wire type	250 to 187	4 to 5.3

* For example, a moving-iron type voltmeter having a 150-volt scale would have a resistance of from $12 \times 150 = 1800$ to $17 \times 150 = 2550$ ohms. The current taken by the instrument would be from 83 to 60 milliamperes. Lower ranges in moving-iron and in electrodynamic voltmeters require larger currents for full-scale deflection, in order to keep down errors due to frequency.

When a d-c. voltmeter is connected to any two points on a network, its reading is in general lower than the previous potential difference between the points, because the voltmeter draws some current from the network. Let the voltmeter reading be V_1. If the voltmeter be now shunted by a resistance equal to its own, a second reading V_2, lower than V_1, will be obtained. The potential difference V can then be found from the formula,

$$V = V_1 V_2/(2V_2 - V_1).$$

The above method gives correct results not only when the voltmeter is connected directly to the points but also when it is connected through "pressure wires" of appreciable resistance.

Temperature and Inductance Errors. — In order that a voltmeter calibrated at one temperature shall read correctly at all other ordinary temperatures, it is necessary that its total resistance shall change with temperature by only the small amount necessary to offset the resultant effect of temperature change in the springs, magnets, or other operative parts. To secure this condition, the resistance included in series with the active winding (which latter is usually made of copper wire) must have a very low temperature coefficient.

Manganin, constantan, and various other alloys may be used for this purpose; see *Wires, Resistance*. The total resistance of a permanent-magnet moving-coil voltmeter, or of a voltmeter of the electrodynamic type should increase about 0.02 per cent per degree C. if its readings are to be independent of temperature.

The added resistance in an a-c. voltmeter is wound non-inductively, either as a bifilar coil or on a card. The total impedance is then only slightly greater than the total resistance for well-designed voltmeters used at ordinary lighting and power frequencies. At higher frequencies the voltmeter will read lower because of its higher impedance. If the reading is correct at the calibration frequency f_1, the reading at frequency f_2 may be corrected by multiplying by the factor $1+2\pi^2L^2(f_2^2-f_1^2)/R^2$, where R is the resistance, L the inductance, and $2\pi^2f_1^2L^2/R^2$ and $2\pi^2f_2^2L^2/R^2$ are each small in comparison with unity. In many cases the term f_1^2 may be neglected.

Multipliers and Potential Transformers. — A voltmeter multiplier is simply an external resistor placed in series with the voltmeter. These resistors are made of high-resistance alloys having a low temperature coefficient, and are wound non-inductively. If r and R are the resistance of the voltmeter and of the multiplier respectively, v and V the value of one scale division in volts without and with multiplier respectively, then

$$V=v(R+r)/r.$$

The ratio $(R+r)/r$ is frequently called the "multiplying power" of the multiplier for the voltmeter in question.

For voltages above 750 volts potential transformers are to be preferred to multipliers for a-c. circuits; see *Transformers, Instrument*.

Electrostatic Voltmeters. — When the current taken by the ordinary form of voltmeter is an appreciable fraction of the load current or when the frequency is very high, as in radio work, an electrometer (*see article on Electrometers*) or electrostatic voltmeter may be used. The ordinary electrostatic voltmeter is in principle the same as an electrometer, but is provided with a pointer and scale and is so constructed as to be more readily portable than the ordinary electrometer which is read by means of a telescope and scale. Repulsion electrostatic voltmeters are also used, the principle of operation being the repulsion of two conductors charged with electricity of the same sign, as in a gold-leaf electroscope.

Condenser multipliers are sometimes used with electrostatic voltmeters; see *Electrometers*.

Checking of Voltmeters. — A voltmeter may be checked by comparing it with a standard voltmeter whose correction curve is known, or it may be checked directly by means of a potentiometer and standard cell; see *Potentiometers*. Hot-wire and electrodynamic voltmeters for alternating-current work are checked on direct or alternating current; a moving-iron voltmeter should be checked by comparing it, on alternating current, with one of the other types which has been previously checked on direct current. Electrostatic voltmeters are usually employed commercially only for high voltages, and therefore must be checked on alternating current.

Precautions in Use and Installation, Costs, Bibliography, etc. — See the article on *Ammeters*.

WATER WHEELS AND THEIR SETTINGS. — (*See also Hydraulics, Principles of; Power Stations, Hydroelectric; Water Wheels, Speed Regulation of.*) —A water wheel is a machine for converting the energy of falling water into mechanical work. The old-fashioned undershot and over-shot wheels are now seldom used. In modern water wheels, usually called hydraulic turbines, the energy of the water is gradually transformed into kinetic energy as the water passes through a set of guides and vanes.

Work is the product of force times distance, and power is the rate of doing work. Let f be the total force in pounds exerted by the water on the runner of a turbine, let L be the distance in feet travelled by a point on the runner located at the center of pressure, and let t be the time in seconds for this point to travel this distance. The power developed by the water is then

$$P = \frac{f}{550} \cdot \frac{L}{t} \qquad \text{horsepower} \qquad (1)$$

The power developed by falling water may also be expressed in terms of the rate of flow in cubic-feet per second, Q, and the *net* effective head h, in feet. The potential energy of Q cubic-feet of water at an elevation of h feet is $62.4\,Qh$ foot-pounds. Hence for a rate of flow of Q cubic feet per second the *theoretical* power developed is:

$$P = \frac{62.4Qh}{550} = \frac{Qh}{8.814} \qquad \text{horsepower} \qquad (2)$$

It is important to note that in this equation h is not the difference in elevation between the intake and discharge levels, but is equal to this gross head H less the sum of the heads corresponding to the friction losses in the conduit system and the residual velocity where the water is returned to the stream.

All the energy developed by the water as it passes through the turbine is not converted into useful work. The ratio, e, of the power output of the turbine, measured on the turbine shaft, to the theoretical power of the water, is the efficiency of the turbine. The output of the turbine may then be expressed by the equation:

$$P_0 = \frac{Qhe}{8.814}. \qquad (3)$$

Speed of Rotation. — In the conversion of energy by an hydraulic turbine the factor $\frac{L}{t}$ in equation (1) is the *linear speed* of the runner at the center of pressure.

If the linear speed is zero, it is evident that the power must be zero. If the linear speed is too great, approximately equal to the spouting velocity of the water corresponding to the head on the turbine, the pressure of the water on the runner is zero and again no power is produced. It is evident, therefore, that at some linear speed between these limits the power and the efficiency will be the greatest. In other words, the linear speed of the runner must be a fixed percentage (theoretically about 50 per cent) of the spouting velocity of the water, in order to obtain the best efficiency. Therefore, since the *speed of rotation* or revolutions per minute (commonly termed "speed" of the runner) varies directly as the linear speed, it increases with the head (and spouting velocity), if the diameter is constant. It also decreases as the diameter (and capacity) increases.

Consequently, for a given type of runner, low heads and large capacities require a low speed and, conversely, high heads and small capacities require high speeds. It is the object of turbine builders to keep the speed within

reasonable limits to suit the characteristics of the generators and other machinery, and for this purpose many types of runners have been developed.

CLASSIFICATION OF TURBINES. — Modern hydraulic turbines are of two types: reaction turbines and impulse turbines. The basic difference between the two types is in the runner.

Reaction Turbines. — A typical low-head reaction turbine and runner are indicated in Figs. 1 and 2. The water enters the runner through the guide

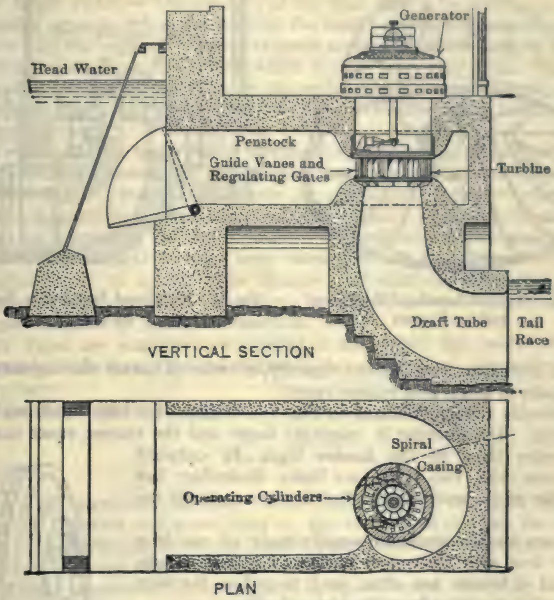

Fig. 1. Low-head Spiral-case Reaction Turbine

vanes and regulating gates. It passes inwards and downwards through the runner vanes to the draft tube.

Power is developed by the pressure and reaction of the water on the runner vanes, the runner being completely filled. Modern reaction turbines are adaptable to all heads up to about 800 feet where the units are large.

Impulse Turbines. — A typical impulse turbine and runner are indicated in Fig. 8. The pressure head is first converted into velocity through the nozzle at the end of the penstock, and power is developed by the impulse of the water jet acting upon the buckets on the rim of the runner.

The impulse turbine must never be submerged, it being necessary to allow the water to fall away freely from the buckets. Small impulse turbines are adaptable to all heads. For all capacities they are used exclusively for heads greater than about 800 feet.

CHARACTERISTICS OF REACTION TURBINES — The Runner. — Reaction runners vary in shape and design depending upon the conditions under which they are to be used. The runner indicated in Fig. 2 is designed for a low-head, large-capacity turbine. The diameter is made as small as

Fig. 2. Typical Low-head Reaction-turbine Runner

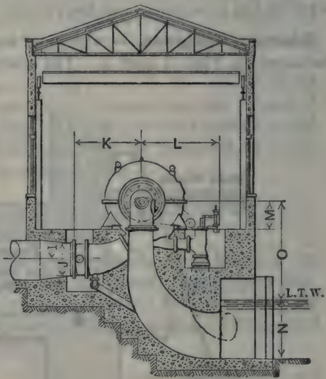

Fig. 3. High-head Spiral-case Reaction-turbine

possible, consistent with desired capacity, in order to obtain the highest speed. This results in large, long runner vanes.

The double runner indicated in Fig. 4 is designed for high heads and small discharges. The diameter is relatively larger and the runner vanes narrower and smaller than in the former type. In outward appearance the two types are quite dissimilar; but the action of the water and pathof flow are basically the same.

Runners for low heads are usually made of cast iron or cast iron with plate-steel blades. For high heads, cast steel or bronze are often used to reduce the wear of the metal resulting from the high velocities.

No uniformity exists among turbine manufacturers in the methods of measuring the diameters of reaction runners. For this reason it is never possible to compare the diameter of a runner with that of another make; or even with a runner of another type of the same manufacturer. Diameters are therefore employed only for comparison of runners of the same type and, as relative diameters only are used, it is never necessary to know the actual points to which the diameter is measured when selecting the type and size of runner for a given installation.

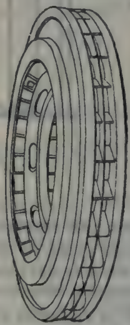

Fig. 4. Typical High-head Double-discharge Reaction-turbine Runner

The capacity of a runner is specified as a certain full gate capacity at a given head and speed.

The Gates. — In modern installation, wicket-gates are used because they present the least resistance to flow when in a part open position. Cylinder gates, consisting of a steel cylinder, moving axially over the opening to the

runner, are sometimes adopted for small installations and those designed to operate normally at full gate. Cylinder gates are not efficient when operated part open, owing to the sharp contraction at the bottom of the cylinder.

Wicket gates are actuated by the governor or hand control through rods and levers. The gates of the turbine indicated in Fig. 1 are controlled by two operating cylinders located in the turbine pit. Oil is admitted by the governor to the cylinders and the resulting movement of the piston rods revolves slightly a shifting ring to which are connected the individual gate-links for moving the gates.

The Draft Tube. — The function of the draft tube is to utilize, by suction or negative pressure below the turbine, the head between the turbine and the tail race. It also serves to reclaim as much as possible of the velocity head of the water issuing from the runner, which is sometimes very great. They are increased gradually in area towards the outlet where the average velocity is usually from 3 to 10 feet per second, the higher velocities being for the higher head installations.

The velocity of the water, when leaving the runner, is not strictly axial but, due to imperfect reaction and skin friction, has a swirling motion in the direction of the rotating runner. It is only the component of this velocity in the axial direction which can be reclaimed by the ordinary form of draft tube. It was for the purpose of reclaiming more of the residual velocity that a spiral type of draft tube has recently been developed. It is, however, adaptable economically only to special cases because of its relatively larger size.

Because of the draft tube, turbines can be set well above tail water in order to be convenient at all times for inspection.

Draft tubes are sometimes constructed of steel plate; but more often, for the larger installations, they are shaped in the concrete of the power house substructure.

The practical limit of the draft head is given by the equation:

$$h' = A + \frac{v_2^2 - v_1^2}{2g} - a \qquad (4)$$

where h' = the maximum allowable draft head in feet, measured from the lowest tail water level to the center line of gates, for vertical turbines; or to the highest point of the discharge space of the runner, for horizontal turbines.

A = the pressure of one atmosphere in feet of water, normally about 34 feet at sea level. It decreases with the altitude. Two feet should be subtracted from the normal weight to allow for reductions during meteorological disturbances.

v_1 = the average velocity of the water in feet per second at the exit of the runner, measured at the point of minimum internal diameter of the runner band.

v_2 = the average velocity at the end of the draft tube.

a = a margin allowed for disturbances during load changes and vapor tension, ordinarily adopted as five feet, but should be larger for long draft tubes, low pressure heads and sudden load changes.

Friction in the draft tube will increase h', but is too small for consideration.

Settings. — Reaction turbines are adaptable to many types of settings, and the choice of type is governed by the relative cost compared with the efficiency of the turbine and ease of operation. Typical examples of turbine settings are indicated in Figs. 1, 3, 5, 6 and 7. The following are some of the more common types of settings, of which many combinations are possible.

a. The turbines may be horizontal or vertical; that is, set with horizontal or vertical shaft.

b. One or more runners on the same shaft may be used.

c. For multiple runner turbines, there may be a separate draft tube for each runner, or each pair of runners may have a single draft tube located between them.

d. The turbines may be set in an open forebay or flume with shaft extending vertically to a generator above water surface, or horizontally through the wall to a generator on the opposite side.

e. The runners may be enclosed in a casing or prolongation of the pipe line. Such casings may be spiral or cylindrically shaped.

The tendency is towards single runner vertical turbines for low heads, and horizontal or vertical turbines with one or two runners for medium and high heads. More than one runner is seldom now used for vertical installations.

The open flume setting is common for very low-head installations utilizing small turbines. Such a setting is indicated in Figs. 5 and 6. In open flumes of ample size, the water approaches the turbine with low velocity and without disturbance. Consequently this type of setting possesses the highest degree of efficiency.

As the head becomes higher or the diameter of the curbine larger, the structural problems of open-flume construction become increasingly difficult. It is finally necessary to effect a reduction in size of conduit and a corresponding increase in velocity of flow, and some type of closed casing is then necessary.

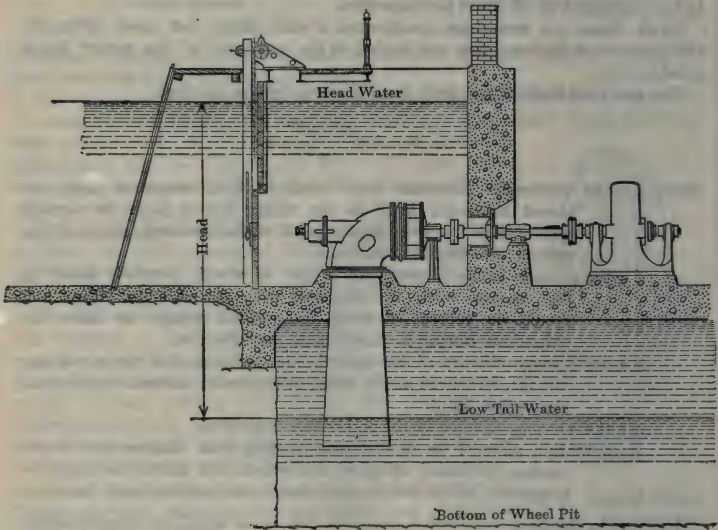

Fig. 5. Open-flume Setting for Horizontal Turbine

A cylindrical casing is indicated in Fig. 7. This type of casing cannot be made hydraulically efficient. Because of its shape, the water cannot enter the turbine with the absence of disturbance, so essential to best efficiency of operation. Cylindrical casings are less expensive than spiral casings. but are seldom

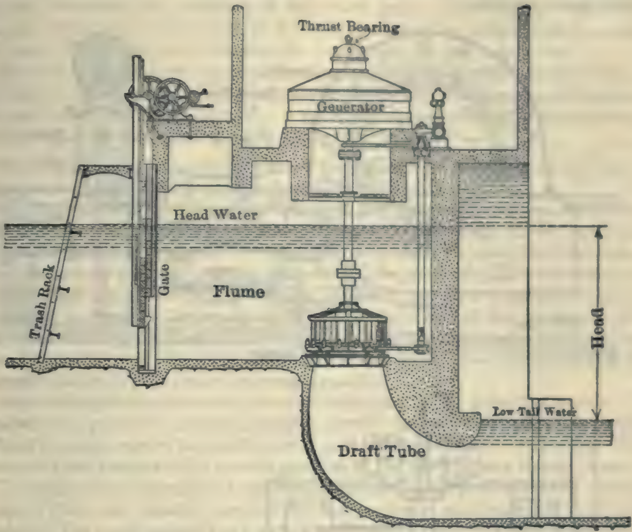

Fig. 6. Open-flume Setting for Vertical Turbine

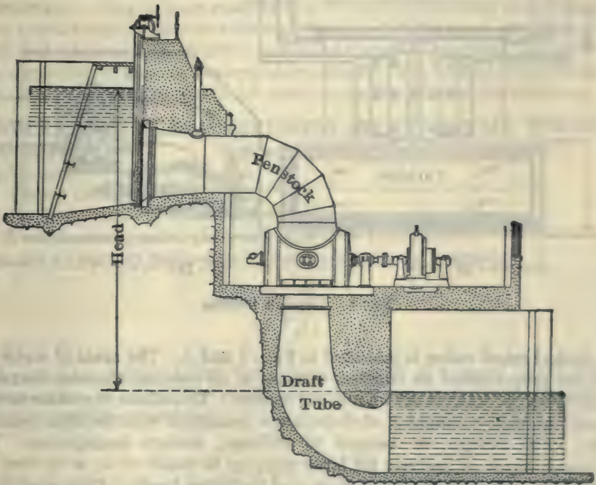

Fig. 7. Horizontal Cylinder-case Double-runner Setting

now used for large units, except to effect a reduction in first cost at the expense of efficiency. They are usually made of plate steel or formed in the concrete of the toe of the dam.

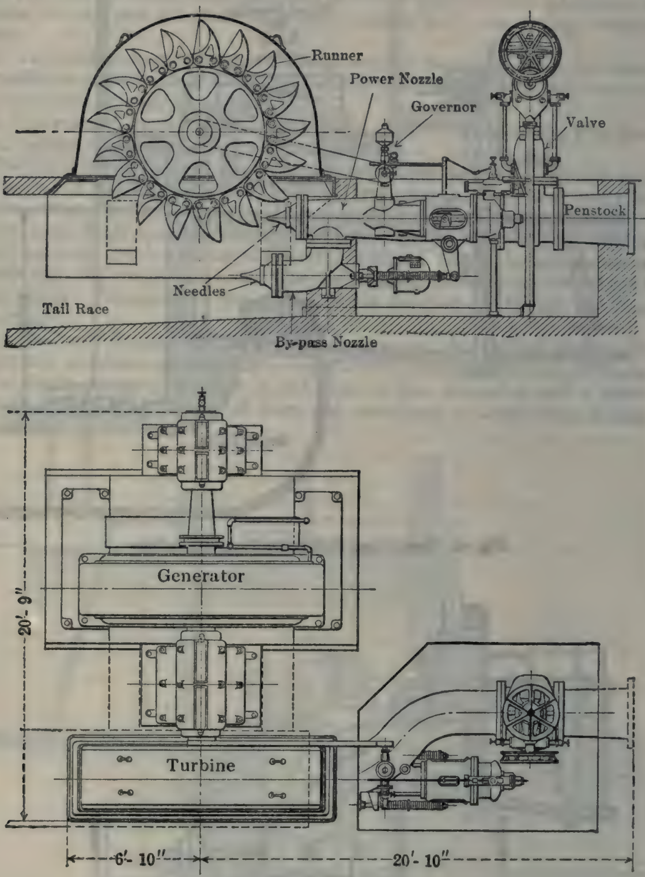

Fig. 8. Impulse Turbine

A typical spiral casing is indicated in Figs. 1 and 3. The areas of water passages are proportioned for constant velocity at all sections, the area decreasing gradually as the water is drawn into the runner. This allows the water to enter the turbine with a minimum of disturbance consistent with the high velocities used. Spiral casings are common to all large, modern installations. They are usually made of concrete for large units up to heads of 60 or 80 feet, and of cast iron, cast steel or riveted plate steel for higher heads. Plate steel is considered inferior to the other materials for casings, but is often less expensive.

CHARACTERISTICS OF IMPULSE TURBINES. — The Runner. — In Fig. 8 is shown a typical impulse runner, consisting of cast-steel or bronze buckets bolted to the periphery of the wheel body. The buckets receive the impulse from the jet and destroys its velocity, the water falling inertly to the tail race. The total head between the runner and the tail race is lost.

For each turbine there is one ratio of runner diameter to jet diameter which results in best efficiency. This ratio varies from about 12 to 15 depending upon the head and capacity of the unit. A ratio as low as 8 is practical under special conditions, with a sacrifice of a few per cent in efficiency.

The Nozzles. — The flow of water through the nozzle is controlled by a governor actuated needle valve as indicated in Fig. 8.

Impulse turbines are commonly used for high-head installations with long penstocks in which the flow cannot be arrested quickly when a sudden reduction in turbine output is required. For this reason deflecting nozzles, by-pass valves or jet deflectors are used in order to eliminate the possibility of water-hammer.

Deflecting nozzles are provided with a ball joint, and when a sudden reduction in output is required, are made to deflect by the action of the governor, with the result that only part of the jet remains in contact with the buckets and the power output is reduced. The needle valve then partly closes slowly, and coincidently the jet is returned to its normal position.

A jet deflector acts on much the same principle as the deflecting nozzle. It consists of a governor-actuated, hinged, curved vane which, upon a reduction in load demand, moves into and deflects part of the jet from the buckets. The deflector then returns to its normal position as the needle valve slowly closes.

The most recent method of regulation consists of by-pass valves of various types. They are usually operated by the governor synchronously with the closing of the needle valve to an extent sufficient to retain a constant penstock velocity, then are slowly closed automatically. Fig. 8 indicates a typical by-pass valve of the needle type.

Settings. — One or more runners may be used on the same shaft and one or more nozzles may be provided for each runner; but the usual arrangement, for small and medium capacity units, consists of a single runner with a single nozzle. Impulse turbines may be horizontal or vertical; but vertical installations are rare, because the water cannot fall away as readily without interference.

OPERATING CHARACTERISTICS OF HYDRAULIC TURBINES. — Unit Speed.

Let R = speed of runner, in r.p.m.
h = net effective head, in feet.
P = theoretical horsepower developed.
K = a constant, called the "unit speed" of the runner.

Then for a **reaction turbine** at a given speed and gate opening,

$$\frac{R\sqrt{P}}{h\sqrt{h^{1/2}}} = K \qquad (5)$$

The reason for calling K the "unit speed" is that K is equal to the speed R corresponding to a head h of 1 foot and a power output P of 1 horsepower.

For an **impulse turbine,**

$$\frac{R\sqrt{P}}{h\sqrt{h^{1/2}}} = 3.67, \qquad (5a)$$

which indicates the usual unit speed of impulse runners to be about 3.67.

Values of Unit Speed. — In equation (5*a*), if h and P are unity, $R = K$, the speed of the runner when generating. Turbine builders use the value of the unit speed at full gate opening as a basis for representing the speed characteristics of the various types of runners. For each type of runner there is one unit speed near which occur the best efficiencies for ordinary conditions of operations. High-speed runners are those requiring a high full-gate unit speed for best efficiencies and vice versa. The practical maximum unit speed for the standard modern reaction-type runner, without considerable sacrifice of efficiency, is about 95. Special runners have been developed recently, both in this country and abroad, which are claimed to operate satisfactorily at a unit speed of 150 or over, and several small units have been installed. With unit speeds greater than 95, considerable saving can be made in the cost of generators. because a large single runner unit operating at 95 unit speed under a low head requires a speed of revolution much too low for economical generator design.

Efficiency. — The methods of calculating, from test data, the performance of an hydraulic turbine are too complicated to be given in the limited space here

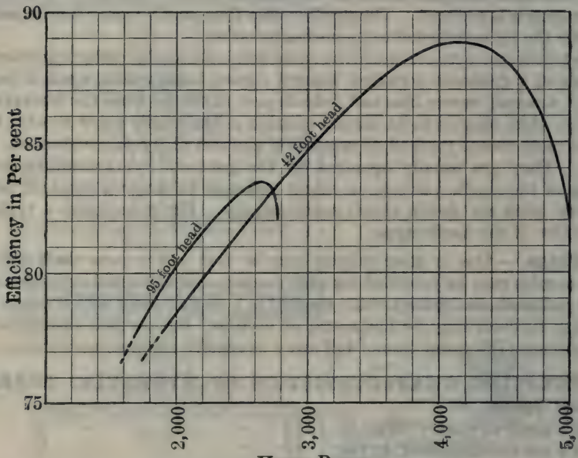

Fig. 9. Efficiency-power Curves of 46 In. Runner

available. Typical efficiency curves of a large reaction-type runner at two different heads are given in Fig. 9, and a typical efficiency curve for an impulse-type runner in Fig. 10.

It will be noted that the efficiency and power falls off quite rapidly as the head reduces. It is not possible to obtain a runner which will have good efficiencies for a wide range of heads for a constant speed.

The use of test data for the determination of probable turbine operating characteristics under practical conditions is influenced by the following considerations:

a. Other conditions being the same, the efficiency and capacity of a runner increases with its size, owing principally to the smaller influence of defects in fabrication and roughness of the water passages.

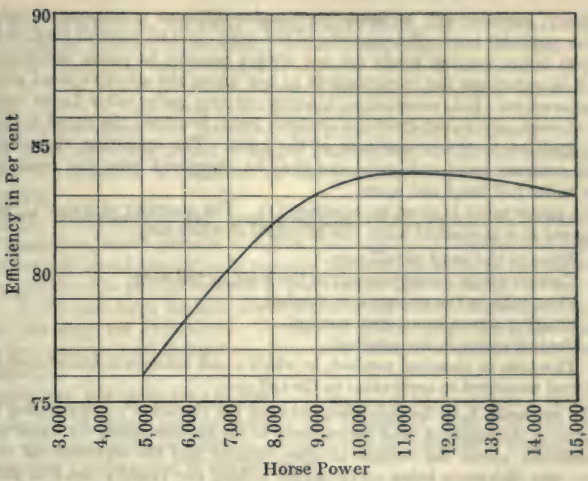

Fig. 10. Efficiency-power Curves of an Impulse Water Wheel

b. The setting of the turbine may differ from that of the test, a condition which often has a marked influence on efficiency and power.

c. The efficiency and capacity decrease with service, particularly if there is much silt in the water to wear the runner and increase clearances.

d. Turbine manufacturers have patterns for only a part of the possible runner sizes. Moreover the best speed of the runner is not always the best speed for the least expensive type of generator. Consequently, in many installations, it is advisable to sacrifice a certain amount of adaptability and efficiency in order to reduce the first cost of the installation.

Factors Affecting Choice of Runner. — Complete data for the final determination of the type and size of runner most suited for a particular installation are in the possession of turbine builders only, and advice from that source should invariably be obtained.

A type of runner and turbine setting should be adopted which will have the best characteristics for the conditions under which it must be operated for the greater portion of the time, and also possess reasonable characteristics for unusual conditions. Among the influences governing the choice of type are:

a. Period of low-stream flow, during which maximum efficiency is essential.

b. Period of high-stream flow, during which maximum capacity only is essential.

c. Range of fluctuation in head. The runner must have sufficient capacity at minimum head and also have reasonable efficiencies at mean and high heads.

d. Load characteristics, or the part-gate-opening periods. The efficiency drops off quite rapidly for small gate openings and best efficiency is not usually at full gate power.

e. Local conditions affecting the type of turbine setting.

TESTING OF HYDRAULIC TURBINES. — The "Testing Code" of the Machinery Builders' Society has been adopted by most hydraulic turbine manufacturers as a basis for comparing actual operating characteristics with the guarantees of the manufacturer. The object of the test is to determine the

efficiency, equation (3), at fixed speed under different conditions of load and (where possible) head; the capacity of the unit; and its general performance.

Tests under varying heads are not always possible in power developments, and it is sometimes not even possible to test under the effective head stated in the guarantee. It is permissible, however, to test under other heads, differing by not over 10 per cent from the contract head, provided the speed is changed in proportion to the square root of the head. The operating characteristics under contract head and speed can then be computed from the result of the tests under test head and speed.

The power input is given by equation (2). The quantity of water Q, may be measured in a number of ways, the principal of which are,

a. Weirs in the head or tail race.

b. Current meter measurements in the head or tail race.

c. Pitot tube in the penstock or conduit.

d. Floating screen in an open conduit.

e. Venturi meter in the penstock.

f. Titration or chemical method, in which a salt solution is introduced at the inlet and measured in centration in the tail race.

The net head, h, is the difference between the elevation corresponding to the pressure in the penstock near the entrance to the turbine casing (or water surface immediately above the turbine in open flumes) and the elevation of the tail water, this difference being corrected by adding the velocity head in the penstock at the point of measurement and subtracting the residual velocity head at the end of the draft tube.

The output, P_0, is usually measured electrically if the turbine-driven generator has been previously tested. One of the many forms of Prony brakes may be used for small units.

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WATER WHEELS, SPEED REGULATION OF.* — (*See also Hydraulics; Power Stations, Hydroelectric; Water Wheels and Their Settings.*) In a few special cases close speed regulation of water wheels is unnecessary, for example, in pulp grinding, certain chemical processes and operating air compressors. Even in these cases experience has shown that better results may be obtained when wheels are controlled by properly designed governors. For factory work, where the power is applied through the medium of belting, the problem of regulation is simplified by the large inertia effect of the pulleys and belting.

SPEED VARIATION IN HYDROELECTRIC PLANTS. — For electrical generators, which perhaps comprise the majority of installations, the utmost nicety of speed regulation is required because the value of the electrical energy produced is greatly lessened if the voltage and frequency are not uniform. In the best large modern power plants speed variations greater than one or two per cent are exceptional. In smaller plants, since there are usually larger proportional load fluctuations, it is more difficult to maintain close speed regulation.

METHOD OF REGULATION. — A change of load requires a change in the quantity of water passing through the turbine in order that the power output will equal the new demand. This is effected through a change in the gate opening by the action of the governor.

If a change in load demand occurs, and the turbine is not able at once to alter its output to correspond, the surplus or deficiency of power will be absorbed or supplied by the inertia of the rotating elements of the turbine and generator. These can give up or receive energy only by means of a change in speed. Thus the first manifestation of a change in load demand is a change in the speed of the unit.

Governors are sensitive to very slight changes of speed and start almost at once to adjust the gate opening; but this cannot be done immediately. Consequently, during the time required to make the adjustment, the change in the inertia of the rotating elements, corresponding to the surplus or deficiency of power, must be made without too great a change in speed. Therefore the necessary inertia of the rotating elements or "fly wheel effect" is fixed by the capacity of the unit, the expected load change and the gate-closing speed of the governor.

The influences to be overcome in obtaining good regulation are:

a. Lack of sensitiveness of the governor, or the fact that the speed must depart from normal before the governor action starts.

b. The governor cannot change the gate opening instantly. A common speed for open flume settings or short penstocks is about two seconds. For plants with long penstocks, the speed of the governor must be much slower to guard against excessive water hammer.

c. In the case of long penstocks, a change in penstock velocity, resulting from a load change, necessitates a variation in head at the turbine to accelerate or retard the velocity. This change in head is exactly opposed to that desired for good regulation.

Open Flume Setting. — In Fig. 1, assume the power demand, for an open flume installation to be represented by the line *ABCD*. After the instant, *X*, of the load change, the output of the unit is insufficient and the deficiency must be supplied by the inertia of the rotating elements. Consequently the speed begins to fall off as indicated. At *Y* the speed change is sufficient to affect the governor and the gates begin to open. The rate of gate opening is not necessarily constant, but will vary according to the characteristics of the governor as with open-flume conditions, the head is practically constant during the change of output, the power output may be represented by the line *EFG*, which, for open

flumes, may be assumed to vary uniformly. The area $BCFE$ represents the total deficiency in energy, ΔK, to be supplied by the fly-wheel effect of the unit. At Z the output is equal to the demand and, as indicated, no further reduction in speed occurs. As the speed is still below normal, the governor will continue to open the gates and the power will further increase. The excess output over demand, after time Z, will add to the speed of the unit until, at time W, the speed is again normal but the output is in excess of the demand. The speed will then rise above normal and the governor will start closing the gates. There is seen to be a tendency for the output to fluctuate above and below the new power demand. This feature is called "hunting" of the governor. In actual practice this does not occur to an objectionable extent, as the governors are provided with a compensating device or "anti-racing" mechanism which has the effect of dampening these fluctuations, and the speed is soon brought permanently to normal. This condition requires, however, that the capacity of the unit be somewhat in excess (usually 5 per cent to 10 per cent) of the maximum output to which the demand may suddenly change, otherwise there would not be available the necessary excess capacity to bring the speed back to normal after a sudden increase in demand to maximum demand occurs.

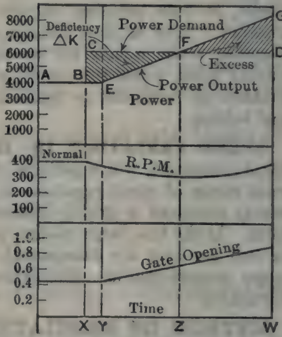

Fig. 1

Long Penstock Settings. — In Fig. 2, assume the power demand for a plant with a long penstock to be represented by the line $ABCD$. As in the previous case, the speed begins to be retarded at X, the moment of the load change, and at Y the governor action starts. The variations of gate opening and penstock velocities are indicated in the diagram. Part of the total head available for power is consumed in increasing the penstock velocity and the net head on the turbine decreases momentarily as indicated. The net result of the slowly accelerating velocity, combined with considerable reduction in net head, may be a momentary *reduction* in output as indicated by the output line. This reduction in net head, at the critical time when an *increase* in power output is most needed, is the chief influence opposed to good regulation with long penstocks. The velocity soon picks up, however, and the power output increases about as indicated by the line EFG.

As in the case of open-flume installations, the speed of the unit is a minimum at Z, when the output equals the demand; but the gate opening, penstock velocity and output continue to increase until the speed reaches normal at time W. The gates then start to close. The time interval between the sharp breaks in the head, penstock velocity, and output lines correspond to the time required for the wave of penstock velocity change to travel from the turbine to the upper end of the penstock and back again.* The deficiency in output ΔK, is indicated by the area $BCFE$.

For sudden *decreases* in load demand the characteristics indicated in Figs. 1 and 2 are simply reversed.

Water Hammer. —Water hammer is a term used to indicate the increase

* See article by N. R. Gibson, referred to in *Bibliography*.

in head caused by a reduction in penstock velocity. Negative water hammer is the decrease in head resulting from an increase in penstock velocity. For sudden load changes the resulting water hammer is very great and, for that reason, the speed of gate movement must be very slow for long penstocks.

It is not safe to use approximate equations for the determination of water hammer. Accurate methods are too intricate for presentation here and the reader is referred to the article by N. R. Gibson mentioned in *Bibliography*.

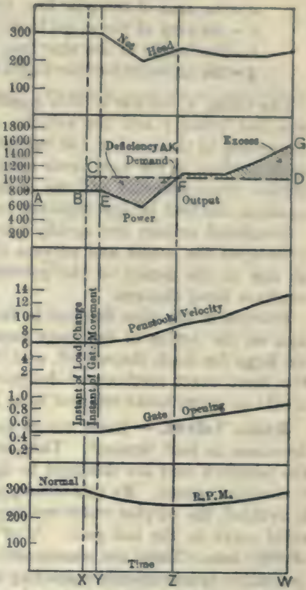

Fig. 2

REGULATION APPLIANCES.—The term "pipe line" is commonly used to indicate a long pipe extending from the reservoir to the brow of the incline leading to the power house. The "penstock" extends from this point to the turbines.

Surge tanks are usually provided at the lower end of long pipe lines in order to restrict the excessive water hammer, or head changes due to sudden load changes. Relief valves are often used at the lower end of long penstocks for the same purpose.

Surge Tanks.—A surge tank consists of a vertical pipe, considerably larger in diameter than the pipe line, resting on the ground and connected to the pipe line, or it may be an elevated tank connected to the pipe line by a riser pipe of the same or smaller diameter than the pipe line. Open fore-bays also serve the same purpose as the surge tank. Surge tanks are feasible only when the difference in level between the lower end of the pipe line and head water is not excessive.

The function of the surge tank is to store or supply water subsequent to a load change pending the gradual adjustment of the flow in the pipe line.

The water surface in the surge tank is normally at the level of the hydraulic gradient of the pipe line at that point. When a sudden demand for more power occurs the deficiency of water is supplied by the tank and the water surface in the tank lowers. The resultant increase in head between the tank and the forebay serves to accelerate the velocity in the pipe line. When a reduction in output occurs, the water surface in the tank rises and the induced head decreases the flow. The action is entirely automatic.

The influence of the rise and fall of water surface in the surge tank on the net head at the turbine during a load change is also opposed to speed regulation; but the head change with a surge tank is considerably less than corresponding water hammer without the tank.

Neglecting the effect of friction, the rise or fall of water surface in the tank may be expressed by the following equation:

$$h = v\sqrt{\frac{aL}{Ag}},$$

where h = the rise or fall, in feet.

v = the velocity change in the pipe line corresponding to the change of load, in feet per second.

a = the area of the pipe line, in square feet.

A = the area of the surge tank, in square feet.

L = the length of the pipe line, in feet.

g = the acceleration due to gravity = 32.16.

The value, v, for use in this equation, is the difference between the original pipe line velocity and the velocity required for the new load, the latter corresponding to the head on the turbine at the time of the maximum deviation of water surface in the tank from normal. Its value therefore depends upon the turbine characteristics and the value of h. Consequently the solution of the equation can best be accomplished by the cut-and-try method.

Restricted orifices are usually provided between the pipe line and the tank to dampen the surges and prevent them from continuing indefinitely or piling up due to the action of the governor in endeavoring to compensate for the resultant head changes.

Friction in the pipe line and in the restricted orifices have considerable influence on the action of the surge tank and the foregoing equation gives results too large for loads thrown off, and considerably too small for loads thrown on. For more exact methods of design the reader is referred to R. D. Johnson's article on surge tanks referred to in *Bibliography*.

Relief Valves. — Relief valves are serviceable only to compensate for sudden decreases in load demand. They are appliances attached to the lower end of the penstock and are designed to open automatically when a sudden decrease in demand occurs. By this means, a sudden large decrease in penstock velocity is avoided, the surplus water not needed for power being by-passed through the relief valve to the tail race. After the output has been adjusted, the valves slowly close. They are sometimes actuated by the rise in pressure; but are preferably synchronously opened by the governor to an extent corresponding to the closure of the turbine gates, the penstock velocity remaining nearly constant during the load change. Relief valves are very wasteful of water. Deflecting nozzles and jet deflectors of impulse turbines, described in *Water Wheels and their Settings*, also serve the same purpose.

CALCULATION OF SPEED REGULATION. — Open Flume Settings. —In Fig. 1, let:

p_0 = the initial horsepower output.

p_1 = the new power demand after time X.

T_1 = the time interval, in seconds, XY, after load change before the governor action starts.

T_2 = the speed of the governor or time, in seconds, YZ, for the governor to adjust the gates to the new load demand.

R_0 = the normal speed of the unit in revolutions per minute.

R_1 = the speed of the unit at time Z corresponding to maximum deviation from normal.

s = the percentage change in speed of the unit.

K = the foot-pounds of kinetic energy in the rotating elements at speed R_0.

ΔK = the energy change in the rotating elements due to change in speed from R_0 to R_1.

I = the fly-wheel effect of the rotating elements in foot-pound units, usually expressed in pounds at one foot radius, i.e., the weight which, at a radius of one foot, would have an equivalent moment of inertia.

N = the regulation constant of the unit.

For open flume settings, the average horsepower deficiency during the time interval XZ, assuming EF to be a straight line, is,

$$p = \frac{(p_1 - p_0)T_1 + (p_1 - p_0)\dfrac{T_2}{2}}{T_1 + T_2}.$$

The total deficiency in work for the interval XZ, which must be supplied from the inertia of the rotating elements, is:

$$\Delta K = 550(T_1 + T_2)p.$$

Therefore,
$$\Delta K = 275(p_1 - p_0)(2T_1 + T_2) \qquad (1)$$

From the laws of mechanics,

$$K = \frac{2\,I\pi^2 R^2}{g\,60^2} = 0.00017\,IR^2,$$

and
$$\Delta K = 0.00017\,I(R_1^2 - R_0^2).$$

Therefore,
$$R_1 = \sqrt{R_0^2 + \frac{\Delta K}{0.00017\,I}}.$$

From definition,
$$s = \frac{100\,(R_1 - R_0)}{R_0}.$$

Therefore,
$$s = \frac{\sqrt{R_0^2 + \dfrac{\Delta K}{0.00017\,I}} - R_0}{0.01 R_0}. \qquad (2)$$

By neglecting T_1, which is small, Meade has shown that equation (2) may be written, for small speed changes, in the following approximate form:

$$s = 81{,}000{,}000\,\frac{T_2}{IR_0^2}(p_1 - p_0) \qquad (3)$$

For full load change, equation (3) may be written,

$$I = \frac{81{,}000{,}000\,T_2 p_F}{S R_0^2} = N\frac{p_F}{R_0^2}.$$

where
$$N = \frac{81{,}000{,}000\,T_2}{S}.$$

Therefore,
$$N = \frac{R_0^2 I}{p_F}. \qquad (4)$$

For open-flume plants or plants with negligible length of penstock as indicated by Fig. 1, of the article on *Water Wheels and Their Setting*, I usually has a value such that N will be at least 5,000,000.

A quick acting governor and a large fly-wheel effect make for good regulation. If the fly-wheel effect is too small for the required regulation, the speed of the governor must be increased or additional fly-wheel effect provided either in the form of a separate fly-wheel or by increasing the weight of the rotating parts of the generator. Either method requires additional expense and the choice should be made to obtain the greatest economy.

Plants with Long Penstocks. — For plants with long penstocks, the output line, BEF, Fig. 2, and hence the deficiency of output, ΔK, cannot be

expressed by a simple equation. The fluctuations in head and penstock velocity are affected by the considerations of water hammer and stand pipe or relief valve characteristics. Each must be computed and plotted, from which the power output line can be determined from the turbine characteristics. The area *BEFC* or deficient work, ΔK, can then be computed for use in equation (2).

HYDRAULIC-TURBINE GOVERNORS. — The governor proper is the device for automatically opening and closing the gates as the load on the turbine varies. The requirements to be met are (1) sensitiveness to small variations in speed, (2) quickness in action, (3) steadiness and accuracy in moving the gates the proper amount, (4) ability to operate continuously with minimum amount of attention.

Types of Governors. — There are two general classes of governors: (1) the "mechanical" type, which takes energy from the turbine itself, as required, through a belt or gearing, and by means of friction clutches applies this energy at suitable times to the operation of the turbine gates; (2) the "hydraulic" type, in which stored energy in the form of compressed air is made available when required to move the turbine gates through the medium of a liquid (usually oil) acting against the piston in a hydraulic cylinder. The energy in the compressed air is thus transmitted by the liquid through a suitably controlled valve to do the work of moving the turbine gates, when necessary, and the energy of the air is afterwards restored by pumping the same liquid back into the air tank. The operation of restoring the energy by pumping is gradual; while the operation of moving the turbine gates, by utilizing the energy of the compressed air, may be as rapid as desired.

Governors of the mechanical type were the first in the field, and, being similar and cheaper to construct, have been very widely used. They have certain objectionable features, however, which have prevented them from being adopted under any conditions which require extreme nicety in speed regulation. Consequently, governors of the hydraulic type are almost exclusively used in large and important power plants. When properly designed, they are capable of giving extremely accurate speed regulation and are thoroughly reliable. Such governors are sometimes operated continuously night and day for weeks, or even months.

Connections between Turbine and Governor. — The connections between a hydraulic turbine governor and the gates must be strong, direct and free from lost motion; otherwise the governor cannot perform its functions properly. Modern governors are built in many different types, so as to be readily adapted for connections to any standard make of turbine. It is generally advantageous to have the hydraulic cylinder of the governor located near the turbine gates, although for very large units the control mechanism of the governor may be more conveniently placed on the floor above. For small turbines, which are occasionally located in comparatively inaccessible places, it is sometimes necessary to place the governors at a considerable distance and connect them with the turbine gates by means of rods and levers. Occasionally several turbines are connected in this manner to a single governor.

Distant Control. — Large governors are frequently provided with suitable electric devices by means of which the speed of the wheel may be adjusted from any distant point, thus permitting alternators driven by independent turbines being brought to the same speed and angular position for parallel connection.

Rating of a Governor. — A hydraulic governor is usually rated in terms of the energy, expressed in foot-pounds, which is developed by one stroke of the piston, and the time, in seconds, required for a complete stroke (i.e., to completely close or open the gates). The size of governor required for a given size

of turbine depends so largely upon the type of gate, head, etc., that no general rules can be given here.

Specifications for Governor. — The following is taken from the standard specifications of one of the large governor manufacturers:

The governors are each guaranteed to develop the energy, expressed in foot-pounds stated below, and if not called upon to develop a greater amount of energy than that named, will completely open or close the gates to which they are attached in the following number of seconds:

Type,........Governor,........Foot-Pounds,........Seconds,.........

Said governors are further guaranteed: (*a*) to stand substantially steady when the speed does not vary; (*b*) to be dead beat in action and not to hunt; (*c*) to correct with maximum promptness for all load changes within the capacity of the turbines to which they are attached; (*d*) to maintain the speed steady within half of one per cent under uniform load upon the turbines to which they are attached; (*e*) to begin to adjust the turbine gates when the speed has varied half of one per cent; (*f*) after sudden decrease of load to bring the speed back to normal in......seconds; (*g*) to operate perfectly in parallel with other governors of the same make.

BIBLIOGRAPHY. — See the *Bibliography* in the article on *Water Wheels and Their Settings.* In addition; Gibson, N. R., *Pressures in Penstock Caused by the Gradual Closing of Turbine Gates*, Trans. A.S.C.E., 1920; Vensano, H.C., *Pulsations in Pipe Lines as Shown by Recent Tests*, Trans. A.S.C.E., 1918; Johnson, R. D., *The Differential Surge Tank*, A.S.C.E., 1915; Lauchli, E., *Tests Check Computed Values of Surge*, Eng. Rec., March, 1915; Hillburg, A. G., *Surge Tank Problems Solved by New Methods*, Eng. Rec., Dec., 1916.

WATTHOUR METERS. — (*See also Ampere-hour Meters; Electrodynamometers; Wattmeters.*) A watthour meter consists essentially of (1) a small electric motor, which may be either of the commutator type, mercury and disc type, or induction type, (2) a brake system composed of a disc of non-magnetic material (usually copper or aluminum) mounted on the armature spindle and so arranged that its edge rotates between the poles of one or more permanent magnets, and (3) a system of gears with numbered dials forming a suitable registering mechanism for indicating the number of revolutions of the armature or disc. One winding, called the "potential" coil, is connected across * (in shunt with) the load and the other winding, called the "current" coil, is connected in series * with the load, the connections being the same as for an indicating wattmeter; see *Wattmeters*.

Principle of Operation. — Motor-type meters are so constructed that the average torque exerted by the motor is proportional to the average power taken by the load. The brake system is so designed that the opposing torque, due to the eddy or Foucault currents induced in the disc as it rotates between the poles of the permanent magnets, is proportional to the speed of the disc. When the disc acquires a given speed the driving torque must be just equal to the opposing torque, and must therefore be proportional to the speed. Hence the speed of the disc is proportional to the average power, and therefore the total number of revolutions which the disc makes during any interval must be proportional to the total energy input during this interval, whether the power remains constant or varies. To determine the energy input to the load in watt-hours or kilowatt-hours it is therefore only necessary to take the difference between the dial readings at the beginning and end of the given interval, and multiply by the proper constant if the meter is not direct reading.

Sources of Error. — In a d-c. watthour meter the chief sources of error are the friction of the brushes (which is more or less variable), friction of the bearings of the motor and gear train, and air friction, the brush friction being by far the most important. In a-c. watthour meters, the lack of exact 90° phase relation between the impressed voltage and the magnetic flux due to the current in the potential coil may cause additional errors which vary with the frequency, power factor and also with the distortion of the wave form; in well-designed modern meters these errors are practically negligible. Instrument transformers (*see Transformers, Instrument*), introduce additional sources of error which may be undesirably large if the transformers are not properly designed. Devices and methods for overcoming, or of correcting for, these errors are described below.

Classification According to Service. — Watthour meters may be classified according to the service in which they are used, viz., (1) service meters for use in residences or factories, (2) switchboard meters for use in central stations, and (3) meters for use in individual power installations and isolated plants. Meters for house service are generally front-connected, separately-sealed devices which can be installed and sealed to prevent tampering. Switchboard meters are generally back connected and of such design as to match other switchboard devices. "Meter-boards," equipped with the necessary auxiliary or protective devices, are frequently used with house meters, in order to insure proper connections and sealing.

COMMUTATOR TYPE OF WATTHOUR METER. — The commutator type of meter is now used only on direct-current circuits, although before

* Either directly or through suitable instrument transformers; see *Transformers, Instrument.*

the induction meter was so highly perfected large numbers were used for alternating current also. The motor consists of a set of stationary coils, commonly called field coils, which carry the current, and an armature wound with small wire which is connected across the terminals of the load or in shunt with the supply circuit; generally a series resistance is used to absorb part of the line voltage. The connection to the armature is by a commutator and brushes, as in an ordinary commutator type of shunt motor. Special attachments are sometimes added for very large capacities, such as double armatures astatically arranged, and damping magnets inclosed in a laminated iron shield in order to reduce stray-field errors where heavy currents are used and heavy short-circuits are frequent.

It should be noted that the operation differs from that of an ordinary shunt motor in that the speed increases with increase of field strength, since its back e.m.f. is much less than 50 per cent of the line voltage.

On account of the inductance of the windings of the commutator meter, it does not record accurately on a-c. circuits unless properly compensated, the percentage error being greater the less the power factor.

Compensation for Friction; Light-load Adjustment. — Friction in a well-designed watthour meter will be very small and will be noticeable only on light loads of 10 per cent and below. To compensate for this friction, a "light-load adjusting coil" is added, which is an auxiliary or compounding coil. This light-load coil is connected in series with the armature. It is placed adjacent to the field coils so that its field strengthens the main field and produces a slight torque independent of the power and just sufficient to compensate for friction.

Details of Construction, Bearings, etc. — In the design of the modern direct-current commutator type of watthour meter, great thought has been given to the mechanical construction in order to obtain small air gaps between the armature and field coils, light-weight armatures and commutators and brushes which will have very small friction and yet stand the wear and carry the current to the armature without undue sparking and pitting of the contact parts. In a watthour meter of the most approved design the field coils are wound with enameled copper strips and the armature winding is enamel-covered wire wound on a paper sphere mounted on a spindle of light steel tubing. The lower bearing carries all of the weight and the upper bearing is a guide bearing, as all meters have vertical shafts. The lower bearing consists of a steel pivot mounted in the end of the spindle or armature shaft, running on a jewel bearing. In most cases a good grade of sapphire is used for the jewel and is cupped and polished so as to provide a bearing with as little friction as possible. Of late years a great deal of attention has been given to the bearings and in some cases, particularly on high-capacity meters with astatic armatures and a correspondingly heavy weight, diamond jewels are being used with excellent results The diamond jewel will last much longer than the sapphire before wearing sufficiently to increase the friction and thereby change the accuracy of the meter, particularly at light loads. See also section below on *Data on House-Type Meters*.

Capacity of Commutator Watthour Meters. — Commutator-type direct-current meters are built in capacities ranging from 5 to 10,000 amperes, 100 to 600 volts. Meters of this type are furnished with double current circuits for three-wire circuits, the maximum ampere capacity for three-wire meters being about 6000 amperes. Special meters have been supplied for 1200- and 2400-volt railway circuits.

Accuracy. — Tests on direct-current commutator meters show that a meter of good design should start on two per cent of full load and should give accu-

racies about as follows: — To within 3½ per cent from 5 per cent to ¼ load, 2 per cent from ¼ to full load. The Meter Code (*see Bibliography*) requires d-c. watthour meters to run continuously on normal voltage and 2 per cent of rated current, and for higher loads to be accurate within the following limits:

Per cent rated current	5	10	20	50	100	150
Maximum deviation, per cent	7.5	3.0	2.0	2.0	2.0	2.0

For a summary of the meter accuracies required by commission regulations, city ordinances, and state statutes, see Circular No. 56 of the Bureau of Standards, 1st ed., pages 234–5.

The Mercury-motor Watthour meter is manufactured for direct-current service. The motor element consists of a mercury chamber in which is submerged a copper armature. The line current enters and leaves the mercury through two electrodes diametrically apart. Since mercury has about fifty-five times the resistivity of copper, the major part of the current traverses the armature. The mercury thus performs the function of the commutator and brushes of the commutator-type meter. A shaft attached to the armature passes through tubes constructed to prevent the mercury from escaping when the meter is inverted. Externally, this shaft carries a worm which engages with the register train, and an aluminum damping disc moving in the field of a permanent magnet. The copper armature is notched radially to direct the current flow in such a manner as to obtain the greatest torque, and carries a float so proportioned that there is a slight upward thrust. In consequence, the lower bearing is a ring-stone jewel, while the upper end of the shaft is carried in a ring-stone end-stone bearing. A laminated iron electromagnet having many turns of fine wire is connected across the line, and sends through the copper armature a magnetic flux which varies in proportion to the line voltage. The driving torque is thus proportional to the power supplied to the load. The damping at small and moderate loads is nearly all caused by the drag disc, but at full load and overloads the fluid friction of the mercury increases at such a rate as to cause a drop in the per cent registration curve unless compensated. Compensation is effected by passing the line current through a few turns of heavy wire around the voltage electromagnet.

Compensation for Friction, Light-load Adjustment. — For effecting the light-load compensation for friction, a thermal device is used in some cases, and it is also possible to obtain compensation by shunting part of the potential current through the armature. The thermal device employed is a thermocouple shunted around the mercury chamber, and heated from a resistance coil connected in series with the potential circuit. The small electromotive force generated in the thermocouple sends a small current through the armature which is sufficient to overcome the friction of the moving parts.

Application of Mercury Meters. — Mercury-motor meters, on account of the low resistance of the current circuit, are particularly well adapted for use with external shunts, since but a small potential drop in the shunt is required. They are therefore used in comparatively large numbers for switchboard work where large currents are used and where external shunts can be conveniently installed. They are much less subject to derangement by vibration than commutator meters.

Details of Construction. — The preceding description applies to the mercury motor meter extensively used in the United States. Other designs differ in mechanical construction and in methods for obtaining light-load adjustments. The chief difference in mechanical construction is in the motor element, where a mercury chamber of non-metallic compound is used in some cases, and a German silver chamber enameled on the inside in other designs. Armatures differ in

that some designs use a copper disc and others a copper drum or thimble. See also section below on *Data on House-Type Meters*.

Capacity of Mercury Watthour Meters. — Direct-current mercury meters are in themselves independent of ampere capacity, as external shunts are used. Shunts giving a capacity as high as 60,000 amperes have been furnished. Meters designed for voltages up to 600 volts are regularly furnished and special meters have been supplied for 1200- to 3000-volt direct-current railway circuits.

Accuracy. — The mercury-motor watthour meter above described is stated to have a load-accuracy curve at normal voltage which deviates from a straight line by less than 1.5 per cent over a range of load of 5 per cent to 200 per cent of rating.

INDUCTION WATTHOUR METER (Fig. 1). — This type of meter, the essential elements of which are illustrated in Fig. 1, has a laminated soft-iron core on which are mounted the current and potential windings. The current winding consists of a few turns of coarse wire, while the potential coil has many turns of fine wire. The flux due to the current coil is in phase with the load current, and the flux due to the potential coil is approximately in quadrature with the voltage across the load, since the potential coil is highly inductive; see *Alternating Currents*. The poles from the two windings are arranged so that the armature disc passes between them and is cut by the alternating flux due to each winding.

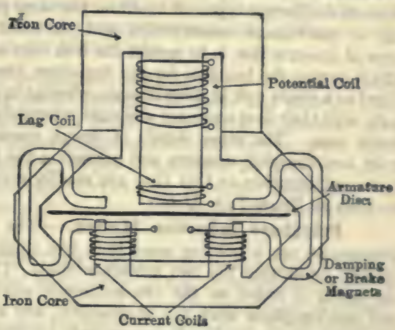

Fig. 1. Diagram of Induction Watthour Meter

Principle of Operation. — There are thus set up in the disc currents which flow about each pole in approximately concentric circles, the induced currents due to the two windings being in quadrature for a load of unity power factor (approximately only, unless a suitable phase compensator is used; see below). Part of the current induced by the current coil passes under the pole of the potential coil and therefore through a magnetic field which is approximately in phase with this current, and similarly part of the current induced by the potential coil passes under the pole of the current coil and therefore through a magnetic field approximately in phase with it; for power factors less than unity the phase difference between these currents and the magnetic fields through which they pass is the same as the difference in phase between the load current and voltage. Hence a torque is produced on the disc having an average value proportional to the load supplied through the meter; see *Electricity and Magnetism. Principles of*. A braking system similar to that employed in the commutator type of meter (see above) causes the speed of the disc to be proportional to this average torque, and therefore the number of revolutions is proportional to the energy supplied.

Phase Compensation; Lag Coil. — In order to make the induction watthour meter record correctly, especially for power factors less than unity, it is necessary to cause the flux at the tip of the potential pole to lag 90 degrees behind

the voltage impressed on the potential coil. A method of securing this condition is to mount on the potential pole a closed winding the resistance of which can be varied by a resistance wire soldered to the terminals. This closed winding is sometimes called the "lag coil." In many designs it is a metal plate in the form of a closed loop.

Compensation for Friction; Starting Plate. — The light-load adjustment can be obtained by placing a short-circuited loop of copper adjacent to the potential pole so that it can be shifted in a plane at right angles to the axis of the potential coil. This loop has induced in it currents which produce a field out of phase with the flux from the potential coil, and the reaction between these two produces on the disc a slight turning moment independent of the current in the current coil. The amount of compensation can be varied by shifting the position of the light-load coil, or starting plate, as it is sometimes called.

Details of Construction. — The induction watthour meter is manufactured in several different types all based on the same principle of operation but differing in mechanical construction and electrical characteristics. The general type of construction is to mount the iron core and windings on a metal frame, which carries the bearings for the armature, the core and windings being combined as a single unit in some designs, whereas in others the current and potential coils with their iron cores are mounted separately on a common frame or on the meter case. The other details of design consist of mounting the registering train and damping magnets and arranging the full-load, light-load and power-factor adjustments so that they are readily accessible. See also section below on *Data on House-Type Meters*.

Polyphase Induction Meters. — The polyphase induction meter in commercial use is nothing more than two single-phase meters with a common spindle connecting the two armature discs. The measurement of power with the meter is based on the two-wattmeter method for three-wire, three-phase and quarter-phase work (*see Wattmeters*), and a slight modification of the two-element meter is used quite extensively for four-wire, three-phase work. The modification consists of making a third current circuit by adding a winding to the current coils of both elements. Such a construction is quite accurate except on badly unbalanced voltages, the most accurate arrangement for such work being three separate single-phase meters, or a single meter which contains three single-phase elements.

Capacity of Induction Watthour Meters. — Induction watthour meters are supplied in standard capacities ranging from 5 to 300 amperes. Meters are generally used with self-contained potential circuits up to 600 volts and for higher voltages and current capacities potential and current transformers are used with 5-ampere 110-volt meters. When transformers are used, either a multiplying constant for the dial alone can be used in connection with the ratio of transformation of the transformers, or the ratio of the transformer can be included in the meter constant (see below), so that the register is direct reading with the transformers of proper ratio. For three-wire, single-phase circuits it is

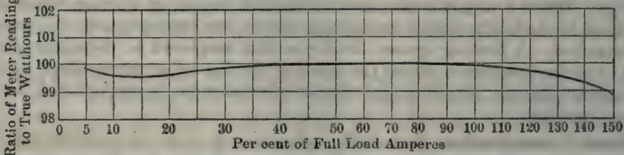

Fig. 2. Calibration Curve of Induction Watthour Meter

possible to obtain a special form of current transformer with double primary windings and a single secondary winding for connection to the meter.

Accuracy. — Modern induction watthour meters are susceptible of a higher degree of accuracy than the commutator or mercury-motor meter. Fig. 2 shows a typical characteristic curve of an induction watthour meter. It is possible to obtain by special calibrations a combination of induction watthour meters and instrument transformers which will be accurate to within 1 per cent over the ordinary range of commercial operation. The Meter Code (see *Bibliography*) requires a-c. watthour meters to run continuously, at rated voltage and frequency and unity power factor, with 2 per cent of rated current, and for higher loads to be accurate within the following limits:

Per cent rated current........	5	10	20	50	100	150
Maximum deviation........	3.0	1.5	2.0	2.0	1.5	3.0

Meters must also be adjusted for measurement on power factors of 75 per cent and 50 per cent to within 2 per cent and 4 per cent respectively. Other requirements of accuracy can be found in the Meter Code.

PREPAYMENT METERS. — Standard watthour meters are sometimes fitted with a device whereby the circuit through the meter and load is closed only upon the insertion of a coin into the device and remains closed only until a certain predetermined amount of energy has been recorded by the meter, when the circuit again opens automatically. When so equipped the meter is called a prepayment meter. The prepayment device may be either inserted in the watthour meter, or it may be placed in a separate attachment electrically connected to the meter.

OTHER TYPES OF WATTHOUR METERS. — The descriptions of watthour meters given above refer particularly to the types manufactured in the United States. Meters of the same general types are manufactured abroad, as well as certain other types. Among the latter, the Aron clock meter is worthy of notice. In this meter the potential coils are carried on two pendulums arranged in such a manner that they swing close to the coils carrying the line current. One pendulum must be retarded as the other is accelerated and to do this connections are so made that the instantaneous polarity of the pendulum coils is always opposite while the current coils are always alike. The meter contains no iron and has no commutator and is, therefore, a very accurate measuring device, although somewhat complicated and delicate of adjustment for ordinary commercial work.

DATA ON SERVICE-TYPE METERS. — General data on service-type watthour meters of different types are tabulated below. These data are approximate, but are representative of modern practice in this country. The weights and dimensions given refer to meters of 5 to 25 amperes rating, and are exceeded by meters of greater rating.

TESTING OF WATTHOUR METERS. — For laboratory tests the meter should be mounted on a firm support and set level. These conditions should also obtain when the meter is permanently mounted for service; see below under *Installation*. For laboratory tests a bank of lamps or some form of rheostat or "phantom load" may be used for a load. The instructions given below apply to both laboratory and service tests.

Testing Instruments. — For d-c. measurements a portable ammeter and voltmeter or a portable watthour meter may be used; for a-c. measurements an indicating wattmeter or portable watthour meter may be used. Two ammeter capacities should be provided, one sufficient to take care of the full

COMMERCIAL SMALL CAPACITY SERVICE-TYPE METERS

Item	Commutator type	Mercury type *	Induction type
Total weight, pounds	13 to 15	10 to 15	8 to 10
Height, inches	13	9	8
Width, inches	7.5	7	6
Depth, inches	7.5	6	5.5
Torque of motor element at full load, millimeter-grams	170	60	30 to 50
Weight of moving element, grams	100	3 †	10 to 16
Watts loss in potential circuit at 110 volts	5	5	1 to 2
Watts loss in current circuit at full load	5.5	0.3 ‡	0.3 to 0.8
Resistance of potential circuit, ohms.	2400	2400	....
Drop across shunt, millivolts	....	60 to 75	....

* The data on the mercury type also apply to meters used with shunt of any capacity.

† Upward thrust.

‡ Not including shunt.

load of the meter and the other small enough to read the light load with a proper degree of accuracy.

The use of a portable watthour meter is now recognized as the standard method for testing watthour meters up to 100-ampere capacity. There is no question as regards the accuracy of this method, since it is not necessary that the load be constant. With the portable watthour meter, indicating instruments and stop watches are unnecessary, and personal errors in reading are thereby reduced.

When indicating instruments are employed, a reliable stop watch is essential for taking the meter speed. As a matter of safety, it is advisable to have at least two stop watches, one to serve as a check on the other in test. Too much care cannot be used in the handling of these watches and in giving them careful checks with some local jeweler's standards. In checking the stop watch it is advisable to check at different points of the dial.

Precautions. — In order to insure accuracy, d-c. meters should not be tested until they have been connected in circuit with the voltage on for fully 15 to 20 minutes before the test is made. This enables the heating effect of the current in the potential circuit to become constant. If the meters are tested before this, consecutive readings may not check.

The testing instruments should not be allowed to measure the losses of the potential circuit of the meter under test, or vice versa. In order to avoid this, the current circuits of the testing instrument should be connected in series with the meter under test, while the potential circuits of the testing instrument and of the meter under test must both be connected across the line at the same place and between the generator and the nearer instrument.

A number of revolutions should be taken so that the time of observation will be at least from 40 to 60 seconds. If the time is materially less than 40 seconds, undesirably large errors in the measurement of time are probable; a time interval longer than 120 seconds is generally unnecessary.

Test Constant. — In checking a watthour meter the speed of the disc is

timed directly, since too long a time would be required to obtain a suitable reading on the dials of the register. The relation between the number of revolutions of the disc and the corresponding dial reading may be expressed by a constant multiplier, called the "test constant," which depends only upon the gearing between the disc and dial. Various manufacturers express this constant differently, viz.:

General Electric Test Constant (K). — The number of watthours indicated by the register per revolution of disc; its value is marked on the disc.

Westinghouse Test Constant (K). — Westinghouse meters (except type CW–6) are all adjusted for a certain speed at rated full load, which is stamped on the nameplate. Meters of modern manufacture have a speed of 25 r.p.m., and older forms of 50 r.p.m. Let N be the number of revolutions per minute corresponding to a rating of P kilowatts, then the test constant K (the number of watt-seconds per revolution of disc), as given in the instruction books issued by this company, is $K = 60{,}000\,P/N$. For type CW–6 (direct-current) meters the test constant is the number of watthours per disc revolution.

Fort Wayne Test Constant (C). — For type K meters the number of watthours for 36 revolutions of the disc; for types K_1, K_2, K_3, K_4 and K_5 the number of watthours for 1 revolution of the disc; the constant is marked on the disc.

Duncan Test Constant (K). — The number of watthours per revolution of the disc; it is marked on the meter name plate.

Sangamo Test Constant (K). — The number of watt-seconds per revolution of disc; it is now marked on the name plate in the D–5 mercury-motor meters and on the register of the type H induction meters.

Calibration Formulas. — Let R = number of revolutions of disc, S or T = number of seconds for R revolutions, K or C = meter constant as defined above: Then the power corresponding to the speed of the meter is:

Type of meter	Meter watts
General Electric	$\frac{3600\,KR}{S}$
Westinghouse	$\frac{KR}{T}$
Fort Wayne, Type K	$\frac{100\,CR}{S}$
Fort Wayne, Types K_1, K_2, K_3, K_4 and K_5	$\frac{3600\,CR}{S}$
Duncan	$\frac{3600\,KR}{S}$
Sangamo	$\frac{KR}{S}$

Per Cent Accuracy. — The meter watts as thus calculated divided by the true watts, as read on the test meter, and multiplied by 100, gives the per cent accuracy. If the per cent accuracy is less than 100 the meter is running slow; if greater than 100 it is running fast. The term "per cent accuracy" is open to

some objection, and "per cent registration" is coming to be preferred. No entirely satisfactory name has yet been found for this quantity.

Adjusting Speed of Meter. — If the error in the meter reading at full load is in excess of the permissible error the speed should be altered by adjusting the position of the permanent magnets or shunting part of their flux by the means provided therefor. If the error at light load, say 5 per cent load, is excessive, the light-load adjustment should be altered to bring the speed to its correct value.

Method of Using the Portable Watthour Meter. — The dial of the portable meter reads directly in revolutions of the meter disc. At the instant the mark on the disc of the service meter passes some arbitrarily chosen fixed point, note the dial reading of the portable meter. Count the number of revolutions of the disc of the service meter, and at the instant when a suitable whole number of revolutions are completed note the dial reading of the portable meter again. Let R_0 be the difference in the two dial readings of the portable meter, K_0 the test constant of the portable meter, R the number of revolutions of the service meter and K its test constant. Then the per cent accuracy is KR/K_0R_0. Of course the test constants of the two meters must be expressed in the same manner, i.e., as watthours per revolution or as watt-seconds per revolution.

Testing of Three-wire and Polyphase Meters. — A three-wire meter can be tested as a two-wire meter by connecting the current coils in series. The three-wire meter will then read twice the number of watts actually supplied to the test load. By using two sets of instruments three-wire meters may be tested as actually used.

A polyphase meter is most readily tested as a single-phase meter with the current circuits in series and the potential circuits in parallel. The reading of the meter is then twice the number of watts actually supplied to the test load.

INSTALLATION OF WATTHOUR METERS. — A watthour meter should always be mounted on a firm support, should be set with the disc level, and in a place where it will not be liable to be injured or subjected to weather. A firm support is particularly essential, as continual vibration of the meter may cause it to "creep," since vibration tends to lessen the friction, and the friction compensation may then be excessive.

Meters should not be installed closer than about fifteen inches between centers, and should not be too close to conductors carrying heavy currents or in the vicinity of iron girders, posts, water or steam pipes.

A check on its accuracy should always be made after the installation of a meter.

Determination of the Capacity of Meter. — Before installing a meter some tests or accurate estimates should be made to determine the correct capacity of meter, as it is not advisable to run a meter continuously above its rated capacity, and too large a meter means in general poor accuracy on light load. As a rough guide, the capacity of the meter should be one-half the total connected load for ordinary residences, three-fourths the total connected load for stores and offices, and 1½ times the connected load for elevator, hoist and other motors taking a large starting current. When several such motors are on one meter, the diversity factor will often permit the use of a meter capacity less than their combined current capacity.

Precautions in Connecting Polyphase Meters. — Direct-current and single-phase alternating-current meters are comparatively simple to install and connect properly, but all installations on polyphase circuits require considerably more care and knowledge to insure proper connections. Polyphase induction meters must be connected properly in regard to phase relations, as otherwise very inaccurate results will be obtained, which may not be apparent on casual examination of the meter. The manufacturer's instructions should be

followed very carefully and the phase relations traced out to make sure that they are correct. Practically all instrument transformers are now marked in some distinctive manner to indicate the relative instantaneous polarity of the primary and secondary windings, in order to facilitate making connections.

MAINTENANCE, REPAIRS AND DEPRECIATION. — It has been found from experience that except for the very smallest central stations or distributing companies it is economy to maintain some systematic method of testing and caring for all meters on the system. For ordinary house service and all except very large installations, readings are taken about once a month and meters are examined and tested as required. Commutator-type and mercury-motor meters should undoubtedly be tested and put in good condition at least once a year; induction meters may be left for two years and perhaps more before examination, as the very slight change in accuracy is not enough to warrant the expense of the test. With very large capacity meters, where the amount of money involved is large, frequent tests are made and in some cases a monthly check and adjustment is made, and whenever necessary the meter is given a complete overhauling and a spare meter installed in the meantime.

Repairs. — In making repairs on watthour meters, it is practically necessary to maintain considerable equipment and a department for this class of work. Unless such a department is available it is preferable to return meters to the manufacturer for all repairs except those of a minor nature, such as cleaning of commutators on direct-current meters and inserting new jewels and pivots for the lower bearing. Manufacturers will usually supply repair parts at reasonable prices and quite extensive repairs can be made by the central stations if they are properly equipped. In service the attention and care necessary vary with the type of meter, induction meters being by far the cheapest to repair and under most conditions such meters require the least attention of any type.

COST OF WATTHOUR METERS. — Ordinary service type commutator meters for direct current range in price from $16 to $170. Switchboard commutator meters in capacities from 800 to 6000 amperes range in price from $280 to $500. Shunted type commutator meters range from $100 for a 100-ampere meter to $200 for one of 5000 amperes capacity.

Mercury-motor meters usually have external shunts. The meter itself is of 10-ampere capacity and sells for about $16 in the service type and $55 to $70 in the switchboard type. Prices of shunts for these meters are approximately $12 for a 100-ampere, $30 for a 500-ampere, $40 for a 1000-ampere and $100 for a 5000-ampere meter.

Small-capacity single-phase service-type induction meters are sold for about $9 to $15 each; larger capacities and polyphase meters range from $35 to $70. Switchboard induction meters sell for $40 to $100; they are 5-ampere, 110-volt meters designed for use with instrument transformers, q.v.

The above prices are approximate, and should be used only as a rough guide.

The cost of installing and connecting the meter varies considerably with local customs or requirements. Where no protective devices are used, and the customer bears the cost of the main line cutout and cutout box, the installation cost to the company may be about $2. Where the company furnishes the main line cutout and box, with protective devices to prevent tampering, the installation cost will be about $10.

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See also an extensive bibliography on electrical instruments and meters in the 1914 report of the Meter Committee of the National Electric Light Association.

WATTMETERS. — (*See also Ammeters; Electrodynamometers; Electrometers; Oscillographs; Voltmeters; Watthour Meters.*) A wattmeter is a device for measuring electric power and is particularly useful for measuring alternating-current power. Usually some form of electrodynamometer is used in which the current in the circuit or a known part of it is sent through one winding, the current coil; and a small current in phase with the e.m.f. across the terminals of the circuit is sent through the other winding, or potential coil; see Figs. 1 and 2. The small current in the potential circuit is drawn through a resistance as in a voltmeter. For voltages above 150 to 750 volts, potential transformers (*see Transformers, Instrument*) are usually employed. In some cases specially-constructed resistances, or multipliers, are employed in testing work up to 25,000 volts or even higher. The moving coil is generally used for the potential circuit as it is more convenient to carry through it a small current than a large one.

Principle of Operation. — In an instrument constructed and used as above described, the current in the current coil is practically proportional to the load current i at each instant and the current in the potential coil is practically proportional to the voltage v across the load at that instant. Hence the instantaneous torque acting on the moving element at this instant is proportional to vi or to the instantaneous power. The average torque acting on the moving element during each cycle of the current and voltage is then proportional to the average power. If the free period of the moving system is large compared with the period of the current and voltage, then the steady reading will also be proportional to the average power.

CONSTRUCTION OF WATTMETERS. — The electrodynamometer type of instrument is built in several forms, e.g., the original Siemens electrodynamometer in various forms in which the moving coil is brought back to zero by means of a torsion head from which the reading is obtained; deflection instruments in which the moving coil is allowed to advance through an angle until the torque exerted on it by the currents in the instrument windings is balanced by the restoring torque developed in a spring or equivalent device. In the latter form of instrument the position of the moving coil is usually read by means of an attached pointer passing over a scale, graduated to read directly in watts or decimal multiples. In very sensitive instruments for laboratory use a beam of light or a telescope and scale like those used with mirror galvanometers is employed; see also *Electrodynamometers*.

In addition to the electrodynamometer type of instrument, induction wattmeters, operating on the same general principle as the induction watthour meter (*see Watthour Meters*), are sometimes used for switchboard and even for portable work, but they are not so generally useful through wide ranges of frequency and voltage as electrodynamometer instruments.

For certain limited conditions electrostatic wattmeters (*see Electrometers*) have been successfully used; in this general class of wattmeters for special purposes may be included the cathode-ray oscillograph or Braun tube (*see Braun Tube*).

Polyphase wattmeters, having two or more sets of current and potential windings contributing torque to a common shaft, are made for use on polyphase circuits; see below under *Measurement of Three-phase Power*.

Range of Different Types of Wattmeters. — Reflecting electrodynamometer wattmeters for laboratory use are made for currents up to 100 or 200 amperes and for use on circuits from a fraction of a volt up to 150 volts. Such instruments have been specially constructed for larger currents and higher voltages. See article on *Electrodynamometers*. Such instruments are chiefly useful for measurements on circuits of low voltage and of low power factor where even

the small power required to operate the instrument makes the use of ordinary portable instruments inconvenient or impossible.

Portable wattmeters are usually made to give a full-scale deflection with about two-thirds of the product of their rated amperes and volts, i.e., an instrument rated as 5 amperes and 150 volts would usually have a 500-watt scale. For determining core losses and other service where the power factor is low, wattmeters may be had giving full-scale reading with one-third or even with one-fifth of the product of the rated amperes and volts. It is customary to supply such wattmeters with a certificate showing the values of the equivalent phase angle α (*see below*) that have been determined, so that they may be used where a high degree of precision is required on low power factor work.

Portable wattmeters are made in ranges up to 200 amperes and 750 volts. Extra potential terminals are often supplied so that the instruments may be used on more than one voltage, e.g., 125 or 250 volts. Double current windings are also supplied, with a switch or links for changing the current range, and it is also usual to construct wattmeters having two current ranges and two voltage ranges. This construction greatly increases the usefulness of the instrument, and is particularly useful when applied to the low power factor wattmeters, as it makes it possible to use them for higher power factors by using say the 10-ampere, 250-volt range for a load of 5 amperes at 125 volts.

Portable wattmeters for currents larger than 200 amperes and voltages higher than 750 volts have been successfully constructed but are not generally used, because the higher ranges can usually be taken care of with 5-ampere 150-volt wattmeters in connection with calibrated instrument transformers.

Switchboard wattmeters are made single-phase up to 200 amperes and 750 volts, and polyphase up to 50 or 100 amperes and up to 750 volts. Higher ranges are usually taken care of by using 5-ampere wattmeters in connection with instrument transformers.

METHODS OF CONNECTING WATTMETER TO LOAD; CORRECTION FOR POWER LOST IN WATTMETER. — A wattmeter may be connected to the load which it is to measure in either of the two ways illustrated in Figs. 1 and 2. When connected as in Fig. 1 the current through the

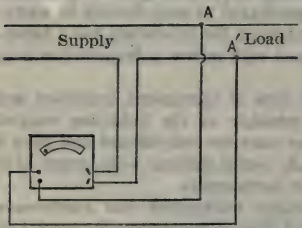

Fig. 1. Includes Loss in Potential Circuit

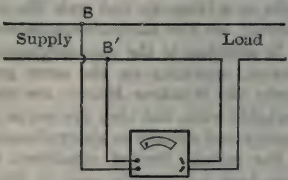

Fig. 2. Includes Loss in Current Coil

current circuit is equal to the sum (vector sum in the case of alternating currents) of the current taken by the load and that taken by the potential circuit of the wattmeter; hence the wattmeter will read the sum of the watts taken by the load and the watts lost in the potential circuit. The loss in the potential circuit is equal to $\frac{E^2}{R_p}$, where R_p is the resistance of the potential circuit and E the voltage across the load. For precise work this correction should always be made, unless the wattmeter is "compensated"; see below.

In the second scheme of connection, Fig. 2, the current through the current circuit is the same as that taken by the load, but the voltage across the potential

circuit is higher than the voltage across the load by the voltage drop through the current circuit of the instrument, and the wattmeter reads too high by an amount equal to the watts lost in the current circuit. This loss is equal to R_cI^2, where R_c is the resistance of the current circuit and I the load current. This loss is usually less than the loss in the potential circuit which occurs when the first scheme of connection is used. If no correction is made for the wattmeter loss the scheme of connections shown in Fig. 2 should therefore be used. If the highest precision is required, especially when a small-capacity wattmeter is used, the connections shown in Fig. 1 should be used, and a correction should be applied. The resistance of the potential coil is usually stated on the case of the instrument.

When a voltmeter is connected to the load across AA' in Fig. 1, while the wattmeter reading is being taken, a correction should also be made, if appreciable, for the power taken by the voltmeter. Calling R_v the resistance of the voltmeter and multiplier, if any, and E the voltage across the load, the loss in the voltmeter is $\frac{E^2}{R_v}$. In most cases the potential circuit losses in the wattmeter and voltmeter are best determined by a direct measurement with the wattmeter, using the test voltage and leaving the current circuit open. If the voltmeter is connected across BB' in Fig. 2, then the power lost in it is not read by the wattmeter, but the voltmeter reading is not exactly equal to the voltage across the load, but equals the load voltage plus the resistance drop (or impedance drop, added vectorially, in the case of an alternating-current circuit) through the current circuit of the wattmeter.

Compensation for Wattmeter Loss. — In the so-called compensated wattmeters a stationary compensating coil inside the instrument is connected in series with the potential coil, this compensating coil being so placed that the current through it produces on the moving element a torque equal and opposite to that produced by this same current when it passes through the current coil. A compensated wattmeter should always be connected to the circuit, as shown in Fig. 1; when so connected no correction for the loss in the wattmeter is necessary.

SOURCES OF ERROR. — Due to the unavoidable inductance in the potential circuit of a wattmeter, the current in the moving coil is not exactly in phase with the e.m.f. impressed on the circuit; in addition there are other small inherent defects in wattmeters, such as eddy currents in windings, supports, etc., which tend to make commercial wattmeters more or less imperfect. These imperfections are more serious as the frequency of the circuit is raised and the power factor is lowered.

For precision testing under service conditions when the best obtainable accuracy is required, it is often desirable to apply corrections to wattmeter observations to eliminate the errors due to phase displacement, or its equivalent, in the potential circuit; when instrument transformers are used, additional phase displacement occurs, rendering such corrections especially important. In other cases where corrections cannot well be applied, it is useful to consider the magnitude of the error that will occur under any given conditions, in order to select suitable instruments and transformers and to arrange the burdens connected to their secondaries in such a way as to produce as near as may be the desired results. In many cases indications that would otherwise be considerably in error may be made to give results of satisfactory accuracy by correcting for phase displacement in the wattmeter, and particularly in the instrument transformers.

There are many ways in which the necessary corrections may be determined and applied but the following has been found satisfactory in practical work.

Equivalent Phase Angle of Wattmeter. — All the errors in wattmeters may be considered as due to a certain phase displacement between the current in the potential coil and the impressed e.m.f. of the circuit. This equivalent angle of phase displacement may be due to a variety of causes, but can be determined for any given instrument from the following relations. Consider a wattmeter in which the current in the potential circuit lags α degrees behind the e.m.f. impressed on the potential circuit, the meter being in all other respects perfect. If such a meter is used to measure the watts P supplied by a current I at a voltage E, this current lagging behind E by an angle θ, the wattmeter will read $P_2 = EI \cos \theta_2$ watts, where $\theta_2 = \theta - \alpha$. Hence if a wattmeter is used to measure a known load of P watts with I amperes and E volts, and the wattmeter reads P_2 watts, the equivalent phase angle of the wattmeter may be defined by the relation

$$\alpha = \cos^{-1}\left(\frac{P}{EI}\right) - \cos^{-1}\left(\frac{P_2}{EI}\right).$$

This angle α is to be taken positive when the wattmeter reads too high on an inductive load and negative when it reads too low on an inductive load.

For a given wattmeter and given potential coil this equivalent phase angle is practically constant for all currents, voltages and load power factors, provided the frequency remains constant.

In the best portable and reflecting instruments α is usually + and very small. In a portable instrument built for a 125- to 150-volt circuit, when used on a 60-cycle circuit, α may have a value of from +2 minutes to as much as +10 minutes. The phase angle varies directly as the frequency and inversely as the voltage rating.

The phase angle of a wattmeter may be compensated by shunting a condenser of capacity C around a portion R of the resistor in the potential circuit. If the equivalent inductance of this circuit be L, the relation for compensation is

$$C = \frac{L}{R^2}.$$

Correction for Phase Angle of Wattmeter and of Instrument Transformers. — Let α = the phase angle of the wattmeter, as defined above, β = the phase angle of the current transformer, γ = the phase angle of the potential transformer (*see Transformers, Instrument*), P_2 = wattmeter reading corrected for scale error and multiplied by the product of the corrected ratios of the current and potential transformers, E = voltmeter reading corrected for scale error and multiplied by the ratio of the potential transformer, and I = ammeter reading corrected for scale error and multiplied by the corrected ratio of the current transformer. Then the apparent power factor is

$$\cos \theta_2 = \frac{P_2}{EI},$$

and the true power is

$$P = P_2 \frac{\cos(\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}.$$

The angle θ_2 is to be taken positive when the current lags behind the voltage, and negative when the current leads. Note that α, β and γ may also be positive or negative angles. Values of the correction factor $\frac{\cos(\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}$ are given in Tables I and II. Note carefully the conditions, stated at the head of each table, to which each table applies.

On three-phase systems (*see pp. 1932 and 1933*), when badly unbalanced,

TABLE I. CORRECTION FACTORS FOR PHASE ANGLE

$$\frac{\cos(\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}$$

For lagging current when $(\alpha + \beta + \gamma)$ is positive
For leading current when $(\alpha + \beta + \gamma)$ is negative

$\alpha+\beta+\gamma$	Apparent power factor $\cos \theta_2$										
	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
5′	0.9855	0.9904	0.9929	0.9944	0.9954	0.9967	0.9975	0.9981	0.9985	0.9989	0.9993
10′	0.9711	0.9808	0.9857	0.9887	0.9907	0.9933	0.9950	0.9961	0.9970	0.9978	0.9986
15′	0.9566	0.9712	0.9786	0.9831	0.9861	0.9900	0.9924	0.9942	0.9955	0.9967	0.9979
20′	0.9421	0.9616	0.9715	0.9775	0.9815	0.9867	0.9899	0.9922	0.9940	0.9956	0.9972
25′	0.9276	0.9520	0.9643	0.9718	0.9768	0.9833	0.9874	0.9903	0.9926	0.9945	0.9965
30′	0.9131	0.9424	0.9572	0.9662	0.9722	0.9800	0.9848	0.9883	0.9911	0.9934	0.9957
40′	0.8842	0.9232	0.9429	0.9549	0.9629	0.9733	0.9798	0.9844	0.9881	0.9912	0.9943
50′	0.8552	0.9040	0.9286	0.9436	0.9536	0.9666	0.9747	0.9805	0.9851	0.9890	0.9929
1° 00′	0.8262	0.8848	0.9143	0.9323	0.9444	0.9599	0.9696	0.9766	0.9820	0.9868	0.9914
10′	0.7972	0.8656	0.9000	0.9209	0.9350	0.9531	0.9645	0.9726	0.9790	0.9845	0.9899
20′	0.7682	0.8464	0.8857	0.9096	0.9257	0.9464	0.9594	0.9687	0.9760	0.9823	0.9885
30′	0.7392	0.8271	0.8714	0.8983	0.9164	0.9397	0.9543	0.9648	0.9730	0.9800	0.9870
40′	0.7102	0.8079	0.8571	0.8869	0.9071	0.9329	0.9492	0.9608	0.9699	0.9778	0.9855
50′	0.6812	0.7886	0.8428	0.8756	0.8978	0.9262	0.9441	0.9568	0.9668	0.9755	0.9840
2° 00′	0.6521	0.7694	0.8284	0.8642	0.8884	0.9194	0.9389	0.9529	0.9638	0.9732	0.9825
10′	0.6231	0.7501	0.8141	0.8529	0.8791	0.9127	0.9338	0.9489	0.9607	0.9709	0.9810
20′	0.5941	0.7308	0.7997	0.8415	0.8697	0.9059	0.9287	0.9449	0.9576	0.9686	0.9795
30′	0.5650	0.7115	0.7854	0.8301	0.8603	0.8991	0.9235	0.9409	0.9545	0.9663	0.9779
40′	0.5360	0.6923	0.7710	0.8187	0.8510	0.8923	0.9183	0.9369	0.9515	0.9640	0.9764
50′	0.5069	0.6730	0.7566	0.8073	0.8416	0.8855	0.9132	0.9329	0.9483	0.9617	0.9748
3° 00′	0.4779	0.6537	0.7422	0.7959	0.8322	0.8787	0.9080	0.9288	0.9452	0.9594	0.9733
10′	0.4488	0.6344	0.7279	0.7845	0.8228	0.8719	0.9028	0.9248	0.9421	0.9570	0.9717
20′	0.4198	0.6151	0.7135	0.7731	0.8134	0.8651	0.8976	0.9208	0.9390	0.9547	0.9701
30′	0.3907	0.5957	0.6991	0.7617	0.8040	0.8583	0.8924	0.9167	0.9359	0.9523	0.9686
40′	0.3616	0.5764	0.6847	0.7503	0.7946	0.8514	0.8872	0.9127	0.9327	0.9500	0.9670
50′	0.3326	0.5571	0.6702	0.7388	0.7852	0.8446	0.8820	0.9086	0.9296	0.9476	0.9654
4° 00′	0.3035	0.5378	0.6558	0.7274	0.7758	0.8377	0.8767	0.9046	0.9264	0.9452	0.9638
10′	0.2744	0.5185	0.6414	0.7160	0.7663	0.8309	0.8715	0.9005	0.9232	0.9429	0.9622
20′	0.2453	0.4991	0.6270	0.7045	0.7569	0.8240	0.8663	0.8964	0.9201	0.9405	0.9605
30′	0.2163	0.4798	0.6125	0.6930	0.7474	0.8171	0.8610	0.8923	0.9169	0.9381	0.9589
40′	0.1872	0.4604	0.5981	0.6816	0.7380	0.8103	0.8558	0.8882	0.9137	0.9357	0.9573
50′	0.1581	0.4411	0.5837	0.6701	0.7285	0.8034	0.8505	0.8841	0.9105	0.9333	0.9556
5° 00′	0.1290	0.4217	0.5692	0.6586	0.7191	0.7965	0.8452	0.8800	0.9073	0.9308	0.9540
10′	0.0999	0.4024	0.5548	0.6472	0.7096	0.7896	0.8400	0.8759	0.9041	0.9284	0.9523
20′	0.0708	0.3830	0.5403	0.6357	0.7001	0.7827	0.8347	0.8717	0.9008	0.9260	0.9507

TABLE II. CORRECTION FACTORS FOR PHASE ANGLE

$$\frac{\cos(\theta_2 + \alpha + \beta + \gamma)}{\cos \theta_2}$$

For lagging current when $(\alpha + \beta + \gamma)$ is negative
For leading current when $(\alpha + \beta + \gamma)$ is positive

$\alpha+\beta+\gamma$	Apparent power factor $\cos \theta_2$										
	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
5′	1.0145	1.0096	1.0071	1.0056	1.0046	1.0033	1.0625	1.0019	1.0015	1.0011	1.0007
10′	1.0289	1.0192	1.0142	1.0113	1.0092	1.0067	1.0050	1.0039	1.0030	1.0022	1.0014
15′	1.0434	1.0288	1.0214	1.0169	1.0139	1.0100	1.0075	1.0058	1.0044	1.0033	1.0021
20′	1.0579	1.0383	1.0285	1.0225	1.0185	1.0133	1.0101	1.0077	1.0059	1.0043	1.0028
25′	1.0723	1.0479	1.0356	1.0281	1.0231	1.0166	1.0126	1.0097	1.0074	1.0054	1.0035
30′	1.0868	1.0575	1.0427	1.0338	1.0277	1.0200	1.0151	1.0116	1.0089	1.0065	1.0042
40′	1.1157	1.0766	1.0569	1.0450	1.0369	1.0266	1.0201	1.0154	1.0118	1.0087	1.0056
50′	1.1446	1.0958	1.0711	1.0562	1.0461	1.0332	1.0251	1.0193	1.0147	1.0108	1.0069
1° 00′	1.1735	1.1149	1.0853	1.0674	1.0553	1.0398	1.0301	1.0231	1.0177	1.0129	1.0083
10′	1.2024	1.1340	1.0995	1.0787	1.0645	1.0464	1.0351	1.0269	1.0206	1.0151	1.0097
20′	1.2313	1.1531	1.1137	1.0898	1.0737	1.0530	1.0400	1.0308	1.0235	1.0172	1.0110
30′	1.2601	1.1722	1.1279	1.1010	1.0829	1.0596	1.0450	1.0346	1.0264	1.0193	1.0123
40′	1.2890	1.1913	1.1421	1.1122	1.0921	1.0662	1.0500	1.0384	1.0292	1.0214	1.0137
50′	1.3178	1.2104	1.1562	1.1234	1.1012	1.0728	1.0549	1.0421	1.0321	1.0235	1.0150
2° 00′	1.3466	1.2294	1.1704	1.1346	1.1104	1.0794	1.0598	1.0459	1.0350	1.0256	1.0163
10′	1.3755	1.2485	1.1845	1.1457	1.1195	1.0859	1.0648	1.0497	1.0379	1.0276	1.0176
20′	1.4043	1.2675	1.1986	1.1569	1.1286	1.0925	1.0697	1.0535	1.0407	1.0297	1.0189
30′	1.4331	1.2866	1.2127	1.1680	1.1377	1.0990	1.0746	1.0572	1.0435	1.0318	1.0202
40′	1.4618	1.3056	1.2268	1.1791	1.1469	1.1055	1.0795	1.0610	1.0464	1.0338	1.0215
50′	1.4906	1.3246	1.2409	1.1902	1.1560	1.1120	1.0844	1.0647	1.0492	1.0359	1.0227
3° 00′	1.5194	1.3436	1.2550	1.2013	1.1650	1.1185	1.0893	1.0684	1.0520	1.0379	1.0240
10′	1.5481	1.3626	1.2691	1.2124	1.1741	1.1250	1.0942	1.0721	1.0548	1.0399	1.0252
20′	1.5768	1.3816	1.2832	1.2235	1.1832	1.1315	1.0990	1.0758	1.0576	1.0419	1.0265
30′	1.6056	1.4005	1.2972	1.2346	1.1923	1.1380	1.1039	1.0795	1.0604	1.0439	1.0277
40′	1.6343	1.4195	1.3113	1.2456	1.2013	1.1445	1.1087	1.0832	1.0632	1.0459	1.0289
50′	1.6630	1.4384	1.3253	1.2567	1.2103	1.1509	1.1136	1.0869	1.0660	1.0479	1.0301
4° 00′	1.6916	1.4573	1.3393	1.2677	1.2194	1.1574	1.1184	1.0906	1.0687	1.0499	1.0313
10′	1.7203	1.4763	1.3533	1.2788	1.2284	1.1638	1.1232	1.0942	1.0715	1.0519	1.0325
20′	1.7489	1.4952	1.3673	1.2898	1.2374	1.1703	1.1280	1.0979	1.0742	1.0538	1.0337
30′	1.7776	1.5141	1.3813	1.3008	1.2464	1.1767	1.1328	1.1015	1.0770	1.0558	1.0349
40′	1.8062	1.5329	1.3953	1.3118	1.2554	1.1831	1.1376	1.1052	1.0797	1.0577	1.0361
50′	1.8348	1.5518	1.4092	1.3228	1.2644	1.1895	1.1424	1.1088	1.0824	1.0596	1.0373
5° 00′	1.8634	1.5707	1.4232	1.3337	1.2733	1.1959	1.1472	1.1124	1.0851	1.0616	1.0384
10′	1.8920	1.5895	1.4371	1.3447	1.2823	1.2023	1.1519	1.1160	1.0878	1.0635	1.0396
20′	1.9205	1.6083	1.4510	1.3557	1.2912	1.2086	1.1567	1.1196	1.0905	1.0654	1.0407

corrections should be applied separately, as above, to each wattmeter. When the circuit is balanced a single correction based on $\cos \theta_2$ for the whole circuit may be used.

Example No. 1. — Given a single-phase circuit with lagging current in which the wattmeter reading corrected for scale error and multiplied by the corrected ratios of current and potential transformers equals 24,520 watts, and the product of the voltmeter and ammeter readings, similarly corrected, equals 35,600 volt-amperes. Then $\cos \theta_2 = \frac{24,520}{35,600} = 0.689$. If the equivalent phase angle α of the wattmeter is $+4'$ and if from examination of characteristic curves the current transformer phase angle β is found to be $+48'$ and the potential transformer phase angle γ is $-10'$, then $\alpha+\beta+\gamma = +42'$, and from Table I, the correction factor is 0.9871. Whence the true power equals $24,520 \times 0.9871 = 24,204$ watts.

Example No. 2. — Given a single-phase circuit with leading current, in which the wattmeter reading, corrected as in example No. 1, equals 12,266 watts and the product of the voltmeter and ammeter readings, similarly corrected, equals 24,532 volt-amperes. Then $\cos \theta_2 = \frac{12,266}{24,532} = 0.5$. If the equivalent phase angle α of the wattmeter is $+5'$, the phase angle β of the current transformer $+2° 33'$ and the phase angle γ of the potential transformer is $+38'$, then $\alpha+\beta+\gamma = +3° 16'$ and therefore from Table II the correction factor to be used is 1.0971. Whence the true power equals $12,266 \times 1.0971 = 13,457$ watts.

Errors Due to Stray Fields. — Stray magnetic fields influence the reading of a wattmeter in much the same way as such disturbances influence the readings of other electrical instruments; see *Ammeters*. In unshielded instruments errors due to stray fields may be quite large. Considerable errors are often caused by the effect of neighboring instruments on one another. To avoid these errors, high-grade American wattmeters are magnetically shielded.

ACCURACY OF WATTMETERS. — The question of the accuracy of electrical instruments is discussed in detail in the article on *Ammeters*. High-grade portable wattmeters may be obtained having a stated accuracy of 0.25 per cent of full-scale value for single-phase or 0.5 per cent for polyphase wattmeters. It is not practicable to obtain as high accuracy in the latter, on account of the impossibility of making both elements of the polyphase wattmeter follow exactly the same scale law. This theoretical disadvantage of the polyphase wattmeter is more than offset in practical use by its greater convenience and the reduction in number of observations necessary, and its service accuracy, especially on fluctuating power, may even exceed that of single-phase wattmeters used for polyphase measurements. High-grade switchboard wattmeters may be obtained having a stated accuracy of 1 per cent of full-scale value. Of course the higher the degree of accuracy demanded the more costly is the meter.

TESTING OF WATTMETERS. — Wattmeters are usually tested by being compared with other standard wattmeters, which have in turn been examined by special methods and their behavior under various conditions of use determined. For examining the effect of low power factor on wattmeters, phase-shifting transformers or generators having shifting fields are conveniently employed. By these means low power factor conditions may be produced on instruments without the necessity of troublesome adjustments of reactances, condensers, etc.

After a given instrument is known with certainty to give no sensible error on alternating current due to structural defects, it is usually more convenient to make subsequent checks to detect any change in the instrument by using direct current. The current and voltage are in this case determined by means of

suitable standard ammeter and voltmeter or by direct reference to potentiometer and standard cells; see *Potentiometers*.

MEASUREMENT OF THREE-PHASE POWER. — (*See also article by L. T. Robinson in General Electric Review, 1912, Vol. 15, p. 350.*) The measurement of the power supplied to a three-phase load may be effected in one or more of the following ways. The connections in each case may be made either directly or through proper instrument transformers, the latter being used for heavy currents or high voltages, or both.

Single (One-element) Wattmeter on Three-wire System. — The current circuit of the wattmeter is connected in series with one of the mains supplying the load and the potential circuit of the wattmeter is connected between the corresponding terminal of the load and the neutral. If the neutral point of the load, or of the transformers supplying the load, is not available, a "Y-box" (see below) can be used to establish an artificial neutral. If the load is perfectly balanced (*see Alternating Currents*) the power input is then three times the wattmeter reading. However, a three-phase load is seldom sufficiently well balanced, even in the case of a three-phase motor load, to render this method of measurement an accurate one. In most practical cases it gives merely a rough approximation to the true power.

Y-Box. — The simplest form of Y-box consists of two equal non-inductive resistors connected in series, each of the free ends and the junction point being connected to a binding post. Each resistor has a resistance equal to that of the potential circuit of the wattmeter. One terminal of the potential circuit of the wattmeter is connected to the junction terminal of the Y-box and the other terminal of the potential circuit to the line wire in which the current circuit of the wattmeter is connected. The other two terminals of the Y-box are connected to the other two line wires respectively.

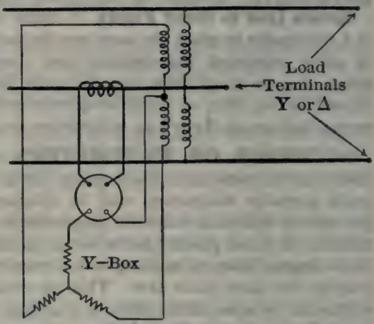

Fig. 3. Single-phase Wattmeter and Y-box for Three-phase Circuit

In the case of wattmeters designed especially for use with a Y-box, part of the resistance of the potential circuit of the wattmeter is placed in the Y-box, being connected permanently to the junction point between the other two resistors. A similar arrangement may be used as a multiplier. The connections of such a Y-box, wattmeter and instrument transformers are shown diagrammatically in Fig. 3.

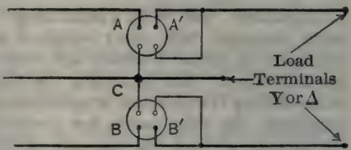

Fig. 4. Two Single-phase Wattmeters for Three-phase Circuits

Two-wattmeter Method for a Three-wire System. — The simple arrangement of two wattmeters shown in Fig. 4 will give exactly the total power in *any three-wire* circuit, provided each wattmeter by itself gives accurate indications. Aside from the sources of error, as noted above, which may affect the accuracy of a wattmeter on a single-phase circuit, the arrange-

ment shown in Fig. 4 will give the true power for any condition of unbalancing, wave-form, frequency, etc. It is also immaterial whether the load be Y or Δ connected. The connections may be made directly as shown or through two current transformers and two potential transformers, the connections being the same as in Fig. 5, except that two separate wattmeters instead of a polyphase wattmeter are used.

Rule for Adding or Subtracting Readings. — With the arrangement shown in Fig. 4 the total power is always the *algebraic* sum of the readings of the two wattmeters. Since a wattmeter reads only in one direction, usually to the right, the two instruments must be so connected to the line that the needle of each instrument is deflected over the scale. For a balanced three-phase load having a power factor greater than 50 per cent, the sum of the two wattmeter readings, when the connections are thus made, gives the total power; for a power factor less than 50 per cent the difference of the two readings must be taken. When the power factor is not known, one can determine whether the sum or the difference of the readings should be taken by interchanging the two wattmeters, leaving unaltered the potential connections to the third wire (C in Fig. 4); if the pointers of the two wattmeters deflect in the same direction as before add the two readings, if the pointers deflect in the opposite direction (i.e., against the stop) take the difference. In making this test, care must be taken to connect the source side of each of the two lines (in which the current coil of each wattmeter is connected in succession) to the same binding post of the given wattmeter in each of the two positions.

When the load is *balanced*, and the two wattmeters are connected, as in Fig. 4, so that each gives a positive reading, the question as to whether the power factor is above or below 50 per cent can be determined by changing the potential lead of the lower reading meter from the common connection C to the line in which the current coil of the other wattmeter is connected. If the reading thus obtained is positive, the power factor is more than 50 per cent and the readings should be added; if the reading is negative the power factor is less than 50 per cent and the readings should be subtracted.

Phase Difference between Voltage and Current in the Wattmeter Windings. — In case the load is balanced it can be shown that if θ is the power-factor angle of the load, I the line current (current per wire) and E the line voltage (voltage between wires), then the power read by one wattmeter is $P_1 = EI \cos(\theta - 30°)$, and the power read by the other is $P_2 = EI \cos(\theta + 30°)$. The performance of the first wattmeter is the same as it would be on a single-phase load having a power factor of $\cos(\theta - 30°)$ and the performance of the second is the same as it would be on a single-phase load having a power factor of $\cos(\theta + 30°)$. These relations should be borne in mind when making any corrections for errors due to low power factor (see above).

Measurement of Power Factor by Two-wattmeter Method. — From the above relations it can be shown that the power factor of the load, when the load is balanced, may be calculated from the two wattmeter readings P_1 and P_2 by the formula

$$\text{Power factor} = 100 \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}\right)\right] \quad \text{per cent,}$$

when the power factor is greater than 50%; and by the formula

$$\text{Power factor} = 100 \cos\left[\tan^{-1}\left(\sqrt{3}\,\frac{P_1 + P_2}{P_1 - P_2}\right)\right] \quad \text{per cent,}$$

when the power factor is less than 50%, P_1 and P_2 both being taken as positive in both cases and P_2 being the smaller reading.

Two-element Polyphase Wattmeter on Three-wire System. — Instead of using two separate wattmeters, as shown in Fig. 4, the two wattmeters may be combined into a single instrument with but one shaft and pointer. The connections for such a two-element wattmeter, when instrument transformers are used, are shown in Fig. 5. For measurements where the power is badly fluctuating and especially when accompanied by low power factor, polyphase wattmeters may be more accurate and convenient. For ordinary polyphase service a polyphase wattmeter is not capable of as high accuracy as two single-phase wattmeters.

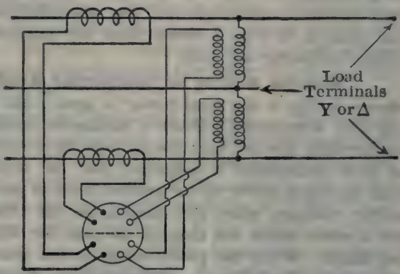

Fig. 5. Two-element Polyphase Wattmeter with Instrument Transformers

Determination of Power Factor with Two-element Wattmeter. — The two-element wattmeter reads directly the sum P_1+P_2, using the notation in the previous paragraph. By reversing the connections of one set of potential terminals to the corresponding potential transformer the meter may be made to read P_1-P_2. Hence by the use of a suitable reversing switch the power factor of the load may be readily determined by taking two readings; see preceding paragraph.

Three Wattmeters for Three-wire System. — As noted above, where two wattmeters, or a two-element polyphase wattmeter, is used on a three-wire system, the phase difference between the voltage and current in each wattmeter or element may differ greatly from the power-factor angle of the load. For example, in the case of a balanced load having a 50 per cent power factor, the current in one wattmeter differs by 90° from the voltage on the potential circuit of this wattmeter, and it should read zero watts, i.e., this wattmeter operates under the worst possible conditions as regards power factor, and consequently the phase-angle error (see above), particularly when instrument transformers are used, may be quite large when expressed as a percentage. It should be noted, however, that the reading of the wattmeter or element in which the large phase difference occurs contributes proportionally less to the total reading as this phase difference approaches 90°, and consequently the percentage error of the total reading will be only a correspondingly small part of the error in this wattmeter or element.

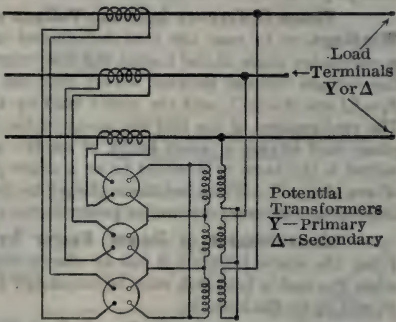

Fig. 6. Y–Δ Connections for Three Wattmeters on Three-wire System

Where the highest accuracy is demanded, it is therefore theoretically advisable to use three wattmeters connected as shown in Fig. 6. It should be noted, however, that for most practical purposes the very slight gain in accuracy by such an

arrangement over the two-element wattmeter does not justify the expense of the additional wattmeter and instrument transformers. Again, a precision measurement of power with the three-wattmeter arrangement requires the accurate calibration and the accurate reading of three wattmeters instead of one.

Measurement of Power in Four-wire Circuits. — When part of the load current returns through a fourth wire, e.g., in an unbalanced three-phase system having a neutral wire, at least three wattmeters or wattmeter elements are required to obtain a theoretically accurate measurement of the power. A two-element wattmeter having three current circuits may be used for the measurement of power on a four-wire system, and if the vectors representing the three voltages to neutral form a triangle will give results having as high an accuracy as can be obtained with a two-element wattmeter on a three-wire system.

When three wattmeters are used, they should be connected in a manner similar to that used for two wattmeters on a three-wire system, i.e., each of the three potential circuits should be connected between a main wire and the neutral wire, and each of the three current circuits should be connected in series with the corresponding main wire. When instrument transformers are used the primaries of the potential transformers are connected in Y, with the neutral point connected to the neutral wire, the three branches being connected to the main wires; the secondaries and the potential circuits of the wattmeters are also connected in Y, with the wattmeters Y-connected to the Y made by the secondaries.

Two-element Wattmeter on Four-wire System. — When used on a four-wire system the current coil of each element is divided into two like sections, and one section of each current coil is connected in series with one section of the other coil, thus forming three groups of current coils. Each group is connected to a current transformer as shown in Fig. 7, which also shows the connections of the potential transformers, only two of which are required. With this arrangement the instrument measures correctly the power supplied by two branches of the system, but for the third branch the power measured is that corresponding to the current in this branch and the resultant (vector sum) of the voltages to neutral in the other two branches. Voltage unbalance or badly distorted current and voltage waves will cause this resultant voltage to differ from the actual voltage to neutral in the third branch, producing an error in the measurement ot this third of the total power. Under ordinary conditions, however, this error is not objectionably large for commercial measurements.

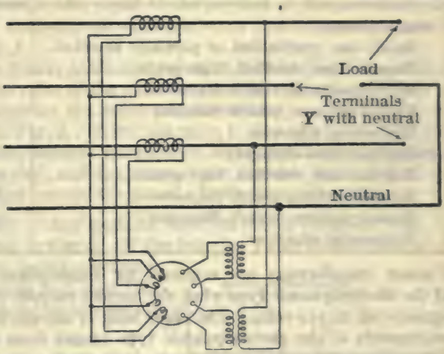

Fig. 7. Connections for Two-element Wattmeter on Four-wire System

GRAPHIC WATTMETERS. — These instruments are designed to draw on a chart a curve of power against time, without any appreciable time lag. All instruments of this type consist of a wattmeter element and of a time element. The curve-drawing pointer is actuated either directly by the motion of

the moving part of the wattmeter element, or indirectly by a system of solenoids, or a small motor, energized through relay contacts on the wattmeter moving part. The purpose of the time element is to produce a steady motion of the paper at the point where the curve-drawing pointer (pen) is in contact with it. The clock, which turns the paper rolls, is either of the hand-wound type or of the electric self-winding type. In several types the rate of the paper feed may be varied from ¾ inch per hour to as high as 6 inches per minute.

Semi-portable forms of graphic wattmeters are on the market, but the more accurate instruments are for switchboard use only. Large errors due to friction between paper and curve-tracing pen or pointer, or due to change in level of the instrument, are not uncommon with certain types of graphic wattmeters.

PRECAUTIONS IN USE OF WATTMETERS. — In addition to the general precautions given under *Ammeters* (p. 37), the following special points must be noted. A wattmeter has three distinct limits, namely, current, voltage, and power, no one of which should be exceeded. The potential of the moving coil must not be greatly different from that of the fixed coil, and the connections must always be such as to ensure this condition, both in the use and in the checking of a wattmeter. This precaution is especially necessary when an external multiplier is used for high voltages.

COSTS, WEIGHTS AND DIMENSIONS OF WATTMETERS. — (As of 1920). — The following figures give a rough idea of the cost of wattmeters:

Indicating wattmeters	Portable	Switchboard
Single-phase:	$	$
Small size, unshielded or partially shielded	45– 72	32– 35
Regular size, shielded, high grade	72–141	....
Round pattern, 7-inch diameter	....	52–100
Round pattern, 9-inch diameter	....	69–103
Horizontal edgewise	....	54– 72
Polyphase (two-element):		
Regular size, shielded, high grade	132–197	....
Round pattern, 7-inch diameter	....	75–115
Round pattern, 9-inch diameter	....	96–124
Horizontal edgewise	....	75– 87

Current transformers are required for single-phase switchboard wattmeters of the electrodynamic type when the current exceeds 200 amperes or the voltage exceeds 600 to 750 volts. The upper limit of current for polyphase switchboard wattmeters of the electrodynamic type ranges from 60 to 200 amperes. For induction wattmeters as now made, both single-phase and polyphase, the current limit is 10 amperes. Potential transformers are required for all current ratings when the voltage exceeds 600 to 750 volts.

Switchboard wattmeters are made in 4-inch, 7-inch, and 9-inch round pattern, and in horizontal edgewise pattern occupying a space on the panel 6 by 8.5 inches and projecting 7.6 inches from the panel.

High-grade shielded portable single-phase wattmeters measure about 8 by 9 by 6 inches high, and weigh from 9 to 12 pounds; polyphase wattmeters of the same grade measure from 8 by 8 by 10 inches high to 9.5 by 10.5 by 8.5 inches high, and weigh from 14 to 17.5 pounds.

BIBLIOGRAPHY. — See the Bibliographies in the articles on *Ammeters* and *Watthour Meters;* also Robinson, L. T., *Metering on Three-phase Systems*, Gen. Elec. Rev., 1912, Vol. 15, p. 350.

WAVE ANALYSIS. — (*See also Alternating Currents.*) Any sine function y of a variable t, that is, any function of the form $y = Y \sin(\omega t + \phi)$, may be plotted as a wave, as shown in Fig. 1. The constant Y is called the

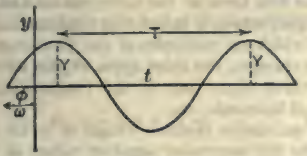

Fig. 1.

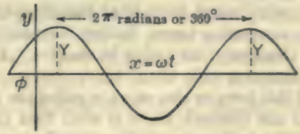

Fig. 2.

"maximum value" of the wave, the constant ϕ the "phase" of the wave, and the constant ω is called the "periodicity" of the wave. The distance between successive positive maxima (or between successive negative maxima) of such a wave is called the "period" of the wave, and is usually represented by the symbol T. The reciprocal of the period is called the "frequency," and is usually represented by the symbol $\frac{1}{T}$.

Period, frequency and periodicity are related as follows:

$$\omega = 2\pi f = \frac{2\pi}{T}.$$

The phase (ϕ) of the wave is also equal to the product of the periodicity by the distance to the left of the origin at which the wave first crosses the t axis in a *rising* direction; see Fig. 1.

In plotting a wave it is usually more convenient to take as the distance along the horizontal axis, not the value of the variable t, but the value of ωt. That is, putting $x = \omega t$, the equation for the wave may be written $y = Y \sin(x + \phi)$ and Fig. 1 then reduces to Fig. 2. When the wave is thus plotted the distance between successive positive maxima (or between successive negative maxima) is equal to 2π radians or 360°, and the phase is the distance to the left of the origin at which the wave first crosses the axis of x in the rising direction.

FOURIER'S SERIES. — Any periodic function y of a variable t may be plotted as a wave as shown in Fig. 3. If the function y and its first derivative with respect to t (that is, dy/dt) are finite and continuous for all values of t, then the function y may be represented by a series of terms of the form

$$y = Y_1 \sin(\omega t + \phi_1) + Y_2 \sin 2(\omega t + \phi_2) + Y_3 \sin 3(\omega t + \phi_3) + \text{etc.},$$

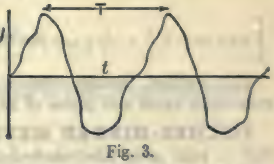

Fig. 3.

in which $\omega = 2\pi/T$, where T is the period of the given wave (i.e., the distance between successive positive, or between successive negative, maxima) and the Y's and ϕ's are constants. That is, the wave, no matter how complex, may be represented by the sum of a number of sine waves whose frequencies are even multiples of the period of the given wave.

This theorem is of great value in the theory of alternating currents (q.v.), since any alternating current or voltage satisfies the above conditions. The series is known as Fourier's Series.

As in the case of simple sine waves, it is usually more convenient in plotting

to take the distance along the horizontal, not the variable t, but the value of ωt. That is, putting $x = \omega t$, the series for y may be written

$$y = Y_1 \sin (x + \phi_1) + Y_2 \sin 2 (x + \phi_2) + Y_3 \sin 3 (x + \phi_3) + \text{etc.}$$

Fundamental and Harmonics. — The first term of the series, which has a frequency equal to that of the given complex wave, is called the fundamental. The succeeding terms, having frequencies which are multiples of the fundamental frequency, are called harmonics. For example, the third term, or the wave representing this term, has a frequency three times that of the fundamental and this term or sine wave is called the third harmonic, etc.

If the complex wave is such that the successive positive values of y for a positive half wave are numerically equal to the successive negative values of y for the succeeding negative half wave, only the *odd* harmonics occur. Such a wave is called a symmetrical wave. Voltage and current waves are usually symmetrical, and therefore contain only the odd harmonics.

DETERMINATION OF MAXIMUM VALUES AND PHASES OF THE HARMONICS BY INTEGRATION. — Consider the n th harmonic in the series

$$y = Y_1 \sin (x + \phi_1) + Y_2 \sin 2 (x + \phi_2) + Y_3 \sin 3 (x + \phi_3) + \text{etc.}$$

Plot the value of $u_n = y \sin nx$ between the limits $x = 0$ and $x = 2\pi$, and integrate the resultant curve, by planimeter, between these limits 0 and 2π. Call this area U_n. Similarly, plot the value of $v_n = y \cos nx$ between the limits $x = 0$ and $x = 2\pi$, and integrate the resultant curve, by planimeter, between these limits 0 and 2π. Call this area V_n. Then

$$Y_n = \frac{1}{\pi} \sqrt{U_n^2 + V_n^2}$$

and

$$\phi_n = \frac{1}{n} \tan^{-1} \left(\frac{V_n}{U_n} \right),$$

where the wave length of the fundamental corresponds to 360°. If the wave contains only the *odd* harmonics, as is usually the case in current and voltage waves (*see preceding paragraph*), U_n and V_n need be plotted only between the limits 0 and π. Calling U_n' and V_n' the corresponding areas, then

$$Y_n = \frac{2}{\pi} \sqrt{(U_n')^2 + (V_n')^2},$$

$$\phi_n = \frac{1}{n} \tan^{-1} \left(\frac{V_n'}{U_n'} \right).$$

This method, though cumbersome, is applicable to the determination of the maximum value and phase of any harmonic.

FISCHER-HINNEN METHOD OF ANALYSIS.— (*Elec. Journal, 1908, Vol. 5, page 386; Electrotechnische Zeitschrift, 1901, Vol. 22, page 396.*) This method is quite simple when the wave contains only the fundamental and the third, fifth and seventh harmonics. When even harmonics are present, or when there exist higher odd harmonics than the seventh, certain corrections must be applied. Since voltage and current waves usually contain only odd harmonics, and seldom contain higher harmonics than the seventh, the simple method without corrections is usually sufficiently accurate.

Waves Containing only the Third, Fifth and Seventh Harmonics. — To determine the nth harmonic (n equals 3, 5 or 7) divide the base of a half

wave into 2 n equal parts and measure the ordinates of the wave at the beginning of each of these sections of the base. Call these ordinates y_1, y_2, y_3 . . . y_{2n}, taking the y's positive if above the base line, negative if below. y_1 will be zero, since the first section begins where the resultant wave crosses the base line.

Then the ordinates of this harmonic at the points 1 and 2 are, respectively,

$$A_n = \frac{1}{n}[(y_5 + y_9 + \ . \ . \ . \ y_{2n-1}) - (y_3 + y_7 + \ . \ . \ . \ y_{2n-3})],$$

$$B_n = \frac{1}{n}[(y_2 + y_6 + \ . \ . \ . \ y_{2n}) - (y_4 + y_8 + \ . \ . \ . \ y_{2n-2})].$$

The maximum value of this harmonic is

$$Y_n = \sqrt{A_n^2 + B_n^2},$$

and the phase angle, calling the wave length of the fundamental 360°, is

$$\phi_n = \frac{1}{n} \tan^{-1} \frac{A_n}{B_n}.$$

These formulas give the third, fifth and seventh harmonics ($n = 3$, 5 and 7 respectively). The fundamental is found by calculating

$$A_1 = -(A_3 + A_5 + A_7),$$
$$B_1 = y_0 + B_3 - B_5 + B_7,$$

where y_0 is the mid-ordinate of the half wave. Then

$$Y_1 = \sqrt{A_1^2 + B_1^2},$$
$$\phi_1 = \tan^{-1} \frac{A_1}{B_1}.$$

The equation of the given wave is then

$$y = Y_1 \sin(x + \phi_1) + Y_3 \sin 3(x + \phi_3) + Y_5 \sin 5(x + \phi_5) + Y_7 \sin 7(x + \phi_7).$$

The *effective* value of the given wave is

$$Y = \sqrt{\frac{Y_1^2 + Y_3^2 + Y_5^2 + Y_7^2}{2}},$$

and the *average* value is

$$Y_{av'ge} = \frac{2}{\pi}\left[Y_1 \cos \phi_1 + \frac{1}{3} Y_3 \cos 3\phi_3 + \frac{1}{5} Y_5 \cos 5\phi_5 + \frac{1}{7} Y_7 \cos 7\phi_7\right].$$

In using the above formulas strict attention must be paid to algebraic signs.

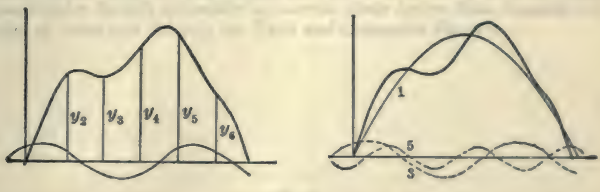

Fig. 4.

Example. — Find the third harmonic in the wave shown in Fig. 4. The values of the six ordinates are found by measurement to be 0, 676, 660, 940, 1004 and 554 respectively. Then

$$A_3 = \tfrac{1}{3}(1004 - 660) = 114.7,$$
$$B_3 = \tfrac{1}{3}(676 + 554 - 940) = 96.7,$$
$$Y_3 = \sqrt{(114.7)^2 + (96.7)^2} = 150,$$
$$\phi_3 = \tfrac{1}{3}\tan^{-1}\frac{114.7}{96.7} = 16.6°.$$

Similarly, for the fifth harmonic,

$$A_5 = -92.8 \text{ and } B_5 = 37.4,$$
$$Y_5 = 100 \text{ and } \phi_5 = -13.6°.$$

For the fundamental,

$$A_1 = -114.7 + 92.8 = -21.9,$$
$$B_1 = 940 + 96.7 - 37.4 = 999.3,$$
$$Y_1 = \sqrt{(21.9)^2 + (999.3)^2} = 1000,$$
$$\phi_1 = \tan^{-1}\frac{-21.9}{999.3} = -1.25°.$$

Hence the complete expression for the given wave, taking as the origin the point at which the resultant wave crosses the base line in the rising direction, is

$$y = 1000 \sin(x - 1.25°) + 150 \sin 3(x + 16.6°) + 100 \sin 5(x - 13.6°).$$

The *effective* value is then 718 and average value 673.

Waves Containing Any Number of Harmonics, Odd or Even. — See *Electric Journal*, 1908, Vol. 5, p. 386; and *Electrotechn. Zeit.*, 1913, Vol. 34, p. 936.

WEIGHTS (MASSES) OF MATERIALS. — (*See also Mechanics, Principles of.*) The density of any substance is the mass of that substance per unit volume. Or, using weight in the ordinary sense as equivalent to mass, the density may also be defined as the *weight* per unit volume. The numerical value of the density of any substance depends upon the unit in which the mass or weight is expressed and also upon the unit of volume used; see *Units and Conversion Factors*. However, it is quite common to state the density of a substance in grams per cubic centimeter, without naming the units, since when so expressed the density is numerically equal (practically) to the specific gravity.

The "specific gravity" of a substance is defined as the ratio of the weight (mass) per unit volume of that substance to the weight (mass), expressed in the same unit of an equal volume of water. To make such a statement exact the temperature of the water should be specified. There is no general agreement as to the temperature of reference, though water at 0° C. is commonly taken as the reference temperature. For gases, air at 0° C. and 760 mm. mercury pressure is frequently taken as the reference substance instead of water.

Variation of Density of Water with Temperature. — The following table gives the results of measurements by Thiesen, Scheel and Diesselhorst (*Landolt, Börnstein and Roth, Physikalisch-chemische Tabellin, 1913*).

DENSITY OF WATER; GRAMS PER CU. CM.

Degrees Cent.	0	1	2	3	4	5	6	7	8	9
0	0.99987	0.99993	0.99997	0.99999	1.00000	0.99999	0.99997	0.99993	0.99988	0.99981
10	0.99973	0.99963	0.99953	0.99940	0.99927	0.99913	0.99897	0.99880	0.99862	0.99843
20	0.99823	0.99802	0.99780	0.99756	0.99732	0.99707	0.99681	0.99654	0.99626	0.99597
30	0.99567	0.99537	0.99505	0.99473	0.99440	0.99406	0.99371	0.99336	0.99299	0.99262
40	0.99224	0.99186	0.99147	0.99107	0.99066	0.99025	0.98982	0.98940	0.98896	0.98852
50	0.98807	0.98762	0.98715	0.98669	0.98621	0.98573	0.98525	0.98475	0.98425	0.98375
60	0.98324	0.98272	0.98220	0.98167	0.98113	0.98059	0.98005	0.97950	0.97894	0.97838
70	0.97781	0.97723	0.97666	0.97607	0.97548	0.97489	0.97429	0.97368	0.97307	0.97245
80	0.97183	0.97121	0.97057	0.96994	0.96930	0.96865	0.96800	0.96734	0.96668	0.96601
90	0.96534	0.96467	0.96399	0.96330	0.96261	0.96192	0.96122	0.96051	0.95981	0.95909
100	0.95838	0.95765	0.95693	.....	.....	.....	.....	.....	.....	.....

* Example: The density of water at 33° C. is 0.99473.

Weights per Cubic Foot and Specific Gravity. — In the following table are given the values of the density in pounds per cubic foot of the more commonly used substances. The specific gravity, or density in grams per cubic centimeter, corresponding to any weight per cubic foot w is equal to $w/62.43$; for the conversion factors necessary to convert these figures into densities for other units of mass and volume, see *Units and Conversion Factors*.

SPECIFIC GRAVITY AND POUNDS PER CUBIC FOOT OF VARIOUS MATERIALS AT ROOM TEMPERATURES

Specific Gravities all referred to water at 0° C.

(See References at end of table)

Material	Lb. per cu. ft. From	Lb. per cu. ft. To	Average spec. grav.
Air* (2)	0.0809	...	0.00129
Acetylene gas* (2)	0.0733	...	0.00117
Aluminum, cast (1)	160	161	2.57
" wire (6)	168	...	2.70
Ammonia* (2)	0.0482	...	0.000771
Antimony (1)	414	...	6.64
Asbestos (2)	125	175	2.40
Asphaltum (1)	69	94	1.30
Basalt (3)	176	181	2.86
Bismuth (2)	604	618	9.78
Brass (1)	511	542	8.45
Brick, red (3)	111	128	1.92
" fire (3)	110	...	1.76
Bronze (1)	545	555	8.80
Carbon (2)	125	144	2.15
" dioxide* (2)	0.124	...	0.00199
" monoxide* (1)	0.0782	...	0.00125
Caoutchouc (1)	57	62	0.955
Cement, loose (1)	72	105	1.42
" set (1)	168	187	2.85
Charcoal (1)	17	35	0.421
Clay, hard (3)	129	133	2.10
" soft (3)	118	...	1.89
Coal, anthracite (5)	81	106	1.50
" anthracite piled loose (5)	47	58	0.84
Coal, bituminous (5)	78	88	1.33
" bituminous, piled loose (5)	44	54	0.79
Coal, lignite (3)	52	...	0.83
Cobalt (2)	530	563	8.77
Coke (1)	62	105	1.34
" piled loose (5)	23	32	0.45
Concrete, 1:2:4 (3)	146	...	2.34
" 1:1½:3 (3)	139	...	2.23
" 1:3:6 (3)	156	...	2.50
Copper, cast (1)	549	558	8.87
" wrought (1)	552	558	8.90
" wire (1) (6)	555	558	8.89†
Cork (5)	15.6	...	0.25
Ebonite (1)	72	...	1.15
Flint (1)	162	...	2.61
German silver (1) ($52\,Cu+26\,Zn+22\,Ni$)	527	...	8.44
Glass, common (1)	150	175	2.6
" flint (1)	180	280	3.7
Gold, cast (2)	1200	...	19.3
Granite (1)	125	187	2.5
Gravel (5)	90	147	1.9
Gutta percha (5)	61.1	...	0.980
Gypsum or plaster of Paris (5)	142	143	2.28
Hydrogen* (2)	0.00562	...	0.0000900
Ice (1)	55	57	0.895
Iridium (2)	1399	...	22.4
Iron, pure (1)	490	492	7.86
" gray cast (1)	439	445	7.08
" white cast (1)	473	482	7.65
" wrought (1)	487	492	7.85
" steel (1)	474	494	7.76
Lead (1)	710	...	11.4
Leather, dry (1)	54	...	0.86
" greased (1)	64	...	1.02
Lime (5)	53	75	1.03
Limestone (3)	156	162	2.55
Loam (3)	65	88	1.23
Marble (1)	157	177	2.68
Masonry (5)	100	165	2.12
Mercury at 0° C. (1)	849	...	13.6
Mercury at 20° C. (1)	846	...	13.5
Mica (1)	165	200	2.9
Molybdenum (2)	524	536	8.50
Mortar, hard (5)	103	...	1.75
Muck (3)	40	74	0.915
Mud (5)	80	130	1.68
Nickel (1)	540	550	8.75
Nitrogen* (2)	0.0782	...	0.00125
Nitrous oxide* (2)	0.0838	...	0.00134

* At a temperature of 0° C. and a pressure of 760 mm. mercury.

† This value has been adopted internationally as representing the average density at 20° C.; see reference.

SPECIFIC GRAVITY AND POUNDS PER CUBIC FOOT OF VARIOUS MATERIALS AT ROOM TEMPERATURES

Specific Gravities all referred to water at ° C.

(See References at end of table)

Material	Lb. per cu. ft. From	Lb. per cu. ft. To	Average spec. grav.
Oil, cotton-seed (1)	60.2	....	0.962
" gasoline (1)	41	43	0.675
" lard (1)	57.4	....	0.920
" linseed (1)	58.8	....	0.942
" mineral, lubricating (1)	56.2	57.7	0.912
Oil, petroleum (1)	54.8	....	0.878
" turpentine (1)	54.2	....	0.873
" whale (1)	57.3	....	0.918
Osmium (2)	1400	....	22.5
Oxygen* (2)	0.0895	....	0.00143
Palladium (2)	686	749	11.4
Paper (1)	44	72	0.92
Paraffine (1)	54	57	0.89
Pitch (1)	67	....	1.07
Platinum (1)	1320	1350	21.4
Porcelain (1)	143	156	2.4
Pumice stone (1)	23	56	0.63
Quartz (1)	165	....	2.65
Rhodium (1)	686	755	11.5
Salt (5)	50	70	0.965
Sand (5)	90	120	1.68
Sandstone (1)	124	200	2.6
Selenium (2)	300	....	4.8
Silver (1)	650	657	10.5
Slate (1)	162	205	2.85
Snow, fresh fallen (5)	5	12	0.136
" wet compact (5)	15	50	0.520
Soapstone (1)	162	175	2.7
Steel *(see Iron)*	...	....	....
Sulphur (1)	120	130	2.05
Tantalum (2)	1040	....	16.7
Tar (5)	62.4	....	1.00

Material	Lb. per cu. ft. From	Lb. per cu. ft. To	Average spec. grav.
Tile, hollow terra cotta, building block (3)	26	38	0.51
Tile, flat and segmental arches (3)	31	45	0.608
Tile partitions† (3)	12	26	....
Tin (1)	455	....	7.29
Trap rock (5)	187	190	3.02
Tungsten (1)	1160	1190	18.8
Turf (5)	20	30	0.400
Water, max. density (2)	62.4	....	1.00
" sea (5)	64.0	64.3	1.03
Wax, bees (1)	60.5	....	0.965
Wood, ash (4)	45	47	0.737
" butternut (4)	28	....	0.448
" cedar (4)	37	38	0.600
" chestnut (4)	38	41	0.633
" cypress (4)	32	37	0.553
" elm (4)	35	36	0.569
" fir (4)	34	35	0.553
" hemlock (4)	25	29	0.432
" hickory (4)	53	58	0.890
" lignum vitae (4)	78	83	1.29
" mahogany (4)	32	40	0.577
" maple (4)	49	50	0.793
" oak (4)	37	56	0.745
" pine, white (4)	24	25	0.392
" " yellow (4)	34	45	0.633
" poplar (4)	24	27	0.424
" red wood (4)	30	....	0.481
" spruce (4)	25	32	0.457
" walnut (4)	38	45	0.649
Zinc (1)	428	448	7.10

* At a temperature of 0° C. and a pressure of 760 mm. of mercury.

† Including air spaces.

REFERENCES. — (1) *Smithsonian Physical Tables*, (2) *Physikalisch-Chemische Tabellen*, Landolt-Börnstein-Roth; (3) *Investigation of Weights of Building Material*, Thesis, Mass. Inst. of Tech., 1913, Orr, S. W., and Mutersbaugh, A. M.; (4) *Publications of Forestry Division, U. S. Dept. of Agriculture*, Bulletin No. 10; Circular No. 32; Circular No. 115; (5) Trautwine, J. C., *Civil Engineers' Pocket Book;* (6) *Copper Wire Tables*, Cir. No. 31, Bureau of Standards.

WELDING, ELECTRIC. — The heat of the electric current is used in various processes for welding metals, both ferrous and nonferrous. These processes may be tabulated as follows:

General designation	Varieties		Kind of current at weld
Resistance welding	Butt welding		A-C.
	Spot welding		A-C.
	Seam (or line) welding		A-C.
Percussive welding	Using discharge from static condenser		D-C.
	Using electro-magnetic discharge		D-C.
Arc welding	Carbon arc		D-C. (though A-C. may be developed)
	Metal arc	With bare electrodes	Usually D-C. but occasionally A-C.
		With flux covered electrodes	Either D-C. or A-C.

SOME APPLICATIONS OF ELECTRIC WELDING. — Arc Welded Boats.— A good many years ago the Seamless Steel Boat Company of Wakefield, England, built successful all-steel life boats in which the joints in the shell plating were welded by the carbon arc process before being pressed into shape. The *Dorothea M. Geary*, a boat with a length of 42 feet and a width of 11 feet and driven by a 50 horsepower gasolene engine, was built with most of the jointing done by arc welding using bare metal electrodes. It was launched on Lake Erie in 1915.

In 1918 the British Admiralty, using flux-covered electrodes, built an arc welded 275-ton Cross Channel barge. It is 125 feet in length and 16 feet wide. In 1919 there was built in France an arc-welded barge 65 feet long by 13 feet wide. The welding work was all done by women. In 1920 the *Fullagar*, a 500-ton coaster built by Messrs. Cammell Laird and Company, and entirely arc welded, was launched at Birkenhead. The *Fullagar*, which is driven by an internal-combustion engine of the type designed by the late Mr. H. F. Fullagar, has a speed of 10 knots and a length of 150 feet.

The above cases are mentioned as being of interest because the displacement of riveting by arc-welding for making the joints in the shell plating, foreshadows an impending immense field for arc welding.

But already there is a very large amount of arc welding in ship construction and repair. In the space of this article their mere enumeration would be impossible. As typical may be mentioned: repairs of boilers, stern frames, stems, tail-end shafts, windlasses, shaft-brackets, rudder gudgeons, and fuel tanks. In battleships, anti-submarine bulges have been attached to hulls and their

own shell plates have been arc welded, such undertakings exceeding in magnitude the welding of complete vessels of moderate tonnage.

In "The Times Engineering Supplement," for April, 1920, Caldwell enumerates as follows the advantages of arc welding in ship construction:

(1) With the elimination of marking off, drilling or punching, countersinking and caulking, a considerable economy in time and labor can be effected.

(2) By the reduction in width of laps and size of butt straps a saving of from 5 to 10 per cent of material can be effected.

(3) Welded joints can be made to withstand greater stresses than similar riveted joints, and can be made equal in strength to the ship plating.

(4) Once the practice is standard a reduction in the weight of the shell plating will be possible without reducing the safety factor.

(5) A welded joint is oil and water tight up to its breaking point, whereas riveted and caulked joints are liable to leak badly at as low as 25 per cent of their ultimate strength.

(6) While these may be described as general advantages, there are specific cases where electric arc welding possesses even greater attractions. For example, the present expensive smith-work involved in making complicated knees, angle collars, frames, etc., can be replaced by notching, bending up, and welding the angles at a fraction of the present cost, and it provides an ideal method of securing fittings, etc., to water-tight bulkheads without piercing them, and of making water-tight joints round pipes, etc., where these have to pass through bulkheads.

Arc Welding in Railway and Other Shops. — Several of America's great railways each employ many dozens of arc welding operators in repair work on locomotives and are thereby effecting great economies. In street railway shops similar economies are obtained; as also in boiler shops, foundries, blast furnaces, breweries, machine shops and mines.

Welding in Manufacturing. — Not alone arc welding, but also (already on a very large scale) resistance welding, are being applied in the manufacturing of all sorts of products of steel and other metals. The use of such methods is increasing at an enormous rate.

Welding Steel Buildings. — In England the steel work of the roofs of several large buildings has been joined entirely by arc welding and the saving of time and money as compared with riveting has been shown to be very great. This bids fair to be a very important field for electric welding, both arc welding and resistance welding.

RESISTANCE WELDING. — The art of welding metals by passing a low-voltage alternating current across the surface of contact of the metals to be joined, was worked out and made a commercial success by Professor Elihu Thomson in the 80's. This process, which is known as resistance welding, is in such wide use and is so familiar to all engineers that there is no occasion for more than the briefest review.

For any given application of resistance welding, the engineering problems usually relate chiefly to the design of machines for holding the parts, bringing them into suitable juxtaposition, regulating the mechanical pressure and rate of advance of the parts toward one another at various stages of the process of heating the metal at the surfaces of contact, arresting the process at the proper instant, and the regulation of the current to the most suitable amount at all stages of the process. Careful consideration requires also to be given to the sizes and shapes of the two surfaces to be brought together and of the proportions of the parts in the neighborhood of these surfaces. For some applications the best results are obtained by welding a third (and otherwise superfluous) piece of metal to each of the two parts to be joined.

Most metals in reasonable states of purity can be joined to one another by the resistance process. Welds can also be made between some alloys. Practically all grades of steel are readily welded. Different metals can also, in some instances, be welded to one another. The current, for resistance welding, is usually supplied from the low-voltage secondary of an alternating current transformer, but direct current might under certain circumstances be used to advantage.

Butt Welding. — When, in the resistance process, the parts to be welded to one another are "butted" together, the current passing through the entire section of metal near the weld, the term butt welding is often employed. In all resistance welding the heat is generated within the metal and not brought in from the outside as in other processes.

Spot Welding. — When, in resistance welding, the current is passed transversely across two plates which it is desired to joint at many places (or spots) the term spot welding is employed. Spot welding has been employed for many years for thin plates of metal. It has recently been applied to the welding of steel plates of thicknesses up to one inch. This latter development is described more fully in a later paragraph.

Seam Welding. — When, in resistance welding, the point of introducing and carrying away the current moves along a line so as to make a continuous weld along that line, the term seam welding, or line welding, is employed.

SPOT WELDING OF THICK PLATES. —Spot welding of thick plates is an alternative to the long established process of riveting and bids fair to be very extensively employed in most kinds of work now using rivets. As important examples may be mentioned the construction of steel ships, steel buildings and, in general, in many uses of structural steel. Where the parts can be brought to the welding machine, the process is especially attractive. But portable spot welders have been developed and have been used with success in ship construction.

Duplex Spot Welder. — (Fig. 1). — The usual construction of commercial spot welders for use in shipbuilding is similar in general appearance to so-called bull-riveters. The largest spot welder yet built for actual use in ship fabrication has a six-foot gap and operates from a 60-cycle circuit. This outfit is a stationary machine in the frame of which two transformers are incorporated and to which the steel plates and shapes are brought. Bulkheads, frames, floors and other parts are constructed with it, and are then transported by cranes to their places in the ship. This six-foot-gap machine has the capacity to weld two three-quarter-inch-thick plates. It provides a pneumatic pressure of 60,000 pounds and a current of 50,000 amperes. It is called a duplex welder and welds simultaneously two spots, each of some 1.5 inch diameter, in about 30 seconds.

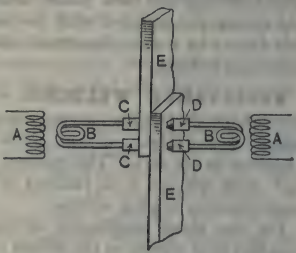

Fig. 1. Duplex Spot Welder

The arrangement is shown diagrammatically in Fig. 1. The outfit comprises two transformers, one located on each side of the plates to be welded. The current crosses the plates in one direction between two electrodes and then back again between two other electrodes. Thus the two secondaries and the two joints make four elements connected in series.

AA represent the two primaries.

BB represent the two secondaries (which, in the actual construction, have only one turn each).

CC and *DD* represent water-cooled, copper electrodes.

EE represent the two plates to be joined.

The above-described duplex feature was introduced in this spot-welder to decrease the inductance of the long loop of conductor which otherwise would have been necessary and which would have involved proportioning the transformer secondary for a higher voltage to overcome the inductive drop.

Portable Spot Welder. — Fig. 2 is a diagram of the circuits of an ordinary (simplex) spot welder which welds only one spot at a time. In this figure:

A represents the primary of the transformer.

B represents its secondary.

C and *D* represent water-cooled copper electrodes.

E and *F* represent the two plates to be welded.

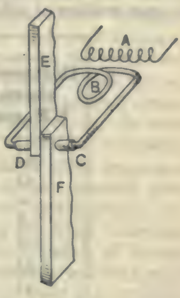

Fig. 2. Simplex Spot Welder

A typical 60-cycle portable welder of this type and having a 27-inch gap weighs only 2800 pounds, including the transformer (which is embodied in the structure), and welds plates of a thickness of half an inch with a current of about 30,000 amperes. The required pressure for such work is some 25,000 pounds.

For experimental work, currents up to 100,000 amperes have been employed in improvised spot welding machines, and pairs of one-inch-thick steel plates have been welded.

Freight Car. — Although until very recently spot welding has generally been employed only for joining thin plates, mention should be made of the 50-ton gondola freight car which was completely spot welded in 1911 by Mr. J. A. Osborne, Chief Engineer of the American Car and Foundry Company of St. Louis, Mo. Ever since that time, this car has been in regular service on the C. B. & Q. Railway. In 1918, Mr. Osborne reported that the welded joints have successfully withstood the severe stresses of continuous hard use. The spot welder built by Mr. Osborne in 1911 for this undertaking was demonstrated to be capable of making sound welds with plates of ¾-inch thickness. It had a 66-inch throat and housed an 85-kw. transformer with a secondary giving 25 volts on open circuit. The welding points of the copper electrodes had a diameter of ¾ inch.

Spot Welding Versus Riveting. — Spot welding would appear to have decided advantages over riveting for a great variety of important work now usually done by riveting. By employing spot-welding great savings are effected through eliminating the laying out and punching of the rivet holes, and the subsequent reaming when the holes do not match up. Furthermore there is avoided the reduction in cross-section and strength occasioned by the necessity for the rivet holes. With spot welding there is a great saving in material through the reduction in overlap made practicable as compared with the two or three rows of rivets which often have to be employed. Tests show that greater strength can be obtained with a single row of spot welds than with the best triple riveting.

Speed of Spot Welding. — At p. 175 of Vol. 38 (1919) of the Trans. A.I.E.E., Mr. John Martin of the American Bureau of Shipping reports as follows:

"In one test performed in the writer's presence, 32,000 amperes with a volt-

age of 4, successfully spot-welded pairs of ⅝ inch to 2.5 inches ship steel plates together, the diameter of the spots being 1 inch, the pitch 3 inches, and the hydraulic pressure 7½ tons. The time of the current was 15 seconds. 15 feet was welded in 30 minutes. The spots sheared at 56,000 pounds, or partly pulled and sheared at 60,000 pounds. Water and oil-tight joints resulted without calking. For plates of the same pitch, calking or arc fillet welds would be required for water-tight joints; though by lessening the pitch and decreasing the diameters of the spots, practically the same water-tight results could be had."

Power Required for Spot Welding. — In a paper entitled *Practical Applications of Electric Welding*, read by F. P. Vaughan on October 15, 1920, before the Engineering Institute of Canada, the following table of data applying to the spot welding of mild steel plates is given:

Thickness of plates, S.W. gage	Total welded thickness, inch	Power, kw.	Time, seconds
24	0.044	4.3	0.6
22	0.056	4.6	0.8
20	0.072	4.7	1.0
18	0.096	5.3	1.2
16	0.128	6.3	1.8
14	0.160	6.6	1.8
12	0.208	8.0	2 5
10	0.256	9.0	3.0

Spot Welding Combined with Arc Welding. — Arc welding (described later) may be used in combination with spot welding, the former being relied upon not only for calking but to contribute to the total strength of the weld. Some tests comparing ordinary riveted joints with this method of spot and arc jointing are described at page 95 of Vol. 38 (1919) of the Trans. A.I.E.E. as follows:

FIG. 3 FILLET AND SPOT WELDED

FIG. 4 FILLET WELDED

FIG. 5 RIVETED AND FILLET WELDED

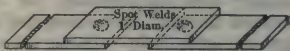

FIG. 6 SPOT WELDED

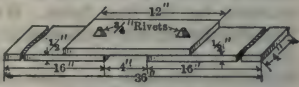

FIG. 7 RIVETED JOINT

The specimens were made up of the following combination:

1. Spot and fillet welded, Fig. 3 (two samples made).
2. Fillet welded—made by welding fillets about two inches in length at the ends of the plates, Fig. 4 (two samples made).
3. Riveted and fillet welded, Fig. 5 (one sample made).
4. Spot welded—made by welding two spots approximately one inch in diameter, on the plates, Fig. 6 (two samples made).
5. Riveted joint, made by riveting a 0.5-inch by 4-inch by 12-inch plate with two plates 0.5-inch by 4-inch by 16-inch, using two ¾-inch rivets and a 4-inch lap. Fig. 7 (one sample made).

The results of the test show the comparative strength of the joints as follows:

	Ultimate load, Pounds
Spot and fillet welded (average of tests on two samples)	50,350
Fillet welded (average of tests on two samples)	37,500
Riveted and fillet welded (only one sample made)	35,000
Spot welded (average of tests on two samples)	28,000
Riveted joint (only one sample made)	13,000

Wide Use of Resistance Welding. — At p. 146 of Vol. 38 (1919) of the Trans. A.I.E.E., Mr. Hermann Lemp, a pioneer in resistance welding, in comparing resistance welding with arc welding (described later), writes as follows:

"When one considers the vast quantities of mild steel products daily welded in the United States alone (by the resistance process of Professor Elihu Thomson) and this, with unskilled persons operating automatic and semi-automatic machinery, with a speed unexcelled by the arc-welding process, the writer may be pardoned for drawing attention to one characteristic feature of the process; namely, that parts to be welded are substantially protected against contact with the oxygen of the air, and what few oxides may be formed near the edges are effectively squeezed out of the joint by the pressure exerted on completing the weld. In addition, all kinds of metals and alloys, some of which are termed unweldable, are successfully welded thereby. Crane chains, harness rings, automobile rims, bicycle tubing, wire fences, carriage tires, metal wheels, etc., are welded daily in such quantities as would astonish any one when confronted with actual figures. In the welding of valve flanges to cylinders of Liberty Aviation motors at Detroit, the Thomson Process (of resistance welding) easily surpassed the gas and electric-arc processes, as regards both speed and excellence of finished product."

SPEED OF BUTT WELDING BY RESISTANCE PROCESS AND AMOUNT OF ELECTRICITY CONSUMED. — Mr. Vaughan gives the following table as applying to the butt welding of "ordinary bar iron, wheel rims, tires, cycle rims, shafting and piping and various forms of rods, etc." by the resistance process.

Section of metal at the weld, square inches	Approximate power consumption		
	Maximum kw. per weld	Duration of weld, seconds	Kw.-hr. per weld
0.25	10	15	0.075
0.50	15	20	0.13
0.70	20	30	0.2
1.0	25	35	0.3
2.0	40	60	0.7
3.0	50	90	1.25
5.0	65	208	3.75
7.0	80	315	7.0
9.0	95	400	10.5
10.	105	428	12.5
11	115	453	14.5
12	125	475	16.5

For butt welding round copper rods by the resistance process, Mr. Vaughan's values are:

Size		Power, kw.	Time, seconds
Diameter, inches	Area, square inches		
¼	0.05	5	4
⅜	0.11	7	5
½	0.20	14	7
⅝	0.31	19.5	9
¾	0.44	28	12
⅞	0.60	35	14
1	0.79	48	17
1⅛	0.99	63	21

SEAM (OR LINE) WELDING. — A seam welder may be so designed as to have the weld made between a wheel rolling on a bar, or between two wheels rolling together. For thick plates, the design with two wheels rolling together is probably the most suitable. Mr. R. E. Wagner, who has had wide experience in the design and use of seam welders, has given the writer the following tabulated data applying to the seam welding of mild steel plates:

Thickness of each plate, inch	Speed making line weld, in feet, per minute	Corresponding current, amperes	Pressure in pounds per square inch
0.0140	5	3500	2,200
0.0625	2.5	7000	13,000
0.0938	2.5	8000	22,500

In the above table the pressure is calculated on the basis of having a contact area in each case of 1/32 square inch.

The voltage of the welding circuit on open circuit may vary from 2 to 10 volts, depending upon the impedance introduced into the welding circuit by its design. The actual drop across the weld will vary between ¼ and 1 volt. The plates at the line of welding must be free from scale and rust. A properly made seam weld has a strength at least equal to that of the plates which are welded together.

PERCUSSIVE WELDING. — Mr. L. W. Chubb invented the process of percussive welding in 1905.

Percussive Welding with Static Condenser. — At that time the method consisted in discharging a static condenser simultaneously with the delivering of a percussive blow in a direction to force together the two metal parts to be welded. The process was applied chiefly to welding together relatively small wires, the largest being No. 12 B. & S. gauge. The ends of the wires are cut with suitable

wire cutters and the resulting chisel edges arranged to make a right-angle intersection, thus starting the discharge of energy at a central point of contact.

The current density is stated to be about 400,000 amperes per square inch. This process is suitable for welding together wires of utterly different material such as platinum to lead, tungsten to aluminum, and aluminum to copper.

Electromagnetic Percussive Welding. — More recently an electromagnetic process of percussive welding has been developed by Mr. Chubb and is adapted to quite heavy work. Stored up electromagnetic energy is released under such conditions as to establish a short arc between the surfaces to be welded. This arc is traversed by a very heavy current for less than a tenth of a second and suffices in that time to melt these surfaces. They are immediately forged together by a hammer blow. The secondary current approximates 30,000 amperes per square inch at an arc drop of from 20 to 40 volts, and it only flows for a small fraction of a second. The heat is so localized that the weld is effected with the expenditure of a minimum of energy. It is claimed that the energy consumed per weld is only a very small part of that required in resistance welding. With the electromagnetic process of percussive welding parts up to ½ inch diameter are readily welded and it is anticipated that the method will soon be developed to be applicable to much larger welds.

Field of Application for Percussive Welding. — It is claimed that percussive welding permits of readily welding together very unequal sections, since the short duration of the process precludes difficulties due to unequal heating conduction or difference in melting points. For example, small rods may be welded to heavy plates. In tool manufacture, steel alloys may be welded to cold rolled steel. Brass and copper parts may, by the percussive process, be welded to steel parts. Percussive welding has been described in more detail in a paper presented by Mr. Chubb before the American Electrochemical Society, October, 1914, and more recent developments have been described by Mr. D. F. Miner before the A.I.E.E., in June, 1920.

ELECTRIC ARC WELDING. — The technique of electric arc welding has practically nothing in common with that of butt and spot welding, although substantially identical objects are in some cases obtained by both methods. Often, however, the associated circumstances will indicate which of the two kinds of electric welding should be employed. While butt welding and spot welding are applicable to almost all kinds of metals, electric arc welding is chiefly confined to the welding of steel.

Before entering upon the consideration of electric arc welding, it should be pointed out that it is one of the two main subdivisions of an art which is often termed **autogenous welding**, but which will here be designated **fusion welding.**

Fusion Welding. — The term fusion welding is employed to comprise gas welding and electric arc welding. Gas welding does not come within the scope of this article, and it must suffice to state that the process in its simplest form consists in simultanelusly fusing with an oxyacetylene flame (1) the material at and near the surfaces which it is desired to join, and (2) some material (which is usually similar in composition) in the form of a rod, the tip of which is subjected to the heat of the flame. The oxyacetylene flame is directed with one hand and the welding rod is manipulated with the other hand.

Kinds of Electric Arc Welding. — Electric arc welding may be subdivided into several classes. The two broadest classes are:

a. Carbon arc welding.
b. Metal arc welding.

In carbon arc welding, an arc (usually several tenths of an inch in length) is established between a carbon or graphite electrode (usually a carbon electrode) and the two pieces of steel which it is desired to join. This carbon electrode is manipulated with one hand and a welding rod is fed into the weld by the other hand. The manual activities in carbon arc welding are seen to be quite similar to those in gas welding. In neither case is it necessary for the material of the welding rod to traverse the arc.

In metal arc welding, there is a fundamental difference in this latter respect, since in metal arc welding of mild steel, instead of having a carbon electrode for one terminal of the circuit, the arc is established between a steel welding rod (or welding electrode) and the two steel parts requiring to be joined. There is always a distance of preferably not over one-tenth of an inch between the end of the welding rod and the work. This distance is bridged by an electric arc.

Kinds of Work Not Requiring Welding Rod. — Both for carbon arc welding and gas welding, the edges of the parts to be joined sometimes may be so designed as to obviate the need for any additional material; in other words. no welding rod is necessary in such cases. In Fig. 8 is shown a case where it is not necessary to use a welding rod, since it suffices to simply melt the adjacent flanges with an arc.

Fig. 8.

Comparison of Cost of Gas Welding with Cost of Arc Welding. — Quite a thorough comparison of gas welding with metal arc welding was made in 1917 by Major James Caldwell, then of the Controller's Department of the British Admiralty. This comparison is reproduced at p. 85 of Vol. 38 (1919) of the Trans. A.I.E.E. Since that date there has been a great deal of careful study given to the subject. For thoroughly first class welds, the speeds formerly credited to gas and arc welding are not obtained even by the most skillful operators. The author has made every effort to arrive at reasonable values for the various data involved and has compiled the results in the following table as representative of his opinion. The table indicates that gas welding has the advantage over metal arc welding for plates of 1/16 inch thickness, both as regards speed and cost. From 1/8 inch to 3/8 inch thickness, the author credits both methods with the same speed, but the advantage as regards lower cost increases rapidly for the metal arc method with increasing thickness of plate. For thicknesses of 7/16 inch and 1/2 inch both speed and cost are very decidedly in favor of metal arc welding. The costs (1921 basis) by the two methods are plotted in the curves in Fig. 9. Plenty of authority can be found for differing with the present author's results, since speeds over twice as great as those in the table have often been claimed by welding specialists as being representative. The speeds in the table are such as can be maintained by a fairly skilled welder during an eight-hour day and under thoroughly favorable conditions. The plates to be welded are assumed to be laid flat and at a convenient height for the operator to work. It is also assumed that the edges have been prepared and that the plates are clamped and all ready for welding. As regards the consumption of electricity it is assumed that the welding is done from a 110 volt circuit with a rheostat in series with the electrode. With a motor generator set the consumption would not be enough different to much affect the aggregate cost of (1) electricity, (2) electrodes, and (3) labor.

Welding Machinery. — The voltage at the arc is only some 15 to 20 volts, consequently **while welding,** some 85 per cent of the electricity consumed is wasted in the rheostat when operating from a 110-volt circuit. By interposing a motor-generator set transforming to lower voltage, the efficiency **while welding** may be considerably increased notwithstanding the losses in the motor-

COMPARATIVE COSTS OF MANUAL BUTT WELDING FOR VARIOUS THICKNESSES OF MILD STEEL

Thickness of metal in inches	Oxy-Acetylene							Metal arc (bare electrodes)					
	Cubic feet of gas per hour		Speed of welding, feet per hour	Component costs, cents per foot run			Total cost, in cents per foot run	Speed of welding, feet per hour	Kw.-hr. per foot of weld	Component costs, cents per foot run			Total cost, cents per foot run
	Oxygen at 2½ cts.	Acetylene at 2½ cts.		Gas	Iron wire 12 cts. per pound	Labor 80 cts. per hour				Electricity 3 cts. kw.-hr.	Electrodes 12 cts. per pound	Labor 80 cts. per hour	
1/16	6	5½	24	1.2	0.3	3.3	4.8	16	0.20	0.6	0.4	5.0	6.0
⅛	13	12	13	4.8	0.6	6.2	11.6	13	0.35	1.1	0.7	6.2	8.0
3/16	19	17	10	9.0	0.9	8.0	17.9	10	0.60	1.8	1.2	8.0	11.0
¼	30	27	8	17.8	1.1	10.0	28.9	8	0.80	2.4	1.6	10.0	14.0
5/16	36	32	6	28.5	1.3	13.3	43.1	6	1.10	3.3	2.2	13.3	18.8
⅜	40	36	4½	42.2	1.5	17.8	61.5	4½	1.45	4.4	2.9	17.8	25.1
7/16	43	39	3	68.3	1.7	26.6	96.8	3½	2.00	6.0	4.0	22.9	32.0
½	45	41	2½	86.0	1.9	32.0	119.9	3	3.00	9.0	6.0	26.7	41.7

generator set. But in the intervals between welds there are **no** losses with the former outfit, while with the latter outfit the running light losses in the motor-generator set detract from the "all-day" efficiency. It will be readily seen

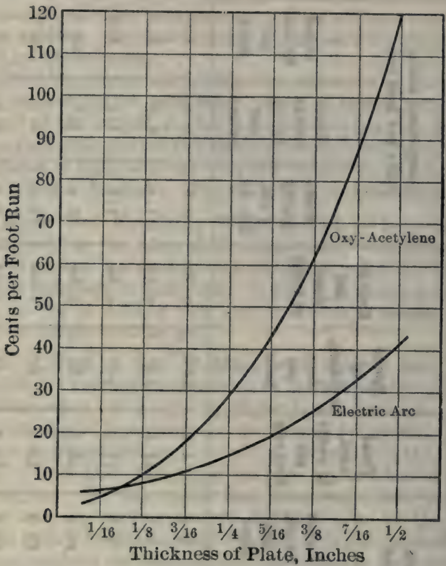

Fig. 9.

that all the attendant circumstances must be taken into consideration in determining upon the most suitable plant to install for arc welding.

Each of several manufacturers has placed on the market types of welding apparatus with characteristics for which advantages are claimed and realized. Even brief descriptions would occupy too much space for this article. Furthermore it is the author's opinion that substantially equally good arc welding can be done with any of the best known types of machines, and it resolves itself down to a question of relative costs and economy of operation. These features should be given careful comparative study before purchasing a welding plant, but it is much more a question of economics than of obtaining the best welds. This latter object depends chiefly on having good operators and good materials.

With a-c. welding a transformer is employed, but while the efficiency is good, the power factor is very bad. This does not disqualify a-c. arc welding, but should be taken into account in balancing the advantages and disadvantages.

Speed of Arc Welding and Energy Consumption. — The speeds of making butt welds by the arc process with various thicknesses of steel set forth in the preceding table are not necessarily applicable to other than the material there employed and all the attendant circumstances. The attainable speed for first class arc welds is a function of (1) the skill and disposition of the operator, (2) the kind of circuit from which he welds and the characteristics of the intermediary welding apparatus and the voltage, (3) the length of the arc which the opera-

tor holds (the shorter the arc, the better the work, but the slower the speed), (4) the kind and quality of the electrodes employed, (5) the positions of the pieces to be joined (whether flat on a bench at a convenient height as the one extreme, or overhead or inaccessible as the other extreme), (6) the kind and quality of the steel to be welded, (7) the thickness or size of the material to be welded, (8) the preparation of the edges and surfaces.

Fig. 10

Fig. 11

Fig. 12

Fig. 13

In an article by Mr. William Spraragen entitled *Speed of Metal Arc Welding*, published at p. 44 of the *Welding Engineer* for June, 1920, the author refers to Figs. 10, 11, 12 and 13 as typical designs for welded joints and, based on some extensive tests which he has made, suggests as average speeds capable of being sustained for hours when welding half-inch thick ship plates, and allowing reasonable time for changing electrodes and making minor adjustments.

Number of figure	Speed of welding
10	1.7 feet per hour
11	1.5 feet per hour
12	2.3 feet per hour
13	2.5 feet per hour

Speed of Arc Welding in Pounds of Electrode Material Deposited per Hour. — Spraragen states that the average results of a vast amount of data show that an operator can deposit about 1.2 pounds of metal per hour when working out in the open (as on ships), with the usually attendant general inconvenience and also the disadvantage of liability of air currents; as against about 1.8 pounds of metal per hour in the shop. Both these figures are based on the assumption that the work has been lined up and is ready for welding.

TIME, METAL AND CURRENT USED WITH WELDS OF DIFFERENT BEVELS

Total opening of bevel in degrees	30	60	90	120
Ultimate strength in pounds per sq. in.	44,000	51,000	45,300	48,800
Angle of bend in degrees at which cracks first start	9	15	13	14
Amperes	160	145	118	125
Weight of electrode used up (pound)	2.56	3.83	4.63	6.63
Weight of metal deposited (pound)	1.70	2.53	3.63	5.08
Weight of metal wasted (pound)	0.86	1.28	0.98	1.55
Pounds deposited per hour	1.82	1.61	1.82	1.81
Feet welded per hour	3.22	1.90	1.50	1.07
Circuit kilowatts	9.91	9.00	7.68	8.25
Kilowatt-hours per foot of weld	3.10	4.70	5.10	7.70
Pounds deposited per foot of weld	0.57	0.85	1.21	1.69
Pounds deposited per kw. hour	0.18	0.18	0.24	0.22
Pounds of electrode used up per kw. hour	0.27	0.27	0.30	0.29

Allowance for Waste of Electrode Material. — Based on researches involving the welding of ten tons of half-inch-thick ship plate, Spraragen has concluded that out of 100 per cent weight of electrode,

70 per cent is deposited in the weld,
12 per cent is burned or vaporized,
18 per cent is wasted as short ends.

Angle of Bevel of Welding Edges. — As to the effect of the preparation of the welding edges on the speed of welding and the quality of the weld, tests were made by Spraragen at the author's request. In each case ½-inch thick ship plate was used, and a weld 3 feet long was made in a flat position with a 5/32-inch diameter bare electrode and a "free distance" of ⅛ inch. The welded plates were cut up and subjected to standard tensile and bending tests. The results are shown in the table at bottom of page 1955.

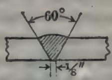

Fig. 14

Fig. 15

Preparation of Bottom Edges. — In Figs. 14 and 15, Spraragen shows two specimens of half-inch plate both prepared with 60-degree edges, and with the same "free distance" of ⅛ inch. In Fig. 14 the bottom edges are sharp, while in Fig. 15 shoulders are provided. The results were:

Fig. No.	Speed of welding, feet per hour	Pounds of electrode wire needed per foot of weld	Pounds of electrode wire used up per hour
14	2.3	1.1	2.5
15	2.8	0.9	2.5

Wagner's Tests of Relation of Speed to Current. — With skilled welders the speed of arc welding may be increased considerably when larger currents are employed. Also, up to a certain value, the strength of the weld is greater the greater the current. This is brought out by Mr. R. E. Wagner in a series of tests on ½-inch tank-steel plates. The plate edges were prepared with a

Current in amperes	Size of electrode, inch	Tensile strength in pounds per sq. in. after grinding off the raised parts of the welds
80	5/32	37,000
125	5/32	50,000
150	5/32	57,000
180	5/32 and 3/16	63,000
220	5/32 and 3/16	66,000
275	3/16	62,000
300	3/16	58,000

double V. Two sizes of Roebling electrodes were used, 5/32-inch and 3/16-inch. The pressure was maintained constant and the d-c. current was varied from 80 amperes up to 300 amperes. The results are shown in the preceding table.

The type of joint is illustrated in Fig. 16 under the curve plotted with currents as abscissæ and tensile strength as ordinates. Wagner stated that "reduction in tensile strength beyond 220 amperes is due apparently to the increased porosity of the deposited metal and difficulty in manipulation." For a more complete record of Mr. Wagner's investigations reference should be made to pages 157 to 160 of Vol. 38 (1919) of Trans. A.I.E.E.

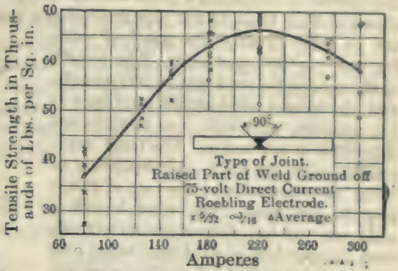

Fig. 16. Tensile Strength versus Current, Arc Welding.

Further Data of Speed and Cost of Arc Welding with Bare Electrodes. — In a 1920 General Electric Company bulletin the following data are given for the speed and cost of metal arc welding with bare electrodes and of gas welding:

Metal arc welding			Comparative cost per foot for acetylene welding
Thickness of plate, inch	Speed, in feet per hour	Cost per foot, cents	
1/16	20	2.1	1.8
1/8	16	3.1	4.7
1/4	10	7.1	13
3/8	6.5	12	36
1/2	4.3	20	Much higher
3/4	2.0	42	Much higher
1	1.4	61	Much higher

Suitable Size of Electrode and Current for Various Thicknesses of Steel. — In the following table Sparagen gives representative values for the size of electrode and the current to be employed in welding mild steel plate.

Electrode diameter, inch	Plate thickness, inch	Amperes
1/16	up to 1/8	25 to 40
3/32	up to 1/4	50 to 90
1/8	3/16 to 1/2	75 to 160
5/32	3/16 up	125 to 200
3/16	1/2 up	175 to 250

Data of Speed and Cost of Welding a Large Steel Tank with Bare Electrodes. — In 1918, Wagner welded a 12-foot-cube tank of half inch steel and purposely employed a great variety of types of joint including those already shown in Figs. 10 to 13 and also those shown below in Figs. 17 to 20, and a number of other types. The speed of welding was 3.0 feet per hour. The weight of the steel in the tank was 16,000 pounds, and the weight of electrode used up was 334 pounds, of which 299 pounds was deposited in the welds. The total welding time was 165 hours, corresponding to using up electrodes at the rate of just 2 pounds per hour. The total length of weld was 501 feet, the weight of electrode used up per foot of weld thus being 0.60 pound. 0.29 pound of welding wire was deposited and 0.33 pound was used up per kilowatt hour of electricity. The diameter of the welding wire was 3/16 inch. Six different operators worked on this job and the average current per operator (when actually welding), was 150 amperes. The per cent of time welder was welding was ascertained to be 60 per cent.

Fig. 17

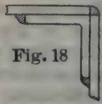

Fig. 18

Fig. 19

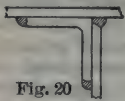

Fig. 20

Automatic Arc Welding. — Machinery has been developed permitting of automatically feeding the welding wire at such a rate as to maintain a steady and very short arc. A direct current type has recently (1920) been developed for the General Electric Company by Mr. P. O. Noble. Any variation in the length of the arc is immediately corrected by an automatic alteration in the speed of the motor controlling the rate of feed of the welding wire. With this machine the speed of welding is from two to five times as fast as with the best hand operators. Time lost in changing electrodes is eliminated by the use of reels of welding wire. This also eliminates the waste of short ends which occurs with hand welding with the usual 18-inch straight welding rods. As further advantages may be mentioned the even deposit of the electrode metal which minimizes defects and produces uniform welds which are stronger and more effective than with the hand process. The uniform arc and uninterrupted feed reduce the power consumption.

At pages 150 to 157 of Vol. 38 (1919) of the Trans. A.I.E.E., Mr. H. D. Morton describes his work in the development of automatic and semi-automatic arc welding machinery and gives illustrations of a semi-automatic machine and of some work done with it.

Flux-covered Electrodes. — In America practically all metal arc welding is done with bare-wire electrodes. In England, on the contrary, flux-covered electrodes are much the most extensively employed. The limits of space will not permit describing the nature and variety of the flux coverings employed.

From the lack of interest displayed in America on the subject of flux-covered electrodes, it may be inferred that for the great bulk of electric arc welding entirely adequate results are obtained by the much cheaper bare electrodes. The author's opinion is, however, that a higher grade of work can be obtained by the use of suitable flux-covered electrodes in the hands of operators skilled in their use. Especially with respect to ability to withstand tests of their ductility and to withstand fatigue tests he believes that welds can be made with flux-covered electrodes which can probably not be equalled with bare wire electrodes. It would not appear that this justifies the greater outlay, for ordinary

work. The use of electrodes with other than the very best flux-coverings may easily lead to welds much inferior to those obtained with bare electrodes of good quality.

The present (1921) cost of flux-covered electrodes ranges from 60 cents per pound for electrodes with steel cores of 1/10-inch diameter down to 40 cents per pound for those of 0.16-inch diameter. Taking a mean value of 50 cents per pound of steel in the electrodes, and assuming for the speed of welding, quantity of electrodes and amount of electricity per foot of weld, the values taken in the table previously given for bare electrodes, there results for the cost per foot in the two cases the following values:

COMPARATIVE COSTS OF MANUAL BUTT WELDING FOR VARIOUS THICKNESSES OF MILD STEEL

Thickness of metal, inches	Speed of welding, feet per hour	Outlay in cents per foot of weld, for (1) electricity, (2) electrodes, and (3) labor	
		Using bare electrodes	Using flux-covered electrodes
1/16	16	6.0	7.3
1/8	13	8.0	10.2
3/16	10	11.0	14.8
1/4	8	14.0	19.0
5/16	6	18.8	25.7
3/8	4½	25.1	34.2
7/16	3½	32.9	45.5
1/2	3	41.7	60.7

D-C. and A-C. Electricity for Arc Welding. — Arc welding is, in America, usually done with d-c. electricity. In England a-c. electricity is in much greater use than in America for arc welding. It is more difficult for the welder when a-c. is used, especially when he employs bare electrodes. But with some kinds of flux-covered electrodes, a-c. may be used without any notable difficulty in manipulation as compared with direct-current. When a-c. is used, every precaution should be taken to guard the operator from shock, notwithstanding the low voltage employed.

Comparison of Strengths of Riveted and Arc Welded Joints. — At p. 870 of the *General Electric Review*, for December, 1918, Mr. H. Jasper Cox of Lloyd's Register of Shipping, published the following tabular comparison of the tensile strengths of riveted and arc welded joints of ship plates of half-inch and three-quarter inch thickness.

Thickness, inches	Diameter of rivet, inches	Total sectional area, square inches	Breaking stress, pounds per square inch	Strength of plain plate, pounds per square inch	Percentage strength of joint
		Triple-riveted lap joints			
0.49	7/8	.....	42,400	61,400	69.0
0.53	3/4	.....	38,300	54,700	62.5
		Lap weld with full fillet at both edges			
0.514	.....	10.02	45,300	63,600	71.0
0.73	.....	8.76	40,330	59,600	68.0
		Butt weld (not strapped)			
0.505	.....	10.66	61,000	63,600	96.0
0.76	.....	9.88	54,680	59,600	91.5

Comparative Costs of Joining Ship Plates by Riveting and Welding. — The following are British figures and would not necessarily apply in America, but they give a good illustration of tendencies. They are taken from an article by Caldwell in *The Times Engineering Supplement* for April, 1920.

Thickness of plate in inches	Description of joint	Diameter of rivets, inches	Cents per foot *
7/16	Single riveted lap	3/4	43
	Light welded lap	....	30
	Double riveted lap	3/4	81
	Heavy welded lap	....	40
1/2	Single riveted lap	7/8	51
	Light welded lap	....	38
	Double riveted lap	7/8	91
	Heavy welded lap	....	47
	Double butt strap	7/8	120
	Heavy butt weld	....	47

* On the basis of one pound sterling = $4.

CARBON ARC WELDING. — Where carbon electrodes are used, the electrode is connected to the negative terminal of the circuit. The process is especially suitable for quick work requiring the melting of large quantities of material, such as in filling blow holes and in salvaging defective castings and forgings. It is also used for much other work, such as welding long seams which may be so designed with abutting edges as to permit of effecting the weld by simply fusing these edges without adding electrode material. An example of such a joint has already been given in Fig. 8.

In carbon arc welding the arc is very long (of the order of an inch or so), and it is considered that this entails the disadvantage that the weld may be badly oxidized.

Current Employed in Carbon Arc Welding and Size of Electrodes. — A reliable trade bulletin gives the accompanying data as suitable values for the current for carbon arc welding.

Class of work	Amperes
Light welding	150 to 250
Medium welding	250 to 350
Heavy welding	400 to 600
Very heavy welding	600 to 1000

The same bulletin gives the accompanying data for the maximum current for carbon electrodes of various sizes.

Diameter of carbon electrode, inch	Maximum current, amperes
¼	100
½	300
¾	500
1	1000

Still higher currents can be used by employing graphite electrodes, but their greater cost usually disqualifies them for use as welding electrodes.

Speed of Carbon Arc Welding. — It is found that for depositing or building up metal on flat surfaces where the work is accessible and under otherwise favorable conditions the following data hold:

Current in amperes	Pounds of metal deposited per hour	
	For continous work	For short jobs of 10 minutes or less
200	1.5	3
300	3.0	6
400	4.5	9
500	6	12

Thus carbon arc welding, in virtue of the higher current customarily employed, is a much faster process than metal arc welding.

Automatic Welding with the Carbon Arc. — Automatic machinery has been in use for several years for welding together abutting flanges with the carbon arc. With the kinds of joint shown in Fig. 8 it is not necessary to supply any additional metal. The carbon simply travels along parallel to the abutting flanges and the arc raises their temperature to the point where they flow together.

Carbon arc welding is usually performed with direct current but sufficient research will probably lead to the development of successful welding with carbon electrodes and an a-c. arc.

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In the Journal of the American Welding Society, first published in Jan., 1922, will be found recorded the most recent developments in Electric Welding.

WIND PRESSURE. — (*See also Structures, Simple; Transmission Lines.*) Wind pressure is a subject upon which little exact information exists, although many experiments have been made and much study given to the subject by engineers and scientists. Among the unsettled questions are:

a. The relation between pressure and velocity.

b. The variation of pressure with size and shape of exposed plane surfaces.

c. The direction and intensity of pressure upon non-vertical surfaces.

d. The intensity of pressure upon non-planar surfaces.

e. The total pressure upon a number of parallel bars or other members placed side by side.

f. The decrease of pressure upon leeward surfaces.

g. The lifting power of the wind.

RELATION BETWEEN INDICATED (U. S. WEATHER BUREAU) WIND VELOCITY AND ACTUAL VELOCITY.—The indications of the anemometers used by the U. S. Weather Bureau do not give the *actual* wind velocity, but give values considerably higher than the actual velocities, as shown in the following table:

RELATION BETWEEN INDICATED AND ACTUAL WIND VELOCITY

Indicated Velocity, mi. per hr.	Actual Velocity, mi. per hr.	Indicated Velocity, mi. per hr.	Actual Velocity, mi. per hr.
10	9.6	60	48.0
20	17.8	70	55.2
30	25.7	80	62.2
40	33.3	90	69.2
50	40.8	100	76.2

In the U. S. Weather Bureau reports the indicated and not the actual wind velocities are given. However, as the anemometers used give the *average* velocity for several minutes, the instantaneous velocities due to sudden gusts may be considerably greater than the indicated velocities; the indicated velocity probably more nearly represents the "gust" velocity than the actual average velocity. In all calculations of maximum wind pressure it is therefore recommended that the *indicated* velocity be used.

RELATION BETWEEN PRESSURE AND VELOCITY. — The pressure varies about as the square of the velocity, the results given by different experimenters for the pressure due to a *normal wind on a plane surface* ranging from

$$P = 0.005\,V^2 \quad \text{to} \quad P = 0.0032\,V^2,$$

where P = pressure in pounds per square foot,
V = actual wind velocity in miles per hour.

The latter of these values represents the results of unusually careful experiments by Stanton (*see Minutes of Proceedings of the Institute of Civil Engineers, Vols. 156 and 171*) upon the intensity of pressure on plates varying in size from 25 to 100 square feet and is probably more nearly correct than the higher value. In the Stanton formula the values are reduced to correspond to a temperature of 60° F. and an atmospheric pressure of 14. 7 pounds per square inch.

The influence of size and shape of exposed surface is an important question and is not well understood, although it is known that the resultant pressure on a large surface may be taken as less per square foot than that on a small

surface, since the maximum intensity of the wind is due to gusts of comparatively small cross-section.

Formulas for Pressure on Plane Surfaces when Wind is not Normal. — The pressure upon vertical plane surfaces may be taken as normal to the surface and equal in intensity to the assumed wind pressure. Upon surfaces which are not vertical, the pressure is usually considered to be normal to the surface but lower in intensity than upon vertical surfaces. The variation in pressure with respect to the slope is not well understood and a number of empirical formulas are in use, among which are the Duchemin formula

$$P_n = P \frac{2 \sin i}{1 + \sin^2 i},$$

and the Hutton formula

$$P_n = P (\sin i)^{(1.84 \cos i - 1)},$$

where P = intensity of normal pressure upon the vertical surface,
P_n = intensity of normal pressure upon the given surface,
i = angle made by surface with the horizontal.

The following theoretical formula results from the assumption that the wind always blows in horizontal lines, and that if the pressure be resolved into normal and tangential components, the tangential component may be neglected:

$$P_n = P \sin^2 i.$$

This formula gives lower values than the empirical formulas and probably gives too low results since it makes no allowance for the reduction in pressure on the leeward side which is known to occur, and which may in part be attributed to the influence of the tangential component. It should also be noted that the wind does not blow uniformly in horizontal lines but may deviate considerably from the horizontal.

The values given by these three formulas are tabulated for comparison, using an assumed value of 30 pounds per square foot for P. In the absence of further experience upon this phase of wind pressure it would seem wise to use one of the empirical formulas instead of the theoretical one. The Hutton formula is used quite generally by structural engineers in England and the United States.

Pressure on Non-Planar Surfaces. — The pressure upon non-planar surfaces is important in the case of chimneys, standpipes, and other similar objects.

Upon the same assumptions as made in the preceding paragraph it may be demonstrated that theoretically the pressure on a cylinder is two-thirds of the total pressure on a plane diametrical section. This value is quite generally used. The pressures thus obtained lack experimental proof but are probably more nearly correct than the pressure obtained by the same method upon plane surfaces.

Effect of Reduction of Pressure on Leeward Side. — The pressure upon the windward side of an exposed surface is a function of the density and velocity of the air currents. The pressure on the leeward side is also a function of the shape of the surface, and has been shown by numerous experiments to be less than the static pressure of the air current. The resultant total pressure upon a surface is in consequence not only a function of the direct pressure on the windward side, but also of the pressure on the leeward side, which in turn is a function of the form of the surface. No algebraic formula can be given which will give the pressure on surfaces of varying shape with any considerable degree of precision.

Wind Pressure on Wires. — H. W. Buck (*Trans. Int. Elec. Cong., St. Louis, 1904, Vol. 2, p. 318*) gives the following formula for the pressure due to a normal wind on a stranded wire:

$$P = 0.0025 V^2,$$

WIND PRESSURE IN POUNDS PER SQUARE FOOT

$P = 30$ pounds per square foot

Angle i, degrees	Theoretical	Duchemin	Hutton
	$P \sin^2 i$	$P \frac{2 \sin i}{1 + \sin^2 i}$	$P(\sin i)^{(1.84 \cos i - 1)}$
5	0.0	5.2	3.9
10	0.9	10.1	7.3
15	2.0	14.6	10.5
20	3.5	18.4	13.7
25	5.3	21.5	16.9
30	7.5	24.0	19.9
35	9.9	25.8	22.6
40	12.4	27.3	25.1
45	15.0	28.3	27.0
50	17.6	29.0	28.6
55	20.1	29.4	29.7
60	22.5	....	....
65	24.6	Above 60 deg.	Above 60 deg.
70	26.4	use 30 lb.	use 30 lb.
75	28.0		
80	29.1		
85	29.7		
90	30.0		

where P is the pressure in pounds per square foot of projected area of wire (length times diameter) and V is the velocity in miles per hour. This formula is based upon tests made on a 950-foot span at Niagara Falls; the wind velocities were measured by a U. S. Weather Bureau anemometer corrected to give actual average velocities.

PRACTICAL RULES FOR WIND PRESSURE ALLOWANCE. — The many uncertainties connected with wind pressure make worthless the attempts to specify with precision its magnitude and direction. In the lack of additional information and further theoretical studies there seems to be no reason for deviating from the common rules which have been used for many years with satisfactory results.

Bridges. — The portal, vertical and horizontal bracing are usually proportioned for a wind pressure of 30 lb. per sq. ft. on the surface of the applied load, and on the exposed surfaces of the floor system and both trusses. The pressure on the applied load is considered as a moving live load, and the other pressure as a dead load. For structures of ordinary spans the wind stresses are computed upon the unloaded structure for a pressure of 50 lb. per sq. ft. In the design the maximum stress computed by either of the above methods is used.

Buildings. — For wind pressure on roofs and buildings it is common practice to allow 30 lb. per sq. ft. acting horizontally upon the sides and ends of buildings, or on the vertical projection of roofs. It is also very important to figure the wind stresses on the steel frame considering it as an independent structure without walls, floors or partitions, since failures often occur in erection.

Transmission Poles and Towers. — For transmission towers a pressure as low as 13 lb. per sq. ft. has been used. This is perhaps warranted by the

fact that such towers are comparatively low and not exposed to the highest wind pressure. In the report of the Joint Committee on Overhead Line Construction (*Trans. N.E.L.A., 1911, Vol. 2, p. 521*) the following is recommended: "The wind pressure on the poles, or towers, shall be assumed at 13 lb. per sq. ft. of the projected area of solid or closed structures and 1½ times the projected area of latticed structures."

Wire Spans. — In the report of the Joint Committee just referred to it is recommended that the spans be designed for "a wind pressure of 8.0 lb. per sq. ft. on the ice-covered diameter (ice coating ½ inch in thickness), at a temperature of 0° F."

LIFTING POWER OF WIND. — In the case of a very rapid reduction of atmospheric pressure, as in a tornado, it is often observed that building roofs are lifted and walls blown outward. This phenomenon is due to the air in the building which is under more or less restraint, changing pressure less rapidly than the outside air and thereby producing a difference in pressure. This lifting action doubtless occurs to a greater or less degree whenever the external pressure is reduced, and should be guarded against by anchoring roofs securely to the walls.

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[C. M. SPOFFORD.]

WIRES AND CABLES, BARE. — (*See also Aluminum; Copper; Gages, Wires; Resistance and Conductance; Standardization Rules of the A.I.E.E.; Wires and Cables, Insulated; Wires, Resistance.*)

The following is a brief table of contents of this article:

A wire may be either solid or stranded, i.e., made up of a number of smaller wires twisted or braided together. A large bare stranded wire is usually called a bare cable. Data on the insulation and protection of wires and cables will be found in the article on *Wires and Cables, Insulated*. Data on resistance wires will be found in the article on *Wires, Resistance*, and data on the properties of the various metals will be found in the articles on *Aluminum*, *Copper*, and *Steel*.

PER CENT CONDUCTIVITY. — The definitions, constants and tables in this article are all in accord with the 1920 edition of the *Standards of the A.I.E.E.* Per cent conductivity refers to the "International Annealed Copper Standard." On the assumption of a resistivity temperature coefficient of 0.00393 at 20° C. this per cent conductivity is only 0.283 per cent higher than the conductivity referred to Matthiessen's Standard (*see Resistance and Conductance*). If the length of a given wire is L cm., its cross-section A sq. cm. and its resistance at 20° C. is R_{20} ohms then the per cent conductivity of this wire is

$$C = \frac{15.328\,L}{88{,}900\,AR_{20}} \quad \text{per cent.}$$

Annealed or soft-drawn copper usually has a conductivity of 100 per cent, hard-drawn copper a conductivity of about 97 per cent. Ordinarily hard-drawn aluminum has a conductivity of 61 per cent. The conductivity of iron or steel wire ranges from 8 to 16 per cent.

DIMENSIONS, RESISTANCE AND WEIGHT. — The following tables for copper and aluminum are compiled from the tables in *Circular No. 31 of the Bureau of Standards*, except that for stranded aluminum on p. 1977. The table for steel wire is compiled from that published by the American Steel and Wire Co.

SOLID COPPER WIRE

A. W. G. or B. & S. Gage; English Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Diameter in mils.	Cross-section		Resistance at 20° C. or 68° F.*		Weight in pounds		Feet per pound
		Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
0000	460.0	211,600	0.1662	0.04901	0.259	640.5	3380	1.561
000	409.6	167,800	0.1318	0.06180	0.326	507.9	2680	1.968
00	364.8	133,100	0.1045	0.07793	0.411	402.8	2130	2.482
0	324.9	105,500	0.08289	0.09827	0.519	319.5	1680	3.130
1	289.3	83,690	0.06573	0.1239	0.654	253.3	1340	3.947
2	257.6	66,370	0.05213	0.1563	0.825	200.9	1060	4.977
3	229.4	52,640	0.04134	0.1970	1.04	159.3	841	6.276
4	204.3	41,740	0.03278	0.2485	1.31	126.4	667	7.914
5	181.9	33,100	0.02600	0.3133	1.65	100.2	529	9.980
6	162.0	26,250	0.02062	0.3951	2.09	79.46	420	12.58
7	144.3	20,820	0.01635	0.4982	2.63	63.02	333	15.87
8	128.5	16,510	0.01297	0.6282	3.32	49.98	264	20.01
10	101.9	10,380	0.008155	0.9989	5.28	31.43	166	31.82
12	80.81	6,530	0.005129	1.588	8.38	19.77	104	50.59
14	64.08	4,107	0.003225	2.525	13.3	12.43	63.3	80.44
15	57.07	3,257	0.002558	3.184	16.8	9.858	52.0	101.4
16	50.82	2,583	0.002028	4.015	21.2	7.818	41.3	127.9
17	45.26	2,048	0.001609	5.064	26.7	6.200	32.7	161.3
18	40.30	1,624	0.001276	6.385	33.7	4.917	26.0	203.4
19	35.89	1,288	0.001012	8.051	42.5	3.899	20.6	256.5
20	31.96	1,022	0.0008023	10.15	53.6	3.092	16.3	323.4
21	28.46	810.1	0.0006363	12.80	67.6	2.452	12.9	407.8
22	25.35	642.4	0.0005046	16.14	85.2	1.945	10.3	514.2
23	22.57	509.5	0.0004002	20.36	108	1.542	8.14	648.4
24	20.10	404.0	0.0003173	25.67	135	1.223	6.46	817.7
25	17.90	320.4	0.0002517	32.37	171	0.9699	5.12	1,031
26	15.94	254.1	0.0001996	40.82	216	0.7692	4.06	1,300
27	14.20	201.5	0.0001583	51.46	272	0.6100	3.22	1,639
28	12.64	159.8	0.0001255	64.90	343	0.4837	2.55	2,067
29	11.26	126.7	0.00009953	81.84	432	0.3836	2.03	2,607
30	10.03	100.5	0.00007894	103.2	545	0.3042	1.61	3,287
31	8.928	79.70	0.00006260	130.1	687	0.2413	1.27	4,145
32	7.950	63.21	0.00004964	164.1	866	0.1913	1.01	5,227
33	7.080	50.13	0.00003937	206.9	1090	0.1517	0.814	6,591
34	6.305	39.75	0.00003122	260.9	1380	0.1203	0.635	8,310
35	5.615	31.52	0.00002476	329.0	1740	0.09542	0.504	10,480
36	5.000	25.00	0.00001964	414.8	2190	0.07568	0.400	13,210
38	3.965	15.72	0.00001235	659.6	3480	0.04759	0.251	21,010
40	3.145	9.888	0.000007766	1049	5540	0.02993	0.153	33,410

*Let C = per cent conductivity, R_{20} = resistance of 100 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C, then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393\,(t - 20)\right]$$

COPPER CABLES, CONCENTRIC-LAY

Cir. Mils and A. W. G. or B. & S. Gage; English Units

100 Per Cent Conductivity; Density 8.89 at 20° C. (*See p. 1982.*)

Circular mils and A. W. G.	Resistance at 25° C. or 77° F.*		Weight in pounds, bare		Standard strands			Flexible strands		
	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	Number of wires	Diameter of wires in mils	Outside diameter, in mils	Number of wires	Diameter of wires, in mils	Outside diameter, in mils
2,000,000	0.00539	0.0285	6180	32600	127	125.5	1631	169	108.8	1632
1,900,000	0.00568	0.0300	5870	31000	127	122.3	1590	169	106.0	1590
1,800,000	0.00599	0.0316	5560	29300	127	119.1	1548	169	103.2	1548
1,700,000	0.00634	0.0335	5250	27700	127	115.7	1504	169	100.3	1504
1,600,000	0.00674	0.0356	4940	26100	127	112.2	1459	169	97.3	1460
1,500,000	0.00719	0.0380	4630	24500	91	128.4	1412	127	108.7	1413
1,400,000	0.00770	0.0407	4320	22800	91	124.0	1364	127	105.0	1365
1,300,000	0.00830	0.0438	4010	21200	91	119.5	1315	127	101.2	1315
1,200,000	0.00899	0.0475	3710	19600	91	114.8	1263	127	97.2	1264
1,100,000	0.00981	0.0518	3400	17900	91	109.9	1209	127	93.1	1210
1,000,000	0.0108	0.0570	3090	16300	61	128.0	1152	91	104.8	1153
950,000	0.0114	0.0600	2930	15490	61	124.8	1123	91	102.2	1124
900,000	0.0120	0.0633	2780	14670	61	121.5	1093	91	99.4	1094
850,000	0.0127	0.0670	2620	13860	61	118.0	1062	91	96.6	1063
800,000	0.0135	0.0712	2470	13040	61	114.5	1031	91	93.8	1031
750,000	0.0144	0.0759	2320	12230	61	110.9	998	91	90.8	999
700,000	0.0154	0.0814	2160	11410	61	107.1	964	91	87.7	965
650,000	0.0166	0.0876	2010	10600	61	103.2	929	91	84.5	930
600,000	0.0180	0.0949	1850	9780	61	99.2	893	91	81.2	893
550,000	0.0196	0.1036	1700	8970	61	95.0	855	91	77.7	855
500,000	0.0216	0.1139	1540	8150	37	116.2	814	61	90.5	815
450,000	0.0240	0.1266	1390	7340	37	110.3	772	61	85.9	773
400,000	0.0270	0.1424	1240	6520	37	104.0	728	61	81.0	729
350,000	0.0308	0.1627	1080	5710	37	97.3	681	61	75.7	682
300,000	0.0360	0.1899	926	4890	37	90.0	630	61	70.1	631
250,000	0.0431	0.228	772	4080	37	82.2	575	61	64.0	576
0000	0.0509	0.269	653	3450	19	105.5	528	37	75.6	533
000	0.0642	0.339	518	2735	19	94.0	470	37	67.3	471
00	0.0811	0.428	411	2170	19	83.7	418	37	60.0	420
0	0.102	0.540	326	1720	19	74.5	373	37	53.4	374
1	0.129	0.681	258	1364	19	66.4	332	37	47.6	333
2	0.162	0.858	205	1082	7	97.4	292	19	59.1	296
3	0.205	1.082	163	858	7	86.7	260	19	52.6	263
4	0.259	1.365	129	680	7	77.2	232	19	46.9	234
5	0.326	1.721	102	540	7	68.8	206	19	41.7	209
6	0.410	2.170	81.0	428	7	61.2	184	19	37.2	186
7	0.519	2.74	64.3	339	7	54.5	164	19	33.1	166
8	0.654	3.45	51.0	269	7	48.6	146	19	29.5	147

*Let C = per cent conductivity, R_{25} = resistance of 100 per cent conductivity cable at 25° C. (from table), R_t = resistance of cable of conductivity C at any temperature t° C., then

$$R_t = R_{25}\left[\frac{100}{C} + 0.00385\,(t - 25)\right].$$

SOLID COPPER WIRE

A. W. G. or B. & S. Gage in Metric Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Diameter in mm.	Cross-section in sq. mm.	Ohms per kilometer, 20° C*	Kilograms per kilometer
0000	11.68	107.2	0.1608	953.2
000	10.40	85.03	0.2028	755.9
00	9.266	67.43	0.2557	599.5
0	8.252	53.48	0.3224	475.4
1	7.348	42.41	0.4066	377.0
2	6.544	33.63	0.5126	299.0
3	5.827	26.67	0.6464	237.1
4	5.189	21.15	0.8152	188.0
5	4.621	16.77	1.028	149.1
6	4.115	13.30	1.296	118.2
7	3.665	10.55	1.634	93.78
8	3.264	8.366	2.061	74.37
10	2.588	5.261	3.277	46.77
12	2.053	3.309	5.211	29.42
14	1.628	2.081	8.285	18.50
15	1.450	1.650	10.45	14.67
16	1.291	1.309	13.18	11.63
17	1.150	1.038	16.61	9.226
18	1.024	0.8231	20.95	7.317
19	0.9116	0.6527	26.42	5.803
20	0.8118	0.5176	33.31	4.602
21	0.7230	0.4105	42.00	3.649
22	0.6438	0.3255	52.96	2.894
23	0.5733	0.2582	66.79	2.295
24	0.5106	0.2047	84.22	1.820
25	0.4547	0.1624	106.2	1.443
26	0.4049	0.1288	133.9	1.145
27	0.3606	0.1021	168.8	0.9078
28	0.3211	0.08098	212.9	0.7199
29	0.2859	0.06422	268.5	0.5709
30	0.2546	0.05093	338.6	0.4527
31	0.2268	0.04039	426.9	0.3590
32	0.2019	0.03203	538.3	0.2847
33	0.1798	0.02540	678.8	0.2258
34	0.1601	0.02014	856.0	0.1791
35	0.1426	0.01597	1079	0.1420
36	0.1270	0.01267	1361	0.1126
38	0.1007	0.007967	2164	0.07083
40	0.07987	0.005010	3441	0.04454

* Let C = per cent conductivity, R_{20} = resistance of 100 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C, then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393\ (t - 20)\right].$$

COPPER CABLES, CONCENTRIC-LAY

Cir. Mils and A.W.G. or B. & S. Gage in Metric Units

100 Per Cent Conductivity; Density 8.89 at 20°C. (*See p. 1982.*)

Circular mils and A.W.G.	Total cross-section in mm.²	Ohms per kilometer at 25°C.*	Kilograms per kilometer, Bare	Standard strands			Flexible strands		
				Number of wires	Diameter of wires, in mm.	Outside diameter, in mm.	Number of wires	Diameter of wires, in mm.	Outside diameter, in mm.
2,000,000	1013	0.0177	9190	127	3.19	41.4	169	2.76	41.4
1,900,000	963	0.0186	8730	127	3.11	40.4	169	2.69	40.4
1,800,000	912	0.0197	8270	127	3.02	39.3	169	2.62	39.3
1,700,000	861	0.0208	7810	127	2.94	38.2	169	2.55	38.2
1,600,000	811	0.0221	7350	127	2.85	37.1	169	2.47	37.1
1,500,000	760	0.0236	6890	91	3.26	35.9	127	2.76	35.9
1,400,000	709	0.0253	6430	91	3.15	34.7	127	2.67	34.7
1,300,000	659	0.0272	5970	91	3.04	33.4	127	2.57	33.4
1,200,000	608	0.0295	5510	91	2.92	32.1	127	2.47	32.1
1,100,000	557	0.0322	5050	91	2.79	30.7	127	2.36	30.7
1,000,000	507	0.0354	4590	61	3.25	29.3	91	2.66	29.3
950,000	481	0.0373	4370	61	3.17	28.5	91	2.60	28.5
900,000	456	0.0393	4140	61	3.09	27.8	91	2.53	27.8
850,000	431	0.0416	3910	61	3.00	27.0	91	2.45	27.0
800,000	405	0.0442	3680	61	2.91	26.2	91	2.38	26.2
750,000	380	0.0472	3450	61	2.82	25.3	91	2.31	25.4
700,000	355	0.0506	3220	61	2.72	24.5	91	2.23	24.5
650,000	329	0.0544	2990	61	2.62	23.6	91	2.15	23.6
600,000	304	0.0590	2760	61	2.52	22.7	91	2.06	22.7
550,000	279	0.0643	2530	61	2.41	21.7	91	1.97	21.7
500,000	253	0.0708	2300	37	2.95	20.7	61	2.30	20.7
450,000	228	0.0786	2070	37	2.80	19.6	61	2.18	19.6
400,000	203	0.0885	1840	37	2.64	18.5	61	2.06	18.5
350,000	177	0.101	1610	37	2.47	17.3	61	1.92	17.3
300,000	152	0.118	1380	37	2.29	16.0	61	1.78	16.0
250,000	127	0.142	1150	37	2.09	14.6	61	1.63	14.6
0000	107	0.167	972	19	2.68	13.4	37	1.93	13.5
000	85	0.211	771	19	2.39	11.9	37	1.71	12.0
00	67.4	0.266	611	19	2.13	10.6	37	1.52	10.7
0	53.5	0.334	485	19	1.89	9.46	37	1.36	9.50
1	42.4	0.423	385	19	1.69	8.43	37	1.21	8.46
2	33.6	0.533	305	7	2.47	7.42	19	1.50	7.51
3	26.7	0.673	242	7	2.20	6.61	19	1.34	6.68
4	21.2	0.849	192	7	1.96	5.88	19	1.19	5.95
5	16.8	1.07	152	7	1.75	5.24	19	1.06	5.30
6	13.3	1.35	121	7	1.56	4.67	19	0.944	4.72
7	10.5	1.70	95.7	7	1.39	4.16	19	0.841	4.20
8	8.37	2.14	75.9	7	1.23	3.70	19	0.749	3.74

* Let C = per cent conductivity, R_{25} = resistance of 100 per cent conductivity cable at 25° C. (from table), R_t = resistance of cable of conductivity C at any temperature t° C., then

$$R_t = R_{25}\left[\frac{100}{C} + 0.00385\,(t - 25)\right].$$

SOLID COPPER WIRE

British Standard Wire Gage; English Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Diameter in mils	Cross-section		Ohms per 1000 feet, 15.6° C. or 60° F.*	Pounds per 1000 feet
		Circular mils	Square inches		
7-0	500	250,000	0.1964	0.04077	756.8
6-0	464	215,300	0.1691	0.04734	651.7
5-0	432	186,600	0.1466	0.05461	564.9
4-0	400	160,000	0.1257	0.06370	484.3
3-0	372	138,400	0.1087	0.07365	418.9
2-0	348	121,100	0.09512	0.08416	366.6
0	324	105,000	0.08245	0.09709	317.8
1	300	90,000	0.07069	0.1132	272.4
2	276	76,180	0.05983	0.1338	230.6
3	252	63,500	0.04988	0.1605	192.2
4	232	53,820	0.04227	0.1894	162.9
5	212	44,940	0.03530	0.2268	136.0
6	192	36,860	0.02895	0.2765	111.6
7	176	30,980	0.02433	0.3290	93.76
8	160	25,600	0.02011	0.3981	77.49
9	144	20,740	0.01629	0.4915	62.77
10	128	16,380	0.01287	0.6221	49.59
11	116	13,460	0.01057	0.7574	40.73
12	104	10,820	0.008495	0.9423	32.74
13	92	8,464	0.006648	1.204	25.62
14	80	6,400	0.005027	1.592	19.37
15	72	5,184	0.004072	1.966	15.69
16	64	4,096	0.003217	2.488	12.40
17	56	3,136	0.002463	3.250	9.493
18	48	2,304	0.001810	4.424	6.974
19	40	1,600	0.001257	6.370	4.843
20	36	1,296	0.001018	7.864	3.923
22	28	784.0	0.0006158	13.00	2.373
24	22	484.0	0.0003801	21.06	1.465
26	18	324.0	0.0002545	31.46	0.9807
28	14.8	219.0	0.0001720	46.54	0.6630
30	12.4	153.8	0.0001208	66 28	0.4654
32	10.8	116.6	0.00009161	87.38	0.3531
34	9.2	84.64	0.00006648	120.4	0.2562
36	7.6	57.76	0.00004536	176.5	0.1748
38	6.0	36.00	0.00002827	283.1	0.1090
40	4.8	23.04	0.00001810	442.4	0.06974
42	4.0	16.00	0.00001257	637.0	0.04843
44	3.2	10.24	0.000008042	995.3	0.03100
50	1.0	1.000	0.0000007854	10,190	0.003027

*Let C = per cent conductivity, R_{60} = resistance of 100 per cent conductivity wire at 60° F. (from table), R_t = resistance of wire of conductivity C at any temperature t° F., then

$$R_t = R_{60}\left[\frac{100}{C} + 0.00223\,(t - 60)\right].$$

SOLID COPPER WIRE

"Millimeter Gage"; Metric Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Diameter in mm.	Cross-section in sq. mm.	Ohms per kilometer, 20° C.*	Kilograms per kilometer
10.0	78.54	0.2195	698.2
9.0	63.62	0.2710	565.6
8.0	50.27	0.3430	446.9
7.0	38.48	0.4480	342.1
6.0	28.27	0.6098	251.4
5.0	19.64	0.8781	174.6
4.5	15.90	1.084	141.4
4.0	12.57	1.372	111.7
3.5	9.621	1.792	85.53
3.0	7.069	2.439	62.84
2.5	4.909	3.512	43.64
2.0	3.142	5.488	27.93
1.8	2.545	6.775	22.62
1.6	2.011	8.575	17.87
1.4	1.539	11.20	13.69
1.2	1.131	15.24	10.05
1.0	0.7854	21.95	6.982
0.90	0.6362	27.10	5.656
0.80	0.5027	34.30	4.469
0.70	0.3848	44.80	3.421
0.60	0.2827	60.98	2.514
0.50	0.1964	87.81	1.746
0.45	0.1590	108.4	1.414
0.40	0.1257	137.2	1.117
0.35	0.09621	179.2	0.8553
0.30	0.07069	243.9	0.6284
0.25	0.04909	351.2	0.4364
0.20	0.03142	548.8	0.2793
0.15	0.01767	975.6	0.1571
0.10	0.007854	2195	0.06982
0.05	0.001964	8781	0.01746

*Let C = per cent conductivity,
R_{20} = resistance of 100 per cent conductivity wire at 20° C. (from table),
R_t = resistance of wire of conductivity C at any temperature t° C.,
then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393\,(t-20)\right].$$

SOLID COPPER WIRE; OHMS PER UNIT WEIGHT

A. W. G. or B. & S. Gage; English and Metric Units

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Ohms per pound			Ohms per kilogram		
	0° C. 32° F.	20° C. 68° F.	50° C. 122° F.	0° C.	20° C.	50° C.
0000	0.00007051	0.00007652	0.00008554	0.0001554	0.0001687	0.0001886
000	0.0001121	0.0001217	0.0001360	0.0002472	0.0002682	0.0002999
00	0.0001783	0.0001935	0.0002163	0.0003930	0.0004265	0.0004768
0	0.0002835	0.0003076	0.0003439	0.0006249	0.0006782	0.0007582
1	0.0004507	0.0004891	0.0005468	0.0009936	0.001078	0.001206
2	0.0007166	0.0007778	0.0008695	0.001580	0.001715	0.001917
3	0.001140	0.001237	0.001383	0.002512	0.002726	0.003048
4	0.001812	0.001966	0.002198	0.003995	0.004335	0.004846
5	0.002881	0.003127	0.003495	0.006352	0.006893	0.007706
6	0.004581	0.004972	0.005558	0.01010	0.01096	0.01225
7	0.007284	0.007906	0.008838	0.01606	0.01743	0.01948
8	0.01158	0.01257	0.01405	0.02553	0.02771	0.03098
9	0.01842	0.01999	0.02234	0.04060	0.04407	0.04926
10	0.02928	0.03178	0.03553	0.06456	0.07006	0.07833
11	0.04656	0.05053	0.05649	0.1026	0.1114	0.1245
12	0.07404	0.08035	0.08983	0.1632	0.1771	0.1980
13	0.1177	0.1278	0.1428	0.2595	0.2817	0.3149
14	0.1872	0.2032	0.2271	0.4127	0.4479	0.5007
15	0.2976	0.3230	0.3611	0.6562	0.7121	0.7961
16	0.4733	0.5136	0.5742	1.043	1.132	1.266
17	0.7525	0.8167	0.9130	1.659	1.800	2.013
18	1.197	1.299	1.452	2.638	2.863	3.201
19	1.903	2.065	2.308	4.194	4.552	5.089
20	3.025	3.283	3.670	6.670	7.238	8.092
21	4.810	5.221	5.836	10.60	11.51	12.87
22	7.649	8.302	9.280	16.86	18.30	20.46
23	12.16	13.20	14.76	26.81	29.10	32.53
24	19.34	20.99	23.46	42.63	46.27	51.73
25	30.75	33.37	37.31	67.79	73.57	82.25
26	48.89	53.06	59.32	107.8	117.0	131.8
27	77.74	84.37	94.32	171.4	186.0	207.9
28	123.6	134.2	150.0	272.5	295.8	330.6
29	196.6	213.3	238.5	433.3	470.3	525.7
30	312.5	339.2	379.2	689.0	747.8	836.0
31	497.0	539.3	602.9	1,096	1,189	1,329
32	790.2	857.6	958.7	1,742	1,891	2,114
33	1,256	1,364	1,524	2,770	3,006	3,361
34	1,998	2,168	2,424	4,404	4,780	5,344
35	3,177	3,448	3,854	7,003	7,601	8,497
36	5,051	5,482	6,128	11,140	12,080	13,510
38	12,770	13,860	15,490	28,150	30,560	34,160
40	32,290	35,040	39,170	71,180	77,260	86,360

SOLID COPPER WIRE; WEIGHT PER OHM

A. W. G. or B. & S. Gage; English and Metric Units

100 Per Cent Conductivity; Density 8 89. at 20° C.

Gage No.	Meters per ohm		Feet per ohm		Pounds per ohm		Grams per ohm	
	20° C.	50° C.	20° C. 68° F.	50° C. 122° F.	20° C. 68° F.	50° C. 122° F.	20° C.	50° C.
0000	6219	5563	20,400	18,250	13,070	11,690	5,928,000	5,302,000
000	4932	4412	16,180	14,470	8,219	7,352	3,728,000	3,335,000
00	3911	3499	12,830	11,480	5,169	4,624	2,344,000	2,097,000
0	3102	2774	10,180	9,103	3,251	2,908	1,474,000	1,319,000
1	2460	2200	8,070	7,219	2,044	1,829	927,300	829,500
2	1951	1745	6,400	5,725	1,286	1,150	583,200	521,700
3	1547	1384	5,075	4,540	808.6	723.3	366,800	328,100
4	1227	1097	4,025	3,600	508.5	454.9	230,700	206,300
5	972.8	870.2	3,192	2,855	319.8	286.1	145,100	129,800
6	771.5	690.1	2,531	2,264	201.1	179.9	91,230	81,610
7	611.8	547.3	2,007	1,796	126.5	113.2	57,380	51,330
8	485.2	434.0	1,592	1,424	79.56	71.16	36,090	32,280
9	384.8	344.2	1,262	1,129	50.03	44.75	22,690	20,300
10	305.2	273.0	1,001	895.6	31.47	28.15	14,270	12,770
11	242.0	216.5	794.0	710.2	19.79	17.70	8,976	8,030
12	191.9	171.7	629.6	563.2	12.44	11.13	5,645	5,050
13	152.2	136.1	499.3	446.7	7.827	7.001	3,550	3,176
14	120.7	108.0	396.0	354.2	4.922	4.403	2,233	1,997
15	95.72	85.62	314.0	280.9	3.096	2.769	1,404	1,256
16	75.90	67.90	249.0	222.8	1.947	1.742	883.1	790.0
17	60.19	53.85	197.5	176.7	1.224	1.095	555.3	496.8
18	47.74	42.70	156.6	140.1	0.7701	0.6888	349.3	312.4
19	37.86	33.86	124.2	111.1	0.4843	0.4332	219.7	196.5
20	30.02	26.86	98.49	88.11	0.3046	0.2725	138.2	123.6
21	23.81	21.30	78.11	69.87	0.1915	0.1713	86.89	77.72
22	18.88	16.89	61.95	55.41	0.1205	0.1078	54.64	48.88
23	14.97	13.39	49.12	43.94	0.07576	0.06777	34.36	30.74
24	11.87	10.62	38.96	34.85	0.04765	0.04262	21.61	19.33
25	9.417	8.424	30.90	27.64	0.02997	0.02680	13.59	12.16
26	7.468	6.680	24.50	21.92	0.01884	0.01686	8.548	7.647
27	5.922	5.298	19.43	17.38	0.01185	0.01060	5.376	4.809
28	4.696	4.201	15.41	13.78	0.007454	0.006668	3.381	3.022
29	3.725	3.332	12.22	10.93	0.004688	0.004193	2.126	1.902
30	2.954	2.642	9.691	8.669	0.002948	0.002637	1.337	1.196
31	2.342	2.095	7.685	6.875	0.001854	0.001659	0.8410	0.7523
32	1.858	1.662	6.094	5.452	0.001166	0.001043	0.5289	0.4731
33	1.473	1.318	4.833	4.323	0.0007333	0.0006560	0.3326	0.2976
34	1.168	1.045	3.833	3.429	0.0004612	0.0004126	0.2092	0.1871
35	0.9264	0.8288	3.040	2.719	0.0002900	0.0002595	0.1316	0.1177
36	0.7347	0.6572	2.410	2.156	0.0001824	0.0001632	0.08275	0.07402
38	0.4621	0.4133	1.516	1.356	0.00007216	0.00006454	0.03273	0.02927
40	0.2906	0.2600	0.9534	0.8529	0.00002854	0.00002553	0.01294	0.01158

SOLID ALUMINUM WIRE

A. W. G. or B. & S. Gage; English Units

61 Per Cent Conductivity; Density 2.70

Gage No.	Diameter in mils	Cross-section		Resistance at 20° C. or 68° F.*		Weight in pounds		Feet per pound
		Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
0000	460.0	211,600	0.1662	0.0804	0.424	195	1027	5.14
000	409.6	167,800	0.1318	0.101	0.535	154	815	6.48
00	364.8	133,100	0.1045	0.128	0.675	122	646	8.17
0	324.9	105,500	0.08289	0.161	0.851	97.0	512	10.31
1	289.3	83,690	0.06573	0.203	1.073	76.9	406	13.00
2	257.6	66,370	0.05213	0.256	1.353	61.0	322	16.39
3	229.4	52,630	0.04134	0.323	1.706	48.4	255	20.7
4	204.3	41,740	0.03278	0.408	2.15	38.4	203	26.1
5	181.9	33,100	0.02600	0.514	2.71	30.4	160.7	32.9
6	162.0	26,250	0.02062	0.648	3.42	24.1	127.4	41.4
7	144.3	20,820	0.01635	0.817	4.31	19.1	101.0	52.3
8	128.5	16,510	0.01297	1.03	5.44	15.2	80.2	65.9
10	101.9	10,380	0.008155	1.64	8.65	9.55	50.4	104.8
12	80.81	6,530	0.005129	2.61	13.76	6.00	31.7	166.6
14	64.08	4,107	0.003225	4.14	21.9	3.78	19.93	265
15	57.07	3,257	0.002558	5.22	27.6	2.99	15.81	334
16	50.82	2,583	0.002029	6.59	34.8	2.37	12.54	421
17	45.26	2,048	0.001609	8.31	43.8	1.88	9.94	531
18	40.30	1,624	0.001276	10.5	55.3	1.49	7.89	670
19	35.89	1,288	0.001012	13.2	69.7	1.18	6.25	844
20	31.96	1,022	0.0008023	16.7	87.9	0.939	4.96	1,065
21	28.46	810.1	0.0006363	21.0	110.9	0.745	3.93	1,343
22	25.35	642.4	0.0005046	26.5	139.8	0.591	3.12	1,693
23	22.57	509.5	0.0004002	33.4	176.3	0.468	2.47	2,130
24	20.10	404.0	0.0003173	42.1	222	0.371	1.961	2,690
25	17.90	320.4	0.0002517	53.1	280	0.295	1.556	3,390
26	15.94	254.1	0.0001996	67.0	353	0.234	1.233	4,280
27	14.20	201.5	0.0001583	84.4	446	0.185	0.978	5,400
28	12.64	159.8	0.0001255	106	562	0.147	0.776	6,810
29	11.26	126.7	0.00009953	134	709	0.117	0.615	8,580
30	10.03	100.5	0.00007894	169	894	0.0924	0.488	10,820
31	8.928	79.70	0.00006260	213	1127	0.0733	0.387	13,650
32	7.950	63.21	0.00004964	269	1421	0.0581	0.307	17,210
33	7.080	50.13	0.00003937	339	1792	0.0461	0.243	21,700
34	6.305	39.75	0.00003122	428	2260	0.0365	0.1929	27,400
35	5.615	31.52	0.00002476	540	2850	0.0290	0.1530	34,510
36	5.000	25.00	0.00001964	681	3590	0.0230	0.1214	43,500
38	3.965	15.72	0.00001235	1080	5710	0.0145	0.0763	69,200
40	3.145	9.888	0.000007766	1720	9080	0.0091	0.0480	110,000

*Let C = per cent conductivity, R_{20} = resistance of 61 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C., then

$$R_t = \frac{61\ R_{20}}{C}\left[1 + 0.004\ (t - 20)\right].$$

ALUMINUM CABLES

The commercial sizes of stranded aluminum cables, made by the Aluminum Company of America, are not circular mil or A. W. G. sizes, but are of such cross-sections as to give the same conductivity as even circular mil and A. W. G. sizes of copper cables of 97 per cent conductivity. In the following table the first four and the seventh columns are taken from a pamphlet entitled "Instructions for Installation and Maintenance of Aluminum Electrical Conductors" issued by the Aluminum Company of America, in 1914.

A. W. G. gage or circular mils		Usual number of strands	Diameter of bare cable, inches	Resistance at 25° C. or 77° F.*		Weight in pounds	
Copper (97 per cent) equivalent	Aluminum 61 per cent			Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile
1,000,000	1,590,000	61	1 7/16	0.0111	0.0588	1462	7719
950,000	1,515,000	61	1 13/32	0.0118	0.0619	1393	7355
900,000	1,431,000	61	1 23/64	0.0124	0.0653	1317	6954
850,000	1,351,500	61	1 21/64	0.0131	0.0691	1243	6563
800,000	1,272,000	61	1 9/32	0.0139	0.0734	1171	6189
750,000	1,192,500	37	1 1/4	0.0148	0.0783	1098	5797
700,000	1,113,000	37	1 13/64	0.0159	0.0839	1025	5412
650,000	1,033,500	37	1 5/32	0.0171	0.0903	950	5016
600,000	954,000	37	1 7/64	0.0186	0.0978	877	4631
550,000	874,500	37	1 1/16	0.0202	0.1068	805	4250
500,000	795,000	37	1 1/64	0.0223	0.1174	732	3865
450,000	715,500	37	31/32	0.0247	0.1305	658	3474
400,000	636,000	37	29/32	0.0278	0.1468	585	3089
350,000	556,500	19	55/64	0.0318	0.1677	512	2703
300,000	477,000	19	25/32	0.0371	0.1958	439	2318
250,000	397,500	19	23/32	0.0444	0.2351	365	1927
0000	336,420	7	21/32	0.0525	0.277	310.2	1638
000	266,800	7	37/64	0.0662	0.350	245.7	1297
00	211,950	7	33/64	0.0836	0.441	195	1030
0	167,800	7	15/32	0.105	0.557	155	818.4
1	133,220	7	13/32	0.133	0.702	122.6	647.3
2	105,530	7	23/64	0.167	0.885	97.2	513.2
3	83,640	7	21/64	0.211	1.116	77	406.6
4	66,370	7	19/64	0.267	1.407	61.2	323.1
5	52,630	7	17/64	0.336	1.774	48.5	256.1
6	41,740	7	15/64	0.423	2.237	38.5	203.3

* These resistances are taken equal to those given on page 1969 divided by 0.97.

SOLID ALUMINUM WIRE

A. W. G. or B. & S. Gage in Metric Units

61 Per Cent Conductivity; Density 2.70; Temperature 20° C. or 68° F.*

Gage No.	Diameter in mm.	Cross-section in sq. mm.	Ohms per kilometer	Kilograms per kilometer
0000	11.68	107.2	0.264	289
000	10.40	85.03	0.333	230
00	9.266	67.43	0.419	182
0	8.252	53.48	0.529	144
1	7.348	42.41	0.667	114
2	6.544	33.63	0.841	90.8
3	5.827	26.67	1.06	72.0
4	5.189	21.15	1.34	57.1
5	4.621	16.77	1.69	45.3
6	4.115	13.30	2.13	35.9
7	3.665	10.55	2.68	28.5
8	3.264	8.366	3.38	22.6
10	2.588	5.261	5.38	14.2
12	2.053	3.309	8.55	8.93
14	1.628	2.081	13.6	5.62
15	1.450	1.650	17.1	4.46
16	1.291	1.309	21.6	3.53
17	1.150	1.038	27.3	2.80
18	1.024	0.8231	34.4	2.22
19	0.9116	0.6527	43.3	1.76
20	0.8118	0.5176	54.6	1.40
21	0.7230	0.4105	68.9	1.11
22	0.6438	0.3255	86.9	0.879
23	0.5733	0.2582	110	0.697
24	0.5106	0.2047	138	0.553
25	0.4547	0.1624	174	0.438
26	0.4049	0.1288	220	0.348
27	0.3606	0.1021	277	0.276
28	0.3211	0.08098	349	0.219
29	0.2859	0.06422	440	0.173
30	0.2546	0.05093	555	0.138
31	0.2268	0.04039	700	0.109
32	0.2019	0.03203	883	0.0865
33	0.1798	0.02540	1110	0.0686
34	0.1601	0.02014	1400	0.0544
35	0.1426	0.01597	1770	0.0431
36	0.1270	0.01267	2230	0.0342
38	0.1007	0.007967	3550	0.0215
40	0.07987	0.005010	5640	0.0135

* Let C = per cent conductivity, R_{20} = resistance of 61 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C, then

$$R_t = \frac{61\ R_{20}}{C}[1 + 0.004\ (t - 20)].$$

The temperature coefficient is approximate only.

SOLID STEEL WIRE

American Steel Wire Gage; English Units

12.5 Per Cent Conductivity; Density 7.78

Am. Steel Wire Gage No.	Diameter		Cross-section		Resistance at 20° C. or 68° F.*		Weight in pounds		Feet per pound
	In.	Mils	Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
	½	500.0	250,000	0.1964	0.332	1.752	662.5	3499	1.51
7–0		490.0	240,100	0.1886	0.346	1.825	636.3	3360	1.57
	15/32	468.8	219,800	0.1726	0.378	1.993	582.4	3075	1.72
6–0		460.0	211,600	0.1662	0.392	2.07	560.8	2961	1.78
	7/16	437.5	191,400	0.1503	0.433	2.29	507.2	2678	1.97
5–0		430.0	184,900	0.1452	0.449	2.37	490.0	2587	2.04
	13/32	406.3	165,000	0.1296	0.503	2.65	436.8	2306	2.28
4–0		393.8	155,100	0.1218	0.535	2.82	411.9	2175	2.42
	⅜	375.0	140,600	0.1104	0.590	3.12	372.6	1967	2.68
3–0		362.5	131,400	0.1032	0.631	3.33	348.2	1839	2.87
	11/32	343.8	118,200	0.09280	0.702	3.71	313.1	1653	3.19
2–0		331.0	109,600	0.08605	0.757	4.00	290.3	1533	3.44
	5/16	312.5	97,660	0.07670	0.850	4.49	258.8	1366	3.86
0		306.5	93,940	0.07378	0.883	4.66	249.0	1315	4.02
1		283.0	80,090	0.06290	1.036	5.47	212.2	1121	4.71
	9/32	281.3	79,100	0.06213	1.049	5.54	209.6	1107	4.77
2		262.5	68,910	0.05412	1.204	6.36	182.6	964.1	5.48
	¼	250.0	62,500	0.04909	1.328	7.01	165.6	874.5	6.04
3		243.7	59,490	0.04665	1.397	7.38	157.4	831.0	6.35
4		225.3	50,760	0.03987	1.635	8.63	134.5	710.2	7.43
	7/32	218.8	47,850	0.03758	1.734	9.15	126.8	669.5	7.89
5		207.0	42,850	0.03365	1.936	10.22	113.6	599.5	8.81
6		192.0	36,860	0.02895	2.25	11.88	97.7	515.8	10.23
	3/16	187.5	35,160	0.02761	2.36	12.46	93.2	491.9	10.73
7		177.0	31,330	0.02461	2.65	13.98	83.0	438.4	12.04
8		162.0	26,240	0.02061	3.16	16.69	69.6	367.2	14.38
	5/32	156.3	24,410	0.01917	3.40	17.95	64.7	341.6	15.46
9		148.3	21,990	0.01727	3.77	19.92	58.3	307.8	17.16
10		135.0	18,200	0.01431	4.55	24.0	48.3	255.0	20.70
	⅛	125.0	15,630	0.01227	5.31	28.0	41.4	218.6	24.15

* Let C = per cent conductivity,
R_{20} = resistance of 12.5 per cent conductivity wire at 20° C. (from table),
R_t = resistance of wire of any conductivity C at any temperature t° C,

then

$$R_t = \frac{12.5 R_{20}}{C}[1 + 0.006\,(t - 20)].$$

The temperature coefficient is approximate only.

SOLID STEEL WIRE — *Continued*

American Steel Wire Gage; English Units

12.5 Per Cent Conductivity; Density 7.78

Am. Steel Wire Gage No.	Diameter		Cross-section		Resistance at 20° C. or 68° F.*		Weight in pounds		Feet per pound
	In.	Mils	Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
11		120.5	14,520	0.01140	5.71	30.2	38.5	203.2	25.98
12		105.5	11,130	0.00874	7.45	39.4	29.5	155.7	33.90
	3/32	93.8	8,789	0.00690	9.44	49.8	23.3	123.0	42.94
13		91.5	8,372	0.00658	9.91	52.3	22.1	117.2	45.16
14		80.0	6,400	0.00503	12.96	68.5	17.0	89.55	58.97
15		72.0	5,184	0.00407	16.01	84.5	13.7	72.53	72.80
16		62.5	3906	0.00307	21.2	112.1	10.4	54.66	96.60
	1/16	62.5	3906	0.00307	21.2	112.1	10.4	54.66	96.60
17		54.0	2916	0.00229	28.5	150.2	7.73	40.80	129.5
18		47.5	2256	0.00177	36.8	194.2	5.98	31.57	167.2
19		41.0	1681	0.00132	49.4	261	4.45	23.52	224.4
20		34.8	1211	0.00095	68.5	362	3.21	16.95	311.5
21		31.8	1008	0.00079	82.3	435	2.67	14.11	374.4
	1/32	31.3	977	0.00076	85.0	449	2.59	13.66	386.5
22		28.6	818	0.00064	101.4	536	2.17	11.45	461.1
23		25.8	666	0.00052	124.6	658	1.76	9.31	567.0
24		23.0	529	0.00042	156.8	828	1.40	7.40	713.5
25		20.4	416	0.00033	199.4	1053	1.10	5.82	907.0
26		18.1	328	0.00026	253	1337	0.87	4.58	1152
27		17.3	299	0.00024	277	1464	0.79	4.19	1261
28		16.2	262	0.00021	316	1669	0.70	3.67	1438
29		15.0	225	0.00018	369	1947	0.60	3.15	1677
30		14.0	196	0.00015	424	2240	0.52	2.74	1925
31		13.2	174	0.00014	476	2510	0.46	2.44	2166
32		12.8	164	0.00013	506	2670	0.43	2.30	2303
33		11.8	139	0.00011	596	3150	0.37	1.95	2710
34		10.4	108	0.00008	767	4050	0.29	1.51	3489
35		9.5	90	0.00007	919	4850	0.24	1.26	4193
36		9.0	81	0.00006	1023	5410	0.21	1.13	4659

*Let C = per cent conductivity,
R_{20} = resistance of 12.5 per cent conductivity wire at 20° C. (from table),
R_t = resistance of wire of any conductivity C at any temperature t° C.,
then

$$R_t = \frac{12.5\,R_{20}}{C}[1 + 0.006\,(t - 20)].$$

The temperature coefficient is approximate only.

Copper-clad Steel Wire. — This wire consists of a steel core and a concentric coat of copper permanently welded thereto. It is used chiefly for long span transmission and telephone wire. It is made in several grades, which differ in the relative amounts of steel and copper. The grades are designated by the corresponding conductivity expressed as per cents of Matthiessen's Standard; e.g., 40 per cent grade has a conductivity of 40 per cent.

COPPER-CLAD STEEL WIRE

A. W. G. or B. & S. Gage; English Units

40 Per Cent Conductivity; Density 8.26

Gage No.	Diameter in mils	Cross-section		Resistance at 23.9° C. or 75° F.*		Weight in pounds		Feet per pound
		Circular mils	Square inches	Ohms per 1000 feet	Ohms per mile	per 1000 feet	per mile	
0000	460.0	211,600	0.1662	0.123	0.649	595	3140	1.68
000	409.6	167,800	0.1318	0.154	0.813	471	2490	2.12
00	364.8	133,100	0.1045	0.195	1.03	374	1970	2.67
0	324.9	105,500	0.08289	0.246	1.30	297	1570	3.37
1	289.3	83,690	0.06573	0.310	1.64	235	1240	4.26
2	257.6	66,370	0.05213	0.390	2.06	186	982	5.38
3	229.4	52,630	0.04134	0.492	2.60	148	781	6.76
4	204.3	41,740	0.03278	0.622	3.28	117	618	8.55
5	181.9	33,100	0.02600	0.782	4.13	92.9	491	10.76
6	162.0	26,250	0.02062	0.987	5.21	73.7	389	13.57
7	144.3	20,820	0.01635	1.25	6.60	58.5	309	17.09
8	128.5	16,510	0.01297	1.57	8.29	46.4	245	21.6
9	114.4	13,090	0.01028	1.98	10.5	36.8	194	27.2
10	101.9	10,380	0.008155	2.50	13.2	29.2	154	34.2
11	90.74	8,234	0.006467	3.15	16.6	23.1	122	43.3
12	80.81	6,530	0.005129	3.97	21.0	18.3	96.6	54.6
13	71.96	5,178	0.004067	5.00	26.4	14.6	77.1	68.5
14	64.08	4,107	0.003225	6.31	33.3	11.5	60.7	87.0

* Let C = per cent conductivity,
$R_{23.9}$ = resistance of 40 per cent conductivity wire at 23.9° C. (from table),
R_t = resistance of wire of conductivity C at temperature t° C.,

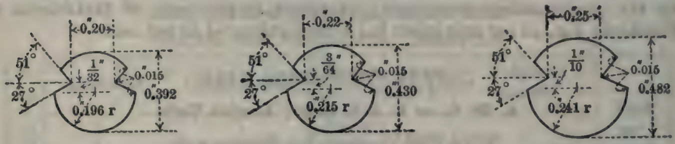
then

$$R_t = \frac{40\,R_{23.9}}{C}\,[1 + 0.00432\,(t - 23.9)].$$

The temperature coefficient is approximate only.

Alloy Wires of High Tensile Strength. — Copper alloys having a low conductivity, but having a tensile strength from 50 per cent to 100 per cent greater than that of copper are sometimes used where strength or hardness is a primary requisite, as in long spans of small wires or for trolley wires. The Bridgeport Brass Co. make a wire known as "phono-electric wire" which has a conductivity of 25 per cent and a tensile strength ranging from 68,000 pounds per square inch for No. 0000 B. & S. gauge to 85,000 pounds per square inch for No. 18 B. & S. gage.

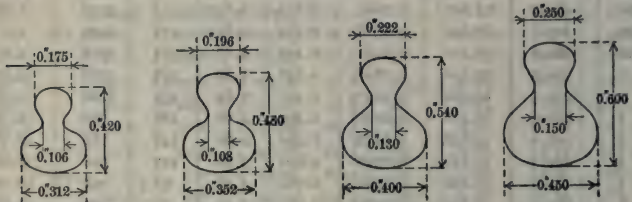
Trolley Wire. — Trolley wires of two different sections are in use in the United States. The sections shown in Fig. 1 are known as the "American Standard" and the sections shown in Fig. 2 as the "Figure 8" sections. The



133,200 C.M. (00) | 168,000 C.M. (000) | 211,600 C.M. (0000)

Fig. 1. American Standard Trolley-wire Sections

American Society for Testing Materials recommend that the sizes be specified in circular mils, and not as gage numbers; the sizes shown in the figure differ in the area of the cross-section from the gage numbers given in parentheses by less than 5 parts in 1000.



105,600 C.M. (0) | 133,200 C.M. (00) | 168,100 C.M. (000) | 211,600 C.M. (0000)

Fig. 2. "Figure 8" Trolley-wire Sections

Trolley wires are usually of hard-drawn copper; the electrical and mechanical properties are the same as for round hard-drawn wire of the same cross-section. Copper alloys, such as phono-electric wire (*see above*), are sometimes used for trolley wires.

FACTORS AFFECTING DIMENSIONS, WEIGHT AND RESISTANCE OF STRANDED WIRES. — Individual stranded wires or cables are of four different types, namely: (a) bunched wire; (b) wire braids; (c) concentric-lay cables and (d) rope-lay cables.

Bunched Wires. — Bunched wires are used especially for those extra flexible cables known as cords, wherein the individual wires are so small that concentric stranding is not necessary to keep them together. The wires are assembled parallel and then generally given a slight twist. Sometimes they are kept together by being wound with soft cotton thread which also serves to prevent adhesion between the insulation and wires.

Wire Braids. — In the flat form, wire braids are used for potential leads, etc., in lighting cables, where a flexible flat conductor is necessary. Tubular wire braids are also frequently formed over the insulation of cables in order to afford mechanical protection. Cables for naval or military purposes and for automobile work are frequently thus protected.

Concentric-lay Cables. — A concentric-lay cable is a stranded conductor composed of a central core surrounded by one or more layers of helically laid

wires. A rope-lay cable is a stranded wire made up in the same manner by using stranded wires instead of individual solid wires for the core and layers. The cores of concentric-lay cables may be composed of one, two, three or four wires of equal diameter. A five or six wire core would not be symmetrical and seven wires would themselves constitute a core and a layer.

Number of Wires in Concentric-lay Cables. — The following table gives all the possible concentric-lay cables with eight or less layers of equal size wires and formulæ for calculating the number of wires with any number of layers.

NUMBER OF WIRES IN CONCENTRIC-LAY CABLES

(All wires of same diameter)

Number of layers over core	Number of wires in core			
	1	2	3	4
0	1	2	3	4
1	7	10	12	14
2	19	24	27	30
3	37	44	48	52
4	61	70	75	80
5	91	102	108	114
6	127	140	147	154
7	169	184	192	200
8	217	234	243	252
n	$3n^2+3n+1$	$3n^2+5n+2$	$3n^2+6n+3$	$3n^2+7n+4$

The number of wires per layer increases by six for each successive layer when the core has one wire, the first layer over the core having six. With cores having more than one wire, the increment per layer is not constant.

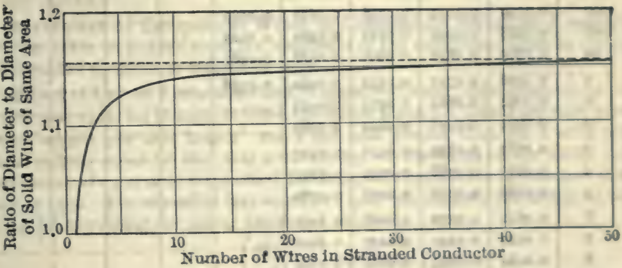


Fig. 3.

Diameter of Concentric-lay Cables. — The diameter of the circumscribing circle of any of the above cables is equal to $(2n + b)$ times the diameter of each wire, where n is the number of layers over the core and b has the following values: 1 wire in core, $b = 1$; 2 wires in core, $b = 2$; 3 wires in core, $b = 2.155$; 4 wires in core, $b = 2.414$.

The relation between the number of component wires and the diameter of the cable is shown in Fig. 3.

COMMERCIAL STRANDED CONDUCTORS

Area of conductor C. M.	Number of wires in the stranded conductor								
	7	19	37	7×7 =49	61	91	127	169	217
	Diameter, in inches, of each wire in the cable								
2,000,000	0.5345	0.3244	0.2325	0.202	0.181	0.1482	0.1255	0.1086	0.096
1,750,000	0.5000	0.3035	0.2175	0.189	0.169	0.1387	0.1174	0.1020	0.090
1,500,000	0.4629	0.2810	0.2013	0.175	0.157	0.1285	0.1087	0.0940	0.083
1,250,000	0.4226	0.2565	0.1838	0.1507	0.143	0.1174	0.0992	0.0860	0.076
1,000,000	0.3779	0.2294	0.1644	0.1429	0.1285	0.1048	0.0887	0.0769	0.0678
950,000	0.3684	0.2236	0.1602	0.1392	0.1247	0.1021	0.0864	0.0749	0.0661
900,000	0.3585	0.2176	0.1559	0.1355	0.1214	0.0995	0.0841	0.0729	0.0644
850,000	0.3484	0.2115	0.1515	0.1317	0.1180	0.0966	0.0818	0.0709	0.0625
800,000	0.3380	0.2050	0.1470	0.1278	0.1145	0.0937	0.0793	0.0687	0.0607
750,000	0.3273	0.1986	0.1423	0.1237	0.1108	0.0907	0.0769	0.0666	0.0588
700,000	0.3163	0.1919	0.1375	0.1195	0.1071	0.0887	0.0742	0.0643	0.0567
650,000	0.3047	0.1849	0.1325	0.1152	0.1032	0.0845	0.0715	0.0620	0.0547
600,000	0.2927	0.1776	0.1273	0.1107	0.0991	0.0812	0.0687	0.0595	0.0525
550,000	0.2803	0.1701	0.1219	0.1060	0.0949	0.0777	0.0658	0.0571	0.0503
500,000	0.2673	0.1622	0.1162	0.1010	0.0905	0.0741	0.0628	0.0543	0.0480
450,000	0.2535	0.1538	0.1103	0.0958	0.0858	0.0703	0.0595	0.0516	0.0455
400,000	0.2390	0.1457	0.1039	0.0904	0.0809	0.0663	0.0561	0.0486	0.0429
350,000	0.2236	0.1357	0.0972	0.0845	0.0757	0.0620	0.0526	0.0455	0.0401
300,000	0.2070	0.1256	0.0903	0.0783	0.0701	0.0574	0.0486	0.0421	0.0371
250,000	0.1889	0.1147	0.0824	0.0714	0.0640	0.0524	0.0443	0.0384	0.0339
Size A. W. G.									
0000	0.1739	0.1055	0.0756	0.0657	0.0589	......	......	......	......
000	0.1548	0.0940	0.0674	0.0586	0.0525	......	......	......	......
00	0.1379	0.0837	0.0600	0.0521	0.0467	......	......	......	......
0	0.1228	0.0745	0.0534	0.0464	0.0416	......	......	......	......
1	0.1094	0.0664	0.0475	0.0413	......	......	......	......	......
2	0.0974	0.0591	0.0424	0.0369	......	......	......	......	......
3	0.0867	0.0525	0.0377	0.0327	......	......	......	......	......
4	0.0772	0.0468	0.0335	0.0291	......	......	......	......	......
5	0.0688	0.0418	0.0299	0.0260	......	......	......	......	......
6	0.0612	0.0372	0.0266	0.0231	......	......	......	......	......
7	0.0545	0.0331	0.0237	0.0206	......	......	......	......	......
8	0.0484	0.0294	0.0211	0.0184					
9	0.0432	0.0263	0.0188	0.0164	......	......	......	......	......
10	0.0386	0.0233	0.0168	......	......	......	......	......	......
12	0.0306	0.0185	0.0133	......	......	......	......	......	......
14	0.0242	0.0148	0.0105	......	......	......	......	......	......

Weights in lb. per 1000 ft. of all bare copper cables are computed by multiplying the circular mils by 0.00309.

Rope-lay Cables. — As already noted, a rope-lay cable is made up in the same way as a concentric-lay cable except that stranded wires are used for the core and layers instead of individual solid wires. Rope strands are used for large conductors which would be too stiff if stranded concentrically. The formulas for regular concentric-lay cables may be readily modified to apply to rope-lay cables, as each stranded wire bears the same relation to the rope as each individual solid wire does to the concentric-lay cable. The following table gives the principal forms of rope-lay cables.

WIRES IN ROPE-LAY CABLES

Number of layers over core	Number of strands*	Total number of wires				
		Wires per strand*				
		7	19	37	61	91
0	1	7	19	37	61	91
1	7	49	133	259	427	637
2	19	133	361	703	1159	1,729
3	37	259	703	1369	2257	3,367
4	61	427	1159	2257	3721	5,551
5	91	637	1729	3367	5551	8,281
6	127	889	2413	4699	7747	11,557
n	$3(n^2+n)+1$	$21(n^2+n)+7$	$57(n^2+n)+19$	$111(n^2+n)+37$	$183(n^2+n)+61$	$273(n^2+n)+91$

* By "strand" is here meant the stranded wires of which the rope is built up.

The number of wires in a rope-lay cable is frequently designated by a product; thus, 7×19 indicates a conductor made up of 7 strands, each strand containing 19 wires.

Hemp-core Cables. — Cables with hemp centers have caused serious trouble and are not being recommended. (D. B. Rushmore, *G. E. Review, June, 1912.*)

Diameters of Component Wires in Commercial Stranded Conductors. — The table on p. 1875 gives the diameters of the component wires in the types of stranded conductors ordinarily used.

Effect of Lay on Resistance and Weight. — In the tables given on pp. 1969, 1977, and 1978, for stranded cables, the values given for "ohms per unit length" and "weight per unit length" are 2 per cent greater than for a solid rod of cross-section equal to the total cross-section of the wires of the cable. This increment of 2 per cent means that the values are correct for cables having a lay of 1 in 15.7. For any other lay, equal to 1 in n, resistance or mass may be calculated by increasing the above tabulated values by

$$\left(\frac{484}{n^2} - 2\right) \text{ per cent.}$$

General Formulas for Properties of Cables in Terms of the Properties of the Constituent Wires. — The following table gives the principal formulas for concentric-lay cable having a core of one wire.

A = total area in circular mils of the component wires measured at right angles to their axes, when laid out straight.

D = diameter of cable over-all, in inches.

d = diameter of each of the component wires, in inches.

d_c = diameter of core, in inches.

d_p = pitch diameter, in inches, of any layer (= mean diameter of the helix made by any layer).
e = elongation, per cent, at which the wires (other than the core) break.
l = number of wires in any layer having pitch diameter d_p.
N = total number of wires except where the core is of special size, in which case N is the number of wires exclusive of the core.
n = number of layers of wire over the core.
P = pitch of any layer of wires = distance in inches measured along the axis of the cable for one complete turn of the helix formed by any wire of this layer.
p = pitch-factor of any layer of wires = ratio of the actual length of a wire to the corresponding axial length of the cable.
R = ratio of wire area to the total area of the circle circumscribing the outside of the conductor.
s = stress in pounds per square inch in the core when the elongation is e.
t = tensile strength of each outer wire, in pounds per square inch.
T = tensile strength of conductor, in pounds.
W = weight of conductor, in pounds per foot.
w = weight of each wire of the cable, in pounds per foot.
w_c = weight of the core of the cable, in pounds per foot.

PROPERTIES OF CONCENTRIC-LAY CABLES

		Regular; $d_c = d$	Special; $d_c \neq d$
I.	Number of wires in terms of number of layers (and core diameter).	$N = 3(n^2+n)+1$ (including core)	$N = 3\left(n\frac{d_c}{d}+n^2\right)$ (excluding core)
II.	Diameter of cable in terms of diameter of wires and number of layers.	$D = d(1+2n)$	$D = d_c + 2nd$
III.	Diameter of cable in terms of total area and number of wires.	$D = 10^{-3}\sqrt{\frac{1}{3}\left(4-\frac{1}{N}\right)} \cdot \sqrt{A}$	
IV.	Ratio of wire area to area of circle circumscribing the outside of cable.	$R = \frac{3(n^2+n)+1}{(2n+1)^2}$	
V.	Weight of cable in terms of weight of wire, number of layers and pitch factors.	$W = w(1+6p_6+12p_{12}+\text{etc.})$	$W = w_c + w(1+6p_6+12p_{12}+\text{etc.})$
VI.	Strength of cable in terms of strength of the component wires and the pitch factors.	$T = \frac{\pi}{4}d^2\left[s+t\left(\frac{6}{p_6}+\frac{12}{p_{12}}+\text{etc.}\right)\right]$	$T = \frac{\pi}{4}\left[sd_c^2+d^2t\left(\frac{6}{p_6}+\frac{12}{p_{12}}+\text{etc.}\right)\right]$

PROPERTIES OF CONCENTRIC-LAY CABLES — *Continued*

		Regular; $d_o = d$	Special; $d_o \neq d$
VII.	Minimum pitch in terms of wire diameter and core diameter.	$\frac{3\pi d_p}{\sqrt{(\pi+3)(\pi-3)}} = 10.1$ times pitch diameter	$\frac{\pi d_p l d}{\sqrt{(\pi d_p)^2-(ld)^2}}$
VIII.	Diameter of wires in terms of total conductor area and number of wires.	$d = \frac{1}{1000}\sqrt{\frac{A}{N}}$	

STRENGTH, ELASTICITY AND EXPANSION COEFFICIENT OF WIRES. — The strength and elasticity of a wire of any material depends to a considerable extent upon the method of manufacture, heat treatment, etc. The tensile strength of soft copper is between 25,000 and 35,000 pounds per square inch, as against 60,000 pounds per square inch for hard-drawn copper. Again, due to the greater relative thickness of the hard "skin" and comparatively soft "core" of small hard-drawn copper wires as compared with large wires, the tensile strength, in pounds per square inch, of a small hard-drawn copper wire is greater than the tensile strength of a large hard-drawn wire. For example, a No. 0000 B. & S. hard-drawn copper wire has a tensile strength of about 50,000 pounds per square inch as against approximately 65,000 pounds per square inch for a No. 18. A similar but smaller variation holds for soft-drawn wires. The tensile strength of steel wire depends to a very great extent upon the composition of the steel.

The following table gives, for a No. 0 A. W. G. wire, representative values of the various quantities stated. These values do not hold, except to a rough approximation, for other sizes of wire. For further information see articles on *Copper, Aluminum,* etc., and the section on *Specifications* below.

STRENGTH, ELASTICITY AND COEFFICIENT OF EXPANSION
Of a No. 0 A. W. G. or B. & S. Wire

Kind of wire	Tensile strength, lb. per sq. in.	Elastic limit, lb. per sq. in.*	Modulus of elasticity, lb.-in. units	Coefficient of linear expansion per ° F.	Coefficient of linear expansion per ° C.
Copper, soft-drawn	36,000	......	....	9.6×10^{-6}	17×10^{-6}
Copper, hard-drawn	54,500	30,000	16×10^{6}	9.6×10^{-6}	17×10^{-6}
Aluminum, soft-drawn	16,000	......	....	12.8×10^{-6}	23×10^{-6}
Aluminum, hard-drawn	25,000	25,000	9×10^{6}	12.8×10^{-6}	23×10^{-6}
Copper-clad steel, 40% grade	60,000	51,000	22×10^{6}	6.7×10^{-6}	12×10^{-6}
Phono-electric	75,800	55,000	18×10^{6}	8.3×10^{-6}	14.9×10^{-6}
Steel, ordinary	68,000	40,000	24×10^{6} to 30×10^{6}	7.0×10^{-6}	12.6×10^{-6}
Steel, Siemens-Martin	90,000	45,000		7.0×10^{-6}	12.6×10^{-6}
Steel, high strength	150,000	82,000		7.0×10^{-6}	12.6×10^{-6}
Steel, extra high strength	225,000	135,000		7.0×10^{-6}	12.6×10^{-6}

* There is no elastic limit for soft annealed copper and the elastic limits of hard-drawn copper and aluminum are doubtful.

The tensile strength in pounds for solid wires from 1/16 inch to 1/2 inch in diameter are given in the following table.

BREAKING LOAD FOR SOLID WIRES IN POUNDS PER WIRE

Gage No. A.W.G. or B. & S.	Diameter Inches	Diameter Mils	Hard-drawn copper (Am. Soc. for Test. Mat.)*	Hard-drawn aluminum (23,000 to 33,300 lb. per sq. in.)	Copper-clad steel, 40 per cent grade	Steel (100,000 lb. per sq. in.) †
	1/2	500	9310	4520	11,400	19,640
0000		460	8140	3820	10,000	16,620
	7/16	437	7500	3460	9,250	15,030
000		410	6720	3030	8,300	13,180
	3/8	375	5800	2540	7,150	11,040
00		365	5540	2400	6,850	10,450
0		325	4520	1910	5,700	8,289
	5/16	312	4220	1770	5,400	7,670
1		289	3680	1530	4,800	6,573
2		258	3000	1240	4,000	5,213
	1/4	250	2830	1170	3,780	4,909
3		229	2420	1000	3,200	4,134
4		204	1950	810	2,600	3,278
	3/16	187	1680	693	2,300	2,761
5		182	1570	655	2,200	2,600
6		162	1270	532	1,800	2,062
7		144	1020	432	1,450	1,635
8		129	822	351	1,200	1,297
	1/8	125	780	335	1,150	1,227
9		114	660	287	975	1,028
10		102	528	234	800	816
11		91	423	191	650	647
12		81	337	155	510	513
13		72	268	126	410	407
14		64	213	103	330	323
	1/16	62	203	98	310	307

* Tensile strength in pounds per square inch ranging from 49,000 for No. 0000 to 66,200 for No. 14; see below.

† For wires having a tensile strength of S pounds per square inch, multiply by $S/100{,}000$. The tensile strength of steel varies from 60,000 to 225,000 pounds per square inch.

Strength, Elasticity and Expansion Coefficient of Cables.—The following is a summary of the results of tests made on stranded copper wires at the Massachusetts Institute of Technology in 1912. Each figure is the average of a number of individual tests. In determining the modulus of elasticity each cable was given a preliminary stretch before readings were taken.

MECHANICAL PROPERTIES OF BARE COPPER CABLES

Designation	Size of cable A. W. G. or cir. mils	Number of wires in cable	Pitch of layers, inches		Modulus, lb.-in. units	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.
			First layer	Second layer			
A	0	7	4.5	...	16.6×10^6	22,000	53,500
B	300,000	19	7	5	16.4×10^6	21,300	59,800
C	300,000	19	9	4.5	16.3×10^6	25,000	56,000
D	300,000	19	9	9	16.5×10^6	24,000	52,600
E	300,000	19	4.5	3	13.6×10^6	25,300	53,800

PROPERTIES OF COMPONENT WIRES OF ABOVE CABLES

Wires from cable	Size of component wires, mils	Modulus, lb.-in. units	Elastic limit, lb. per sq. in.	Tensile strength, lb. per sq. in.
B	125	16.9×10^6	27,000	62,800
C	125	18.9×10^6	27,000	61,900
D	125	16.6×10^6	26,000	61,800
E	125	17.5×10^6	26,000	59,300

Tests by B. Welbourne (Jour. I.E.E., 1917, Vol. 56, p. 53) indicate that the elastic modulus of copper cables varies with the number of strands, as shown in the accompanying table.

Strands	Modulus, lb.-in. units
7	20.0×10^6
19	17.5×10^6
37	15.5×10^6
61	14.0×10^6
91	12.5×10^6

Steel Cable for Catenary Construction. — A modulus of 22×10^6 pound-inch units is representative of ordinary steel messenger cable.

The following tables are compiled from tables published by the General Electric Co. (*Bulletin 4538*). "High" and "Extra high" strength steel should only be used where absolutely necessary, as, on account of its stiffness, it requires special mechanical fastenings.

STEEL CABLE FOR CATENARY SUSPENSION

Extra-galvanized Siemens-Martin Steel Strand, 90,000 Pounds Per Square Inch.				
Diameter, inches	Tensile strength, pounds	Elastic limit, pounds	Elongation, per cent	Lay, inches
1/4	3,060	1,830	6 to 9	3
5/16	4,860	2,910	6 to 9	3 1/2
3/8	6,800	4,080	5 to 8	4
7/16	9,000	5,300	5 to 8	4 1/2
1/2	11,000	6,600	5 to 8	4 1/2
5/8	19,000	11,400	4 to 6	5

STEEL CABLE FOR CATENARY SUSPENSION — (*Continued*)

Extra-galvanized High-strength Crucible Steel Strand

Diameter, inches	Tensile strength, pounds	Elastic limit, pounds	Elongation, per cent	Lay, inches
1/4	5,100	3,315	3 to 5	3 1/2
5/16	8,100	5,265	3 to 5	4
3/8	11,500	7,475	3 to 5	4 1/2
7/16	15,000	9,500	3 to 5	5
1/2	18,000	11,700	3 to 5	5
5/8	25,000	16,250	2 to 4	5 1/2

Extra-galvanized Extra-high-strength Plow Steel Strand.

Diameter, inches	Tensile strength, pounds	Elastic limit pounds	Elongation, per cent	Lay, inches
1/4	7,600	5,700	2 1/2 to 4	4
5/16	12,100	9,075	2 1/2 to 4	4 1/2
3/8	17,250	12,930	2 1/2 to 4	5
7/16	22,500	16,800	2 1/2 to 4	5 1/2
1/2	27,000	20,250	2 1/2 to 4	5 1/2
5/8	42,000	31,500	1 1/2 to 3	6

COMPARISON OF COPPER AND ALUMINUM FOR EQUAL LENGTH AND EQUAL RESISTANCE. — (*See Aluminum.*)

ALUMINUM-STEEL CABLES. — Cables made of a central core of steel, solid or stranded, surrounded by one or more layers of aluminum wire are now finding great favor for aerial lines. Their great advantage is that they may be erected not only with less sag than aluminum but with from 60 to 70 per cent of that of hard-drawn copper.

The modulus of elasticity and temperature coefficient of expansion depend upon the relative cross-section of aluminum and steel. The anchor clamps hold both the aluminum and steel, so that at low temperatures the load is divided between the two metals. As the temperature rises, the aluminum expands faster than the steel and the load gradually shifts over to the steel which in turn stretches more to keep pace with the expanded length of aluminum, so that there is no relative motion between the two metals. Eventually the steel carries the total load, and the expansion follows the coefficient for steel only.

Let H_a = fraction of area covered by aluminum,
H_s = fraction of area covered by steel,
M_a = modulus of elasticity of aluminum (9,000,000),
M_s = modulus of elasticity of steel (30,000,000),
M = modulus of elasticity of aluminum-steel cable,
G_a = coefficient of expansion of aluminum (0.0000128),
G_s = coefficient of expansion of steel (0.0000064),
G = coefficient of expansion of cable.

Then, $$M = M_aH_a + M_sH_s = (9 \times H_a + 30H_s)10^6,$$

$$G = \left\{ \frac{1}{1+\dfrac{M_aH_a}{M_sH_s}} \right\} G_s - \left\{ \frac{1}{1+\dfrac{M_aH_a}{M_sH_s}} - 1 \right\} G_a$$

$$= 0.0000064 \left\{ \frac{1}{1+0.3\dfrac{H_a}{H_s}} \right\} - 0.0000128 \left\{ \frac{1}{1+0.3\dfrac{H_a}{H_s}} - 1 \right\}.$$

CURRENT-CARRYING CAPACITY OF BARE WIRES. — (*See also articles on Rheostats; Wires, Resistance.*) Let d = diameter of conductor in inches, T = permissible temperature rise in ° C. above surrounding medium (air, earth, or water), r = resistance of conductor in ohms per mil-foot at final temperature, I = current per conductor in amperes. Assuming that the rate of heat radiation per unit length of wire is proportional to the difference of temperature between the conductor and the surrounding medium and also proportional to the surface of the conductor, then for solid conductors

$$I = K\sqrt{\frac{Td^3}{r}},$$

and for stranded conductors

$$1.2\,K\sqrt{\frac{Td^3}{r}},$$

where K is a constant, which depends upon the condition of the surface of the wire, and upon the amount of heat convection due to air currents. Values of the constant K for air given by different authorities vary from 800 to 1000, the former referring to still air and the latter to open air.

TESTS OF BARE WIRES AND CABLES. — The usual tests on bare wires are gaging diameter, measuring tensile strength, elongation, modulus, elastic limit and electrical conductivity.

Gaging Diameter. — (*See also section on Specifications, below.*) The best type of gage for measuring wire diameters is that shown in Fig. 4. The wire is placed between the measuring surfaces and the screw adjusted until a click occurs. The number of large divisions exposed on the axis is multiplied by 100; the number of small divisions exposed on the axis is multiplied by 25; and the sum of these two items added to the number indicated on the revolving scale. The sum will be the diameter of the wire in mils.

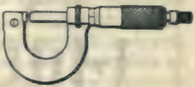


Fig. 4.

Tensile Strength, Elongation, Modulus and Elastic Limit. — The essential features of a wire-testing machine are a means of applying a measurable pulling force to the wire, and a means of taking up the elongation. Accordingly, the usual testing machine consists of two pairs of jaws for gripping the wire, one pair being connected to a balance lever and the other pair to a power-driven mechanism which draws it in the direction of the axis of the wire. A typical machine is shown diagrammatically in Fig. 5, where A and B are the two pairs of jaws between which the wire is stretched. The machine is operated by setting in motion the mechanism which makes the jaw A move steadily in the direction indicated. The operator then moves the counterpoise C by hand, in the direction in-

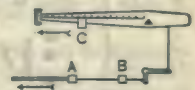


Fig. 5.

ALUMINUM-STEEL CABLE

A. C. S. R.				Copper of equivalent resistance	Usual stranding		Elastic limit, pounds	Ultimate strength pounds	Ohms per 1000 feet (61 per cent)	Diameter, inches	Weight-pounds Per 1000 feet			Per cent of total weight	
B. & S. gage No.	Aluminum area														
	Circular mils	Square inches	Square millimeters		Aluminum	Steel					Total	Al.	Steel	Per cent Al.	Per cent steel
....	605,000	.5656	364.9	380,500	54 × .1059	7 × .1059	14,675	21,270	.0286	.953	780.0	568.0	212.0	72.8	27.2
....	500,000	.3927	253.3	314,500	30 × .1291	19 × .0775	17,140	23,750	.0347	.904	777.0	469.0	308.0	60.4	39.6
....	336,400	.2642	170.5	No. 4/0	30 × .1059	7 × .1059	11,715	16,200	.0515	.741	528.0	316.0	212.0	59.8	40.2
....	266,800	.2094	135.1	No. 3/0	6 × .2108	7 × .0705	6,470	9,385	.0648	.633	343.0	250.0	93.0	72.8	27.1
No. 4/0	211,600	.1662	107.2	No. 2/0	6 × .1880	1 × .1880	5,940	8,435	.0816	.564	295.0	199.0	96.0	67.6	32.4
No. 3/0	167,805	.1318	85.03	No. 1/0	6 × .1670	1 × .1670	4,690	6,660	.1026	.501	232.5	157.0	75.5	67.6	32.4
No. 2/0	133,079	.1045	67.42	No. 1	6 × .1490	1 × .1490	3,730	5,300	.1294	.447	185.0	125.0	60.0	67.6	32.4
No. 1/0	105,534	.0829	53.48	No. 2	6 × .1327	1 × .1327	2,960	4,200	.1639	.398	147.0	99.5	47.5	67.6	32.4
No. 1	83,694	.0657	42.39	No. 3	6 × .1182	1 × .1182	2,355	3,340	.2070	.355	117.0	79.0	38.0	67.6	32.4
No. 2	66,373	.0521	33.61	No. 4	6 × .1052	1 × .1052	1,860	2,660	.2610	.316	92.4	62.5	29.9	67.6	32.4
No. 3	52,634	.0413	26.65	No. 5	6 × .0938	1 × .0938	1,480	2,100	.3291	.281	73.4	49.7	23.7	67.6	32.4
No. 4	41,742	.0328	21.16	No. 6	6 × .0834	1 × .0834	1,170	1,665	.4150	.250	58.0	39.3	18.7	67.6	32.4
No. 5	33,102	.0260	16.77	No. 7	6 × .0743	1 × .0743	930	1,315	.5217	.223	46.0	31.0	15.0	67.6	32.4
No. 6	26,250	.0206	13.29	No. 8	6 × .0661	1 × .0661	735	1,045	.6577	.198	36.4	24.6	11.8	67.6	32.4
No. 7	20,816	.0163	10.52	No. 9	6 × .0586	1 × .0586	575	820	.8293	.176	28.5	19.3	9.2	67.6	32.4
No. 8	16,509	.0130	8.39	No. 10	6 × .0525	1 × .0525	465	660	1.045	.158	23.0	15.6	7.4	67.6	32.4

dicated, so as to keep the beam balanced. This operation is continued until the wire breaks, when the elongation of the sample is measured by the travel of the jaw *A* and its breaking strength by the weight indicated on the balance beam at the counterpoise *C*.

Measurement of Strain. — The amount by which the wire is stretched is measured by means of an extensometer which consists of a pair of clamps to grip the wire at points a definite distance apart, and a magnifying scale for measuring the increase of distance between these clamps as the wire stretches. The stress-strain curve obtained by plotting the elongations thus measured against the stresses measured by the machine described above is not a true one, as there is initially an abnormal elongation due to the straightening of the wire, as shown by curve *OA* in Fig. 6. The standard method of overcoming this, is described in Section 8 of the A.S.T.M. *Specification for Hard-drawn Copper Wire*.

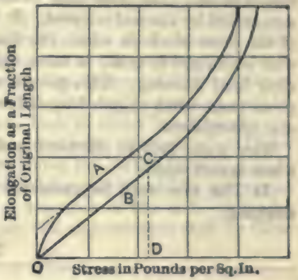


Fig. 6.

Modulus of Elasticity. — The modulus of elasticity is obtained from the slope of the straight part of the corrected curve. In Fig. 6 the modulus of elasticity is *OD/CD* pound-inch units.

Elastic Limit. — The true elastic limit can be obtained only by applying a series of increasing loads, releasing the load (leaving, however, a sufficient load to keep the wire straight) and measuring the elongation between successive loads. The load at which a permanent elongation begins is the elastic limit.

Conductivity. — In order to maintain a wire at a uniform and known temperature, it must be short. Unless the wire is very small the test sample will therefore have a very low resistance, and an ordinary Wheatstone bridge will not be sufficiently accurate to measure it. This difficulty is avoided by using a bridge of either the Kelvin, Hoop, Willyoung or Reeves type (*see Bridges for Electrical Measurements; Resistance and Conductance*).

SPECIFICATIONS FOR COPPER WIRE AND CABLE. — The following specifications for bare copper wire and cable are those prepared by the American Society for Testing Materials, 1915 (hard-drawn copper), and 1915 (soft or annealed copper), and 1916 (copper cable). See also the article on *Specifications and Contracts*, and the section on *Specifications* in the article on *Wires and Cables, Insulated*.

SPECIFICATION FOR SOFT OR ANNEALED COPPER WIRE.

1. General. — The copper shall be of such quality and purity that, when drawn and annealed, it shall have the properties and characteristics herein required.

2. This specification covers untinned, drawn and annealed round wire.

3. (*a*) The wire must be free from all imperfections not consistent with the best commercial practice.

(*b*) Necessary brazes in soft or annealed wire must be made in accordance with the best commercial practice.

4. **Shipment; Coils, Spools and Reels.** — (*a*) Wire may be shipped in coils or on reels as agreed upon by the purchaser and manufacturer. In Table I (*below*) there are stated the maximum and minimum weights of wire of the stated

sizes which may be shipped in any one package, whether coil, reel or spool; in the case of wire larger than 0.010 inch in diameter, the maximum and minimum package weights are net, and in the case of wire 0.010 inch and less in diameter, the maximum package weights are gross, and the minimum package weights are net. The table also states the limiting dimensions of the coils, reels and spools on which wire may be shipped. The length and diameter stated for reels and spools are to be measured over-all and are maximum sizes; reels or spools smaller than these may be used provided the minimum weights called for are carried by the reel or spool. In the table, there are also stated the diameters of the draw-block on which the final drawing of the wire is to be made, when wire is shipped in coils; it being understood that the wire is not to be rewound after final drawing. This provision is made to insure that coils of wire of a given gage, when supplied by different manufacturers, will be of the same general dimensions.

Wire 0.204 inch in diameter and larger may be shipped in larger packages when agreed upon.

(*b*) The wire shall be protected against damage in ordinary handling and shipping.

TABLE I

Diameters, inches	Package weights, pounds		Diam. of draw-block, inches	Dimensions of reels and spools, inches		
	Max.	Min.		Max. diam.	Max. length	Diameter of hole for rod
0.460 to 0.360	520	290	24	32	21	1½ to 2½
0.359 to 0.258	430	290	24	32	21	1½ to 2½
0.257 to 0.129	290	140	22	24	12	1½ to 2½
0.128 to 0.102	230	95	22	24	12	⅝ to 1½
0.101 to 0.083	230	75	22	24	12	⅝ to 1½
0.082 to 0.081	200	75	16	24	12	⅝ to 1½
0.080 to 0.064	200	50	16	24	12	⅝ to 1½
0.063 to 0.051	120	50	16	24	10	⅝ to 1½
0.050 to 0.041	100	50	16	24	10	⅝ to 1½
0.040 to 0.032	50	20	8	24	8	⅝ to 1⅛
0.031 to 0.020	25	15	8	10	6½	⅝ to ⅞
0.019 to 0.011	10	5	8	5½	4	⅜ to 11/16
0.010 to 0.008	5	2½	8	4	4	⅜ to 11/16
0.007 to 0.0056	2½	1	6	2½	4	⅜ to 11/16
0.005	1½	⅝	6	2½	4	⅜ to 11/16
0.004	1½	⅜	6	2½	4	⅜ to 11/16
0.003	1	¼	6	2½	4	⅜ to 11/16

5. **Specific Gravity.** — For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be taken as 8.89.

6. **Size and Gaging.** — (*a*) Size shall be expressed as the diameter of the wire in decimal fractions of an inch.

(*b*) Wire shall be accurate in diameter; permissible variations from nominal diameter shall be:

For wire 0.010 inch in diameter and larger, 1 per cent over or under.

For wire less than 0.010 inch in diameter, 0.1 mil (0.0001 inch) over or under.

(*c*) Each coil shall be gaged at three places, one near each end and one approximately at the middle; from spools, approximately twelve feet shall be reeled off, the wire shall be gaged in six places between the second and twelfth foot from the end. The coils or spools will be rejected if the average of the measurements obtained is not within the limits specified in (*b*).

7. **Tensile Strength and Elongation.** — Wire shall be so drawn and annealed that its tensile strength shall not be greater and its elongation not less than the values stated in Table II. Tensile tests shall be made upon fair samples, and the elongation shall be determined as the permanent increase in length, due to the breaking of the wire in tension, measured between bench marks placed upon the wire originally 10 inches apart. The fracture shall be between the bench marks and not closer than 1 inch to either bench mark. If, upon testing a sample from any coil, reel or spool of wire, the results are found to be below the stated value in elongation or above the stated value in tensile strength, tests upon two additional samples shall be made, and the average of the three tests shall determine acceptance or rejection of the coil. For wire whose nominal diameter is between listed sizes, the requirements shall be those of the next larger size included in the table.

TABLE II. — ANNEALED COPPER WIRE

Diameter, inches	Tensile strength, lb. per sq. in.	Elongation in 10 in., per cent
0.460–0.290	36,000	35
0.289–0.103	37,000	30
0.102–0.021	38,500	25
0.020–0.003	40,000	20

8. **Resistivity.** — Electric resistivity shall be determined upon fair samples by resistance measurements at a temperature of 20° C. (68° F.) and it shall not exceed 891.58 pounds per mile-ohm (0.15614 ohm per meter-gram).

9. **Testing and Inspection.** — All testing and inspection shall be made at the place of manufacture. The manufacturer shall afford the inspector representing the purchaser all reasonable facilities to satisfy him that the material conforms to the requirements of these specifications.

SPECIFICATION FOR HARD-DRAWN COPPER WIRE

1. **General.**— The material shall be copper of such quality and purity that, when drawn hard, it shall have the properties and characteristics herein required.

2. These specifications cover hard-drawn round wire, grooved trolley wire, and figure-eight trolley wire, as hereinafter described.

3. (*a*) The wire, in all shapes, must be free from all imperfections not consistent with the best commercial practice.

(*b*) Necessary brazes in hard-drawn wire must be made in accordance with best commercial practice, and tests upon a section of wire containing a braze must show at least 95 per cent of the tensile strength of the unbrazed wire. Elongation tests are not to be made upon test sections including brazes.

4. **Shipment.** — (*a*) Package sizes for round wire and for cable shall

be agreed upon in the placing of individual orders; standard packages of grooved trolley wire shall be shipped upon reels holding about 2500 pounds each.

(*b*) The wire shall be protected against damage in ordinary handling and shipping.

5. **Specific Gravity.** — For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be taken as 8.89 at 20° C.

6. **Testing and Inspection.** — All testing and inspection shall be made at the place of manufacture. The manufacturer shall afford the inspector representing the purchaser all reasonable facilities to enable him to satisfy himself that the material conforms to the requirements of these specifications.

Hard Drawn Round Wire

7. **Dimensions and Permissible Variations.** — (*a*) Size shall be expressed as the diameter of the wire in decimal fractions of an inch, using not more than three places of decimals; i.e., in mils.

(*b*) Wire is expected to be accurate in diameter; permissible variations from nominal diameter shall be:

For wire 0.100 inch in diameter and larger, one per cent over or under;
For wire less than 0.100 inch in diameter, one mil over or under.

(*c*) Each coil is to be gaged at three places, one near each end, and one approximately at the middle; the coil may be rejected if, two points being within the accepted limits, the third point is off gage more than 2 per cent in the case of wire 0.064 inch in diameter and larger, or more than 3 per cent in the case of wire less than 0.064 inch in diameter.

8. **Physical Test.** — Wire shall be so drawn that its tensile strength and elongation shall be at least equal to the value stated in Table I. Tensile tests shall be made upon fair samples, and the elongation of wire larger in diameter than 0.204 inch shall be determined as the permanent increase in length, due to the breaking of the wire in tension, measured between bench marks placed upon the wire originally 10 inches apart. The elongation of wire 0.204 inch in diameter and smaller shall be determined by measurements made between the jaws of the testing machine. The zero length shall be the distance between the jaws when a load equal to 10 per cent of the required ultimate breaking strength shall have been applied, and the final length shall be the distance between the jaws at the time of rupture. The zero length shall be as near 60 inches as possible. The fracture shall be between the bench marks in the case of wire larger than 0.204 inch in diameter and between the jaws in the case of smaller wire, and not closer than 1 inch to either bench mark or jaw. If, upon testing a sample from any coil of wire, the results are found to be below the values stated in the table, tests upon two additional samples shall be made, and the average of the three tests shall determine acceptance or rejection of the coil. For wire whose nominal diameter is between listed sizes, the requirements, shall be those of the next larger size included in Table I.

9. **Electric Resistivity.** — Electric resistivity shall be determined upon fair samples by resistance measurements at a temperature of 20° C. (68° F.).

The wire shall not exceed the following limits:

For diameters 0.460 inch to 0.325 inch, 900.77 pounds per mile-ohm at 20° C.
For diameters 0.324 inch to 0.040 inch, 910.15 pounds per mile-ohm at 20° C.

Grooved Trolley Wire

10. **Sections.** — Standard sections shall be known as the "American Standard" grooved trolley-wire sections, the shape and dimensions of which are as shown in Fig. 1, above.

TABLE I.—HARD-DRAWN COPPER WIRE

Approx. Gage No., B. & S.	Diameter, inches	Area, circular mils	Tensile strength, lb. per sq. in.	Elongation, per cent
				in 10 in.
0000	0.460	211,600	49,000	3.75
000	0.410	168,100	51,000	3.25
00	0.365	133,225	52,800	2.80
0	0.325	105,625	54,500	2.40
1	0.289	83,520	56,100	2.17
2	0.258	66,565	57,600	1.98
3	0.229	52,440	59,000	1.79
				in 60 in.
4	0.204	41,615	60,100	1.24
5	0.182	33,125	61,200	1.18
	0.165	27,225	62,000	1.14
6	0.162	26,245	62,100	1.14
7	0.144	20,735	63,000	1.09
	0.134	17,956	63,400	1.07
8	0.128	16,385	63,700	1.06
9	0.114	12,995	64,300	1.02
	0.104	10,815	64,800	1.00
10	0.102	10,404	64,900	1.00
	0.092	8,464	65,400	0.97
11	0.091	8,281	65,400	0.97
12	0.081	6,561	65,700	0.95
	0.080	6,400	65,700	0.94
13	0.072	5,184	65,900	0.92
	0.065	4,225	66,200	0.91
14	0.064	4,096	66,200	0.90
15	0.057	3,249	66,400	0.89
	0.051	2,601	66,600	0.87
16	0.045	2,025	66,800	0.86
18	0.040	1,600	67,000	0.85

11. **Dimensions and Permissible Variations.** — (*a*) Size shall be expressed as the area of cross-section in circular mils, the standard sizes being as follows:

211,600 circular mils, weighing 3386 pounds per mile.
168,100 circular mils, weighing 2690 pounds per mile.
133,200 circular mils, weighing 2132 pounds per mile.

(*b*) Grooved trolley wire may vary 4 per cent over or under in weight per unit length from standard, as determined from the nominal cross-section.

12. **Physical Tests.** — The physical tests shall be made in the same manner as those upon round wire. The tensile strength of grooved wire shall be at least 95 per cent of that required for round wire of the same sectional area; the elongation shall be the same as that required for round wire of the same sectional area.

13. **Electric Resistivity.** — The requirements for electric resistivity shall be the same as those for round wire of the same sectional area.

Figure-eight Trolley Wire

14. **Sections.** — Standard sections of figure-eight trolley wire shall be as shown in Fig. 2, above.

15. **Requirements.** — The requirements for weight, physical properties and electric resistivity of figure-eight trolley wire shall be the same as for the same sizes of grooved trolley wire.

SPECIFICATION FOR BARE CONCENTRIC-LAY COPPER CABLE: HARD, MEDIUM-HARD, OR SOFT

1. (*a*) **Products Covered.** — These specifications cover bare concentric-lay cables made from round copper wires laid helically around a central core in one or more layers. The central core shall be made of wire having the same quality and temper as the concentric layers, unless otherwise especially provided for in separate specifications governing the individual case.

(*b*) **Classes.** — The purpose for which the several classes of concentric-lay cables are generally used are as follows:

Class *A*, for bare, weatherproof, slow-burning, and slow-burning weatherproof cable for aerial use;
Class *B*, for various insulated cable, such as rubber, paper, varnished cloth, etc.
Class *C*, for cable where greater flexibility is required than in Class *B*.

2. **Requirements of Wires.** — The copper wires entering into the construction of standard concentric-lay cable shall, before stranding, meet all the requirements of that one of the Standard Specifications of the American Society for Testing Materials for Hard-Drawn, Medium Hard-Drawn, or Soft or Annealed Copper Wire (Serial Designations: *B* 1, *B* 2, or *B* 3), which applies.

3. **Brazes.** — Brazes may be made in the wire when finished and ready for cabling. Such brazes shall be made in accordance with the best commercial practice. No brazes in cable made from hard, or medium hard-drawn copper wire may be closer together than 50 feet.

4. **Pitch and Lay.** — The pitch of standard cable shall not be less than 12 nor more than 16 diameters of the cable, and the lay may be right or left-handed, unless one direction of lay is specified by the purchaser.

5. **Testing.** — Tests for the physical and electrical properties of the wire composing the cables may be made before, but not after stranding. Experience indicates that the tensile strength of concentric-lay copper cable of standard pitch is at least 90 per cent of the total strength required of the wires forming the cable.

6. **Weights and Area.** — For the purpose of calculating weights, cross-sections, etc., the specific gravity of copper shall be taken as 8.89 at 20° C. The resistance and mass of a stranded conductor are greater than in a solid conductor of the same cross-sectional area, depending on the lay (that is, the pitch of the twist of the wires). Two per cent shall be taken as the standard increment of resistance and of mass. In cases where the lay is definitely known, the increment shall be calculated and not assumed.

7. **Variation in Area.** — The area of cross-section of the completed cable shall not be more than 2 per cent below the area specified, as determined by weight.

8. **Construction.** — The area of cross-section, number and diameter of wires, in standard cable Classes *A*, *B* and *C*, shall be as specified in Table I.

TABLE I

Area of cross-section Circular mils.	Approximate A. W. G. or B. & S. gage sizes	Class A		Class B		Class C	
		Number of wires	Diameter of wires mils.	Number of wires	Diameter of wires mils.	Number of wires	Diameter of wires mils
2,000,000	......	91	148.2	127	125.5	169	108.8
1,900,000	......	91	144.5	127	122.3	169	106.0
1,800,000	......	91	140.6	127	119.1	169	103.2
1,700,000	......	91	136.6	127	115.7	169	100.3
1,600,000	......	91	132.6	127	112.2	169	97.3
1,500,000	......	61	156.8	91	128.4	127	108.7
1,400,000	......	61	151.5	91	124.0	127	105.0
1,300,000	......	61	146.0	91	119.5	127	101.2
1,250,000	......	61	143.2	91	117.2	127	99.2
1,200,000	......	61	140.3	91	114.8	127	97.2
1,100,000	......	61	134.3	91	109.9	127	93.1
1,000,000	......	61	128.0	61	128.0	91	104.8
950,000	......	61	124.8	61	124.8	91	102.2
900,000	......	61	121.5	61	121.5	91	99.4
850,000	......	61	118.0	61	118.0	91	96.6
800,000	......	61	114.5	61	114.5	91	93.8
750,000	......	61	110.9	61	110.9	91	90.8
700,000	......	61	107.1	61	107.1	91	87.7
650,000	......	61	103.2	61	103.2	91	84.5
600,000	......	37	127.3	61	99.2	91	81.2
550,000	......	37	121.9	61	95.0	91	77.7
500,000	......	37	116.2	37	116.2	61	90.5
450,000	......	37	110.3	37	110.3	61	85.9
400,000	......	19	145.1	37	104.0	61	81.0
350,000	......	19	135.7	37	97.3	61	75.7
300,000	......	19	135.7	37	90.0	61	70.1
250,000	......	19	114.7	37	82.2	61	64.0
212,000	4/0	7–19	173.9–105.5	19	105.5	37–61	75.6–58.9
168,000	3/0	7–19	155.0– 94.0	19	94.0	37–61	67.3–52.5
133,000	2/0	7	138.0	19	83.7	37	60.0
106,000	1/0	7	122.8	19	74.5	37	53.4
83,700	1	7	109.3	19	66.4	37	47.6
66,400	2	7	97.4	7	97.4	19	59.1
52,600	3	7	86.7	7	86.7	19	52.6
41,700	4	7	77.2	7	77.2	19	46.9
33,100	5	7	68.8	7	68.8	19	41.7
26,300	6	7	61.2	7	61.2	19	37.2
20,800	7	7	54.5	7	54.5	19	33.1
16,500	8	7	48.6	7	48.6	19	29.5

Note. — Class A cable, sizes 4/0 and 3/0, is usually 7-strand when bare and 19-strand when weatherproof, etc.

9. (*a*) **Packing and Shipping.** — Packing sizes for cable shall be agreed upon in the placing of individual orders.

(*b*) The cable shall be protected against damage in ordinary handling and transportation.

10. (*a*) **Inspection.** — All testing and inspection, both of individual wires entering into the construction of the cable, and of the completed cable, shall be made at the place of manufacture. Tests on individual wires shall be made on samples taken before cabling, and not on wires removed from the completed cable.

(*b*) The manufacturer shall afford the inspector representing the purchaser all reasonable facilities to satisfy him that the material conforms to the requirements of these specifications.

INSTALLATION OF BARE WIRES AND CABLES. — For the installation of wires and cables in buildings see article on *Wiring of Buildings for Light and Power*. Below is given a brief description of modern practice in erecting bare wires and cables on pole or tower lines. For the construction of the latter see articles on *Distribution Lines; Transmission Lines*. Catenary contruction is described in the article on *Trolley Systems, Overhead*.

Simple Span Construction with Pin Insulators. — Starting at an anchored pole, a rope about twice the length of the span is put over the cross arm and the wire pulled up by means of it. The rope is then put over the cross arm of the next pole and the wire drawn up in the same way. This is repeated from pole to pole until the reel, which remains at the starting point, is exhausted. The pulling may be done by men, horses, automobile, or locomotive. Care should be taken to prevent the wire unwinding too rapidly from the reel and the end of the wire must be prevented from slipping away.

The wire is placed on the insulators by means of a block and tackle attached either to the arm above or to a temporary boom. The next step is to draw the wires to the proper tension. Commencing at the first pole after the anchorage, the wire is gripped by a clamp attached to a rope which is pulled until the wire is drawn to the sag indicated by a table or curve showing the proper sag for different temperatures and spans (*Spans, Wire*). The sag is gauged by sighting from pole to pole, using a gauge or sight on each pole, and drawing the wire until the bottom of the span is tangential to the sight line.

Tying to Insulator.— (*See also Insulators for Overhead Lines.*) The wires are tied to their insulators by small wires, usually of the same metal, as shown in Fig. 7. The two ends of the tie wire (not visible in Fig. 7) are twisted together for three or four turns. The tie wire must be held quite taut while it is being installed.

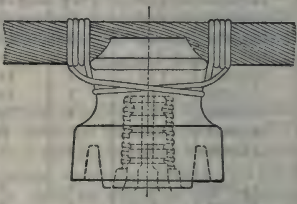


Fig. 7.

Simple Span Construction with Suspension Insulators.— Where suspension insulators are used, the wire, instead of being initially placed over the cross arms, is temporarily suspended therefrom on snatch blocks provided with wooden rollers. Prior to running the cables, linemen are sent ahead to attach the snatch blocks to the arms of the towers. The cables are then strung loosely through the snatch blocks, in much the same way as they are strung over the cross arms in the construction described above. The cables are anchored to the first pole and the necessary tension applied to the cable between the first and second towers, a dynamometer being used to indicate when the required stress point has been reached. The cable is then anchored

to the second tower and the operation repeated for the different spans until the cable length is exhausted. The cable is then transferred to the insulator clamps and the anchorages removed except at the ends.

Clamping to Insulator. — The usual type of insulator clamp (Fig. 8) consists of a curved, malleable-iron wire seat about nine inches long, a saddle which fits over the wire and a pair of hook bolts to hold the wire tightly between the saddle and wire seat. The wire seat is supported from the insulator through a swivel joint. Copper wire or cable is laid in the wire seat, the saddle placed on it and the hook bolts placed in position. In the case of aluminum wire a 1/16-inch aluminum sleeve should be placed around the wire in the clamp. The entire clamp is then attached to the insulator by a bolt and socket or pin joint.

Fig. 8.

Jointing Conductors. — The following methods are used:

Western Union Joint. — The two ends of the wire are brought together so that they lap from 3 to 8 inches, depending upon the size. Then beginning half-way between the two ends, each wire is wound around the other wire in a tight helix. With hard-drawn copper, excessive stress is avoided by giving the helix a long pitch for the first turn or two and then gradually reducing the pitch until a tight helix is obtained.

It would appear from tests on galvanized-iron wire by C. T. Rashman (*E. W., 1910, Vol. 56, p. 1187*), that in order to make the joint as strong as the wire, it should have the form shown in Fig. 9 which may be described as consisting of a neck of five turns and five end turns at each end.

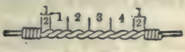


Fig. 9.

Fig. 10.

Fig. 11.

In order to obtain five turns in the neck, the following length of neck was found necessary:

Size wire, B.W.G.	Length of neck, inches
14	3-1/2
12	3-3/4
10	4
8	6
6	7

The form of splice shown in Fig. 10 should be avoided.

Dovetailing Strands. — The cables are unwound for say three or four feet, dovetailed or interlaced, and the strands individually wound round the unopened part of the cable. The joint may be soldered or not, as desired.

Aluminum Tube Joint. — Aluminum conductors of sizes up to about one-half inch diameter are usually jointed by inserting the two ends side by side into a flat aluminum tube, which is then given 3 or 4 twists as shown in Fig. 11 by means of a special kind of tongs called connectors.

Pressed Aluminum Joint. — Aluminum conductors larger than one-half inch in diameter are more often jointed by dovetailing the strands, or by mechanical clamps such as the Dossert connector (*see below*), or by inserting the wires into a cast aluminum sleeve which is squeezed hydraulically until the conductors and sleeve flow into a solid homogeneous mass.

Dossert Joint and Connector. — The Dossert joint, shown in Fig. 12, consists of a compression sleeve which is slipped over the conductor, a screw on

Fig. 12.

Fig. 13.

the lug proper and a nut threaded with a taper thread. The sleeve containing the conductor is thrust into the lug, with the nut over it. The nut is then tightened until the reaction of the tapered surface and the sleeve gives rise to a pressure of several thousand pounds per square inch, thereby making good electrical contact.

The Dossert connector or terminal is similar in construction, and is shown in Fig. 13.

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WIRES AND CABLES, INSULATED. — *(See also Aluminum; Capacity and Charging Current; Copper; Distribution Lines; Electrolysis; Inductance and Inductive Reactance; Insulating Materials; Resistance; Skin Effect; Standards of the A.I.E.E.; Transmission Lines; Wires and Cables, Bare; Wires, Resistance; Wiring of Buildings.)*

The following is a brief table of contents of this article:

TERMINOLOGY. — The terminology employed in referring to wires and cables has been in a very confused state, the same term being used with several different meanings. The following terminology is quoted from the *Standards of the A.I.E.E.*

Wire. — A slender rod or filament of drawn metal.

The definition restricts the term to what would ordinarily be understood by the term "solid wire." In the definition the word "slender" is used in the sense that the length is great in comparison with the diameter. If a wire is covered with insulation, it is properly called an insulated wire. While primarily the term "wire" refers to the metal, nevertheless when the context shows that the wire is insulated the term "wire" will be understood to include the insulation.

Conductor. —A wire, or combination of wires not insulated from one another, suitable for carrying a single electric current.

The term "conductor" is not to include a combination of conductors insulated from one another, which would be suitable for carrying several different electric currents.

Rolled conductors (such as bus-bars) are, of course, conductors, but are not considered under the terminology here given.

Stranded Conductor. — A conductor composed of a group of wires or any combination of groups of wires.

The wires in a stranded conductor are usually twisted or braided together.

Strand. — One of the wires or groups of wires of any stranded conductor.

The majority of American copper wire manufacturers have used the word "strand" to designate a group of bare wires twisted together.

Cable. — (1) A stranded conductor (single-conductor cable; or (2) a combination of conductors insulated from one another (multiple-conductor cable).

The component conductors of the second kind of cable may be either solid or stranded, and this kind of cable may or may not have a common insulating covering. The first kind of cable is a single conductor, while the second kind is a group of several conductors. The term "cable" is applied by some manufacturers to a solid wire heavily insulated and lead-covered; this usage arises from the manner of the insulation, but such a conductor is not included under this definition of "cable." The term "cable" is a general

one and in practice it is usually applied only to the larger sizes. A small cable is called a "stranded wire" or a "cord," both of which are defined below. Cables may be bare or insulated, and the latter may be armored with lead or steel wires or bands.

Stranded Wire. — A group of small wires, used as a single wire.

A wire has been defined as a slender rod or filament of drawn metal. If such a filament is subdivided into several smaller filaments or strands and is used as a single wire, it is called a "stranded wire." There is no sharp dividing line of size between a "stranded wire" and a "cable." If used as a wire, for example, in winding inductance coils or magnets, it is called a stranded wire and not a cable. If it is substantially insulated, it is called a "cord," defined below.

Cord. — A small cable, very flexible and substantially insulated to withstand wear.

There is no sharp dividing line in respect to size between a "cord" and a "cable," and likewise no sharp dividing line in respect to the character of insulation between a "cord" and a "stranded wire."

Concentric Strand. — A strand composed of a central core surrounded by one or more layers of helically laid wires or groups of wires.

Concentric-lay Cable. — A single-conductor cable composed of a central core surrounded by one or more layers of helically laid wires.

Rope-lay Cable. — A single-conductor cable composed of a central core surrounded by one or more layers of helically laid groups of wires.

This kind of cable differs from the preceding in that the main strands are themselves stranded.

N-Conductor Cable. — A combination of N conductors insulated from one another.

It is not intended that the name as here given be actually used. One would instead speak of a "3-conductor cable" and a "12-conductor cable," etc. In referring to the general case, one may speak of a "multiple-conductor cable" (as in definition § 324 above.)

N-Conductor Concentric Cable. — A cable composed of an insulated central conducting core with (N-1) tubular stranded conductors laid over it concentrically and separated by layers of insulation.

This kind of cable usually has only 2 or 3 conductors. Such cables are used in carrying alternating currents. The remarks on the expression "N-Conductor" given for the preceding definition apply here also.

Duplex Cable. — Two insulated single-conductor cables twisted together.

They may or may not have a common insulating covering.

Twin Cable. — Two insulated single-conductor cables laid parallel, having a common covering.

Twin Wire. — Two small insulated conductors laid parallel, having a common covering.

Triplex Cable. — Three insulated single-conductor cables twisted together.

They may or may not have a common insulating covering.

Twisted Pair. — Two small insulated conductors twisted together without a common covering.

The two conductors of a "twisted pair" are usually substantially insulated, so that the combination is a special case of a "cord."

Sector Cable. — A sector cable is a multiple-conductor cable in which the

cross-section of each conductor is substantially a sector, an ellipse, or a figure intermediate between them.

Sector cables are used in order to obtain decreased overall diameter and thus permit the use of larger conductors in a cable of given diameter.

Round Conductor. — A round conductor is either a solid or stranded conductor of which the cross-section is substantially circular.

Split Conductor. — A split conductor is a conductor which is divided into two or more parts, separated from one another by insulation which is thin compared with the insulation around the conductor.

The term split conductor usually designates a conductor in two parts or splits, which may be either concentric or external to one another.

Factor of Assurance. — The factor of assurance of wire or cable insulation is the ratio of the voltage at which it is tested to that at which it is used.

Insulation Resistance.— The insulation resistance of an insulated conductor is the electrical resistance offered by its insulation, to an impressed voltage tending to produce a leakage of current through it.

Circular Mil. — A circular mil is a unit of area equal to $\frac{\pi}{4}$ ($=0.7854\ldots$) of a square mil. The cross-sectional area of a circle in circular mils is therefore equal to the square of its diameter in mils. A circular inch is equal to a million circular mils.

Lay. — The lay of any helical element of a cable is the axial length of a turn of the helix of that element.

Among the helical elements of a cable may be each strand in a concentric-lay cable, or each insulated conductor in a multiple conductor cable.

Direction of Lay.—The direction of lay is the lateral direction in which the strands of a cable run over the top of the cable as they recede from an observer looking along the axis of the cable.

CONSTRUCTION OF INSULATED WIRES AND CABLES. — Data on the manufacture and properties of insulating materials will be found in the articles on *Cambric, Varnished; Gutta-percha; Paper, Impregnated; Rubber; Insulating Materials, Miscellaneous*. Data on the conductor itself will be found in the articles on *Copper; Aluminum; Wires and Cables, Bare*.

Conductors. — Round copper wire, solid or stranded, is almost invariably used for insulated wires and cables. Aluminum, requiring a larger cross-section for the same conductance per unit length, requires more insulating material for the same thickness of insulation. Aluminum cables, however, are sometimes provided with a weatherproof covering.

Stranding. — Conductors are stranded in order to make them more flexible. The cable can then be drawn into a conduit more readily, following the bends of the conduit without injury during installation. In the case of large cables, flexibility is required first in order to permit them to be put on reels; second, to permit easy installation, and third to permit them to be bent around the walls of splicing chambers. Cords, elevator cables, mining machine cables, etc., must have considerable flexibility in order to permit them to be readily shifted from place to place in service.

For conduit work it is found unnecessary to strand conductors smaller than No. 6 A. W. G., although some contractors use stranded conductors as small as No. 12.

Sector-shaped Conductors. — Two, three and four conductor cables,

with sector-shaped conductors are in successful use, the three-conductor type being most common. It has been shown experimentally that if the conductors are properly shaped there will be lower dielectric stresses than in a round conductor cable of the same cross-section. The principal advantage of this type of cable, however, is the greater carrying capacity for a given outside diameter.

Preparation of Conductors for Insulation. — Where rubber insulation is used it is necessary to cover the conductor with a thin film of tin or with a layer of soft cotton threads.

Tinning. — Copper and rubber when brought into contact react upon one another chemically. It was formerly thought that this action was due to the sulphur in the rubber compound combining with the copper and in the old days, it was common practice to place a layer of pure rubber next to the copper and the regular vulcanized compound over that. It was found, however, that pure rubber reacted with the copper even more vigorously than vulcanized rubber containing free sulphur. The pure rubber broke down into a gluey, sticky mass. It was later found that tinning the copper afforded the necessary protection as tin does not react with rubber.

Separators. — Small stranded conductors are usually covered with a winding of soft cotton threads for the following purposes:

1. To protect the copper and rubber from mutual chemical action where a coating of tin on the copper cannot be used.
2. To hold together a group of fine wires so that individual wires will not stand up and penetrate the insulation during the manufacturing process.
3. To prevent adhesion between the rubber and copper in order that the copper may be easily bared for making connections.

A separator composed of a wind of soft cotton is used on cords of various types for the first and second reasons. A similar cotton wind on small conductors, or a paper or dry muslin tape on large conductors is used on cables for car wiring for the third reason given above.

Insulation. — The materials used for insulation are vulcanized rubber, gutta-percha, varnished cloth, impregnated paper, asbestos, cotton and silk thread, enamel, etc.

Rubber Insulation. — There are two processes by which rubber compound is put on wire, the strip and the tube or seamless processes. By the former, the compound is first made into long narrow strips and then pressed around the wire; by the latter, the wire is run through a die through which the compound is pressed on to the wire. Insulation made by the former process shows a seam or ridge where the sides of the strip have united, unless a tape is applied before vulcanization, while that made by the tube process is seamless.

Rubber insulating compounds are prepared by mixing rubber gum with sulphur, wax and fillers which may be mineral substances or rubber substitutes. Experience has shown that from 25 per cent to 30 per cent of rubber must be used in order to obtain compounds which will resist oxidation when exposed to the air.

The various grades of rubber insulation on the market may be divided into three classes, known respectively as 30 per cent Hevea, Intermediate and Code compounds. The majority of commercial 30 per cent Hevea compounds probably fall within the following limits of composition:

Ingredient	Percentage by weight
Rubber	30 to 32
Whiting	0 to 30
Zinc oxide	28 to 67

Ingredient	Percentage by weight
Litharge	1 to 12
Ozokerate	2 to 4
Sulphur	2 to 4

Compounds of intermediate grade usually contain less than 30 per cent of Hevea rubber mixed with reclaimed rubber, to make a total of at least 30 per cent of rubber gum. The other ingredients are similar to those used in 30 per cent Hevea compounds.

Code compounds contain a mixture of new and reclaimed rubber, mineral rubber or black hydrocarbon, sulphur and mineral fillers.

Gutta-Percha is applied to the wire in the same manner as rubber. It is used only for deep-sea submarine cables.

Varnished Cloth. — The prepared cloth is applied to the conductor in the form of tape wound on helically and reversed every two layers, with overlapping joints staggered in successive layers. A thin layer of a non-hardening viscous filler is applied between layers.

Impregnated Paper. — Strips of Manila paper are applied helically, then thoroughly dried and impregnated with an oily compound.

Asbestos. — Asbestos is used for low-voltage wires to be used in very hot places. It is applied in the form of threads or tape, usually in several layers. One or more layers of varnished cloth are sometimes used to separate the groups of asbestos tapes, and the whole is covered with a protecting braid.

Cotton and Silk. — Cotton or silk insulation consists of one to three layers of threads spun on to the wire.

Enamel. — This type of insulation is put on wires from No. 8 to No. 40 A.W.G., its most important use being for magnet windings. It is made by passing specially prepared copper wire through successive baths of oily compound which is oxidized by exposure to warm air and thereby hardened. This insulation is not sensibly softened by mineral oil at 212° F. Enameled wire can be bent around a mandrel twice to six times its base diameter without cracking, the lower figure referring to sizes smaller than No. 20 A.W.G. and the higher figure to sizes greater than No. 12 A.W.G.

Fillers.— Dry jute or paper is used as a filler between the insulated conductors of multiple-conductor cables, being placed directly in contact with the insulation. Tarred or asphalted jute is used as a filler between the sheath and armor of armored cables.

Protective Coverings. — Coverings of treated cotton, hemp, paper, reinforced rubber, and lead are used to protect cables from mechanical injury and arcing.

Cotton Braid. — The most common covering for rubber-insulated conductors is a cotton braid saturated with a compound consisting of asphaltic materials and waxy hydrocarbons (principally paraffine wax). The hydrocarbon paraffine is used to carry the asphaltic material into the fibers.

The purpose of this saturation is to protect the cotton against natural deterioration.

The melting point of saturating compound is from 118 to 124° F.

The saturated cotton is covered with a finishing wax, the purpose of which is to fill the interstices between threads and to afford a smooth surface which will exclude moisture and permit the wire to be drawn into a conduit with the least

friction. Finishing wax is usually composed of asphaltic material and higher grade waxes, such as Carnauba and Montan wax, which are used to give finish or lustre.

There are two important grades of finishing wax, one of which has a melting point between 156 and 166° F. and a higher grade which has a melting point between 180 and 190° F.

Weatherproofing. — Treated cotton braid is also put on uninsulated hard-drawn copper wire for overhead service, in order to protect the wire from destructive arcing due to accidental contact with tree branches and other foreign bodies.

Lead Sheath. — The only thoroughly waterproof cable covering yet devised is a lead sheath. It is put over the insulation by passing the cable through a die while hot lead is pressed hydraulically around it through an annular die, forming a continuous and close-fitting pipe. Were it not for electrolytic corrosion, it would be practically permanent. Unfortunately lead is very subject to electrolytic and even chemical corrosion. It is also rendered brittle and eventually breaks into pieces when exposed to vibration, this effect being due to crystallization. Pure lead is suitable for cables which are hard and compact. Other cables require a small quantity of tin to harden the lead. For a given thickness pure lead is cheaper, but for a given tensile strength a 3 per cent tin alloy is cheaper.

Antimony has been used instead of tin with very satisfactory results and a material saving in cost, especially with telephone cable.

Armor Wire and Tape. — Submarine cables are usually covered with steel wire armor and cables to be buried direct in the ground are usually covered with galvanized steel tape armor. In either case the armor is covered with asphalted jute.

APPLICATIONS OF VARIOUS TYPES OF CABLES. — The type of insulation and protection to employ in any instance depends upon the purpose for which the conductor is to be used and the place in which it is to be installed.

Power Cables, that is, cables for the transmission or distribution of electric energy, may be installed in buildings, in cars, in underground conduits, on pole lines, or under water.

Wires and cables for buildings are usually rubber insulated and covered with a saturated braid. Varnished cambric is often used for the larger sizes.

Wires and cables for power houses are usually varnished cambric insulated with either a saturated or a flame-proof braid.

Wires and cables for underground conduit lines are almost invariably paper-insulated, but rubber or varnished cambric are sometimes used.

Wires for mining machinery and for railway signals are invariably rubber insulated.

Submarine Power Cables are usually insulated with rubber, sometimes sheathed in lead and armored with wire or steel bands. In recent years, however, paper-insulated lead-sheathed and armored submarines have come into extensive use.

Communication Wires and Cables, that is, wires and cables for telephone and telegraph service, are rubber insulated where only two or three conductors are required. Cables composed of many pairs are generally insulated with dry paper and are known as dry-core cables.

Submarine telegraph cables are insulated either with gutta-percha or vulcanized rubber, the former if for deep-sea use. Such cables are protected by jute and steel wire armor.

Insulated Conductors for Instrument and Machine Windings. — Enameled wire or wire insulated with cotton or silk is used for the former, while varnished cambric, mica and asbestos compounds are used for the latter.

INSULATION THICKNESS. — The thickness of insulation required for low-voltage cables is merely that necessitated by inevitable irregularities of manufacture and roughness in handling. At voltages between 500 and 1000, the dielectric stress begins to have an influence and when 2000 volts are reached, the dielectric stress becomes the dominant factor. If certain assumptions are made, the thickness of insulation corresponding to a given dielectric stress may be calculated, but these assumptions are only partially realized in practice. These assumptions are:

a. That the radial depth of the insulation or dielectric is the same at all points, i.e., that the cross-section of the cable is a perfect circle with the conductor section (also a perfect circle) exactly in the center of the insulation. Due, however, to the crinkling of tape, the pressure of the braid, or other accidents of manufacture, this is never the case. The eccentricity of the conductor with respect to the insulation may, however, be allowed for by adding to the theoretical thickness of insulation an additional thickness, known as the "error thickness" or excess thickness.

b. That the dielectric is perfectly homogeneous throughout. This is never realized in practice. It is probable that even in the most carefully made insulation there is a minute amount of air and moisture, but sufficient to modify considerably any conclusions based on absolute homogeneity of the dielectric. In the case of impregnated paper, the lack of homogeneity results in important departures from the following law.

c. That when the electric stress, or potential gradient at any part of the dielectric exceeds a certain value F_s known as its dielectric strength, that part of the dielectric becomes a partial conductor even though there is no actual rupture or puncturing.

This assumption is made only in the case of rubber insulation, and then only when the electric stress is applied for a considerable time. The internal breakdown of the rubber appears to be due to the ionization of the air between the rubber and conductor, the rubber being oxidized, split internally, and showing other signs of disintegration. (See C. A. Adams, Trans. A.I.E.E., 1917, Vol. 36, pp. 514–51.7.)

Theoretical Thickness of Insulation. — Let

F = potential gradient, in kilovolts per centimeter, at any point P in the dielectric at a distance x centimeters from the center of the wire, Fig. 1,

E = kilovolts between wire and sheath (or outside surface of insulation).

r = radius of wire, in centimeters,

R = outside radius of insulation, in centimeters,

F_s = dielectric strength of the insulation, in kilovolts per centimeter.

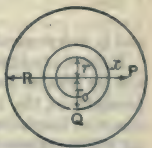


Fig. 1.

Then, on the assumption of a perfectly homogeneous dielectric and perfect symmetry between conductor and insulation,

$$F = \frac{E}{x \log_e \left(\frac{R}{r}\right)}.$$

That is, the potential gradient is the greatest at the surface of the conductor

and decreases toward the outer surface of the insulation as shown in Fig. 2. The potential gradient at the surface of the conductor is

$$F_{max} = \frac{E}{r \log_\epsilon \left(\frac{R}{r}\right)}.$$

For the same outside diameter of the insulation this stress at the surface varies with the radius of the conductor as shown in Fig. 3, and has a *minimum* value theoretically when $r = R/2.72$, but actually when R/r is somewhat greater than 2.72.

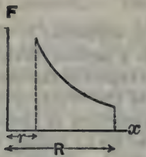


Fig. 2.

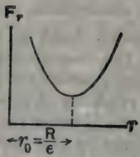


Fig. 3.

Consequently, if the breakdown of a cable is a progressive action, i.e., if the insulation breaks down close to the wire and then the breakdown spreads out through the insulation, it is evident, from Fig. 3, that if r is greater than $R/2.72$, and the impressed voltage E is such as to produce a stress F_r at the surface of the conductor in excess of F_s, then as successive layers of the insulation break down, F_r at each new conducting surface thus formed will be greater than F_s, and the breakdown will rapidly extend to the sheath. If, however, r is less than $R/2.72$, then a value of F_r in excess of F_s will break down only a sufficient layer of insulation to render the value of F_r at the new conducting surface thus formed just equal to F_s.

Therefore, on the basis of the assumptions noted in the preceding section, the maximum voltage which a cable can withstand is equal to

$$E_m = F_s r \log_\epsilon \left(\frac{R}{r}\right),$$

provided R/r is less than 2.72, but when R/r is greater than 2.72 the maximum voltage is

$$E_m = F_s r_0 \log_\epsilon \left(\frac{R}{r_0}\right) = \frac{RF_s}{2.72},$$

and is *independent of the radius of the conductor.*

If there are films or globules of air in the insulation the dielectric stress in them will be greater than in the surrounding insulation in the ratio of the dielectric constant of the insulation to that of the air. As the dielectric constants of common insulating materials are between twice and five times that of air, the air will be stressed between two and five times as much as the insulation. As air ionizes at a much lower stress than the dielectric strength of the insulation, the limiting working voltage of a cable may depend upon the effects of air ionization. In the case of rubber insulation the principal effect is oxidation by ozone, but in the case of paper or varnished cambric the energy loss due to ionization is the most serious matter.

Practical Determination of Insulation Thickness. — On account of the numerous assumptions which must be made in determining the necessary thickness of insulation from any simple theory, such as outlined above, a theory

is of little value in the generality of cases and reliance must be placed upon experience, as embodied in the standards of various associations.

RUBBER INSULATION

Thickness of Insulation in 64ths of an Inch

Working pressure, volts between wires	Nat. Elec. Code * and A.I.E.E. Standard: Size of wire, A. W. G. or B. & S. and Cir. Mils.	Nat. Elec. Code * and A.I.E.E. Standard: Insulation thickness, on each wire
600	18–16	2
600	14–8	3
600	7–2	4
600	1–0000	5
600	225,000–500,000	6
600	525,000–1,000,000	7
600	1,100,000–2,000,000	8
1500	14–8	6
1500	7–2	7
1500	1–0000	8
1500	225,000–500,000	9
1500	525,000–1,000,000	10
1500	1,100,000–2,200,000	10
2500	14–8	8
2500	7–2	9
2500	1–2,000,000	10
3500	14–0000	10
3500	225,000–500,000	11
3500	525,000–2,000,000	12
5000	14–1,000,000	12
5000	1,100,000–2,000,000	14
7000	14–1,000,000	16
7000	1,100,000–2,000,000	18

Working pressure, volts, a-c.	A.I.E.E. Standard,† 30–40 per cent hevea: Size of wire,	A.I.E.E. Standard,† 30–40 per cent hevea: Insulation thickness
8,000	All	18
9,000	All	20
10,000	All	22
11,000	All	24

Railway Signal Association ‡ 666 volts or less

Single conductor		Multi-conductor	
Size of wire, A. W. G. or B. & S.	Insulation thickness, 64ths inch	Size of wire, A. W. G. or B. & S.	Insulation thickness, 64ths inch
18–16	4	16	3
14–9	5	14–10	4
8–4	6	9–6	5
2–0	8	4	6

* The rubber to comply with the rubber specification of the National Board of Fire Underwriters, which assumes the use of 20 per cent of rubber gum.

† The A.I.E.E. Standards for 600 to 7000 volts are identical with the code except that they cover only 30 to 40 per cent hevea insulation and that for multiple conductor cables of any voltage the thickness of insulation on each conductor shall be based on the highest r.m.s. voltage between the conductor and the outside of the insulation.

‡ The rubber insulation to comply with the specifications of the R.S.A., which calls for a very high-grade 30 per cent Para compound.

VARNISHED CLOTH AND IMPREGNATED PAPER

Insulation Thickness

(Wall thickness for single-conductor cables and thickness between conductors for multiple-conductor cables.)

(*G. E. Co. Practice*)

Working pressure, volts between wires	Varnished cloth		Impregnated paper	
	Size, A. W. G. or B. & S. and Cir. Mils	Insulation thickness,* 64ths inch	Size, A. W. G. or B. & S. and Cir. Mils	Insulation thickness,* 64ths inch
600†	6–2	6	6–2	4
600	1–0000	8	1–0000	5
600	250,000–500,000	10	225,000–500,000	6
600	550,000 and over	12	550,000–1,000,000	7
1,000	6–2	4	12–2	5
1,000	1–0000	5	1–0000	6
1,000	250,000–500,000	6	....	..
1,000	550,000–1,000,000	7	225,000–500,000	7
1,000	1,100,000 and over	8	550,000–2,000,000	8
2,000	6–0000	6	10–0000	7
2,000	250,000–500,000	7	225,000–500,000	8
2,000	550,000–2,000,000	8	550,000–2,000,000	9
3,000‡	All sizes	8	8 and larger	10
4,000	All sizes	10	8 and larger	12
5,000	All sizes	12	6 and larger	14
......	....	......	6 and larger	16
7,000	All sizes	14	5 and larger	18
8,500	All sizes	16	....	..
......	....	......	5 and larger	20
11,500	All sizes	20	4 and larger	22
13,500	All sizes	24	....	..
14,000	All sizes	24	4 and larger	24
15,000	All sizes	26	3 and larger	26
16,000	All sizes	26	....	..
17,000	All sizes	28	3 and larger	28
18,000	All sizes	28	....	..
19,000	All sizes	30	2 and larger	30

* Above working voltages are based on all conductors of the circuit being insulated. For d-c 600-volt railway single-conductor, leaded cables, use 2,000-volt class. For three-phase "Y"-connected circuits with grounded neutral and three-conductor cables, thickness of insulation between conductors and ground need only be 7/10 of that between conductors.

† For three-conductor cables only, use 1,000-volt class for single-conductors from 0 to 1,000 volts.

‡ For three-conductor cables use 4/64 in. for No. 6 to No. 0000, 5/64 in., for 250,000 to 500,000 cir. mils and 6/64 in. for larger sizes.

PAPER INSULATED CABLE

Insulation Thicknesses of Cables of Various Light and Power Companies

3 and 4 Conductor Cables

Name of company	Operating voltage	Size A. W. G. or Cir. Mil.	Insulation, Conductor	64ths inch Belt
Chicago	33,000	350,000	19	7
Milwaukee	26,400	#00	18	18
Baltimore	26,000	#0000	18	10
Boston	25,000	....	18	12
New York Edison	24,000	350,000	17	8
Detroit	23,000	#00	18	14
Toledo	23,000	300,000	18	14
Cleveland	22,000	#2	20	20
Chicago	22,000	350,000	19	7
Pittsburgh	22,000	250,000	18	10
Los Angeles	20,000	#0000	16	16
New York Edison	15,000	350,000	14	14
San Francisco	15,000	#0000	13	13
Milwaukee	13,200	....	14	14
Louisville	13,200	250,000	14	14
Toledo	13,200	133,000	12	10
Washington	13,200	#0000	12	12
Baltimore	13,000	#0000	16	4
Chicago	12,000	500,000	13	6
Los Angeles	12,000	#0000	14	14
Cleveland	11,000	#0000	16	4
Pittsburgh	11,000	#0000	14	12
New York Edison	8,000	350,000	10	10
Louisville	7,500	#8	8	8
Baltimore	6,600	#0000	10	8
Toledo	6,600	250,000	8	8
San Francisco	5,000	#0000	8	8
Detroit	4,800	200,000	10	10
Cleveland	4,600	#0000	8	8
Louisville	4,000	250,000	10	10
Baltimore	4,000	#0	8	6
Chicago	4,000	#0	6	5
Louisville	4,000	#0000	6	6
Los Angeles	2,500	#0000	6	6
San Francisco	2,500	#0000	6	6
Cleveland	2,300	#0	6	4
Pittsburgh	2,300	#0000	8	6
San Francisco	750	#0000	4	4

Cotton, Silk and Enamel. — The thicknesses rated by various manufacturers as "single," "double," and "triple" covered differ by several mils; see article on *Electromagnet Windings*. Enamel insulation will stand a working potential of about 500 volts alternating per mil of thickness. Silk or cotton will stand about one-quarter as much.

WEIGHT OF INSULATED WIRES AND CABLES. — So many variables enter into the weight of an insulated wire or cable, and there are so many forms in use, that it is impossible to give comprehensive tables here. The reader is referred to the catalogues and circulars of the manufacturing companies. When such sources of data are not available the weight may be calculated from the dimensions of the cable by finding the sum of the weights of the conductors and the weights of the insulation, braid, tape, sheath, etc. The weight of the conductor may be found in the wire tables under *Wires and Cables, Bare*. The weight of the insulation, braid, tape or sheath, may be found by calculating the cross-section of each of these materials and multiplying by the length of the cable and a factor proportional to the density of the material; see table accompanying.

The cross-section of a tube having an internal diameter d and thickness t is $\pi t(d+t)$. When the diameter and thickness are in inches and the specific gravity of the material forming the tube is δ, then the weight in pounds per 1000 feet of tube is

$$W = 1362\,\delta t(d+t).$$

The values of the specific gravity δ, the product $1362\,\delta$, and the weight per cu. in., for the various materials used in the construction of cables are given in the accompanying table. From this relation the weight per 1000 feet of any tubular (circular cross-section) layer of insulation, braid, tape or sheath may be readily calculated. The formula is, therefore, directly applicable to single-conductor cables.

The weight of duplex or triplex cables is calculated as for a group of single-conductor cables, except with regard to the fillers. The cross-section of the filling material is most readily calculated by subtracting from the cross-section of the entire cable the cross-sections of the individual conductors and the tubular insulation, sheath, etc. The weight of twin cables can also be readily found by calculating the cross-section of the various parts in inches and multiplying by the length and the proper density factor.

The weight of separators in pounds per 1000 feet is usually between 5 and

WEIGHTS OF CABLE INSULATION

Material	Specific gravity, δ	Pounds per cubic inch, $0.03613\,\delta$	$1362\,\delta$
Rubber compound, 30 per cent para:			
Organic base	1.2	0.0434	1635
Mineral base	1.5 to 2.0	0.054 to 0.072	2043 to 2724
Varnished cloth	1.26	0.455	1720
Paper, impregnated	1.5	0.054	2043
Lead	11.4	0.411	15,530
Braid, untreated	1.11	0.0402	1515
Braid, saturated	1.33	0.0480	1809
Tarred jute, in cable	0.534	0.0193	728
Dry jute, in cable	0.267	0.00965	364
Dry hemp, in cable	0.267	0.00965	364
Gutta percha	1.0	0.0361	1362
Rubber filled tape	1.0	0.0361	1362

10 times the mean diameter, expressed in inches, of the conductor and separator.

The weight of saturated braids per 1000 feet is from 12 to 30 times the diameter over the insulation expressed in inches.

The corresponding multiplier for 12 mil tape with ¼ lap, is 24.

INSULATION RESISTANCE. — The insulation resistance of a cable is usually expressed in megohm-miles, sometimes erroneously called megohms per mile. The total insulation resistance of a cable varies inversely as its length, e.g., a cable two miles long has half the resistance between conductor and sheath of a length of one mile of this cable. The formulas for insulation resistance of various types of cables are given below.

Single-conductor Cable. — In absolute units the insulation resistance of a length of l centimeters of such a cable is

$$R' = \frac{\rho}{2\pi l} \log_e \frac{D}{d},$$

where d = diameter of conductor, D = outside diameter of insulation, R = insulation resistance, l = axial length, ρ = specific resistance. From the above formula the megohm-miles are

$$R = k \log_{10} \frac{D}{d},$$

where $k = 0.43 \times$ (specific resistance in millions of megohms per centimeter cube) and the two diameters are expressed either in inches or in centimeters. The value of k for various types of insulation at 60° *F.*, after an electrification of approximately one minute under a constant *d-c.* voltage, is given in the accompanying table. k varies both with the time of electrification and with the temperature.

Insulation	k at 60° F.	
	Limits	Usual values
Vulcanized rubber.	780 to 23,000	3000 to 6000
Gutta-percha......	500 to 4,000	2500
Varnished cloth....	400 to 1,200	700
Impregnated paper.	1000 to 3,000	1500

The tables on the following pages give the value of $\log_{10} \frac{D}{d}$ for various sizes of wire and thicknesses of insulation.

If the insulation resistance be measured with alternating current, it will appear to be much less than with direct current because of the dielectric loss.

Two- and Three-conductor Cable in Lead Sheath. — The insulation resistance between the two conductors of a two-conductor cable is practically double that between each conductor separately and the sheath.

The insulation resistance of a three-conductor cable between any one conductor and the other two conductors and the sheath is approximately one-half of that between each conductor separately and the sheath. Simple formulas for these resistances in terms of the dimensions of the cable are not available, for the various parts of the coverings (insulation, fillers and braids) have different resistivities.

LOG$_{10}$ $\frac{D}{d}$ FOR SOLID WIRES

Size wire, A. W. G. or B. & S.	d inches	Insulation thickness, 64ths of an inch							
		3	4	5	6	7	8	9	10
14	0.064	0.393	0.470	0.537	0.595	0.645	0.691	0.732	0.770
12	0.081	0.334	0.405	0.467	0.521	0.568	0.613	0.650	0.687
10	0.102	0.283	0.348	0.403	0.453	0.498	0.538	0.574	0.610
8	0.129	0.238	0.294	0.344	0.391	0.431	0.468	0.502	0.535
6	0.162	0.199	0.248	0.292	0.334	0.371	0.405	0.436	0.467
4	0.204	0.164	0.207	0.246	0.283	0.316	0.346	0.377	0.403
2	0.258	0.134	0.170	0.207	0.238	0.267	0.294	0.320	0.344
1	0.289	0.124	0.155	0.188	0.217	0.243	0.270	0.294	0.318
0	0.325	0.111	0.140	0.170	0.199	0.223	0.248	0.270	0.292
00	0.365	0.100	0.127	0.155	0.182	0.204	0.225	0.248	0.270
000	0.410	0.090	0.114	0.140	0.164	0.185	0.207	0.228	0.246
0000	0.460	0.079	0.104	0.127	0.149	0.170	0.188	0.207	0.225

Size wire, A. W. G. or B. & S.	d inches	Insulation thickness, 64ths of an inch							
		11	12	14	16	18	20	22	24
14	0.064	0.804	0.836	0.894	0.945	0.991	1.032	1.070	1.104
12	0.081	0.720	0.751	0.807	0.856	0.900	0.941	0.977	1.011
10	0.102	0.640	0.670	0.723	0.771	0.814	0.853	0.889	0.922
8	0.129	0.565	0.592	0.642	0.688	0.729	0.766	0.803	0.833
6	0.162	0.494	0.520	0.568	0.611	0.650	0.687	0.720	0.751
4	0.204	0.428	0.453	0.498	0.538	0.575	0.609	0.640	0.670
2	0.258	0.367	0.389	0.431	0.468	0.502	0.534	0.565	0.592
1	0.289	0.340	0.362	0.401	0.436	0.470	0.500	0.529	0.556
0	0.325	0.314	0.332	0.371	0.405	0.436	0.465	0.496	0.520
00	0.365	0.288	0.307	0.342	0.377	0.405	0.433	0.461	0.486
000	0.410	0.265	0.281	0.316	0.346	0.375	0.401	0.428	0.452
0000	0.460	0.243	0.260	0.290	0.320	0.346	0.373	0.398	0.420

d = diameter of wire; D = outside diameter of insulation.

$LOG_{10} \frac{D}{d}$ FOR CABLES (STRANDED WIRES)

Size wire, A. W. G. or B. & S. and C.M.	d inches	Insulation thickness, 64ths of an inch							
		3	4	5	6	7	8	9	10
14	0.073	0.360	0.433	0.497	0.553	0.602	0.646	0.686	0.724
12	0.092	0.307	0.373	0.431	0.483	0.529	0.571	0.607	0.643
10	0.116	0.258	0.318	0.369	0.418	0.461	0.498	0.534	0.568
8	0.146	0.215	0.267	0.316	0.360	0.398	0.433	0.470	0.497
6	0.184	.....	0.225	0.267	0.305	0.340	0.373	0.403	0.431
4	0.232	.....	0.188	0.223	0.258	0.288	0.316	0.344	0.371
2	0.292	.....	0.152	0.185	0.215	0.241	0.267	0.290	0.314
1	0.332	.....	0.140	0.167	0.196	0.220	0.243	0.267	0.288
0	0.373	.....	0.127	0.152	0.179	0.201	0.225	0.246	0.267
00	0.419	.....	.....	0.137	0.161	0.182	0.201	0.223	0.243
000	0.471	.....	.....	0.124	0.146	0.167	0.185	0.204	0.220
0000	0.529	.....	.....	0.114	0.130	0.149	0.167	0.185	0.201
250,000	0.575	.....	.....	0.104	0.124	0.140	0.158	0.173	0.190
300,000	0.632	.....	.....	0.097	0.114	0.130	0.146	0.161	0.176
350,000	0.681	.....	.....	0.090	0.104	0.121	0.137	0.149	0.164
400,000	0.728	.....	.....	0.083	0.100	0.114	0.127	0.143	0.155
450,000	0.772	.....	.....	0.079	0.093	0.107	0.121	0.137	0.149
500,000	0.815	.....	.....	0.076	0.090	0.104	0.117	0.130	0.140
600,000	0.893	.....	.....	.....	0.086	0.097	0.111	0.121	0.130
700,000	0.964	.....	.....	.....	0.079	0.090	0.100	0.111	0.124
750,000	0.998	.....	.....	.....	0.074	0.085	0.097	0.107	0.118
800,000	1.031	.....	.....	.....	0.073	0.084	0.094	0.104	0.115
900,000	1.094	.....	.....	.....	0.069	0.079	0.089	0.099	0.109
1,000,000	1.153	.....	.....	.....	0.066	0.076	0.085	0.095	0.104
1,250,000	1.289	.....	.....	.....	0.059	0.068	0.077	0.086	0.094
1,500,000	1.413	.....	.....	.....	0.054	0.063	0.071	0.078	0.087
1,750,000	1.526	.....	.....	.....	0.050	0.058	0.066	0.073	0.081
2,000,000	1.632	.....	.....	.....	0.047	0.055	0.062	0.069	0.076

$\mathrm{LOG}_{10}\frac{D}{d}$ FOR CABLES (STRANDED WIRES)

Size wire, A.W.G. or B. & S. and C. M.	d inches	Insulation thickness, 64ths of an inch							
		11	12	14	16	18	20	22	24
14	0.073	0.757	0.788	0.845	0.895	0.940	0.980	1.018	1.051
12	0.092	0.676	0.705	0.760	0.809	0.852	0.892	0.928	0.961
10	0.116	0.598	0.626	0.679	0.725	0.767	0.806	0.841	0.873
8	0.146	0.525	0.553	0.602	0.645	0.686	0.723	0.757	0.788
6	0.184	0.458	0.483	0.529	0.571	0.609	0.643	0.676	0.705
4	0.232	0.394	0.418	0.461	0.498	0.534	0.567	0.598	0.626
2	0.296	0.334	0.356	0.394	0.430	0.462	0.493	0.521	0.548
1	0.332	0.310	0.328	0.365	0.398	0.431	0.459	0.487	0.513
0	0.373	0.286	0.305	0.338	0.371	0.401	0.430	0.455	0.480
00	0.419	0.260	0.276	0.310	0.340	0.369	0.396	0.422	0.438
000	0.471	0.238	0.255	0.286	0.314	0.342	0.367	0.391	0.415
0000	0.529	0.217	0.233	0.260	0.290	0.314	0.338	0.362	0.384
250,000	0.575	0.204	0.217	0.246	0.272	0.297	0.320	0.342	0.364
300,000	0.632	0.190	0.204	0.230	0.255	0.276	0.299	0.320	0.340
350,000	0.681	0.176	0.190	0.215	0.238	0.262	0.283	0.303	0.322
400,000	0.728	0.167	0.182	0.204	0.228	0.250	0.270	0.290	0.307
450,000	0.772	0.161	0.173	0.196	0.217	0.238	0.258	0.276	0.294
500,000	0.815	0.152	0.164	0.188	0.210	0.228	0.248	0.267	0.283
600,000	0.833	0.143	0.152	0.173	0.193	0.212	0.230	0.248	0.265
700,000	0.964	0.134	0.143	0.164	0.182	0.201	0.217	0.236	0.250
750,000	0.998	0.129	0.138	0.158	0.176	0.196	0.211	0.228	0.243
800,000	1.031	0.125	0.135	0.154	0.172	0.189	0.206	0.222	0.237
900,000	1.094	0.118	0.128	0.146	0.163	0.180	0.196	0.212	0.227
1,000,000	1.153	0.113	0.122	0.140	0.157	0.173	0.188	0.203	0.217
1,250,000	1.289	0.103	0.111	0.127	0.142	0.157	0.172	0.186	0.199
1,500,000	1.413	0.094	0.102	0.117	0.132	0.146	0.159	0.172	0.185
1,750,000	1.526	0.088	0.096	0.110	0.123	0.136	0.149	0.162	0.173
2,000,000	1.632	0.083	0.090	0.103	0.116	0.129	0.141	0.153	0.164

CAPACITY AND CHARGING CURRENT. — See article on *Capacity and Charging Current.*

INDUCTANCE, REACTANCE AND IMPEDANCE. — The formulas for bare wires and cables (*see Inductance and Inductive Reactance*) are also applicable to lead-sheathed insulated cables, the insulation and sheath producing practically no effect on the self inductance.

Resistance and Impedance of Three-conductor Cables. — The following table, published by the G. E. Co., is based upon 100 per cent conductivity copper at 75° F., with an allowance of 3 per cent for spiral path of the component wires, 60 cycles per second, and standard thicknesses of varnished cambric insulation.

RESISTANCE AND IMPEDANCE OF THREE-CONDUCTOR COPPER CABLES

Size, A. W. G. or B. & S. and Cir. Mils	Resistance ohms of each conductor per mile R	Impedance of each conductor at 60 cycles, ohms per mile, z					
		Working voltage between wires					
		3000	5000	7000	10,000	15,000	20,000
2	0.850	0.858	0.859	0.863	0.867	0.872	0.884
1	0.674	0.692	0.696	0.700	0.706	0.712	0.724
0	0.535	0.545	0.547	0.552	0.558	0.565	0.580
00	0.424	0.436	0.439	0.444	0.452	0.460	0.478
000	0.336	0.352	0.352	0.357	0.365	0.374	0.396
0000	0.267	0.280	0.283	0.288	0.296	0.306	0.332
250,000	0.227	0.245	0.245	0.252	0.261	0.272	0.299
300,000	0.188	0.210	0.210	0.217	0.227	0.241	0.270
350,000	0.161	0.187	0.187	0.194	0.204	0.217	0.250
400,000	0.141	0.166	0.166	0.174	0.185	0.199	0.234
450,000	0.127	0.148	0.148	0.156	0.167	0.182	0.221
500,000	0.113	0.137	0.137	0.144	0.156	0.172	0.212

The reactance is $x = \sqrt{z^2 - r^2}$; the inductance is $L = x \div 2\pi f$ where f is the frequency in cycles per second.

Resistance and Reactance of a Pair of Armored Single-conductor Cables. — Dr. J. B. Whitehead (*Trans. A.I.E.E., 1909, Vol. 28, p. 737*), gives the following formula for the self inductance of each of two parallel armored single-conductor cables:

$$L = 0.0805 + 0.741 \left[\log_{10} \frac{D}{r} + (k - 1) \log_{10} \frac{D_2}{D_1} \right],$$

where L = millihenrys per mile of each conductor,
D = distance between centers of conductors, any unit,
r = radius of each conductor, in same unit as D,
k = 14 to 36, depending upon the permeability of the armor wires.
D_1 and D_2 = diameter over and under the armor respectively.

The above-cited paper concludes as follows: 1. Theoretically the reactance of the usual type of single-conductor, iron-armored power cable may be from 2 to 4 times that of unarmored cable, depending on the distance to return conductor; the factor increases with decreasing distance. 2. In cables as manufactured, the reactance is about 0.7 of the maximum theoretical value as calculated by the above formula. 3. The effective resistance at 25 and 60 cycles is about 1.6 and 2 times respectively the d-c. resistance for a current density of 1 ampere per 1000 circular mils, and varies little with the distance between conductors. 4. The impedance for distances under 12 inches is about 3 times the d-c. resistance at 60 cycles and 1.7 to 2 times the d-c. resistance at 25 cycles. With the sheathing and the armor grounded at both ends the impedance is somewhat lessened and is largely of the nature of resistance. 5. The impedance of a double-conductor iron-armored cable with conductors located 0.873 inch apart between centers and at the current density given above is about 5 per cent greater at 60 cycles and 3 per cent greater at 25 cycles than the calculated value. The increase is in effective resistance, due to eddy currents in the lead sheathing.

H. W. Fisher (*Trans. A.I.E.E., 1909, Vol. 28, p. 747*) says that the impedance of armored cables cannot be calculated with any degree of certainty owing to the effect of slight differences in the air gaps between armor wires. He shows that a single-conductor No. 0 B. & S. cable armored with two flat bands of steel (0.39×1.5) may have a maximum effective resistance 8 times the d-c. resistance when the lead and armor are disconnected, and that under practical conditions of operation the ratio is about 3.5. With larger cables the ratio will be greater.

Unless the armor and lead of single-conductor cables are bonded together, arcs may occur between them and under water they would be injured by alternating-current electrolysis. Single-conductor steel armored cables cannot be used with 60-cycle currents greater than 8 or 10 amperes because of the inductance and losses due to hysteresis and eddy currents.

CURRENT-CARRYING CAPACITY OF WIRES AND CABLES.— The maximum permissible temperature rise of insulated conductors is limited by the effect of heat upon the insulation. The highest temperatures which may be safely attained continuously are given in the table below.

Insulation	Maximum permissible temperature centigrade	
	Low voltage	At E kv. between conductors
Impregnated paper...	85	$85-E$
Varnished cambric...	75	$75-E$
Rubber insulation....	60	$60-0.25E$

When rubber is heated, the usual order of events is as follows: Oxidation is accelerated if air is present, "devulcanization" commences, i.e., the rubber molecule breaks up without, however, liberating any sulphur; vulcanization proceeds until all the free sulphur has combined with the rubber or mineral matter. In the case of varnished cambric, both the insulation resistance and dielectric strength fall quite rapidly after the limiting temperature is attained. The same applies to impregnated paper in a less degree. Furthermore, at high voltages, the dielectric loss is often considerable, and if neglected in calculating carrying capacity, is likely to make one overlook possibilities of temperatures, far in excess of those to be expected on the basis of I^2R losses alone. The A.I.E.E. rule for high voltage cables may be assumed to refer to temperatures calculated on the assumption that the only losses are ohmic losses in the conductor. Hence, if the A.I.E.E. temperatures are used, dielectric losses may be neglected and the carrying capacities derived from these temperatures may be used with safety. The A.I.E.E. rule is equivalent to assuming that the dielectric loss in watts per foot are equal to $12\frac{E}{R}$ (see below).

Where, however, cables of either very low or very high dielectric loss are used, it may be advisable to calculate the carrying capacity, taking the dielectric loss into account, as in the former case a higher, and in the latter case, a lower carrying capacity will be obtained than when the A.I.E.E. rule is followed.

Formula for Carrying Capacity, Using A.I.E.E. Rule. — Where the cable construction or operating conditions are unusual, the carrying capacity may be calculated from the following formula.

$$I = 110\sqrt{\frac{T}{rR}},$$

where I = amperes,

T = permissible temperature rise, degrees centigrade,

r = electrical resistance per 1000 feet of each conductor in ohms (the factor 110 would become unity if r were expressed in ohms per inch),

R = heat resistance, from conductor to air, in degrees, centigrade rise per watt lost in each inch of conductor (see below).

Formula for Carrying Capacity, Using Actual Dielectric Loss. — When it is desired to take into account the dielectric loss, an approximation may be made by assuming the entire dielectric loss to occur at the surfaces of the conductors. Then:

$$I = 110\sqrt{\frac{1}{r}\left(\frac{T}{R} - W\right)}.$$

Where W = dielectric loss per inch of cable for each phase at the ultimate temperature. This is 1/36 of the dielectric loss in watts per foot of triplex cable.

Example. — In the case of a No. 0000 paper insulated cable for 12,000 volt service, with a dielectric loss of one watt per foot at 85° C., the carrying capacity will be about 5 per cent less than if the dielectric loss were zero.

Formulas for Heat Resistance of Cables. — The heat resistance of single conductor cables may be calculated from the formula given below.

Let D = overall diameter of cable, inches,

Q = ratio of the outer to the inner diameter of each layer of heat insulation,

K = heat resistivity of the material of each layer of heat insulation in degrees centigrade rise per watt per inch cube.

= 285 for rubber,
= 300 for varnished cambric,
= 300 for impregnated cotton,
= 465–485 for impregnated paper,
= 100 for asphalted jute,
= 300 for impregnated cotton braid.

H = heat resistivity from cable surface to room, in degrees centigrade per watt dissipated from each square inch of surface.

= 120 for impregnated braid,
= 120 for painted steel braid,
= 180 for lead bright,
= 155 for lead dull.

Then,

$$R = 0.318\frac{H}{D} + 0.367\,\Sigma\,K\log_{10}Q.$$

A corresponding formula for multiple conductor cables has been developed by Mie and is quoted by Powell, *Trans. A.I.E.E., 1916, Vol. 35–2, p. 1035.* The heat resistances of triplex paper, lead cables are given below, based upon Mie's formula and the following heat resistivities,

$$K = 475 \quad\text{and}\quad H = 155.$$

These tables give the heat resistance from each conductor to air and are used with values of r for one conductor to get the carrying capacity of each conductor.

HEAT RESISTANCE OF THREE-CONDUCTOR CABLES

Each Conductor to Outside of Sheath

Thickness of Belt Insulation = ½ × Thickness of Conductor Insulation

Size of conductor or A. W. G. or circular inch	Heat resistances, degrees centigrade rise, per watt dissipated, per inch length of cable							
	6 × 3	8 × 4	10 × 5	12 × 6	14 × 7	16 × 7	18 × 9	20 × 10
6	253	257	259	263	266	269	272	272
4	230	235	237	242	244	249	252	254
2	206	210	216	220	230	228	235	236
1	194	200	204	210	212	217	220	225
0	182	190	192	198	202	208	211	219
00	174	178	183	187	194	197	201	205
000	159	166	172	178	183	187	191	196
0000	151	157	162	168	172	178	183	185
0.25	144	150	155	161	164	170	174	179
0.30	137	142	148	154	158	163	168	172
0.35	131	136	142	145	153	157	161	165
0.40	125	131	136	140	146	150	155	160
0.45	120	127	132	137	143	147	151	155
0.50	117	123	128	133	139	142	147	150

HEAT RESISTANCE OF THREE-CONDUCTOR CABLES

Each Conductor to Outside of Sheath

Thickness of Belt Insulation = Thickness of Conductor Insulation

Size of conductor or A. W. G. or circular inch	Heat resistances, degrees centigrade rise, per watt dissipated, per inch length of cable							
	6 × 6	8 × 8	10 × 10	12 × 12	14 × 14	16 × 16	18 × 18	20 × 20
6	266	272	277	284	287	291	296	300
4	242	250	255	261	260	271	275	282
2	217	225	232	238	240	249	254	259
1	204	213	219	225	231	237	242	248
0	252	102	206	215	221	228	232	236
00	184	190	194	204	211	215	221	226
000	171	179	186	193	198	205	210	216
0000	160	166	176	183	188	195	200	205
0.25	153	160	168	175	181	187	192	198
0.30	145	152	160	166	172	180	185	190
0.35	138	145	154	159	167	173	178	183
0.40	132	140	147	153	161	167	172	177
0.45	128	136	143	150	156	163	167	172
0.50	124	131	138	145	152	157	162	167

If the cable is in a duct, the heat resistance of the duct should be taken into account. For a single duct, this is usually from 1 to 20 degrees C. per watt lost in each inch of duct. Clark and Shanklin, *Trans. A.I.E.E., 1920*, give the average heat resistance from the conductors in a cable, to the earth, as 69° C. per watt lost in each inch of duct, or 207° C. per watt lost in each inch of each conductor of a triplex cable. R.W. Atkinson, *Jour. A.I.E.E., Sept., 1920*, gives 12° C. per watt-inch as a safe average figure for a conduit containing up to 16 ducts surrounded by earth with a normal amount of moisture.

FACTOR "A" TO REDUCE CARRYING CAPACITY PER CABLE WHEN SEVERAL EQUALLY LOADED CABLES FILL A DUCT LINE

Number of ducts horizontally or vertically	Number of ducts vertically or horizontally							
	1	2	3	4	5	6	7	8
1	1	0.86	0.81	0.79	0.77	0.76	0.76	0.75
2	0.86	0.71	0.64	0.61	0.59	0.58	0.57	0.56
3	0.81	0.64	0.58	0.54	0.52	0.50	0.49	0.48
4	0.79	0.61	0.54	0.50	0.47	0.46	0.44	0.43
5	0.77	0.59	0.52	0.47	0.45	0.43	0.42	0.40
6	0.76	0.58	0.50	0.46	0.43	0.41	0.39	0.38
7	0.76	0.57	0.49	0.44	0.42	0.39	0.38	0.36
8	0.75	0.56	0.48	0.43	0.40	0.38	0.36	0.35
9	0.74	0.55	0.47	0.42	0.39	0.37	0.36	
10	0.74	0.55	0.46	0.42	0.39	0.37		
11	0.74	0.54	0.46	0.41	0.38			
12	0.73	0.54	0.46	0.41	0.38			

E.g., in a group of ducts 3×6 each of the 18 cables could carry only 0.5 of the current which could be carried by one cable alone.

CARRYING CAPACITY PER CONDUCTOR OF MULTIPLE CONDUCTOR CABLES

(Use Carrying Capacity of Single Conductor Having Same Total Thickness of Insulation between Conductor and Air.)

Number of conductors	Type of cable	Multiply carrying capacity for single conductor cable by
2	Flat (twin)	0.87
2	Round (duplex)	0.80
2	Concentric *	0.75
3	Round (triplex)	0.72
3	Modified sector	0.75
4	Round	0.67

* Also D-shaped cables.

CARRYING CAPACITIES, IN AMPERES, ALLOWED BY THE REGULATIONS OF THE NATIONAL BOARD OF FIRE UNDERWRITERS FOR INTERIOR COPPER CONDUCTORS.

(For Aluminum 84 Per cent of These Currents is Allowed)

Single Conductor Cables or Each Conductor of Multiple Conductor Cable

A. W. G.	Area in circular mils	Table A. Rubber insulation	Table B. Varnished cloth	Table C. Other insulation
18	1,624	3	....	5
16	2,583	6	....	10
14	4,107	15	(18)	20
12	6,530	20	(25)	25
10	10,380	25	(30)	30
8	16,510	35	(40)	50
6	26,250	50	60	70
5	33,100	55	65	80
4	41,740	70	85	90
3	52,630	80	95	100
2	66,370	90	110	125
1	83,690	100	120	150
0	105,500	125	150	200
00	133,100	150	180	225
000	167,800	175	210	275
	200,000	200	240	300
0000	211,600	225	270	325
	250,000	250	300	350
	300,000	275	330	400
	400,000	325	390	500
	500,000	400	480	600
	600,000	450	540	680
	700,000	500	600	760
	800,000	550	660	840
	900,000	600	720	920
	1,000,000	650	780	1000
	1,100,000	690	830	1080
	1,200,000	730	880	1150
	1,300,000	770	920	1220
	1,400,000	810	970	1290
	1,500,000	850	1020	1360
	1,600,000	890	1070	1430
	1,700,000	930	1120	1490
	1,800,000	970	1160	1550
	1,900,000	1010	1210	1610
	2,000,000	1050	1260	1670

Varnished cloth smaller than No. 6 may be used by special permission only.

MAXIMUM CONTINUOUS CARRYING CAPACITIES IN OPEN AIR, AMPERES

Assuming A.I.E.E. Ultimate Temperatures and 40° C. Air Temp.

Single Conductor Cables

Size	Rubber			Varnished cambric			Paper		
A. W. G. or circular	Braid		Lead	Braid		Lead	Lead		
	0–600 V.	7000 V.	13500 V.	0–600 V	7000 V.	13500 V.	0–600 V.	7000 V.	13500 V.
14	20	......	......	......	......	......	30		
12	25	......	......	......	......	......	35		
10	30	......	......	......	......	......	50		
8	45	......	......	......	......	......	65		
6	60	60	55	75	75	60	85	75	65
4	85	80	70	105	100	80	120	100	90
2	110	110	100	140	130	110	160	140	120
1	130	130	115	165	160	130	185	160	140
0	155	150	130	190	185	150	215	180	165
00	180	180	160	225	210	180	250	220	195
000	210	205	180	265	255	205	290	250	225
0000	250	240	210	310	290	240	335	295	260
.25	280	265	240	345	320	270	380	330	290
.35	350	320	300	430	400	335	470	410	365
.50	450	410	330	540	505	435	600	520	460
.75	600	555	500	720	660	560	790	670	590
1.00	720	670	600	880	795	670	970	820	725

This table is based on the following heat resistivities:

Material	C° per watt per inch cube
Rubber..................	285
Varnished cambric..........	300
Impregnated paper..........	475
Saturated braid............	300
	C° per watt per square inch
Lead (to air)..............	155
Braid (to air).............	120

Cables Buried Underground, Under Water, etc. — A cable buried in earth will carry 25 per cent more current with a given rise in temperature than it will when run in a dry duct. depending upon the character of the soil; immersed in water it will carry about 50 per cent more.

Current-carrying Capacity of Lead Sheath. — The carrying capacity of a lead sheath is given approximately by the following formula:

$$C = 128\sqrt{dp\,(d-p)T},$$

where

d = outside diameter of sheath in inches,
p = thickness of sheath in inches,
T = temperature rise, °C.

Rise of Temperature with Time. — Fig. 5 shows the rise of temperature with time for a single conductor, 0.5 square inch in cross-section, carrying 750 amperes. Curves I to VI are from a paper by C. Beaver (*Jour. I.E.E., 1911*) and curve VII from Del Mar's *Electric Power Conductors*.

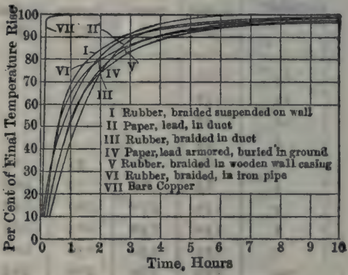


Fig. 5.

TESTS AND INSPECTION OF INSULATED WIRES AND CABLES. — (*See also section on Specifications, below.*) Wires and cables are submitted to factory tests to ascertain whether they meet specified requirements. For the tests on the conductors themselves, see the article on *Wires and Cables, Bare*. The usual factory tests of insulation are the high-potential test and the measurement of insulation resistance and capacity. In the case of rubber insulation, the tensile strength and elasticity of the insulation are also measured. Insulated wires and cables are also tested after installation to discover incipient faults and to locate existing faults.

The tests described below in greatest detail are those employed by the majority of American cable manufacturers and operating engineers. Methods of measuring insulation resistance are described in the article on *Resistance and Conductance*, and methods of measuring capacity and inductance in the articles on *Capacity* and *Inductance*.

A.I.E.E. Rules for Tests. — Electrical tests of insulation on wires and cables shall be made on the entire lengths to be shipped.

The outer surface of the insulation of complete insulated wires and cables shall be grounded while being electrically tested. If the insulation is not provided with a conducting covering and if the covering is not liable to injury by water, the ground shall be obtained by immersing the insulated wire or cable for at least twelve hours and testing at the end of that period while immersed. If the outer covering is susceptible to injury by immersion, the insulated conductor shall be tested before the application of such covering.

Dry core paper-insulated lead-covered cables, such as telephone and telegraph cables, for use in water, shall be tested after at least twelve hours' immersion.

In the case of multiple-conductor cables, without waterproof overall jacket of insulation, no immersion test should be made on finished cables, but only on the individual conductors before assembling.

High-voltage Tests. — High-voltage tests are intended to detect weak spots in the insulation and to determine whether its dielectric strength is suffi-

cient for enabling it to withstand the voltage to which it is likely to be subjected in service, with a suitable factor of assurance.

High-voltage tests shall be made at the factory by applying an alternating voltage between the conductor and sheath or water. The initially applied voltage must not be greater than the working voltage, and the rate of increase shall be approximately uniform and not over 100 per cent in 10 seconds.

Except in the case of communication cables and wires insulated with code rubber compound, the duration of the test voltage shall be five consecutive minutes. Code wires are tested for one minute.

If a multiple-conductor cable is designed for the same operating voltage between conductors and sheath or water as between conductors, each conductor shall be tested against the other conductors connected together and to the sheath or water. If the cable is designed for an operating voltage between conductors and ground different from that between conductors, the test between conductors and the sheath or water shall be made separately and shall be based on the normal operating voltage between conductors and sheath or water as prescribed in the accompanying table.

This table is based upon the minimum thickness of insulation specified by engineers and operating companies. The following tables are much used where the insulation thickness is greater.

HIGH-VOLTAGE TESTS FOR VARNISHED CAMBRIC OR IMPREGNATED PAPER INSULATED CABLES, MINIMUM VALUES A.I.E.E. STANDARD

Operating kilovolts	Test kilovolts
Below 0.5*	2.5
0.5	3
1	4
2	6.5
3	9
4	11.5
5	14
7.5	19.5
10	25
Over 10	2½ times operating pressure

*Minimum thickness shall be 1/16 inch.

While alternating current is used for cable testing in America, the use of direct current is finding great favor in Europe. The source of current is usually a mechanical rectifier and voltages as high as 150 kilovolts have been attained. It is claimed for this method that it enables a very high voltage to be applied without heating the insulation. (J. Delon.)

Insulation Resistance Test. — The purposes of the insulation resistance test are to ascertain whether the high-voltage test has caused a break-down, to ensure the dryness of the insulation and to reveal lack of uniformity in manufacture. The test is accordingly made after the high-voltage test, by measuring the leakage current after a one-minute electrification, with a continuous e.m.f. of from 100 to 500 volts, the conductor being maintained negative to the sheath or water. In the case of multiple conductor cables, the insulation resistance is measured from each conductor to the other conductors in multiple with the sheath or water.

Mechanical Tests of Rubber. — The test piece should be carefully prepared avoiding incisions and irregularities. The best way to obtain a test piece from a small conductor is to wet a sharp knife and run it tangentially along the wire so as to cut off a strip of rubber of segmental cross-section. In the case of large conductors, the best plan is probably to remove the insulation, spread it out flat and cut off a piece of uniform width by means of a special cutter consisting of a pair of parallel knife blades attached to a common frame. The diagonal projections of rubber which occur on the inside of insulation

POTENTIAL TESTS FOR VARNISHED-CLOTH INSULATION

Potentials in Kilovolts

(G. E. Co. Bulletin 4787)

Size wire, A. W. G. or B. & S. and circular mils	Thickness of insulation, 64ths inch	At factory	After installation		
		5 min.	5 min.	30 min.	60 min.
6–2	4	2.5	2	1.6	1.3
1–0000	5	2.5	2	1.6	1.3
250,000–500,000	6	2.5	2	1.6	1.3
550,000–1,000,000	7	2.5	2	1.6	1.3
1,100,000 and over	8	2.5	2	1.6	1.3
6–0000	6	5.0	4	3.2	2.6
250,000–500,000	7	5.0	4	3.2	2.6
550,000–2,000,000	8	5.0	4	3.2	2.6
All sizes	9	7.5	6	4.8	3.8
All sizes	10	10.0	8	6.4	5.1
All sizes	12	12.5	10	8.0	6.4
All sizes	14	15.0	12	9.6	7.7
All sizes	16	17.5	14	11.2	9.0
All sizes	18	20.0	16	12.8	10.2
All sizes	18	22.5	18	14.4	11.5
All sizes	20	25.0	20	16.0	12.8
All sizes	22	27.5	22	17.6	14.1
All sizes	24	30.0	24	19.2	15.4
All sizes	24	32.5	26	20.8	16.6
All sizes	26	35.0	28	22.4	17.9
All sizes	26	37.5	30	24.0	19.2
All sizes	28	40.0	32	25.6	20.5
All sizes	28	42.5	34	27.2	21.7
All sizes	30	45.0	36	28.8	23.0
All sizes	30	47.5	38	30.4	24.3
All sizes	32	50.0	40	32.0	25.5
All sizes	32	52.5	42	33.6	26.8
All sizes	34	55.0	44	35.2	28.1
All sizes	34	57.0	46	36.8	29.4
All sizes	36	60.0	48	38.4	30.7
All sizes	36	62.5	50	40.0	31.9

Tests on three-conductor cables for circuits with grounded neutral, in proportion to thickness of insulation: Example, three-phase, 12,000-volt circuit "Y," neutral grounded, insulation on each conductor 3⁄16 inch (total between conductors 3⁄8 inch), outer belt 3⁄32 inch (total 9⁄32 inch); test pressure at factory for 5 minutes, between conductors 30,000 volts each conductor to earth 22,500 volts. For mechanical reasons, thickness of insulation on individual conductors of three-conductor cables 3000 volts and less is made somewhat greater than required by working pressure on some sizes.

POTENTIAL TESTS FOR PAPER INSULATION

Potentials in Kilovolts

(G. E. Co. Bulletin 4787)

Size wire, A. W. G. or B. & S. and circular mils	Thickness of insulation, 64ths inch	At factory	After installation		
		5 min.	5 min.	30 min.	60 min.
14–2	4	2.0	1.6	1.3	1
1–0000	5	2.0	1.6	1.3	1
225,000–500,000	6	2.0	1.6	1.3	1
550,000–1,000,000	7	2.0	1.6	1.3	1
12–2	5	2.5	2	1.6	1.3
1–0000	6	2.5	2	1.6	1.3
225,000–500,000	7	2.5	2	1.6	1.3
550,000–2,000,000	8	2.5	2	1.6	1.3
10–0000	7	5.0	4	3.2	2.5
225,000–500,000	8	5.0	4	3.2	2.5
550,000–2,000,000	9	5.0	4	3.2	2.5
8 and larger	10	7.5	6	4.8	3.8
8 and larger	12	10.0	8	6.4	5.1
6 and larger	14	12.5	10	8.0	6.4
6 and larger	16	15.0	12	9.6	7.7
5 and larger	18	17.5	14	11.2	9.0
5 and larger	20	22.5	18	14.4	11.5
4 and larger	22	27.5	22	17.6	14.1
4 and larger	24	32.5	26	20.8	16.6
3 and larger	26	37.5	30	24.0	19.2
3 and larger	28	42.5	34	27.2	21.7
2 and larger	30	47.5	38	30.4	24.3
2 and larger	32	52.5	42	33.6	26.8
1 and larger	34	57.0	46	36.8	29.4
0 and larger	36	62.5	50	40.0	31.9

Tests on three-phase cables for grounded neutral in proportion to thickness of insulation: Example, three-phase 13,000-volt circuit "Y," neutral grounded, insulation on each conductor 3/16 inch (total between conductors 3/8 inch), outer belt 3/32 inch (total 9/32 inch); test pressure at factory for five minutes, between conductors 32,500 volts, each conductor to earth 17,500 volts.

HIGH-VOLTAGE TESTS FOR RUBBER COVERED WIRES AND CABLES

(a) National Electric Code, One Minute Test

Size	Working voltage					
	600	1500	2500	3500	5000	7000
14 to 8 A. W. G.	1500	6000	8000	10,000	12,500	17,500
6 to 2	2000	7000	9000	10,000	12,500	17,500
1 to 0000	2500	8000	10000	10,000	12,500	17,500
225,000 to 500,000 cir. mils	3000	9000	10000	11,250	12,500	17,500
600,000 and over cir. mils	3500	10000	10000	12,500	12,500	17,500

(b) A.I.E.E. Standards, Five Minutes Test

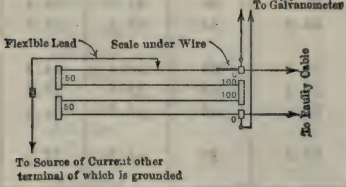
One kilovolt per 64th inch insulation thickness up to 10/64 in. Above 10/64 in. 10 kilovolts plus 1.5 kv. per 64th inch up to 30/64. Where thickness is 16/64 in., or over, rule applies only to conductors as big as No. 6 A.W.G. The A.I.E.E. allows lower test voltage for working voltages up to 600 v. a-c. In this case, the rule is as follows:

14–8 A.W.G.	3.0 kv.
7–0000	3.5
250,000 and larger	4.0

stripped from stranded conductors are neglected in computing the cross-sectional area.

In order to measure its tensile strength the sample of rubber is held in some sort of grip and stretched in a machine similar to that used for wire (*see Bureau of Standards Bulletin No. 37*). It is usual to reduce the results thus obtained to pounds per square inch.

Tests after Installation. — A high-potential-test with the voltages given above in the tables is usually applied to the full length of cable immediately after installation and at regular intervals to detect incipient faults. Double line voltage is frequently used for the periodic tests. Grounds or crosses of relatively low resistance may be detected by bridge methods or by exploration, as described below.



Arrangement of Bridge
Fig. 6.

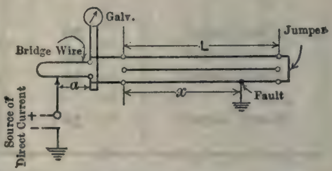


Fig. 7.

Some engineers make periodic insulation-resistance tests and plot the results as a curve. If the curve manifests any decided downward tendencies the cause is immediately sought for. The methods of making these tests are given below.

Testing for Faults by Murray Bridge Methods. — The bridge used in the Murray loop tests is shown diagrammatically in Fig. 6. It consists essentially of a resistance wire graduated into two equal scales each reading from 0 to 100, the zero points being at the ends of the wire. A galvanometer

is connected across the zero points and a flexible lead connected to the positive terminal of a source of direct current is put in sliding contact with the wire. The negative terminal of the direct-current source is grounded.

One Conductor Grounded. — When one conductor of a multiple-conductor cable is grounded, the bridge is connected as shown in Fig. 7, a jumper being placed between the faulty conductor and one of the others. The sliding contact is moved along the bridge wire until the galvanometer deflection becomes zero. Let a = distance of sliding contact, on bridge scale, to the terminal of the bridge connected to faulty wire, as a percentage of the length of half the bridge wire, and L = length of cable; then if x is the distance from the bridge to the fault,

$$x = \frac{aL}{100}.$$

Two Conductors Crossed. — When one conductor of a three-conductor cable is crossed with another, the bridge is connected as shown in Fig. 8, a jumper

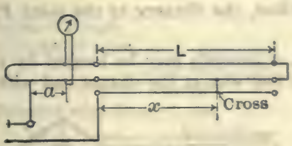


Fig. 8.

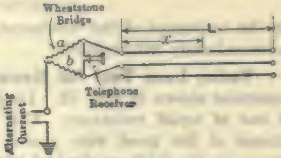


Fig. 9.

being again used, although connected differently. Zero reading is obtained on the galvanometer as before, and, using the same notation,

$$x = \frac{aL}{100}.$$

One Conductor Open. — When one conductor is open, the bridge is connected as shown in Fig. 9, a telephone receiver being substituted for the galvanometer, and alternating current for the direct current. The sliding contact is moved along the bridge until silence is obtained in the telephone.

Let a = distance from sliding contact to the terminal of bridge connected to open conductor,

b = distance from sliding contact to the terminal of bridge connected to whole conductor: then,

$$x = \frac{b}{a} L.$$

Ayrton's Method to Detect Crosses. — In Fig. 10, a, b and r represent three arms of a Wheatstone bridge, the fourth arm being the conductor N, which is in contact with a second conductor M of the same cable at the point P. Ground the conductor N at the far end and let x and y be the resistances

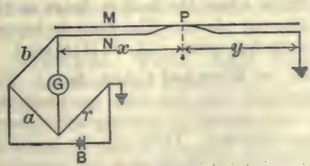


Fig. 10.

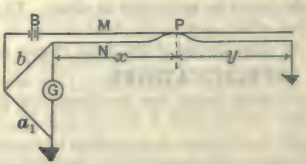


Fig. 11.

of the grounded conductor from the bridge to P and from P to the grounded end respectively. Let a, b and r be the resistances of the other three arms of the bridge. Make $a = b$, and adjust the variable resistance r until the galvanometer shows no deflection; then

$$x + y = r,$$

neglecting the resistance of the ground.

Then connect battery to M instead of to earth as shown in Fig. 11, and adjust a to a value a_1 such that the galvanometer shows no deflection. Then $a_1 x = by$ or

$$\frac{x}{x+y} = \frac{b}{b+a_1}.$$

But from first arrangement $x + y = r$, whence

$$x = \frac{br}{b+a_1}.$$

From the resistance of the conductor per foot, the distance to the point P is then immediately determined.

Varley Loop Test for Grounds. — Grounds may be located by the arrangement shown in Fig. 12. Loop the bad wire at the distant end with a good one of equal resistance. Call the resistance of the good wire c and the resistance to the fault on the bad wire x. Adjust the rheostat until the galvanometer balances. Then

$$x = \frac{2\,bc - ar}{a+b},$$

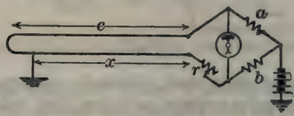


Fig. 12.

a, b and r being the values of the bridge arms and rheostat respectively when the galvanometer shows no deflection. (*S. G. McMeen.*)

Varley Loop Test for Crosses. — This is made by substituting one of the crossed wires for the earth path of Fig. 12, using one good wire as before.

Cable Tests by Exploring Coil. — Grounds on wires in cables can be located by the use of a telephone receiver connected to an exploring coil. To do this, connect a source of interrupted current to one end of the wire in trouble, the other terminal of the source being grounded. Remove from the wire in trouble all the grounds other than the fault. Then move along the cable a coil such as one spool of an 80-ohm telephone ringer (*see Telephony*), across whose terminals a receiver is connected. Hold the exploring coil with its iron core at right angles to the cable. A tone from the interrupter should be heard when the exploring coil is between the fault and the source of sound, but not when it is beyond the fault. (*S. G. McMeen.*)

Listening Tests on Telephone Lines. — A long telephone line will be noisy if one of its wires is grounded, open or crossed with a third wire, whether the latter be grounded or clear. To determine where the fault is, listen on the line while assistants short-circuit it at different places. The fault is beyond the short-circuit if the short-circuit makes the line quiet (*S. G. McMeen.*)

SPECIFICATIONS. — (*See also article on Wires and Cables, Bare.*) The following forms are recommended for insulated wire and cable specifications. The general idea is for the purchaser to have printed or mimeographed forms and to fill them in according to specific requirements. These tabular specifications are supplemented by general specifications for workmanship and materials, which should be included when bids are sought or orders given. This arrange-

ment minimizes the chance of accidental omission on the part of the purchaser and manufacturer and saves the bidder time in reading long specifications.

Items to be Covered in Detail Specification. —

Specification No.,
Title,
State briefly the service for which the conductor is intended,
Number of separately insulated conductors,
Material of conductors,
Type of stranding (parallel, concentric or rope),
Number of strands per conductor,
Number of wires per strand (if rope strand),
Total area of conductor or A.W.G. No.,
Conductor shall *not* be tinned (omit the word "not" as required),
Type of separator, if any,
Insulation material, each conductor,
belt,
Minimum thickness of insulation, each conductor,
belt,
Tape, material,
Number of tapes, each conductor,
over each layer,
over-all,
Pressure wire (if any), size,
insulation,
location,
Servings, material,
number,
thickness, inches,
Braid, material,
number on each conductor,
number over-all,
Tracers (if required), material,
number,
arrangement,
Laterals (if required), material,
Filling under sheath,
Sheath material,
Thickness of sheath,
Filling between sheath and armor, material,
thickness, measured after armoring,
Armor, material, wire or band,
Jute over armor,
Maximum over-all diameter (if limit is necessary),
Length, maximum,
minimum,
Whether to be supplied in coils or on reels,

All workmanship and materials shall conform with stated general specifications such as the following:

GENERAL WIRE AND CABLE SPECIFICATIONS. — Purchasers who are desirous of obtaining wires or cables of adequate design and good construction, use specifications which describe the major details of construction to be followed and tests to be applied.

Formerly every customer who purchased on specification wrote his own specifications, but this led to endless difficulty with the manufacturers who had to make special compounds or follow special rules for each customer. In recent

years various engineering societies have adopted standard specifications which are now largely used.

American Institute of Electrical Engineers. — (*a*) *Standards of the A.I.E.E.*, which cover definitions of terms, methods of test, insulation thickness for high grade rubber insulation, test voltages, etc.

(*b*) Marine Code, which covers merchant ship wiring.

American Railway Association. — Signal wire specifications.

American Railway Engineering Association. — General wire specifications.

American Society for Testing Materials. — Specifications for bare copper wires and cables. Tentative specifications for rubber-insulated conductors.

Association of Railway Electrical Engineers. — Specifications for practically all types of wires and cables used by railways. (Now becoming obsolete.)

Joint Rubber Insulation Committee (superseded in 1921 by A.S.T.M.). — Specification for 30 per cent hevea rubber insulation.

National Board of Fire Underwriters. — Specification for wires and cables to be used in buildings.

National Electric Light Association. — Specifications for paper-insulated, lead-covered underground cable.

Rubber-insulated Wire and Cable. — Most of the characteristics of rubber compounds are so good from the point of view of electrical insulation, that the efforts of purchasers are mostly concentrated on obtaining life, or permanency.

There are three recognized standard specifications for rubber insulation, the National Electrical Code specification which is the least severe, the Joint Rubber Insulation Committee specification which has severe chemical requirements, and the American Railway Engineering Association (Signal Division), specification which specifies a formula and requires rigid inspection during manufacture. The idea of chemical clauses is that by specifying close limits upon the following analytical results (obtained by a standard method of analysis), the use of non-permanent substances will be prevented.

Analytical results upon which limits are placed	Deleterious substances whose use is prevented by limits upon analytical results
Total sulphur	Reclaimed rubber
Alcoholic potash extract	Rubber substitutes
Chloroform extract	Tars
Mineral matter	Carbon, cellulose, etc.
Saponifiable acetone extract	Unextracted high resin rubbers
Unsaponifiable acetone extract	Deresinated rubber

The chemical requirements are supplemented by mechanical and electrical requirements, the former for the purpose of checking vulcanization and the latter to ensure proper insulating qualities.

Paper Cable. — The principal characteristic which is insisted upon for low-voltage paper insulated cables is good condition of the paper in regard to mechanical strength, in order that the cable may not be injured during installation. When cables are installed they o ten have to be bent to a very short radius, and unless the paper is in very good condition, it will crack, usually on the outside of the bend. If the paper is wrinkled, scorched or improperly lapped, it will break open when it is installed.

Purchasers, therefore, require that a high grade of manila rope paper be used and that it be free from wrinkles, tears and scorches.

In the case of high-voltage cables, in addition to the requirements for low-voltage cable, it is usual to put a limit upon the dielectric losses in order to ensure the cable having ample carrying capacity.

Severe bending tests, followed by high voltage tests, are often specified to show up defects in the paper, and lack of uniformity in drying or impregnating is revealed by the megohms test.

Varnished Cambric Cable. — Most of the specifications for varnished cambric cables are not very severe, except in requiring a smooth application of the tape and proper overlap.

Dielectric loss requirements are not applied to varnished cambric cables, but some recent specifications call for certain characteristics of the varnished cloth itself, which make it necessary to use only the highest grade of cloth. These specifications call for soaking the cambric in petroleum at a high temperature to make sure that it does not become tacky as a result of reaction with the "slipper" compound.

INSTALLATION OF INSULATED WIRES AND CABLES. — For the installation of wires and cables in buildings see articles on *Wiring of Buildings for Light and Power; Wiring of Buildings for Miscellaneous Devices.* Below is given a brief description of modern practice in installing insulated wires and cables on messenger wires and in underground conduit systems. For the construction of the pole lines carrying the messenger wire see articles on *Poles for Overhead Lines; Towers, Transmission.* For the construction of conduit systems see article on *Conduits and Conduit Lines.* See also the articles on *Distribution Lines* and *Transmission Lines.*

Messenger Construction. — The messenger wire is erected in the same manner as an ordinary line wire; see article on *Transmission Lines.* A "leading-up" wire is stretched from the bottom of one pole to the messenger wire on the starting pole, forming an incline on which to pull up the cable. A pulling rope is then fastened to the end of the cable by means of a cable grip and carried alongside of the messenger to the point where the cable is to reach, thence through a snatch-block down to the terminal pole to a second block at the bottom and thence to a capstan, winch, locomotive or whatever is to be used for pulling. Either temporary rollers should be provided on the poles over which the rope runs, or the rope should be suspended from the messenger by wire hooks. The cable is then slowly drawn up the inclined wire and along the messenger, attaching temporary carriers to the cable as it is paid out and hooking them over the messenger to carry the weight of the cable. Linemen must be stationed on each pole to pass these carriers around the messenger clamp or insulator. The final suspension of the cable may be accomplished in either of the following ways: (1) When the end of the cable arrives at the beginning of the last span, the lineman on each pole replaces the temporary carriers by permanent hangers, spacing them regularly along the cable, so that when the last span is pulled, all the hangers will be in place. (2) When the cable has been pulled all the way, a lineman rides along the messenger wire in a carriage replacing each temporary carrier by a hanger. This plan is preferable as the hangers may be attached more tightly to the messenger wire, and are less likely to slip on the cable.

Installation of Cables in Ducts. — The conduit system having been constructed, it must be prepared for the reception of the cable by being cleaned out or rodded as described in the article on *Conduits and Conduit Lines.*

If there are several ducts available, the choice of the particular duct to be used should be governed by the following considerations: 1. Avoiding un-

necessary crossings of cable in the splicing chambers, substations, etc.; 2. Avoiding the obstructing of empty ducts; 3. Keeping the cables cool; and 4. Keeping d-c. cables away from others. The following statement in the catalogue of the A. S. & W. Co. is pertinent with regard to keeping the cables cool. "Usually the coolest and best heat-radiating ducts are those located at the lower corners of the system, next are those nearest to the outside of the system and lastly the middle and top ducts which not only take up heat from the lower cables, but must dissipate heat through adjoining ducts. Attention to these points when planning a new system may prove very profitable in the end."

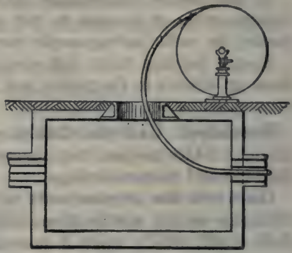


Fig. 13.

The next step is to set the cable reel on the shaft of a pair of wheels of slightly greater diameter than the reel itself or on jacks and raise it slightly above the ground, taking care to locate it as shown in Fig. 13, so that the cable will unreel into the manhole without making a reverse bend.

Fig. 14.

The pulling rope having been left in the duct after rodding, the cable is unreeled sufficiently to bring its end close to the mouth of the latter and a wire pulling grip (Fig. 14) is drawn over its end. The end of the grip is hooked to the rope and the rope pulled from the other end. Fig. 15 shows an arrangement of pulleys for guiding the pulling rope. The pulling may be done by capstan, winch, motor truck, horse or by hand, depending upon the size and amount of cable to be pulled, and upon local conditions. The cable should be carefully guided into the duct so as to avoid sharp bends and abrasions, and a small quantity of grease may with advantage be spread over the cable as it enters the duct. R. J. Robb (*Journ. I.E.E., 1911, Vol. 47, p. 359*) says that 112 pounds of petroleum jelly is required per mile of cable. It is also necessary to cover the edges of the duct with pieces of lead to prevent abrasion of the cable as it passes into the mouth of the duct.

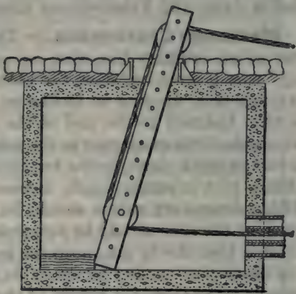


Fig. 15.

Protection of Cables in Splicing Chambers. — (*See also article on Conduits and Conduit Lines.*) Wherever there are several large cables in a splicing chamber, there is always danger that a burn-out of one cable will involve some or all of the remainder. Hence it is usual to protect such cables by means of one or more of the following methods:

1. Concrete shelves,
2. Open-face conduits,
3. Cement coating with ¼-inch rope bond,
4. Asbestos tape saturated with silicate of soda,
5. Asbestos tape covered with soft steel-tape armor,
6. Asbestos rope,
7. Split tile duct.

Methods Nos. 1 and 2 are applicable only to chambers containing few cables.

No. 3 has given good results in many cases, but the cement is liable to crack and fall away from the cable unless carefully bonded. On the other hand, the location of a burn-out is plainly indicated by the cement covering being blown off. No. 4 combined with No. 5 is very generally used and gives very satisfactory results. No. 6 is but little used. No. 7 is very satisfactory, but is very clumsy, and cannot be used where there are many cables in the chambers.

The necessity for such protection (according to the Committee on Underground Construction of the N. E. L. A., 1911) is equally great with low-tension cables as with high-tension cables, for, while a breakdown of the insulation of a high-tension cable is often attended with a violent explosion and the generation of a very high temperature, the duration of the trouble is limited to one or two seconds; whereas in the case of a short circuit on a low-tension cable, it may continue to burn with a considerable flame for several minutes, which would be quite long enough to damage adjacent unprotected cables.

In the case of direct-current feeders for electric railways, where the return is grounded, the danger of burn-outs spreading is much greater than with the insulated lighting systems referred to in the N. E. L. A. report. This subject is treated at greater length in the author's *Electric Power Conductors*, from which the following is quoted: "If it is necessary to put direct- and alternating-current cables in the same duct line, it is well to isolate the direct-current cables as much as possible in the splicing chambers. The racks on which direct-current cables are supported should not be in metallic contact with other racks. If, however, this is unavoidable, the cables should not lay directly on the racks, but on insulating pads or blocks."

Jointing Insulated Conductors. — (*See also article on Wires and Cables, Bare.*) Having selected the corresponding cable ends, they should be inspected for mechanical defects along the entire exposed length and bushings placed over them at the ducts so that the sheath will not be cut by the edges of the conduits. The cables should then be bent until the ends overlap, the bends being of ample radius throughout, and the position of the cable such that the joint, when completed, will not have to bear any elastic stress or weight.

Drying Out Cable Ends. — In the case of paper- or cambric-insulated cables, the ends should be examined for moisture before they are finally cut short and if any exists or is suspected, heat should be applied to the sheath, beginning at the duct end and slowly working to the open end. This is usually done with a gasoline torch but sometimes by pouring on hot insulating compound.

Overlap. — This operation having been completed, the cable ends are cut so as to leave an overlap, depending upon the method of joining the conductors (*see below*) and upon the number of separately insulated conductors in the cable. If the cable has but a single conductor, and a butt joint is to be used, the overlap should be merely sufficient to allow for cutting off the ends, whereas if an interlaced joint is contemplated, the overlap should be greater by about 3 or 4 times the diameter of the conductor.

Removal of Sheath. — The next operation is to cut off a sufficient length of lead sheath to permit the joint to be made. This length may be judged from the size of the sleeve to be used (*see below*). The operation is performed with a chipping knife and hammer, or with a special tool designed for the purpose, and the greatest care should be exercised to avoid cutting the insulation to the slightest degree. It is usual to make a cut around the sheath and gradually increase its depth until the lead is cut through. The lead must then be cut lengthwise from the circular cut to the end, injury to the insulation being avoided by holding the knife tangent thereto. The lead may then be pulled off with a pair of pliers and loose particles carefully removed. In the

case of high-tension cables the ends of the lead should then be turned up slightly to a bell-mouth shape by means of a fibre wedge.

Putting on Lead Sheath. — Before jointing the conductors, the lead sleeve should be slipped over one of the cable ends and pushed out of the way. The ends of the sleeve and the corresponding surfaces of the sheath should have been previously scraped for a length of about a couple of inches along the outside and the cleaned surfaces smeared with stearine.

Removal of Insulation. — The next step is to cut back the insulation for a length between ¼ and ½ inch greater than half the length of the connector to be used, and taper the remainder uniformly for a distance of 1 to 1¼ inches using a small sharp knife. In the case of multiple-conductor cables having a belt, the belt must be cut back sufficiently to expose all the joints, the greatest care being exercised to avoid cutting the inner insulation in doing so. The paper tapes should be tied with string to prevent unwrapping. The surface of each taper should be covered temporarily with a strip of dry cloth.

Jointing the Conductors. — The next process is to prepare the conductor or conductors for jointing. The most usual type of joint is made by butting the ends of the conductors and enclosing them in a cylindrical copper connector. The conductors are first cleaned with gasoline and then tinned by pouring molten solder over them, using stearine for flux. All burrs should be removed with a file and the ends smoothed so that they will butt together perfectly. The connector is then put over one conductor end and the other end slipped in until the two ends butt. Solder is then poured over the joint until it is thoroughly saturated, when the surplus is wiped off so as to leave no sharp projections.

Connectors. — The connector should have a cross-section not less than that of the conductor and a length about four times the diameter of the conductor.

From points ½ inch back from each end, it should be pencilled down to thin rounded edges at the ends, and it should be split the entire length.

It should be finished so as to eliminate all sharp points and edges. The opening should not be over 3/64 inch in width.

Jointing Stranded Conductors. — While the connector joint is by far the most common, stranded conductors may be joined by cutting the wires alternately long and short, and fitting the two conductor ends into one another. The joint is then bound with small wire. Stranded conductors are sometimes joined by cutting the wires of successive layers alternately long and short, and telescoping the ends into one another.

Insulation of Joint. — The next step is to insulate the joint. The depression at each end of the copper sleeve should be filled flush with impregnated 4-strand cotton twine, which should be thoroughly impregnated with compound. The usual process is to cover the joint with a tape of similar material to the remainder of the cable insulation, although this method is not universal. If of oiled paper or varnished cambric, the tape should be cut on the bias. Narrow strips of impregnated tape should first be wrapped back and forth in the space between the two ends of the original paper insulation until it is built up to the level of this insulation. Compound should be applied on each layer. Wider tape is then wound on until a thickness about 40 per cent greater than that of the cable insulation is reached.

In the case of three-conductor high-tension cables the wrapping should commence at a point on the covered conductor 3 inches from the outer belt and extend to a corresponding point at the other end. The wrapping is then continued back and forth, stopping each successive layer ½ inch short of the end of the preceding layer. In applying the tape, each turn should be drawn tight

to exclude air and should overlap the preceding turn by two-thirds of its width. It is important that the tape should be applied tightly and evenly. In the case of rubber tape the tension should be such as to stretch it to about half its width. In this process care must be taken to have everything perfectly clean and dry.

"Boiling Out." — Where cotton or linen tape is used, each layer must be "boiled out" by pouring hot compound over it until all the moisture is expelled and the tape wrapping is thoroughly saturated. Before applying the compound the jointer should assure himself that it is *not* hot enough to ignite a piece of paper dipped in it; otherwise the tape is likely to be charred. This should be done before entering the splicing chamber, as the accidental ignition of a pot of compound is dangerous in such a confined space.

Vulcanizing. — Where rubber tape containing sulphur is used, it should be partially vulcanized by the application of heat from a spirit lamp, care being taken to apply the heat evenly and to avoid burning the insulation. This process is usually complete in about one minute with conductors up to one-half inch diameter.

Spreaders for Three-conductor Cables. — In the case of three-conductor cables, after all the conductors are joined and insulated as described, a roll of tape ½-inch diameter is inserted between them at the center of the joint to serve as a spreader. A band of tape is then wrapped around all three conductors to such a diameter that the whole will slide nicely into the lead sleeve, as shown in Fig. 16. Instead of this, a band of tape may be built around each conductor at the center of the splice.

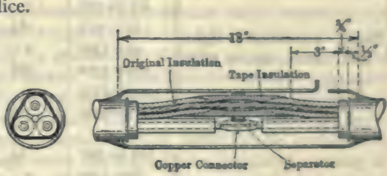


Fig. 16.

Use of Insulating Tubes. — Another way of insulating joints is to slip an insulating tube over the conductors before soldering and bring it over the joint when the latter is completed. In order to get the sleeve out of the way during soldering, it must be large enough to slip easily over the insulation. Such tubes are made of prepared paper, varnished cloth, or micanite. The jointed conductor should first be wound with cotton tape up to the level of the original insulation, "boiled out" as described above, and then the sleeve slipped on. A further "boiling out" with hot compound completes the job. In the case of belted multiple-conductor cables, in addition to the insulating tube over each conductor a large tube must be slipped over the belt before splicing the conductors.

Insulating Baffles. — Baffles of micanite, bakelite, etc., are now commonly used between conductors of triplex cables. They replace part of the wrapped insulation around each conductor and the whole of the belt. The Conducell and Commonwealth Edison Co. splices are of this type.

Lead Sleeves. — The joint having been insulated, the lead sleeve should be brought symmetrically over it, and the ends beaten down into contact with the sheath, taking care to make the sleeve concentric with the cable.

The sleeve and sheath are then joined by pouring solder over the ends from a ladle and wiping the joint with a cloth. This process should be continued until perfectly air-tight joints are obtained, the under side being examined for defects by means of a hand mirror.

When the sleeve is well wiped on, two small holes should be made in the top of it and hot compound poured in one hole until it appears at the other. If any frothing appears the compound should be poured in one hole and allowed

to escape from the other until this defect ceases. The joint should then be allowed to cool for about an hour, and if then the compound has settled, more should be added until the sleeve is full. The sleeve is then closed by soldering small patches of lead over the holes. When the joint has thoroughly cooled and solidified, it may be pushed gently into its permanent place.

The following data on lead sleeves are given by the Standard Underground Cable Co.

DATA ON LEAD SLEEVES

	Outside diam. of cable, mils	Inside diam. of sleeve, inches	Length of sleeve, inches	Gals. of compound per joint	Wiping solder per joint, lbs.
Single-conductor, light and power, up to 6600 volts.	Up to 550	1	8	0.05	0.9
	551–950	1½	10	0.1	1.7
	951–1350	2	12	0.2	2.8
	1351–1750	2½	12	0.3	4.2
	1751–2150	3	14	0.5	5.5
	2151–2550	3½	14	0.6	6.8
Single conductor, light and power, above 6600 volts.	Up to 550	1	10	0.05	0.9
	551–950	1½	12	0.1	1.7
	951–1350	2	14	0.2	2.8
	1351–1750	2½	16	0.4	4.2
	1751–2150	3	18	0.6	5.5
	2151–2550	3½	18	0.8	6.8
Multi-conductor, light and power, all voltages.	Up to 800	1½	14	0.2	1.5
	801–1200	2	16	0.25	2.5
	1201–1600	2½	16	0.35	3.7
	1601–2000	3	18	0.6	5.0
	2001–2400	3½	18	0.8	6.3
	2401–2800	4	18	1.0	7.6
	2801–3200	4½	20	1.4	8.3

Compounds for Filling Lead Sleeves. — Various compounds have been used for filling the sleeves, such as conduline, G. E. No. 227 compound, E. P. Trotter's "B–7," ozite, etc. The Commonwealth Edison Co. of Chicago uses a compound developed by the engineers of the company. It is poured in at a temperature of 150° C.

A good compound should have a high melting point, adhesiveness, high dielectric strength, low coefficient of contraction and should not be brittle at ordinary temperatures.

Stamping the Joint. — After the joint is completed the jointer should stamp his initials at each end of the sleeve.

Taped Joints. — Where the cable has no lead sheath, but is merely braided, it is usual to finish the joint with friction tape.

Shielded Splice. — A joint used for 25,000-volt cables is described by P. Torchio, Elec. W., 1916, Vol. 67, p. 873. The conductors are wrapped with oil-saturated paper-tape, and over the three conductors is provided an outer jacket pierced with holes to allow a free circulation of oil. Over the jacket is a

copper gauze sleeve and around it a heavy lamp wick. The gauze protects against sharp potential gradients and the wick retains the insulating oil within the splice. When the joint is completed, it is subjected to a vacuum of 27–29 inches and then filled with liquid petrolatum.

Terminals. — Conductors may be soldered to lugs or terminals or they may be clamped mechanically by means of a Dossert or similar connector.

In the former case the insulation is cut from the end of the conductor, which is then brightened by scraping or sandpapering and smeared with soldering flux. The conductor is then tinned by plunging into molten solder. The lug must also be tinned internally and heated so that it will hold some solder in the molten state. The conductor, also heated above the melting point of solder, is then pushed into the lug and the latter cooled by the application of wet waste. When cool the shreds and globules of solder are filed off and the surfaces brightened with sandpaper. Some jointers prefer to hold the lug in the molten solder until it attains the same temperature as the latter. In this case, if the lug has been previously treated with flux, its outside as well as its inside surface will be tinned and it will make better contact with its accompanying lug or terminal. If, however, the copper or brass surface is desired for appearances, it may be preserved by coating the outside of the lug in a light oil of high flash point, before dipping it into the solder.

In either case it is advisable to wrap the end insulation in a rag previously wrung out in cold water to prevent it being melted or charred.

Grounding Sheaths. — The conductors of high-tension cables induce electrostatic charges on the sheaths, often raising the latter to dangerously high potentials. It is, therefore, customary to ground the sheaths of cables at suitable points in order to carry off the "static," as these induced charges are called. The sheaths of low-tension cables do not have to be grounded to drain off static electricity. The sheaths of direct-current railway feeders which are used in conjunction with a grounded return system should, however, be grounded to the negative return system, except when the tracks have insulated sections for automatic block signals, in order to afford a low-resistance path for a short-circuit current. Unless this is done, the escaping current will return through devious paths, inflicting damage without attaining sufficient strength to trip the station circuit breakers.

BURN-OUTS OR PUNCTURING OF INSULATION. — Cable burn-outs may be caused by mechanical injury, exposure to excessive heat or cold, by cumulative heating, by chemical deterioration, or by transient high-voltage phenomena.

Burn-outs Due to Mechanical Injury. — Paper insulation differs from rubber and varnished cambric in depending upon the integrity of the lead sheath which incloses it. Hence in the case of paper-insulated conductors, a puncture of the lead sheath will sooner or later result in a burn-out even though it may take a week or longer for the moisture to penetrate the insulation sufficiently to accomplish this. (*Burch, Trans. A.I.E.E., 1903, Vol. 22, p. 433.*)

Mechanical injury often occurs in the process of installing underground cables. A slight projection in a duct will cut a groove in the lead sheath, thereby reducing its effective thickness and rendering it liable to crack open. The bending of cable in splicing chambers may crack the insulation, especially if tightly wound paper insulation is used. A loosely insulated cable is also liable to injury because bending will flatten the cable, compress the insulation in one direction and flare it out in the other, causing voids or air spaces between the layers, which under the influence of high voltages will cause electric discharges, heating ozone and consequent chemical action. Mechanical injury after installation

may be due to settlement of the conduit line, to careless stepping upon the cable, to vibration setting up crystallization of the lead sheath, etc.

Exposure to alternate heat and cold leads to expansion and contraction which may introduce mechanical stresses into the insulation and thereby injure it. Excessive heat (120 to 150° F.), such as would result from direct exposure to summer sunlight, is likely to dangerously reduce the insulating qualities of varnished cambric and to a less extent that of impregnated paper. A higher degree of heat (200° F.), such as would occur in the proximity of steam pipes, has the effect of making rubber insulation become brittle.

Chemical deterioration may be due to electrolysis (q.v.) or merely to ordinary chemical reaction between the sheath and the material in contact with it, the former cause being the more common.

Burn-outs Due to Static Discharges. — Failure of high-tension rubber-insulated conductors is sometimes due to electrostatic discharges from the charged conductor to its supports, the irregular potential gradient giving rise to local static discharges with consequent formation of ozone and oxidation of the rubber.

Burn-outs Due to Imperfect Manufacture or Splicing. — Insulated conductors also burn out because of defects of manufacture or of splicing. The former class of defects is happily rare, but dirt, moisture and jagged edges of metal are frequently responsible for the failure of joints, especially on high-tension cables.

Transient High Voltage and Currents. — "Surges," i.e., transient high voltages and heavy currents, are sometimes responsible for very serious cable failures, such as described in *Trans. A.I.E.E., Vol. 24, p. 207*. To determine the origin and cause of high-voltage disturbances, so as to be able to guard against their recurrence, the most important thing seems to be to very carefully observe and record all the details of the phenomena, even those which appear unessential. The existence of static (i.e., high-voltage discharges of small currents) on switchboards, lines, etc., and the existence of voltages and currents different from those which may be expected require special attention. Either of these is sufficient to raise the suspicion of some dangerous fault in the system or some dangerous arrangement of apparatus, which requires consideration. The severity of the phenomena depends almost entirely upon the power momentarily available in the system and very rapidly increases with the size of the generating stations (*C. P. Steinmetz*).

After considering the likelihood and severity of such potential disturbances, P. Junkersfeld and E. Schweitzer reached the following conclusions with respect to their influence upon the use of high-tension cables.

1. Where local and commercial conditions justify, pressures as high as 25,000 volts can be satisfactorily used even for systems aggregating as much as a hundred miles of cable. No single line of such a system would be much longer than twenty miles. If higher voltages are needed to meet operating requirements and can be justified commercially, special construction will be necessary to overcome limitations in paper, rubber or varnished-cabric insulation, and also in the standard forms of underground conduit or subways used in this country.

2. On comparatively short lengths, underground or under water, as a part of a long overhead transmission line, cables operating at 40,000 volts can be used,

3. Potential rises of 50 per cent and 100 per cent are not uncommon in large underground cable systems, although this fact may not always be manifest, due to the high factor of safety in the insulation.

Occurrence of Cable Burn-outs. — The following table gives the number of cable burn-outs reported to the Underground Systems Committee of the N.E.L.A. in 1921.

CLASSIFICATION OF CABLE FAILURES ACCORDING TO VOLTAGE FOR YEAR 1920

	Voltages						
	6,000 to 9,000	9,000 to 12,000	12,000 to 15,000	15,000 to 18,000	18,000 to 21,000	21,000 to 24,000	24,000 to 27,000
Number of companies reporting	6	5	7	1	1	5	3
Miles of cable	1151.4	1207.3	1093.1	5.7	80	408.7	37.1
Cable failures	48	98	108	0	8	17	5
Joint failures	11	32	90	0	6	28	0
Total failures	59	130	198	0	14	45	5
Failures per 100 miles of cable	5.1	10.8	18.1	0	17.5	11.0	13.5
Failures per 100 miles of cable in 1919	2.6	7.9	10.5	0	17.7	19.3	24.2

Life. — The life of cables depends so much upon the type, excellence of manufacture and conditions of service, that specific figures are of little use. Furthermore, many of the earliest cables made are yet in satisfactory operation. For example, there are at Buffalo, rubber-covered 11,000-volt cables that were installed in 1897, and in 1921 they were in perfect condition.

COSTS. — There are so many types of wires and cables and the prices fluctuate so greatly that it is impossible to give adequate cost data. The following examples will give a general idea of the cost of a few common types and sizes:

Kind of wire or cable	Cost per 1000 ft.
No. 14 code wire	\$6.00 to \$ 12.00
Single conductor, 2500 volt, rubber ins., Park cable.	\$85.00 to \$115.00
Single conductor, 1 circ. inch, 5/32″ paper and 1/8″ lead	\$725.00 to \$1350.00
Three conductor, 0.35 circ. inch, 6/32 by 6/32, 1/8″ lead	\$1070.00 to \$1710.00

BIBLIOGRAPHY. — Bauer, K., *Elektrische Kabel*, Berlin; Coyle, D., and Howe, F. J., *Electric Cables*, London; Del Mar, W. A., *Electric Power Conductors*, New York; Meyer, E. B., *Underground Transmission and Distribution*, New York; Russel, A., *Theory of Electric Cables and Networks*, London; see also Standards of the A.I.E.E.; National Electrical Code, Transactions of A.I.E.E., and Committee Reports of A.E.R.A., A.R.A., A.R.E.E., and N.E.L.A.

WIRES, RESISTANCE. — (*See also Electromagnet Windings; Resistance and Conductance, Electric; Rheostats and Resistors; Wires and Cables, Bare, Wires and Cables, Insulated.*) Metals and alloys having high specific resistance or low temperature coefficient of resistance are largely used for resistors. Some of the principal metals used for this purpose are listed below. They can usually be obtained in the form of wires, ribbon or sheets. The data given for Nichrome, Climax, Advance, Therlo, Yankee Silver, Ferro-nickel, Monel Metal, Kromore, No. 193 Alloy, and Nickel Silver were supplied by the Driver-Harris Co., those on Calido, Ideal, Akbar, Comet, Karma, Lucero, Nickel, Phenix and Rayo were supplied by the Electrical Alloys Co., those on Excello, Ia Ia, and Superior resistance metals were taken from the catalogue of H. Boker & Co., those on Acme, Electris, Excelsior, Eureka, Peerless, Premier Nickel Chrome, Sichrome, Superior Nickel Chrome and Tarnac were supplied by the Alloy Metal Wire Co., those on Chromel were supplied by the Hoskins Manufacturing Co., and those on Manganin by Baker & Co.

Acme is a nickel-steel-chromium alloy and is used in heat radiating elements. It is similar to No. 193 Alloy and Comet.

Advance is a copper-nickel alloy, containing no zinc. It is uniform in its composition and constant in its resistance under all conditions of service. Because of its high thermal e.m.f. against copper, it cannot be used in electrical apparatus in which the generation of an electromotive force would cause errors, as for example, in a wheatstone bridge where a few microvolts would cause an error in the galvanometer deflection. It is used in switchboard voltmeters, street car heaters, and other forms of heating devices provided its operating temperature is not exceeded. For ordinary temperature measurements, the commercial grade may be used as a thermo-couple element. For accurate temperature measurements, it should be purchased on specification.

It is similar to Excelsior and Ideal.

Akbar is an alloy of copper, nickel and manganese. It is used for heating purposes.

Calido is a high percentage nickel-chromium alloy. The melting point is about 1530° C. It is recommended for electrically heated devices at temperatures up to 1000° C.

Chromel is furnished in three grades: Chromel "A" is an alloy containing about 80 per cent nickel and 20 per cent chromium. It is used in heating devices such as hot plates at temperatures from 875° C. to 1100° C. Chromel "B" contains about 85 per cent nickel and 15 per cent chromium. It is less durable than Chromel "A" but is suitable for the same class of work where conditions are less severe. Chromel "C" is a nickel-chromium-iron alloy containing about 25 per cent iron and 11 per cent chromium. Its iron content somewhat lowers its durability at high temperatures. It is used extensively in flatirons where it is enclosed and thus protected to a certain degree against oxidation.

Climax is a high-resistance nickel-steel alloy. It is especially well suited for use in rheostats. It is one of the cheapest resistance metals, but has a tendency to rust and scale and should therefore not be used in smaller sizes than No. 20 A.W.G. It is similar to Electris and Phenix.

Comet is an alloy of nickel, chromium and steel. It is low in cost and used extensively in heating devices at temperatures below red heat. This alloy with Ideal wire produces a thermocouple of high e.m.f. and as such is more durable than iron or copper. It is similar to Acme and No. 193 Alloy.

Constantan is a copper-nickel alloy. It is used primarily as an element for

thermocouples. Constantan, Advance and Ideal are very similar and used for similar purposes. When thermocouples are to be made up for use with calibrated apparatus, the thermoelements should be purchased on specification as impurities in very small quantities affect the thermal e.m.f. (see article on *Pyrometers*).

Electris is used as resistance element at medium low temperatures in such service as controlling devices and rheostats. It can be soldered with any standard flux and is similar to Climax and Phenix.

Eureka is a nickel-copper alloy and is claimed to be superior to German silver because it is free of zinc, which produces brittleness when the alloy is overheated. It is used for general heating purposes such as car heating. In general it is similar to Lucero.

Excelsior is an alloy of nickel and copper. It has characteristics similar to Advance, Ideal and Constantan and is used for similar purposes. The manufacturer recommends it for use in heating pads and other light duty service.

Excello is adapted for use in electric heating devices.

Ferro-nickel has a high current-carrying capacity, on account of its low specific resistance. As it will rust, it can only be used where it is not attacked by moisture.

German Silver is an alloy of copper, nickel and zinc. The "grade" of the wire designates the percentage of nickel. The 18 per cent grade is the most common. The resistance of any particular grade depends upon the degree of annealing; hard wire is slightly higher in resistance than soft. German silver was for many years the only resistance alloy obtainable, but it is now being generally displaced by materials of the same specific resistance but of superior qualities. (See article on *Alloys*.)

Ia Ia is recommended for use in instruments and electrical devices where a low temperature coefficient is desired.

Ideal is an alloy of nickel and copper, and contains no zinc. The manufacturers state that its temperature coefficient is "nil." It may be used at an incipient red heat of 520° C. Its use in general is the same as that of Advance. When used with Comet wire it forms a very durable and desirable thermocouple. In general it is similar to Advance and Excelsior.

Karma is a nickel-chromium alloy used for heating purposes at temperatures up to 1100° C.

Kromore is a nickel-chromium alloy. It is used at temperatures up to 1150° C. in heat radiating elements. It is similar to Peerless and Rayo.

Lucero is a nickel-copper alloy and is intended for use in place of German silver. It is used in car heaters and other similar heating devices. In general it is similar to Eureka.

Manganin is a material developed by the Reichsanstalt, for use in instruments and standards. The alloy which was shown to be the best for ordinary purposes is one containing 85 per cent of copper, 12 per cent of manganese and 3 per cent of nickel. Prior to 1918 it was imported but now it is being manufactured in the United States and is known as Baker manganin. The Baker manganin contains a small amount of iron and is claimed to be an improvement over that formerly obtainable.

Manganin should be used with a double silk covering saturated with shellac and then baked in an oven at 140° C. for at least 24 hours. The baking process anneals the wire and sets the shellac so as to exclude air from the wire which would otherwise become slightly oxidized in use. Oxidation increases the temperature coefficient of resistance. When used in resistance standards and

precision apparatus the shellacked coils of wire should be sealed to exclude air from the shellac, otherwise the shellac will absorb varying amounts of moisture from the air under a variation of humidity. When the shellac absorbs moisture it expands and changes the tension of the wire and thus changes its resistance. The resistance element should be adjusted to its desired value after the annealing process, as the resistance of the wire decreases by a few tenths of one per cent upon annealing.

The change of resistance with temperature is not a straight line but a curve represented by the formula:

$$R_t = R_{20}[1 + \alpha(t - 20) - \beta(t - 20)^2],$$

in which R_{20} is the resistance at 20° C., R_t is the desired resistance at the temperature t in degrees centigrade, α is a constant given in Table I, β is a constant usually equal to 4×10^{-7}.

Monel Metal contains approximately three parts nickel to one of copper. In smelting and refining the ore from which monel metal is made, the nickel and copper are not separated, and, therefore, appear in the finished alloy in the same relative proportions. The treatment of this ore consists merely in eliminating the impurities, excepting a small percentage of reduced iron. As a result the metal is tough, strong, as non-corrosive as pure nickel, and is the same in appearance; whereas nickel, as a pure metal, is relatively expensive, owing to the difficulty of isolating it. This alloy is produced at a cost which permits favorable competition with German silver, etc. The resistance varies somewhat in different lots, and according to temper. The variation is, however, no greater than that of 18 per cent German silver.

Nichrome is practically *non-corrosive*, has an extremely high melting point (about 1550° C.) and is far superior to nickel in its ability to withstand high temperatures. It is especially recommended for use in electrically heated appliances and resistance elements generally where extreme conditions are encountered.

Nichrome II is strongly resistant to oxidation. It has been especially developed for use in carbon combustion furnaces, and other laboratory furnaces where the more extreme temperatures are to be met.

Nickel, due to its high temperature coefficient, is very efficient for use in resistance thermometers and owing to its non-corrosive qualities it may be employed for rheostats where acid fumes are to be met with.

Nickel Silver, as commonly known, contains 18 per cent of nickel. Its resistance varies somewhat in different lots and according to temper.

Peerless is an alloy of nickel and chromium used in such service as electric ovens and furnaces. It does not deteriorate when operating continuously at 1100° C. with exposure to air. It is similar to Kromore and Rayo.

Phenix is a nickel-steel material suitable for resistances at comparatively low temperatures. It is readily soldered with any standard flux. It is used extensively in arc lamps, rheostats, resistors for controllers and other similar heavy duty apparatus. It is similar to Electris and Climax.

Premier Nickel Chrome has a high resistivity and used in heating appliances at temperatures up to 1000° C. It is similar to Nichrome and Calido.

Rayo is an alloy of nickel and chromium in such proportions that it will withstand high temperatures and mechanical rubbing. It can be used continuously at temperatures up to 1100° C. without deterioration. It is used in electric ranges and ovens. It is similar to Kromore and Peerless.

Sichrome is a new alloy consisting of silicon, chromium and iron. It is

TABLE I—PROPERTIES OF RESISTANCE METALS

Material	Maximum working temperature, ° C.	Microhm cm. at 20° C., ρ	Temperature coefficient per ° C. at 20° C., α	Microvolts per ° C. against copper	Specific gravity, δ	Tensile strength, lbs. per sq. in. (annealed)	Linear expansion coefficient per ° C.
							$10^{-6}\times$
Acme	......	87.3	0.00072	......	8.15	......	......
Advance	535	48.8	0.000018	43.2	8.9	60,000	14.4
Akbar	600	83.1	0.0006	25	......	80,000	14
Calido (see p. 2048)	1000	110.0	0.00025	12	8.15	90,000	16
Climax	425	87.0	0.00098	......	8.15	75,000	17.1
Chromel "A"	1090	103.0	0.00011	......	8.23	......	......
Chromel "B"	980	89.0	0.00011	......	8.40	......	......
Chromel "C"	......	108.0	0.00018	......	8.1	......	......
Comet	600	87.0	0.0007	5	8.15	80,000	12
Electris	......	83.2	0.0011	......	8.1	......	......
Eureka	......	46.5	0.00072	......	8.8	......	......
Excelsior	......	49.2	0.0000	......	8.9	......	......
Ferro-nickel	340	28.2	0.00207	......	7.8	......	......
German silver 18% *	260	33.3	0.00031	25	8.5	......	17.3
Ia Ia (soft)	......	47.1	0.000005	......	8.92	......	......
Ia Ia (hard drawn)	......	50.2	0.000011	......	8.92	......	......
Ideal	500	49.2	0.000005	40	8.9	65,000	14
Karma	1100	103.0	0.00016	15	......	90,000	15
Kromore	1100	94.6	0.000242	......	8.9	80,000	......
Lucero	600	46.5	0.0007	30	8.9	75,000	14
Manganin	100	49.9	0.00001	1.4	8.4	......	......
Monel metal	425	42.6	0.00191	......	8.15	44,000	12.6
Nichrome	980	109.5	0.000171	......	8.9	98,000	......
Nichrome II	1040	109.5	0.000172	......	8.02	98,000	16.4
Nickel	700	10.6	0.00400	25	8.8	70,000	14.0
Nickel silver, 18%	260	33.3	0.00027	......	8.5	70,000	17.3
Nickel silver, 30%	......	48.2	0.00020	......	8.5	75,000	......
Peerless	1100	95.5	0.00018	......	8.05	......	......
Phenix	400	83.1	0.0011	4	8.10	75,000	14
Premier nickel chrome	1000	103.0	0.00036	......	8.15	......	......
Rayo	1100	95.7	0.00018	14	8.05	90,000	15
Sichrome	1100	113.0	0.000025	......	7.63	......	......
Superior	550	87.2	0.00081	......	8.04	......	......
Superior nickel chrome	1100	103.0	0.00011	......	8.2	......	......
Tarnac	......	41	0.000025	2.0	8.89	......	......
Therlo	200	46.7	0.000006	— 1.5	8.15	78,000	19.4
Yankee silver	480	33.0	0.000155	......	8.6	......	15.9
No. 193 alloy	650	87.2	0.00072	......	8.15	60,000	17.1

* 30 per cent German silver has substantially the same properties as Advance metal.

claimed to have a less tendency to corrode than the nickel chromium alloys. It is intended for use in heating elements at temperatures up to 1100° C. under rather severe conditions. Its specific gravity is lower than that of any other resistance wire.

Superior is a nickel-steel alloy recommended for use in rheostats, arc lamp resistances, etc.

Superior Nickel Chrome is an alloy containing approximately 80 per cent nickel and 20 per cent chromium. It is used in heating devices at temperatures up to 1100° C.

Tarnac is an English product which is very similar to manganin and is used for the same purposes. For precision purposes it should be treated in the same manner as manganin. The variation of its resistance with temperature is similar to that of manganin. The value of β in this case is usually 2.5×10^{-7}. It has a maximum resistance at 70° C.

Therlo is an alloy of copper, manganese and aluminum for work where low thermo-electric effect against copper is demanded. Compared with manganin, this alloy gives a higher specific resistance, does not oxidize so fast, and is more stable in its electrical and mechanical behavior. This material is especially suitable for shunts. Temperature coefficient is +0.0000031 per 1° F. It cannot be soldered but has to be brazed. This characteristic is favorable to its proper use. It should be treated in the same way as manganin to produce accurate resistance coils.

Yankee Silver. — This is a new alloy with most of the qualities of "18 per cent German Silver." It will withstand repeated heating and cooling and often gives satisfactory service where German silver fails.

No. 193 Alloy consists of nickel, iron and chromium. It serves as resistance elements for elevator controllers and rheostats at temperature up to 640° C. It is similar to Acme and Comet.

TABLE II. — RESISTANCE, WEIGHT AND CURRENT-CARRYING CAPACITY OF WIRES

$$\text{Resistance, ohms per foot at } 20^\circ \text{ C.} = \frac{K\rho}{1000},$$

where ρ is the resistivity of the metal in microhms per centimeter cube, taken from Table I, and K is given in the table below.

$$\text{Weight, pounds per foot} = \frac{H\delta}{1000},$$

where δ is the specific gravity of the metal, taken from Table I, and H is given in the table below.

$$\text{Current for given temperature rise, amperes} = \frac{10\, I_0}{\sqrt{\rho}},$$

where ρ is the resistivity of the metal in ohms per centimeter cube, taken from Table I, and I_0 is given in the table below. This formula is based upon the following multipliers to convert the resistivity at 20° C. to that at the temperatures stated:

100° C.	1.031
200° C.	1.052
500° C.	1.081
1000° C.	1.118

These are correct for Calido wire. For any other wire, the current derived from this formula must be multiplied by the factor $\sqrt{\frac{m_c}{m_x}}$, where m_c is a multiplier from the above table and m_x the corresponding multiplier for the wire in question.

A. W. G. or B. & S. Gage	Diameter, inches	Resistance factor (a) K	Weight factor (b) H	Current-carrying Cap. I_0 for $\rho = 100$ (c)			
				100° C.	200° C.	500° C.	1000° C.
6	0.162	0.229	8.94	24.7	38.5	73.3	146.0
8	0.128	0.364	5.62	17.6	27.5	52.3	105.0
10	0.102	0.579	3.54	12.7	19.7	37.7	75.4
12	0.0808	0.921	2.22	9.04	14.2	28.2	53.8
14	0.0641	1.46	1.40	6.51	10.2	19.3	38.7
16	0.0503	2.33	0.880	4.66	7.35	13.9	27.8
18	0.0403	3.70	0.553	3.36	5.25	9.97	20.0
20	0.0320	5.89	0.348	2.40	3.76	7.15	14.3
22	0.0254	9.36	0.219	1.72	2.59	5.10	10.2
24	0.0201	14.9	0.138	1.24	1.94	3.69	7.35
26	0.0159	23.7	0.0865	0.89	1.39	2.64	5.27
28	0.0126	37.6	0.0544	0.65	1.01	1.92	3.84
30	0.01003	59.9	0.0342	0.47	0.75	1.42	2.92
32	0.00795	95.2	0.0215	0.35	0.56	1.04	2.08
34	0.00631	151	0.0135	0.26	0.41	0.79	1.54
36	0.00500	241	0.00851	0.19	0.30	0.57	1.13
38	0.00396	383	0.00537	0.14	0.22	0.41	0.84
40	0.00314	608	0.00337	0.10	0.16	0.30	0.61

(*a*) The factor K is the resistance per 1000 feet for a resistivity of 1 microhm per cm.[3]
(*b*) The factor H is the weight per 1000 feet for a specific gravity of 1.
(*c*) Amperes for a rise of temperature above 20° C. equal to the temperatures stated at head of column, for a wire stretched straight and freely exposed to air.

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WIRING OF BUILDINGS FOR LIGHT AND POWER. — (*See also Distribution of Electric Energy; Distribution Lines; Transmission Lines; Wires and Cables; Wiring of Buildings for Miscellaneous Devices.*)

The following is a brief table of contents of this article:

General Requirements. — Wires and fittings designed to conduct electricity in a building should be selected as to size and insulation and installed in such a manner that: (1) the attending fire risk and the possibility of an electric shock to the inhabitants shall be a minimum; (2) the electric power efficiency of the system shall be reasonable; (3) the voltage at the receiver shall approximate the rated voltage of the receivers and shall remain sensibly constant; (4) the mechanical arrangement of the system shall be simple and convenient for inspection and use; (5) the conductors shall be mechanically protected from external injury; (6) the service shall not be interrupted under normal load; (7) the cost of the materials, labor of installation and replacement due to depreciation shall not be excessive, and (8) the entire wiring system shall conform to the rules and regulations of any authority having jurisdiction over the building in question.

SYSTEMS OF WIRING. — For direct-current and single-phase distribution the two-wire system (Fig. 1) and the three-wire system (Fig. 2) are

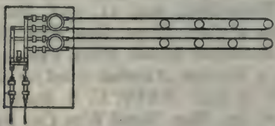

Fig. 1.

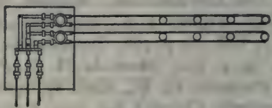

Fig. 2.

employed. For two-phase distribution either three or four wires are used. For three-phase distribution three wires are usually employed, although a fourth or neutral wire is sometimes installed. Fig. 3 shows typical methods of controlling a group of lamps from two or more switches.

Direct-current and Single-phase Distribution. — In direct-current or single-phase alternating-current circuits, the two-wire or the three-wire system is used. The two-wire system is used for the greater part of interior wiring, the three-wire system being used chiefly for feeders and mains. The three-wire system possesses the advantage over the two-wire system that for the

same power transmitted at the same efficiency to receivers of the same voltage, the three-wire system requires less weight of conductor. The neutral wire of a three-wire system is usually of the same size as either outside wire, the saving in copper over the two-wire system then being ⅝ or 62.5 per cent (*see Distribution of Electric Energy*). In some cases, buildings are wired with the three-wire system in which the neutral wire is made twice the size of either outside wire so that if necessary, the system may be operated either as a two-wire or a three-wire system; in the former case the two outside wires are connected in parallel. Power may then be supplied to the building from a local two-wire source of supply (isolated plant in the building) or from an emergency three-wire street service.

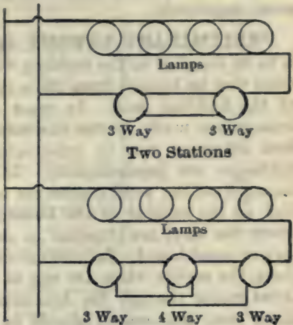

Fig. 3.

In this case wires of such size would be used as would give normal efficiency and regulation when used as a two-wire system; the loss when used as a three-wire system would then be only half as great.

The first cost of a three-wire system may not be less than that of a two-wire system because of the increased cost of the fittings, insulation and the labor of installation. The three-wire system is not as simple as the two-wire system, and is subject to more disturbances unless the load is kept balanced. Some electric power companies limit the power which they will supply to a two-wire system and in such cases in new installations buildings taking power above the two-wire limit must be wired with the three-wire system. The two-wire system is used for either lighting or power loads while the three-wire system is used for either lighting or mixed lighting and power loads, motors in the latter case being connected between the outside wires.

Two-phase and Three-phase Distribution. — For distributing two-phase alternating currents, either three or four wires are employed. A four-wire two-phase system may be treated as two separate two-wire systems, which cannot in any case be connected in parallel. A single wire 41 per cent larger than either of the wires it displaces may be substituted for any two of the wires of the four-wire two-phase system thus making a three-wire two-phase system. Either three or four wires may be used for two-phase lighting or power loads.

Three wires are usually employed on three-phase alternating-current circuits supplying power to lighting or power loads. If the neutral is accessible, a neutral wire may also be used, making a four-wire three-phase system for lighting loads, the lamps being connected between any outside wire and the neutral wire.

STANDARD VOLTAGES FOR LIGHTING AND POWER LOADS. — On all standard lighting loads the voltage across the lamps ranges from 100 to 125 volts in accordance with the rating of the lamps. In a few cases, lamps rated between 200 and 250 volts are used with the corresponding voltages. The voltage between the service wires of a constant-current series-arc or series-incandescent lamp system must not be greater than 5000 volts, the maximum voltage for inside work allowed by the National Electrical Code (see p. 2063). Constant-current systems are used chiefly in lighting large areas as in mills, factories, armories, etc. On power loads standard voltages of approximately 110, 220 or 550 volts are used depending on the magnitude of the load supplied.

STANDARD FREQUENCIES ON ALTERNATING-CURRENT CIRCUITS. — Alternating currents are usually supplied to lighting loads at a frequency of 60 cycles and to power loads at 25 cycles. In some cases, a frequency of 40 cycles is used when the lighting and power loads are of about the same magnitude.

PRELIMINARY LAYOUT OF THE WIRING SYSTEM. — If power is to be supplied to the building from some outside source, the service entrance must first be located from plans or by an inspection of the building itself. In most cases, service entrances are made in the basement, although first-floor entrances may be substituted when basement entrances are impracticable. In overhead-service connections, the service wires are usually run in conduit on the outside of the building from the point where the wires are attached to the building to the service panel-board. In underground-service connections, the service wires are run in conduit from the street mains through the basement walls. In the latter case, the conduit should be tightly closed at the outlet to prevent gases from entering the building. Fig. 4 shows two types of service connections.

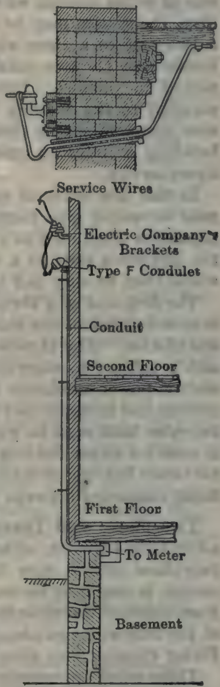

Fig. 4.

Location of Panel-Board. — The service panel-board should be placed securely in an accessible location and should contain space for the service cut-outs, service switch and meter. When power is supplied from within the building, the switchboard of the power plant must be located as a starting point in the design of the feeder system.

Location of Outlets, Switches, etc. — Outlets, control switches and cut-out cabinets should then be located throughout the building. Great care must be exercised in locating outlets and control switches in such positions as to accommodate the receiving devices planned for the building. Cut-out cabinets, which constitute the local distributing centers, should be located as near as possible to the dependent receiving devices and should at the same time be easily connected by feeders to the main switchboard. Fig. 5 shows a riser diagram and Fig. 6 a diagram of the wiring for one floor of an office building.

Number of Feeders. — Having located the outlets, control switches, distribution centers and the main switchboard, a feeder system must be planned to suit the conditions under which the system is to operate. The number of feeder sets required depends upon: (1) the power taken by the receiving devices: (2) the degree of control desired at the main switchboard; (3) the number of receiving devices which may rely upon one set of feeders; (4) the desired uniformity of voltage at the receiving devices, and (5) the character of the receiving devices. If the total current supplied to the building is roughly calculated and is found to exceed 650 amperes (the allowable carrying capacity of a 1,000,000-circular-mil rubber-insulated conductor), more than one set of feeders should be used, since it is impracticable to install feeders larger than one inch in diam-

(*Text continued on p. 2055.*)

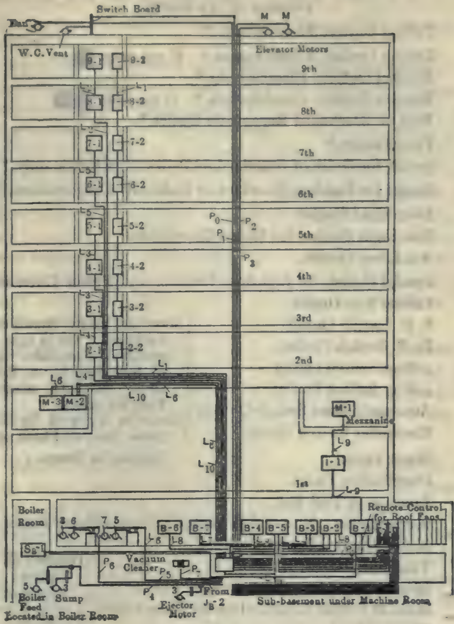
Switch Board
Fan
W.C. Vent
Elevator Motors
M M
9th
8th
7th
6th
5th
4th
3rd
2nd
1st
Mezzanine
Boiler Room
Remote Control for Roof Fans
Vacuum Cleaner
Boiler Feed
Sump
Located in Boiler Room
Ejector Motor
From
Sub-basement under Machine Room

Fig. 5.

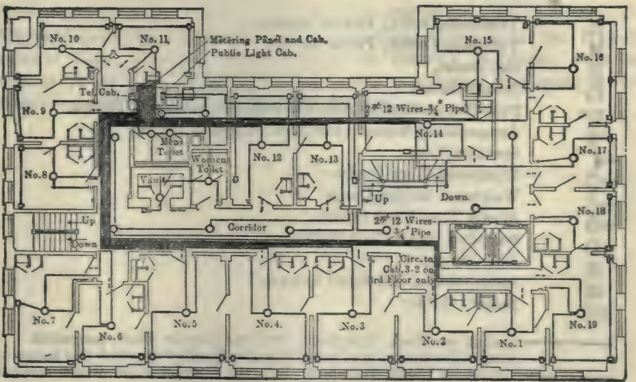
Metering Panel and Cab.
Public Light Cab.
Tel. Cab.
2 # 12 Wires-3/4" Pipe
Mens Toilet
Womens Toilet
Vault
Corridor
Up
Down
Circ. to Cab. 3-2 on 3rd Floor only
No. 1
No. 2
No. 3
No. 4
No. 5
No. 6
No. 7
No. 8
No. 9
No. 10
No. 11
No. 12
No. 13
No. 14
No. 15
No. 16
No. 17
No. 18
No. 19

Fig. 6.

STANDARD WIRING SYMBOLS

(Adopted by the National Electrical Contractors' Association and the American Institute of Architects.)

Ceiling Outlet; Electric only.*
Ceiling Outlet; Combination.† If gas only
Bracket Outlet; Electric only.*
Bracket Outlet; Combination.† If gas only
Wall or Baseboard Receptacle Outlet.*
Floor Outlet.*
Outlet for Outdoor Standard or Pedestal; Electric only.*
Outlet for Outdoor Standard or Pedestal; Combination.†
Drop Cord Outlet.
One Light Outlet, for Lamp Receptacle.
Arc Lamp Outlet.
Special Outlet, for Lighting, Heating and Power Current, as described.
Ceiling Fan Outlet.

S^1 S. P. Switch Outlet.
S^2 D. P. Switch Outlet.
S^3 3-Way Switch Outlet.
S^4 4-Way Switch Outlet.
S^D Automatic Door Switch Outlet.
S^E Electrolier Switch Outlet.

Show as many Symbols as there are Switches. Or in case of a very large group of Switches, indicate number of Switches by a Roman numeral, thus; S^1 XII; meaning 12 Single Pole Switches.
Describe Type of Switch in Specifications, that is, Flush or Surface, Push Button or Snap.

Meter Outlet.
Distribution Panel.
Junction or Pull Box.
Motor Outlet; Numeral in center indicates Horse-Power.
Motor Control Outlet.
Transformer.
Main or Feeder concealed under Floor.
Main or Feeder concealed under Floor above.
Main or Feeder run exposed.
Branch Circuit concealed under Floor.
Branch Circuit concealed under Floor above.
Branch Circuit run exposed.
Pole Line.
Riser.

Heights of Center-of-wall Outlets (unless otherwise specified):	
Living Rooms	5 ft. 6 in.
Chambers	5 ft. 0 in.
Offices	6 ft. 0 in.
Corridors	6 ft. 3 in.
Height of Switches (unless otherwise specified)	4 ft. 0 in.

Telephone Outlet; Private Service.
Telephone Outlet; Public Service.
Bell Outlet.
Buzzer Outlet.
Push Button Outlet; Numeral indicates number of Pushes.
Annunciator; Numeral indicates number of Points.
Speaking Tube.
Watchman Clock Outlet.
Watchman Station Outlet.
Master Time Clock Outlet.
Secondary Time Clock Outlet.
Door Opener.
Special Outlet; for Signal Systems.
Battery Outlet.

Circuit for Clock, Telephone, Bell, etc.§ under Floor, concealed.
Kind of Service wanted ascertained by Symbol to which line connects.
Circuit for Clock, Telephone, Bell, etc.§ under Floor above, concealed.

* Numeral indicates number of standard 16 C. P. incandescant lamps.
† Upper numeral indicates number of standard 16 C. P. incandescent lamps, lower numeral number of gas burners, e.g., $\frac{4}{2}$ indicates 4 incandescent lamps and 2 gas burners.
§ Kind of service wanted ascertained by symbol to which line connects.

eter. Unless controlled by remote control switches, separate feeders must be installed from the switchboard to receiving devices which are controlled at the main switchboard.

If a single set of feeders is used on a lighting load, the possible extinction of all the lights in a building by an open circuit in the main feeders may cause a panic; on a power load an interruption of all the machinery for any length of time may cause a considerable loss of money. To avoid such a discontinuance of the service, it is customary to connect separate feeders to sectionalized parts of the load, so that the entire load may not be interrupted at one time.

The voltage at receiving devices may be made more uniform by the use of separate feeders than by the use of one set of feeders, except when a single set of feeders may be run to a distributing panel to which branch feeders of nearly equal length are connected. When the power taken by certain receiving devices varies extensively and rapidly, the voltage at all other devices connected to the same feeder will vary accordingly, and in such cases lamps must be connected to separate feeders to avoid undesirable flickering.

Wiring Diagram. — After deciding upon the number of feeders to be used in any installation, a diagram should be made showing the location and length of all feeders and branch circuits. When the wiring system is sectionalized, each section should be treated by itself as a complete system. Figs. 5 and 6 are typical wiring diagrams for an office building.

WIRING CALCULATIONS. — The proper size of wire for any feeder is determined by three factors: (1) mechanical strength; (2) current-carrying capacity, and (3) the allowable potential drop in the feeder.

Minimum Size of Wire. — The National Electrical Code specifies No. 14 A. W. G. as the minimum allowable size for all classes of rubber-insulated wiring, except that in fixture work and for pendant cords wires as small as No. 18 A. W. G. may be used. The minimum allowable size of varnished cloth insulated wire is No. 6 A. W. G., except by special permission.

Kind of Insulation. — Rubber insulated wire protected with an outside braided covering and varnished-cloth insulated wire also with an outside braided covering may be used where not exposed to moisture.

Lead Sheaths. — Lead-covered wire should be used when installed in tile ducts for mechanical protection against abrasion and in underground runs in cellars, etc., where condensed moisture may be present.

Current-carrying Capacity. — The following table, from the National Electrical Code, gives the allowable carrying capacity of copper wires and cables of 98 per cent conductivity, and must be followed in placing interior conductors. See also *Wires and Cables, Insulated*.

CURRENT-CARRYING CAPACITY OF COPPER WIRES
National Electrical Code Standard

Size of wire, A.W.G.	Maximum allowable amperes			Size of wire, circular mils	Maximum allowable amperes *		
	Rubber insulation	Varnished cambric insulation	Other* insulations.		Rubber insulation	Varnished cambric insulation	Other* insulations.
18	3	....	5	200,000	200	240	325
16	6	....	10	250,000	250	300	350
14	15	18	20	300,000	275	330	400
12	20	25	25	400,000	325	390	500
10	25	30	30	500,000	400	480	600
8	35	40	50	600,000	450	540	680
6	50	60	70	700,000	500	600	760
5	55	65	80	800,000	550	660	840
4	70	85	90	900,000	600	720	920
3	80	95	100	1,000,000	650	780	1000
2	90	110	125	1,100,000	690	830	1080
1	100	120	150	1,200,000	730	880	1150
0	125	150	200	1,300,000	770	920	1220
00	150	180	225	1,400,000	810	970	1290
000	175	210	275	1,500,000	850	1020	1360
0000	225	270	350	1,600,000	890	1070	1430
.....	......	....	....	1,700,000	930	1120	1490
.....	......	....	....	1,800,000	970	1160	1550
.....	......	....	....	1,900,000	1010	1210	1610
.....	......	....	....	2,000,000	1050	1260	1670

* For insulated aluminum wire the maximum allowable current is 84 per cent of that given in the table for the corresponding type of insulation.

† This includes paper, slow-burning insulation, etc.

Currents Taken by Various Receiving Devices. — The current taken by each receiving device if not stated in the specifications may be determined roughly from the following tables.

CURRENT TAKEN BY INCANDESCENT LAMPS AT 114 VOLTS

Candle power	Amperes per lamp		
	Carbon	Gem	Tungsten
16	0.5	0.35	....
20	....	....	0.22
32	1.0	0.7	....
48	....	....	0.53
50	1.4	1.1	....
80	....	....	0.88
100	2.7	2.2	....

CURRENT TAKEN BY ORDINARY ARC LAMPS

½-inch carbons

Type	Multiple*		Series	
	Volts	Amperes	Volts per lamp	Amperes
Direct current	110 220	5 to 6.5 3	70	5 to 6.6
Alternating current	110 220	6 to 8.5 6	70	6.6 to 7.5

* On multiple-arc-lamp circuits, the conductors must be designed to carry 150 per cent of the normal current taken by the lamps.

CURRENT TAKEN BY DIRECT-CURRENT MOTORS*

Horse-power	Amperes at		
	110 volts	220 volts	500 volts
½	4.5	2.2	1
¾	6.8	3.4	1.5
1	9.0	4.5	2
1½	13.6	6.8	3
2	16.9	8.5	3.8
3	25.4	12.7	5.6
4	33.8	16.9	7.5
5	42.3	21.1	9.3
7½	56.5	32.2	12.4
10	75.3	37.6	16.6
15	113	56.5	24.9
20	150	75.3	33.1
25	188	94.1	41.6
30	226	113	49.7
40	301	150	66.3
50	376	188	82.8
60	452	226	99.4
70	527	263	116
80	602	301	132
90	678	339	149

* For single-phase a-c. motors divide current given by power factor of motor.

CURRENT TAKEN BY THREE-PHASE INDUCTION MOTORS

Power factor taken as 80 per cent

Horse-power	Amperes per wire at line voltages of		
	110 volts*	220 volts*	550 volts*
1	6	3	1.2
2	12	6	2.4
3	18	9	4
4	24	12	5
5	28	14	6
10	56	28	11
15	85	42	17
20	112	56	22
25	140	70	28
30	167	83	33
40	222	110	44
50	278	140	58
60	330	165	66
70	385	192	77
80	440	220	88
90	490	245	98
100	550	275	110
150	790	395	158
200	1050	525	210
250	1320	660	264
300	1580	790	316

* These are volts between wires; the corresponding volts to neutral are 63.5, 127 and 318, respectively.

WATTS TAKEN BY HOUSEHOLD APPLIANCES

Device	Watts	Device	Watts
Flatirons	350–500	Curling iron	90
Immersion heater	300–500	Vibrators	50
Water heaters	600–5000	Vacuum cleaner	150
Chafing dishes	420	Stove (4 burner and oven)	5720
Grill	660	Bake oven	5–80 kw.
Coffee percolaters	420	Dish washer	200
Toasters	500	Washing machine	600
Heating pads	60	Ironing machine	2.5–6 kw.
Cigar lighters	70	Hot plates (2)	2300
Glue pots	440–880	Hot plates (3)	3100
Luminous radiators	750–1500	Laboratory plate	860–4300
Reflection heater	600	Industrial disc heater	1000–1800
Air heater	500–1000	Soldering iron	100–500
Sewing machine	50	Office fans	30–60

Potential Drop. — In most installations the potential drop *per conductor* between service-entrance or switchboard and the farthest receiver is taken as approximately 3 per cent of the *voltage to neutral* at the service-entrance or switchboard. This is equivalent to a *total drop* (both wires) of 3 per cent of the *voltage between* wires in the case of a two-wire direct-current or single-phase system. In three-wire systems the two outside wires are to be regarded as the feeders.

In direct-current or single-phase alternating-current systems the maximum potential drop *per conductor* will then be $0.015 \times 110 = 1.65$ volts for a 110-volt system, 3.3 volts for a 220-volt system, etc. In three-phase systems the potential drop *per conductor* will be $0.03 \times \frac{110}{\sqrt{3}} = 1.91$ volts for a 110-volt system, 3.82 volts for a 220-volt system, etc. In feeders and branch circuits the drop in the feeders is usually made two-thirds and the drop in the branch circuits one-third of the total drop.

Note that in the case of a lamp load the allowable drop of 3 per cent is based on the current corresponding to the *total connected load*, i.e., this is the maximum drop when *all* lamps are burning. In ordinary buildings the actual maximum load is seldom more than one-third the connected load; consequently when the wiring is designed on this basis the voltage at the lamps will seldom be more than 1 per cent lower than the voltage at the service connection.

Calculation of Size of Wire. — Let

I = current per conductor in amperes,
v = allowable potential drop *per conductor* in volts,
l = length of the conductor in feet,*

then the required cross-section of a copper wire in circular mils is

$$A = \frac{KlI}{v}, \tag{1}$$

where K is a factor depending upon the specific resistance of the wire, the size and spacing of the wires, the frequency and the power factor of the receiver. The factor K for alternating-current circuits is therefore not a constant but for preliminary calculations its value for the sizes of wires and spacings (1 to 6 inches) ordinarily used for interior wiring is approximately as given in the following table. The alternating-current values apply to single-phase, 2-phase 4-wire

System	Power factor	Values of K for copper
Direct current............	....	11*
Alternating current........	....	....
Lighting load only.......	1.00	11
Lighting and power loads	0.95	12
Lighting and power loads.	0.90	13.5
Power loads............	0.85	15
Power loads............	0.80	17

* This value is practically exact, the others are approximate only.

and 3-phase 3-wire systems at any frequency from 25 to 60 cycles per second. For a direct-current or single-phase 3-wire system proceed as for a 2-wire system, neglecting the presence of the neutral wire; the neutral wire should then be made the same size as each outside wire as thus calculated.

* Note that l is the length of *each conductor;* the total length of wire for a two-wire line is $2\,l$, for a three-wire line $3\,l$, etc.

Commercial sizes of wire differ successively by about 25 per cent in the larger sizes and 60 per cent in the smaller sizes; the odd-numbered sizes smaller than No. 1 are not generally manufactured, although No. 3, 5 and 9 are sometimes made. From the table below * select the size of wire corresponding to the calculated area. Unless the calculated area in circular mils agrees very closely with the area of one of the conductors given in the table, the next larger size of wire should be selected.

600-VOLT RUBBER-INSULATED COPPER WIRE

(Single Braid, N.E.C. Standard; Conductivity of Copper, 98 per cent)

Size, A.W.G.	Cross-section of copper, circular mils	Over-all diameter, inches	Ohms per 1000 feet at 77° F.	Pounds per 1000 feet of insulated wire	Reactance per 1000 feet of each wire: Wires in contact †		6 inches between wires ‡	
					25 cycles	60 cycles	25 cycles	60 cycles
		stranded	stranded	stranded				
	1,000,000	1.50	0.0110	3556	0.0130	0.0313	0.0283	0.0679
	900,000	1.44	0.0122	3223	0.0132	0.0317	0.0288	0.0692
	800,000	1.37	0.0138	2888	0.0134	0.0320	0.0292	0.0702
	700,000	1.31	0.0157	2554	0.0136	0.0328	0.0298	0.0717
	600,000	1.24	0.0184	2215	0.0138	0.0332	0.0306	0.0732
	500,000	1.09	0.0220	1805	0.0135	0.0324	0.0312	0.0749
	400,000	1.00	0.0275	1477	0.0141	0.0340	0.0322	0.0773
	300,000	0.90	0.0367	1133	0.0144	0.0347	0.0333	0.0800
	250,000	0.84	0.0441	962	0.0148	0.0354	0.0341	0.0820
0000	212,000	0.77	0.0520	807	0.0144	0.0347	0.0348	0.0835
000	168,000	0.71	0.0656	656	0.0149	0.0358	0.0358	0.0860
00	133,000	0.66	0.0827	535	0.0151	0.0362	0.0368	0.0883
0	106,000	0.61	0.104	438	0.0159	0.0381	0.0378	0.0908
1	83,700	0.57	0.133	350	0.0165	0.0396	0.0390	0.0936
		solid	solid	solid				
1	83,700	0.53	0.126	340	0.0159	0.0381	0.0388	0.0933
2	66,400	0.47	0.159	268	0.0160	0.0385	0.0398	0.0958
3	52,600	0.44	0.201	221	0.0166	0.0400	0.0410	0.0986
4	41,700	0.41	0.253	184	0.0168	0.0403	0.0421	0.1011
5	33,100	0.39	0.320	153	0.0176	0.0422	0.0432	0.1037
6	26,300	0.37	0.403	129	0.0179	0.0430	0.0442	0.1061
8	16,500	0.27	0.641	76	0.0181	0.0434	0.0464	0.1112
10	10,400	0.23	1.02	54	0.0193	0.0464	0.0495	0.1165
12	6,530	0.21	1.62	40	0.0210	0.0505	0.0507	0.1218
14	4,110	0.20	2.58	30	0.0221	0.0532	0.0529	0.1270
16	2,580	0.16	4.10	20	0.0231	0.0554	0.0550	0.1320
18	1,620	0.14	6.51	13	0.0245	0.0588	0.0572	0.1372

* More extended wire tables will be found in the article on *Wires* and *Cables*.

† Two insulated wires side by side, the insulation of the two wires in contact.

‡ Measured from insulation to insulation, equals distance between centers minus twice the thickness of insulation on either wire.

Check on Calculation of Size of Wire. — Calculation of Actual Potential Drop. — (*See also Alternating Currents.*) From the table, corresponding to the size of wire selected, take

r = the resistance per 1000 feet of conductor,
x = the reactance per 1000 feet of conductor,

and put

l = length of each conductor in feet,
I = amperes per conductor,
V = volts to neutral at receiver,*
$\cos\phi$ = power factor of receiver,

$$R = \frac{rl}{1000} = \text{total resistance per conductor,}$$

$$X = \frac{xl}{1000} = \text{total reactance per conductor.}$$

Then the actual volts drop *per conductor* is

$$v = \sqrt{(V\cos\phi + RI)^2 + (V\sin\phi + XI)^2} - V. \tag{2}$$

To a very close approximation in all ordinary cases this is equal to

$$v = RI\cos\phi\left(1 + \frac{X}{R}\tan\phi\right). \tag{3}$$

If the drop as calculated does not check within a reasonable value with the drop assumed in calculating the size of the wire, select another size of wire and recalculate, etc. This refinement will be found necessary, only in the case of low power factors and when the size of wire as calculated is either exceptionally large or exceptionally small, in which case the values given for the factor K in the formula for A, equation (1), may be in error by a large amount (in limiting cases 50 per cent or more).

In addition to making sure that the drop will be within a reasonable amount, one should also note whether the conductor selected is large enough mechanically and has the proper current-carrying capacity (*see above*), remembering also that the National Electrical Code requires that conductors through which power is supplied to a motor shall have a current-carrying capacity equal to 125 per cent of the full-load current taken by the motor.

Example of Calculation for a D-C. or Single-phase System. — Incandescent lamps taking 0.5 ampere each are supplied with power from a 115-volt 60-cycle service through a set of feeders 200 feet in length which terminate at a cut-out cabinet as shown in Fig. 7. The lamps are grouped in sets of 10 lamps each (only one set is shown) and each set is connected to the cut-out cabinet by branch circuit conductors averaging 25 feet in length. Assuming a total potential drop of 3 volts, the drop per conductor will be 1.5 volts. The drop per conductor in the branch circuit should then be one-third of 1.5 or 0.5 volt and the drop per conductor in the feeder circuit should be two-thirds of 1.5 or

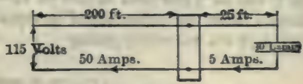

Fig. 7.

* For d-c. or a-c. single-phase or a-c. 2-phase, 2-wire systems $V = \frac{1}{2} \times$ (volts between wires); for a 3-phase system $V = \frac{1}{\sqrt{3}} \times$ (volts between wires); for a 2-phase 3-wire system $V = \frac{1}{2} \times$ (volts between either outside wire and middle wire). In calculating the normal drop in a single-phase 3-wire system pay no attention to the middle wire, i.e. assume a balanced load.

1 volt. The respective calculations of the areas of the conductors in the feeder and branch circuits may then be carried on independently. In the branch circuit, referring to equation (1), $I = 10 \times 0.5 = 5$, $l = 25$ and $v = 0.5$, whence for copper wire

$$A = \frac{11 \times 5 \times 25}{0.5} = 2750 \text{ circular mils.}$$

From the wire table it is found that the next larger commercial size is No. 14 A. W. G., which has a resistance of 2.58 ohms per 1000 feet, and a reactance (assuming a 2.5-inch spacing) of 0.106 ohms per 1000 feet.

Referring to equation (3), $V = 57.5$, $\cos\phi = 1$, $\tan\phi = 0$, $R = 2.58 \times 25/1000 = 0.0645$, $X = 0.106 \times 25/1000 = 0.00265$, whence

$$v = 0.0645 \times 5 = 0.323.$$

Equation (2) gives the same value. This voltage is 35.4 per cent less than the assumed voltage of 0.5 volt, but since No. 13 wire is not a commercial size a closer realization of the assumed voltage is impracticable.

In the same manner, the required area of the conductors used in the feeder circuit is determined by substituting the following values in equation (1): $I = 100 \times 0.5 = 50$ amperes, $l = 200$ feet, and $v = 1$ volt. Hence

$$A = \frac{11 \times 50 \times 200}{1} = 110{,}000 \text{ circular mils.}$$

From the wire table it is found that a No. 00 A. W. G. wire must be used.

The above calculations although made for a lighting load apply equally well to a motor load, except that in the final selection a wire must be chosen which will carry 125 per cent of the full-load current of the motor.

Example of Calculation for a Three-phase System. — Power is to be supplied by three feeders each 400 feet in length to a three-phase, 60-cycle, alternating-current motor (power factor 80 per cent), the voltage between any two line wires at the service entrance being 550 volts. At full load the current taken by the motor is 50 amperes. The voltage to neutral at the service entrance is $\frac{550}{\sqrt{3}}$ or 318 volts. The wires are to be spaced 6 inches apart.

Allowing a drop in each conductor of 3 per cent of the voltage to neutral, the drop will be 3 per cent of 318 or 10 volts approximately. The following values should then be substituted in equation (1): $K = 17$, $I = 50$ amperes, $l = 400$ feet and $v = 10$ volts, giving

$$A = \frac{17 \times 50 \times 400}{10} = 34{,}000 \text{ circular mils.}$$

From the wire table it is found that a No. 5 A. W. G. wire might be used considering the potential drop alone but since the carrying capacity of the conductor must be 1.25×50 or 62.5 amperes, it would be necessary to use a No. 4 wire.

Adopting a No. 4 wire and referring to equation (3), $V = 318$, $\cos\phi = 0.8$, $\tan\phi = 0.75$, $R = 0.253 \times 400/1000 = 0.101$, $X = 0.101 \times 400/1000 = 0.040$, whence

$$v = 0.101 \times 50 \times 0.8 \left(1 + \frac{0.04 \times 0.75}{0.101}\right) = 5.3 \text{ volts,}$$

which is a little more than half the allowable drop of 10 volts. Equation (2) also gives 5.3 volts.

AUTHORITIES GOVERNING THE INSTALLATION OF WIRING IN BUILDINGS. — As noted at the beginning of this article all interior wiring must be installed in accordance with the regulations of the authorities having jurisdiction over the building in question. In general, these authorities are: (1) the fire underwriters; (2) the municipal authorities; and (3) the power company supplying the current.

National Electrical Code. — This code is a set of instructions published in even years by the National Board of Fire Underwriters. A List of Electrical Fittings approved by the Underwriters' Laboratories, Inc. is published by the National Board in April and October of each year. In order that fire insurance on any building wired for electric light or power may be obtained from an insurance company, the wiring must be installed in accordance with the rules of the National Electrical Code or such modifications of it as are required by law in certain municipalities (*see Municipal Regulation, below*). Permission must also be obtained to use any fitting not included in the list of approved fittings.

Both the Code and the List of Approved Fittings may be obtained gratis by applying to the National Board of Fire Underwriters, 135 William St., New York City. In view of this fact, and the frequent revisions of the rules and list it is not deemed advisable to give them in detail here. The general requirements, however, are covered in the section below on *Methods of Installing Wiring*.

Municipal Inspection. — Many state legislatures have passed laws regulating the installation of electric wiring and empowering the appointment of municipal inspectors of wiring. In some states fines are imposed by legislative enactment upon those who violate the inspector's rules. Although free to decide upon the proper installation of electric wires, municipal inspectors have generally adopted the National Electrical Code as a standard with modifications suited to their desires. A copy of these modified rules can usually be obtained gratis from the municipal authorities.

Regulations of the Light and Power Companies. — The National Electrical Code is usually adopted by electric power companies with the addition of specific requirements pertaining to service connections, placing of meters, type of system, etc.

METHODS OF INSTALLING WIRING. — The National Electrical Code approves the use of the following methods of wiring: open work, moulding (wooden moulding and metallic raceways), concealed knob and tube work, rigid conduit, flexible conduit, armored cable and flexible tubing (in short runs). These methods are described in detail below, but first are given the more important general requirements regarding joints, protection, cut-outs, switches, etc., which apply to all classes of wiring for voltages not exceeding 600.

General Requirements. — All joints must be mechanically and electrically secure without solder and unless made with some form of approved splicing device must be soldered and covered with an insulation equal to that on the conductor. Wires must be separated from contact with walls, floors, timbers or partitions through which they may pass by non-combustible, non-absorptive insulating tubes, such as glass or porcelain, except at outlets where flexible tubing is required. Unless inclosed in conduit or moulding, wires must be separated from any conducting material or from any other electric wire not more than two inches away by some continuous and firmly fixed non-conductor creating a permanent separation. In damp places wires must be separated from pipes by an air space and should be run over rather than under pipes upon which

moisture may gather. Wires must be protected from mechanical injury by running-boards or guard-strips on low ceilings and by wooden boxing or metal conduit on side walls. Protection on side walls must extend not less than five feet from the floor. No method other than open wiring may be used when the difference of potential between any two wires exceeds 550 volts.

Automatic Cut-outs (fuses or circuit breakers) must be placed in all service wires (i.e., the leading-in wires from the street circuit), and at every point where a change is made in the size of wire unless the cut-out in the larger wire will protect the smaller wire. The cut-out in the neutral of a three-wire system may be omitted if the neutral wire is grounded and is of the same size as the outside wires. No set of incandescent lamps requiring more than 660 watts, whether grouped on one fixture or on several fixtures must be dependent upon one cut-out. Cut-outs must in general be inclosed in an approved cabinet although circuit breakers and inclosed fuses may be placed in plain sight in dry places where there is no danger of igniting any combustible material.

Heating devices, which singly or in groups require more than 660 watts, must be protected by a cut-out and the supply conductors should preferably be installed in rigid conduit. Such conductors should be insulated with rubber.

Switches which disconnect *all* wires (i.e., switches having 2, 3 or more poles) must be placed in the service wires as near as possible to the point where they enter the building and in all circuits supplying current to motors, heating devices or lamps; except that in the case of motors of one-fourth horse-power or less in circuits where the voltage does not exceed 300 volts, and in circuits supplying not more than 660 watts of power to heating devices or lamps, single-pole switches may be used in two-wire systems or branches. Switches must always be placed in dry, accessible places and must be grouped as far as possible. When used in rooms where combustible flyings are liable to exist, switches must be placed in dust-tight cabinets and when flush switches are used, they must be inclosed in an approved steel box.

Open Wiring. — When the appearance of the wiring is not important, exposed surface wiring supported on cleats or insulators furnishes one of the safest and best methods of wiring. The wires must be insulated in dry places with rubber, varnished cloth, slow-burning weatherproof or slow-burning insulation, in damp places with rubber insulation, and in locations where the wires are exposed to corrosive vapors with weather-proof, varnished cloth or rubber insulation.

Voltage	Inches from surface to insulation on wire		Inches between wires* from insulation to insulation
	Dry	Damp	
0–300	0.5	1	2.5
301-600	1	1	4

* In the case of 3-wire d-c. or single-phase systems the neutral may be placed between the two outers, these latter being 2.5 inches apart.

Spacing of Wires and Supports. — Wires must be separated from each other and spaced from the surface wired over as given in the accompanying table.

Wires must be supported under ordinary conditions at least every 4.5 feet, except that in mill construction, wires of not less than No. 8 A. W. G. gage, if separated about six inches, may be run from timber to timber and be supported at each timber only. Open wiring cannot be used in elevator shafts.

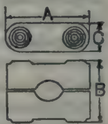

Fig. 8

Cleats. — Cleats are made in a variety of forms. The following are the dimensions of cleats of the form shown in Fig. 8, all dimensions being in inches.

DIMENSIONS AND COSTS OF CLEATS

Size of wire, circular mils or A. W. G.	1-wire cleats			
	Dimensions, inches			Size of screw, inches
	A	B	C	
14–12	2	1¼	¾	3×8
10–8	2	1⅝	¾	3×8
6– 3	2¼	1¾	1	3×10
2	2½	1¾	1⅛	3½×12
1– 0	2½	2	1⅛	3½×12
00	2¾	2⅛	1¼	3½×12
000	3	2¼	1¼	4×14
0000	3	3⅜	1¼	4×14
225,000- 500,000	3¼	2¾	1½	4½×16
600,000–800,000	4	3¼	2	5×18 or ⅜×5 lag
900,000–1,000,000	4¾	3⅝	2	
2- and 3-wire cleats				
14-12	3⅜	1⅜	⅝	3×8
10- 6	3⅜	1⅜	⅝	3×8

Wooden Moulding. — When exposed wiring is not desired, and when it is impracticable in old buildings or too expensive in new buildings to install concealed wiring, wooden moulding may be used, provided the difference of potential between any two wires in the same moulding does not exceed 300 volts. Its use is forbidden in elevator shafts and in concealed or damp places. Wooden moulding is manufactured in the following sizes. See table on next page.

Moulding may be made inconspicuous by matching the wood of the moulding to the finish of the room and by using a capping, which will conceal the purpose of the moulding. In this manner, moulding may be made to simulate picture moulding, and on ceilings a panel effect can be obtained by the use of dead moulding. All wires used in wooden moulding must have a rubber insulation and must not be jointed or tapped in the moulding. Branch taps may be made with tap fittings designed for the purpose. Many other moulding fittings are manufactured, which, if used, reduce the labor of installation and improve the general appearance of the work. Wooden moulding is fastened to walls by means of thin screws or toggle-bolts and the capping is nailed on with brads. "Kicking boxes" are usually placed around the end of moulding on floors to protect the wires and porcelain tubes from possible injury.

Metallic Moulding. — Metallic moulding may also be used to conceal and protect wires when the difference of potential between any two wires in the moulding is not more than 300 volts and the power transmitted through the wires contained in the moulding does not exceed 1320 watts. Metallic moulding is manufactured in one width only, namely, 1 inch wide by ⅜ to ½ inch deep, the depth depending upon the type of moulding. The two types of metal

STANDARD WOODEN MOULDING

No. of wires	Size of groove, inches	Dimensions, inches		Size of wire	
				A. W. G. or circular mils	
		Width over all	Depth, cap to base	Solid, 1 braid	Stranded, 1 braid
2	1/4	1 1/2	13/16	14–12	
2	5/16	1 11/16	15/16	10–8	8
2	7/16	2	1	6–4	6–5
2	9/16	2 5/16	1 1/8	3–2	4–2
2	3/4	3	1 3/8		1–000
2	7/8	3 7/8	1 3/4		0000–250,000
2	1	4 3/4	2 1/8		300,000–400,000
3	1/4	2 1/8	13/16	14–12	
3	5/16	2 7/16	15/16	10–8	8
3	7/16	2 13/16	1 1/8	6–4	6–5
3	9/16	3 1/2	1 1/4	3–2	4–2
3	3/4	4 3/16	1 7/16		1–000
3	7/8	5 1/8	1 3/4		0000–250,000
3	1	6 11/16	2 3/16		300,000–400,000

moulding in common use differ principally in the method of attaching the capping. In the "Lutz" metal moulding the capping consists of a flexible metal strip, which is slid into the two grooves that form the upper edge of the moulding. In the "National" metal moulding the capping is snapped over the base. "Lutz" moulding is made in 10-foot lengths while the National moulding is made in 8-foot 4-inch lengths.

Metallic moulding cannot be used in concealed or damp places except that, in passing through a floor, such moulding may be used if carried through an iron pipe extending from the ceiling below up through the floor to a point 5 feet above the floor or to a point at least 3 inches above the floor where appearance is an essential feature. Such moulding may be carried through walls and partitions provided it is in one continuous length and that the location is dry. All parts of a metallic moulding system including outlet boxes, junction boxes and cabinets must be electrically connected and grounded. Fittings employed with metallic moulding must be so constructed as to protect the insulation of the wires from abrasion. Wires placed in metallic moulding must be insulated and installed as in wooden moulding with the exception that in alternating-current systems two or more wires of the same circuit must be installed in the same moulding. It is suggested that this also be done for direct-current systems when there is a possibility that an alternating-current system may be used at some future time.

Concealed Knob and Tube Work. — This method of wiring is forbidden by the inspectors in many large cities, but when approved, it is employed in buildings of frame construction when it is desired to install the wiring at a

minimum first cost. The wires when running parallel to beams or studding are supported on knobs and when running through beams, studding or floors are insulated by porcelain tubes. When passing through floors at the bottom of plastered partitions or through braces, the porcelain tubes insulating the wires must project at least four inches above the floor or brace. Split knobs must be used for the support of conductors smaller than No. 8 A. W G. except at the end of runs where a solid knob or strain insulator must be used. For conductors larger than No. 8 A. W. G., solid knobs may be used. Wires must have a rubber insulation and tie wires when used must have an insulation equal to that of the conductors they confine.

Distance between Wires. — Wires must be installed in such a manner that the distance between *any* two wires is at least 5 inches and each wire must be separated from the surface wired over by a distance of at least 1 inch. If, in any place, the 5-inch separation cannot be maintained, each wire must be separately incased in a continuous length of approved flexible tubing. At outlets, wires must be protected by flexible tubing extending in continuous lengths from the last porcelain support to at least 1 inch beyond the outlet.

Distance between Supports. — Wires must be supported under ordinary conditions every 4.5 feet and if the wires are liable to be disturbed the distance between the supports must be shortened.

Knobs and Tubes. — Two types of knobs are used, namely, the solid knob and split knob. The following table gives the dimensions of one type of split knobs and 4-inch tubes:

Size of wire, A. W. G.	Split knobs *		Tubes	
	Height, inches	Diameter, inches	Inside diameter, inches	Outside diameter, inches
14–5	1⅞	1⅛–1½	5/16	9/16
4–00	2	2	⅜	11/16
			⅝	15/16
			¾	1 3/16

* Holding single wire 1 inch away from wall.

Rigid Conduit. — Although the most expensive method, rigid conduit is considered to be the best method of wiring and its use is required by inspectors in certain districts of many large cities. Two kinds of conduit are manufactured: lined conduit, having a lining of insulating material, and unlined conduit, having an inner coating of insulating enamel. Unlined conduit is used more often than lined conduit because unlined conduit is cheaper to buy and to install and because wires are drawn in unlined conduit more easily than in lined conduit. In lined conduit corrosive action on the conductors due to a possible leak in the conduit is prevented by the insulating lining. The conductors used in lined conduit cost slightly less than the conductors used in unlined conduit, because the conductors used in unlined conduit must have an additional layer of braid to allow for abrasion in drawing in the wires. Rigid conduit is manufactured in the following sizes:

STANDARD RIGID CONDUIT

Made in 10-foot lengths

Standard size of pipe, inches	Internal diameter, inches	External diameter, inches	Nominal weight per 100 feet, lb.	No. of threads per in. of screw	Maximum size of double-braided wire used in a single conduit A. W. G. or cir. mils		
					1 wire	2 wires	3 wires
½	0.62	0.84	85	14	8	14	....
¾	0.82	1.05	112	14	2	10	12
1	1.04	1.31	167	11½	00	6	8
1¼	1.38	1.66	224	11½	200,000	3	5
1½	1.61	1.90	268	11½	400,000	1	3
2	2.06	2.37	361	11½	800,000	200,000	00
2½	2.46	2.87	574	8	1,300,000	350,000	250,000
3	3.06	3.50	754	8	2,000,000	500,000	400,000
3½	3.54	4.00	900	8	....	800,000	500,000
4	4.02	4.50	1066	8	....	1,200,000	850,000

Bushings, Bends, etc., in Rigid Conduit. — All parts of the conduit system including outlet or junction boxes must be mechanically secured in place and must be electrically connected and grounded. At outlets or junction boxes rigid conduits must be provided with bushings or nipples to protect the wires from abrasion. The radius of curvature of the inner edge of an elbow or bend must not be less than 3.5 inches and there must not be more than the equivalent of 4 quarter bends from outlet to outlet, the bends at the outlet not being counted. Bends should be used whenever possible in place of elbows as the wires will pass more easily around a bend than an elbow. The various bending tools that are on the market consist of some form of lever, the end of which may be slipped over the conduit while it is in position. When the number of bends between outlets becomes excessive or when a large number of conduits placed side by side must be deflected, pull boxes are installed so that the direction of the conduit may be changed and the wires may be drawn in more easily. In fireproof buildings conduit is frequently installed just after the steel work has been erected and is then covered up by concrete or tiling.

Insulation of Wires Used in Rigid Conduits. — Wires used in rigid conduit must have an approved rubber insulation and must not be spliced or tapped within the conduit. Varnished cloth insulation may be used in permanently dry locations. In alternating-current systems the two or more wires of a circuit must be drawn in the same conduit. It is suggested that this be done for direct-current systems also when there is a possibility that an alternating-current system may be used at some future time. The same conduit must not contain more than four two-wire or three three-wire circuits of the same system and circuits of different systems must be run in separate conduits.

Wires should not be drawn in until all mechanical work on the building has been completed. For short runs wires may be pushed in at one opening until they come out at the other opening, but on long runs or where there are many bends, a spring-steel "fish-wire" or "snake" is first pushed through the conduit and a piece of sash cord attached to the "fish-wire" is pulled through. The

conductor is then attached to the sash cord and pulled through. When the wires cannot be pulled through a conduit easily, powdered soapstone is blown into the conduit to reduce the friction of the conductors.

Supports for Wires. — In vertical runs conductors must be supported in the conduit in accordance with the following table:

No. 14 A. W. G. to 0,	every	100	feet
No. 00 A. W. G. to 0000,	"	80	"
0000 A. W. G. to 350,000 cir. mils,	"	60	"
350,000 cir. mils to 500,000 cir. mils,	"	50	"
500,000 cir. mils to 750,000 cir. mils,	"	40	"
750,000 cir. mils,	"	35	"

Conductors may be supported by a turn of 90 degrees in the conduit system or by junction boxes designed for the purpose.

In addition to the above, varnished cambric insulated conductors must be supported at the upper ends of vertical runs by securely attaching the copper, from which the insulation has been removed, to a strain insulator.

See pages 2072 and 2073 for tables, covering number of wires in a conduit.

Flexible Conduit. — In distinction to flexible tubing, a non-metallic tubing, flexible conduit is a *metallic* tubing built up of spiral, convex and concave steel strips which interlock in such a manner as to form a fairly smooth surface, externally and internally. "Flexible conduit," which is metallic, is usually distinguished from "flexible tubing" (*see below*), which is non-metallic tubing. Flexible conduit possesses the advantage over rigid conduit in that it can be installed continuously and quickly from outlet to outlet and if necessary may be fished between partitions or floors. Its chief disadvantages are that it is neither nail-proof nor moisture-proof. Flexible conduit is manufactured in the following sizes:

STANDARD FLEXIBLE CONDUIT

Size, inches	Approx. outside diameter, inches	Weight per 100 feet, pounds	Approx. feet in coil	Maximum size of double-braided wire used in single conduit A. W. G. or cir. mils		
				1 wire	2 wires	3 wires
5/16	0.485	20	250	14	....	....
3/8	0.61	34	250	12	....	....
1/2	0.92	68	100	8	12	....
3/4	1.18	95	50	2	10	12
1	1.49	144	50	00	6	8
1 1/4	1.75	182	50	200,000	3	5
1 1/2	2.06	217	25–50	400,000	1	3
2	2.56	265	25–50	800,000	200,000	00

All requirements as to installation and wires used in rigid conduit apply as well to flexible conduit.

Armored Cable. — In place of flexible conduit in which wires must be drawn in after the conduit is installed, armored cable consisting of rubber-insulated conductors protected by interlocking spiral steel strips may be used. When installed in damp places, leaded armored cable, which has a lead covering between the insulation and the steel armor, must be used. Armored cable is employed extensively in wiring old buildings where it would be impossible to install rigid conduit without cutting up the walls and floors. Armored cable for interior wiring is manufactured in the following sizes:

STANDARD ARMORED CONDUCTORS

Size of each conductor A. W. G.	Number of bunched conductors		Approximate outside diameter, inches	Weight per 100 feet, pounds	Approximate feet in coil
14	1		0.37	20	250
12	1		0.40	21.5	250
10	1		0.435	23	250
8	1	Steel armored	0.500	28	250
6	1	Steel armored	0.640	54	250
10*	1	Steel armored	0.435	23	250
8*	1	Steel armored	0.500	28	250
6*	1	Steel armored	0.640	54	250
4*	1	Steel armored	0.690	59	200
2*	1	Steel armored	0.770	71	200
1*	1	Steel armored	0.860	98	100
10*	1	Lead covered and steel armored	0.555	48	250
8*	1	Lead covered and steel armored	0.620	54	200
6*	1	Lead covered and steel armored	0.760	81	200
4*	1	Lead covered and steel armored	0.820	90	150
2*	1	Lead covered and steel armored	0.900	120	150
1*	1	Lead covered and steel armored	0.985	165	100

* Stranded.

All requirements as to mechanical and electrical connections to outlet boxes, grounding, splicing and tapping of wires, assembling of conductors in alternating-current systems, etc., applying to rigid conduit also pertain to armored cable. Bends in armored cable may be of smaller radius than in rigid conduit, however, the minimum allowable radius of a bend being 1.5 inches for armored cable.

Flexible Tubing. — In addition to its use in all places where it is necessary to add to the insulation of the conductors, if the difference of potential between the wires is not over 300 volts, continuous flexible tubing may be used to protect individual wires when fished for short distances in dry places. Although not allowed by some inspectors, flexible tubing is extensively used in this manner in conjunction with moulding or knob and tube work, especially in old buildings where it is necessary to fish the wires to avoid

STANDARD ARMORED CONDUCTORS — Continued

Size of each conductor, A. W. G.	Number of bunched conductors		Approximate outside diameter, inches	Weight per 100 feet, pounds	Approximate feet in coil
14	2	Steel armored	0.630	44	150–250
12	2		0.640	45	150–250
10	2		0.675	52	150–250
8*	2		0.818	81	100–150
6*	2		1.066	114	100
14	2	Lead covered and steel armored	0.636	75	100–200
12	2		0.688	78	100–200
10	2		0.861	108	100–150
8*	2		0.900	125	100–150
14	3	Steel armored	0.648	53	150–250
12	3		0.675	55	150–250
10	3		0.755	66	100–200
8*	3		0.900	99	100–150
14	3	Lead covered and steel armored	0.694	79	100–150
12	3		0.766	88	100–150
10	3		0.901	117	100–150

* Stranded.

cutting into walls, floors, or ceiling Flexible tubing is manufactured in the following sizes:

STANDARD FLEXIBLE TUBING

Inside diameter, inches	Feet in coil	Weight per 1000 feet, pounds	Largest size of wire accommodated, A. W. G.
¼	250	75	14
⅜	250	110	12
½	200	125	8
⅝	200	155	4
¾	150	200	2
1	100	275	00
1¼	100	360	200,000
1½	100	400	400,000
1¾	100	440	600,000
2	Odd lengths	600	800,000
2¼	Odd lengths	700	1,100,000

SIZE OF CONDUITS FOR WIRES AND CABLES

Electrical Trade Sizes, Inches

I. All Conductors of Same Size

Size of conductor, A. W. G. or circular mils	Number of conductors in a conduit			
	1	2	3	4
14	½	½	½	¾
12	½	¾	¾	¾
10	½	¾	¾	1
8	½	1	1	1
6	½	1	1¼	1¼
4	¾	1¼	1¼	1½
3	¾	1¼	1¼	1½
2	¾	1¼	1½	1½
1	¾	1½	1½	2
0	1	1½	2	2
00	1	2	2	2½
000	1	2	2	2½
0000	1¼	2	2½	2½
200,000	1¼	2	2½	2½
250,000	1¼	2½	2½	3
300,000	1¼	2½	2½	3
400,000	1¼	3	3	3½
500,000	1½	3	3	3½
600,000	1½	3	3½	....
700,000	2	3½	3½	....
800,000	2	3½	4	....
900,000	2	3½	4	....
1,000,000	2	4	4	....
1,250,000	2½	4½	4½	....
1,500,000	2½	4½	5	....
1,750,000	3	5	5	....
2,000,000	3	5	6	....

EXCEPTION. — For sizes not greater than No. 10 A. W. G. one more conductor than permitted by the above table may be installed, provided the conduit is not longer than 30 feet and has not more than the equivalent of two right-angle bends from outlet to outlet, the bends at the outlets not being counted.

II. Three Conductors

Two numbers and	One number	Size conduit inches
14	10	¾
12	8	¾
10	6	1
8	4	1
6	2	1¼
5	1	1¼
4	0	1½
3	00	1½
2	000	1½
1	0000	2
0	250,000 Cir. mils	2
00	350,000	2½
000	400,000	2½
0000	550,000	3
250,000 Cir. mils	600,000	3
300,000	800,000	3
400,000	1,000,000	3½
500,000	1,250,000	4
600,000	1,500,000	4
700,000	1,750,000	4½
800,000	2,000,000	4½

General Rule for any Number of Wires

The size of conduit for a given combination of wires may also be estimated as follows: Find the total *circular inches* * of area of insulated wire to be put in the conduit and divide by the following factor:

1 conductor	0.56
2 conductors	0.32
3 conductors	0.42
4 conductors	0.40
Over 4 conductors	0.37

The result will be the circular inches inside area of conduit required. The inside diameter of conduit wili be the square root of this figure.

For example, what size conduit is required for six No. 8 and four No. 2 wires? The overall diameter of a No. 8 is 0.27 in. and of a No. 2 is 0.47 in. Their areas in circular inches are $0.27 \times 0.27 = 0.073$ and $0.47 \times 0.47 = 0.22$, respectively. Six times 0.073 is 0.436 and four times 0.22 is 0.88. The total is $0.436 + 0.880 = 1.316$ circular inches, which divided by 0.37 gives 3.56 circular inches. The square root of this is 1.89 which is the diameter of pipe required The next standard size is 2 inches.

* The area in circular inches of any circle is equal to the square of its diameter in inches.

COST OF WIRING. — The total cost of wiring a building is made up of the three elements: materials, labor and engineering. The labor and engineering charge will depend upon local conditions, and these costs may be estimated accurately only by reference to similar costs in completed work of the same general character. In regard to labor, the local conditions must include the factor of wages paid in the vicinity and the amount of work completed in a given time by the class of labor available. In determining the engineering charge the contractor must estimate the amount for each contract which should be charged to office expense and solicitation of business.

Office Estimates. — The cost of the materials is usually determined from a supply catalogue. This may be accomplished by finding (1) the costs of the wiring materials between outlets and (2) the costs of the special devices such as outlet boxes, switches, cut-out cabinets, etc., and adding the two together.

Cost of Complete Installations. — According to 1920 prices of materials, the cost of wiring various types of buildings per outlet (switches counting as outlets) is as follows:

Residences wired with armored cables	\$5.50–\$7.50
Office buildings, hotels, lofts, factories, etc., with wires installed in rigid conduit	8.00– 9.00

Knob and tube and cleat wiring is not now much less in cost than conduit work on account of the high labor cost.

Field Estimates. — In order to enable its representative to make a rapid estimate of the cost of wiring a building while he is interviewing the owner or agent, the contract department of a lighting company usually provides their solicitors with a schedule of wiring costs, based (1) upon the number of lamp sockets and *switch* outlets to be provided, or (2) upon the *total* number of outlets (both lamp and switch), or (3) upon the number of lamp sockets and the number of *lamp* outlets required. Such wiring schedules are usually applicable only to finished buildings; the cost of wiring a building while it is being erected is usually much less than the cost of wiring the same building after it has been completed.

First Method. — The buildings to be wired are divided into classes and the cost of wiring a building in any class is based upon the number of lamp sockets, the number of *switch* outlets, and the type of switches and receptacles installed. The disadvantage of this method is that it does not take into account the number of lamp outlets. In two buildings in which the same number of lamp sockets are installed, the number of lamp outlets may differ widely, and as a result the labor and material required for the work will differ in the two buildings although the cost of the wiring may be the same. In most instances, however, this defect is not serious, because of the close relationship between the number of lamp sockets and the number of lamp outlets which are usually installed in the buildings for which the schedule is intended.

Second Method. — The various openings for switches, receptacles, drop cords and fixtures are considered as outlets and on the number of these is based the cost of work in houses of different classes of construction. An extra charge is made for hardware and for work installed under double or hardwood floors. With this method the number of lamp sockets installed in two houses for which the cost of wiring is the same may differ considerably. For schedule of cost see *Elec. World, 1910, Vol. 56, p. 1134.*

Third Method. — The buildings to be wired are divided into classes and the cost of wiring a building in any class is based upon the number of lamp outlets and lamp sockets and the type of switches installed. This method is

an improvement on Methods I and II as it does take into account the average number of lamp sockets per lamp outlet, and while the cost of the wiring in two buildings requiring the same labor and material may differ slightly, such differences will not be as frequently encountered as in the other two methods. As an example of this type of schedule see article in *Elec. World, 1912, Vol. 59, p. 548,* descriptive of the method used by the Consolidated Gas, Electric Light and Power Company of Baltimore, Md.

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WIRING OF BUILDINGS FOR MISCELLANEOUS DEVICES. — (*See also Wiring of Buildings for Light and Power.*) In addition to the wires designed to conduct electrical energy to lamps, motors and heating devices in a building, wires may be installed in connection with telephone (q.v.), telegraph (q.v.), district messenger and call-bell circuits, fire and burglar alarms, door-opening devices, gas lighters, watchman's clocks and electric clocks. Since all of these devices are operated at low voltage, it is unnecessary to use the same care in selecting and installing the wires as in the higher voltage systems, except that in all low-voltage systems care must be exercised that the conductors shall not become crossed with light and power circuits.

Protection of Low-voltage Wiring. — When the conductors of any low-voltage system are brought into a building from the outside, an approved protective device must be located as near as possible to the entrance of the wires to the building. With the exception of instrument circuits of telegraph systems, where cut-outs only are required, protective devices must contain a lightning arrester with a ground connection and a cut-out or heat-coil (*see Telephone Instruments and Circuits*). The conductors beyond the protective device in low-voltage systems need be insulated only sufficiently to prevent short-circuits and the consequent interruption of service. When bunched together in vertical runs the wires must be inclosed in a fire-resisting covering to prevent the wires from carrying fire from floor to floor. Low-voltage circuits may be run in the same shaft with light and power circuits, provided the two classes of wires are separated by at least two inches or one of the classes is run in a non-combustible tubing. Low-voltage wires may not be run in any case in the same tube with lighting or power wires.

Telephone Wiring. — (*See also Telephone Instruments and Circuits.*) Except in large buildings, rubber-insulated twisted pairs of wire are used extensively for telephone work. The twisted wires may be fished between partitions or concealed in moulding and if open wiring is not objectionable, the wires are carried along the finish of the room and fastened with insulating staples or tacks. In new buildings the wires are frequently installed in conduit and local connections are made at connection boxes designed for the purpose. In large buildings, when the number of telephones in use becomes very great, lead-covered insulated cable containing multiple pairs of conductors is installed in wire-ways provided for the purpose. Distributing connection boxes to which branch circuits may be connected are then installed on each floor.

Bell Wiring. — In its simplest form a bell-wiring system consists of a battery (usually a Leclanché type or dry battery), a bell-push, a bell or buzzer and the connecting wire. Paraffine-impregnated double-cotton covered wires ranging in size from No. 16 to No. 22 A.W.G. gauge are usually employed, either singly or in the form of twin wires. The wires may be fished, concealed in moulding or run exposed along the finish of the room. When a large number of bell-wires must be run together through a building as in the case of a hotel annunciator system, the wires are frequently inclosed in a cotton braid or in a tube made of zinc or impregnated paper. In many buildings supplied with an alternating current source of power a small step-down transformer is used in place of the batteries, thus effecting a saving in the cost of the installation and eliminating the possibility of run-down bat-

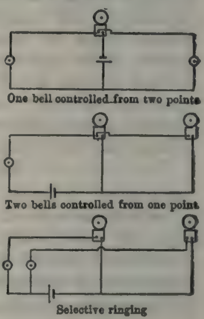
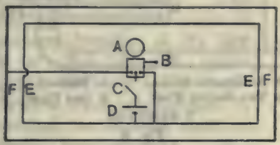


Fig. 1.

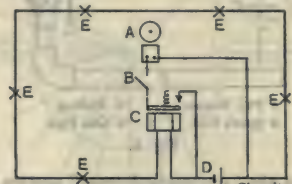
teries. In Fig. 1 are shown several systems of bell wiring; these figures are self-explanatory.

Burglar Alarms. — In place of the bell-push used in bell-wiring systems, the bell circuit may be closed by some circuit-closing device attached to windows, doors, etc., so that the opening of any window or door in the building will be made known by the ringing of a bell. As such a system has the objection that it will not operate if the battery is run down or if the wires are cut, a closed-circuit system is preferable, in which the opening of a window or door opens the circuit



A Bell B Constant Ringing Drop
C Day Switch D Battery
E and F Wires to which closing springs are connected

Fig. 2. Open-circuit Burglar Alarm



A Bell C Relay E Circuit-opening Springs
B Day Switch D Battery

Fig. 3. Closed-circuit Burglar Alarm

and a relay in turn closes the bell circuit. In Fig. 2 is shown a typical open-circuit and in Fig. 3 a typical closed-circuit system. In Fig. 4 are shown a door spring *A* and a window spring *B*.

Fire Alarms. — In the manually-operated system glass disks are placed at convenient points throughout the building and with each disk is provided a hammer, with which the disk may be broken in case of fire. The breaking of the disk opens or closes an electric circuit by means of which electric gongs are rung throughout the building. In the automatic system the expansion or fusion by heat of elements in circuit-closing or opening devices usually placed on ceilings, causes gongs to ring throughout the building or in the watchman's office.

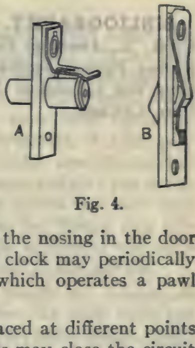


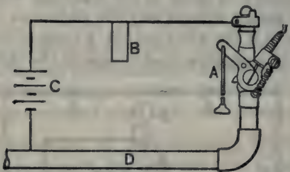
Fig. 4.

Door-opening Devices and Electric Clocks. — Doors may be opened by pressing a button at some distant point and thereby closing a circuit, through which current flows, energizing a magnet, which releases the nosing in the door frame. In the same manner, a moving disk on a master clock may periodically close a circuit, through which a magnet is energized which operates a pawl attached to the hands of another clock.

Watchman's Clock. — Circuit-closing devices are placed at different points throughout a building so that a watchman on his rounds may close the circuit by means of a handle or special key. The closing of the circuit at any station makes a record on a dial in the watchman's clock indicating the time at which the circuit was closed at each particular station, the stations being designated by numbers. To obviate the possibility of run-down batteries, magneto systems are often installed, a small magneto being placed at each station.

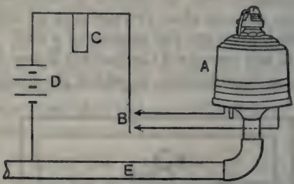
Gas Lighters. — In the "pull-burner" and "automatic-burner" systems, when the gas is turned on, an electric circuit is made and broken by means of contacts placed at the tip of the burner. To intensify the spark at the burner, a spark coil is always connected in series with the circuit. In the "pull-burner" system (Fig. 5) the pulling of a chain or the turning of a key on the gas jet turns on the gas and at the same instant makes and breaks the circuit by contacts

placed near the burner. In this system one wire connects the battery in series with a spark coil to the contact device, the other terminal of the contact device being grounded to the gas pipe through which connection is made to the other terminal of the battery.



A Pull Burner C Battery
B Spark Coil D Gas Pipe

Fig. 5.



A Automatic Burner D Battery
B Two-Button Push E Gas Pipe
C Spark Coil

Fig. 6.

In the "automatic-burner" system (Fig. 6) three wires are run from a two-button push to one pole of the battery and one terminal each of the two magnets on the automatic burner, the other terminals of the battery and the two magnets being grounded on the gas pipe. The two push buttons are usually made black and white respectively, the white button being pushed to light the gas and the black one to extinguish it. When the white button is pushed, a circuit is closed connecting the battery, the magnet which turns on the gas, the vibrating contactor and the spark coil in series. When the black button is pushed, a circuit is closed connecting the battery and the magnet, which turns off the gas, in series.

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X-RAYS. — (*See also Generators, Static; Induction Coils.*) X-rays or Röntgen rays are generated when the cathode rays of a vacuum tube impinge upon any solid substance. The form of X-ray tube commonly used, known as a "focus tube," is shown in Fig. 1. When a high unidirectional e.m.f. is impressed between the anode (A) and the cathode (C), the cathode rays emitted from (C) strike upon the anti-cathode (B) from which the X-rays originate. The anode and cathode are made of aluminum because of its low rate of disintegration under vacuous discharge. The anti-cathode is made of platinum to withstand the intense heat of cathodic bombardment. The anti-cathode must be electrically connected to the anode to prevent the possible emanation of cathode rays from its surface. The cathode is concaved spherically so that its rays are focussed upon the anti-cathode, the rays falling within a spot not more than 0.08 inch in diameter. Owing to the repulsion between the particles constituting the cathode rays, the optical focus point of the cathode should fall just in front of the surface of the anti-cathode, so that the actual focus point will fall upon the surface of the anti-cathode. The vacuum tube, made of thin glass to reduce the absorption of the X-rays, is blown in spherical form to withstand the external pressure most effectively.

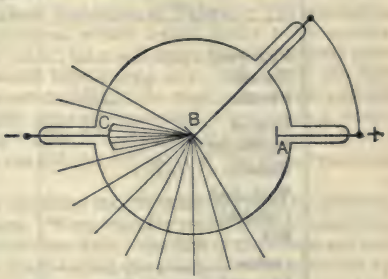

Fig. 1. X-ray Tube

Character of X-Rays. — Numerous attempts made to prove that X-rays are of undulatory form, similar to that of light, have been unsuccessful. It has been found that X-rays are neither reflected nor refracted and are not deflected like cathode rays by a magnetic force. Barkla (*Phil. Trans., 1905, p. 467*) found that such rays may be polarized and his conclusions have been confirmed by other investigators. According to the existing information, it may only be presumed that, when the negatively electrified particles constituting the cathode rays strike any solid substance, they lose their charge and give rise to the extremely high vibrations known as X-rays. It has been shown that such rays do not emanate from the impact spot on the anti-cathode but from a ring surrounding the spot, and travel in all directions from the surface of the anti-cathode, the densest emanation being in a direction normal to the surface.

Properties of X-Rays. — All substances, many of which are opaque to ordinary light, transmit X-rays to a greater or less degree. In most cases the transparency of any substance to such rays is (roughly) inversely proportional to its specific gravity. In the accompanying table are given the relative transparencies of various substances and their specific gravities, the transparency of water being taken as unity.

When X-rays are passed through gases, the gases become ionized and their electrical conductivity is accordingly increased. This property has been used as a means of measuring the relative strengths of X-rays.

Certain substances fluoresce under the action of X-rays making the existence of X-rays visible to the eye. The fluorescent screens ordinarily used for anatomical examinations are coated with barium platino-cyanide, potassium platino-cyanide or calcium tungstate.

X-rays are to a considerable degree chemically active and photographs may be made of objects generally opaque to the eye but built up of substances of

TRANSPARENCY OF VARIOUS SUBSTANCES TO X-RAYS

Material	Specific gravity, water = 1	Transparency, water = 1
Pine wood	0.56	2.21
Walnut	0.66	1.50
Paraffin	0.87	1.12
Rubber	0.93	1.10
Cardboard	....	0.80
Ebonite	1.14	0.80
Wool-cloth	....	0.76
Celluloid	....	0.76
Silk	....	0.74
Cotton	....	0.70
Charcoal	....	0.63
Bone	1.9	0.56
Aluminum	2.67	0.38
Glass	2.6	0.34
Tin	7.28	0.118
Zinc	7.20	0.116
Iron	7.87	0.101
Nickel	8.67	0.095
Brass	8.70	0.093
Copper	8.96	0.084
Silver	10.5	0.070
Lead	11.38	0.055
Mercury	13.59	0.044
Gold	19.36	0.030
Platinum	22.07	0.020

different transparencies to the X-rays, so that the internal structure of the object is revealed in the photograph. Such photographs made with highly sensitive plates are called radiographs or skiagraphs.

Considerable use is also made of X-rays in the electro-therapeutic treatment of certain diseases especially those of the skin.

Manipulation of the Tube. — It is important that the tube be connected in the proper polarity as a reverse current in the tube will cause it to blacken. The voltage used with the tube depends upon its size and the degree of vacuum attained. The ordinary tube requires a voltage of from 50,000 to 100,000 volts, or, as usually stated, a voltage which will puncture a needle gap of from 3 to 10 inches in air. The source of e.m.f. used with the tube is usually an induction coil (q.v.) and in some cases a static generator (q.v.). The voltage required to puncture a tube is materially reduced if a sheet of tinfoil electrically connected to the cathode terminal is wrapped around the outside of the tube at the cathode end and covering about one-third of its surface. The current taken by a tube is extremely small so that the connecting wires need only be large enough to withstand breakage but must be heavily insulated. Under continuous use the internal pressure of the gas in the tube decreases due to the occlusion of the gas content by the metals and glass of which the tube is constructed.

As the internal pressure in the tube decreases, it may become inoperative and should then be heated in an oven to release the occluded gases and reëstablish the proper pressure. Certain "regenerative" tubes contain palladium which occludes gas at ordinary temperatures and gives off gas under heat, thereby maintaining a constant pressure within the tube.

"Hard" and "Soft" Tubes; Condition of Tubes.—The penetrating power of an X-ray tube is proportional, within certain limits, to its vacuity. A high-vacuum tube, known as a "hard" tube, will emit a few rays of great penetrating power, whereas a low vacuum tube, known as a "soft" tube will emit a greater number of rays of less penetrability. In most cases better contrast is obtained upon the photographic plate or fluorescent screen when a "soft" tube is used than when a "hard" tube is used, since the rays emitted by a "hard" tube may penetrate all the constituent parts of the object examined with nearly equal strength.

The condition of a tube under operation may be easily detected by observing the color of the vacuous discharge. When running under normal conditions, the tube will glow with a greenish-yellow light; the face of the anode will emit a canary-yellow light, but in the space behind the anode, a violet glow will appear. The question of correct polarity may then be checked by this appearance of the anode. A pinkish light within the tube indicates that the vacuum is too low, and the absence of any glow indicates too high a vacuum. A needle gap connected in parallel with the tube will show when the voltage is too high and protects the tube from excessive voltages.

The Coolidge Tube.—In the ordinary X-ray tube, shown in Fig. 1, the emanation from the cathode is due to the bombardment of the cathode by positive ions. The stability of operation of the tube is therefore dependent upon the maintenance of a constant gas pressure within the tube. In the Coolidge tube (Fig. 2), use is made of the principle of emission of electrons from hot bodies, the cathode being constructed of an electrically heated spiral of tungsten (*c*) surrounded by a focussing surface (*S*) of tungsten or some other refractory material. The anti-cathode or target (*A*), which also serves as anode, consists of a piece of wrought tungsten. The pressure within the tube is made as low as possible, usually not more than a few hundredths of a micron. The tungsten spiral is heated by current supplied from a storage battery, while the terminals of the tube are connected to a high-potential source of direct or alternating current. The operating temperature of the tube is lower with direct than with alternating current.

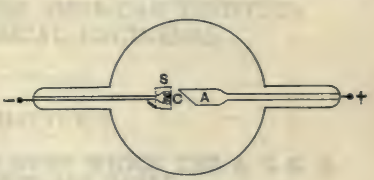

Fig. 2.

The principal advantages of the Coolidge tube over the ordinary Röntgen tube are (1) the use of either direct or alternating current, (2) the independent control of the penetrating power and intensity of the X-rays by adjusting the impressed voltage or the temperature of the cathode, (3) the continuous stable operation of the tube under any adjustment for several hours, (4) the reduction in temperature and absence of the fluorescence of the glass and (5) the production of intense X-rays of any desired penetrating power.

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STANDARDS OF THE A. I. E. E.

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APPENDIX

STANDARDS OF THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CHAPTER I

GENERAL PRINCIPLES UPON WHICH THE A. I. E. E. STANDARDS ARE BASED

(All temperatures in this and the following chapters are given in centigrade degrees.)

HEATING

1000. General Principles.— The General Principles by reference to which the ratings of electrical machines are fixed, so far as their heating is concerned, admit that the life of insulating materials depends upon the temperatures to which these materials are subjected. Taking, as a basis, the results of experience with machinery in practical service and the results of laboratory tests of various insulating materials, limiting " hottest-spot " temperatures have been established for various classes of insulation *for purposes of standardization*. Limiting " observable " temperatures are deduced from these limiting " hottest-spot " temperatures by subtracting therefrom a specified number of degrees which, *for purposes of standardization* represents the margin fixed between the limiting hottest spot and the limiting observable temperatures.

This margin may be designated as the " *conventional allowance*."

1001. Methods of Temperature Measurement. — There are three fundamental methods of temperature measurement, namely:

1. The Thermometer Method.
2. The Resistance Method, and,
3. The Embedded-Detector Method.

The General Principles stated in Section 1000 permit of the use of whichever method is best suited to the class of machine, or part thereof, to be tested, by introducing appropriate values for the limiting observable temperature by each method. All the values of the observable temperatures are based upon the "hottest-spot" limitation adopted *for purposes of standardization* for the class of insulation employed.

1002. Methods of Temperature Measurement Defined. — These three fundamental methods of making temperature measurements are designated Methods 1, 2 and 3, and are defined as follows:

TABLE 100

Methods of Temperature Measurement

Method	Description of Method
1.	Thermometer Method This method consists in the measurement of the temperature, by mercury or alcohol thermometers, by resistance thermometers, or by thermocouples, any of these instruments being applied to the hottest accessible part of the completed machine. This method does not include the use of thermocouples or resistance coils embedded in the machine as described under Method No. 3.
2.	Resistance Method This method consists in the measurement of the temperature of windings by their increase in resistance. In the application of this method, *thermometer measurements shall also be made whenever practicable without disassembling the machine,** in order to increase the probability of obtaining the highest observable temperature. The measurement indicating the higher temperature shall be taken as the "observable" temperature.
3.	Embedded Temperature-Detector Method This method consists in the measurement of the temperature by thermocouple or resistance temperature detectors, located as nearly as possible at the estimated hottest spot. When Method No. 3 is used, it shall, when required, be checked by Method No. 2. The highest observable temperature obtained from the readings of the embedded detectors shall not exceed the values permitted by the Rules for Method 3, and the highest observable temperature obtained by Method 2 shall not exceed the values permitted by the Rules for Method 2.

* Note. As one of the few instances in which the thermometer check cannot be applied in Method II, the rotor of a turbo alternator may be cited.

1003. Conventional Allowances for the Three Methods of Temperature Measurement. — The specified differences by which the "observable" temperatures shall, *for purposes of standardization*, be assumed to be lower than the "hottest spot" temperatures, (which may be designated the "Conventional Allowances"), are as follows:

Method 1 .... 15° C.
" 2 .... 10° C.
" 3 .... (See following table)

TABLE 101

Conventional Allowance for Method 3

Method 3	
For windings with two coil-sides per slot with detectors between top and bottom coil-sides (and between coil-sides and core).	5° C.
For windings with one coil-side per slot for 5000 volts or less, with detectors between coil-side and core and between coil-side and wedge.	10° C.
For windings with one coil-side per slot for more than 5000 volts, with detectors between coil-side and core and between coil-side and wedge.	10° C. plus 1° C. for every kv. of terminal pressure of the machine above 5 kv.

1004. Classification of Insulating Materials. — The insulations employed in Electrical Machinery are subdivided into four main classes, designated O, A, B and C and defined as follows:

TABLE 102

Classification of Insulating Materials

Class	Description of Material
O.	Cotton, silk, paper and similar materials when neither impregnated nor immersed in oil.
A.	Cotton, silk, paper and similar materials when so treated or impregnated as to increase the thermal limit, or when permanently immersed in oil; also enamelled wire.
B.	Mica, asbestos and other materials capable of resisting high temperatures, in which any Class A material or binder is used for structural purposes only, and may be destroyed without impairing the insulation or mechanical qualities of the insulation. (The word "impair" is used in the sense of causing any change which could disqualify the insulation for continuous service.)
C.	Materials capable of resisting higher temperatures than Class B, such as pure mica, porcelain, quartz, etc.

1005.* **Limiting "Hottest Spot" Temperatures.** — The limiting "hottest spot" temperatures are, *for purposes of standardization*, taken at the following values:

For Class O material.............. 90° C.
For Class A material.............. 105° C.
For Class B material.............. 125° C. (See Note)
For Class C material.............. no limit yet specified

(1005) The Institute recognizes the ability of manufacturers to employ Class B insulation successfully at maximum temperatures of 150° C. or even higher. However, as sufficient data covering experience over a period of years at such temperatures are at present unavailable, the Institute adopts 125° C. as a conservative limit for this class of insulation, and any increase above this figure should be the subject of special guarantee by the manufacturer.

If different insulating materials are used on various parts of one winding (for instance in the slot and for the end windings) the temperature of each material shall not exceed the limit set for that material.

When insulation consists of layers of materials having different temperature-limits (for instance high-temperature limit material adjacent to the copper and lower-temperature limit material adjacent to the iron or to the air) the temperature of each material shall not exceed the limit set for that material.

1006. Limiting Observable Temperatures. — The limiting observable temperatures for use with Methods 1, 2 and 3, are arrived at by subtracting the "conventional allowances" from the limiting "hottest-spot" temperatures for insulating materials. They are set forth as follows:

TABLE 103

Limiting Observable Temperatures

		Class O Material	Class A Material	Class B* Material
Method 1		75° C.	90° C.	110° C.
Method 2		80° C.	95° C.	115° C.
Method 3	For windings with two coil-sides per slot with detectors between top and bottom coil-sides and between coil-sides and core.	85° C.	100° C.	120° C.
	For windings with one coil-side per slot with detectors between coil-side and core and between coil-side and wedge.	80° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts.)	95° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts).	115° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts).

* See also Note, Section 1005.

1007. Limiting Observable Temperature of Oil. — The oil in which apparatus is permanently immersed shall, in no part, have a temperature, observable by thermometer, in excess of 90° C.

1008. Standard Ambient Temperatures of Reference. — The following values are adopted for the standard ambient temperatures of reference:

For Air.......................... 40° C.
For Water........................ 25° C.

These values for the standard ambient temperatures of reference apply to all conditions where the actual ambient temperature does not exceed them.

1009. Limiting Observable Temperature Rises. — The limiting observable temperature rises in the following Table 104 are obtained by subtracting the standard ambient temperatures of reference given in § 1008 from the limiting observable temperatures given in Table 103. The limiting observable temperature rises to be used in practice are given later in the Rules. They are in some cases greater and in other cases smaller than those given in Table 104. See § 1010.

For the purpose of rating, the limiting observable temperature rise must not be increased even when the ambient temperature is lower than the standard ambient temperature of reference.

TABLE 104

Limiting Observable Temperature Rises

		Air Cooled		
		Class O	Class A	Class B*
Method 1		35° C.	50° C.	70° C.
Method 2		40° C.	55° C.	75° C.
Method 3	For windings with two coil-sides per slot, with detectors between top and bottom coil-sides and between coil-sides and core.	45° C.	60° C.	80° C.
	For windings with one coil-side per slot with detectors between coil-side and core and between coil-side and wedge.	40° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts.)	55° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts).	75° C. (minus 1 degree for every 1000 volts of terminal pressure of the machine above 5000 volts).

*See also Note, Section 1005.

1010. General Comments on Special and Specific Cases. — In the foregoing it has been assumed for the purpose of presenting a comprehensive, logical and consistent plan, that the Rules actually used in the industry are exactly in accord with the General Principles. Practical experience indicates the necessity of establishing definite Rules to cover Special as well as Specific Cases. These Cases are set forth in later chapters. Any case not specifically dealt with may come under the General Principles.

1011. Comments on the Method of Measurement to be Employed. — In the absence of definite Rules, the manufacturer may, on the occasion of the acceptance test, use any of the three methods for the temperature measurements. In most cases, however, restrictions on the choice of method are imposed. These are set forth in the Rules.

1012. Comments on Temperature Limits in Special Cases. — Temperature limits are prescribed in the Rules for special cases where conditions determined by practice, by experience, or by agreements, require departures (often arbitrary) from the limits of temperature rise corresponding to the General Principles.

1013. Hottest Spot Temperature the Primary Point of Reference. — The hottest spot temperature is the primary point of reference, or the "benchmark" used as the basis for the foregoing scheme or temperature delimitation. It is not employed in commercial transactions or in the ordinary course of testing or operation of electrical machinery.

1014. Observable Temperature Rise the Working Standard. — The observable temperature rise is the working standard. A summary of working data with explanatory notes, will be found in Table 200.

1015. Duration of Temperature Test and Correction to Time of Shut Down. — Whatever method of temperature measurement be employed, it is required that

(a) operation shall be continued until constant temperatures are determined if the machine has a continuous rating, or for the full period if the machine has a short time rating, and

(b) when measurements cannot be made while the machine is loaded, appropriate corrections to raise the temperature readings to the time of shut down shall be applied. See Chapter II.

MECHANICAL AND COMMUTATION LIMITATIONS

These limitations are set forth in subsequent Chapters dealing with specific kinds of machines.

WAVE SHAPE

1200. The sine wave shall be considered as standard except where the departure therefrom is inherent in the operation of the system of which the machine forms a part.

DIELECTRIC STRENGTH AND INSULATION RESISTANCE

(See §§ 2350 to 2380 incl.)

1300. The injury produced by dielectric stress applied to insulation is related to the time during which the stress is applied. A stress up to a certain limit may be applied for an indefinite period without injury to the insulation. A somewhat greater stress will cause heating of the insulation and a progressive deterioration, eventually resulting in breakdown. Higher values of stress cause more rapid deterioration and a quicker breakdown. It is customary to determine whether machinery will withstand the voltage stresses met in practice by a preliminary test for a definite period of time at a voltage considerably higher than the normal voltage to which the machinery is to be subjected, but not high enough to produce injury to the insulation during the period of test.

1301. The test voltage which shall be applied to determine the suitability of insulation for commercial operation is dependent upon the kind and size of the machine, and its operating voltage, upon the nature of the service in which it is to be used, and upon the severity of the mechanical and electrical stresses to which it may be subjected. The voltages and other conditions of test which are recommended have been determined as reasonable and proper for the great majority of cases, and are proposed for general adoption, except when specific reasons make a modification desirable.

1400. The insulation resistance of machinery is of doubtful significance as compared with the dielectric strength. It is subject to wide variation with temperature, humidity and cleanliness of the parts. When the insulation resistance falls below prescribed values it can, in most cases of good design and where no defect exists, be brought up to the required standard by cleaning and drying the machine. The insulation resistance therefore may afford a useful indication as to whether the machine is in suitable condition for application of the dielectric test.

EFFICIENCY

(See §§ 2331 to 2333 incl.)

1500. The conditions under which efficiency is determined are those normal to the operation of the machine. These include voltage, current, power factor, frequency, wave shape, speed, temperature, or such of them as may apply in each particular case.

1501. The efficiency at all loads of all apparatus shall be corrected to a reference temperature of 75° C.

1502. In the case of machinery, two efficiencies are recognized, conventional efficiency (§ 3524) and directly measured efficiency. Unless otherwise specified, the conventional efficiency is to be employed. When the efficiency of a machine is stated without specific reference to the load conditions, rated load is always to be understood, whether the efficiency be the conventional or directly measured efficiency.

RATING

(See §§ 2202 to 2232 incl.)

1600. Principle of Machine Rating. — (a) *Rating by Temperature Rise:* The principle upon which machine rating is based, so far as relates to thermal characteristics, has been stated in earlier sections.

(b) *Rating by Limitations Other than Temperature Rise:* In some machines, the rating is limited by other than thermal considerations. In such cases, the principle upon which machine rating is based is that the rated load applied continuously or for a stated period, shall not cause the various limitations specified

in later chapters; *e.g.*, §§ 4250–4252 inclusive, to be exceeded. The rating shall be based upon the capacity as limited by heating unless the capacity as limited by other characteristics, is less.

CHAPTER II

GENERAL RULES

The expressions "machinery" and "machines" are here employed in a general sense, in order to obviate the constant repetition of the words "machinery or induction apparatus."

To ensure satisfactory results, electrical machinery should be specified to conform to the Institute Standards, in order that it shall comply in operation, with approved limitations in the following respects, so far as they are applicable.

Operating temperature	Efficiency
Mechanical strength	Power factor
Commutation	Wave shape
Dielectric strength	Regulation
Insulation resistance	

OPERATION

Temperature Limits

2104. Permissible Temperatures with Insulations of More than One Class. — (a) If different insulating materials are used on various parts of one winding (for instance in the slot and for the end windings) the temperature of each material shall not exceed the limit set for that material.

(b) When insulation consists of layers of materials having different temperature limits (for instance high-temperature limit material adjacent to the copper and lower-temperature-limit material adjacent to the iron or to the air) the temperature of each material shall not exceed the limit set for that material.

2116. Temperatures of Metallic Parts of Machines. — (a) *Parts Adjacent to Insulating Material:* Metallic parts of machines in contact with or adjacent to any kind of insulation, shall not attain a temperature in excess of that allowed for the adjacent insulation.

(b) *Parts not Adjacent to Insulating Material:* All parts of machines other than those covered by § 2116 (a) may be operated at such temperatures as shall not be injurious in any other respect.

2120. Protection against Short Circuit. — The Institute recognizes the self-destructibility, both mechanical and thermal, of certain sizes and types of machines, when subjected to severe short-circuits, and recommends that ample protection be provided in such cases, external to the machine if necessary.

RATING

General

2202. Expression of Rating. — Except where otherwise specified the machines shall be rated in terms of their available *output*. For exceptions see §§ 4223, 5203, 6204 and 6223.

2204. Institute Rating. — The Institute Rating of a machine shall be its rating when operating with a cooling medium of the ambient temperature of reference specified in §§ 2211 and 2212 and with barometric conditions within the range given in § 2215. See §§ 2300, 2310, 2311, 4110 and 4300.

Ambient Temperature of Reference and Altitude Correction

2211. Ambient Temperature of Reference for Air. — The standard ambient temperature of reference, when the cooling medium is air, shall be 40° C.

2212. Ambient Temperature of Reference for Water-Cooled Machinery. — For water-cooled machinery, the standard temperature of reference for incoming cooling water shall be 25° C., measured at the intake of the machine.

2213. Machines Cooled by Other Means. — Machines cooled by means other than air or water shall receive special consideration.

2214. Outdoor Machinery Exposed to Sun's Rays. — Outdoor machinery not protected from the sun's rays at times of heavy load, shall receive special consideration.

2215. Altitude. — Increased altitude has the effect of increasing the temperature rise of some types of machinery. In the absence of information in regard to the height above sea level at which a machine is intended to work in ordinary service, this height is assumed not to exceed 1000 meters (3300 feet). For machinery operating at an altitude of 1000 meters or less, a test at any altitude less than 1000 meters is satisfactory, and no correction shall be applied to the observed temperatures. Machines intended for operation at higher altitudes shall be regarded as special. It is recommended that when a machine is intended for service at altitudes above 1000 meters (3300 feet) the permissible temperature rise at sea level, shall be reduced by 1 per cent for each 100 meters (330 feet) by which the altitude exceeds 1000 meters.

Kinds of Rating

There are various kinds of rating such as:

2220. Continuous Rating. — A machine rated for continuous service shall be able to operate continuously at its rated output, without exceeding any of the limitations established herein.

In the absence of any specification as to the kind of rating, the continuous rating shall be understood.

2221. Short-Time Rating. — A machine rated for discontinuous or short-time service (*i.e.*, service including runs alternating with stops of sufficient duration to ensure substantial cooling), shall be capable of operating at its rated output during a limited period, to be specified in each case, without exceeding any of the limitations established herein. Such a rating is a short-time rating.

2222. Duty-Cycle Operation. — Many machines are operated on a cycle of duty which repeats itself with more or less regularity. For purposes of rating, either a continuous or a short-time equivalent load may be selected, which shall simulate as nearly as possible the thermal conditions of the actual duty-cycle.

2223*. Standard Short-Time Ratings. — The following periods shall be used for short-time ratings: 5, 10, 15, 30, 60 and 120 minutes.

2224. A. I. E. E. and I. E. C. Ratings. — When the prescribed conditions of test are those of the A. I. E. E. Standards the rating of the machine is the Institute Rating. (See § 2401.) When the prescribed conditions of the test are those of the I. E. C. Rules, the rating of the machine is the I. E. C. rating. A machine so rated in either case may bear a distinctive sign upon its rating plate. I. E. C. stands for "International Electrotechnical Commission."

2225. Continuous Rating Implied. — Machines marked "A. I. E. E. Rating," or "I. E. C. Rating" shall be understood to have a continuous rating, unless otherwise marked in accordance with §§ 2223, 5201 or 5202.

Rating by Temperature Rise

2230. Limiting Observable Temperature Rises. — The following limiting observable temperature rises have been adopted.

(2223) When, for example, a short-time rating of 10 minutes duration is adopted, and the thermally equivalent load is 25 kw. for that period, then such a machine shall be stated to have a 10-minute rating of 25 kw.

In every case the equivalent short-time test shall commence only when the windings and other parts of the machine are within 5° C. of the ambient temperature at the time of starting the test.

TABLE 200

Limiting Observable Temperature Rises for Machines for Operation in Locations where the Ambient Temperature will not Exceed 40° C. for Air or 25° C. for Water.

For Class A insulation use the values in the table.
For Class O insulation use 15° C. lower values.
For Class B insulation use 20° C. higher values (or 45° C. higher in the cases covered by the Note in § 1005).
For Class C insulation no limits yet specified.

	Items	Method 1	Method 2	Method 3	
				For windings with two coil-sides per slot with detectors between top and bottom coil-sides and between coil sides and core	For windings with one coil-side per slot with detectors against core and against wedge
Windings on Stators	1. Insulated windings other than 2.3 Note 1	50° C. Note 1	55° C. Note 1	60° C. Note 1	55° C. minus 1° for every 1000 volts by which the terminal pressure of the machine exceeds 5000 volts. Note 1
Windings on Stators	2. Single layer field windings with exposed surfaces uninsulated	60° C.	60° C.		
Windings on Stators	3. Short-circuited insulated windings	60° C.			
Windings on Rotors	4. Field Windings (other than 5)		55° C.		
Windings on Rotors	5. Single layer field windings with exposed surfaces uninsulated	60° C.	60° C.		
Windings on Rotors	6. Windings in slots	50° C.	55° C.		
Windings on Rotors	7. Short-circuited insulated windings	60° C.			
	8. Transformers and Induction Regulators		55° C.		

Note 1 — (a) The temperature of the windings of transformers and induction regulators is always to be ascertained by Method 2.
(b) In measuring the temperature of air-blast transformers, the air supply shall be shut

off immediately at the end of the temperature run and air intake shall be closed to prevent further admission of cooling air. In checking the temperatures ascertained by resistance, the readings of thermometers well distributed and in good contact with the coils shall be noted and the maximum temperature indicated by them, if higher than that determined by resistance, shall be taken as the maximum observable temperature of the windings. With the above procedure, the observable temperature rise for air-blast transformers may attain a value not in excess of 60° C. as determined by thermometer, although it must not exceed 55° C. as determined by resistance.

(c) Method 3 shall be applied to all stators of machines with cores having a width 50 cm. and over; it shall also be applied to all machines of 5000 volts and over if of over 500 kv-a. regardless of core width.

(d) Method 2 shall not be used for circuits of low resistance (other than transformer windings), such as interpole windings, where external joints and connections form a considerable part of the total resistance.

(e) For all other cases it is optional to employ either Method 1 or Method 2. (This is equivalent to authorizing Method 1 with a 5° C. lower limit of observable temperature than is permitted for Method 2).

Note 2. — For enclosed machines (rotating) the limiting observable temperature rise shall be taken as 5 degrees higher than the values set forth in the Table for Items 1 and 6.

Note 3. — A further limitation to this Table relates to the restriction of its application to machinery for operation in locations whose altitude is not more than 1000 meters above sea level. Recommendations relating to the limiting temperature rise for machines for operation at higher altitudes are given in §§ 2215 and 2231.

Note 4. — If different insulating materials are used on various parts of one winding (for instance in the slot and for the end windings) the temperature of each material shall not exceed the limit set for that material.

When insulation consists of layers of materials having different temperature-limits (for instance high-temperature-limit material adjacent to the copper and lower-temperature-limit material adjacent to the iron or the air) the temperature of each material shall not exceed the limit set for that material.

2231. Exceptions to Table 200. — (a) When the thermometers are applied directly to the surfaces of bare windings, such as an edgewise strip conductor, or a cast copper winding, the limiting observable temperature rise shall be 10° C. higher than given for Method 1 in the table.

(b) For commutators, collector rings, or bare metallic surfaces not forming part of a winding, the limiting observable temperature rise shall be 15° C. higher than given for Method 1 in the table.

(c) Any machinery destined for use with higher ambient temperatures of cooling mediums, and also any machinery for operation at altitudes for which no provision is made in § 2215, should be the subject of special guarantee by the manufacturer. The methods of test and performance set forth in these Rules, will, however, afford guidance in such cases.

2232. Limiting Observable Temperature of Oil. — The oil in which apparatus is permanently immersed shall in no part have a temperature, observable by thermometer, in excess of 90° C.

TESTS

Ambient Temperature

2300.* Measurement of the Ambient Temperature During Tests of Machinery. — (a) *General:* The ambient temperature is to be measured by means of several thermometers placed at different points around and half-way up the machine at a distance of 1 to 2 meters (3 to 6 feet), and protected from drafts, and abnormal heat radiation, preferably as in § 2301.

(b) *Mean Temperature:* The value to be adopted for the ambient temperature during a test, is the mean of the readings of the thermometers (placed as above), taken at equal intervals of time during the last quarter of the duration of the test.

(c) *Use of Idle Units:* It is sometimes desirable to avoid errors due to time lag in temperature changes, by employing an idle unit of the same size and sub-

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(2300) The cooling fluid may either be led to the machine through ducts, or through pipes, or merely surround the machine freely. In the former case the ambient temperature is to be measured at the intake of the machine itself.

jected to the same conditions of cooling as the unit under test, for obtaining the ambient temperature.

2301. Oil Cup. — In order to avoid errors due to the time lag between the temperature of large machines and the variations in the ambient air, all reasonable precautions must be taken to reduce these variations and the errors arising therefrom. Thus, the thermometer for determining the ambient temperature shall be immersed in a suitable liquid, such as oil, in a suitably heavy metal cup. This can be made to respond to various rates of change, by proportioning the amount of oil to the metal in the containing cup. A convenient form for such an oil cup consists of a massive metal cylinder, with a hole drilled partly through it. This hole is filled with oil and the thermometer is placed therein with its bulb well immersed. The larger the machine under test, the larger should be the metal cylinder employed as an oil cup in the determination of the ambient temperature. The smallest size of oil cup employed in any case shall consist of a metal cylinder 25 mm. in diameter and 50 mm. high (1 in. in diameter and 2 in. high).

Machine Temperatures

2310. Temperature Rise for Any Ambient Temperature. — A machine may be tested at any convenient ambient temperature, preferably not below 10° C., but whatever be the value of this ambient temperature, the permissible rises of temperature must not exceed those given in Table 200.

2311. Correction for the Deviation of the Ambient Temperature of the Cooling Medium, at the Time of the Heat Test, from the Standard Ambient Temperature of Reference. — Numerous experiments have shown that deviation of the temperature of the cooling medium from that of the standard of reference, at the time of the heat run, has a negligible effect upon the temperature rise of machines; therefore, no correction shall be applied for this deviation.

2312. Duration of Temperature Test of Machine for Continuous Service. — The temperature test shall be continued until sufficient evidence is available to show that the maximum temperature and temperature rise would not exceed the requirements of the rules, should the test be prolonged until the attainment of a steady final temperature.

2313. Duration of Temperature Test of Machine with a Short-Time Rating. — The duration of the temperature test of a machine with a short-time rating shall be the time required by the rating. In every case the equivalent short-time test shall commence only when the windings and other parts of the machine are within 5° C. of the ambient temperature at the time of starting the test. See § 2235.

2314. Duration of Temperature Test for Machine Having More than One Rating. — The duration of the temperature test for a machine with more than one rating shall be the time required by that rating which produces the greatest temperature rise. In cases where this cannot be determined beforehand, the machine shall be tested separately under each rating.

2315. Temperature Measurements During Heat Run. — When possible, temperature measurements shall be taken during operation, as well as when the machine is stopped. The highest figures thus obtained shall be adopted. In order to abridge the long heating period, in the case of large machines, reasonable overloads of current during the preliminary period are suggested for them.

2316. Rules for Correcting to Time of Shut-Down. — (a) Whenever a sufficient time has elapsed between the instant of shut-down and the time of the final temperature measurement to permit the temperature to fall, suitable corrections shall be applied, so as to obtain as nearly as practicable the temperature at the instant of shut-down. This can sometimes be approximately effected by plotting a curve with temperature readings as ordinates and time as abscissas, and extrapolating back to the instant of shut-down. In other instances, acceptable correction factors can be applied; *e.g.*, in the case of machines manu-

factured in large quantities, the correction obtained from tests made on representative machines may be used.

(b) Exception. In cases where successive measurements show *increasing* temperatures after shut-down, the highest value shall be taken.

Details of Testing Methods

2320. Covering of Thermometer. — Thermometers used for taking temperatures of machinery shall be covered by felt pads 4 cm. ×5 cm. (1½ in. ×2 in.), 3 mm. (⅛ in.) thick cemented on; oil putty may be used for stationary and small apparatus.

2321.* Temperature Coefficient of Copper. —The temperature coefficient of copper shall be deduced from the formula $1/(234.5+\theta)$. Thus, at an initial temperature $\theta=40°$ C., the temperature coefficient of increase in resistance per degree centigrade rise is $1/(274.5=0.00364)$. The following table, deduced from the formula, is given for convenience of reference.

TABLE 201

Temperature Coefficients of Copper Resistance

Temperature of the winding, in degrees C. at which the initial resistance is measured	Increase in resistance of copper per °C., per ohm of initial resistance
0	0.00427
5	0.00418
10	0.00409
15	0.00401
20	0.00393
25	0.00385
30	0.00378
35	0.00371
40	0.00364

2322. Temperature Measurement of Low Resistance Circuit. — In circuits of low resistance, where joints and connections form a considerable part of the total resistance, the measurement of temperature by the resistance method shall not be used. (Except transformers, for which see § 6320.)

2323.* Location of Embedded Temperature Detectors. — Embedded temperature detectors should be placed in at least two sets of locations. One of these should be between a coil-side and the core and one between the top and bottom coil-sides where two coil-sides per slot are used. Where only one coil-side per slot is used, one set of detectors shall be placed between coil-side and core, and one set between coil-side and wedge. A liberal number of detectors shall be employed, and all reasonable efforts, consistent with safety, shall be made to locate them at the various places where the highest temperatures are likely to occur. See § 1002.

(2321) Temperature by Resistance: The temperature by resistance may be calculated by the following formula:

Let R_θ=resistance at $\theta°$ C., R_{θ_2}=resistance at $\theta_2°$ C. Then

$$\theta_2=\frac{R_{\theta_2}}{R_\theta}(234.5+\theta)-234.5.$$

(2323) **A coil side is one of the two active sides of the coil lying in a slot.**

Efficiency

2331.* Efficiencies Recognized. — Two efficiencies are recognized, conventional efficiency and directly measured efficiency. Unless otherwise specified, the conventional efficiency is to be employed. See §§ 3514 and 3524.

Input and output determinations of efficiency may be made directly, measuring the output by brake, or equivalent, where applicable. Within the limits of practical application, the circulating power method, sometimes described as the Hopkinson or "loading-back" method, may be used.

2332.* Normal Conditions for Efficiency Tests. — (a) *General:* The efficiency shall correspond to, or be corrected to, the normal conditions herein set forth, which shall be regarded as standard. These conditions include voltage, current, power-factor, frequency, wave-shape, speed, temperature, or such of them as may apply in each particular case.

(b) *Load:* When the efficiency of a machine is stated without specific reference to the load conditions, rated load is always to be understood whether the efficiency be the conventional or directly measured efficiency.

(c) *Wave Shape:* The sine wave shall be standard, unless a different wave form is inherent in the operation of the system. See § 2350.

*(d) *Temperature of Reference:* The efficiency of all apparatus at all loads shall be corrected to a reference temperature of 75° C., but tests may be made at any convenient ambient temperature, preferably not less than 15° C.

(e) *Power Factor:* The efficiency of alternators and transformers shall be stated at the rated power factor.

2333. Direct Measurement of Efficiency. — (a) *General:* Electric power shall be measured at the terminals of the apparatus.

(b) *Polyphase Machines:* In polyphase machines, sufficient measurements shall be made on all phases to avoid errors of unbalance.

(c) *Mechanical Power:* Mechanical power delivered by machines shall be measured at the pulley, gearing or coupling, on the rotor shaft, thus excluding the loss of power in the belt or gear friction. See, however, an exception in § 5202.

Wave Shape

2340. Standard Wave Shape. — The Sine Wave shall be considered as standard, except where departure therefrom is inherent in the operation of the system of which the electrical machine forms a part.

Tests of Dielectric Strength

2350. Condition of Machine to be Tested. — Commercial tests shall, in general, be made with the completely assembled machine and not with individual parts. The machine shall be in good condition, and high-voltage tests, unless otherwise specified, shall be applied before the machine is put into commercial service, and shall not be applied when the insulation resistance is low due to dirt or moisture. High-voltage tests to determine whether specifications are fulfilled are admissible on new machines only.

2351. Where High-Voltage Tests are to be Made. — Unless otherwise agreed upon, high-voltage tests of machines shall be made at the factory.

2352. Temperature at which High-voltage Tests are to be Made. — High-voltage tests shall be made at the temperature assumed under normal operation or at the temperature attained under the conditions of commercial testing.

(2331) The need for assigning conventional values to certain losses arises from the fact that some of the losses in electrical machinery are practically indeterminable, and must, in many cases, either be approximated by an approved method of test, or else values recommended by the Institute and designated "conventional" values shall be employed for them in arriving at the "conventional efficiency."

(2332 d) In calculating plant or system efficiency it may be desirable to calculate the losses in each individual machine or part of the system at the actual temperature of that transformer or part during the specified interval. These losses may be appreciably different from the losses at 75° C., which latter shall be the standard temperature of reference for all efficiency guarantees.

2353. Points of Application of Voltage. — (a) *General:* The test voltage shall be successively applied between each electric circuit and all other electric circuits and metal parts grounded.

(b) *Interconnected Polyphase Windings:* Interconnected polyphase windings shall be considered as one circuit. All windings except that under test shall be connected to ground.

2354. Frequency and Wave Shape of Test Voltage. — The frequency of the testing voltage shall be not less than the rated frequency of the machine tested. A sine wave shape is recommended (see §§2340 and 4351). The test shall be made with alternating voltage having a crest value equal to $\sqrt{2}$ times the specified test voltage.

2355. Duration of Application of Test Voltage. — (a) *General:* The testing voltage for machines shall be applied continuously for a period of 60 seconds. See exception § 2355 (b).

(b) *Standard Machines and Devices produced in large quantities:* Standard machines and devices produced in large quantities for which the standard test pressure is 2500 volts or less, may be tested for one second with a test pressure 20 per cent higher than the one-minute test pressure.

2356.* Standard Test Voltage. — (a) *General:* The standard test voltage for all machines, except as otherwise specified, shall be twice the normal voltage of the circuit to which the machine is connected plus 1000 volts. See exceptions §§ 2357, 4361, 6361, 7323 (b).

2357. Assembled Apparatus. — Where a number of pieces of apparatus are assembled together and tested as an electrical unit they shall be tested with 15 per cent lower voltage than the lowest required on any of the individual pieces of apparatus.

2358. Measurement of Voltage in Dielectric Strength Tests. — There are two methods of measuring the voltage used in making dielectric strength tests, namely:

1. The voltmeter method.
2. The spark-gap method, using either the sphere spark-gap or the needle spark-gap.

2359.* Use of Voltmeters and Spark-gaps in Dielectric Tests. — When making high-voltage tests on electrical machinery every precaution must be taken against the occurrence of spark-gap discharges in the circuits from which the machine is being tested. A non-inductive resistance of about one ohm per volt of test pressure shall be inserted in series with one terminal of the spark-gap. If the test is made with one electrode grounded, this resistance shall be inserted directly in series with the non-grounded electrode; if neither terminal is grounded one-half shall be inserted directly in series with each electrode. In either case this resistance shall be as near the measuring gap as possible and not in series with the tested apparatus. A water tube is the most suitable form of resistor.

2360. Use of Spark-gap with Machines of Low Capacitance. — When the machine under test does not require sufficient charging current to distort the high-voltage wave shape, or change the ratio of transformation, the spark-gap should be set for the required test voltage and the testing apparatus adjusted to give a voltage at which this spark-gap just breaks down. This adjustment should be made with the machine under test disconnected. The machine should then be connected, and with the spark-gap about 20 per cent longer, the testing apparatus again adjusted to give the voltage of the former breakdown,

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(2356) All dielectric strength tests shall be based on the normal voltage of the system, even if apparatus is applied on a part of the system which ordinarily operates somewhat below the normal voltage.

(2359) The resistance will damp high-frequency oscillations at the time of breakdown and limit the resulting current.

Carbon resistors should not be used because their resistance may become very low at high voltages.

which is the assumed voltage of test. This voltage shall be maintained for the required interval.

2361.* Use of Spark-gap with Machines of High Capacitance.— When the charging current of the machine under test may appreciably distort the voltage wave or change the effective ratio of the testing transformer, the first adjustment of voltage with the gap set for the test voltage shall be made with the machine under test connected to the circuit and in parallel with the spark-gap.

2362. Measurements with Voltmeter. — In measuring the voltage with a voltmeter, the instrument should preferably derive its voltage from the high-pressure circuit, either directly, or by means of a voltmeter coil placed in the testing transformer, or through an auxiliary *ratio transformer*. It is permissible to measure the voltage at other places such as the transformer primary, provided corrections can be made for the variations in ratio caused by the charging current of the machine under test, or provided there is no material variation in this ratio. In any case when the capacitance of the machine to be tested is such as to cause wave distortion, the testing voltage must be checked by a spark-gap as set forth in §§2364 and 2366 or by a crest-voltage meter. If the crest-voltage meter is calibrated in crest volts, its readings must be reduced to the corresponding r.m.s. sinusoidal value by dividing by $\sqrt{2}$.

2363. Measurements with Spark-gaps. — (a) *General:* If proper precautions are taken, spark-gaps may be used to advantage in checking the calibration of voltmeters for high-voltage tests of machines.

(b) *Range of Voltages:* For the calibrating purposes set forth above, the sphere gap shall be used for voltages above 50 kv., and is preferred down to 30 kv. The needle spark-gap may, however, be used for voltages from 10 to 50 kv.

2364. Needle Spark-gap. — The needle spark-gap shall be between new sewing needles, supported axially at the ends of linear conductors, which are at least twice the length of the gap. There must be a clear space around the gap for a radius at least twice the gap length.

2365. Needle-gap Sparking Distances. — The sparking distances in air between No. 00 double long sewing needle points for various root-mean-square sinusoidal voltages shall be assumed to be as shown in Table 202.

TABLE 202

Needle-Gap Spark-Over Voltages

(At 25° C. and 760 mm. barometer)

R. M. S. Kilovolts	Millimeters	R. M. S. Kilovolts	Millimeters
10	11.9	35	51
15	18.4	40	62
20	25.4	45	75
25	33	50	90
30	41		

The values in Table 202 refer to a relative humidity of 80 per cent. Variations from this humidity may involve appreciable variations in the sparking distance.

(2361) When making arc-over tests of large insulators, leads, etc., partial arc-over of the tested apparatus may produce oscillations which will cause the measuring gap to discharge prematurely. The measured voltage will then appear too high. In such tests the "equivalent ratio" of the testing transformer should be measured by gap to within 20 per cent of the arc-over voltage of the tested apparatus with the tested apparatus in circuit. The measuring gap should then be greatly lengthened out and the voltage increased until the tested apparatus arcs over. This arc-over voltage should then be determined by multiplying the voltmeter reading by the equivalent ratio found above. Direct measurement of the spark-over voltage over one gap by another gap should always be avoided.

2366.* Sphere Spark-gap. — The standard sphere spark-gap shall be between two suitably mounted spheres. No extraneous body, or external part of the circuit, shall be nearer the spheres than twice their diameter.

The shanks shall be not greater in diameter than ⅕th the sphere diameter. Metal collars, etc., through which the shanks extend, shall be as small as practicable and shall not, during any measurement, come closer to the sphere than the maximum gap length used in the measurement.

The sphere diameter should not vary more than 0.1 per cent, and the curvature, measured by a spherometer, should not vary more than 1 per cent from that of a true sphere of the required diameter.

2367.* Use of Spherometer. — In using the spherometer to measure curvature, the distance between the points of contact of the spherometer feet shall be within the limits as indicated in Table 203.

TABLE 203

Spherometer Specifications

Diameter of sphere in mm.	Distance between contact points in mm.	
	Maximum	Minimum
62.5	35	25
125	45	35
250	65	45
500	100	65

2368. Sphere-gap Sparking Distances. — The sparking distance between spheres for various r.m.s. sinusoidal voltages shall be assumed to be as shown in Table 204.

2369.* Correction of Gap Spacing for Air-density. — The spacing at which it is necessary to set a gap to spark over at some required voltage, is found as follows. Divide the required voltage by the correction factor given in Table 205 and use the new voltage thus obtained, to find the corresponding spacing from Table 204, using a graph of the latter, if more convenient.

2370.* Correction of Voltage for Air-density. — The voltage at which a gap sparks over is derived from the voltage corresponding to the spacing in Table 204 by multiplying by the correction factor.

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* (2366) When used as specified, the accuracy obtainable should be approximately 2 per cent.

(2367) In using sphere-gaps constructed as indicated in § 2366 and § 2367, it is assumed that the apparatus will be set up for use in a space comparatively free from external dielectric fields. Care should be taken that conducting bodies forming part of the circuit, or at circuit potential, are not so located with reference to the gap that their dielectric fields are superposed on the gap, *e.g.*, the protecting resistance should not be arranged so as to present large masses or surfaces near the gap, even at a distance of two sphere diameters.

In case the sphere is grounded, the spark point of the grounded sphere should be approximately five diameters above the floor or ground.

(2369 and 2370) Effect of Air-density on Spark-over Voltage. The spark-over voltage, for a given gap, decreases with decreasing barometric pressure and increasing temperature. This variation may be considerable at high altitudes. When the variation from sea level is not great, the relative air density may be used as the correction factor; when the variation is great, or greater accuracy is desired, the correction factor corresponding to the relative air density should be taken from Table 204 in which

$$\text{Relative air-density} = \frac{0.392b}{273+t}$$

b = barometric pressure in mm.

t = temperature in deg. C.

Corrected curves may be plotted for any given altitude, if desired. It will be noted in Table 213 that for values of relative air density above 0.9 the correction factor does not differ greatly from the relative air density.

TABLE 204

Sphere-gap Spark-over Voltages

(At 25° C. and 760 mm. barometric pressure)

Kilovolts	Sparking Distance in Millimeters							
	62.5 mm. spheres		125 mm. spheres		250 mm. spheres		500 mm. spheres	
	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated
10	4.2	4.2						
20	8.6	8.6						
30	14.1	14.1	14.1	14.1				
40	19.2	19.2	19.1	19.1				
50	25.5	25.0	24.4	24.4				
60	34.5	32.0	30	30	29	29		
70	46.0	39.5	36	36	35	35		
80	62.0	49.0	42	42	41	41	41	41
90		60.5	49	49	46	45	46	45
100			56	55	52	51	52	51
120			79.7	71	64	63	63	62
140			108	88	78	77	74	73
160			150	110	92	90	85	83
180				138	109	106	97	95
200					128	123	108	106
220					150	141	120	117
240					177	160	133	130
260					210	180	148	144
280					250	203	163	158
300						231	177	171
320						265	194	187
340							214	204
360							234	221
380							255	239
400							276	257

The sphere-gap is more sensitive than the needle-gap to momentary rises of voltage and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage, or by slowly raising the voltage with a fixed setting of the gap. Open arcs should not be permitted in proximity to the gap during its operation, as they may affect its calibration.

TABLE 205

Air-density Correction Factors for Sphere Gaps

Relative air-density	Diameter of standard spheres in mm.			
	62.5	125	250	500
0.50	0.547	0.535	0.527	0.519
0.55	0.594	0.583	0.575	0.567
0.60	0.640	0.630	0.623	0.615
0.65	0.686	0.677	0.670	0.663
0.70	0.732	0.724	0.718	0.711
0.75	0.777	0.771	0.766	0.759
0.80	0.821	0.816	0.812	0.807
0.85	0.866	0.862	0.859	0.855
0.90	0.910	0.908	0.906	0.904
0.95	0.956	0.955	0.954	0.952
1.00	1.000	1.000	1.000	1.000
1.05	1.044	1.045	1.046	1.048
1.10	1.090	1.092	1.094	1.096

Insulation Resistance

2380. General. — The insulation resistance test shall be made with all circuits of equal voltage above ground connected together. Circuits or groups of circuits of different voltage above ground shall be tested separately.

2381. Voltage for Insulation Resistance Test. — Insulation resistance tests shall, if possible, be made at a d-c. pressure of 500 volts. Since the insulation resistance varies with the pressure, it is necessary that, if a pressure other than 500 volts is to be employed in any case, this other pressure shall be clearly specified.

2382.* Minimum Values. — The insulation resistance of a machine at its operating temperature shall be not less than that given by the following formula:

$$\text{Insulation Resistance in megohms} = \frac{\text{voltage at terminals}}{\text{rating in kv-a.} + 1000}.$$

The formula applies only to dry apparatus. Such high values are not attainable in oil-immersed apparatus.

Regulation

2390. Conditions for Tests of Regulation. — (a) *Speed and Frequency:* The regulation of generators shall be determined at constant speed, and that of alternating-current machines at constant frequency.

(b) *Wave Form:* A sine wave of voltage shall be assumed in determining the regulation of alternating-current machinery receiving electric power, except where expressly specified otherwise. See § 2340.

(2382) The order of magnitude obtained by this rule is shown in the following table:

TABLE 206

Insulation Resistance of Machines Excluding Oil-immersed Apparatus

Rated voltage of machine	Megohms		
	100 kv-a	1000 kv-a.	10,000 kv-a.
100	0.091	0.05	
1,000	0.91	0.50	0.091
10,000	9.1	5.0	0.91
100,000	......	50	9.1

(c) *Temperature:* It is desirable that all parts of the machine affecting the regulation be maintained at constant temperature between the two loads and where the influence of temperature is of consequence, a reference temperature of 75° C. shall be considered as standard. If change of temperature should occur during the tests the results shall be corrected to the reference temperature of 75° C.

CONSTRUCTION

Rating Plates

2401. Marking of Rating Plate. — (a) *Distinctive Marking:* It is recommended that the rating plate of machines which comply with the Institute Rules shall carry a distinctive special sign, such as "A.I.E.E. 1922 Rating" or "A22" Rating.

(b) *Significance of Marking:* The absence of any statement to the contrary on the rating plate of a machine implies that it is intended for continuous service and for the standard altitude and ambient temperature. See §§ 2211, 2212, 2215, and 2220.

(c) *Marking for Various Ratings:* The rating plate of a machine intended to work under various kinds of rating must carry the necessary information in regard to those kinds of ratings.

CHAPTER III

GENERAL DEFINITIONS

In this chapter are given definitions which are of general application to electric circuits, machines and systems. Definitions pertaining to a specific class of apparatus are given in the chapter on the class of apparatus in question. The definitions here given are primarily descriptive rather than scientifically precise.

The definitions given below for currents are also applicable, in most cases, to electromotive forces, potential differences, magnetic fluxes, etc.

DEFINITIONS

General

3000. Ambient Temperature. — The ambient temperature is the temperature of the air or water which comes into contact with the heated parts of a machine and carries off its heat. See §§ 2300 and 2301.

3010. Electrical Tension.—Electrical tension is electromotive force, or difference of potential.

3011. Voltage.—Voltage is electrical tension expressed in volts.

3020. Resistivity. — The resistivity of a material is the resistance expressed in ohms between two opposite faces of a centimeter cube of the material, and is usually coupled with a statement of the temperature. See § 9050.

Apparatus

3064. Resistor. — A resistor is a device used primarily because it possesses the property of electrical resistance. Resistors are used in electric circuits for purposes of operation, protection, or control. See § 7017.

3070. Inductor. — An inductor is a device used primarily because it possesses the property of inductance.

3078. Reactor. — A reactor is a device used primarily because it possesses the property of reactance. Reactors are used in electric circuits for purposes of operation, protection or control. See § 7018.

Kinds of Currents

3104. Direct Current. — A direct current is a unidirectional current. As ordinarily used, the term designates a practically non-pulsating current.

3108. Pulsating Current. — A pulsating current is a current which has regularly recurring variations in magnitude. As ordinarily employed the term refers to a unidirectional current.

3112. Continuous Current. — A continuous current is a practically non-pulsating direct current.

3116. Alternating Current. — An alternating current is a current the direction of which reverses at regularly recurring intervals. Unless distinctly otherwise specified, the term *alternating current* refers to a periodically varying current with successive half waves of the same shape and area. See § 3212.

3120. Oscillating, or Free Alternating Current. — An oscillating, or free alternating current is the current following any electro-magnetic disturbance in a circuit having capacity, inductance, and less than the critical resistance. When the critical resistance of a circuit is reached the current becomes aperiodic.

Alternating Currents

3204. Cycle. — A cycle is one complete set of positive and negative values of an alternating current.

3206. Period. — The period of an alternating current is the time required for the current to pass through one cycle.

3208. Frequency. — The frequency of an alternating current is the number of cycles through which it passes per second, that is, the reciprocal of the period.

3212. Wave Shape. — The wave shape, or wave form, of an alternating current is the shape of the curve obtained when the instantaneous values of the current are plotted against time in rectangular coordinates.

Two alternating quantities are said to have the same wave shape when their ordinates of corresponding phase bear a constant ratio to each other. The wave shape, as thus understood, is therefore independent of the frequency of the current and of the scale to which the curve is plotted.

3214. Sine Wave, or Simple Alternating Current. — A sine wave, or simple alternating current, is a current whose wave shape is sinusoidal.

3218.* Root-Mean-Square or Effective Value. — The root-mean-square or effective value of an alternating current is the square root of the mean of the squares of the instantaneous values for one complete cycle. It is usually abbreviated r.m.s. Unless otherwise specified, the numerical value of an alternating current refers to its r.m.s. value. The word "virtual" is sometimes used in place of r.m.s., particularly in Great Britain.

3222. Phase. — Phase is the fraction of the period of an alternating current which has elapsed since the current passed through the zero position of reference.

This fraction is usually expressed in angular measure, and the period corresponding to one complete cycle is taken as representing 2π radians or 360 degrees. The angles are frequently called electric angles, and the degrees electric degrees.

In the usual equation $$i = I_m \sin(\omega t + \phi)$$
the quantity $(\omega t + \phi)$ is the phase and ϕ is the phase angle of the current.

3224.* Phase Difference; Lead and Lag. — The phase difference of two alternating quantities of the same frequency is the difference between their phases at any instant. That quantity whose maximum occurs first in time is said to lead the other, and the latter is said to lag behind the former.

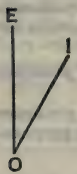

Fig. 3–1.

3228. Vector Representation and Angular Velocity. — A sine-wave current or voltage may be represented by a vector of constant length rotating counter-clockwise at a constant angular velocity ($\omega = 2\pi f$); this angular velocity is frequently termed the angular velocity of the current or voltage.

3230.* Counter-clockwise Convention. — It is recommended that, in any vector diagram, the leading vector be drawn counter-clockwise with respect to the lagging vector, as in Fig. 3-1 where *O I* rep-

(3218) The r.m.s. value of a sine wave (see Section 3214) is equal to its maximum, or crest value, divided by $\sqrt{2}$.

(3224) When the two alternating quantities do not have the same wave form, the phase difference as here defined may not be identical with equivalent phase difference as defined in Section 3262.

(3230) See Publication 12 of the International Electrotechnical Commission (Report of Turin meeting, Sept., 1911, p. 78).

resents the vector of a current in a simple alternating-current circuit lagging behind the vector $O\,E$ of impressed electromotive force.

3234. Power. — Power is the rate of transfer of energy. In the case of an alternating-current circuit the word power is generally used to denote the average value of the power over a cycle. The power in an electric circuit at any instant is equal to the product of the values of the current and voltage at that instant, and is generally called the instantaneous power.

3238. Apparent Power or Volt-amperes. — The apparent power, or volt-amperes, in an alternating-current circuit is the product of the r.m.s. value of the voltage across the circuit by the r.m.s. value of the current in the circuit. Apparent power is also expressed in kilovolt-amperes, abbreviated kv-a.

3242.* Power Factor. — Power factor is the ratio of the power to the apparent power.

3243. Power Factor in Polyphase Circuits.—The power factor of a polyphase circuit, either balanced or unbalanced, is the ratio of the total active power in watts to the total vector volt-amperes.

The total vector volt-amperes is the square root of the sum of the squares of the total active power and the total reactive power.

The total reactive power is the algebraic sum of the reactive powers corresponding to the separate harmonic components of the system.

3244. Momentary and Average Power Factors.—Power factors are either momentary power factors or integrated "average power factors"; i.e., extended over a specific time interval by averaging the active and reactive powers over the interval. Thus, a power factor may be an integrated average five-minute power factor, or an integrated average hour power factor, or an integrated average monthly power factor, etc.

The difference between a polyphase power factor (momentary) and a polyphase power factor (integrated average) lies only in the period of time over which the active and reactive powers are taken.

3246.* Reactive Volt-amperes. — The reactive volt-amperes in a circuit is the square root of the difference between the square of the apparent power and the square of the power.

3250.* Reactive Factor. — The reactive factor is the ratio of the reactive volt-amperes to the total volt-amperes.

3254.* Active Component. — The active component of the current in a circuit is the average power divided by the voltage.

3256.* Reactive Component. — The reactive component of the current in a circuit is the square root of the difference between the square of the current and the square of the active component of the current.

3260. Equivalent Sine Wave. — An equivalent sine wave is a sine wave which has the same frequency and the same r.m.s. value as the actual wave.

3262. Equivalent Phase Difference. — The equivalent phase difference (applicable to non-sinusoidal currents and voltages) is the phase difference between the equivalent sine waves of current and voltage when so related as to have the same power factor as the non-sinusoidal quantities.

There are cases, however, where this equivalent phase difference is misleading,

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(3242) The power factor when both the current and voltage are sinusoidal is equal to the cosine of the angle which expresses their difference in phase (see Section 3224).

(3246) The reactive volt-amperes, when both current and voltage are sinusoidal, is equal to the volt-amperes times the sine of the angle which expresses the phase difference between current and voltage.

(3250) The reactive factor, when both current and voltage are sinusoidal, is equal to the sine of the angle which expresses their phase difference.

(3254) The active, or in-phase component of the current in a circuit corresponds to average power passing in a given direction through the circuit. With sine-wave voltage and current, the active component of the current is in phase with the voltage.

(3256) The reactive, or quadrature, component of the current in a circuit corresponds to power alternating in direction in the circuit so that the average value of the power transferred in a given direction through a cycle is zero. With sine-wave current and voltage the reactive component of the current is in quadrature with the voltage.

since the presence of harmonics in the voltage wave, current wave, or in both, may reduce the power factor without producing a corresponding displacement of the two wave forms with respect to each other; *e.g.*, the case of an a-c. arc. In such cases, the components of the equivalent sine waves, the equivalent reactive factor and the equivalent reactive volt-amperes may have no physical significance.

3266.* Crest Factor or Peak Factor. — The crest factor or peak factor of a wave is the ratio of the crest, or maximum, value to the r.m.s. value.

3270.* Form Factor of a Wave. — The form factor of a wave is the ratio of the r.m.s. to the algebraic mean ordinate taken over a half-cycle beginning with the zero value. If the wave passes through zero more than twice during a single cycle, that zero shall be taken which gives the largest algebraic mean for the succeeding half-cycle.

3274. Deviation Factor of a Wave. — The deviation factor of a wave is the ratio of the maximum difference between corresponding ordinates of the wave and of the equivalent sine wave to the maximum ordinate of the equivalent sine wave when the waves are superposed in such a way as to make this maximum difference as small as possible.

3278. Telephone Interference Factor of a Wave. (See § 4352.) — The telephone interference factor is the ratio of the square root of the sum of the squares of the weighted values of all the sine-wave components (including in alternating waves both fundamental and harmonics) to the r.m.s. value of the wave.

Circuits and Phases

3304. Electric Circuit. — An electric circuit is a path in which an electric current may flow. Strictly speaking, an electric circuit is a complete circulatory path, but the term circuit is commonly employed to designate a specific part of a complete path. When part of a complete path is referred to, such as a branch circuit, a derived circuit, or a conductor, both the terminals and the conductor which form that path should be specified in order to avoid ambiguity; *e.g.*, the circuit *a-b-c*. When the whole circuit is referred to, it may be designated as a complete or closed circuit.

3324.* Single-phase Circuit. — A single-phase circuit is a circuit energized by a single alternating electromotive force.

3326.* Three-phase Circuit. — A three-phase circuit is a combination of circuits energized by alternating electromotive forces which differ in phase by one-third of a cycle; *i.e.*, 120 degrees.

3328.* Quarter-phase or Two-phase Circuit. — A quarter-phase or two-phase circuit is a combination of circuits energized by alternating electromotive forces which differ in phase by a quarter of a cycle; *i.e.*, 90 degrees.

3330.* Six-phase Circuit. — A six-phase circuit is a combination of circuits energized by alternating electromotive forces which differ in phase by one-sixth of a cycle, *i.e.*, 60 degrees.

3332. Polyphase Circuit. — A polyphase circuit is a circuit of more than a single phase. This term is ordinarily applied to symmetrical systems.

3344. Symmetrical Voltages and Currents. — Polyphase voltages or currents are symmetrical when the voltages or currents have the same wave shape and r.m.s. value and differ in phase each from the next by the same angle.

3348. Symmetrical Polyphase System. — A symmetrical polyphase system is a polyphase system in which the voltages are symmetrical.

(3266) The crest factor of a sine wave is $\sqrt{2}$.

(3270) The form factor of a sine wave is $\frac{\pi}{2\sqrt{2}}$ or 1.11.

(3324) A single-phase circuit is usually supplied through two wires. The currents in these two wires, counted outwards from the source, differ in phase by 180 degrees or a half cycle.

(3326) In practise the phases may vary several degrees from the specified angle.

(3328) In practise the phases may vary several degrees from the specified angle.

(3330) In practise the phases may vary several degrees from the specified angle.

3352.* Balanced Polyphase System. — A balanced polyphase system is a polyphase system in which both the currents and voltages are symmetrical.

Loads

3404. Reactive Load. — A reactive load is a load in which the current lags behind or leads the voltage across the load.

3406. Non-reactive Load. — A non-reactive load is a load in which the current is in phase with the voltage across the load. (The term non-inductive load is sometimes used for non-reactive load.)

3408. Inductive Load. — An inductive load is a reactive load in which the current lags behind the voltage across the load.

3410. Condensive Load. — A condensive load is a reactive load in which the current leads the voltage across the load.

3414.* Balanced Polyphase Load. — A balanced polyphase load is a load to which symmetrical currents are supplied when it is connected to a system having symmetrical voltages.

3424. Connected Load. — The connected load on any system, or part of a system, is the combined continuous rating of all the receiving apparatus on consumers' premises which is connected to the system, or part of the system under consideration.

3434. Peak Power. — The peak power is the average power during a time interval of specified duration occurring within a given period of time, that interval being selected during which the average power is greatest.

3438. Load Factor. — The load factor is the ratio of the average power to the peak power.

In each case, the interval of maximum load and the period over which the average is taken should be definitely specified, such as a " half-hour monthly " load factor. The proper interval and period are usually dependent upon local conditions and upon the purpose for which the load factor is to be used.

3442. Plant Factor. — The plant factor is the ratio of the average load to the rated capacity of the power plant; *i.e.*, to the aggregate ratings of the generators.

3454. Demand of an Installation or System. — The demand of an installation or system is the load which is drawn from the source of supply at the receiving terminals averaged over a suitable and specified interval of time. Demand is expressed in kilowatts, kilovolt-amperes, amperes, or other suitable units.

3458. Maximum Demand. — The maximum demand of an installation or system is the greatest of all the demands which have occurred during a given period. It is determined by measurement, according to specifications, over a prescribed time interval.

3460. Demand Factor. — The demand factor of any system or part of a system, is the ratio of the maximum demand of the system, or part of a system, to the total connected load of the system, or of the part of the system under consideration.

3464. Diversity Factor. — The diversity factor of any system, or part of a system, is the ratio of the sum of the maximum power demands of the subdivisions of the system, or part of a system, to the maximum demand of the whole system, or part of the system under consideration, measured at the point of supply.

(3352) The term balanced polyphase system is applied also to a quarter-phase (or two-phase) system in which the voltages have the same wave form and r.m.s. value and in which the currents have the same wave form and r.m.s. value and differ in phase by ninety electrical degrees.

(3414) The term balanced polyphase load is applied also to a load to which are supplied two currents having the same wave form and r.m.s. value and differing in phase by ninety electrical degrees when it is connected to a quarter-phase (or two-phase) system having voltages of the same wave form and r.m.s. value.

Machinery and Apparatus

3504. Capacity (or Properly, Capability). — The word "capacity" is frequently used in the general sense of "capability." It is also used in a more exact sense to denote the load which, when carried by a machine, apparatus, or device will, under specified conditions of test, cause it to reach *any one of its physical limitations*, such for example, as operating temperature or ability to maintain required voltage.

Capacity should be distinguished from rating. On account of the different senses in which it has been employed (see § 3508), capacity is less used than it formerly was, rating being more useful commercially.

3508.* Rating. — A rating of a machine, apparatus or device is an arbitrary designation of an operating limit.

(The rating of a machine is the output marked on the rating plate, and shall be based on, but shall not exceed the maximum load which can be taken from the machine under prescribed conditions of test. This is also called the rated output. *Maximum possible rating obviously corresponds with capability as defined in* § 3504).

3514. Efficiency. — The efficiency of an electric machine or apparatus is the ratio of its useful output to its total input. Unless otherwise specified the above output and input shall mean the power output and the power input respectively

3524.* Conventional Efficiency. — The conventional efficiency of an electric machine or apparatus is the ratio of the output to the sum of the output and the losses, or of the input minus the losses to the input, when, in either case conventional values are assigned to one or more of these losses.

3534. Plant, or System, Efficiency. — Plant, or system, efficiency is the ratio of the energy delivered from the plant or system to the energy received by it in a specified period of time. In calculating plant, or system, efficiency it may be desirable to calculate the losses in each individual machine, or part of the system, at the actual temperature of that machine, or part, during the specified interval. These losses may be appreciably different from the losses at 75° C., which latter shall be the standard temperature of reference for all efficiency guarantees. This definition is not applicable to storage batteries. See § 2332.

3535. Regulation. — The regulation of a machine in regard to some characteristic quantity (such as terminal voltage or speed) is the change in that quantity occurring between any two loads. Unless otherwise specified, the two loads considered shall be zero load and rated load, and at the temperature attained under normal operation. The regulation may be expressed by stating the numerical values of the quantity at the two loads, or it may be expressed by the "percentage regulation," which is the percentage ratio of the change in the quantity occurring between the two loads, to the value of the quantity at either one or the other load, taken as the normal value. The normal value may be either the no-load value, as the no-load speed of induction motors; or it may be the rated-load value, as in the voltage of a-c. generators.

It is assumed that all parts of the machine affecting the regulation maintain constant temperature between the two loads, and where the influence of temperature is of consequence a reference temperature of 75° C. shall be considered as standard.

Systems

3560. Normal Voltage.—In systems employing transformers, the normal voltage of the system or circuit is defined as the highest rated voltage of the secondaries of transformers supplying the system or circuit. This voltage rating applies to all parts of that particular system or circuit.

(3508) The term maximum load does not refer to loads applied solely for mechanical, commutation, or similar tests.

(3524) The need for assigning conventional values to certain losses arises from the fact that some of the losses in electric machinery are practically indeterminable, and must, in many cases, either be approximated by an approved method of test, or else values recommended by the Institute and designated "conventional" values shall be employed for them in arriving at the "conventional efficiency."

TABLE 301

3604* SYMBOLS AND ABBREVIATIONS

Name of quantity	Symbol for the quantity	Unit	Abbreviation for the unit
Acceleration due to gravity	g	centimeter per second per second	cm. per sec. per sec.
Admittance	Y, y	mho	......
Angular velocity	ω	radian per second	......
Capacitance (Electrostatic capacity)	C	farad	......
Conductance	g	mho	......
Conductivity	γ	*mho per centimeter	mho per cm.
Current	I, i	ampere	......
Dielectric constant	K	......	......
Efficiency	η	per cent	......
Electrical tension	E, e or V, v	volt	......
Electromotive force, abbreviated e.m.f.	E, e	volt	......
Electrostatic field intensity	F	......	......
Electrostatic flux	Ψ	......	......
Electrostatic flux density	D	......	......
Energy, in general	U or W	joule, watt-hour	......
Frequency	f	cycle per second	$\sim$
Impedance	Z, z	ohm	......
Inductance (or coefficient of self-induction)	L	henry	......
Intensity of magnetization	J	......	......
Length	l	centimeter	cm.
Magnetic field intensity	$H, \mathcal{H}$	gilbert per centimeter	gilbert per cm.
Magnetic flux	Φ, ϕ	maxwell	......
Magnetic flux density	$B, \mathfrak{B}$	gauss	......
Magnetic reluctance	$\mathcal{R}$	oersted	....
Magnetomotive force, abbreviated m.m.f.	$\mathfrak{F}$	gilbert*	......
Mass	m	gram	g
Mutual Inductance (or coefficient of mutual induction)	M	henry	......
Number of conductors or turns	N	convolution or turn of wire	......
Permeability	$\mu = B/H$	......	......
Phase displacement	θ, ϕ	degree or radian	°
Potential difference, abbreviated p. d.	V, v or E, e	volt	......
Power	P, p	watt	......
Quantity of electricity	Q, q	coulomb, ampere-hour	......
Reactance	X, x	ohm	......
Resistance	R, r	ohm	......

(3604) An additional unit for magnetomotive force is the "ampere-turn," for flux the "line," for magnetic flux-density "maxwells per sq. in."

The numerical values of resistivity and conductivity are *ohms resistance* and *mhos conductance* between two opposite faces of a cm. cube of the material in question, but the correct names are as given, not ohms and mhos per cm., cube, as commonly stated.

The value 980.665 for g_0 has been the accepted standard value for many years and was formerly considered to correspond accurately to 45° latitude and sea-level. Later researches, however, have shown that the most reliable value for 45° and sea-level is slightly different; but this does not affect the standard value given above.

TABLE 301—*Continued*

Name of quantity	Symbol for the quantity	Unit	Abbreviation for the unit
Resistivity..................	ρ	*ohm-centimeter	ohm-cm.
Standard acceleration due to gravity (at about 45 deg. latitude and sea level) equals 980.665*...........	g_0	centimeter per second per second	cm. per sec. per sec.
Susceptance..................	b	mho	......
Susceptibility...............	$\kappa = J/H$	......	......
Temperature..................	θ	degree centigrade	°C.
Time.........................	t	second	sec.
Velocity of rotation.........	n	revolution per second	rev. per sec.
Voltage......................	E, e or V, v	volt	......

3608. Symbols for Maximum, Instantaneous and R.M.S. Values. — E_m, I_m and P_m should be used for maximum cyclic values, e, i and p for instantaneous values, E and I for r.m.s. values (see §3218) and P for the average value of the power, or the active power. These distinctions are not necessary in dealing with direct-current circuits. In print, vector quantities should be represented by bold-face capitals.

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CHAPTER IV

STANDARDS FOR ROTATING MACHINES (OTHER THAN RAILWAY MOTORS, RAILWAY SUBSTATION MACHINERY CARRYING TRACTION LOADS, AND AUTOMOBILE PROPULSION MACHINES)

The A. I. E. E. Standards for Rotating Machines are the General Standards shown in Chapters II and III and the Standards in other chapters which are applicable to the devices involved, together with the modifications and extensions given in this chapter.

DEFINITIONS

General

Certain rules applying exclusively to railway machinery have, for convenience, been placed in Chapter V, with cross-references in all cases in this chapter. The rules of Chapter IV apply to railway machinery except as they are modified by rules of Chapter V.

4000. Classification of Electric Rotating Machinery. — Rotating electric machinery may be classified in various ways, these classifications over-lapping or interlocking in considerable degree. *First*, Rotating electric machinery may be classified as Direct-Current and Alternating-Current; *Second*, according to the function of the machines; *e.g.*, Motors, Generators, Boosters, Motor-Generators, Dynamotors, Double-Current Generators, Converters and Phase Advancers; *Third*, according to construction or principle of operation; *e.g.*, Commutating, Synchronous, Induction, Unipolar, Rectifying. Obviously, some of these machines could be rationally included in either classification, *e.g.*, Motor-Generators and Rectifying Machines.

In the following, self-evident definitions have for the most part been omitted.

Functional Classification of Rotating Electric Machines

4001. Generator. — A generator is a machine which transforms mechanical power into electric power.

4002. Motor. — A motor is a machine which transforms electric power into mechanical power.

4003. Booster. — A booster is a generator inserted in series in a circuit to change its voltage. A booster may be driven by an electric motor (in which case it is termed a motor-booster) or otherwise.

4004. Motor-generator Set. — A motor-generator set is a transforming device consisting of one or more motors mechanically coupled to one or more generators.

4005. Dynamotor. — A dynamotor is a transforming device combining both motor and generator action in one magnetic field, either with two armatures, or with one armature having two separate windings and independent commutators.

4006. Direct-current Compensator or Balancer. — A direct-current compensator or balancer is a machine which comprises two or more similar direct-current machines (usually with shunt or compound excitation) directly coupled to each other and connected in series across the outer conductors of a multiple-wire system of distribution, for the purpose of maintaining the potentials of the intermediate wires of the system, which are connected to the junction points between the machines.

4007. Double-current Generator. — A double-current generator is a machine which supplies both direct and alternating currents from the same armature-winding.

4008. Converter. — A converter is a machine which employs mechanical rotation in changing electric energy from one form into another. There are several types of converters, as defined in §§ 4009 to 4013 below.

4009. Direct-current Converter. — A direct-current converter is a machine which converts from a direct current to a direct current, usually with a change of voltage. Such a machine may be either a motor-generator set or a dynamotor.

4010. Synchronous Converter. — A synchronous converter (sometimes called a rotary converter) is a machine which converts from an alternating to a direct current, or vice-versa. It is a synchronous machine with a single closed-coil armature winding, a commutator and slip rings.

4011. Cascade Converter. — A cascade converter (also called a motor converter) is a combination of an induction motor with a synchronous converter, the secondary circuit of the former feeding directly into the armature of the latter; *i.e.*, a synchronous converter concatenated with an induction motor.

4012. Frequency Converter. — A frequency converter is a machine which converts the power of an alternating-current system from one frequency to another, with or without a change in the number of phases, or in the voltage.

4013. Rotary Phase-converter. — A rotary phase-converter is a machine which converts from an alternating-current system of one or more phases to an alternating-current system of a different number of phases, but of the same frequency.

4014. Phase Advancer. — A phase advancer is a machine which supplies reactive volt-amperes to the system to which it is connected. Phase advancers may be either synchronous or asynchronous.

4015. Synchronous Condenser or Synchronous Phase Advancer. — A synchronous condenser or synchronous phase advancer is a synchronous machine, running either idle or with load, the field excitation of which may be varied so as to modify the power factor of the system, or through such modification to influence the load voltage.

Constructional Classification of Rotating Electric Machines

4016. Direct-current Commutating Machines. — A direct-current commutating machine comprises a magnetic field of constant polarity, an armature, and a commutator connected therewith. Specific types of direct-current commutating machines are: Direct-Current Generators; Direct-Current Motors; Direct-Current Boosters; Direct-Current Motor-Generator Sets and Dynamotors; Direct-Current Compensators or Balancers; and Arc Machines.

4017. Alternating-current Commutating Machine. — An alternating-current commutating machine comprises a magnetic field of alternating polarity, an armature, and commutator connected therewith. See §§ 4071 to 4074.

4018. Synchronous Commutating Machine. — Synchronous commutating machines include synchronous converters, cascade-converters, and double-current generators.

4019. Synchronous Machine. — A synchronous machine comprises a constant magnetic field and an armature receiving or delivering alternating-currents in synchronism with the motion of the machine; *i.e.*, having a frequency strictly proportional to the speed of the machine. Specific types of synchronous machines are defined in §§ 4020 to 4023 below.

4020. Alternator. — An alternator is a synchronous alternating-current generator, either single-phase or polyphase.

4021. Polyphase Alternator. — A polyphase alternator is a polyphase synchronous alternating-current generator, as distinguished from a single-phase alternator.

4022. Inductor Alternator. — An inductor alternator is an alternator in which both field and armature windings are stationary, and in which masses of iron or inductors, by moving past the coils, alter the magnetic flux through them. It may be either single-phase or polyphase.

4023. Synchronous Motor. — A synchronous motor is a machine structurally identical with an alternator, but operated as a motor.

4024. Induction Machine. — An induction machine is a machine wherein primary and secondary windings rotate with respect to each other; *e.g.*, induction motors, induction generators, certain types of frequency converters and certain types of rotary phase-converters.

4025. Induction Motor. — An induction motor is an alternating-current motor, either single-phase or polyphase, comprising independent primary and secondary windings, one of which, usually the secondary, is on the rotating member. The secondary winding receives power from the primary by electromagnetic induction.

4026. Induction Generator. — An induction generator is a machine structurally identical with an induction motor, but driven above synchronous speed as an alternating-current generator.

4027. Engine Type Generator. — An engine type generator is one coupled to an engine in such a way that it cannot be run independently of the engine.

4028. Unipolar or Acyclic Machine. — A unipolar, or acyclic machine, is a direct-current machine, in which the voltage generated in the active conductors maintains the same direction with respect to those conductors.

Speed Classification of Motors

4035. Constant-speed Motor. — A constant-speed motor is one whose speed is either constant or does not materially vary; such as a synchronous motor, an induction motor with small slip, and an ordinary direct-current shunt motor.

4036. Multispeed Motor (or Change Speed Motor). — A multispeed motor is a motor which can be operated at any one of several distinct speeds (these speeds being practically independent of the load), but which cannot be operated at intermediate speeds.

4037. Adjustable-speed Motor. — An adjustable-speed motor is one in which the speed can be varied gradually over a considerable range, but when once adjusted remains practically unaffected by the load; such as a shunt motor designed for a considerable range of speed variation.

4038. Base Speed of an Adjustable-speed Motor. — The base-speed of an adjustable-speed motor is that speed of the motor obtained with full field under full load with no resistor in the armature circuit.

4039. Varying-speed Motor. — A varying speed motor is one whose speed varies with the load, ordinarily decreasing when the load increases; such as a series motor, a compound-wound motor, and a series-shunt motor. As a subclass of varying-speed motors, may be cited adjustable varying-speed motors, or motors in which the speed can be varied over a considerable range at any given load, but when once adjusted, varies with the load; *e.g.*, compound-wound motors arranged for adjustment of speed by varying the strength of the shunt field.

Classification of Rotating Electric Machines Relative to their Degree of Enclosure

4041. Open Machine. — An open machine is of either the pedestal-bearing or end bracket type where there is no restriction to ventilation, other than that necessitated by good mechanical construction.

4042. Protected Machine. — A protected machine is one in which the armature, field coils, and other live parts are protected mechanically from accidental or careless contact, while free ventilation is not materially obstructed.

4043. Enclosed Ventilated Machine. — An enclosed ventilated (or semi-enclosed machine is one in which the ventilating openings in the frame are protected with wire screen, expanded metal, or other suitable perforated covers, having apertures not exceeding ½ square inch (3.2 sq. cm.) in area. See § 4316.

4044. Totally Enclosed Machine. — A totally enclosed machine is one so enclosed as to prevent circulation of air between the inside and the outside of the case, but not sufficiently to be termed air-tight.

4045. Separately Ventilated Machine. — A separately ventilated machine has its ventilating air supplied by an independent fan or blower external to the machine.

4046. Self-ventilated Machine. — A self-ventilated machine differs from a separately ventilated machine only in having its ventilating air circulated by a fan, blower, or centrifugal device integral with the machine.

If the heated air expelled from the machine is conveyed away through a pipe attached to the machine, this should be so stated.

4047. Water-cooled Machine. — A water-cooled machine is one which mainly depends on water circulation for the removal of its heat.

4048. Drip-proof Machine. — A drip-proof machine is one so protected as to exclude falling moisture or dirt. A drip-proof machine may be either open or semi-enclosed, if it is provided with suitable protection integral with the machine, or so enclosed as to exclude effectively falling solid or liquid material.

4051. Explosion-proof Machine (or Flame-proof Machine). — An explosion-proof machine is a machine in which the enclosing case can withstand, without injury, any explosion of gas that may occur within it, and will not transmit the same to any inflammable gas outside it.

4052. Machine with Explosion-proof Slip-ring Enclosure. — A machine in which the slip rings and brushes alone are included within an explosion-proof case should not be described as an explosion-proof machine, but as a machine with explosion-proof slip-ring enclosure.

Classification of Alternating-Current Commutator Motors

(An alternating-current commutator motor may be classified under more than one of the following groups)

Classification by Phases of Energy Supply.

4061. Single-phase Commutator Motor. — A single-phase commutator motor is one that receives the whole of its energy from only one phase of an alternating-current supply system, without requiring external phase-converting apparatus.

4062. Polyphase Commutator Motor. — A polyphase commutator motor is one that receives its energy from a plurality of phases of an alternating-current supply system, or from a single-phase system through phase-converting apparatus external to the motor.

Classification by Speed Characteristics.

4063. General. — For convenience, alternating-current commutator motors may be classified with reference to their speed characteristics as (1) constant-speed motors, (2) multi-speed motors, (3) adjustable-speed motors, and (4) varying-speed motors. Definitions of these terms as given in §§ 4035 to 4039 for motors in general, should be adopted for alternating-current commutator motors, in so far as they are applicable.

Classification by Excitation.

4064. Stator-excited Commutator Motor. — A stator-excited commutator motor is one in which the torque-producing field is due to a current in a winding

located on the stator. By the "torque-producing field" is meant that component of the magnetic field which, with the in-phase component of the current, produces the torque of the motor.

4065. Rotor-excited Commutator Motor. — A rotor-excited commutator motor is one in which the torque-producing field is due to a current in a winding located on the rotor. See § 4064.

4066. Stator- and Rotor-excited Commutator Motor. — A stator- and rotor-excited commutator motor is one in which the torque-producing field is due to currents in windings located on the stator and on the rotor. See § 4064.

4067.* Constant-field Commutator Motor. — A constant-field commutator motor is one in which the torque-producing field remains practically constant, independent of the load. See § 4064.

4068.* Varying-field Commutator Motor. — A varying-field commutator motor is one in which the torque-producing field varies in some proportion with the current in the armature (which latter is generally the rotor). See § 4064.

Classification by Neutralization and Compensation.

4069. Neutralized Commutator Motor. — A neutralized commutator motor is one in which use is made of a winding for producing a magnetizing force which at each instant and at each point in the air-gap under the pole face is practically equal and opposite to the magnetizing force due to the armature current.

4070. Compensated Commutator Motor. — A compensated commutator motor is one in which means, other than a neutralizing winding, are provided within the motor for improving the power-factor.

Classification by Energy Reception.

4071. Conduction Commutator Motor. — A conduction commutator motor is one in which the working energy is supplied to only one of the members, and is conveyed to it by conduction. By "working energy" is meant the energy which is directly converted into mechanical energy, and which includes the shaft energy output plus core losses and friction.

4072. Transformer Commutator Motor. — A transformer commutator motor is one in which the working energy is transmitted from one member to the other by transformer action.

A motor in which the energy required by its armature (which is generally the rotor) is conveyed to it by electromagnetic induction or transformer action, may properly be referred to either as an "induction motor," or as a "transformer motor." Although it is equally applicable to a motor having a commutator, the term "induction motor" is usually applied to a motor without a commutator. The term "transformer commutator motor" is therefore recommended for use with motors of the induction, or transformer type, having commutators.

4073. Transformer-conduction Commutator Motor. — A transformer-conduction commutator motor is one in which the energy required by its armature (which is generally the rotor) is conveyed to it by both conduction and electromagnetic induction.

4074. Repulsion Commutator Motor. — A repulsion commutator motor is a transformer commutator motor in which use is made of brushes for short-circuiting a number of coils of the commutated winding.

Miscellaneous Definitions

4085. Saturation Factor. — The saturation factor of a machine is the ratio of a small percentage increase in field excitation to the corresponding percentage increase in voltage thereby produced. Unless otherwise specified, the saturation factor of a machine refers to the no-load excitation required at normal rated

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(4067) Alternating-current commutator motors of this class will in general have load-speed characteristics similar to those of the direct-current shunt motor, but not all alternating-current commutator motors having such load-speed characteristics are constant-field machines.

(4068) Such a motor will in general have load-speed characteristics similar to those of the direct-current series motor.

speed and voltage. It is determined from measurements of saturation made on open circuit at rated speed.

4086. Percentage Saturation. — The percentage saturation of a machine at any excitation may be found from its saturation curve (generated voltage as ordinates, against excitation as abscissas), by drawing a tangent to the curve at the ordinate corresponding to the assigned excitation, and extending the tangent to intercept the axis of ordinates drawn through the origin. The ratio of the intercept on this axis to the ordinate at the assigned excitation, when expressed in per cent, is the percentage saturation, and is independent of the scales selected for excitation and voltage. This ratio as a fraction, is equal to the reciprocal of the saturation-factor at the same excitation, deducted from unity; or, if f be the saturation factor and p the percentage saturation,

$$p = 100\left(1 - \frac{1}{f}\right).$$

4088.* Variation in Alternators. — The variation in alternators, or alternating-current circuits in general is the maximum angular displacement, expressed in electrical degrees (see § 3222) of corresponding ordinates of the voltage wave and of a wave of absolutely constant frequency equal to the average frequency of the alternator or circuit in question, and may be due to the variation of the prime mover. See §§ 14010 and 14011.

4089. Per cent Resistance Drop. — The per cent resistance drop in an electric machine is the ratio of the internal resistance drop at 75° C. to the terminal voltage expressed in per cent.

Unless otherwise specified this per cent drop shall be referred to rated load and rated power factor.

The per cent resistance drop in an induction motor is expressed in terms of the internally induced electromotive force.

4090. Per cent Reactance Drop. — The per cent reactance drop in an electric machine or apparatus is the ratio of the internal reactance drop to the terminal voltage, expressed in per cent.

Unless otherwise specified this per cent drop shall be referred to rated load and rated power factor.

The per cent reactance drop in an induction motor is expressed in terms of the internally induced electromotive force.

4091. Per cent Impedance Drop. — The per cent impedance drop in an electric machine is the ratio of the internal impedance drop at 75° C. to the terminal voltage, expressed in per cent.

Unless otherwise specified this per cent drop shall be referred to rated load and rated power factor.

The per cent impedance drop in an induction motor is expressed in terms of the internally induced electromotive force.

4092. Magnetic Degree. — A magnetic degree is the 360th part of the angle subtended, at the axis of a machine, by a pair of its field poles. One *mechanical degree* is thus equal to as many magnetic degrees as there are pairs of poles in the machine.

4094. Regulation of D-C. Generators. — The regulation of a d-c. generator is usually stated by giving the numerical values of the voltage at no load and rated load, and in some cases it is advisable to state regulation at intermediate loads. The regulation of d-c. generators refers to changes in voltage corresponding to gradual changes in load, and does not relate to the comparatively large momentary fluctuations in voltage that frequently accompany instantaneous changes in load.

4095. Regulation of Constant-potential A-C. Generators. — In constant-potential a-c. generators, the regulation is the rise in voltage (when the specified load at specified power factor is reduced to zero) expressed in per cent of normal rated-load voltage.

(4088) If p is the number of pairs of poles, the variation of an alternator is p times the variation of its prime mover, if direct-connected, and pn times the variation of the prime mover if rigidly connected thereto in such a manner that the angular speed of the alternator is n times that of the prime mover.

4096. Regulation of Constant-current Machines. — In constant-current machines the regulation is the ratio of the maximum difference of current from the rated-load value (occurring in the range from rated-load to short-circuit, or minimum limit of operation), to the rated-load current.

4097. Regulation of Constant-speed Motors. — In constant-speed direct-current motors and induction motors, the regulation is the ratio of the difference between full-load and no-load speeds to the no-load speed.

4098. Regulation of Converters, Dynamotors, Motor-Generators and Frequency Converters. — In converters, dynamotors, motor-generators, and frequency converters, the regulation is the change in the terminal voltage of the output side between the two specified loads. This may be expressed by giving the numerical values, or as the percentage of the terminal voltage at rated load.

OPERATION

Temperature Limits

4105. Exceptions to General Temperature Limits Given in Chapter II.

(a) Railway Motors: See §§ 5202 and 5101.

(b) Automobile Propulsion Machines: See § 5205.

(c) Railway Substation Machines: See §§ 5201 and 5102.

(d) Squirrel Cage and Amortisseur Windings. The temperature may attain any value such as will not occasion mechanical injury to the machine.

(e) Field Control Railway Motors: See § 5204.

4106. Collector Rings. — The observable temperature of collector rings shall not be permitted to exceed the values set forth in § 2231 (b) for the insulations employed either in the collector rings themselves or in adjacent insulations whose life would be affected by the heat from the collector rings.

4107. Commutators. — The observable temperature shall in no case be permitted to exceed the values given in § 2231 (b) for the insulation employed either in the commutator or in an insulation whose life would be affected by the heat of the commutator. These temperature limits are intended only to protect the insulation of the commutator and of the adjacent parts and are not intended as a criterion of successful commutation.

4108. Cores. — The observable temperature of those parts of the iron core in contact with insulating materials shall in no case be permitted to exceed the values given in §§ 2231 (b) for the insulation employed.

4109. Other Parts (Such as Brush-holders, Brushes, Bearings, Pole-tips, Cores, etc.). — All parts of electrical machinery other than those whose temperature affects the temperature of the insulating material may be operated at such temperatures as shall not be injurious in any other respect.

4110. Maximum Temperature Rise in Service. — Whatever may be the ambient temperature when the machine is in service, the limits of the maximum observable temperature or of temperature rise specified in the rules should not be exceeded in service; for, if the maximum temperature be exceeded, the insulation may be endangered, and if the rise be exceeded, the excess load may lead to injury, by exceeding limits other than those of temperature; such as commutation, stalling load and mechanical strength. For similar reasons, loads in excess of the rating should not be taken from a machine.

RATING

Units in Which Rating Shall be Expressed

4220. Rating of D-C. Generators.—The rating of direct-current generators shall be expressed in kilowatts (kw.) available at the terminals at a specified voltage.

4221. Rating of Alternators.— The rating of alternators shall be expressed in kilovolt-amperes (kv-a.) available at the output terminals, at a specified voltage and power factor.

4222.* Rating of Motors. — It is strongly recommended that the rating of

(4222) Since the input of machinery of this class is measured in electrical units and since the output has a definite relation to the input, it is logical and desirable to measure the delivered power in the same units as are employed for the received power. Therefore, the output of motors should be expressed in kilowatts instead of in horsepower

motors shall be expressed in kilowatts (kw.) available at the shaft. (An exception to this rule is made in the case of railway motors, which, for some purposes, are also rated by their *input*.) See § 5203.

4223. Rating of Auxiliary Machinery. — Auxiliary machinery, such as regulators, balancer sets, synchronous-condensers, etc., shall have their ratings appropriately expressed. It is also essential to specify the voltage (and frequency, if a-c.), of the circuits on which the machinery may appropriately be used.

Limitations Other than Temperature Rise

4250.* Mechanical Limitations. — *(a) *General:* All types of rotating machines shall be so constructed that they will safely withstand an over-speed of 25 per cent, except in the case of steam turbines, which, when equipped with emergency governors, shall be constructed to withstand 20 per cent over-speed.

(b) *Generators:* Water-wheel generators shall be constructed for the maximum runaway speed which can be attained by the combined unit.

(c) *Motors:* Motors for continuous service shall, except when otherwise specified, be required to develop running torque at least 175 per cent of that corresponding to the running torque at their rated load, without stalling. Obviously, duty-cycle machines must carry their peak loads without stalling.

4251. Commutation Limitations. — (a) *Continuously Rated Machines:* Continuously rated machines shall be required to commutate successfully momentary loads of 150 per cent of the amperes corresponding to the continuous rating, keeping the rheostat set for rated-load excitation. Successful commutation is such that neither brushes nor commutator are injured by the test. See §§ 2220 and 5203.

(b) *Machines for Duty-cycle Operation:* Machines for duty-cycle operation with widely fluctuating loads shall commutate successfully under their specified operating conditions. See §§ 2222 and 2223.

4252. Limitations of Stability. — Continuously rated machines shall be required to carry momentary loads of 150 per cent of the amperes corresponding to the continuous rating, keeping the rheostat set for rated-load excitation.

In the case of direct-connected generators, this clause is not to be interpreted as requiring the prime mover to drive the generator at this overload.

TESTS

Ambient Temperature

4300. Measurement of the Ambient Temperature During Tests of Machines. — (See § 2300) (a) *Machines Cooled by Forced Draught:* In the testing of rotating machines, cooled by forced draught, a conventional weighted mean shall be employed, a weight of four being given to the temperature of the circulating air supplied through ducts and a weight of one to the surrounding room air. See §2300 Note.

(b) *Machines Below Floor Line:* Where machines are partly below the floor line in pits, the temperature of the rotor shall be referred to a weighted mean of the pit and room temperatures, the weight of each being based on the relative proportions of the rotor in and above the pit. Parts of the stator constantly in the pit shall be referred to the ambient temperature in the pit.

Machine Temperatures

4316. Machines with Small Ventilating Apertures. — Machines having ventilating openings smaller than 0.02 sq. in. (0.13 sq. cm.) in area, when intended to be operated in locations or under conditions where the openings are liable to become clogged, should be considered as totally enclosed machines and tested as

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However, on account of the hitherto prevailing practise of expressing mechanical output in horsepower, it is recommended that for machinery of this class the rating should, for the present, be expressed both in kilowatts and in horsepower; as follows:

Kw.———————Approx. equiv. h.p.——————

For the purposes of these rules the horsepower shall be taken as 746.0 watts.

In order to lay stress upon the preferred future basis, it is desirable that on rating plates, the rating in kilowatts shall be shown in larger and more prominent characters than the rating in horsepower.

One kilowatt is equal to 102 kilogrammeters per second.

(4250 a) In the case of series motors, it is impracticable to specify percentage values for the guaranteed overspeed, on account of the varying service conditions.

such with openings closed, and in all cases the rating on this basis should be indicated on the rating plate.

4319. Exception to Temperature Limits Used in Method 1. — In the case of enclosed motors and generators, the limits of the observable temperature rise shall be 5° C. higher than allowed by the general rule. This rule does not apply to those types of machines defined in §§ 4043, 4045 and 4046.

4320. Exception to Temperature Limits Used in Method 2. — In the case of enclosed motors and generators, the limits of the observable temperature rise shall be 5° C. higher than allowed by the general rule. This rule does not apply to those types of machines defined in §§ 4043, 4045 and 4046.

4321. Method of Temperature Measurement Used in Determining Temperature of Stators of Machines. — Method 3 should be applied to all stators of machines with cores having a width of 50 cm. (20 in.) or over. It should also be applied to all machines of 5000 volts or more, if rated over 500 kv-a., regardless of core width.

Efficiency

4334.* Classification of Losses. — Losses are classified as shown in Table 401.

4335.* Losses to be Considered in Machines. — Conventional efficiencies shall be based upon the losses listed in Table 402, and these losses shall be measured as specified in §§ 4336–4342 inclusive.

TABLE 401

Classification of Losses in Machinery

Accurately Measurable	Approximately Measurable or Determinable	Indeterminable
No-load core losses including eddy-current losses in conductors at no-load	Brush Friction loss	Iron loss due to flux distortion
Load I^2R losses in windings No-load I^2R losses in windings	Brush-contact loss	Eddy-current losses in conductors due to transverse fluxes occasioned by the load currents
	Losses due to windage and to bearing friction	Eddy-current losses in conductors due to tooth saturation resulting from distortion of the main flux
		Tooth-frequency losses due to flux distortion under load
	Dielectric losses	Short-circuit loss of commutation

(4334) The losses in constant-potential machinery, either of the stationary type, or of the constant-speed rotary type, are of two classes; namely, those which remain substantially constant at all loads, and those which vary with the load. The former include iron losses, windage and friction, also I^2R losses in any shunt windings. The latter include I^2R losses in series windings. The constant losses may be determined by measuring the power required to operate the machine at no load, deducting any series I^2R losses. The variable loss at any load may be computed from the measured resistance of the series windings and the given load current.

4336. I^2R **Loss.** — (a) *General:* The I^2R loss shall be based upon the current and the measured resistance.

(b) *Polyphase Induction-motor Rotor:* The I^2R loss in the rotors of polyphase induction motors should be determined from the slip, whenever the latter is accurately determinable, using the following equation:

$$\text{Rotor } I^2R \text{ loss} = \frac{\text{output} \times \text{slip}}{1 - \text{slip}}.$$

TABLE 402

Losses in Rotating Electric Machines

(References are to Sections)

	I^2R loss windings	Friction and windage	Brush friction	Core loss	Brush contact I^2R loss	Stray load losses	Miscellaneous losses when present
D-C. commutating machines (Note 1)	4336(a)	4337(a)	4338	4339	4341 5341	Note 5	4343(a)(b)(c)
A-C. commutating machines (Note 1)	4336(a)	4337(a)	4338	4339	4341	Note 5 5339	4343(b)
Railway motors	4336(a)	5337 5338	5338 5339	5339	4341	Note 5 5339	4343(b)
Synchronous motors and generators (Note 4)	4336(a)	4337(a)	4338 Note 3	4339 4339	4341 Note 3	4342(a)	4343(a)(b)(c)
Synchronous converters	4336(c)	4337(a)	4338	4339(a) 4339(b)		Note 5	4343(a)
Induction machines	4336(b)	4337(a)	4338	4339(a) 4339(c)	4341 Note 2	4342(b)	4343(b)

Notes: (1) Except railway motors.

(2) When there are collector rings.

(3) Brush friction and brush contact losses are negligible except in the case of revolving armature machines.

(4) For the booster type of synchronous converter, where the booster forms an integral part of the unit, its losses shall be included in the total converter losses in estimating the efficiency.

(5) These losses, while usually of low magnitude, are erratic, and the Institute is not at this time prepared to make recommendations for approximating them.

In large slip-ring motors, in which the slip cannot be directly measured by loading, the rotor I^2R loss shall be determined by direct resistance measurement; the rotor full-load current to be calculated by the following equation:

$$\text{Current per ring} = \frac{\text{watts output}}{\text{rotor voltage at stand-still} \times \sqrt{3} \times K}.$$

(4335) This simple method of determining the losses and hence the efficiency is only approximate, since the losses which are assumed to be constant do actually vary to some extent with the load, and also because the actual loss in the copper windings is sometimes appreciably greater than the calculated I^2R loss. The difference between the approximate losses, as above determined, and the actual losses, is termed "stray-load losses." These latter are due to distortions in electric or magnetic fluxes from their no-load distributions or values, brought about by the load current. They are usually only approximately measurable, or may be indeterminable, but certain of them reach values in various kinds of machinery, which require that they should be taken into account.

Dielectric losses are usually negligible.

The stray-load losses include the items in the column of Table 401 headed "Indeterminable" but do not include the increased core losses due to increased excitation for compensating internal drop under load.

This equation applies to three-phase rotors. For rotors wound for two-phase, use 2 instead of the $\sqrt{3}$. K may be taken as 0.95 for motors of 150 kw. or larger. The factor K usually decreases as the size of motor is reduced, but no specific value can be stated for smaller sizes.

(c) *Synchronous Converters:* The I^2R losses in the armature winding shall be derived from those corresponding to its use as a direct-current generator, by using suitable factors.

4337. Bearing Friction and Windage. — (a) *General:* Drive the machine from an independent motor, the output of which shall be suitably determined. The machine under test shall have its brushes removed and shall not be excited. This output represents the bearing friction and windage of the machine under test.

(b) *Induction Motors:* The bearing friction and windage of induction motors may be measured by running motors free at the lowest voltage at which they will rotate continuously at approximately rated speed; the watts input, minus I^2R loss, under these conditions being taken as the friction and windage.

(c) *Engine-type Generators:* In the case of engine-type generators (see §4027), the windage and bearing friction loss is ordinarily very small, amounting to a fraction of one per cent of the output. This loss shall be neglected owing to its small value and the difficulty of measuring it.

(d) *D. C. Railway Motors:* See § 5337.

4338. Brush Friction of Commutator and Collector Rings.—(a) *General:* Drive the machine from an independent motor, the output of which shall be suitably determined. The brushes shall be in contact with the commutator or collector rings, but the machine shall not be excited. The difference between the output obtained in the test in § 4337 and this output shall be taken as the brush friction. The surfaces of the commutator and brushes should be smooth and glazed from running when this test is made.

(b) *D. C. Railway Motors:* See § 5338.

4339. Core Losses. — (a) *General:* Drive the machine from an independent motor, the output of which shall be suitably determined. The brushes shall be in contact, and the machine shall be excited, so as to produce at the terminals a voltage corresponding to the calculated internal voltage for the load under consideration. The difference between the output obtained by this test and that obtained by test under § 4338 (a) shall be taken as the core loss.

(b) *Synchronous Machines:* The internal voltage of synchronous machines shall be determined by correcting the terminal voltage for the resistance drop only.

(c) *Induction Motors:* The core loss of an induction motor may be determined by measuring the watts input to the motor when running free at rated voltage and frequency and subtracting therefrom the no-load copper loss, bearing friction and windage.

(d) *D. C. Railway Motors:* See § 5339.

4341.* Brush-contact I^2R Loss. — (a) *General:* One volt drop per brush shall be considered as the Institute standard drop corresponding to the I^2R brush-contact loss, for carbon and graphite brushes with pigtails attached.

(4341) The brush-contact I^2R loss depends largely upon the material of which the brush is composed.

As indicating the range of variation the following table will be of interest:

TABLE 403

Brush-contact Drop

Grade of brush	Volts drop across one brush-contact (Average of positive and negative brushes)
Hard carbon	1.1
Soft carbon	0.9
Graphite	0.5 to 0.8
Metal-graphite types	0.15 to 0.5 (The former for largest proportion of metal)

One and one-half volts per brush shall be allowed where pigtails are not attached. Metal-graphite brushes shall be considered as special.

(b) *Automobile Motors:* See § 5341.

4342.* Stray Load-losses.—(a) *Synchronous Machines:* These include iron losses, and eddy-current losses in the copper, due to fluxes varying with load and also to saturation.

Stray load-losses shall be determined by operating the machine on short circuit and at rated-load current. This, after deducting the windage and friction and I^2R loss, gives the stray load-loss for polyphase generators and motors. These losses in single-phase machines are large; but the Institute is not yet prepared to specify a method for measuring them.

(b) *Induction Machines:* These include eddy-current losses in the stator copper, and other eddy-current losses due to fluxes varying with the load. In windings consisting of relatively small conductors, these eddy-current losses are usually negligible.

With rotor removed, measure the power input to the stator with different values of current at the rated frequency. The curve plotted with these values gives the combined I^2R and stray load-losses due to eddy-currents in the stator copper. Deduct the I^2R loss determined from the resistance, and the difference will represent the stray load-losses corresponding to the various currents. While this method is not accurate for some types of motors it usually represents a sufficiently good approximation.

4343. Miscellaneous Losses. — (a) *Field-rheostat Losses:* Field-rheostat losses shall be included in the generator losses where there is a field rheostat in series with the field magnets of the generator, even when the machine is separately excited.

(b) *Ventilating Blower:* When a blower is supplied as part of a machine set, the power required to drive it shall be charged against the complete unit, but not against the machine alone.

(c) *Other Auxiliary Apparatus:* Auxiliary apparatus, such as a separate exciter for a generator or motor, shall have its losses charged against the plant of which the generator and exciter are a part, and not against the generator. An exception should be noted in the case of turbo-generator sets with direct-connected exciters, in which case the losses in the exciter shall be charged against the generator. The actual energy of excitation and the field-rheostat losses, if any, shall be charged against the generator. See § 4343 (a).

Wave Shape

4351. Deviation Factor of a Wave. — The deviation factor of the open circuit terminal voltage wave of synchronous machines shall not exceed ten per cent unless otherwise specified. See § 3274.

4352. Telephone Interference Factor of a Wave. — (For trial only.) (See § 3278.) — (a) *Conditions of Test.* The weighting of the sine-wave components of different frequencies shall be as given in Fig. 4-1.

The telephone interference factor of a voltage wave, corresponding to this weighting, may be measured by the use of the network shown in Fig. 4-2.

With this network the telephone interference factor of a voltage wave is the ratio of the current I in micro-amperes in the meter branch of the network to the voltage E applied to the external terminals of the network. The measurement may be made on the low-tension side of a potential transformer. A sensitive vacuum thermo-couple provided with a shunt, and a direct-current milammeter have been found convenient for measuring the current.

(b) The appropriate limiting value of the telephone interference factor of a wave (see §3278), either for machines or for circuits, has not yet been determined, and cannot now be specified. The whole matter of interference, including reasonable requirements for both power and communication systems, is under discussion, in consultation with power, telephone, and other interests concerned.

(4342) Values of the indeterminate losses may also be obtained by brake or other direct test and used in estimating actual efficiencies of similar machines by the separate-loss method.

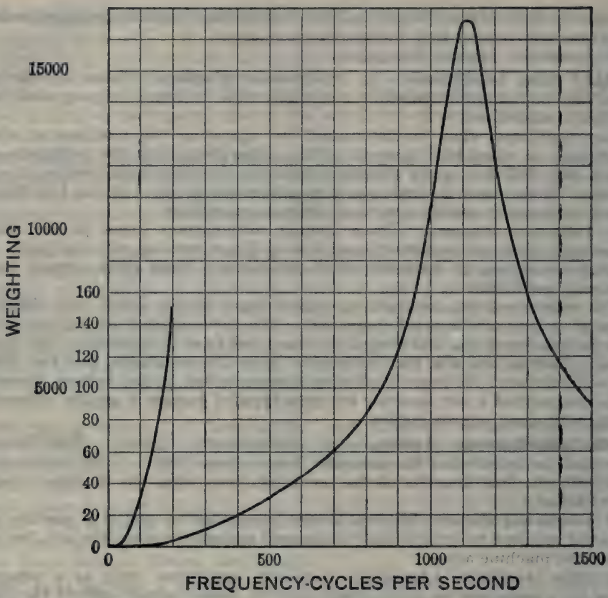
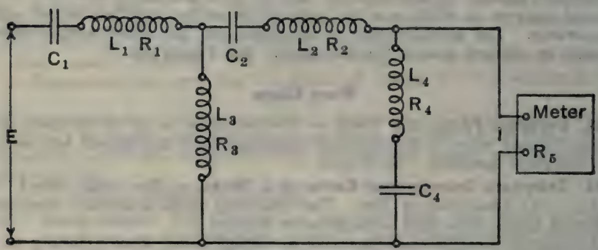


Fig. 4-1



Network Constants

$C_1=0.9$ mf.	±0.5%	$L_1=0.023$ henry	±0.5%	$R_1=5$ ohms	±2%
$C_2=0.9$ "		$L_2=0.0205$ "		$R_2=12$ "	±2%
		$L_3=0.068$ "		$R_3=73$ "	±1%
$C_4=7.5$ "		$L_4=0.019$ "		$R_4=22.5$ "	±2%
				$R_5=43$ "	±1%

Fig. 4-2

Tests of Dielectric Strength

4358. Frequency of Test Voltage. — In d-c. machines, and in general commercial application of a-c. machines, the testing frequency of 60 cycles per second is recommended.

4361.* Exceptions to Standard Test Voltage Given in Section 2356.

(a) *Field Windings of Alternating-current Generators:* Field windings of alternating-current generators shall be tested with 10 times the exciter voltage, but in no case with less than 1500 volts nor more than 3500 volts.

*(b) *Field Windings of Synchronous Machines:* Field windings of synchronous machines including motors and converters which are to be started with alternating current are to be tested as follows:

When machines are to be started with field short-circuited, the field windings shall be tested as specified in § 4361 (a).

When machines are to be started with fields open-circuited and sectionalized while starting, the field windings shall be tested with 5000 volts.

When machines are to be started with fields open-circuited and connected all in series while starting, the windings shall be tested with 5000 volts for less than 275 volts excitation and 8000 volts for excitation of 275 volts to 750 volts.

*(c) *Phase-wound Rotors of Induction Motor:* The secondary windings of wound rotors of induction motors shall be tested with twice their normal induced voltage, plus 1000 volts. When induction motors with phase-wound rotors are to be reversed, while running at approximately normal speed, by reversing the primary connections, the test shall be four times the normal induced voltage plus 1000 volts.

(d) *Small Motors and Generators:* Small machines taking not over 660 watts or having an output not exceeding ½ h. p. (373 watts), such as fractional horsepower motors, and intended solely for operation on supply circuits not exceeding 275 volts, shall be tested with 900 volts.

(e) *Alternating-current Machines Connected to Permanently Grounded Single-phase Systems:* Alternating-current machines connected to permanently grounded single-phase systems, for use on permanently grounded circuits operating at more than 300 volts shall be tested with 2.73 times the voltage of the circuit to ground, plus 1000 volts. This does not refer to three-phase machines with grounded star neutral.

*(f) *Machines for Use on Circuits of 25 Volts or Lower:* Machines for use on circuits of 25 volts or lower, such as bell-ringing apparatus, electric machines used in automobiles, machines used on low-voltage battery circuits, etc., shall be tested with 500 volts.

Regulation

4390. Conditions for Tests of Regulation (see § 2390). — (a) *Power Factor:* In alternating-current generators the power factor of the load to which the regulation refers should be specified. Unless otherwise specified, it shall be understood as referring to non-inductive load, that is, to a load in which the current is in phase with the e.m.f. at the terminals of the machine.

(b) *Excitation:* In commutating machines, rectifying machines and synchronous machines, the regulation shall be determined under such conditions as to maintain the field adjustment constant at a value which gives rated-load voltage at rated-load current. These conditions are as follows:

In the case of separately excited fields: constant excitation.

In the case of shunt machines: constant resistance in the shunt-field circuit.

In the case of series or compound machines: constant resistance shunting the series field windings.

4394.* Tests and Computation of Regulation of A-C. Generators. — (a) *Methods Available:* The regulation of alternating-current generators may be determined by any one of the three following methods, which are given in the order of preference:

(b) *Method I. By Loading:* The regulation can be measured directly, by loading the generator at the specified output and power factor, then reducing the load to zero, and measuring the terminal voltage, with speed and excitation adjusted to the same values as before the change. This method is not generally applicable for shop tests, particularly on large generators and it becomes necessary to determine the regulation from such other tests as can be readily made.

*(c) *Method II. From Test Curves:* This method consists in computing the

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(4361 b) Series-field windings should be regarded as part of the armature circuit and tested as such.

(4361 c) By normal induced voltage is here meant the voltage between slip rings on open circuit at standstill with normal voltage impressed on the primary.

(4361 f) The present National Electrical Code limit for a single outlet is 660 watts.

(4394 c) Method II for deducing the load-saturation curve, at any assigned power factor, from no-load and zero power-factor saturation curves obtained by test, must be

regulation from experimental data of the open-circuit saturation curves and the zero-power-factor saturation curve. The latter curve, or one approximating very closely to it, can be obtained by over-exciting the generator while carrying a load of idle-running under-excited synchronous motors. The power factor under these conditions is very low and the load-saturation curve approximates very closely the zero-power-factor saturation curve. From this curve and the open-circuit curve, points for the load-saturation curve for any specified power factor can be obtained by means of vector diagrams.

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regarded as empirical. Its value depends upon the fact that experience has demonstrated the reasonable correctness of the results obtained by it.

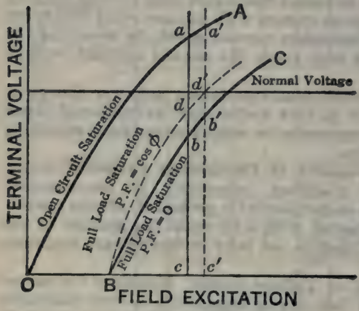

Fig. 4-3

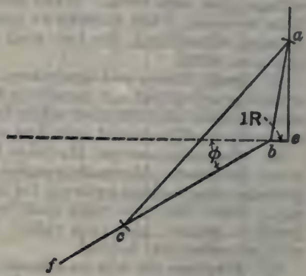

Fig. 4-4

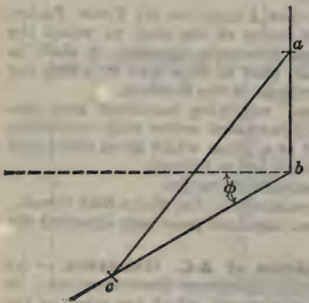

Fig. 4-5

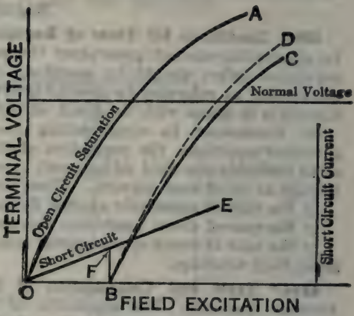

Fig. 4-6

To apply Method II, it is necessary to obtain from test the open-circuit saturation curve Fig. 4-3, and the load-saturation curve $B\ C$ at zero power factor and rated-load current. At any given excitation $O\ c$, the voltage that would be induced on open circuit is $c\ a$, the terminal voltage at zero power factor is $c\ b$ and the apparent internal drop is $a\ b$. The terminal voltage $c\ d$ at any other power factor can then be found by drawing an e.m.f. diagram as in Fig. 4-4, where ϕ is an angle such that $\cos\phi$ is the power factor of the load, $b\ e$ the resistance drop ($I\ R$) in the stator winding, $b\ a$ the total internal drop and $a\ c$ the total induced voltage; $b\ a$ and $a\ c$ being laid off to correspond with the values obtained from Fig. 4-3. The terminal voltage at power factor $\cos\phi$ is then $c\ b$ Fig. 4-4, which when laid off in Fig. 4-3 gives point d. By finding a number of such points, the curve $B\ d\ d'$ for power factor $\cos\phi$ is obtained and the regulation at this power factor (expressed in per cent) is $\frac{100 \times a'\ d'}{c'\ d'}$ since $a'\ d'$ is the rise in voltage when the load at power factor $\cos\phi$ is thrown off at normal voltage $c'\ d'$.

Generally, the ohmic drop can be neglected as it has little influence on the regulation, except in very low-speed machines where the armature drop is relatively high or in some

*(d) *Method III. From Estimated Zero-power-factor Curve.* Where it is not possible to obtain by test a zero-power-factor curve as in Method II this curve can be estimated closely from open-circuit and short-circuit curves, by reference to tests at zero power factor on other machines of similar magnetic circuit. Having obtained the estimated zero-power-factor curve, the load saturation for any other power factor is obtained as in Method II, § 4394 (c).

4395. Compound Wound D-C. Generator. — In determining the regulation of a compound-wound d-c. generator, two tests shall be made, one bringing the load down and the other bringing the load up, between no-load and rated load. These may differ somewhat, owing to residual magnetism. The mean of the two results shall be used.

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cases where regulation at unity power factor is being estimated. For low power factors its effect is negligible in practically all cases. If resistance is neglected, the simpler diagram Fig. 4-5, may be used.

(4394 d) Method III is the same as Method II except that the zero-power-factor curve must be estimated. This may be done as follows. In Fig. 4-6, $O\,A$ is the open-circuit saturation curve and $O\,E$ the short-circuit line as obtained from test. The zero-power-factor curve corresponding to any current $B\,F$ will start from point B, and for machines designed with low saturation and low reactance, will follow parallel to $O\,A$ as shown by the dotted curve $B\,D$, which is $O\,A$ shifted horizontally parallel to itself by the distance $O\,B$. In high-speed machines, or in others having low reactance, and a low degree of saturation in the magnetic circuit, the zero-power-factor curve will be quite close to $B\,D$ particularly in those parts that are used for determining the regulation. This is the case with many turbo-generators and high-speed water-wheel generators.

In many cases, however, the zero-power-factor curve will deviate from $B\,D$, as shown by $B\,C$ and the deviation will be most pronounced in machines of high reactance, high saturation and large magnetic leakage. The position of curve $B\,C$ with relation to $B\,D$ can be approximated with sufficient exactness by investigating the corresponding relation as obtained by test at zero power factor on machines of similar characteristics and magnetic circuit. Curve B C can also be calculated by methods based on the results of test at zero power factor. After curve $B\,C$ has been obtained, the load-saturation curve and regulation for any other power factor can be derived as in Method II, § 4394 (c).

CHAPTER V

STANDARDS FOR ELECTRIC RAILWAYS AND FOR AUTOMOBILE PROPULSION MACHINES

The A. I. E. E. Standards for Electric Railways and for Automobile Propulsion Machines are the General Standards shown in Chapters II and III, and the Standards in other Chapters which are applicable to the devices involved together with the modifications and extensions given in this Chapter.

DEFINITIONS

General

5000. Contact Conductors. — A contact conductor is that part of the distribution system other than the traffic rails, which is in immediate electrical contact with the circuits of the cars or locomotives.

Contact Rails

5003.* Contact Rail. — (a) *General:* A contact rail is a rigid contact conductor.

*(b) *Overhead Contact Rail:* An overhead contact rail is a contact rail which is above the elevation of the maximum equipment line.

(c) *Third Rail:* A third rail is a contact conductor placed at either side of the track, the contact surface of which is a few inches above the level of the top of the track rails.

(c) *Center Contact Rail:* A center contact rail is a contact conductor placed between the track rails, having its contact surface above the ground level.

(e) *Underground Contact Rail:* An underground contact rail is a contact conductor placed beneath the ground level.

(f) *Gage of Third Rail:* The gage of a third rail is the distance, measured parallel to the plane of running rails, between the gage line of the nearer track rail and the inside gage line of the contact surface of the third rail.

(g) *Elevation of Third Rail:* The elevation of a third rail is the elevation of the contact-surface of the third rail, with respect to the plane of the tops of running rails.

(h) *Third Rail Protection:* A third rail protection is a guard for the purpose of preventing accidental contact with the third rail.

Trolley Wires

5004. Trolley Wire. — A trolley wire is a flexible contact conductor, customarily supported above the cars.

5005. Messenger Wire or Cable. — A messenger wire or cable is a wire or cable running along with and supporting other wires, cables or contact conductors.

A primary messenger is directly attached to the supporting system. A secondary messenger is intermediate between a primary messenger and the wires, cables or contact conductors.

5006. Classes of Construction. — (a) *General:* Overhead trolley constructions are classed as *Direct Suspension* and *Messenger or Catenary Suspension.*

(b) *Direct Suspension:* A direct suspension is the form of overhead trolley construction in which the trolley wires are attached, by insulating devices, directly to the main supporting system.

(c) *Messenger or Catenary Suspension:* A messenger or catenary suspension is the form of overhead trolley construction in which the trolley wires are attached, by suitable devices, to one or more messenger cables, which in turn may be carried either in *Simple Catenary, i.e.,* by primary messengers, or in *Compound Catenary, i.e.,* by secondary messengers.

(5003 b) The maximum equipment line is the contour which embraces cross-sections of all rolling stock under all normal operating conditions.

5007. Supporting Systems. — (a) *General:* Supporting systems for trolley wires shall be classed as follows:

(b) *Simple Cross-Span Systems:* Simple cross-span systems are those having at each support a single flexible span across the track or tracks.

(c) *Messenger Cross-span Systems:* Messenger cross-span systems are those having at each support two or more flexible spans across the track or tracks, the upper span carrying part or all of the vertical load of the lower span.

(d) *Bracket Systems:* Bracket systems are those having at each support an arm or similar rigid member, supported at only one side of the track or tracks.

(e) *Bridge Systems:* Bridge systems are those having at each support a rigid member, supported at both sides of the track or tracks.

5030.* Transmission System. — When the current generated for an electric railway is changed in kind or voltage, between the generator and the cars or locomotives, that portion of the conductor system carrying current of a kind or voltage substantially different from that received by the cars or locomotives, constitutes the *transmission system.*

5031.* Distribution System. — That portion of the conductor system of an electric railway which carries current of the kind and voltage received by the cars or locomotives, constitutes the *distribution system.*

5032. Substation. — A substation is a group of apparatus or machinery which receives current from a transmission system, changes its kind or voltage, and delivers it to a distribution system.

OPERATION

Temperature Limits

5101.* Railway Motors in Continuous Service. — The following maximum observable temperatures are permissible in the windings of railway motors, when in continuous service.

TABLE 501

Temperatures of Railway Motors in Continuous Service

Class of material See § 1004	Temperature	
	By Thermometer See § 1002	By Resistance See § 1002
A	85° C.	110° C.
B	100° C.	130° C.

5120. Railway Substation Machines and Transformers. — Under conditions specified in § 5201, the windings of railway substation machines and transformers carrying traction loads may have observable temperature rises 5° C. in excess of the limiting observable temperature rises specified in Table 200.

5130.* Automobile Propulsion Machines. — On stand test, the *observable* temperature rises shall not exceed the limits specified in § 5205.

RATING

Ratings of Railway Substation Machinery and Transformers

5201.* Nominal Rating of Railway Substation Machines and Transformers. — The nominal rating of a substation machine or transformer carrying traction

(5030 and 5031) These definitions are identical in sense, although not in words, with those of the Interstate Commerce Commission, as given in their Classification of Accounts for Electric Railways.

(5101) Under extreme ambient temperatures it is permissible to operate, for short infrequent periods, at 15° C. higher temperature than specified in this rule.

(5101 and 5130) Owing to space limitations and the cost of carrying dead weight on vehicles, it is considered good practise to operate propulsion machinery at higher temperatures than would be advisable in stationary machines. (See Table 501.)

loads shall be the kv-a. output at a rated power factor input, which, having produced a constant temperature in the machine or transformer, may be increased 50 per cent for two hours, without producing temperature rises exceeding by more than 5° C. the limiting values given in Table 200. These machines or transformers should be capable of carrying a load of twice their nominal rating for a period of one minute, without disqualifying them for continuous service. The name plate should be marked "nominal rating."

Ratings of Railway Motors

5202.* Nominal Rating of Railway Motors. — The nominal rating of a railway motor shall be the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90° C. at the commutator, and 75° C. at any other normally accessible part after one hour's continuous run at its rated voltage (and frequency in the case of an alternating-current motor) on a stand with the motor covers arranged to secure maximum ventilation without external blower. The rise in temperature as measured by resistance shall not exceed 100° C. The statement of the nominal rating shall include the corresponding voltage and armature speed.

5203.* Continuous Ratings of Railway Motors. — The continuous ratings of a railway motor shall be the *inputs* in amperes at which it may be operated continuously at ½, ¾ and full voltage respectively, without exceeding the observable temperature rises specified in Table 502, when operated on stand test with motor covers and cooling system, if any, arranged as in service. Inasmuch as the same motor may be operated under different conditions as regards ventilation, it will be necessary in each case to define the system of ventilation which is used. In case motors are cooled by external blowers, the flow of air on which the rating is based shall be given.

TABLE 502

Stand-test Temperature Rises of Railway Motors

Class of material See § 1004	Temperature rises of windings	
	By Thermometer See § 1002	By Resistance See § 1002
A	65° C.	85° C.
B	80° C.	105° C.

5204. Field-control Railway Motors. — The nominal and continuous ratings of field-control motors shall relate to their performance with the operating field which gives the maximum motor rating. Each section of the field windings shall be adequate to perform the service required of it, without exceeding the specified temperature rises.

Ratings of Automobile Propulsion Machines

5205. Automobile Propulsion Machines. — The rating of automobile motors and generators shall be based upon temperature rise, on a stand test and with motor covers arranged as in service, fifteen degrees by thermometer or twenty-five degrees by resistance, above those of Table 200.

(5201 and 5202) In the absence of any specification as to the kind of rating the "nominal" rating shall be understood.
(5203) The temperature rise in service may be very different from that on stand-test. See § 5502 for the relation between stand-test and service temperatures as affected by ventilation.

Ratings of Electric Locomotives

5210. Rating. — Locomotives shall be rated in terms of the weight on drivers, nominal one-hour tractive effort, continuous tractive effort and corresponding speeds.

5211. Weight on Drivers. — The weight on drivers, expressed in pounds, shall be the sum of the weights carried by the drivers and of the drivers themselves.

5212. Nominal Tractive Effort. — The nominal tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers when the motors are operating at their nominal (one-hour) rating.

5213. Continuous Tractive Effort. — The continuous tractive effort, expressed in pounds, shall be that exerted at the rims of the drivers when the motors are operating at their full-voltage continuous rating, as indicated in § 5203.

In the case of locomotives operating on intermittent service, the continuous tractive effort may be given for ½ or ¾ voltage, but in such cases the voltage shall be clearly specified.

5214. Speed. — The rated speed, expressed in miles per hour, shall be that at which the continuous tractive effort is exerted.

TESTS

Efficiency

Losses in D-C. Railway Motors

5337. Losses in Gearing and Axle Bearings. — The losses in gearing and axle bearings for single-reduction single-geared motors, varies with the type, mechanical finish, age and lubrication. The following values, based upon accumulated tests, shall be used in the comparison of single-reduction single-geared motors § 5339.

TABLE 503

Losses in Axle Bearings and Single-reduction Gearing of Railway Motors

Per cent of input at nominal rating	Loses as per cent of input
200	3.5
150	3.0
125	2.7
100	2.5
75	2.5
60	2.7
50	3.2
40	4.4
30	6.7
25	8.5

Note. — Further investigation may indicate the desirability of giving separate values of the losses for full and tapped fields, or low- and high-speed motors.

5338. Brush Friction, Armature-bearing Friction and Windage. — The brush friction, armature-bearing friction and windage shall be determined as a total under the following conditions:

In making the test, the motor shall be run without gears. The kind of brushes and the brush pressure shall be the same as in commercial service. Drive the machine idle as a series motor on low voltage. The product of armature counter-

electromotive-force and amperes at any speed shall be the sum of the above losses at that speed. See § 5339.

5339.* No-load Core Loss, Brush Friction, Armature-bearing Friction and Windage. — The no-load core loss, brush friction, armature-bearing friction and windage shall be determined as a total under the following conditions:

In making the test, the motor shall be run without gears. The kind of brushes and the brush pressure shall be the same as in commercial service. With the field separately excited, such a voltage shall be applied to the armature terminals as will give the same speed for any given field current as is obtained with that field current when operating at normal voltage under load. The sum of the losses above-mentioned, is equal to the product of the counter-electromotive force and the armature current.

The no-load core loss is obtained by deducting from the total losses thus obtained the power required to drive the motor at corresponding speeds as determined under § 5338.

The core loss under load shall be assumed to have the values given in Table 504.

TABLE 504

Core Loss in D-C. Railway Motors at Various Loads

Per cent of input at nominal rating	Loss as per cent of no-load core loss
200	165
150	145
100	130
75	125
50	123
25 and under	122

Note. — With motors designed for field control the core losses shall be assumed as the same for both full and permanent field. It shall be the mean between the no-load losses at full and permanent field, increased by the percentages given in the above table.

5341. Automobile Motors. — When automobile motors are of low voltage, the great influence of brush-contact losses on the efficiency requires that these losses be determined experimentally for the type of brush used.

(5339) In comparing projected railway motors, and in case it is not possible or desirable to make tests to determine mechanical losses, the following values of these losses, determined from the averages of many tests over a wide range of sizes of single-reduction single-geared motors, will be found useful, as approximations. They include axle-bearing, gear, armature-bearing, brush-friction, windage, and stray-load losses.

TABLE 505

Approximate Losses in D-C. Railway Motors

Input in per cent of that at nominal rating	Losses as per cent of input
100 or over	5.0
75	5.0
60	5.3
50	6.5
40	8.8
30	13.3
25	17.0

The core loss of railway motors may also be determined as specified for other machines.

CHARACTERISTIC CURVES OF RAILWAY MOTORS

5401. General. — The Characteristic Curves of railway motors shall be plotted with the current as abscissas and the tractive effort, speed and efficiency as ordinates. In the case of a-c. motors, the power factor shall also be plotted as ordinates.

5402. Voltage. — Characteristic curves of direct-current motors shall be based upon full voltage, which shall be taken as 600 volts, or a multiple thereof.

5403. Field-Control Motors. — In the case of field-control motors, characteristic curves shall be given for all operating field connections.

SELECTION OF RAILWAY MOTOR FOR SPECIFIED SERVICE

5501. Data Required in Selecting Motor. — The following information, relative to the service to be performed, is required, in order that an appropriate motor may be selected.

(a) Weight of total number of cars in train (in tons of 2000 lb.) exclusive of electrical equipment and load.
(b) Average weight of load and durations of same, and maximum weight of load and durations of same.
(c) Number of motor cars or locomotives in train, and number of trailer cars in train.
(d) Diameter of driving wheels.
(e) Weight on driving wheels, exclusive of electrical equipment.
(f) Number of motors per motor car.
(g) Voltage at train with power on the motors—average, maximum and minimum.
(h) Rate of acceleration in miles per hour per second.
(i) Rate of braking (in miles per hour per second).
(j) Speed limitations, if any (including slowdowns).
(k) Distances between stopping points.
(l) Average duration of stops.
(m) Schedule speed, including stops, in miles per hour.
(n) Train resistance in pounds per ton of 2000 pounds at stated speeds.
(o) Moment of inertia of revolving parts, exclusive of electrical equipment.
(p) Profile and alignment of track.
(q) Distance coasted as a percentage of the distance between stopping points.
(r) Duration of layover at end of run, if any.

5502.* Method of Comparing Motor Capacity with Service Requirements. — When it is not convenient to test motors under actual specific service conditions, recourse may be had to the following method of determining temperature rise from the stand-tests.

The essential motor losses affecting temperatures in service are those in the motor windings, core and commutator. The mean service conditions may be expressed, as a close approximation, in terms of that continuous current and core loss which will produce the same losses and distribution of losses as the average in service.

(5502) Calculation for comparing motor capacity with service requirements. The heating of a motor should be determined, wherever possible, by testing it in service, or with an equivalent duty-cycle. When the service or equivalent duty-cycle tests are not practicable, the ratings of the motor may be utilized as follows to determine its temperature rise.

The motor losses which affect the heating of the windings are as stated above, those in the windings and in the core. The former are proportional to the square of the current. The latter vary with the voltage and current, according to curves which can be supplied by the manufacturers. The procedure is therefore as follows:

(a) Plot a time-current curve, a time-voltage curve, and a time-core loss curve for the duty-cycle which the motor is to perform, and calculate from these the root-mean-square current and the average core loss.

(b) If the calculated r.m.s. service current exceeds the continuous rating, when run with average service core loss and speed, the motor is not sufficiently powerful for the duty-cycle contemplated.

A stand-test with the current and voltage which will give losses equal to those in service, will determine whether the motor has sufficient capacity to meet the service requirements. In service, the temperature rise of an enclosed motor (§ 4044), well exposed to the draught of air incident to a moving car or locomotive, will be from 75 to 90 per cent (depending upon the character of the service) of the temperature rise obtained on a stand-test with the motor completely enclosed and with the same losses. With a ventilated motor (§ 4045 and § 4046), the temperature rise in service will be 90 to 100 per cent of the temperature rise obtained on a stand-test with the same losses.

In making a stand-test to determine the temperature rise in a specific service, it is essential in the case of a self-ventilated motor (§ 4046) to run the armature at a speed which corresponds to the schedule speed in service. In order to obtain this speed it may be necessary, while maintaining the same total armature losses, to change somewhat the ratio between the I^2R and core-loss components.

CONSTRUCTION

5601.* Standard Height of Trolley Wire on Street and Interurban Railways. — It is recommended that supporting structures shall be of such height that the lowest point of the trolley wire shall be at a height of 18 feet (5.5 m.) above the top of rail under conditions of maximum sag, unless local conditions prevent. On trackage operating electric and steam road equipment and at crossings over steam roads, it is recommended that the trolley wire shall be not less than 21 feet (6.4 m.) above the top of rail, under conditions of maximum sag.

5602. Standard Gage of Third Rails. — The gage of third rails shall be not less than 26 inches (66 cm.) and not more than 27 inches (68.6 cm.).

5603. Standard Elevation of Third Rails. — The elevation of third rails shall not be less than 2¾ inches (7 cm.) and not more than 3½ inches (8.9 cm.).

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(c) If the calculated r.m.s. service current does not exceed the continuous rating, when run with average service core loss and speed, the motor is ordinarily suitable for the service.

In some cases, however, it may not have sufficient thermal capacity to avoid excessive temperature rises during the periods of heavy load. In such cases a further calculation is required, the first step of which is to compute the equivalent voltage which, with the r.m.s. current, will produce the average core loss. Having obtained this, determine, as follows, the temperature rise due to the r.m.s. service current and equivalent voltage.

Let $\left.\begin{array}{l}\theta_2 = \text{temperature rise} \\ p_0 = IR \text{ loss, kw.} \\ p_c = \text{core loss, kw.}\end{array}\right\}$ with r.m.s. service current, and equivalent service voltage.

$\left.\begin{array}{l}\theta_2 = \text{temperature rise} \\ P_0 = I^2R \text{ loss, kw.} \\ P_c = \text{core loss, kw.}\end{array}\right\}$ with continuous load current corresponding to the equivalent service voltage.

Then

$$\theta = \theta_2 \frac{p_0 + p_c}{P_0 + P_c}, \text{ approximately.}$$

(d) The thermal capacity of a motor is approximately measured by the ratio of the electrical loss in kw. at its nominal (one-hour) capacity, to the corresponding maximum observable temperature rise during a one-hour test starting at ambient temperature.

(e) Consider any period of peak load and determine the electrical losses in kilowatt-hours during that period from the *electrical* efficiency curve. Find the excess of the above losses over the losses with r.m.s. service current and equivalent voltage. The excess loss, divided by the coefficient of thermal capacity, will equal the extra temperature rise due to the peak load. This temperature rise added to that due to the r.m.s. service current, and equivalent voltage, gives the total temperature rise. If the total temperature rise in any such period exceeds the safe limit, the motor is not sufficiently powerful for the service.

(f) If the temperature reached, due to the peak loads, does not exceed the safe limit, the motor may yet be unsuitable for the service, as the peak loads may cause excessive sparking and dangerous mechanical stresses. It is, therefore, necessary to compare the peak loads with the short-period overload capacity. If the peaks are also within the capacity of the motor, it may be considered suitable for the given duty-cycle.

(5601) A. E. R. A. Standard.

Foreign

Engineering Standards Committee of Great Britain: Standardization Rules for Electrical Machinery (exclusive of motors for traction purposes). Standard Method of Specifying the Resistance of Steel Conductor Rails; Standard for Trolley Groove and Wire.

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CHAPTER VI

STANDARDS FOR TRANSFORMERS AND OTHER STATIONARY INDUCTION APPARATUS

Wherever the General Standards in Chapters II and III apply to transformers they are referred to in the following Chapter by cross references.

Certain rules applying exclusively to railway machinery have, for convenience, been placed in Chapter V with cross references in all case to this Chapter. Rules in Chapter VI apply to railway machinery except as they are modified by the rules in Chapter V.

Note: The word "Transformer" will be used throughout this Chapter as an abbreviation of "Transformer or other stationary induction apparatus."

DEFINITIONS

Apparatus

6000. Stationary Induction Apparatus. — For the purpose of these Standards, stationary induction apparatus is defined as electric apparatus which changes electric energy to electric energy through the medium of magnetic energy, without mechanical motion. It comprises several forms, as defined in §§ 6001 and 6010 to 6015.

6001. Transformer. — A transformer is a form of stationary induction apparatus in which the primary and secondary windings are ordinarily insulated one from another.

6010. Auto-transformer. — An auto-transformer is one which has a part of its turns common to both primary and secondary circuits.

6011. Voltage-regulator. — A voltage-regulator is a form of stationary induction apparatus which has turns in shunt and turns in series with the circuit, so arranged that the voltage ratio of the transformation, or the phase relation between the circuit-voltages, is variable at will.

6012. Contact Voltage-regulator. — A contact voltage-regulator is a voltage regulator in which the number of turns in one or both of the coils is adjustable.

6013. Induction Voltage-regulator. — An induction voltage-regulator is one in which the relative position of the primary and secondary coils is adjustable.

6014. Magneto Voltage-regulator. — A magneto voltage-regulator is one in which the direction of the magnetic flux with respect to the coils is adjustable.

6015. Reactor. — A reactor is a device used primarily because it possesses the property of reactance. Reactors are used in electric circuits for purposes of operation, protection or control.

Parts of Apparatus

6020. High-voltage and Low-voltage Winding. — The terms "high voltage" and "low voltage" are used to distinguish the winding having the greater from that having the lesser number of turns.

6021. Primary and Secondary Windings. — The terms "primary" and "secondary" serve to distinguish the windings in regard to energy flow, the primary being that which receives the energy from the supply circuit, and the secondary that which receives the energy by induction from the primary.

6022. Full Capacity Tap.—A full capacity tap is a tap from a transformer winding on which the unit may be operated at rated kv-a. capacity without exceeding the specified temperature rise.

6023. Reduced Capacity Tap.—A reduced capacity tap is one on which the unit may not be operated at full capacity without exceeding the specified temperature rise.

Properties of Apparatus

6031. Rated Current of a Constant-potential Transformer. — The rated current of a constant-potential transformer is that secondary current which, multiplied by the rated-load secondary voltage, gives the kv-a. rated output. That is, a transformer of given kv-a. rating must be capable of delivering the rated output at rated secondary voltage, while the primary impressed voltage is increased to whatever value is necessary to give rated secondary voltage.

6032. Full or Reduced Capacity Taps.—Where transformers are provided with taps from either the high-voltage or low-voltage windings, a definite statement shall be made as to whether such are full capacity or reduced capacity taps.

6033. Rated Primary Voltage of a Constant-potential Transformer. — The rated primary voltage of a constant-potential transformer is the rated secondary voltage multiplied by the turn ratio.

6034. Ratio of a Transformer. — The ratio of a transformer, unless otherwise specified, shall be the ratio of the number of turns in the high-voltage winding to that in the low-voltage winding; *i.e.*, the "turn-ratio."

6035. Voltage Ratio of a Transformer. — The voltage ratio of a transformer is the ratio of the r.m.s. primary terminal voltage to the r.m.s. secondary terminal voltage, under specified conditions of load.

6036. Current Ratio of a Transformer. — The current ratio of a current-transformer is the ratio of the r.m.s. primary current to the r.m.s. secondary current, under specified conditions of load.

6037. Volt-ampere Ratio of Transformer. — The volt-ampere ratio, which should not be confused with real efficiency, is the ratio of the volt-ampere output to the volt-ampere input of a transformer at any given power factor.

6050.* Per Cent Resistance Drop. — The per cent resistance drop in a transformer is the ratio of the internal resistance drop at 75° C. to the secondary terminal voltage expressed in per cent.

6051.* Per Cent Reactance Drop. — The per cent reactance drop in a transformer is the ratio of the internal reactance drop to the secondary terminal voltage expressed in per cent.

6052.* Per Cent Impedance Drop. — The per cent impedance drop in a transformer is the ratio of the internal impedance drop at 75° C. to the secondary terminal voltage expressed in per cent.

6053. Regulation of Constant-Potential Transformer. — In constant-potential transformers, the regulation is the difference between the no-load and rated-load values of the secondary terminal voltage at the specified power factor (with constant primary impressed terminal voltage) expressed in per cent of the rated-load secondary voltage, the primary voltage being adjusted to such a value that the apparatus delivers rated output at rated secondary voltage. See § 6390.

Ambient Temperature. — See § 3000.

(6050-6051-6052) The internal drop in a transformer is the sum of the primary drop (reduced to secondary terms) and the secondary drop.

RATING

General

6201 TABLE 601

Limiting Observable Temperatures and Temperature Rises for Transformers Using Class A* Insulation

	†Air cooled and air blast	Oil cooled	Water cooled
Limiting observable temperature	95° C.	95° C.	80° C.
Standard ambient temperature	40° C.	40° C.	25° C.
Limiting observable temperature rise	**55° C.**	**55° C.**	**55° C.**

* For cotton, silk, paper and similar materials when neither treated, impregnated nor immersed in oil, the limits of the observable temperature rise shall be 15° C. below the limits fixed for these materials when impregnated.

† For exceptions in the case of Airblast Transformers, see § 6320 (b).

The temperature of the windings of transformers is always to be ascertained by Method 2.

6202. Limiting Observable Temperature of Oil (From § 2232). — The oil in which apparatus is permanently immersed shall, in no part, have a temperature, observable by thermometer, in excess of 90° C.

Permissible Temperatures of Insulations of More than One Class. — See § 2104.

Temperatures of Metallic Parts of Transformers. — See § 2116.

Protection Against Short Circuit. — See § 2120.

Nominal Rating of Railway Substation Transformers. — See § 5201.

Expression of Rating. — See § 2202.

Institute Rating. — See § 2204.

6204.* Rating of Protective Reactors. — Protective reactors shall be rated by the following characteristics:

(a) Kilovolt-amperes absorbed by normal current.

(b) Normal current, frequency and line (delta) voltage.

(c) Current which the device is required to stand under short-circuit conditions.

Ambient Temperature of Reference

Ambient Temperature of Reference for Air. — See § 2211.

Ambient Temperature of Reference for Water-cooled Transformers. — See § 2212.

Transformers Cooled by Other Means. — See § 2213.

Outdoor Transformers Exposed to Sun's Rays. — See § 2214.

Altitude Correction

Altitude. — See § 2215.

6215. Exception to "Altitude." — See § 2215. — Water-cooled oil-immersed transformers are exempt from this reduction.

Units in Which Rating Shall be Expressed

6221. Rating of Transformers. — The rating of transformers shall be expressed in kilovolt-amperes (kv-a.) available at the output terminals, at a specified frequency and voltage.

6223. Rating of Other Stationary Induction Apparatus. — Other stationary induction apparatus, such as auto-transformers, regulators, reactors, etc., shall have their ratings appropriately expressed. It is also essential to specify the voltage and frequency of the circuits on which the apparatus may be used.

(6204) Reactors shall be so designed as to be capable of withstanding the sudden application, without mechanical injury, of rated current at normal frequency.

Kinds of Rating

Continuous Rating. — See § 2220.
Short-time Rating. — See § 2221.
Duty-cycle Operation. — See § 2222.
Standard Short-time Ratings. — See § 2223.
A. I. E. E. and I. E. C. Ratings. — See § 2224.
Continuous Rating Implied. — See § 2225.

6236. Nominal Ratings. — Nominal ratings are ratings which do not conform with §§ 2220 and 2221. They are sometimes used for railway substation transformers carrying traction loads. Transformers with nominal rating shall be capable of operating under the conditions enumerated in § 5201.

Rating by Temperature Rise

Permissible Temperature Rises for Various Ambient Temperatures above Standard. — See § 2231 (c).

TESTS

Ambient Temperature

Measurement of Ambient Temperatures during Tests of Transformers. — See § 2300.

6300. Measurement of the Ambient Temperature During Tests of Water-cooled Transformers. — The temperature rise of water-cooled transformers shall be based entirely upon the temperature of the cooling water and it is not necessary to take into account the heat carried off by the air, unless it exceeds the amount specified below. If, under assumed standard conditions of water at 25° C., and air at 40° C., the amount of heat which would be carried off by the air is 15 per cent or more of the total, the temperature of the cooling water, during test, should be maintained within 5° C. of that of the surrounding air. Where this is impracticable the ambient temperature should be determined from the change in the resistance of the windings, using a disconnected transformer, supplied with the normal amount of cooling water, until the temperature of the windings has become constant.

Oil Cup. — See § 2301.

Transformer Temperatures

Temperature Rise for Any Ambient Temperature. — See § 2310.

Correction for the Duration of the Ambient Temperature of the Cooling Medium, at the Time of the Heat Test, from the Standard Ambient Temperature of Reference. — See § 2311.

6311.* Correction for the Deviation of the Ambient Temperature of the Cooling Medium, at the Time of the Heat Test of Air-blast Transformers from the Standard Ambient Temperature of Reference. — A correction shall be applied to the observed temperature rise of the windings of air-blast transformers due to difference in resistance, when the temperature of the ingoing cooling air differs from that of the standard of reference. This correction shall be the ratio of the inferred absolute ambient temperature of reference to the inferred absolute temperature of the ingoing cooling air, *i.e.*, the ratio $274.5/(234.5+\theta)$; where θ is the ingoing cooling-air temperature.

Duration of Temperature Test of Transformers for Continuous Service. — See § 2312.

Duration of Temperature Test of Transformer with a Short-time Rating. — See § 2313.

Duration of Temperature Test for Transformer Having More than One Rating. — See § 2314.

Temperature Measurements During Heat Run. — See § 2315.

(6311) Thus, a cooling-air room temperature of 30° C. would correspond to an inferred absolute temperature of 264.5° on the scale of copper resistivity, and the correction to 40° C. (274.5° inferred absolute temperature) would be $274.5/264.5 = 1.04$, making the correction factor 1.04; so that an observed temperature rise of say 50° C. at the testing ambient temperature of 30° C. would be corrected to $50 \times 1.04 = 52$° C., this being the temperature rise which would have occurred had the test been made with the standard ingoing cooling-air temperature of 40° C.

6317. Methods of Loading Transformers for Temperature Tests. — (a) *General:* Whenever practicable, transformers should be tested under conditions that will give losses approximating as nearly as possible to those obtained under normal or specified load conditions, maintained for the required time. The maximum temperature rises measured during this test should be considered as the observable temperature rises for the given load. See §§ 2312 to 2314.

An approved method of making these tests is the *loading-back* method. The principal variations of this method are given in § 6317 (b), (c) and (d).

(b) *Loading-back with duplicate single-phase transformers:* Duplicate single-phase transformers may be tested in banks of two, with both primary and secondary windings connected in parallel. Normal magnetizing voltage should then be applied and the required current circulated from an auxiliary source. One transformer can be held under normal voltage and current conditions while the other may be operating under slightly abnormal conditions.

(c) *Loading-back with one three-phase transformer:* One three-phase transformer may be tested in a manner similar to § 6317 (b) provided the primary and secondary windings are each connected in delta for the test. Normal three-phase magnetizing voltage should be applied and the required current circulated from an auxiliary single-phase source.

(d) *Loading-back with three single-phase transformers:* Duplicate single-phase transformers may be tested in banks of three in a manner similar to that described in § 6317 (c), by connecting both primary and secondary windings in delta, applying normal three-phase magnetizing voltage and circulating the required current from an auxiliary single-phase source.

(e) *Other Methods:* Among other methods that have a limited application and can be used only under special conditions may be mentioned:

Applying dead load by means of some form of rheostat.

Running alternately for certain short intervals of time on open circuit and then on short-circuit, alternating in this way until the transformer reaches a steady temperature. In this test, the voltage for the open-circuit interval and the current for the short-circuit interval shall be such as to give the same integrated core loss, and the same integrated copper loss as in normal operation.

6320. Method of Temperature Measurement. — (a) *Description:* The temperature of transformer windings shall be measured by their increase in resistance, corrected to the instant of shut-down when necessary, and by thermometers. Whichever measurement yields the higher temperature, that temperature shall be taken as the highest observable temperature by Method 2.

(b) In the case of air-blast transformers, it is important to have the thermometers well distributed and in good contact with the coils, and it is especially important to note the temperature near the air outlet. In measuring the temperature of air-blast transformers, the air supply shall be shut off immediately at the end of the temperature run and the air intake closed to prevent further admission of cooling air. With the above procedure, the observable temperature rise for air-blast transformers may attain a value not in excess of 60° C. as determined by thermometer, although it must not exceed 55° C. as determined by resistance.

(c) *Temperature Correction for Cooling of Transformer Windings after Shut-down:* Since a drop in temperature occurs in a winding between the instant of shut-down and the time of measuring the hot resistance, a correction shall be applied to the temperature determined from this measurement so as to obtain, as nearly as practicable, the temperature at the instant of shut-down. This correction may be determined approximately by plotting a time-temperature curve with temperatures as ordinates and times as abscissas and extrapolating back to the instant of shut-down.

In cases where successive measurements show increasing temperatures after shut-down the highest value shall be taken.

In certain cases, however, other correction factors may be applied as follows:

Oil-Immersed Transformers: For the purpose of simplifying the application of the rule to transformers when the weight of copper in each winding is known and the copper loss as determined by wattmeter measurement does not exceed 30 watts per pound, the extrapolation method has been reduced to the following form which is recommended on account of the greater accuracy obtainable

under ordinary conditions of testing. The correction in degrees C. shall be the product of the watts loss per pound of copper for each winding multiplied by a factor depending upon the time elapsed between shut-down and the time of the temperature reading as given in the following table:

Time in minutes	Factor
1	0.19
2	0.32
3	0.43
4	0.50

For intermediate times, the value of the factor can be obtained by interpolation.

When the copper loss, measured by wattmeter, does not exceed 7 watts per pound an arbitrary correction of one degree per minute may be used provided the time elapsed between the instant of shut-down and the measurement of the hot resistance does not exceed four minutes.

For determining the copper loss in watts per pound, the total loss in both windings as measured by the wattmeter should be apportioned between the high- and low-voltage windings in the same ratio as their respective I^2R losses.

Air-blast Transformers: An arbitrary correction of one degree per minute may be used provided the time elapsed between the instant of shut-down and the measurement of the hot resistance does not exceed four minutes.

(d) *Covering of Thermometers:* Thermometers used for taking the temperature of air-cooled or air-blast transformers shall have their bulbs covered for protection from air currents. This shall be done by felt pads, approximately 4 cm. × 5 cm. (1½ in. × 2 in.) and 3 mm. (⅛ in) thick, except that where pads are inconvenient, as in ventilating ducts between coils, grooved wooden sticks may be used.

Temperature Coefficient of Copper. — See § 2321.

Efficiency

Efficiencies Recognized. — See § 2331.

Normal Conditions for Efficiency Tests. — See § 2332.

Direct Measurement of Efficiency. — See § 2333.

6334. Classification of Losses. — (a) *General:* Losses are classified as shown below.

(b) *No-load Losses:* No-load losses include the core loss, the I^2R loss due to the exciting current and the dielectric loss in the insulation.

(c) *Load Losses:* Load losses include I^2R losses, and stray load-losses due to eddy-currents caused by fluxes varying with load.

6335. Losses to be Considered in Transformers. — Conventional efficiencies shall be based upon the losses listed in § 6334 and these losses shall be measured as specified in §§ 6336 and 6337.

6336. No-load Losses. — The no-load losses shall be measured with open secondary circuit at the rated frequency, and with an applied primary voltage giving the rated secondary voltage plus the IR drop which occurs in the secondary under rated-load conditions.

6337. Load Losses. — The load losses include I^2R and stray load-losses. They shall be measured by applying a primary voltage, at rated frequency, sufficient to produce rated load current in the windings, with the secondary windings short-circuited.

Wave Shape

Standard Wave Shape. — See § 2340.

Tests of Dielectric Strength

Condition of Transformers to be Tested. — See § 2350.

Where High-voltage Tests are to be Made. — See § 2351.

Temperature at which High-voltage Tests are to be Made. — See § 2352.

Points of Application of Voltage. — See § 2353.

Frequency and Wave Form of Test Voltage. — See § 2354.

Duration of Application of Test Voltage. — See § 2355.

6356. Standard Test Voltage. — (From § 2356.) *General:* The standard test voltage for all machines, except as otherwise specified, shall be twice the normal voltage of the circuit to which the machine is connected plus 1000 volts. See exception § 6361.

6360. Transformers for Star Connection. — Transformers which may be used in star connection on three-phase circuits shall be tested on the basis of the line to line voltage for which they are rated. See § 6361(f).

6361.* Exceptions to Standard Test Voltage Given in Section 6356. — (a) *Distributing Transformers:* Transformers for primary pressures from 550 to 4500 volts, the secondaries of which are directly connected to consumer's circuits and commonly known as distributing transformers, shall be tested with 10,000 volts from primary to core and secondary combined. The secondary windings shall be tested with twice their normal voltage plus 1000 volts.

(b) *Auto-transformers:* Auto-transformers used for starting purposes, shall be tested with the same voltage as the test voltage of the apparatus to which they are connected.

(c) *Household Devices:* Transformers taking not over 660 watts and intended solely for operation on supply circuits not exceeding 275 volts, shall be tested with 900 volts.

*(d) *Transformers for Use on Circuits of 25 Volts or Lower:* Transformers for use on circuits of 25 volts or lower, such as bell-ringing apparatus, shall be tested with 500 volts.

(e) *Alternating-current Transformers Connected to Permanently Grounded Single phase Systems, for use on Permanently Grounded Circuits of more than 300 Volts:* Transformers used under these conditions shall be tested with 2.73 times the voltage of the circuit to ground plus 1000 volts. This does not refer to three-phase transformers operating with grounded neutral.

(f) *Transformers to be used on Star-connected Three-phase Circuits:* Transformers which may be used in star connection on three-phase circuits shall have the line to line (as distinguished from line to neutral) voltage of the circuits on which they may be used indicated on the rating plate and the test shall be based on the line to line voltage. See § 6360.

(g) *Testing Transformers:* Rules do not apply to testing transformers, for which no definite rule has yet been established.

(h) *Protective Reactors:* Protective reactors shall be tested from conductors to ground with 2000 volts plus 2¼ times the line voltage.

6362.* Testing Transformers by Induced Voltage. — Under certain conditions it is permissible to test transformers by inducing the required voltage in their windings in place of using a separate testing transformer. By " required voltage " is meant a voltage such that the line end of the winding shall receive a test to ground equal to that required by the general rules.

6363. Transformers with Graded Insulation. — Where transformers have graded insulation they shall be so marked. They shall be tested by inducing the required test voltage in the transformer and connecting the successive line leads to ground.

Transformer windings permanently grounded within the transformer shall be tested by inducing the required test voltage in such windings. See § 6361.

Use of Voltmeter and Spark-gaps in Dielectric Tests. — See § 2359.

Use of Spark-gap with Transformers of Low Capacitance. — See § 2360.

Use of Spark-gap with Transformers of High Capacitance. — See § 2361.

Measurements with Voltmeter. — See § 2362.

Measurements with Spark-gap. — See § 2363.

(6361 d) This rule does not include bell-ringing transformers of ratio 125 to 6 volts.

(6362) This test can be made by connecting the windings of two or more transformers in series, with one end of the series grounded and a voltage impressed such as will give the test from the free end to ground required by the above rule.

(6391 a) This method is not generally applicable for shop tests, particularly on large transformers.

Regulation

Conditions for Tests of Regulation. — See § 2390.

6390. Conditions for Test of Regulation. — (a) *Frequency:* The regulation of transformers is to be determined at constant frequency.

(b) *Power Factor:* In transformers, the power factor of the load to which the regulation refers should be specified. Unless otherwise specified, it shall be understood as referring to non-inductive load, that is, to a load in which the current is in phase with the e.m.f. at the output side of the transformer. See § 2390.

6391.* Tests and Computation of Regulation. — (a) *Method I. By Loading:* The regulation of a constant potential transformer can be determined by loading the transformer and measuring the change in voltage with change in load, at the specified power factor.

(b) *Method II. From Impedance Watts and Volts:* The regulation of a constant potential transformer for any specified load and power factor can be computed from the measured impedance watts and impedance volts as follows:

Let P = impedance watts, as measured in the short-circuit test,

E_z = impedance volts, as measured in the short-circuit test,

P_{75} = impedance watts, as measured in the short-circuit test, and corrected to 75° C.,

IX = Reactance Drop in Volts,

I = Rated Primary Current,

E = Rated Primary Voltage,

qr = per cent drop in phase with current,

qx = per cent drop in quadrature with current,

$$IX = \sqrt{E_z^2 - \left(\frac{P}{I}\right)^2},$$

$$qr = 100\frac{P_{75}}{EI},$$

$$qx = 100\frac{IX}{E}.$$

Then:

For unity power factor, we have approximately,

$$\text{Per cent regulation} = qr + \frac{qx^2}{200}.$$

For *inductive loads* of power-factor m and reactive-factor n,

$$\text{Per cent regulation} = mqr + nqx + \frac{(mqx - nqr)^2}{200}.$$

CONSTRUCTION

Rating Plates

Marking of Rating Plates. — See § 2401.

Transformer Connections

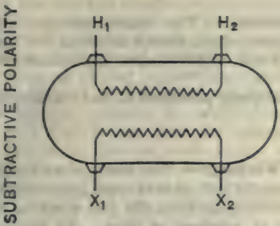
(*These rules do not apply to auto-transformers*)

General:

6402.* Scope. — These rules specify the markings of leads brought out of the case but not the markings of winding terminals inside of the case, except that these terminals shall be marked with numbers in any manner that will permit of convenient reference and that cannot be confused with the markings of the leads brought out of the case.

(6402) It is recognized that special cases will arise from time to time that these rules will not cover and that it would be very difficult to cover by any set of general rules.

TRANSFORMER LEAD MARKINGS SINGLE-PHASE TRANSFORMERS



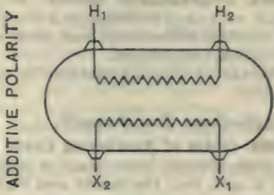
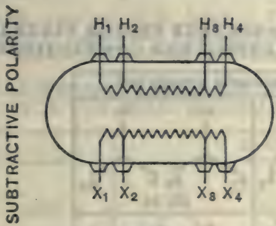


Fig. 6-1 Fig. 6-2

Simple High- and Low-voltage Windings without Taps



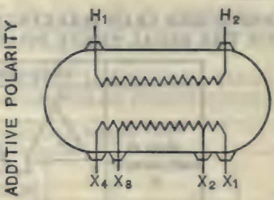


Fig. 6-5 Fig. 6-4

Simple High- and Low-voltage Windings with Taps

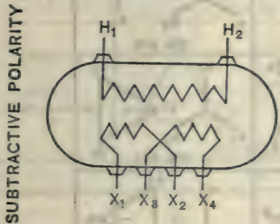


Fig. 6-3

Series Multiple Low-voltage Winding without Taps

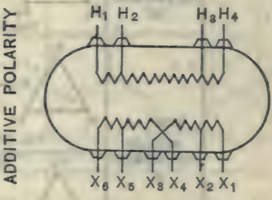


Fig. 6-6

Series Multiple Low-voltage Winding with Taps

Note. — The above figures illustrate the application of the rules on lead markings to transformers having subtractive and additive polarity.

6403.* Markings of Leads. — *(a) *General:* The leads shall be distinguished from one another by marking each lead with a capital letter followed by a number. The letters to be used are: *H* for high-voltage leads, *X* for low-voltage leads and *Y* for tertiary winding leads. The numbers to be used are 1, 2, 3, etc.

(6403 a) By "tertiary winding" is meant a third winding that, compared with both of the other two windings, has smaller kv-a. rating than either or, if the kv-a. rating is the same as one or both of the other two, has lower voltage. *E.g.*, if a transformer has three separate windings, one for 1000 kv-a., 33,000 volts, one for 600 kv-a., 550 volts and one for 400 kv-a. 6,600 volts, the 400 kv-a. winding is the tertiary winding; or, if a transformer has three separate windings each with a capacity of 1,000 kv-a., and with voltages of 33,000, 6,600 and 550 respectively, the 550-volt winding is the tertiary winding.
According to this definition neither one of two similar windings arranged for series-parallel connection is to be classed as a tertiary winding.

*(b) *Neutral Lead:* A neutral lead shall be marked with the proper letter followed by *O*, *e.g.*, *HO*, *XO*.

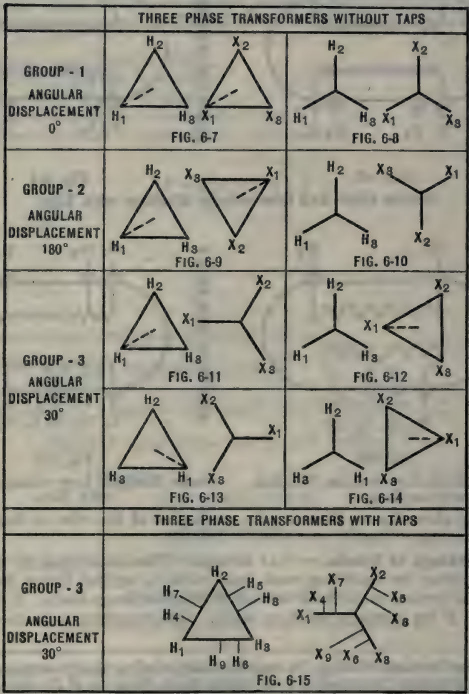
6404. Diagrammatic Sketch of Connections. — The manufacturer shall furnish with each transformer a complete diagrammatic sketch showing the leads and internal connections and their markings and the voltages obtainable with the various connections.

This sketch should preferably be on a metal plate attached to the transformer case.

Single-phase Transformers:

6405. Order of Numbering Leads in any Winding. — The leads of any winding (high-voltage, low-voltage or tertiary) brought out of case shall be numbered 1, 2, 3, 4, 5, etc., the lowest and highest numbers marking the full winding and the intermediate numbers marking fractions of winding or taps. All numbers shall be so applied that the potential difference from any lead having a lower number toward any lead having a higher number shall have the same sign at any instant.

TRANSFORMER LEAD MARKINGS AND VOLTAGE VECTOR DIAGRAMS FOR THE USUAL THREE-PHASE TRANSFORMER CONNECTIONS



Note. — The above figures are included to illustrate the method of marking transformer leads that are brought out of the case and are not intended to standardize connections, vector diagrams or polarity.

(6403 b) A lead brought out from the middle of a winding for some other use, than that of neutral lead, *e.g.*, a 50 per cent starting tap, shall be marked as a tap lead.

If a winding is divided into two or more parts for series-parallel connections, and the leads of these parts are brought out of case, the above rule shall apply for the series connection with the addition that the leads of each portion of winding shall be given consecutive numbers. See Figs. 6-5 and 6-6.

6406. Relation of Order of Numbering Leads of Different Windings. — The numbering of the high-voltage and low-voltage leads shall be so applied that when H_1 and X_1 are connected together and voltage applied to the transformer, the voltage between the highest numbered H lead and the highest numbered X lead shall be less than the voltage of the full high-voltage winding.

The same relation shall apply between high-voltage and tertiary and low-voltage and tertiary winding.

6407. Polarity. — When leads are marked in accordance with the above rules, the polarity of a transformer is:

Subtractive when H_1 and X_1 are adjacent. See Figs. 6-1, 6-3 and 6-5.

Additive when H_1 is diagonally located with respect to X_1. See Figs. 6-2, 6-4 and 6-6.

6408. Location of H_1 Lead. — To simplify the work of connecting transformers in parallel it is recommended that the H_1 lead shall be brought out on the right-hand side of the case, facing high-voltage side of the case.

TRANSFORMER LEAD MARKINGS AND VOLTAGE VECTOR DIAGRAMS FOR THE USUAL SIX-PHASE TRANSFORMER CONNECTIONS

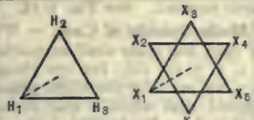
<table>
<tr><td></td><td colspan="2">SIX PHASE TRANSFORMERS WITHOUT TAPS</td></tr>
<tr><td>GROUP - 4
ANGULAR DISPLACEMENT
0°</td><td>

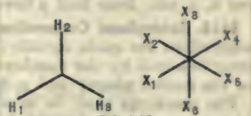
FIG. 6-16</td><td>

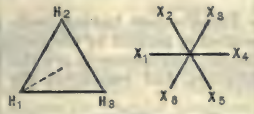
FIG. 6-17</td></tr>
<tr><td>GROUP - 5
ANGULAR DISPLACEMENT
30°</td><td>

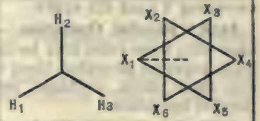
FIG. 6-18</td><td>

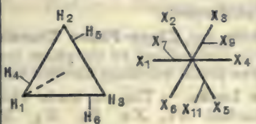
FIG. 6-19</td></tr>
<tr><td></td><td colspan="2">SIX PHASE TRANSFORMERS WITH TAPS</td></tr>
<tr><td>GROUP - 5
ANGULAR DISPLACEMENT
30°</td><td>

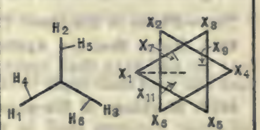
FIG. 6-20</td><td>

FIG. 6-21</td></tr>
</table>

Note. — The above figures are included to illustrate the method of marking transformer leads that are brought out of the case and are not intended to standardize connections, vector diagrams or polarity.

6409.* Parallel Operation. — Transformers having leads marked in accordance with these rules may be operated in parallel by connecting similarly marked leads together, provided their ratio, voltages, resistances and reactances are such as to permit parallel operation.

(6409) In some cases design may be such as to permit parallel operation, although due to the difference in the number of tap leads, the leads to be connected together may not have the same number.

Three-phase Transformers:

6410. Marking of Full Winding Leads. — The three high-voltage leads and the three low-voltage leads which connect to the full-phase windings, shall be marked H_1, H_2, H_3, and X_1, X_2, X_3. The full-phase winding of a tertiary winding shall be marked Y_1, Y_2, Y_3.

6411.* Relation between High-voltage and Low-voltage Windings. — (*a*) *General:* The markings shall be so applied that if the phase sequence of voltage on the high-voltage side is in the time order H_1, H_2, H_3 it is in the time order of, X_1, X_2, X_3 on the low-voltage side and Y_1, Y_2, Y_3 for a tertiary winding.

*(b) *Angular Displacement:* In order that the markings of lead connections between phases shall indicate definite phase relations, they shall be made in accordance with one of the three three-phase groups as shown. The angular displacement between the high-voltage and low-voltage windings is the angle in each of the voltage vector diagrams (Figs. 6-7 to 6-14 inclusive) between the lines passing from its neutral point through H_1 and X_1 respectively.

6412. Tap Leads. — (a) *General:* Where tap leads are brought out of the case (neutral lead excepted) they shall be marked with the proper letter followed by the numbers, 4, 7, etc., for one phase, 5, 8, etc., for another phase and 6, 9, etc., for the third phase. See Fig. 6-15.

(b) *Delta Connection:* The order of numbering tap leads shall be as follows: 4, 7, etc., from lead 1 toward lead 2; 5, 8, etc., from lead 2 toward lead 3; and 6, 9, etc., from lead 3 toward lead 1. See Fig. 6-15.

(c) *Star Connection:* The order of numbering tap leads shall be as follows: 4, 7, etc., from lead 1 towards neutral; 5, 8, etc., from lead 2 towards neutral; and 6, 9, etc., from lead 3 towards neutral. See Fig. 6-15.

6413. Interphase Connection Made Outside of Case. — Where the interphase connections are made outside of case, the leads shall be marked with the proper letter followed by the numbers 1, 4, 7, 10, etc., for one phase; 2, 5, 8, 11, etc., for the second phase; and 3, 6, 9, 12, etc., for the third phase.

The markings shall be so applied that when a star connection is made by joining together the highest numbered leads of each phase, all rules here given, excepting § 6403 (b) apply.

6414.* Parallel Operation. — Transformers having leads marked in accordance with these rules may be operated in parallel by connecting similarly marked leads together provided their angular displacements are the same and provided also their ratios, voltages, resistances, and reactances are such as to permit parallel operation.

6415. Location of H1 Lead. — To simplify the work of connecting transformers in parallel it is recommended that the $H1$ lead shall be brought out on the right-hand side of the case, facing the high-voltage side of the case.

Three-Phase to Six-phase Transformers:

6416. Rules that are Applicable for Three-Phase Transformers. — Sections 6411 (b) and 6413 shall apply to three-phase to six-phase transformers. Rules 6410 and 6412 shall apply to three-phase windings but not to six-phase windings.

6417. Markings of Six-phase Leads. — The six leads which connect to the full-phase windings shall be marked $X1$, $X2$, $X3$, $X4$, $X5$, $X6$. See Figs. 6-16 to 6-19 inclusive.

6418. Relation between Three-phase and Six-phase Windings. — (a) *General:* The markings shall be so applied that if the phase sequence of voltage

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(6411 b) Any three-phase transformer having a delta-Y connection may be represented by voltage vector diagram either in accordance with Fig. 6–11 or Fig. 6–13. Any three-phase transformer having Y-delta connection may be represented by voltage vector diagram either in accordance with Fig. 6–12 or Fig. 6–14. Since these voltage vector diagrams are equivalent, it is recommended that the terminal markings for three-phase transformers having delta-Y connection be always made in accordance with Fig. 6–11 and that the terminal markings for three-phase transformers having Y-delta connection be always made in accordance with Fig. 6–12.

(6414) In some cases designs may be such as to permit parallel operation although, due to a difference in the number of tap leads, the leads to be connected together are not similarly marked.

on the three-phase side is in the time order $H1$, $H2$, $H3$, it is in the time order of $X1$, $X2$, $X3$, $X4$, $X5$, $X6$ on the six-phase side.

(b) *Angular Displacement:* In order that the markings of lead connections between phases shall indicate definite phase relations, they shall be made in accordance with one of the four six-phase groups shown in Figs. 6-16 to 6-19 inclusive. The angular displacement between the high-voltage and low-voltage windings is the angle in each of the voltage vector diagrams from its neutral through $H1$ and $X1$ respectively.

6419.* Tap Leads. — (a) *General:* Where tap leads from low-voltage windings are brought out of the case (neutral lead excepted), they shall be marked as follows:

(b) *Diametrical Connection:* Diametrical connection tap leads shall be marked from the two ends of each phase winding towards the middle or neutral point in the following order; $X7$, $X13$, etc., from $X1$ towards neutral; $X8$, $X14$, etc., from $X2$ towards neutral; $X9$, $X15$, etc., from $X3$ towards neutral; $X10$, $X16$, etc., from $X4$ towards neutral; $X11$, $X17$, etc., from $X5$ towards neutral; $X12$, $X18$, etc., from $X6$ towards neutral. See Fig. 6-20.

A tap from the middle point of any phase winding, not intended as a neutral, shall be given a number determined by counting from $X1$, $X2$ or $X3$ and not from $X4$, $X5$, or $X6$; *e.g.*, if the only taps brought out are 50 per cent starting taps, they shall be numbered $X7$, $X8$, and $X9$.

*(c) *Double Delta Connection:* Tap leads shall be marked in the following order; $X7$, $X13$, etc., from $X1$ towards $X3$; $X8$, $X14$, etc., from $X2$ towards $X4$; $X9$, $X15$, etc., from $X3$ towards $X5$; $X10$, $X16$, etc., from $X4$ towards $X6$; $X11$, $X17$, etc., from $X5$ towards $X1$; $X12$, $X18$, etc., from $X6$ towards $X2$. See Fig. 6-21.

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CHAPTER VII

STANDARDS FOR SWITCHING, CONTROL AND PROTECTIVE APPARATUS

The A. I. E. E. Standards for Switching Control and Protective Apparatus are the General Standards shown in Chapters II and III and the Standards in other Chapters which are applicable to the devices involved, together with the modifications and extensions given in this Chapter.

(6419 c) For starting purposes it is generally customary to bring out only two taps from one delta and start three-phase.

DEFINITIONS

Devices—General

7000.* Switching and Control Apparatus. — For the purpose of these Standardization Rules switching and control apparatus is defined as electric apparatus whose function is primarily to control or protect in some predetermined manner electric apparatus to which it is connected.

7001. Switch. — A switch is a device for making, breaking or changing the connections in an electric circuit.

7002. Master-switch. — A master-switch is a switch which serves to govern the operation of contactors and auxiliary devices of an electric controller.

7003. Control Switch. — A control switch is a switch for controlling electrically-operated switches and circuit breakers.

7004. Auxiliary Switch. — An auxiliary switch is a switch actuated by some main device, for signalling, interlocking, etc.

7005. Circuit Breaker. — A circuit breaker is a device (other than a fuse) constructed primarily for the interruption of a circuit under *infrequent abnormal* conditions.

7006. Contactor. — A contactor is a device for repeatedly establishing and interrupting an electric circuit under normal conditions.

7007.* Electric Controller. — An electric controller is a device, or group of devices, which is designed to control in some predetermined manner the operation of the apparatus to which it is connected.

7008.* Motor-starter. — A motor-starter is an electric controller designed for accelerating a motor to normal speed in one direction of rotation.

7009. Automatic Motor-starter. — An automatic motor-starter is a motor-starter designed to automatically control the acceleration of a motor.

7010. Auto-transformer Motor-starter. — An auto-transformer motor-starter is a motor-starter having an auto-transformer to furnish a reduced voltage for starting. The device includes the necessary switching mechanism, and is frequently called a Compensator or Auto-starter.

7015.* Fuse. — A fuse is an element designed to melt or dissipate at a predetermined current value, and intended to protect against abnormal conditions of current.

7016. Electric Protective Relay.—An intermediate device, equipped with contacts to open or close an auxiliary circuit, by means of which one circuit is indirectly controlled by a change in conditions in the same or other circuits.

7017. Rheostat. — A rheostat is a resistor which is provided with means for readily varying its resistance. See § 3064.

7018. Protective Reactor. — A protective reactor (See § 3078) is a device for protecting circuits by limiting the current flow and localizing the disturbance under short-circuit conditions.

(7000) The "National Electrical Code" of the National Fire Protection Association deals with certain circuit breakers up to 550 volts rating and switches and fuses up to 600 volts rating fuses.

(7007) A switch (see § 7001) should not be called a controller.

(7008) A device designed for starting a motor in either direction of rotation is called a controller (see § 7007).

(7015) Any terminals, tubes, etc., integral with this element are included as part of the fuse.

Fuses may be divided into two classes:

(a) Those designed to protect the circuit and apparatus both against short-circuit and against definite amounts of overload (*e.g.* fuses of the National Electric Code which open on overload in excess of 10 per cent).

(b) Those designed to protect the system only against short-circuits; (*e.g.*, expulsion fuses, which blow at several times the current which they are designed to carry continuously). The line separating these two classes is not definitely fixed.

7019.* Lightning Arrester. — A lightning arrester is a device for protecting circuits and apparatus against lightning or other abnormal potential rises of short duration.

7020. Under-voltage or Low-voltage Release Switching and Control Apparatus. — Under-voltage or low-voltage release switching and control apparatus is apparatus which, on the reduction or failure of voltage, operates to cause the interruption of power to the main circuit, but which does not prevent the re-establishment of the main circuit on return of voltage.

7021. Under-voltage or Low-voltage Protection Switching and Control Apparatus. — Under-voltage or low-voltage protection switching and control apparatus is apparatus which, on the reduction or failure of voltage, operates to cause and maintain the interruption of power to the main circuit.

7022. Phase-failure Protection Switching and Control Apparatus. — Phase-failure protection switching and control apparatus is apparatus which, on the failure of power in one wire of a polyphase circuit, operates to cause and maintain the interruption of power on the circuit.

7023. Phase-reversal Protection Switching and Control Apparatus. — Phase-reversal protection switching and control apparatus is apparatus which, on the reversal of the phase relations in a polyphase circuit, operates to cause and maintain the interruption of power on the circuit.

Electric Protective Relays—Classified According to Functions

7026. Directional Relay.—Any relay which functions in conformance with direction of power or voltage or current or phase rotation, etc.

7027. Power-Directional Relay.—Any relay which functions in conformance with direction of power.

Note.—This includes both unidirectional relays with single-throw contacts and duodirectional relays with double-throw contacts. The reason this name is preferred to "reverse power" is that the device is frequently used to function under normal direction of power. Furthermore, in some cases, the normal condition of the system may permit power to flow in either direction. Relays for use in either alternating or direct-current circuits are to be classed as power-directional relays.

7028. Polarity-directional Relay.—Any relay which functions by reason of a change of the direction of polarity.

7029. Phase-rotation Relay. — Any relay which functions by reason of a change of the direction of phase rotation.

7030. Current Relay. — Any relay which functions at a predetermined value of the current. These may be either over-current relays or under-current relays.

7031. Voltage Relays. — Any relay which functions at a predetermined value of the voltage. These may be either over-voltage relays or under-voltage relays.

7032. Power Relay. — Any relay which functions at a predetermined value of the power. These may be either over-power relays or under-power relays.

7033. Frequency Relay. — Any relay which functions at a predetermined value of the frequency. These may be either over-frequency relays or under-frequency relays.

7034. Temperature Relay. — Any relay which functions at a predetermined temperature in the apparatus protected.

7035. Open-phase Relay. — Any relay which functions by reason of the opening of one phase of a polyphase circuit.

(7019) Lightning arresters may be divided into two classes:
(a) Those intended to discharge for a very short time.
(b) Those intended to discharge for a period of several minutes.

7036. Differential Relay. — Any relay which functions by reason of the difference between two quantities, such as current or voltage, etc.

NOTE.—This term includes relays heretofore known as "ratio balance relays," "biased," and "percentage differential relays."

Electric Protective Relays—Classified According to Applications

7040. Locking Relay. — Any relay which renders some other relay or other device inoperative under predetermined values of current or voltage, etc.

7041. Trip-free Relay. — Any relay which prevents holding in an electrically operated device, such as a circuit breaker, while an abnormal condition exists on the circuit.

7042. Auxiliary Relay. — Any relay which assists another relay in the performance of its function and which operates in response to the opening or closing of its operating circuit.

7043. Signal Relay. — An auxiliary relay which operates an audible or a visible signal.

Electric Protective Relays—General Qualifying Terms

7050. Notching. — A qualifying term applied to any relay, indicating that a number of separate impulses are required to complete operation.

7051. Inverse Time. — A qualifying term applied to any relay indicating that there is purposely introduced a delayed action, which delay decreases as the operating force increases.

7052. Definite Time. — A qualifying term applied to any relay indicating that there is purposely introduced a delayed action, which delay remains substantially constant regardless of the magnitude of the operating forces. (For forces slightly above the minimum operating value, the delay may be inverse.)

7053. Instantaneous. — A qualifying term applied to any relay indicating that no delayed action is purposely introduced.

7054. Use of Terms. — Where relays operate in response to changes in more than one condition, all functions should be mentioned.

Characteristics of Devices—General

7060. "Air" as a Prefix. — The prefix "air" applied to a device which interrupts an electric circuit indicates that the interruption occurs in air.

7061. "Oil" as a Prefix. — The prefix "oil" applied to a device which interrupts an electric circuit indicates that the interruption occurs in oil.

7062. Fume-resisting. — Fume-resisting switching and control apparatus is apparatus so constructed that it will not be readily injured by the specified fumes.

7063.* Drip-proof. — Drip-proof switching and control apparatus is apparatus so protected as to exclude falling moisture or dirt.

7064. Dust-proof. — Dust-proof switching and control apparatus is apparatus so constructed or protected that the accumulation of dust within or without the device will not interfere with its successful operation.

7065. Dust-tight. — Dust-tight switching and control apparatus is apparatus so constructed that the dust will not enter the enclosing case.

7066. Explosion-proof. — Explosion-proof switching and control apparatus is apparatus so constructed that explosions of gas within the casing will not injure it or ignite inflammable gas outside it.

7067. Gas-proof. — Gas-proof switching and control apparatus is apparatus so constructed or protected that the specified gas will not interfere with its successful operation.

(7063) Drip-proof apparatus may be either open or semi-enclosed, if it is provided with suitable protection integral with the apparatus, or so enclosed as to exclude effectively falling solid or liquid material.

7068. Gas-tight. — Gas-tight switching and control apparatus is apparatus so constructed that the specified gas will not enter the enclosing case.

7069. Moisture-resistant. — Moisture-resistant switching and control apparatus is apparatus so constructed or treated that it will not be readily injured by moisture. (Such apparatus shall be capable of operating in a very humid atmosphere, such as found in mines, evaporating rooms, etc.).

7070. Splash-proof. — Splash-proof switching and control apparatus is apparatus so constructed or protected that external splashing will not interfere with its successful operation.

7071. Submersible. — Submersible switching and control apparatus is apparatus so constructed that it will operate successfully when submerged in water under specified conditions of pressure and time.

7072. Sleet-proof. — Sleet-proof switching and control apparatus is apparatus so constructed or protected that the accumulation of sleet will not interfere with its successful operation.

Parts of Devices—General

7080. Conducting Parts. — Conducting parts of switching and control apparatus are those designed to carry current or which are conductively connected therewith.

7081. Contact. — A contact is a surface common to two conducting parts, united by pressure, for the purpose of carrying current.

7082. Magnet Brake. — A magnet brake is a friction brake controlled by electro-magnetic means.

7083. Grounded Parts. — Grounded parts are those parts which may be considered to have the same potential as the earth.

Properties of Devices—General

7090. Interrupting Rating. — Interrupting (breaking or rupturing) rating is a rating based upon the r.m.s. current at normal voltage which the device can interrupt under prescribed conditions at stated intervals a specified number of times.

OPERATION

Temperature Limits

7101.* Circuit Breakers, Relays and Switches. — The maximum observable temperature rises of the various parts of circuit breakers, relays and switches shall not exceed the following limits for ambient temperatures up to and including but not greater than 40° C. See § 7301.

Contacts in air when clean and bright	30° C.
Contacts in oil	30° C.
Oil	40° C.
Potential Coils, Class O insulation	35° C.
Series Coil, Class O insulation	50° C.
Series and Potential Coils, Class A insulation	50° C.
Series and Potential Coils, bare or Class B insulation	70° C.
All other parts	70° C.

7102. Magnetic Contactors. — The maximum observable temperature rises of the various parts of magnetic contactors shall not exceed the following limits

(7101) The Institute calls attention to the inherent decrease in current which can be carried by switch and circuit-breaker contacts in air, due to oxidization of the contact surfaces. The rating of air switches and circuit breakers is, therefore, based on sufficient maintenance to keep the temperature rise within the specified limits. Relays which form part of controllers are to have the temperature limits specified in § 7102.

for ambient temperatures up to and including but not greater than 40° C. See § 7302.

Laminated contacts	65° C.
Operating coils	70° C.
Solid contacts	100° C.
Current-carrying parts insulated with asbestos or other fireproof material	150° C.

7105.* Fuses. — The maximum observable temperature rise of coils or windings, measured by thermometer, shall not exceed the following limits for ambient temperatures up to and including but not greater than 40° C.

If insulation is of unimpregnated fibrous material	35° C.
If insulation is of fibrous material treated to withstand heat	50° C.
If insulation is of asbestos, mica or similar heat-resisting material with a cotton binder	70° C.

7106. Cast Grid Resistors. — The maximum observable temperature rises of cast grids used as resistors shall not exceed 350° C. for ambient temperatures up to and including but not greater than 40° C.

RATING

Expression of Rating

7201. Rating of Circuit Breakers and Switches. — The rating of a circuit breaker or switch shall include the following items:

(a) the normal r.m.s. current which it is designed to carry.

(b) the normal r.m.s. voltage of the circuit on which it is intended to operate.

(c) the normal frequency of the current.

(d) the interrupting rating of the device. See § 7060.

7202. Continuous Current-carrying Capacity of Fuses. — Fuses shall be so constructed that they will carry continuously 110 per cent of their rated current.

7205. Rating of Lightning Arresters. — The rating of a lightning arrester shall be the voltage of the circuit on which it is to be used.

TESTS

Heat Tests

7301. Circuit Breakers, Relays and Switches. — The rated current of circuit breakers, relays and switches at rated frequency shall be applied continuously until the temperature becomes constant The temperature rises measured by thermometer or thermocouple (Method No. 1) placed on the surface of the part to be measured shall not exceed the limits specified in § 7101.

The ambient temperature shall be determined by taking the average of the readings of three thermometers placed as follows:

One 12 inches above;
One 12 inches below;
One midway but 12 inches from the breaker as installed.

7302. Magnetic Contactors.— The rated current of magnetic contactors at rated frequencies shall be applied continuously or until the temperature becomes constant when continuous duty is specified. It shall be applied for the specified length of time when given a short time rating. The temperature rises measured by thermometer shall not exceed the limits specified in § 7102.

Tests of Dielectric Strength

7323.* Standard Test Voltage. — (a) *Apparatus rated at* 600 *volts or less:* The standard test voltage for all switching and control apparatus rated at 600 volts or less shall be twice the normal voltage of the circuit to which the apparatus is to be connected plus 1000 volts.

(7105) Coils or windings such as accompany fuses of the magnetic blow-out type.

(7323) This assumes a precipitation of 1/10th inch (2.54 mm.) per minute at an angle of 45° from the perpendicular with water having a resistivity as low as 7000 ohm-centimeters.

(b) *Apparatus rated above 600 volts:* Apparatus rated above 600 volts shall be tested at $2\frac{1}{4}$ times rated voltage, plus 2000 volts, at a specified altitude.

*As a supplementary test, devices for outdoor use should be capable of withstanding for 10 seconds a dielectric wet test at twice rated voltage plus 1000 volts.

(c) *Auto-Transformers for Motor-starters:* Auto-transformers for motor-starters shall be tested with the same voltage as the test voltage of the apparatus to which they are to be connected.

Tests of Lightning Arresters.

7371. Resistance. — The resistance of the arrester at double potential and also at normal potential, shall be determined by observing the discharge currents through the arrester.

7372. Arrester with Gap. — In the case of any arrester using a gap, a test shall be made of the spark potential on either direct-current or 60 cycle a-c. excitation.

7373. Equivalent Sphere Gap. — The equivalent sphere gap under disruptive discharge shall be measured, using a considerable quantity of electricity.

7374. Continuous Surges. — The endurance of the arrester to continuous surges shall be tested.

7375. Dielectric Strength. — See §§ 2355 and 7323.

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CHAPTER VIII

STANDARDS FOR METERS, INSTRUMENTS AND INSTRUMENT TRANSFORMERS

The A.I.E.E. Standards for Meters, Instruments and Instrument Transformers are the General Standards shown in Chapters II and III, and the Standards in other Chapters which are applicable to the devices involved, together with the modifications and extensions given in this Chapter.

DEFINITIONS

8000.* Meter. — A meter is a device which registers through a totalizing mechanism, the integral, with respect to time, of the electrical quantity to which it responds. (This definition does not preclude the general use of " meter " as a suffix or in compound words, to mean a " measuring device.")

8001.* Instrument. — An instrument is a device which indicates or records the present value of the quantity under observation.

8002. General Nomenclature. — In general, the names of meters and instruments are self-defining. The following names are preferred to others some-

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(8000 and 8001) While the word "instrument" is a general term which may properly include indicating, integrating and recording devices, there is a tendency to restrict its use to indicating devices and to recording (graphic or curve drawing) devices. Integrating devices are then denoted by the word "meter." This distinction gives rise to the above general definitions.

times used for the same devices: Reactive-factor Meter, Power-factor Meter, Watthour Meter, Reactive Volt-Ammeter (or Reactive Volt-ampere Indicator) etc.

8003. Recording Instruments. — Recording ammeters, voltmeters, wattmeters, etc., are instruments which record graphically, upon time charts, the values of the quantities they measure.

8004.* Crest Voltmeter. — A crest voltmeter is a voltmeter depending for its indications upon the crest, or maximum value of the voltage of the system to which it is connected. Crest voltmeters shall be marked in true crest volts and also in the r.m.s. value of the sinusoidal wave having the same crest value. (See § 2362.)

8005. Synchronoscope (also called a Synchroscope or Synchronism Indicator). — A synchronoscope is a device which indicates synchronism between two machines, and in addition shows whether the incoming machine is fast or slow.

8006. Line-drop Voltmeter Compensator. — A line-drop voltmeter compensator is a device used in connection with a voltmeter which causes the latter to indicate the voltage at some distant point of the circuit.

8007. Demand-meter. — (a) *General:* A demand-meter is a device which indicates or records the demand or maximum demand. In practise, two types are recognized. See §§ 3454, 3458, 3460 and 3464.

(b) *Integrated-Demand-Meter:* An integrated-demand-meter is a demand-meter which indicates or records the maximum demand obtained through integration.

(c) *Lagged-Demand-Meter:* A lagged-demand-meter is a demand-meter in which the indication of maximum demand is subject to a characteristic time lag.

8020.* Period of an Instrument. — The period of an instrument, sometimes called the "periodic time," is the time taken for the pointer to make one complete oscillation (two consecutive swings). A swing is a complete movement in either direction.

8030. Instrument Transformer. — An instrument transformer is a transformer suitable for use with measuring instruments; that is, one in which the conditions of phase and of current or potential in the primary circuit, are represented with acceptable accuracy in the secondary circuit. An instrument transformer may be either an instrument current transformer or an instrument potential (voltage) transformer.

8031.* Secondary Burden. — The secondary burden of a current transformer is an expression in ohms and henrys of the resistance and inductance of the external circuit connected to the secondary of that transformer.

8032. Voltage Ratio of Instrument Transformer. — The voltage ratio of an instrument potential transformer is the ratio of the r.m.s. primary terminal voltage to the r.m.s. secondary terminal voltage, under specified secondary burden.

8033. Current Ratio of Instrument Transformer. — The current ratio of an instrument current transformer is the ratio of r.m.s. primary current to r.m.s. secondary current, under specified secondary burden.

8034. Marked Ratio of Instrument Transformer. — The marked ratio of an instrument transformer is the ratio which the apparatus is designed to give under average conditions of use. When a precise ratio is required, it is necessary to specify the voltage or current, frequency, load and secondary burden.

(8004) Corona voltmeter (see Transactions of the A. I. E. E., Volume XXXIX, page 1057) is recognized as a satisfactory form of crest voltmeter.

(8020) In strongly damped instruments, the period is influenced by the amplitude of the movement.

(8031) Considerable uncertainty of meaning has been occasioned by the use of the terms, load, secondary load, and secondary connected load for this quantity, and such use is discouraged.

OPERATION

8101. Permissible Temperature in Shunts. — (a) *General:* The limiting observable temperature of shunts measured by Method I shall not exceed 120° C.

(b) *Exceptions:* The above rule shall not apply to shunts having no soldered joint and made of material which is not permanently changed in resistance if continuously subjected to a higher temperature.

8110. Grounding of Meters and Instruments. — The covers of meters and instruments, which are used with current and potential transformers, shall be connected to the grounded sides of the secondary circuits of such transformers in all cases where the indications of the instrument are liable to be influenced by electrostatic action.

8111. Instrument Current Transformers on Open Secondary Circuit.— Under conditions of open secondary circuit, current transfomers shall be capable of carrying continuously rated primary current without damage to the primary insulation and without interruption of service.

8112. Instrument Current Transformers on Closed Secondary Circuit. — Under conditions of closed secondary circuit, current transformers shall withstand 40 times rated current applied for 1 second, without injury.

RATING

8200. General. — The rating of a meter is a designation assigned by the manufacturer to indicate its operating limitations. The full scale marking of an instrument does not necessarily correspond to its rating, but if the rating differs from the full scale marking, the rating shall be marked on the instrument.

8201. Standard Ambient Temperature.— For purposes of rating meters and shunts, the standard ambient temperature shall be 40° C. See §§ 8301 and 2211.

8202. Rating Limitation of the Circuits of Meters and Instruments. — No circuit of a meter or instrument shall be given a rating higher than that corresponding to the maximum current or voltage to which it may be continuously subjected.

8203.* Temperature Rise of Meter and Instrument Windings. — The permissible temperature rises in meters and instruments shall be based upon the temperatures specified in § 1005 and the standard ambient temperature of 40° C.

8204. Temperature Rise in Shunts. — Shunts shall be rated in accordance with their observable temperature rise by Method 1, assuming the ultimate temperatures specified in § 8101 and an ambient temperature specified in § 8201.

TESTS

8300. Measurement of Temperature Rise of Shunts. — Observable temperature shall be measured in such a manner as not to cause local change of temperature.

8301. Standard Temperature of Reference for Meter and Instrument Characteristics. — The standard temperature of reference for meter and instrument characteristics shall be 20° C. See §§ 8201, 2211.

8302. Damping. — The pointer being at zero before any load is applied, damping shall be measured by suddenly applying and maintaining a load which will give a steady deflection of one-half full angular scale, and observing the following quantities:

(a) The number of swings taken by the pointer in coming to rest.

(8203) Heating is frequently an immaterial consideration in determining the rating of meters and instruments. Losses, impairment of accuracy and other factors often determine the rating.

(b) The time, in seconds, required for the pointer to come to rest.
(c) The overshooting, in per cent of the angular displacement due to the disturbance.

Dielectric Strength of Instrument Transformers.

8310. Test Voltage Instrument Potential Transformers. — The test voltage for instrument potential transformers shall be twice the normal voltage of the circuit to which it is connected plus 1000 volts.

8311. Test Voltage of Instrument Current Transformers. — The test voltage of instrument current transformers shall be 2¼ times the rated voltage plus 2000 volts.

8312. Test Voltage for Meters and Instruments. — Meters and instruments which have a potential circuit shall withstand a test of twice potential of the circuit plus 1000 volts. Instruments without a potential circuit shall withstand a test of 1000 volts.

SPECIFICATION OF CHARACTERISTICS

8500. Errors of Indicating Instruments. — In specifying the accuracy of an indicating instrument, the error at any point on the scale shall be expressed as a percentage of the full scale reading.

8501. Torque. — The torque of meters and instruments shall be expressed in millimeter-grams.

8502. Damping. — The damping of an instrument shall be expressed in terms of the quantities enumerated in § 8302, all three of which are essential to a complete description.

8503.* Marking of Switchboard Shunts. — The marking of switchboard shunts shall include the rating in amperes, the drop in volts at that rating, and the serial number of any instrument in connection with which the shunt may be calibrated. When shunts are designed to be used with devices taking sufficient current to be an appreciable proportion of the whole, this fact shall be indicated.

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(8503) For example, if with 100 amperes rated load in the main circuit, a measuring device takes 10 amperes, leaving 100 less 10 amperes in the shunt with a drop of 0.050 volts, the shunt shall be marked: Volts 0.050. Amperes 100 less 10.

CHAPTER IX

STANDARDS FOR WIRES AND CABLES

DEFINITIONS

9000.* Wire. — A wire is a slender rod or filament of drawn metal.

9001.* Conductor. — A conductor is a wire or combination of wires not insulated from one another, suitable for carrying a single electric current.

9002.* Stranded Conductor. — A stranded conductor is a conductor composed of a group of wires, or of any combination of groups of wires.

9003. Strand. — A strand is one of the wires, or groups of wires, of any stranded conductor.

9004.* Cable. — A cable is either a stranded conductor (single-conductor cable), or a combination of conductors insulated from one another (multiple-conductor cable).

9005.* Stranded Wire. — A stranded wire is a group of small wires, used as a single wire.

9006.* Cord. — A cord is a small, flexible, insulated cable.

9007. Concentric Strand. — A concentric strand is a strand composed of a central core surrounded by one or more layers of helically-laid wires or groups of wires.

9008. Concentric-lay Cable. — A concentric-lay cable is a single-conductor cable composed of a central core surrounded by one or more layers of helically-laid wires.

9009.* Rope-lay Cable. — A rope-lay cable is a single-conductor cable composed of a central core surrounded by one or more layers of helically-laid groups of wires.

(9000) The definition restricts the term to what would ordinarily be understood by the term "solid wire." In the definition, the word "slender" is used in the sense that the length is great in comparison with the diameter. If a wire is covered with insulation, it is properly called an insulated wire; while primarily the term "wire" refers to the metal, nevertheless when the context shows that the wire is insulated, the term "wire" will be understood to include the insulation.

(9001) The term "conductor" is not to include a combination of conductors insulated from one another, which would be suitable for carrying several different electric currents. Rolled conductors (such as bus-bars) are, of course, conductors, but are not considered under the terminology here given.

(9002) The wires in a stranded conductor are usually twisted or braided together.

(9004) The first kind of cable is a single conductor, while the second kind is a group of several conductors. The component conductors of the second kind of cable may be either solid or stranded, and this kind of cable may or may not have a common insulating covering. The term "cable" is applied by some manufacturers to a solid wire heavily insulated and lead-covered; this usage arises from the manner of the insulation, but such a conductor is not included under this definition of "cable." The term "cable" is a general one, and in practise, it is usually applied only to the larger sizes. A small cable is called a "stranded wire" or a "cord," both of which are defined below. Cables may be bare or insulated, and the latter may be armored with lead, or with steel wires or bands.

(9005) A wire has been defined as a slender rod or filament of drawn metal. If such a filament is subdivided into several smaller filaments or strands, and is used as a single wire, it is called a "stranded wire." There is no sharp dividing line of size between a "stranded wire" and a "cable." If used as a wire, for example in winding inductance coils or magnets, it is called a stranded wire and not a cable. If it is substantially insulated, it is called a "cord," defined below.

(9006) There is no sharp dividing line in respect to size between a "cord" and a "cable," and likewise no sharp dividing line in respect to the character of insulation between a "cord" and a "stranded wire." Rubber is used as the insulating material for many classes of cords.

(9009) This kind of cable differs from the preceding in that the main strands are themselves stranded.

9010.* N-conductor Cable. — An N-conductor cable is a combination of N conductors insulated from one another.

9011.* N-conductor Concentric Cable. — An N-conductor concentric cable is a cable composed of an insulated central conductor with (N-1) tubular stranded conductors laid over it concentrically and separated by layers of insulation.

9012.* Duplex Cable. — A duplex cable is a cable composed of two insulated stranded conductors twisted together.

9013. Twin Cable. — A twin cable is a cable composed of two insulated stranded conductors laid parallel, having a common covering.

9014. Twin Wire. — A twin wire is a cable composed of two small insulated conductors laid parallel, having a common covering.

9015.* Triplex Cable. — A triplex cable is a cable composed of three insulated single-conductor cables twisted together.

9016.* Twisted Pair. — A twisted pair is a cable composed of two small insulated conductors, twisted together, without a common covering.

9017.* Sector Cable. — A sector cable is a multiple-conductor cable in which the cross-section of each conductor is substantially a sector, an ellipse, or a figure intermediate between them.

9018. Round Conductor. — A round conductor is either a solid or stranded conductor of which the cross-section is substantially circular.

9019.* Split Conductor. — A split conductor is a conductor which is divided into two or more parts, separated from one another by insulation which is thin compared with the insulation around the conductor.

9030. Factor of Assurance. — The factor of assurance of wire or cable insulation is the ratio of the voltage at which it is tested to that at which it is used.

9031. Insulation Resistance. — The insulation resistance of an insulated conductor is the electrical resistance offered by its insulation, to an impressed voltage tending to produce a leakage of current through the same.

9032.* Circular Mil. — A circular mil is a unit of area equal to $\frac{\pi}{4}(=0.7854\ldots)$ of a square mil. The cross-sectional area of a circle in circular mils is therefore equal to the square of its diameter in mils. A circular inch is equal to a million circular mils.

9033.* Lay. — The lay of any helical element of a cable is the axial length of a turn of the helix of that element.

9034. Direction of Lay. — The direction of lay is the lateral direction in which the strands of a cable run over the top of the cable as they recede from an observer looking along the axis of the cable.

(9010) It is not intended that the name as here given be actually used. One would instead speak of a "3-conductor cable," a "12-conductor cable" etc. In referring to the general case, one may speak of a "multiple-conductor cable" (as in § 9004 above).

(9011) This kind of cable usually has only two or three conductors. Such cables are used particularly for alternating currents. The remark on the expression "N-conductor" given for the preceding definition also applies here.

(9012) They may or may not have a common insulating covering.

(9015) They may or may not have a common insulating covering.

(9016) The two conductors of a "twisted pair" are usually substantially insulated, so that the combination is a special case of a "cord."

(9017) Sector cables are used in order to obtain decreased overall diameter and thus permit the use of larger conductors in a cable of given diameter.

(9019) The term split conductor usually designates a conductor in two parts or splits, which may be either concentric or external to one another.

(9032) A mil is the one-thousandth part of an inch. There are 1974 circular mils in a square millimeter.

(9033) Among the helical elements of a cable may be each strand in a concentric-lay cable, or each insulated conductor in a multiple-conductor cable.

ANNEALED COPPER STANDARD

9050.* Standard Annealed Copper. (a) *General:* The following shall be taken as normal values for standard annealed copper.

(b) *Resistance:* At a temperature of 20° C., the resistance of a wire of standard annealed copper one meter in length and of a uniform section of 1 square millimeter is 1/58 ohm = 0.017241 . . . . ohm.

(c) *Density:* At a temperature of 20° C., the density of standard annealed copper is 8.89 grams per cubic centimeter.

(d) *Temperature Coefficient of Resistance:* At a temperature of 20° C., the "constant mass" temperature coefficient of resistance of standard annealed copper, measured between two potential points rigidly fixed to the wire, is 0.00393 = 1/254.45 . . . . per degree centigrade.

(e) *Resistance of Standard Annealed Copper at 20° C.:* As a consequence, it follows from (b) and (c) that, at a temperature of 20° C. the resistance of a wire of standard annealed copper of uniform section, one meter in length and weighing one gram, is $(1/58) \times 8.89 = 0.15328$ . . . . ohm.

OPERATION

Temperature Limits

9100.* Maximum Temperatures. — The temperature of the insulation of a wire or cable at the surface of the conductor shall not be allowed to exceed the following values.

Let t = maximum safe temperature.

E = r.m.s. operating electromotive force in kilovolts between conductors.

Impregnated paper, $t = 85 - E$

Varnished cambric, $t = 75 - E$

Rubber insulation, $t = 60 - \frac{E}{4}$.

DESIGNATION

9200. Designation of Wires by Diameter or Gage Number. — The sizes of wires shall be stated by their diameters in mils, the American Wire Gage (Brown & Sharpe) sizes being taken as standard. For brevity, in cases where the most careful specification is not required, the sizes of wires may be stated by the gage number in the American Wire Gage.

9201. Designation of Cables by Cross-sectional Area. — The sizes of stranded conductors shall be stated by their cross-sectional area in circular mils or circular inches, except in the case of flexible stranded conductors, for

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(9050) See I. E. C. Publication No. 28, "International Standard of Resistance for Copper," March, 1914.

Paragraphs (b) and (e) define what are sometimes called "Volume Resistivity" and "Mass Resistivity," respectively. This may be expressed in other units as follows:

Volume Resistivity = 1.7241 microhms-cm. (microhms in a centimeter cube) at 20° C.

Mass Resistivity = 875.20 ohms (mile, pound) at 20° C.

For detailed specifications of commercial copper see the Standard Specifications of the American Society for Testing Materials.

(9100) For example: At a working pressure of 3.3 kv., the maximum safe limiting temperature at the surface of the conductor, or conductors, in a cable would be as follows:

For impregnated paper 81.7° C.
For varnished cambric 71.7° C.
For rubber insulation 59.2° C.

The life of the insulation of a cable depends in a great measure upon the actual temperature attained by the insulation. The result of operating at temperatures in excess of the safe limit is to shorten the life of the insulating material. When the safe limits are exceeded, deterioration is rapid and permanent, the damage increasing with the length of time that the excessive temperature is maintained and with the amount of excess temperature until finally the insulation breaks down.

Some of the older types of cable for voltages above 7500 have a dielectric loss that is so high that it may add considerably to the heating that would otherwise result. In such cases the dielectric loss is a material factor in determining the safe load to be carried by the cable, and the safe operating temperature will be determined by the temperature at which cumulative heating occurs under the conditions of service, if this occurs at a lower temperature than that at which the insulation deteriorates.

which see § 9402. The cross-sectional area of a cable shall be considered to be the sum of the cross-sectional areas of its component wires, when measured perpendicular to their axes. The sizes of stranded conductors smaller than 250,000 circular mils (*i.e.*, No. 0000 A. W. G. or smaller) may be stated by means of the gage number of a solid wire having the same cross-sectional area.

9202.* Conductivity. — The conductivity of the metal of wires shall be expressed in terms of the conductivity of the Annealed Copper Standard, as defined in § 9050.

9203.* Copper-wire Tables. — The copper-wire tables published by the Bureau of Standards in Circular No. 31 are adopted. Table VI therein gives the values of diameters and cross-sections of A. W. G sizes to four significant figures. These tables are based upon the Annealed Copper Standard described in § 9050.

TESTS

General

9300. Cable Lengths Tested. — Electrical tests of insulation on wires and cables shall be made on the entire lengths to be shipped.

9301. Immersion in Water. — (a) *General:* The outer surface of the insulation of complete insulated wires and cables shall be grounded while being electrically tested. If the insulation is not provided with a conducting covering, and if the covering is not liable to injury by water, the ground shall be obtained by immersing the insulated wire or cable in water for at least twelve hours and testing at the end of that period while immersed. If the outer covering is susceptible to injury by immersion, the insulated conductor shall be tested before the application of such covering.

Dry core paper insulated lead covered cables, such as telephone and telegraph cables, for use in water, shall be tested after at least twelve hours immersion.

(b) *Multiple-conductor Cable:* In the case of multiple-conductor cables, without waterproof overall jacket of insulation, no immersion test should be made on finished cables, but only on the individual conductors before assembling.

Tests of Dielectric Strength

9310. Object of Tests. — High-voltage tests are intended to detect weak spots in the insulation and to determine whether its dielectric strength is sufficient for enabling it to withstand the voltage to which it is likely to be subjected in service, with a suitable factor of assurance.

9311. Nature of Tests. — High-voltage tests shall be made at the factory, by subjecting the insulation to an alternating voltage. The initially applied voltage must not be greater than the working voltage, and the rate of increase shall be approximately uniform and not over 100 per cent in 10 seconds, and not less than 100 per cent in 60 seconds.

9312.* Magnitude and Duration of the Test Voltage. — (a) *General:* Wires and cables shall be tested at the place of manufacture for five consecutive minutes, except as provided in § 9312 (b) and (f).

(b) *Rubber Insulation, National Electrical Code:* Rubber covered wires and cables for working pressures up to 600 volts alternating, insulated in accordance with the requirements of the National Electrical Code, shall be tested in accordance with that Code.

(9202) For any given wire, let

k = conductivity, in per cent of Annealed Copper Standard;
l = length, meters;
R = resistance, ohms;
W = weight, grams;
θ = temperature, degrees centigrade.

Then the conductivity may be derived from the following formula:

$$k = \frac{15.328}{\frac{WR}{l^2} + 0.000597\,(20-\theta)}.$$

(9203) For detailed specifications of commercial copper, see the Standard Specifications of the American Society for Testing Materials.

(c) *Thirty per cent to 40 per cent Hevea Rubber Insulation for Pressures up to 600 Volts, a-c.:* Wires and cables for working pressures up to 600 volts alternating, insulated with 30 per cent to 40 per cent Hevea rubber compound, unless the insulation thickness is less than specified in § 9405, shall be tested in accordance with Table 901.

TABLE 901

High-voltage Tests for Rubber Insulated Wires and Cables

(30 per cent to 40 per cent Hevea Rubber Insulation for working pressures up to 600 Volts a-c.)

Size A. W.G. or Cir. Mils.	Size Sq. mm.	Test-pressure kilovolts
14–8	2.081– 8.366	3.0
7–0000	10.55 –107.2	3.5
250,000 and larger	127 and larger	4.0

(d) *Thirty per cent to 40 per cent Hevea Rubber Insulation for Pressures over 600 Volts, a-c.:* Wires and cables insulated with 30 per cent Hevea rubber compound for working pressures over 600 volts alternating, shall be tested with one kilovolt per 64th inch of thickness (2.53 kv., per mm.) up to 10/64th inch (3.96 mm.). Above 10/64ths inch, (3.96 mm.), the test pressure shall be 10 kilovolts plus 1.5. kilovolts per 64th inch (3.79 kv. per mm.) additional up to 30/64ths inch (11.89 mm.). Where the insulation thickness is 16/64ths inch (6.34 mm.) or over, this rule shall apply only to conductors over 26,000 cir. mils (13.2 sq. mm.) area.

(e) *Varnished Cambric and Impregnated Paper Insulation:* Varnished cambric and impregnated paper insulated wires or cables shall be tested in accordance with Table 902.

TABLE 902

High-voltage Tests for Varnished Cambric or Impregnated Paper Insulated Cables

(Minimum Values)

Operating kv.	Test kv.	Operating kv.	Test kv.
Below 0.5	2.5*	5	14
0.5	3	7.5	19.5
1	4	10	25
2	6.5	over 10	2½ times operating pressure
3	9		
4	11.5		

* The minimum thickness of insulation shall be 1/16 in. (1.6 mm.).

For intermediate working voltages, the test voltage shall be interpolated.

(f) *Telephone, Telegraph and Annunciator Wires and Cables:* Section 9312 shall not apply to wires and cables for telephone, telegraph, annunciator and similar devices.

9313. Frequency of Test Voltage. — The frequency of the test voltage shall not exceed 100 cycles per second, and should approximate as closely as possible to a sine wave. The source of energy should be of ample capacity.

9314. Dielectric Strength Tests. — Ultimate dielectric strength tests, when required, shall be made on samples not more than 6 meters (20 ft.) long. The

(9312 c) Hevea rubber is rubber from the Hevea Brasiliensis tree. Compounds containing 30 to 40 per cent of Hevea rubber have electrical and mechanical properties superior to compounds insulated in accordance with the requirements of the National Electrical Code.

(9312 e) Different engineers specify different thickness of insulation for the same working voltages. Therefore, at the present time the test kv. corresponding to working kv. given in Table 902, are based on the *minimum* thickness of insulation specified by engineers and operating companies.

maximum allowable temperature, at which the test is made, for the particular type of insulation and the particular working pressure, shall be not greater than the temperature limits given in § 9100.

9315. Multiple-conductor Cables. — If a multiple-conductor cable is designed for the same operating voltage between conductors and sheath or water as between conductors, each conductor shall be tested against the other conductors connected together and to the sheath or water. If the cable is designed for an operating voltage between conductors and ground different from that between conductors, the test between conductors and the sheath or water shall be made separately and shall be based on the normal operating voltage between conductors and sheath or water as prescribed in § 9312.

Insulation Resistance

9320.* Expression of Insulation Resistance. — Insulation resistance shall be expressed in megohms. Linear insulation resistance, or the insulation resistance of unit length, shall be expressed in terms of the megohm-kilometer, or the megohm-mile, or the megohm-thousand-feet, and shall be corrected to a temperature of 15.5° C., using a temperature coefficient determined experimentally for the insulation under consideration.

9321. Megohms Constant. — The megohms constant of an insulated conductoı shall be the factor K in the following equation:

$$R = K \log_{10} \frac{D}{d},$$

where R = insulation resistance, in megohms, for a specified unit length,
D = outside diameter of insulation,
d = diameter of conductor.

Unless otherwise stated, K will be assumed to correspond to the mile unit of length.

9322. Measurement of Insulation Resistance. — The apparent insulation resistance should be measured after the high-voltage test, measuring the leakage current after a one-minute electrification, with a continuous e.m.f. of from 100 to 500 volts, the conductor being maintained negative to the sheath or water.

9323. Insulation Resistance of Multiple-conductor Cables. — The insulation resistance of each conductor of a multiple-conductor cable shall be the insulation resistance measured from each conductor to all the other conductors in multiple with the sheath or water.

Capacitance or Electrostatic Capacity

9330.* Expression of Capacitance. — Capacitance shall be expressed in microfarads. Linear capacitance, or the capacitance of unit length, shall be expressed in microfarads per unit length (kilometer, or mile, or one thousand feet), and shall be corrected to a temperature of 15.5° C., using a temperature coefficient determined experimentally for the insulation under consideration.

9331. Microfarads Constant. — The microfarads constant of an insulated conductor shall be the factor K in the following equation:

$$C = \frac{K}{\mathrm{Log}_{10} \frac{D}{d}}$$

where C = capacitance in microfarads per unit length,
D = outside diameter of insulation,
d = diameter of conductor.

Unless otherwise stated, K will be assumed to refer to the mile unit of length.

(9320) In the case of dry-core paper-insulated cables, the temperature coefficient of insulation resistance cannot be closely determined on account of variations in design and manufacture. Therefore no temperature corrections shall be applied to insulation resistance tests. Tests should be made at a temperature of 15.5° C. or higher.

(9330) In the case of dry-core paper-insulated cables, the temperature coefficient of capacitance cannot be determined closely on account of variations in design and manufacture. Therefore no temperature corrections shall be applied to capacitance tests. Tests should be made at a temperature of 15.5° C. or higher.

TABLE 903

Proposed Standard Cables

(This table is offered for consideration but will not be recommended for final adoption until ratified by other societies interested.)

Strands			Total nominal cross-section circular mils	Total diameter, inches
Number and size. See Note 4	Individual wires			
	Nominal diameter mils	Nominal circular mils		
127 No. 8	128.5	16,510	2,097,000	1.671
127 No. 9	114.4	13,090	1,662,000	1.487
91 No. 8	128.5	16,510	1,502,000	1.414
91 No. 9	114.4	13,090	1,191,000	1.258
61 No. 8	128.5	16,510	1,007,000	1.157
61–121 mils	121.0	14,641	893,100	1.089
61 No. 9	114.4	13,090	798,500	1.030
61–107 mils	107.0	11,449	698,400	.963
61 No. 10	101.9	10,380	633,200	.917
37–116 mils	116.0	13,456	497,900	.812
37 No. 10	101.9	10,380	384,100	.713
37–97 mils	97.0	9,409	348,100	.679
37 No. 11	90.74	8,234	304,700	.635
19 No. 9	114.4	13,090	248,700	.572
19–107 mils	107.0	11,449	217,500	.535
19 No. 11	90.74	8,234	156,400	.454
19 No. 12	80.81	6,530	124,100	.404
19 No. 13	71.96	5,178	98,380	.360
19 No. 14	64.08	4,107	78,030	.320
7 No. 10	101.9	10,380	72,660	.306
7 No. 11	90.74	8,234	57,640	.272
7 No. 12	80.81	6,530	45,710	.242
7 No. 14	64.08	4,107	28,750	.192
7 No. 16	50.82	2,583	18,080	.152
7 No. 18	40.30	1,624	11,370	.121
7 No. 20	31.96	1,022	7,154	.096
7 No. 22	25.35	642.4	4,497	.076
7 No. 24	20.10	404.0	2,828	.060

Note 1. — Nominal diameters and circular mils of the individual wires are taken from Table VI, circular No. 31 of the Bureau of Standards.

Note 2. — The variation of the mean diameter of any wires shall not exceed 1 per cent above or below the nominal diameter.

Note 3. — The variation of the total cross-section of the cable shall not exceed 1 per cent above or below the nominal cross-section.

Note 4. — Sizes are expressed as A. W. G. numbers except where diameters are given in mils.

9332. Measurement of Capacitance. — The capacitance of cable shall be measured with alternating current by comparison with a standard condenser. It is preferable that the measurement be made either at a frequency approximating that of operation or at a frequency giving results approximating those corresponding to the operating frequency or frequencies.

9333. Capacitance of Paired Cables. — The capacitance of paired cables shall be measured between the two conductors of any pair, the other wires being connected to the sheath or ground.

9334. Capacitance of Multiple-conductor Cables (not paired). — The capacitance of multiple-conductor (not paired) cables shall be measured between conductors, and also between each conductor and the other conductors connected to the sheath or ground.

CONSTRUCTION

Stranding

9400.* Proposed Standard Cables. — Insulated cables not requiring special flexibility shall be made of the number and size strands specified in Table 903.

9401. Cables not Requiring Special Flexibility. — Cables not requiring special flexibility and not made in accordance with § 9400 shall be stranded in accordance with Table 904.

TABLE 904

Standard Stranding of Concentric-Lay Cables

Size (see Note 1)	Square mm.	Number of Wires (See note 2)	
		A Bare, insulated or weatherproof cables for aerial use	B Insulated cables for other than aerial use
2.0 Cir. Inches	1013	91	127
1.5 "	760	61	91
1.0 "	507	61	61
0.6 "	304	37	61
0.5 "	253	37	37
0.4 "	203	19	37
0000 A. W. G.	107	19 or 7 (See note 3)	19
00 "	67.4	7	19
2 "	33.6	7	7
7 and smaller	10.5	..	7

Note 1. — For intermediate sizes, use stranding for next larger size.
Note 2. — Conductors of 0000 A. W. G. and smaller are often made solid and this table of stranding should not be interpreted as excluding this practice.
Note 3. — Class A cable, sizes 0000 and 000 A. W. G., is usually made of 7 strands when bare and 19 strands when insulated or weatherproof.

9402.* Flexible Cables. — Conductors of special flexibility should ordinarily be made with wires of regular A. W. G. sizes, and rated by the number and size of wires. The stranding of flexible cables is given in Table 905.

(9400) The basis of this rule is the use of strands of American Wire Gage sizes. To meet existing operating conditions, four sizes of strands other than American Wire Gage sizes have been deemed necessary and their diameters are shown in mils.
(9402) Where necessary to closely approximate a regular size cable, the strands may be made of half-size wires from No. 15 to No. 30 A. W. G.

TABLE 906

Stranding of Flexible Cables

Nearest A. W. G. size (see Note 1)	Circular mils (see Note 2)	Diam. of cable, mils	No. of wires	Size of each wire		Construction (see Note 3)
				A. W. G.	Diam., mils.	
....	2,039,000	1885	703	15.5	53.9	37 × 19
....	1,816,000	1779	"	16.0	50.8	"
....	1,617,000	1679	"	16.5	48.0	"
....	1,440,000	1584	"	17.0	45.3	"
....	1,284,000	1496	"	17.5	42.7	"
....	1,103,000	1372	427	16.0	50.8	61 × 7
....	874,600	1222	"	17.0	45.3	"
....	693,600	1088	"	18.0	40.3	"
....	550,000	969	"	19.0	35.9	"
....	436,200	863	"	20.0	32.0	"
....	345,900	768	"	21.0	28.5	"
....	274,300	684	"	22.0	25.3	"
....	264,600	671	259	20.0	32.0	37 × 7
0000	209,800	598	"	21.0	28.5	"
000	171,300	538	133	19.0	35.9	19 × 7
00	135,900	479	"	20.0	32.0	"
0	107,700	427	"	21.0	28.5	"
1	82,780	332	91	20.5	30.2	Concentric
2	65,650	295	"	21.5	26.9	"
3	52,060	263	"	22.5	23.9	"
4	39,190	228	61	22.0	25.3	"
5	31,080	203	"	23.0	22.6	"
6	24,650	181	"	24.0	20.1	"
8	17,410	152	"	25.5	16.9	"
10	10,560	118	37	25.5	16.9	"
12	6,640	94	"	27.5	13.4	"
14	4,176	74	"	29.5	10.6	"
Smaller	....	....	To equal Required Size	30.0	.....	Bunched

Note 1. — The A.W.G. cross-sectional areas except for 61 strands, are approximated within 2 per cent. In the case of 61 strand cables the approximation is 6 per cent.

Note 2. — Circular mils are based on theoretical diameters of A. W. G. sizes, which vary above or below values given in table by less than 0.1 mil.

Note 3. — "61 × 7" in the rating of a rope-lay cable signifies 61 strands of 7 wires each.

9403.* Correction for Lay. — Two per cent shall be taken as the standard increment of resistance and of mass, due to stranding. In cases where the lay is definitely known, the increment should be calculated and not assumed.

(9403) The resistance and mass of a stranded conductor are greater than in a solid conductor of the same cross-sectional area, depending on the lay (*i.e.*, the pitch of the twist of the wires).

Thickness of Insulation

9405. Thickness of Insulation for Rubber Insulated Wires and Cables. — Unless special conditions warrant departures from this rule, the thickness of insulation for rubber compounds containing from 30 to 40 per cent of Hevea rubber, shall be in accordance with Table 906.

TABLE 906
Thickness of Insulation
30 to 40 per cent Hevea Rubber Compound
Recommended Walls of Insulation, 64ths, Inch

Size A. W. G. or Cir. Mils	Square mm.	Working pressure, volts alternating										
		600 or less	1500	2500	3500	5000	6000	7000	8000	9000	10000	11000
14–8	2.08–8.37	3	6	8	10	12	14	16	18	20	22	24
7–2	10.6–33.6	4	7	9	10	12	14	16	18	20	22	24
1–0000	42.4–107	5	8	10	10	12	14	16	18	20	22	24
250,000–500,000	127–253	6	9	10	11	12	14	16	18	20	22	24
550,000–1,000,000	279–507	7	10	10	12	12	14	16	18	20	22	24
1,250,000–2,000,000	633–1013	8	10	10	12	14	16	18	18	20	22	24

Notes. — In multiple-conductor cables, the thickness of insulation on each conductor shall be based on the highest r.m.s. voltage between the conductor and the outside of this insulation. The above table is based upon alternating voltages of commercial frequencies. For voltages over 600, the insulation thickness for direct-current cable has not been established. For intermediate sizes the insulation thickness should be the same as for the next larger sizes.

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CHAPTER X

STANDARDS FOR STORAGE BATTERIES

DEFINITIONS

10001. Battery. — A group of cells electrically connected. Storage batteries are conveniently divided in two groups: stationary batteries and portable batteries. Stationary batteries include line batteries, load-regulating batteries, peak-load batteries, oil switch and exciter-reserve batteries, stand-by batteries, telephone exchange batteries and farm light batteries. Portable batteries include starting and lighting batteries, vehicle batteries, car lighting batteries, electric locomotive batteries, mine-lamp batteries, and ignition batteries.

10002. Cell. — A single element with electrolyte and container.

10003. Element. — The positive and negative groups with separators assembled.

10004. Group. — Assembly of a set of positive or a set of negative plates for one cell.

10005. Jar or Tank. — The container for a cell.

NOTE. — The term tank is generally confined to lead containers supported by wood.

10006. Separator. — A device for preventing metallic contact between the plates of opposite polarity within the cell.

10007. Electrolyte. — A solution of sulphuric acid for lead batteries and of certain hydroxides for nickel-iron batteries.

10008. Positive Plate. — The grid and active material from which the current flows to the external circuit when the battery is discharging.

10009. Negative Plate. — The grid and active material to which the current flows from the external circuit when the battery is discharging.

10010. Grid. — A metallic framework for conducting current and holding the active material or active material units of the plate.

10011. Active Materials. — Substances reacting chemically to produce electrical energy during discharge of the cell, but subject to oxidation or reduction when on charge in the reverse direction, transforming electrical energy into chemical energy. In the charged condition, these are lead peroxide on the positive plates and sponge lead on the negative plates of lead-acid cells, but for nickel-iron cells they are nickel peroxide in the positive tubes and iron in the negative pockets.

10012. Couple. — One positive and one negative plate connected together as a unit for adjacent cells.

10013. Counter Cells. — Cells of practically no capacity used to oppose the line voltage.

10014. End Cells. — Those cells of a battery which may be cut in or out of the circuit for adjusting the voltage.

10015. Terminals. — Fittings to provide electrical connections to storage cells or batteries.

10016. Pilot Cell. — A selected cell whose temperature, voltage and gravity are assumed to indicate the state of charge of the entire battery.

MISCELLANEOUS

10030. Ampere-hour Capacity. — The number of ampere-hours which can be delivered by a cell or battery under specified conditions as to temperature, rate of discharge and final voltage.

10031. Ampere-hour Efficiency. — The ratio of the ampere-hours output of a cell or battery under specified conditions of temperature, rate of discharge and final voltage, to the ampere-hours of input required for complete charge. See "efficiency."

10032. Average Voltage. — The average value of the voltage during the period of charge or discharge. It is conveniently obtained from the time integral of the voltage curve.

10033. Boost Charge. — A partial charge, usually at a high rate for a short period.

10034. Charge. — The restoration of the active materials in a battery by passing a direct current through it in the opposite direction to that of the discharge.

10035. Charging Rate. — The current, expressed in amperes, at which a battery is charged.

10036. Closed Circuit Voltage. — The voltage at a cell or terminals of the battery when current is flowing.

10037. Constant-current Charge. — A charge in which the current is maintained at constant value. For portable lead batteries this usually involves two rates called the starting and the finishing rates.

10038. Constant-potential Charge. — A charge in which the voltage at the terminals of the battery is held at a constant value. A modified constant potential system is one in which a fixed resistance is inserted in the charging circuit to limit the initial current.

10039. Discharge. — The conversion of the chemical energy of the battery into electrical energy.

10040. Efficiency. — The ratio of the useful output to the required input. The efficiency from an electro-chemical standpoint is the ampere-hour efficiency. It represents the ratio of the quantity of electricity delivered by the battery, to the quantity required for a complete recharge, expressed in ampere-hours. The energy efficiency is the watt-hour efficiency. It represents the ratio of the energy delivered by the battery to the energy expended in charging it, expressed as watt-hours measured at the terminals of the battery. If the measurements are made at the busses, the watt-hour efficiency of the installation may be obtained. The so-called volt efficiency is the ratio of the average voltage on discharge to the average voltage on charge. The watt-hour efficiency of the battery is customarily taken as the product of the ampere-hour efficiency and the volt efficiency.

10041. Final Voltage. — The prescribed voltage upon reaching which the discharge is considered complete. The final voltage is usually chosen so that the useful capacity of the cell is realized. Final voltages vary with the kind of battery and the rate of the discharge.

10042. Equalizing Charge. — A charge given to a lead battery to insure the complete reduction of the lead sulphate. The equalizing charge is ordinarily at the finishing rate or less.

10043. Finishing Rate. — The rate of charge expressed in amperes to which the charging current for lead batteries is reduced near the end of charge to prevent excessive gassing.

10044. Gassing. — The evolution of oxygen or hydrogen, or both.

10046. Polarity. — An electrical condition determining the direction in which current tends to flow. By common usage the discharge current is said to flow from the peroxide plate or position through the external circuit. In a nickel iron battery the positive plate is that containing nickel peroxide.

10047. Reversal. — Change in normal polarity of a storage cell.

10048. Specific Gravity. — The ratio of the weight of a given volume of the liquid electrolyte to the weight of the same volume of water at a specified temperature. It is usually measured in storage battery work by a hydrometer.

10049. Trickle Charge. — A continuous charge at low rate for batteries in wet storage. It is approximately 1 per cent of the finishing rate.

10050. Volt Efficiency. — The ratio of the average voltage of the cell or battery on discharging to the average voltage on charge. See "efficiency."

10051. Watt-hour Capacity. — The ampere-hour capacity multiplied by the average voltage during discharge. It may, however, be measured directly.

10052. Watt-hour Efficiency. — The ratio of the watt-hours of useful output to the watt-hours of input expended in charging. See "efficiency."

TEMPERATURE

10101. The capacity obtained from a storage battery on discharge varies with the temperature of the electrolyte. In order to compare various batteries, standard reference temperatures are necessary; the following standard reference temperatures are established:

1. The temperature of electrolyte at beginning of discharge shall be 25° C. (77° F.).
2. The ambient temperature on discharge shall be from 5° C. to 8° C. lower than the temperature of the electrolyte, on the beginning of discharge. The ambient temperature shall be kept constant throughout the discharge.

Note. (a) No limit is placed on the temperatures obtained by the electrolyte during discharge.

(b) In lead acid batteries, the limit of temperature of electrolyte permitted on charge depends upon the specific gravity of the electrolyte, higher temperatures being permitted on low gravity electrolyte. In general, it is desirable that the temperature of the electrolyte on charge shall not exceed 45° C. (113° F.).

RATINGS

General

10201. The capacities of storage batteries for different classes of service are not directly comparable. The capacity depends upon the discharge rate, reference temperature of electrolyte (which has been standardized as 25° C.), specific gravity of electrolyte, and final voltage allowed.

10202. Storage batteries shall be rated in accordance with the following tabulated capacity specifications:

(a) Ampere-hour capacity.
(b) Discharge rate.
(c) Reference temperature of discharge—25° C.
(d) Specific gravity of electrolyte at 25° C.
(e) Final voltage limit per cell on discharge.

10203. The tabulated data required by the capacity specifications shall either be given on a name-plate attached to battery, or, where this is not practicable, shall be furnished separately.

TESTS

General

10301. Batteries shall be tested as required, to determine if the rated capacity is obtained under the conditions as given in capacity specifications.

BATTERIES FOR SPECIAL SERVICE

10400. In certain definite classes of service, some or all of the variables given in the capacity specifications (No. 10202) can be definitely stated and standardized.

CHAPTER XI

STANDARDS FOR ILLUMINATION

This chapter consists of extracts from the Report of the Committee on Nomenclature and Standards of the Illuminating Engineering Society. It is here included by permission.

General

11000. Radiant flux, ϕ, is the rate of flow of radiation evaluated with reference to energy, and is expressed in ergs per second or in watts.

11001.* **Luminous flux** is the rate of flow of radiant energy evaluated with

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(11001), (11006), (11007), (11008), (11010), (11011) These are the international definitions as agreed on at the Paris meeting of the International Commission on Illumination in 1921.

reference to visual sensation. Although luminous flux must strictly be defined as above, it may be regarded for practical photometric purposes as an entity, since the rate of flow is for such purposes invariable.

11002. Visibility, K_λ, of radiation of a particular wave-length is the ratio of the luminous flux at that wave-length to the corresponding radiant flux.

Defining equation: $$K_\lambda = \frac{F_\lambda}{\Phi_\lambda}.$$

11003.* The mechanical equivalent of light is the ratio of radiant flux to luminous flux for the wave-length of maximum visibility, and is expressed in ergs per second per lumen, or in watts per lumen. It is the reciprocal of the maximum visibility.

11004. Luminosity of a particular wave-length is the product of the visibility of that wave-length and the corresponding ordinate of the spectral curve of radiant flux, and is represented by the ordinate of the spectral curve of luminous flux. This curve is called the spectral luminosity curve and is different with different sources.

11005. The **Luminous efficiency** of any source is the ratio of the luminous flux to the radiant flux from the source and is expressed in lumens per watt.

11006.* Luminous intensity of a point source in any direction is the flux per unit solid angle (one steradian) emitted by the source in that direction. (The flux from any source of dimensions which are negligibly small by comparison with the distance at which it is observed, may be treated as if it were emitted from a point.)

11007*. Illumination at any point of a surface is the luminous flux density at that point, or when the illumination is uniform, the flux per unit of intercepting area.

11008.* International Candle. — The unit of luminous intensity is the International Candle, such as has resulted from international agreement between the three national standardizing laboratories of France, Great Britain and the U. S. A. in 1909.

This unit has been conserved since then by means of incandescent lamps in the laboratories which continue (or remain) charged with conservation.

11009. Candlepower, cp., is luminous intensity expressed in candles.

11010.* Lumen. — The unit of luminous flux is the lumen. It is equal to the flux emitted in a unit solid angle by a uniform point source of one international candle.

11011.* Lux, Phot, Foot-candle. — The practical unit of illumination is the lux. It is equal to one lumen per square meter, or it is the illumination at the surface of a sphere of one meter radius due to a uniform point source of one international candle placed at its center.

As a consequence of certain recognized usages, the illumination can also be expressed by means of the following units:

Using the centimeter as the unit of length, the unit of illumination is one lumen per square centimeter, and is called the phot. Using the foot as the unit of length, the unit of illumination is one lumen per square foot, and is called the foot-candle.

11012. Brightness of an element of a luminous surface may be expressed in either of two ways: (a) in terms of intensity, I, (b) in terms of flux, F.

(a) Brightness in terms of the luminous intensity I (or candle-power) per unit of projected area of the surface (candlepower brightness) corresponds to the

defining equation, $$b_I = \frac{dI}{dS \cos \theta}$$

where θ is the angle between the normal to the surface and the line of sight.

(b) Brightness in terms of the flux, F, proceeding from a unit area of the sur

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(11003) This term has been used in a variety of senses. As here defined it refers only to the minimum mechanical equivalent of light. The reciprocal of this quantity is sometimes called the luminous equivalent of radiation.

(11010) A uniform source of one candlepower emits 4π lumens.

face, on the assumption that the surface is a perfect diffuser; *i.e.*, that it obeys the cosine law of emission or reflection, (lumen brightness) corresponds to the

defining equation, $b_F = \frac{dF}{dS}$

(perfect diffusion assumed).

The units in which brightness is measured according to (a) and (b) differ only in numerical value. See § 11013.

11013. Lambert, *L*, is the unit of brightness in the lumen system. The lambert is the brightness of a perfectly diffusing surface emitting or reflecting one lumen per square centimeter. For most purposes the **millilambert,** 0.001 lambert, is the preferable practical unit.

To say that the brightness of a surface as viewed from a given point is *n* lamberts, signifies that its brightness is the same as that of a perfectly diffusing surface emitting or reflecting *n* lumens per square centimeter.

In practice no surface obeys exactly the cosine law of emission or reflection; hence the brightness of a surface generally is not uniform but varies somewhat with the angle at which it is viewed.

A perfectly diffusing surface emitting one lumen per square foot will have a brightness of 1.076 millilamberts.

Brightness expressed in candles per square centimeter may be reduced to lamberts by multiplying by $\pi = 3.14$.

Brightness expressed in candles per square inch may be reduced to lamberts by multiplying by $\pi/6.45 = 0.487$.

Surfaces and Media Modifying Luminous Flux

11020. Diffusing surfaces and media are those which break up the incident flux and distribute it more or less in accordance with the cosine law, as for example, white plaster and opal glass.

11021. Redirecting surfaces and media are those which change the direction of the luminous flux in a definite manner; as for example, a mirror or a lens.

11022. Scattering surfaces and media are those which redirect the luminous flux and break it up into a multiplicity of separate pencils; as for example, ripple glass, reflecting or transmitting.

11023.* Reflection factor, of a body ρ, is the ratio of the flux reflected by the body to the flux incident upon it. The reflection from a body may be regular, diffuse or mixed. In regular reflection the flux is reflected at an angle of reflection equal to the angle of incidence. In diffuse reflection the flux is reflected in all directions. In perfectly diffuse reflection, the distribution of the reflected flux is in accordance with Lambert's cosine law. In most practical cases, there is a superposition of regular and diffuse reflection.

11024.* Absorption factor, of a body a, is the ratio of the flux absorbed by the body to the flux incident upon it.

11025.* Transmission factor, of a body τ, is the ratio of the flux transmitted by the body to the flux incident upon it.

$$\rho + a + \tau = 1.$$

Illumination

11030. Unidirectional illumination on a surface is that produced by a single light source of relatively small dimensions. It is characterized by the fact that a small opaque object placed near the illuminated surface casts a sharp shadow.

11031. Multidirectional illumination on a surface is that produced by several separated light sources of relatively small area. It is characterized by the fact that a small opaque object placed near the illuminated surface casts several shadows.

(11023) (11024) (11025) These terms are introduced to replace the more commonly used terms, Coefficient of reflection, Coefficient of absorption, Coefficient of transmission, which latter terms refer to the specific properties of materials rather than to the behavior of bodies under specified conditions, such as angle of incidence, etc.

11032. Diffused illumination is that produced either by primary or secondary light sources having dimensions relatively large with respect to the distance from the point illuminated, and scattering light in all directions. It is characterized by relative lack of shadow. Diffused illumination may be derived principally from a single direction as in the light from a skylit window or from all directions as in the open air. Perfectly diffused illumination on a surface is shadowless.

In any practical case of illumination on a surface there is usually a mixture of the above types.

11033. Coefficient of utilization of an illumination installation on a given plane is the total flux received by that plane divided by the total flux from the lamps illuminating it. When not otherwise specified, the plane of reference is assumed to be a horizontal plane 30 inches (76 cm.) from the floor.

11034. Variation factor of an illumination installation is the ratio of either the maximum or minimum illumination on a given plane to the average illumination on that plane.

11035. Variation range of illumination on a given plane is the ratio of the maximum illumination to the minimum illumination on that plane.

11036. Hemispherical ratio for a given lighting unit is the ratio of the luminous flux in the upper hemisphere to that in the lower hemisphere.

11037. Brightness ratio is the ratio of the brightness of any two surfaces. When the two surfaces are opposed, the brightness ratio is commonly called the "brightness contrast."

Illuminants

11040. The **output** of all illuminants should be expressed in lumens.

11041. Illuminants should be rated upon a lumen basis rather than a candle-power basis.

11042. Lamp efficiency is the ratio of the luminous flux output to the power input.

11043. The lamp efficiency or **specific output of electric lamps** should be stated in terms of lumens per watt and that of illuminants depending upon combustion should be stated in lumens per British thermal unit per hour.

11044. The power consumption of auxiliary devices which are necessarily employed in circuit with a lamp should be included in the input of the lamp. For example, the watts lost in the ballast resistance of an arc lamp are properly chargeable to the lamp.

11045. The specific consumption of an electric lamp is its watt consumption per lumen. "Watts per candle" is a term used commercially in connection with electric incandescent lamps, and denotes watts per mean horizontal candle.

11046. Life Tests. — Electric incandescent lamps of a given type may be assumed to operate under comparable conditions only when their lumens per watt consumed are the same. Life test results, in order to be compared, must be either conducted under, or reduced to, comparable conditions of operation.

11047. In comparing different luminous sources not only should their candle-power be compared, but also their relative form, brightness, distribution of illumination and character of light.

Lamp Accessories

11048. A **reflector** is an appliance the chief use of which is to redirect the luminous flux of a lamp in a desired direction or directions.

11049. A **shade** is an appliance the chief use of which is to diminish or to interrupt the flux of a lamp in certain directions where such flux is not desirable. The function of a shade is commonly combined with that of a reflector.

11050. A **globe** is an enclosing appliance of clear or diffusing material the chief use of which is either to protect the lamp or to diffuse its light.

Photometry

11060. Performance curve is a curve representing the behavior of a lamp in any particular (candlepower, consumption, etc.) at different periods during its life.

11061. Characteristic curve is a curve expressing a relation between two variable properties of a luminous source, as candlepower and volts, candlepower and rate of fuel consumption, etc.

11062. Mean horizontal candlepower of a lamp is the average candlepower in the horizontal plane passing through the luminous center of the lamp.

It is here assumed that the lamp (or other light source) is mounted in the usual manner, or, as in the case of an incandescent lamp, with its axis of symmetry vertical.

11063. Mean spherical candlepower of a lamp is the average candlepower of a lamp in all directions in space. It is equal to the total luminous flux of the lamp in lumens divided by 4π.

11064. Mean hemispherical candlepower of a lamp (upper or lower) is the average candlepower of a lamp in the hemisphere considered. It is equal to the total luminous flux emitted by the lamp in that hemisphere divided by 2π.

11065. Mean zonal candlepower of a lamp is the average candlepower of a lamp over the given zone. It is equal to the total luminous flux emitted by the lamp in that zone divided by the solid angle of the zone.

11066.* Spherical reduction factor of a lamp is the ratio of the mean spherical to the mean horizontal candlepower of the lamp.

TABLE 1100

11067 **Photometric Units and Abbreviations**

Photometric quantity	Name of unit	Symbols and defining equations	Abbreviation for name of unit
1. Luminous flux	Lumen	F. ψ	l.
2. Luminous intensity	Candle	$I=\frac{dF}{d\omega}$, $\Gamma=\frac{d\psi}{d\omega}$	cp.
3. Illumination	Phot, foot-candle, lux	$E=\frac{dF}{dS}=\frac{I}{r^2}\cos\theta$	ph. fc.
4. Exposure	Phot-second; Micro phot-second	Et	phs. μ phs.
5. Brightness	Apparent candle per sq. cm.; Apparent candle per sq. in.	$b_t=\frac{dI}{dS\cos\theta}$	
	Lambert	$b_l=\frac{dF}{dS}$	L. mL.
6. Reflection factor		ρ	
7. Absorption factor		α	

8. Transmission factor	τ
9. Mean spherical candlepower	scp.
10. Mean lower hemispherical candlepower	lcp.
11. Mean upper hemispherical candlepower	ucp.
12. Mean zonal candlepower	zcp.
13. Mean horizontal candlepower	mhc.

(11066) In the case of a uniform point-source, this factor would be unity, and for a straight cylindrical filament obeying the cosine law it would be $\pi/4$.

* Perfect diffusion assumed.

14. 1 lumen is emitted by 0.07958 spherical candlepower.
15. 1 spherical candlepower emits 12.57 lumens.
16. 1 lux = 1 lumen incident per square meter = 0.0001 phot = 0.1 milliphot.
17. 1 phot = 1 lumen incident per square centimeter = 10,000 lux = 1,000 milliphots = 1,000,000 microphots.
18. 1 milliphot = 0.001 phot = 0.929 foot-candle.
19. 1 foot-candle = 1 lumen incident per square foot = 1.076 milliphots = 10.76 lux.
20. 1 lambert = 1 lumen emitted per square centimeter of a perfectly diffusing surface.
21. 1 millilambert = 0.001 lambert.
22.*1 lumen, emitted, per square foot = 1.076 millilamberts.
23.*1 millilambert = 0.929 lumen, emitted, per square foot.
24. 1 lambert = 0.3183 candle per square centimeter = 2.054 candles per square inch.
25. 1 candle per square centimeter = 3.1416 lamberts.
26. 1 candle per square inch = 0.487 lambert = 487 millilamberts.

CHAPTER XII

STANDARDS FOR TELEPHONY AND TELEGRAPHY

Many of the following definitions are tentative and not yet fully established. Criticisms and suggestions, addressed to the Secretary of the Standards Committee, will be welcomed. Some of the definitions are specific to telephony, and differ in detail from similar definitions appearing in other parts of the rules.

DEFINITIONS

Line Circuits

12000. Ground-return Circuit. — A ground-return circuit is a circuit consisting of one or more metallic conductors in parallel, with the circuit completed through the earth.

12001. Metallic Circuit. — A metallic circuit is a circuit of which the earth forms no part.

12002. Two-wire Circuit. — A two-wire circuit is a metallic circuit formed by two parallel conductors insulated from each other.

12003. Superposed Circuit. — A superposed circuit is an additional circuit obtained from a circuit normally required for another service, and in such a manner that the two services can be given simultaneously without mutual interference.

12004. Phantom Circuit. — A phantom circuit is a superposed circuit, each side of which consists of the two conductors of a two-wire circuit in parallel.

12005. Side Circuit. — A side circuit is a two-wire circuit forming one side of a phantom circuit.

12006. Non-phantomed Circuit. — A non-phantomed circuit is a two-wire circuit, which is not arranged for use as the side of a phantom circuit.

12007. Simplexed Circuit. — A simplexed circuit is a two-wire telephone circuit, arranged for the superposition of a single ground-return signalling circuit operating over the wires in parallel.

12008. Composited Circuit. — A composited circuit is a two-wire telephone circuit, arranged for the superposition on each of its component metallic conductors of a single independent ground-return signalling circuit.

12009. Quadded or Phantomed Cable. — A quadded or phantomed cable is a cable adapted for the use of phantom circuits.

12010. Simplex Circuit. — A simplex circuit in telegraphy is one arranged for operation in one direction at one time.

12011. Duplex Circuit. — A duplex circuit in telegraphy is one arranged for simultaneous operation in opposite directions.

12012. Diplex Circuit. — A diplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in the same direction.

12013. Quadruplex Circuit. — A quadruplex circuit in telegraphy is one arranged for the simultaneous transmission of two messages in each direction.

12014. Multiplex Circuit. — A multiplex circuit in telegraphy is one arranged for the simultaneous transmission of one or more messages in both directions. Both duplex and quadruplex are examples of multiplex whereas diplex is not.

12015. Linear Electrical Constants. — The linear electrical constants of a line are the electrical constants per unit length of the line, *e.g.*, linear resistance, linear inductance, etc.

12016. Smooth Line. — A smooth line is a line whose electrical elements are all continuously and uniformly distributed throughout its length.

12017.* Periodic Line. — A periodic line is a line consisting of successive similar sections in each of which one or more electric elements are not distributed uniformly. As examples of periodic lines are (1) loaded lines and (2) artificial lines consisting of successive similar sections of lumped constants.

12018. Equivalent Smooth Line. — An equivalent smooth line of a periodic line is a smooth line having the same electrical behavior as the periodic line, at a given single frequency, when measured at terminals or at corresponding section junctions.

12019. Equivalent Periodic Line. — An equivalent periodic line of a smooth line is a periodic line having the same electrical behavior, for an assumed single frequency, as the smooth line, when measured at terminals or at corresponding section junctions. The terms conjugate smooth line and conjugate periodic line are also sometimes used.

12020. Composite Line. — A composite line is a line consisting of a plurality of successive sections having different linear electrical constants, as in the case where an underground cable section is joined to an overhead open-wire section.

12021. Loaded Line. — A loaded line is one in which the normal reactance of the circuit has been altered for the purpose of increasing its transmission efficiency.

12022. Series Loaded Line. — A series loaded line is one in which the normal reactance has been altered by reactance serially applied.

12023. Shunt Loaded Line. — A shunt loaded line is one in which the normal reactance of the circuit has been altered by reactance applied in shunt across the circuit.

12024. Continuous Loading. — A continuous loading is a series loading in which the added inductance is uniformly distributed along the conductors.

12025.* Coil Loading. — A coil loading is one in which the normal inductance is altered by the insertion of lumped inductance in the circuit at intervals.

Circuit Constants and Characteristics

12050. Damping of a Circuit. — The damping at a given point in a circuit from which the source of energy has been withdrawn, is the progressive diminution in the effective value of electromotive force and current at that point resulting from the withdrawal of electrical energy.

12051.* Damping Constant. — The damping constant of a circuit is a measure

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(12017) The term periodic in this definition refers to the line constants and not to time relations.

(12025) This lumped inductance may be applied either in series or in shunt.

As commonly understood, coil loading is a series loading, in which the lumped inductance is applied at uniformly spaced recurring intervals.

(12051) Applied to the admittance of a condenser or other simple circuit having capacity reactance, the damping constant for a harmonic electromotive force of given frequency is the ratio of the conductance G, of the condenser or simple circuit at that frequency to twice the capacitance, C, of the condenser at the same frequency, $(G/2C)$.

Applied to the reactance of a coil or other simple circuit having inductive reactance, the damping constant for a harmonic current of a given frequency is the ratio of the resistance, R, of the coil or circuit at that frequency to twice the inductance, L, at the same frequency $(R/2L)$.

of the ratio of the dissipative to the reactive component of its admittance or impedance.

12052.* Mutual Impedance. — The mutual impedance for single frequency alternating currents, between a pair of terminals and a second pair of terminals of a network, under any given condition, is the negative ratio of the electromotive force produced between either pair of terminals on open circuit to the current flowing between the other pair of terminals.

12053.* Self-impedance. — The self-impedance between a pair of terminals of a network, under any given condition, is the ratio of the electromotive force applied across the terminals to the entering current.

12054.* Characteristic Impedance. — The characteristic impedance of a line is the ratio of the applied electromotive force to the resulting steady-state current upon a line of infinite length and uniform structure, or of periodic recurrent structure.

12055.* Sending-end Impedance. — The sending-end impedance of a line is the ratio of the applied electromotive force to the resulting steady-state current at the point where the electromotive force is applied.

12056.* Propagation Constant. — The propagation constant of a uniform line, or section of a line of periodic recurrent structure, is the natural logarithm of the ratio of the steady-state currents at various points separated by unit length in a uniform line of infinite length, or at successive corresponding points in a line of recurrent structure of infinite length. The ratio is determined by dividing the value of the current at the point nearer the transmitting end by the value of the current at the point more remote.

12057. Attenuation Constant. — The attenuation constant for a single frequency is the real part of the propagation constant taken at that frequency.

12058. Wave-length Constant. — The wave-length constant is the imaginary part of the propagation constant.

12059. Standard Cable. — A standard cable is an ideal uniform line in terms of which the attenuation of a line or network may be specified. It is characterized by the following constants: Linear resistance, 88 ohms per loop mile (54.7 ohms per loop km.). Linear capacitance between wires 0.054 microfarad per loop mile (0.03355 microfarad per loop km.). Linear inductance and linear leakance, 0.

Equivalent Circuits

12102.* Equivalent Circuit. — An equivalent circuit is a simple network of series and shunt impedances, which, at a given frequency, is the approximate electrical equivalent of a complex network.

12103.* " *T* " Equivalent Circuit. — A " *T* " equivalent circuit is a triple-star or " *Y* " connection of three impedances externally equivalent to a complex network. See Fig. 12-1 for symbol.

(12052) A receiving-end impedance is an example of a mutual impedance.
Single-frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

(12053) Single-frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

(12054) In practise, the terms (1) line impedance, (2) surge impedance, (3) iterative impedance, (4) sending-end impedance, (5) initial sending-end impedance, (6) final sending-end impedance, (7) natural impedance and (8) free impedance, have apparently been more or less indefinitely and indiscriminately used as synonyms with what is here defined as "characteristic impedance."
Single-frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

(12055) See note under "Characteristic Impedance." In case the line is of infinite length of uniform structure or of periodic recurrent structure, the sending-end impedance and the characteristic impedance are the same.
Single-frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

(12056) Single-frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

(12102 to 12106 Incl.) As ordinarily considered, the simple networks, as defined, are the electrical equivalents of complex networks only with respect to definite pairs of terminals.

12104.* " *I* " Equivalent Circuit. — An " *I* " equivalent circuit is a connection of five impedances in the form shown in Fig. 12-2, which is externally equivalent to a complex network. It differs from the " *T* " equivalent circuit in that the impedances are arranged symmetrically on the two sides of the circuit, which is often desirable in connection with practical problems, as indicating that the circuit is balanced with respect to ground.

12105.* " Π " Equivalent Circuit. — A " Π " equivalent circuit is a delta connection of three impedances externally equivalent to a complex network. It is also called a " *U* " equivalent circuit. See Fig. 12-3 for symbol.

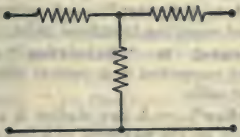

Fig. 12-1

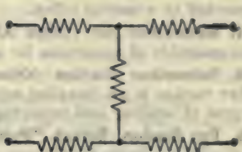

Fig. 12-2

12106.* " *O* " Equivalent Circuit. — An " *O* " equivalent circuit is a connection of four impedances in the form shown in Fig. 12-4, externally equivalent to a complex network. It differs from the Π equivalent circuit in that the impedances are arranged symmetrically on the two sides of the circuit, which is often desirable in connection with practical problems, as indicating that the circuit is balanced with respect to ground.

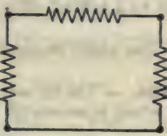

Fig. 12-3

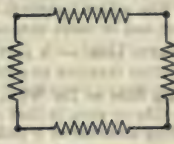

Fig. 12-4

Telephony

12200. Manual Telephone System. — A manual telephone system is one in which the calling party gives his order to an operator who completes the call directly by hand, either with or without the assistance of one or more additional operators.

12201. Automatic or Full Mechanical Telephone System. — An automatic or full mechanical telephone system is one in which the calling party is enabled to complete a call by remote-control switches without the aid of an operator.

12202. Semi-automatic or Semi-mechanical Telephone System. — A semi-automatic or semi-mechanical telephone system is one in which the calling party gives his order to an operator who completes the call through remote-control switches.

12203. Telephone Exchange. — A telephone exchange consists of one or more central offices with associated plant, by means of which telephone service is rendered in a specified local community.

12204. Telephone Exchange Area or District. — A telephone exchange area or district is the area or district served by a telephone exchange.

12205. Central Office (British "Exchange"). — A central office is a switching center for inter-connecting lines terminating therein.

12206. Toll Central Office. — A toll central office is one in which toll and long distance lines terminate.

12207. Local Central Office. — A local central office is one in which subscriber's lines terminate.

12208. Private Branch Exchange (Generally Abbreviated "P. B. X."). — A private branch exchange is a telephone system generally installed on the premises of a subscriber, including a switchboard and extension sets, and connected to a central office, affording intercommunication between the extension sets and also between these sets and the central office.

12209. Private Exchange. — A private exchange is one which serves one business organization or individual, and is not connected to a central office.

12210. Private Automatic Exchange. — A private automatic exchange is an automatic exchange which serves one business organization or individual, and is not connected to a central office.

12211. Subscriber Set (Often Abbreviated to "Subset"). — A subscriber set is an assembly of apparatus for sending and receiving telephone calls.

12212. Subscriber Station (Often Abbreviated to "Substation"). — A subscriber station is an installed subscriber set connected to a central office for the purpose of sending and receiving telephone calls.

12213. Pay Station (British "Public Call Office"). — A pay station is a subscriber station available for the use of the public on the payment of a fee. The fee may be either deposited in a coin box or paid to an attendant.

12214. Toll Station. — A toll station is a pay station located outside of a local service area and affording toll and long distance service only.

12215. Subscriber Line or Subscriber Loop. — A subscriber line or subscriber loop is the wire connection between a subscriber station and the central office.

12216. Subscriber Line Circuit. — A subscriber line circuit is a subscriber line with its associated individual central office apparatus.

12217. Individual Line (British "Direct Line"). — An individual line is a subscriber line which connects one subscriber station to a central office, though it may have one or more extension sets.

12218. Party Line. — A party line is a subscriber line which connects two or more subscriber stations to a central office.

12219. Tip Side or Tip Wire, Ring Side or Ring Wire. — The tip side or wire, or the ring side or wire, is that conductor of a circuit which is associated with the corresponding member of a jack.

12220. Negative Side or Negative Wire, Positive Side or Positive Wire. — The negative side or wire, or the positive side or wire, is that conductor of a circuit which is normally connected to the corresponding pole of a battery.

12221. Main Distributing Frame. — A main distributing frame is a structure for terminating the permanent inside and outside wires of a central office and for effecting flexible junctions between them. It generally carries central office protective devices and functions as a test point between line and office.

12222. Intermediate Distributing Frame. — An intermediate distributing frame is a structure for terminating permanent inside wires of a central office and for effecting flexible junctions between them.

12223. Switchboard. — A switchboard is an assemblage of apparatus in a coordinate structure for switching talking and signaling circuits.

12224. Switchboard Section. — A switchboard section is an element or unit one or more of which constitutes a complete manual switchboard.

12225. Operating Room. — An operating room is a room which contains a manual switchboard and associated apparatus.

12226. Combination Current. — A combination current consists of two or more currents of different characteristics in the same circuit. As ordinarily used the term refers to currents whose characteristics are steadily maintained, as for example, a combination of direct current and an alternating current.

12227. Manual Ringing. — Manual ringing is ringing which is affected by and continues with the operation of a key.

12228. Machine Ringing. — Machine ringing is intermittent and is caused to act periodically by the apparatus itself.

12229. Superimposed Ringing Current. — A superimposed ringing current is a combination current for ringing, consisting of a direct and an alternating current.

12230. Pulsating Ringing Current. — A pulsating ringing current is a current for ringing in which the succeeding impulses are separated by intervals approximately equal to those of the impulses themselves.

12231. Harmonic Selective Signaling. — Harmonic selective signaling employs devices tuned mechanically or electrically to the frequency of the ringing current, so that each device will not operate when receiving current intended to operate another device.

12232. Multiple Harmonic Signaling. — Multiple harmonic signaling employs frequencies which are integral multiples of the lowest frequency.

12233. Non-multiple Harmonic Signaling. — Non-multiple harmonic signaling employs frequencies which are not integral multiples of the lowest frequency.

12234. "To Call." — "To call" is to originate a telephone call.

12235. "To Dial." — "To dial" a number is to use a dial type of calling device in order to control automatic switches.

12236. "To Set Up." — "To set up" a number is to use a key type or multiple lever type of calling device in order to control automatic switches.

12237. Calling Device. — A calling device is an apparatus by means of which automatic switches are controlled for the purpose of establishing a connection.

12238. Calling Party. — A calling party is a person who originates a telephone call.

12239. Called Party. — A called party is the person who answers when a station is called.

12240. Reverting Call. — A reverting call is one between two stations on the same subscriber line.

12241. Telephone Traffic. — Telephone traffic is the aggregate volume of communication handled in a given time.

12242. "Busy." — "Busy" is the condition of a line or an apparatus when it is in use.

12243. Free. — Free is the condition of a line or an apparatus when it is not in use. Free is the opposite of busy.

12244. "To Make Busy." — "To make busy" is to cause a line or an apparatus to appear to be busy.

12245. "To Release" or to "Disconnect." — "To release" or "to disconnect" is to terminate a telephone connection by disengaging the apparatus.

12246. "To Clear." — "To clear" is to restore a line or an apparatus to the free condition.

12247. Trunk. — A trunk is the wire connection between switching devices or central offices.

12248. Trunk Circuit. — A trunk circuit is a trunk with its associated individual apparatus.

12249. Trunked Call. — A trunked call is one which employs an inter-office trunk or a trunk between two switchboard positions.

12250. Relay. — A relay is a device by means of which contacts in one circuit are operated by a change in conditions in the same circuit or in one or more associated circuits. (See Rule 7016.)

12251. Polar Relay. — A polar relay is a relay which operates in response to a change in the direction of the current in the controlling circuit.

12252. Quick Operating Relay. — A quick operating relay is one which operates its contacts within a specified brief time limit.

12253. Quick Release Relay. — A quick release relay is one which releases its contacts within a specified brief time limit.

12254. Quick Acting Relay. — A quick acting relay is one which has the properties of both a quick operating and a quick release relay.

12255. Slow Operating Relay. — A slow operating relay is one which will not operate until after a specified delay.

12256. Slow Release Relay. — A slow release relay is one which when operated will not release until after a specified delay.

12257. Slow Acting Relay. — A slow acting relay is one which has the properties of both a slow operating and a slow release relay.

12258. Line Relay. — A line relay is one whose coil is normally in the line circuit.

12259. Cut-off Relay. — A cut-off relay is one which when operated disconnects from a line apparatus normally connected to it.

12260. Relay Coil Section. — A relay coil section is one of two or more windings of a coil on one and the same core. The several sections may be concentric or placed side by side on the core.

12261. Tension Spring. — A tension spring is one which functions to exert mechanical pressure but does not carry an electrical current.

12262. Contact Spring. — A contact spring is one which takes an electrical part in switching a circuit.

12263. Main Contact Spring. — A main contact spring is one which may switch a circuit between two or more other contact springs.

12264. Armature Spring. — An armature spring is the first of a group to be moved by the armature. It may or may not be a main contact spring.

12265. Plunger Spring. — A plunger spring is the first of a group to be moved by the plunger.

12266. Impulse Springs. — Impulse springs are those which act to make or break a circuit for the purpose of sending impulses.

12267. Make-Before-Break Contact Springs (Abbreviation "M. B. B."). — Make-before-break contact springs are those in which the main spring touches the front contact before it breaks away from the back contact. Also called a continuity preserving contact.

12268. Back Contact Spring. — A back contact spring is one against which the main contact spring rests when in the normal position.

12269. Front Contact Spring. — A front contact spring is one against which the main contact spring rests when in the operated position.

12270. Automatic Signaling. — Automatic signaling is effected without the aid of an operator.

12271. Automatic Switch. — An automatic switch is a remote control device for controlling talking or signaling circuits.

12272. Finder Switch. — A finder switch is a switch connected to one of a smaller number of circuits and which finds automatically a circuit out of a larger number of circuits from whence the signal comes.

12273. Line Switch. — A line switch is a switch connected to one of a larger number of circuits from which a signal comes and which finds automatically a circuit out of a smaller number of circuits.

12274. Selector Switch. — A selector switch is a switch whose duty is to select a particular group of trunks and one trunk of the group selected. In particular cases, one of these functions may be omitted.

12275. Connector Switch or Final Selector. — A connector switch or final selector is a switch whose duty is to establish a connection with the called line. It is usually operated by the last digit or digits of the call number.

12276. Switch Frame. — A switch frame is a structure for mounting an assembly of switching apparatus which may be integral therewith.

12277. Section of Switches. — A section of switches, considered from a trunking standpoint, is a group of adjacent switches whose banks are multipled together.

12278. Switchroom. — A switchroom is a room which contains an assemblage of automatic switches and associated apparatus.

12279. Bank Wires. — Bank wires are those wires which multiple adjacent switch banks to each other.

12280. Bank Cable. — A bank cable is one which connects a switch bank to a terminal rack.

12281. Multiple Cable. — A multiple cable is one which multiples together two or more sections of switch banks by connecting together their terminals.

12282. Impulse. — An impulse is any sudden change of brief duration produced in the current of a circuit.

12283. Make Impulse. — A make impulse is an impulse due to a temporary flow of current.

12284. Break Impulse. — A break impulse is an impulse due to a temporary interruption of current.

12285. Impulse Frequency. — The impulse frequency is the number of impulses occurring per second. The reciprocal of this is the impulse period.

12286. Impulse Period. — The impulse period is the period of time included between the corresponding points in periodically recording impulses. It thus corresponds to the period of alternating current.

12287. Impulse Ratio. — Impulse ratio is the ratio of duration of an impulse to the impulse period.

12288. Impulse Circuit. — An impulse circuit is one through which impulses are transmitted.

12289. Telephone Impulse Repeater. — A telephone impulse repeater is a device for repeating impulses from one line circuit into another and for performing other duties.

12290. Supervisory Signal. — A supervisory signal is a device for attracting attention of an attendant to a duty in connection with switching apparatus or its accessories. This includes cord supervisory lamps on a manual switchboard and the supervisory lamps in an automatic exchange which indicates that a switch has been occupied but has not completed its function.

12291. Tell-tale Signal. — A tell-tale signal is a device for locating the failure of some apparatus; for example, the blowing of a fuse, the continued drawing of heavy current by apparatus intended to receive only momentary current, etc.

12292. Alarm Signal. — An alarm signal is a sound-producing device for attracting attention to either a supervisory or a tell-tale signal.

12293. Amplifier. — See § 13040.

12294. Telephone Repeater. — A telephone repeater is a device for amplifying a voice current from one line circuit into another line circuit.

12300. Telephone Receiver. — A telephone receiver is an electrically operated device designed to produce sound waves or vibrations which correspond to the electromagnetic waves or vibrations actuating it.

12301. Microphone. — A contact device designed to have its electrical resistance directly and materially altered by slight differences in mechanical pressure.

12302. Telephone Transmitter. — A telephone transmitter is a sound-wave-operated or vibration-operated device designed to produce electromagnetic waves or vibrations which correspond to the sound waves or vibrations actuating it.

12303.* Coefficient of Coupling of a Transformer. — The coefficient of coupling of a transformer at a given frequency is the ratio of the mutual impedance between the primary and secondary of the transformer, to the square root of the product of the self-impedances of the primary and of the secondary.

12304. Repeating Coil. — A term used in telephone practice meaning the same as transformer, and ordinarily a transformer of unity ratio.

(12303) Single-frequency voltages and currents are here supposed to be represented by complex numbers. Their ratio is therefore a complex number.

12305.* Retardation Coil. — A retardation coil is a reactor (reactance coil) used in a circuit for the purpose of selectively reacting on currents which vary at different rates.

12306. Manual Switchboard. — A manual switchboard is one in which the switching operations are performed by hand.

12307. Multi Office Exchange (British "Multi Exchange Systems"). — A multi office exchange is one which is composed of more than one office.

12308. Trunk Hunting. — Trunk hunting is the operation of an automatic switch in moving its wipers or brushes to an idle set of terminals or contacts in a chosen group of terminals or contacts.

12309. Wiper. — A wiper is that portion of the moving member of a selector which engages with a bank contact.

12310. Bank. — A bank is an assembly of fixed contacts with which the moving members of a selector engage. Banks are usually multiplied.

Telegraphy

12500. Relay. — A relay is a device by means of which contacts in one circuit are operated by a change in conditions in the same circuit or in one or more associated circuits.

12501. Polar Relay. — A polar relay is a relay which operates in response to a change in the direction of the current in the controlling circuit.

12502. Non-polar Relay, or Neutral Relay. — A non-polar relay is a relay which operates in response to a change in the strength of the current in the controlling circuit, irrespective of the direction of the current.

12503. Neutral Relay. — See non-polar relay.

12504. Selector. — A selector is a device which performs certain functions such as causing an electric lamp to light, or an electric bell to sound, in response to a definite signal or group of successive signals received over a controlling circuit.

12505. Direct-point Repeater. — A direct-point repeater is a repeater in which the receiving relay controlled by the signals received over a line repeats these signals into another line or lines without the interposition of any other repeating or transmitting apparatus.

12506. Concentrator. — A concentrator is a traffic distributing device by means of which a number of telegraph or telephone lines, and connections to operating instruments are brought together at one point to facilitate their interconnection at such times as signals or messages are to be transmitted from one to the other.

12507. Transmitter. — A transmitter is a device for effecting electrical changes in a controlled circuit. The term transmitter is commonly applied principally to devices which in response to a controlling means effects in a main line telegraph circuit electrical changes necessary to send signals over the line.

12508. Synchronous System. — A synchronous system of telegraphy is one in which the proper transmission and reception of signals is dependent upon the synchronous operation of similar commutators or other devices located at the sending and receiving stations of a circuit.

12509. Differential Duplex. — A differential duplex is a duplex system in which at each station one of two portions of the receiving instrument is connected in series with the line wire and the other in series with an artificial line of such electrical characteristics that the effects upon the receiver of currents passing through the main and artificial lines, as a result of outgoing signals, are neutralized.

12510. Bridge Duplex. — A bridge duplex is a duplex system in which the receiving instruments at each station are connected across two impedances, one

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(12305) In telephone and telegraph usage, the terms "impedance coil," "inductance coil," "choke coil" and "reactance coil" are sometimes used in place of the term "retardation coil."

in series with the line wire and the other in series with the artificial line in such manner that no electrical change in the receiver circuit is effected by outgoing signals.

12511. Half-set Repeater. — A half-set repeater is a repeater used for connecting together a simplex circuit and a duplexed circuit converting them into the equivalent of a single simplex circuit.

12512. Intermediate Current Supply. — An intermediate current supply is an ungrounded source of current connected in series with a line wire at a station other than a terminal on a ground return telegraph circuit.

12513. Phantoplex Circuit. — A phantoplex circuit is a superposed circuit operated by alternating current over a simplex, duplex or quadruplex circuit operated from direct-current sources.

12514. Spark Condenser. — A spark condenser is a condenser, with or without associated non-inductive resistance, connected with a pair of instrument contact points for the purpose of diminishing sparking at these points.

12515. Current Margin. — In a non-polar simplex system, the difference between the current flowing through a receiving instrument when operated, to that flowing when not operated.

12516. Margin Ratio. — In a non-polar simplex system, the ratio of the current flowing through a receiving instrument when operated, to that flowing when not operated.

12517. Percentage Margin. — In a non-polar simplex, the current margin expressed as a percentage of the current flowing through the relay when operated.

12518. Main Circuit. — A main circuit is a major electrical circuit of a telegraph system and includes both transmitting and receiving devices.

12519. Local Circuit. — A local circuit is a circuit, within the limits of the station, usually controlled by a receiving instrument in a main circuit or controlling a transmitter effecting changes in a main line circuit.

CHAPTER XIII

STANDARDS FOR RADIO COMMUNICATION

General

This chapter has been mainly abstracted from the report of the Standardization Committee of the Institute of Radio Engineers, and is here included by permission, until further revised. For full particulars, see the I. R. E. Standardization Committee report.

13000. Acoustic Resonance Device. — One which utilizes, in its operation, resonance to the audio frequency of the received signals.

13001. Antenna. — A system of conductors designed for radiating or absorbing the energy of electromagnetic waves.

13002. Atmospheric Absorption. — That portion of the total loss of radiated energy due to atmospheric conductivity.

13003. Audio Frequencies. — Frequencies corresponding to the normally audible vibrations. These are assumed to lie below 10,000 cycles per second.

13004. Capacitive Coupler. — An apparatus which, by electric fields, joins portions of two radio-frequency circuits, and which is used to transfer electrical energy between these circuits through the action of electric forces.

13005. Coefficient of Coupling (Inductive). — The ratio of the effective mutual inductance of two circuits to the square root of the product of the effective self-inductances of each of these circuits.

13006. Direct Coupler. — A coupler which magnetically joins two circuits having a common conductive portion.

13007. Counterpoise. — A system of electrical conductors forming one portion of a radiating oscillator, the other portion of which is the antenna. In land stations a counterpoise forms a capacitive connection to ground.

13008. Damped Alternating Current. — A damped alternating current is an alternating current whose amplitude progressively diminishes.

13009. Damping Factor. — The damping factor of an exponentially damped alternating current is the product of the logarithmic decrement and the frequency.

Let I_0 = initial amplitude,
I_t = amplitude at the time t,
ϵ = base of Napierian logarithms,
a = damping factor.

Then $I_t = I_0 \epsilon^{-at}$.

13010. Detector. — That portion of the receiving apparatus which, connected to a circuit carrying currents of radio frequency, and in conjunction with a self-contained or separate indicator, translates the radio-frequency energy into a form suitable for operation of the indicator. This translation may be effected either by the conversion of the radio-frequency energy, or by means of the control of local energy by the energy received.

13015. Electromagnetic Wave. — A periodic electromagnetic disturbance progressing through space.

13016. Forced Alternating Current. — A current, the frequency and damping of which are equal to the frequency and damping of the exciting electromotive force.

13017. Free Alternating Current. — The current following any electromagnetic disturbance in a circuit having capacitance, inductance, and *less* than the critical resistance.

13018. Critical Resistance of a Circuit. — That resistance which determines the limiting condition at which the oscillatory discharge of a circuit passes into an aperiodic discharge.

13019. Group Frequency. — The number per second of periodic changes in amplitude or frequency of an alternating current.

Note 1. Where there is more than one periodically recurrent change of amplitude or frequency, there is more than one group frequency present.

Note 2. The term "group frequency" replaces the term "spark frequency."

13020. Inductive Coupler. — An apparatus which, by magnetic forces, joins portions of two radio-frequency circuits and is used to transfer electrical energy between these circuits, through the action of these magnetic forces.

13025. Logarithmic Decrement. — The logarithmic decrement of an exponentially damped alternating current is the logarithm of the ratio of successive current amplitudes in the same direction.

Note: Logarithmic decrements are standard for a complete period or cycle.

Let: I_n and I_{n+1} be successive current amplitudes in the same direction.

d = logarithmic decrement.

Then: $$d = \log_\epsilon \frac{I_n}{I_{n+1}}.$$

13026. Radio Frequencies. — The frequencies higher than those corresponding to the normally audible vibrations, which are generally taken as 10,000 cycles per second. See also Audio Frequencies.

Note: It is not implied that radiation cannot be secured at lower frequencies and the distinction from audio frequencies is merely one of definition based on convenience.

13027. Resonance. — Resonance of a circuit to a given exciting alternating e.m.f. is that condition due to variation of the inductance or capacity in which the resulting effective current (or voltage) in that circuit is a maximum.

13028. Standard Resonance Curve. — A standard resonance curve is a curve the ordinates of which are the ratios of the square of the current at any frequency to the square of the resonant current, and the abscissas are the ratios of the corresponding wave length to the resonant wave length; the abscissas and ordinates having the same scale.

13029. Sustained Radiation. — Sustained radiation consists of waves radiated from a conductor in which an alternating current flows.

13030. Tuning. — The process of securing the maximum indication by adjusting the time period of a driven element. (See Resonance.)

13035. Wave-Meter. — A wave-meter is a radio-frequency measuring instrument, calibrated to read wave lengths.

13036. Decremeter. — An instrument for measuring the logarithmic decrement of a circuit or of a train of electromagnetic waves.

13037. Attenuation, Radio. — The decrease with distance from the radiating source, of the amplitude of the electric and magnetic forces accompanying (and constituting) an electromagnetic wave.

13038. Attenuation, Coefficient of (Radio). — The coefficient which, when multiplied by the distance of transmission through a uniform medium, gives the natural logarithm of the ratio of the amplitude of the electric or magnetic forces at that distance, to the initial value of the corresponding quantities.

13039. Coupler. — An apparatus which is used to transfer radio-frequency energy from one circuit to another by associating portions of these circuits.

13040. Amplifier. — An amplifier is an instrument which modifies the effect of a local source of energy in substantial accordance with the wave-form of the received energy, and gives out a wave of greater amplitude than that which it receives.

13042. Phase Angle Defect. — The phase angle defect of a condenser is the departure from quadrature of the phase difference between potential and current at terminals. This is sometimes called the phase angle of a condenser: although strictly speaking the phase angle of a condenser is 90° less the phase angle defect, and is therefore exactly 90° when the phase angle defect is zero.

13043. Impulse E.m.f. — An e.m.f. the effective value of which becomes small compared with its maximum value in a time which is short compared with the duration of the current which it causes.

13044. Directive Coefficient. — The directive coefficient of a transmitting antenna at a given distance therefrom on the surface of the earth or sea, for a given wave length, is the ratio of average field intensity within an angle of stated degrees centered about the direction of maximum radiation, to the average field intensity in all directions.

13045. Directional Selectivity. — The directional selectivity of a receiving antenna at a given wave length is the ratio of the average e.m.f. induced in that antenna for waves of equal intensity coming from directions comprised within an angle of stated degrees centered about the direction of best reception, to the average e.m.f. induced in the antenna for waves of equal intensity coming from all directions.

13046. Radiation Efficiency. — The radiation efficiency of an antenna at a given wave length is the ratio of radiation resistance to the antenna resistance.

13047. Selectivity. — The (overall) selectivity of a receiving system is the product of the several selectivities of that system.

13048. Average Selectivity. — The average selectivity of a receiving system is the *n*th root of the product of the *n* selectivities of that system.

13049.* Radio-Frequency Selectivity. — The radio-frequency selectivity of a simple element * of a receiving system is the ratio of resonant response (in terms of effective voltage or current measured at the indicator) to the non-resonant response when the radio-frequency portions of the elements of that system are detuned by one per cent of the resonant frequency.

(13049) A simple element as referred to a combination of an inductance, a capacitance and optionally a resistance; or their mechanical equivalent.

CHAPTER XIV

STANDARDS FOR PRIME MOVERS AND GENERATOR UNITS

General

14000. Regulation of Steam Engines, Steam Turbines and Internal Combustion Engines. — In steam engines, steam turbines and internal combustion engines, the percentage speed regulation is usually expressed as the percentage ratio of the maximum variation of speed, to the rated-load speed in passing slowly from rated-load to no-load (with constant conditions at the supply.)

14001. Fluctuation of Steam Engines, Steam Turbines and Internal Combustion Engines. — The percentage fluctuation of a steam engine, steam turbine or internal combustion engine, is the immediate percentage speed regulation corresponding to a sudden change from rated-load to no-load.

14002. Regulation of Hydraulic Turbines. — In a hydraulic turbine or other water motor, the percentage speed regulation is expressed as the percentage ratio of the maximum variation in speed in passing slowly from rated-load to no-load (at constant head of water), to the rated-load speed.

14003. Regulation of Generator Units. — In a generator unit, consisting of a generator combined with a prime mover, the speed or voltage regulation shall be based upon constant conditions of the prime mover; *i.e.*, constant steam-pressure, head, etc. It includes the inherent speed variations of the prime mover. For this reason, the regulation of a generator unit is to be distinguished from the regulation of either the prime mover, or of the generator combined with it, when taken separately.

14010.* Variation in Prime Movers. — The variation in prime movers which do not give an absolutely uniform rate of rotation or speed, as in reciprocating steam engines, is the maximum angular displacement in position of the revolving member expressed in degrees, from the position it would occupy with uniform rotation, and with one revolution taken as 360 degrees. See § 4088.

14011. Pulsation in a Prime Mover, or in the Alternator Connected Thereto — The pulsation in a prime mover, or in the alternator connected thereto, is the ratio of the difference between the maximum and minimum velocities in an engine-cycle to the average velocity.

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Foreign

British Engineering Standards Association: Standard Electrical Pressures for New Systems and Installations (Low and Medium Pressures); Steam Turbines for Electrical Plants; Report on Reciprocating Steam Engines for Electrical Purposes.

(14010) If p is the number of pairs of poles, the variation of an alternator is p times the variation of its prime mover, if direct connected, and $p\,n$ times the variation of the prime mover if rigidly connected thereto in such a manner that the angular speed of the alternator is n times that of the prime mover.

CHAPTER XV

STANDARDS FOR TRANSMISSION LINES AND DISTRIBUTION LINES

Note: For flash-over tests of insulators, see foot-note to § 2361.

General

15000. Regulation of Transmission Lines, Feeders, etc. — The regulation of transmission lines, feeders, etc., is the change in the voltage at the receiving end between rated non-inductive load and no-load, with constant impressed voltage upon the sending end. The percentage regulation is the percentage change in voltage to the normal rated voltage at the receiving end.

CHAPTER XVI

MISCELLANEOUS STANDARDS

HEATING DEVICES

16000. Value of A-C. Test Voltage for Household Devices. — Heating devices taking not over 660 watts, intended solely for operation on supply circuits not exceeding 275 volts, shall be tested with 500 volts at operating temperature.

INTERNATIONAL ELECTROTECHNICAL COMMISSION

I. E. C. RULES FOR ELECTRICAL MACHINERY

VOLUME I

(*Adopted at the Plenary Meeting, held in London, October,* 1919)

These rules apply to rotating machines of which the terminal pressure does not exceed 5000 volts or of which the rated output does not exceed 750 kv.-a., or of which the stator cores do not exceed 50 cm. in length axially, and to all transformers which are not water-cooled.

PART I. GENERAL

I. Scope of Rules

1. *Rotating Machines.* — The rules of the I. E. C. contained in this publication apply to rotating machines, of which the terminal pressure does not exceed 5000 volts or of which the rated output does not exceed 750 kv.-a., or of which the stator cores do not exceed 50 cm. in length axially.

2. *Transformers.* — These rules also apply to all transformers which are not water-cooled. (For water-cooled transformers, *see* Appendix II.)

3. *Altitude.* — In the absence of any information in regard to the height above sea level at which the machine is intended to work in ordinary service, this height is assumed not to exceed 1000 meters. If the machine is intended to work at an altitude above 1000 meters a correction to the temperature rise should be applied. The value for this correction has not yet been fixed by the I. E. C.

4. *Temperature.* — In the absence of any information to the contrary, it is assumed that the temperature of the cooling air shall not exceed 40° C.

II. Definitions

5. *Rating.* — (See Appendix IV.)

6. *Use of the Term "Machine."* — The term "machine" is used in these rules in its most general sense so as to avoid the constant repetition of the words "machines, transformers and other electromagnetic induction apparatus."

7. *Use of the Term "Power."* — It is usual to speak of a machine by its *power*. It is necessary to note that the term "power" should be used in the following way:

(a) For direct-current generators, the electric power at the terminals expressed in watts (w.) or kilowatts (kw.).

(b) For alternators, the apparent power at the terminals, expressed in volt-amperes (v.-a.) or kilovolt-amperes (kv.-a.).

(c) For motors, the mechanical power available at the shaft, expressed in watts (w.) or kilowatts (kw.).

(d) For transformers, apparent output at the secondary terminals, expressed in volt-amperes (v.-a.) or kilovolt-amperes (kv.-a.).

III. I. E. C. Rating

8. *Test Rating.* — The I. E. C. rating has been established as a *test* rating which will enable an exact comparison to be made between machines of different makes.

9. *Classes of Rating.* — There are two classes of I. E. C. rating:

(a) The I. E. C. continuous raing (*see* Clause 10).

(b) The I. E. C. short-time rating or limited-time rating (*see* Clause 12).

10. *Continuous Rating.*—The I. E. C. continuous rating is the load which can be carried on test, under the conditions of that rating, for an unlimited period without the limits of the I. E. C. rules, as regards temperature rise, being exceeded.

11. *Service Thermally Equivalent to the Continuous Rating.* — Any machine intended for continuous service on fluctuating load may be given for test purposes a thermally equivalent I. E. C. continuous rating, provided that the service for which it is intended shall not cause, in any of its parts, temperatures or temperature rises in excess of those allowed by the I. E. C. rules when the machine is tested under the conditions of its continuous rating.

General Note re Classes of Fluctuating Load Service, inserted by the Editing Committee. — It is desirable to distinguish between two kinds of fluctuating load service:

(*i*) *That in which the overload peaks can be sustained by a machine of ordinary construction, without modification, and without exceeding the limits of temperature rise allowed by these rules for the machine when tested under its I. E. C. continuous rating.*

(*ii*) *That in which the overload peaks involve special provisions in design or construction either for mechanical or for electrical reasons.*

To designate this second class of service it is customary in Great Britain and the United States of America to employ the term "duty cycle rating," and the word "cycle" in this case signifies a period of time sufficiently long to include all the variations of load which might influence either the electrical construction or the mechanical construction of the machine.

12. *Short-time Rating.*—The I. E. C. short-time rating is the load which can be carried on test for the time specified in the rating, the test being started with the machine cold and carried out under all the conditions of the rating, without the limits fixed by these I. E. C. rules, as regards temperature rise, being exceeded.

13. *Service Thermally Equivalent to the Short-time Rating.* — Any machine intended for service on loads which vary considerably may be given for test purposes a thermally equivalent I. E. C. short-time rating, provided that the service for which it is intended shall not occasion, in any of its parts, temperatures or temperature rises in excess of those allowed by these rules when the machine is tested under the conditions of its short-time rating.

PART II. INFORMATION TO BE GIVEN WITH ENQUIRIES AND ORDERS FOR ELECTRICAL MACHINES

IV. General Information

Note. — The term *machine* is used in these rules in its most general sense so as to avoid the constant repetition of the words *machines*, *transformers* and other *electromagnetic induction apparatus*.

14. *General Information.* — The enquiry or order for an electrical machine should give the following general information:

(a) The service output.

(b) The class of service required.

In the absence of any indication to the contrary, continuous service is understood.

(c) The maximum temperature of the cooling air in which the machine is intended to work when it exceeds 40° C.

In the absence of any definite information, it is understood that the temperature of the cooling air will not exceed 40° C.

(d) The altitude of the place where the machine is intended to work, if it exceeds 1000 meters. In regard to altitude, *see* Clause 3.

(e) Any special requirements, if necessary, with regard to windings, methods of connection, neutral points and special tapping points, etc.

(f) When the apparatus is intended to operate in parallel with other apparatus, the fact should be stated.

(g) Any special requirements in regard to electrical and mechanical details such as protective devices, cooling arrangements, etc.

(Specific recommendations regarding these details will be made at a later date by the I. E. C.)

V. Supplementary Information

15. *Supplementary Information.* — The above general information should be completed by the following supplementary information in regard to the particular machine forming the subject of the order:

16. *Direct-current Generator.*
 Output at the terminals, in watts (w.) or kilowatts (kw.).
 Pressure between terminals, in volts.
 Current, in amperes.
 Speed, in revolutions per minute.
 Method of excitation.

17. *Direct-current Motor.*
 Output at the shaft, in watts (w.) or in kilowatts (kw.).
 Pressure between terminals, in volts.
 Current, approximate, in amperes.
 Speed at rated output, approximate, in revolutions per minute.
 Method of excitation.

18. *Alternating-current Transformer.*
 Frequency, in periods per second.
 Number of phases.
 Output, in voltamperes (v.-a.), or in kilovoltamperes (kv.-a.).
 Primary pressure between terminals, in volts.
 Secondary pressure between terminals, in volts, at no-load and at rated output with statement as to the power factor of the circuit fed by the secondary. If the power factor is not specified it shall be taken as 0.8.
 Secondary current, in amperes.
 For transformers intended to work in parallel, the primary pressure, current, and power factor on short-circuit test, shall also be stated.
 For three-phase transformers the method of connection shall also be indicated in accordance with the vector diagrams (*see* Appendix I).
 Any special requirements as to the accessibility of neutral points and special tapping points shall be indicated.

Note. — Whatever may be the nature of the transformers (step-up or step-down) the primary terminals are those which are connected to the source of electrical energy and the secondary terminals those which receive the electrical energy.

19. *Synchronous Alternator for Alternating Currents, Single or Polyphase.*
 Frequency, in periods per second.
 Number of phases.
 Output between terminals, in volt-amperes (v.-a.) or kilovolt-amperes (kv.-a.).
 Pressure between terminals, in volts, corresponding to the rated output.
 Power factor of the system to be supplied. If this is not specified it shall be taken as 0.8.
 Current, in amperes.
 Speed, in revolutions per minute.
 Excitation pressure, in volts (if the alternator is not provided with a special exciter).
 Maximum exciting current available, in amperes (if the alternator is not provided with a special exciter).

20. *Synchronous Motor for Alternating Currents, Single or Polyphase.*
Frequency, in periods per second.
Number of phases.
Mechanical output at the shaft, in watts (w.) or in kilowatts (kw.).
Current, approximate, in amperes.
Pressure, in volts, of supply available.
Speed, in revolutions per minute.
Unless otherwise specified, the motor must be capable of giving its rated mechanical output at unity power factor.
If the motor is required to act as a device for improving the power factor, the value of the reactive power required shall be stated.
Excitation pressure, in volts (if the motor is not provided with a special exciter).
Method of starting to be employed and source of power available for this purpose.
Maximum exciting current available, if limited.

21. *Non-synchronous Motor for Alternating Currents, Single or Polyphase.*
Frequency, in periods per second.
Number of phases.
Mechanical power at the shaft, in watts (w.) or in kilowatts (kw.).
Pressure between terminals, in volts.
Current, approximate, in amperes.
Speed, in revolutions per minute, approximate, at the rated output.
Rotor, whether wound or squirrel cage.
Method of starting.
Unless otherwise specified it is assumed that the stator receives the supply current.
Starting torque in kilograms at one meter.
Ratio of the starting current to the current corresponding to the rated output.
Ratio of the starting torque to the torque corresponding to the rated output.
The last three items are to be stated for the motor with its starting accessories.

PART III. CONDITIONS TO BE FULFILLED BY ELECTRICAL MACHINERY

VI. General Remarks

22. *General.* — This section deals with the conditions to be fulfilled by a machine purporting to comply with the I. E. C. Rules.

VII. Limits of Temperature and Temperature Rise

Temperature Limits

23. *Table of Temperature Limits.* — The following table gives the limits for the observable temperatures and temperature rises of windings and of certain parts of machines.

The permissible temperature limits are indicated in Column 1 of the table.
The permissible limits of temperature rise are given in Column 2. The temperature rises measured on any machine which has worked for the specified time at the output corresponding with its I. E. C. rating shall not exceed in any of its parts the limiting values given in Column 2 of the table. The highest permissible temperature given in Column 1 and the temperature rises given in Column 2 of the table should never be exceeded by a machine operating in service.
(For exception *see* Clause 27.)

Temperature Limits

Item No.	Nature of the insulation of the winding or name of part.	Column 1 Highest permissible observable temperature	Column 2 Highest permissible observable temperature rise for the purpose of fixing the international rating
		Degrees C.	Degrees C.
1	Cotton, paper or silk, non-impregnated	80	40
2	Cotton, paper or silk, impregnated (*see* Clause 24)	95	55
3	Cotton, paper or silk, immersed in oil	95	55
4	Enamelled wire (*see* Clause 25)	95	55
5	Mica, asbestos, glass, porcelain, micanite and similar compositions	115	75
6	Insulated windings permanently short-circuited	100	60
7	Non-insulated windings permanently short-circuited	110	70
8	Oil (for temperature limits, *see* Appendix II).	..	..
9	Commutators, slip rings (*see* Cl. 27)	90	50
10	Bearings	80	40
11	Iron core immersed in oil	95	55
12	Iron core in contact with windings	Same as the windings	
13	Iron core not in contact with windings nor immersed in oil. The temperature and temperature rise shall not exceed that allowed for the windings themselves, and in no case shall the temperature and temperature rise exceed 110° C. and 70° C. respectively.		
14	Single layer windings: An increase of 5° C. above the temperatures given for items 1, 2 and 4 shall be permitted in the case of coils, revolving or stationary, with single layer windings when not immersed in oil.		

24. *Impregnated Cotton, Paper or Silk.*—An insulation is considered to be "impregnated" when a suitable substance replaces the air between its fibers, even if this substance does not completely fill the spaces between the insulated conductors. The impregnating substance, in order to be considered suitable, must have good insulating properties; must entirely cover the fibers and render them adherent to each other and to the conductor; must not produce interstices within itself as a consequence of evaporation of the solvent or through any other cause; must not flow during the operation of the machine at full working load at the temperature limit specified; must not deteriorate under prolonged action of heat.

25. *Enamelled Wire.* — When employing the temperature limits in the table for enamelled wire the maker must satisfy himself that the enamel employed is of good quality.

26. *Compound Insulations Made Up of Different Materials.* — When the insulation consists of layers of several different materials, the lowest of the temperatures permitted for the different insulating materials employed (*see* Clause 23) is to be adopted as the limiting temperature. The insulating material, even when forming the support, shall always be assumed as forming part of the winding.

27. *Commutators and Slip Rings.* — The observable temperature and temperature rise of commutators and slip rings may exceed the values given in Item 9 of the table, provided that the three following conditions are fulfilled:

(a) The temperatures of the insulating materials in the commutator and on the adjoining windings shall not exceed those allowed in the table for the insulating materials of those parts.

(b) The manufacturer shall give a special guarantee that the high temperature attained shall not impair the commutation.

(c) The temperature shall not be so high as to affect the quality of the soldered joints and the connections.

Reference Temperature of Cooling Medium

28. *Reference Temperature of Cooling Medium.* — (a) *Temperate Climates.* — In the absence of any indication to the contrary the maximum temperature of the air in which the machine is intended to operate in service shall be deemed to be 40° C.

(b) *Cold Climates.* — In cold climates, when the actual temperature of the air in which the machine is intended to operate in service is not much different from 40° C. it is recommended that this conventional reference temperature of 40° C. should be adopted.

(c) *Tropical Climates.* — The question of a reference temperature for cooling air for machines intended to operate in service in tropical climates will be dealt with by the I. E. C. at a later date.

(d) *Water Cooling* (*see* Appendix II).

Permissible Limits for Temperature Rise

29. *Permissible Limits for Temperature Rise.* — The limits permitted for temperature rise are deduced from the values allowed for the highest permissible observable temperature (*see* Clauses 23–27) by subtracting therefrom 40° C. (the value assumed as that of the maximum cooling air temperature of the place in which the machine may be required to work in service (*see* Clause 28).

Temperature Measurements

30. *Value of Temperature of Cooling Medium.* — A machine may be tested at any convenient cooling air temperature less than 40° C., but whatever be the value of this cooling air temperature the permissible rises of temperature shall not exceed those given in Column 2 of the table (*see* Clause 23).

Corrections for variations in the cooling air temperature are not considered necessary within the limits of cooling air temperature obtaining in general practise.

In the case of cooling by means of forced ventilation, the temperature of the air, measured where it enters the machine, shall be considered as the cooling air temperature.

For all machines cooled by other means, special rules will be necessary. (For water cooling, *see* Appendix II).

31. *Measurement of Cooling Air Temperature During Tests.* — The cooling air temperature shall be measured by means of several thermometers placed at different points around and half-way up the machine at a distance of one to two meters, and protected from all heat radiation and draughts.

The value to be adopted for the temperature of the cooling air during a test shall be the mean of the readings of the thermometers (placed as mentioned above), taken at equal intervals of time during the last quarter of the duration of the test.

In order to avoid errors due to the time lag between the temperature of large machines and the variations in the cooling air, all reasonable precautions shall be taken to reduce these variations and the errors arising therefrom.

Methods of Measurement of the Temperatures of machines

32. *Measurement of the Temperatures of Machines.* — Two methods of determining the temperature of windings and other parts of machines are recognized:

(a) Thermometer method.
(b) Resistance method.*

33. *Thermometer Method.* — In this method the temperature is determined by thermometers applied to the accessible surfaces of the completed machine. The term "thermometer" also includes thermocouples and resistance-thermometers.

34. *Resistance Method.* — In this method the temperature rise of the windings is determined by the increase in the resistance of the windings themselves and checked by thermometers applied to the accessible surfaces of the windings to ascertain whether there is any higher local temperature. The highest of the temperatures thus found shall be taken as the observable temperature.

35. *Temperature of Windings.* — The temperature of windings, as a rule, shall be measured by the resistance method. The thermometer method alone is permitted in the following cases:

(a) When it is not practicable to determine the temperature rise by the resistance method, as for example with low resistance commutating coil and compensating windings, and in general in the case of low resistance windings, especially when the resistance of joints and connections forms a considerable portion of the total resistance. In this case the temperature limits given in the table apply without correction.

(b) Single layer windings, revolving or stationary, when not immersed in oil. In this case an increase of 5° C. above the limits of temperature and of temperature rise given in the table is permitted.

(c) When, for reasons of manufacturing in quantity, the thermometer method is used alone, although the resistance method would be possible. In this case the value of the highest permissible observable temperature and temperature rise given in the table shall be reduced by five degrees except in the case of stationary field coils, when the values given in the table shall be reduced by the difference between resistance and thermometer measurements as determined on similar machines, but in no case shall such reduction be less than 5° C.

36. *Corrections of Measurements Taken after the Machine Has Shut Down.* — If the temperature is measured only after shut-down, the highest temperature attained while running shall be deduced by extrapolation on the time-temperature curve.

37. *Measuring Temperature of Direct-current Generators and Motors.* — The temperature of field windings shall be measured in the manner described in Clauses 35 and 36.

The temperature of the armature shall be determined as a rule by thermometers placed on the windings at the hottest accessible parts, and when this method is employed the value of the highest permissible observable temperature and temperature rise shown in the table shall be reduced by 5° C.

38. *Measuring Temperature of Transformers.* — The temperature of transformer windings shall always be ascertained by resistance.

39. *Measuring Temperature of Synchronous Alternators and Motors.* — The temperature of the field windings shall always be ascertained by resistance. The temperature of stator windings shall be ascertained either by resistance or by thermometer in the manner described in the preceding clauses.

40. *Measuring Temperature of Non-synchronous Motors without Commutators.* — The temperatures of the stator and rotor shall be ascertained in the same manner as those of the stator of a synchronous alternator (*see* Clause 39), except in the case of a permanently short-circuited winding, when the thermometer method shall be employed.

41. *Coefficients of Variation of Resistance of Copper with Temperature.* — In the case of resistance measurements, the temperature coefficient of copper shall be taken from the values stated in the accompanying table, which have been

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* Note. — With a view to brevity, the expression "method of variation of resistance of the winding" is replaced by the term "resistance method," or simply "by resistance."

deduced from the formula $1/(234.5+t)$. Thus, at an initial temperature $t = 30°$ C. the temperature coefficient or increase in resistance per degree Centigrade rise is $1/(264.5) = 0.00378$.

Temperature of the windings in degrees C., at which the initial resistance is measured.	Copper—increase in resistance per ohm per degree C.
0	0.00427
5	0.00418
10	0.00409
15	0.00401
20	0.00393
25	0.00385
30	0.00378
35	0.00371
40	0.00364

42. When the temperature of a winding is to be determined by resistance, the temperature of the winding before the test measured by thermometer shall not differ much from that of the cooling air.

43. *Duration of Temperature Test for Continuous Rating.* — For machines with I. E. C. continuous rating, the temperature test shall be continued until it is evident that the maximum temperature rise attained would not exceed the limits given in the table (*see* Clause 23), if the test were to be prolonged until the final steady temperature were attained. If possible, the temperature shall be measured both while running and after shut-down.

44. *Duration of Temperature Test for Short-time Rating.* — For machines with I. E. C. short-time rating the duration of the temperature test shall be that corresponding to the short-time test rating as indicated upon the rating plate.

At the commencement of the test the temperature of the machine must be practically that of the cooling air.

VII. Dielectric Tests

(*See* Appendix III for proposals.)

IX. Mechanical Tests

(Not yet prepared.)

X. Commutation

(Not yet prepared.)

PART IV. MARKINGS

XI. Rating Plates

45. *Rating Plate.* — Every machine shall bear the information necessary to define the limitations of the service for which it is intended.

For this purpose it shall have, in all cases, a rating plate and also such diagrams and terminal markings as may be necessary.

46. *Information on Rating Plate.* — The rating plate of a machine complying with the I. E. C. rules shall have a distinctive special sign and give the following information:

(a) The name of the maker.

(b) The maker's machine number.

(c) The class of rating or the necessary information if the machine is intended to operate under more than one class of rating.

(d) The altitude at which the machine is intended to work if such altitude exceeds 1000 meters.

(e) The following technical information, according to the character of the machine:

In the absence of any indication in regard to the class of rating it is understood that the machine is intended for continuous service.

47. *Direct-current Generator.*

Generator — Direct-current.
Output, in watts (w.) or in kilowatts (kw.), with statement as to the class of rating.
Pressure between terminals, in volts.
Current, in amperes.
Speed, in revolutions per minute.

48. *Direct-current Motor.*

Motor — Direct-current.
Output, in watts (w.) or in kilowatts (kw.), with statement as to the class of rating.
Pressure between terminals, in volts.
Current, approximate, in amperes.
Speed, in revolutions per minute.

49. *Transformer.*

Frequency, in periods per second.
Number of phases.
Apparent output at the secondary, in volt-amperes (v.-a.) or in kilovolt-amperes (kv.-a.), with statement as to the class of rating.
Primary pressure between terminals, in volts.
Secondary pressure, in volts, at no load and at rated load, with statement as to the power factor.
Short-circuit pressure, in volts.
Secondary current, in amperes.
In addition, for three-phase transformers, a vector diagram indicating the method of connection of the windings in accordance with the figures. (*See* Appendix I.)

50. *Alternator.*

Frequency, in periods per second.
Number of phases.
Apparent output, in volt-amperes (v.-a.) or kilovolt-amperes (kv.-a.), with statement as to the class of rating.
Pressure between terminals, in volts, corresponding to the rated output.
Current, in amperes.
Power factor corresponding to the rated output.
Speed, in revolutions per minute.
Excitation pressure, in volts.
Maximum exciting current, in amperes.

51. *Synchronous Motor.*

Frequency, in periods per second.
Number of phases.
Mechanical output, in watts (w.) or in kilowatts (kw.), with statement as to the class of rating.
Pressure between terminals, in volts, corresponding to the rated output.
Current, approximate, in amperes.
If the motor is intended to work with a power factor different from unity, the necessary information to be given.
Speed, in revolutions per minute.
Excitation pressure, in volts.
Maximum exciting current, in amperes.

52. *Non-synchronous Motor.*

Frequency, in periods per second.
Number of phases.
Mechanical output, in watts (w.) or in kilowatts (kw.), with statement as to the class of rating.
Pressure between terminals, in volts.
Current, approximate, in amperes.
Speed, in revolutions per minute, at rated output.
Maximum pressure between slip rings, in volts.

APPENDIX I

At the meeting of the Advisory Committee on Rating held in September, 1913, the question of terminal marking and vector diagrams was referred to Messrs. Everest and la Cour. The following rules were forwarded to the Central Office, but as they have not been submitted to the National Committees, the Editing Committee decided to include them as an Appendix only. They will, therefore, be submitted to the next Plenary Meeting for ratification.

Terminal Markings for Transformers

1. *Single-phase Transformers.* — The terminals of all single-phase transformers shall be marked with the letter T for the high-pressure side and *t* for the low-pressure side.

The neutral terminal, if provided, shall be marked by N or *n*.

The letters T and *t* should be accompanied by subscripts 0, 1, 2, etc., arranged in order of progression in the same direction as the electromotive force in each circuit at the same instant, as shown in the diagram.

If the transformer has two or more windings intended to be coupled in series or in parallel, the subscripts shall be single numbers for the first of such windings (1, 2, etc.), double numbers for the second windings (11, 22, etc.), and so on.

2. *Polyphase Transformers.* — The terminals of all polyphase transformers shall be marked as follows:

(a) Phase Identification. The terminals of the high-pressure and low-pressure windings of any one phase shall be marked with the same letter, using capital letters on the high-pressure terminals and small type letters on the low-pressure terminals.

The letters A, B, C, *a*, *b*, *c* shall be used.

The neutral terminal, if provided, shall be marked N or *n*.

(b) Polarity Identification. The relative polarity of the corresponding high-pressure and low-pressure windings in each phase shall be indicated by the addition of subscripts (0) and (1) after the phase letters, so placed that at the instant when (in, for instance, phase A) the high-pressure terminal marked A_1 is positive to terminal A_0, the low-pressure terminal a_1 shall be simultaneously positive to that marked a_0.

Vector Diagrams for Polyphase Transformers

3. *Polyphase Transformers Connected Together.* — When two or more polyphase transformers are to be grouped together with their windings connected to the same primary and secondary systems, it is essential that the transformers shall correspond, not only as regards the pressures for which they are intended, but also as regards the exact phase relation of the secondary winding to the primary winding.

4. *Scope of Vector Diagrams.* — All polyphase transformers shall bear a vector diagram which shows accurately the phase relation between primary and secondary terminals. To secure the correctness of such a diagram the following requirements shall be complied with:

(a) The identification of each phase of the secondary winding with the corresponding phase of the primary winding shall be clearly indicated by the same terminal marks (*see* Paragraph 2 above).

(b) To avoid error arising from differences in methods of winding, the relative instantaneous polarity of primary and secondary windings in each phase shall be indicated at the terminals of the various phase windings (*see* Paragraph 3 above).

5. *Vector Diagrams.* — The vector diagram of connections shall show the phase letter and the polarity marking for each phase of the windings, and shall show correctly how the various phases are connected together. But, to avoid the complexity of marking both phase letter and polarity mark at each end of every winding vector, it shall be sufficient to show the phase letter once, together with an arrow head to indicate polarity, the arrow head in every case pointing away from that end of the phase which has the polarity mark (o) on its terminal.

6. The following are some typical vector diagrams which embody the principles laid down in these rules:

APPENDIX II

This Appendix contains proposals prepared by the Advisory Committee on Rating for submission to the National Committees, but not yet presented to a Plenary Meeting for ratification.

1. *Temperature Limits for Oil.*

Temperature limit for oil (measured by thermometer) . . . 90° C.
Temperature limit for the windings (measured by the increase of resistance) and other parts immersed in oil . . 95° C.

Note. — The adoption of these temperatures implies the employment of a good oil of which the quality may be verified by ascertaining the flash point and measuring the deposit produced by heating.

The I. E. C. has not yet sufficient data to fix limiting values for the flash point or to prescribe the methods of measurement of the deposit.

2. *Reference Temperature of Cooling Water.* — For water-cooled apparatus, in the absence of any indication to the contrary, the maximum temperature of the cooling water in service shall be deemed to be 25° C. at the point of entry.

3. *Temperature Limits for Water-cooled Transformers.* — In the case of oil-immersed water-cooled transformers, the limits of highest permissible observable temperature given in the table are to be reduced by 10° C.; therefore, the corresponding limits of temperature rise shown in the table may be increased 5° C.

APPENDIX III

Dielectric Tests

This Appendix contains proposals in regard to Dielectric Tests prepared by the Advisory Committee on Rating, for submission to the National Committees, but not yet presented to the Plenary Meeting for ratification.

Dielectric Tests. — The high-pressure test shall be applied between the winding and the frame, with the core connected to the frame and to the winding not under test, and shall be applied only to a new and completed machine with all its parts in place under conditions equivalent to normal working conditions, and, unless otherwise specified, the test shall be carried out at the maker's works at the conclusion of the temperature test of the machine.

The test pressure shall be alternating, preferably of sine wave form.

The test shall be commenced at a pressure of less than one-third the test pressure and shall be increased to the full test pressure as rapidly as is consistent with its value being correctly indicated by the measuring instrument. The full test pressure shall then be maintained for one minute in accordance with the values as indicated in the following table:

Item No.	Machine or part	Test pressure (R. M. S.)
1	Rotating machines of size less than 1 kw.	500 volts + twice the rated pressure.
2	Rotating machines of size 1 kw. to 3 kw.	1000 volts + twice the rated pressure.
3	Rotating machines of size above 3 kv.-a.	1000 volts + twice the rated pressure with minimum, 2000 volts.
4	Field windings for synchronous generators when the excitation pressure does not exceed 750 volts.	10 times the excitation pressure. Minimum, 2000 volts. Maximum, 3500 volts.
5	Field windings for synchronous motors:	
	(a) When intended to be started up with the field windings short-circuited.	10 times the excitation pressure. Minimum, 2000 volts. Maximum, 3500 volts.
	(b) When intended to be started up with the field windings separated by a break-up switch.	5000 volts.
	(c) When intended to be started up with the fields on open circuit and without a break-up switch.	5000 volts when the excitation pressure is less than 275 volts. 8000 volts when the excitation pressure is equal to or exceeds 275 volts.
6	Exciter	Not yet decided.
7	Transformers in general	1000 volts + twice the rated pressure.
8	Transformers for primary pressures over 550 volts, the secondaries of which are for direct connection to public or private distribution systems or public or private consumers (i.e., secondary pressures less than 550 volts).	Primary windings: 1000 volts + twice the rated primary pressure with minimum, 10,000 volts (adopted as a protection to human life). Secondary windings: 1000 volts + twice the rated secondary pressure.
9	Secondary (rotor) windings of induction motors not permanently short-circuited.	For non-reversing motors: 1000 volts + twice the maximum pressure which could be induced between the slip rings. For reversing motors: 1000 volts + 4 times the pressure between the slip rings at standstill on open circuit with full primary pressure applied to stator windings.
10	Alternating-current apparatus connected to a single-phase system of more than 300 volts pressure permanently earthed.	Not yet decided.
11	Assembled apparatus	When the test is made on an assembled group of several pieces of new apparatus each one of which has previously passed its high pressure test, the test on such assembled group shall not exceed 85 per cent. of the lowest test pressure appropriate for any part of the group.

APPENDIX IV

Definitions dealing with the subject of "Rating":

Great Britain. — The rating of an electrical machine is the output assigned to it by the maker, together with the associated conditions, all of which are marked on the Rating Plate.

Note. — A machine may have a test rating or a service rating, or both, assigned to it, and marked on the rating plate.

United States of America. — The rating of a machine is the output marked on the Rating Plate and shall be based on, but shall not exceed the maximum load which can be taken from the machine under prescribed conditions of test. This is also called the Rated Output.

(The term "maximum load" does not refer to loads applied solely for mechanical, commutation, or similar tests.)

France. — The rating of a machine is determined by the conditions of working, such as speed, pressure, current, power factor, etc., as indicated on the Rating Plate.

Italy. — The output of a machine is the normal or average output, that is to say, the load at which the machine can work under normal conditions.

INDEX

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(Numbers refer to pages)